Operational Decisions under the Influence of Government Regulation

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Abstract

This dissertation focuses on examining firms' operational decisions under the influence of government regulation, including the regulation of anti-trust agencies and environmental protection agencies.

In the first chapter, I investigate a merger between price-setting newsvendors in an oligopolistic market. It is well-known that inventory pooling can greatly reduce inventory costs in a centralized distribution system because it helps reduce aggregate demand uncertainty. Although such statistical economies of scale are important benefits of a retail merger, the extant literature models cost savings from a merger only through reduction in a post-merger firm's marginal cost. In this paper, I develop a model of a retail merger under uncertain demand that distinguishes between cost savings from conventional economies of scale and those from statistical economies of scale. I show that these two sources of cost savings have substantially different impacts on firms' decisions in a post-merger market. Specifically, and contrary to the existing theory of mergers developed under deterministic demand, I find that although inventory pooling enables the post-merger firm to achieve cost savings, it always induces firms to raise their prices, and that marginal cost reduction induces firms to lower their prices only when it is substantial – consequently, larger marginal cost reduction can benefit even nonparticipant firms when it induces the post-merger firm to raise its price. Finally, even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels under uncertain demand.

In the second chapter, I investigate firms' development and adoption decisions of green technology. This work is motivated by the observation that while enforcing a stricter standard on a pollutant, a government agency often takes into account the proportion of firms that are able to meet the new standard (I refer to this proportion as a "capability index"). Despite this fact, existing research assumes that a government agency might move to a stricter standard regardless of the industry's capability index. Additionally, the literature also assumes that a firm's benefit from developing a new green technology to reduce pollution is deterministic. By contrast, I develop a novel model in which the probability of enforcing a stricter standard increases with the capability index, and in which the benefit of a new green technology is uncertain and correlated for all firms. Thus, one firm's adoption decision can affect the adoption decisions of

other firms through enforcement interactions with the government (via the capability index), requiring a firm to conjecture on other firm's decisions using its own payoff information. Given the interactions among firms' decisions and the correlated uncertain payoffs, I use the global game framework to analyze this model; this framework was recently developed in economics to analyze similar problems. My analysis shows that regulation based on a capability index, compared with regulation that ignores it, has a substantially different impact on firms' decisions for new green technology development. The latter effectively motivates a firm to develop a green technology when the first-mover advantage of that technology is high. Regulation based on the capability index, on the other hand, works well when the first-mover advantage is low. Surprisingly, I also find that the uncertainty about the benefit of the technology can promote a firm's development of a green technology.

In the third chapter, I examine firms' quality and variety decisions after a merger. Existing research focuses mainly on price changes in mergers, and predicts that the cost synergies achieved in mergers benefit consumers because cost synergies can reduce prices. By analyzing a merger in a market where firms sell vertically-differentiated goods, I show that cost synergies achieved in a merger might also induce merging firms to reduce their product variety and quality levels. Such reductions can be harmful to consumers, even when a merger reduces the prices of all products.

Acknowledgements

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Chapter 1

Newsvendor Mergers

1.1 Introduction

Potential benefits of mergers and acquisitions (M&As) are manifold. Through M&As, firms could increase their revenues by utilizing stronger market power, and achieve cost savings by utilizing economies of scale and improving operational efficiency. M&As have been an important aspect of management strategy in modern economies – 26,409 M&As that were worth 1.9 trillion dollars occurred in 2013 (WilmerHale 2014). Our focus in this paper is on studying the effects of a horizontal merger between retailers in an oligopolistic market.

In a retail business, price is a natural strategic variable in competing against other retailers. In their seminal study, Deneckere and Davidson (1985) show that in the absence of cost synergies, a merger of price-setting retailers will induce both merging and nonparticipant firms to raise their prices: by determining prices jointly that were set independently prior to the merger, merging parties will raise their prices, and this initial price increase will be followed by price increase of their competitors; the merging parties then further raise their prices, and so on until all prices in the market will have risen, and all firms will be better off.

Since mergers that lead to such price increase will be unlikely to be approved by antitrust authorities, merging firms often argue that they can achieve cost savings through the merger, which in turn will be passed on to consumers. Several empirical studies (e.g., Houston et al. 2001, DeLong 2003) suggest that operating synergies are the most important determinant of successful M&As. According to Bascle et al. (2008), cost efficiency is the main rationale behind more than 70% of M&As. Because a post-merger firm can pool its resources at the two retail outlets that were managed independently prior to the merger, cost savings from a merger can come from pooling inventory as well as from conventional economies of scale (e.g., lower procurement cost from higher purchasing power, lower R&D cost, lower cost of capital) as illustrated in the following examples:

– "Zipcar agreed to sell itself to Avis Budget Group Inc., for about $500 million ... Avis expects the deal to lower the companies' combined costs by $50 million to $70 million a year. Mr. Nelson [CEO of Avis] said the synergies were tied to three components: lower fleet costs, better fleet utilization and increased revenue ... The deal would allow Avis to reduce the number of cars at Zipcar locations during the week, but also to use Avis's excess weekend inventory to meet Zipcar's strong weekend demand." (Kell 2013)

– "Hertz would acquire Dollar Thrifty for about $2.3 billion in cash ... they expect the merger's synergies to include annual savings of $65.6 million in fleet costs, in part through sharing of vehicles across rental brands." (Sawyers 2012)

Savings in inventory costs from a merger are also substantial in various other industries such as auto dealerships (Gattorna et al. 1998), office-supply stores such as Office Depot and OfficeMax (Office Depot 2013), and airline industry (Seidenman and Spanovich 2011).

In the operations literature (e.g., Eppen 1979, Corbett and Rajaram 2006), it is well-known that inventory pooling can greatly reduce inventory costs in a centralized distribution system because it helps reduce aggregate demand uncertainty. While such *statistical* economies of scale are important benefits of a retail merger, the extant literature on mergers (e.g., Williamson 1968, Farrell and Shapiro 1990, Cho 2014) models cost synergies only through reduction in a post-merger firm's marginal cost; in other words, prior models of mergers treat both statistical economies of scale and conventional economies of scale in the same manner. A primary reason for this modeling choice is that researchers have analyzed the effects of a merger under *deterministic* demand, whereas a careful examination of statistical economies of scale requires a merger analysis under *uncertain* demand. Under uncertain demand, a merger analysis becomes more complex because firms use inventory as well as price as their strategic variables. Nevertheless, a retail business always entails uncertainty in consumer demand, and therefore demand uncertainty has been one of the most fundamental features in the literature of operations management (OM).

The objective of this paper is to study the effects of a merger on firms' prices and expected profits as well as consumer welfare under uncertain demand. Specifically, we consider a merger of two firms in an oligopolistic market in which firms determine their prices and inventory levels under uncertain demand. In the OM literature, such firms are often called price-setting competitive "newsvendors." Our focus is on examining the following three effects of a merger. First, the "collusion effect" arises due to the ability of a post-merger firm to set its prices jointly at the two retail outlets which were independent prior to the merger. Second, a merger creates the "pooling effect" when a post-merger firm can manage its inventory in a centralized manner. In order to save its inventory cost by utilizing statistical economies of scale, a post-merger firm may manage a single safety stock in a central warehouse that serves two retail outlets or in two warehouses by allowing transshipment between them. Third, the "synergy effect" exists when a post-merger firm can reduce its marginal cost; for example, a post-merger firm may spread fixed costs over a larger number of sales units through economies of scale or reduce the cost of capital from lower securities and transaction costs.

Our analysis highlights the important role of demand uncertainty and inventory pooling in evaluating a retail merger. As discussed above, the existing literature models cost savings from a merger through marginal cost reduction under deterministic demand, and it does not distinguish inventory cost savings due to statistical economies of scale from marginal cost reduction. The conventional wisdom that cost savings from a merger will drive firms' prices down has been proven by many economists – notably, Williamson (1968), Perry and Porter (1985) and Farrell and Shapiro (1990), and it has been regarded as the de facto standard result in the theory of mergers (see Whinston 2007 for a comprehensive review). As a result, firms justify their proposed mergers by emphasizing that their cost savings will be passed on to consumers, and it appears that antitrust agency views such cost savings positively (c.f. Horizontal Merger Guidelines of the U.S. Department of Justice and the Federal Trade Commission). However, our results indicate that neither marginal cost reduction (from conventional economies of scale) nor inventory cost savings (from statistical economies of scale) will always induce firms to lower their prices. Furthermore, although both conventional and statistical economies of scale enable merging firms to reduce their expected costs, their impacts on firms' prices and expected profits are substantially different. Counter-intuitively, consumer price is more likely to rise after a merger when the benefit of pooling is more significant, and larger cost synergies from a merger can benefit nonparticipant firms. Contrary to the previous literature studied under deterministic demand, we find that even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels.

The rest of this paper is organized as follows. In §2 we review the related literature. In §3 we describe our pre-merger model. In §4 we present our post-merger model and analysis. In §5 we study several extensions of our base model. We conclude our paper in §6. Proofs are presented in Appendix A.

1.2 Related Literature

In this section, we first review the economic theory of a merger, and then we review the related operations management (OM) literature on competitive models of newsvendors, inventory pooling, operational models of mergers, and cooperative networks.

Economists and antitrust agency have long studied mergers, in particular focusing on how a merger affects price. Stigler (1950) considers the formation of a cartel among firms that make a collusive decision in a competitive market, and he shows that a cartel is not stable because an increase of a market price will benefit external firms more than cartel members. To explain the observed formation of cartels or mergers in practice, starting from Williamson (1968), economists have taken into account cost synergies of mergers that may induce merging firms to lower their prices. Most notably, Farrell and Shapiro (1990) show that if the amount of marginal cost reduction from a merger exceeds a certain threshold, then price will fall after a merger. Whereas these papers and their subsequent extensions adopt the Cournot model of quantity competition among homogeneous goods (e.g., see a comprehensive review by Whinston 2007), Deneckere and Davidson (1985) analyze a merger in a differentiated market where firms engage in price competition. The analysis of a merger in such a market is particularly important because firms, especially retailers, are often price-setters, and the nature of price competition is different from quantity competition (e.g., Vives 1999). For this reason, numerous papers have constructed their models by building on Deneckere and Davidson (1985), including Werden and Froeb (1994) and Davidson and Ferrett (2007). Since our work deals with retail mergers, we use Deneckere and Davidson (1985) as our benchmark model of deterministic demand. To the best of our knowledge, our paper is the first that evaluates a merger under uncertain demand and characterizes statistical economies of scale from a merger. Contrary to the existing results in this literature, our results show that marginal cost reduction from conventional economies of scale induces merging firms to lower their prices only when they are sufficiently large, and that larger statistical economies of scale always induce both merging and nonparticipant firms to raise their prices.

In order to evaluate the effect of a merger under uncertain demand, we need a benchmark in which firms compete *before* the merger takes place. For this benchmark, our paper builds on the OM literature that studies competition among newsvendors. Traditional research on newsvendor models considers a monopolistic firm's decision on inventory under uncertain demand, while taking demand and price as given exogenously. A major extension to this traditional approach is to consider a monopolistic newsvendor who sets its price and inventory simultaneously (e.g., Petruzzi and Dada 1999, Kocabıyıkoğlu and Popescu 2011). Another important extension is to introduce competition among newsvendors (e.g., Lippman and McCardle 1997, Netessine and Rudi 2003). While these papers focus on the inventory decisions of competitive newsvendors, Zhao and Atkins (2008) consider a more general case in which each competitive newsvendor determines both price and inventory simultaneously. Our paper builds on Zhao and Atkins

(2008) for the pre-merger model, and examines the effect of a merger between two competitive newsvendors on merging firms, nonparticipant firms, and consumers.

Research on inventory pooling has a long tradition in operations management. The seminal paper by Eppen (1979) considers a multi-location newsvendor problem with normal demand at each location. He shows that inventory costs in a centralized system increase with the correlation between uncertain demands in different locations. Numerous extensions have followed Eppen (1979), including among others: a decentralized system with one manufacturer and multiple retailers owned and operated by a single entity who can transship inventory between them (Dong and Rudi 2004), arbitrary dependence structure with non-normal distributions (Corbett and Rajaram 2006), capacity-sharing joint ventures (Roels et al. 2012), and procurement contracts between two buyers and one common supplier (Hu et al. 2013). While these papers consider the centralization of inventory among warehouses of a single firm or among monopolistic firms, Anupindi and Bassok (1999) and Wang and Gerchak (2001) analyze the centralization of inventory in a supply chain with one supplier and two competitive retailers, and compare its performance with the decentralized system. There are two important differences between these papers and our work. First, whereas the previous papers consider the centralization of inventory (or stocking decisions) of all retailers in a market, in the context of a merger, such *complete* centralization will create a monopolist, and hence will not be approved a priori by antitrust authorities. Instead, a merger usually involves only two firms, and it affects other nonparticipant firms in an oligopoly market – in this sense, a merger may be referred to as *partial* centralization. The essence of a merger analysis is to examine the competitive reactions of nonparticipant firms to the proposed merger, which in turn affect the decision of the post-merger firm, and so on; hence, the merger analysis is substantially different from the previous analyses that compare centralization with decentralization. Second, these papers assume *fixed* consumer prices of all retailers, but consider stock-out substitution among retailers (i.e., a fraction of consumers who do not find the good at their local retailers look for the good at other retailers). In contrast, a central question in the analysis of a merger is how a merger affects consumer price. Therefore, in our work, we consider a *price-setting* competitive newsvendor model as our pre-merger model, which itself is hard to analyze. To maintain tractability, we assume initially that firms compete only through prices, while examining stock-out substitution in a later extension numerically.

Despite the importance of mergers in practice, there is scant literature on mergers in the OM literature. Prior research in this literature mainly focuses on quantifying operating synergies from a merger in monopolistic markets (e.g., Gupta and Gerchak 2002, Nagurney 2009) or on vertical integration under deterministic demand (e.g., Corbett and Karmarkar 2001, Lin et al.

2014). Recently, Cho (2014) studies a horizontal merger in a multitier decentralized supply chain in which firms engage in quantity competition at each tier. He characterizes the impact of a merger at one tier on strategic decisions of firms at different tiers of the supply chain. Unlike Cho (2014) who considers a deterministic setting, the main research question of the current paper is to characterize the impact of demand uncertainty and inventory pooling in a merger of price-setting firms. Different from the analytical papers reviewed above, Zhu et al. (2011) empirically study the effects of a horizontal merger on the financial and inventory-related performance of firms. Although they discuss the potential impact of demand uncertainty on firms' performance, they develop their hypotheses mainly from Deneckere and Davidson (1985) who characterize only the collusion effect under deterministic demand. Our paper complements their empirical work by providing theoretical results about the collusion, pooling and synergy effects of a merger under uncertain demand.

Lastly, in another stream of research, researchers use cooperative game theory to study the formation of resource-pooling or inventory-transshipment coalitions among firms. In this literature, firms maintain their *independence* but consider forming coalitions to obtain synergies or to reduce financial risk (e.g., Kemahlıoğlu-Ziya and Bartholdi 2011, Fang and Cho 2014, Huang et al. 2015). This literature focuses primarily on examining how stable coalitions can be formed by allocating the benefit from collaboration to independent firms appropriately. In contrast, the benefit from a merger need not be allocated between merging firms, since merging firms become a *single* entity after the merger. Our focus is on analyzing the effect of a merger on prices, expected profits, and consumer welfare, provided that such a merger occurs.

