ESSAYS ON MANDATORY DISCLOSURE AND ENFORCEMENT

Wenjie Xue

Dissertation Committee:

Pierre Jinghong Liang (co-chair)
Carlos Corona (co-chair)
Austin C. Sudbury
Ariel Zetlin-Jones

A thesis presented for the degree of
Doctor of Philosophy
in the field of Industrial Administration (Accounting)

Carnegie Mellon University, May 2019
Acknowledgement

I am indebted to my dissertation committee members (Pierre Jinghong Liang (co-chair), Carlos Corona (co-chair), Austin Sudbury, Ariel Zetlin-Jones) in developing this thesis. The chapters 1 and 2 of this thesis are based on a co-authored paper with Jeremy Bertomeu and Igor Vaysman. I also benefited from the seminar participants at Carnegie Mellon University, University of California San Diego, University of Chicago, University of Texas Austin, National University of Singapore, University of Alberta, ARW conference at the University of Basel, and University of Sydney.
## Contents

1 Efficient disclosure laws .............................................. 6
   1.1 Introduction .................................................. 6
   1.2 Literature review ............................................. 10
   1.3 The Model ..................................................... 12
   1.4 Main result .................................................... 15
   1.5 Further analyses ..............................................
      1.5.1 Verification cost of non-disclosure ................. 20
      1.5.2 Bounded support ....................................... 22
   1.6 Conclusion ................................................... 23
   1.7 Appendix for chapter 1 ....................................... 25

2 Efficient disclosure laws with real effects ........................ 33
   2.1 Introduction .................................................. 33
   2.2 Real effects of information ...................................
      2.2.1 Convex payoff .......................................... 34
      2.2.2 Optimal liquidations .................................. 36
      2.2.3 Market for lemons ..................................... 39
   2.3 Conclusion ................................................... 44
   2.4 Appendix of chapter 2 ....................................... 47

3 Towards a positive theory of regulatory enforcement of financial reporting ........................................... 58
   3.1 Introduction .................................................. 58
   3.2 The model .................................................... 65
      3.2.1 Set-up .................................................. 65
      3.2.2 The reporting subgame ................................ 69
      3.2.3 The regulator’s enforcement decision ............... 71
   3.3 Benchmarks .................................................... 74
      3.3.1 The first-best ........................................... 74
      3.3.2 The commitment problem without coordination .... 76
   3.4 Main results .................................................. 78
      3.4.1 The unique investment equilibrium ................. 78
   3.5 Comparative statics ........................................... 83
      3.5.1 Enforcement intensity and market-size ............ 83
The purpose of this thesis is to address a fundamental question: what explains the nature and scope of the observed disclosure regulation and enforcement in securities markets? The first chapter asks the question: why is it desirable to have disclosure regulation even when sellers of the securities (firms) can credibly and voluntarily disclose information? A theoretical model is developed to show that a law that mandates disclosure of unfavorable events reduces socially excessive voluntary disclosures when the credible disclosure is costly (e.g., verification cost). Absent the optimal mandatory disclosure, firms have incentives to disclose too much because non-disclosure is perceived to be bad news. The efficient law takes the form of a threshold such that only unfavorable events are subject to mandatory disclosure. Both the voluntary and efficient mandatory disclosure increase when information is more precise, or when disclosure costs decrease. Hence, we should expect a positive association between mandatory disclosure and voluntary disclosure from a cross-sectional perspective. The threshold-type regulation is also consistent with various observed accounting standards (e.g., asset impairment).

The second chapter extends the theory in the first chapter by considering various environments in which information per se has real effects (social value). The settings considered include (i) information can facilitate optimal post-sale decision making by the buyers, (ii) information can facilitate optimal liquidation of assets, and (iii) information can prevent market break-down in a “market for lemons”. The optimality of the threshold-type regulation is robust in those settings. The model thus provides a coherent framework
to understand the information environment in securities markets with various insights which are not available in models with the provision of information being exogenous or the mandatory disclosure and voluntary disclosure being considered independently.

The first two chapters examine the disclosure regulation problems while assuming away the enforcement problem (disclosure is truthful although costly). The third chapter develops a positive theory of regulatory enforcement in a multi-firm setting where enforcement and investments are jointly determined by economic fundamentals. The purpose of this theory is to explain (i) why enforcement of securities laws varies significantly across jurisdictions, and (ii) why there is a positive association between enforcement intensity and capital market development. By extending the classic problem that public agency cannot commit to any long-term policy (Kydland and Prescott, 1977) to a heterogeneous-agent setting in which entrepreneurs’ private information of their heterogeneous projects serves as correlated signals of the aggregate state of the economy, the theory explains how the discretionary enforcement policy induces a coordination problem among entrepreneurs (firms) when making investment decisions. The model offers a sharp characterization of the unique equilibrium of the “global game” in which the market can be either over-sized and over-regulated or under-sized and under-regulated depending on the primitives of the financial market. In addition to contributing to the extensive literature of accounting and law enforcement, the theory also adds to the macroeconomics theory by showing a novel mechanism through which a coordination problem can arise in economic development.
Chapter 1

Efficient disclosure laws

1.1. Introduction

Should accounting standards force firms to disclose bad news? Many facets of accounting standards answer this question positively; examples include imposing impairments on assets having lost value and deferring recognition on the balance sheet of various transactions (research activities, revenue, etc.) that carry value but are not yet complete. Falling under the broad concept of conservatism (Moonitz, 1951), asymmetric reporting practices emphasize, all other things being equal, reporting of unfavorable news. Recently, however, the convergence of accounting standards across the globe challenges the role of asymmetric reporting, with international accounting standards emphasizing a more symmetric recognition of news. Considerations of the market benefits of asymmetric financial reporting have not yet been fully incorporated into these debates.¹

This study examines optimal disclosure regulation when sellers can use a voluntary, but costly, disclosure technology (Aghamolla and An, 2015; Jorgensen and Kirschenheiter, 2015, while the terminology of conservatism is proper to accounting research, the debates are broader than only financial reporting. Regulations that organize what type of news must be disclosed are ubiquitous, especially with respect to news with potentially negative content: pharmaceutical companies must test drugs and disclose the results of clinical trials and drugs’ side-effects (Ma, Marinovic and Karaca-Mandic, 2015); the Affordable Care Act mandates quality reporting by hospitals (Dranove and Jin, 2010); and federal education initiatives, and many states’ laws, require schools to disclose performance statistics.

¹While the terminology of conservatism is proper to accounting research, the debates are broader than only financial reporting. Regulations that organize what type of news must be disclosed are ubiquitous, especially with respect to news with potentially negative content: pharmaceutical companies must test drugs and disclose the results of clinical trials and drugs’ side-effects (Ma, Marinovic and Karaca-Mandic, 2015); the Affordable Care Act mandates quality reporting by hospitals (Dranove and Jin, 2010); and federal education initiatives, and many states’ laws, require schools to disclose performance statistics.
2015; Jovanovic, 1982; Marinovic and Varas, 2016; Verrecchia, 1983). With voluntary disclosure an option, the existence of reporting laws is theoretically puzzling given that markets efficiently price non-disclosure and firms internalize price consequences after a non-disclosure: in models where acquisition or distribution of information is costly, voluntary disclosure tends to exhibit over-provision of information (Bertomeu and Cianciaruso, 2015; Shavell, 1994).

We ask three questions. When does a seller benefit ex-ante, i.e., obtains a higher market value, by being bound to a mandatory disclosure before receiving information? Are there any characteristics of optimal mandatory disclosures that make them different from voluntary disclosure, in particular, as to asymmetric consideration of good versus bad news? Lastly, what are the determinants of the desirable scope for mandatory disclosure? In summary, our model provides a framework addressing why and how we should design financial reporting rules.

Our model is an extension of costly voluntary disclosure theory under the assumptions that (i) the law can pre-commit firms to disclosing a subset of events, and (ii) voluntary and mandatory disclosures carry the same costs. We elaborate here in slightly more detail about a possible interpretation of the information and disclosure process. The firm receives a piece of soft unverified information, which at some cost - the preparation of a record, documentation, and audit - can be entered into the accounting system (as a transaction), becoming hard. We refer to the law as the rules that state which information should be considered as a transaction and which information should not be a part of the accounting system. Making information hard and disseminating it credibly is costly and involves the use of accounting and legal teams, audits, investor relations, and certification bodies such as banks or rating agencies (more generally, information intermediaries); we represent this in reduced form as a cost when the firm submits verifiable information to the market.

---

2 Of course, there are also pieces of information that cannot be made credible and these are not the focus of this study; see Bertomeu and Marinovic (2015) for a discussion of voluntary reporting of "soft" news.
A key assumption in our model is that the law cannot censor voluntary disclosures or control voluntary disclosure costs. A firm may voluntarily leak information to the market using alternative communication channels that are not controlled by the regulator. As an example, consider debt ratings and suppose, hypothetically, that regulators deem that debt ratings do not have social value and prohibit the practice of rating debt. The market would then engineer other ways to certify debt, possibly in the form of an announcement of a private placement with an investor with expertise. For another example, consider the market for audits. A subset of private firms voluntarily submit to costly audits (Minnis, 2011). Accounting standards govern formal reporting channels: standards prescribe who must disclose and the scope of disclosure. But accounting standards do not effectively control disclosures via alternative channels and firms’ various certification actions.\(^3\)

We demonstrate that laws that only mandate disclosing all sufficiently unfavorable news increase the expected net selling price. Surprisingly, this result holds regardless of productive benefits of disclosure (in section 2.2, we also show that the value of asymmetric mandatory disclosure is magnified in a general model with real effects). Rather, mandatory disclosure of unfavorable news is valuable because it reduces the level of socially-excessive voluntary disclosure of intermediate news. The intuition for this result has three components. First, consistent with Shavell (1994), a voluntary disclosure equilibrium features excessive disclosure because disclosers do not internalize the negative externality of their own disclosure of good news on the non-disclosure price.\(^4\) Second, a lower non-disclosure price, reflecting greater skepticism (Rappoport, 2017), magnifies excessive voluntary disclosure as firms with better news incur the disclosure cost to differentiate themselves.

\(^3\)Note also that, in the extended version of the baseline model with real productive uses of information (section 2.2), mandating no-disclosure would be socially undesirable and optimal disclosure prohibitions would have to censor conditional on the information known to the firm. We use the pure-exchange model to make the case that the role of mandatory disclosure in our approach is not just caused by productive benefits of information - for which there are various modelling choices that could be made. As we show in section 3.1, productive benefits actually magnify the desirability of mandatory disclosure over bad news.\(^4\)This feature is immediate without productive decisions given that disclosures are costly, but the intuition holds more generally in the context of productive decisions where some disclosure is desirable. In section 3.1, we show that the same results hold with productive effects.
Third, bad news, withheld without regulation, decrease the non-disclosure price the most. Put together, mandatory reporting over bad news provides the maximum reduction in socially-excessive voluntary disclosure.

Our theory applies to reporting events, in which a mandatory disclosure is conditional on the realization of certain events, so we would see variation in disclosure policies, i.e., line items in financial statements, recognition of a particular transaction, or special filings such as significant events (8K). An application is in the context of asymmetries in financial accounting standards: impairment of an asset when certain conditions indicating a decrease in value are met; recognition of contingent liabilities; the immediate expensing of investments in intangibles (such as R&D, marketing investments, and personnel training), jointly with deferrals of their associated future revenues; and a wide span of voluntary forecasts and other news releases. In the model, accounting standards should mandate more provision of information over potentially adverse news while leaving it to voluntary disclosure channels to report favorable information.

We also provide a simple theoretical foundation to explain why a greater degree of certainty is required to report relatively better news. As the news known to the manager becomes more precise, relative to what the market initially knows, the mandatory disclosure threshold increases, increasing the span of news subject to mandatory disclosure. In other words, a greater level of certainty is required for given news to be subject to mandatory disclosure.

The theory has further implications for the statistical relationships between prices, news, and the channels through which the news is reported. Our main result requires few assumptions on the distribution of news, but additional properties can be derived under plausible assumptions. Consider single-peaked distributions of news; this would be implied

\footnote{We do not intend the model as a representation of the release of entire financial statements (e.g., an earnings announcement), because financial statements are aggregated across many transactions: if some transactions do not involve cost, we would always observe some earnings number including at least for the transactions that do not involve costs.}
by logconcavity of the density function, satisfied by most of the common distributions (Bagnoli and Bergstrom, 2005). In this case, the mandatory disclosure threshold is below the peak, while all voluntary disclosure is above the peak. Events near the peak are not disclosed. These "near-peak" events have the least effect on posterior beliefs, and it is suboptimal to incur disclosure costs to reveal them.

Interestingly, this property implies a new form of unravelling. As the cost of disclosure becomes small, both the voluntary and the mandatory disclosure thresholds converge to the peak so that, indeed, the probability of disclosure converges to one; but the last to be disclosed are the "near-peak" events, unlike the extremely negative news in classic unravelling theory (Milgrom, 1981). We also show that, if the distribution is sufficiently symmetric (including in the special case of normals), the unconditional probability of mandatory disclosure is smaller than the unconditional probability of voluntary disclosure. Both mandated disclosure and non-disclosure trigger negative expected price reactions, while voluntary disclosure provides positive expected price reactions.

1.2. Literature review

To our knowledge, few studies have shown that pre-commitments are desirable in the context of communication games. Like us, Heinle and Verrecchia (2015) examine a model in which firms can pre-commit not to issue costly biased signals (e.g., a news release or a forecast); our approach differs from their model: in our analysis, firms commit to issue more information. In Jiang and Yang (2016), the firm pre-commits to an information system, such that the information carried by the information system cannot exceed an entropy constraint. The recent studies by Gao (2015) and Armstrong, Taylor and Verrecchia (2015) analyze different aspects of asymmetric information systems. Their research questions are different from ours: they consider the benefits of information systems, while our focus is on the trade-offs between private and public provision of information. In the first paper, the
asymmetric verification of good reports improves the efficiency of debt contracts. In the second paper, a greater precision on low outcomes improves the efficiency of risk-sharing in competitive markets. The recent study by Friedman, Hughes and Michaeli (2018) shares with us the interaction between mandatory and voluntary disclosure. Their setting and research questions are different from ours: their focus is on the interaction between a regulator’s public-information-system design decisions and a firm’s private-information-acquisition strategy.

While the friction in our model is a costly voluntary disclosure, the recent study by Glode, Opp and Zhang (2018) examines a post-disclosure auction where both parties have bargaining power. They show that a suitably-chosen partial disclosure can maximize the revenue of the seller and implement the efficient allocation. Another related study is Huddart, Hughes and Brunnermeier (1999) who examine the optimal choice by private parties (exchanges) to set up mandatory disclosure requirements to maximize revenues; however, their focus is not on the interaction between mandatory and voluntary disclosure channels, and so both the objectives and the trade-offs are different from ours.

Several other studies have analyzed the effects of mandatory disclosure. Einhorn (2005) shows that the nature of mandatory disclosure can change the interpretation of a given voluntary disclosure signal as good or bad news. Gao (2015) assumes that the manager has access to a given set of distribution of reports and will choose the reports as a function of the binary classification imposed by the standard. In Bertomeu and Magee (2015), voluntary disclosure reduces political demands for more mandatory disclosure. The substitution between voluntary and mandatory disclosure is also implicit in Verrecchia (1990), in that a more informative information environment (due to, say, more transparent mandatory disclosures) is equivalent to a reduction in the dispersion of the manager’s private information and thus causes less disclosure. In this area, a study related to ours is Heinle, Samuels and Taylor (2017) who, like us, consider an environment in which a mandated signal can reduce the cost to be incurred in a latter voluntary stage. Their focus is different, however, in
that the mandatory and voluntary components in their model are about different elements of information and may carry different costs, both aspects that we do not have in our model. Our emphasis, by contrast, is on the design of the efficient mandatory-disclosure law that requires certain events to be reported, even though these same events could have been reported voluntarily.

1.3. The Model

For expositional purposes, we use the analogy of a seller (firm) of an asset in a competitive market. The timeline is as follows. At date 1, the seller privately observes a signal $\tilde{s}$ regarding the asset value $\tilde{y}$ which implies a posterior expectation $\tilde{\pi} = E(\tilde{y} | \tilde{s})$, drawn from a distribution with continuously-differentiable p.d.f. $f(.)$ and c.d.f. $F(.)$ with support $\mathbb{R}$ and finite mean. Assume that the c.d.f. is strictly log-concave, and the p.d.f. is single-peaked with its peak (or mode) denoted $m$.

At date 2, a public signal $r(x) \in \{x, nd\}$ is issued, where $r(.)$ is a function such that $r(x) = x$ is interpreted as a verifiable report that $\tilde{x} = x$, and $r(x) = nd$ is a non-disclosure. At date 3, conditional on the report $r$, the asset is sold for a price $P(r)$, and the seller achieves a payoff $P(r) - 1(r \neq nd)c$ net of a verification cost $c > 0$ (Verrecchia, 1983).

Note that it is important for our analysis that at least some of the cost is a social cost and carries a deadweight loss (e.g., verification or certification, excess competition that deters entry or investment, or wealth transfers to foreign companies that are not internalized by a representative regulator). If the entire cost were redistributive, that is, fully recovered

---

6Strict log-concavity distribution ensures that we can characterize the problem using the first-order condition, an assumption that plays a similar role as the monotone likelihood ratio property (MLRP) for using the first-order approach in Rogerson (1985). Specifically, we need to ensure that the voluntary disclosure threshold under consideration is continuous in the mandatory disclosure rule. The single-peakedness assumption ensures that the characterizing equation uniquely identifies the optimal solution.

7In the benchmark model, there is no cost to verify that a firm is not subject to mandatory disclosure, as this verification occurs ex-post, when private information is public. Below, in section 1.5.1, we consider the possibility that verifying that a firm is not subject to mandatory disclosure is costly. Also, while our benchmark analysis considers constant disclosure cost $c$, the main results hold if the cost is a non-decreasing function $c(x)$, as long as $[x - c(x)]$ is increasing (the proof of this is available upon request).
by another party, then optimal regulations would require full disclosure.\textsuperscript{8}

The price $P(r)$ is a function of the buyers’ rational expectations about the value of the asset given the buyers’ information set. We initially focus on the simplest pure-exchange setting, with $P(x) = x$, and, when required by Bayes rule, $P(nd) = \mathbb{E}(\tilde{x} | r(\tilde{x}) = nd)$.\textsuperscript{9} The reporting function $r(\cdot)$ is determined by two disclosure channels. First, the law prescribes that all $x \in D_m$ must be reported, where $D_m$ is a finite union of open intervals. Hence $r(x) = x$ if $x \in D_m$. Second, for any $x \notin D_m$, the seller chooses to report voluntarily or withhold, so that $r(x) \in \{x, nd\}$ maximizes $P(r) - 1(r \neq nd)c$, withholding when indifferent.

We next formalize the definition of a voluntary disclosure equilibrium in the presence of mandatory disclosure.

**Definition 1** For any mandatory-disclosure law $D_m$, an equilibrium is denoted $E(D_m) = (P, r)$ with two functions $P : \mathbb{R} \cup \{nd\} \to \mathbb{R}$ and $r : \mathbb{R} \to \mathbb{R} \cup \{nd\}$ that satisfy:

(i) the seller withholds when it is in accordance with the law and would yield a higher price, that is, $r(x) = nd$ if and only if $x \notin D_m$ and $P(nd) \geq P(x) - c$;

(ii) whenever possible, Bayes rule applies, that is, $P(nd) = \mathbb{E}(\tilde{x} | r(\tilde{x}) = nd)$ and $P(x) = x$.

Our objective is to characterize the properties of an efficient disclosure law, that is, a law that maximizes the ex-ante surplus of the seller.

**Definition 2** $D_m$ is an efficient disclosure law if there exists an equilibrium $E(D_m) = (P, r)$ such that

\textsuperscript{8}We employ a model of costly disclosure instead of the alternative approach involving uncertainty about information endowment, because it is non-trivial to think about forcing firms to disclose in a model where managers cannot credibly prove that they informed, see, e.g., Ebert, Simons and Stecher (2014) for a recent example. An alternative approach is in Dye (2017), which requires a formal model of verification by a fact checker; we conjecture that many insights in the current model would carry over to a related setting in which the firm has access to a costly fact-checking technology.

\textsuperscript{9}In section 2.2 we extend our analysis to a more general setting, in which $P(r)$ is a convex function of posterior expectations which implies that information has social value.
(i) for any other law $D_m^a$ and any equilibrium $\mathcal{E}(D_m^a) = (P^a, r^a)$, $\mathbb{E}[P(r(\hat{x})) - 1(r(\hat{x}) \neq nd)c] \geq \mathbb{E}[P^a(r^a(\hat{x})) - 1(r^a(\hat{x}) \neq nd)c]$;

(ii) for any $x$ such that $P(x) - c > P(nd)$, it holds that $x \notin D_m$.

Part (i) states that an efficient disclosure law provides a higher total surplus relative to any other equilibrium that could be achieved with a different law. Disclosure games with costly disclosure can have multiple equilibria, but there is a unique equilibrium that is weakly preferred by all sellers regardless of their information. By construction, the efficient disclosure law must attain the highest surplus within this preferred equilibrium.

Part (ii) is imposed assures that mandatory disclosures are a constraining legal requirement, over news that the seller would not disclose voluntarily. This implies that we can hereafter restrict attention to $\sup D_m < \infty$ since all sufficiently favorable events will be disclosed voluntarily.

Before proceeding to our main result, we point to a few key assumptions in our research design:

- **Seller regulation preference.** Our analysis pertains only to seller-preferred regulations, as we do not explicitly model welfare consequences to other parties that do not make strategic decisions. In the special case of a perfectly competitive market with homogenous price-protected buyers, competitive buyers price the asset at its expected value. Thus, buyers would be indifferent to disclosure. More generally, the effect of information in product markets has been the object of an extensive prior literature (Friedman, Hughes and Saouma, 2016; Suijs and Wielhouwer, 2018; Wagenhofer, 1990) that falls somewhat beyond our current objective and, unfortunately, is a non-trivial endeavor without deeper knowledge of the market structure.

- **Perfect enforcement.** We focus only on the disclosure choice (which events will be reported) and assume that the law is perfectly enforceable. This is an important
reason to focus on costly disclosure, as models where the seller can claim to be un-informed (Dye, 1985) require more structure on the ex-post verification game, with interactions between disclosure and enforcement. For example, private information may be revealed ex-post, and sellers who did not follow the law are penalized. Later, we explore a richer version of the model in which sellers incur a non-disclosure verification cost, the magnitude of which depends on the scope of the disclosure regulation (see Section 4.1).

- Disclosure technology. A key assumption in our setting is that the regulator does not directly control all communication channels through which firms’ disclosures may occur. In particular, the law cannot forbid voluntary disclosures since, for most practical settings, a seller could leak information through alternative channels. Similarly, the regulator cannot arbitrarily increase voluntary disclosure costs without creating incentives for sellers to use cheaper alternative unmonitored channels to make their voluntary disclosures. We thus view the law as requiring disclosure within a particular regulated channel. In addition, mandatory disclosures may, in practice, be more or less costly than voluntary disclosure. To focus on non-cost considerations, and to make a fair conceptual comparison between the two disclosure channels, we abstract away from any technological advantage or disadvantage of voluntary disclosure, and assume that the cost incurred is the same for both channels.

