Essays on Technical Change and Labor Markets

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Essays on Technical Change and Labor Markets

Abstract

This dissertation consists of three chapters exploring the impact of technical change on labor markets. In the first chapter I conduct a forensic analysis of wage sorting: an observed tendency for high-earning workers to match with high-paying employers, which over time has contributed to rising wage inequality. I evaluate competing causal mechanisms using matched employer-employee data from Germany and find that (1) wage sorting is entirely a between-industry/-occupation phenomenon, and inconsistent with models of assortative matching within markets; (2) increased wage sorting over time can be fully accounted for by declining employment in low-skill, high-paying manufacturing sectors, and rising wages in skill-intensive jobs more common in high-paying firms; and (3) wage sorting is poorly predicted by measures of anti-competitive rents in output and labor markets, but strongly associated with job and workplace characteristics related to technology, which proxy for two well-known dimensions of wage variation: worker skill and employer scale.

In chapter 2, I study the quantitative implications of firm-wage premia for theories of skill-biased demand. I develop a search-based, assignment model of labor markets that accounts for equilibrium interactions between labor demand and supply, skill premia, and firm premia, while remaining sufficiently tractable that the key distributional parameters can be non-parametrically identified from empirical wage effects. I structurally estimate using matched data from West Germany, and find that more than half of the rise in wage variance associated with industry and occupation demand is the result of interactions with firm premia. I show that the “firm-bias” of demand confounds the relationship between skill-biased shocks and wages, and I show in addition that because skill and firm premia are highly correlated across labor markets, policies that seek to reduce wage dispersion by targeting firm premia are generally skill-biased, partially offsetting their aggregate wage impact.
In the final chapter I study the effects of task-level automation when jobs consist of multiple tasks. I consider an environment with endogenous assignment of workers to occupations, and of worker time across tasks, and I characterize the aggregate effects of a technology that replaces labor at a low-skill task. The model predicts a reverse pattern of automation: the low-skill task is first automated in high-skill occupations, where labor costs are higher. In the short-run this creates wage and employment polarization. In the long-run, automation has ambiguous implications for wage inequality and employment. I use panel survey data on occupational tasks and computerization to test the model’s short-run predictions, and I estimate a structural version of the model in order to obtain long-run labor market predictions. Further declines in IT-related costs are predicted to have little effect on wages, but to increase employment in low- and middle-skill occupations.
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Introduction

This dissertation consists of three chapters exploring the impact of technical change on labor markets. The first chapter is a forensic analysis of sorting patterns in labor markets, and how these patterns have contributed to the post-1980’s rise in wage inequality. Past studies find that much of observed wage dispersion is due to differences in pay between employers, and that over time there is an increasingly strong association between high-paying firms and high-earning workers. I utilize matched German employer-employee data to characterize wage sorting over the 1993-2017 period, and to evaluate alternative causal mechanisms. Group decompositions indicate that wage sorting is entirely a between-industry/-occupation phenomenon, and I find little evidence to support assortative matching within narrowly-defined labor markets. The greater contribution from wage sorting over time is not driven by changes to the wages firms pay, but is instead accounted for by a shift in employment from low-skill, high-paying manufacturing firms to low-skill, low-paying service establishments; and by rising wages in skill-intensive jobs, which in all periods are more common in high-paying firms. To uncover the dimensions of worker and firm heterogeneity most closely associated with wage sorting, I turn to establishment and occupational survey data, which I use to conduct a partial correlation analysis. I find that wage sorting is poorly predicted by measures of anti-competitive rents in output and labor markets, but strongly associated with technology-related job characteristics, which are highly effective at capturing two well-known dimensions of wage variation: worker skill and employer scale.

In chapter 2, I study firm-bias: how changes to industry and occupation labor demand interact with the firm-specific component of wages, impacting the distribution of firm premia and their relationship with skill premia. I develop a search-based, assignment model of labor markets that accounts for equilibrium interactions between labor demand, labor supply, skill premia, and firm premia, and in particular the behavioral response of agents to changes in wage premia. The model nests as a reduced form the canonical empirical wage regression,
which allows the key distributional parameters to be non-parametrically identified from empirical wage effects. I structurally estimate the model from matched German employer-employee data, and conduct counterfactual experiments quantifying (1) the contribution of firm-bias to historical wage trends, and (2) the implications of firm-bias for policies targeting firm rents and rent-sharing. I find that in the absence of firm-bias, historical demand shocks would have increased wage inequality by only half as much, and that by itself the skill-bias of demand is an unreliable predictor of wage outcomes. I then show that, because skill and firm premia are highly correlated, policies targeting firm-side wage gaps are generally skill-biased, partially offsetting their aggregate impact on wage inequality.

In the final chapter, I study the short-term and long-run effects of task automation when jobs consist of multiple tasks. Leveraging German survey data, I show that task variety is ubiquitous at the job level, and that computerization over the 1979-2018 period is associated with intra-occupational shifts away from lower-skill and routine task content. I explore the implications of task automation in a model that combines occupational assignment with a time allocation problem where workers must perform multiple tasks. The model predicts a reverse pattern of automation: low-skill tasks are automated first in high-skill occupations, where labor costs are higher. In the short-term this creates wage and employment polarization. In the long-run, low-skill automation has ambiguous implications for wage inequality and employment, with outcomes for low-skill workers generally improving as the costs of automation decrease. I test the model’s short-run predictions against the historical time paths of computerization and occupational employment, and estimate a structural version of the model to obtain long-run predictions for German labor outcomes. Further declines in computer-related costs are expected to have little effect on wages, but to substantially increase employment in low- and middle-skill occupations.
Chapter 1

What Drives West German Wage Sorting?

1.1 Introduction

Two stylized facts emerge from recent empirical studies of the wage distribution. First, a considerable amount of wage dispersion is due, not to differences between workers, but to differences between firms.1 Second, in many countries high-earning individuals are more likely to work for high-paying employers. This phenomenon, known as wage sorting, represents an increasingly important source of wage inequality. It has been found to contribute substantially to rising wage variance in Germany, the United States, and the Scandinavian countries,2 and is related to the broader and well-known result that most of the increase in OECD wage inequality is the result of growing wage gaps between firms, rather than within them.3 Wage sorting nevertheless remains a puzzle. Because the matched employer-employee datasets used to study it are generally poor in worker and firm observables, we know little about the underlying dimensions of heterogeneity that matter for wage sorting, and little progress has been made in uncovering the causal origins of either the cross-sectional pattern or its increased importance over time. Wage sorting represents a challenge to conventional explanations of wage inequality that focus on one side of the labor market, and abstract from the question of “who works where?”; but the nature of this challenge, and the implications of wage sorting for our understanding of wage dispersion, are not yet clear.

In this paper I study West German wage sorting: its mechanical drivers, its relation-

1Following Abowd, Kramarz, and Margolis (1999), many papers have replicated this result; see Card, Cardoso, Heining, and Kline (2018) for a survey.
3See Dunne, Foster, Haltiwanger, and Troske (2004), Simon (2010), Barth, Bryson, Davis, and Freeman (2014), and Tomaskovic-Devey et al. (2020).
ship with observable characteristics of workers and firms, and its consistency with various explanations explored in the theoretical literature. I draw on a unique dataset that pairs administrative records for 1993-2017 with an annual survey of business establishments, offering an unusual level of detail on both workers and their employers. This dataset is combined with an updated version of the AKM wage effects estimated by Card, Heining, and Kline (2013), who leveraged population-level matched data to show that wage sorting - or the covariance of person and firm wage effects - is an important and growing source of German wage dispersion. I use decomposition methods to address the question: to what extent is wage sorting occurring within labor markets versus between them? The purpose of this exercise is to differentiate between two main classes of theories: those that study worker-firm sorting in the context of match-level complementarities and unobservable heterogeneity, and those concerned with industry or geographic variation in the technical demand for different types of labor. Focusing on the between-market component of wage sorting, which by nature is more amenable to study, I conduct a series of reduced-form experiments to uncover how changes to market composition and wage premia have contributed to wage sorting over time. Finally, I utilize establishment- and person-level survey data to study the statistical relationship between wage sorting and agent heterogeneity, for the purpose of evaluating alternative causal stories relating to technological skill-bias and market structure.

The first main finding is that wage sorting in West Germany is entirely a between-market phenomenon, apparently unrelated to geographic region, but fully accounted for by aggregated industry and occupation groups. Conditional on the broad industry of the employer and the occupation associated with the job spell, the covariance of person and firm wage effects is small and negative over the duration of the sample. If wage sorting is between-market, then the relevant dimensions of person and firm heterogeneity should be observable and not require any learning by agents; and consistent with this prediction, I find that wage sorting is just as strong among new job spells, new establishments, and workers entering the labor force. With respect to what has changed over the years 1993-2017, I find that inter-industry differences in the employer component of pay have been largely stable, while widening intra-industry pay gaps appear to be entirely within-occupation, indicating that changes to market structure are unlikely to account for increased wage sorting during this
Figure 1.1: AKM Decomposition of West German Wage Variance, 1993-2017

Source: German linked employer-employee dataset (LIAB). Note: AKM wage effects estimated following Card, Heining, and Kline (2013) over four panels: 1993-99, 1998-04, 2003-10, and 2010-17. From bottom to top, regions correspond to the covariance of person and firms age effects, the variances of the firm and person effects, and the regression residual. Results are averaged in years with overlapping panels.

Period. Counterfactual experiments indicate that, instead, the upward trend in wage sorting can be almost entirely accounted for by (1) changes to industry employment shares and (2) rising wages (i.e. person effects) in labor markets associated with high-paying employers. The first of these channels is associated with a shift of employment away from high-paying firms in commodities and crafts manufacturing, and towards low-paying employers in the personal service, food and accommodation, and temp agency sectors. The accompanying decline in employer pay has predominantly affected low-wage workers, thereby contributing to wage sorting. The second channel has arisen because, in all periods, high-earning occupations are more common at high-paying firms. As the mean person effect in these occupations rises, so too does the overall covariance of the person and firm effects, amplifying the impact on wage variance.

The second main finding is that wage sorting is closely and intuitively related to the observable characteristics of industries and jobs. At the industry-occupation level, differences in employer pay are well-explained by the scale and capital intensity of the establishment, while variation in the person wage effect is strongly associated with worker education and skill-intensive job tasks. Despite this, scale and skill are poorly predictive of wage sorting;
controlling for these characteristics has little effect on the correlation of wage effects. On the other hand, measures of technology adoption and innovation in the workplace - which are highly correlated with both establishment scale and worker skill - can substantially account for the wage sorting patterns observed at the industry-occupation level. I find that measures of competitiveness in output markets are only weakly informative about either wage effect, and are unrelated to their covariance; while collective bargaining agreements, although predictive of employer pay, are unrelated to the person wage component and are consequently uninformative with respect to wage sorting. These results suggest that market institutions are no more explanatory than match complementarities in context of West German wage sorting, and that technological linkages between worker skill and employer scale present a more plausible mechanism.

This paper contributes first and foremost to the empirical literature on wage sorting. In recent years a number of researchers have studied wage trends by performing panel implementations of the regression decomposition proposed by Abowd, Kramarz, and Margolis (AKM, 1999). In this approach, wages are regressed on person and employer fixed effects, and the wage variance is decomposed into a sum of the variances and covariance of the so-called “AKM effects.” Card, Heining, and Kline (2013) estimate that one-third of the post-1980’s rise in West German wage variance is due to increased covariance of the person and firm effects. Song et al. (2019) find a similar rise in the covariance for the United States, but a decline in the variance of firm effects. Through one or both channels, firm wage effects have been found to contribute to rising wage inequality in Denmark (Bagger, Sorensen, and Vejlin 2013) and Sweden (Hakanson, Lindqvist, and Vlachos 2021, appendix D). On the other hand, negative contributions have been shown for several countries with stable or declining wage inequality (Torres et al. 2018; Alvarez, Benguria, Engbom, and Moser 2018). A key challenge for these studies is their interpretation: wage effects are endogenous objects that tell us little about the mechanism that generated them, and consequently the implications of wage sorting for studies of the wage distribution are unclear. I show that in Germany, wage sorting can be closely tied to observable characteristics of agents, and that this is sufficient to rule out several explanations that past researchers have proposed. Card et al. suggest that declining coverage of collective bargaining agreements may explain
West German wage trends. Song et al. review different theoretical mechanisms and argue that match complementarities (worker-firm or worker-worker) may drive wage sorting in the United States. The evidence I present here, while descriptive and short of conclusive, is not consistent with either of these mechanisms.

The second contribution of this paper is to the macroeconomic literature on wage inequality, which can be divided into two strands. The skill-bias literature has focused on changes to industrial and occupational labor demand, most commonly due to technological change (and information technology in particular) and to outsourcing and offshoring associated with globalization. Representative work in this area includes Acemoglu and Autor (2011) and Goos, Manning, and Salomons (2014).\(^4\) At the same time, a number of theoretical studies extend Becker’s theory of matching to labor markets, positing complementarities between workers and their peers (e.g. Kremer and Maskin (1996)), or between workers and firms as in Eeckhout and Kircher (2011).\(^5\) Whereas skill-bias is associated with changes to the market-clearing “price” of skill, match complementarities are thought to affect wages through sharing of match rents, arising due to market frictions. I find that the characteristics of German wage sorting are inconsistent with match complementarities: it is absent within narrow labor markets, tightly associated with observable features of a job match, and there is no evidence that assortative matches are “selected” over time through on-the-job search or match destruction. On the other hand wage sorting is well-explained by technological differences across workplaces, while the same trends studied in the skill-bias literature - declining manufacturing employment and rising wages in skilled occupations - are also capable of rationalizing the increase in German wage sorting over 1993-2017.

This paper also contributes to the literature on industry and occupation wage gaps and their role in wage trends. Previous studies have looked at the between-industry and between-occupation components of rising wage dispersion from firm effects (Torres et al. 2018; Abowd et al. 2012; Akerman et al. 2013; Card, Heining, and Kline 2013), but have not directly explored their relationship to, or implications for, the theoretical literature. In a closely


\(^5\)Other work in this area includes Lise, Meghir, and Robin (2016), Hagedorn, Law, and Manovskii (2017), Bagger and Lentz (2019), and Bonhomme, Lamadon, and Manresa (2019).
related paper, Haltiwanger and Spletzer (2020) examine matched data from the United States and show that industrial and occupational classifications explain the majority of rising wage variance over the years 1996-2015. These authors find, as I do, that the principal drivers are changes to occupation mean wages and occupation-industry sorting. They do not decompose earnings into firm and worker components, however, and so their results are silent on whether these trends can also explain the rise in U.S. wage sorting observed by Song et al. (2019). Given the similarity of the wage trends in the two countries, it is likely that the results shown in this paper will also hold for the United States.

Next I give a brief and somewhat heuristic description of the theoretical mechanisms motivating this paper. In section 2 I provide a brief background on the AKM wage decomposition and German wage sorting, and results attesting to the robustness of the upward trend in wage sorting over time. Section 3 contains the main results on between- versus within-market components of wage sorting, and the observable correlates of between-market wage sorting are explored in section 4. Section 5 concludes this paper.

1.1.1 Theories of Wage Sorting

Wage sorting is a natural prediction of two classes of models. The first class of models studies the technical demand for skilled and unskilled labor, which is generally supposed to vary across regions or industries. An example of this is Autor and Dorn (2013), who consider how a decline in demand for manufacturing labor affects skill premia. Although this paradigm usually imposes competitive labor markets, and firm premia are absent, extension to the case of imperfect competition is straightforward. The second class of models studies worker-firm or worker-worker complementarities, which lead to sorting and - when markets are frictional - can create match rents that pass through to wages. The canonical example is Becker’s (1973) model of marriage markets, which was extended to frictional markets by Shimer and Smith (2000). In this framework, assortative matching describes a tendency for types to seek out similar types; and given search frictions and bargaining, assortative matching will result in match rents that are not arbitraged away and are shared between agents.

A key differentiator between these classes of models is the role they allow for market segmentation: across space, across output markets (i.e. industries), and across markets
Figure 1.2: Labor Market Segmentation

for different types of labor (i.e. occupations). Figure 1.2 provides a simple representation of this structure, that is meant to be illustrative rather than exhaustiv. A key feature of market segmentation is that prices and quantities adjust. Labor supply and demand are substitutable across markets, and differences in expected wages or profits are subject to arbitrage. This is in contrast to models of assortative matching, which assume a homogeneous output and fixed type distributions, precluding quantity and price adjustments.

The distinction between observed and priced heterogeneity, and unobserved and unpriced heterogeneity, is a critical one. In the first place the nature of the underlying sorting is different. Whereas assortative matching results from agents’ pursuit of greater match rents, sorting between markets is most naturally understood to reflect technical (and technological) differences between markets, that impact the demand for skilled labor, economies of scale, and other factors favoring particular agent types or characteristics. Second, the normative and policy implications of wage sorting are likely to be different. When wage differentials reflect marginal products and profits reflect expected costs, and when sorting represents an efficient allocation of labor across firms, there is less scope for wage policies to redistribute wage income without significant adverse effects. Because prices and quantities may adjust,
they may also become distorted.

A more practical difference between these two theories of wage sorting is the extent to which they can be empirically characterized. Because market segmentation is observed, differences between markets can be characterized, even if only in terms of quantities and prices. But it is also more likely that when differences between workers and firms are realized across markets, they reflect observable rather than idiosyncratic heterogeneity, responding to differences in skill demand, production requirements, and market institutions. There is more hope in such a case of evaluating specific causal mechanisms, and identifying which features of markets, and which dimensions of heterogeneity, are key to wage sorting. If instead wage sorting is a story about match complementarities, and unobservable differences in worker ability or firm productivity, then the empirical challenges are grave. Indeed the wage decomposition shown in figure 1.1 becomes difficult to interpret in this case, because such decompositions are mis-specified when match complementarities are present.\textsuperscript{6} Hence our ability to make empirical progress on the causal origins of wage sorting depends, to a large extent, on whether it is a between-market or a within-market phenomenon.

\textsuperscript{6}See e.g. Lopes de Melo (2018).
1.2 West German Wage Sorting, 1993-2017

In this section I give a brief overview of the data used in the analysis, and the AKM wage decomposition employed to identify worker and firm wage effects. I then introduce the trend that is the focus of this paper: a rise in the covariance of person and firm effects over the 1993-2017 period, similar to previous findings by Card, Heining, and Kline (2013). Disaggregated results show the trend to be broad-based, though there is substantial variation in the extent of wage sorting across demographic groups and different subsets of the labor market. An issue that has gone unexplored in the literature is the extent to which trends in wages sorting are affected by the so-called “limited-mobility bias” known to affect cross-sectional studies of the wage distribution. I show that in this case, concern is justified; German labor mobility patterns are time-varying, and depressed job transition rates during the 2000’s lead to an apparent downward bias of the wage effects covariance during this period. The 2010’s and the 1990’s exhibit similar mobility patterns and therefore comparisons over the full sample period are less problematic, but the results shown in this section suggest that caution be taken when comparing AKM variance decompositions from different periods of time, and that such analyses be accompanied by summary statistics describing job transition rates within the sample.

1.2.1 Data and Decomposition

LIAB linked employer-employee dataset. The data and methods discussed in this section are similar to those previously studied by Card, Heining, and Kline (2013) - henceforth CHK - and therefore I provide only a brief description below and leave detailed steps and summary statistics to the appendix. The dataset used in this analysis is the German linked employee-employer dataset (LIAB), provided by the Institute for Employment Research (IAB). Every year since 1993 the IAB has conducted a stratified survey of German business establishments, collecting information on operational, investment, and hiring activities. An establishment is defined as a physical workplace, however locations may be aggregated when they share the same corporate ownership, industry classification, and municipal code.

A matched employee-employer dataset is constructed by pulling administrative social security records for all individuals employed by the surveyed established on June 30 of the survey year.

---

7An establishment is defined as a physical workplace, however locations may be aggregated when they share the same corporate ownership, industry classification, and municipal code.
The social security records include the occupation and (top-coded) daily wage associated with the employment spell, along with basic information on demographics and education. In total, the individuals in the LIAB represent approximately 5% of the German workforce, and in any given year there are between 1.5 and 2.5 million workers and between 4 and 15 thousand establishments.

In this paper I limit attention to full-time West German workers aged 20-60. Partly this is to facilitate comparison with the prior literature, but also in response to data limitations. Part-time work is recorded inconsistently over the sample, due to changes in the applicability of social security taxes. Apprentices are paid both in wages and non-wage benefits (e.g. certification) and are therefore excluded; and for similar reasons I also exclude workers at the extremes of the wage distribution. Finally, East German establishments are only observed beginning in 1996, and are omitted for consistency; however in the appendix I present results separately for the East, as well as for East and West combined over the 1996-2017 period.

For analyses involving industry and occupation, I rely on aggregate groupings that preserve - to the extent possible - inter-industry differences in the AKM wage effects discussed below. On the one hand this allows me to avoid problems associated with over-fitting, and to better satisfy confidentiality requirements by maintaining large group sample sizes; while on the other hand, aggregation minimizes the impact of coding changes over time, the most substantial of which is a shift to a new occupational coding system in 2011. The appendix contains details on the industry and occupation groups used, as well as (in many cases) results using less-aggregated codes.

**AKM wage variance decomposition.** Provided with the LIAB are updated versions of the person and employer wage effects estimated by CHK, following the now-standard approach of Abowd, Kramarz, and Margolis (1999). These effects are obtained from the panel wage

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8Wages are top-coded at the social security contribution thresholds. Tobit regressions are used to impute affected values following CHK; see the appendix for details.
9Card et al. also consider this subset of the West German workforce, though for computation reasons their main results are further limited to male workers.
10Time-consistent codes are provided by IAB, but in some cases these rely on imputation; therefore I propagate industry and occupation codes forwards or backwards (as applicable) when a job spell is observed on both sides of a coding change.
where $w(i,t)$ is the log daily wage of person $i$ in year $t$, $\pi$ is a time-invariant wage effect associated with person $i$, $\phi$ is a time-invariant effect associated with $i$’s employer $j$, and $x$ is a vector containing year fixed-effects and a cubic polynomial in worker age, interacted with dummies for educational attainment. Estimation is performed on the population-level datasets from which the LIAB is extracted, in four partially-overlapping panels spanning 7-8 years.\textsuperscript{11} From equation (1), the wage variance can be decomposed as a sum of the variances and covariances of the estimated regression effects, which allows one to see how the underlying sources of wage dispersion have evolved over time. Panel decompositions for West Germany are shown in table 1.4.

\begin{table}[h]
\centering
\begin{tabular}{lccccc}
\hline
\hline
$\text{Var}(w)$ & 0.1684 & 0.1997 & 0.2321 & 0.2316 \\
$\text{Var}(\pi)$ & 0.1088 & 0.1238 & 0.1399 & 0.1415 \\
$\text{Var}(\phi)$ & 0.0310 & 0.0381 & 0.0518 & 0.0399 \\
$\text{Var}(x'\beta)$ & 0.0039 & 0.0054 & 0.0054 & 0.0131 \\
$\text{Var}(\epsilon)$ & 0.0126 & 0.0149 & 0.0157 & 0.0181 \\
$2\times\text{Cov}(\pi,\phi)$ & 0.0160 & 0.0228 & 0.0250 & 0.0338 \\
$2\times\text{Cov}(\pi,x'\beta)$ & 0.0018 & 0.0000 & 0.0000 & -0.0186 \\
$2\times\text{Cov}(\phi,x'\beta)$ & 0.0014 & 0.0016 & 0.0024 & 0.0008 \\
\hline
Observations & 10,645,769 & 9,185,412 & 9,511,130 & 7,080,688 \\
Persons & 3,351,593 & 3,301,936 & 3,097,049 & 2,347,598 \\
Establishments & 8,151 & 18,518 & 19,989 & 17,684 \\
\hline
\end{tabular}
\caption{AKM Variance Decomposition, 1993-2017}
\end{table}

\textbf{Note:} Daily wage $w$ denoted in log 1995 euros. Variance components sum to equal $\text{Var}(w)$.

The variance moments in table 1.4 are subject to several sources of error and bias. First, wages are top-coded at the upper contribution levels for social security, with the affected values imputed \textit{via} Tobit regressions. Second, time-varying effects are not provided and I

\textsuperscript{11}I find in practice that while direct estimation on the LIAB subsample yields qualitatively similar trends, loss of sample is severe, with the number of identified establishments falling by more than half relative to the IAB-provided effects. Identification of the wage effects $\pi$ and $\phi$ comes from individuals who move between employers, hence in smaller samples it is more likely that one of these employers is unobserved.
therefore estimate them *ex post*; moments based on these effects are provided for reference only, and should be interpreted with caution. Third, as mentioned above, the data is limited to a subset of the employed German workforce. While this allows for greater comparability over time, it also implies that results may differ from those that would be obtained for the full population of German workers. Finally, it is well-known that the variances $\text{Var}(\pi)$ and $\text{Var}(\phi)$ are biased upwards, while $\text{Cov}(\pi, \phi)$ is biased downwards. As this bias directly affects the moment under study - that is, $\text{Cov}(\pi, \phi)$ - I discuss it separately at the end of this section.

### 1.2.2 West German Wage Sorting

Wage sorting - the covariance of the AKM person and establishment wage effects - accounts for 15% of West German wage variance over 2010-2017, and 29% of the increase in wage variance during the 1993-2017 period. In other words, a substantial portion of German wage inequality is due to a tendency for highly-paid individuals into high-paying firms, and this association has become quantitatively more important over time. The trend in wage sorting is shown graphically in figure 1.5.

![Figure 1.5: West German Wage Sorting, 1993-2017](image)

*Note:* Solid (dashed) line indicates panel (annual) value of $2 \times \text{Cov}(\pi, \phi)$. Panels span 1993-99, 1998-04, 2003-10, and 2010-17, with annual values reflecting the average in years for which panels overlap.

We observe $\text{Cov}(\pi, \phi)$ rising both across panels and within them, though the 2003-2010 panel sees a lower covariance than the panels before and after. This deviation from trend
appears to be due, at least in part, to statistical bias, which I discuss in the next section.

Note that as the covariance of the wage effects has experienced a proportionally greater rise than either of the variances, the correlation of $\pi$ and $\phi$ also rises over this period, from .14 to .22; results in terms of correlations are provided in the appendix. The trend in figure 1.5 is somewhat smaller than that shown by CHK, reflecting the inclusion of female workers in the same. Because the rise in $Cov(\pi, \phi)$ is more pronounced among males, inclusion of both sexes in the sample tends to reduce the overall trend. In addition, there is some wage sorting across genders - male workers tend to be both higher-earning and to work for higher-paying firms - but this pattern has weakened over time, which also tends to weaken the aggregate increase in $Cov(\pi, \phi)$.

<table>
<thead>
<tr>
<th>Table 1.6: Wage Sorting By Demographic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>1993-99</td>
</tr>
<tr>
<td>---------------------------------------------------</td>
</tr>
<tr>
<td>Male</td>
</tr>
<tr>
<td>Female</td>
</tr>
<tr>
<td>Aged 20-30</td>
</tr>
<tr>
<td>Aged 31-40</td>
</tr>
<tr>
<td>Aged 41-50</td>
</tr>
<tr>
<td>Aged 51-60</td>
</tr>
<tr>
<td>Lower secondary ed.</td>
</tr>
<tr>
<td>Apprenticeship</td>
</tr>
<tr>
<td>Upper secondary ed.</td>
</tr>
<tr>
<td>University degree</td>
</tr>
<tr>
<td>Schleswig-Holstein</td>
</tr>
<tr>
<td>Hamburg</td>
</tr>
<tr>
<td>Lower Saxony</td>
</tr>
<tr>
<td>Bremen</td>
</tr>
<tr>
<td>North Rhine-Westphalia</td>
</tr>
<tr>
<td>Hesse</td>
</tr>
<tr>
<td>Rhineland-Palatinate</td>
</tr>
<tr>
<td>Wurttemberg-Baden</td>
</tr>
<tr>
<td>Bavaria</td>
</tr>
<tr>
<td>Saarland</td>
</tr>
<tr>
<td>Berlin</td>
</tr>
</tbody>
</table>

**Note:** Value shown is $2 \times Cov(\pi, \phi)$ where $\pi$ ($\phi$) is the person (establishment) AKM wage effect.

How broad-based is wage sorting, and the upward trend observed over 1993-2017? Table 1.6 gives results dis-aggregated by demographic group, educational attainment, and German
state. Wage sorting is cross-sectionally robust, and not limited to one or several subsets of the workforce. The time trend is also shared across groups, though some patterns are evident. In particular we see that increases in $\text{Cov}(\pi, \phi)$ are concentrated among lower-educated and male workers. The results shown later in the paper suggest an explanation for this. I will show that wage sorting is predominantly a between-industry phenomenon, and that industries associated with highly-paid workers tend also to pay more. The exception to this trend is commodities and crafts manufacturing, where firms are relatively high-paying but the typical worker is lower-earning, less-educated, and male. The large number of high-paying production jobs tends to reduce wage sorting among this demographic, while the decline over time in such jobs accounts for the larger increase in $\text{Cov}(\pi, \phi)$.

### 1.2.3 Limited-Mobility Bias

A well-known issue with the AKM wage decomposition is that, due to the incidental parameters problem, the variance components in table 1.4 are biased. $^{12}$ Wage effects are identified from job-movers - workers who move between establishments in the sample - and the number of job-movers associated with a given employer may be quite small, even when that employer is observed over a period of 7-8 years. This is particularly the case for small establishments. In these cases the law of large numbers does not apply, and estimation of the AKM effects is unbiased but inconsistent. And because wages are additive, positive errors in the estimation of $\phi$ show up negatively in $\pi$, biasing downwards $\text{Cov}(\pi, \phi)$ and biasing upwards $\text{Var}(\pi)$ and $\text{Var}(\phi)$. This “limited-mobility bias” is reduced when using population-level data, as both employers in a job transition will be observed; but it is not altogether eliminated. Therefore an important question is whether this bias varies over time, as any time variation it would directly impact the wage sorting trend shown in figure 1.5.

The identification statistics in table 1.7 indicate important differences between panels in terms of the quality of the AKM wage effects. A rough measure of the estimation error associated with these effects is the proportion of the sample that cannot be identified - i.e. observations linked to establishments that cannot be connected to the identified sample through job-movers. This number is slightly above 2% for the 1993-1999 and 2010-2017

---

$^{12}$See for example Andrews et al. (2008) and Bonhomme et al. (2020).
Table 1.7: AKM Identification Statistics

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unidentified Sample (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All establishments</td>
<td>0.0219</td>
<td>0.0278</td>
<td>0.0296</td>
<td>0.0226</td>
</tr>
<tr>
<td>1-9 employees</td>
<td>0.1038</td>
<td>0.1239</td>
<td>0.1328</td>
<td>0.1181</td>
</tr>
<tr>
<td>10-24 employees</td>
<td>0.0052</td>
<td>0.0065</td>
<td>0.0131</td>
<td>0.0047</td>
</tr>
<tr>
<td>25-99 employees</td>
<td>0.0039</td>
<td>0.0046</td>
<td>0.0043</td>
<td>0.0029</td>
</tr>
<tr>
<td>100-499 employees</td>
<td>0.0035</td>
<td>0.0040</td>
<td>0.0049</td>
<td>0.0040</td>
</tr>
<tr>
<td>500+ employees</td>
<td>0.0029</td>
<td>0.0047</td>
<td>0.0046</td>
<td>0.0028</td>
</tr>
<tr>
<td><strong>Job Transitions (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job entry</td>
<td>0.151</td>
<td>0.150</td>
<td>0.133</td>
<td>0.154</td>
</tr>
<tr>
<td>Job exit</td>
<td>0.172</td>
<td>0.168</td>
<td>0.153</td>
<td>0.166</td>
</tr>
</tbody>
</table>

**Note:** All proportions use sample weights. Job entry (exit) is new (ending) job spells as percent of total employment at continuing establishments, for full-time West German workers.

panels, but closer to 3% for the two intervening panels. This could in principle be due to a greater preponderance of small employers during the early 2000’s, but a similar pattern is observed when we focus solely on employers with less than 10 workers. Rather the explanation appears to be a decline in the number of job-movers. The rates at which job spells begin and end are markedly lower during the 2000’s, indicating a general decline in job transition rates.\(^{13}\) Fewer job movers means that wage effects are less precisely estimated, and taken together, the evidence indicates that limited-mobility bias is likely to be more severe during the 1998-2004 and 2003-2010 panels.

Indeed the decline in wage sorting observed over 2003-2010 is entirely concentrated in small employers, where we would expect variation in job transition rates to have the greatest impact on estimation error, and the largest impact on $\text{Cov}(\pi, \phi)$. This discrepancy between small and large establishments, readily visible in figure 1.8, suggests that the aggregate decline in wage sorting during this period is an artifact of statistical bias.\(^{14}\) It does not constitute hard proof, as wage trends may evolve differently among small and large employers, and in fact we can see that no trend is evident over the full sample period for employers

\(^{13}\)This decline in entry/exit rates is concurrent with the Hartz reforms (2003-2005), and lasts until the early 2010’s.

\(^{14}\)The trend for establishments with 100-499 employees is similar to that shown for the 25-99 size class; for very large establishments wage sorting is negligible in all panels. A key challenge in drawing inferences from differences across size classes is that very large employers, like very small ones, tend to be concentrated in particular industries.
with less than 10 workers. Nevertheless bias is the simplest explanation for the observed deviation from trend, and it is consistent with the decline in job transition rates discussed above.

Figure 1.8: Wage Sorting By Establishment Size

Note: line indicates value of $2 \times Cov(\pi, \phi)$.

To summarize: lower job transition rates during the first decade of the 2000’s appear to have exacerbated limited mobility bias over this period, and are likely account for the deviation from trend observed in figure 1.5. Bias does not, however, explain the trend itself; identification statistics are comparable for the 1993-1999 and 2010-2017 panels, indicating that the change in $Cov(\pi, \phi)$ over 1993-2017 requires some other explanation. The results shown in this section suggest that caution should be exercised when comparing AKM variance moments across time. Labor mobility is an economic outcome - endogenous to the macroeconomic environment - and the expectation should therefore be that limited-mobility bias will vary over time. While there is no direct way to measure this bias, summary statistics like those in table 1.7 can provide an informal test of whether comparisons between two periods of time are likely to be valid.

1.3 Is Wage Sorting Within or Between Markets?

What are the drivers of wage sorting, cross-sectionally and over time? I explore this question below. I employ group decomposition methods to disentangle between- and within-market sources of wage sorting, and I show that in the West German case, wage sorting is entirely
a between-market phenomenon; conditional on the employer’s industry (i.e. output market) and the worker’s occupation (i.e. labor input market), $\text{Cov}(\pi, \phi)$ is small, negative, and trend-less over the 1993-2017 period. To the extent that match-based sorting is a within-market phenomenon, it does not appear to play an important distributional role in Germany. Consistent with this interpretation I find that wage sorting is not more prevalent among older firms and workers, or among job spells with longer tenure, indicating a lack of selection into matches with large complementarities realized $\textit{ex post}$. I then consider the drivers of rising between-market wage sorting, using reduced-form counterfactual experiments to distinguish between different sources of wage effect covariance. I find that virtually the entire trend can be accounted for by (1) changes to unconditional industry employment shares and (2) a rising mean person effect among jobs associated with high-paying firms. This would seem to rule out explanations based on changing market structure (i.e. firm pay), as establishment wage effects are largely stable over time at the market level.

1.3.1 Between-Group Wage Sorting

The starting point for group decompositions is the law of total covariance, which states that any covariance can be decomposed into two components: the average covariance within each of a set of groups, and the covariance of the group means. More formally, for any two variables $x$ and $y$ and a partition of the sample $g = \{1, ..., G\}$, where $\omega_g$ is the employment share of group $g$, we have

$$\text{Cov}(x, y) = \sum_{g \in G} \omega_g \left( x - \mathbb{E}_g[x] \right) \left( y - \mathbb{E}_g[y] \right) + \sum_{g \in G} \omega_g \left( \mathbb{E}_g[x] - \mathbb{E}[x] \right) \left( \mathbb{E}_g[y] - \mathbb{E}[y] \right). \tag{2}$$

If for example $G$ corresponds to occupational classifications, the within-group component of $\text{Cov}(\pi, \phi)$ would be positive if high-paying employers tend to hire better-skilled workers within a particular type of job; while the between-group component would be positive if skilled occupations comprise a large portion of the workforce at high-paying employers. One advantage of this approach is that group means may be consistently estimated even when person and firm wage effects are inconsistent. Thus the between-group variance components...
will be less affected by the limited-mobility bias discussed in the previous section, provided that groups are sufficiently well-populated.

### Table 1.9: Between-Group Wage Sorting (% Total)

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry (12)</td>
<td>0.845</td>
<td>0.823</td>
<td>1.002</td>
<td>0.829</td>
</tr>
<tr>
<td>Occupation (15)</td>
<td>0.945</td>
<td>0.841</td>
<td>0.981</td>
<td>0.734</td>
</tr>
<tr>
<td>State (11)</td>
<td>0.067</td>
<td>0.037</td>
<td>0.051</td>
<td>0.027</td>
</tr>
<tr>
<td>Ind. × Occ.</td>
<td>1.155</td>
<td>1.066</td>
<td>1.285</td>
<td>1.035</td>
</tr>
<tr>
<td>Ind. × State</td>
<td>0.893</td>
<td>0.852</td>
<td>1.032</td>
<td>0.827</td>
</tr>
<tr>
<td>Occ. × State</td>
<td>0.981</td>
<td>0.867</td>
<td>1.020</td>
<td>0.758</td>
</tr>
<tr>
<td>Ind. × Occ. × State</td>
<td>1.182</td>
<td>1.091</td>
<td>1.305</td>
<td>1.037</td>
</tr>
</tbody>
</table>

**Note:** Value shown is the between-group component of $\text{Cov}(\pi, \phi)$ divided by the total.

Results in table 1.9 show that while geography is largely unexplanatory,\(^{15}\) aggregate industry and occupation each account for the preponderance of West German wage sorting. The intersection of industry and occupation accounts for more than the entire positive value of $\text{Cov}(\pi, \phi)$ in each panel, with the negative within-group component likely reflecting limited-mobility bias as discussed previously. Hence industries with high-paying establishments tend also to employ high-earning workers; and likewise, occupations with large average person effects are relatively more concentrated within high-paying employers. Note that the ability of industry and occupation to account for the covariance of the wage effects is not due to their overall explanatory power: most of the variation in $\pi$ and $\phi$ is within industry-occupation pairs. Nor does using disaggregated industry and occupation codes substantially affect results; I show in the appendix that while detailed groups explain significantly more of the variation in the wage effects separately, the between-group covariance is largely unchanged.

The between- and within-group components for industry and occupation are plotted in figure 1.10, from which it is evident that the deviation from trend in the early 2000’s is entirely a within-industry, within-occupation phenomenon - further evidence that limited-mobility bias is the likely cause. The between-group trend is linear over the first half of the sample, but diminishes somewhat in the second half. The evolution of overall wage variance

\(^{15}\)Note that German states are roughly comparable in size to counties in the United States. The data provider possesses more granular data on establishment location, but this is censored in the LIAB.
Figure 1.10: Between-Group Wage Sorting, Industry and Occupation

Note: Between- and within-group values of $2 \times Cov(\pi, \phi)$ for 15 KLDB 1988 occupational groups, 12 WZ 2008 industry groups, and 180 occupation × industry groups.

exhibits a similar pattern, suggesting that these trends are more tightly related than would be apparent from table 1.4. It is also evident that there is substantial overlap in the wage sorting “explained” by industry and occupation, reflecting the fact that occupations are often industry-specific, while different industries employ workforces that differ substantially in terms of their occupational mix.

Mean wage effects by industry and occupation are shown in figure 1.11, from which two additional points are evident. First, the between-group correlations are very high, generally falling in the .6-.8 range. This is necessary to explain why industry and occupation are highly predictive of $Cov(\pi, \phi)$ despite explaining little of the overall variance of $\pi$ and $\phi$. Second, the correlations would be even higher were it not for the fact that manual labor occupations associated with goods production tend to earn a higher establishment premium than comparable service sector jobs, while manufacturing sectors pay a .1 log wage premium relative to service sectors with a similar mean person effect. Hence the manufacturing-services wage gap tends to reduce wage sorting, which will be important in the next section for explaining the mechanics of the rise in wage sorting.

The conclusion of this section is stark: wage sorting in West Germany is a story about industrial sectors and occupational labor. Different types of jobs are associated with different types of employers, and along this dimension there is a strong association between high-earners and high-payers. Based on the limited information available for this analysis,
geographic sorting does to appear to play a role. In addition, sorting on unobservable characteristics would appear to be an unlikely explanation for the observed trend, notwithstanding the interest that theoretical models of match-based sorting have generated. I provide further evidence on this in the next section, in which I ask whether there is any evidence of sorting on match quality.