1.3 Pre-Merger Model and Analysis

Consider n symmetric firms that sell products through different retail locations. Let p_i denote the price of firm i $(= 1, 2, ..., n)$, $\mathbf{p} = (p_1, ..., p_n)$ denote the price vector, and $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$ denote the price vector of all firms but firm i. The demand of firm i is $D_i(\mathbf{p}) = L_i(\mathbf{p}) + \widetilde{\varepsilon}_i$, where $L_i(\mathbf{p})$ is the deterministic part of the demand and $\widetilde{\varepsilon}_i$ is the random part of the demand. Following our benchmark model of deterministic demand in Deneckere and Davison (1985), we assume

$$L_i(\mathbf{p}) = a - bp_i + \gamma \left(\frac{1}{n} \sum_{j=1}^{n} p_j - p_i \right), \tag{1.1}$$

where a (> 0) is the deterministic demand when all firms' prices are zero, and b (> 0) captures the sensitivity of demand to firm i's own price p_i. The parameter γ (≥ 0) captures competition among firms in the following sense. When γ is close to zero, competition among firms is low, so that the difference between a firm's own price and other firms' prices has little impact on the demand; whereas when γ is large, competition is intense. We assume that $\widetilde{\varepsilon} = (\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, ..., \widetilde{\varepsilon}_n)$ follows a multivariate normal distribution $N(0, \sum)$, where $\sum$ represents the covariance structure of demand with $Var(\widetilde{\varepsilon}_i) = \sigma_i^2 = \sigma^2$. We denote by $\phi(\cdot)$ and $\Phi(\cdot)$ the density and the cumulative distribution function of the standard normal random variable, respectively. To ensure positive demand, we require $a >> \sigma$ so that $\Pr(a + \widetilde{\varepsilon}_i < 0) \approx 0$. Each firm i incurs a marginal cost $w_i = w$. We assume there is no salvage value for unsold goods.

Each firm i decides its price p_i and inventory q_i simultaneously with other firms before the random demand is realized. Let $q_i = L_i(\mathbf{p}) + y_i$, where y_i is firm i's safety stock to hedge against demand uncertainty. As in Petruzzi and Dada (1999) and Zhao and Atkins (2008), it can be shown that a game with decision variables (p_i, q_i) is equivalent to a game with decision variables (p_i, y_i). With (p_i, y_i) as decision variables, a firm's strategy set can be unbounded. To ensure the compactness of a firm's strategy set for the existence of Nash equilibrium, we assume $p_i \in [\underline{p}, \overline{p}]$ and $y_i \in [-\overline{y}, \overline{y}]$, where $\underline{p} > w$, and $\overline{p}$ and $\overline{y}$ are sufficiently large numbers that do not constrain firms' decisions. Given $\mathbf{p}_{-i}$, firm i chooses p_i and y_i to maximize its expected profit given as

$$\pi_i(\mathbf{p}, y_i) = (p_i - w)L_i(\mathbf{p}) - wy_i - p_i E\left(\widetilde{\varepsilon}_i - y_i\right)^+ = (p_i - w)L_i(\mathbf{p}) - wy_i - p_i \sigma R\left(\frac{y_i}{\sigma}\right), \quad (1.2)$$

where $R(x) = \int_x^{\infty}(u - x)\phi(u)du$ is the standard normal loss function and the expected lost sale of firm i having safety stock y_i is given as $E\left(\widetilde{\varepsilon}_i - y_i\right)^+ = \int_{y_i}^{\infty}(u - y_i)\frac{\phi(u/\sigma)}{\sigma}du = \sigma R\left(\frac{y_i}{\sigma}\right)$. Since there are no lost sales under deterministic demand, for convenience, we define $\sigma R\left(\frac{y}{\sigma}\right) = 0$ when $\sigma = 0$. Let $\pi_i^d(\mathbf{p}) = (p_i - w)L_i(\mathbf{p})$ and $c_i(p_i, y_i) = wy_i + p_i\sigma R\left(\frac{y_i}{\sigma}\right)$, representing the profit from the deterministic demand and the expected cost caused by demand uncertainty, respectively. Then $\pi_i(\mathbf{p}, y_i) = \pi_i^d(\mathbf{p}) - c_i(p_i, y_i)$. Note that $-c_i(p_i, y_i)$ can also be interpreted as the expected profit of a newsvendor who faces the demand of $\widetilde{\varepsilon}_i$. In our subsequent analysis, we will focus on the case in which all firms earn positive expected profits.

Following Netessine and Rudi (2003) and Zhao and Atkins (2008), we can show that a unique pure-strategy Nash equilibrium exists under a certain condition (see Lemma A1 in Appendix). The symmetric equilibrium price $p_1^{pre} = p_2^{pre} = ... = p_n^{pre}$ is the unique solution of the following equation:

$$-\left(2b + \frac{n-1}{n}\gamma\right)p_1^{pre} - \sigma R\left(\Phi^{-1}\left(1 - \frac{w}{p_1^{pre}}\right)\right) + a + \left(b + \frac{n-1}{n}\gamma\right)w = 0. \quad (1.3)$$

The equilibrium safety stock $y_1^{pre} = y_2^{pre} = ... = y_n^{pre}$ is equal to $\sigma\Phi^{-1}\left(1 - w/p_1^{pre}\right)$, and it is also unique. The corresponding expected profit of firm i in equilibrium is denoted by π_i^{pre}.

Before proceeding to our post-merger analysis, we remark on our assumptions and later extensions. First, we consider symmetric firms in the pre-merger market in §§3-4, while extending the analysis to asymmetric firms in §5.1. Our analysis in §§3-4 enables us to isolate the effect of demand uncertainty on the celebrated result of Deneckere and Davison (1985) who also consider symmetric firms in the pre-merger market, and to compare the effect of a merger on merging firms with that on nonparticipant firms. Second, we assume that random demands follow a multivariate normal distribution, which is widely used in the literature that studies the effect of inventory pooling (e.g., Eppen 1979, Anupindi and Bassok 1999, Dong and Rudi 2004, Hu et al. 2013). Even in this case, no closed-form expressions for p_i^{pre}, y_i^{pre} and π_i^{pre} exist, and our subsequent analysis deals with the implicit functions such as (1.3) that define these equilibrium outcomes. Nevertheless, we show in §5.2 that our results hold under a more general class of distributions. In §5.3, we consider a demand model with a general uncertainty structure. Lastly, in §5.4, we examine the impact of stock-out substitution on our results.

1.4 Post-Merger Model and Analysis

In §4.1, we present our post-merger model, and describe the collusion, pooling and synergy effects of a merger. We then characterize these effects of a merger on firms' prices and expected profits in §4.2, and on firms' service levels and expected consumer welfare in §4.3.

1.4.1 Post-Merger Model

Suppose firm 1 and firm 2 in the pre-merger market described in §3 have merged. We refer to these two firms that are merged as the *merging* firms. When the two merging firms become a single firm in the post-merger market, we refer to this firm as the *post-merger* firm, while referring to the other firms as the *nonparticipant* firms. We index the post-merger firm by $i = m$, and the nonparticipant firms by $i = 3, 4, ..., n$. We consider an oligopolistic market with $n \geq 3$ because a merger that creates a monopolist (i.e., $n = 2$) is unlikely to be approved by antitrust authorities.

The post-merger firm faces the demand of $L_m(p) + \widetilde{\varepsilon}_m$, where $L_m(p) = L_1(\mathbf{p}) + L_2(\mathbf{p})$ represents the deterministic part of the demand and $\widetilde{\varepsilon}_m$ represents the random part of the demand. The random part $\widetilde{\varepsilon}_m$ is given as $\widetilde{\varepsilon}_m = \widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2$. Since the linear combination of the components of the multivariate normal random vector $\widetilde{\varepsilon}$ is normally distributed, $\widetilde{\varepsilon}_m$ follows $N(0, \sigma_m)$, where σ_m represents the post-merger firm's aggregate volatility of the uncertain demand. Letting ρ ($\in [-1, 1]$) denote the correlation coefficient between $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$, we obtain $\sigma_m = \sigma\sqrt{2 + 2\rho}$. Prior to the merger, firm 1 and firm 2 set prices p_1 and p_2 at their respective retail outlet independently, whereas the post-merger firm can set its prices p_1 and p_2 *collusively* at these two retail outlets. We call the effect of such price collusion on equilibrium as the "collusion effect." To examine the collusion effect, we assume that the post-merger firm maintains their two retail outlets after the merger – the same assumption is made by Deneckere and Davidson (1985) and subsequent extensions (e.g., Werden and Froeb 1994, Davidson and Ferrett 2007).

In addition to setting its prices at two retail outlets collusively, the post-merger firm may also manage its inventories at these two locations in a *centralized* manner. We model this "pooling effect" by allowing the post-merger firm to sell the same product at both retail outlets and to use a single safety stock y_m to hedge against the aggregate demand volatility σ_m. This is essentially the same as the centralization of inventory in the literature (e.g., Eppen 1979, Anupindi and Bassok 1999, Wang and Gerchak 2001, Corbett and Rajaram 2006). In contrast, when each retail outlet sells a different product and/or manages its stock *separately* even after the merger, there is no pooling effect. This corresponds to the centralization of stocking decisions in the literature (as compared to the centralization of physical inventory) (e.g., Netessine and Rudi 2003, Netessine and Zhang 2005).[1] We can prove that this case is identical to the special case when the post-merger firm pools its inventories under perfectly-correlated demand (i.e., $\rho = 1$) so that $\sigma_m = 2\sigma$. In the presence of the pooling effect (i.e., $\rho < 1$), when the demands of the two merging firms are highly correlated (i.e., high ρ), the post-merger firm faces high demand volatility σ_m due to the *low* pooling effect.

The post-merger firm often achieves cost synergies by utilizing economies of scale. Following the common approach in the merger literature (e.g., see Cho (2014) and references therein), we model the "synergy effect" of a merger by reducing the marginal cost of the post-merger firm from w to w_m ($\in (0, w]$). Let $s \equiv \frac{w - w_m}{w}$ ($\in [0, 1)$) denote a percentage of marginal cost reduction after a merger. When $s = 0$, there is no synergy effect. When $s > 0$, the synergy effect exists, and as s increases, the merger entails larger cost synergies. The synergy level s is an aggregate measure for cost synergies from various areas of operations, marketing and administration, and its estimation often requires an industry-specific detailed analysis.

As in the pre-merger market, each firm i ($= m, 3, 4, ..., n$) in the post-merger market decides its price p_i and inventory q_i simultaneously before the random demand is realized. The post-merger firm decides its prices p_1 and p_2 as well as its safety stock y_m to maximize its expected profit π_m, which can be expressed similarly to (1.2) as follows:[2]

$$\pi_m(\mathbf{p}, y_m) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - w_m y_m - (p_1 + p_2)\frac{\sigma_m}{2} R\left(\frac{y_m}{\sigma_m}\right). \quad (1.4)$$

[1] In other words, the pooling effect does not exist when two merging firms sell two distinct products after the merger. Although this is quite plausible for a merger of *manufacturers*, it may not be common for a merger of *retailers*, which is the main focus of this paper.

[2] As is common in the literature, we do not consider a fixed cost of a merger. However, we can easily incorporate this cost into (1.4). Since the fixed cost does not affect the functional characteristic of π_m, it has no impact on subsequent analyses.

Let $\pi_m^d(\mathbf{p}) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p})$ and $c_m(p_1, p_2, y_m) = w_m y_m + (p_1 + p_2)\frac{\sigma_m}{2} R\left(\frac{y_m}{\sigma_m}\right)$, so that $\pi_m(\mathbf{p}, y_m) = \pi_m^d(\mathbf{p}) - c_m(p_1, p_2, y_m)$. The expected profit of nonparticipant firm i $(= 3, 4, ..., n)$ remains the same as $\pi_i(\mathbf{p}, y_i)$ given in (1.2). Similar to the pre-merger market, we can show that equilibrium prices p_m^{post} (where $p_1^{post} = p_2^{post} = p_m^{post}$) and p_3^{post} (where $p_3^{post} = p_4^{post} = ... = p_n^{post}$) are the unique solutions that satisfy two first-order conditions, while safety stocks in equilibrium are $y_i^{post} = \sigma \Phi^{-1}(1 - w/p_i^{post})$ for nonparticipant firm i and $y_m^{post} = \sigma_m \Phi^{-1}(1 - w_m/p_m^{post})$ for the post-merger firm. The corresponding expected profit of firm i is π_i^{post} for $i = m, 3, 4, ..., n$.

1.4.2 Post-Merger Analysis: Price and Expected Profit

The prior literature on a merger of price-setting firms focuses on the collusion effect of the merger under *deterministic* demand. In this section, we will first investigate the collusion effect under *uncertain* demand. We then examine the impact of two sources of cost savings for a post-merger firm – namely, inventory pooling and cost synergies – on firms' prices and expected profits. Lastly, by combining the collusion, pooling and synergy effects of a merger, we examine the aggregate effect of a merger on firms' prices and expected profits.

To isolate the impact of demand uncertainty on the *collusion* effect, we examine the same setting as Deneckere and Davidson (1985) except that firms face uncertain demand in our model. In this special case, no pooling and synergy effects exist; i.e., a post-merger firm decides on its prices at its two retail outlets, but the merger entails no cost savings through inventory pooling or marginal cost reduction (i.e., $\rho = 1$ and $w_m = w$).

Lemma 1.1 *When $\rho = 1$ and $w_m = w$, the collusion effect of a merger leads to the following results:*

(a) The post-merger price of any firm is higher than its pre-merger price (i.e., $p_m^{post} > p_1^{pre}$ and $p_i^{post} > p_i^{pre}$ for $i = 3, 4, ..., n$). In addition, the price of the post-merger firm is higher than that of a nonparticipant firm (i.e., $p_m^{post} > p_i^{post}$ for $i = 3, 4, ..., n$).

(b) The post-merger expected profit of any firm is higher than its pre-merger expected profit (i.e., $\frac{1}{2}\pi_m^{post} > \pi_1^{pre}$ and $\pi_i^{post} > \pi_i^{pre}$ for $i = 3, 4, ..., n$). Furthermore, the post-merger expected profit of a merging firm is lower than that of a nonparticipant firm (i.e., $\frac{1}{2}\pi_m^{post} < \pi_i^{post}$ for any $i = 3, 4, ..., n$).

Lemma 1.1 shows that the price collusion of the merging parties induces all firms to raise their prices and thereby earn higher expected profits. These results verify that the collusion effect of a merger on prices and profits under deterministic demand remains valid under uncertain demand. This suggests that the nature of competition that drives these results is unaffected by demand uncertainty. Specifically, a merging firm has an incentive to raise its price after

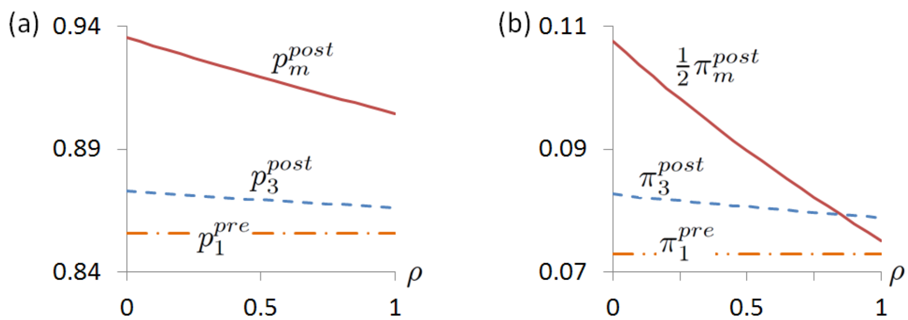

Figure 1-1: The Pooling Effect of a Merger on (a) Prices and (b) Expected Profits. (*Note*. The following parameter values are used: $n = 3$, $a = 1$, $b = 0.6$, $\gamma = 0.5$, $w = w_m = 0.5$, and $\sigma = 0.3$. These values are motivated by the U.S. rental car industry; see Appendix B.)

the merger because its higher price has a positive externality on the other merging party. The increased prices of the merging firms in turn benefit nonparticipant firms by raising their demands (see (1.1)). Consequently, nonparticipant firms also raise their prices after the merger. However, they raise prices less so than the post-merger firm due to the (technical) reason that a nonparticipant firm's best response function to the merged firm's price is upward sloping with a slope less than one (see Appendix).

