DISSERTATION PROPOSAL

Christian Tjandraatmadja

Friday, December 18, 2015
1:30 pm
324 GSIA (West Wing)

Relaxed Decision Diagrams and Integer Programming

Mixed-integer programming (MIP) is often a practitioner's primary approach when tackling hard combinatorial optimization problems. This important role was enabled by decades of theory and practical experience poured into modern MIP solvers. However, many problems are still challenging for MIP solvers, which motivates the need for novel perspectives to enhance MIP technology.

This dissertation investigates new approaches with the goal of promoting the continued advancement of MIP solvers. We examine two research directions. The first and main part studies how MIP solvers can benefit from discrete relaxations known as relaxed decision diagrams. The second part considers the question of how to identify an effective set of cutting planes from a practical perspective.

Relaxed decision diagrams compactly encode a relaxation of the feasible set of a discrete optimization problem. They can be constructed from a dynamic programming formulation of the problem. Recent research has provided evidence that relaxed decision diagrams yield strong bounds for many applications. The objective of the first part of the dissertation is to investigate methodologies to integrate this relatively new concept of relaxed decision diagrams into the mature machinery of MIP solvers.

We examine two methods to integrate relaxed decision diagrams into integer programming. In the first chapter of the dissertation, we computationally investigate the natural approach of incorporating decision diagram bounds into the branch-and-bound tree. Strong primal and dual bounds are crucial for the solver to keep the size of the tree small and thus find an optimal solution faster. We also propose the use of additive bounding to further improve the bound.

The second chapter considers decision diagrams from a polyhedral perspective. We exploit a connection between decision diagrams and polar sets to generate strong cuts. In particular, these cuts are facet-defining with respect to the convex hull of the relaxation. Computational experiments suggest that these cuts can be effective for certain classes of instances for the maximum independent set problem.

In the second part of this dissertation and third chapter, we seek to measure the practical importance of a given set of cutting planes in solving a mixed-integer problem. Introducing a large number of cuts may lead to a high overhead in computing time, which motivates the need to select a smaller subset of cuts that continues to ensure a tight relaxation of the problem.

A major difficulty in improving cut selection is the computing time involved in comparing strategies. To tackle this obstacle, we investigate the use of a proxy for solving time: a measure of quality of a set of cuts that is computable from a single run of the solver. Such a proxy allows us to compare a large number of cut selection procedures with relatively few executions of the solver. In this work, we computationally test the hypothesis that cut activity is a good measure of cut quality for a large set of real-world instances. The activity of a cut is the number of LPs in which the cut is tight with respect to their optimal solutions throughout the solution process. We find that highly active cuts are important to the speed of the MIP solver.

As an overarching goal of the first part of the dissertation, we plan to develop and release an open-source package that integrates the proposed decision diagram-based techniques into a MIP solver and provides a flexible platform for future advancements in this research direction. This package will include a high-level interface that allows a user to supply dynamic programming formulations for decision diagrams or let them be automatically constructed from IP formulations, with the purpose of making these methods more easily accessible as a generic framework.