1.4. Main result

It is convenient to rewrite the voluntary disclosure strategy in terms of a threshold above which news is voluntarily disclosed.\(^{10}\) Given an arbitrary mandatory disclosure set

\[ \{ \tilde{x} | \tilde{x} \notin D_m \} \]

A complication that arises in our setting is that \( \{ \tilde{x} | \tilde{x} \notin D_m \} \) may not be log-concave even if \( \tilde{x} \) is log-concave, so that a voluntary disclosure equilibrium is not unique. Technically, when \( D_m \) is a lower interval, that is, at the optimal solution, the voluntary disclosure game will satisfy logconcavity because truncation preserve logconcavity (Bagnoli and Bergstrom, 2005), so that uniqueness of the voluntary disclosure equilibrium holds at the solution. However, suboptimal choices of \( D_m \) (such as when \( D_m \) is not
$D_m$, an equilibrium $E$ features a voluntary disclosure threshold $\tau(D_m|E) \in \mathbb{R}$ such that all sellers with $x > \tau(D_m|E)$ disclose voluntarily. Since it is optimal to disclose if and only if $x - c > P_{D_m,E}(nd)$, where $P_{D_m,E}(nd)$ is the non-disclosure price in the voluntary disclosure equilibrium $E$ given the law $D_m$, one can rewrite this threshold in terms of a standard indifference condition, for the marginal discloser at $x = \tau(D_m|E)$, equating payoffs from disclosing and from non-disclosing:

$$\tau(D_m|E) - c = \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq \tau(D_m|E)).$$  \hspace{1cm} (1.4.1)

If the solution $\tau(D_m|E)$ exists and is unique, we denote it as $\tau(D_m)$. If the solution is not unique, the largest threshold is associated with the lowest expected disclosure cost. With a slight abuse in notation, we then denote $\tau(D_m) = \max_E \tau(D_m|E)$ as the seller-preferred voluntary disclosure threshold.\footnote{The equilibrium that achieves the lowest disclosure cost is well-defined as long as an equilibrium exists. The reason is that the function $t - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t)$ is continuous in $t$, so that for any convergent sequence $\{t_i\}$ that satisfies $t_i - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t_i) = 0$ for all $i$, its limit $t_\infty$ satisfies $t_\infty - c - \mathbb{E}(\tilde{x}|\tilde{x} \notin D_m, \tilde{x} \leq t_\infty) = 0$ and, therefore, $t_\infty = \max_E \tau(D_m|E)$.}

We focus next on the form of the efficient disclosure law $D_m^*$. The ex-ante surplus of the seller $W(D_m) = \mathbb{E}[P(r(\tilde{x})) - 1(r(\tilde{x}) \neq nd)]$ is the expected price net of disclosure costs. The law of iterated expectations implies that the expected price is the expected news $\mathbb{E}[P(r(\tilde{x}))] = \mathbb{E}(\tilde{x})$, so that an efficient law maximizes

$$W(D_m) = \mathbb{E}(\tilde{x}) - c \int_{D_m \cup \{\tau(D_m), \infty\}} f(x)dx,$$

which amounts to minimizing the probability of disclosure.

Mandatory disclosure thus presents a trade-off. Expanding the mandatory disclosure set $D_m$ directly increases the disclosure cost for any event that was not disclosed; indirectly,
however, expanding $D_m$ can increase the voluntary disclosure threshold $\tau(D_m)$, reducing disclosure costs. The next lemma shows that this trade-off is best resolved with a threshold mandatory disclosure law: the efficient law can be fully characterized as a threshold $\theta^*$ below which events are subject to mandatory disclosure.

**Lemma 1** An efficient disclosure law $D_m^*$ has the form $(-\infty, \theta^*)$, where $\theta^* \in \mathbb{R}$.

Lemma 1 simplifies the characterization of the voluntary disclosure threshold. Rewriting (1.4.1) with $D_m = (-\infty, \theta)$, the equilibrium voluntary disclosure threshold $\tau$ makes the marginal voluntary disclosers indifferent between disclosing and not disclosing, satisfying

$$\Gamma_1(\tau, \theta) = 0$$

where

$$\Gamma_1(t, \theta) \equiv \int_{\theta}^{t} f(x) dx - (t - c) \int_{\theta}^{t} f(x) dx.$$ (1.4.2)

Define the function $\tau_{\theta} \in \mathbb{R}$ as the unique solution in $t$ to $\Gamma_1(t, \theta) = 0$ for any $\theta \in [-\infty, \infty)$. For any mandatory disclosure threshold $\theta \in [-\infty, \infty)$, we use $P_{\theta}(nd) = \tau_{\theta} - c$ to denote the equilibrium non-disclosure price.

The next proposition is our main result: an efficient mandatory disclosure law requires the disclosure of sufficiently unfavorable events.

**Proposition 1** There exists an efficient disclosure law $D_m^* = (-\infty, \theta^*)$ such that $\theta^* \in \mathbb{R}$ is uniquely given by

$$F(\tau_{\theta^*}) - F(\theta^*) - f(\tau_{\theta^*})(\tau_{\theta^*} - \theta^*) = 0.$$ (1.4.3)

Proposition 1 builds on the following economic intuition. A low non-disclosure price induces excessive voluntary disclosure and disclosure cost. To increase the non-disclosure price and reduce buyers’ “skepticism” and disclosure cost, the efficient law filters out unfavorable news. The more unfavorable the news, the more it decreases the non-disclosure price. Indeed, in the limit, an infinitely unfavorable event has an infinite marginal effect.

---

12Uniqueness follows directly from strict logconcavity of c.d.f. The distribution truncated below at $\theta$ also has this property because the c.d.f. of the truncated distribution is a linear transformation of the original c.d.f. (Bagnoli and Bergstrom, 2005).
on the non-disclosure price and, therefore, would be disclosed at any finite cost. It then follows mandatory reporting requirements over sufficiently unfavorable events are socially valuable.\footnote{As can be seen from the intuition here, the main result that the mandatory disclosure set is a non-empty lower interval does not depend on the assumption that the c.d.f. is strictly log-concave and the p.d.f. is single-peaking. Please see Note 1 in the Appendix for details.}

To expand on this intuition, we plot in Figure 1 the two thresholds for the special case of the Normal distribution. The relative placements of the mandatory and voluntary disclosure thresholds have a geometric interpretation that can be used to derive additional properties. For any $\theta^*$, the voluntary disclosure threshold $\tau_{\theta^*}$ that satisfies (1.4.3) is obtained at the point where the line that crosses $(\theta^*, F(\theta^*))$ is tangent to the c.d.f. of the news. For any placement of $\theta^*$, this tangency point is always above the peak of the distribution. We generalize this property to any strictly log-concave distribution with peak $m$ in the next corollary.

Figure 1.1: Mandatory and voluntary thresholds with standard Normal and $c = 1$.

**Corollary 1** The mandatory disclosure threshold $\theta^*$ satisfies $-\infty < \theta^* < m < \tau_{\theta^*}$. If the distribution of $\tilde{x}$ is symmetric, the probability of voluntary disclosure is always greater than the probability of mandatory disclosure.
The disclosure thresholds $\theta^*$ and $\tau_{\theta^*}$ are located at each side of the peak of the distribution. That is, voluntary disclosures are bounded from below by the peak for any non-zero cost. By contrast, absent efficient mandatory disclosure, the voluntary disclosure threshold becomes arbitrarily small and the voluntary disclosure equilibrium unravels to full disclosure as the disclosure cost becomes small (Verrecchia, 1983).

To reconcile this observation to unravelling theory, let us examine the two thresholds as the cost $c$ becomes small. The unravelling theorems imply that $\tau_{\theta^*} - \theta^*$ converges to zero for any given $\theta$, as all events that are not subject to the law are voluntarily disclosed. Hence, in equation (1.4.3), $F(\tau_{\theta^*}) - F(\theta^*)$ converges to zero which implies that both $\tau_{\theta^*}$ and $\theta^*$ must converge to the peak $m$. The equilibrium features unravelling, but via both the voluntary and the mandatory disclosure channels.

**Corollary 2** The optimal mandatory disclosure threshold $\theta^*$ decreases in disclosure cost $c$; the equilibrium voluntary disclosure threshold $\tau_{\theta^*}$ increases in disclosure cost $c$.

An interpretation of the cutoff $\theta^*$ is in terms of a level of conservatism, since events below the cutoff typically yield unfavorable market reactions and increasing this threshold implies a wider disclosure over these unfavorable events. In other words, firms with higher levels of disclosure costs should be less conservative in their enforcement of disclosures of bad news.

Next, we consider how changes in the variance of the distribution of $\tilde{x}$ affect mandatory and voluntary disclosure, extending the comparative static of Verrecchia (1990) to efficient mandatory disclosure laws, in the special case where $F(.)$ is Normal.

**Corollary 3** With $\tilde{x} \sim N(\mu, \sigma^2)$, the unconditional probabilities of mandatory and voluntary disclosures both increase in the variance $\sigma^2$.

Recall that we conduct most of our analysis using the induced posterior expectation $\tilde{x}$ – the expected cash flow conditional on observing the private signal. To interpret Corollary 3
in terms of an information structure, it is helpful to return to the fundamental information structure: recall that \( \tilde{x} = \mathbb{E}(\tilde{y}|\tilde{s}) \), where \( \tilde{y} \sim N(\mu, \sigma_y^2) \) is the future cash flow, and \( \tilde{s} \) is the signal privately observed by the seller. Suppose \( \tilde{s} = \tilde{y} + \tilde{\epsilon} \), with i.i.d. noise \( \tilde{\epsilon} \sim N(0, \sigma_{\epsilon}^2) \).

Because there is a one-to-one mapping between \( \tilde{s} \) and \( \tilde{\epsilon} \), disclosing the signal is equivalent to disclosing the posterior expectation. Then the variance \( \sigma^2 \) is given by

\[
\sigma^2 = \operatorname{Var}(\mathbb{E}(\tilde{y}|\tilde{s})) = \frac{\sigma_y^4}{\sigma_\epsilon^2 + \sigma_y^2}.
\]

(1.4.4)

Hence, Corollary 3 states that events that tend to generate greater variation in cash flows (higher \( \sigma_y^2 \)) will tend to feature more mandatory disclosure. Also, the probability of mandatory disclosure increases as the signal becomes more precise, that is, \( \sigma_\epsilon^2 \) decreases. In other words, for a given economic cash flow risk \( \sigma_y^2 \), the law requires more disclosure when the signal is closer to fundamentals. Hence, the efficient standard should only mandate accounting for transactions which convey sufficiently precise information about future cash flows.

1.5. Further analyses

1.5.1. Verification cost of non-disclosure

In the benchmark model, verification costs are incurred only by disclosing firms because verification of non-disclosure occurs ex-post, when private information is publicly revealed. As in Bertomeu and Magee (2015), we can alternatively assume that it is costly to verify that a type is not subject to mandatory disclosure. We denote the verification cost of proving that a non-disclosure does not violate the law by \( H(\operatorname{Prob}(\tilde{x} \in D_m)) \in [0, c] \), assumed to be increasing in the probability of mandatory disclosure. The reason why the verification cost of non-disclosure (the type not being in \( D_m \)) should be lower than that of the disclosure (the exact type) is because less information is verified.
Lemma 1 readily holds under this specification of the cost function because all events contribute equally to the total disclosure cost per unit of probability mass. Thus, the mandatory disclosure is again a threshold $\theta$ below which firms must disclose, and the total cost of non-disclosure can be written $h(\theta) \equiv H(\text{Prob}(\tilde{x} < \theta))$.

In this case, the efficient law’s threshold $\theta$ minimizes

$$C(\theta) = c(F(\theta) + (1 - F(\tau_{\theta}))) + h(\theta)(F(\tau_{\theta}) - F(\theta)), \quad (1.5.1)$$

where $\tau_{\theta}$ is given by

$$\int_{\theta}^{\tau_{\theta}} xf(x)dx - (\tau_{\theta} - c + h(\theta)) \int_{\theta}^{\tau_{\theta}} f(x)dx = 0. \quad (1.5.2)$$

The additional term $h(\theta)$ captures verification cost of non-disclosure, i.e., the cost to confirm that a non-disclosing firm is compliant with the law. It affects the equilibrium via two channels. First, the non-disclosure cost reduces total surplus by imposing this cost on non-disclosers, as seen in the second term in the right-hand side of (1.5.1). Second, the non-disclosure cost reduces the incremental disclosure cost to $c - h(\theta)$ in (1.5.2) and provides additional incentives to disclose.

Our main result – the efficient disclosure law features a non-empty lower interval – holds in this setting, as formalized in the following proposition.

**Proposition 2** If the verification cost of non-disclosure $H(\text{Prob}(\tilde{x} \in D_{m}))$ is an increasing function and bounded above from the cost of disclosure $c$, the efficient disclosure law is of the form $D_{m}^{*} = (-\infty, \theta^{*})$. If $\lim_{\theta \to -\infty} \max(h(\theta), h'(\theta)) = 0$, the mandatory disclosure set is not empty, that is, $\theta^{*} > -\infty$.

The optimality of mandatory disclosure in the model with non-disclosure costs requires an additional condition. The first part of that condition is mild, as $\lim_{\theta \to -\infty} h(\theta) = 0$ simply states that there would be no verification cost when there is almost nothing that
must be disclosed. The second part of the assumption is more restrictive. The condition 
\[ \lim_{\theta \to -\infty} h'(\theta) = 0 \]
states that, starting for low levels of verification, it is not very costly to slightly increase the scope of verification. These assumptions hold, for example, if we specify the cost as a power function of the probability of disclosure.

### 1.5.2. Bounded support

If \( \bar{x} \) has a support bounded from below \([x, \infty)\), with \( x > -\infty \), it is possible that no event may be sufficiently unfavorable to be subject to efficient mandatory disclosure, and thus the efficient disclosure law would prescribe a zero probability of mandatory disclosure. As we will show next, there are known classes of distributions with bounded support in which the economy would rely exclusively on voluntary disclosure.

**Proposition 3** If the support of \( \bar{x} \) is \([x, +\infty)\), the optimal mandatory disclosure threshold \( \theta^* \) satisfies \( \theta^* > x \) and is uniquely given by equation (1.4.3), if and only if
\[
F(\tau_x) - (\tau_x - x)f(\tau_x) < 0.
\]

Because \( \tau_\theta \) is decreasing in \( c \), we can restate the above condition as an upper bound on disclosure cost: define \( \overline{c} \) as the solution to
\[
F(\tau_x) - (\tau_x - x)f(\tau_x) = 0 \quad \text{if the solution exists,}
\]
or \( \overline{c} = 0 \) otherwise. Then mandatory disclosure is efficient if and only if \( c < \overline{c} \). That is, mandatory disclosure is desirable provided the disclosure cost is sufficiently small.

Interestingly, there are well-known classes of bounded distributions for which no mandatory disclosure is efficient regardless of the disclosure cost (i.e., \( \overline{c} = 0 \)). Recall that the efficient disclosure threshold is below the peak of the distribution; then, if the distribution has a non-increasing density (e.g., the exponential distribution or some Beta distributions including the Uniform), Proposition 3 implies that \( \theta^* = \bar{x} \), and the efficient law features a zero probability of mandatory disclosure.

Similar intuition applies when \( \bar{x} \) is bounded from above by \( \overline{x} \), except that it is now the voluntary disclosure region that may be empty. As long as the solution to equation (1.4.3)
involves some voluntary disclosure, that is, $\tau_{\theta^*} < \bar{x}$, the solution $\theta^*$ remains the efficient mandatory disclosure threshold. Otherwise, the efficient voluntary disclosure threshold is $\tau_{\theta^*} = \bar{x}$.

1.6. Conclusion

A fundamental question in accounting research is why regulations force public firms to make disclosures, even though firms bear the negative price consequences of withholding and may disclose voluntarily. Before the Securities and Exchange Commission was established in 1934 in the US, state laws granted firms wide discretion to decide which information would be reported. For the last century or so, various arguments have been made against the growing encroachment of regulations into disclosure, a self-regulated activity disciplined by markets and not obviously within the scope of government regulators (Zeff, 2002).

In this paper, we provide analysis that gives credit to this argument in an environment where there is no obvious non-tradeable externality, and firms can do voluntarily, at the same cost, what regulations require. In all settings that we explore, we show that the market outcome features both excess disclosure of good news and insufficient disclosure of bad news. A regulator cannot easily address the problem of excess disclosure if it cannot control all means of communication available to firms, so forbidding disclosure is not a feasible solution. However, regulators can mandate more disclosure of bad news, with required filings containing a minimum amount of information for certain verifiable adverse events, to directly increase the non-disclosure price and to indirectly reduce excess disclosure of good news. This regulatory role points to a surprising role for regulation:

\footnote{Although the externality argument is a clear reason for intervention (Dye, 1990), it is not entirely uncontroversial. Information externalities are not as obvious as pollution or health externalities, and their economic significance is still the object of ongoing research. Theoretically, information intermediaries may act to collect and trade information, and in a world where such externalities are significant, the market will find solutions to disseminate information (i.e., firms will sell their information or pool their information into industry associations).}
assuming a voluntary disclosure mechanism, the role of mandatory disclosure is not necessarily to increase the amount of information available to investors but to dissuade the excess signalling costs that would occur in an unregulated market.
1.7. Appendix for chapter 1

Proof of Lemma 1: Let \( D^*_m \) be an efficient disclosure law with \( \tau \equiv \tau(D^*_m) \) and \( \lambda \equiv \{ x : x > \tau \} \). Suppose that there exists \( (a_1, a_2) \subset D^*_m \) where \( -\infty < a_1 < a_2 \leq \tau \) and \( (-\infty, a_1] \notin D^*_m \). Below, we construct an alternative disclosure set with a lower probability of disclosure.

Define \( \epsilon_1, \epsilon_2 > 0 \) such that (i) \( a_1 + \epsilon_2 < a_2 \) and (ii) \( F(a_1 + \epsilon_2) - F(a_1) = F(a_1) - F(a_1 - \epsilon_1) \) and an alternative mandatory disclosure set \( D'_m \) by \( D'_m = D^*_m \cup (a_1 - \epsilon_1, a_1) \setminus (a_1, a_1 + \epsilon_2) \).

By construction, \( t - c - \mathbb{E}(\tilde{x} | \tilde{x} \leq t, \tilde{x} \notin D'_m) < t - c - \mathbb{E}(\tilde{x} | \tilde{x} \leq \tau, \tilde{x} \notin D^*_m) = 0 \). Moreover, \( t - c - \mathbb{E}(\tilde{x} | \tilde{x} \leq t, \tilde{x} \notin D'_m) \) is positive for \( t > \mathbb{E}(\tilde{x} | \tilde{x} \notin D'_m) + c \). Hence, \( \tau' = \tau(D'_m) \) exists by intermediate value theorem, and it is true that \( \tau' > \tau \). Denote \( \lambda' \equiv \{ x : x > \tau' \} \). It then follows that

\[
\int_{x \in D'_m \cup \lambda'} f(x) dx = \int_{x \in D^*_m \cup \lambda} f(x) dx < \int_{x \in D^*_m \cup \lambda} f(x) dx.
\]

Therefore, the set \( D'_m \) would feature lower expected disclosure costs than \( D^*_m \), a contradiction. \( \square \)

Proof of Proposition 1: Strict log-concavity of c.d.f. implies that \( t - \mathbb{E}(\tilde{x} | \theta \leq x \leq t) \) is strictly increasing in \( t \) (Bagnoli and Bergstrom, 2005). Hence,

\[
\frac{\partial (t - \mathbb{E}(\tilde{x} | \theta \leq x \leq t))}{\partial t} \bigg|_{t=\tau_0} = 1 - \frac{f(\tau_0) \int_{\theta=\tau_0}^{\tau_0} f(x) dx - f(\tau_0) \int_{\theta=\tau_0}^{\tau_0} x f(x) dx}{\left( \int_{\theta=\tau_0}^{\tau_0} f(x) dx \right)^2} = 1 - \frac{f(\tau_0)}{\int_{\theta=\tau_0}^{\tau_0} f(x) dx} \left( \tau_0 - \frac{\int_{\theta=\tau_0}^{\tau_0} x f(x) dx}{\int_{\theta=\tau_0}^{\tau_0} f(x) dx} \right) > 0, \tag{1.4.2}
\]

which can be rewritten as \( F(\tau_0) - F(\theta) - cf(\tau_0) > 0 \).
It then follows from the implicit function theorem on $\Gamma_1(\tau_\theta, \theta) = 0$,

$$
\frac{\partial \tau_\theta}{\partial \theta} = \frac{\partial \Gamma_1(t, \theta)}{\partial \theta} \left|_{t=\tau_\theta} \right. = \frac{(\tau_\theta - \theta - c)f(\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} > 0
$$

(1.7.1)

because $\tau_\theta - c = P_\theta(nd) > \theta$.

Let $C(\theta)$ denote the expected disclosure cost as a function of $\theta$,

\[
C(\theta) = F(\theta)c + (1 - F(\tau_\theta))c, \\
C'(\theta) = \left( f(\theta) - \left( \frac{\partial \tau_\theta}{\partial \theta} \right) f(\tau_\theta) \right) c \\
= \left( f(\theta) - \frac{(\tau_\theta - \theta - c)f(\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} f(\tau_\theta) \right) c \\
= f(\theta)c \left( \frac{F(\tau_\theta) - F(\theta) - cf(\tau_\theta) - (\tau_\theta - \theta - c)f(\tau_\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} \right) \\
= f(\theta)c \left( \frac{F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} \right).
\]

(1.7.2)

Because the denominator in the right-hand side of (1.7.2) is positive, the sign of $C'(\theta)$ is equal to the sign of $F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)$. It has been shown that $\tau_\theta$ increases in $\theta$. If $\tau_\theta$ is bounded below from $\tau > -\infty$, it is obvious that $F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)$ is negative for $\theta$ sufficiently small. If $\tau_\theta$ is not bounded from below, it is true that $\tau_\theta < m$ when $\theta$ is sufficiently small, which suggests that $F(\tau_\theta) - F(\theta) \equiv \int_\theta^{\tau_\theta} f(x)dx < \int_\theta^{\tau_\theta} f(\tau)dx \equiv (\tau_\theta - \theta)f(\tau_\theta)$ when $\theta$ is sufficiently small. Hence, $\theta^* = -\infty$ is not efficient, so that $\theta^* \in \mathbb{R}$.

By equation (1.7.2), the mandatory disclosure threshold $\theta^*$ must satisfy the following necessary first-order condition

\[
F(\tau_{\theta^*}) - F(\theta^*) - (\tau_{\theta^*} - \theta^*)f(\tau_{\theta^*}) = 0.
\]

(1.7.3)
From the mean value theorem, there exists $y^* \in (\theta^*, \tau_{\theta^*})$ such that:

$$f(y^*) = \frac{F(\tau_{\theta^*}) - F(\theta^*)}{\tau_{\theta^*} - \theta^*} = f(\tau_{\theta^*}). \quad (1.7.4)$$

Since the p.d.f. is single-peaked, $y^*$ and $\tau_{\theta^*}$ must be on opposite sides of the mode $m$, i.e., $y^* < m < \tau_{\theta^*}$, which implies: $\theta^* < m < \tau_{\theta^*}$ and

$$f(\theta^*) < f(\tau_{\theta^*}). \quad (1.7.5)$$

We prove next that the solution to equation (1.7.3) is unique. For any $\theta < m$, define $\tau_2(\theta)$ as a solution to $\Gamma_2(t, \theta) = 0$ with $\theta < m < t$ and $f(\theta) < f(t)$ where

$$\Gamma_2(t, \theta) \equiv F(t) - F(\theta) - (t - \theta)f(t).$$

Note that (i) $F(m) - F(\theta) - (m - \theta)f(m) < 0$ because $F(m) - F(\theta) \equiv \int_{\theta}^{m} f(x)dx < \int_{\theta}^{m} f(m)dx \equiv (m - \theta)f(m)$, (ii) $\lim_{t \to \infty} \Gamma_2(t, \theta) = \lim_{t \to \infty} F(t) - F(\theta) - \lim_{t \to \infty} tf(t) + \lim_{t \to \infty} \theta f(t) = 1 - F(\theta) > 0$ because $\lim_{t \to \infty} tf(t) = 0$, and (iii) $\partial \Gamma_2 / \partial t = -(t - \theta)f'(t)$ is strictly positive for $t > m$. Hence, $\tau_2(\theta)$ is unique and is a function from $(-\infty, m)$ to $(m, \infty)$. Further, the voluntary threshold $\tau_{\theta^*}$ must satisfy the cost-minimizing optimality condition $\tau_2(\theta^*)$, which implies that $\tau_2(\theta^*) = \tau_{\theta^*}$. We know from (1.7.1) that $\tau_0$ is increasing in $\theta$ and, applying the implicit function theorem to $\Gamma_2(t, \theta)$,

$$\tau'_2(\theta) = \frac{(\tau_2(\theta) - \theta)f'(\tau_2(\theta))}{f(\tau_2(\theta)) - f(\theta)} < 0, \text{ for } \tau_2(\theta) > m > \theta \text{ and } f(\tau_2(\theta)) > f(\theta).$$

This implies that $\tau_\theta$ and $\tau_2(\theta)$ cross once and, hence, the solution to $\tau_{\theta^*} = \tau_2(\theta^*)$ is unique.