Having characterized the collusion effect of a merger under uncertain demand, we next examine how inventory pooling and cost synergies affect post-merger equilibrium. Since a merger always enables merging parties to collude on their prices, we examine these effects in the presence of the collusion effect. We first examine the pooling effect and then the synergy effect. As discussed in §4.1, the post-merger firm faces the aggregate volatility in its total demand, $\sigma_m = \sigma\sqrt{2+2\rho}$, which is increasing in the correlation coefficient ρ between the demands of two merging firms. Thus, when ρ is low (resp., high), σ_m is low (resp., high) due to the high (resp., low) pooling effect.

Proposition 1.1 *For any $w_m \in (0, w]$, the inventory pooling of a post-merger firm affects post-merger equilibrium as follows:*
(a) The post-merger price of any firm i, p_i^{post} $(i = m, 3, 4, ..., n)$, is decreasing in ρ.
(b) The post-merger expected profit of any firm i, π_i^{post} $(i = m, 3, 4, ..., n)$, is decreasing in ρ.

Proposition 1.1(a) states that as the pooling effect becomes more substantial with lower ρ (i.e., the post-merger firm faces a lower aggregate volatility), the post-merger firm charges a higher price; see Figure 1-1(a). Because the post-merger firm saves its inventory cost from pooling inventories, one might anticipate that such cost savings will be passed on to consumers through reduced prices. In fact, the existing theory of mergers developed under deterministic demand posits that marginal cost reduction through cost synergies will induce firms to reduce their prices (see §2). However, our result indicates that although inventory pooling enables the post-merger firm to achieve cost savings, it always induces firms to raise their prices.

We can explain this result as follows. In order to prove that p_m^{post} decreases with σ_m as well as ρ (since $\sigma_m = \sigma\sqrt{2+2\rho}$), we apply the implicit function theorem to two first-order conditions for p_m^{post} and p_3^{post}, and obtain the following after simplifications (see the proof):

$$\frac{dp_m^{post}}{d\sigma_m} = \frac{-\left.\frac{\partial^2 \pi_m(\mathbf{p},\sigma_m)}{\partial p_1 \partial \sigma_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}}{\frac{\partial}{\partial p_m^{post}}\left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) + \frac{\partial}{\partial p_3^{post}}\left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right)\frac{dp_3^{post}}{dp_m^{post}}}. \tag{1.5}$$

In (1.5), the first term in the denominator captures the shape of the post-merger firm's profit function π_m with respect to its own price, and the second term captures the competitive dynamics between post-merger and nonparticipant firms. Using Lemma 1.1 (as well as Lemma A1 in Appendix that shows $0 < dp_3^{post}/dp_m^{post} < 1$), we show in the proof that the denominator of (1.5) is negative. Thus, it suffices to show that the numerator of (1.5) is positive: i.e., $\frac{\partial^2 \pi_m}{\partial p_1 \partial \sigma_m} = \frac{\partial^2 \pi_m^d}{\partial p_1 \partial \sigma_m} - \frac{\partial^2 c_m}{\partial p_1 \partial \sigma_m} < 0$ at $\mathbf{p} = \mathbf{p}^{post}$, which follows from $\frac{\partial^2 \pi_m^d}{\partial p_1 \partial \sigma_m} = 0$ and $\frac{\partial^2 c_m}{\partial p_1 \partial \sigma_m} = R(\Phi^{-1}(1 - w_m/p_m^{post})) > 0$ at $\mathbf{p} = \mathbf{p}^{post}$. Note that the numerator is computed for fixed prices of all nonparticipant firms, and hence it is consistent with the result of the price-setting monopolistic newsvendor models (cf. Mills 1959, Petruzzi and Dada 1999). Its intuition is as follows. Although demand volatility σ_m does not affect the profit from the deterministic demand, π_m^d, it does affect the expected cost due to demand uncertainty, c_m. It can be shown that c_m increases with price p_m^{post} as well as volatility σ_m. The result that $\partial^2 c_m/\partial p_1 \partial \sigma_m\big|_{\mathbf{p}=\mathbf{p}^{post}} > 0$ suggests that the marginal cost of a higher demand volatility σ_m increases with price p_m^{post} because the lost revenue due to demand uncertainty increases with price p_m^{post}. Likewise, the marginal cost of a higher price p_m^{post} increases with volatility σ_m because more demand will be lost with a higher volatility σ_m. Taken as a whole, considering the impact of σ_m on its own expected profit π_m as well as the competitive response of nonparticipant firms, the post-merger firm raises its price in equilibrium when facing a lower σ_m. In response to the increased price of the post-merger firm, as discussed earlier in Lemma 1.1(a), nonparticipant firms raise their prices as well.

Proposition 1.1(b) shows, as expected, that a post-merger firm will obtain a higher expected profit by pooling its inventories. One might expect that the cost advantage of the post-merger firm may hurt its competitors. In contrast to this first intuition, Proposition 1.1(b) shows that

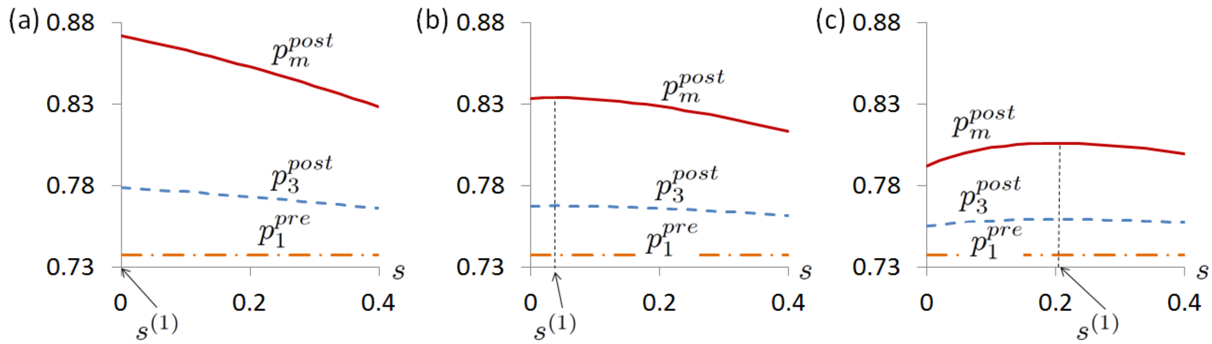

Figure 1-2: The Synergy Effect of a Merger on Prices under Uncertain Demand at (a) $\rho = 0$, (b) $\rho = 0.5$, and (c) $\rho = 1$. (*Note.* The same parameter values are used as in Figure 1 except $\sigma = 0.5$.)

the high pooling effect also benefits nonparticipant firms. This happens because the increased price of the post-merger firm will allow nonparticipant firms to raise their prices as well, and to earn higher expected profits. Although the pooling effect benefits both post-merger and nonparticipant firms, Figure 1-1(b) illustrates that the pooling effect is more beneficial to a merging firm than a nonparticipant firm, and hence when ρ is sufficiently low, a merging firm earns a higher expected profit than that of a nonparticipant firm. This is contrary to the result of Deneckere and Davidson (1985) who show that a merger is always more beneficial to a nonparticipant firm under deterministic demand.

We next examine the impact of marginal cost reduction from merger synergies on post-merger equilibrium. Although both inventory pooling and cost synergies enable a post-merger firm to reduce its expected cost, the following proposition shows that the synergy effect on post-merger equilibrium differs substantially from the pooling effect presented earlier in Proposition 1.1.

Proposition 1.2 *For any $\rho \in [-1, 1]$, there exists a threshold $s^{(1)} \in [0, 1)$, which is nondecreasing in σ_m with $s^{(1)} = 0$ at $\sigma_m = 0$, such that:*
(a) The post-merger price of any firm i, p_i^{post} $(i = m, 3, 4, ..., n)$, is decreasing in s if and only if $s > s^{(1)}$.
(b) The expected profit of the post-merger firm, π_m^{post}, is always increasing in s, whereas the expected profit of a nonparticipant firm, π_i^{post} $(i = 3, 4, ..., n)$, is decreasing in s if and only if $s > s^{(1)}$.

Proposition 1.2(a) states that larger cost synergies of a merger do not necessarily induce firms to lower their prices under uncertain demand. This bears important implications for antitrust policies, since firms often use cost synergies to justify their proposed merger to antitrust authorities. Note that this result is not obtained under deterministic demand (since $s^{(1)} = 0$) as is the case in the existing literature. This result can be explained similarly to Proposition 1.1(a). In particular, p_m^{post} decreases with s if and only if $\partial^2 \pi_m / \partial p_1 \partial w_m > 0$ at $\mathbf{p} = \mathbf{p}^{post}$, which is proven by showing that:

$$\left.\frac{\partial^2 \pi_m^d}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = b + \gamma \frac{n-2}{n} > 0; \quad \left.\frac{\partial^2 c_m}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = \frac{\sigma_m w_m}{2\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^2} \geq 0. \tag{1.6}$$

Under deterministic demand with $\sigma_m = 0$, the expected cost c_m is zero, so is $\partial^2 c_m / \partial p_1 \partial w_m$. In this case, since $\partial^2 \pi_m / \partial p_1 \partial w_m > 0$, p_m^{post} decreases with s. This means that without demand uncertainty, cost synergies enable the post-merger firm to lower its price. However, under uncertain demand with $\sigma_m > 0$, observe from (1.6) that $\partial^2 c_m / \partial p_1 \partial w_m > 0$, and consequently $\partial^2 \pi_m / \partial p_1 \partial w_m$ can be either positive or negative. Proposition 1.2(a) provides the necessary and sufficient condition for $\partial^2 \pi_m / \partial p_1 \partial w_m < 0$ at the equilibrium point, so that p_m^{post} increases with s. This condition requires that the synergy level s is lower than the threshold $s^{(1)}$.[3] The threshold $s^{(1)}$ is nondecreasing with the aggregate demand volatility σ_m. This implies that when the post-merger firm faces a higher demand volatility, it is more likely to observe the counter-intuitive result that p_m^{post} increases with s; see Figure 1-2. The same condition applies to nonparticipant firms as well, since nonparticipant firms change their prices in the same direction as the post-merger firm (see Lemma 1.1(a)).

Interestingly, Proposition 1.2(b) shows that when the post-merger firm achieves larger cost synergies, nonparticipant firms can (but not always) also earn higher expected profits. We can explain this result in the same manner as the non-monotonic change of the post-merger prices in Proposition 1.2(a). Although the price of a post-merger firm changes non-monotonically with the synergy level s, Proposition 1.2(b) shows that as the synergy level s increases, the post-merger firm earns larger expected profit. This result is intuitive and also holds for the case under deterministic demand.

Finally, by combining Lemma 1.1 with Propositions 1.1 and 1.2, we examine the aggregate (collusion, pooling and synergy) effect of a merger, and compare pre-merger equilibrium with

[3] Intuitively, when a post-merger firm decides on its price p_1, it considers a tradeoff between a marginal gain in a deterministic profit from increasing p_1 and a marginal gain from hedging against uncertainty by reducing p_1. It turns out that the former is increasing linearly with w_m, while the latter is increasing convexly with w_m. Thus, when w_m is high, a small reduction of w_m has a larger impact on the marginal gain from hedging against uncertainty.

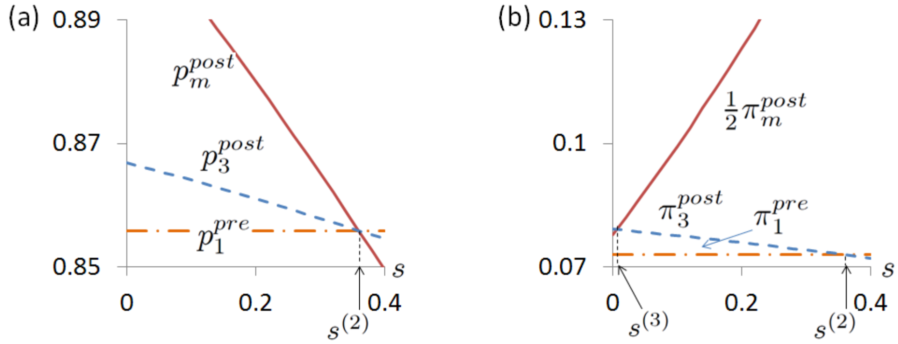

Figure 1-3: The Aggregate Effect of a Merger on: (a) Prices and (b) Expected Profits. (*Note.* The same parameter values are used as in Figure 1 except $\rho = 0.9$ to better illustrate $s^{(2)}$ and $s^{(3)}$.)

post-merger equilibrium. See Figure 1-3 for illustration.

Proposition 1.3 *For any $\rho \in [-1, 1]$ and $w_m \in (0, w]$, there exist thresholds $s^{(2)} \in (s^{(1)}, 1]$ and $s^{(3)} \in [0, s^{(2)}]$ such that:*
(a) The post-merger price of any firm is higher than its pre-merger price (i.e., $p_m^{post} > p_1^{pre}$ and $p_i^{post} > p_i^{pre}$ for $i = 3, 4, ..., n$) if and only if $s < s^{(2)}$. The price of the post-merger firm is higher than that of a nonparticipant firm (i.e. $p_m^{post} > p_i^{post}$ for $i = 3, 4, ..., n$) if and only if $s < s^{(2)}$. Furthermore, $s^{(2)}$ is nonincreasing in ρ.
(b) The post-merger expected profit of a merging firm is higher than its pre-merger expected profit (i.e., $\frac{1}{2}\pi_m^{post} > \pi_1^{pre}$) for any s, whereas the post-merger expected profit of a nonparticipant firm is higher than its pre-merger expected profit (i.e., $\pi_i^{post} > \pi_i^{pre}$ for $i = 3, 4, ..., n$) if and only if $s < s^{(2)}$. Moreover, the post-merger expected profit of a merging firm is higher than that of a nonparticipant firm (i.e. $\frac{1}{2}\pi_m^{post} > \pi_i^{post}$ for $i = 3, 4, ..., n$) if $s > s^{(3)}$.

Proposition 1.3(a) states that a merger will cause firms' prices to drop only when the synergy level s is higher than $s^{(2)}$. This result combines the collusion effect (which causes prices to rise as shown in Lemma 1.1(a)), the pooling effect (which causes prices to rise as shown in Proposition 1.1(a)), and the synergy effect (which causes prices to drop only when $s > s^{(1)}$ as shown in Proposition 1.2(a)). Since the synergy effect causes prices to drop only when $s > s^{(1)}$, even in the absence of the pooling effect, the threshold $s^{(2)}$ in Proposition 1.3(a) is higher than $s^{(1)}$. It is also possible that $s^{(2)} = 1$, implying that a merger will increase firms' prices for any synergy level s. This extreme case may happen when the pre-merger marginal cost w is so low that further reduction of the marginal cost from synergies does not outweigh the collusion and pooling effects of the merger on prices. Proposition 1.3(a) also reveals that when the synergy level is so high that the merger decreases prices (i.e., $s > s^{(2)}$), the price of the post-merger firm becomes lower than the price of nonparticipant firms; see Figure 1-3(a).

In addition, Proposition 1.3(a) shows that the threshold $s^{(2)}$ is nondecreasing in ρ. This happens because the pooling effect drives prices upward as shown in Proposition 1.1(a). This result suggests that as the pooling effect becomes more significant, larger cost synergies are required for prices to drop after a merger. Therefore, *ceteris paribus*, consumer price is less likely to rise after a merger in an industry where firms' uncertain demands are highly correlated (e.g., household furniture, home appliances, and motor vehicle dealerships, where demand is closely related to business cycles (Berman and Pfleeger 1997)).

Since the post-merger firm benefits from each of the collusion, pooling and synergy effects, a merger will increase the expected profit of a merging firm (Proposition 1.3(b)). On the other hand, a merger will increase the expected profit of a nonparticipant firm only when the merger induces all firms to raise their prices. This is consistent with our previous lemma and propositions. As illustrated in Figure 1-3(b), when the synergy level s is larger than $s^{(3)}$, the expected profit of a merging firm exceeds that of a nonparticipant firm.