1.3.2 Testing For Match-Based Sorting

A possible explanation for the trend in figure 1.5 is that there are complementarities between skilled workers and productive firms, or between skilled coworkers in the same firm. For this to generate wage sorting, we also require some mechanism that (1) preserves match rents and (2) ensures rents are shared with workers, and do not simply accrue to the employer. By far the most common assumption is that labor markets are characterized by random
search, with bargaining over the match surplus. Such models will generally predict that over time there is a selection effect: “bad” matches are dissolved in favor of better ones. In the simple framework of Shimer and Smith (2000), selection occurs immediately through match acceptance sets; one or both parties will reject the match if the match product is too low. But in general we would expect selection to be a more gradual process, due to learning over time about match products, culling of poor matches during market downturns, and dynamic behaviors such as on-the-job search. For example in the framework of Lentz (2010) and Bagger and Lentz (2019), sorting becomes stronger as workers move up the job ladder.

![Figure 1.12: Wage Sorting and Match Selection, 2010-2017](image)

**Figure 1.12:** Wage Sorting and Match Selection, 2010-2017

*Note:* Time in workforce dated from first payment into social security.

If such a theory were to explain wage sorting, we would expect sorting to be greatest among agents with a longer labor market history, and job spells of a longer tenure. In figure 1.12 we see that this is not the case. Wage sorting is most pronounced among workers that have recently entered the labor force, establishments that recently opened, and job spells more recently begun. The negative slopes in the figure are explained, to a large extent, by compositional differences between agents and matches with long tenure and those with short; but as the differences in composition are closely related to industry and occupation, it is not possible to control for them without also controlling for wage sorting, per the previous section. Hence it is difficult to reconcile match-based sorting with observed wage trends in West Germany. One must be prepared to argue that industry and occupation not only capture unobservable heterogeneity of workers and establishments, but are themselves *ex ante* unknown to market participants - i.e. they do not capture differentiated output and
input markets. But in this case it becomes difficult to avoid the conclusion that there is negative selection of assortative matches as agents become informed, which is inconsistent with the notion of match complementarities as described above.

1.3.3 Drivers of the Between-Group Trend

Why has between-group wage sorting increased over time? A causal answer to this question is beyond the scope of the paper, but noting that the between-group covariance in equation (2) is a function of group weights and average wage premia, it is possible to answer a more limited question: what changes at the industry/occupation level are associated with the upward trend in figure 1.10? In this section I conduct a set of simple counterfactual experiments that isolate the relative importance of changes to the three terms entering the between-group component in equation (2). I focus on the between-group covariance for 180 industry-occupation cells, and I consider how this covariance changes when group shares and/or wage effects are held constant over time. Moments are held constant in three ways: for each the 180 industry-occupation cells; for the 12 industry groups, while allowing for within-industry, between-occupation variation over time; and for the 15 occupational groups, allowing for within-occupation but between-industry variation. While these experiments are out-of-equilibrium, and they do not cleanly separate price and quantity changes since within-group composition is unobserved, they may nevertheless provide descriptive evidence as to which causal mechanisms are most likely.

The results of the reduced-form experiments are given in table 1.13. The 1993-2017 trend decreases by 1/3 when group mean person effects are held constant, by 1/2 when group weights are held constant, and by a negligible amount when mean establishments effects are fixed. This result suggests that changes to $\phi$ have not played a key role; on average we do not observe employer wage differentials increasing in jobs associated with highly-paid workers. Breaking down changes to the person effect $\pi$, we see that the rise in $Cov(\pi, \phi)$ is more strongly associated with inter-industry than with inter-occupation differentials. In other words, it is less important that wages have risen in highly-paid occupations, than that they have risen in industries associated with high-paying employers. Finally, the rise in covariance associated with group shares is almost entirely due to industry shares. Though
the occupational distribution has evolved over this period, in and of itself this evolution has contributed little to wage sorting. More subtly, we see little indication of a role for changes to the conditional distribution of occupations across industries - that is, controlling for industry shares, we do not see increased sorting of high-paying jobs into high-paying sectors.

The last four rows of 1.13 explore the combined effects of \( \pi \) and \( \omega \). When industry shares and group mean person effects are held fixed, there is no longer a significant trend over the full sample period, further indicating a limited role for changes to employer wage differentials. This conclusion changes somewhat when looking at the early 2000’s; there is considerable movement in the group mean employer effects over these years, which may reflect short-term changes to behavior or within-group composition. A similar trend is also seen in the between-group variance of \( \phi \),\(^{16}\) which is likely due in part to the greater incidence of limited-mobility bias during this period. Note, however, that at the group level the effect changes to \( \phi \) has been to increase wage sorting, whereas if it were the result of bias then we should see the opposite effect. Hence, the rise in \( Var(\phi) \) is likely to reflect real changes, if

\(^{16}\)The between-group variances are shown in the appendix.
apparently temporal ones, to the wages paid by different establishments.

Figure 1.14: Establishment Effects by Industry and Size

Note: Axes indicate group mean wage effects, averaged across 1993-1999 and 2010-2017. Green (red) bubbles indicate an increase (decrease) in occupation occupation mean person effect and industry employment shares, measured as the difference between 1993-1999 and 2010-2017 values.

The key developments are shown graphically in figure ???. The pattern across occupations, which also holds for industry-occupation pairs, is that wage growth in terms of \( \pi \) has been concentrated in high-earning jobs. At the same time, high-earning jobs are in all periods more common at high-paying establishments. This is true at the industry level, where for example we see technology-intensive sectors being both high-paying and relatively more intensive in skilled occupations; and it is also true within industries, where larger establishments tend to both pay more and to employ more highly-paid types of occupations. Hence a rise in wages (\( \pi \)) among high-earning occupations tends to strengthen the aggregate relationship between \( \pi \) and \( \phi \), contributing to wage sorting. The pattern across industries is less subtle, and can be characterized as a kind of two-sided polarization: growth has been concentrated in industries that are either low-\( \pi \) and low-\( \phi \), or high-\( \pi \) and high-\( \phi \). In particular, employment has shifted away from crafts and commodities sectors that employ a relatively lower-paid workforce, but are associated with relatively high-paying firms. Here as well, the overall effect of these changes is to increase the degree of wage sorting present.

While changes to employer wage effects have contributed little to the aggregate trend, at a disaggregate level as in figure ?? a more complicated picture emerges. The pay gap between small and large firms has increased in a number of industries, most notably those
in manufacturing. This is consistent with the argument made by Card, Heining, and Kline (2013) that the increase in \( \text{Var}(\phi) \) for West Germany is related to the decline in collective bargaining coverage that occurred following reunification, as this development mostly affected smaller establishments.\(^{17}\) Large firms tend also to employer workers with a higher average \( \pi \) - a pattern that holds within industries as well as between them - and so one might expect a widening size-wage gap to contribute to wage sorting. This is offset, however, by a rise in \( \phi \) associated with employers in the hospitality and temp agency industry group, which overwhelmingly employs lower-paid workers. Hence it appears that between- and within-industry changes to employer wage effects have had offsetting effects over the 1993-2017 period.

### 1.3.4 Discussion

Summarizing this section, West German wage sorting is entirely a between-industry, between-occupation phenomenon, and there is little evidence of sorting on match effects. Cross-section patterns at the occupation and industry level are qualitatively similar between the 1993-99

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\(^{17}\) I find in addition that the wage gap between covered and non-covered employers has also increased, likely reflecting a decline in the extent to which industry bargaining agreements are observed by establishments not formally subject to them.
and 2010-17 periods, with the primary changes being a widening of group $\pi$-differentials, and a shift in composition - primarily industry-related - favoring low-$\pi$, low-$\phi$ and high-$\pi$, high-$\phi$ jobs. Establishment wage effects are not entirely stable at the industry-occupation level, but in the aggregate they contribute little to the increase in wage sorting, suggesting that changes related to rents, rent-sharing, and market structure more generally are not key drivers of the trend. The most likely explanation for the observed changes would seem to structural and technological change, which are widely believed to have impacted labor markets over this period, and have direct implications for the composition of labor markets and for relative wages (i.e. $\pi$).

These results provide definition for the trend discovered by Card et al. (2013), and they indicate that wage sorting - a trend in unobservable wage effects - is fundamentally a story about observed dimensions of heterogeneity. In the next section I give descriptive results on the characteristics of workers and their employers most predictive of wage sorting. The ultimate goal, which lies beyond the scope of this paper, is a causal one: to uncover the causes of sorting in labor markets, and of the wage differentials that interact with sorting to yield a large and increasingly important source of wage dispersion.

1.4 Descriptive Evidence: Skill and Scale

The results shown thus far say little about the underlying causes of wage sorting. Industries and occupations are theoretically meaningful only as measures of differentiation in output and input markets, though for that reason they are also likely to capture agent characteristics more directly relevant to wages, such as worker skill and firm size. The patterns in figure 1.11 suggest that such characteristics are relevant: industries that are anecdotally associated with greater scale tend to exhibit higher mean values of both wage effects, as do occupations requiring greater skill. In this section I quantify the role of such characteristics, using the rich employer survey data present in the LIAB and supplementing this with person-level survey data on occupational tasks. The purpose of this exercise is first to assess whether, and to what extent, wage sorting is associated with meaningful observable characteristics of workers and firms; and second, to evaluate possible theoretical mechanisms that might explain these associations.
The approach I take is to study how the correlation $Cor(\pi, \phi)$ is changed when controlling for the observable characteristics of agents. Correlation is preferred in this case because, as a normalized measure, it is less influenced by changes to the dispersion of wage effects (e.g. due to rising skill premia) and better reflects the strength of underlying worker-firm sorting. I use partial and semi-partial correlations to measure the ability of worker and establishment variables to explain wage sorting at the industry-occupation levels. Letting $X$ denote an observable characteristic, linear regressions of the wage effects on $X$ yield residuals $\pi_X$ and $\phi_X$. For a partial correlation this step is performed for both wage effects, whereas for semi-partial correlation only one of the two wage effects is regressed on $X$. These measures have different interpretations. If for example $X$ is education, then the semipartial correlations will tell us (1) how person-level wage gaps unrelated to education are correlated with firm premia, and (2) how person wage effects are correlated with the component of firm premia unrelated to the educational composition of the firm. The partial correlation employs both controls, and indicates the relationship between person wage gaps orthogonal to education and employer wage gaps unrelated to educational composition. Intuitively, while partial correlations provide information about the overall ability of $X$ to explain the correlation, semipartial correlations are informative about where this explanatory power comes from - i.e. whether it tells us about worker traits or employer characteristics associated with wage sorting.

1.4.1 Wage Sorting and Establishment Characteristics

I begin with an analysis of establishment characteristics, drawing on the employer survey data provided with the LIAB. I focus on the 2003-2010 period, as this corresponds to one of the panels of estimation for the AKM wage effects, and it is also the period in which survey questions - which are not in general consistent across years - are most relevant for this project. This is also in keeping with the focus of the question, which is not to study the cross-sectional relationship In most cases, only a small percentage of employers decline to answer a particular question, but this proportion can become considerable when using several survey responses to calculate a statistic (e.g. value-added). For this reason I impute missing answers, for each of the variables below, by running a regression with fixed effects
for detailed industry groups, the interaction of five employment size categories with three sectoral groupings, log employment, and year dummies. Logistic regression is used when the response variable is binary. I then aggregate at the industry-occupation level by running a second-stage regression with industry-occupation fixed effects and year dummies, with the predicted values for 2003 forming the basis of the results below.

I focus on two sets of variables. First, I consider employer characteristics related to scale - a well-known covariate of higher pay - and activities related to technology adoption and innovation, which are widely thought to increase skill requirements at the firm level. Scale, technology, and innovation are not independent; the first is often reflective of fixed costs, while the latter two are key sources of such costs. There is, for example, a robust between firm size and information technology adoption,¹⁸ and this type of linkage is consistent with a mechanism in which high-tech firms are simultaneously more intensive in skilled labor, and characterized by larger match rents or more severe forms of capital hold-up that manifest as higher wages. The second set of variables I examine are related to market structure. We would expect a less competitive output market, or a labor market characterized by wage agreements, to result in higher wages. It may be the skill-intensive industries are by nature less competitive, or that bargaining agreements drive firms to employ a higher-skilled workforce, which could explain the wage sorting patterns observed.

The relationships between establishment characteristics, wage effects, and wage sorting are described in table 1.¹⁶ The first two columns give the unconditional correlations between the observable and the AKM wage effects; the third and fourth columns give the semi-partial correlation of the wage effects, as described above; and the fifth and final column gives the partial wage effects correlation. We see that measures of market competitiveness are only weakly related to either wage effect, but it should be noted that these variables reflect categorical survey responses and are therefore quite limited. Firm premia are higher in industry-occupation groups with a higher propensity of collective bargaining agreements, but the correlation is relatively low at one-third. Measures of establishment size, on the other hand, are highly predictive of firm premia at the industry-occupation level,¹⁹, with

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¹⁸See e.g. the meta-analysis by Lee and Xia (2006).
¹⁹I note in passing that size, and observable employer characteristics more broadly, are much less effective at explaining the intra-industry dispersion of firm effects, suggesting a different underlying mechanism.
Table 1.16: Wage Effects and Establishment Characteristics, 2003-2010

<table>
<thead>
<tr>
<th>Variable (X)</th>
<th>Cor(X,π)</th>
<th>Cor(X,φ)</th>
<th>Cor(πX,φ)</th>
<th>Cor(π,φX)</th>
<th>Cor(πX,φX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No control</td>
<td></td>
<td></td>
<td>0.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log employment</td>
<td>0.404</td>
<td>0.751</td>
<td>0.267</td>
<td>0.370</td>
<td>0.404</td>
</tr>
<tr>
<td>Log value-added</td>
<td>0.461</td>
<td>0.844</td>
<td>0.179</td>
<td>0.296</td>
<td>0.334</td>
</tr>
<tr>
<td>Log investment/worker</td>
<td>0.400</td>
<td>0.861</td>
<td>0.221</td>
<td>0.399</td>
<td>0.435</td>
</tr>
<tr>
<td>Log ICT inv./worker</td>
<td>0.647</td>
<td>0.809</td>
<td>0.032</td>
<td>0.041</td>
<td>0.054</td>
</tr>
<tr>
<td>Multi-estab.</td>
<td>0.490</td>
<td>0.536</td>
<td>0.327</td>
<td>0.338</td>
<td>0.387</td>
</tr>
<tr>
<td>Collective barg. (%)</td>
<td>0.070</td>
<td>0.336</td>
<td>0.525</td>
<td>0.556</td>
<td>0.558</td>
</tr>
<tr>
<td>Product dev. (%)</td>
<td>0.427</td>
<td>0.782</td>
<td>0.236</td>
<td>0.343</td>
<td>0.379</td>
</tr>
<tr>
<td>Process impr. (%)</td>
<td>0.413</td>
<td>0.805</td>
<td>0.237</td>
<td>0.363</td>
<td>0.399</td>
</tr>
<tr>
<td>Positive profits (%)</td>
<td>0.261</td>
<td>0.019</td>
<td>0.562</td>
<td>0.543</td>
<td>0.562</td>
</tr>
<tr>
<td>Competitive mkt. (%)</td>
<td>-0.110</td>
<td>0.265</td>
<td>0.580</td>
<td>0.598</td>
<td>0.602</td>
</tr>
</tbody>
</table>

Note: All variables first aggregated at the industry-occupation level via a first-stage regression on fixed effects for industry-occupation and year. Terms π_x and φ_x indicate residuals from a regression of wage effects on establishment characteristics. All results weighted by employment; see appendix for confidence intervals.

correlations as high as .84 for value-added and .86 for capital investment per full-time worker. Critically, we observe that person wage effects are strongly correlated with IT investment, and with the percentage of employers engaging in product development or process innovation; and that these same variables are strongly correlated with the firm effect as well.

The semi-partial and partial correlations provide further evidence that wage sorting is strongly associated with technological gradients across employers. Product/process development are particularly explanatory of the variation on π associated with wage sorting, though less informative about φ. A limitation of these variables is that they are highly correlated ∼ .9) with measures of establishment scale. This is not true of investment in information technology, which is less correlated with size and is poorer at explaining the firm effects overall, but nevertheless succeeds in capturing virtually all of the variation in firm effects associated with wage sorting. The reason for this difference is that employer scale varies less between service sectors than between manufacturing sectors, whereas ICT investment per capita exhibits a more consistent relationship and is therefore better able to simultaneously explain wage sorting within manufacturing, within services, and between manufacturing and services.
Figure 1.17: Establishment Scale and Wage Effects

Note: First two rows show industry mean values. Last two rows show industry-occupation mean values, demeaned by industry. Shaded areas indicate 95% confidence intervals.

The conditional linear relationships, plotted in figure 1.17, provide additional clarity on why ICT investment is more informative than establishment scale. The former is strongly predictive of $\pi$, both across industries and within them. Industries that are more intensive in information technology tend to be more skill-intensive as well, and the same relationship hold for establishments within industries. On the other hand scale-related measures are only
predictive of the person wage effect between industries; intra-sector wage sorting does not appear to be explained by size \emph{per se.}

These results cannot speak to causality, though they do suggest that \emph{some} causal mechanism leads capital-intensive firms and industries to also be skill-intensive, and this relationship to be particularly strong when the capital in question is particularly skill-biased. The most obvious explanation is capital-labor substitution. Alternatively, it may be that capital intensity is a function of scale, and that industries in which knowledge is intensive tend to benefit from greater economies of scale, for example due to spillovers. This would be consistent with the superstar firm hypothesis as stated by Autor, Dorn, Katz, Patterson, and Van Reenen (2020). Yet another possibility is that in sectors with greater efficient scale, there are correspondingly larger knowledge hierarchies, for example as proposed by Garicano (2000), and consequently a stronger propensity to invest in information-related technologies. The challenge to this story, and to others, is that if technology investment is only incidental, then it is unclear why it would appear more explanatory than observables directly linked to scale.

\subsection*{1.4.2 Wage Sorting and Job Characteristics}

I next consider the association of match-level characteristics with wage sorting. For this exercise I supplement data on educational attainment and experience from the LIAB with task data from the 2005-2006 BIBB employment survey, collected by the Federal Institute for Vocational Education and Training (BIBB) in partnership with the Federal Institute for Occupational Health and Safety. The BIBB surveys draw on a random sample of the employed German labor force, and asks respondents a range of questions concerning job task content, the use of technology and tools, and other aspects of the job environment and the individual’s work history. In this section I focus on questions concerning tasks performed on the job, which are answered on a frequency scale ("never", "sometimes", "always") from which I impute numerical values (0, .5, 1). For some industry-occupation cells I observe few or no workers, so I estimate task values by performing a fixed-effects regression on dummy variables for industry and occupation separately, and interactions between three industry groups and four occupation groups. The BIBB surveys are conducted every 6 years and in
principle the 2005-06 survey might be supplemented with earlier and later surveys, however in practice this is the last version of the BIBB survey containing occupational codes consistent with the LIAB, while the questions concerning job tasks are different in previous versions of the survey. As in the LIAB I limit attention to full-time workers aged 20-60, but it is important to note that workers who do not pay social security taxes, and are absent from the LIAB, may nevertheless show up in the BIBB surveys. Hence there is likely to be some mismatch present when making comparisons between these datasets.

Three groups of variables are included in this analysis. First, I consider several statistics related to worker experience and tenure, as these are usually associated with higher wages, and many models (especially those with on-the-job search) would predict that they are also associated with higher-paying employers. Second I include variables related to education and skill. We would expect these to be strongly related to person wage effects, and it is possible that skilled workers, for various reasons, are more likely to sort into high-paying employers; one example would be a complementarity between skilled labor and high-paying (e.g. large-scale) employers, which if labor types are substitutes would tend to result in greater skill-intensity. Finally, I consider a representative set of BIBB task measures. The value of job tasks is that they describe the type of work being done, and are therefore indicative of both the employer’s type and the worker’s type. This makes tasks particularly useful for the study of sorting in labor markets.

Unconditional and partial correlations are shown in table 1.18. Worker experience and job tenure are highly correlated with firm premia, as would be expected if high-paying employers are sought-out over time, and they are also associated with a higher person effect. Wage sorting is equally strong, however, between the portions of the firm and worker effects unrelated to these variables. Education yields a similar story. As expected there is a strong correlation between education and the person effect, as well as a positive but smaller correlation with the firm effect, but there is also substantial wage sorting between when considering only that part of the person effect unrelated to education. The task variables are more informative. Consistent with results in the previous section, PC use is explanatory of both wage effects, and wage sorting is substantially weaker when it is controlled for. This, too, suggests that the technological capacity of the firm is closely related to the sorting patterns affected
Table 1.18: Wage Effects and Job Characteristics, 2003-2010

<table>
<thead>
<tr>
<th></th>
<th>Cor($X,\pi$)</th>
<th>Cor($X,\phi$)</th>
<th>Cor($\pi_X,\phi$)</th>
<th>Cor($\pi,\phi_X$)</th>
<th>Cor($\pi_X,\phi_X$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LIAB Person Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No control</td>
<td>0.548</td>
<td>0.548</td>
<td>0.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>College degree (%)</td>
<td>0.823</td>
<td>0.253</td>
<td>0.598</td>
<td>0.351</td>
<td>0.618</td>
</tr>
<tr>
<td>Years of education</td>
<td>0.873</td>
<td>0.271</td>
<td>0.638</td>
<td>0.323</td>
<td>0.663</td>
</tr>
<tr>
<td>Job tenure</td>
<td>0.162</td>
<td>0.778</td>
<td>0.427</td>
<td>0.671</td>
<td>0.680</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>0.193</td>
<td>0.625</td>
<td>0.435</td>
<td>0.547</td>
<td>0.558</td>
</tr>
<tr>
<td><strong>BIBB Task Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No control</td>
<td>0.541</td>
<td>0.541</td>
<td>0.541</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analyze information</td>
<td>0.823</td>
<td>0.318</td>
<td>0.491</td>
<td>0.294</td>
<td>0.518</td>
</tr>
<tr>
<td>Use PC</td>
<td>0.703</td>
<td>0.402</td>
<td>0.364</td>
<td>0.282</td>
<td>0.397</td>
</tr>
<tr>
<td>Manage others</td>
<td>0.577</td>
<td>-0.008</td>
<td>0.668</td>
<td>0.545</td>
<td>0.668</td>
</tr>
<tr>
<td>Buy/sell</td>
<td>0.040</td>
<td>-0.401</td>
<td>0.557</td>
<td>0.608</td>
<td>0.608</td>
</tr>
<tr>
<td>Control machines</td>
<td>-0.245</td>
<td>0.226</td>
<td>0.615</td>
<td>0.612</td>
<td>0.631</td>
</tr>
<tr>
<td>Weigh/measure</td>
<td>-0.038</td>
<td>0.251</td>
<td>0.551</td>
<td>0.569</td>
<td>0.569</td>
</tr>
<tr>
<td>Care for others</td>
<td>-0.068</td>
<td>-0.237</td>
<td>0.526</td>
<td>0.540</td>
<td>0.541</td>
</tr>
<tr>
<td>Clean</td>
<td>-0.707</td>
<td>-0.372</td>
<td>0.393</td>
<td>0.299</td>
<td>0.423</td>
</tr>
</tbody>
</table>

Note: All variables first aggregated at the industry-occupation level via a first-stage regression. Years of education is imputed following Card et al. (2013). Task values imputed from verbal frequencies. Terms $\pi_x$ and $\phi_x$ indicate residuals from a regression of wage effects on job characteristics. All results weighted by employment; see appendix for confidence intervals.

Wages. Tasks related to cleaning are also explanatory, though in the opposite direction. These appear to proxy not just for a lower level of skill and hence wages on the worker side, but also for smaller, less sophisticated establishments, where simple manual tasks comprise a greater proportion of the work to be done.

In figure 1.19 we can see the reasons for the weak explanatory power of variables that solely capture worker skill. At the occupation level, education is highly predictive of occupation-level differences in the person effect, but only weakly associated the firm effect, and this association vanishes entirely within-occupation. A job focus on analytical tasks performs somewhat better, while the two task measures discussed above are best at accounting for both between- and within-occupation variation in wage effects. That is, not only are more computer-intensive occupations associated with both higher-paying firms and higher-earning individuals, but this relationship also holds strongly within occupations; and this is impor-
Figure 1.19: Worker Skill and Wage Effects

NOTE: First two rows show occupation mean values. Last two rows show industry-occupation mean values, demeaned by occupation. Shaded areas indicate 95% confidence intervals.

Tant because a substantial portion of wage sorting is within-occupation.

These results bear on a large literature relating labor outcomes to job tasks. The virtue of task-level data is that it allows us to describe labor markets not in terms of the agents

20See for example Autor, Levy, Murnane (2003), Goos and Manning (2007), Acemoglu and Autor (2011), and Autor and Handel (2013), and for studies specific to Germany Spitz-Oener (2006) and Gathmann and Schonberg (2010).
involved, but in terms of the work being done, which is more closely linked to changes e.g. in the technological environment. It is nevertheless evident from the graphs shown above that (1) tasks may capture substantially heterogeneity in firm capabilities, in addition to worker skill, and (2) much of this variation is within-occupation but between-industry. This implies that studies relating tasks to wage outcomes may not be measuring returns to skill but, in part, returns to scale - e.g. firm rents. It also indicates that industry codes can provide a useful supplement to occupation when considering job tasks, as there is important variation in important dimensions.

1.5 Conclusion

I conclude by summarizing what we know about wage sorting. It is, first of all, a robust feature of the German labor market. Although I find that longitudinal analyses can be adversely impacted by statistical bias, which varies over time with labor market conditions, this does not appear to explain growing contribution of wage sorting to German wage inequality over 1993-2017. Secondly we know that in the German case, wage sorting is entirely a between-industry, between-occupation phenomenon. Although theoretical studies of sorting in labor markets have typically focused on match-level complementarities, these are not consistent with the aggregate nature of wage sorting in Germany, which is more consistent with a mechanism based on heterogeneity factor demand (i.e. for skill) across sectors. In addition there is little evidence that changes to market structure have contributed to wage sorting; inter-industry firm premia differentials are stable over the full sample period. A novel explanation does not seem necessary for the rise in wage sorting, which is well-accounted for by developments associated with technical and structural change: a shift of low-skill labor from manufacturing to services, and rising wages in skilled jobs that, due to occupational sorting patterns, are more likely to locate at high-paying firms. We know, finally, that observable characteristics of agents are surprisingly effective at accounting for person and firm wage effects at the industry-occupation level; and that technological differences between workplaces - being strongly associated with both worker premia and employer scale - are statistically able to capture much of observed wage sorting patterns. This last result provides further evidence of an important role for technical and technological differences between markets,
and suggests that a useful direction of future research will be to explore how these differences relate to the organization structure (and in particular the wage policies) of firms.

One implication of these results, explored in the next chapter, is a likely connection between wage sorting and skill-bias. If technology adoption is associated not only with skilled labor but also with high-paying employers - either causally or indirectly through an association with firm scale - then there are several channels by which skill-biased technical change may affect wages. Conversely, because industry and occupation capture substantial differences in employer pay as well as worker-firm sorting, technology adoption at the industry-occupation level is unlikely to provide clean results on the presence or magnitude of skill-bias. This is potentially important for empirical studies exploiting such variation.

A second implication concerns the competitiveness of labor and goods markets: if wage sorting reflects fundamental (e.g. technological) differences between markets rather than lack of competition, then the trade-offs associated with firm-side wage policies becomes more adverse. This problem is highlighted by the positive correlation between the firm component of wages and the (admittedly noisy) measures of market competitiveness considered above. For a policy targeting firm rents or rent-sharing to have a large effect, it will have to target high-paying, skill-intensive sectors; but if these sectors are high-paying not as a result of greater market imperfections but due, for example, to greater economies of scale, then the trade-offs associated with such policies will be more adverse. This further highlights the need for a better understanding not only of wage sorting, but of labor sorting patterns more broadly.
Chapter 2

Skill-Bias, Firm-Bias, and Wage Inequality

2.1 Introduction

Rising OECD wage inequality is widely attributed to changes in the relative demand for different types of labor. Technological change is thought to have complemented knowledge-intensive occupations, increasing the demand for “skilled” workers, while a structural shift away from manufacturing has reduced demand for goods-production jobs, and for the less-skilled parts of the labor force traditionally employed by industry.\(^{21}\) The effect of a *skill-biased* demand shock in a competitive labor market is widely-studied, and well-known: an increase in the relative wage paid to skill workers, and a general rise in wage inequality. This explanation for observed wage trends is straightforward and intuitively compelling, but also challenged by a large body of evidence attesting that labor markets are not perfectly competitive. Different employers are found to pay different wages, controlling for worker type - a phenomenon usually attributed to differences in firm or match rents, coupled with market frictions that prevent arbitrage.\(^{22}\) Differences in employer pay are found, moreover, to be related to occupational and industry wage gaps, an important example being the manufacturing wage premium observed in many developed countries.\(^{23}\) The presence of large, firm-specific wage premia implies that demand shocks may be *firm-biased* as well as skill-biased; they may affect employer composition and hence the distribution of firm premia,


\(^{22}\)Card, Cardoso, Heining, and Kline (2018) survey this literature. A typical estimate is that 15% of observed wage variance is due to employer wage differentials.

and they may change the incidence of firm premia across the wage distribution, and the extent to which labor market rents are captured by high wage-earners. By failing to account for these dynamics, canonical models of skill-bias may fundamentally mis-characterize the sources of rising wage inequality.

In this chapter, I study theoretically and quantitatively how skill-bias and firm-bias jointly determine the effect of demand shocks on the wage distribution. I develop a model with (1) explicit notions of industry and occupation demand, (2) labor supply formalized as an assignment problem, in which differentially skilled workers sort into different occupations, (3) firms that earn positive flow rents due to fixed hiring costs, and (4) search frictions that result in rent-sharing between firms and workers. Skill premia and firm premia are equilibrium outcomes, but are also inputs to the hiring decisions of firms and to the job search behavior of workers. The model allows for rich interactions between prices and quantities, but is also both theoretically and empirically tractable. The wage and policy functions yield closed-form solutions that map directly into empirical wage effects, allowing the key distributional model parameters to be non-parametrically identified. I estimate the model using matched employer-employee data from Germany spanning the period 1993-2017, and I conduct counterfactual experiments that address two questions. First, what is the historical role of firm-bias in explaining German wage trends? And second, what are the implications of firm-bias for policies that affect the wages firms pay?

I find that industry and occupation demand account for two-thirds of the rise in West German wage variance, of which at least one-half is explained by firm-bias and interactions between firm-bias and skill-bias. To obtain this result I conduct counterfactual experiments in which firm-bias is “shut down” by equalizing the match rents earned by firms. Firm-bias is especially important for industry demand, which I estimate to explain one-sixth of the overall trend but which would have exerted a small, negative impact on wage inequality if firm premia were homogeneous. Disaggregating further, I find that the effect of firm-bias varies substantially between individual industries and occupations, and I show that conditioning on worker skill does not allow one to predict the direction of the effect on wage variance; an increase in demand for low-skill jobs can increase wage inequality if it occurs in low-paying sectors, while demand for skilled jobs can have the opposite effect. I show in
addition that the variability introduced by firm-bias can help to account for regional trends in wage dispersion, and that differences in the distribution of firm premia across industries and occupations can substantially explain the smaller increase in wage variance observed in East Germany.

Finally, I consider the joint implications of skill-bias and firm-bias for policies that target firm premia. I conduct two experiments. The first, motivated by attempts to extend collective bargaining coverage to temp agency workers, involves eliminating the wage gap between temp agencies and the firms that employ temp worker. In the second experiment I consider how wage inequality is affected by a compression of firm entry costs, rationalized as the outcome of an exogenous (and un-modeled) reduction in anti-competitive barriers to entry. I show that in equilibrium, demand and supply responses largely offset, but on net tend to dampen the effects of such policies on wage inequality. The reason is that firm-side wage policies are firm-biased by design, and when firm premia and skill premia are correlated, they will also be skill-biased.

The main contribution of this paper is to the macroeconomic literature on wage inequality, and the contributing role of skill-biased demand shocks due to technological change and trade. Models of skill-bias commonly assume frictionless labor markets: for example Krusell et al. (2000), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018) on technological change, and Costinot and Vogel (2010), Autor and Dorn (2013), and Burstein and Vogel (2017) on trade. On the other hand studies that allow for imperfectly competitive labor markets, such as Helpman, Itskhoki, and Redding (2010), abstract from relative labor demand and the price of skill in order to study residual wage inequality within narrow labor markets. In this paper I consider interactions between the demand for skill and firm premia, which with few exceptions (discussed below) have not been studied in the literature. In doing so I combine and generalize the models of Acemoglu and Restrepo (2018) and Autor and Dorn (2013), and extend the skill-bias paradigm to the case of imperfectly competitive labor markets with search frictions. In this environment there are rich interactions between occupational choice and employer choice, and wage premia are not simply an equilibrium outcome but a motivating force behind the job search behavior of workers and the hiring decisions of firms.
A second key contribution of this paper is the development of an empirically tractable equilibrium model of the wage distribution. This is difficult to achieve because neither skill nor firm premia are observable, and equilibrium models with imperfectly competitive labor markets do not, in general, map into additive empirical wage effects. Type distributions must be parameterized, which in this case would result in loss of information about the underlying wage and employment structures that constitute the key objects of study. I achieve tractability through two strong but reasonable modeling assumptions. First, I assume that labor search is directed across industries and occupations, and not random as is commonly studied in the literature. Under an efficiency restriction on the unemployment insurance payout, directed search results in a closed-form, log-additive wage function. Second, I assume that workers choose their occupations, formalized as an assignment problem building on Costinot and Vogel (2010). Assignment yields skill premia that are monotonically increasing in type, allowing worker skill to be identified from empirical wage effects in a time-consistent manner. Although tractable, the model allows for rich interactions between prices and quantities. Workers may seek out high-paying firms and occupations associated with high-paying firms, though I allow for the possibility of offsetting non-pecuniary amenities. In addition, I incorporate a non-participation labor state to capture supply shocks stemming from the secular decline in German unemployment rates.

This approach contrasts with extent work in the literature. Wage accounting models such as Feenstra and Hanson (1999), Goos, Manning, and Salomons (2014), and Lee and Wolpin (2010) are highly stylized, and are either out-of-equilibrium or limited to a small set of agent types. The equilibrium model I develop allows for an arbitrary number of industries and a continuum of skill types and occupations. Closer to this study are papers by Card, Cardoso, Heining, and Kline (2018), who develop a reduced-form model in which demand is differentiated across skill types and firms face upward-sloping supply curves, and that by Haanwinckel (2021) who extends the Card et al. framework to an equilibrium setting. Their approach yields a separable wage function when firms have homogeneous demand for skill, but becomes analytically and empirically intractable when skill demand is heterogeneous.

24Directed search is not, in general, efficient when (1) the environment is dynamic and (2) submarkets produce differentiated outputs. A dynamic setting will tend to create congestion in particular submarkets, which in turn creates inefficiency when submarket output is not perfectly substitutable.
The model studied here retains wage separability under arbitrary industry-occupation sorting patterns, and is therefore better-suited to the quantitative study of interactions between skill demand and firm premia. Finally, while a number of studies have used occupational assignment as a framework modeling skill premia, this is also one of the only papers to structurally estimate such a model. The only other study to do so - Ales, Kurnaz, and Sleet (2015) - imposes parametric assumptions on match production, adversely affecting the ability of the model to replicate wage and employment patterns.

Finally, this paper contributes to the literature on the empirical and theoretical sources of wage dispersion, and bridges the gap between two empirical literatures. On the one hand are studies that attribute wage trends to skill-biased demand, such as Acemoglu and Autor (2011), Autor and Dorn (2013), Hummels et al. (2014), Autor, Dorn, and Hanson (2013), Ebenstein et al. (2014), Frey and Osborne (2017), and Acemoglu and Restrepo (2020). These papers focus on industry- and occupation-level outcomes, and the role of market-clearing prices in explaining a general widening of skill premia. Separately, a number of labor researchers have employed the wage decomposition proposed by Abowd, Kramarz, and Margolis (AKM, 1999) to study, in a reduced form setting, the mechanical sources of rising wage dispersion. Closely related to this paper, Card, Heining, and Kline (2013) estimate that one-fourth of the post-1980’s rise in West German wage variance is due to greater dispersion of firm AKM wage effects, and one-third to increased covariance of the person and firm effects.\footnote{See also Song et al. (2019), Bagger, Sorensen, and Vejlin (2013), and Hakanson, Lindqvist, and Vlachos (2021) for similar results in other countries, and Torres et al. (2018) and Alvarez, Benguria, Engbom, and Moser (2018) for cases in which firm wage effects are associated with a decline in wage dispersion.} I show that the different sources of wage dispersion studied in these papers are not independent, but have interacted over time. The results I present suggest that the effect of a skill-biased demand shock is sensitive to the distribution of firm premia across industries and occupations, whereas the presence of equilibrium interactions between supply, demand, and wage premia indicates that reduced-form models of the wage distribution will not be able to disentangle wage dispersion related to market-clearing prices, from dispersion associated with firm premia.

The outline of this paper is as follows. After a brief discussion of motivation, I develop the equilibrium model in section 2. The estimation procedure is described in depth in section 3,
as this is a key contribution of the paper. Quantitative experiments and results are described in section 4.

2.1.1 Motivation

The motivation for this paper comes from an empirical literature on the sources of wage dispersion, that builds on the regression approach of Abowd, Kramarz, and Margolis (AKM, 1999) to show that firm heterogeneity is a key source of wage variation. The AKM framework assumes that wage is the product of a person-specific and an employer-specific term, each representing both observable and unobservable agent heterogeneity. Wage effects are estimated from matched employer-employee data, and identified from wage changes associated with job transitions between employers. Variants of this basic approach have been applied in many different contexts and countries. The general finding is that firm premia constitute a small but important source of wage dispersion, directly accounting for between 10% and 20% of wage variance (Card, Cardoso, Heining, and Kline, 2018). As discussed in the introduction, a number of studies find that the contribution of firm premia has grown over time, most notably due to a strengthening pattern of association between high-earning workers and high-paying firms.

In contrast, the literature on skill-bias has focused almost exclusively on worker heterogeneity in explaining wage trends, with strong emphasis on changes to labor demand at the industry and occupation level. This approach is justified if firm premia are entirely a within-industry, within-occupation phenomenon, but past research suggests this is not the case. Inter-industry wage gaps have been found to reflect not only differences in worker ability, but in firm pay as well. The role of firm heterogeneity in driving occupational wage gaps is not as well-studied, but in their analysis of German wage dispersion Card, Heining, and Kline (2013) find large inter-occupation differences in firm premia, while Torres, Portugal, Addison, and Guimaraes (2018) estimate an AKM regression with occupation fixed

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27 See Gibbons and Katz (1992) for an early study, and Abowd, Kramarz, Lengermann, McKinney, and Roux (2012) for more recent work. The important of firm premia varies by country; Goux and Maurin (1999) found them to be unimportant in the case of France, but for Germany Card, Heining, and Kline (2013) found that the variance of the between-industry component of firm premia is large and growing over time.
effects on Portuguese data, which they find to strongly covary with firm premia. Hence changes to industry or occupation demand are not just of importance for skill premia, but are likely to affect the distribution of firm premia as well.

Figure 2.1: Wage Effects by Occupation and Industry, 2010-2017

**Source:** German linked employer-employee dataset (LIAB). **Note:** Weighted average of AKM wage effects by aggregated KLDB 1988 occupation and WZ 2008 industry. Wage effects estimated following specification of Card, Heining, and Kline (2013).

The extent of this problem can be seen in figure 2.1, which plots occupation- and industry-mean wage effects for West Germany over the years 2010-2017. There are substantial differences in firm premia across industries and occupations. Considered jointly, industry and occupation are as good as explaining firm premia as they are skill premia - they account for one-third of either variance - indicating that firm heterogeneity is just as important at the industry and occupation level as in the aggregate. Moreover, it is evident from the figure that these two sources of wage dispersion - person and employer - are not independent across either occupations or industries, but are strongly correlated.

The implication of the patterns observed in figure 2.1 is that changes to labor demand can affect wage dispersion in one (or all) of three ways. First, through the well-known mechanism of skill-bias, labor demand can impact the relative wages earned by skilled workers. Second, labor demand can disproportionately or disparately affect high-paying and low-paying firms, thereby changing the distribution of labor across firms and hence the dis-

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28This assumes that firm premia are not market-clearing prices but reflect e.g. rents insensitive to the demand for labor. This assumption is consistent with the model I develop in this paper.
tribution of firm premia across job matches. I refer to this mechanism as firm-bias. Finally, if there is sorting of skilled labor into high-paying firms, skill-bias and firm-bias may interact. Changes to skill premia may be concentrated in high-paying firms, thereby having a greater affect on wage variance than they otherwise would have; and shifts in the composition of employers can affect the incidence of firm premia across the wage distribution, and the extent to which high-skill workers also benefit from a high-paying employers. The third channel in particular depends on the equilibrium responses of wages and quantities, and is the primary motivation for the equilibrium model developed next.

