DISSERTATION PROPOSAL

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"Effective Linear Relaxations for Combinatorial Optimization"

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This dissertation concerns the study of relaxations for computationally hard combinatorial problems. We study two combinatorial optimization problems for which we investigate semidefinite relaxations. We propose a new technique to enhance linear programming approaches to solve this class of problems.

In the *first chapter*, we present a simple yet flexible systematic way to significantly improve linear relaxations of semidefinite optimization programs (SDPs) using instance-specific information. We are inspired by the celebrated SDP relaxation for max cut due to Poljak and Rendl and analyzed by Goemans and Williamson. By using the instance at hand, we provide a pair of closely related compact linear programs that sandwich the optimal value of the max cut semidefinite program. The instance-specific information allows us to trivially avoid results of hardness of approximation of semidefinite programs using linear ones. We give sufficient conditions that guarantee that the optimal value of both our programs match the optimal value of the semidefinite relaxation. We extensively test our methodology on synthetic and real graphs and show how our ideas perform in practice. Even though our methodology is inspired on the max cut SDP relaxation, we show experimentally that our ideas can be applied successfully to obtain good solutions to other computationally hard SDP's such as sparse PCA and the Lovasz Theta number.

In the *second chapter*, we propose to extend the previous ideas to the case of Mixed-integer semidefinite programs by noticing that our linear programs are outer approximations of the region over which one solves the semidefinite program. The performance of outer approximation algorithms for mixed integer convex programs crucially depends on the tightness of the corresponding outer approximation polyhedra and so the encouraging results of the previous chapter motivate our approach.

In the *third chapter*, we study the vector clock problem, where one seeks to minimize a certain measure of delay of messages being transmitted over a network under a telephone model. We introduce a continuous mathematical program to approximate the problem and prove that it approximates it within polylogarithmic factors. We then show that the mathematical program can be written as a positive semidefinite integer program.

In the *fourth chapter*, we propose to study the problem of peer grading in large online classes where students are evaluated on questions they create. Question design enhances learning, while correctly ranking students is a desired output of any course. Although these two problems are usually studied individually, we are not aware of any system that transparently blends the two. We approach the question from the perspective of matrix completion, where the aim is to recover a matrix capturing student's interactions with the question, under the hypothesis that the matrix satisfies certain regularity conditions. Matrix recovery is amenable to semidefinite programming relaxations.