1.4.3 Post-Merger Analysis: Service Level and Expected Consumer Welfare

So far we have focused on the effect of a merger on firms' prices and their expected profits, following the tradition of prior work on mergers studied under deterministic demand. However, when demand is uncertain, firms determine their stocking levels which can affect the availability of products to consumers. In this section, we examine how the pooling and synergy effects of a merger affect firms' service levels and ultimately expected consumer welfare.

Following the convention of the operations management literature, we define firm i's service level l_i as its in-stock probability: $l_i = \Pr(D_i \leq q_i) = \Pr(\widetilde{\varepsilon}_i \leq y_i) = \Phi(y_i/\sigma_i)$. The following proposition shows the pooling and synergy effects of a merger on firms' service levels (in the presence of the collusion effect), and compares service levels between pre-merger and post-merger markets.

Proposition 1.4 *(a) For any fixed $w_m \in (0, w]$, the post-merger service levels of all firms, l_m^{post} and l_i^{post} $(i = 3, 4, ..., n)$, are decreasing in ρ.*
(b) For any fixed $\rho \in [-1, 1]$, the service level of the post-merger firm, l_m^{post}, is always increasing in s, whereas the service level of a nonparticipant firm, l_i^{post} $(i = 3, 4, ..., n)$, is increasing in s if and only if $s < s^{(1)}$ (where $s^{(1)}$ is defined in Proposition 2)
(c) The service level of the post-merger firm is always higher than its pre-merger service level (i.e., $l_m^{post} > l_1^{pre}$). The service level of a nonparticipant firm is higher than its pre-merger

service level (i.e. $l_i^{post} > l_i^{pre}$ for $i = 3, 4, ..., n$) if and only if $s < s^{(2)}$ (where $s^{(2)}$ is defined in Proposition 3).

Proposition 1.4(a) shows that when the pooling effect of a merger is significant with low ρ, firms raise their service levels. To understand this result, recall from §4.1 that the optimal safety stock is $y_i^{post} = \sigma_i \Phi^{-1}\left(1 - w_i/p_i^{post}\right)$, at which the service level is $l_i^{post} = 1 - w_i/p_i^{post}$. Because a higher pooling effect (i.e., a lower ρ) raises all firms' prices p_i^{post} $(i = m, 3, 4, ..., n)$ for any fixed w_i (Proposition 1(a)), it also raises their service levels l_i^{post} $(i = m, 3, 4, ..., n)$. With a higher service level, a firm's lost sales are decreased; i.e., $E\left(D_i - q_i\right)^+ = \sigma_i R\left(\Phi^{-1}\left(l_i\right)\right)$ is decreasing with l_i.

Unlike the pooling effect, the synergy effect affects the service level of the post-merger firm $l_m^{post} = 1 - w_m/p_m^{post}$ via changes in both w_m and p_m^{post}. When the synergy level s is significant with $s > s^{(1)}$, the result is not straightforward because a higher s means a lower w_m but it induces the post-merger firm to lower its price p_m^{post} (cf. Proposition 2). It turns out that the price drop is always less than the cost reduction (i.e., $dp_m^{post}/dw_m < 1$; see the proof of Proposition 1.4(b)), so the service level l_m^{post} increases for any synergy level s. For nonparticipant firm i $(i = 3, 4, ..., n)$, the synergy effect affects its service level $l_i^{post} = 1 - w_i/p_i^{post}$ only through a change in its price p_i^{post}. Since p_i^{post} increases with the synergy level s if and only if $s < s^{(1)}$ (cf. Proposition 2), so does l_i^{post}.

The aggregate effect (collusion, pooling and synergy effects) of a merger on a firm's service level is shown in Proposition 1.4(c). It always increases the post-merger firm's service level, but it increases the nonparticipant firm's service level only when the synergy level is low (i.e., $s < s^{(2)}$).

This result raises an interesting point. When the synergy level $s < s^{(2)}$, a merger not only induces all firms to raise their prices (cf. Proposition 3(a)), but also induces all firms to raise their service levels. The former affects consumer negatively, whereas the latter affects consumers positively. Note that the latter effect exists only when demand is uncertain. A similar trade-off also exists when $s > s^{(2)}$ because consumers will benefit from lower prices as well as a higher service level of a post-merger firm, but hurt from a lower service level of nonparticipant firms. Then how can we measure the overall impact of a merger on consumers? Now we propose expected consumer welfare as the aggregate measure that antitrust agency and other interested parties may use in evaluating a merger. As compared to the standard approach of computing consumer welfare as an area under the demand curve of a single firm, special care must be taken to account for price-competing oligopoly as well as for potential stock-outs. We derive expected consumer welfare in the following two steps. First, we present the consumer utility function of a representative consumer that leads to our demand function in the oligopolistic market. Second, we use this utility function to define expected consumer welfare that takes into account potential stock-outs.

Following Shubik (1980), we can show that the following utility function of a representative consumer generates the demand function $D_i = L_i(\mathbf{p}) + \widetilde{\varepsilon}_i$ (where $L_i(\mathbf{p})$ is given in (1)):

$$u(\mathbf{D}) = \sum_{i=1}^{n} \left\{ \frac{1+\frac{\gamma}{nb}}{b+\gamma} \left(a + \widetilde{\varepsilon}_i - \frac{1}{2} D_i \right) + \frac{\gamma}{nb(b+\gamma)} \sum_{j \neq i}^{n} \left(a + \widetilde{\varepsilon}_j - \frac{1}{2} D_j \right) \right\} D_i, \tag{1.7}$$

where $D = (D_1, D_2, ..., D_n)^T$ denotes a consumption bundle. In (1.7), the first term in the bracket represents the (direct) marginal utility from the product sold by firm i (hereinafter, product i in short), and the second term in the bracket is the marginal utility from substitution.

Total expected consumer welfare (or surplus) from the consumption bundle is denoted by $E[cs(\mathbf{D})]$, which is the sum of expected consumer surplus from product i, $E[cs_i(\mathbf{D})]$; i.e., $E[cs(\mathbf{D})] = \sum_{i=1}^{n} E[cs_i(\mathbf{D})]$. To derive $cs_i(\mathbf{D})$, we consider two different cases. In the first case when a realization of random demand component $\widetilde{\varepsilon}_i$ is smaller than or equal to safety stock $y_i = \sigma_i \Phi^{-1}(l_i)$, all demand will be satisfied. In this case, by substituting the demand $D_i = L_i(\mathbf{p}) + \widetilde{\varepsilon}_i$ to (1.7) and then subtracting the price paid $\sum_{i=1}^{n} p_i D_i$, we obtain the following *ex-post* consumer surplus:

$$cs_i(\mathbf{D}) = cs_i(\mathbf{p}, \widetilde{\boldsymbol{\varepsilon}}) = \frac{1}{2} \left\{ \frac{1+\frac{\gamma}{nb}}{b+\gamma} (a + \widetilde{\varepsilon}_i) + \frac{\gamma}{nb(b+\gamma)} \sum_{j \neq i}^{n} (a + \widetilde{\varepsilon}_j) - p_i \right\} \{L_i(\mathbf{p}) + \widetilde{\varepsilon}_i\}. \tag{1.8}$$

In the second case when a realization of $\widetilde{\varepsilon}_i$ is greater than y_i, some demand will be lost. Similar to the monopoly case of Cohen et al. (2014) and Ovchinnikov and Raz (2015), we assume that customers are first-come-first-served, so that every customer faces the same probability of not getting a product, $\{L_i(\mathbf{p}) + y_i\} / \{L_i(\mathbf{p}) + \widetilde{\varepsilon}_i\}$. In this case, the *ex-post* consumer surplus is given as $cs_i(\mathbf{p}, \widetilde{\varepsilon}_i) \{L_i(\mathbf{p}) + y_i\} / \{L_i(\mathbf{p}) + \widetilde{\varepsilon}_i\}$. Putting the two cases together, we can express the expected consumer surplus from product i, $E[cs_i(\mathbf{p}, \widetilde{\boldsymbol{\varepsilon}})]$, as follows:

$$\begin{aligned} E[cs_i(\mathbf{p}, \widetilde{\boldsymbol{\varepsilon}})] &= \int_{-\infty}^{\sigma_i \Phi^{-1}(l_i)} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} cs_i(\mathbf{p}, \boldsymbol{\varepsilon}) f(\boldsymbol{\varepsilon}) d\varepsilon_1 \cdots d\varepsilon_{i-1} d\varepsilon_{i+1} \cdots d\varepsilon_n d\varepsilon_i \quad (1.9) \\ &+ \int_{\sigma_i \Phi^{-1}(l_i)}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} cs_i(\mathbf{p}, \boldsymbol{\varepsilon}) \frac{L_i + \sigma_i \Phi^{-1}(l_i)}{L_i + \varepsilon_i} f(\boldsymbol{\varepsilon}) d\varepsilon_1 \cdots d\varepsilon_{i-1} d\varepsilon_{i+1} \cdots d\varepsilon_n d\varepsilon_i, \end{aligned}$$

where $f(\boldsymbol{\varepsilon}) = e^{-\boldsymbol{\varepsilon}' \Sigma^{-1} \boldsymbol{\varepsilon} / \mathbf{2}} / (2\pi)^{n/2} / |\Sigma|^{1/2}$ is the n-dimensional joint density of the multivariate normal random variable $\widetilde{\boldsymbol{\varepsilon}}$ and Σ is its covariance matrix. After a merger between firms 1 and

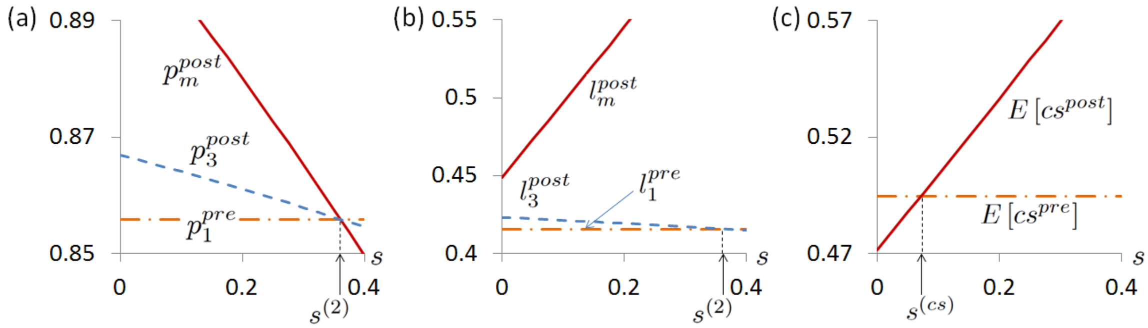

Figure 1-4: The Aggregate Effect of a Merger on: (a) Prices, (b) Service Levels, and (c) Expected Consumer Welfare. (*Note.* The same parameter values are used as in Figure 3.)

2, $cs_m(\mathbf{p}, \boldsymbol{\varepsilon})$ in (1.9) needs to be integrated over the aggregate uncertain component, $(\varepsilon_1 + \varepsilon_2)$.

Using the total expected consumer welfare $E[cs(\mathbf{D})]$ defined above, we now examine the aggregate impact of a merger on consumers (which essentially combines *all* the effects we have examined separately, including the collusion, pooling and synergy effects on prices and service levels of both post-merger and nonparticipant firms).

Proposition 1.5 *Suppose* $b \geq \frac{(n-2)\sigma w}{n\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)\left(p_3^{post}\right)^2}$. *Then there exists a threshold* $s^{(cs)} \in \left[0, s^{(2)}\right]$ *such that for any* $s > s^{(cs)}$, *the expected consumer welfare after a merger,* $E[cs^{post}]$, *is greater than that before the merger,* $E[cs^{pre}]$.

The existence of the threshold $s^{(cs)}$ is intuitive. As discussed above, when $s > s^{(2)}$, consumers benefit from lower prices of all firms and a higher service level of a post-merger firm, although they hurt from a lower service level of nonparticipant firms. When the synergy level s is sufficiently high, the former positive effect outweighs the latter negative effect. However, as compared to the earlier result in Proposition 3(a) that $s^{(2)}$ is nonincreasing in ρ, we observe $s^{(cs)}$ as well as $E[cs^{post}]$ change non-monotonically in ρ. This is because the high pooling effect hurts consumers through increased prices, but at the same time it benefits consumers through increased service levels. More importantly, even if a merger induces all firms to raise their prices, it can still improve expected consumer welfare by increasing firms' service levels. This is illustrated in Figure 1-4 when the synergy level s falls between $s^{(cs)}$ and $s^{(2)}$.[4]

[4]The technical condition given in Proposition 1.5 is a sufficient condition for $s^{(cs)} \leq s^{(2)}$, which is observed in all of our extensive numerical experiments with the following parameter values: n=3, a=1, $b \in \{0.1, 0.6, 1, 2\}$, $\gamma \in \{0.1, 0.5, 1, 2\}$, $w \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, $\sigma \in \{0.05, 0.1, 0.3, 0.5\}$ and $\rho \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. These scenarios include a set of the parameter values used in Figure 1, and also cover various possible scenarios.

The main takeaway from the above analysis is as follows. The extant literature has measured the impact of a merger on consumers via price changes, assuming consumer demand is deterministic. In reality, consumer demand is fundamentally uncertain. Our result indicates that under uncertain demand, it is crucial to take into account how a merger affects consumers via firms' service levels as well as their prices.

1.5 Extensions

This section examines four extensions of our base model. In §5.1, we consider asymmetric firms in a pre-merger market. In §5.2, we extend our results to non-normal distributions. In §5.3, we analyze a demand model with a general uncertainty structure. Lastly, in §5.4, we analyze the impact of stock-out substitution on mergers. For brevity, we focus on the pooling effect of a merger on prices and the synergy effect on nonparticipants' profits. The effects on service levels and expected consumer welfare can also be shown similarly.

1.5.1 Asymmetric Firms

So far we have analyzed a merger in the pre-merger market in which firms are symmetric. Such a merger results in asymmetric competition between a post-merger firm and nonparticipant firms in the post-merger market. In this section, we examine the impact of a merger in the pre-merger market in which firms are asymmetric, and we demonstrate that our main results continue to hold.

Consider a pre-merger market in which firms differ in demand and cost parameters. Specifically, firm i $(= 1, 2, \cdots, n)$ faces its demand $D_i(\mathbf{p}) = L_i(\mathbf{p}) + \widetilde{\varepsilon}_i$, where $L_i(\mathbf{p}) = a_i - b_i p_i + \gamma(\sum_{j=1}^{n} p_j/n - p_i)$ and $\widetilde{\varepsilon} = (\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, \cdots, \widetilde{\varepsilon}_n)$ follows a multivariate normal distribution $N(0, \sum)$ with $Var(\widetilde{\varepsilon}_i) = \sigma_i^2$; and firm i incurs a marginal cost w_i. We consider a situation where firms sell homogeneous products, and differentiation among firms occurs at the retail level due to the reasons such as locations, consumer characteristics, and store characteristics. After a merger between firms 1 and 2, the deterministic part of the post-merger firm's demand becomes $L_m(\mathbf{p}) = L_1(\mathbf{p}) + L_2(\mathbf{p})$, assuming that two retail outlets maintain their differentiation. As before, we measure the pooling effect of a merger in terms of the correlation coefficient ρ between $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$. Unlike the symmetric case, the post-merger firm may set two different prices at retail outlets 1 and 2 *in equilibrium*, so we denote firm i's post-merger price by p_i^{post} for $i = 1, 2, ..., n$ (instead of using subscript m). As for the synergy effect, when firms are symmetric in the pre-merger market, in §4 we have defined the synergy level as $s \equiv (w - w_m)/w$. However, when asymmetric firms compete in the pre-merger market, merging firms' marginal costs w_1 and w_2 may differ, so we refine our previous definition of s to $s \equiv (\min\{w_1, w_2\} - w_m)/\min\{w_1, w_2\}$. The synergy level s is non-negative when the marginal cost of the post-merger firm w_m is at least as small as $\min\{w_1, w_2\}$. Note that when w_1 and w_2 are different, a merger leads to a change in the marginal cost of at least one merging firm. For this reason, we cannot isolate the collusion effect from the synergy effect as we did in Lemma 1.1 for the symmetric case. The following corollary shows that the pooling and synergy effects of a merger in the pre-merger market of asymmetric firms are consistent with those in the pre-merger market of symmetric firms.