2.2 Model

In this section I develop a model that formalizes, in a tractable way, the linkages between labor demand, skill premia, and firm premia. Labor demand is differentiated by industry and occupation, generalizing an approach taken previously by Acemoglu and Autor (2011) and Autor and Dorn (2013). Firm premia arise due to the presence of search frictions, which create a rent-sharing motive and a wage gap between high-rent and low-rent firms. Skill premia are modeled as the result of an occupational assignment problem following Costinot and Vogel (2010), adapted here to the case where firms are heterogeneous and labor markets are frictional. A key prediction of the model is that under a set of restrictions that may be interpreted as efficiency conditions, the wage function is separable in worker and firm type, providing a rationale and a microfoundation for the common empirical assumption of log-additivity. Skill premia and firm premia are not, however, independent; the model predicts a set of potentially important interactions between supply, demand, and wage premia, indicating that an equilibrium framework is necessary for disentangling the roles of skill- and firm-biased changes to demand. One implication of this, explored further in the quantitative portion of the paper, is that wage policies targeting firm premia will in general have implications for skill premia, suggesting that the distributional effects of such policies are not straightforward. I close the section with a set of numerical comparative statics that illustrate the equilibrium properties of the model.
2.2.1 Environment

The economy is set in continuous time. There exists a continuum of infinitely-lived workers, heterogeneous in a scalar skill variable $s \in [s, \bar{s}]$ whose distribution is given by the continuously differentiable and strictly increasing function $\nu(s)$. Firms are endogenous in measure and exist in two types. Employers are single-vacancy firms that hire workers to produce an $(i, j)$-specific labor output, where $i \in \{1, ..., I\}$ is industry and $j \in [0, 1]$ denotes occupation.\textsuperscript{29} Industry aggregators (or simply aggregators) combine labor outputs within an industry to produce an $i$-specific final good, which is then sold to, and consumed by, a representative household. All agents are risk-neutral and discount the future at rate $\rho$. As I will focus on the steady-state equilibrium, time subscripts are suppressed in this section.

Employers. An employer enters the market by paying a vacancy flow cost $C(i, j)$ and posting a wage offer $w$, with entry assumed to be otherwise free. Vacancies are filled at a rate $q(\theta(i, j))$ with $\theta$ denoting market tightness. An $(i, j)$ vacancy filled by an $s$-worker generates match output $m(j, s)$, which may be sold to $i$-aggregators at a unit price of $p(i, j)$.

To preclude the screening of workers by firms, I assume that the wage offer $w$ is denominated in units of match output, so that the amount paid to a hired worker is $wm(j, s)$.\textsuperscript{30} Existing matches dissolve at an exogenous rate $\delta$ that is the same across types.\textsuperscript{31} I assume that match output is increasing in skill, and that skilled workers are relatively better at high-$j$ (i.e. skill-intensive) occupations:\textsuperscript{32}

\textsuperscript{29}The assumption of a discrete number of industries is empirically motivated, and may be relaxed without affecting the results shown below.

\textsuperscript{30}With search frictions, the worker-optimal and firm-optimal occupational assignments are generally different. This assumption imposes the worker-optimal assignment, which I favor for its intuitive appeal. From a quantitative standpoint there is no loss of generality, as the two approaches yield different parameter estimates but identical model relationships.

\textsuperscript{31}My rational for this assumption is that I am not attempting to replicate empirical worker flows. While it will be important that the model captures the response of labor supply to posted wages, this requires only a single margin of worker choice (job search), and it is therefore convenient to assume that $\delta$ is exogenous and to account for variation in separation rates when estimating the empirical elasticity of labor supply. A similar point may be made regarding the absence of on-the-job (OTJ) search, which has implications primarily for worker flows and not wages given that wages in this environment are not the outcome of bargaining.

\textsuperscript{32}Comparative advantage is required for uniqueness of the optimal assignment, while the ranking of $j$ is simply a normalization. That skill possesses an absolute advantage is intuitive in this context, and a similar assumption is made by Teulings (1995). The optimal assignment is simplified by having $m$ independent of $i$, and there is little loss of generality given that skill is one-dimensional.
Assumption 1 (Match productivity): \( m(j, s) \) is a continuously differentiable function, and for any \( j \) and \( s \) we have \( \frac{\partial}{\partial s} m(j, s) > 0 \) and \( \frac{\partial^2}{\partial j \partial s} \log m(j, s) > 0 \).

The problem facing an employer is therefore to choose the wage that maximizes the expected value of a vacancy posting, taking as given equilibrium prices and market tightness:

\[
\max_w S^V(i, j, w) \\
\text{s.t. } \rho S^V(i, j, w) = -C(i, j) + q(\theta(i, j)) \left[ E_s S^J(i, j, s, w) - S^V(i, j, w) \right] \\
\rho S^J(i, j, s, w) = m(j, s)(p(i, j) - w) - \delta S^J(i, j, s, w).
\]

Here \( S^V \) is the net present value of a posted vacancy and \( S^J \) the NPV of a filled vacancy. Worker skill is an expectation as the firm does not choose \( s \) and it is \textit{ex ante} possible that multiple worker types could apply to submarket \((i, j)\), though in equilibrium this will not be the case.

\textbf{Workers.} At any point in time workers are either employed or unemployed. Employed workers earn the wage \( w \times m \) and realize an industry-specific, non-pecuniary amenity \( A(i) \), for a total flow value from employment of \( u(wm, A) \). Unemployed workers become employed by directing their search across occupations \((j)\) and industries \((i)\). While searching for a job they receive an insurance payment \( B(u, \theta) \) that is allowed to depend on market tightness and the flow utility of employment, subject to the constraint that any match must be accepted (i.e. there is a perfectly enforced work-search requirement).\(^{33}\) Jobs are found at a rate \( f(\theta(i, j)) \). I focus on the symmetric equilibrium in which workers choose a mixed strategy \( \phi(i, j, s) \) over submarkets that solves the problem:

\[
\max_{\phi(i, j)} \sum_i \phi(i, j) S^U(i, j, s) \\
\text{s.t. } \rho S^U(i, j, s) = B(u(i, s, j), \theta(i, j)) + f(\theta(i, j)) \left[ S^W(i, j, s) - S^U(i, j, s) \right]
\]

\(^{33}\)The motivation for \( B(u, \theta) \) is discussed in more detail later in this section; for now I simply give the intuition that because employment flow payoffs, job-finding rates, and \( B \) will jointly determine workers’ job-seeking behavior, a policymaker will optimally take \( u \) and \( \theta \) into account when setting unemployment benefits.
\[ \rho^{SW}(i, j, s) = u(w(i, j)m(j, s), A(i)) + \delta \left[ \max_{i', j', s'} S^U(i', j', s) - S^{SW}(i, j, s) \right] \]
\[ \sum_i \int \phi(i, j)dj = 1, \]

where the terms \( S^U \) and \( S^{SW} \) are the net present values of job search and a matched job, respectively.\(^{34}\)

**Aggregators and household.** Match output is sold to industry aggregators as an intermediate good \( y(i, j) \). Aggregators then produce an industry-specific final good, using a constant returns CES technology.\(^{35}\) All industries face the same elasticity of substitution \( \sigma > 0 \) across \( j \)-goods, but occupational shares \( \alpha(i, j) \) are allowed to be industry-specific. Final goods are sold to the representative household at a price \( P(i) \). Aggregator profits will be maximized by the bundle of intermediate goods that solves the problem

\[ \max_{y(j)} \left( \int \alpha(i, j)y(j)^{\frac{\sigma+1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} - \int p(i, j)y(j)dj. \] (5)

Here \( \int \alpha(i, j)dj = 1 \), and I assume that \( \alpha \) is continuous in \( j \). The household is assumed to have CES preferences with elasticity \( \tau > 0 \), and will maximize utility by solving

\[ \max_{Y(i)} \left( \sum_i \beta(i)Y(i)^{\frac{\tau+1}{\tau}} \right)^{\frac{\tau}{\tau-1}} - \sum_i P(i)Y(i). \] (6)

I abstract from the household’s budget constraint as it affects neither relative prices nor quantities, but note in passing that the model may be easily closed by repatriating all wages and profits to the household. For similar reasons I omit the government’s budget constraint, with unemployment payouts \( B \) assumed to take the form of an endowment.

\(^{34}\)Although the supply of skill is assumed to be exogenous, it may easily be made endogenous to reflect e.g. the influence of wages on labor market participation or human capital investment. The main challenge in doing so is empirical: because wages are generally used to identify worker skill - a latent variable - any causal relationship between wages and skill is thereby lost.

\(^{35}\)For simplicity I omit capital from production, though the entry costs \( C(i, j) \) may be thought of as reflecting capital or any other fixed costs incurred prior to job creation, in addition to competitive barriers to entry.
2.2.2 Equilibrium

Markets. Goods markets are perfectly competitive. Free entry of employers implies that the value of a vacancy is non-positive:

$$\rho S^V(i, j) \leq 0,$$

with equality in any submarket for which entry is positive. Subject to perfect competition, aggregators earn no profits.

Labor markets are frictional in the sense that, at any point in time, some vacancies go unfilled and some workers fail to find an employer. Given $N$ unemployed workers and $V$ vacant employers, matches will form at a rate $M(N, V)$. For tractability reasons I assume this function to be Cobb Douglas:

**Assumption 2 (Matching function):** $M(N, V) = \zeta V^n N^{1-n}$.

With Cobb Douglas matching the vacancy-filling and job-finding rates will be $q(\theta) = \zeta \theta^{n-1}$ and $f(\theta) = \zeta \theta^n$, respectively.

The price of the final good may be normalized, and a convenient normalization is $P = \delta / Y$ where $Y$ is household utility (or equivalently, “aggregate output”). I maintain this assumption in what follows.

Equilibrium. In equilibrium, market tightness must be consistent with the optimal job search behavior of workers and employer vacancy policies. Defining $N(s)$ to be the unemployed mass of $s$-workers and $V(i, j)$ the total number of $(i, j)$-vacancies in steady-state, we require that

$$\theta(i, j) = \frac{V(i, j)}{\int \phi^*(i, j, s)N(s)ds}.$$  \hspace{1cm} (8)

It must also be that the markets for intermediate and final goods each clear:

$$y^*(i, j) = \frac{q(\theta(i, j))V(i, j)}{\delta} \times \frac{\int N(s)\phi^*(i, j, s)m(j, s)ds}{\int N(s)\phi^*(i, j, s)ds}.$$  \hspace{1cm} (9)
\[ Y^*(i) = \left( \int \alpha(i, j) y^*(i, j) \frac{\sigma}{\sigma - 1} dj \right)^{\frac{\sigma}{\sigma - 1}}, \] 

where the two terms on the right-hand side of (9) correspond to total employment multiplied by average labor productivity in submarket \((i, j)\). Finally, in steady-state we must have parity between flows into and out of unemployment:

\[ q(\theta(i, j)) V(i, j) = \delta(\nu(s) - N(s)). \] 

A steady-state equilibrium in this economy is employer wage posting decisions \(w^*(i, j)\) that solve problem (21), search behavior \(\phi^*(i, j, s)\) solving (19), and output quantities \(y^*(i, j)\) and \(Y^*(i)\) that solve problems (5)-(6), together with market tightness \(\theta(i, j)\), unemployment and vacancy distributions \(N(s)\) and \(V(i, j)\), and prices \(p(i, j)\) and \(P(i)\) such that conditions (7) and (8)-(11) hold.

### 2.2.3 The Restricted Model

Further characterization of the model requires that assumptions be placed on the worker flow payoffs \(u\) and \(B\). These are important because they determine the relative values that workers place on the different forms of “payment” associated with a given submarket: wages, amenities, and job-finding rates. In this section I impose conditions on \(u\) and \(B\) such that worker preferences are homogeneous. Homogeneity is an attractive property in this case for several reasons. First, it allows the equilibrium to be described in closed-form. Second, it is a natural assumption in this environment, insofar as it is a necessary condition for the allocative efficiency usually associated with directed search. And third, homogeneous preferences are a sufficient condition for the model to generate a separable wage function - which provides both a direct path to empirical identification of the model primitives, and a microfoundation for the common empirical assumption of a log-additive wage function.\(^{36}\)

\(^{36}\)The AKM wage regression assumes log-additivity, but many authors have observed that separability is not consistent with random search models - see for example the criticism by Lopes de Melo (2018). The monopsony framework of Card, Cardoso, Heining, and Kline (2018) does generate a separable wage function, but only when firms have identical technical demand for skill; separability is lost when there is sorting as in Haanwinckel (2021). The key advantage of wage separability is that it allows agent types and model primitives to be identified separately, which in turn allows for a non-parametric estimation approach as employed in this paper.
Restrictions. The conditions to be imposed are that (1) flow utility \( u \) is a homogeneous function of wages, and (2) unemployment payoffs \( B \) are set in such a way that the value of labor search (i.e. \( S^U \)) is a homogeneous function of \( u \). I implement these conditions through the following assumptions:

**Assumption 3 (Flow payoffs):** The flow payoffs of employment and unemployment take the functional forms

\[
\begin{align*}
    u(\omega, A) &= \omega^{\psi} A^{1-\psi}, \\
    B(u, \theta) &= \left( \frac{\rho + \delta + f(\theta)}{\rho + \delta} \right) u^{\kappa-1} - 1 f(\theta) u.
\end{align*}
\]

where \( \psi \in (0, 1] \) and \( \kappa \geq 0 \).

The restriction on \( u \) lacks an intuitive interpretation, insofar as amenities are exogenous and play a structural rather than a theoretical role in this paper.\(^{37}\) Condition (13), on the other hand, says that workers with a high job-finding rate are incentivized to place a greater value on additional increases in market tightness, since due to the dynamic nature of the environment they would otherwise be less sensitive to changes in the job-finding rate.

**Interpretation of \( B \).** The restriction on \( B \) has an efficiency interpretation: it is a necessary condition for an efficient allocation of labor across industries and occupations.\(^{38}\) Perhaps surprisingly, directed search in this environment is not generally efficient; inefficiency arises due to the fact that with differentiated output and labor markets, labor demand cannot be perfectly substituted across submarkets. When preferences are non-homogeneous, the effective “price” of skill will vary across industries, which distorts labor shares and reduces output due to the quasi-concavity of the aggregator’s production function. Condition (13)

\(^{37}\)It is straightforward also to include occupational amenities, though characterization of the optimal assignment becomes problematic unless these are continuously differentiable.

\(^{38}\)In the appendix I provide an informal proof for the simplified case in which occupational assignment is exogenous and amenities are homogeneous. Efficiency is not key to the results in this paper, and is discussed here for purpose of providing intuition for condition (13).
does not eliminate this congestion externality, but constrains it to be independent of skill. Workers may still “over-apply” to industries offering a high wage ($\kappa > 1$), or to low-paying firms offering a higher job-finding rate ($\kappa < 1$), but all workers will behave similarly regardless of $s$. When $\kappa = 1$ the externality is removed altogether, and this can be interpreted as the special case in which the efficiency properties of directed search are preserved under differentiated labor and product markets.

**Occupational assignment.** An immediate implication of restrictions (12) and (13) is that the assignment of workers to occupations will be independent of industry. Following Costinot and Vogel (2010) in defining a correspondence $\lambda : i, s \rightarrow j$ that gives the set of jobs $j$ chosen by $s$-workers in industry $i$, and provided that $\alpha(i, j)$ and $C(i, j)$ are continuous, we can show the following:

**Proposition 1.** Under assumptions 1-3 $\lambda(s)$ is a strictly increasing, differentiable function independent of $i$, that satisfies the system of equations

$$\frac{dU(s)}{ds} = \kappa \psi \frac{m_s(\lambda(s), s)}{m(\lambda(s), s)} U(s)$$

$$\frac{d\lambda(s)}{ds} = \frac{m(\lambda(s), s) N(s)U(s)}{\sum_i \left[ \left( \frac{\kappa \psi (1-\eta)}{\eta + \kappa \psi (1-\eta)} p(i, \lambda(s)) m(\lambda(s), s) \right)^{\psi} A(i)^{\psi} \right]^{1-\psi} y(i, \lambda(s))}$$

where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$, and where $U(s) \equiv \rho \max_{i,j} S^U(i, j, s)$ is the reservation value of type $s$.

Equation (14) states that at any point, the wage function and the match production function increase at the same rate in $s$, because only when this is true will workers not realize a larger payoff by moving to a higher or a lower $k$. Equation (15) describes the sorting of workers across occupations. Higher-skilled workers are assigned to higher-$j$ jobs, at which they have a comparative advantage; and this assignment is constant across industries. Note that assignment depends not just on labor demand (through $y$), but also on wages (through $p$) and industry amenities.
2.2.4 Demand, Supply, and Wages

The model developed thus far makes two important predictions: that the wage function is separable, and that wage premia have behavioral implications. In this section I briefly discuss both predictions. I then illustrate, by way of a numerical exercise, how the skill-bias and firm-bias of a demand shock jointly determine the effect on wages.

Wage separability. As a consequence of conditions (12) and (13), the equilibrium wage function can be shown to be a multiplicative function of a firm premium that depends on \( i \) and \( j \), and a worker or skill premium that depends on \( s \):

\[
w^*(i, \lambda(s)) = \left( \frac{\kappa \psi (1 - \eta) (\rho + \delta) C(i, \lambda(s))}{\eta \xi^\frac{1}{\eta} A(i)} \right)^{\frac{\eta}{\eta + \kappa \psi (1 - \eta)}} U(s)^{\frac{1 - \psi}{\eta + \kappa \psi (1 - \eta)}} \equiv FP(i, \lambda(s)) SP(s). \tag{16}
\]

The firm premium is a function of entry costs and amenities; because these increase or reduce the cost of hiring a worker, they affect firm entry, and hence output prices and wages as well. The skill premium is a function of the worker’s outside option \( U \), which is determined in equilibrium and reflects the demand for different types of jobs as well as the substitutability of skill types across these jobs.

Behavioral response to wage premia. The supply of labor to a given submarket will depend on the matching function - for which there is no closed-form solution - and on the search probability \( \phi \):

\[
\phi^*(i, j, s) = \frac{V(i, \lambda(s)) \left( FP(i, \lambda(s)) \psi A(i)^{1-\psi} \right)^{\frac{\xi}{\eta}}}{\sum_k V(k, \lambda(s)) \left( FP(k, \lambda(s)) \psi A(k)^{1-\psi} \right)^{\frac{\xi}{\eta}}}. \tag{17}
\]

Equation (17) states that workers are more likely to apply to a submarket when there are more vacancies, and when firms in this submarket are higher-paying or have relatively larger
amenities. Less obvious is that, through the matching function (15), workers will also tend to seek out high-paying occupations. Hence labor demand $V$ and the distribution of firm premia will jointly determine occupational assignment and therefore distribution of skill premia $SP$. A key role here is played by the term $\kappa/\eta$, which gives the $\lambda$-conditional elasticity of labor supply with respect to the flow payoff $FP(1-\psi).$ When $\kappa$ is large, workers will respond more strongly to firm-specific payoffs, and when $\kappa = 0$ they will not respond at all. Similarly, a low value of $\eta$ means that the job-finding rate is unresponsive to vacancies, and as a result workers will respond relatively less to $V$ and relatively more to $FP$ and $A$.

The natural measure of labor demand in this environment is the mass of vacancies in submarket $(i,j)$, which is given by the equation

$$V(i, \lambda(s)) = \alpha(i, \lambda(s))^{\sigma} \beta(i)^{\tau} \frac{m(\lambda(s), s)^{\sigma-1}}{A(i)^{\lambda(1-\psi)(1-\eta)}} \frac{FP(i, \lambda(s))^{\tau+\kappa(1-\psi)}}{SP(s)^{\sigma-1}}$$

$$\times \left( \int \alpha(i, \lambda(k))^{\sigma} \left[ \frac{m(\lambda(k), k)}{SP(k)} \right]^{\sigma-1} dk \right)^{\frac{\tau-\sigma}{\sigma-1}} V,$$  \hspace{1cm} (18)

where $V$ is a function of aggregate parameters. Aside from the technical demand parameters $\alpha$ and $\beta$, vacancies will also depend on the industry amenity and the firm premium, which both tend to increase the vacancy-filling rate and hence to reduce output prices given $V$. Two substitution effects are also possible: across occupations within an industry whenever $\sigma \neq 1$, and across industries when $\tau \neq \sigma$. Both substitution effects depend inversely on the term $SP/m$, or the unit labor cost associated with $s$-workers. This ratio is the outcome of equation (14), and will only be constant in $s$ if $\kappa = 1/\psi$. In other words changes to skill premia will generally exert an equilibrium effect on demand, though the direction and magnitude of that effect will depend critically on the demand elasticities $\sigma$ and $\tau$.

Comparative statics. To better illustrate the model’s predictions regarding wages, I consider the effects of a skill-biased demand shock in the context of a simple, two-industry version of the model. I assume that share functions and amenities are initially the same in both industries; that type distributions are uniform, and match production is an exponential
function of $s$ and $j$; and that $\kappa = \sigma = \tau = 1$. Parameter values are roughly calibrated to match wage variance moments for West Germany over the period 1993-1999. The demand shock is modeled as a change to occupational demand $\alpha_{\text{low}}$ and $\alpha_{\text{high}}$ such that the aggregate vacancy shares of skilled jobs increases by a fixed amount: that is, for any $j'$, the mass of vacancies where $j > j'$ is now greater than the mass of vacancies for which $j < j'$ by a proportion held constant across experiments. Whether this change in demand is also firm-biased depends upon whether it is driven by changes to $\alpha_{\text{low}}$ or $\alpha_{\text{high}}$, with different cases represented in different columns of figure 2.2. The first row of plots gives results as a function of the worker’s wage percentile, to show the effects of the shock on the wage distribution; and the second row as a function of the worker’s skill percentile, to better illustrate the implications for wages earned by a particular type.

![Figure 2.2: Simulated Effects of a Skill-Biased Demand Shock](image)

When the increase in demand for skilled jobs is identical across industries (first column), there is no change to the average firm premium earned by different worker types. Because the optimal occupational assignment will now have each skill type assigned to a higher-$j$ job than before, the return to skill and hence $\text{Var}(SP)$ increases, as shown by the upward-sloping dashed line. Critically, however, wage variance $\text{Var}(w)$ increases by a smaller amount than $\text{Var}(SP)$, reflecting the fact that wage and skill are imperfectly correlated; for this

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reason there is incomplete “pass-through” from the distribution of skill premia to the wage
distribution. The extent to which pass-through is incomplete will depend on the initial
distribution of jobs across industries, and will become greater when high-\textit{j} occupations are
initially more common in the high-paying sector.

The second and third columns show wage outcomes when the skill-biased shock is con-
centrated in the low-paying and the high-paying industry, respectively. If rising skill demand
is driven by the low-paying industry, then skill premia and firm premia become negatively
correlated and there are offsetting effects on wages. On the one hand, $Var(SP)$ increases
as in the case with no firm-bias. On the other hand, $Cov(SP,FP)$ now declines. In this
particular example the sorting effect dominates, and wage variance decreases overall. The
third column shows the effects of skill-bias when these two channels are complementary, and
skilled workers are sorted into the high-paying industry. We now have $Var(w)$ increasing by
a larger amount than $Var(SP)$, with firm-bias amplifying the effects of the demand shock
on wages.

By assumption only the supply side responds behaviorally to the change in demand, and
this response may be seen by examining the effect on wages by skill percentile (row 2). A
given increase in skilled vacancies will have a larger effect on worker search behavior, and
hence on occupational assignment and the distribution of skill premia, when this increase
occurs in the high-paying industry. This is because, as shown in equation (17), vacancies
and firm premia play a similar role in determining workers’ job choice. An increase in the
average firm premium associated with a particular occupation acts like increase in vacancies.
Hence the “effective” skill-bias of the demand shock will depend upon the industry that it
predominantly affects.

2.2.5 Discussion

Summarizing the results shown above, the model developed in this paper suggests three
ways in which a change to the environment (such as a demand shock) might impact the
wage distribution:

1. skill-bias: by affecting the occupational assignment of workers
2. **firm-bias**: by affecting the magnitude of firm premia or their incidence across skill types, conditional on occupational assignment

3. **interactions of skill- and firm-bias**: by generating behavioral supply and/or demand responses that are themselves skill- or firm-biased

The first channel is the standard one considered in the literature. The second is non-standard but straightforward: any change that impacts the distribution of labor across firms may also impact the distribution of firm premia across skill types. The third channel is not straightforward, and arises when occupations are unevenly distributed across industries, or when employer rents are different for skilled and unskilled occupations (which may be true even if occupational demand is homogeneous across industries). In this case a firm-biased or skill-biased change to the environment is likely to elicit behavioral responses by workers, firms, or both, for the reasons explained above; and these behavioral responses may themselves be firm- or skill-biased.

While the prediction of a separable wage function is somewhat unique among models of this class, and will be useful in the next section, the predictions regarding agent behavior are objectively more important. They suggest that skill-bias and firm-bias are *not separable*, either in theory or in practice, when different occupations are associated with different firms. As the numerical exercise in this section indicates, it is possible for a skill-biased shock to reduce the wage gap between low-earners and high-earners - an outcome not only quantitatively but also qualitatively inconsistent with standard models of skill-bias. At the same time, a strong role for a behavioral response to wage premia is also inconsistent with common empirical experiments, in which wages are decomposed into firm- and worker-specific effects, and the wage distribution is then manipulated along one axis (e.g. firm effects) but held fixed along the other. Such experiments cannot reliably measure the impact of firm premia on the wage distribution; and they are likely to be even more mis-specified when used to draw inferences about the wage impacts of policies that target firm rents or the wages that firms pay. These issues are explored in more depth in the quantitative portion of this paper.
2.3 Structural Estimation

This section describes the structural estimation procedure for the model developed above. I begin with a description of the German matched data used for this exercise. I then start with the non-parametric identification of the latent type distributions, which is made possible by the log separability of the model-predicted wage function. I then discuss estimation of the match primitives, followed by the distributional parameters describing demand, entry costs, and amenities. Lastly I discuss the aggregate parameters, and the approach I take to the elasticity terms $\sigma$, $\tau$, and $\kappa$, which are unidentified. The model moments used in estimation are shown in the appendix, along with a characterization of model performance and goodness-of-fit.

2.3.1 Data

*German linked employee-employer dataset (LIAB).* The principal dataset used in estimation is provided by the Institute for Employment Research (IAB), and contains matched data on German establishments and their employees over the years 1993-2017. Each year the IAB conducts a large, stratified survey of German private sector establishments, which is then matched with social security records of all employees of record as of the survey date. The establishment portion of the dataset contains a wealth of data, with key variables including detailed industry codes, current vacancy postings, and cross-sectional weights to correct for stratification. Establishments are surveyed in waves lasting several years, making it possible to construct measures of establishment-level job flows. Employee data include information on daily earnings, the occupational code assigned by the employer, and basic demographic characteristics including sex and coarse educational attainment. Workers that do not pay social security taxes (e.g., those in marginal employment) are not observed, and as part-time employees are recorded inconsistently over the sample period, I exclude them from the sample as well. Following past literature I further restrict the sample to individuals aged 20-60, leaving a dataset that consists annually of between .75 and 2.2 million individuals and

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39 An establishment is defined as a physical workplace, however locations may be aggregated when they share the same corporate ownership, industry classification, and municipal code.

40 See Chapter 1 for a description of how industry and occupation codes are aggregated.
between 4 and 9 thousand establishments.

Provided with the LIAB are updated versions of the person and employer wage effects estimated by Card, Heining, and Kline (2013), following the now-standard approach of Abowd, Kramarz, and Margolis (1999). These effects are obtained from the panel wage regression

\[ w_{i,t} = \pi_i + \phi_{j(i,t)} + x_i \beta + \epsilon_{i,t}, \]

where \( w(i,t) \) is the log daily wage of person \( i \) in year \( t \), \( \pi \) is a time-invariant wage effect associated with person \( i \), \( \phi \) is a time-invariant effect associated with \( i \)'s employer \( j \), and \( x \) is a vector containing year fixed-effects and a cubic polynomial in worker age, interacted with dummies for educational attainment. Estimation is performed on the population-level datasets from which the LIAB is extracted, in four partially-overlapping panels spanning 7-8 years. While it is possible to estimate AKM wage effects from the LIAB, I prefer the data provider’s estimates for two reasons. First, because wage effects are identified from job transitions, they will be more precisely estimated in datasets that contain the full universe of establishments. And second, because better identification means that wage effects can be estimated for a greater number of establishments, less approximation is needed when estimating sample distributions in order to satisfy confidentiality restrictions.

For consistency with the model, I average both the person and the firm wage effect at the industry-occupation level using detailed (3-digit) codes. As there is substantial and apparently idiosyncratic variation in the extreme tails of the person effect distribution, even after averaging, I therefore filter observations lying in the first and last percentiles. See the appendix to Chapter 1 for additional details on data cleaning.

**Aggregate data.** All nominal variables in LIAB are converted to real values using German CPI data from the OECD. As the LIAB contains data only on employed individuals, I obtain annual employment, unemployment, and vacancy data from the Federal Employment Agency. Numbers are reported separately for West and East Germany. Measures of unemployment and vacancies are not fully consistent over time, however these data inform only

---

41 See Bellman (2020) for a complete description of this procedure.
42 Monthly CPI of all items, averaged annually with a base year of 1995.
two aspects of the model - the match function efficiency $\zeta$, and aggregate market tightness (through the overall level of entry costs $C$) - neither of which is influential with respect to the wage and employment distributions.

### 2.3.2 Type Distributions

For the 2010-2017 panel I normalize $\lambda(s)$ and $\nu(s)$, which serves to “fix” $s$ and $j$ in all panels. In earlier panels, these distributions are estimated as follows. Using equation (??) and the estimated match productivity function $m$ (discussed next), the empirical matching function is backed out from percentiles of AKM person effects, which is smoothed to ensure that the matching function has no discontinuities. Once the empirical analogue of $\lambda$ is estimated, other model primitives can then be estimated over $j$.

![Figure 2.3: Estimated Type Distributions](image)

The skill distribution is allowed to vary across panels, as the 1993-2017 period saw labor migration due to reunification and the implementation of the European Common Market, a general increase in German educational attainment, and a secular rise in the participation rate and fall in the unemployment rate following the Hartz reforms of 2003-2005. To account as best as possible for these changes, I use AKM person effects, together with the model prediction of a strictly increasing skill premium $SP(s)$, to assign workers to wage percentiles calculated over each panel. I then estimate the annual distributions of labor across the

---

43 The match function is assumed to take the form of a truncated normal distribution, and skill to be uniformly distributed over $[0, 1]$. Both assumptions are WLOG since the match productivity function $m$ is as yet unrestricted. Some care is required when fixing $\lambda(s)$, however, as a poor choice may result in non-monotonic matching functions for earlier panels. In practice the monotonicity condition arising from occupational assignment must be checked and verified, and the normalization for 2010-2017 adjusted if necessary.
(panel) wage percentiles in order to capture within-panel changes to the distribution of skill; and I leverage the years of overlap between panels - 1998-1999, 2003-2004, and 2010 - to estimate the between-panel changes to this distribution. In this way I am able to map the skill distribution $\nu$ backwards from 2010-2017 into earlier panels. I find that each successive panel sees an increase in the supply of high-skill labor, consistent with increased workforce education; and following the early 2000’s there is an upwards trend in the supply of very low-skill labor, as would be expected from increased participation of marginal workers.

2.3.3 Match Productivity

A functional form is required for the match production function, and as match production must be log-supermodular, a natural approach is to assume that $m$ is an exponential function.\footnote{See for example Teulings (1995) and Ales, Kurnaz, and Sleet (2015).} I generalize this assumption as follows.

**Assumption 4** (Functional form of $m$): $m(j, s) = e^{G(s)\gamma sj + H(j)}$, with $G$ and $F$ time-invariant.

Fixing an initial guess for unemployment $N(s)$, I estimate $SP(s)$ from percentiles of the distribution of empirical person effects in the LIAB. Fitting a quadratic spline and differentiating, I obtain $SP'(s)$, which may then be substituted into equation (14) to non-parametrically obtain $G'(s)$ for the 2010-2017 period under the normalization $\gamma_{sj}^{2010-17} = 1$. The term $G(s)$ is then recovered by integration.\footnote{To prevent division by 0 when solving equation (14) for $G'$, it is necessary to add an intercept $G(s)\gamma s$ where I set $\gamma s = .01$.} The occupational productivity multiplier $H(j)$ is also estimated over the 2010-2017 panel, using the restriction $\frac{SP(s)}{m(\lambda(s), s)} = 1$.\footnote{The intent behind this assumption is to equalize unit labor costs, the levels of which are unconstrained, prior to conducting forward-looking experiments that shift the occupational composition of industries. Note, however, that unit labor costs will deviate from 1 in the earlier panels, and may deviate from 1 in the policy experiments if the occupational assignment is affected.} Once $G$ is estimated it is then possible to back out $\lambda(s)$ for panels prior to 2010-2017, as described above. The term $\gamma sj$ is identified in each panel from the boundary condition $\lambda(\bar{s}) = 1$, and increases over the sample period from .72 in 1993-1999 to the normalized value of 1 in 2010-2017.

Note that all estimates are conditional on $N(s)$, because empirical wages are calculated as...
percentiles of the *employed* distribution and must be mapped into \( s \). As the unemployment distribution is not initially known but must be solved for, this necessitates an iterative estimation procedure, which I describe in greater detail below.

### 2.3.4 Demand, Amenities, and Costs

The demand functions \( \alpha \) and \( \beta \) are estimated in each panel from industry (wage) cost shares and the intra-industry distributions of AKM person effects, averaged by 3-digit industry and occupation as described previously.\(^{47}\) Entry costs are estimated from the inter- and intra-industry distributions of AKM establishment effects, with the level of entry costs calibrated so that market tightness is equal to its empirical counterpart over 1993-2017, which I estimate at .146 using annual unemployment and vacancy statistics. Industry amenities are estimated from industry-mean AKM effects and vacancy-filling rates. The equilibrium equations used for estimation are provided in the appendix.

Estimation is an iterative process for two reasons. First, unemployment \( N(s) \) is *ex ante* unknown, but appears in the equilibrium equations used to identify the technical parameters and those governing match production. Second, the coarseness of the empirical distributions, together with the interpolation steps used in estimating the distributional parameters, adversely affects the model’s goodness of fit. I therefore begin with an initial guess for unemployment, after which estimation is performed based on this guess, and the model

\(^{47}\)All intra-industry distributions are measured over 20 quantiles. A finer resolution is generally not possible due to data confidentiality requirements that require each statistic to represent at least twenty unique establishments.
Figure 2.5: Estimated Technical Parameters

is solved conditional on the parameter estimates.\textsuperscript{48} The guess for \( N(s) \) is then updated, the model parameters are re-estimated, \( \alpha \) is adjusted to close the gap between the empirical and model-generated matching functions, and the process is repeated until convergence. In addition to this, an outer loop is performed in which industry demand and entry costs are adjusted to minimize the distance between empirical and predicted (1) market tightness, (2) industry employment shares, and (3) industry mean firm premia.

2.3.5 Aggregate Parameters

The amenity exponent \( \psi \) is not separately identified from the amenity terms \( A \), and so I impose the normalization \( \psi = \eta/(1 - \eta) \), which has the convenient implication that the elasticity of job applicants with respect to \( FP \) will be exactly equal to \( \kappa \). The discount rate \( \rho \), which serves as a scaling variable and is not influential, is assigned a value corresponding to a discount rate of 0.96. Estimation of the matching elasticity \( \eta \) is possible, using LIAB vacancies and hires together with aggregate unemployment flows; however this results in an

\textsuperscript{48}In particular a numerical solution must be obtained to the system of differential equations (??)-(15).
implausibly high value.\footnote{In their survey, Petrongolo and Pissarides (2001) suggest a plausible range of .3 to .5. I obtain a value of approximately .6, likely due to attenuation bias caused by noise in the LIAB-based hiring measures.} I therefore set $\eta = .35$ as estimated for Germany by Kohlbrecher et al. (2016). Match efficiency $\zeta$ is then estimated from aggregate unemployment, vacancies, and hires. The separation rate, which also plays a nominal role, is set equal to the proportion of workers observed in year $t$ but not year $t + 1$, conditional on the establishment being observed in both years.

Table 2.6: Aggregate Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>Amenity exponent</td>
<td>.538</td>
<td>Equal to $\frac{\eta}{1-\eta}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>.042</td>
<td>Discount factor of .96</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Separation rate</td>
<td>.165</td>
<td>Total hires</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Match elasticity</td>
<td>.35</td>
<td>Kohlbrecher et al. (2016)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Match efficiency</td>
<td>2.39</td>
<td>Predicted hires</td>
</tr>
</tbody>
</table>

It remains to discuss the key elasticities in the model: of occupational demand ($\sigma$), industry demand ($\tau$), and labor supply ($\kappa$). The demand elasticities $\sigma$ and $\tau$ are fundamentally unidentified. In part this is because unit labor costs $SP(s)/m(\lambda(s), s)$ are unobserved and cannot be measured,\footnote{Wage data does not allow one to disentangle the components of wages - here $p$ and $m$ - and productivity measures are unhelpful because, as indexes, they can only be meaningfully aggregated in the context of changes and not levels.} but also because $\alpha$ and $\beta$ are allowed to vary by panel, and it is not possible to disentangle the role of changes to technical demand from that of substitution effects (e.g. in response to changing skill premia or entry costs). For this reason, I assume below that labor demand is unit elastic for all experiments spanning the 1993-2017 sample period. For forward-looking policy experiments, I relax this assumption and consider the implications of elastic and inelastic demand, as in these cases firm vacancy postings will respond to changes in the “price” of skill. For all quantitative experiments I impose the following simplification:

Assumption 5 (Common demand elasticity): $\sigma = \tau$ and $\sigma \in \{.5, 1, 2\}$.

A common value allows for a smaller set of results to be considered, which avoids the need for a 3-dimensional grid of results given that $\kappa$ is also set-valued. Unit elasticities provide a
natural baseline, while the values of .5 and 2 are arbitrary and chosen for convenience. Note that because $H(j)$ is chosen so that $SP/m$ is constant for the 2010-2017 panel, counterfactual experiments involving occupational demand $\alpha$ will only result in substitution effects if there are changes to $FP$ or to $SP/m$. In particular, any changes to the distribution of skill types across industries will be neutral with respect to industry shares - a “neutrality” assumption that will be important for many of the counterfactual experiments considered below.

The elasticity of supply $\kappa$ is also problematic for identification. Although I am able to estimate vacancy-filling rates from the LIAB, there are likely to be unobserved amenities that affect the job search behavior of workers, as assumed in the model. In regressions of vacancy-filling rates on skill premia and firm premia, I estimate an elasticity of labor supply with respect to firm premia of between .96 (East Germany) and 2.04 (West Germany). This is consistent with past studies, which suggest a plausible range of $1−4$ when non-homogeneous separation rates are also taken into account.\(^{51}\) I therefore consider values lying in the set $\kappa \in \{.5, 1, 2, 4\}$, though for reasons of space I omit results for $\kappa = 4$ in many of the results that follow.

### 2.4 Quantitative Results

In this section I present the simulated results of counterfactual experiments for the model developed and estimated as described previously. I first consider historical experiments intended to address the question: given changes to labor demand over the 1993-2017 period, how did the presence of firm-bias affect the resulting wage trend? The second set of results concerns the predictiveness of skill-bias, and the extent to which firm-bias confounds the effect of a skill-biased demand shock on wage variance. Third, I consider the extent to which firm-bias can account for regional variation in wage trends, through a comparison of East and West Germany. Lastly I present several experiments that quantify the the response of wages to wage policies, while accounting for the equilibrium effects on worker job search and employer labor demand.

\(^{51}\)See for example Bo, Finan, and Rossi (2013), Sokolova and Sorensen (2021), and Bassier, Dube, Naidu (2020).
2.4.1 Historical demand

To what extent does the contribution of labor demand to the historical trend in wage inequality reflect firm-bias, as opposed to skill-bias alone? To answer this question I consider how wage variance would have evolved under the counterfactual scenario in which all firms pay the same wage conditional on skill. This is done by manipulating the entry costs faced by firms so that $FP(i, j)$ is constant. As entry costs also affect the mass of vacancies posted, I “compensate” changes to entry costs by also adjusting $\alpha$ and $\beta$ so as to preserve the distribution of vacancies in the 1993-99 panel. From this starting point, the proportional changes to industry and occupation demand are preserved for subsequent periods. Note that this experiment does not hold constant labor supply, which is endogenous to the distribution of firm premia. Hence there will be a shift of labor away from submarkets that used to be high-paying, and towards formerly low-paying ones.