$\Box$

**Note 1:**
The main result that $D^*_m$ is a non-empty lower interval holds even without assuming strict log-concave c.d.f. $F(.)$ and single-peaking p.d.f. $f(.)$. The following is a proof.

If $\Gamma_1(t, \theta) = 0$ has multiple solutions of $t$, we can redefine $\tau_\theta$ as the maximal solution which is the most efficient one. Since $\Gamma_1(t, \theta) = 0$ is negative for sufficiently large $t$, it has to be true that

$$\frac{\partial \Gamma_1(t, \theta)}{\partial t}|_{t=\tau_\theta} \leq 0.$$ 

By implicit function theorem, the case in which $\frac{\partial \Gamma_1(t, \theta)}{\partial t}|_{t=\tau_\theta} < 0$ resembles the baseline case above, that is

$$C'(\theta) = f(\theta) c \left( \frac{F(\tau_\theta) - F(\theta) - (\tau_\theta - \theta)f(\tau_\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} \right) < 0,$$

for sufficiently small $\theta$.

The only complexity here is the case in which $\frac{\partial \Gamma_1(t, \theta)}{\partial t}|_{t=\tau_\theta} = 0$, which makes the implicit function theorem fail. However, we still have right-derivative:

$$\lim_{\varepsilon \to 0^+} \frac{\Gamma_1(\tau_\theta + \varepsilon, \theta + \varepsilon) - \Gamma_1(\tau_\theta, \theta)}{\varepsilon} = 0,$$

which implies that,

$$\lim_{\varepsilon \to 0^+} \frac{\Gamma_1(\tau_\theta + \varepsilon, \theta + \varepsilon) - \Gamma_1(\tau_\theta, \theta)}{\varepsilon} = \lim_{\varepsilon \to 0^+} \frac{\Gamma_1(\tau_\theta + \varepsilon - \tau_\theta, \theta) - \Gamma_1(\tau_\theta, \theta)}{\varepsilon} = \frac{\partial \Gamma_1(t, \theta)}{\partial \theta}|_{t=\tau_\theta} + \lim_{\varepsilon \to 0^+} \frac{\tau_\theta + \varepsilon - \tau_\theta}{\varepsilon} \cdot \lim_{\varepsilon \to 0^+} \frac{\Gamma_1(\tau_\theta + \varepsilon, \theta) - \Gamma_1(\tau_\theta, \theta)}{\varepsilon} = 0.$$

Hence it must be true that $\lim_{\varepsilon \to 0^+} \frac{\tau_\theta + \varepsilon - \tau_\theta}{\varepsilon} = \infty$. In this case,
\[
\lim_{\varepsilon \to 0^+} \frac{C(\theta + \varepsilon) - C(\theta)}{\varepsilon} = \left( f(\theta) - \left( \lim_{\varepsilon \to 0^+} \frac{\tau_{\theta + \varepsilon} - \tau_{\theta}}{\varepsilon} \right) f(\tau_{\theta}) \right) c = -\infty.
\]

As a result, \( \theta^* = -\infty \) is not optimal.

**Proof of Corollary 1:** The fact that \(-\infty < \theta^* < m < \tau_{\theta^*} \) is shown in the proof of Proposition 1. For the special case in which the distribution of \( \tilde{x} \) is symmetric, \( \tau_{\theta^*} - m = m - y^* \), where \( y^* \) is given by equation (1.7.4). Thus, \( \text{Prob}(\tilde{x} < y^*) = \text{Prob}(\tilde{x} > \tau_{\theta^*}) \), implying that the probability of mandatory disclosure \( \text{Prob}(\tilde{x} < \theta^*) < \text{Prob}(\tilde{x} < y^*) \) is lower than the probability of voluntary disclosure. \( \Box \)

**Proof of Corollary 2:** Applying implicit function theorem to \( \Gamma_1(\tau_{\theta^*}, \theta^*) = 0 \) and \( \Gamma_2(\tau_{\theta^*}, \theta^*) = 0 \):

\[
\begin{pmatrix}
\frac{\partial \theta^*}{\partial c} \\
\frac{\partial \tau_{\theta^*}}{\partial c}
\end{pmatrix}
= \left( \begin{array}{cc}
\frac{\partial \Gamma_1(t,\theta)}{\partial \theta} |_{\theta=\theta^*, t=\tau_{\theta^*}} & \frac{\partial \Gamma_1(t,\theta)}{\partial t} |_{\theta=\theta^*, t=\tau_{\theta^*}} \\
\frac{\partial \Gamma_2(t,\theta)}{\partial \theta} |_{\theta=\theta^*, t=\tau_{\theta^*}} & \frac{\partial \Gamma_2(t,\theta)}{\partial t} |_{\theta=\theta^*, t=\tau_{\theta^*}}
\end{array} \right)^{-1}
\begin{pmatrix}
\frac{\partial \Gamma_1(t,\theta)}{\partial t} |_{\theta=\theta^*, t=\tau_{\theta^*}} \\
\frac{\partial \Gamma_2(t,\theta)}{\partial t} |_{\theta=\theta^*, t=\tau_{\theta^*}}
\end{pmatrix}
\begin{pmatrix}
\Delta \\
0
\end{pmatrix}
\]

\[
= \frac{F(\tau_{\theta^*}) - f(\theta^*)}{Z}(\tau_{\theta^*} - \theta^*)f'(\tau_{\theta^*})
\]

where \( \Delta = F(\tau_{\theta^*}) - F(\theta^*) \) and

\[
Z = (f(\tau_{\theta^*}) - f(\theta^*))F(\tau_{\theta^*}) - F(\theta^*) - c f(\tau_{\theta^*}) + (\tau_{\theta^*} - \theta^*)(c - \tau_{\theta^*} + \theta^*) f(\theta^*) f'(\tau_{\theta^*}). \quad (1.7.6)
\]

We know that (i) \( f'(\tau_{\theta^*}) < 0 \) from Corollary 1 and (ii) \( f(\tau_{\theta^*}) - f(\theta^*) > 0 \) from (1.7.5)
so that $\text{Sign}(\frac{\partial \tau}{\partial c}) = -\text{Sign}(\frac{\partial \theta^*}{\partial c}) = \text{Sign}(Z)$, and $Z > 0$ is implied by (iii) $\tau^* - c > \theta^*$ from (1.4.1), and (iv) $F(\tau^*) - F(\theta^*) - cf(\tau^*) > 0$ from (iii) and (1.4.3). □

**Proof of Corollary 3:** Without loss of generality, we set $\mu = 0$. Denote the p.d.f. and c.d.f. of the standard normal distribution as $\phi(.)$ and $\Phi(.)$ respectively. The probability of mandatory and voluntary disclosure can be written respectively as:

$$
\text{Prob}(x < \theta^*) \equiv \Phi\left(\frac{\theta^*}{\sigma}\right),
$$

$$
\text{Prob}(x > \tau^*) \equiv 1 - \Phi\left(\frac{\tau^*}{\sigma}\right).
$$

We do the following change of variable to $\Gamma_1(\tau^*, \theta^*) = 0$ and $\Gamma_2(\tau^*, \theta^*) = 0$:

$$
\int_{\theta^*/\sigma}^{\tau^*/\sigma} \phi(u) \, du - \left(\frac{\tau^*}{\sigma} - \frac{c}{\sigma}\right) \int_{\theta^*/\sigma}^{\tau^*/\sigma} \phi(u) \, du = 0,
$$

$$
\Phi\left(\frac{\tau^*}{\sigma}\right) - \Phi\left(\frac{\theta^*}{\sigma}\right) - \left(\frac{\tau^*}{\sigma} - \frac{\theta^*}{\sigma}\right) \phi\left(\frac{\tau^*}{\sigma}\right) = 0.
$$

From the proof of Corollary 2, we have that $\frac{\theta^*}{\sigma}$ decreases in $\hat{\xi}$ and $\frac{\tau^*}{\sigma}$ increases in $\hat{\xi}$. Hence, it proves that $\frac{\theta^*}{\sigma}$ increases in $\sigma$ and $\frac{\tau^*}{\sigma}$ decreases in $\sigma$. □

**Proof of Proposition 2** As in the benchmark, there is a unique equilibrium for any $\theta$ because one can think of this problem as the original problem with cost of disclosure $c - h(\theta) > 0$. It then follows from the implicit function theorem that

$$
\frac{\partial \tau_{\theta}}{\partial \theta} = \frac{(\tau_{\theta} - \theta - (c - h(\theta)))f(\theta) - h'(\theta)(F(\tau_{\theta}) - F(\theta))}{F(\tau_{\theta}) - F(\theta) - (c - h(\theta))f(\tau_{\theta})}. \quad (1.7.7)
$$
The denominator is positive from log-concavity. The sign of the numerator is undetermined. As \( \theta \) increases, the net cost of disclosure decreases, which may yield more voluntary disclosure.

Then the first order condition is

\[
\frac{\partial C(\theta)}{\partial \theta} = c \left( f(\theta) - \frac{\partial \tau_0}{\partial \theta} f(\tau_0) \right) + h'(\theta)(F(\tau_0) - F(\theta)) + h(\theta) \left( \frac{\partial \tau_0}{\partial \theta} f(\tau_0) - f(\theta) \right),
\]

\[
= (c - h(\theta)) \left( f(\theta) - \frac{\partial \tau_0}{\partial \theta} f(\tau_0) \right) + h'(\theta)(F(\tau_0) - F(\theta)).
\]

Since \( \lim_{\theta \to -\infty} h(\theta) = 0 \) and \( \lim_{\theta \to -\infty} h'(\theta) = 0 \), the first order condition converges to that in the benchmark as \( \theta \to -\infty \). Hence \( \theta = -\infty \) is never optimal. \( \square \)

**Proof of Proposition 3:** It can be easily checked that Lemma 1 applies. We prove the if and only if parts separately.

**If:** As shown in the proof of Proposition 1, the sign of the F.O.C. for minimizing the expected disclosure cost when evaluated at \( \theta = \bar{\tau} \) is determined by the sign of \( [F(\tau_x) - F(\bar{\tau}) - (\tau_x - \bar{\tau})f(\tau_x)] \). The set \( \{ (\cdot, c) | F(\tau_x) - F(\bar{\tau}) - (\tau_x - \bar{\tau})f(\tau_x) < 0 \} \) is not empty, as shown below.

Assuming \( \bar{x} < m < +\infty \), \( F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x) < 0 \) if \( \bar{x} < \tau_x \leq m \). If \( \tau_x > m \), \( F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x) \) is increasing in \( \tau_x \), i.e., \( \frac{\partial(F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x))}{\partial \tau_x} = -(\tau_x - \bar{x})f'(\tau_x) \geq 0 \). Also \( \lim_{\tau_x \to +\infty} F(\tau_x) - F(\bar{x}) + (\bar{x} - \tau_x)f(\tau_x) = F(+\infty) - F(\bar{x}) + (\bar{x} - \infty)f(+\infty) = 1 - F(\bar{x}) > 0 \). By intermediate value theorem, the exists a unique \( \hat{\tau}_x > m \) such that \( F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x) \geq (\leq) 0 \) if \( \tau_x \geq (\leq) \hat{\tau}_x \). Since \( \tau_x \) increases in \( c \), there exists a unique \( \bar{c} > 0 \) such that \( F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x) < 0 \) if \( \tau_x > m \) and only if \( c < \bar{c} \).

\( F(\tau_x) - F(\bar{x}) - (\tau_x - \bar{x})f(\tau_x) < 0 \) implies that \( \bar{x} < m < +\infty \). The proof of uniqueness is identical to the uniqueness proof in Proposition 1, with the the support replaced by \([\bar{x}, +\infty)\).
Only if: $F(\tau_x) - F(x) - (\tau_x - x)f(\tau_x) \geq 0$ contradicts the fact there exists a unique solution to equation (1.4.3) in $[x, +\infty)$. □
Chapter 2

Efficient disclosure laws with real effects

2.1. Introduction

In this chapter, we extend our analysis in several directions where information has social value. A unique feature of the solution to the benchmark problem is that, although the efficient disclosure law reduces the probability of disclosure, it can increase the total amount of information being disclosed. It is well-known that news that moves the posteriors further away from the unconditional mean can have larger social value. However, the problem of how those news are generated in an economy by different disclosure channels are understudied.

Specifically, we show that the main result generalizes to environments where information serves post-sale productive purposes or enhances risk-sharing. Post-sale investment, production, and other decisions, or risk-averse buyers render the price convex in the posterior belief. The socially-preferred outcome features disclosure over both sufficiently favorable and sufficiently unfavorable news, and we formally demonstrate that the former is resolved via the voluntary channel, while the latter is resolved via the mandatory channel. We also explore disclosure in the context of optimal liquidations and demonstrate that the efficient law induces excess liquidation because firms can liquidate to reveal bad
news before incurring a disclosure cost. Lastly, we consider a market for lemons: because of adverse selection in the asset market, better assets are retained at a social cost. The mandatory disclosure requirement serves to remove the lowest-quality assets out of the lemons’ market; consequently, the efficiency of the lemons’ market is enhanced by disclosure regulation.

2.2. Real effects of information

2.2.1. Convex payoff

In the baseline model, the price is a linear function of the posterior expectation of the asset value. One way of incorporating social efficiency implications of information is to generalize this to $P(x) = \phi(x)$ when a disclosure is made and, when possible, $P(nd) = \phi(\mathbb{E}(\tilde{x}|r(\tilde{x}) = nd))$, where $\phi(.)$ is strictly increasing ($0 < \phi'(.) < \infty$). If $\phi(.)$ is strictly convex, the expected selling price is always greater in the presence of more precise information. For example, price-protected buyers may make additional decisions conditional on public information.\(^1\)

This structure is similar to the assumption of convex payoffs used in the persuasion literature with non-linear receiver’s utility.\(^2\) For this analysis, we rule out pathological cases by imposing a regularity condition: we assume that the voluntary disclosure indifference constraint satisfies the constraint qualification for Kuhn-Tucker conditions to be necessary for optimality, which allows us to use standard differential methods to analyze

\(^1\)It should be noted that this formulation focuses on an investment decision by a manager with no conflict of interest with shareholders, apart from horizon issues. Managing the information environment presents many interesting issues when information may also be used as an incentive device (Arya, Glover and Sivaramakrishnan, 1997), and it remains an ongoing question as to when insights from persuasion theory extend to stewardship problems.

\(^2\)To illustrate, in our setting, suppose that a receiver makes an investment $I$ to maximize $\mathbb{E}(\tilde{x}I - I^2/2 | r(\tilde{x}))$, choosing $I = \mathbb{E}(\tilde{x}|r(\tilde{x}))$, which implies total value $\mathbb{E}(\tilde{x}|r(\tilde{x}))^2/2$, i.e., mapping to a function $\phi(x) = x^2/2$. See Kamenica and Gentzkow (2011) and Kartik, Lee and Suen (2017), supplementary appendix B.3.
Define \( x_{nd} = \mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \tilde{x} \not\in D_m) \). The equilibrium \( \mathcal{E}(D_m) \), if it exists, features a threshold \( \tau \) such that \( x \) is voluntarily disclosed if and only if \( x > \tau \) where \( \tau \) is defined as the indifference point between disclosing voluntarily and not disclosing:

\[
\phi(\tau) - c - \phi(x_{nd}) = 0. \tag{2.2.1}
\]

To characterize an efficient disclosure law, we adapt the Lagrangian approach used in Bertomeu and Cheynel (2015): we replace the mandatory reporting set \( D_m \) with an indicator function \( \Theta(x) \in \{0, 1\} \), where \( \Theta(x) = 1 \) if and only if \( x \not\in D_m \). There exists a one-to-one mapping between \( D_m \) and \( \Theta(x) \) and, hence, it is equivalent to solve for this indicator function.

An efficient disclosure law is then characterized by a solution \( (\Theta^*(.), x_{nd}^*, \tau^*) \) to the following constrained optimization program:

\[
\text{(Q1)} \quad \max_{\Theta(.,) \in \mathbb{R}, \tau \in \mathbb{R}} \int_{-\infty}^{\tau} (\phi(x_{nd})\Theta(x) + (\phi(x) - c)(1 - \Theta(x)))f(x)dx + \int_{\tau}^{+\infty} (\phi(x) - c)f(x)dx \\
\text{subject to (2.2.1) and} \\
x_{nd} \int_{-\infty}^{\tau} \Theta(x)f(x)dx - \int_{-\infty}^{\tau} x\Theta(x)f(x)dx = 0. \tag{2.2.2}
\]

Constraint (2.2.2) states that \( x_{nd} = \mathbb{E}(\tilde{x}|\tilde{x} \leq \tau, \Theta(\tilde{x}) = 1) \) must be consistent with Bayes rule and parallels (ii) in Definition 1. As we prove next, the problem has a bang-bang solution such that the mandatory disclosure takes the form of a threshold.

**Proposition 4** If the pricing function \( \phi(.) \) is convex, an efficient disclosure law mandates the disclosure of all sufficiently unfavorable information, that is, \( D_m^* = (-\infty, \theta^*) \), where

\footnote{The constraint qualification ensures that the problem can be solved with Lagrangian multipliers. We were not able to find any examples in which the qualification did not hold.}
\(-\infty < \theta^* < \tau^* < \infty.\)

In an economy with value of information, the value of each piece of information is larger if it moves the posterior further away from the unconditional mean. As a result, when generating each piece of information is costly, it is optimal to only generate more extreme posteriors. However, the unregulated voluntary disclosure only generates more extreme posteriors on the upper-side leaving the under-supply of information on the lower-side. As in the benchmark case with linear pricing, the voluntary disclosure also features over-provision of news the value of which is not as significant as its cost. Hence, mandating disclosing bad news here not only serves as a mechanism to mitigate the excessive voluntary disclosure but also supplements the provision of unfavorable news. From a mathematically perspective, with the pricing function \(\phi(.)\) strictly convex, the seller prefers more variability in price because the expectation of a convex function is greater than the convex function of an expectation. Disclosing bad news now creates additional social value by increasing the variability of posterior expectations over events that were previously withheld.

2.2.2. Optimal liquidations

We examine next a version of the model where the firm can liquidate the asset after the law is passed but before the voluntary disclosure stage, adding a decision node at this point of the timeline.\(^4\)

We model liquidations similar to Lizzeri (1999), Gigler, Kanodia, Sapra and Venugopalan (2009), and Cheynel (2013), assuming that liquidation yields a fixed payoff which we normalize to zero. The firm liquidates if it gets a strictly higher payoff by doing so. There are two important observations we need to note to characterize the equilibrium. Firstly, only an asset with \(x < c\) can possibly be liquidated because the seller always has the option to sell with disclosure. Secondly, if there is sale with non-disclosure on equi-

\(^4\)We thus assume that liquidation is no longer available once the voluntary disclosure is made - this is entirely innocuous to the extent that the firm can anticipate the price that follows the disclosure.
librium path, the non-disclosure price has to be strictly positive because the seller always has the option to liquidate.

Hence, given a particular $D_m$, there are two classes of equilibria we need to consider. (i) In the first class, a firm continues and voluntarily discloses if $x > c$ and liquidates otherwise; the off-equilibrium belief for a firm that continues but does not disclose is negative (thus, ruling out a deviation of the sort). (ii) In the second class, there is a strictly positive non-disclosure price, and only those with $x < c$ and who are mandated to disclose liquidate.

We first show that it is always optimal for the law to induce $x \in (-\infty, 0)$ to be liquidated, and the associated efficient equilibrium has to feature a positive non-disclosure price.

**Lemma 2** If $D_m^*$ is an efficient law, it must be true that $D_m^* = (-\infty, \theta^*)$, where $\theta^* \geq 0$, and the most efficient equilibrium induced by the law features a positive non-disclosure price.

The intuition of this lemma is very simple, that is, it is socially inefficient to let $x < 0$ to be traded, and as a consequence of liquidating them, the non-disclosure price is also higher, which mitigates the over-disclosure of the positive types as well.

As a result liquidating all negative types, the most efficient equilibrium can only be in the second class because those in the first class features over-liquidating positive types and over-disclosure by positive types as well. Hence there is a strictly positive non-disclosure price and only those who are mandated to disclose and has $x < c$ liquidate. If we refer to the liquidation threshold as $k$, it must be true that $k \leq \theta < \tau_\theta$. Hence, the model becomes similar to a limiting case of the convex payoff setup; the only difference is that, if disclosure is mandated for a firm with $x < c$, the firm will liquidate rather than incur a disclosure cost. In the next proposition, we derive the nature of the efficient law.

37
Proposition 5  With an efficient law, \(0 < k^* \leq \theta^* < \tau_{\theta^*}\). Further, there exists a threshold \(c_0\), which uniquely satisfies \((\tau_{c_0} - c_0)f(\tau_{c_0}) - (F(\tau_{c_0}) - F(c_0)) = 0\), such that

(i) If \(c < c_0\), \(\theta^*(> k^* = c)\) is uniquely given by the baseline equation in (1.4.3).

(ii) If \(c \geq c_0\), \(\theta^*(= k^* \leq c)\) is given by

\[
c(\tau_{\theta^*} - c)f(\tau_{\theta^*}) - \theta^*(F(\tau_{\theta^*}) - F(\theta^*)) = 0, \tag{2.2.3}
\]

and, holding all other parameters constant, is larger than the mandatory threshold in Proposition 1.

Liquidations allow firms to exit before they incur the disclosure cost. For firms with \(x < 0\), such liquidations also maximize social surplus and, therefore, information can be elicited with only a regulatory threat without an effective deadweight loss from carrying out the regulation (Suijs and Wielhouwer, 2018). In fact, strengthening this argument, firms with \(x \approx 0\) carry almost no social surplus but induce costly voluntary disclosures at the top. So increasing the mandatory disclosure threshold \(\theta\) strictly above zero is desirable, implying our first result that \(\theta^* > 0\).

If the disclosure cost is sufficiently low, the efficient law is determined as in the baseline model, and we thus have firms, subject to mandatory disclosure, that do not liquidate:
\(c = k^* < \theta^* < \tau_{\theta^*}\). The mandatory and voluntary disclosure thresholds satisfy the same equations as in the baseline model.

If the disclosure cost is high, there are significant differences from the baseline - all firms that would have been subject to the law will liquidate: \(\theta^* = k^* \leq c\). The threat of mandated disclosure regulation disciplines liquidations, but no actual mandatory disclosure is observed on the equilibrium path. This changes the welfare trade-off when choosing the mandatory reporting threshold \(\theta\), since the effect of increasing the threshold is no longer to impose a social disclosure cost, but to liquidate a firm with \(x \in (0, c)\). This entails a loss of
welfare strictly less than $c$ for $x < c$, which implies that the cost of mandatory disclosure is lower. Hence, the possibility of liquidations increases the mandatory disclosure threshold relative to the baseline model.

2.2.3. Market for lemons

We extend our model to a “lemons” market, where firms consider selling an asset to a buyer who values the asset more highly than the firm. The firm has private information about the asset’s value, creating adverse selection in the asset market; firms with better assets may then find it desirable not to sell (Akerlof, 1970; Levin, 2001). Assume that the asset’s value $\tilde{x}$ has support over $\mathbb{R}^+$. Firms have the option to retain the asset, which earns them value $\delta x$, where $\delta \in (0, 1)$; each untraded asset thus carries an opportunity loss of $(1 - \delta)x$. Firms retaining the asset do not disclose. The proof that the efficient mandatory disclosure law takes the form of threshold $\theta$ below which information must be disclosed is identical to the baseline model; we thus continue to use $\theta \geq 0$ to describe the mandatory disclosure threshold, and $\tau_\theta$ to denote the voluntary disclosure threshold.

The lemons’ problem is conceptually similar to our baseline model, but with an added feature: in addition to the disclosure cost $c$ as in the baseline disclosure model, here firms retaining the asset incur the cost $(1 - \delta)x$. Leaving aside for now the interpretation of the trade and no-trade regions, we can view the lemons’ problem as the baseline disclosure problem with type-dependent cost $\min((1 - \delta)x, c)$.

Define $\kappa(\theta) > \theta$ as the root of $\Lambda(k, \theta) = \delta k - \mathbb{E}(x|x \in [\theta, k])$. We assume the following (natural) regularity condition: the root of $\Lambda(k, \theta)$ exists and is unique for any $\theta$.

