D I S S E R T A T I O N P R O P O S A L

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“New bounds for integrality gaps by gluing convex combinations”

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Abstract: In combinatorial optimization the aim is to find the optimal solution in a finite and usually discrete set of solutions. Many important combinatorial optimization problems are NP-hard to solve to optimality. A common approach to deal with such problems is relaxing the discrete solution set into a continuous set, where the optimization problem becomes tractable. In this dissertation, our focus is on linear relaxation of combinatorial optimization problems. Turning continuous (fractional) solutions into discrete (integer) solutions requires rounding procedures. However, the intergrality gap of the linear relaxation is a limit for this approach: rounding a fractional solution into an integer solution incurs a multiplicative cost proportional to the integrality gap. In this dissertation we study integrality gap for different combinatorial optimization problems and introduce new rounding algorithms that imply bounds on their respective integrality gaps.

In the first chapter, we study the traveling salesman problem (TSP). A problem related to TSP is the 2-edge-connected subgraph problem (2EC). TSP and 2EC share their linear relaxation. It is conjectured that the integrality gap of TSP with respect to its linear relaxation is 4/3. For 2EC the conjectured gap is 6/5. However, we only know an upper bound of 3/2 on the gap for both problems. For a special case of uniform covers for TSP, proposed by Sebo, we present the first integrality gap result that is below 3/2. We also use the same idea to achieve efficiently computable uniform covers for 2EC with improved bounds. Both results are obtained by gluing two convex combination together. Then, we use the properties of integral polyhedra (1-tree polytope and cycle cover polytope) together with a coloring procedure to obtain new efficiently computable bounds for uniform cover problem.

In the second chapter, we study approximation algorithms for node-weighted instances of TSP and 2EC. We show that the ideas in chapter 1 can be applied to achieve improved approximation factors for special cases of this problem. Then, we introduce a decomposition algorithm, that allows us to get a 4/3-approximation algorithm for node-weighted subcubic graphs.

In chapter three, we study fundamental extreme points of TSP relaxation. Fundamental extreme points are classes of fractional extreme points such that showing any result on the integrality gap for them implies a result for the integrality gap for the TSP. Fundamental extreme points are also used for 2EC. We show a very interesting class of fundamental extreme points called half-square-points have gap at most 9/7. Our proof uses the concept of uniform covers and coloring introduced in chapter 1.

Finally, in the last chapter, we show a general purpose tool for evaluating integrality gaps. We introduce an algorithm that given a 0,1-MILP either finds a feasible integer solution or conclude that the linear relaxation of the MILP has unbounded integrality gap. We also present an algorithm that given a fractional extreme point rounds is by losing a multiplicative factor of at most C^n where C is the integrality gap of the relaxation and n is the dimension. We show through experiments that in practice this factor is much lower.