In figure 2.7 I show the model-predicted wage trend and the counterfactual trend with homogeneous $FP$, for different values of the worker search exponent $\kappa$ and while changing different sets of model parameters over time. In the first row all time-varying parameters - $\alpha$ (which includes $\gamma_{i,j}$), $\beta$, $C$, and $\nu$ - are set at their contemporaneous values. Row 2 contains results when only the demand parameters are changed; shown is the average outcome from two experiments, in which other parameters are fixed at their 1993-99 and 2010-17 values. In the last two rows, industry and occupation demand are considered separately. Tabulated results are given in the appendix.

The predicted trend in wage variance is slightly less than two-thirds of the empirical increase, with the remainder due to widening residual wage inequality that is not captured by industry and occupation. When all parameters are allowed to vary, eliminating firm-bias reduces the predicted trend by between one-half and three-fifths. By themselves, the demand parameters $\alpha$ and $\beta$ account for most but not all of the model-predicted trend, the rest driven mainly by changes to the skill distribution $\nu$.\footnote{Over time, labor shifts towards the extremes of the skill distribution; see the discussion in the previous section.} Hence the role of firm-bias is somewhat greater when we consider only the effect of changing labor demand: the predicted trend falls by between 60% and 70%. The interpretation of this result is that, by itself,
skill-bias explains at most two-fifths of the effect of industry and occupation labor demand on the wage trend. The difference between the counterfactual and predicted trends reflects both firm-bias - changes to the distribution of $FP$ - and interactions between skill-bias and firm-bias, due to a higher covariance of skill premia and firm premia. The differences between the columns are substantial, indicating that the response of labor supply to firm premia is an important determinant of the overall impact of firm-bias. When $\kappa$ is large, the elimination of differences in $FP$ results in more workers shifting their job search towards lower-skill sectors. This further reduces the effect of a skill-biased shock to demand, because the impact of a
proportional change in skill demand depends on the labor share - and not the vacancy share - of skilled jobs.

When occupation and industry demand are considered separately, it is evident that they are affected to different extents by the presence of heterogeneous firm premia. The increase in wage variance associated with occupational demand is between one-third and one-half smaller under the counterfactual scenario. The effect of industry demand changes signs; whereas $\beta$ accounts for roughly one-quarter of the model-predicted trend (equal to one-sixth of the empirical trend), this contribution becomes negative when firm premia are homogeneous. Hence through the lens of the model, the shift in employment from manufacturing to service sectors has not been skill-biased. Rather it has contributed to wage inequality through firm-bias, and through the interaction of firm-bias with pre-existing and largely stable differences in worker skill across industries.

Figure 2.8: Contribution to Change in Wage Variance, 1993-2017
The model allows for further disaggregation of industries and occupations, and so I therefore plot the individual contributions in figure 2.8.\textsuperscript{53} In the first column are shown the predicted effects on wage variance of changes to $\alpha$ and $\beta$ between the 1993-99 and 2010-17 panels. Counterfactual effects with homogeneous firm premia are given in the second. Comparing the two columns, we can see that firm-bias has tended to amplify the (positive) effects of industry-level demand on wage variance. This is especially true for crafts and material manufacturing, in which employment declines over this period, and for the hospitality/temp, personal services, and information sectors, which have grown. In the absence of heterogeneous firm premia, the increased use of temp labor would have exerted a large, positive effect on wage variance, compared to an even larger negative effect when accounting for the low firm premia associated with this industry group. Turning to occupations, we can see a similar overall pattern: firm-bias tended to amplify the wage effect of skill-biased shocks, and reduce the effect of shocks favoring unskilled workers. The differences between the predicted and counterfactual scenarios are smaller, however, due to the fact that occupations are dispersed across industries. This is also reflected in the insensitivity of results to the job search elasticity $\kappa$.

These results have several implications. First, the impact of firm-bias is large; hence it would seem to be of first-order importance for understanding wage trends. Whether this conclusion extends further than West Germany is unclear, though Song et al. (2019) document a mechanical contribution of firm premia to U.S. wage trends similar to that observed in Germany by Card, Heining, and Kline (2013). On the other hand, if the importance of firm-bias varies across regions then this is also important, because it would imply that firm-bias is important for understanding regional differences in wage trends - an idea I revisit in the comparison of East and West Germany later in this section. Second, the evident differences in firm-bias across industries and occupations suggests that, depending on the part of the labor market in which a skill-biased shock takes place, there may be substantially different effects on wages. In this respect, skill-bias may be poorly predictive of wage outcomes, a possibility I explore next.

\textsuperscript{53}I define occupations as panel-specific distributions over $i$ and $j$. In practice, confidentiality restrictions prevent me from obtaining the joint distribution for each occupation, so instead I impute this from the unconditional distributions of occupations over industry and worker type.
2.4.2 How Well Does Skill-Bias Predict Wage Outcomes?

A large literature considers the effects of a skill-biased demand shock on wages, under the assumption of competitive labor markets in which firm heterogeneity is absent or irrelevant. Is this approach justified? In this section I consider the implications of firm-bias for the predictive ability of simple models of skill-bias. The industry- and occupation-level results in the previous section indicate that even among similarly-skilled labor markets, there is substantial variation in firm-bias. Below I formalize this intuition, by considering an experiment in which the occupational distribution is divided into ten deciles, ranked according to worker skill, and the aggregate effect of a one-percentage point increase in vacancy share is simulated for each of 120 industry-decile cells. Results are shown in figure 2.9, with the first panel giving the effect on wage variance when firm premia are homogeneous (i.e. no firm-bias). The remaining three panels correspond to different values of the supply elasticity $\kappa$.

**Figure 2.9:** Effect of a Skill-Biased Demand Shock

Note: Effect on wage variance of a 1% increase in vacancy share, by skill decile and aggregate industry group. Columns (bars) indicate the mean (standard deviation) of the change in $Var(w)$. In the case of homogeneous firm premia the value of $\kappa$ is set to 1.
Firm-bias can be seen to have two broad effects: it weakens the overall relationship between the skill-bias of a shock (moving from left to right on the graph) and the change in wage variance, and it increases the variance of outcomes associated with demand shocks pertaining to any given level of skill. As the work response to firm premia ($\kappa$) grows larger, the predictive power of skill-bias grows weaker and the variance of outcomes correspondingly greater. Even in the case where labor supply is unresponsive ($\kappa = .5$), however, the influence of firm-bias is substantial; even here it is possible for an increase in demand in the lowest skill decile to increase wage variance, and an increase in the highest skill decile to increase it.

The two effects discussed above would seem to have two substantive implications. First, firm-bias may act as a confounder in studies relating wage outcomes to environmental changes related to skill-bias. This is potentially important insofar as past researchers have argued that observable skill-biased shocks, such as aggregate adoption of information technology, are only weakly correlated with contemporaneous changes to wage inequality. The patterns in figure 2.9 suggest that, if firm-bias is as quantitatively important in other countries as in Germany, then the relationship between skill-biased demand shocks and wages will be noisy, and only apparent over large samples or long periods of time. The second implication concerns wage variation across regions. The variation and structure of firm premia is found to vary geographically, perhaps reflecting differences in market institutions or the organizational structure or characteristics of firms. Industry- and occupation-level shocks are also likely to differ across country, even when resulting from a common technological trend or change to the trade environment. Hence firm-bias may be able to account for regional variation in wage trends, which is useful not only for our retrospective understanding of wage inequality but also for quantifying the importance of local institutions like collective bargaining. This insight in turn motivates the next section.

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2.4.3 How Well Does Firm-Bias Explain Regional Variation?

In this section I turn to East Germany, where wage variance has increased at a markedly lower pace over the 1996-2017 period. Can firm-bias account for this difference? To explore this I estimate the model for East Germany. The model-predicted trend in wage variance is only three-fourths as large as that in West Germany, which nevertheless understates the aggregate divergence between these regions: the variance of empirical wages has risen by less than half as much in the East, though much of this is due to a decrease in residual wage dispersion. While this rules out the possibility that firm-bias might account for all of the regional differences in trend, it can indeed explain the portion predicted by the model. As shown in figure 2.10, the counterfactual trends with homogeneous firm premia are nearly identical between East and West, particularly for larger values of \( \kappa \).

![Figure 2.10: Predicted and Counterfactual Wage Variance, East Germany](image)

In some respects the result in figure 2.10 is serendipitous, as changes to labor demand

\[55\] West German establishments are only included in the LIAB beginning in 1996.
and the skill distribution are sharply different in East Germany. While new entrants to the West German labor force have tended to concentrate in very lower or very high wage percentiles, polarizing the estimated skill distribution \( \nu \), the opposite has taken place in the East. Meanwhile the shift in employment towards service industries is less pronounced, and declining labor demand in the crafts sector has been offset to a larger extent by gains in other manufacturing industries. Hence there is no reason to expect that, in the absence of firm-bias, the two regions would show a similar increase in wage variance. This nevertheless turns out to be the case, and the greater rise in West German wage dispersion entirely disappears when firm premia are made constant.

The lesser incidence of firm-bias in East Germany is evidently the result of differences in the distribution of firm premia across occupations and industries, and of changes to this distribution in the years following reunification. To show this I conduct several simulated experiments, in which East German firms face different entry costs and therefore pay different wages, but demand parameters are compensated so as to hold fixed the 1993-99 distribution of vacancies. First I consider counterfactual scenarios in which firm premia are held constant at their 1993-99 and 2010-17 values. I then consider the model-predicted trend when East German firms pay West German wages. Results are plotted in figure 2.11.

![Figure 2.11: Counterfactual East German Firm Premia](image)

What we observe is that, if East German firms paid West German wages, or if the wage structure in East Germany were the same in the 1990’s as it is currently, the rise in
wage variance would have been similar between the two regions. From this result it is not surprising that, by the end of the sample period, the distribution of $FP$ is similar between East and West. Whereas in the years after reunification there was little difference in pay between manufacturing and service industries, by the 2010’s there is a wage gap similar in magnitude to that observed among West German firms. The changes to firm premia over time have tended to further dampen wage inequality in the East, as manufacturing workers in that region are comparatively lower-earning, and consequently an increase in industrial wages disproportionately benefits low-earners.

In conclusion, firm-bias can in part account for the slower rise in wage variance in East Germany, owing to differences in the distribution of firm premia that have largely disappeared by the end of the sample period. This result raises two questions of interest. First, what explains the different wage structure observed in East Germany during the 1990’s? And second, what accounts for the convergence with West Germany over time? The most likely answer to both questions is an institutional one. It was recognized after reunification that there were large differences in labor representation and bargaining between the two regions, and there was a concerted effort to “export” West German trade associations, work councils, and bargaining agreements to East Germany.\footnote{See for example Burda and Funke (1993) and Snower and Merkl (2006) on extension of West German bargaining agreements to East Germany.} It is also likely that access to capital and exposure to external markets had an impact on the organizational structure (and especially the size) of East German establishments. Such explanations nevertheless suggest that there is an important idiosyncratic component to the distribution of firm premia within an economy, in which case firm-bias should be expected to vary across countries as well as over time.

### 2.4.4 Firm Premia and Wage Policies

As a final exercise I consider the quantitative effects of wage policies targeting firm premia. As empirical evidence accumulates on the contribution of firm premia to wage inequality and to wage trends, it has become more common to see such proposals, which most commonly take the form of (1) policies targeting competition in output markets, so as to reduce anti-competitive rents, and (2) policies supporting worker bargaining power, to allow for greater
pass-through of rents to workers.\textsuperscript{57} On the one hand, such policies might be expected to adversely impact labor demand, and if they target labor markets associated with low-skill labor then any demand response is likely to be skill-biased. On the other hand labor supply will also adjust, and if workers respond to an increase in firm pay by increasing their likelihood of submitting a job application, then the responses of supply and demand may offset. The ability to quantify these equilibrium responses is a unique advantage of the structurally estimated model developed in this paper.

I consider two experiments, one directly related to German policy discussion and one that is somewhat more speculative. In the first experiment I consider the impact of a policy that enacts “equal pay” for temp agency workers. That temp workers receive lower pay than their full-time coworkers is well-known,\textsuperscript{58} a wage gap that in Germany is usually attributed to differences in collective bargaining coverage. Temp worker pay has become a policy issue as the share of temp labor has tripled since the 1990’s, almost entirely due outsourcing of manufacturing labor.\textsuperscript{59} Current laws mandate that temp workers receive equal pay after 9 months, and have been strengthened over time, but over the sample period there is a large and persistent difference in firm premia associated with temp agencies, on the order of 20-30\%. I consider a policy in which rent-sharing ratios are set exogenously so that this gap in firm premia is eliminated - a policy that, while stylized, can be intuitively thought of as the extension of collective bargaining agreements to temp workers. To perform this experiment I use LIAB establishment data to identify the industry-specific demand for temp labor, so that the effects of an equal pay policy will reflect the sectoral distribution of workplaces in which temp agency employees work.

In the second experiment I consider how wage inequality is affected by compression of the distribution of firm entry costs, implemented as a concave, monotonic, and rank-preserving transformation of $C(i, j)$.\textsuperscript{60} This experiment is intended to capture the effects of pro-competitive policies that reduce the costs of entry, without taking a stand on which

\textsuperscript{57}See for example Tomaskovic-Devey et al. (2020) and the OECD (2021) report “The Role of Firms in Wage Inequality: Policy Lessons from a Large Scale Cross-Country Study.”

\textsuperscript{58}See for example Garz (2013) and Jahn and Pozzoli (2013).

\textsuperscript{59}See appendix for details.

\textsuperscript{60}Specifically I set entry costs equal to $\hat{C} = C^\nu$ where $\nu \in (0, 1)$; in the experiment shown below I set $\nu = .95$. Entry costs are then re-scaled so as to maintain the overall level of market tightness.
submarkets are most likely to be affected. As argued by Autor, Dorn, Katz, Patterson, and Van Reenen (2020), firm-level rents may reflect economies of scale that are greater when output markets are competitive. Nevertheless the purpose of this experiment is not evaluate a particular policy but to quantify the equilibrium effects of such a policy on the wage distribution, under the assumption that the policy has its intended effect.

Figure 2.12: Simulated Effect of Equal Pay for Temp Workers

Results for the first experiment are shown in figure 2.12. As the response of labor demand is a key effect of interest, and as I am considering only the 2010-2017 panel for this exercise and not making comparisons across time, I consider several different values for the demand elasticities $\rho$ and $\sigma$. For each value of the demand and supply elasticities, three results are shown: (1) the change in wage variance when there is no supply or demand response, which is the value that one would obtain from a “reduced-form” empirical experiment; (2) the change in wage variance once temp agencies reduce labor demand in response to the policy, which has effectively raised their wage costs; and (3) the final effect on wage variance when labor supply also responds, and job-seekers increase the likelihood at which they apply to temp agencies.

From the figure it is evident that the equilibrium responses do offset, though not entirely; and in conjunction they tend to dampen the the aggregate decline in wage inequality, with this tendency becoming greater as the supply and demand elasticities become larger. The magnitude of the offset is modest, ranging from a negligible amount to approximately 20%.
The effect of a larger demand is straightforward. When \( \rho \) is large, demand for temp workers falls by a greater amount, and by extension demand for low-skill workers falls as well. This reduces employment at temp agencies that are now high-paying, while raising the variance of skill premia. The labor supply response in turn pushes temp agency employment close to its pre-policy level, but some of the increase in the variance of \( SP \) persists.

**Figure 2.13:** Simulated Effect of Entry Cost Compression

The effects of a compression of entry costs are shown in figure 2.13. In this case the demand and supply responses work to opposite effect: a reduction of entry costs in high-\( FP \) markets encourages more vacancy postings, which reduces output prices and therefore further reduces firm premia. At the same time this increases the demand for skilled labor, due to the positive correlation of \( SP \) and \( FP \), and consequently the demand response is skill-biased. On the other hand labor supply responds by shifting job search towards lower-paying submarkets, which offsets some of the skill-biased change to demand, and pushes firm premia back towards their original level as it is now requires more vacancies to hire a given number of workers, effectively raising entry costs.

In both experiments, most of the remaining effect after both supply and demand responses are factored in is the result of greater dispersion of skill premia. The labor distribution is largely unaffected, since the two responses offset. Hence the combined equilibrium response is does not reflect skill-bias or firm-bias individually, but the *interaction* of skill-bias and
firm-bias. The policies are firm-biased by nature, insofar as they affect different submarkets and different types of firms; and it is only because these submarkets are also associated with different levels of worker skill that the equilibrium response is dampened.

2.5 Conclusion

In this paper I have presented theoretical and quantitative evidence on the firm-bias of labor demand, its contribution to wage trends in West Germany, and its implications for the study of wage inequality and for policies intended to address inequality. The model I develop provides a rich but empirically tractable framework for studying the relationships between labor demand, supply, skill premia, and firm premia. I estimate that by itself, skill-bias accounts for less than half of the impact of changes to industry and occupation labor demand on German wage inequality, and explains none the rise in wage variance due to changing industry employment. I show that because jobs of all skill levels are stratified across industries, there is considerable variability in the quantitative effect of a change in job demand conditional on skill, implying poor predictive power of models that abstract from firm premia. I also show that the effectiveness of labor market policies that target firm premia depends to an important extent on the equilibrium responses of labor demand and labor supply, and on the extent to which skill and firm premia are positively associated across industries and occupations.

Broadly, these results attest to an important role played by firms: they intermediate how changes to macroeconomic environment affect labor markets. The traditional approach to studying subjects like skill-bias is to abstract from firm heterogeneity, but this approach is problematic for several reasons. First, if the structural linkages between workers and firms vary across time and place, then so too will the impact of environmental change. I find that the slower rise of wage inequality in East Germany owes much to differences in the distribution of firm premia across industries and occupations. Hence one benefit of paying closer attention to heterogeneity on the demand side of labor markets is that it may allow us to better understand regional wage trends, and better quantify the role of local labor

These differences are often attributed to location institutions; see for example Dustmann et al. (2009) and Antonczyk et al. (2010).
market institutions like collective bargaining. The equilibrium model developed in this paper provides a means for studying the comprehensive effects of changes to labor demand, and may be readily applied to other countries for which matched employer-employee data are available, which includes most nations in the OECD.

A second key implication of the findings presented here is for policy. Wage inequality is an issue that generates widespread concern, but effective policy requires a good understanding of the underlying mechanisms generating wage gaps. Ignoring the role played by firms and firm premia may lead one to over-estimate the importance of skill, and the quantitative impact of policies that address rising skill premia such as re-training programs. On the other hand, policies targeting the wages set by firms have two issues to tackle. First, the trade-offs associated with these policies will be less adverse when firm premia represent anti-competitive rents in output markets, but firm premia are not themselves proof of anti-competitive rents; the environment studied in this paper features competitive output markets. Second, because skill premia and firm premia are strongly associated across labor submarkets, policies that impact the distribution of firm premia are also likely to affect the distribution of skill premia, and in that sense to generate skill-biased demand responses. I show that the extent of this response depends on the elasticities of labor demand and supply, suggesting that these are important objects of interest when considering such policies.
Chapter 3

Task Automation and Labor Polarization

3.1 Introduction

In both the popular and the academic literature, automation is associated with the wholesale replacement of human workers with machines. It is common to read predictions of robots “taking jobs”.\(^{62}\) This notion of one-to-one substitution has been formalized in theoretical models of labor-substituting technology and in empirical studies that estimate occupation-level automation risk.\(^{63}\) Historical examples of job-level automation are nevertheless difficult to find,\(^{64}\) and case studies of technology adoption generally paint a far different picture, in which new technologies substitute for labor at particular tasks within jobs, changing but not eliminating the role of human labor.\(^{65}\) Despite this, little is known about the macroeconomic implications of task-level automation when tasks are distinct from jobs.

In this paper, I study how labor markets are impacted by task automation when jobs consist of multiple tasks. I begin with an empirical analysis that draws on four decades of German survey data to show two motivating results. First, virtually all workers report performing a variety of tasks on the job, including both routine and non-routine work. Second, the most important technological shift over this time period - computerization - is associated with an intra-occupational increase in the proportion of time spent on non-


\(^{63}\)Notable studies include Acemoglu and Autor (2011), Frey and Osborne (2017), and Acemoglu and Restrepo (2018).

\(^{64}\)See Autor (2015) and Bessen (2016).

routine tasks, consistent with the case study literature. To study the implications of task automation, I develop an assignment-based model of labor markets in which occupation-specific jobs involve multiple tasks, and workers solve a time allocation problem. Automation is formalized as the replacement of labor with capital at a single, low-skill task. In this environment, automation has multiple effects: a standard *labor-substitution* effect in which less labor is needed to produce a given amount of output, a *skill-complementarity* effect arising from the reallocation of workers' time towards more skilled task content, and a *demand* effect that depends on the responses of worker productivity and unit labor costs, together with the elasticity of substitution across occupations.

The model delivers several main predictions. In the short-run, when the automating technology is expensive, it will tend to be adopted only in skilled occupations for the reason that skilled labor has a comparative disadvantage at the unskilled task. The result is polarization of the wage and employment distributions: job losses from automation are concentrated around the “marginal” occupation where firms are indifferent between automating or not, which in turn drives polarization of wages. Post-adoption, employment in affected occupations recovers provided that occupations are not perfect complements, since declining capital rents tend to reduce unit labor costs and increase relative demand. In the long-run, after adoption is universal and as the cost of the automating technology approaches zero, employment in low-skill occupations rises and wage inequality falls; hence the effects of automation are less severe in the long-run than in the short-run.

In the final part of the paper I test the model’s short-run predictions, and structurally estimate a continuous version of the model in order to obtain long-run forecasts. Consistent with model predictions, computerization in West Germany exhibited a top-down pattern, with significant PC adoption in low-skill occupations only occurring in the 2000’s. PC adoption is associated with contemporaneous declines in occupational employment, while high rates of pre-existing PC use are predictive of occupational growth. The estimated structural model predicts that job losses in middle-skill occupations have peaked, as has the 90/50 wage ratio, with future computerization anticipated to drive wage inequality in the lower half of the wage distribution. In the very long-run, employment in high-skill occupations is anticipated to decline relative to current levels, and the 90/10 wage ratio to increase slightly.
As a last exercise I use the quantitative model to explore implications for two commonly-studied elasticities. First, I show that the capital-labor elasticity of substitution is a complex function of rental rates and worker skill, and neither constant nor exogenous as is commonly assumed. Second, higher values of the labor-labor elasticity of substitution are associated with greater short-run (but not long-run) changes to occupational employment. This elasticity is therefore important for understanding the persistence of job loss from automation, and the efficacy of supply-side policies such as retraining programs.

The main contribution of this paper is to the macroeconomic literature on the effects of automation. Substitution of capital and labor is traditionally modeled in simple fashion as depending on the parameters of an aggregate production function, a tradition carried over to the case of automation by Zeira (1998). Subsequent work has also taken this approach, including Acemoglu and Autor (2011), Peretto and Seater (2013), Acemoglu and Restrepo (2018), Aghion, Jones, and Jones (2019), and Hemous and Olsen (2020). Theoretical models of capital-labor substitution yield a simple, negative relationship between technology adoption and employment, that has been used to generate forecasts for future disemployment as in Frey and Osborne (2017). Several strands of criticism exist. A number of authors have observed that history offers few examples of wholesale automation of jobs, which are rarely reducible to a single, automatable task. In addition, there have been significant within-occupation changes to task content that are difficult to explain through job-level automation. I reconcile these strands of literature with the notion of labor-substituting capital by relaxing the assumption of one-to-one substitution, and by providing both empirical evidence and theory on the automation of tasks at a sub-job level.

This paper also contributes to the literature on how occupational structure intermediates the effects of macroeconomic change. The framework developed here represents an intermediate point between models in which occupations consist of task bundles requiring task-specific skills, such as Gathmann and Schonberg (2010), Yamaguchi (2012), and Autor and Handel (2013), and models that abstract from occupational task structure and assign to each occup-

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66 For example Autor (2015), Bessen (2016), Arntz, Gregory, Zierahn (2017), and Dengler and Matthes (2018)

pation an exogenous value (e.g. an index) describing the relationship between productivity and skill (or a skill composite), such as Costinot and Vogel (2010) and Acemoglu and Autor (2011). I am able to maintain the tractability in general equilibrium of the second class of models, while incorporating the occupational task structure studied by the first group. By explicitly modeling the allocation of workers’ time across tasks, I am able to incorporate the time-reallocation effect often mentioned in anecdotal and case study accounts of automation, and to estimate task production shares from empirical task frequencies. This approach is more general than that taken in the task literature, where occupational output is usually constrained to be a linear function of task output.

The structure of this paper is as follows. In section 2 I show the main descriptive results: that virtually all workers report performing both routine and non-routine tasks, and computerization is associated with greater time spent on non-routine tasks. In section 3 I develop a model of task-based automation with multi-task occupations. I begin with a simple environment in which all jobs are homogeneous within an occupation, which in turn yields a set of precise analytical predictions. I then allow for intra-occupational heterogeneity, which weakens the analytical results but allows the model to be taken more directly to the data. Results on the qualitative model predictions are shown in section 5, and quantitative estimation and predictions are discussed in section 6.

3.2 Descriptive Analysis

In this section I present motivating evidence on within-job task content. After describing the data and the task measures used in the analysis, I show two main facts. First, the vast majority of jobs involve both routine and non-routine tasks, indicating that “whole sale” automation of jobs is \textit{ex ante} unlikely. Second, computerization of occupations is associated with a shift towards more non-routine task content, suggesting that \textit{ex post}, technological automation has had an impact on the within-job distribution of task content. These facts are inconsistent with stylized models of automation that assume an equivalence between tasks and jobs, and they constitute evidence that technological change has interacted in a substantive way with the distribution of tasks within occupations, as well as between them.
3.2.1 Occupational Routineness in the BIBB Surveys

This analysis draws on the seven BIBB Employment Surveys, collected by the Federal Institute for Vocational Education and Training (BIBB) over the period 1979-2018 in partnership with the Institute for Employment Research (IAB, 1979-1999) and the Federal Institute for Occupational Health and Safety (2006-2018). Each survey draws on a random sample of the employed German labor force, and asks respondents a range of questions concerning job task content, the use of technology and tools, and other aspects of the job environment and the individual’s work history. Also contained is information on monthly wage, which I convert to an hourly number using reported weekly hours.\textsuperscript{68} The first two surveys do not contain data on East German workers and consequently, for comparability over time, I limit the analysis in this paper to West German workers between the ages of 18 and 65.

<table>
<thead>
<tr>
<th>Table 3.1: Summary statistics for BIBB employment surveys, 1979-2018</th>
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<td>Observations <em>with task data</em></td>
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<td>Observations <em>with wage data</em></td>
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<tr>
<td>PC use (%)</td>
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<td>weighted</td>
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<td>3-digit occupations</td>
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The detailed information contained in the BIBB surveys, together with the long time-frame over which they have been collected, makes them especially well-suited to research on the impact of technological change. Past examples of such research include Spitz-Oener (2006) and Bachmann, Cim, and Green (2019). There are nevertheless several challenges that must be addressed when using the BIBB surveys to study questions relating technology to labor markets. First, survey questions are often inconsistent across panels. For this project I rely on two sets of survey questions: those concerning PC use on the job, and those on task performance. Questions about PC use are broadly consistent over time, with one substantial change in format and wording occurring between 1992 and 1999. Questions regarding task content - for example, whether and how often workers “control machines”,

\textsuperscript{68}Wage data are highly aggregated for surveys prior to 2005/06, and consequently of limited usefulness in this study.
“advise others”, and so forth - vary substantially across most survey years. For example, the 1979 survey considered more than 80 such tasks, while the 1999 survey contained only 13. Past research has relied on aggregation methods in order to obtain a smaller set of task categories that can be compared across time, but such methods are essentially *ad hoc* and *Rohrbach-Shmidt and Tiemann* (2013) show that different approaches can have a substantial impact on measures of occupational routineness.

A second issue is that job tasks do not map directly into conceptual frameworks like skillfulness or routineness, and researcher interpretation is therefore required when relating empirical results to hypotheses. This is especially problematic so when combined with the aggregation issues discussed above. Methods like factor analysis that provide a principled approach to dimensionality reduction are, by nature, unlikely to result in easily-interpreted task groups. Approaches based on intuition - combining tasks that “seem” similar - are perhaps more meaningful, but provide no guidance on precisely how tasks should be aggregated.

Table 3.2: Routine task content in the BIBB surveys

<table>
<thead>
<tr>
<th>Survey years</th>
<th>Repeat tasks</th>
<th>Follow instructions</th>
<th>Adapt to new tasks</th>
<th>Improve procedures</th>
<th>Solve problems</th>
<th>Make decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979-1992</td>
<td>.491</td>
<td>.654</td>
<td>.647</td>
<td>.455</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>.530</td>
<td>.779</td>
<td>.585</td>
<td>.517</td>
<td>.583</td>
<td>.441</td>
</tr>
<tr>
<td>Vocational</td>
<td>.533</td>
<td>.767</td>
<td>.690</td>
<td>.619</td>
<td>.711</td>
<td>.587</td>
</tr>
<tr>
<td>University</td>
<td>.383</td>
<td>.546</td>
<td>.821</td>
<td>.752</td>
<td>.838</td>
<td>.766</td>
</tr>
<tr>
<td>Wage pct.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-25</td>
<td>.500</td>
<td>.780</td>
<td>.615</td>
<td>.554</td>
<td>.644</td>
<td>.490</td>
</tr>
<tr>
<td>26-50</td>
<td>.542</td>
<td>.749</td>
<td>.694</td>
<td>.624</td>
<td>.717</td>
<td>.586</td>
</tr>
<tr>
<td>51-75</td>
<td>.504</td>
<td>.703</td>
<td>.758</td>
<td>.680</td>
<td>.764</td>
<td>.655</td>
</tr>
<tr>
<td>76-100</td>
<td>.421</td>
<td>.586</td>
<td>.803</td>
<td>.732</td>
<td>.812</td>
<td>.750</td>
</tr>
</tbody>
</table>

Table notes. Survey answers re-coded as frequencies and means calculated using survey weights. Results by education and wage percentile are for the 2006 survey. See appendix for details.

For both of these reasons I depart from past literature and limit attention to a set of survey questions about *task characteristics*. In all seven panels, and with minimal changes in wording and response categories, respondents are asked how often they find themselves (1) repeating the same work process, (2) following detailed instructions, (3) adapting to
new tasks, and (4) improving existing procedures or trying something new. These questions would seem to bear directly on the amount of routine task content present in the job, which is typically described in the literature as some combination of task repetition and the ability to codify task performance into a set of steps that a machine or computer might execute. I also consider questions from the 2006-2018 panels on whether respondents must (5) react to problems and solve them or (6) make difficult decisions on their own. These last three panels also feature a consistent set of questions regarding job tasks, for which I provide reference results in the appendix. Responses consist of between three and five verbal frequencies - e.g. “often” or “never” - which I interpret numerically and assign in even intervals to [0, 1].

Table 3.2 shows conditional mean values after splitting the sample by year and, for the 2006 year survey, by educational attainment and wage quartile. Less education and a lower wage is associated as expected with greater task repetition and instruction, and less adaptive and cognitive content. Jobs do not appear less routine in later years, although there is a shift towards greater cognitive content that is likely the result of increased employment in professional and technical occupations. Comparisons with task content (see appendix) indicate that repetition and detailed instructions are associated with manual labor tasks, while non-routine characteristics are associated with tasks relating to analysis, instruction, advising, and sales and marketing. Notably, production-related tasks are associated with both routine and non-routine characteristics, suggesting a potentially complicated relationship between tasks and task characteristics like routineness.

### 3.2.2 Task Variety Within Jobs

Are all tasks within a job equally susceptible to automation? In figure 3.3 I show, for each wage percentile, the distribution of survey responses for the 2006 survey.\(^69\) Across all wage percentiles, the majority of respondents report performing both routine and non-routine tasks. Although higher-earning respondents perform non-routine content more frequently, and routine content less so, much of the variation in responses is within rather than between wage percentiles. In terms of task repetition, the 25th and 75th wage percentiles look similar;
and even in the lowest quartile, three-quarters of respondents report that their job involves decision-making, problem-solving, and other task content that previous literature asserts is difficult to automate.

**Figure 3.3:** Routine and non-routine activity by wage percentile, 2006

**Figure notes.** LOESS interpolation of mean response by wage percentile. Gray regions indicate 95% confidence intervals.

Similar results can be shown for the type of tasks performed on the job, summarized in figure 3.4. Of the sixteen task categories present in the 2006 survey, four-fifths of respondents report performing at least 5. Of six broad task groups represented, four-fifths of respondents perform tasks in at least 3 of these groups. Similarly, when asked how often they must perform many different tasks at the same time, virtually all individuals report that this is at least sometimes true, and two-thirds that it is often the case. This and the previous figure offer a stark rebuttal to the assumption that jobs and tasks are equivalent. There exists a variety of tasks not just within a given occupation but within individual jobs, and in most cases this variety includes both routine and non-routine task content.

One notable aspect of the survey responses shown in figure 3.3 is the lack of any discernible hump-shape in measures of routineness. Although Germany, like the United
States, has experienced labor market polarization,\textsuperscript{70} and employment has generally declined in “middle-skill” jobs, there is little evidence in the BIBB surveys that jobs in the middle of the wage distribution involve greater routine task content. Measures of routineness peak in the first quartile, while measures of non-routine task content are generally increasing in wage. If routineness were the only predictor of employment effects from technology adoption, then it is difficult to avoid the conclusion that disemployment should be greatest at the bottom of the wage distribution. More broadly, these results would seem to be inconsistent with theories that pose a simple relationship between task content and automation.

\subsection{Computerization and Task Content}

If automation is task-specific and jobs vary in their task content, then we would expect automation to affect the within-job distribution of tasks. A simple test of this hypothesis is to see whether technology adoption is associated with changes in the overall routineness of job tasks. As my measure of technological automation I focus on the use of personal computers in the workplace. A large body of literature argues that computerization reduces time spent on routine tasks, and hence we should observe less frequent routine task content in this case. I begin with a cross-sectional analysis. In table 3.5 I show the marginal effect of computerization on task characteristics, estimated by performing fractional logit regressions.

\textsuperscript{70}See e.g. Dustmann, Ludsteck, and Schonberg (2007) and Bachmann, Cim, and Green (2019).
of task content on computer use. All regressions control for 3-digit occupation, 1-digit industry, and year. Because the wording and format of the survey question on computer use changes between 1992 and 1999, I split the sample into two subsamples covering the 1979-1992 and 1999-2018 periods. Conditional on the type of work being done, computerization is associated with relatively less routine task content, and relatively more non-routine content. This result holds within educational groups and wage quantiles, as well as across panels.

### Table 3.5: Task characteristics and PC use, 1979-2018

<table>
<thead>
<tr>
<th>Sample</th>
<th>Repeat tasks</th>
<th>Follow instructions</th>
<th>Adapt to new tasks</th>
<th>Improve procedures</th>
<th>Solve problems</th>
<th>Make decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Survey years 1979 - 1992</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full sample</td>
<td>-.054 (.005)</td>
<td>-.046 (.005)</td>
<td>.103 (.004)</td>
<td>.094 (.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>-.043 (.021)</td>
<td>-.043 (.016)</td>
<td>.147 (.019)</td>
<td>.107 (.017)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocational</td>
<td>-.038 (.006)</td>
<td>-.033 (.006)</td>
<td>.098 (.005)</td>
<td>.093 (.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>University</td>
<td>-.039 (.009)</td>
<td>-.039 (.010)</td>
<td>.051 (.007)</td>
<td>.062 (.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage pct.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-25</td>
<td>-.024 (.012)</td>
<td>-.016 (.011)</td>
<td>.099 (.011)</td>
<td>.072 (.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26-50</td>
<td>-.046 (.011)</td>
<td>-.011 (.010)</td>
<td>.096 (.010)</td>
<td>.071 (.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51-75</td>
<td>-.034 (.010)</td>
<td>-.036 (.009)</td>
<td>.080 (.009)</td>
<td>.072 (.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>76-100</td>
<td>-.044 (.009)</td>
<td>-.056 (.010)</td>
<td>.064 (.007)</td>
<td>.084 (.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Survey years 1999 - 2018</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full sample</td>
<td>-.013 (.006)</td>
<td>-.045 (.006)</td>
<td>.102 (.004)</td>
<td>.116 (.005)</td>
<td>.098 (.005)</td>
<td>.145 (.007)</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>.009 (.017)</td>
<td>-.016 (.016)</td>
<td>.119 (.014)</td>
<td>.130 (.014)</td>
<td>.128 (.018)</td>
<td>.162 (.020)</td>
</tr>
<tr>
<td>Vocational</td>
<td>.003 (.007)</td>
<td>-.039 (.006)</td>
<td>.098 (.005)</td>
<td>.102 (.006)</td>
<td>.098 (.006)</td>
<td>.133 (.008)</td>
</tr>
<tr>
<td>University</td>
<td>-.020 (.013)</td>
<td>-.041 (.015)</td>
<td>.090 (.008)</td>
<td>.103 (.010)</td>
<td>.082 (.009)</td>
<td>.147 (.018)</td>
</tr>
<tr>
<td>Wage pct.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-25</td>
<td>.004 (.012)</td>
<td>-.032 (.010)</td>
<td>.107 (.009)</td>
<td>.102 (.009)</td>
<td>.089 (.011)</td>
<td>.134 (.013)</td>
</tr>
<tr>
<td>26-50</td>
<td>-.005 (.012)</td>
<td>-.016 (.011)</td>
<td>.079 (.009)</td>
<td>.083 (.010)</td>
<td>.079 (.011)</td>
<td>.082 (.014)</td>
</tr>
<tr>
<td>51-75</td>
<td>-.025 (.013)</td>
<td>-.058 (.013)</td>
<td>.077 (.010)</td>
<td>.088 (.010)</td>
<td>.086 (.013)</td>
<td>.095 (.018)</td>
</tr>
<tr>
<td>76-100</td>
<td>-.030 (.019)</td>
<td>-.050 (.021)</td>
<td>.100 (.012)</td>
<td>.121 (.014)</td>
<td>.068 (.017)</td>
<td>.165 (.025)</td>
</tr>
</tbody>
</table>

**Table notes.** Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance. See appendix for details.

To better the answer the question of how changes in computer use and task content are related, I aggregate survey responses by 3-digit occupation and perform a series of difference-
in-difference regressions, shown in figure 3.6. Regressions are performed for consecutive panels in order to minimize confounding from time-variation in other variables that may affect task content; results for 1992-1999 are shown, but indicated in italics due to the change in question format. The results are comparable to those shown previously: when significant, the coefficients on routine task content are negative and those on non-routine content are positive. Marginal effects are large, and indicate that a 10% increase in occupational computer use is associated with a 1-2% change in task frequencies. Note that even if the presumed direction of causality is correct - i.e. from computerization to task frequencies - there are two ways in which this effect might occur. Task content may change within individual jobs, or employment may substitute away from jobs in the same occupation but with less routine task content. Either story would nevertheless yield a similar implication: automation is unlikely to replace occupational labor wholesale, and employment outcomes would depend both on the degree to which labor is automated, and on any changes to occupational productivity.

Table 3.6: Occupation mean task characteristics and PC use (two-way FE)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Repeat tasks</th>
<th>Follow instructions</th>
<th>Adapt to new tasks</th>
<th>Improve procedures</th>
<th>Solve problems</th>
<th>Make decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLDB 1988</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979-1986</td>
<td>-.049 (.034)</td>
<td>-.077 (.035)</td>
<td>-.022 (.052)</td>
<td>.073 (.036)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986-1992</td>
<td>-.220 (.024)</td>
<td>-.069 (.033)</td>
<td>.072 (.027)</td>
<td>.004 (.027)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1992-1999</td>
<td>.046 (.030)</td>
<td>.028 (.028)</td>
<td>-.025 (.025)</td>
<td>.069 (.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1999-2006</td>
<td>-.118 (.031)</td>
<td>-.106 (.030)</td>
<td>.111 (.019)</td>
<td>.164 (.023)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KLDB 1992</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1999-2006</td>
<td>-.077 (.028)</td>
<td>-.086 (.025)</td>
<td>.114 (.018)</td>
<td>.144 (.020)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006-2012</td>
<td>-.018 (.036)</td>
<td>-.037 (.033)</td>
<td>.053 (.025)</td>
<td>.078 (.026)</td>
<td>.095 (.023)</td>
<td>.176 (.033)</td>
</tr>
<tr>
<td>2012-2018</td>
<td>-.068 (.040)</td>
<td>-.051 (.042)</td>
<td>.053 (.030)</td>
<td>.152 (.030)</td>
<td>.064 (.029)</td>
<td>.161 (.040)</td>
</tr>
</tbody>
</table>

Table notes. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include occupation and year dummies. Bold results indicate 95% significance. Italic results indicate a change in change in question wording between the two samples. See appendix for details.

These findings, like those in the previous section, are difficult to reconcile with standard notions of automation as the wholesale substitution of capital for labor. Past authors have noted that it is difficult to find examples of jobs (i.e. occupations) eliminated by automa-

71 This would be consistent with case studies such as Autor, Levy, and Murnane (2002).
The results shown in this section indicate that it is difficult to find examples of jobs that could be entirely automated. The technological change most studied in the literature - widespread adoption of the PC - is associated with changes in occupational task content, indicating a reallocation of labor across tasks within occupations, and not just between occupations. These facts form the basis for the model developed in the next section.