Define $\kappa(\theta) > \theta$ as the root of $\Lambda(k, \theta) = \delta k - \mathbb{E}(x|x \in [\theta, k])$. We assume the following (natural) regularity condition: the root of $\Lambda(k, \theta)$ exists and is unique for any $\theta$. This guarantees that, ignoring any mandatory disclosure, there is a unique solution $\kappa$ above which firms prefer to retain the asset conditional on non-disclosure. We also assume that

---

5This also implies that our main lemons’ market results can be easily adapted to generalize the optimal liquidation problem in 2.2.2 to the case in which the liquidation payoff depends linearly on the firm’s type.

6This assumption is with little loss of generality for our main results. If there are multiple solutions, it can be shown that the largest root is the preferred equilibrium which implies the maximal amount of trade.
\frac{\partial \Lambda(k, \theta)}{\partial k} |_{k=\kappa(\theta)} \neq 0, \text{ which enables us to apply the first-order approach. For later comparisons,}

we use \( \tau_0^\theta \) to describe the voluntary disclosure threshold in the baseline model of section 1.3 without lemons; recall that \( \tau_0^\theta \) is given by \( \Gamma_1(\tau_0^\theta, \theta) = 0 \) in (1.4.2).

Unlike in the baseline setting, here firms that are not subject to mandatory disclosure may prefer to disclose voluntarily even before \( \kappa \) is attained and, similarly, firms subject to mandatory disclosure with sufficiently low types may prefer to retain the asset. We describe the three main cases of the model in Figure 2.1, and Figure 2.2 provides a numerical example with the beta distribution.

Figure 2.1: Mandatory disclosure and the market for lemons.

If \( \theta < \frac{c}{(1 - \delta)} \) is small, in cases 1 and 2 of Figure 2.1, all firms subject to mandatory disclosure retain their asset, so that no firm is actually making a mandatory disclosure and the law serves entirely a deterrence role by inducing firms with low values to retain their asset (deterrence roles of regulation appears in a different setting in Suijs and Wielhouwer (2018)).

In case 1, \( \kappa(\theta) \leq \tau_0^\theta \) is below the voluntary disclosure threshold absent lemons; consequently, there is some breakdown of trade in the non-disclosure region. The voluntary
Figure 2.2: Lemons’ problem with a beta distribution with $\alpha = 2, \beta = 5, \delta = 0.6$. 

**Expected payoff** 

- **Case 1:** $c = 0.25$  
  - $\tau = c/(1-\delta)$ 
  - $\kappa(\theta^*)$ 

- **Case 2:** $c = 0.1$  
  - $\tau = c/(1-\delta)$ 

- **Case 3:** $c = 0.01$  
  - $\tau = c/(1-\delta)$
disclosure threshold is then given by the point of indifference between disclosure followed by trade, versus no-trade, i.e., \( \tau_\theta - c = \delta \tau_\theta \). This yields the voluntary disclosure threshold

\[
\tau_\theta = \frac{c}{1 - \delta}.
\]  

(2.2.4)

In contrast to our baseline model, this voluntary disclosure threshold is no longer a function of the mandatory disclosure threshold \( \theta \) (which affects the non-disclosure price but not the utility of the marginal voluntary discloser). There is, however, a new role for \( \theta \) in the lemons’ market: more mandatory disclosure increases trade. Case 2 is similar, except that \( \kappa(\theta) > \tau_\theta^0 \), and all non-disclosers choose to trade. In other words, the mandatory disclosure threshold is set to resolve the lemons’ problem fully for values above \( \theta \). The voluntary disclosure threshold is given as in the baseline by \( \tau_\theta = \tau_\theta^0 \); the key difference is that the cost of mandatory disclosure is effectively \((1 - \delta)\theta\). In the appendix, we show that \( \kappa(\theta) \) is increasing in \( \theta \). Hence, the market will typically switch from case 1 to case 2 as the mandatory disclosure threshold \( \theta \) is increased.

In case 3, when \( \theta \) exceeds \( c/(1 - \delta) \), some firms subject to mandatory disclosure are better-off disclosing and selling, and all non-disclosers are better-off selling; as in case 2 above, mandatory disclosure fully solves the lemons’ problem for \( x > \theta \). In fact, this case becomes identical to the baseline model, as the marginal mandatory discloser incurs a cost equal to \( c \).

We show next that, consistent with the analysis of the baseline model, efficient mandatory disclosure in the lemons’ setting requires a strictly positive threshold below which firms must disclose.

**Proposition 6** The efficient disclosure law features \( \theta^* > 0 \). If the peak of the distribution is \( m > 0 \), then there exits a threshold \( c_1 > 0 \) such that:

(i) if \( c \leq c_1 \), then \( \theta^* \geq c/(1 - \delta) \) is given by equation (1.4.3), and \( \tau_{\theta^*} = \tau_\theta^0 \);
(ii) if $c > c_1$, then $0 < \theta^* < c/(1 - \delta)$, that is, the probability of mandatory disclosure is 0.

If the peak of the distribution is $m = 0$, then $0 < \theta^* < c/(1 - \delta)$, that is, the probability of mandatory disclosure is 0.

The intuition for Proposition 6 is related to the analysis of the optimal-liquidation problem in section 2.2.2. As in optimal liquidation, it is desirable to remove firms with sufficiently low types from the market; in the lemons’ setting, this can be either because of a market breakdown where too many firms retain their assets, or because low-value firms cause excessive voluntary disclosure.

When the distribution is single-peaked at $m > 0$, the worst type has a relatively low probability mass. In this case, the efficient mandatory disclosure threshold is set at the mandatory threshold of the baseline model if and only if the disclosure cost is sufficiently small ($c < c_1$), because at the margin, the firm with $x$ just below $\theta^*$ discloses rather than retains. More generally, the lemons’ market inefficiency results from the inability of firms to communicate information. Hence, when communication cost is sufficiently low, the lemons’ problem is fully resolved, and the optimal solution resembles our baseline of section 1.3, with one exception: firms with $x < c/(1 - \delta)$ retain the asset rather then disclose.

However, when the low types have a relatively large probability mass, e.g., the distribution has a downward sloping density, as in the exponential distribution, the efficient disclosure law always sets the mandatory disclosure threshold below $c/(1 - \delta)$. The reason is that, in this particular case, there are relatively too many low types such that the lemons problem can never be fully resolved. On the margin, the efficient law has to liquidate low types to prevent market break-down or excessive disclosure of intermediate types. Since firms that are subject to the mandatory disclosure requirements have the option not to sell (and thus not to incur the cost of disclosure), we do not observe mandatory disclosure.
even with disclosure regulation.

2.3. Conclusion

To conclude this chapter, we develop and expand upon several empirical implications of our analysis. As any study on optimal regulations, this requires an assumption on observational data, namely, that regulations are sufficiently close to efficiency to allow the use our model’s prediction of the efficient mandatory disclosure threshold $\theta^*$. Nevertheless, it is worth noting that any implications stated here are joint tests of the model proper, and of the efficiency of observed regulations.

We predict that, for a given event, mandatory releases are associated with negative market reactions, while voluntary disclosures are associated with positive market reactions. In practice, we are unlikely to see this for all observations, if only because there is noise in market prices; still, even in the presence of other forces, the prediction is likely to hold in expectation. This is why we suggest testing this prediction by considering events where we can partly separate the voluntary and mandatory channels, and can verify whether, in expectation, news that first appear in a voluntary channel trigger on average positive market reactions, while the same event triggers a negative market reaction if it appears for the first time in a mandatory reporting channel. In practice, regulatory filings such as 8-K’s include a number of identifiable events of interest: auditor changes, changes in management or material contracts (Li, 2013). The model predicts different directions in market reactions to similar events, depending on whether an event is first reported in a regulatory filing or in a voluntary report prior to the regulatory filing. Novel datasets from news wires, along with our model, provide opportunities to explore how the market reacts to voluntary versus regulated filings (Li, Ramesh and Shen, 2011).

Management forecasts are another research area where our model may help organize the empirical evidence. About half of publicly traded firms make occasional voluntary
forecasts about next-quarter or next-year earnings, and there is an extensive literature documenting that markets respond positively to long-term forecasts (Beyer, Cohen, Lys and Walther, 2010). We know less of the interaction of voluntary and actual (regulated) earnings reports. An important puzzle in this research area is: why do firms disclose bad news? Our model suggests that certain disclosures connected to loss-making transactions are not voluntary but may reflect essentially regulated disclosures since non-disclosure potentially increases the risk of a lawsuit (Skinner, 1994). A minor extension of our model would also suggest that factors that increase the strength of this regulatory channel, such as a pre-existing history of making forecasts or a pre-announcement where information is more likely to be known, would also make the firm more willing to disclose bad news.

There is also a large literature on conservatism developing various hypotheses about the optimal choice of conservatism as a function of the firm’s economic environment (Watts, 2003). With a few exceptions (Armstrong et al., 2015), this literature focuses on the contracting benefits of conservatism. We show here that a key feature of conservatism – mandated reporting asymmetry – provides a social benefit by avoiding excessive voluntary reporting. Our theory links the quality of the information received by management, a notion of accounting quality, to the reporting asymmetry inherent in conservatism, and we note that the reporting system becomes more conservative as the manager has more information to report to the market. This comparative static can also be examined from the perspective of disclosure cost as a deadweight proprietary cost lost to outsiders (i.e., foreign companies or consumers). While a greater cost makes it more important to reduce excess voluntary disclosure, we show that a lower cost affects both mandatory and voluntary disclosure channels in the same direction, causing a reduction in conservatism and more delay in the release of news.

There are institutional settings in which we can observe firms choosing to be listed in a particular jurisdiction, as an attempt to bind to a set of disclosure regulations that is, presumably, consistently enforced (Christensen, Hail and Leuz, 2013). Voluntary adoptions
of accounting standards (Leuz and Verrecchia, 2000), cross-listings as ADRs, or IPOs by non-domestic firms are various examples in which a foreign firm can make a commitment to follow the laws of a jurisdiction. From the perspective of a foreign firm considering this choice, the ex-ante cost may generally reflect both verification and proprietary costs, with the objective of reducing how much information may be revealed voluntarily in an unregulated economy. Therefore, we predict that firms and industries with more proprietary cost are more willing to commit to mandatory disclosure.

The flavors of the model with social value of information connect to empirical settings in which a decision to exit or liquidate was affected by new legislation. Bushee and Leuz (2005) document the consequences of an expansion of disclosure requirements by the SEC to firms traded in over-the-counter markets. In response to the significant increase in disclosure costs, many firms went private, especially firms with weaker financials. The study also documents the benefits of the newly imposed disclosure requirements: firms that remained and started to comply with the new requirements experienced significant increases in liquidity, and firms that had previously been voluntarily compliant with the new requirements experienced positive abnormal returns and increases in liquidity. Our model predicts that some amount of forced exit by firms with worse information benefits not just firms that remain, but the value of all firms in expectation. Put differently, we argue that the regulation, when set optimally, is not purely redistributive but increases total surplus.
2.4. Appendix of chapter 2

Proof of Proposition 4: We prove this result in several steps.

Step 1: We argue that $\tau^* = \max\{\tau : \phi(\tau) - c - \phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau, \Theta^*(\tilde{x}) = 1)) = 0\}$, i.e., $\tau^*$ is the maximal threshold that can be sustained in equilibrium. To see this, note that, for any two solutions $\tau'' > \tau'$ to $\phi(\tau) - c - \phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau, \Theta^*(\tilde{x}) = 1)) = 0$, it holds that (i) $\phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau'' \leq \tau, \Theta^*(\tilde{x}) = 1)) > \phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau', \Theta^*(\tilde{x}) = 1))$, and (ii) $\phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau '', \Theta^*(\tilde{x}) = 1)) \geq \phi(x) - c$ for any $x \in (\tau', \tau'')$. This implies that the objective function is greater under $\tau''$ than under $\tau'$.

Step 2: In this step, we show that it is not optimal to disclose an interval adjacent to the left of $\tau^*$. By contradiction, suppose that $\Theta^*(x) = 0$ for $x \in (\tau^* - \varepsilon, \tau^*)$ for some $\varepsilon > 0$. Set $\Theta^{**}(x) \equiv \Theta^*(x)$ except that $\Theta^{**}(x) = 1$ for $x \in (\tau^* - \varepsilon, \tau^{**})$, where $\tau^{**}$ corresponds to the new rule $\Theta^{**}(x)$. $\Theta^{**}(x)$ is more efficient than $\Theta^*(x)$ because:

(i) $\phi(\tau^*) - c - \phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau^*, \Theta^{**}(\tilde{x}) = 1)) < 0$ implies $\tau^{**} > \tau^*$;

(ii) $\phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau^{**}, \Theta^{**}(\tilde{x}) = 1)) > \phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau^*, \Theta^*(\tilde{x}) = 1))$;

(iii) for any $x \in (\tau^* - \varepsilon, \tau^{**})$, $\phi(\mathbb{E}(\tilde{x} | \tilde{x} \leq \tau^{**}, \Theta^{**}(\tilde{x}) = 1)) \geq \phi(x) - c$.

Step 3: We show a necessary property of $\tau^*$:

From step 1, $\tau^*$ is the maximal solution to the constraint (2.2.1) at optimality. Also,

$$
\lim_{\tau \to +\infty} \phi(\tau) - c - \phi \left( \frac{\int_{-\infty}^{\tau} \Theta^*(x) f(x) \, dx}{\int_{-\infty}^{\tau} \Theta^*(x) f(x) \, dx} \right) = +\infty.
$$

Thus the following inequality holds:

$$
\frac{\partial \phi(\tau) - c - \phi \left( \frac{\int_{-\infty}^{\tau} \Theta^*(x) f(x) \, dx}{\int_{-\infty}^{\tau} \Theta^*(x) f(x) \, dx} \right)}{\partial \tau} \bigg|_{\tau = \tau^*} > 0. \tag{7}
$$

7The inequality is strict because we have assumed that $\tau^*$ is not a critical point.
Expanding the above inequality, we get

\[
\frac{\partial}{\partial \tau} \left( \phi(\tau) - c - \phi \left( \frac{\int_{-\infty}^{\tau^*} \Theta(x) f(x) dx}{\int_{-\infty}^{\tau^*} \Theta(x) f(x) dx} \right) \right) \bigg|_{\tau = \tau^*} \\
= \phi'(\tau^*) - \phi'(x_{nd}^*) \left( \frac{\Theta^*(\tau^*) f(\tau^*)}{\int_{\tau^*}^{\tau} \Theta^*(x) f(x) dx} \right) \left( \tau^* - \frac{\int_{-\infty}^{\tau^*} \Theta^*(x) x f(x) dx}{\int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx} \right) \\
= \phi'(\tau^*) - \phi'(x_{nd}^*) \left( \frac{\Theta^*(\tau^*) f(\tau^*)}{\int_{-\infty}^{\tau} \Theta^*(x) f(x) dx} \right) (\tau^* - x_{nd}^*) \\
= \frac{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*)(\tau^* - x_{nd}^*)}{\int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx} > 0.
\]

The above implies that \( \tau^* \) satisfies:

\[
\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*)(\tau^* - x_{nd}^*) > 0. \tag{2.4.1}
\]

**Step 4:** Taking the F.O.C.s of the Lagrangian, we get:

\[
\frac{\partial L}{\partial \Theta(x)} = f(x) (\phi(x_{nd}^*) - \phi(x) + \gamma_2(x_{nd}^* - x)) \text{, for each } x \leq \tau^*,
\]

\[
\frac{\partial L}{\partial \tau} \bigg|_{\tau = \tau^*} = \gamma_1 \phi'(\tau^*) + \gamma_2 f(\tau^*) (x_{nd}^* - \tau^*) = 0,
\]

\[
\frac{\partial L}{\partial x_{nd}} \bigg|_{x_{nd} = x_{nd}^*} = \phi'(x_{nd}^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \gamma_1 \phi'(x_{nd}^*) + \gamma_2 \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx = 0.
\]

Define

\[
S \equiv \frac{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx}{\phi'(\tau^*) \int_{-\infty}^{\tau^*} \Theta^*(x) f(x) dx - \phi'(x_{nd}^*) f(\tau^*)(\tau^* - x_{nd}^*)} > 1, \text{ from inequality (2.4.1)}.
\]

Solving for \( \gamma_2 \), we get:

\[
\gamma_2 = -S \phi'(x_{nd}^*).
\]
Hence, the sign of $\frac{\partial L}{\partial \Theta(x)}$ depends on the sign of

$$\Sigma(x) \equiv \phi(x^*_nd) - \phi(x) - S\phi'(x^*_nd)(x^*_nd - x) + c.$$  

(2.4.2)

$\phi(.)$ is convex, i.e., $\phi''(x) \geq 0$, which implies that:

$$\frac{\partial \Sigma(x)}{\partial x} = S\phi'(x^*_nd) - \phi'(x) > 0 \text{ for } x \leq x^*_nd, \text{ (from } S > 1 \text{ and } \phi''(.) \geq 0),$$

$$\frac{\partial^2 \Sigma(x)}{\partial x^2} = -\phi''(x) \leq 0.$$

Hence, in this case, $\Sigma(x)$ is concave and increasing for $x < x^*_nd \in \mathbb{R}$, with $\Sigma(x^*_nd) = c > 0$, which implies that there exists a $\theta^* \in (-\infty, x^*_nd)$ such that $\Sigma(x) < 0$ for $x < \theta^*$. In step 2, we have shown that it is not optimal to disclose an interval adjacent to the left of $(\tau^*, +\infty)$. It proves that an efficient disclosure law features $D^*_m = (-\infty, \theta^*)$, where $-\infty < \theta^* < \tau^* < +\infty$. $\square$

**Proof of Lemma 2:** By contradiction, suppose that there exists $l > l$ such that $(l, l) \subseteq (-\infty, 0)$ and $(l, l) \notin D^*_m$. Denote the set of of types liquidated as $L$. For any $x < c$, disclosure by mandate would induce liquidation. Also, non-disclosure price has to be positive if there is non-disclosure in equilibrium.

**Case (1):**

The efficient equilibrium is such that (i) the voluntary disclosure set is $(c, \infty)$, and (ii) $L = (-\infty, c)$. In this equilibrium, there is no non-disclosure on equilibrium path. Define $D'_m \equiv (-\infty, 0) \cup D^*_m$. Then, given this $D'_m$, those with $x < 0$ liquidate. In the efficient equilibrium there is a positive non-disclosure price $P_{D'_m}(nd) = \tau(D'_m) - c > 0$ such that those with $x > \tau(D'_m)$ voluntarily discloses, those with $x \in [0, c] \cap D'_m$ liquidates, and those with $x \in [0, \tau(D'_m)] \setminus D'_m$ sell without disclosure at price $P_{D'_m}(nd) = \mathbb{E}(\tilde{x} | \tilde{x} \notin D'_m, \tilde{x} \leq \tau(D'_m))$. This equilibrium features weakly more trade for positive types (i.e., those in
[0, c) \ {D^*_m} now trade) and less disclosure (i.e., \( \tau(D'_m) > c \)). Hence \( D'_m \) is more efficient than \( D^*_m \), a contradiction.

**Case (2):**

The efficient equilibrium is such that there is a positive non-disclosure price \( P_{D^*_m}(nd) = \tau(D^*_m) - c > 0 \) such that (i) the voluntary disclosure set is \( (\tau(D^*_m), \infty) \), where \( \tau(D^*_m) > c \), (ii) the liquidation set is \( L \equiv (-\infty, c) \cap D^*_m \), and (iii) others sell at \( P_{D^*_m}(nd) \) without disclosure with the price being determined by \( P_{D^*_m}(nd) = \mathbb{E}(\hat{x}|\hat{x} \notin D^*_m, \hat{x} \leq \tau(D^*_m)) > 0 \).

In this case, no sellers would voluntarily liquidate unless mandated to disclose for the reason that a positive non-disclosure price is always a better alternative to liquidation. Define \( D'_m \equiv (-\infty, 0) \cup D^*_m \). Given \( D'_m \), \( x \in (L, \overline{L}) \) would be liquidated, so are others with \( x \in (-\infty, 0) \). Moreover, the efficient equilibrium given this \( D'_m \) must feature a positive non-disclosure price \( P_{D'_m}(nd) > 0 \) such that \( P_{D'_m}(nd) > P_{D^*_m}(nd) \). Hence, the law \( D'_m \) features less trade for \( x < 0 \) and less disclosure, which makes it more efficient than \( D^*_m \), a contradiction.

We have proved that \( (-\infty, 0) \subseteq D^*_m \). For \( x_2 > x_1 \geq 0 \), liquidating \( x_1 \) implies lower loss in trade efficiency than liquidating \( x_1 \). Hence the argument in the proof for lemma 1 holds here as well, that is, one can always construct a more efficient law if the \( D^*_m \) is not a threshold type.\( \square \)

**Proof of Proposition 5:** For \( \theta > c \), \( \theta \) should maximize

\[
\mathcal{P}_1(\theta) \equiv \int_{-\infty}^{c} 0f(x)dx + \int_{c}^{\theta} (x-c)f(x)dx + \int_{\theta}^{\tau_\theta} P_{\theta}(nd)f(x)dx + \int_{\tau_\theta}^{+\infty} (x-c)g(x)dx
= \int_{c}^{+\infty} xf(x)dx - \int_{c}^{\theta} cf(x)dx - \int_{\tau_\theta}^{+\infty} cf(x)dx
= \int_{c}^{+\infty} xf(x)dx + \int_{-\infty}^{c} cf(x)dx - \left( \int_{-\infty}^{\theta} cf(x)dx + \int_{\tau_\theta}^{+\infty} cf(x)dx \right).
\]

\( \mathcal{C}(\theta) \) in the proof of Proposition 1
where $\tau_\theta$ uniquely solves equation (1.4.2) and $P_\theta(nd) = \frac{\int_{\theta}^{\tau_\theta} xf(x)dx}{\int_{\theta}^{\tau_\theta} f(x)dx}$.

Hence, for $\theta > c$, the objective function is equivalent to the objective function in the baseline, i.e.,

$$P_1(\theta) \equiv \text{Constant} - C(\theta).$$

From equation (1.7.2),

$$P'_1(\theta) = -C'(\theta)$$

$$= cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - cf(\theta)$$

$$= \frac{cf(\tau_\theta) \left( (\tau_\theta - \theta - c)f(\theta) \right)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} - cf(\theta)$$

$$= \frac{f(\theta)c((\tau_\theta - \theta)f(\tau_\theta) - (F(\tau_\theta) - F(\theta)))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}$$

For $0 \leq \theta \leq c$, $\theta$ should maximize

$$P_2(\theta) \equiv \int_{-\infty}^{0} 0f(x)dx + \int_{\theta}^{\tau_\theta} P_\theta(nd)f(x)dx + \int_{\tau_\theta}^{+\infty} (x - c)f(x)dx$$

$$= \int_{\theta}^{\tau_\theta} xf(x)dx + \int_{\tau_\theta}^{+\infty} (x - c)f(x)dx$$

$$= \int_{\theta}^{+\infty} xf(x)dx - \int_{\tau_\theta}^{+\infty} cf(x)dx$$

where $\tau_\theta$ uniquely solves equation (1.4.2) and $P_\theta(nd) = \frac{\int_{\theta}^{\tau_\theta} xf(x)dx}{\int_{\theta}^{\tau_\theta} f(x)dx}$.

---

8See the proof of Proposition 1.
Taking the first order derivative of the above objective function, we get

\[ P_2'(\theta) = cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - \theta f(\theta) \]
\[ = cf(\tau_\theta) \left( \frac{(\tau_\theta - \theta - c)f(\theta)}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} \right) - \theta f(\theta) \text{ (from (1.7.1))} \]
\[ = \frac{f(\theta)(c(\tau_\theta - c)f(\tau_\theta) - \theta(F(\tau_\theta) - F(\theta)))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}. \]

From the proof in Proposition 1, we have shown that \( F(\tau_\theta) - F(\theta) - cf(\tau_\theta) > 0 \). Hence,

\[ P_2'(0) = \frac{f(0)c(\tau_0 - c)f(\tau_0)}{F(\tau_0) - F(0) - cf(\tau_0)} > 0 \text{ (because } \tau_0 - c > 0) \], which proves that \( 0 < k^* \leq \theta^* \).