Corollary 1.1 *(a) For any fixed $w_m \in (0, w]$, all post-merger prices p_i^{post} $(i = 1, 2, ..., n)$ are decreasing in ρ.*
(b) For any fixed $\rho \in [-1, 1]$, there exists a threshold $s_{asym}^{(1)} \in [0, 1)$ such that a nonparticipant firm's profit π_i^{post} $(i = 3, 4, ..., n)$ as well as all post-merger prices p_i^{post} $(i = 1, 2, ..., n)$ is increasing in s if $s < s_{asym}^{(1)}$.

1.5.2 Non-Normal Distributions

In this section, we consider a case in which the demand of a firm follows a general distribution. We denote by $f(\cdot)$ and $F(\cdot)$ the density of $\widetilde{\varepsilon}_i$ with $E(\widetilde{\varepsilon}_i) = 0$ and its cumulative distribution respectively, and we denote by $f_m(\cdot)$ and $F_m(\cdot)$ the density of $\widetilde{\varepsilon}_m = \widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2$ and its cumulative distribution. In the pre-merger market, the expected profit of firm i can be expressed as: $\pi_i(\mathbf{p}, y_i) = (p_i - w)L_i(\mathbf{p}) - wy_i - p_i R_f(y_i)$, where $R_f(y_i) \equiv \int_{y_i}^{\infty} (t - y_i) f(t)\, dt$ represents the expected lost sales of firm i having safety stock y_i. Similarly, the expected profit of the post-merger firm is given as $\pi_m(\mathbf{p}, y_m) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - w_m y_m - \frac{p_1+p_2}{2} R_{f_m}(y_m)$.

We next present the definition of dispersive ordering and a failure rate (e.g., see Müller and Stoyan 2002), and then use these properties to generalize our results in §4.

Definition *(a) A random variable X is smaller than Y in dispersive ordering, written as $X \preceq_{disp} Y$, if $F^{-1}(\tau_2) - F^{-1}(\tau_1) \leq G^{-1}(\tau_2) - G^{-1}(\tau_1)$ for all $0 < \tau_1 < \tau_2 < 1$, where F and G are the distribution functions of X and Y, respectively.*
(b) Let X be a random variable with density f and cumulative distribution F. The failure rate of X is defined as $h(x) = f(x)/\{1 - F(x)\}$. The random variable X has an increasing failure rate (IFR) if $h(x)$ is increasing for all x such that $F(x) < 1$.[5]

Corollary 1.2 *(a) Let $\widetilde{\varepsilon}_m$ and $\widetilde{\xi}_m$ be two random variables with $E[\widetilde{\varepsilon}_m] = E[\widetilde{\xi}_m] = 0$. Let p_i^{post} or $\widehat{p}_i^{post}$ $(i = m, 3, 4, ..., n)$ be the equilibrium price of firm i when the post-merger firm faces*

[5] Dispersive ordering and an IFR have the following relation. Let X be a random variable with support on (a, ∞) where $a \geq -\infty$. For any $t \in (a, \infty)$, let $X_t = [X - t | X \geq t]$ denote the residual life time. The following statements are equivalent (Pellerey and Shaked 1997): (i) X has an IFR; (ii) $X_t \preceq_{disp} X$ for all t; (iii) $X_t \preceq_{disp} X_s$ for all $s < t$.

random demand component $\widetilde{\varepsilon}_m$ *or* $\widetilde{\xi}_m$*, respectively. Then,* $\widetilde{\varepsilon}_m \preceq_{disp} \widetilde{\xi}_m$ *implies* $p_i^{post} \geq \widehat{p}_i^{post}$*.*
(b) If $\widetilde{\varepsilon}_m$ *follows an IFR distribution, then there exists a threshold* $s_{non}^{(1)} \in [0,1]$ *such that a nonparticipant firm's profit* π_i^{post} $(i = 3, 4, ..., n)$ *as well as all post-merger prices* p_i^{post} $(i = m, 3, 4, ..., n)$ *is increasing in* s *if and only if* $s < s_{non}^{(1)}$*.*

Corollary 1.2(a) generalizes the pooling effect of a merger presented earlier in Proposition 1.1(a). The condition $\widetilde{\varepsilon}_m \preceq_{disp} \widetilde{\xi}_m$ implies that the expected lost sales $R_{f_m}\left[F_m^{-1}\left(1 - w_m/p_m\right)\right]$ $(= 2\partial c_m/\partial p_1)$ is smaller for a less dispersive demand. Because a marginal loss from raising a price is lower for a less dispersive demand, the post-merger firm charges a higher price in equilibrium, which is followed by an increase in prices of nonparticipant firms. This is consistent with Proposition 1(a), since a larger ρ, or equivalently a larger σ_m under a normally distributed demand, results in a more dispersive $\widetilde{\varepsilon}_m$. Corollary 1.2(b) shows that the synergy effect of a merger presented earlier in Proposition 1.2 holds for an IFR distribution which includes many commonly used distributions such as normal, uniform, gamma and Weibull (with a shape parameter greater than 1 for gamma and Weibull); see, e.g., Kocabıyıkoğlu and Popescu (2011).

1.5.3 Demand Function with a General Uncertainty Structure

In this section, we consider a more general model in which firm i's demand is $D_i\left(\mathbf{p}\right) = L_i(\mathbf{p}) + \delta_i(\mathbf{p})\widetilde{\varepsilon}_i$, where $L_i(\mathbf{p})$ is given in (1.1), $\delta_i(\mathbf{p}) = \alpha - \beta p_i + \theta(\frac{1}{n}\sum_{j=1}^{n} p_j - p_i)$ ($\alpha \geq 0$, $\beta \geq 0$ and $\theta \geq 0$), and $\widetilde{\varepsilon} = (\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, ..., \widetilde{\varepsilon}_n)$ follows a multivariate normal distribution with $E\left(\widetilde{\varepsilon}_i\right) = 0$ and $Var\left(\widetilde{\varepsilon}_i\right) = 1$. This model takes the following two commonly-used models in the literature as special cases: (1) when $(\beta, \theta) = (0, 0)$, $D_i\left(\mathbf{p}\right) = L_i(\mathbf{p}) + \alpha\widetilde{\varepsilon}_i$, which is the additive demand function in the main body with $\sigma = \alpha$; and (2) when $(\beta, \theta) = (b\alpha/a, \gamma\alpha/a)$, $D_i\left(\mathbf{p}\right) = L_i(\mathbf{p})(1 + \alpha\widetilde{\varepsilon}_i/a)$, which is a multiplicative demand function.[6] After a merger between firm 1 and firm 2 takes place, the post-merger firm faces the demand of $D_m\left(\mathbf{p}\right) = L_1(\mathbf{p}) + L_2(\mathbf{p}) + \delta_1(\mathbf{p})\widetilde{\varepsilon}_1 + \delta_2(\mathbf{p})\widetilde{\varepsilon}_2$.

Corollary 1.3 *(a) For any fixed* $w_m \in (0, w]$ *and* $\theta \geq 0$*, there exists a threshold* $\widehat{\beta}$ (≥ 0) *such that if* $\beta < \widehat{\beta}$*, all post-merger prices* p_i^{post} $(i = m, 3, 4, ..., n)$ *are decreasing in* ρ*.*
(b) Suppose $\gamma > \theta\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_3^{post}}\right)\right)\left(1 - \frac{w}{p_3^{post}}\right)^{-1}$*. Then, for any fixed* $\rho \in [-1, 1]$*, there exists a threshold* $s_{gen}^{(1)} \in [0, 1)$ *such that a nonparticipant firm's profit* π_i^{post} $(i = 3, 4, ..., n)$ *as well as all post-merger prices* p_i^{post} $(i = m, 3, 4, ..., n)$ *is increasing in* s *if and only if* $s < s_{gen}^{(1)}$*.*

[6] Young (1978) and Petruzzi and Dada (1999) use a similar demand function for a monopoly case: $D\left(p\right) = L\left(p\right) + \delta\left(p\right)\widetilde{\varepsilon}$. Since we analyze an oligopoly model, we replace the price p with a vector of prices of all firms, $\mathbf{p}$. To be consistent with the main body, we assume that $L_i(\mathbf{p})$ is given in (1.1), and that $\delta_i\left(\mathbf{p}\right)$ takes a similar form to $L_i\left(\mathbf{p}\right)$.

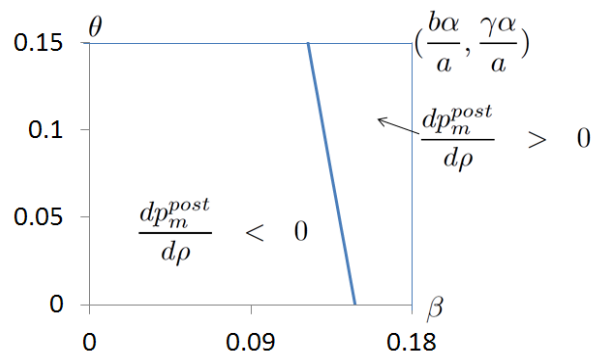

Figure 1-5: The Pooling Effect of a Merger under a General Uncertainty Model. (*Note.* The same parameter values are used as in Figure 1.)

Corollary 1.3(a) shows that the pooling effect of a merger on firms' prices is the same as that in our base model as long as the impact of demand uncertainty on price sensitivity is sufficiently small (i.e., β and θ are small). See Figure 1-5 for illustration, in which the lower left corner at $(\beta, \theta) = (0, 0)$ corresponds to the additive demand case and the upper right corner at $(\beta, \theta) = (b\alpha/a, \gamma\alpha/a)$ corresponds to the multiplicative demand case. The intuition from this result is as follows. To hedge against the risk of uncertainty due to high ρ, in the additive case, a post-merger firm wants to reduce its price to decrease the coefficient of variance without affecting the variance; whereas in the multiplicative case, a post-merger firm wants to increase its price to reduce the variance without affecting the coefficient of variance (cf. Petruzzi and Dada 1999). When β and θ are sufficiently small, the effect of controlling the coefficient of variance dominates the effect of controlling the variance, hence inducing a post-merger firm to decrease its price with ρ as in the additive case. Next, Corollary 1.3(b) shows that the non-monotonic relationship between post-merger firm's prices p_i^{post} and the synergy level s is preserved in the general demand model. Consequently, when synergy level s is small, larger cost synergies from a merger benefit nonparticipant firms. The condition on γ guarantees that a higher price of a merging firm has a positive externality on a nonparticipant firm as in the base model.[7]

[7] A demand function with additive uncertainty is more amenable to modeling consumer behavior from stock-out substitution as we discuss next in §5.4. It is also worth noting that a demand function with multiplicative uncertainty does not satisfy the conditions necessary to guarantee utility maximization by a representative consumer (Krishnan 2010), and hence it cannot be used for welfare analysis in §4.3.

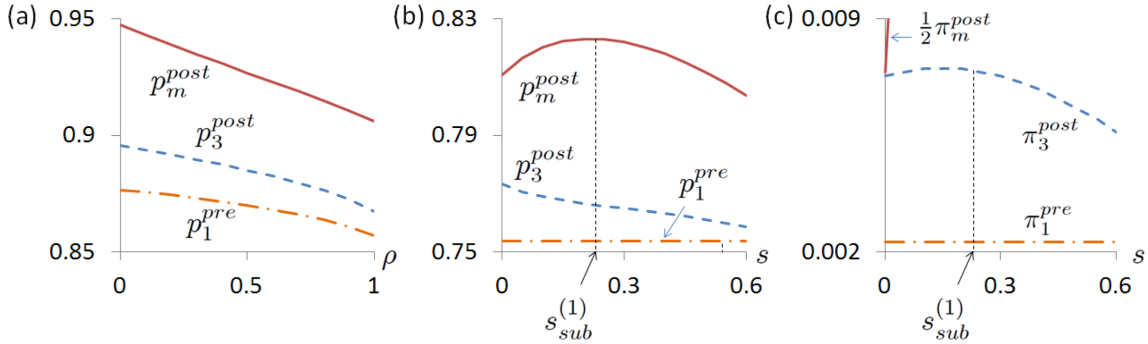

Figure 1-6: Under Stock-Out Substitution: (a) the Pooling Effect on Prices, (b) The Synergy Effect on Prices, and (c) The Synergy Effect on Expected Profits.

1.5.4 Stock-Out Substitution

This section examines the case in which a fraction of consumers who do not find a product at their local retailers look for the product at other retailers. We follow the standard approach of modeling this stock-out substitution (Netessine and Rudi 2003, Zhao and Atkins 2008, and references therein) by assuming that a part of excess demand is reallocated to other retailers in deterministic proportions, and that the sales is lost if reallocated demand cannot be satisfied. Then, the total demand of firm i is the sum of its original demand and a fraction of lost demands from other firms; i.e., $\widehat{D}_i\left(\mathbf{p}, \mathbf{y}_{-i}\right) = L_i(\mathbf{p}) + \widetilde{\varepsilon}_i + \frac{\kappa}{n-1}\sum_{j\neq i}(D_j - q_j)^+$, where κ (≤ 1) is a fraction of the excess demand from firm j spilled to other firms, and $\frac{\kappa}{n-1}\sum_{j\neq i}(D_j - q_j)^+$ is referred to as "spill demand." Random variable $\widehat{D}_i$ is the sum of n random variables – a normal $\widetilde{\varepsilon}_i$ and $(n-1)$ truncated normal $\frac{\kappa}{n-1}(D_j - q_j)^+$ for $j \neq i$ – which are correlated with each other in a complex manner. Thus the analytical characterization of even pre-merger equilibrium is intractable (Netessine and Rudi 2003, Zhao and Atkins 2008). For this reason, we study stock-out substitution numerically.

We first examine how stock-out substitution affects the pooling effect of a merger. Figure 1-6(a) uses the same parameter values as in Figure 1-1 except $\kappa = 0.2$. By comparing these two figures, we observe that under stock-out substitution inventory pooling continues to induce all firms to raise their prices (i.e., p_m^{post} and p_3^{post} decrease with ρ). This is because the main driver for the pooling effect (i.e., the demand volatility of the post-merger firm is increasing with ρ) exists with or without stock-out substitution. Although the correlation between firms' demands does not affect *pre-merger* equilibrium in Figure 1-1, we observe in Figure 1-6(a) that p_1^{pre} is decreasing with ρ under stock-out substitution. To understand this result, suppose that ρ is high. Then when the demand of one firm, say firm 1, is high, there is a high chance that the demands of the other firms are also high. In this case, when the other firms experience stock-outs, it is likely that firm 1 also experiences a stock-out and cannot satisfy the spill demand from the other firms. As a result, higher ρ reduces the expected spill demand a firm can satisfy.[8] Therefore, with higher ρ, firms compete more intensely by reducing their prices.

We next examine how stock-out substitution affects the synergy effect of a merger. Figure 1-6(b)-(c) use the same parameter values as in Figure 1-2 except $\kappa = 0.2$. From these two figures, we observe that the synergy effect on the post-merger firm is consistent in both cases with or without stock-out substitution: larger cost synergies increase the post-merger firm's price only when s is lower than a certain threshold (denoted by $s_{sub}^{(1)}$ in Figure 1-6(b)), although it always increases the post-merger firm's expected profit. For the nonparticipant firm, when s is small, π_3^{post} increases with s, confirming that larger cost synergies can benefit non-participant firms. However, unlike Proposition 1.2 (showing that without stock-out substitution p_3^{post} and π_3^{post} decrease in s if and only if $s > s^{(1)}$), Figure 1-6(b)-(c) show that p_3^{post} and π_3^{post} can decrease in s even when $s < s_{sub}^{(1)}$. We can explain this result intuitively as follows. With larger cost synergies, the post-merger firm increases its safety stock, which not only reduces its own expected lost sales due to stock-outs, but also reduces its spill demand to the nonparticipant firm. Therefore, with stock-out substitution, larger cost synergies create an additional force that induces the nonparticipant firm to charge a lower price and to stock less, causing π_3^{post} to be decreasing in s further.

