D I S S E R T A T I O N   P R O P O S A L

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"Valid Inequalities for Mixed-Integer Linear and Conic Programs"

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Valid inequalities, or cuts, are essential to the success of practical solution methods for mixed-integer programs. They speed up the process of finding an optimal solution by providing a tighter mathematical formulation of the set of feasible solutions. In this thesis, we develop and study general-purpose valid inequalities for unstructured mixed-integer linear and conic programs.

Cut-generating functions are a priori formulas for generating a cut from the data of a mixed-integer program. This concept has its origin in the work of Balas, Jeroslow, Gomory, and Johnson from the 1970s. It has received renewed attention in the past few years. Recently Conforti, Cornuejols, Daniilidis, Lemarechal, and Malick proposed a general framework for studying cut-generating functions. However, they gave an example showing that not all cuts can be produced by cut-generating functions in this framework. They conjectured a natural condition under which cut-generating functions might be sufficient. In our first contribution, we settle this open problem and show that their conjecture is true.

Gomory and Johnson studied cut-generating functions in the context of Gomory's corner relaxation, which is obtained by ignoring the nonnegativity of the basic variables in a tableau formulation. In our second contribution, we consider what happens when we do not relax these nonnegativity constraints. We generalize a classical result of Gomory and Johnson characterizing minimal cut-generating functions in terms of subadditivity, symmetry, and periodicity. Our result is based on a new concept, the notion of generalized symmetry condition. We also extend the celebrated Gomory-Johnson 2-Slope Theorem, which gives a sufficient condition for a cut-generating function to be extreme, to our setting.

In our third contribution, we turn to a natural generalization of mixed-integer linear programming, mixed-integer conic programming. Balas introduced disjunctive cuts in the 1970s for mixed-integer linear programs. Several recent papers have attempted to extend this work to mixed-integer conic programming. We study the structure of the convex hull of a two-term disjunction applied to the second-order cone and develop a methodology to derive closed-form expressions for convex inequalities describing the resulting convex hull. Our approach is based on first characterizing the structure of minimal and tight valid linear inequalities for the disjunction and then using conic duality to derive a family of convex, possibly nonlinear, valid inequalities that correspond to these linear inequalities. We identify and study the cases where these valid inequalities can equivalently be expressed in conic quadratic form and where a single inequality from this family is sufficient to describe the convex hull.

In our fourth contribution, we extend our previous work on two-term disjunctions from the second-order cone to its affine cross-sections. We derive a closed-form expression for a convex inequality that is valid for a two-term disjunction on an affine cross-section of the second-order cone and give sufficient conditions which guarantee that this inequality characterizes the resulting convex hull. These conditions are satisfied in particular by all two-term disjunctions on ellipsoids and paraboloids, a large class of two-term disjunctions on hyperboloids, and all split disjunctions on all cross-sections of the second-order cone.

We conclude the proposal with a discussion of possible directions for future research.