An immediate observation is that \( P_1(c) = P_2(c) \) and \( P_1'(c) = P_2'(c) \). Further

\[ P_1'(c) = P_2'(c) = f(c) \left( \frac{\chi(c)}{F(c) - F(\tau_\theta) - cf(\tau_\theta)} \right), \]

where

\[ \chi(c) \equiv (\tau_\theta - c)f(\tau_\theta) - (F(\tau_\theta) - F(c)). \]

**Case (1) \( \chi(c) > 0 \):** We know from the proof of uniqueness in Proposition 1 that

\[ P_1'(\theta) = cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - cf(\theta) > 0, \text{ for } \theta \leq c. \]

It implies that

\[ P_2'(\theta) = cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - \theta f(\theta) \geq cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - cf(\theta) > 0, \text{ for } \theta \leq c. \]

In this case, the solution to \( \theta^* > c \) is given by equation (1.4.3).
**Case (2)** \( \chi(c) < 0 \): We know from the proof of uniqueness in Proposition 1 that

\[
\mathcal{P}_1'(\theta) = cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - cf(\theta) < 0, \text{ for } \theta \geq c.
\]

It implies that

\[
P_2'(\theta) = cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - \theta f(\theta) \leq cf(\tau_\theta) \left( \frac{\partial \tau_\theta}{\partial \theta} \right) - cf(\theta) < 0, \text{ for } \theta \geq c.
\]

In this case, the solution to \( \theta^* < c \) is given by equation (2.2.3). We have shown that \( 0 = \mathcal{P}_2'(\theta^*) > \mathcal{P}_1'(\theta^*) \) because \( \theta^* < c \). Hence \( \theta^* \) in this case is greater than the baseline mandatory disclosure threshold.

From Corollary 2, we have that \( \chi(c) \geq (\leq) 0 \) when \( c \leq (\geq) c_0 \) where \( c_0 \) uniquely solves \( \chi(c_0) = 0. \Box \)

**Proof of Proposition 6:** The proof that the efficient law is a threshold type of rule follows along the lines of Lemma 1.

\( \Lambda(k, \theta) \) is positive for sufficiently large \( k \):

\[
\begin{align*}
\frac{\partial \Lambda(k, \theta)}{\partial k} |_{k=\kappa(\theta)} &= \delta - \frac{\kappa(\theta)f(\kappa(\theta)) \int_{\theta}^{\kappa(\theta)} f(x)dx - \int_{\theta}^{\kappa(\theta)} x f(x)dx f(\kappa(\theta))}{\left( \int_{\theta}^{\kappa(\theta)} f(x)dx \right)^2} \\
&= \delta - \frac{\kappa(\theta)f(\kappa(\theta)) - \left( \frac{\int_{\theta}^{\kappa(\theta)} f(x)dx}{\int_{\theta}^{\kappa(\theta)} f(x)dx} \right) f(\kappa(\theta))}{\int_{\theta}^{\kappa(\theta)} f(x)dx} \\
&= \delta - \frac{\kappa(\theta)f(\kappa(\theta)) - \delta \kappa(\theta)f(\kappa(\theta))}{\int_{\theta}^{\kappa(\theta)} f(x)dx} \\
&= \delta \frac{F(\kappa(\theta)) - F(\theta) - f(\kappa(\theta))\kappa(\theta)(1 - \delta)}{F(\kappa(\theta)) - F(\theta)} > 0.
\end{align*}
\]
Hence, we can conclude that \( \delta(F(\kappa(\theta)) - F(\theta)) - f(\kappa(\theta))\kappa(\theta)(1 - \delta) > 0 \). Applying the implicit function theorem to \( \Lambda(k, \theta) \), we get

\[
\frac{\partial \kappa(\theta)}{\partial \theta} = - \frac{\partial \Lambda/\theta}{\partial \Lambda/k} \bigg|_{k=\kappa(\theta)} = \frac{f(\theta)(\delta \kappa(\theta) - \theta)}{\delta(F(\kappa(\theta)) - F(\theta)) - f(\kappa(\theta))\kappa(\theta)(1 - \delta)} > 0,
\]

because \( \delta \kappa(\theta) = \mathbb{E}(x|x \in [\theta, \kappa(\theta)]) > \theta \).

First, if \( \kappa(\theta) < c/(1 - \delta) \), those with \( x < \theta \) would retain the asset to avoid disclosure because \( \theta < \kappa(\theta) < c/(1 - \delta) \). Those with \( x \in [\theta, \kappa(\theta)] \) would sell without disclosure, those with \( x \in (\kappa(\theta), c/(1 - \delta)) \) would retain the asset, and those with \( x > c/(1 - \delta) \) would sell with voluntary disclosure.

The objective function in this case is

\[
\mathcal{W}_3(\theta) = \int_{0}^{\theta} f(x) \delta x dx + \int_{\theta}^{\kappa(\theta)} f(x) dx + \int_{\kappa(\theta)}^{c/(1-\delta)} f(x) \delta x dx + \int_{c/(1-\delta)}^{+\infty} f(x)(x-c) dx.
\]

Taking the first order condition yields:

\[
\mathcal{W}_3'(\theta) = f(\theta)\delta + \kappa(\theta)f(\kappa(\theta)) \frac{\partial \kappa(\theta)}{\partial \theta} - \theta f(\theta) - f(\kappa(\theta))\delta \kappa(\theta) \frac{\partial \kappa(\theta)}{\partial \theta},
\]

\[
= (1 - \delta)\kappa(\theta)f(\kappa(\theta)) \frac{\partial \kappa(\theta)}{\partial \theta} - (1 - \delta)f(\theta)\theta,
\]

\[
= \left( \frac{(1 - \delta)\kappa(\theta)f(\kappa(\theta))f(\theta)(\delta \kappa(\theta) - \theta)}{\delta(F(\kappa(\theta)) - F(\theta)) - f(\kappa(\theta))\kappa(\theta)(1 - \delta)} \right) - (1 - \delta)f(\theta)\theta,
\]

\[
= (1 - \delta) \left( \left( \frac{\kappa(\theta)f(\kappa(\theta))(\kappa(\theta) - \theta) - \theta(F(\kappa(\theta)) - F(\theta))}{\delta(F(\kappa(\theta)) - F(\theta)) - f(\kappa(\theta))\kappa(\theta)(1 - \delta)} \right) f(\theta)(1 - \delta) \right),
\]

Evaluating the numerator at \( \theta = 0 \) yields:

\[
\kappa(0)f(\kappa(0))(\kappa(0) - 0) - 0(F(\kappa(0)) - F(0))
\]

\[
= \kappa(0)f(\kappa(0))\kappa(0) > 0.
\]
We denote the $\theta$ that solves the first order condition as $\theta_3$, that is

$$
\kappa(\theta_3)f(\kappa(\theta_3))(\kappa(\theta_3) - \theta_3) - \theta_3(F(\kappa(\theta_3)) - F(\theta_3)) = 0.
$$

Second, if $\theta < c/(1 - \delta)$ but $\kappa(\theta) > c/(1 - \delta)$, the firm retains in the middle. The objective function in this case is:

$$
W_2(\theta) = \int_0^\theta f(x)dx + \int_{\tau_\theta}^\theta x f(x)dx + \int_{\tau_\theta}^{+\infty} f(x)(x - c)dx.
$$

where $\tau_\theta$ is given by equation (1.4.2).

Taking the first-order-condition yields:

$$
W_2'(\theta) = f(\theta)\delta\theta + \tau_\theta f(\tau_\theta)\frac{\partial \tau_\theta}{\partial \theta} - \theta f(\theta) - f(\tau_\theta)(\tau_\theta - c)\frac{\partial \tau_\theta}{\partial \theta},
$$

$$
= f(\theta)\delta\theta + cf(\tau_\theta)\left(\frac{\tau_\theta - \theta - c}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}\right) - \theta f(\theta),
$$

$$
= f(\theta)\frac{cf(\tau_\theta)(\tau_\theta - c) - \theta(F(\tau_\theta) - F(\theta)) + \delta\theta(F(\tau_\theta) - F(\theta) - cf(\tau_\theta))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)},
$$

$$
= f(\theta)\frac{cf(\tau_\theta)(\tau_\theta - c - \delta\theta) - (1 - \delta)\theta(F(\tau_\theta) - F(\theta))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}.
$$

Evaluating the numerator at $\theta = 0$:

$$
cf(\tau_0)(\tau_0 - c - \delta\theta) - (1 - \delta)0(F(\tau_0) - F(0)),
$$

$$
= cf(\tau_0)(\tau_0 - c) > 0.
$$

We denote $\theta$ that solves the first order condition as $\theta_2$, that is,

$$
cf(\tau_{\theta_2})(\tau_{\theta_2} - c - \delta\theta_2) - (1 - \delta)\theta_2(F(\tau_{\theta_2}) - F(\theta_2)) = 0.
$$
The third case to consider is when \( \theta > \frac{c}{1 - \delta} \). The objective function is

\[
\mathcal{W}_1(\theta) = \int_0^{\frac{c}{1 - \delta}} f(x) \delta x + \int_{\frac{c}{1 - \delta}}^{\theta} f(x)(x - c)dx + \int_{\frac{c}{1 - \delta}}^{\tau_\theta} x f(x)dx + \int_{\tau_\theta}^{+\infty} f(x)(x - c)dx
\]

The first order condition is the same as the baseline model

\[
\mathcal{W}'_1(\theta) = f(\theta) \frac{c((\tau_\theta - \delta)(\tau_\theta) - (F(\tau_\theta) - F(\theta))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)}
\]

Denote the solution to the first order condition as \( \theta_1 \).

It can be easily shown that \( \mathcal{W}_2(\frac{c}{1 - \delta}) = \mathcal{W}_1(\frac{c}{1 - \delta}) \). Further,

\[
\mathcal{W}'_2(\frac{c}{1 - \delta}) = \mathcal{W}'_1(\frac{c}{1 - \delta})
\]

\[
= f(\frac{c}{1 - \delta}) \frac{\psi(\frac{c}{1 - \delta})}{F(\tau_{c/(1 - \delta)}) - F(\frac{c}{1 - \delta}) - cf(\tau_{c/(1 - \delta)})}
\]

where

\[
\psi(\frac{c}{1 - \delta}) \equiv c((\tau_{c/(1 - \delta)} - \frac{c}{1 - \delta})f(\tau_{c/(1 - \delta)}) - (F(\tau_{c/(1 - \delta)}) - F(\frac{c}{1 - \delta})).
\]

**Case (i):** \( \psi(\frac{c}{1 - \delta}) > 0 \). We know from the proof of uniqueness in Proposition 1 that \( \mathcal{W}'_1(\theta) = f(\theta) \frac{c((\tau_\theta - \theta)(\tau_\theta) - (F(\tau_\theta) - F(\theta))}{F(\tau_\theta) - F(\theta) - cf(\tau_\theta)} > 0 \) for \( \theta \leq \frac{c}{1 - \delta} \). It implies that

\[
\mathcal{W}'_2(\theta) \geq \mathcal{W}'_1(\theta) > 0 \quad \text{for} \quad \theta \leq \frac{c}{1 - \delta}.
\]

Consider the numerator of \( \mathcal{W}'_3(\theta) \) for \( \theta < \kappa(\theta) \leq \frac{c}{1 - \delta} \):

\[
\kappa(\theta)f(\kappa(\theta))(\kappa(\theta) - \theta) - \theta(F(\kappa(\theta)) - F(\theta))
\]

\[
> \theta \left( f(\kappa(\theta))(\kappa(\theta) - \theta) - (F(\kappa(\theta)) - F(\theta)) \right)
\]

From corollary 1, \( \frac{c}{1 - \delta} < \theta_1 < m \), which implies that \( \theta < \kappa(\theta) < m \). Hence
\[ f(\kappa(\theta))(\kappa(\theta) - \theta) - (F(\kappa(\theta)) - F(\theta)) > 0, \]
that is, \( W_3''(\theta) > 0 \) for \( \theta < \kappa(\theta) \leq c/(1 - \delta) \).

It proves that the optimal threshold is the baseline one, that is, \( \theta^* = \theta_1 \).

**Case (ii):** \( \psi(c/(1 - \delta)) < 0 \). We know from the proof of uniqueness in Proposition 1 that 
\[ W_1''(\theta) = f(\theta) \frac{c(\tau_0 - \theta) - (F(\tau_0) - F(\theta))}{F(\tau_0) - F(\theta) - cf(\tau_0)} < 0 \] for \( \theta \geq c/(1 - \delta) \). It implies that 
\[ W_2''(\theta) \leq W_1''(\theta) < 0 \] for \( \theta \geq c/(1 - \delta) \).

In this case, the optimal threshold is \( \theta^* < c/(1 - \delta) \).

When \( m > 0 \), from the proof of corollary 2, \( \psi(c/(1 - \delta)) > (\leq)0 \) when \( c < (\geq)c_1 \), where \( c_1 \) solves \( \psi(c_1/(1 - \delta)) = 0 \). When \( m = 0 \), it must be true that \( \psi(c/(1 - \delta)) < 0 \). \( \square \)
Chapter 3

Towards a positive theory of regulatory enforcement of financial reporting

3.1. Introduction

The accounting standard alone does not determine financial reporting outcomes, and, like any regulation, its effectiveness depends on its enforcement (Holthausen, 2009). More surprisingly, the empirical literature has documented significant variations in regulatory enforcement across markets and over time.\(^1\) While an extensive theoretical literature has examined firms’ economic decisions in response to different enforcement policies, we know little of the economic problems faced by a regulator choosing the enforcement. Due to the absence of a coherent theory of enforcement choices, most empirical studies remain agnostic about the causes of the empirical variations in enforcement.

In this study, we provide a first step toward addressing this gap in the literature. We do so within a model that contains a simple ingredient: the regulator is benevolent and makes an economic trade-off between the aggregate benefit and cost of enforcement.

For example, a well-cited benefit of more stringent enforcement of securities laws is greater capital market development and thus economic growth (Coffee, 2007). However, given that government agencies are subject to commitment problem and have incentives to choose the best action given the current situation (Kydland and Prescott, 1977), it is also possible that a larger market leads to more stringent enforcement by the regulator.

To formalize this two-way linkage between regulatory enforcement and capital market development, we consider a model in which a regulator has limited commitment and optimally chooses costly enforcement intensity after firms make long-term investments. The investments are made by owner-managers with a consumption horizon shorter than that of the generated cash flows, which induces them to bias their reports of investment outcomes when financing their liquidity from a capital market. The market discounts the price of each firm accordingly by rationally anticipating the magnitude of the bias and any cost of misreporting to the long-term value of the firm. Hence, the regulatory enforcement, by disciplining the misreporting, reduces the magnitude of the bias as well as the market discount, which then increases each firm’s stand-alone investment efficiency. The regulator is more willing to stringently enforce in a larger market with a larger such benefit in aggregate. Anticipating this choice of the regulator, each owner-manager cares about others’ investment choices when making his/her own investment decision; the more the others invest, the more likely there will be stringent enforcement, and the higher the payoff of making investments. In other words, the ex-post optimal enforcement policy (“discretionary policy”) induces an investment coordination problem among firms.

The theory thus provides a novel interpretation of the well-known positive empirical association between public enforcement intensity of securities laws and capital market development (Jackson and Roe, 2009). Although the most popular inference from this empirical observation is that public enforcement of securities laws has a positive impact on capital market development, our theory suggests that the causal relationship is actually two-way. It is not only that firms make more investments when expecting more stringent
enforcement, but also that more investments actually cause more stringent enforcement. Any primitive variables that affect either of these two endogenous objects can only affect the other in the same direction through this two-way relationship. From an efficiency standpoint, a discretionary policy is known to be sub-optimal from the \textit{ex-ante} perspective and tends to distort economic decisions. In the problem considered by this paper, firms have incentives to preempt the regulator with a market larger than the socially optimal one through over-investing because they do not internalize the social cost of public enforcement. However, a larger market has to be achieved collectively through investment coordination by the firms. As a result, the market can be over-sized and over-regulated when the coordination is relatively easy but under-sized and under-regulated otherwise.

Now we briefly describe the structure of the model. There are three types of strategic players: owner-managers, investors and a benevolent regulator. Each of the continuum of owner-managers is endowed with an investment project and decides whether or not to invest in the project. When making the investment decision, each owner-manager knows the forthcoming productivity (“fundamental”) of his/her own project and can infer about other projects because their heterogeneous fundamentals are correlated along the business cycle. After the investments and the fundamentals are publicly observed, the owner-managers finance their liquidity through selling their firms in a competitive market before the projects’ cash flows are realized. The equity transactions give rise to a demand for financial reporting because of a moral hazard problem in which the cash flow of each project invested also depends on an unobservable productive action by the owner-manager in the sense of Holmström and Tirole (1993). More precise information in the report can be desirable because it renders the price more responsive to the report and thus the choice of productive action more efficient. However, to deliver more information in practice, the accounting process has to incorporate more soft (subjective and manipulable) information from the owner-managers, which gives them more opportunities to introduce bias into the financial reports (Dye and Sridhar, 2004). Misreporting can be costly to the firms’ long-
term values, as well as to the owner-managers personally. Since the market rationally prices all consequences of the bias, the regulatory enforcement, which discourages the misreporting, increases the owner-managers’ returns on investment by economizing on both the personal and real costs of misreporting.

There is a wide variety of enforcement activities by regulators (e.g., regulatory reviews, monitoring auditors, market surveillance, investigation, imposing penalties, consulting). In this paper, we do not attempt to model any detail of enforcement mechanisms adopted by regulators in different markets nor any interaction among firms after an enforcement budget is fixed in the short term. Instead, the regulatory enforcement is modeled as a binary choice after the investments and the aggregate fundamental are observed but before the financial reporting occurs. Specifically, strong regulatory enforcement incurs a social cost but restrains the firms from misreporting. Weak regulatory enforcement allows the firms to misreport at a private cost; even when regulatory enforcement fails, non-regulatory (private) enforcement (e.g., shareholders litigation, board monitoring, audit) can function as a mechanism which makes misreporting costly to managers. The social cost of enforcement is not internalized by the firms.

Ex-post, the regulator finds it optimal to choose the strong enforcement when the market is sufficiently large so that the social benefit of the strong enforcement outweighs its cost. In equilibrium, firms with a sufficiently favorable project invest because a project with a more favorable fundamental not only yields higher price in expectation but also signals that other firms also have more favorable fundamentals, and that other firms believe that other firms also have favorable fundamentals, and so on. More firms investing induces the strong enforcement, which in turn increases the stand-alone investment efficiency of each firm. Hence, the ex-post optimal enforcement policy (“discretionary policy”) endogenously

2Such costs can be due to possible litigation by shareholders, diversion of managerial efforts, the costs incurred to ship inventories to third-party warehouses in order to book fictitious sales, and loss of future business opportunities due to the reputational effects of shareholders litigation.

3For related topics, see, e.g., Liang (2004), Nagar and Petacchi (2016), Ewert and Wagenhofer (2016), Laux and Stocken (2018), and Schantl and Wagenhofer (2018).
gives rise to this investment coordination motive.

If, ideally, (i) the regulator is able to commit to an optimal enforcement policy as a function of the aggregate fundamental, and (ii) the investment projects are owned by a representative owner-manager who then perfectly knows the the aggregate fundamental when making investments, the investment decisions would be conditional on the forthcoming enforcement and the regulator commits to the “first-best” policy which makes the trade-off between the ex-ante aggregate benefit and cost of enforcement given each state. However, if the regulator cannot make such a commitment, the representative owner-manager wants to over-invest in order to induce over-enforcement because the firms do not bear the social cost of enforcement. If possible, the owner-managers actually want to delegate the investments to the representative owner-manager. But, in reality, the investment decision is made by each individual firm and more investments in aggregate have to be achieved through more firms coordinately investing. As a result, the firms can possibly end up under-investing with the regulator who under-enforces relative to the “first-best” state-contingent enforcement plan; the need to speculate about other firms’ decision-relevant information makes the investment decisions more conservative.

In addition to explaining the variation of regulatory enforcement with respect to the business cycle, the theory also yields predictions about the cross-sectional variation of enforcement intensity with respect to changes in the legal environment and reporting standards being enforced. The model predicts that public enforcement is more stringent in a legal regime where private enforcement is less effective (e.g., less stringent liability standard, less accessible class-action procedure, more corruptive court system). For example, private litigation is less popular in the United Kingdom than in the United States, and the former focuses more on public enforcement (Armour, 2008; Jackson, 2006). In the model, the owner-managers (preparers) actually prefer weaker private enforcement as private enforcement substitutes for public enforcement. However, weaker private enforcement reduces the aggregate efficiency by increasing the public enforcement costs. As the
accounting aggregation process incorporates more precise but soft information from the owner-managers, both the aggregate investment and the enforcement intensity increase. The owner-managers prefer more weight on their soft-information, although the aggregate efficiency may decrease due to the higher enforcement costs. Hence, aggregate investment (market-size) may not be a good measure of aggregate efficiency to evaluate alternative accounting policies. The current theoretical literature on regulatory enforcement mainly studies the effects of enforcement on economic agents’ decisions at the firm level, with the enforcement being modeled as an exogenous variable. For example, Liang (2004) studies the effect of risk-sharing on the optimal structure of penalty imposed on earnings manipulation. Specifically, less penalty on misreporting to the direction consistent with the manager’s private information enhances risk-sharing. Ewert and Wagenhofer (2016) study the relationship between audit and regulatory investigation. They show that, if the regulatory investigation by nature is a less informative technique than the auditing, a greater investigation effort can actually crowd out the audit effort and reduce both earnings quality and firm value. Bertomeu, Darrough and Xue (2017) show that conservative accounting measurement should be preconditioned with more stringent enforcement because conservative measurement induces more earnings manipulation efforts by making the optimal incentive contracts steeper. Laux and Stocken (2018) study the effect of penalty structure (fixed versus proportional to the magnitude of violation) on the ex-ante standard setting when both the enforcement and the endogenous standard have effects on the magnitude of compliance. They show that a fixed penalty would induce a relaxed standard as the optimal one which is fully complied with, while greater sensitivity of the penalty to the magnitude of violation would induce a more stringent standard as the optimal one which is not fully

4 Aggregate investment has been widely used to evaluate the effects of International Financial Reporting Standards (IFRS) adoption (see, e.g., DeFond, Hu, Hung and Li, 2011; Florou and Pope, 2012; Gordon, Loeb and Zhu, 2012; Shima and Gordon, 2011). IFRS is believed to contain more fair-value elements than European GAAPs (Larson and Street, 2004; Schipper, 2005), and, as argued by Christensen et al. (2013), enforcement is an important omitted correlated variable when evaluating the effects of IFRS adoption.
complied with. More recently, Schantl and Wagenhofer (2018) argue that since private enforcement serves a dual role in monitoring public regulators as well as deterring frauds, private enforcement and public enforcement could be either substitutes or complements.\(^5\)

This paper, however, abstracts from modeling a specific enforcement mechanism and focuses on the coordination problem at the macro level. The only first-order assumption we make about enforcement is that it reduces the owner-managers’ ability to bias the reports. Hence, the contribution of the paper to the literature is to explicitly consider a possible interaction among reporting firms, and such interaction is endogenously caused by the discretionary enforcement policy of the regulator who has limited commitment. The coordination problem has non-trivial implications for the aggregate efficiency of the market. As argued by Kydland and Prescott (1977), “even if there is an agreed-upon, fixed social objective function and policymakers know the timing and magnitude of the effects of their actions, discretionary policy, namely, the selection of that decision which is best, given the current situation and a correct evaluation of the end-of-period position, does not result in the social objective function being maximized.” In other words, the economic agent who rationally anticipates the \textit{ex-post} optimal policy chooses an action which distorts the policy and thus the economic outcome. What they fail to consider, however, is that economic planning is not a game of the policymaker against a single (representative) economic agent, but, rather, a game against a large group of economic agents. In our example, firms want to distort enforcement policy through over-investing, while they can end up in under-investing if they have to speculate about others’ actions.\(^6\)

The rest of the paper proceeds as follows. Section 2 discusses the model set-up and

\(^{5}\)A common feature of this literature is that, in their models, the reporting constraints are set before the economic decisions (e.g., contracts, investments) are made. As argued by Schipper (1989), this timing assumption implies a temporary rigidity. In the model considered by this paper, the regulator responds to the investment decisions due to its limited commitment.

\(^{6}\)The coordination problem has been recognized by a large literature in speculative currency attacks and confidence crisis. For a review, please see Tabellini (2005). However, that literature had emphasized the indeterminacy of equilibrium due to the self-fulfilling nature of belief, until the theory of “global games” (Carlsson and Van Damme, 1993; Morris and Shin, 1998) pointed out that heterogeneity in beliefs among agents yield unique equilibrium in this class of coordination problem.
some preliminary results. Section 3 discusses some important benchmarks. Section 4 and Section 5 are about the main predictions of the model. Section 6 concludes the paper. All proofs are in the appendix.