1.6 Conclusion

M&As have been employed by many firms as major strategies to create competitive advantages. Not only does a merger enable merging parties to cooperate with each other in their decision-making, but also to achieve cost savings by improving operational efficiencies. Whether such competitive advantages created by a merger will benefit consumers is a central concern of antitrust agency. Popular defensive arguments used by firms have been that merger synergies will lower the cost of a post-merger firm and thus be ultimately passed on to consumers. Whereas the existing theory of mergers has been supportive of those arguments, this paper shows they are not necessarily true.

While building on the competitive models established in the rich literature on mergers, our

[8] Higher ρ also increases $Var(\widehat{D}_1)$, since $Var(\widehat{D}_1) = Var(\widetilde{\varepsilon}_1) + \frac{\kappa^2}{4}\sum_{j=2}^{3} Var((\widetilde{\varepsilon}_j - y_j)^+) + \kappa \sum_{j=2}^{3} Cov(\widetilde{\varepsilon}_1, (\widetilde{\varepsilon}_j - y_j)^+) + \frac{\kappa^2}{2} Cov((\widetilde{\varepsilon}_2 - y_2)^+, (\widetilde{\varepsilon}_3 - y_3)^+)$, where both $Cov(\widetilde{\varepsilon}_1, (\widetilde{\varepsilon}_j - y_j)^+)$ and $Cov((\widetilde{\varepsilon}_2 - y_2)^+, (\widetilde{\varepsilon}_3 - y_3)^+)$ are increasing in ρ.

model features two novel operational elements: uncertain demand and statistical economies of scale. Clearly, a retail business entails uncertainty in consumer demand, and therefore demand uncertainty has been one of the most fundamental features in the literature of operations management. Under uncertain demand, a merger can create statistical economies of scale by reducing the aggregate volatility of combined demands in addition to conventional economies of scale that lead to marginal cost reduction. Such statistical economies of scale allow a post-merger firm to reduce inventory costs by managing their stocks in a centralized manner.

Our analysis shows that cost savings from statistical economies of scale have substantially different impacts on firms' prices and expected profits as compared to cost savings from conventional economies of scale. First, we find that statistical economies of scale (i.e., pooling effect) indeed reduce the expected cost of a post-merger firm (hence increasing its expected profit), but contrary to a common belief, they always induce both post-merger and nonparticipant firms to raise their prices. Second, although the existing theory has shown that cost synergies due to conventional economies of scale (i.e., synergy effect) lead to price reduction under deterministic demand, our analysis shows that this is no longer true under uncertain demand. When a post-merger firm faces highly uncertain demand or its cost synergies are not significant, it is better off raising its price. Interestingly, larger cost synergies of a post-merger firm can benefit nonparticipant firms when accompanying a price increase. Finally, when a post-merger firm can utilize both conventional and statistical economies of scale, consumer price is less likely to rise after a merger in the industries that exhibit higher correlation among firms' uncertain demands. Furthermore, a merger may induce firms to raise their service levels, and ultimately benefit consumers even if prices are increased.

We have considered various extensions of our base model and analysis. However, due to inherent complexity of analyzing competitive price-setting newsvendors and their mergers, we have made some simplifying assumptions such as a linear demand function, no supply chain consideration, and a single product with no economies of scope. Relaxing these assumptions will enrich our findings, but the incorporation of these features may require simplification of other parts. Our results also provide several important theoretical findings that may be tested empirically.

Chapter 2

Green Technology Development and Adoption: Competition, Regulation, and Uncertainty – A Global Game Approach

2.1 Introduction

When a government agency considers tightening a standard on a pollutant, it usually takes into account the proportion of firms in the industry that are able to meet the new standard. For example, in a regulatory impact analysis of proposed greenhouse gas (GHG) emission standards, the United States Environmental Protection Agency (EPA) stated the following (EPA 2012): "the vast majority of technology we project as being utilized to meet the GHG standards is commercially available and already being used to a limited extent across the fleet, although far greater penetration of these technologies into the fleet is projected as a result of both the MYs [model years] 2012-2016 rule and this final rule." The Tier 3 Gasoline Sulfur Standard is another example of industry capability influencing regulation. This standard requires gasoline sold in the U.S. to have an annual average of no more than 10 parts per million of sulfur. When proposing the Tier 3 Standard, the EPA demonstrated its feasibility by claiming that 40 out of 108 gasoline refineries were already able to meet this standard (EPA 2014a). Such consideration is not just a recent development: in early 1999, BP Amoco announced that it would lower the sulfur level in its gasoline in 40 cities around the world; it is believed that this encouraged the EPA to set tougher standards (Kendall and Grossman 1999), as later that year the EPA proposed the Tier 2 Gasoline Standard, which required a 90% reduction of the sulfur level in gasoline by 2004. This pattern of looking at early technology adopters before mandating universal adoption is also common in other parts of the world. For example, in Europe, newer emission control technologies became mandatory only after their feasibility had been demonstrated in practice by early adopters (Faiz et al. 1996).

We call the proportion of firms within an industry who are currently able to meet a proposed stricter standard the industry's "capability index." A higher capability index indicates that more firms have the technology to meet the standard, thereby potentially encouraging the government agency to tighten regulations. This relationship between capability and potential future regulation can in turn affect firms' decisions on technology adoption: if a firm expects that many other firms will adopt a new technology to reduce a pollutant, the firm is also likely to adopt this new technology because mandated adoption is more likely, and may be more costly. For example, after the Tier 3 Gasoline Sulfur Standard became effective, refineries that had not met the standard were required to purchase credits to offset their pollution until they updated their technology to bring them into full compliance with the standard (EPA 2014b). As a result, regulation which considers an industry's capability index makes firms' actions *strategic complements*.

Despite the fact that regulation often takes industry capability into account, there has been disappointingly little research on evaluating the impact of the interrelation of capability and regulation on firms' adoption decisions: existing research assumes that government agencies move to stricter standards with fixed probabilities regardless of industry capability (e.g., Farzin and Kort 2000 and Kraft and Raz 2015). Under this assumption of fixed regulation probabilities, firms' actions are no longer strategic complements, and an important driving force behind firms' decisions may be missed. In addition, most existing research assumes that the benefits of adopting new green technologies to reduce pollutants are deterministic (e.g., Baker and Shittu 2006 and Kraft et al. 2013). In practice, the benefit of a new technology is often highly uncertain – a firm is not likely to know the precise payoff of a new technology before it is developed. We aim to provide insight into how the uncertainty of a new technology's payoff and the strategic complementarity induced by regulation based on industry capability jointly affect firms' incentives to develop or adopt a new green technology.

Specifically, we consider three factors that may affect a firm's decision to innovate or adopt a green technology: the benefits from the technology, the costs of developing, adopting and using this technology, and other firms' decisions. The benefits of a green technology are often the primary incentive for a firm to innovate or adopt it. For example, Steve Percy, the former Chairman and CEO of BP America, Inc., summarized the benefits from a proactive sustainability move by BP as follows: "an enhanced reputation that provided exclusive access through partnerships and relationships to new ideas, natural resources, business opportunities, the best employees, and probably most importantly, a seat at the public policy table," "market share gains from attracting customers with concerns about the environment," and "reduced risks from unforeseen liabilities [primarily regulatory fines]" (Percy 2013). All of these benefits are highly uncertain, although the sources of uncertainty are different: the uncertainty concerning an enhanced reputation and demand gain is primarily due to the market and the new green technology itself, whereas the uncertainty of the reduced risks from regulatory fines stems from the uncertainty of government regulation. Our model captures these benefits, distinguishing between the two different types of benefit uncertainty. The costs of a green technology also consist of two parts: a fixed cost of developing or adopting it, and typically a higher production cost caused by using the new green technology, since a greener product is often more expensive to produce. For example, when BP Amoco promised to reduce the sulfur level in its gasoline in 1999, they estimated that such a reduction would increase production costs by 5 or 6 cents per gallon (Kendall and Grossman 1999). Finally, other firms' decisions play an important role in a firm's decision process. On the one hand, as more firms adopt the new technology, the reputation enhancement and demand gain for a particular firm become smaller. In this case, other firms' adoption decisions discourage a firm from adopting the new technology; firms' actions exhibit *strategic substitutability.* On the other hand, as more firms adopt the new technology, the probability of the government enforcing a stricter standard will increase, yielding a higher incentive to adopt a new technology so as to avoid a higher cost of later adoption. In this case, firms' actions exhibit *strategic complementarity.*

To analyze firms' adoption decisions in equilibrium, taking into account these complex interactions, we utilize the *global game* framework recently developed in economics. This framework is appropriate for addressing two important features of our model: the strategic complementarity among firms' actions and the uncertainty concerning the new green technology's payoff. Because of the strategic complementarity, a firm must take into account other firms' actions when deciding its own action. And, due to uncertainty, firms do not know the payoff of adopting the technology exactly; instead, they observe noisy private signals about the payoff. These noisy private signals imply that a firm cannot predict other firms' actions exactly; the best it can do is to use its own private signal to form a belief on other firms' signals since the other firms are considering the same technology. Since every firm acts based on its belief on other firms' signals, in order to conjecture on other firms' actions a firm must also form a belief on other firms' beliefs (on other firms' signals), a belief on other firms' belief on other firms' beliefs, and so on. When all firms rely on such higher-order beliefs to decide their actions, an equilibrium can be reached in a global game. Crucially, global games differ from typical Bayesian games in which firms' signals are independently distributed: In such Bayesian games one's own signal does not reveal any information about other firms' signals. Thus higher-order beliefs do not play an important role in those games (see, e.g., Morris and Shin 2003 for a more detailed discussion).

Our analysis highlights the importance of taking into account the interplay of industry capability and uncertainty about a new green technology's payoff in a firm's development decision. We find that regulation that considers industry capability, compared with regulation that ignores it, more effectively motivates a firm to develop a new green technology when the first-mover advantage from developing this new technology is small. Therefore, for an industry in which firms can easily catch up with a new technology (thus reducing a firm's first-mover advantage), a government agency may wish to use the regulation scheme that considers industry capability to encourage innovation. Surprisingly, we also find that uncertainty of the payoff can help promote a firm's development of a new green technology when competition is intense and the first-mover advantage is small, or when competition is mild and the first-mover advantage is large. Finally, we find that more stringent regulation (which implies a higher probability of enforcing a stricter standard for a given capability index) encourages more firms to adopt a green technology once the technology becomes available, but may discourage a firm from developing it in the first place when facing intense competition. Therefore, for an industry with intense competition, a government agency should caution against enforcing too stringent regulation that may stifle innovation.

The rest of this paper is organized as follows. In §2 we review the related literature. In §3 we describe our model. In §4 we analyze the equilibrium behaviors of all firms. In §5 we compare our findings in §4 with two benchmarks: the case in which a government agency might enforce a stricter standard with a fixed probability, and the case in which the benefit of the new technology is common knowledge. In §6 we study two extensions of our base model. We conclude our paper in §7. Proofs are presented in the Appendix.

2.2 Related Literature

We first review the literature discussing the impact of government regulation on firms' environmental decisions, and then we discuss the literature on technology adoption under network effects. We finally present the literature related to global games.

Research in the impact of government regulation on firms' environmental decisions has received significant attention recently. The issues studied in this literature include mandatory disclosure (e.g., Kalkanci et al. 2014), allocation of emission responsibilities (e.g., Granot et al. 2014), and financial incentives such as taxes and subsidies (e.g., Tarui and Polasky 2005, Krass et al. 2013, Cohen et al. 2014, and Sunar and Plambeck 2016). Innes and Bial (2002) and Puller (2006) examine how a firm can influence regulation to increase its rivals' compliance costs and thereby gain competitive advantages. Our model incorporates such competitive advantages into the benefit of a new green technology, and furthermore also studies the crucial role of uncertainty in the technology's benefit and effects of regulation.

Our work is particularly related to a stream of research that analyzes the impact of regulatory uncertainty on firms' environmental decisions. Farzin and Kort (2000) study how an uncertain tax rate increase affects firms' adoption of more efficient pollution abatement technologies, Baker and Shittu (2006) study a firm's R&D response to an uncertain carbon tax, Kraft et al. (2013) investigate a monopolist's decision to replace potentially hazardous substances in anticipation of possible regulation, and Kraft and Raz (2015) study firms' replacement decisions for potentially hazardous substances under competition and potential government regulation. Finally, Hoen et al. (2015) study the impact of an uncertain future emission price on a monopolist's investment decision regarding a cleaner technology and production capacity decision. Like these works, our paper also investigates how uncertainty of government regulation affects firms' environmental decisions. However, whereas prior work assumes that the probability of government regulation is fixed, our model incorporates the fact that such a probability often increases with industry capability. We find that the consideration of industry capability in regulation can make a firm more likely to develop a new green technology when the first-mover advantage from the technology is low. In addition, whereas most prior work assumes that the benefits of adopting new technologies are deterministic, we model the uncertainty of these benefits, finding–counterintuitively–that this uncertainty can actually promote development of a green technology.

Our work is also related to the literature on technology adoption under network effects in economics and operations management. Network effects arise when a user's utility increases with the number of other users, as is the case for technology standards. There is extensive literature on various issues related to network effects, including compatibility choices for products (e.g., Katz and Shapiro 1986, Regibeau and Rockett 1996, and references therein), coordination failures (see Farrell and Klemeperer 2007 for a summary of the literature), and price competition (e.g., Argenziano 2008 and references therein). Our consideration of strategic complementarity among firms' decisions in developing or adopting a new technology places us within this framework. However, different from the previous literature, our focus is on the effect of regulatory uncertainty on firms' investment decisions; government regulation is seldom studied in this literature.

The notion of global games was originally defined by Carlsson and van Damme (1993); they refer to global games as games of incomplete information in which players receive noisy private signals about a fundamental of the real world, and decide their actions based on their correlated signals. They solve a two-player global game in which players' actions are strategic complements, showing that uncertainty about the payoffs leads to a unique equilibrium because of players' consideration of higher-order beliefs. Morris and Shin (1998) extend global games to the case of a continuum of players, and Morris and Shin (2005) extend global games to the case in which players' actions are strategic substitutes. Karp et al. (2007) analyze a problem in which players' actions are strategic complements in one region, and strategic substitutes in the other region. Global games have been used to analyze various problems of decentralized coordination among players, including financial crises (Angeletos and Werning 2006), accounting standards comparison (Plantin et al. 2008), network analysis (Argenziano 2008), and business cycles (Schaal and Taschereau-Dumouchel 2014). In operations management, Chen and Tang (2015) apply the related concept of higher-order beliefs to study the economic values of private and public information in farmers' production process. To the best of our knowledge, our paper is among the first papers that apply the theory of global games to sustainable operations. The use of the global game framework enables us to analyze firms' decisions when both strategic complementarity and substitutability are co-present, due to government regulation and firms' competition, respectively.

2.3 Model

We consider a game of incomplete information between a leading firm and multiple other following firms. We assume a continuum of firms indexed by the interval $[0, 1]$. This assumption is common in the literature on global games; it is reasonable in our context in which a government agency's decision is based on consideration of an entire industry consisting of many firms. For example, there are about 40 car brands in the U.S. in 2008 (Marks 2008), and the EPA considered 108 gasoline refineries when proposing the Tier 3 Gasoline Sulfur Standard (EPA 2014).