### 3.3 A Model of Partial Automation

In this section I develop a model of automation where jobs consist of multiple tasks. As is common in the literature, automation is formalized as the substitution of capital for labor at a particular task. In contrast to one job-one task models like Acemoglu and Autor (2011), however, the effects of automation are not limited to a reduction in the demand for labor. Automation will also affect workers’ time allocation across tasks, and hence occupational returns to skill. And because automation will tend to reduce unit costs within a given occupation, the overall effect on employment is ambiguous, and will depend on the relative magnitudes of countervailing substitution effects.

#### 3.3.1 Environment

The environment is static and consists of workers and firms. Workers are heterogeneous in a continuous skill variable \( s \in [0, 1] \) with distribution \( F(s) \). Intermediate good producers are heterogeneous in a continuous variable \( j \in [0, 1] \) indicating the type of labor output they produce. Here, \( j \) is interpreted as referring to an “occupation”, with intermediate producers acting as aggregators of occupation-specific labor. The output of \( j \)-producers is in turn aggregated by final good producers that require no labor input.

**Occupational output.** Workers in a \( j \)-firm produce labor output by allocating a unit of time between a low-skill and a high-skill task, denoted \( l \) and \( h \). Task output depends on worker skill, and is given by the functions \( \gamma_l(s) \) and \( \gamma_h(s) \). I assume that there are no agency problems: workers choose their time allocation so as to maximize firm output. The unskilled task may be partially automated, in which case a portion \( \kappa \) or less of task output

---

\(^{72}\) Examples include Bessen (2016) and Autor (2015).
may be performed by capital at a per-unit rental cost of $r$. Tasks are assumed to be perfect
complements, with worker output given by the Leontief output function

$$ y(j, s, K) = \min \left\{ \frac{t\gamma_h(s)}{\alpha(j)}, \frac{(1-t)\gamma_l(s) + K_l}{1 - \alpha(j)} \right\}. $$

In this equation $t$ is the proportion of the worker’s time allocated to the skilled task, and $K_l$
the (per-worker) capital allocated to the unskilled task. The automation feasibility constraint
implies that if $K$ total capital is provided to the worker, then

$$ K_l = \min \left\{ K, \frac{\kappa}{1 - \kappa} [1 - t] \gamma_l(s) \right\}. $$

The task share function $\alpha(j) > 0$ is assumed to be continuously differentiable and to lie
strictly between 0 and 1 for all $j$, implying that there are no jobs consisting of a single
task. Wages are denoted $w(s)$ and intermediate output prices $p(j)$. Per-worker profits of
intermediate good producers can be written as

$$ \pi_i(j, s, K) = p(j) y(j, s, K) - w(s) - rK. $$

I assume there is free entry of producers, and for now I abstract from producer scale.

**Aggregate technology.** Final good producers aggregate $j$-output into a consumption
good using the technology

$$ Y = \left( \int \beta(j) Y(j)^{\frac{\rho - 1}{\rho}} dj \right)^{\frac{\rho}{\rho - 1}}. $$

I assume that production of the final good requires only $j$-inputs, and normalize the price
of the final good to 1. Given these assumptions, the profit of the representative final good
producer will be

$$ \pi_f = Y - \int p(j) Y(j) dj. $$

Final good markets are assumed to be perfectly competitive.
Agents’ problems. I now formalize the problems of workers and producers. Beginning
with workers, the time spent on tasks is chosen to maximize output, given assigned capital
\( K_l \):

\[
\max_{t \in [0,1]} \min \left\{ \frac{t \gamma_h(s)}{\alpha(j)}, \frac{(1-t) \gamma_l(s) + K_l}{1 - \alpha(j)} \right\}, \tag{19}
\]

where \( K_l \) is subject to the feasibility constraint above. Intermediate good producers choose
worker skill \( s \) and per-worker capital \( K \) in order to maximize profits:

\[
\max_{K \geq 0, s \in [s, \bar{s}]} p(j) y^*(j, s, K) - w(s) - rK \tag{20}
\]

\[
\text{s.t. } y^*(j, s, K) \text{ solves (19)}.
\]

Final good producers then choose the mix of \( j \)-goods that maximizes profits:

\[
\max_{Y(j)} \left( \int \beta(j) Y(j)^{\frac{1-\rho}{\rho}} dj \right)^{\frac{\rho}{1-\rho}} - \int p(j) Y(j) dj. \tag{21}
\]

Note that because capital costs are fixed when workers choose their time allocation, no ineffi-
ciency is introduced by separating the worker’s time allocation problem from the intermediate
producer’s choice of capital.

3.3.2 Equilibrium

The worker’s time allocation problem (19) yields two solutions, depending on whether the
technological constrain \((1-\kappa)K \leq \kappa[1-t] \gamma_l(s)\) is binding:

\[
t^*(j, s, K) = \begin{cases} \frac{\alpha(j)}{\gamma_l(s)} + \frac{1+K}{\frac{1}{\gamma_l(s)} + \frac{1-\alpha(j)}{\gamma_l(s)}}, & K \leq \frac{\kappa(1-\alpha(j))}{\gamma_h(s)} + \frac{1-\alpha(j)}{\gamma_l(s)} \\ \frac{\alpha(j)}{\gamma_l(s)} + (1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}, & K > \frac{\kappa(1-\alpha(j))}{\gamma_h(s)} + \frac{1-\alpha(j)}{\gamma_l(s)} \end{cases}. \]

Both costs and output are linear in capital, and so profit maximization by producers (20)
will entail a corner solution: producers will choose either \( K = 0 \) or \((1-\kappa)K = \kappa[1-t] \gamma_l(s)\),
depending on whether it is cheaper to hire additional labor or to automate the low-skill task.
Formally,

\[
K^*(j, s) = \begin{cases} 
0 & w(s) \leq r\gamma_l(s) \\
\kappa \frac{(1-\alpha(j))}{\gamma_h(s) + (1-\kappa) \frac{1-\alpha(j)}{\gamma_l(s)}} & w(s) > r\gamma_l(s)
\end{cases}.
\]  

(22)

On the other hand the first-order condition for worker skill implies that in equilibrium we must have

\[
w'(s) = \begin{cases} 
\frac{d}{ds} \left( \frac{\alpha(j)}{\gamma_h(s)} + \frac{1-\alpha(j)}{\gamma_l(s)} \right)^{-1} & w(s) \leq r\gamma_l(s) \\
\frac{d}{ds} \left( \frac{\alpha(j)}{\gamma_h(s)} + (1-\kappa) \frac{1-\alpha(j)}{\gamma_l(s)} \right)^{-1} & w(s) > r\gamma_l(s)
\end{cases}.
\]

The optimal choices of \(s\) and \(K\) will depend on the wage function. This is a potential problem, as it may not be possible to characterize the optimal assignment (and hence the wage function) without knowing producers’ automation decisions. Nevertheless I show below that in this simple environment, \(K^*\) can be characterized \(ex\ ante\) to a degree sufficient for the optimal assignment and wage functions to be characterized and solved numerically without difficulty.

The final good producer’s problem yields the first-order condition

\[Y(j) = \left( \frac{\beta(j)}{p(j)} \right)^\rho Y.\]

Free entry, on the other hand, has the implication that \(p(j) = \frac{w(j)+rK(j)}{y(j)}\). Hence we can write total \(j\)-employment as

\[L(j, s) = \frac{y^*(j, s, K)^{\rho-1}}{(w(s) + rK^*(j, s))^{\rho}} \beta(j)^\rho Y.\]

Fixing skill and wages, automation will increase labor demand whenever

\[
\left( \frac{\alpha(j)}{\gamma_h(s)} + \frac{1-\alpha(j)}{\gamma_l(s)} \right)^{1-\rho} > \left( \frac{w(s) \left( \frac{\alpha(j)}{\gamma_h(s)} + (1-\kappa) \frac{1-\alpha(j)}{\gamma_l(s)} \right) + r(1-\alpha(j))\kappa}{w(s) \left( \frac{\alpha(j)}{\gamma_h(s)} + (1-\kappa) \frac{1-\alpha(j)}{\gamma_l(s)} \right)} \right)^\rho.
\]

(23)
The right-hand side of (23) will always be greater than one, and hence for employment to increase it must be that automation decreases unit output costs, and that \( \rho \) is sufficiently large. Unit output costs will decrease by a greater amount when \( \alpha(j) \) is large but automation has a large effect on labor productivity, e.g. when workers spend a large amount of time on the unskilled task because they have a comparative disadvantage at that task. Note, however, that \( w(s) \) will be endogenous to \( r \) in equilibrium and so a critical factor in determining the employment effects of automation is the response of the wage function. On the other hand the capacity for automation \( \kappa \) will inform the overall magnitude of the employment effect. This is a departure from past, well-known models in that automation does not necessarily reduce employment, and the set of workers affected is an endogenous outcome depending on relative costs.

**Optimal assignment.** I now turn to the optimal assignment of workers to jobs, which will only be well-defined when different types of workers enjoy a comparative advantage at different types of jobs. I therefore impose the condition:

**Assumption 6:** \( \frac{d}{ds} \log \gamma_h(s) > \frac{d}{ds} \log \gamma_l(s) > 0 \) and \( \alpha'(j) > 0 \).

Task output is increasing in skill, and this increase is proportionally greater for skill-intensive tasks. That \( \alpha(j) \) is increasing implies that jobs are ordered in terms of their return to skill, from which we can predict that in equilibrium, higher \( j \) will be associated with higher \( s \). In order for the optimal assignment to be a smooth function, I further impose the conditions that \( \beta(j) \) and \( F(s) \) are continuously differentiable, and that \( F(s) \) is a strictly increasing function. With these restrictions, the model can be shown to possess two properties that allow automation decisions (i.e. \( K^* \)) to be characterized separately from the wage function, allowing for a precise characterization of the equilibrium. Denoting \( K \) to be the optimal level of capital conditional on automation, the model satisfies the following conditions:

1. **rank-preserving:** for any two occupations \( j \) and \( j' \), if \( \frac{d}{ds} \log y^*(j', s, 0) > \frac{d}{ds} \log y^*(j, s, 0) \) then \( \frac{d}{ds} \log \left[ p(j)y^*(j', s, K) - rK \right] > \frac{d}{ds} \log \left[ p(j)y^*(j, s, K) - rK \right] \)

2. **bias-consistent:** for all \( j \) and all \( s \) we have \( \frac{d}{ds} \log y^*(j, s, 0) > \frac{d}{ds} \log \gamma_l(s) \)

Under the first property, the ordering of occupations from least to most skill-intensive is un-
changed by automation, which greatly facilitates characterization of the optimal assignment. The second property implies that automation either everywhere increases the occupational return to skill, or everywhere decreases it, which simplifies the equilibrium pattern of automation across producers and allows it to be precisely characterized independent of the optimal assignment. These properties are satisfied in the present case due to the assumption that there are only two tasks; in an environment with three or more, they would require additional restrictions on worker productivity and task shares.

Now let $\lambda : s \to j$ indicate the set of jobs at which at least one $s$-worker is employed. Given assumption 1, $\lambda$ will be a strictly increasing, piece-wise differentiable function. There will exist a unique $s^*$ separating automated and non-automated labor types, where $s^*$ may take an interior value or, in the cases where no or all employers automate, may be equal to $\underline{s}$ or $\overline{s}$. The wage and market tightness functions will satisfy the differential equations

$$
\frac{w'(s)}{w(s)} = \begin{cases} \\
\frac{d}{ds} \log y(\lambda(s), s, 0) & s < s^* \\
\frac{d}{ds} \log y(\lambda(s), s, \overline{K}) & s > s^*
\end{cases}
$$

(24)

$$
\lambda'(s) = \begin{cases} \\
\frac{y(\lambda(s), s, 0)^{1-\rho} F'(s)}{\beta(\lambda(s))^\rho w(s)^\rho} & s < s^* \\
\frac{y(\lambda(s), s, \overline{K})^{1-\rho} F'(s)}{\beta(\lambda(s))^\rho Y(\lambda(s))^{1-\rho} w(s) + \kappa [1 - \alpha(\lambda(s))] r y(\lambda(s), s, \overline{K})} & s > s^*
\end{cases}
$$

(25)

where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$. If $s^*$ lies in the interior of $[0, 1]$ then the wage and matching functions will be continuous but not differentiable at $s^*$, and continuously differentiable for all $s \neq s^*$.

All together, equilibrium in this environment is time and capital allocations $t^*$ solving (19)-(20), intermediate good bundles $Y^*(j)$ satisfying the final good producer’s problem (21), prices $p(j)$ such that zero-profit conditions hold and goods markets clear, and wage and matching functions satisfying the system of differential equations (24)-(25).

### 3.3.3 Automation, Wages, and Employment

In this section I characterize the short-run and long-run effects of automation, where (in this static setting) I interpret the short-run as describing scenarios where $r$ is sufficiently large.
that \( s^* > s \), and the long-run corresponding to the case where \( r \to 0 \).

The first property of the model requires no additional proof and is shown in the system of differential equations (24)-(25): in the short-run, the automating technology is only adopted in high-skill jobs. Skilled labor has a comparative advantage at the skilled task, which in turn implies that it is more costly to have skilled workers performing the low-skill task. Automation therefore reduces unit costs by a greater amount when the worker is skilled. Declines in \( r \) will tend to lower the automation threshold \( s^* \) and, with \( w(s) > 0 \), it is evident that for sufficiently low rental rates we will have \( s^* = s \) and all jobs will adopt the technology.

A second property is that in the short-run where \( s^* > 0 \), automation will tend to polarize the wage and employment distributions:

Theorem 1 (short-run polarization). For \( s^* \in (0, 1) \), low-skill employment \( \int_0^{\lambda(s^*)} L(j) dj \) is greater under automation. If \( \rho \) is sufficiently small and whenever \( \rho < 1 \), there will exist \( j' > \lambda(s^*) \) such that \( \int_{j'}^{\infty} L(j) dj / \int_{j'}^{\lambda(s^*)} L(j) dj \) is greater under automation. Moreover for \( s < s^* \) we will have \( w'(s)/w(s) \) strictly smaller under automation, whereas for \( s \in (s', \bar{s}] \) for \( s' \) sufficiently large we will have \( w'(s')/w(s') \) greater under automation.

For low values of \( \rho \) disemployment effects will be strongest among automated jobs ‘close’ to the threshold skill level \( s^* \), while a smaller share in the unskilled task and greater reductions in unit costs result in better employment outcomes for skilled jobs close to \( \bar{s} \). As \( \rho \) becomes large, the short-run effects on employment are more difficult to characterize because changes to the wage function will have a larger impact on labor demand. With respect to wages, the immediate effect of automation is that low-skill workers shift towards lower-skill jobs, reducing \( w'(s)/w(s) \) in the bottom part of the wage distribution. Workers at the upper end of the wage distribution see an increase in \( w'(s)/w(s) \) as, even if \( \lambda(s) \) shifts downwards (i.e. employment falls in skilled jobs), this will be more than compensated for by the increased return to skill at automated jobs. Assumption 1 and continuity then predict the existence of a region for which \( w'(s)/w(s) \) is increasing, with the lower bound of this regional potentially equal to \( s^* \) but in all cases smaller than \( \bar{s} \).

Long-run effects of automation will depend critically on the behavior of the rental rate \( r \), the magnitude of the elasticity of substitution \( \rho \), and on how \( t^*(j, s, K) \) varies with \( s \).
I impose the condition, consistent with the empirical results shown previously, that \( t^* \) is increasing in \( s \).

**Assumption 7:** \( t^*(j, s, K) \) is increasing in \( s \).

A general characterization is not possible due to the lack of a closed-form solution to (24) and (25), but results can be derived for the two cases \( \rho \in \{0, 1\} \), which are helpful for developing intuition as to the implications of automation in this environment. First I define \( \tau = \sup \{r | s^* = 0\} \) to be the highest rental rate consistent with ‘full’ automation. In the comparatively simple case where \( \rho = 0 \), declines in the rental rate below \( \tau \) will have no effect on labor demand or on wages and we can show that:

**Theorem 2** (long-run effects of automation: \( \rho = 0 \)). If \( r \leq \tau \) and \( \rho = 0 \), then relative to the case where \( r \to +\infty \), \( w'(s)/w(s) \) will be greater for all \( s \) and for any \( j' \in (0, 1) \) we will have \( \int_0^{j'} L(j)ds \) smaller. In this case the wage and matching functions will be independent of \( r \in [0, \tau] \).

The perfect complements case yields an intuitive outcome: low-skill automation reduces employment at low-skill jobs and increases wage inequality. This is consistent with standard notions of labor-substituting automation, and notwithstanding the distinction between short- and long-run effects of automation, this may be said to be the case where the model’s
predictions are closest to those of the one task-one job environment studied in the extant literature.

**Figure 3.8:** Long-run effects of automation, $\rho = 0$

**Figure notes.** Linear task shares where $\gamma_l(s) = \exp(.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = .5$, and $\rho \approx 0$. Changes are relative to the case where $s^* = \bar{s}$; wages are demeaned prior to calculating change.

Long-run effects of automation become considerably more complicated as we move away from the limiting case where $\rho = 0$, because there are two offsetting effects. On the one hand automation increases the intensity of the skilled task and therefore tends to raise wages of skilled workers, leading to substitution away from skilled occupations. On the other hand the cost of automation is smaller relative to wage costs for higher $s$, resulting in substitution in the opposite direction. The result is that while wage inequality will tend to increase overall, the implications for employment are ambiguous.

**Theorem 3** (long-run effects of automation: $\rho = 1$). *If $r \leq \tau$ and $\rho = 1$, then for any $s$ we will have $w(s)/w(\bar{s})$ greater under automation. If $r = 0$ then there will exist a $j' > 0$ such that $\int_0^{j'} L(j)ds$ is also greater under automation. Finally, comparing any two rental rates $r' < r''$, we will have $w'(s)/w(s)$ smaller under $r'$ and, for any $j' \in (0,1)$ we will have $\int_0^{j'} L(j)ds$ larger.*

When $r = \tau$ the effect of automation on employment is unclear, but further declines in $r$ will tend to both reduce wage inequality and increase low-skill employment, and as $r$ goes to zero the overall effect will be to raise employment in low-skill occupations. Hence technological disemployment will depend critically on the long-run behavior of $r$ and the value of $\rho$. 

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Figure 3.9: Long-run effects of automation, $\rho = 1$

Figure notes. Linear task shares where $\gamma_l(s) = \exp(0.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = 0.5$, and $\rho = 1$. Changes are relative to the case where $s^* = \bar{s}$; wages are demeaned prior to calculating change.

For values of $\rho$ greater than one characterization of the wage and matching functions becomes difficult. In general, larger values of $\rho$ and smaller values of $r$ will be associated with greater compression of the wage distribution and more employment in low-skill occupations. The extent to which automation affects wages rather than prices will also depend on the functional forms of $\gamma_l$ and $\gamma_h$, which together with task shares $\alpha$ determine how easy it is to substitute one skill type for another at a given job, and hence the response of occupational labor supply.

3.3.4 Continuous Model

The model developed thus far is of limited quantitative usefulness, as empirical adoption patterns are not binary at the occupation level, and consequently there is no empirical analogue of the automation threshold $s^*$. Intra-occupational heterogeneity must be taken into account if the model is to yield quantitatively meaningful predictions. Adding such heterogeneity will tend to weaken the theoretical results shown in the previous section, but they will hold approximately so long as the proportion of jobs automated is increasing in $j$. In this section I allow for a simple form of intra-occupation heterogeneity, and derive the equilibrium for a continuous model that will form the basis for the quantitative results shown below.

I assume that producers face idiosyncratic shocks $\epsilon$ to their rental cost, with total costs
equal to $\epsilon r$ per unit of capital. I further assume that these shocks are realized only after hiring labor, a simplification which has the implication that all $j$-firms will continue to hire the same worker type. I assume that $\epsilon$ is drawn from a continuously differentiable distribution $G(\epsilon)$, and that producers are constrained to a unit of output, so that $G(\epsilon)$ also gives the proportion of output attributable to producers with costs $\epsilon$ or lower.\footnote{Because firms will now enjoy greater or lesser rents depending on the value of $\epsilon$, some restriction on scale is required for the equilibrium to be well-defined.} I assume that workers are paid wages upon hiring and hence, firms have no incentive to leave the market upon discovering their rental costs as they can always choose not to automate and still receive positive revenue. Capital costs paid by automating firms will then be

$$
\epsilon r K^*(j, s, z) = \kappa \frac{1 - \alpha(j)}{z} y^*(j, s, K),
$$

and producers will automate whenever

$$
w(s) > \epsilon r \gamma_l(s).
$$

I assume that $G$ has full support over $\mathbb{R}^{++}$, in which case for each $s$ there will exist a $\epsilon^*(s)$ such that producers employing $s$ automate whenever $\epsilon > \epsilon^*(s)$. The policy functions and optimal assignment are similar to before and so I provide them in the appendix but omit them here.

Although equilibrium in the continuous model is more difficult to characterize, two general predictions are preserved from static case. The first prediction is that $\epsilon^*(s)$ is increasing: a larger proportion of jobs will be automated in high-skill occupations. The reason for this is the same as before, that skilled workers have a comparative advantage at skilled tasks and therefore it is more costly when they must spend time on the unskilled task. The second prediction is that, holding $r$ fixed, an increase in the automation threshold $\epsilon^*$ will tend to reduce occupational employment; whereas holding $\epsilon^*$ fixed, a decrease in $r$ will tend to increase employment in occupations with a high $\epsilon^*$. A lower cost of capital will tend to reduce expected costs in highly automated occupations and incentivize job creation, whereas an increase in the proportion of jobs being automated will result in less demand for labor.
3.4 Qualitative Analysis

In this section I test the main qualitative predictions of model developed above, one concerning the cross-sectional distribution of technology adoption, and the other the effects of declining capital costs on employment. Although the model also delivers predictions regarding wages, these are difficult to test retrospectively, because in the first place they cannot be precisely characterized without restricting the model parameters, and in the second, wage data in the BIBB surveys are highly aggregated prior to the 2005/06 survey. I therefore defer consideration of the model’s wage predictions until the quantitative section of this paper.

3.4.1 Time Path of Automation

The first qualitative prediction of the model is that automation of a low-skill task is “top-down”, and at any given point in time is more likely to take place in high-skill occupations. In terms of model primitives:

**Prediction #1**: the automated proportion of jobs $G(\epsilon^*(s))$ is increasing in $s$. 

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**Figure 3.10**: Short-run effects of automation, continuous model

**Figure Notes**: Linear task shares where $\gamma_l(s) = \exp(0.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = 0.5$, $\rho = 0.5$, and $\epsilon$ log-linear with mean 0 and standard deviation 0.1. Changes are relative to the case where $s^* = \bar{s}$; wages are demeaned prior to calculating change.

Hence as before, the effect of a change in $r$ on $j$-employment will depend on the relative magnitude of these two effects.
This prediction is straightforward to test in the context of the BIBB surveys. Numerous case studies suggest that computers reduce the need for labor at low-skill tasks, for example by facilitating automation of manual tasks or allowing for faster execution of simple calculation and data manipulation tasks. To analyze computer adoption patterns, I calculate for each KLDB occupation the percentage of workers using a computer on the job and the mean log wage associated with the occupation. Taking a similar approach to Acemoglu and Autor (2011), I use wage data to construct a time-invariant ranking of occupations into percentiles, allowing for a more straightforward comparison across survey years. Results are shown in figure 3.11, and are consistent with model predictions. Between 1979 and 1999, PC adoption occurs primarily in high-paying and presumably high-skill occupations. By 1999 adoption rates are close to 100% in the highest-paying quartile of jobs, and only during the years after 1999 is there substantial PC adoption in the lower half of the occupation wage distribution.

One concern with these results is that, as a general purpose technology, PCs may substitute for labor at a variety of tasks, and do not necessarily reflect a distinct technological innovation as assumed in the model. I therefore consider two use cases for PCs - CNC machining in production jobs, and word processing for clerical tasks - for which data is available during the survey years 1986-1999. These applications are not relevant for all jobs, and so I therefore omit observations for which workers do not report using machinery (computer-controlled or not) and typing equipment (word processors or typewriters). Figure 3.12 shows

\[ \text{Figure 3.11: Computer use by occupation wage percentile} \]

\textbf{Figure notes.} Log wage and PC use averaged by 1988 KLDB occupation for 1979-1999, and by 1992 KLDB occupation for 1999 (left panel) and 1999-2018 (right panel). Percentiles are time-invariant and reflect 1979 and 1999 wages. Shaded regions indicate 95% confidence intervals.
similar adoption patterns to that for PCs as a whole, with early-stage use concentrated in high-paying jobs. A caveat is that most adoption of word processors occurs between two panels in the 1990’s, and consequently I do not observe intermediate levels of adoption.

![Figure 3.12: Technology penetration by wage percentile](image)

**Figure notes.** First panel: use of CNC machines as percent of workers who report using heavy machinery. Second panel: use of word processors as percent of workers who report using typing equipment. Occupation-mean wages and weights calculated from equipment-using subsample. Shaded regions indicate 95% confidence intervals.

Summarizing these results, PC adoption over 1979-2018 exhibited a top-down pattern consistent with model predictions. While intuitive, this result is not explained by models of computerization based on capital-skill complementarity, which abstract from job-level interactions of labor and technology. It is also inconsistent with the simple labor-substitution framework of Acemoglu and Autor, in which technology adoption is associated with low-skill or routine jobs - which, given the empirical results shown in this paper, would suggest a “bottom-up” pattern of automation. The inability of extant models to predict adoption patterns is important, because these patterns can influence wage and employment trends irrespective of the long-run effects of the technology. In other words, the skill-bias of technological change at any point in time is likely to reflect both adoption patterns and the fixed characteristics of the technology, and disentangling these two factors is critical for predicting the long-run effects on labor markets.
3.4.2 Employment Effects of Automation

The second prediction of the model developed in this paper is that automation reduces contemporaneous employment, but will tend to increase employment as the cost of the technology declines further.

**Prediction #2**: employment is decreasing in $G(\epsilon^*(s))$ given $r$, and decreasing in $r$ for occupations with relatively high $G(\epsilon^*(s))$.

I test this prediction through a difference-in-difference approach relating occupational growth rates to levels of, and changes to, occupational PC use. A key limitation is that the BIBB surveys are not designed to measure occupational employment. Although the surveys are randomly sampled, non-response rates are not random and in addition, sample sizes are not large enough to ensure precise measures at the 3-digit occupational level. Survey weights correct for demographics but not occupation, and I find that in practice the weights introduce additional noise when calculating occupation growth rates. I therefore opt for the simplest approach and use raw counts to calculate occupational employment. Results for the pooled panel regressions are shown in table C.7, and indicate that high levels of PC use are associated with employment growth, while increases in the rate of PC are generally accompanied by declining employment. This pattern is robust to controlling for occupation-mean wage. Inclusion of occupation fixed effects results in a somewhat stronger pattern for the years 1979-1999, but over the 1999-2018 period PC use contains no additional information over the occupational effects, which may reflect the fact that there is less cross-sectional and time variation in PC use after 2006.

To further assess the robustness of these results, in table 3.14 I show the estimated coefficients when considering only consecutive survey panels. The predicted employment patterns are present in four out of six regressions. Results are insignificant for the remaining two regressions, one of which (1992-1999) is compromised by changes to the wording and format of the survey question concerning PC use. As in the previous table, the magnitude of the coefficients is large: a 10% point increase in PC use is associated initially with a

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74Future plans for this project include the use of administrative employment data to calculate occupational wage and employment data.

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Table 3.13: Regression: change in log occupational employment share

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Regression coefficients</th>
<th>Years 1979-1999</th>
<th>Years 1999-2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC use</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.268</td>
<td>.226</td>
<td>.497</td>
</tr>
<tr>
<td></td>
<td>(.077)</td>
<td>(.085)</td>
<td>(.174)</td>
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<tr>
<td>△ PC use</td>
<td>-.462</td>
<td>-.472</td>
<td>-.520</td>
</tr>
<tr>
<td></td>
<td>(.186)</td>
<td>(.185)</td>
<td>(.192)</td>
</tr>
<tr>
<td>Log(wage)</td>
<td>.079</td>
<td>-.064</td>
<td>.223</td>
</tr>
<tr>
<td></td>
<td>(.085)</td>
<td>(.272)</td>
<td>(.084)</td>
</tr>
<tr>
<td>Observations</td>
<td>830</td>
<td>830</td>
<td>830</td>
</tr>
</tbody>
</table>

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable. Employment shares calculated from raw survey counts. All regressions include year fixed effects.

2-7% decline in occupational employment share, while a pre-existing PC use rate of 10% is associated with a 2-7% increase in share.

Table 3.14: Regression: change in log occupational employment share, by year

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PC use</td>
<td>.681</td>
<td>.205</td>
<td>.128</td>
<td>.704</td>
<td>-.005</td>
<td>.530</td>
</tr>
<tr>
<td></td>
<td>(.183)</td>
<td>(.099)</td>
<td>(.099)</td>
<td>(.135)</td>
<td>(.120)</td>
<td>(.170)</td>
</tr>
<tr>
<td>△ PC use</td>
<td>-1.396</td>
<td>-.470</td>
<td>.097</td>
<td>-.380</td>
<td>.135</td>
<td>-.635</td>
</tr>
<tr>
<td></td>
<td>(.392)</td>
<td>(.195)</td>
<td>(.231)</td>
<td>(.174)</td>
<td>(.222)</td>
<td>(.244)</td>
</tr>
<tr>
<td>Observations</td>
<td>277</td>
<td>277</td>
<td>276</td>
<td>291</td>
<td>292</td>
<td>291</td>
</tr>
</tbody>
</table>

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable. Employment shares calculated from raw survey counts.

These results suggest an intuitive explanation for why PCs have, in various cases, both complemented and substituted for occupational labor. Past studies such as Autor, Levy, and Murnane (2002) have made note of these disparate effects, and attributed them to fundamental (i.e. unexplained) differences in how technology interacts with different skill types, but the patterns shown in table C.7 are present even if one restricts attention to skilled occupations. There is no mystery if only portions of jobs are automated, because this naturally introduces a distinction between the marginal (labor-substituting) and the long-run (labor-complementing) effects of automation.

75 Tabulated results for high-wage occupations are provided in the appendix, and are qualitatively and quantitatively similar to those for the full sample.
3.5 Quantitative Analysis

In this section I estimate the continuous model of partial automation developed above, and derive long-run predictions for wages and employment. I begin with an overview of the estimation procedure. Next I consider the model’s long-run predictions regarding the wage and employment distributions. I close the section with results characterizing the capital-labor and labor-labor elasticities of substitution in this environment.

3.5.1 Estimation

Estimation is performed on the 2017/18 survey panel. As a first step, I reduce the dimensionality of the task space from 6 task variables (i.e. those used in the empirical analysis) to 2 variables, using a factor analysis approach. Of the two composite variables, one loads principally on the routine task characteristics and the other on the non-routine characteristics. For each observation I divide factor scores by their sum and take the resulting values as measures of workers’ time allocation $t^*$ and $1 - t^*$.

I assume that the task production functions $\gamma_l(s)$ and $\gamma_h(s)$ are exponential functions taking the form $\gamma_k(s) = \exp(G(s)\gamma_k)$ with the normalization $\gamma_l = .5$. Once a value for $\gamma_h$ is fixed, task shares can be estimated by solving the worker’s time allocation policy function $t^*(j, s, K)$ for $\alpha(j)$, taking the expectation over all workers in $j$, substituting empirical values for $t^*$ and the percentage of workers using PCs $G(\epsilon^*(s))$, and then numerically solving the resulting quadratic. The empirical values of $t^*$ and $G(\epsilon^*(s))$ are averaged by occupation wage percentile and are generally increasing but, in order to ensure that equilibrium monotonicity conditions are met, I interpolate the empirical values subject to a non-decreasing constraint. The return to skill function $G(s)$ is then estimated by imposing the normalization $\lambda(s) = s$, solving the differential equation describing the wage function for $G'(s)$, and solving this system using empirical wages. The empirical and predicted distributions are shown for reference in figure 3.15.

The aggregate model parameters are estimated by using an indirect inference approach.

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 Factor analysis is appropriate in this case because of the assumption of an underlying two-task structure; by preserving only the off-diagonal elements of the covariance matrix, factor analysis effectively ignores the idiosyncratic variance associated with individual survey questions.
The distribution $G(\epsilon)$ is assumed to be log-normal with standard deviation $\sigma$, and values of $\{r, \kappa, \sigma, \rho, \gamma_h/\gamma_l\}$ are obtained by minimizing the distance between empirical and model-predicted outcomes. The technological parameters $r$ and $\sigma$ are chosen to minimize the (squared) distance between the empirical and model-predicted distributions of PC use by wage percentile $G(\epsilon^*(s))$. To estimate $\kappa$, I match the empirical and model-predicted coefficients from a diff-in-diff regression of occupation-mean skilled task share $E[t^*|s]$ on PC use $G(\epsilon^*(s))$. The elasticity of demand $\rho$ can only be obtained by comparing outcomes over time, so I implement a second diff-in-diff regression of occupational employment shares on (1) PC use and (2) the time trend in PC use, and minimize the distance between the predicted and empirical coefficients on PC use. For each of the two regressions, I use data from the 2006-2018 surveys as they are identical in terms of questions and coding. Finally, the task skill differential $\gamma_h$ is not separately identified from task shares $\alpha(j)$ in cross-sectional data, but will determine the wage response to automation. I therefore estimate $\gamma_h$ by matching predicted and observed changes to the 90/10 wage ratio over 1979-2018.77 Estimates are given in table B.10.

The most problematic aspect of model estimation is identification of $\rho$ and $\gamma_h$. Changes over time to employment shares and wages are likely to reflect a number of factors, some of

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77Wage data for the 1979 survey are aggregated into bins, and so prior to calculations I perform similar aggregation on the wage data from the 2018 survey after adjusting for purchasing power using data on purchasing power comparisons of historical monetary amounts obtained from Deutsche Bundesbank. Wage ratios are calculated from occupation-mean log wage, and the model-predicted change is obtained by finding the rental rate for which aggregate PC adoption is equal to the observed 1979 value.
Table 3.16: Aggregate parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Obj. function</th>
<th>Survey years</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>rental rate</td>
<td>5.40</td>
<td>((G^<em>(s) - \hat{G}^</em>(s))^2)</td>
<td>2018</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>std. dev. G((\epsilon))</td>
<td>.157</td>
<td>((G^<em>(s) - \hat{G}^</em>(s))^2)</td>
<td>2018</td>
</tr>
<tr>
<td>(\rho)</td>
<td>elasticity of subs.</td>
<td>1.72</td>
<td>((\beta^L_G - \hat{\beta}^L_G)^2)</td>
<td>2005-18</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>automation feas.</td>
<td>.44</td>
<td>((\beta^t_G - \hat{\beta}^t_G)^2)</td>
<td>2005-18</td>
</tr>
<tr>
<td>(\gamma_h/\gamma_l)</td>
<td>task skill diff.</td>
<td>2.82</td>
<td>((\Delta WR_{90}^{10} - \Delta \hat{WR}_{90}^{10})^2)</td>
<td>1979, 2018</td>
</tr>
</tbody>
</table>

which may also be associated with PC use. The value of 1.72 is high relative to the “best guess” range of 1.4-1.5 proposed by Johnson (1997) for demand elasticity of substitution across skill types, but here the estimation procedure is influenced by other parameter values (in particular \(\kappa\) and \(\gamma\)) and interpretation of this parameter is therefore difficult. The value of \(\gamma_h\) determines the elasticity of labor supply, and is similarly unconstrained by past studies. I therefore consider alternative values of \(\rho\) and \(\gamma_h\) later in this section. For similar reasons the estimation of \(\kappa\) is also potentially problematic, but as \(\kappa\) is primarily a scaling parameter, over- or under-estimation should not bias comparison between different points in time (i.e. different values of \(r\)) that will be the focus of the analysis in this section.

3.5.2 Employment, Wages, and Technology Adoption

The model-predicted wage and employment distributions are shown in figure 3.17. The second panel shows how automation exerts a staggered effect on labor shares: employment declines initially in high-skill occupations, followed by middle-skill and finally low-skill occupations. Wages initially polarize, with the 90/50 ratio increasing at a faster rate than the 50/10 rate. As technology adoption approaches 100% for all \(j\), polarization gives way to a general increase in wage dispersion. The main implication of partial automation, however, is the trajectory of wages and employment after the technology has been everywhere adopted. As \(r\) approaches zero, employment increases in low-skill jobs and wage inequality declines. The model predicts that employment in low-skill occupations ultimately increases, although automation tends also to increase the skill-intensity of these jobs in that workers spend a greater proportion of their time on the skilled task.

The distributional effects of automation are shown in greater detail in figure 3.18. Wage
Figure 3.17: Technology adoption and labor outcome

Figure notes. Smoothed model output over a grid of rental rates $r$. Occupational groups correspond to the $j$-intervals $[0, 1/3)$, $[1/3, 2/3)$, and $[2/3, 1]$.

Variance peaks as PC adoption approaches 100% (i.e. $r$ approaches $\bar{r}$), and then declines slightly with further decreases in $r$. The small magnitude of this decline reflects the fact that the wage effects of automation are largely associated with the intensive task margin: automation increases the return to skill within a given automation because it results in workers spending a greater proportion of their time on the skilled task. The extensive margin - changes to the sorting of workers across jobs - is less important from a wage standpoint, with the caveat that job transitions in this environment are friction-less. If there are costs to switching jobs, for example due to non-transferable human capital or to search frictions, then the extensive margin is likely to exert a stronger effect on wages.

Figure 3.18: Short-run and long-run distributional effects

Figure notes. The value $\bar{r}$ is defined as the highest value of $r$ for which $G(c^*(s)) \geq .99$ for all $s$. 

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Summarizing these results, the general prediction of the quantitative model is that automation of the low-skill task results in a persistent increase in wage inequality but non-persistent employment effects at the occupation level. In many respects this is a negative result. The undesirable outcome - wage inequality - is not expected to abate in the long-run, while the short-run nature of occupational disemployment suggests that policies facilitating job transitions may be counterproductive. Overall, figures 3.17 and 3.18 indicate that long-run effects of automation are important, but it is clear that the implications of this result will depend on the nature and magnitude of the labor market frictions affecting occupational choice.

3.5.3 Capital-Labor Substitution

The capital-labor elasticity of substitution $\partial \log L / \partial \log r$ is a common object of study in the skill-bias literature. A distinction is often made between technologies that are complementary with skill (i.e. “capital-skill complementarity”) and those that substitute for low-skilled labor, while Autor, Levy, and Murnane (2003) make the case that both of characteristics are true of computers. A major departure of the model studied in this paper is that whether capital and labor are gross substitutes is not the result of assumptions imposed on technology, but is endogenous to occupational task structure and the rental rate. In figure 3.19 I plot cross-price elasticity labor demand over skill types and rental rates. In the early stages of computerization, capital substitutes for skilled labor, although it tends also to increase wage inequality as skilled workers devote a greater share of their time to the skilled task, and hence the return to skill rises. For intermediate values of the rental rate, capital complements labor in occupations at the extremes of the distribution, and substitutes for those in the middle. And in the final stages of adoption, capital substitutes for low-skill labor.

What determines this elasticity? It will generally be positive when changes to $r$ have a large effect on $G(\epsilon^*(s))$ - when adoption is responsive to costs - and negative when $G(\epsilon^*(s))$ is large and relatively insensitive to the rental rate, which will be the case once the technology has been adopted in most jobs within an occupation. Note that for any given skill level, there exist rental rates for which capital is a gross substitute and rental rates for which capital complements labor. This result illustrates a main implication of task-level automation,
which is that labor outcomes will depend critically on the time frame under consideration, and therefore the time-path of capital costs will be important for predicting labor outcomes in the long-run. These outcomes will also depend on the distribution of tasks across and within occupations - a point that is likely to hold with even more force under more general specifications of the model, in which more than two tasks are allowed.

### 3.5.4 Supply and Demand Elasticities

Two key elasticities in the model are the demand elasticity of substitution $\rho$, and the elasticity of occupational labor supply that, given the functional assumptions on match production, will depend on the ratio $\gamma_h/\gamma_l$. Together, these elasticities determine the response of wages and employment to automation, and it is for this reason that they are difficult to identify in practice. Historical changes to wages and employment are likely to reflect a multitude of other factors and therefore to be unreliable as measures of the effects of automation. In this section I consider the sensitivity of model predictions to alternative values of these parameters.