3.2. The model

3.2.1. Set-up

In this section, we will first describe the production technologies by various agents of the model. Then we will discuss the timeline and the information structure. There is a continuum of risk-neutral owner-managers (firms) with mass normalized to 1. Each of them has an investment project and they decide whether or not to invest. The decision is coded as $d_i = 1(0)$ for investing (not investing). Each investment project requires a capital of one dollar which is paid by its owner-manager. Conditional on investment $d_i = 1$ and the mean $x_i + 2a_i$, the distribution of the cash flow (gross of the real cost incurred by misreporting) follows:

$$\tilde{\omega}_i = x_i + 2a_i + \tilde{\delta}_i,$$

where $\tilde{\delta}_i \sim N(0, \nu^{-1})$ is an i.i.d. random shock. Conditional on not-investing $d_i = 0$, the cash flow produced is normalized to 0. We call $x_i$ the investment fundamental of the firm $i$, and $a_i$ is an unobservable productive action privately chosen by the owner-manager at a private cost $a^2$. Before each firm’s $x_i$ is realized, it is a common knowledge that $\tilde{x}_i$ is subject to an i.i.d. uniform distribution with support on $[\tilde{\theta} - \eta, \tilde{\theta} + \eta]$, where $\eta > 0$ captures the dispersion of the investment fundamentals and $\tilde{\theta}$ is the aggregate fundamental (aggregate state of the economy) which is subject to a standard uniform distribution on $[0, 1]$.\footnote{As in Morris and Shin (1998), the uniform distribution assumption is made for tractability of the coordination problem. Although the model is not fully dynamic, one can still think of $\theta$ and $x_i$ as the real
The owner-manager does not directly consume the cash flow but sells it to a competitive market of risk-neutral investors for exogenous reasons, e.g., life-cycle considerations (Bertomeu and Magee, 2011; Dye, 1988). This assumption gives rise to a financial reporting problem which is specified in the following. Before selling the firm to the market, the investing owner-manager privately learns $\omega_i$. The accounting report $e_i$ aggregates the owner-manager’s report of his private information $\hat{\omega}_i$ and a hard signal collected by the accountant $s_i$ by using the weight $\lambda$ in the sense of Dye and Sridhar (2004):

$$e_i = \lambda \hat{\omega}_i + (1 - \lambda) s_i.$$ 

The hard signal is known by the owner-manager but not the market, and is correlated with the true cash flow:

$$\tilde{s}_i = \tilde{\omega}_i + \tilde{\epsilon}_i,$$

where $\tilde{\epsilon}_i \sim \mathcal{N}(0, \nu^{-1}_\epsilon)$ is an i.i.d. random noise. Only the accounting report $e_i$ is publicly known and the two primitive signals $s_i$ and $\hat{\omega}_i$ are latent inputs to the aggregation process.\(^8\)

So the accounting variable $\lambda$ regulates the weight on relevance (more precise managerial input which is potentially biased) versus reliability (more reliable but less precise hard signal). As will be shown, the model is not continuous at $\lambda = 0$ or $\lambda = 1$, thus we assume that $\lambda \in (0, 1)$.\(^9\)

How $\omega_i$ is reported by the owner-manager is affected by the enforcement intensity chosen by the regulator. Denote by $r = 1$ strong enforcement (hereafter “enforcement”) business cycle shock.

\(^8\)We take this aggregation process as exogenous. However, as shown by Dye and Sridhar (2004), the aggregated reporting regime can dominate the disaggregated reporting regime in a single firm setting. The assumption that the owner-manager knows $\omega_i$ perfectly is a way to operationalize the reliability-relevance trade-off with simplicity.

\(^9\)Our purpose is not to characterize the optimal $\lambda$ in this model and the standard-setting is a much more complicated process than an optimization problem. So we assume $\lambda$ is exogenous for now. Later, we provide comparative statics of the change in $\lambda$ on efficiency as well as various endogenous objects in the model.
and \( r = 0 \) weak enforcement (hereafter “non-enforcement”). When the regulator enforces \( r = 1 \), the owner-manager’s report has to be truthful, that is, \( \hat{\omega}_i = \omega_i \). When the regulator does not enforce, the owner-manager can bias the report at a private cost \( \frac{\beta}{2}(\omega_i - \hat{\omega}_i)^2 \), where \( \beta > 0 \) captures the effectiveness of private enforcement (e.g., liability standard, class-action rules, court independence and efficiency, audit effectiveness). Thereafter, we call \( \beta \) private enforcement effectiveness. Earnings manipulation is not only costly to the owner-manager but also costly to the long-term value of the firm (Bertomeu, 2013; Gao and Zhang, 2019; Kedia and Philippon, 2007; Strobl, 2013). Hence, we assume that the real cost of earnings manipulation is \( \gamma |\hat{\omega}_i - \omega_i| \) such that the net cash flow consumed by investors is \( \omega_i - \gamma |\hat{\omega}_i - \omega_i| \).\(^{10}\)

Enforcement \( r = 1 \) costs \( \kappa > 0 \) from the social perspective and the cost of non-enforcement \( r = 0 \) is normalized to 0. The enforcement cost includes both the resources spent directly by the regulator and the opportunity cost of not allocating the budget to other public areas. By assumption in the model, as well as in practice to a large extent, these costs are not internalized by the firms. The regulator is benevolent and maximizes the aggregate output net of the enforcement cost. The timeline of the model goes as follows:

- \( t = 1 \), each owner-manager privately observes his/her investment fundamental \( x_i \).
  Given \( x_i \), each of them publicly makes the investment decisions \( d_i \in \{0, 1\} \).

- \( t = 2 \), the aggregate fundamental \( \theta \) as well as the investing firms’ fundamentals \( \{x_i\} \) are publicly known, and then the regulator publicly chooses the enforcement intensity \( r \in \{0, 1\} \).

- \( t = 3 \), each investing owner-manager privately chooses productive action \( a_i \).

\(^{10}\)As will be shown, the functional form of the real cost is inconsequential as long as it is increasing in the amount of bias. Such costs can be due to diversion of managerial efforts, the costs incurred to ship inventories to third-party warehouses in order to book fictitious sales, and loss of future business opportunities due to the reputational effects of shareholders litigation.
• $t = 4$, each investing owner-manager privately learns $s_i$ and $\omega_i$, and reports $\hat{\omega}_i$ to
the accounting aggregation process. The accounting reports $e_i$ for investing firms are
disclosed and the capital market opens.

In reality, there is never full commitment or complete lack of commitment. In the
securities market setting, it can be argued that the regulator has commitment in the
short term by setting up the enforcement institutions in a particular way while may lack
commitment in the long term. In the model, we implement this “limited commitment” case
by assuming that the regulator cannot commit to a policy before the investment decisions
are made but can optimally choose a policy after the investment decisions are made while
before the reporting occurs.

Uncertainties about the aggregate fundamental are resolved after the investment deci-
sions are made, resembling the “time-to-build” assumption in Kydland and Prescott (1982),
that is, “it takes time to build a factory”. In other words, when making investment de-
cision, a firm does not know the state of the economy in the long term, the time when
its investment project becomes productive. The model also attempts to capture that, a
firm, when making the investment decision, knows more about its own productivity than
the aggregate productivity in the long term. Hence, we assume that firm $i$ knows $x_i$ when
making investment decision while the aggregate state $\theta$ only becomes public knowledge
after all firms make the investment decisions.

Investing firms’ fundamentals are publicly observed after the investments are made. For
example, the market can form a more precise belief of the cash flow produced by the project
after observing some characteristics of the project (e.g., types of production technology
used). Non-investing’s firms’ fundamentals do not affect the market’s perception of their
cash flows, which are normalized to be 0. The cash flows are realized after $t = 4$ and are
consumed by the investors.\textsuperscript{11}

\textsuperscript{11}We interpret the “fundamental” in the model as the part of the cash flow variation (e.g., technological
advancement) which is learned by the public through sources other than the financial reports.
3.2.2. The reporting subgame

The equity transaction connects the investment problem with a financial market problem because the owner-managers care about the market’s perception of their investment outcomes. In this section, we describe the reporting subgame equilibrium for an investing firm given enforcement $r$. For a non-investing firm, its price is always 0 because investment decision is observable. We suppress the firm index $i$ for ease of exposition. It is well known that the signal-jamming problem can admit a plethora of equilibria (Guttman, Kadan and Kandel, 2006; Riley, 1979; Stein, 1989). For tractability, we make the following assumption about the equilibrium which is widely used in prior literature.

There is a linear pricing function $P(e, r) = b_0(r) + b_1(r)e$, reporting function $\hat{\omega}(\omega, r)$, and optimal productive action $a(r)$ such that

(i) given $P(e, r)$, $s$, $\omega$ and $r$, $\hat{\omega}(\omega, r)$ maximizes

$$P(\lambda \hat{\omega} + (1 - \lambda)s, r) - \frac{\beta}{2}(\hat{\omega} - \omega)^2, \text{s.t. } \hat{\omega} = r\omega + (1 - r)\hat{\omega};$$

(ii) given $P(e, r)$ and $\hat{\omega}(\omega, r)$, the productive action $a(r)$ maximizes

$$E[P(\lambda \hat{\omega}(\hat{\omega}, r) + (1 - \lambda)s, r) - \frac{\beta}{2}(\hat{\omega}(\hat{\omega}, r) - \hat{\omega})^2] a - a^2;$$

(iii) given $a(r)$ and $\hat{\omega}(\omega, r)$, the pricing function satisfies

$$P(e, r) = E[\hat{\omega} - \gamma|\hat{\omega}(\hat{\omega}, r) - \hat{\omega}| |e].$$

The firm index $i$ is suppressed here because each firm faces an identical problem although with different fundamental $x$. We can summarize the equilibrium strategy of this

\textsuperscript{12}Since $s$ does not affect the $\hat{\omega}$ in equilibrium, we do not include $s$ in the reporting strategy function $\hat{\omega}(\omega, r)$ for notational simplicity. The result holds even when the owner-manager does not observe $s$. 

subgame in the following lemma.

**Lemma 3** The equilibrium of the subgame is given by

(i) the pricing function $P(e, r) = b_0(r) + b_1(r)e$ where,

\[
b_1(1) = b_1(0) = \frac{\nu^{-1}}{\nu^{-1} + (1 - \lambda)^2 \nu^{-1}} \equiv b_1,
\]

\[
b_0(r) = (1 - b_1)(x + 2a(r)) - (1 - r) \left( \frac{\lambda^2 b_1^2}{\beta} + \frac{\gamma \lambda b_1}{\beta} \right),
\]

(ii) the reporting strategy and productive action are

\[
\hat{\omega}(\omega, r) = r\omega + (1 - r) \left( \omega + \frac{\lambda b_1}{\beta} \right), \quad \text{and} \quad a(r) = b_1,
\]

(iii) the expected return on investment as of $t = 2$ given enforcement decision $r$ is

\[
x - (1 - b_1)^2 - (1 - r) \left( \frac{\gamma \lambda b_1}{\beta} + \frac{\beta}{2} \left( \frac{\lambda b_1}{\beta} \right)^2 \right).
\]

The expected payoff for an investing owner-manager as of $t = 2$ is the expected selling price net of the earnings manipulation cost, the productive action cost and the one dollar of capital investment. It can be decomposed into three parts. The first part is the fundamental of the investment project $x$, which is already public knowledge at the reporting stage. The second part $(1 - b_1)^2$ is the loss due to the moral hazard problem (hereafter, “moral hazard loss”). This loss decreases in $b_1$; the more informative the report, the closer the productive action is to the first-best choice $a^{f\beta} = 1$ because it helps the owner-manager internalize the real consequence of the productive action. The third part is the loss due to misreporting (hereafter “earnings manipulation loss”); the owner-manager bears the real cost to the long-term value of the firm $\frac{\gamma \lambda b_1}{\beta}$ as well as a private cost $\frac{\beta}{2} \left( \frac{\lambda b_1}{\beta} \right)^2$. The owner-managers do not gain from the ability to manipulate reports from the *ex-ante*
perspective, because the competitive market correctly prices all the consequences of the manipulation, that is, the market discounts the price by \( \frac{\lambda^2 \beta^2}{\beta} + \frac{\gamma \lambda b_1}{\beta} \), the first part of which is the effect of the bias on the report multiplied by the pricing coefficient \( b_1 \), and the second part of which is the real cost of earnings manipulation. Hence, enforcement increases the expected returns on investment, i.e., owner-managers are more willing to invest if they know that (believe that) the regulator enforces (with greater probability).

To economize on notation when discussing the investment and enforcement problem, we can define

\[
\ell \equiv (1 - b_1)^2, \quad c \equiv \gamma \left( \frac{\lambda b_1}{\beta} \right) + \frac{\beta}{2} \left( \frac{\lambda b_1}{\beta} \right)^2.
\]

For ease of exposition, define two thresholds \( x_L \equiv \ell \) and \( x_H \equiv \ell + c \); given enforcement \( r = 1 \), \( x > x_L \) guarantees a positive expected return on investment, and given non-enforcement \( r = 0 \), \( x > x_H \) guarantees a positive expected return on investment. Since we have restricted the support of \( \theta \) to be on \([0, 1]\), we make the following assumption to ensure interior solution of the investment equilibrium.

**Technical assumption 1:** \( \eta \leq \min \{ x_L, 1 - x_H \} \).

Given this assumption, if the aggregate economy is at its worst state \( \theta = 0 \), no firm invests, and, if the aggregate economy is at its best state \( \theta = 1 \), all firms invest.

### 3.2.3. The regulator’s enforcement decision

To specify how the owner-managers form an expectation of the enforcement decision when making the investment decisions, we need to first specify the problem of the regulator who optimally chooses the enforcement to maximize the aggregate output net of

---

13We can have two interpretations of the enforcement mechanism working here. The first is that enforcement prevents misreporting. For example, regulators directly monitor the reporting process on a continuous basis (e.g., regulatory reviews, monitoring the auditors who monitor the firms). The second is that enforcement increases the misreporting cost coefficient \( \beta \); regulators investigate and penalize misreporting after it occurs. In effect, our assumption is equivalent to that enforcement makes \( \beta \) to \( \infty \). But our results hold qualitatively even if we assume that enforcement only increases \( \beta \) by a finite amount.
the enforcement cost $\kappa$ at $t = 2$ after the investments are made. Each firm’s investment strategy $d_i(x)$ can be potentially asymmetric and we denote by $\pi(x)$ the proportion of firms that would invest given a particular fundamental $x$, that is, $\pi(x) \equiv \int_0^1 d_i(x) di$.\textsuperscript{14}

The \textit{ex-post} aggregate surplus as a function of $\{\theta, \pi(x), r\}$ can be written as:

$$r \left( \int_{\theta-\eta}^{\theta+\eta} \frac{1}{2\eta} \pi(x) (x - \ell) \, dx - \kappa \right) + (1 - r) \int_{\theta-\eta}^{\theta+\eta} \frac{1}{2\eta} \pi(x) (x - \ell - c) \, dx.$$

$\{\theta, \pi(x)\}$ determines the distribution of investing firms’ fundamentals. After observing the distribution, the regulator can compute the aggregate surplus of the market as a function of its enforcement decision $r$. When enforcement is $r = 1$ which costs $\kappa$, the expected payoff for each owner-manager is $x - \ell$. When enforcement is $r = 0$, each firm incurs the earnings manipulation loss $c$ but the regulator does not incur the enforcement cost $\kappa$.

Define a decision-relevant value function (marginal benefit) for the regulator

$$v(\theta, \pi(x)) \equiv c \times \frac{1}{2\eta} \pi(x) dx \quad n(\theta, \pi(x)) \text{: proportion of investing firm}$$

The value function, which equals the proportion of investing firm $n$ multiplied by the earnings manipulation loss $c$, captures the contribution of enforcement to the aggregate surplus of the market. The regulator enforces if and only if this marginal benefit is greater than the cost of enforcement, that is, $v(\theta, \pi(x)) > \kappa$. Hence the regulator enforces if and only if the market is large enough, that is, $n > \frac{\kappa}{c}$.

\textbf{Technical assumption 2}: It is optimal to enforce when all firms invest, that is, $v(\theta, 1) = c > \kappa$.

\textsuperscript{14}Firms’ strategy can be potentially asymmetric. However, in equilibrium, firms receiving the same fundamental $x$ would choose the same action. So $\pi(x)$ is in fact an indicator function in equilibrium.
When all firms invest, the contribution of enforcement is significantly large so that the regulator would enforce. Also observe that when no firm invests, it is not optimal to enforce because $\kappa > 0$. For ease of exposition, assume that the firms, when indifferent, would not invest, and the regulator, when indifferent, would not enforce.

It can be seen here that the optimal enforcement rule will be similar if the enforcement cost also includes a part which is an increasing function of the market-size with an increasing rate lower than $c$. Hence, what drives the decision rule is the enforcement’s economy of scale. By assumption, strong enforcement disciplines all firms’ misreporting.

In practice, regulators engage in enforcement activities that help monitor all firms (e.g., sampling firms to review, monitoring auditors). In the long term, a benevolent regulator makes choices about what institutional arrangements and how many resources are put in place to carry out those activities.

Given the optimal enforcement rule of the regulator and the subsequent reporting equilibrium in Lemma 3, we can define a reduced-form game of investment:

**Definition 3** A Bayesian-Nash-Equilibrium of investment is a strategy profile $\{d_i(x)\}$ such that:

(i) the strategy of each owner-manager $d_i(x) : [-\eta, 1 + \eta] \rightarrow \{0, 1\}$ is the best response to other owner-managers’ strategies;

(ii) whenever possible, owner-managers apply the Bayes’ rule.

It has been assumed that the regulator cannot make an announcement of an enforcement policy and commit to it before investments are made. Full commitment is impossible for a wide variety of public policies from monetary policies to flood controls for the reason that the policymakers are political entities who choose the best-action given the current situation (Kydland and Prescott, 1977). Hence it is not only that enforcement intensity increases firms’ stand-alone investment efficiency, but also that more investments actually
induce more stringent enforcement. The discretionary enforcement policy gives rise to a coordination problem among owner-managers because it is the aggregate investment that determines the enforcement.

3.3. Benchmarks

3.3.1. The first-best

Given the reporting friction and the moral hazard problem, we can characterize the socially-optimal choice of the investments and enforcement by solving a social planner’s problem.\footnote{Note that this is not the first-best of the overall problem because of the moral-hazard and misreporting friction.} For a given $\theta$, consider the combination of investment rule and enforcement rule $\{\pi^{fb}(x), r^{fb}(\theta)\}$ which solves

$$\max_{r \in \{1, 0\}, \pi(x) \in [0, 1]} r \left( \int_{\theta-\eta}^{\theta+\eta} \frac{1}{2\eta} \pi(x)(x - \ell) dx - \kappa \right) + (1 - r) \int_{\theta-\eta}^{\theta+\eta} \frac{1}{2\eta} \pi(x)(x - \ell - c) dx.$$

For $x \leq x_L$, it is always optimal not to invest. For $x_L < x \leq x_H$, it is optimal to invest if and only if $r = 1$. For $x > x_H$, it is always optimal to invest. Given this optimal investment decision, the \textit{ex-ante} aggregate marginal benefit of enforcement given a particular $\theta$ is

$$mb(\theta) \equiv \int_{[\theta-\eta, \theta+\eta] \cap (x_L, 1+\eta]} \frac{1}{2\eta} (x - \ell) dx - \int_{[\theta-\eta, \theta+\eta] \cap (x_H, 1+\eta]} \frac{1}{2\eta} (x - \ell - c) dx$$

$$= \int_{[\theta-\eta, \theta+\eta] \cap (x_L, x_H]} \frac{1}{2\eta} (x - \ell) dx + \int_{[\theta-\eta, \theta+\eta] \cap (x_H, 1+\eta]} \frac{1}{2\eta} c dx.$$

For $x \in (x_L, x_H]$, the marginal benefit of enforcement is to make infeasible investment feasible. For $x \in (x_H, 1 + \eta]$, the marginal benefit is to increase the return of the already feasible investment by $c$. The social planner cares about the sum (integration) of the
marginal benefits of enforcement for all firms. Observe that \( x - \ell < c \) for \( x < x_H \), so it must be true that \( mb(\theta) \) is an strictly increasing function for \( \theta \in (x_L - \eta, x_H + \eta) \) and is constant otherwise. For \( \theta \leq x_L - \eta \), \( mb(\theta) = 0 \), and for \( \theta \geq x_H + \eta \), \( mb(\theta) = c \). Moreover, \( mb(\theta) \) is continuous in \( \theta \).

The above features of the marginal benefit function can yield a simple threshold characterization of the first-best solution to the enforcement decision.

**Proposition 7** The first-best enforcement policy \( r^{fb}(\theta) \) is that \( r^{fb}(\theta) = 1 \) for \( \theta > \theta^{fb} \) and \( r^{fb}(\theta) = 0 \) for \( \theta \leq \theta^{fb} \), where \( \theta^{fb} \in (x_L - \eta, x_H + \eta) \) solves

\[
mb(\theta^{fb}) = \kappa.
\]

The infeasibility of the above first-best choices is due to the limited commitment problem by the regulator and the fact that investment opportunities are decentralized.

**Corollary 4** The first-best investments and enforcement choice can be implemented as a unique equilibrium if (i) the regulator can publicly commit to \( r^{fb}(\theta) \) before investments are made, and (ii) all investments projects are owned by a representative owner-manager.

This corollary is self-evident because, given the commitment which stipulates enforcement as a function of the aggregate state, the first-best investment choices are always incentive compatible for the representative owner-manager who perfectly knows the aggregate state when making the investment decisions.

Figure 3.1 is a graphical representation of the solution. When the realized \( \theta \) is below \( \theta^{fb} \). The regulator should not enforce, and only the green projects above \( x_H \) are invested. When the realized \( \theta \) is above \( \theta^{fb} \), the regulator should enforce, and all the green projects above \( x_L \) are invested.
3.3.2. The commitment problem without coordination

To understand the role of commitment in this problem, we consider the case in which the regulator has limited commitment but the investment projects are owned by a representative agent. The reporting equilibrium is not affected by the assumption because firms issue their own reports. This set-up assumes away the coordination problem, and resembles the applications originally considered by Kydland and Prescott (1977).

The solution to this problem is rather simple. The representative owner-manager knows that the regulator will enforce given that the market is larger than $\kappa c$. Hence, if the aggregate fundamental $\theta$ is sufficiently large such that the projects with fundamental $x > x_L$ amount to a market larger than $\kappa c$, the representative owner-manager will invest in those projects with $x \in (x_L, x_H]$.

**Proposition 8** A representative owner-manager with all the investment projects invests
in those with fundamental \(x > x_L\) if the aggregate fundamental \(\theta > \bar{\theta}\) where

\[
\theta \equiv x_L + \left(\frac{2\kappa}{c} - 1\right) \eta.
\]

Otherwise, the owner-manager only invests in projects with fundamental \(x > x_H\).

When \(\theta = \bar{\theta}\), the proportion of invested projects equals the cut-off market-size \(\frac{k}{c}\). It would be interesting to compare \(\bar{\theta}\) with \(\theta^{fb}\) to understand the role of commitment.

**Corollary 5** With a representative owner-manager, the regulator can over-enforce, i.e., \(\bar{\theta} < \theta^{fb}\).

The reason for the over-enforcement is that the owner-manager does not internalize the social cost of enforcement and preempts the regulator with a large market, which the regulator has no choice but to enforce. Hence, associated with the over-enforcement problem is an over-investment problem because some projects with \(x \in (x_L, x_H)\), which are invested in, should not be invested in in the first-best when \(\theta \in (\bar{\theta}, \theta^{fb})\). Next, we solve the baseline model where investments are decentralized; owner-managers choose their own investments knowing only their own fundamentals.