The game proceeds as follows: In period 1, a leading firm decides whether to develop a new green technology, which enables the firm to reduce a certain pollutant in its product. If the technology is not developed, the game ends. If it is developed, the game proceeds to period 2 in which other firms decide whether to adopt this technology. In period 3, a government agency announces whether to enforce a stricter standard on the pollutant. The enforcement of the stricter standard occurs with a probability which increases with the proportion of firms that have installed this technology, i.e., the industry's capability index. Once the new regulation is enforced, those firms that have installed the technology can meet the stricter standard immediately, while the rest of the firms have to incur extra cost to adopt the technology. The sequence of decisions and events is illustrated in Figure 2-1.

We next present the details of our model in each period: In period 1 the leading firm, denoted as firm 1, has an opportunity to develop a new green technology. Let a_1 denote firm 1's action: $a_1 = 1$ if firm 1 chooses to develop the new technology, or $a_1 = 0$ otherwise. If the firm chooses to develop the new technology (i.e., $a_1 = 1$), then it incurs a fixed cost f_1 (> 0), while also enjoying the first-mover advantage over other firms. The existence of the first-mover advantage is evident in the example of BP mentioned in §1 (Percy 2013): "BP believed that if it were to move proactively on the topic, it would be much more valuable to be the first rather than second, especially in terms of the reputational benefits." Let λ (≥ 0) denote the expected payoff from the first-mover advantage.[1] In addition, firm 1 privately observes a signal about the benefit of the new green technology during period 2, at which time other firms as followers can also adopt the technology. We assume that firm 1's payoff during period 2 is

[1] We focus on uncertainty in the value of the new technology as well as uncertainty in regulation, while abstracting away from uncertainty in various other dimensions such as lead time and development cost. Our model could be extended to include these, but its analysis would be much more complicated.

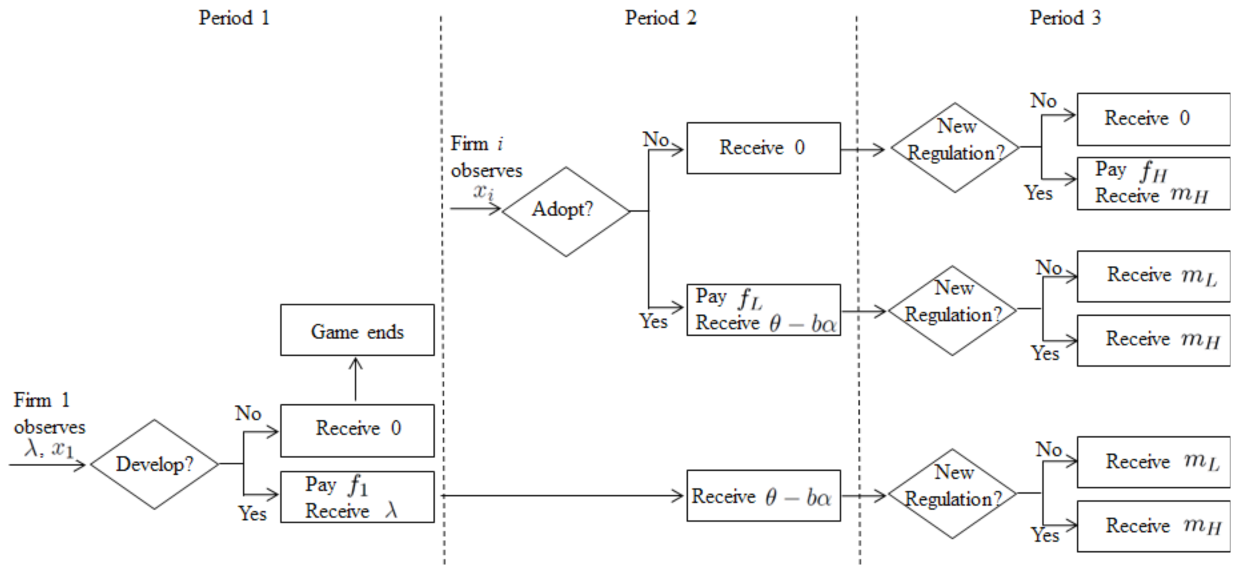

Figure 2-1: Sequence of Decisions and Events

given as $\theta - b\alpha$, where b (≥ 0) captures the competition intensity and α ($\in [0,1]$) represents the proportion of firms that adopt this technology in period 2, i.e., the capability index. The unknown parameter θ, called a "fundamental" of the new technology, represents the maximum payoff firm 1 can get from the new technology if no other firms adopt the technology in period 2. Firm 1 cannot observe θ directly, but instead it observes a noisy private signal $x_1 = \theta + \widetilde{\varepsilon}_1$, where $\widetilde{\varepsilon}_1$ is distributed uniformly on $[-\epsilon, \epsilon]$ with $\epsilon > 0$ (ϵ is common knowledge). We assume that firm 1's prior belief on θ is very noisy such that firm 1 relies fully on its signal x_1 to estimate θ. Such a prior is often called an "improper prior." Note that x_1 as well as θ can be negative, meaning that the technology can cause a loss to firm 1's profit. If firm 1 decides not to develop the new technology (i.e., $a_1 = 0$), the game ends.[2]

In period 2, if firm 1 has developed the technology in period 1 (i.e., $a_1 = 1$), each firm i ($\in [0,1)$) decides whether to adopt the new green technology ($a_i = 1$) or not ($a_i = 0$). If $a_i = 1$, then firm i ($\in [0,1)$) incurs a cost f_L (> 0), while receiving the payoff of $\theta - b\alpha$ during period 2. Like firm 1, firm i ($\in [0,1)$) observes a noisy private signal $x_i = \theta + \widetilde{\varepsilon}_i$, where $\widetilde{\varepsilon}_i$ is uniformly

[2]This implies that a stricter standard will not be enforced if there is no extant green technology to meet the elevated standard. In §6, we consider a case in which a stricter standard may still be enforced even if a new green technology has not been developed. We show that our findings continue to hold in this case.

distributed on $[-\epsilon, \epsilon]$ with $\epsilon > 0$. All $\widetilde{\varepsilon}_i$ are mutually independent as well as independent of $\widetilde{\varepsilon}_1$. If $a_i = 0$, then firm i ($\in [0, 1)$) receives zero payoff during this period.

In period 3, a government agency enforces a stricter standard on the pollutant with a probability α^r, where α is the capability index and r is a positive constant. Since $\alpha \in [0, 1]$, the larger r is, the less likely the new standard is to be enforced. We refer to the probability α^r as the "regulation probability." Firms' payoffs differ depending on whether or not the new regulation is enforced: firms who have installed the new technology, including both firm 1 (who developed the new technology in period 1) and any firm $i \in [0, 1)$ who adopted the new technology in period 2 receive m_H when the new regulation is enforced, and m_L when it is not. We assume that these payoffs are non-positive (i.e., $m_H \leq 0$ and $m_L \leq 0$), since it is typically more costly to reduce the pollutant (see §1). In addition, we assume that these firms receive lower payoffs in the absence of the more stringent standard (i.e., $m_L \leq m_H$) because their production costs would typically be higher than those of their competitors who have not installed the green technology.[3] Such a cost disadvantage to those firms who have installed the green technology does not exist when the new regulation is enforced, because then all firms must install the green technology.[4]

Next we consider the period-3 payoffs of the firms who have not installed the new technology. If the stricter standard is not enforced, then these firms' payoffs are zero. This is consistent with our earlier assumption that a firm's payoff is zero during period 1 or 2 when the green technology is not installed. On the other hand, if the new regulation is enforced, these firms must adopt the new green technology at a cost f_H in order to meet the stricter standard. We assume that the cost of this later adoption is no less than that in period 2 (i.e., $f_H \geq f_L$).[5] For example, as mentioned in §1, after the Tier 3 Gasoline Sulfur Standard became effective, the refineries that could not meet the new standard had to purchase credits to offset their pollution until they became fully compliant (EPA 2014b). Once these firms install the new technology, they will earn the same payoff m_H as other firms who have previously installed the technology.

[3] Our model can be extended to the case in which $m_L \geq m_H \geq 0$, indicating that the production cost using a green technology is lower than that using a conventional technology. In this case the only cost of installing a green technology is a fixed cost, and the problem becomes much simpler than the one we analyze.

[4] In periods 1 and 2, the potential cost disadvantage of a firm who has installed the green technology can be captured in λ and θ, respectively.

[5] We assume $f_H \geq f_L$ for consistency with observations in practice. Relaxing this constraint will not change the intuitions of our results because there is an additional force that incentivizes early adoption: An early adopter will receive a higher payoff in period 3 with new regulation than that without new regulation (i.e., $m_H \geq m_L$).

Table 1 Summary of Notation

Symbol	Definition
a_i	Firm i's action ($\in \{0,1\}$)
π_i	Firm i's total expected payoff
u_i	Firm i's gain from developing or adopting the new technology
λ	Firm 1's expected payoff from the new technology during period 1 (≥ 0)
θ	Firm's maximum payoff from the new technology during period 2 ($\in \mathbb{R}$)
b	Competition intensity (≥ 0)
α	Proportion of firms that adopt the technology (i.e., capability index) ($\in [0,1]$)
r	Regulation probability parameter (> 0)
α^r	Probability that a stricter standard will be enforced (i.e., regulation probability)
x_i	Firm i's private signal about θ
$\widetilde{\varepsilon}_i$	Noise term in x_i which is uniformly distributed on $[-\epsilon, \epsilon]$
f_1	Firm 1's cost for developing the new technology in period 1 (> 0)
f_L	Cost of firm $i\,(\neq 1)$ for adopting the new technology in period 2 (> 0)
f_H	Cost of firm $i\,(\neq 1)$ for adopting the new technology in period 3 ($\geq f_L$)
m_L	Payoff of firms having the new technology installed during period 3 if the stricter standard is not enforced
m_H	Payoff of all firms during period 3 if the stricter standard is enforced ($m_L \leq m_H \leq 0$)

Based on our model described above, we now derive firm i's total expected payoff $\pi_i\left(a_i; \theta, \alpha\right)$. For any given θ and α, if firm i ($\in [0,1)$) adopts the new green technology in period 2 (i.e., $a_i = 1$ in period 2), then its total expected payoff is given as $\pi_i\left(1; \theta, \alpha\right) = -f_L + \theta - b\alpha + \alpha^r m_H + \left(1 - \alpha^r\right) m_L$; and if firm i chooses $a_i = 0$, its total expected payoff is given as $\pi_i\left(0; \theta, \alpha\right) = \alpha^r\left(m_H - f_H\right)$. Thus, the expected gain from adopting the technology, $u_i(\theta, \alpha)$, is

$$u_i(\theta, \alpha) \equiv \pi_i\left(1; \theta, \alpha\right) - \pi_i\left(0; \theta, \alpha\right) = \theta - b\alpha + \alpha^r\left(f_H - m_L\right) - \left(f_L - m_L\right). \qquad (2.1)$$

Since firm i can use its private signal x_i to estimate θ and α, we can also write $u_i(\theta, \alpha)$ as a function of x_i, $u_i\left(x_i\right)$. Firm i will adopt the technology (i.e., $a_i = 1$) if and only if $u_i\left(x_i\right) \geq 0$. Similarly, we can derive firm 1's total expected payoff $\pi_1\left(a_1; \theta, \alpha\right)$, and then write $u_1\left(\theta, \alpha\right)$ as follows:

$$u_1(\theta, \alpha) \equiv \pi_1(1; \theta, \alpha) - \pi_1(0; \theta, \alpha) = \lambda + \theta + m_L - f_1 - b\alpha + \alpha^r (m_H - m_L). \tag{2.2}$$

Firm 1 will develop the new green technology in period 1 if and only if $u_1(\theta, \alpha) = u_1(x_1) \geq 0$. Table 1 summarizes our notation.

2.4 Equilibrium Analysis

In this section we analyze firms' decisions in equilibrium. We derive a perfect Bayesian equilibrium as follows. In §4.1, assuming that the new green technology has been developed, we first analyze the decision of firm i ($\in [0, 1)$) regarding the adoption of the new green technology in period 2. In §4.2, we then analyze the decision of the leading firm 1 regarding whether to develop the new green technology in period 1. For convenience, we use superscripts (1) and (2) to denote periods 1 and 2, respectively.

2.4.1 Period 2: Adoption of the New Green Technology

In this section we examine the conditions under which firm i ($\in [0, 1)$) adopts the new green technology developed by firm 1. We analyze the game among a continuum of firms using the global game framework. The basic idea of a global game (e.g., see Carlsson and van Damme 1993 and Morris and Shin 2003) is as follows: When the fundamental θ is uncertain, firm i uses its private signal $x_i = \theta + \widetilde{\varepsilon}_i$ to form its belief on the fundamental. Since all firms' signals are correlated, firm i also uses its signal to form its belief on every other firm's signal. As every firm forms a belief on other firms' signals, in order to estimate other firms' actions, firm i must also form a belief on other firms' beliefs, a belief on other firms' beliefs on other firms' beliefs, and so on. When all firms rely on such "higher-order" beliefs to decide their actions, an equilibrium can be reached.

In addition to the correlated signals among firms, our setting has the unique feature that there exist both strategic complementarity and strategic substitutability among firms' adoption decisions. Strategic complementarity exists because as more firms adopt the technology (i.e., capability index α increases), the new regulation is more likely to be enforced, and firms that have adopted the technology will earn higher payoff in period 3 under the new regulation than in the case without new regulation (i.e., $m_H \geq m_L$). In addition, later adoption may be more costly (i.e., $f_H \geq f_L$). For firm $i \in [0, 1)$, the magnitude of complementarity is captured in (2.1) by $f_H - m_L$ as the coefficient of α^r. At the same time, strategic substitutability exists because as more firms adopt the technology, the marketing and sales effect of the green technology—through reputation enhancement—will be reduced. This is captured in our model as a firm's payoff in period 2, $\theta - b\alpha$, decreases with α.

We now present firms' adoption decisions in equilibrium, and discuss the factors that affect their decisions.

Lemma 2.1 *There exists threshold $\epsilon^{(2)}$ (≥ 0) such that if $\epsilon \geq \epsilon^{(2)}$, there exists a pure-strategy equilibrium. The threshold $\epsilon^{(2)}$ is nondecreasing with b and nonincreasing with $f_H - m_L$. In this equilibrium, firm i ($\in [0, 1)$) adopts the green technology if and only if $x_i \geq x^{(2)}$, where $x^{(2)} = \frac{1}{2}b + f_L - \frac{rm_L + f_H}{r+1}$.*

Lemma 2.1 provides a sufficient condition for the existence of a pure-strategy equilibrium ($\epsilon \geq \epsilon^{(2)}$). To understand the intuition for this condition, we first discuss the scenario in which there exists no equilibrium. The nonexistence of a pure-strategy equilibrium is common for games in which strategic substitutability is present among more than two players (see, e.g., Vives 2000). Similarly, in our model when the substitutability effect is sufficiently strong, there does not exist a pure-strategy equilibrium. When a firm expects that all other firms would adopt the technology, the firm may be better off not to adopt it under a strong substitutability effect. Likewise, when a firm expects that no other firms would adopt the technology, the firm may be better off to adopt it. As a result, in this case, there exists no pure-strategy equilibrium. This suggests that in order to ensure the existence of a pure-strategy equilibrium, the substitutability effect needs to be moderate.