I begin with the demand elasticity $\rho$, which has some parallel to the labor-labor elasticity studied by previous authors, and typically formalized as the elasticity of substitution across
skill types (typically education). Recent estimates of this elasticity include those by Ciccone and Peri (1.5, 2005) and Autor, Katz, and Kearney (1.57, 2008), and values in the neighborhood of 1.5 are common in this literature; but Gechert et al. (2021) suggest that published estimates are biased, and that the actual value of the labor-labor elasticity is as low as 1. In this paper, the elasticity of substitution across skill types is not a technological parameter but will depend on $\gamma_h/\gamma_l$ as well as $\rho$, and so it is \textit{ex ante} unclear what a plausible range for $\rho$ would be. I therefore choose the somewhat arbitrary boundary points $\rho \in \{1, 2\}$, for which short-run and long-run employment predictions are shown in figure 3.20. When $\rho$ is larger, dis-employment from automation tends to be quickly reversed, as affected occupations enjoy greater benefits from further declines in $r$. For this reason, at the point $\tau$ where adoption is complete, occupational labor shares are largely unchanged. As the rental rate continues to fall, however, low-skill occupations see growth, and this post-adoption effect is much stronger than in the case where $\rho$ takes on a smaller value. Therefore a wide range of long-run employment outcomes are defensible, given uncertainty as to the true value of $\rho$. Wage predictions, on the other hand, are unaffected by this parameter and instead reflect the non-parametric skill productivity term $H(s)$ and the task skill differential $\gamma_h/\gamma_l$, discussed next.

Turning to the task skill ratio $\gamma_h/\gamma_l$, a larger value of this ratio will imply that skilled workers have a stronger comparative advantage at high-$j$ occupations, and consequently oc-

Figure 3.20: Varying the elasticity of labor demand

\textbf{Figure notes.} The value $\tau$ is defined as the highest value of $r$ for which $G(\epsilon^*(s)) \geq .99$ for all $s$. Occupational shares are re-estimated conditional on $\rho$. 

---

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ocupational labor supply will be less elastic. For this reason wages will exhibit a stronger response to automation. To show this I plot results in figure 3.21 for two alternative parameter specifications where $\gamma_h/\gamma_l = 2$ and $\gamma_h/\gamma_l = 4$. The choice of these points is again somewhat arbitrary, but is sufficient for a demonstration of the sensitivity of model predictions. The first column of figure 3.21 shows occupational employment shares, from which we can see that a larger value of $\gamma_h$ is associated with greater divergence over time in the share of high-skill and low-skill occupations. This in turn reflects the greater increase in the relative wages of skilled workers, indicated in the second column of the figure. Skilled workers experience a larger increase in wages in this case because the comparative advantage profile is steeper, and therefore as automation increases the portion of time spent on the skilled task, the return to skill increases by a greater amount. The long-run decline in wage inequality is somewhat greater in this case: from its peak, the 90/10 wage ratio falls by approximately 15% as capital costs fall towards zero. Uncertainty about the value of $\gamma_h$ therefore suggests a wide range of plausible values for long-run employment shares, as well

![Figure 3.21: Varying the elasticity of labor supply](image)

**Figure notes.** Smoothed model output over a grid of rental rates $r$. All model parameters other than $\rho$ are re-estimated conditional on $\gamma$. 

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as some margin of error for wages, although in all cases the estimated model predicts only weak reversals of wage inequality in the long-run.

### 3.6 Conclusion

In this paper I have presented theoretical and quantitative results on the labor market effects of automation, when technology does not eliminate jobs but replaces labor at particular tasks within jobs. I show descriptively using German survey data that the vast majority of jobs contain both routine and non-routine task content, and that computerization over the 1979-2018 period was associated with intra-occupational changes in the frequency of routine content. The theoretical model developed in this paper predicts that in the short-run, automation of a low-skill task reduces employment as predicted by standard models; but in the long-run, as technology costs continue to fall, employment in low-skill occupations recovers and it is possible for long-run job growth to fully offset short-term losses. I show that the experience of West Germany is consistent with short-run model predictions concerning occupational computerization patterns and employment dynamics. A structurally estimated version of the model predicts that while increases in wage inequality associated with automation will be persistent, employment in middle-skill and low-skill jobs will recover as information technology costs continue to decline.

These results have two broad implications. First, the “traditional” effect of labor-substituting technology - dis-employment - is not an inevitable long-run outcome when automation is gradual, and jobs are not fully but only partially automated. Predictions of future occupational outcomes such as Frey and Osborne (2017) are generally based on the assumption of full automation, for which the empirical and historical support is weak. Second, because job outcomes in the immediate aftermath of automation are substantially different from job outcomes in the long-run, policies focused on worker retraining and occupational up-skilling are likely to be more costly over a long time-frame. The extent to which this is true will depend on how large and how fast are the declines in technology costs, and on the elasticity of labor supply and demand across occupations. On the other hand, policies intended to dampen job losses, such as employer subsidies, are likely to be more effective and less costly than they would be in the case of full automation.
A key shortcoming of this study is that I abstract from frictions affecting occupational labor supply, such as costs associated with retraining and re-schooling, barriers to entry such as licensure, and search frictions associated with finding a new job and, in many cases, a new employer. The effects of such frictions are unclear, as they will affect the response of both wages and employment to automation; but it is reasonable to expect that they would lead to greater variability of wages in the short-run. More broadly, the results in this paper reinforce the notion that factors influencing occupation transitions are key to understanding the quantitative effects of, and the policy trade-offs associated with automation.
References


**LIAB Linked-Employer-Employee-Data of the IAB.** This study uses the LIAB cross-sectional model 2, version 1993-2017, of the Linked-Employer-Employee Data (LIAB) from the IAB. Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently through remote data access. AKM wage estimates also provided by IAB, following the methodology of Card et al. (2013) and as detailed in Bellman et al. (2020). DOI: 10.5164/IAB.LIABQM29317.de.en.v1

**BIBB/BAuA-Employment Survey 2006.** Appendix results make use of task data from the 2006 BIBB/BAuA employment survey, made available by GESIS as a scientific use file. These data were aggregated by 3-digit KLDB 1988 occupation and subsequently merged with LIAB. DOI: 10.4232/1.11072


**Consumer Price Index of All Items in Germany,** Organization for Economic Co-operation and Development [DEUCPIALLMINMEI]. Retrieved from FRED, Federal Reserve Bank of St. Louis, on March 18, 2022.
Appendix A

Supplementary Materials For Chapter 1

A.1 Datasets and Data Preparation

A.1.1 LIAB Matched Dataset

The LIAB cross-sectional database spans the years 1993-2017 in four waves: 1993-1999, 2000-2006, 2007-2013, and 2014+. Establishments in the IAB establishment survey are linked with workers through the administrative social security data. The number of sampled establishments varies between approximately 4,000 and 15,000 per year, increasing during the 1990’s and decreasing thereafter; while the number of employed individuals is between 1.6 and 2.5 million per year, representing a (non-random) 5% sample of the German employed labor force. The IAB establishment survey is a representative sample of establishments with at least one employee subject to social security taxes as of June, with the survey taking place in the third quarter of the following year. The sample is stratified by size, industry, and state, making population weights necessary for inferences about the aggregate economy; smaller establishments, industries, and states are over-sampled. Establishments are aggregated when operating in the same industry and location and under the same ownership, and at the discretion of the interviewer. The same establishment may be interviewed in different waves and assigned a different identification number.

Data cleaning is as follows. For better comparability across individuals, and following past work, I drop part-time workers and apprentices from the sample. These observations, representing 3-5% of the total, are problematic because they introduce variation in earnings due to hours worked and job status, which are not the focus of this paper. In addition, part-time work is measured sporadically after 2011. Workers performing temporary “mini-jobs”
<table>
<thead>
<tr>
<th>Table A.1: LIAB summary statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Persons</td>
</tr>
<tr>
<td>Establishments</td>
</tr>
<tr>
<td><strong>Identified sample</strong></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Persons</td>
</tr>
<tr>
<td>Establishments</td>
</tr>
<tr>
<td>% weighted sample</td>
</tr>
</tbody>
</table>

**Table notes.** Sample consists of full-time employees (male and female) aged 20-60 in surveyed West German establishments. Person/establishment fixed effects estimated by IAB on the underlying administrative dataset following Card, Heining, Kline (2013). Daily wage in log 1995 euros, censored values imputed.

are missing from the sample altogether, as these jobs are exempt from social security taxes. Time-consistent industry and occupation codes are provided for different coding systems. I use 1988 KLDB occupation codes as these are the original codes for the largest portion of the sample period (1993-2010); and 2008 WZ industry codes as these are the most detailed and hence require the least imputation when cross-walking. To address inconsistencies and missing values, I use forward and backward imputation within job spells (occupation) and sets of observations within the same establishment (industry). Non-specific job categories (“disabled”, “rehabilitant”, and codes 971 and above) are set to missing. Forward and backward imputation are also used to populate missing education codes, where educational categories consists of a lower secondary education, a completed apprenticeship, an upper secondary education, a university degree, and a ‘missing’ category that represents 12% of the sample in the early 1990’s and 6% by the late 2010’s. Subsequent to these steps, observations are condensed by person-year by either combining job spells when they occur at the same employer and in the same occupation, or selecting the job spell associated with the greatest amount of income.

Earnings are converted to 1995 Euro values, with daily values below 10 dropped from the sample. Earnings are top-coded at social security thresholds, which can affect upwards of 10% of the sample in any given year. I therefore impute wages using a Tobit approach identical to that used by CHK, with the difference that regressions are not performed separately by
age due to sample size limitations. Regressions are performed separately by year, gender, and educational attainment, with dependent variables including age (nominal and grouped), a dummy variable for establishments with 11 or more workers, another dummy for only a single worker, employment (nominal and squared), the leave-one-out mean wage and censored percentage conditional on the establishment, the leave-one-out mean wage and censored percentage conditional on the person, average years of schooling within the establishment (years imputed following CHK), the percent of workers within the establishment with a college education, and a dummy variable for persons only observed once. Imputed values greater than 1000 are then capped at 1000.

A.1.2 Industry and Occupation Aggregations

Industry and occupational classifications are based off of the WZ 2008 and KLDB 1988 classifications, respectively. For years prior to 2008, the data provider imputes industry using extrapolation when possible and, when not, correspondence tables. This is primarily an issue for observations prior to 1999, which were recorded with a much older classification system (WZ 1973). Occupation codes after 2010 are recorded as KLDB 2010 values, for which there exists no direct correspondence with the earlier system. The data provider imputes KLDB 1988 occupation for these years but with substantial inaccuracies; comparing matches observed both before and after 2010, roughly 1/3 experience a change in occupational code, generally involving a ‘move’ to a similar occupation or from a detailed occupation to a broad N.E.C. category.

To minimize errors from recoding, I conduct the empirical analysis in this paper using aggregated industry and occupational groups. Doing so introduces a second problem: the KLDB 1988 system lacks hierarchical categories, and aggregate WZ 2008 classifications (which follow the NACE system) do an unsatisfactory job of preserving industry wage differentials. In both cases I find it preferable to aggregate classifications manually by combining neighboring industries (occupations) that exhibit similar mean establishment (person) wage effects. This is necessarily an arbitrary approach, but it is successful in producing industry and occupation classifications that are comparable over time while preserving as much as possible of the underlying wage structure.
<table>
<thead>
<tr>
<th>Industry</th>
<th>Mean person wage effect</th>
<th>Mean estab wage effect</th>
<th>WZ 2008 industry codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food mfg.</td>
<td>4.38</td>
<td>-.15</td>
<td>11-32, 161, 101-103, 107</td>
</tr>
<tr>
<td>Crafts mfg.</td>
<td>4.53</td>
<td>.01</td>
<td>310-332, 370-439</td>
</tr>
<tr>
<td>Materials mfg.</td>
<td>4.54</td>
<td>.04</td>
<td>104-106, 108-152, 162-182, 221-239, 251, 255-259, 292, 331-332, 370-390, 411-429,</td>
</tr>
<tr>
<td>Durables mfg.</td>
<td>4.67</td>
<td>.12</td>
<td>51-99, 241-245, 252-254, 264-275, 281-289</td>
</tr>
<tr>
<td>High-tech mfg.</td>
<td>4.71</td>
<td>.18</td>
<td>191-212, 261-263, 279, 290-291, 293-309</td>
</tr>
<tr>
<td>Wholesale</td>
<td>4.62</td>
<td>.01</td>
<td>461-469</td>
</tr>
<tr>
<td>Retail/transport</td>
<td>4.52</td>
<td>-.05</td>
<td>451-454, 471-532</td>
</tr>
<tr>
<td>Hospitality/temp</td>
<td>4.34</td>
<td>-.34</td>
<td>561, 563, 781-783</td>
</tr>
<tr>
<td>Personal svc.</td>
<td>4.40</td>
<td>-.18</td>
<td>472-473, 476-478, 551-559, 562, 801-822, 829, 920, 931-932, 960</td>
</tr>
<tr>
<td>Professional svc.</td>
<td>4.53</td>
<td>-.13</td>
<td>691-692, 741-774, 791-822, 829, 855-856, 862-889, 951-952</td>
</tr>
<tr>
<td>Commercial svc.</td>
<td>4.68</td>
<td>.00</td>
<td>661-683, 711, 731, 823, 841-854, 860-861, 900, 910, 941-949</td>
</tr>
<tr>
<td>Information svc.</td>
<td>4.82</td>
<td>.09</td>
<td>581-653, 701-702, 712-722, 732</td>
</tr>
</tbody>
</table>

**Table Notes.** Mean wage effects calculated first by panel, and then averaged across the four panels.

**A.1.3 Additional Mobility Statistics**
Table A.3: Aggregate occupation classifications

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Mean person wage effect</th>
<th>Mean estab wage effect</th>
<th>KLDB 1988 occupation codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanics</td>
<td>4.57</td>
<td>.10</td>
<td>71-72, 141-142, 191-211, 213-226, 263, 273-274, 284-291, 312-314, 521, 541-549, 711</td>
</tr>
<tr>
<td>Artisans</td>
<td>4.47</td>
<td>-.05</td>
<td>11-21, 51, 163, 175-177, 391-401, 451-453, 470-491, 501-513, 804, 834</td>
</tr>
<tr>
<td>Unskilled labor</td>
<td>4.28</td>
<td>-.21</td>
<td>41-44, 53, 351-358, 411, 792, 856, 901-902, 912-921, 923-937</td>
</tr>
<tr>
<td>Semi-skilled service</td>
<td>4.52</td>
<td>-.07</td>
<td>791, 793-801, 805, 814, 851-852, 854-855, 861, 864, 911</td>
</tr>
<tr>
<td>Skilled service</td>
<td>4.67</td>
<td>-.02</td>
<td>681, 683, 701-705, 753, 772, 782-783, 811, 822-833, 835-838, 842-844, 853, 857, 862-863, 875-877, 891-893, 922</td>
</tr>
<tr>
<td>Sales clerks</td>
<td>4.39</td>
<td>-.10</td>
<td>682, 684-686, 688, 706, 733-734, 773, 784, 781</td>
</tr>
<tr>
<td>Office specialists</td>
<td>4.61</td>
<td>.00</td>
<td>32, 52, 61, 283, 621-623, 625-629, 632-633 691-694, 721-722, 726, 771, 774, 802-803, 922</td>
</tr>
<tr>
<td>Technicians</td>
<td>4.84</td>
<td>.09</td>
<td>32, 52, 61, 283, 621-623, 625-629, 632-633 691-694, 721-722, 726, 771, 774, 802-803, 922</td>
</tr>
<tr>
<td>Engineers</td>
<td>5.07</td>
<td>.11</td>
<td>601-612</td>
</tr>
<tr>
<td>Managers</td>
<td>5.03</td>
<td>.05</td>
<td>687, 751-752, 761-763</td>
</tr>
<tr>
<td>Doctors</td>
<td>5.04</td>
<td>.02</td>
<td>813, 821, 841, 871-874, 881-883</td>
</tr>
</tbody>
</table>

Table notes: Mean wage effects calculated first by panel, and then averaged across the four panels.
### Table A.4: AKM Identification Statistics

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Job Entry (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-time, reweighted</td>
<td>0.171</td>
<td>0.168</td>
<td>0.150</td>
<td>0.165</td>
</tr>
<tr>
<td>Full-time, males-only</td>
<td>0.150</td>
<td>0.143</td>
<td>0.121</td>
<td>0.134</td>
</tr>
<tr>
<td>Full sample</td>
<td>0.171</td>
<td>0.168</td>
<td>0.150</td>
<td>0.165</td>
</tr>
</tbody>
</table>

|                  |         |         |         |         |
| **Job Exit (%)**  |         |         |         |         |
| Full-time, reweighted | 0.184  | 0.179  | 0.162  | 0.168  |
| Full-time, males-only | 0.166  | 0.159  | 0.135  | 0.141  |
| Full sample       | 0.184  | 0.179  | 0.162  | 0.168  |

**Source:** German linked employer-employee dataset (LIAB). **Note:** All proportions use sample weights. Job entry (exit) is new (ending) job spells as percent of total employment at continuing establishments. Re-weighted rates hold fixed the employment shares of industry-size pairs, for 12 aggregate industry groups and 5 size (employment) groups. The full sample includes part-time workers and apprentices, which introduces some inconsistencies over time.
A.2 Additional Empirical Results

A.2.1 West German wage variance

Figure A.5: West German Wage Variance, 1993-2017

Source: German linked employer-employee dataset (LIAB). Note: Variance of log daily wage, full-time West German workers.

Figure A.6: West German Wage Sorting (Correlation), 1993-2017

Source: German linked employer-employee dataset (LIAB) and IAB-provided wage effects. Note: Solid (dashed) line indicates panel (annual) value of $\text{Corr}(\pi, \phi)$.

A.2.2 East German Wage Variance

A.2.3 Is Wage Sorting Within or Between Markets?

A.2.4 Testing For Match-Based Sorting

A.2.5 Drivers of the Between-Group Trend
### Table A.7: Wage Sorting By Demographic (Correlation)

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>0.067</td>
<td>0.145</td>
<td>0.155</td>
<td>0.249</td>
</tr>
<tr>
<td>Female</td>
<td>0.104</td>
<td>0.096</td>
<td>0.041</td>
<td>0.129</td>
</tr>
<tr>
<td>Aged 20-30</td>
<td>0.051</td>
<td>0.095</td>
<td>0.061</td>
<td>0.167</td>
</tr>
<tr>
<td>Aged 31-40</td>
<td>0.129</td>
<td>0.164</td>
<td>0.142</td>
<td>0.244</td>
</tr>
<tr>
<td>Aged 41-50</td>
<td>0.168</td>
<td>0.175</td>
<td>0.165</td>
<td>0.236</td>
</tr>
<tr>
<td>Aged 51-60</td>
<td>0.153</td>
<td>0.187</td>
<td>0.148</td>
<td>0.211</td>
</tr>
<tr>
<td>Lower secondary ed.</td>
<td>0.219</td>
<td>0.194</td>
<td>0.146</td>
<td>0.266</td>
</tr>
<tr>
<td>Apprenticeship</td>
<td>0.095</td>
<td>0.120</td>
<td>0.097</td>
<td>0.203</td>
</tr>
<tr>
<td>Upper secondary ed.</td>
<td>0.108</td>
<td>0.147</td>
<td>0.095</td>
<td>0.149</td>
</tr>
<tr>
<td>University degree</td>
<td>0.180</td>
<td>0.155</td>
<td>0.153</td>
<td>0.099</td>
</tr>
<tr>
<td>Schleswig-Holstein</td>
<td>0.096</td>
<td>0.170</td>
<td>0.107</td>
<td>0.152</td>
</tr>
<tr>
<td>Hamburg</td>
<td>0.181</td>
<td>0.192</td>
<td>0.226</td>
<td>0.012</td>
</tr>
<tr>
<td>Lower Saxony</td>
<td>0.118</td>
<td>0.105</td>
<td>0.080</td>
<td>0.300</td>
</tr>
<tr>
<td>Bremen</td>
<td>0.231</td>
<td>0.209</td>
<td>0.198</td>
<td>0.302</td>
</tr>
<tr>
<td>North Rhine-Westphalia</td>
<td>0.111</td>
<td>0.167</td>
<td>0.120</td>
<td>0.210</td>
</tr>
<tr>
<td>Hesse</td>
<td>0.151</td>
<td>0.198</td>
<td>0.132</td>
<td>0.207</td>
</tr>
<tr>
<td>Rhineland-Palatinate</td>
<td>0.085</td>
<td>0.136</td>
<td>0.105</td>
<td>0.270</td>
</tr>
<tr>
<td>Wurttemberg-Baden</td>
<td>0.131</td>
<td>0.187</td>
<td>0.149</td>
<td>0.241</td>
</tr>
<tr>
<td>Bavaria</td>
<td>0.141</td>
<td>0.138</td>
<td>0.175</td>
<td>0.224</td>
</tr>
<tr>
<td>Saarland</td>
<td>0.112</td>
<td>0.044</td>
<td>0.140</td>
<td>0.209</td>
</tr>
<tr>
<td>Berlin</td>
<td>0.221</td>
<td>0.223</td>
<td>0.206</td>
<td>0.248</td>
</tr>
</tbody>
</table>

**Source:** German linked employer-employee dataset (LIAB) and IAB-provided wage effects. 
**Note:** Value shown is Cor(π, φ) where π (φ) is the person (establishment) AKM wage effect.

### Figure A.8: East German Wage Variance, 1993-2017

**Source:** German linked employer-employee dataset (LIAB). 
**Note:** Variance of log daily wage, full-time East German workers.
Table A.9: AKM Variance Decomposition, East Germany 1996-2017

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Var}(w)$</td>
<td>0.1664</td>
<td>0.1813</td>
<td>0.2076</td>
<td>0.1925</td>
</tr>
<tr>
<td>$\text{Var}(\pi)$</td>
<td>0.0780</td>
<td>0.0966</td>
<td>0.1031</td>
<td>0.1122</td>
</tr>
<tr>
<td>$\text{Var}(\phi)$</td>
<td>0.0381</td>
<td>0.0454</td>
<td>0.0582</td>
<td>0.0438</td>
</tr>
<tr>
<td>$\text{Var}(x'\beta)$</td>
<td>0.0034</td>
<td>0.0052</td>
<td>0.0050</td>
<td>0.0105</td>
</tr>
<tr>
<td>$\text{Var}(\epsilon)$</td>
<td>0.0101</td>
<td>0.0106</td>
<td>0.0121</td>
<td>0.0120</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\pi, \phi)$</td>
<td>0.0294</td>
<td>0.0266</td>
<td>0.0254</td>
<td>0.0362</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\pi, x'\beta)$</td>
<td>0.0006</td>
<td>-0.0040</td>
<td>-0.0018</td>
<td>-0.0218</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\phi, x'\beta)$</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0026</td>
<td>-0.0016</td>
</tr>
<tr>
<td>Observations</td>
<td>1,914,798</td>
<td>2,602,427</td>
<td>2,123,684</td>
<td>1,560,271</td>
</tr>
<tr>
<td>Persons</td>
<td>885,622</td>
<td>888,080</td>
<td>669,038</td>
<td>500,815</td>
</tr>
<tr>
<td>Establishments</td>
<td>6,555</td>
<td>8,175</td>
<td>8,039</td>
<td>7,183</td>
</tr>
</tbody>
</table>

Table A.10: AKM Variance Decomposition, East & West Germany 1996-2017

<table>
<thead>
<tr>
<th></th>
<th>1996-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Var}(w)$</td>
<td>0.1865</td>
<td>0.2125</td>
<td>0.2436</td>
<td>0.2383</td>
</tr>
<tr>
<td>$\text{Var}(\pi)$</td>
<td>0.1015</td>
<td>0.1204</td>
<td>0.1357</td>
<td>0.1393</td>
</tr>
<tr>
<td>$\text{Var}(\phi)$</td>
<td>0.0416</td>
<td>0.0482</td>
<td>0.0608</td>
<td>0.0448</td>
</tr>
<tr>
<td>$\text{Var}(x'\beta)$</td>
<td>0.0035</td>
<td>0.0054</td>
<td>0.0054</td>
<td>0.0127</td>
</tr>
<tr>
<td>$\text{Var}(\epsilon)$</td>
<td>0.0119</td>
<td>0.0142</td>
<td>0.0151</td>
<td>0.0173</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\pi, \phi)$</td>
<td>0.0270</td>
<td>0.0296</td>
<td>0.0316</td>
<td>0.0400</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\pi, x'\beta)$</td>
<td>0.0020</td>
<td>-0.0008</td>
<td>-0.0006</td>
<td>-0.0190</td>
</tr>
<tr>
<td>$2\times\text{Cov}(\phi, x'\beta)$</td>
<td>0.0000</td>
<td>0.0014</td>
<td>0.0020</td>
<td>0.0004</td>
</tr>
<tr>
<td>Observations</td>
<td>6,939,650</td>
<td>11,787,839</td>
<td>11,634,814</td>
<td>8,640,959</td>
</tr>
<tr>
<td>Persons</td>
<td>2,961,442</td>
<td>4,182,882</td>
<td>3,759,847</td>
<td>2,844,816</td>
</tr>
<tr>
<td>Establishments</td>
<td>13,061</td>
<td>26,687</td>
<td>28,012</td>
<td>24,863</td>
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Table A.11: Between-Group Wage Moments (% Total)

<table>
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<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cov((\pi,\phi))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry (46)</td>
<td>0.884</td>
<td>0.853</td>
<td>1.018</td>
<td>0.843</td>
</tr>
<tr>
<td>Occupation (75)</td>
<td>0.885</td>
<td>0.812</td>
<td>0.958</td>
<td>0.746</td>
</tr>
<tr>
<td>Ind. (\times) Occ</td>
<td>1.187</td>
<td>1.086</td>
<td>1.300</td>
<td>1.077</td>
</tr>
<tr>
<td>Education (5)</td>
<td>0.244</td>
<td>0.275</td>
<td>0.353</td>
<td>0.356</td>
</tr>
<tr>
<td>Occ. (\times) Educ.</td>
<td>0.962</td>
<td>0.876</td>
<td>1.022</td>
<td>0.798</td>
</tr>
<tr>
<td>Number of employees (5)</td>
<td>0.737</td>
<td>0.639</td>
<td>0.716</td>
<td>0.463</td>
</tr>
<tr>
<td>Ind. (\times) Size</td>
<td>1.068</td>
<td>1.026</td>
<td>1.246</td>
<td>0.989</td>
</tr>
<tr>
<td><strong>Var((\pi))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry (12)</td>
<td>0.081</td>
<td>0.102</td>
<td>0.112</td>
<td>0.166</td>
</tr>
<tr>
<td>Industry (46)</td>
<td>0.095</td>
<td>0.115</td>
<td>0.124</td>
<td>0.177</td>
</tr>
<tr>
<td>Occupation (15)</td>
<td>0.300</td>
<td>0.307</td>
<td>0.303</td>
<td>0.357</td>
</tr>
<tr>
<td>Occupation (75)</td>
<td>0.323</td>
<td>0.326</td>
<td>0.319</td>
<td>0.376</td>
</tr>
<tr>
<td>Ind. (\times) Occ (12(\times)15)</td>
<td>0.326</td>
<td>0.334</td>
<td>0.329</td>
<td>0.394</td>
</tr>
<tr>
<td>Ind. (\times) Occ (46(\times)75)</td>
<td>0.380</td>
<td>0.379</td>
<td>0.371</td>
<td>0.438</td>
</tr>
<tr>
<td><strong>Var((\phi))</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry (12)</td>
<td>0.296</td>
<td>0.319</td>
<td>0.323</td>
<td>0.311</td>
</tr>
<tr>
<td>Industry (46)</td>
<td>0.339</td>
<td>0.350</td>
<td>0.345</td>
<td>0.331</td>
</tr>
<tr>
<td>Occupation (15)</td>
<td>0.160</td>
<td>0.159</td>
<td>0.163</td>
<td>0.152</td>
</tr>
<tr>
<td>Occupation (75)</td>
<td>0.213</td>
<td>0.211</td>
<td>0.215</td>
<td>0.188</td>
</tr>
<tr>
<td>Ind. (\times) Occ (12(\times)15)</td>
<td>0.337</td>
<td>0.352</td>
<td>0.356</td>
<td>0.342</td>
</tr>
<tr>
<td>Ind. (\times) Occ (46(\times)75)</td>
<td>0.427</td>
<td>0.433</td>
<td>0.424</td>
<td>0.412</td>
</tr>
</tbody>
</table>

**Source:** German linked employer-employee dataset (LIAB) and IAB-provided wage effects. **Note:** Value shown is the between-group covariance divided by the total covariance.

Figure A.12: Annual Between-Group Wage Sorting, 1993-2017

**Source:** German linked employer-employee dataset (LIAB). **Note:** Solid (dashed) line indicates total (between-group) value of \(2 \times Cov(\pi, \phi)\).
Figure A.13: Wage Effects by Detailed Occupation and Industry

Source: German linked employer-employee dataset (LIAB) and IAB-provided wage effects. Note: Weighted average of AKM wage effects by 75 KLDB 1988 occupation groups and 46 WZ 2008 industry groups.

Figure A.14: Wage Sorting and Match Selection (Correlations), 2010-2017

Source: German linked employer-employee dataset (LIAB) and IAB-provided wage effects. Note: Time in workforce dated from first payment into social security.
### Table A.15: Counterfactual Between-Group Wage Sorting (Correlation)

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total between-group</td>
<td>0.473</td>
<td>0.515</td>
<td>0.551</td>
<td>0.633</td>
</tr>
<tr>
<td>Constant group $\pi$</td>
<td>0.506</td>
<td>0.534</td>
<td>0.553</td>
<td>0.587</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.517</td>
<td>0.535</td>
<td>0.549</td>
<td>0.591</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.462</td>
<td>0.512</td>
<td>0.549</td>
<td>0.632</td>
</tr>
<tr>
<td>Constant group $\phi$</td>
<td>0.481</td>
<td>0.523</td>
<td>0.557</td>
<td>0.617</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.474</td>
<td>0.525</td>
<td>0.573</td>
<td>0.628</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.514</td>
<td>0.513</td>
<td>0.506</td>
<td>0.579</td>
</tr>
<tr>
<td>Constant group $\omega$</td>
<td>0.494</td>
<td>0.524</td>
<td>0.531</td>
<td>0.605</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.508</td>
<td>0.528</td>
<td>0.537</td>
<td>0.599</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.471</td>
<td>0.511</td>
<td>0.541</td>
<td>0.638</td>
</tr>
<tr>
<td>Constant group $(\pi, \omega)$</td>
<td>0.535</td>
<td>0.545</td>
<td>0.541</td>
<td>0.563</td>
</tr>
<tr>
<td>By (occupation, industry)</td>
<td>0.493</td>
<td>0.520</td>
<td>0.538</td>
<td>0.599</td>
</tr>
<tr>
<td>By (occupation, both)</td>
<td>0.491</td>
<td>0.524</td>
<td>0.536</td>
<td>0.612</td>
</tr>
<tr>
<td>By (both, industry)</td>
<td>0.54</td>
<td>0.543</td>
<td>0.543</td>
<td>0.554</td>
</tr>
</tbody>
</table>

**Note:** Value shown is the counterfactual value of $Cor(\pi, \phi)$. Groups are 180 industry × occupation pairs. Person effects ($\pi$), establishment effects ($\phi$), and employment shares ($\omega$) are held constant at their 1993-99 and 2010-17 values, as indicated, with the average correlation reported.

### Figure A.18: Establishment Effects by Industry and Size

**Note:** Value shown is the weighted average of the establishment wage effect $\phi$. Size categories based on full-time employees.
Table A.16: Counterfactual Between-Group Wage Sorting (% Trend)

<table>
<thead>
<tr>
<th></th>
<th>1993-99 Values</th>
<th></th>
<th></th>
<th>2010-17 Values</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total between-group</td>
<td>0.361</td>
<td>0.831</td>
<td>1.000</td>
<td>0.000</td>
<td>0.361</td>
<td>0.831</td>
</tr>
<tr>
<td>Constant group (\pi)</td>
<td>0.253</td>
<td>0.590</td>
<td>0.494</td>
<td>0.313</td>
<td>0.639</td>
<td>1.108</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.253</td>
<td>0.590</td>
<td>0.518</td>
<td>0.373</td>
<td>0.675</td>
<td>1.096</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.313</td>
<td>0.711</td>
<td>0.771</td>
<td>0.096</td>
<td>0.446</td>
<td>0.916</td>
</tr>
<tr>
<td>Constant group (\phi)</td>
<td>0.241</td>
<td>0.494</td>
<td>0.964</td>
<td>0.060</td>
<td>0.301</td>
<td>0.554</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.277</td>
<td>0.639</td>
<td>1.108</td>
<td>0.012</td>
<td>0.277</td>
<td>0.578</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.169</td>
<td>0.349</td>
<td>0.554</td>
<td>0.265</td>
<td>0.482</td>
<td>0.711</td>
</tr>
<tr>
<td>Constant group (\omega)</td>
<td>0.193</td>
<td>0.386</td>
<td>0.434</td>
<td>0.482</td>
<td>0.843</td>
<td>1.072</td>
</tr>
<tr>
<td>By industry only</td>
<td>0.217</td>
<td>0.458</td>
<td>0.470</td>
<td>0.518</td>
<td>0.795</td>
<td>1.048</td>
</tr>
<tr>
<td>By occupation only</td>
<td>0.337</td>
<td>0.723</td>
<td>0.904</td>
<td>0.060</td>
<td>0.422</td>
<td>0.843</td>
</tr>
<tr>
<td>Constant group ((\pi, \omega))</td>
<td>0.108</td>
<td>0.229</td>
<td>0.060</td>
<td>0.964</td>
<td>1.181</td>
<td>1.373</td>
</tr>
<tr>
<td>By (occupation, industry)</td>
<td>0.157</td>
<td>0.361</td>
<td>0.301</td>
<td>0.627</td>
<td>0.892</td>
<td>1.157</td>
</tr>
<tr>
<td>By (occupation, both)</td>
<td>0.145</td>
<td>0.313</td>
<td>0.289</td>
<td>0.663</td>
<td>0.976</td>
<td>1.181</td>
</tr>
<tr>
<td>By (both, industry)</td>
<td>0.108</td>
<td>0.265</td>
<td>0.084</td>
<td>0.940</td>
<td>1.133</td>
<td>1.361</td>
</tr>
</tbody>
</table>

Note: Value shown is the counterfactual between-group component of \(\text{Cov}(\pi, \phi)\) divided by the total between-group trend for 1993-2017. Groups are 180 industry \times occupation pairs. Person effects \((\pi)\), establishment effects \((\phi)\), and employment shares \((\omega)\) are held constant at their 1993-99 and 2010-17 values, as indicated. For results using constant 1993-99 values, a column of zeros is omitted for 1993-99, and likewise a column of ones for 2010-17 with constant 2010-17 values.

Table A.17: Between-Group Variances

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th></th>
<th></th>
<th>1998-04</th>
<th></th>
<th></th>
<th>2003-10</th>
<th></th>
<th></th>
<th>2010-17</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Var}(\pi))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry (12)</td>
<td>0.0088</td>
<td>0.0127</td>
<td>0.0156</td>
<td>0.0235</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Occupation (15)</td>
<td>0.0326</td>
<td>0.0379</td>
<td>0.0423</td>
<td>0.0504</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ind. \times Occ</td>
<td>0.0355</td>
<td>0.0412</td>
<td>0.0459</td>
<td>0.0556</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Var}(\phi))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry (12)</td>
<td>0.0092</td>
<td>0.0121</td>
<td>0.0167</td>
<td>0.0124</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Occupation (15)</td>
<td>0.0050</td>
<td>0.0060</td>
<td>0.0084</td>
<td>0.0061</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ind. \times Occ</td>
<td>0.0104</td>
<td>0.0134</td>
<td>0.0184</td>
<td>0.0136</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: German linked employer-employee dataset (LIAB) and IAB-provided wage effects. Note: Value shown is the between-group variance.
A.3 Descriptive Evidence: Skill and Scale

A.3.1 Establishment Characteristics

A.3.2 Person Characteristics
Appendix B
<table>
<thead>
<tr>
<th></th>
<th>VAD</th>
<th>Inv</th>
<th>ICT</th>
<th>Multi</th>
<th>Barg</th>
<th>Prod</th>
<th>Proc</th>
<th>Compete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log employment</td>
<td>0.018</td>
<td>-0.36</td>
<td>0.775</td>
<td>0.117</td>
<td>0.275</td>
<td>0.745</td>
<td>0.565</td>
<td>0.858</td>
</tr>
<tr>
<td>Log value-added</td>
<td>0.837</td>
<td>0.823</td>
<td>0.732</td>
<td>0.486</td>
<td>0.881</td>
<td>0.939</td>
<td>0.169</td>
<td>0.255</td>
</tr>
<tr>
<td>Log investment/worker</td>
<td>-0.383</td>
<td>-0.391</td>
<td>0.275</td>
<td>0.834</td>
<td>0.843</td>
<td>-0.085</td>
<td>0.226</td>
<td></td>
</tr>
<tr>
<td>Log ICT inv./worker</td>
<td>0.698</td>
<td>0.150</td>
<td>0.843</td>
<td>0.838</td>
<td>0.236</td>
<td>0.049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multi-establish.</td>
<td></td>
<td>0.339</td>
<td>0.612</td>
<td>0.662</td>
<td>-0.031</td>
<td>-0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collective barg. (%)</td>
<td></td>
<td>0.297</td>
<td>0.403</td>
<td>-0.397</td>
<td>0.017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product dev. (%)</td>
<td></td>
<td>0.975</td>
<td>0.052</td>
<td>0.361</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Process impr. (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive profits (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log ICT inv./worker</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log employment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log value-added</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.19: Cross-Correlations, Establishment Characteristics
### Table A.20: Wage Effects and Establishment Characteristics, Confidence Intervals

<table>
<thead>
<tr>
<th>Variable (X)</th>
<th>Cor(X,π)</th>
<th>Cor(X,φ)</th>
<th>Cor(π,X,φ)</th>
<th>Cor(π,φ,X)</th>
<th>Cor(π,X,φ,X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log employment</td>
<td>(0.27, 0.52)</td>
<td>(0.68, 0.81)</td>
<td>(0.13, 0.4)</td>
<td>(0.24, 0.49)</td>
<td>(0.27, 0.52)</td>
</tr>
<tr>
<td>Log value-added</td>
<td>(0.34, 0.57)</td>
<td>(0.8, 0.88)</td>
<td>(0.03, 0.32)</td>
<td>(0.16, 0.42)</td>
<td>(0.2, 0.46)</td>
</tr>
<tr>
<td>Log investment/worker</td>
<td>(0.27, 0.52)</td>
<td>(0.82, 0.89)</td>
<td>(0.08, 0.36)</td>
<td>(0.27, 0.52)</td>
<td>(0.31, 0.55)</td>
</tr>
<tr>
<td>Log ICT inv./worker</td>
<td>(0.55, 0.72)</td>
<td>(0.75, 0.85)</td>
<td>(-0.12, 0.18)</td>
<td>(-0.11, 0.19)</td>
<td>(-0.09, 0.2)</td>
</tr>
<tr>
<td>Multi-estab.</td>
<td>(0.37, 0.59)</td>
<td>(0.42, 0.63)</td>
<td>(0.19, 0.45)</td>
<td>(0.2, 0.46)</td>
<td>(0.26, 0.51)</td>
</tr>
<tr>
<td>Collective barg. (%)</td>
<td>(-0.08, 0.21)</td>
<td>(0.2, 0.46)</td>
<td>(0.41, 0.62)</td>
<td>(0.45, 0.65)</td>
<td>(0.45, 0.65)</td>
</tr>
<tr>
<td>Product dev. (%)</td>
<td>(0.3, 0.54)</td>
<td>(0.72, 0.83)</td>
<td>(0.09, 0.37)</td>
<td>(0.21, 0.47)</td>
<td>(0.25, 0.5)</td>
</tr>
<tr>
<td>Process impr. (%)</td>
<td>(0.28, 0.53)</td>
<td>(0.75, 0.85)</td>
<td>(0.09, 0.37)</td>
<td>(0.23, 0.48)</td>
<td>(0.27, 0.51)</td>
</tr>
<tr>
<td>Positive profits (%)</td>
<td>(0.12, 0.39)</td>
<td>(-0.13, 0.17)</td>
<td>(0.45, 0.65)</td>
<td>(0.43, 0.64)</td>
<td>(0.45, 0.65)</td>
</tr>
<tr>
<td>Competitive mkt. (%)</td>
<td>(-0.25, 0.04)</td>
<td>(0.12, 0.4)</td>
<td>(0.47, 0.67)</td>
<td>(0.5, 0.68)</td>
<td>(0.5, 0.69)</td>
</tr>
</tbody>
</table>

**Note:** All variables first aggregated at the industry-occupation level via a first-stage regression on fixed effects for industry-occupation and year. Terms π_x and φ_x indicate residuals from a regression of wage effects on establishment characteristics. All results weighted by employment. Confidence intervals calculated using Fisher transformation, and are approximate as they do not account for survey error, error due to imputation, or error from the AKM wage effects regression.
<table>
<thead>
<tr>
<th></th>
<th>College degree (%)</th>
<th>Years of education</th>
<th>Job tenure</th>
<th>Analyze information</th>
<th>Manage others</th>
<th>Use PC</th>
<th>Buy/sell</th>
<th>Control machines</th>
<th>Weigh/measure</th>
<th>Care for others</th>
</tr>
</thead>
<tbody>
<tr>
<td>College degree (%)</td>
<td>0.976</td>
<td>-0.163</td>
<td>0.702</td>
<td>-0.070</td>
<td>0.827</td>
<td>0.703</td>
<td>0.876</td>
<td>0.876</td>
<td>0.467</td>
<td>-0.623</td>
</tr>
<tr>
<td>Years of education</td>
<td>-0.179</td>
<td>0.822</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
<td>0.747</td>
</tr>
<tr>
<td>Job tenure</td>
<td>-0.123</td>
<td>0.101</td>
<td>0.073</td>
<td>0.073</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
</tr>
<tr>
<td>Analyze information</td>
<td>0.725</td>
<td>0.717</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
<td>0.376</td>
</tr>
<tr>
<td>Manage others</td>
<td>0.621</td>
<td>0.515</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
<td>0.327</td>
</tr>
<tr>
<td>Use PC</td>
<td>0.599</td>
<td>0.599</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
<td>0.414</td>
</tr>
<tr>
<td>Buy/sell</td>
<td>0.054</td>
<td>-0.476</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
<td>0.406</td>
</tr>
<tr>
<td>Control machines</td>
<td>-0.370</td>
<td>-0.253</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
</tr>
<tr>
<td>Weigh/measure</td>
<td>-0.141</td>
<td>0.070</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
<td>-0.021</td>
</tr>
<tr>
<td>Care for others</td>
<td>0.073</td>
<td>-0.738</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Table A.21: Cross-Correlations, Person/Job Characteristics.
### Table A.22: Wage Effects and Person Characteristics, Confidence Intervals

<table>
<thead>
<tr>
<th>Variable (X)</th>
<th>Cor((X, \pi))</th>
<th>Cor((X, \phi))</th>
<th>Cor((\pi_X, \phi))</th>
<th>Cor((\pi, \phi_X))</th>
<th>Cor((\pi_X, \phi_X))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LIAB Person Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College degree (%)</td>
<td>(0.77, 0.87)</td>
<td>(0.11, 0.38)</td>
<td>(0.50, 0.68)</td>
<td>(0.22, 0.47)</td>
<td>(0.52, 0.70)</td>
</tr>
<tr>
<td>Years of education</td>
<td>(0.83, 0.90)</td>
<td>(0.13, 0.40)</td>
<td>(0.54, 0.72)</td>
<td>(0.19, 0.45)</td>
<td>(0.57, 0.74)</td>
</tr>
<tr>
<td>Job tenure</td>
<td>(0.02, 0.30)</td>
<td>(0.71, 0.83)</td>
<td>(0.30, 0.54)</td>
<td>(0.58, 0.74)</td>
<td>(0.59, 0.75)</td>
</tr>
<tr>
<td>Years in labor force</td>
<td>(0.05, 0.33)</td>
<td>(0.53, 0.71)</td>
<td>(0.31, 0.55)</td>
<td>(0.44, 0.64)</td>
<td>(0.45, 0.65)</td>
</tr>
<tr>
<td><strong>BIBB Task Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analyze information</td>
<td>(0.79, 0.88)</td>
<td>(0.22, 0.47)</td>
<td>(0.35, 0.58)</td>
<td>(0.13, 0.40)</td>
<td>(0.38, 0.60)</td>
</tr>
<tr>
<td>Use PC</td>
<td>(0.72, 0.83)</td>
<td>(0.25, 0.50)</td>
<td>(0.27, 0.52)</td>
<td>(0.13, 0.40)</td>
<td>(0.31, 0.55)</td>
</tr>
<tr>
<td>Manage others</td>
<td>(0.51, 0.70)</td>
<td>(-0.17, 0.12)</td>
<td>(0.63, 0.78)</td>
<td>(0.45, 0.66)</td>
<td>(0.63, 0.78)</td>
</tr>
<tr>
<td>Buy/sell</td>
<td>(-0.20, 0.10)</td>
<td>(-0.61, -0.39)</td>
<td>(0.41, 0.62)</td>
<td>(0.50, 0.69)</td>
<td>(0.50, 0.69)</td>
</tr>
<tr>
<td>Control machines</td>
<td>(-0.37, -0.10)</td>
<td>(0.05, 0.33)</td>
<td>(0.51, 0.70)</td>
<td>(0.50, 0.69)</td>
<td>(0.52, 0.71)</td>
</tr>
<tr>
<td>Weigh/measure</td>
<td>(-0.22, 0.07)</td>
<td>(0.03, 0.31)</td>
<td>(0.45, 0.66)</td>
<td>(0.46, 0.66)</td>
<td>(0.46, 0.66)</td>
</tr>
<tr>
<td>Care for others</td>
<td>(-0.17, 0.13)</td>
<td>(-0.28, 0.01)</td>
<td>(0.43, 0.64)</td>
<td>(0.44, 0.64)</td>
<td>(0.44, 0.64)</td>
</tr>
<tr>
<td>Clean</td>
<td>(-0.82, -0.69)</td>
<td>(-0.52, -0.28)</td>
<td>(0.23, 0.49)</td>
<td>(0.12, 0.39)</td>
<td>(0.27, 0.52)</td>
</tr>
</tbody>
</table>

**Note:** All variables first aggregated at the industry-occupation level via a first-stage regression. Years of education is imputed following Card et al. (2013). Task values imputed from verbal frequencies. Terms \(\pi_x\) and \(\phi_x\) indicate residuals from a regression of wage effects on job characteristics. All results weighted by employment; see appendix for confidence intervals. Confidence are intervals calculated using Fisher transformation, and are approximate as they do not account for survey error, error due to imputation, or error from the AKM wage effects regression.