Figure 3.2 is a graphical representation of the solution. In the case above, the realized \(\theta\) is below \(\bar{\theta}\) and all the projects are below \(x_H\). The representative owner-manager invests in no project anticipating no enforcement. In the case below, the realized \(\theta\) is above \(\bar{\theta}\). The representative owner-manager invests in all projects above \(x_L\) anticipating enforcement. However, since \(\theta\) is also below \(\theta^{fb}\), all the red projects in \((x_L, x_H)\) are over-investment which should not be invested in the first-best. The regulator over-enforces as a result of the over-investment.
3.4. Main results

3.4.1. The unique investment equilibrium

In this section, we show that there is a unique equilibrium of the reduced-form game of investment as defined in definition 3, given the linear reporting equilibrium. When an owner-manager only knows his own fundamental, he uses it for two purposes. First, it determines the baseline investment return $x - \ell$. However, the investment return is also dependent on other firms’ investments because the market-size affects the enforcement decision. Hence the signal is also useful for updating about others’ beliefs, and also others’ beliefs about others’ beliefs, and so on. Despite of this complicated beliefs system, we can show that the unique equilibrium strategy is of a simple threshold type by using a similar approach of Morris and Shin (1998). We prove it in several steps.

Given a strategies profile $\pi(x)$, we can define the set of aggregate fundamentals which will NOT induce enforcement:

$$\Theta(\pi) \equiv \{ \theta | v(\theta, \pi(x)) \leq \kappa \}.$$
Hence, the *ex-post* payoff of an investing owner-manager given his own fundamental $x$, the aggregate fundamental $\theta$, and the strategies of others is:

$$ h(x, \theta, \pi) \equiv \begin{cases} 
  x - \ell, & \text{if } \theta \notin \Theta(\pi), \\
  x - \ell - c, & \text{if } \theta \in \Theta(\pi).
\end{cases} $$

Since the owner-manager does not know $\theta$ and has to infer it from his own fundamental $x$, the expected payoff of investing can be written as

$$ u(x, \pi) \equiv \mathbb{E}[h(x, \theta, \pi)|x] \equiv \begin{cases} 
  x - \ell - \int_{[0,x+\eta]\cap\Theta(\pi)} \frac{c}{x+\eta} d\theta < 0, & \text{for } x \in (-\eta, \eta), \\
  x - \ell - \int_{[x-\eta,x+\eta]\cap\Theta(\pi)} \frac{c}{x+\eta} d\theta, & \text{for } x \in [\eta, 1-\eta], \\
  x - \ell - \int_{[x-\eta,1]\cap\Theta(\pi)} \frac{c}{1-x+\eta} d\theta > 0, & \text{for } x \in (1-\eta, 1+\eta).
\end{cases} $$

**Lemma 4** If $\pi(x) \geq \pi'(x)$ for all $x$, it must be true that $u(x, \pi) \geq u(x, \pi')$ for all $x$.

This lemma formalizes the intuition that investments are strategic complements, that is, the more the other owner-managers invest, the higher the payoff for an owner-manager to invest. This strategic complementarity is not driven by any assumption about the production technology, but rather, the discretionary enforcement policy by the regulator.

Next we conjecture a special case of $\pi(x)$, that is, all owner-managers invest if and only if $x > t$, for $t \in [x_L, x_H]$. Denote this $\pi(x)$ as an indicator function

$$ D_t(x) \equiv \begin{cases} 
  1, & \text{if } x > t, \\
  0, & \text{if } x \leq t.
\end{cases} $$

The investment payoff for the project with fundamental $t$ must have the following property.

---

16For $x \notin [x_L, x_H]$, the firm has a dominant strategy of either investing or not investing.
Lemma 5 \( u(t, D_t) \) is continuous and strictly increasing in \( t \). Moreover, there exists a unique \( x^* = \ell + \kappa \in (x_L, x_H) \) such that \( u(x^*, D_{x^*}) = 0 \), and \( D_{x^*}(x) \) is indeed an equilibrium.

If all other owner-managers follow the threshold strategy \( D_t \), then the expected payoff of the marginal owner-manager to invest is increasing in the threshold \( t \). Lemma 4 and Lemma 5 jointly yield the following result, that is, the strategy \( D_{x^*}(x) \) is actually the unique equilibrium investment strategy.

**Proposition 9** The unique investment equilibrium is that owner-managers with fundamental \( x > x^* = \ell + \kappa \in (x_L, x_H) \) invest and those with \( x \leq x^* \) do not invest.

The owner-managers invest given sufficiently high fundamental because (i) a high fundamental implies more cash flow generated by investment and thus higher price in expectation, and (ii) a high fundamental implies that it is more likely that the others also have high fundamentals and invest more, which in turn induces enforcement. Hence financial reporting in a market with more favorable investment projects is more likely to be enforced.

**Corollary 6** The size of the market is sufficiently large to induce enforcement when \( \theta > \theta^* \), where

\[
\theta^* = x^* + \left( \frac{2\kappa}{c} - 1 \right) \eta.
\]

The aggregate fundamental \( \theta \) is a sufficient statistic of the distribution of all firms’ fundamentals. So in equilibrium, enforcement is positively correlated with \( \theta \). An important observation here is that the the regulator is less likely to enforce with the coordination problem because \( \theta^* > \underline{\theta} \), that is, for \( \theta \in (\underline{\theta}, \theta^*) \), the regulator enforces with centralized investment but does not enforce with decentralized investment. It is also obvious that the
owner-managers are better-off if they are able to delegate the investment decisions to a representative owner-manager. The reason is that, in the latter case, (i) more enforcement is provided, that is, \( \theta^* > \theta \), (ii) and more information is available for decision making, that is, the representative owner-manager knows the realization of \( \theta \).

We can also compare this equilibrium threshold \( \theta^* \) with the threshold \( \theta^{fb} \) in the first-best case to understand to what extent the discretionary enforcement policy is sub-optimal in this heterogeneous firms setting, and how it distorts the investment decisions from the social perspective.

**Proposition 10** The regulator can (i) over-enforce, i.e., \( \theta^* < \theta^{fb} \leq x^* \), if \( 2\kappa < c \), or (ii) under-enforce, i.e., \( x^* \leq \theta^{fb} < \theta^* \), if \( 2\kappa > c \).

Decentralization of investments has two consequences. First, an owner-manager does not know the investment choices of other owner-managers. Second, an owner-manager does not know the decision-relevant information of other owner-managers. Hence, the owner-managers have to collectively distort the enforcement decision through investment coordination by speculating about others’ investment fundamentals. When making the investment decision, an owner-manager has to consider the possibility that his/her fundamental is better than most of others’ fundamentals such that most of others do not invest. As a result, when all the owner-managers are contemplating in this way, there can be under-investment and under-enforcement when the coordination among firms is difficult.

If \( 2\kappa < c \) and thus \( \theta^* < \theta^{fb} \leq x^* \), for the state \( \theta \in (\theta^*, \theta^{fb}) \), the regulator enforces in the equilibrium while it should not enforce in the first-best. The reason is that some projects with \( x \in (x^*, x_H) \) are invested in in the equilibrium while they should be forgone in the first-best. The over-enforcement is actually caused by over-investment. Similarly, if \( 2\kappa > c \) and thus \( x^* \leq \theta^{fb} < \theta^* \), for the state \( \theta \in (\theta^{fb}, \theta^*) \), the regulator does not enforce in the equilibrium while it should enforce in the first-best. The reason is that some projects with \( x \in (x_L, x^*) \) are forgone in the equilibrium while they should be invested in in the
first-best. The under-enforcement is actually caused by under-investment. In summary, the market can be over-sized and over-regulated or under-sized and under-regulated given some aggregate state $\theta$.

The ratio $\frac{2\kappa}{c}$ is indexing the difficulty of the coordination among firms because the greater the ratio, the larger the market needs to be to induce enforcement. The cost of enforcement $\kappa$ hinders the coordination but, surprisingly, the earnings manipulation loss $c$ improves the coordination. The reason is related to the limited commitment by the regulator; a potentially more serious earnings manipulation problem gives the regulator no choice \textit{ex-post} but to enforce it more stringently. Anticipating more stringent enforcement, the firms have more incentives to invest which in turn further reinforces the expectation of stringent enforcement and thus the coordination among firms.

Figure 3.3: The coordination problem

Figure 3.3 is a graphical representation of the problem. In the case above, enforcement cost is small relative to earnings manipulation loss. When the realized $\theta$ is in between $\theta^*$
and $\theta^{fb}$. The regulator enforces in equilibrium while it should not enforce in the first-best. In equilibrium, all projects above $x^*$ are invested while in the first-best only those above $x_H$ should be invested. The over-enforcement is caused by over-investment. However, in the case below, the enforcement cost is large relative to earnings manipulation loss. When the realized $\theta$ is in between $\theta^{fb}$ and $\theta^*$, the regulator does not enforce in equilibrium while it should enforce in the first-best. In equilibrium, all projects above $x^*$ are invested while in the first-best those above $x_L$ should all be invested. The under-enforcement is caused by under-investment.

3.5. Comparative statics

3.5.1. Enforcement intensity and market-size

Although the realized enforcement intensity and market-size are also determined by the aggregate fundamental $\theta$, we can examine how the primitive variables of the model affect the thresholds $\theta^*$ and $x^*$ to understand the cross-sectional variation of enforcement and market-size. The cross-sectional predictions are not only helpful in terms of understanding the difference in enforcement intensity across markets, but also helpful in terms of understanding the change in enforcement of a market with respect to regime changes (e.g., change of private litigation rules, IFRS adoption).

**Proposition 11** (i) The enforcement threshold $\theta^*$ increases in enforcement cost $\kappa$, private enforcement $\beta$, and cash-flow precision $\nu_δ$, decreases in real cost of earnings manipulation $\gamma$, hard signal precision $\nu_c$, and soft information weight $\lambda$, and is non-monotonic in firms dispersion $\eta$. (ii) The investment threshold $x^*$ increases in enforcement cost $\kappa$ and cash flow precision $\nu_δ$, decreases in hard signal precision $\nu_c$ and soft information weight $\lambda$.

Generally speaking, the enforcement intensity increases in the variables that decrease the moral hazard loss $\ell$ and (or) increase the earnings manipulation loss $c$. The reason
is that the moral hazard loss $\ell$ reduces the investments since the market rationally price it; a smaller market is less likely to induce enforcement. A higher earnings manipulation loss $c$ implies a greater marginal benefit of enforcement given a market-size; it increases the enforcement intensity holding constant the cost of enforcement. It is not surprising to note that private enforcement $\beta$ substitutes for public enforcement because stronger private enforcement reduces the marginal benefit of public enforcement. In reality, the reason why there exists public enforcement is because private enforcement is imperfect. For example, it has been observed that the United Kingdom relies more on public enforcement relative to the United States due to the absence of private enforcement (Armour, 2008; Jackson, 2006). A greater weight on the more precise soft information $\lambda$ induces a larger market as well as more earnings manipulation, and hence greater enforcement intensity. This is consistent with the concern of Schipper (2005) that the fair-value elements in IFRS can cause reliability problems and the observation that the adoption of IFRS in European countries was typically associated with simultaneous enhancement of public enforcement institutions (Christensen et al., 2013).

The effect of firms dispersion $\eta$ on the enforcement threshold depends on if $2\kappa > c$ or $2\kappa < c$. When $2\kappa > c$, the enforcement threshold $\theta^*$ is greater than $x^*$ and there is a under-enforcement problem. In this case, an increase in $\eta$ intensifies this problem by making $\theta^*$ larger. Similarly, when $2\kappa < c$, an increase in $\eta$ intensifies the over-enforcement problem by making $\theta^*$ smaller.

The investment threshold $x^*$ is not affected by the earnings manipulation loss $c$ for the reason that although it signifies an increase in the misreporting problem, it also increases enforcement intensity which solves the misreporting problem. We summarize these predictions into the Table 3.1. It can be seen that a primitive variable which affects $\theta^*$ can only affect $x^*$ in the same direction if it does affects $x^*$, and vice versa. Two markets can differ in those primitive variables as well as in the aggregate fundamental $\theta$. Hence, we have the following observation.
Table 3.1: Cross-sectional predictions

<table>
<thead>
<tr>
<th></th>
<th>Enforcement threshold $\theta^*$</th>
<th>Investment threshold $x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms dispersion: $\eta$</td>
<td>non-monotonic</td>
<td>no-effect</td>
</tr>
<tr>
<td>Enforcement cost: $\kappa$</td>
<td>increase</td>
<td>increase</td>
</tr>
<tr>
<td>Private enforcement: $\beta$</td>
<td>increase</td>
<td>no-effect</td>
</tr>
<tr>
<td>Real cost: $\gamma$</td>
<td>decrease</td>
<td>no-effect</td>
</tr>
<tr>
<td>Cash-flow precision: $\nu_\delta$</td>
<td>increase</td>
<td>increase</td>
</tr>
<tr>
<td>Hard info. precision: $\nu_\varepsilon$</td>
<td>decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>Accounting weight: $\lambda$</td>
<td>decrease</td>
<td>decrease</td>
</tr>
</tbody>
</table>

**Corollary 7** Given the aggregate fundamental $\theta$, the change of any primitive variable, holding constant others, can only change market size $n^*$ and enforcement $r^*$ in the same direction, if it affects both of them.\(^{17}\)

Two markets can differ in both the aggregate fundamental $\theta$ and any of these primitive variables. Given the empirically-documented positive association between market-size and enforcement intensity in Jackson and Roe (2009), the most popular inference is that more stringent enforcement of securities laws causes greater capital market development. Our model, however, offers a theory of a possible two-way causal relationship. It is not only that firms make more investments when anticipating more stringent enforcement, but also that more investments actually cause more stringent enforcement. Any primitive variable that affects either of these two endogenous variables can only affect the other in the same direction through this two-way relationship.

\(^{17}\)This argument can be directly seen from the Table 1 except for $\eta$. Even though firms dispersion $\eta$ does not affect the investment threshold, it does affect the size of the market. It can be shown (see the proof) that the direction of the firms dispersion’s effect on enforcement intensity always coincide with that on the market-size.
3.5.2. Owner-managers’ preference and aggregate efficiency

In this section, we separately examine the effect of primitives on the own-managers’ welfare and the aggregate efficiency of the market. Since the owner-managers are preparers in the model, the analysis helps understand the preparers’ preference of legal environment and accounting policy. As will be shown, the owner-managers’ preference differs from that of a benevolent social planner.

First, we compute the aggregate welfare of the owner-managers (or equivalently the expected welfare of an owner-manager before he knows $x$) conditional on the realization of $\theta$.

$$
\mathcal{M}(\theta) \equiv \begin{cases} 
0, & \text{for } 0 \leq \theta \leq x^* - \eta, \\
\int_{x^* - \eta}^{\theta + \eta} (x - \ell - c) \frac{1}{2\eta} \, dx, & \text{for } x^* - \eta \leq \theta \leq \theta^*, \\
\int_{x^*}^{\theta + \eta} (x - \ell) \frac{1}{2\eta} \, dx, & \text{for } \theta^* \leq \theta \leq x^* + \eta, \\
\int_{\theta - \eta}^{x^* + \eta} (x - \ell) \frac{1}{2\eta} \, dx, & \text{for } x^* + \eta \leq \theta \leq 1.
\end{cases}
$$

Hence, unconditionally, the expected welfare of the owner-managers is

$$
\mathbb{E}[\mathcal{M}(\theta)] \equiv \int_{0}^{1} \mathcal{M}(\theta) \, d\theta.
$$

**Proposition 12** The expected welfare of the owner-managers increases in real cost of earnings manipulation $\gamma$, hard information precision $\nu_{\epsilon}$, and soft information weight $\lambda$, decreases in enforcement cost $\kappa$, private enforcement $\beta$, and cash flow precision $\nu_{\delta}$, and is non-monotonic in firms dispersion $\eta$.

Generally speaking, any primitive variable that reduces the moral hazard loss $\ell$ and (or) increases the earnings manipulation loss $c$ make the owner-managers better-off from the ex-ante perspective. The latter observation may seem counter-intuitive. It is due
to the limited commitment by the regulator and the fact that owner-managers do not internalize the enforcement cost; owner-managers benefit from over-enforcement and any variable that makes the regulator’s enforcement intensity stronger is preferred. Hence, the owner-managers prefer weaker private enforcement and more weight on the input of their soft information. Less effective private enforcement induces more public enforcement by the regulator, and more soft information not only reduces the moral hazard problem but also induces more public enforcement.

We can also compute the aggregate efficiency condition on $\theta$:

$$W(\theta) \equiv \begin{cases} 
0, & \text{for } 0 \leq \theta \leq x^* - \eta, \\
\int_{x^* - \eta}^{\theta + \eta} (x - \ell - c) \frac{1}{2\eta} \, dx, & \text{for } x^* - \eta \leq \theta \leq \theta^*, \\
\int_{x^* + \eta}^{\theta + \eta} (x - \ell) \frac{1}{2\eta} \, dx - \kappa, & \text{for } \theta^* \leq \theta \leq x^* + \eta, \\
\int_{\theta - \eta}^{\theta + \eta} (x - \ell) \frac{1}{2\eta} \, dx - \kappa, & \text{for } x^* + \eta \leq \theta \leq 1.
\end{cases}$$

Hence, unconditionally, the expected aggregate efficiency is

$$\mathbb{E}[W(\theta)] = \int_0^1 W(\theta) \, d\theta.$$

Note that the only difference between $\mathbb{E}[W(\theta)]$ and $\mathbb{E}[M(\theta)]$ is the enforcement cost $\kappa$ when $\theta > \theta^*$. It is the reason why the owner-managers’ preference differ from that of a social planner.

**Proposition 13** The expected aggregate efficiency increases in private enforcement $\beta$, decreases in enforcement cost $\kappa$ and real cost of earnings manipulation $\gamma$, and is non-monotonic in firms dispersion $\eta$, cash flow precision $\nu_5$, hard signal precision $\nu_6$, and soft information weight $\lambda$.

Any variable that reduces the moral hazard loss $\ell$ and (or) reduces the earnings manipu-
lation loss $c$ will increase the aggregate efficiency. Stronger private enforcement $\beta$ helps the regulator economize on the enforcement cost and thus increases the aggregate efficiency, while it has been shown that the owner-managers do not prefer strong private enforcement.\footnote{This prediction should be interpreted with caution because we do not model what determines the private enforcement and the social costs of it.} As has been shown, cash flow precision $\nu$, hard information precision $\nu_c$, and soft information weight $\lambda$ have opposing effects on the moral hazard loss $\ell$ and the earnings manipulation loss $c$. Hence, the effect of them on the aggregate efficiency can be either positive or negative depending on which effect dominates.\footnote{It is possible that only a monotonic part of the relationship is feasible given some values of other primitive variables.} Although the owner-managers always prefer more weight on their soft information because they do not internalize the enforcement cost, more precise but soft information may be efficiency-reducing for the market due to higher enforcement cost. It is worth comparing this observation with that of Dye and Sridhar (2004) which does not model enforcement choices. In their set-up, the representative owner-manager internalizes the reliability-relevance trade-off; more soft information is more precise but induces greater earnings manipulation loss. In our set-up, the more severe the misreporting problem, the more effort a benevolent regulator puts into solving the problem which actually leaves the owner-managers better-off.

Another important implication of this prediction is that aggregate investment (market-size) may not be a good measure of aggregate efficiency to evaluate alternative accounting policies, although it has been shown that soft information weight $\lambda$ always increases the market-size. After the adoption of IFRS by European countries, some forms of aggregate investment measure have been adopted to test the capital market effects of IFRS adoption.\footnote{See, e.g., DeFond et al. (2011), Shima and Gordon (2011), Florou and Pope (2012), and Gordon et al. (2012).} We should interpret the efficiency implication of those results with caution.

Through out the paper, we take the feature of the accounting standard $\lambda$ as something determined by forces outside the model. The reason is that the set-up in the model is
not idea for characterizing the optimal choice of $\lambda$ by a benevolent standard setter. The reliability-relevance trade-off examined here can yield the expected aggregate efficiency a U-shaped function in $\lambda$. And the technical assumptions made by the paper to generate interior solution to the enforcement and investment problem require the value of $\lambda$ to be away from 0 and 1.\textsuperscript{21}

There is a huge literature in accounting that examines the political and economic forces that affect the accounting standards (Watts and Zimmerman, 1978,9). This paper, however, provides a positive theory of accounting standard enforcement. Despite of the fact that $\lambda$ is exogenous in the model, the problem examined in this paper does provide some normative implications for standard setting, because, in most cases, the decision facing the standard setter is about choosing between a couple of alternatives, rather than maximizing over a continuous variable. As has been shown, the preparers have a preference over alternative standards which can be in conflict with that of a benevolent standard-setter. In particular, the preparers have incentives to promote accounting standards that induce more investment and excessive enforcement cost.

3.6. Conclusion

Economic planning is not a game against nature but, rather, a game against a large group of economic agents of rational expectations. Enforcement policy affects a large number of agents who prepare and use financial reports. The regulator provides the preparers a public good which solves a commitment problem for them. However, the regulator itself is subject to its own commitment problem which gives the preparers opportunities to distort the enforcement decision through investment coordination. Yet, surprisingly, the existing research has largely avoided the question of enforcement as an economic choice. As a result,

\textsuperscript{21}Suppose $\lambda = 0$, there is no misreporting problem at all and thus there is no enforcement and coordination problem. Suppose $\lambda = 1$, the moral hazard problem is completely resolved and does not affect investment decisions at all.
we do not have a good framework to understand the link between the observed choices in enforcement and economic decisions. The notion that enforcement is an endogenous choice is the core of this study. We develop several keys to interpret in economic sense the association between enforcement and investments in aggregate.

Although the theory of this paper provides many predictions of both positive and normative implications, we also need to put some caveats on it. The model assumes too simple an enforcement mechanism; enforcement is just one piece of problem which needs economy of scale to solve. However, in reality, enforcement involves many such problems because there are many different ways firms can misreport and there are many possible ways firms can hide their misreporting. What a regulator faces on the margin is to decide whether or not to solve each of these problems through various different mechanisms. The model focuses exclusively on a stylized investment choice problem as affected by the expectation of enforcement to keep the intuitions as simple and transparent as possible. However, it assumes away many other channels that determine firms’ investment choices. The model keeps the interaction among firms being primarily driven by a regulatory choice, while muting other possible informational and operational externalities among firms. Moreover, the most important element of this strategic interaction in reality is dynamics. We only consider the problem as a simple game of one period business cycle. Economic planning, however, involves a game of infinite periods of business cycle. While these channels should not be all considered in a single model, the problem of the endogenous regulatory choice offers a simple framework to obtain new insights about various empirical observations.

