Mathematical programs featuring uncertain parameters gained significant attention in the last decade. These programs are powerful modeling tools for decision makers, because they provide the flexibility to represent inherent uncertainty that must be accounted for. Stochastic and robust optimization techniques have been widely used to obtain tractable solution methods within this modeling paradigm. In this dissertation, we study three such decision-making problems under uncertainty: planning and scheduling; fair allocation of scarce resources; and portfolio optimization. Our goal is to offer methodological analyses and develop efficient computational methods to solve these problems.

In the first part of this dissertation, we focus on the two-stage stochastic planning and scheduling problem, in which we assign tasks to facilities and then schedule them at each facility, under the uncertainty that affects the processing times of the tasks. We treat the scheduling decisions as recourse decisions since they are made after the uncertainty is revealed. Consequently, traditional Benders decomposition-based algorithms such as the L-Shaped method cannot be used to solve our problem. We consider three variants of the stochastic planning and scheduling problem, derive strong optimality cuts for the newly introduced makespan minimization problem, and develop a logic-based Benders decomposition algorithm to solve them exactly. We benchmark our algorithm against a commercial mixed-integer linear programming solver and the improved integer L-Shaped method. Our computational study illustrates that our proposed logic-based Benders decomposition algorithm significantly outperforms the state-of-the-art benchmark methods. Our study exemplifies the importance of exploiting problem structure and using decomposition-based hybrid methods in solving two-stage stochastic programs with integer recourse.

In the second part, we focus on fair allocation of scarce resources. Combining the conflicting objectives of efficiency and fairness is important for good policymaking concerning the use of scarce resources. We address this issue by employing the Hooker-Williams social welfare function that combines utilitarianism and Rawlsian equity. We analyze the Hooker-Williams social welfare function in the presence of linear budget constraints. Linear budget constraints are useful to represent resource scarcity when the utilities of the players are linear functions of the resources allocated. We begin our analysis by focusing on a case with a single linear budget constraint. We study the polyhedral structure of this formulation and show that there is a closed-form solution. We then extend our analysis to a case that incorporates individual lower and upper bounds on the utilities of the players. We analyze the structure of the optimal solution and show that it follows a certain pattern. The results on the structure of the optimal solutions provide managerial insights to decision-makers. Our next goal is to extend our analyses to the stochastic version of this problem that incorporates uncertainty in the description of the budget constraints.

In the third part, we study portfolio optimization in the presence of estimation errors. It is well known that the classical Markowitz model has a tendency to amplify estimation errors on the expected returns. Robust
optimization overcomes this issue by accounting for these estimation errors. We focus on ellipsoidal uncertainty sets around the point estimates of the expected asset returns. We investigate the performance of diagonal estimation error covariance matrices in the description of the uncertainty set. We show that the class of diagonal estimation error covariance matrices can achieve an arbitrarily small loss in the expected portfolio return as compared to the optimum. We then formulate the problem of finding the best estimation error covariance matrix as a bilevel program. The bilevel model allows us to numerically analyze the loss when there are multiple estimates for the expected return and/or when there are additional restrictions on the structure of the estimation error covariance matrix. The computational results show that diagonal estimation error covariance matrices do have a potential to achieve a very small loss in the expected portfolio return. Our next goal is to investigate data-driven techniques for constructing the uncertainty set for the robust portfolio optimization problem.