### Table A.23: Wage Effects and Job Characteristics, Additional Tasks

<table>
<thead>
<tr>
<th>Variable (X)</th>
<th>Cor((X, \pi))</th>
<th>Cor((X, \phi))</th>
<th>Cor((\pi_X, \phi))</th>
<th>Cor((\pi, \phi_X))</th>
<th>Cor((\pi_X, \phi_X))</th>
</tr>
</thead>
<tbody>
<tr>
<td>No control</td>
<td>0.548</td>
<td>0.548</td>
<td>0.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produce goods</td>
<td>-0.316</td>
<td>0.138</td>
<td>0.623</td>
<td>0.597</td>
<td>0.629</td>
</tr>
<tr>
<td>Repair</td>
<td>-0.253</td>
<td>0.076</td>
<td>0.586</td>
<td>0.569</td>
<td>0.588</td>
</tr>
<tr>
<td>Transport goods</td>
<td>-0.619</td>
<td>-0.266</td>
<td>0.488</td>
<td>0.397</td>
<td>0.506</td>
</tr>
<tr>
<td>PR</td>
<td>0.481</td>
<td>-0.245</td>
<td>0.759</td>
<td>0.687</td>
<td>0.783</td>
</tr>
<tr>
<td>Research/design</td>
<td>0.723</td>
<td>0.455</td>
<td>0.317</td>
<td>0.246</td>
<td>0.356</td>
</tr>
<tr>
<td>Teach others</td>
<td>0.598</td>
<td>0.158</td>
<td>0.565</td>
<td>0.459</td>
<td>0.573</td>
</tr>
<tr>
<td>Advise others</td>
<td>0.599</td>
<td>-0.014</td>
<td>0.694</td>
<td>0.556</td>
<td>0.694</td>
</tr>
<tr>
<td>Care for</td>
<td>-0.273</td>
<td>-0.632</td>
<td>0.390</td>
<td>0.484</td>
<td>0.503</td>
</tr>
<tr>
<td>Protect</td>
<td>-0.137</td>
<td>0.075</td>
<td>0.563</td>
<td>0.559</td>
<td>0.565</td>
</tr>
</tbody>
</table>

**Note:** All variables first aggregated at the industry-occupation level via a first-stage regression. Years of education is imputed following Card et al. (2013). Task values imputed from verbal frequencies. Terms \(\pi_x\) and \(\phi_x\) indicate residuals from a regression of wage effects on job characteristics. All results weighted by employment; see appendix for confidence intervals.
**Table A.24: BIBB Task Descriptions**

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyze information</td>
<td>Gather information, research, document</td>
</tr>
<tr>
<td>Use PC</td>
<td>Work with computers</td>
</tr>
<tr>
<td>Manage others</td>
<td>Organize, plan, prepare work processes for others</td>
</tr>
<tr>
<td>Buy/sell</td>
<td>Buy, procure, sell</td>
</tr>
<tr>
<td>Control machines</td>
<td>Monitor, control machines, systems, technical processes</td>
</tr>
<tr>
<td>Weigh/measure</td>
<td>Measure, check, perform quality control</td>
</tr>
<tr>
<td>Care for others</td>
<td>Nurture, take care of, heal</td>
</tr>
<tr>
<td>Clean</td>
<td>Clean, dispose of waste, recycle</td>
</tr>
<tr>
<td>Produce goods</td>
<td>Produce goods and wares</td>
</tr>
<tr>
<td>Repair</td>
<td>Repair, install</td>
</tr>
<tr>
<td>Transport goods</td>
<td>Transport, store, ship</td>
</tr>
<tr>
<td>PR</td>
<td>Advertising, marketing, public relations</td>
</tr>
<tr>
<td>Research/design</td>
<td>Develop, research, create</td>
</tr>
<tr>
<td>Teach</td>
<td>Educate, teach</td>
</tr>
<tr>
<td>Advise</td>
<td>Advise and inform</td>
</tr>
<tr>
<td>Accommodate</td>
<td>Host, accommodate, prepare meals</td>
</tr>
<tr>
<td>Protect</td>
<td>Secure, protect, guard, monitor, regulate traffic</td>
</tr>
</tbody>
</table>
Supplementary Materials For Chapter 2

B.1 Proofs of Main Theoretical Results

B.1.1 UI and Efficiency (Informal)

In this section I take as fixed worker assignment and suppress $j$-notation. In addition I set
\[ \psi = 1, \]
which simplifies but does not substantially change the proof.

Writing the worker’s flow value of unemployment as
\[
U(s) = \frac{(\rho + \delta)B(i,s)\psi(i,s)\eta}{\rho + \delta + \zeta \theta(i,s)\eta}w(i,s)
\]
\[ \equiv G(i,s)w(i,s), \]
and defining the elasticity of $G$ with respect to $\theta$ as $\epsilon_G(i,s)$ (which may depend on $\theta(i,s)$),
we can show that total $(i,s)$-vacancies satisfy

\[
\frac{\epsilon_G(i,s)}{\epsilon_G(i,s) + \psi(1-\eta)} \sigma
\]

Fixing any two labor types $s$ and $s'$, the vacancy ratio $\frac{V(i,s')}{V(i,s)}$ will be equal to

\[
\frac{\alpha(i,s')C(i,s')}{\alpha(i,s)C(i,s')} \sigma \left( \frac{[\theta(i,s')\eta^{-1}]^{-1} - \frac{\epsilon_G(i,s')}{\epsilon_G(i,s) + \psi(1-\eta)}}{[\theta(i,s)\eta^{-1}]^{-1} - \frac{\epsilon_G(i,s)}{\epsilon_G(i,s) + \psi(1-\eta)}} \right)^\sigma.
\]

Efficiency requires that the term in parenthesis be independent of $i$, which will only be the case when $\epsilon_G(i,s)$ is a constant.

To see this, consider a central planner that directly allocates vacancies and job applicants across submarkets in order to maximize the utility of the representative household. I simplify
by assuming $C(i, j) = C(i)$. Writing the planner’s Bellman equation:

$$\rho S^V(N, Y) = \left( \sum_i \beta(i) Y(i) \right)^{\frac{\eta}{\tau + 1}} +$$

$$\max_{\phi, v} \left[ \left( \frac{\delta[k(s) - N(s)]}{N(s)} - \sum_i \zeta v(i, s)^{\eta}(N(s)\phi(i, s))^{1-\eta} \right) \frac{\partial V}{\partial N(s)} - \sum_i \int C(i)v(i, s) + \right.$$  

$$\left. \left( \int \alpha(i, s) \left( \zeta v(i, s)^{\eta}(N(s)\phi(i, s))^{1-\eta} m(s) \right)^{1-\eta} ds \right)^{\frac{\eta}{\eta + 1}} - \delta Y(i) \right] \frac{\partial V}{\partial Y(i)} \right].$$

As the planner is free to shift workers across $i$, we can define a shadow price $\omega(s)$, and by taking the first-order conditions of the planner’s problem we may derive the following condition for market tightness:

$$\theta^P(i, s) = \frac{\eta}{1 - \eta} \frac{\omega(s)}{C(i)N(s)}.$$

At the planner’s solution, market tightness is multiplicatively separable in industry and worker type. But in the market equilibrium, $\theta$ will only be separable if $\epsilon_G$ is a constant. More explicitly, if we define $H(\theta) = \frac{G(i, s)}{\zeta \theta^{\eta - 1} \epsilon_G(i, s)}$ we can show that

$$\theta^M(i, s) = H^{-1} \left( \frac{U(s)}{(1 - \eta)(\rho + \delta)C(i)} \right).$$

For this to be separable it must be that $H$ has a constant elasticity with respect to market tightness, which in turn implies the same for $G$ and hence that $\epsilon_G$ is constant.

Supposing then that $\epsilon_G(i, s) = D$ for some constant $D$, the market equilibrium will result in market tightness equal to

$$\theta^M(i, s) = \frac{\sum_k V^M_k(s)C(k)^{\frac{1}{\tau + 1 - \eta}}}{N(s)C(i)^{\frac{1}{\tau + 1 - \eta}}} ,$$

and search probabilities will take the form

$$\frac{\phi^M(i', s)}{\phi^M(i, s)} = \frac{V(i')^M(s)C(i')^{\frac{1}{\tau + 1 - \eta}}}{V(i)^M(s)C(i)^{\frac{1}{\tau + 1 - \eta}}}.$$
From here it is straightforward to show that by setting \( D = \eta - 1 + 1/\kappa \) we arrive at the functional form for \( B(i, s) \) given in assumption 3 of the main text. But from the planner’s problem we have that

\[
\frac{\phi^P(i', s)}{\phi^P(i, s)} = \frac{V(i')^P(s)C(i')}{V(i)^P(s)C(i)}.
\]

Therefore we must have \( D = \eta \) at the efficient allocation, or correspondingly \( \kappa = 1 \).

**B.1.2 Occupational Assignment (Proposition 1)**

This proof is an adaptation of that given by Costinot and Vogel (2010) to the case of search frictions and firm heterogeneity. I begin with the following notation:

- let \( \omega(s) \) denote the set of jobs chosen by skill type \( s \) in at least one industry: \( \omega(s) = \{j \mid \exists i \text{ s.t. } \phi^*(i, j, s) > 0\} \).
- let \( \mu(j) \) denote the set of skill types \( s \) that choose job \( j \) in at least one industry: \( \mu(j) = \{s \mid \exists i \text{ s.t. } \phi^*(i, j, s) > 0\} \).

Lemma 1 establishes the correspondence between skill and occupation, and lemmas 2 and 3 the differential equations characterizing optimal assignment.

**Lemma 1.** There exists a continuous and strictly increasing function \( \lambda : [\underline{s}, \bar{s}] \to [0, 1] \), independent of \( i \), such that \( \phi^*(i, j, s) > 0 \) if and only if \( \lambda(s) = j \), and where \( \lambda(\underline{s}) = 0 \) and \( \lambda(\bar{s}) = 1 \).

**Proof.** That \( \omega(s) \) is non-empty follows from \( N(s) > 0 \), which will be true if \( \delta > 0 \) and \( \nu(s) > 0 \), which I assume. Because all workers are assumed to search we must have \( \phi^*(i, j, s) > 0 \) for at least one \( (i, j) \) pair.

Regarding non-emptiness of \( \mu(j) \) suppose that \( \mu(j) \) is empty for some \( j \). By assumption we have \( \alpha_i(j) > 0 \), and so it must be that if \( \omega(s) \) is non-empty and prices are strictly positive, then \( V(i, j) > 0 \) for all \( i \) and \( j \) provided that firms rationally expect that \( \phi(i, j, s) > 0 \) for at least one \( s \).

At the same time \( \omega(s) \) non-empty implies that for any \( s \), there exists at least one \( i' \) and \( j' \) for which \( \phi^*(i', j', s) > 0 \). Now if \( \phi^*(i, j, s) = 0 \) for all \( i \) and all \( s \), then \( \int_{\phi^*(i', j', s)N(s)ds} \int_{\phi^*(i, j, s)N(s)ds} = 0 \). However, defining
\( \bar{m}(i, j) > 0 \) to be the firm’s expected worker productivity, from workers’ first-order condition we have that

\[
\int \phi^*(i, j, s)N(s)\,ds = \frac{V(i, j)\left( (C(i, j)\frac{m(j, s)}{m(i, j)})^{\kappa\psi} A(i)^{\kappa(1-\psi)} \right)^{\frac{1}{\eta + \kappa\psi(1-\eta)}}}{V(i', j')\left( (C(i', j')\frac{m(j', s)}{m(i', j')})^{\kappa\psi} A(i')^{\kappa(1-\psi)} \right)^{\frac{1}{\eta + \kappa\psi(1-\eta)}}} > 0 ,
\]

a contradiction.

Third, \( \mu(j) \) is non-decreasing. Suppose otherwise. From the worker’s first-order condition, we must have

\[
0 \geq \theta(i, j)^\eta + \kappa\psi(1-\eta) \left( \frac{\kappa\psi(1-\eta)}{\eta} \left( \rho + \delta \frac{C(i, j)}{m(i, j)} \right)^{\kappa\psi} \left( \zeta A(i)^{\kappa(1-\psi)} m(j, s)^{\kappa\psi} - U(s) \right) \right) = R(i, j)m(j, s)^{\kappa\psi} - U(s) ,
\]

with equality if \( \phi^*(i, j, s) > 0 \). Supposing that there exist two industries \( i \) and \( i' \), two jobs \( j^+ > j^- \), and two worker types \( s^+ > s^- \) such that \( \phi^*(i, j^+, s^-) > 0 \) and \( \phi^*(i', j^-, s^+) > 0 \), it must be that

\[
0 = R(i, j^-)m(j^-, s^+)\psi - U(s^+) \\
\geq R(i, j^-)m(j^-, s^-)\psi - U(s^-) \\
= \frac{m(j^-, s^-)^\psi}{m(j^-, s^+)^\psi} U(s^+) - U(s^-) \\
> \frac{m(j^+, s^-)^\psi}{m(j^+, s^+)^\psi} U(s^+) - U(s^-) \\
= -\frac{U(s^-)}{m(j^+, s^+)^\psi} R(i', j^+) \left( R(i', j^+)m(j^+, s^+)\psi - U(s^+) \right) \\
\geq 0 ,
\]
a contradiction for \( \psi > 0 \). Note that the result holds across industries due to assumption 1, without which there would be a non-separable term depending on \( (i, j, s) \).

Fourth, \( \omega \) and \( \mu \) are single-valued almost everywhere. The proof is unchanged from CV and so I provide only the intuition: if \( \omega \) (or \( \mu \)) has positive measure over a domain with positive measure, then from the previous result the range of the correspondence will have measure greater than the measure of \([0, 1]\) (or \([s, \bar{s}]\)), a contradiction.
Fifth, $\mu(j)$ is single-valued. If this is not the case, then from step 3 there exists a non-degenerate interval $[s, s']$ in which all workers choose job $j$. Step 4 implies that there exists another job $j'$ that is chosen by a single worker type. From the firm’s first-order condition and market clearing it must be that

$$\sum_i \left( \int \phi^*(i, j, s) N(s) ds \right) \zeta \theta(i, j)^n = \sum_i \frac{\left[ \alpha(i, j) \zeta \theta(i, j)^{\eta-1} \right]^{\sigma} \left[ \bar{m}(i, j) \right]^{\sigma-1}}{\int \alpha(i, k) \left[ \alpha(i, k) \zeta \theta(i, k)^{\eta-1} \bar{m}(i, k) \right]^{\sigma-1} dk} \frac{Y(i)^*}{\sigma - 1}. $$

But then

$$\sum_i \frac{\left[ \alpha(i, j') \zeta \theta(i, j')^{\eta-1} \right]^{\sigma} \left[ \bar{m}(i, j') \right]^{\sigma-1}}{\int \alpha(i, k) \left[ \alpha(i, k) \zeta \theta(i, k)^{\eta-1} \bar{m}(i, k) \right]^{\sigma-1} dk} = 0,$$

which violates the assumptions that $\alpha(i, j)$ is strictly positive and continuous (and therefore finite), that $m(j, s) > 0$, and that $C(i, j)$ is finite.

From the last step we have $\mu(j)$ single-valued; from the third step, weakly increasing; from the first step, continuous and such that $\mu(0) = \underline{s}$ and $\mu(1) = \bar{s}$; and from the fourth step, $\mu$ is strictly increasing. Hence we have a continuous, strictly increasing bijection $\lambda(s) = \omega(s) = \{j \mid \exists i \text{ s.t. } \phi^*(i, j, s) = 1\} = \mu^{-1}(s).$

\[\square\]

**Lemma 2.** *Reservation values satisfy the equation*

$$\frac{d \log U(s)}{ds} = \frac{m_s(s, \lambda(s))}{m(s, \lambda(s))} \kappa \psi. $$

**Proof.** From lemma 1, and with $R(i, j)$ defined as above, for any $\phi^*(i, j, s) > 0$ we must have

$$0 \geq R(i, j) m(j, s)^{\kappa \psi} - U(s),$$

and following CV the following two inequalities must hold:

$$R(i, \lambda(s)) - \frac{U(s)}{m(\lambda(s), s)^{\kappa \psi}} \geq \frac{U(s + ds)}{m(\lambda(s), s + ds)^{\kappa \psi}} - R(i, \lambda(s)) - \frac{U(s + ds)}{m(\lambda(s), s + ds)^{\kappa \psi}}$$
\[
R(i, \lambda(s + ds)) - \frac{U(s + ds)}{m(\lambda(s + ds), s + ds)^{\kappa \psi}} \geq R(i, \lambda(s + ds)) - \frac{U(s)}{m(\lambda(s + ds), s)^{\kappa \psi}},
\]

and therefore
\[
R(i, \lambda(s)) \left[ m(\lambda(s), s + ds)^{\kappa \psi} - m(\lambda(s), s)^{\kappa \psi} \right]
\leq U(s + ds) - U(s)
\leq R(i, \lambda(s + ds)) \left[ m(\lambda(s + ds), s + ds)^{\kappa \psi} - m(\lambda(s + ds), s)^{\kappa \psi} \right].
\]

Regarding continuity in \(s\): \(U\) is continuous given continuity of \(\theta\), which in turn follows from continuity of \(\alpha, \nu, \) and \(\lambda\). If \(\theta\) is continuous then continuity of \(R(i, j)\) also follows, and hence we can divide by \(ds\) and take the limit of the previous inequalities to show that

\[
U'(s) = R(i, \lambda(s)) \kappa \psi m_k(\lambda(s), s)^{\kappa \psi - 1} m(\lambda(s), s)^{\kappa \psi} \frac{m(\lambda(s), s)}{m(\lambda(s), s)}
\]

and the result follows.

\[\square\]

**Lemma 3.** The matching function satisfies

\[
\frac{d\lambda(s)}{ds} = \sum_i \left[ \left( \frac{\kappa \psi (1 - \eta)}{\eta + \kappa \psi (1 - \eta) p(i, \lambda(s))} m(\lambda(s), s)^{\psi} A(i)^{\psi} \right) \right] m(\lambda(s), s)^{\kappa \psi} U(s),
\]

\[
\frac{d\lambda(s)}{ds} = \sum_i \left[ \left( \frac{\eta + \psi (1 - \eta)}{\psi (1 - \eta)} \right) m(\lambda(s), s) U(s) N(s) \right] m(\lambda(s), s)^{\psi} A(i)^{\psi} \frac{\alpha_i(\lambda(s)) / p_i(\lambda(s))}{\sum_k \frac{\alpha_k(\lambda(s)) / p_k(\lambda(s))}{\beta_k}} \frac{\frac{\beta_i}{P_i}}{\sum_k \frac{\alpha_k(\lambda(s)) / p_k(\lambda(s))}{\beta_k}} Y,
\]

where \(\lambda(\underline{s}) = 0\) and \(\lambda(\overline{s}) = 1\).

**Proof.** Total demand for \(j\)-output is given by

\[
\sum_i y^D(i, j) = \sum_i y(i, j),
\]

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while from market-clearing and lemma 1 we have supply of \( j \)-output as
\[
y^S(i, j) = \sum_i m(\lambda(s), s) \phi^*(i, j, s) \zeta \theta(i, \lambda(s))^\eta N(s) \delta[j - \lambda(s)],
\]
with \( \delta \) the Dirac function. Combining these equations:
\[
\sum_i y^D(i, j) = \int \int \sum_i \phi^*(i, j, \lambda(s)) \zeta \theta(i, \lambda(s))^\eta N(\lambda(s)) \delta[j - \lambda(s)] \frac{1}{N(s)} dj.
\]
This simplifies to
\[
\lambda'(s) = \frac{m(\lambda(s), s) N(s) \sum_i \phi^*(i, \lambda(s), s) \zeta \theta(i, \lambda(s))^\eta}{\sum_i y(i, \lambda(s))},
\]
where
\[
\phi^*(i, \lambda(s), s) = \frac{y(i, \lambda(s))}{m(\lambda(s), s)} \left[ m(\lambda(s), s) p(i, \lambda(s)) \right]^{\kappa \psi} A(i)^{\kappa(1-\psi)}
\]
\[
\sum_k \frac{y(k, \lambda(s))}{m(\lambda(s), s)} \left[ m(\lambda(s), s) p(k, \lambda(s)) \right]^{\kappa \psi} A(k)^{\kappa(1-\psi)}
\]
\[
\theta(i, \lambda(s)) = \left[ \frac{U(s)}{\zeta \left( \frac{\kappa \psi(1-\eta)}{\eta + \kappa \psi(1-\eta)} m(\lambda(s), s) p(i, \lambda(s)) \right)^{\kappa \psi} A(i)^{\kappa(1-\psi)} } \right]^{\frac{1}{\eta}}.
\]
Substitution into the previous equation gives us the final result.

\[\square\]

**B.2 Equilibrium Characterization**

Taking the worker’s problem and substituting out \( S^U \) and \( S^W \), we obtain an equation for the worker’s outside option:
\[
\rho U(i, j, s) = f(\theta(i, j)) u(w(i, j)m(j, s), A(i))^\kappa.
\]

From the firm’s first-order condition we then have the wage function
\[
w(i, j) = \frac{\kappa \psi(1-\eta)}{\eta + \kappa \psi(1-\eta)} p(i, j),
\]
which allows us to then solve for the worker’s search probability,

\[
\frac{\phi^*(i', j', s)}{\phi^*(i, j, s)} = \frac{p(i', j')y(i', j')}{C(i', j')} \left[ \left( \frac{p(i', j')m(j', s)}{C(i', j')} \right)^{\kappa \psi} A(i')^{\kappa(1-\psi)} \right]^{\frac{1}{\eta}},
\]

and to characterize market tightness:

\[
\theta(i, j) = \left( \frac{U(s)}{\zeta [(1 - \epsilon(i, j))m(j, s)p(i, j)]^{\kappa \psi} A(i)^{\kappa(1-\psi)}} \right)^{\frac{1}{\eta}}.
\]

Free entry yields solutions for the prices, which are standard, while likewise the optimal bundles \(y^*\) and \(Y^*\) are straightforward to derive.

The system of equations describing the equilibrium can be written more concisely by defining the skill premium and the firm premium as

\[
SP(s) = U(s)^{\frac{1-\eta}{\eta + \kappa \psi (1-\eta)}},
\]

\[
FP(i, j) = \left( \frac{\kappa \psi (1 - \eta)(\rho + \delta)C(i, j)}{\eta \zeta^{\frac{1}{\eta}} A(i)^{\kappa(1-\psi)}} \right)^{\frac{1}{\eta + \kappa \psi (1-\eta)}}.
\]

Prices and wages are then given by the equations

\[
p(i, j) = \frac{\eta + \kappa \psi (1 - \eta) SP(\lambda^{-1}(j))FP(i, j)}{\kappa \psi (1 - \eta) m(j, \lambda^{-1}(j))},
\]

\[
P(i) = \left( \int \alpha(i, j) \left( \frac{\alpha(i, j)}{p(i, j)} \right)^{\sigma-1} \frac{1}{\sigma-1} \right),
\]

while the quantities that characterize labor submarkets and goods markets are

\[
\phi^*(i, j, \lambda^{-1}(j)) = \frac{V(i, j)FP(i, j)^{\frac{\kappa \psi}{\eta}} A(i)^{\frac{\kappa(1-\psi)}{\eta}}}{\sum_k V(k, j)FP(k, j)^{\frac{\kappa \psi}{\eta}} A(k)^{\frac{\kappa(1-\psi)}{\eta}}},
\]

\[
V(i, j) = \frac{1}{\zeta^{\frac{1}{\eta}}} \left[ FP(i, j)^{\kappa \psi} A(i)^{\kappa(1-\psi)} \right]^{\frac{1-\eta}{\eta}} m(j, \lambda^{-1}(j)) \]

\[
y(i, j) = \left( \frac{\alpha(i, j)}{p(i, j)} \right)^{\sigma} \beta(i)^{\tau} P(i)^{\sigma-\tau}
\]
\[ \theta(i, j) = \frac{SP(s)}{\zeta F P_i(\lambda(s))^{\frac{n}{\eta}} A_i^{\frac{n-1}{\eta}}} . \]

Note that we may also write the system of differential equations describing the optimal assignment in terms of wage premia:

\[
\frac{SP'(s)}{SP(s)} = \frac{\kappa \psi (1 - \eta) m_s(\lambda(s), s)}{\eta + \kappa \psi (1 - \eta) m(\lambda(s), s)}
\]

\[
\lambda'(s) = \frac{m(\lambda(s), s) N(s) SP(s)^{\frac{n}{\eta}}}{\sum_k FP(k, j)^{\kappa \psi} A(i)^{\kappa (1 - \psi)} y(i, j)} .
\]

### B.3 Structural estimation

#### B.3.1 Moment Conditions

*Match parameters.* The empirical match function \( \hat{\lambda} \) and the skill productivity function \( G(s) \) are identified by inverting the differential equation for \( U(s) \):

\[
G'(s) \left( \gamma_s + \gamma_{sj} \hat{\lambda}(s) \right) = \frac{\eta + \kappa \psi (1 - \eta) SP'(s)}{\kappa \psi (1 - \eta) SP(s)} ,
\]

where \( \gamma_{sj} = 1 \) for the 2010-2017 panel, while in the earlier panels we have from the boundary condition that

\[ \gamma_{sj} = 1/\hat{\lambda}(\pi) . \]

The term \( H(j) \) is found by solving the equation

\[
H(j) = \log \left( \frac{SP(s)}{e^{G(s)(\gamma_s + j)}} \right) ,
\]

which may be solved by inverting \( \hat{\lambda} \).

*Amenities and costs.* Given \( \kappa \), industry amenities are estimated from empirical vacancy-filling rates calculated as total industry hires over vacancies. Given industry vacancies \( V \)
and applicants $N$, the predicted vacancy-filling rate will be

$$M(V, N) = S A \frac{(1-\psi)(1-\eta)}{\eta} \int \left( \frac{\alpha(\lambda(s))}{FP(\lambda(s))} \right)^{\sigma} \frac{m(\lambda(s), s)^{\sigma-1}}{m(\lambda(s), s)} ds \int \frac{FP(\lambda(s))^{\eta}}{\eta} ds,$$

where $S$ is a constant term that may be dropped as amenities are only identified up to a multiplicative constant. We may then write amenities as a function of the vacancy-filling rate and a labor-weighted ratio of wage premia:

$$A = \left( \frac{M(V, N)}{V} \frac{\int L(\lambda(s)) \frac{SP(s)^{\eta(1-\eta)}}{FP(\lambda(s))^{\eta}} ds}{\int L(\lambda(s)) ds} \right)^{\frac{1}{\eta(1-\psi)(1-\eta)}}.$$

When estimating this equation, $SP$ and $FP$ are averaged for each of the 20 within-industry estimation quantiles, and a weighted average is then taken of the ratio above.

Industry entry costs are obtained using the free entry condition

$$C(i, j) = \frac{FP(i, j)^{\eta \psi \kappa (1-\eta)}}{\kappa \psi (1-\eta)(\rho + \delta)} A(i)^{\frac{\kappa(1-\psi)(1-\eta)}{\eta}}.$$  

Taking the ratio of $C(i', j')$ to $C(i, j)$, we obtain

$$\frac{C(i', j')}{C(i, j)} = \left( \frac{FP(i', j')}{FP(i, j)} \right)^{\eta \psi \kappa (1-\eta)} \left( \frac{A(i')}{A(i)} \right)^{\frac{\kappa(1-\psi)(1-\eta)}{\eta}},$$

which gives us entry costs up to a multiplicative constant, conditional on amenities, firm premia $FP$, and the parameters $\psi$ and $\eta$ which are normalized as described in the text.

**Demand functions.** The occupational demand function $\alpha(i, j)$ is identified from the intra-industry distribution of person effects $SP(s)$, calculated over 20 quantiles. Denoting sub-
market labor (or alternatively, flow matches) as \( L(i, j) \), the model predicts that

\[
\frac{w(i, \lambda(s)) L(i, \lambda(s))}{\int w(i, \lambda(k)) L(i, \lambda(k)) dk} = \frac{\alpha(i, \lambda(s))^{\sigma} \left( \frac{m(\lambda(s), s)}{FP(i, \lambda(s)) SP(s)} \right)^{\sigma-1}}{\int \alpha(i, \lambda(k))^{\sigma} \left( \frac{m(\lambda(k), s)}{FP(i, \lambda(k)) SP(k)} \right)^{\sigma-1} dk}.
\]

Demand is therefore estimated by solving for \( \alpha \) and substituting empirical wage effects, intra-industry labor shares, and the estimated matching function, and finally normalizing to ensure that \( \int \alpha(i, j) dj = 1 \).

To obtain industry shares \( \beta(i) \), we may integrate over total wages \( w(i, \lambda(s)) L(i, \lambda(s)) \) to obtain

\[
\int w(i, \lambda(s)) L_i(\lambda(s)) ds = \left( \beta(i) \frac{\psi(1 - \eta)}{\eta + \kappa \psi(1 - \eta)} \right)^\sigma \times \left( \int \alpha(i, \lambda(k))^{\sigma} \left[ \frac{m(\lambda(k), k)}{FP(i, \lambda(k)) SP(k)} \right]^{\sigma-1} dk \right)^{\frac{1}{\sigma - 1}}.
\]

By solving for \( \beta \) and substituting empirical values as well as \( \alpha \) (estimated previously), we may then estimate industry demand, which is normalized through division by \( \sum_i \beta(i) \).

**Matching function.** At each point in time the flow of matches will be equal to

\[
M(V, N) = \int \zeta \left( \frac{\delta y^*(i, \lambda(s))}{\zeta \theta(i, \lambda(s))^{(\eta-1)} m(\lambda(s), s)} \right)^\eta \left( \psi^*(i, \lambda(s), s) N(s) \right)^{1-\eta}.
\]

I do not observe \( s \)-unemployment and so I assume that changes over time in \( N \) are in equal proportion for all skill types: that is, \( N(s) \equiv N(s)U \) where \( U \) is aggregate unemployment and \( N(s) \) is constant. We can rewrite the previous equation as

\[
M(V, N) = SVU^{1-\eta} A(i)^{\frac{\kappa(1-\psi)(1-\eta)}{\eta}} \left( \frac{\int L(i, \lambda(s))^{\frac{SP(s)}{FP(i, \lambda(s))^{\frac{1}{\sigma}}}}}{\int L(i, \lambda(s)) ds} \right)^{\frac{1}{\sigma - 1}},
\]

where \( S \) is a constant. If wage premia are constant in the short-run, then by taking differences
we obtain an equation in $V$ and $U$, allowing $\eta$ and $\zeta$ to be estimated from industry vacancies and hires and aggregate unemployment.

However, estimates obtained in this fashion result in values of $\eta$ that are implausibly large (in the .6-.7 range) relative to estimates in the literature, likely due to the short length of the time series and noise in the estimates of $V$ and hires. Attenuation bias downwards estimates of $1 - \eta$, resulting in upwardly biased estimates of $\eta$. I therefore use the value $\eta = .35$ estimated by Kohlbrecher et al. (2016). Matching efficiency $\zeta$ is then found by taking the ratio of predicted to actual hires.

B.3.2 Goodness-of-fit
### Table B.1: Empirical and Predicted Wage Moments

<table>
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<tr>
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<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
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<tr>
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<td>0.0167</td>
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<td><strong>East Germany</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empirical Variance</td>
<td>0.1590</td>
<td>0.1712</td>
<td>0.1955</td>
<td>0.1816</td>
</tr>
<tr>
<td>Var(person)</td>
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<td>0.0889</td>
<td>0.0948</td>
<td>0.1031</td>
</tr>
<tr>
<td>Var(firm)</td>
<td>0.0380</td>
<td>0.0451</td>
<td>0.0574</td>
<td>0.0432</td>
</tr>
<tr>
<td>Covariance</td>
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<td>0.0127</td>
<td>0.0123</td>
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<tr>
<td>Between-ind./-occ.</td>
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<td>0.1013</td>
<td>0.1101</td>
<td>0.1188</td>
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<tr>
<td>Var(person)</td>
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<td>0.0555</td>
</tr>
<tr>
<td>Var(firm)</td>
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<td>Var(person)</td>
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</table>

**Note:** For between-group moments, wage and wage effects are averaged by industry-occupation cells using 3-digit codes.

### B.4 Quantitative Results

#### B.4.1 Counterfactual Experiments

**Homogeneous** $FP$. The experiment performed in the text is to set firm premia equal so that $FP(i,j)$ is constant, without affecting the distribution of vacancies *conditional on*
occupational assignment in panel 0 (indicated by subscripts). Vacancies are given by:

\[ V(i, j) = \frac{1}{\zeta \frac{1}{\eta} \left[ FP(i, j)^{\kappa \psi} A(i)^{\kappa (1-\psi)} \right]^{\frac{1}{\eta}}} \frac{SP(\lambda^{-1}(j))}{SP(\lambda^{-1}(j))FF(i,j)} \left( \frac{\alpha(i,j)\lambda^{-1}(j)(1-\psi(i,j))}{m(j, \lambda^{-1}(j))} \right)^{\sigma} \beta(i)^{\tau} P(i)^{\sigma-\tau}. \]

By assumption we have \( \sigma = \tau \), and therefore consolidating terms that are unchanged or ignored (i.e. \( SP \)) we have

\[ V(i, j) = \hat{V} \frac{\alpha(i, j)^{\sigma} \beta(i)^{\tau}}{C(i, j)^{\frac{\eta \kappa \psi (1-\eta)}{\eta \kappa \psi (1-\eta)}}}. \]

Now for firm premia to be constant we must have

\[ C(i, j) = \hat{C} A(i)^{\frac{\kappa (1-\psi)(1-\eta)}{\eta \kappa \psi (1-\eta)}}. \]

Writing \( R(i, j) \equiv C(i, j)/A(i)^{\frac{\kappa (1-\psi)(1-\eta)}{\eta \kappa \psi (1-\eta)}} \), in order for occupational shares to be constant we must have

\[ \hat{\alpha}(i, j) = \frac{\alpha(i, j)/R_0(i, j)}{\int \alpha(i, k)/R_0(i, k)^{\frac{\sigma \eta + \kappa \psi (1-\eta)}{\sigma \eta + \kappa \psi (1-\eta)}} dk}. \]

It then follows that

\[ \hat{\beta}(i) = \frac{\beta(i) \left( \int \alpha_0(i, j)/R_0(i, j)^{\frac{\sigma \eta + \kappa \psi (1-\eta)}{\sigma \eta + \kappa \psi (1-\eta)}} dj \right)}{\sum_k \beta(k) \left( \int \alpha_0(k, j)/R_0(k, j)^{\frac{\sigma \eta + \kappa \psi (1-\eta)}{\sigma \eta + \kappa \psi (1-\eta)}} dj \right)}. \]

Finally, maintaining the overall level of vacancies requires in the initial panel that

\[ \hat{C}_0 = \left( \sum_i \beta_0(i) \left( \int \alpha_0(i, j)/R_0(i, j)^{\frac{\sigma \eta + \kappa \psi (1-\eta)}{\sigma \eta + \kappa \psi (1-\eta)}} dj \right) \right)^{\frac{-\sigma \eta + \kappa \psi (1-\eta)}{\sigma \eta + \kappa \psi (1-\eta)}}.

and as there is no simple way to implement a similar condition on subsequent panels, I hold total vacancies approximately constant by assuming that

\[
\hat{C} = \left( \frac{\sum_i \beta(i) \left( \int \alpha(i, j) \left( \frac{R(i, j)}{R_0(i, j)} \right)^{\frac{\sigma(1 - \eta)}{\sigma + \kappa + \psi(1 - \eta)}} dj \right)}{\sum_i \beta(i) \left( \int \alpha_0(i, j) / R_0(i, j)^{\frac{\sigma(1 - \eta)}{\sigma + \kappa + \psi(1 - \eta)}} dj \right)} \right) \left( \frac{\tau(\eta + \kappa + \psi(1 - \eta))}{\tau(\eta + \kappa + \psi(1 - \eta))} \right),
\]

The counterfactual experiment is then to replace estimated parameter values with \(\hat{C}, \hat{\alpha}, \hat{\beta}\) as defined above.