3.7. Appendix for chapter 3

Proof of Lemma 3

First, consider the case where $r = 0$. Given the price function, the report $\hat{w}(\omega, 0)$
maximizes

\[ b_0(0) + b_1(0)(\lambda \hat{\omega} + (1 - \lambda)s) - \frac{\beta}{2} (\hat{\omega} - \omega)^2. \]

Hence \( \hat{\omega}(\omega, 0) = \omega + \frac{\lambda b_1(0)}{\beta} \). Substituting the reporting strategy into the aggregation process yields \( \hat{e} = \frac{\lambda^2 b_1(0)}{\beta} + \hat{\omega} + (1 - \lambda)\hat{\epsilon} \). Applying the Bayes’ rule, the price should be

\[ p(e, 0) = \left( 1 - \frac{\nu^{-1}}{\nu^{-1} + (1 - \lambda)^2 \nu^{-1}_e} \right) (x + 2a(0)) + \left( \frac{\nu^{-1}}{\nu^{-1} + (1 - \lambda)^2 \nu^{-1}_e} \right) \left( e - \frac{\lambda^2 b_1(0)}{\beta} \right) - \frac{\gamma \lambda b_1(0)}{\beta}. \]

Reorganizing the above equation gives \( b_1(0) = \frac{\nu^{-1}}{\nu^{-1} + (1 - \lambda)^2 \nu^{-1}_e} \) and \( b_0(0) = (1 - b_1(0))(x + 2a(0)) - \left( \frac{\lambda^2 b_1(0)^2}{\beta} + \frac{\gamma \lambda b_1(0)}{\beta} \right). \)

The objective function when choosing the optimal productive action is:

\[
E[P(\lambda \hat{\omega}(\hat{\omega}, 0) + (1 - \lambda)\hat{s}, 0) - \frac{\beta}{2} (\hat{\omega}(\hat{\omega}, r) - \hat{\omega})^2 | a] - a^2
= E \left[ b_0(0) + b_1(0) \left( \frac{\lambda^2 b_1(0)}{\beta} + x + 2a + \hat{\delta} + (1 - \lambda)\hat{\epsilon} \right) - \frac{\beta}{2} \left( \frac{\lambda b_1(0)}{\beta} \right)^2 \right] - a^2.
\]

Hence the optimal productive action is \( a(0) = b_1(0) \). Substituting the solution into the above objective function and subtracting the capital requirement of one dollar give the expected return on investment. The case of enforcement \( r = 1 \) is equivalent to making \( \beta \) to \( \infty \). \( \square \)

**Proof of Proposition 8**

Given that the regulator enforces if the market is larger than \( \frac{x - \xi}{c} \), the projects with \( x > x_L \) are feasible if all invested project amounts to a market larger than \( \frac{x - \xi}{c} \). For this to happen, it has to be that

\[
\frac{\theta + \eta - x_L}{2\eta} > \frac{\kappa}{n}.
\]

Hence, \( \theta \) has to be larger than \( \theta \equiv x_L + (\frac{2\kappa}{c} - 1) \eta \). \( \square \)
Proof of Corollary 5

We have \( \int_{x_L}^{\theta + \eta} \frac{1}{2\eta} \, dx = \kappa \). Substituting \( \theta \in (x_L - \eta, x_L + \eta) \) into \( mb(\theta) \) yields

\[
mb(\theta) = \int_{[\theta - \eta, \theta + \eta] \cap (x_L, x_H]} \frac{1}{2\eta} (x - \ell) \, dx + \int_{[\theta - \eta, \theta + \eta] \cap (x_H, 1 + \eta]} \frac{1}{2\eta} \, dx < \int_{x_L}^{\theta + \eta} \frac{1}{2\eta} \, dx = \kappa,
\]

because \( x - \ell < c \) for \( x \in (x_L, x_H) \). \( \square \)

Proof of Lemma 4

\( \pi(x) \geq \pi'(x) \) for all \( x \) implies that \( v(\theta, \pi) \geq v(\theta, \pi') \) for all \( \theta \), which further implies that \( \Theta(\pi) \subseteq \Theta(\pi') \). Then \( \int_{[x - \eta, x + \eta] \cap \Theta(\pi)} \frac{c}{2\eta} \, d\theta \leq \int_{[x - \eta, x + \eta] \cap \Theta(\pi')} \frac{c}{2\eta} \, d\theta \) for all \( x \in [\eta, 1 - \eta] \). The same can be shown for \( x \notin [\eta, 1 - \eta] \). \( \square \)

Proof of Lemma 5

Given the strategies profile \( D_t(x) \), we can write the proportion of investing firms as

\[
n(\theta, D_t) \equiv \begin{cases} 
0, & \text{for } 0 \leq \theta \leq t - \eta, \\
\frac{\theta + \eta - t}{2\eta}, & \text{for } t - \eta \leq \theta \leq t + \eta, \\
1, & \text{for } t + \eta \leq \theta \leq 1.
\end{cases}
\]

The regulator's decision-relevant value function becomes

\[
v(\theta, D_t) \equiv n(\theta, D_t)c.
\]

The regulator enforces if \( n(\theta, D_t)c > \kappa \). Observe that \( n(\theta, D_t) \) is weakly increasing and continuous in \( \theta \). Hence the cut-off for the regulator is \( \theta^* \), which uniquely solves

\[
\left( \frac{\theta^* + \eta - t}{2\eta} \right) c - \kappa = 0.
\]
Given \( D_t \), the regulator enforces if and only if
\[
\theta > \theta^*(t) \equiv t + \left( \frac{2\kappa}{c} - 1 \right) \eta \in (t - \eta, t + \eta).
\]

Now consider the expected payoff of investment of an owner-manager with the marginal signal \( t \),
\[
u(t, D_t) = t - \ell - \int_{|t-\eta,t+\eta|\cap[0,\theta^*(t)]} \frac{c}{2\eta} d\theta,
\]
which is continuous and strictly increasing in \( t \). In addition, it has been known that
\[
u(x_L, D_{x_L}) = x_L - \ell - \kappa = -\kappa < 0,
\]
\[
u(x_H, D_{x_H}) = x_H - \ell - \kappa > x_H - \ell - c = 0.
\]

By intermediate value theorem, there exists a unique \( x^* = \ell + \kappa \in (x_L, x_H) \) which solves \( \nu(x^*, D_{x^*}) = 0 \). Next we show that \( D_{x^*}(x) \) is indeed an equilibrium.
\[
u(x, D_{x^*}) \equiv \begin{cases} 
  x - \ell - c, & \text{for } x \leq \theta^*(x^*) - \eta, \\
  x - \ell - \left( \frac{\theta^*(x^*) - x + \eta}{2\eta} \right) c, & \text{for } \theta^*(x^*) - \eta \leq x \leq \theta^*(x^*) + \eta, \\
  x - \ell, & \text{for } x \geq \theta^*(x^*) + \eta.
\end{cases}
\]

Hence \( \nu(x, D_{x^*}) \) is strictly increasing in \( x \), that is, \( \nu(x, D_{x^*}) > (\leq) 0 \) for \( x > (\leq) x^* \). □

**Proof of Proposition 9**
To show that $D_{\pi^*}(x)$ is the unique equilibrium, consider any equilibrium of the game $\pi(x)$. Define

$$x = \inf\{x : \pi(x) > 0\},$$

$$\overline{x} = \sup\{x : \pi(x) < 1\}.$$

By construction,

$$\overline{x} \geq \sup\{x : 0 < \pi(x) < 1\} \geq \inf\{x : 0 < \pi(x) < 1\} \geq x.$$

For all $x$ such that $\pi(x) > 0$, that is, some owner-managers choose to invest, it must be true that $u(x, \pi) > 0$. By continuity, $u(x, \pi) \geq 0$. Comparing the strategy $\pi(x)$ with $D_x(x)$, we must have $D_x(x) \geq \pi(x)$. By Lemma 4, $u(x, D_x(x)) \geq u(x, \pi)$, which further implies that $u(x, D_x(x)) \geq 0$. By Lemma 5, $x \geq x^*$. 

For all $x$ such that $\pi(x) < 1$, that is, some owner-managers choose not to invest, it must be true that $u(x, \pi) \leq 0$. By continuity, $u(\overline{x}, \pi) \leq 0$. Comparing the strategy $\pi(x)$ with $D_{\overline{x}}(x)$, we must have $D_{\overline{x}}(x) \leq \pi(x)$. By Lemma 4, $u(\overline{x}, D_{\overline{x}}(x)) \leq u(\overline{x}, \pi)$, which further implies that $u(\overline{x}, D_{\overline{x}}(x)) \leq 0$. By Lemma 5, $\overline{x} \leq x^*$. 

In summary, the above arguments yields $\overline{x} \leq x^* \leq x$, which further implies that

$$\overline{x} = x^* = x.$$

□

**Proof of Proposition 10**

This proof is very involved because the functional form of the $mb(\theta)$ has to be discussed case by case.

**Scenario 1** ($\eta < \frac{x_u - x_l}{2} = \frac{\xi}{2}$):
\[ mb(\theta) = \begin{cases} 
0, & \text{for } 0 \leq \theta \leq x_L - \eta, \\
\int_{x_L}^{\theta + \eta} \frac{1}{2\eta} (x - \ell) \, dx, & \text{for } x_L - \eta \leq \theta \leq x_L + \eta, \\
\int_{\theta - \eta}^{\theta + \eta} \frac{1}{2\eta} (x - \ell) \, dx, & \text{for } x_L + \eta \leq \theta \leq x_H - \eta, \\
\int_{\theta - \eta}^{x_H} \frac{1}{2\eta} (x - \ell) \, dx + \int_{\frac{\theta + \eta}{2\eta}}^{1} \, dx, & \text{for } x_H - \eta \leq \theta \leq x_H + \eta, \\
c, & \text{for } x_H + \eta \leq \theta \leq 1. 
\end{cases} \]

Case 1.1 \((0 < \kappa < \eta)\):

\( \theta^{fb} \) solves \( \frac{(\theta - (\ell - \eta))^2}{4\eta} = \kappa \), which yields

\[ \theta^{fb} = \ell - \eta + 2\sqrt{\eta}\sqrt{\kappa} \in (\theta^*, x^*). \]

Note that

\[ \theta^{fb} > \ell - \eta + 2\kappa > \ell - \eta + \kappa + \frac{2\eta}{c} \kappa = \theta^*, \text{ and} \]

\[ \theta^{fb} < \ell + \kappa = x^* \text{ because } -\eta + 2\sqrt{\eta}\sqrt{\kappa} \text{ decreases in } \eta \text{ for } \eta > \kappa. \]

Case 1.2 \((\eta \leq \kappa \leq c - \eta)\):

\[ \theta^{fb} = \ell + \kappa = x^*. \]
Hence

$$\theta^* \leq (\geq) \theta^{fb} \text{ if } 2\kappa \leq (\geq) c.$$ 

Case 1.3 ($c - \eta < \kappa < c$):

$\theta^{fb}$ solves

$$2c(\theta - \ell + \eta - (\ell + \eta - \theta)^2 - c^2) = \kappa,$$

which yields

$$\theta^{fb} = \ell + c + \eta - 2\sqrt{\eta(c - \kappa)} \in (x^*, \theta^*).$$

Note that

$$\theta^{fb} < \ell + c + \eta - 2(c - \kappa) < \ell + \kappa + (\kappa - c + \eta) < \ell + \kappa - \eta + \frac{2\kappa}{c} \eta = \theta^*,$$

because

$$-\eta + \frac{2\kappa}{c} \eta - (\kappa - c + \eta) = \frac{(c - 2\eta)(c - \kappa)}{c} > 0,$$

and

$$\ell + c + \eta - 2\sqrt{\eta(c - \kappa)} > \ell + c + c - \kappa - 2(c - \kappa) = \ell + \kappa = x^*,$$

because

$$\eta - 2\sqrt{\eta(c - \kappa)} \text{ increases in } \eta \text{ for } \eta > c - \kappa.$$
Scenario 2 \((\eta \geq \frac{x_H - x_L}{2} = \frac{\epsilon}{2})\):

\[
m_b(\theta) = \begin{cases} 
0, & \text{for } 0 \leq \theta < x_L - \eta, \\
\int_{x_L}^{\theta + \eta} \frac{1}{2\eta} (x - l) \, dx, & \text{for } x_L - \eta \leq \theta \leq x_H - \eta, \\
\int_{x_L}^{x_H} \frac{1}{2\eta} (x - l) \, dx + \int_{x_H}^{\theta + \eta} \frac{1}{2\eta} \, c \, dx, & \text{for } x_H - \eta \leq \theta \leq x_L + \eta, \\
\int_{\theta - \eta}^{x_H} \frac{1}{2\eta} (x - l) \, dx + \int_{x_H}^{\theta + \eta} \frac{1}{2\eta} \, c \, dx, & \text{for } x_L + \eta \leq \theta \leq x_H + \eta, \\
c, & \text{for } x_H + \eta \leq \theta \leq \ell.
\end{cases}
\]

Case 2.1 \((0 < \kappa < \frac{\epsilon^2}{4\eta})\):

\(\theta^{fb}\) solves \(\frac{(\theta - \ell - \eta)^2}{4\eta} = \kappa\), which yields

\[
\theta^{fb} = \ell - \eta + 2\sqrt{\eta} \sqrt{\kappa} \in (\theta^*, x^*).
\]

To prove that \(\theta^{fb} > \theta^*\), it is sufficient to show that \(2\sqrt{\eta} \sqrt{\kappa} > (1 + \frac{2\eta}{c})\kappa\). Note that

\[
2\sqrt{\eta} \sqrt{\kappa} = 2\sqrt{\frac{\eta}{\kappa}} \kappa > \left(\frac{2\eta}{c} + \frac{2\eta}{c}\right) \kappa \geq \left(1 + \frac{2\eta}{c}\right) \kappa, \text{ for } \kappa < \frac{c^2}{4\eta} \text{ and } c \leq 2\eta.
\]

Also, \(\ell - \eta + 2\sqrt{\eta} \sqrt{\kappa} < \ell + \kappa = x^*\) because \(-\eta + 2\sqrt{\eta} \sqrt{\kappa}\) decreases in \(\eta\) for \(\eta \geq \frac{c^2}{4\eta} > \kappa\).

Case 2.2 \((\frac{c^2}{4\eta} \leq \kappa \leq c - \frac{c^2}{4\eta})\):
\( \theta^{fb} \) solves \( \frac{c(2\theta+2\eta-2\ell-c)}{4\eta} = \kappa \), which yields

\[
\theta^{fb} = \ell + \frac{c}{2} + \left( \frac{2\kappa}{c} - 1 \right) \eta.
\]

Note that

\[
\theta^{fb} - \theta^* = \frac{c}{2} - \kappa;
\]
\[
\theta^{fb} - x^* = \frac{(c-2\eta)(c-2\kappa)}{2c}.
\]

Since \( c \leq 2\eta \), we have

\[
\theta^* \leq \theta^{fb} \leq x^* \text{ if } 2\kappa \leq c, \text{ and }
\]
\[
\theta^* \geq \theta^{fb} \geq x^* \text{ if } 2\kappa \geq c.
\]

Case 2.3 (\( c - \frac{c^2}{4\eta} < \kappa < c \)):
\( \theta^{fb} \) solves \( \frac{2c(\theta - \ell + \eta) - (\ell + \eta - \theta)^2 - c^2}{4\eta} = \kappa \), which yields

\[
\theta^{fb} = \ell + c + \eta - 2\sqrt{\eta(c-\kappa)} \in (x^*, \theta^*).
\]

To prove that \( \theta^{fb} < \theta^* \), note that \( c - \frac{c^2}{4\eta} < \kappa \) implies that \( \sqrt{\frac{c^2}{c-\kappa}} > \frac{2\eta}{c} \).

Hence

\[
\theta^{fb} = \ell + c + \eta - 2\sqrt{\frac{\eta}{c-\kappa}}(c-\kappa)
\]
\[
< \ell + c + \eta - \frac{4\eta}{c}(c-\kappa)
\]
\[
\leq \ell + c + \eta - \frac{4\eta}{c}(c-\kappa) + (\kappa-c) \left( 1 - \frac{2\eta}{c} \right) = \theta^*.
\]
Also, \( c - \eta \leq c - \frac{c^2}{4}\eta < \kappa \) implies that

\[
\ell + c + \eta - 2\sqrt{\eta(c - \kappa)} > \ell + c + c - \kappa - 2(c - \kappa) = \ell + \kappa = x^*. 
\]

\( \square \)

**Proof of Proposition 11**

The effect of primitives on \( \ell \) is:

\[
\frac{\partial \ell}{\partial \nu} = \frac{2(1 - \lambda)^4\nu_5\nu_e}{((1 - \lambda)^2\nu_5 + \nu_e)^3} > 0; \\
\frac{\partial \ell}{\partial \nu_e} = \frac{\gamma(1 - \lambda)^2\nu_6 + (\gamma + \lambda)\nu_e}{((1 - \lambda)^2\nu_6 + \nu_e)^3} < 0; \\
\frac{\partial \ell}{\partial \lambda} = \frac{4(1 - \lambda)^3\nu_5^2\nu_e}{((1 - \lambda)^2\nu_5 + \nu_e)^3} < 0. 
\]

The effect of primitives on \( c \) is:

\[
\frac{\partial c}{\partial \beta} = \frac{-\lambda\nu_e(2\gamma(1 - \lambda)^2\nu_6 + (2\gamma + \lambda)\nu_e)}{2\beta^2((1 - \lambda)^2\nu_6 + \nu_e)^2} < 0; \\
\frac{\partial c}{\partial \gamma} = \frac{-\gamma(1 - \lambda)^2\nu_6 \nu_e}{2\beta((1 - \lambda)^2\nu_6 + \nu_e)^2} > 0; \\
\frac{\partial c}{\partial \nu_6} = \frac{-\lambda\nu_e(\gamma(1 - \lambda)^2\nu_6 + (\gamma + \lambda)\nu_e)}{\beta((1 - \lambda)^2\nu_6 + \nu_e)^3} < 0; \\
\frac{\partial c}{\partial \nu_e} = \frac{(1 - \lambda)^2\nu_5(\gamma(1 - \lambda)^2\nu_6 + (\gamma + \lambda)\nu_e)}{\beta((1 - \lambda)^2\nu_6 + \nu_e)^3} > 0; \\
\frac{\partial c}{\partial \lambda} = \frac{(\nu_e + (1 - \lambda)^2\nu_6)\nu_e(\gamma(1 - \lambda)^2\nu_6 + (\gamma + \lambda)\nu_e)}{\beta((1 - \lambda)^2\nu_6 + \nu_e)^3} > 0. 
\]
Hence

\[ \frac{\partial \theta^*}{\partial \eta} = \frac{2\kappa}{c} - 1 > (\kappa)0 \text{ if } 2\kappa > (\kappa)c; \]
\[ \frac{\partial \theta^*}{\partial \kappa} = 1 + \frac{2\eta}{c} > 0; \]
\[ \frac{\partial \theta^*}{\partial \beta} = \frac{2\eta\kappa \frac{\partial c}{\partial \beta}}{c^2} > 0; \]
\[ \frac{\partial \theta^*}{\partial \gamma} = \frac{2\eta\kappa \frac{\partial c}{\partial \gamma}}{c^2} < 0; \]
\[ \frac{\partial \theta^*}{\partial \nu_\delta} = \frac{\partial \ell}{\partial \nu_\delta} - \frac{2\eta\kappa \frac{\partial c}{\partial \nu_\delta}}{c^2} > 0; \]
\[ \frac{\partial \theta^*}{\partial \nu_\epsilon} = \frac{\partial \ell}{\partial \nu_\epsilon} - \frac{2\eta\kappa \frac{\partial c}{\partial \nu_\epsilon}}{c^2} < 0; \]
\[ \frac{\partial \theta^*}{\partial \lambda} = \frac{\partial \ell}{\partial \lambda} - \frac{2\eta\kappa \frac{\partial c}{\partial \lambda}}{c^2} < 0, \]

and

\[ \frac{\partial x^*}{\partial \eta} = 0; \]
\[ \frac{\partial \theta^*}{\partial \kappa} = 1; \]
\[ \frac{\partial x^*}{\partial \beta} = 0; \]
\[ \frac{\partial x^*}{\partial \gamma} = 0; \]
\[ \frac{\partial x^*}{\partial \nu_\delta} = \frac{\partial \ell}{\partial \nu_\delta} > 0; \]
\[ \frac{\partial x^*}{\partial \nu_\epsilon} = \frac{\partial \ell}{\partial \nu_\epsilon} < 0; \]
\[ \frac{\partial x^*}{\partial \lambda} = \frac{\partial \ell}{\partial \lambda} < 0. \]

\[ \square \]

Proof of Corollary 7
Note that primitive variables except $\eta$ only affect enforcement intensity and market-size through $\theta^*$ and $x^*$. However, the effect of $\eta$ is more subtle. When $2\kappa > ( \leq )c$, $\eta$ increases (decreases) $\theta^*$. Although $\eta$ does not affect $x^*$, it affects market-size as well because for $\theta \in [x^* - \eta, x^* + \eta]$, the market size is:

$$n = \frac{\theta + \eta - x^*}{2\eta} = \frac{1}{2} + \frac{\theta - x^*}{2\eta}.$$ 

For $2\kappa > c$, it is known that $\theta^* > x^*$. Hence, there is only variation in enforcement intensity when $\theta > x^*$. When $\theta > x^*$, market-size actually decreases in $\eta$. Similarly, for $2\kappa < c$, it is known that $\theta^* < x^*$. Hence, there is only variation in enforcement intensity when $\theta < x^*$. When $\theta < x^*$, market-size actually increases in $\eta$. $\square$

**Proof of Proposition 12**

Some algebra yields

$$\mathbb{E}[\mathcal{M}(\theta)] = \frac{c(3(1 - \ell)^2 + \eta^2) - 3(c + 2\eta)\kappa^2}{6c}.$$
\[
\frac{\partial E[M(\theta)]}{\partial \eta} = \frac{\eta}{3} - \frac{\kappa^2}{c} > (\leq) 0 \text{ if } \kappa < (>) \sqrt{\frac{c\eta}{3}};
\]
\[
\frac{\partial E[M(\theta)]}{\partial \kappa} = -\frac{(c + 2\eta)\kappa}{c} < 0;
\]
\[
\frac{\partial E[M(\theta)]}{\partial \beta} = \frac{\eta\kappa^2}{c^2} < 0;
\]
\[
\frac{\partial E[M(\theta)]}{\partial \gamma} = \frac{\eta\kappa^2}{c^2} > 0;
\]
\[
\frac{\partial E[M(\theta)]}{\partial \nu_i} = \frac{\eta\kappa^2}{c^2} - (1 - \ell) \frac{\partial \ell}{\partial \nu_i} < 0;
\]
\[
\frac{\partial E[M(\theta)]}{\partial \nu_e} = \frac{\eta\kappa^2}{c^2} - (1 - \ell) \frac{\partial \ell}{\partial \nu_e} > 0;
\]
\[
\frac{\partial E[M(\theta)]}{\partial \lambda} = \frac{\eta\kappa^2}{c^2} - (1 - \ell) \frac{\partial \ell}{\partial \lambda} > 0.
\]

\begin{proof}

Some algebra yields

\[
\mathbb{E}[W(\theta)] = \frac{c(3(1 - \ell)^2 + \eta^2) - 6c(1 - \ell + \eta)\kappa + 3(c + 2\eta)\kappa^2}{6c}.
\]

\end{proof}
\[
\frac{\partial E[W(\theta)]}{\partial \eta} = \frac{\eta - \kappa(c - \kappa)}{c} > (\lt)0 \text{ if } c < (\gt) \frac{3\kappa^2}{3\kappa - \eta}; \\
\frac{\partial E[W(\theta)]}{\partial \kappa} = \ell + \kappa + \left(\frac{2\kappa}{c} - 1\right) \eta - 1 = \theta^* - 1 < 0; \\
\frac{\partial E[W(\theta)]}{\partial \beta} = -\frac{\eta \kappa^2}{c^2} > 0; \\
\frac{\partial E[W(\theta)]}{\partial \gamma} = -\frac{\eta \kappa^2}{c^2} < 0; \\
\frac{\partial E[W(\theta)]}{\partial \nu_\delta} = -\frac{\eta \kappa^2}{c^2} \frac{\partial c}{\partial \nu_\delta} - (1 - x^*) \frac{\partial \ell}{\partial \nu_\delta} > (\lt)0 \text{ if } -\partial c > (\lt) \frac{c^2}{\eta \kappa^2} (1 - x^*) \frac{\partial \ell}{\partial \nu_\delta}; \\
\frac{\partial E[W(\theta)]}{\partial \nu_\epsilon} = -\frac{\eta \kappa^2}{c^2} \frac{\partial c}{\partial \nu_\epsilon} - (1 - x^*) \frac{\partial \ell}{\partial \nu_\epsilon} > (\lt)0 \text{ if } -\partial c > (\lt) \frac{c^2}{\eta \kappa^2} (1 - x^*) \frac{\partial \ell}{\partial \nu_\epsilon}; \\
\frac{\partial E[W(\theta)]}{\partial \lambda} = -\frac{\eta \kappa^2}{c^2} \frac{\partial c}{\partial \lambda} - (1 - x^*) \frac{\partial \ell}{\partial \lambda} > (\lt)0 \text{ if } -\partial c > (\lt) \frac{c^2}{\eta \kappa^2} (1 - x^*) \frac{\partial \ell}{\partial \lambda}. 
\]
References


Ewert, Ralf, and Alfred Wagenhofer (2016) ‘Effects of increasing enforcement on firm value
and financial reporting quality.’ Working Paper

107


Kartik, Navin, Frances Xu Lee, and Wing Suen (2017) ‘Investment in concealable infor-
mation by biased experts.’ *The Rand Journal of Economics* 48(1), 24–43


Li, Wei (2013) ‘A theory on the discontinuity in earnings distributions.’ *Contemporary Accounting Research*


Ma, Paul, Iván Marinovic, and Pinar Karaca-Mandic (2015) ‘Drug manufacturers’ delayed disclosure of serious and unexpected adverse events to the US food and drug administration.’ *JAMA Internal Medicine* 175(9), 1565–1566


——— (2005) ‘The introduction of international accounting standards in europe: Implica-
tions for international convergence.’ European Accounting Review 14(1), 101–126