**Alternative FP.** Several experiments involve setting firm premia to alternative (non-constant) values, holding constant the 1993-99 distribution of vacancies conditional on \(\lambda\). As above we have

\[
V(i, j) = \hat{V} \frac{\alpha(i, j)^{\sigma \beta(i)^{\tau}}}{C(i, j)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta + \kappa + \psi(1 - \eta)}}}.
\]

Defining \(\hat{C}\) to be the value of entry costs that implements the alternative firm premia (and noting \(\hat{C}\) may need to account for differences in the parameters \(\zeta\) and \(\delta\), and with \(R(i, j) \equiv C(i, j)/A(i)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta}}\) as above, in order for occupational shares to be constant we must have

\[
\hat{\alpha}(i, j) = \frac{\alpha(i, j) \left( \int \alpha(i, k) / R_0(i, k) \right)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta + \kappa + \psi(1 - \eta)}} dk}{\int \alpha(i, k) / R_0(i, k) \left( \int \alpha(i, j) / R_0(i, j) \right)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta + \kappa + \psi(1 - \eta)}} dj},
\]

where \(A\) denotes the target economy and 0 the initial panel. Likewise industry shares must be set so that

\[
\hat{\beta}(i) = \frac{\beta(i) \left( \int \alpha_0(i, j) / R_0(i, j) \right)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta + \kappa + \psi(1 - \eta)}} dj}{\sum_k \beta(k) \left( \int \alpha_0(k, j) / R_0(k, j) \right)^{\frac{\sigma(1 - \eta)(1 - \eta)}{\eta + \kappa + \psi(1 - \eta)}} dj}.
\]
Maintaining the overall level of vacancies requires in the initial panel that

\[ \hat{C}_0 = \left( \sum_i \beta_0(i) \left( \int \alpha_0(i, \cdot, j) \left( \frac{R_{A,0}(i, \cdot, j)}{R_0(i, \cdot, j)} \right)^{\frac{\sigma \eta + \kappa \psi (1 - \eta)}{\sigma (\eta + \kappa \psi (1 - \eta))}} dj \right) \right)^{-1}, \]

and as there is no simple way to implement a similar condition on subsequent panels, I hold total vacancies approximately constant by assuming that

\[ \hat{C} = \left( \sum_i \beta(i) \left( \int \alpha(i, \cdot, j) \left( \frac{R(i, \cdot, j) R_{A,0}(i, \cdot, j)}{R_0(i, \cdot, j)} \right)^{\frac{\sigma \eta + \kappa \psi (1 - \eta)}{\sigma (\eta + \kappa \psi (1 - \eta))}} dj \right) \right)^{-1}, \]

The counterfactual experiment is then to replace estimated parameter values with \( \{\hat{C}, \hat{\alpha}, \hat{\beta}\} \) as defined above.

**Alternative rent-shares.** Given a target set of firm premia \( FP_A \), we can implement \( FP_A \) by setting the firm’s share of rent equal to

\[ \hat{\eta}(i, \cdot, j) = \frac{C(i, \cdot, j)}{A(i, \cdot, j)}^{\frac{1}{\eta - (1 - \eta) \psi}} + \frac{\zeta \hat{\eta} FP_A}{\rho + \hat{\beta}}. \]

Doing so affects not only \( FP \), but also output price \( p(i, \cdot, j) \) and vacancies \( V(i, \cdot, j) \).

**B.4.2 Historical Results**

**B.4.3 Skill-Bias**

**B.4.4 East Germany**

**B.4.5 Wage Policies**
### Table B.3: Empirical and Predicted Wage Moments

<table>
<thead>
<tr>
<th></th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Predicted Variance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneous $FP$</td>
<td>0.0694</td>
<td>0.0860</td>
<td>0.1021</td>
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<td>$\alpha, \beta$ only</td>
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<td>0.0480</td>
<td>0.0527</td>
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</tr>
<tr>
<td>Homogeneous $FP$</td>
<td>0.0736</td>
<td>0.0862</td>
<td>0.0946</td>
<td>0.1066</td>
</tr>
<tr>
<td>$\kappa = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Predicted Variance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneous $FP$</td>
<td>0.0436</td>
<td>0.0511</td>
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<tr>
<td><strong>Predicted Variance</strong></td>
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<td></td>
</tr>
<tr>
<td>Homogeneous $FP$</td>
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<td><strong>Predicted Variance</strong></td>
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<tr>
<td>$\kappa = 4$</td>
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<td></td>
</tr>
</tbody>
</table>

**Note:** For between-group moments, wage and wage effects are averaged by industry-occupation cells using 3-digit codes.

### Appendix C
Figure B.4: Predicted and Counterfactual Wage Trend, 1993-2017

Figure B.7: Estimated Type Distributions, East Germany

Figure B.8: Estimated Match Production Parameters
### Table B.5: Empirical and Predicted Wage Moments, By Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1993-99</th>
<th>1998-04</th>
<th>2003-10</th>
<th>2010-17</th>
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<td>0.0894</td>
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<td>0.0945</td>
</tr>
<tr>
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<td>$\beta$ only</td>
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<tr>
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<td>0.0856</td>
<td>0.0893</td>
<td>0.0925</td>
<td>0.0945</td>
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<td>0.0336</td>
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<td>$\alpha$ only</td>
<td>0.0766</td>
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<td>0.0909</td>
<td>0.1007</td>
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Figure B.9: Estimated Technical Parameters, East Germany

Table B.10: Aggregate Parameters, East Germany

<table>
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<th>Value</th>
<th>Source</th>
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<td>$\psi$</td>
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<td>Equal to $\frac{1}{1-\eta}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
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<td>Discount factor of .96</td>
</tr>
<tr>
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<td>Separation rate</td>
<td>.183</td>
<td>Annual hires</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Match elasticity</td>
<td>.35</td>
<td>Total employment</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Match efficiency</td>
<td>1.29</td>
<td>Kohlbrecher et al. (2016)</td>
</tr>
</tbody>
</table>

Figure B.11: Industry Average Firm Premia, East and West Germany
Figure B.12: Temp Agency Employment, West Germany

Figure B.13: Simulated Effect of Equal Pay for Temp Workers
Figure B.14: Simulated Effect of Entry Cost Compression
Supplementary Materials For Chapter 3

C.1 Empirical Appendix

The first set of variables used in this analysis are those concerning job routineness, and consist of answers to the following questions:

1. repeat tasks: in all survey years, how often that “one and the same operation is repeated down to the last detail”

2. follow instructions: in all survey years, how often the execution of work is “prescribed down to the last detail”

3. adapt to new tasks: the 1979 survey asks respondents how often they must adapt to “new situations.” Beginning with the 1985/86 survey this changes to ”new tasks that you must first think about and become familiar with.”

4. improve procedures: in all survey years, how often respondents must “improve on previous procedures or try something new”

5. solve problems: for survey years 2006-2018, how often respondents must “react to and solve unforeseen problems”

6. make decisions: for survey years 2006-2018, how often respondents must “make difficult decisions independently and without guidance”

Response categories for 1979-1999 consist of five verbal frequencies: “practically always”, “often”, “every now and then”, “seldom”, and “practically never”. For 2006-2018, response categories for variables 1-4 are changed to four frequencies: “often”, “sometimes”, “seldom”, and “never”. Variables 5-6, which are only present for 2006-2018, are coded in three frequencies: “often”, “sometimes”, and “never”. I code the responses “practically always” and “often” as 1, “every now and then” and “sometimes” as 2/3 for variables 1-4 and 1/2 for
variables 5-6, “seldom” as 1/3, and “never” and “practically never” as 0. Some discrepancies are observed between the 1989-99 and 2005-06 surveys, but they are inconsistent across variables and do not suggest a systematic bias due to the change in response frequencies.

The other principal variable used in this analysis concerns personal computers. In the 1979 survey respondents are asked whether they often work with computers, EDV equipment, terminals, or screened devices on the job. In 1985/86 and 1991/92 this question is split across manufacturing and office roles and by device type, and so I assign a value of “yes” (1) if the answer is affirmative for any of these roles or devices. There is a major change in 1998/99, with workers simply being asked whether they “work with computers and data processing equipment” in their professional activity. There is again a slight change for the 2006-2018 surveys, with respondents asked simply how often they “work with computers”, the options being “often”, “sometimes”, and “never”; assign a value of “yes” to the first two responses.

With respect to other miscellaneous variables, occupation consists of 1988 KLDB 3-digit codes for the years 1979-2006, and 1992 KLDB 3-digit codes for 1999-2018. For consistency, the results in section two use 1988 KLDB codes for 1979-1992 results and 1992 KLDB codes for 1999-2018 results. Some occupations are not observed in all panels, and prior to performing difference-in-difference regressions I aggregate these codes into neighboring occupations. The same aggregated groupings are employed in the quantitative section. Occupational tasks from the 2006-2018 surveys are coded in the same frequencies as “solve problems” and “make decisions” above, and I assign them numerical values in identical fashion. The six aggregate groupings are composed as:

1. **analyze information**: “developing, researching, constructing [designing]”; “gathering information, investigating, documenting”

2. **advise others**: “organizing, planning, and preparing work processes [for others]”; “training, instructing, teaching, educating”; “providing advice and information”

3. **market goods**: “purchasing, procuring, selling”; “advertising, marketing, public relations”

4. **manual labor**: “measuring, testing, quality control”; “repairing, refurbishing”; “transporting, storing, shipping”; “cleaning, removing waste, recycling”
5. **produce goods**: “manufacturing, producing goods and commodities”; “monitoring, control of machines, plants, processes”

6. **care for others**: “entertaining, accommodating, preparing food”, “nursing, caring, healing”, “protecting, guarding, patrolling, directing traffic”

Numerical values are averaged by task group, and divided by the sum across task groups in order to remove variation due to individuals who report performing all tasks more frequently.

**Table C.1**: Task characteristics and task frequencies, 2006-2018

<table>
<thead>
<tr>
<th>Sample</th>
<th>Analyze information</th>
<th>Advise others</th>
<th>Market goods</th>
<th>Manual labor</th>
<th>Produce goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year fixed effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repetition</td>
<td>-.469 (.023)</td>
<td>-.179 (.024)</td>
<td>-.255 (.025)</td>
<td>.129 (.026)</td>
<td>.003 (.026)</td>
</tr>
<tr>
<td>Instructions</td>
<td>-.267 (.024)</td>
<td>-.139 (.025)</td>
<td>-.370 (.026)</td>
<td>-.002 (.003)</td>
<td>.155 (.027)</td>
</tr>
<tr>
<td>Adaptation</td>
<td>.522 (.020)</td>
<td>.205 (.020)</td>
<td>.153 (.020)</td>
<td>-.119 (.018)</td>
<td>.176 (.019)</td>
</tr>
<tr>
<td>Improvement</td>
<td>.281 (.022)</td>
<td>.039 (.023)</td>
<td>.085 (.023)</td>
<td>-.286 (.022)</td>
<td>.053 (.023)</td>
</tr>
<tr>
<td>Solve problems</td>
<td>.110 (.023)</td>
<td>.041 (.024)</td>
<td>-.048 (.023)</td>
<td>-.344 (.020)</td>
<td>-.106 (.021)</td>
</tr>
<tr>
<td>Make decisions</td>
<td>-.013 (.026)</td>
<td>.040 (.028)</td>
<td>.048 (.028)</td>
<td>-.509 (.026)</td>
<td>-.203 (.026)</td>
</tr>
</tbody>
</table>

| Industry, occupation, and year fixed effects |                     |               |              |              |               |
| Repetition     | -.269 (.030)        | -.164 (.029)  | -.282 (.031) | .057 (.032)  | .044 (.034)   |
| Instructions   | -.179 (.031)        | -.171 (.031)  | -.306 (.032) | -.089 (.032) | -.058 (.035)  |
| Adaptation     | .263 (.023)         | .127 (.022)   | .135 (.023)  | -.083 (.022) | .043 (.024)   |
| Improvement    | .208 (.025)         | .074 (.025)   | .135 (.026)  | -.167 (.025) | .029 (.027)   |
| Solve problems | .083 (.025)         | .110 (.024)   | .098 (.025)  | -.163 (.023) | -.017 (.025)  |
| Make decisions | .010 (.030)         | .131 (.030)   | .187 (.032)  | -.332 (.030) | -.128 (.032)  |

**Table notes**: Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance.
Table C.2: Task frequencies and PC use, 2006-2018

<table>
<thead>
<tr>
<th>Sample</th>
<th>Analyze information</th>
<th>Advise others</th>
<th>Market goods</th>
<th>Manual labor</th>
<th>Produce goods</th>
<th>Care for others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>.068 (.004)</td>
<td>.031 (.003)</td>
<td>.035 (.003)</td>
<td>-.053 (.003)</td>
<td>-.011 (.002)</td>
<td>-.014 (.002)</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>.048 (.009)</td>
<td>.022 (.008)</td>
<td>.030 (.007)</td>
<td>-.065 (.010)</td>
<td>-.007 (.008)</td>
<td>-.010 (.006)</td>
</tr>
<tr>
<td>Vocational</td>
<td>.057 (.004)</td>
<td>.027 (.004)</td>
<td>.023 (.003)</td>
<td>-.052 (.003)</td>
<td>-.009 (.003)</td>
<td>-.012 (.002)</td>
</tr>
<tr>
<td>University</td>
<td>.072 (.015)</td>
<td>.012 (.15)</td>
<td>.068 (.010)</td>
<td>-.044 (.006)</td>
<td>-.015 (.005)</td>
<td>-.016 (.004)</td>
</tr>
<tr>
<td>Wage pct.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-25</td>
<td>.058 (.006)</td>
<td>.017 (.005)</td>
<td>.020 (.005)</td>
<td>-.048 (.006)</td>
<td>.000 (.005)</td>
<td>-.021 (.004)</td>
</tr>
<tr>
<td>26-50</td>
<td>.050 (.006)</td>
<td>.029 (.006)</td>
<td>.023 (.005)</td>
<td>-.041 (.006)</td>
<td>-.013 (.005)</td>
<td>-.010 (.003)</td>
</tr>
<tr>
<td>51-75</td>
<td>.050 (.009)</td>
<td>.017 (.008)</td>
<td>.059 (.007)</td>
<td>-.045 (.006)</td>
<td>-.017 (.007)</td>
<td>-.002 (.005)</td>
</tr>
<tr>
<td>76-100</td>
<td>.044 (.019)</td>
<td>.042 (.019)</td>
<td>.097 (.016)</td>
<td>-.040 (.007)</td>
<td>-.027 (.008)</td>
<td>.001 (.005)</td>
</tr>
</tbody>
</table>

Table notes. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance.

Table C.3: Occupation mean task frequencies and PC use (D-in-D)

<table>
<thead>
<tr>
<th>Years</th>
<th>Analyze information</th>
<th>Advise others</th>
<th>Market goods</th>
<th>Manual labor</th>
<th>Produce goods</th>
<th>Care for others</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-2012</td>
<td>.027 (.013)</td>
<td>.033 (.011)</td>
<td>.054 (.013)</td>
<td>-.049 (.008)</td>
<td>-.017 (.008)</td>
<td>.006 (.009)</td>
</tr>
<tr>
<td>2012-2018</td>
<td>.060 (.012)</td>
<td>.041 (.014)</td>
<td>.015 (.013)</td>
<td>-.042 (.011)</td>
<td>-.012 (.010)</td>
<td>-.011 (.008)</td>
</tr>
</tbody>
</table>

Table notes. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include occupation and year dummies. Bold results indicate 95% significance.

C.2 Theoretical Appendix

C.2.1 Assignment and Wages

The environment is similar to that studied by Costinot and Vogel (CV, 2010), with the difference that unit output costs for automated firms are \( \frac{u(s) + rK}{y(j,s,K)} \) and not \( \frac{u(s)}{y(j,s,0)} \). Following CV, I define \( \omega(s) \) to be the set of jobs \( j \) for which at least one producer hires an \( s \)-worker, and \( \sigma(j) \) the worker types assigned to \( j \). I begin with a lemma establishing that non-automated jobs are always associated with lower-skill workers than automated job:

**Lemma 4.** If \( s' \geq s \) and \( K(j) > 0 \) for some firm employing \( s \), then \( K(j') > 0 \) for any firm \( j' \) employing \( s' \).

**Proof.** This result is shown by establishing that \( w(s) \) and \( \gamma_u(s) \) satisfy the single-crossing: there
exists at most one $s$ such that $w(s) = r\gamma_u(s)$, and that for any $s' > s$ we will have $w(s') > r\gamma_u(s')$. Suppose the contrary: there exists an $s' > s$ such that $w(s') < r\gamma_u(s')$ but $w(s) > r\gamma_u(s)$. From free entry we have $p(j)y^*(j, s, K) \geq w(s)$ for any $j$ employing worker $s$, and by assumption we have $y^*(j, s, K)$ increasing more quickly in $s$ than $\gamma_u(s)$. But then $p(j)y^*(j, s', K) - w(s') > p(j)y^*(j, s, K) - w(s)$, violating the producer’s profit-maximizing problem. The result then follows from (22), which states that $K > 0$ when $w > r\gamma_u$ and $K = 0$ otherwise.

With this result it is possible to derive the main result:

**Lemma 5.** There exists a continuous and strictly increasing function $\lambda : [\underline{s}, \overline{s}] \to [0, 1]$ such that $L(j, s) > 0$ if and only if $\lambda(s) = j$, and where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$.

**Proof.** In the proof below, I provide an abbreviated description whenever the steps follow closely those described by CV.

First, $\omega(s)$ and $\sigma(j)$ are non-empty. That $\omega(s)$ is non-empty follows from market-clearing, as labor is supplied inelastically and worker output is strictly positive. If there existed an $s$ that was not assigned to any producer, it must be that $w(s)$ is sufficiently large that no firms wish to employ $s$-workers; but then market-clearing would imply that $w(s) \to 0$, a contradiction. Non-empty $\sigma(j)$ follows from the profit-maximizing condition for intermediate producers:

$$p(j) - \frac{w(s) + rK^*(j, s)}{y^*(j, s, K)} \leq 0$$

with equality for $s \in \sigma(j)$. If $\sigma(j)$ is empty for some $j$, then from $\omega(s)$ non-empty there must be a $j'$ with $\sigma(j')$ non-empty and $p(j')/p(j) = \frac{A}{p(j)} = 0$ for any finite $A$. But then from the previous condition it must be that

$$\frac{p(j')}{p(j)} + r\frac{K^*(j', s) - K^*(j, s)}{p(j)y^*(j', s)} \geq \frac{y^*(j, s, K)}{y^*(j', s, K)} > 0$$

a contradiction since we must have the left-hand side equal to 0.

Second, $\sigma(j)$ is a non-empty interval of $[\underline{s}, \overline{s}]$, and if $j' > j$, then $s' > s$ for any $s' \in \sigma(j')$ and $s \in \sigma(j)$. Suppose instead that for some $j^+ > j^-$ and $s^+ > s^-$, we have $s^- \in \sigma(j^+)$ and $s^+ \in \sigma(j^-)$. Then from intermediate producers’ first-order condition we must have

$$0 \geq p(j^-) - \frac{w(s^-) + rK^*(j^-, s^-)}{y^*(j^-, s^-)}$$
\[
\begin{align*}
\frac{w(s^+)}{y^*(j^-, s^+)} & + rK^*(j^+, s^+) \left(1 - \frac{w(s^-)+rK^*(j^-, s^-)}{y^*(j^-, s^-)}\right) \\
\frac{w(s^+)}{y^*(j^-, s^+)} & > \frac{w(s^-)+rK^*(j^-, s^-)}{y^*(j^-, s^-)} \\
\frac{w(s^+)}{y^*(j^-, s^+)} & = \frac{w(s^+)+rK^*(j^+, s^+)}{y^*(j^+, s^+)} \\
& = \frac{w(s^+)+rK^*(j^+, s^+)}{w(s^+)+rK^*(j^-, s^+)} \left(\frac{w(s^+)}{y^*(j^+, s^+)} - p(j^+)\right) \\
& \geq 0
\end{align*}
\]

a contradiction, with the strict inequality following from assumption 1 and lemma 1. The inequality holds trivially when either \( K > 0 \) for both \( s^- \) and \( s^+ \) or when \( K = 0 \) for both types. From lemma 1 the only other possibility is that \( K > 0 \) for \( s^+ \) but \( K = 0 \) for \( s^- \), in which case \( \frac{w(s^+)+rK^*(j^-, s^+)}{y^*(j^-, s^+)} = w(s^+)y(s^+, j^-, 0) - \kappa[1 - \alpha(j^-)] \gamma_u(s^+)[w(s^+) - r\gamma_u(s^+)] \). From \( \alpha'(j) > 0 \) we can then see that the inequality holds with even greater force than under the other two cases.

Fourth, \( \omega \) and \( \sigma \) are single-valued almost everywhere. The proof is unchanged from CV and so I provide only the intuition: if \( \omega \) (or \( \sigma \)) has positive measure over a domain with positive measure, then from the previous result the range of the correspondence will have measure greater than the measure of \([s, \bar{s}]\), a contradiction.

Fifth, \( \sigma(j) \) is single-valued. If this is not the case, then from step 3 there exists a non-degenerate interval \([s, s']\) for which all workers are assigned to job \( j \). Step 4 implies that there exists another job \( j' \) that is assigned to a single worker type. But then \( p(j)/p(j') = 0 \), contradicting the free entry condition that \( p(j)y(j, s'') \geq p(j')y(j', s'') \) for \( s'' \in [s, s'] \), a contradiction given that \( y > 0 \).

From the last step we have \( \sigma(j) \) single-valued; from the third step, weakly increasing; from the first step, continuous and such that \( \sigma(0) = s \) and \( \sigma(1) = \bar{s} \); and from the fourth step, \( \sigma \) is strictly increasing. Hence we have a continuous, strictly increasing bijection \( \lambda(s) = \omega(s) = \{j \mid L(j, s) == 1\} = \sigma^{-1}(s) \).

\begin{lemma}
There exists a single threshold skill level \( s^* \), which may be equal to \( s \) or \( \bar{s} \), for which producers automate when \( s > s^* \) and do not automate when \( s > s^* \). The wage
functions satisfies the differential equation

$$\frac{w'(s)}{w(s)} = \begin{cases} \frac{d}{ds} \log y(\lambda(s), s, 0) & s < s^* \\
\frac{d}{ds} \log y(\lambda(s), s, K) & s > s^* \end{cases}$$

where \( w(s) \) is continuous but not differentiable at \( s^* \).

**Proof.** Market-clearing, free entry, and continuity of \( y(j, s) \) imply that \( w(s) \) is continuous, while from lemma 1 we have that there exists at most one \( s^* \) satisfying \( w(s^*) = r \gamma u(s^*) \). Let \( \mathbb{I}[s > s^*] \) be an indicator function taking the value 1 when \( s > s^* \), and 0 otherwise.

From lemma 2, for any producer employing \( s \)-labor we must have

$$p(\lambda(s)) - \frac{w(s)}{y(\lambda(s), s, K)} + \mathbb{I}[s > s^*] \kappa [1 - \alpha(\lambda(s))] r \leq 0$$

For any \( s \neq s^* \), there will exist a neighborhood around \( s \) such that \( \mathbb{I}[s' > s^*] \) takes the same value for any \( s' \in [s - ds, s + ds] \). In the case where \( s > s^* \) the following inequalities must hold:

$$\left[p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] r \right] - \frac{w(s)}{y(\lambda(s), s, K)} \geq \left[p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] r \right] - \frac{w(s + ds)}{y(\lambda(s), s + ds, K)}$$

$$\left[p(\lambda(s + ds)) - \kappa [1 - \alpha(\lambda(s + ds))] r \right] - \frac{w(s + ds)}{y(\lambda(s + ds), s + ds, K)} \geq \left[p(\lambda(s + ds)) - \kappa [1 - \alpha(\lambda(s + ds))] r \right] - \frac{w(s + ds)}{y(\lambda(s + ds), s + ds, K)}$$

For \( s < s^* \) we must have the following inequalities hold:

$$p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] r \left[ y(\lambda(s), s + ds, 0) \right] \geq p(\lambda(s)) - \frac{w(s + ds)}{y(\lambda(s), s + ds, 0)}$$

$$p(\lambda(s + ds)) - \frac{w(s + ds)}{y(\lambda(s + ds), s + ds, 0)} \geq p(\lambda(s + ds)) - w(s) \frac{w(s)}{y(\lambda(s + ds), s, 0)}$$

It follows that in the first case we will have

$$\left[p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] r \right] \left[ y(\lambda(s), s + ds, K) - y(\lambda(s), s, K) \right] \leq w(s + ds) - w(s)$$

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\[ \leq \left[ p(\lambda(s + ds)) - \kappa [1 - \alpha(\lambda(s + ds))] \right] \left[ y(\lambda(s + ds), s + ds, \overline{K}) - y(\lambda(s + ds), s, \overline{K}) \right] \]

and in the second
\[
\begin{align*}
p(\lambda(s)) \left[ y(\lambda(s), s + ds, 0) - y(\lambda(s), s, 0) \right] \\
\leq w(s + ds) - w(s) \\
\leq p(\lambda(s + ds)) \left[ y(\lambda(s + ds), s + ds, 0) - y(\lambda(s + ds), s, 0) \right]
\end{align*}
\]

We must have \( p(j) \) continuous and therefore we can divide by \( ds \) and take the limit of the previous inequalities to show that
\[
w'(s) = \begin{cases} 
p(\lambda(s))y(\lambda(s), s + ds, 0) & s < s^* \\
\left[ p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] \right] y(\lambda(s + ds), s, \overline{K}) & s > s^*
\end{cases}
\]

Substitution for \( p(\lambda(s)) \) and \( p(\lambda(s)) - \kappa [1 - \alpha(\lambda(s))] \) and division by \( w(s) \) yields the final result. □

**Lemma 7.** The matching function satisfies
\[
\lambda'(s) = \begin{cases} 
g\left(\lambda(s), s', 0\right)^{1-\rho} F'(s) w(s)^{\rho} & s < s^* \\
\frac{g\left(\lambda(s), s', \overline{K}\right)^{\rho}}{\beta'(\lambda(s))} \left( w(s) + \kappa [1 - \alpha(\lambda(s))] r y(\lambda(s), s', \overline{K}) \right)^{\rho} & s > s^*
\end{cases}
\]

where \( \lambda(\overline{s}) = 0 \) and \( \lambda(\overline{s}) = 1 \).

**Proof.** This portion of the proof follows closely Costinot and Vogel (2010). Total supply of \( j \)-labor is given by
\[
L(j, s) = F'(s) \delta [j - \lambda(s)]
\]

From market-clearing we have
\[
Y(\lambda(s)) = \int_{s<s^*} y(\lambda(s), s', 0) L(\lambda(s), s') ds' + \int_{s>s^*} y(\lambda(s), s', \overline{K}) L(\lambda(s), s') ds'
\]

and following CV we may derive the differential equation for the matching function, which is defined
piece-wise:

\[
\lambda'(s) = \begin{cases} 
\frac{y(\lambda(s),s',0)F'(s)}{Y(\lambda(s))} & s < s^* \\
\frac{y(\lambda(s),s',K)F'(s)}{Y(\lambda(s))} & s = s^* \\
\frac{y(\lambda(s),s,K)F'(s)}{Y(\lambda(s))} & s > s^* 
\end{cases}
\]

From final good profit-maximization and free entry we have

\[
Y(\lambda(s)) = \begin{cases} 
\beta(\lambda(s))^{\rho}Y \left[ \frac{y(\lambda(s),s',0)}{w(s)} \right]^\rho & s < s^* \\
\beta(\lambda(s))^{\rho}Y \left[ \frac{y(\lambda(s),s',K)}{w(s)+rK} \right]^\rho & s > s^* 
\end{cases}
\]

Substitution of \(Y(\lambda(s))\) and \(y(\lambda(s),s)\) in the previous equation then yields the result. \(\square\)

**C.2.2 Theorem 1**

**Proof.** To establish the first part of the result I show that \(\lambda(s^*)\) is strictly smaller under automation. Suppose otherwise. It can then be shown that for all \(s\), \(\lambda(s')\) is greater under automation and hence from (24) \(w'(s)/w(s)\) is everywhere larger. But then \(\frac{w(s)^\rho}{Y} = \frac{w(s)^\rho}{\int [F'(s)w(s)+rK^*(\lambda(s),s)] ds}\) must be strictly smaller for \(s < s^*\), implying that these workers are assigned to a smaller set of jobs and hence that \(\lambda(s^*)\) is smaller, a contradiction.

For the second part of the proof, for any \(j' > j\) and holding \((y(j',\lambda^{-1}(j'),0)/y(j,\lambda^{-1}(j),0))^{\rho-1}\) we will have \(L(j')/L(j)\) greater under automation both because \(\alpha'(j) > 0\) and because \(\lambda(s)\) and \(w(s) - r\gamma_h(s)\) are both increasing in \(s\). Now if \(\rho < 1\) and employment decreases for all automated jobs then we must have \((y(1,\vec{s},0)/y(\lambda(s),s,0))^{\rho-1}\) smaller under automation for any \(s < \vec{s}\) and therefore \(L(j')/L(j)\) greater. On the other hand if employment increases for some \(j'' > \lambda^{-1}(s^*)\), then it must increase for all \(j > j''\) and the result is shown. In both cases the result is shown for \(\rho < 1\). Finally, from continuity the result must also hold for \(\rho > 1\) but sufficiently small.

Finally, from above we will have \(w'(s)/w(s)\) smaller for \(s < s^*\), whereas \(\lambda(\vec{s}) = 1\) and (24) imply that \(w'(s)/w(s)\) will be greater around \(\vec{s}\). Assumption 1 and continuity of all arguments is then sufficient to establish that either \(w'(s)/w(s)\) is strictly greater for \(s > s^*\) and smaller for \(s < s^*\), in which case I define \(s' = s^*\); or that there exists at least one value of \(s\) at which \(w'(s)/w(s)\) is unchanged under automation, in which case I define \(s'\) equal to the largest such value and the proof is completed. \(\square\)
C.2.3 Theorem 2

Proof. Both results may be shown by establishing that $\lambda(s)$ is strictly greater under automation for all $s$, which would in turn imply both that $w'(s)/w(s)$ is strictly greater and that $\int_0^j L(j, \lambda^{-1}(j))$ is strictly smaller. Now if $t^*$ is increasing in $s$ then holding $\lambda$ fixed, $\frac{d}{ds} \log y^*(\lambda(s), s, K)$ will be smaller under automation and therefore $\frac{d}{ds} \log Y/y^*(\lambda(s), s, K)$ will be greater; and in particular, we must have $Y/y^*(\lambda(s), s, K)$ smaller and $Y/y^*(\lambda(\bar{s}), \bar{s}, K)$ bigger. It follows that $\lambda'(s)$ is greater under automation and $\lambda'(\bar{s})$ smaller, while labor market-clearing implies that we cannot have $\lambda(s)$ the same under automation for any $s$ in $(s, \bar{s})$. Continuity of the matching function then establishes that $\lambda(s)$ is everywhere and the results follow.

C.2.4 Theorem 3

Proof. I begin by noting that in the case where $\rho = 1$ we will have labor demand equal to

$$ L(\lambda(s), s) = \frac{\beta(\lambda(s))Y}{(w(s) + r\kappa[1 - \alpha(\lambda(s))]y^*(\lambda(s), s, K))} $$

Making use of equation (24) it can be shown that

$$ \frac{d}{ds} \log \left( w(s) + r\kappa[1 - \alpha(\lambda(s))]y^*(\lambda(s), s, K) \right) = \frac{y^*_r(\lambda(s), s, K)}{y^*(\lambda(s), s, K)} + \lambda'(s)r\kappa y^*(\lambda(s), s, K) \frac{[1 - \alpha(\lambda(s))]y^*_r(\lambda(s), s, K)}{w(s) + r\kappa[1 - \alpha(\lambda(s))]y^*(\lambda(s), s, K)} - \alpha'(\lambda(s)) $$

(26)

where the last term is strictly negative.

Suppose now that $w(\bar{s})/w(\bar{s})$ is smaller under automation. We must have $w'(s)/w(s)$ and $w'(\bar{s})/w(\bar{s})$ greater, which implies that there exists at least one $s$ for which $w(s)/w(\bar{s})$ is unchanged under automation. For each of the smallest and largest such values of $s$, it must be that $w'(s)/w(s)$ is smaller, and hence that $\lambda(s)$ is strictly lower. But then between the lower of the two $s$ values and $\bar{s}$ we must have average wage higher under automation, while between the higher value of $s$ and $\bar{s}$ we must have average wage higher. Higher wages in the lower region imply a larger value of $\lambda'(s)$, and lower values of wages in the upper region imply a smaller value of $\lambda'(s)$, implying that at the points where $w(s)$ is unchanged we must have $\lambda(s)$ larger, a contradiction.

Now suppose that, for any $r \in (0, \bar{r})$, between any two points $s'$ and $s''$ we have $\lambda(s)$ weakly greater relative to the case where $r = \bar{r}$. Then $w'(s)/w(s)$ must be strictly greater in this interval,
and hence from (25) it must be that almost everywhere in \([s', s'']\) we have \(\lambda(s)\) strictly greater. Suppose without loss of generality that \(\lambda(s)\) is strictly greater in this interval. Then it must be that \(\lambda'(s')\) is greater than before while \(\lambda'(s'')\) is smaller. But since \(w'(s)/w(s)\) is strictly larger between \(s'\) and \(s''\), whereas rental costs decrease proportionally in \(r\). it must be that the sum of wages and rental costs has increased for \(s''\) relative to \(s'\), which from (25) implies that \(\lambda'(s')\) has decreased relative to \(\lambda'(s'')\), a contradiction. Hence over any interval we must have \(\lambda(s)\) smaller almost everywhere, and the result follows.

Now when \(r = 0\), the negative term in (26) drops out and so from the previous result we must have \(\lambda'(s)\) smaller relative to \(\lambda'(\bar{s})\). But then either the first of these terms is strictly smaller, or the second strictly larger, implying in either case that for some subset of \([s, \bar{s}]\) we will have \(\lambda(s)\) strictly smaller under automation, and the result follows.

\[\square\]

\textbf{C.2.5 Continuous model}

Free entry implies that

\[p(j, s) = \frac{w(s) + G(\epsilon^*(s))r\kappa[1 - \alpha(j)]y^*(j, s, \bar{K})\int_0^{\epsilon^*(s)} G'(k)k \, dk}{y^*(j, s, 0) + G(\epsilon^*(s))\left[y^*(j, s, \bar{K}) - y^*(j, s, 0)\right]} .\]

while labor demand (i.e. firm entry) will be

\[L(j, s', s) = \left(\frac{\beta(j)}{p(j, s', s)}\right)^\rho \frac{Y}{y^*(j, s, 0) + G(\epsilon^*(s))\left[y^*(j, s, \bar{K}) - y^*(j, s, 0)\right]}\]

\[= \beta(j)^\rho Y \left(\frac{y^*(j, s, 0) + G(\epsilon^*(s))\left[y^*(j, s, \bar{K}) - y^*(j, s, 0)\right]}{w(s) + G(\epsilon^*(s))r\kappa[1 - \alpha(j)]y^*(j, s, \bar{K})\int_0^{\epsilon^*(s)} G'(k)k \, dk}\right)^{\rho - 1}\]

Optimal choice of skill, on the other hand, implies that

\[w'(s) = \left[G(\epsilon^*(s))p(j) - r\kappa[1 - \alpha(j)]\int_0^{\epsilon^*(s)} G'(k)k \, dk\right] \frac{\partial}{\partial s} y^*(j, s, \bar{K}) + [1 - G(\epsilon^*(s))]p(j) \frac{\partial}{\partial s} y^*(j, s, 0)\]

\[+ \frac{d}{ds} \epsilon^*(s) \frac{\partial}{\partial \epsilon^*} \pi(j, s)\]

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where profit maximization implies that the last term $\frac{\partial}{\partial \epsilon} \pi(j, s)$ is equal to zero. Hence we must have

$$
\frac{w'(s)}{w(s)} = \frac{A(j, s) \frac{\partial}{\partial s} y^*(j, s, \overline{K}) + B(j, s) \frac{\partial}{\partial s} y^*(j, s, 0)}{A(j, s) y^*(j, s, \overline{K}) + B(j, s) y^*(j, s, 0)}
$$

$$
A(j, s) = G(\epsilon^*(s)) p(j) - r \kappa [1 - \alpha(j)] \int_0^\epsilon^*(s) G'(k) k \, dk
$$

$$
B(j, s) = [1 - G(\epsilon^*(s))] p(j),
$$

where since match output is log-supermodular and $\alpha$ is increasing, $\frac{A(j, s) \frac{\partial}{\partial s} y^*(j, s, \overline{K}) + B(j, s) \frac{\partial}{\partial s} y^*(j, s, 0)}{A(j, s) y^*(j, s, \overline{K}) + B(j, s) y^*(j, s, 0)}$ must be increasing in $j$ and therefore $A(j, s) y^*(j, s, \overline{K}) + B(j, s) y^*(j, s, 0)$ will also be log-supermodular. Optimal assignment in the continuous model will be characterized by the differential equations

$$
\frac{w'(s)}{w(s)} = \frac{A(j, s) \frac{\partial}{\partial s} y^*(j, s, \overline{K}) + B(j, s) \frac{\partial}{\partial s} y^*(j, s, 0)}{A(j, s) y^*(j, s, \overline{K}) + B(j, s) y^*(j, s, 0)}
$$

(27)

$$
\lambda'_m(s) = \left( y^*(\lambda(s), s, 0) + G(\epsilon^*(s)) \left[ y^*(\lambda(s), s, \overline{K}) - y^*(\lambda(s), s, 0) \right] \right)^{1-\rho}
$$

$$
\times \left( w(s) + G(\epsilon^*(s)) r \kappa [1 - \alpha(\lambda(s))] y^*(\lambda(s), s, \overline{K}) \int_0^\epsilon^*(s) G'(k) k \, dk \right)^{\rho}
$$

(28)

where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$ as before, $\epsilon^*(s) = \frac{w(s)}{r \gamma_l(s)}$, and where

$$
A(j, s) = G(\epsilon^*(s)) p(j) - r \kappa [1 - \alpha(j)] \int_0^\epsilon^*(s) G'(k) k \, dk
$$

$$
B(j, s) = [1 - G(\epsilon^*(s))] p(j),
$$

In contrast to static model, (27) and (28) will be everywhere continuous.

### C.3 Qualitative Results
Figure C.4: Computer use by occupation wage percentile, 1979-1999

**Figure notes.** Log wage and PC use averaged by 1988 KLDB occupation. Percentiles are time-invariant and reflect 1979 wages. Shaded regions indicate 95% confidence intervals.

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Figure C.5: Computer use by occupation wage percentile, 1999-2018

**Figure notes.** Log wage and PC use averaged by 1992 KLDB occupation. Percentiles are time-invariant and reflect 1999 wages. Shaded regions indicate 95% confidence intervals.
Table C.6: Regression: change in log occupational employment share, high-skill

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<td>PC use</td>
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Table notes. Difference-in-difference regression with occupational employment share as the dependent variable, occupation wage percentiles between .5 and 1. Employment shares calculated from raw survey counts. All regressions include year fixed effects.

Table C.7: Regression: change in log occupational employment share, low-skill

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<td>.557 (.266)</td>
<td>.585 (.262)</td>
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<td>△ PC use</td>
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<td>-.044 (.296)</td>
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Table notes. Difference-in-difference regression with occupational employment share as the dependent variable, occupation wage percentiles between 0 and .5. Employment shares calculated from raw survey counts. All regressions include year fixed effects.
C.4 Quantitative Results

C.4.1 Estimation

To be completed.

Figure C.8: Distributional parameter estimates

C.4.2 Automation, Wages, and Employment

Figure C.9: Automation of skilled task

Figure notes. Smoothed model output over a grid of rental rates $r_s$. Model parameters are unchanged from above and reflect the limiting case where the skill-biased technology is cost-less ($r = 0$).