We can show that the magnitude of the substitutability effect increases with b and decreases with ϵ: When the competition intensity b is large, the substitutability effect is strong because firm i's payoff in period 2 is very sensitive to other firms' decisions. When the level of uncertainty captured by ϵ is large, firm i's signal is a very noisy indicator of other firms' signals. As a result, firm i's belief of other firms' decisions does not change much as firm i's signal changes, and hence it does not have a large impact on firm i's decision. Moreover, the substitutability effect can be mitigated by the complementarity effect, which increases with $f_H - m_L$ as discussed above. Taken together, when the substitutability effect is small with large ϵ or small b, or when the complementarity effect is large with large $f_H - m_L$, there exists a pure-strategy equilibrium. In the rest of the paper, we assume the conditions in Lemma 2.1 is satisfied such that there exists a pure-strategy equilibrium.[6]

When a pure-strategy equilibrium exists, Lemma 2.1 shows that firms will adopt the technology if and only if their privately observed signal about the technology's fundamental is sufficiently large (i.e., $x_i \geq x^{(2)}$). Such a strategy is referred to as a "switching" strategy around $x^{(2)}$ in the literature on global games. From the expression of $x^{(2)}$, we observe that $x^{(2)}$ increases with b, f_L, and r, and that it decreases with m_L and f_H. This can be interpreted as follows. Firms are more likely to adopt the green technology (i.e., $x^{(2)}$ is lower) when: (i) competition among firms is less intense (i.e., smaller b); (ii) the fixed cost of adopting the technology in period 2 is lower (i.e., lower f_L); (iii) the likelihood of the new regulation being enforced in period 3 is higher (i.e., smaller r); (iv) the competitive disadvantage in period 3 from higher marginal cost is lower (i.e., less negative m_L); and (v) the penalty due to late adoption in period 3 is larger (i.e., higher f_H).

2.4.2 Period 1: Development of the New Green Technology

In this section we analyze the leading firm 1's decision regarding whether to develop the new green technology. Similar to other firms' adoption decisions in period 2, we characterize firm 1's decision as a function of its private signal x_1 about the technology's fundamental θ. A higher signal x_1 indicates a higher fundamental θ in expectation, which increases the maximum payoff in period 2. In addition, a higher signal x_1 affects the substitutability effect and the complementarity effect: With a higher fundamental θ, other firms' signals x_i for $i \in [0, 1)$ are more likely to be higher as well. This implies from Lemma 2.1 that more firms can be expected to adopt the technology in period 2 (i.e., the capability index α will increase), decreasing firm 1's payoff in period 2, $\theta - b\alpha$. This substitutability effect creates a negative incentive for firm 1 to develop the new green technology. However, a higher capability index α also creates a positive incentive for firm 1 due to the complementarity effect: With a higher α it is more likely that the new regulation will be enforced in period 3. Under the new regulation, firm 1 will earn a higher payoff m_H, rather than m_L without the new regulation. In (2.2), the complementarity effect is captured by $(m_H - m_L)\,\alpha^r$.

In order to characterize these two effects on firm 1's decision, we first examine the case in

[6] The value of $\epsilon^{(2)}$ is within a reasonable range: for example, from the expression for $\epsilon^{(2)}$ in the Appendix (B.2), it is easy to show $\epsilon^{(2)} \leq b/2$. Moreover, when the effects of substitutability and complementarity are of similar magnitude, $\epsilon^{(2)}$ is generally smaller than $0.2b$. When the effect of complementarity is strong, indicating that government regulation plays an important role, $\epsilon^{(2)}$ can be close to zero.

which only the substitutability effect is present. This effect can be isolated by setting $m_H = m_L$. Next, we study the case in which only the complementarity effect exists by setting $b = 0$. Lastly, by combining both effects, we examine the aggregate effect on firm 1's decision.

The Substitutability Effect

We first consider the case in which only the substitutability effect exists, setting $m_H = m_L$. Firm 1 will develop the new green technology if and only if the expected gain from this technology $u_1(x_1) \geq 0$. Since $u_1(x_1)$ depends on the capability index α, we first derive the expression for α using Lemma 2.1. From Lemma 2.1, any other firm i ($\in [0,1)$) will adopt the technology when it observes its signal x_i ($= \theta + \widetilde{\varepsilon}_i$) higher than $x^{(2)}$. Thus, for any given θ, we can express α as follows:

$$\alpha = \begin{cases} 0 & \text{if } \theta < x^{(2)} - \epsilon; \\ \frac{\theta + \epsilon - x^{(2)}}{2\epsilon} & \text{if } x^{(2)} - \epsilon \leq \theta \leq x^{(2)} + \epsilon; \\ 1 & \text{if } \theta > x^{(2)} + \epsilon. \end{cases} \tag{2.3}$$

Since firm 1's signal is $x_1 = \theta + \widetilde{\varepsilon}_1$, where $\widetilde{\varepsilon}_1$ is uniformly distributed on $[-\epsilon, \epsilon]$, the posterior distribution of the fundamental θ is uniformly distributed on $[x_1 - \epsilon, x_1 + \epsilon]$ for any given x_1. We show in the Appendix that for any given x_1, the posterior distribution of x_i ($= \theta + \widetilde{\varepsilon}_i$) is a symmetric triangular distribution on $[x_1 - 2\epsilon, x_1 + 2\epsilon]$. Using this property, we can derive the following expression for $u_1(x_1)$ from (2.2) and (2.3) (see the proof of Proposition 2.1 for details):

$$u_1(x_1) = \begin{cases} -f_1 + \lambda + m_L + x_1 & \text{if } x_1 \leq x^{(2)} - 2\epsilon; \\ -f_1 + \lambda + m_L + x_1 - \frac{b}{2}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)^2 & \text{if } x^{(2)} - 2\epsilon < x_1 < x^{(2)}; \\ -f_1 + \lambda + m_L + x_1 - \frac{b}{2}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)^2\right\} - b\left(\frac{x_1 - x^{(2)}}{2\epsilon}\right) & \text{if } x^{(2)} \leq x_1 < x^{(2)} + 2\epsilon; \\ -f_1 + \lambda + m_L + x_1 - b & \text{if } x_1 \geq x^{(2)} + 2\epsilon. \end{cases} \tag{2.4}$$

In (2.4), $-f_1 + \lambda$ is the sum of the development cost and the expected payoff from the first-mover advantage during period 1. Since $m_H = m_L$, the expected payoff during period 3 is m_L regardless of whether or not the new regulation will be enforced. Next, x_1 represents expected value of the first term of the payoff $\theta - b\alpha$ during period 2 because the posterior distribution of θ is uniformly distributed on $[x_1 - \epsilon, x_1 + \epsilon]$. The last term in (2.4) is the expected value of $-b\alpha$, capturing the substitutability effect: It is zero if $x_1 \leq x^{(2)} - 2\epsilon$ (because $\alpha = 0$), and it is

$-b$ if $x_1 \geq x^{(2)} + 2\epsilon$ (because $\alpha = 1$); in the two middle intervals of x_1 in (2.4), $\alpha \in (0, 1)$.

We can further examine the impact of a greater signal x_1 on firm 1's incentive by computing $du_1(x_1)/dx_1$ as follows:

$$\frac{du_1(x_1)}{dx_1} = \begin{cases} 1 & \text{if } x_1 \leq x^{(2)} - 2\epsilon; \\ 1 - \frac{b}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right) & \text{if } x^{(2)} - 2\epsilon < x_1 < x^{(2)}; \\ 1 - \frac{b}{2\epsilon}\left(\frac{2\epsilon - x_1 + x^{(2)}}{2\epsilon}\right) & \text{if } x^{(2)} \leq x_1 < x^{(2)} + 2\epsilon; \\ 1 & \text{if } x_1 \geq x^{(2)} + 2\epsilon. \end{cases} \tag{2.5}$$

As we can see from (2.5), if $\epsilon \geq b/2$, then $du_1(x_1)/dx_1 \geq 0$ for every interval of x_1 (as illustrated in Figure 2-2(a)), whereas if $\epsilon < b/2$, then $u_1(x_1)$ first increases, then decreases, and finally increases again with x_1 (as illustrated in Figure 2-2(b)). This means that for large enough uncertainty ($\epsilon \geq b/2$), the expected gain from development for firm 1 is nondecreasing in the expected benefit. Surprisingly, if uncertainty is small ($\epsilon < b/2$), there may be regions in which a larger expected benefit reduces the incentive for firm 1 to develop the technology.

We now explain the intuition behind the shape of $u_1(x_1)$. A higher signal about the fundamental (x_1) has two effects on firm 1's payoff in period 2, $(\theta - b\alpha)$: it increases the expected value of the fundamental ($E[\theta]$), and it creates the substitutability effect by increasing the capability index (α). The marginal effect of a higher signal on the former is constant (i.e., the first term of $du_1(x_1)/dx_1$ in (2.5) is the constant 1), whereas the marginal effect on the latter varies with x_1 (i.e., the second term of $du_1(x_1)/dx_1$ in (2.5), if it exists, is a function of x_1). This latter effect depends on how the expected value of the capability index α changes with x_1, which in turn depends on the relative value of x_1 compared to $x^{(2)}$ as shown in (2.5). This is because the proportion of other firms $i \in [0, 1)$ who will adopt the technology in period 2 (i.e., capability index α) depends on firm 1's signal x_1, as the posterior distribution of x_i is a symmetric triangular distribution on $[x_1 - 2\epsilon, x_1 + 2\epsilon]$, as mentioned above.

Specifically, (2.5) illustrates the following three cases. First, when firm 1's signal is very low (i.e., $x_1 < x^{(2)} - 2\epsilon$), firm 1 believes that other firms will not observe signals higher than the threshold $x^{(2)}$, and thus $\alpha = 0$ by Lemma 1. In this case, there is no substitutability effect, and firm 1's expected gain from the technology, $u_1(x_1)$, increases with x_1. Second, when firm 1's signal is close to other firms' adoption threshold $x^{(2)}$ (i.e., $x^{(2)} - 2\epsilon \leq x_1 \leq x^{(2)}$ or $x^{(2)} \leq x_1 \leq x^{(2)} + 2\epsilon$), firm 1 believes that some firms will observe signals higher than

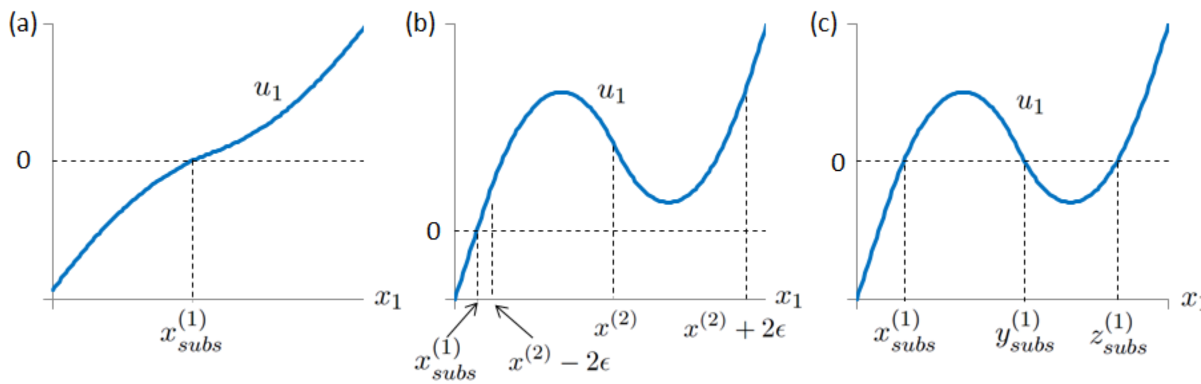

Figure 2-2: Firm 1's Expected Gain from Developing the Technology when: (a) $\epsilon > b/2$, (b) $\epsilon < b/2$ and $\lambda > \lambda_H$, and (c) $\epsilon < b/2$ and $\lambda_L < \lambda < \lambda_H$.

the threshold $x^{(2)}$, and thus $\alpha > 0$. In this case an increase of x_1 is likely to increase α by causing more firms to observe signals larger than $x^{(2)}$. The marginal effect of a higher signal on substitutability is particularly large when x_1 is close to $x^{(2)}$ and the density of x_i is less diffuse (which happens when the level of uncertainty captured by ϵ is lower) due to larger probability mass centered around $x^{(2)}$ in the triangular distribution of x_i. As a result, when $\epsilon < b/2$ and x_1 is sufficiently close to the threshold $x^{(2)}$, the marginal effect of a higher signal on substitutability is dominant, and the expected gain $u_1(x_1)$ *decreases* with x_1. Lastly, when firm 1's signal is very high (i.e., $x_1 > x^{(2)} + 2\epsilon$), firm 1 believes that other firms will also observe high signals, and hence all firms will adopt the technology, resulting in $\alpha = 1$. In this case, a higher x_1 will not change α, so $u_1(x_1)$ again increases with x_1.

This property of $u_1(x_1)$ leads to the following proposition that characterizes firm 1's equilibrium decision.

Proposition 2.1 *In the case of $m_H = m_L$, the following results hold in equilibrium:*
(a) If $\epsilon \geq \frac{b}{2}$, then There exists threshold $x^{(1)}_{subs}$ such that firm 1 develops the green technology if and only if $x_1 \geq x^{(1)}_{subs}$. If $\epsilon < \frac{b}{2}$, then there exist real numbers $\underline{\lambda}_{subs}$, $\overline{\lambda}_{subs}$, $x^{(1)}_{subs}$, $y^{(1)}_{subs}$, and $z^{(1)}_{subs}$ (where $\underline{\lambda}_{subs} \leq \overline{\lambda}_{subs}$ and $x^{(1)}_{subs} \leq y^{(1)}_{subs} \leq z^{(1)}_{subs}$) such that: (i) when $\lambda < \underline{\lambda}_{subs}$ or $\lambda > \overline{\lambda}_{subs}$, firm 1 develops the green technology if and only if $x_1 \geq x^{(1)}_{subs}$; and (ii) when $\underline{\lambda}_{subs} \leq \lambda \leq \overline{\lambda}_{subs}$, firm 1 develops the green technology if and only if $x_1 \in [x^{(1)}_{subs}, y^{(1)}_{subs}] \cup [z^{(1)}_{subs}, \infty)$.
(b) The thresholds $x^{(1)}_{subs}$ and $z^{(1)}_{subs}$ are nonincreasing with r, and the threshold $y^{(1)}_{subs}$ is nonde-

creasing with r.

Proposition 2.1(a) makes explicit how uncertainty about the fundamental of the technology (θ) affects firm 1's strategy. When the fundamental θ of the new technology is highly uncertain (i.e., $\epsilon \geq b/2$), the firm's expected gain is increasing with its signal about the fundamental (x_1). Therefore, when firm 1 observes a sufficiently high signal, it will undertake the development of the new green technology. This equilibrium strategy takes the same form (a switching strategy) as that of the following firms' adoption decisions (See Lemma 2.1).

By contrast, when the level of uncertainty about the fundamental θ is moderate (i.e., $\epsilon < b/2$), firm 1's equilibrium strategy can take two different forms depending on the magnitude of the first mover advantage in period 1 (λ). Recall from (2.4) that the expected payoff $u_1(x_1)$ increases with λ. When the first-mover advantage is very large ($\lambda > \overline{\lambda}_{subs}$), firm 1's strategy is again a switching strategy because $u_1(x_1)$ crosses zero only once as illustrated in Figure 2-2(b). The intuition for the case when λ is very small ($\lambda < \underline{\lambda}_{subs}$) is similar. However, when the first-mover advantage is moderate ($\underline{\lambda}_{subs} \leq \lambda \leq \overline{\lambda}_{subs}$), firm 1 develops the technology in equilibrium when it observes a moderate signal between $x^{(1)}_{subs}$ and $y^{(1)}_{subs}$ or a sufficiently high signal above $z^{(1)}_{subs}$. In this case, as illustrated in Figure 2-2(c), $u_1(x_1)$ crosses zero three times due to the substitutability effect. This illustrates our counterintuitive result that a higher signal on the technology's fundamental may not necessarily lead firm 1 to develop the technology.

Proposition 2.1(b) characterizes the impact of r on firm 1's incentive to develop the new green technology. Recall that the smaller r is, the more likely the new standard is to be enforced. The result that $x^{(1)}_{subs}$ and $z^{(1)}_{subs}$ are nonincreasing in r and $y^{(1)}_{subs}$ is nondecreasing in r implies that a larger chance of a stricter standard being enforced discourages *development* of the new green technology. Interestingly, this result is contrary to Lemma 2.1 which shows that a greater chance of the stricter standard being enforced encourages other firms to *adopt* the green technology later. With a smaller r, the regulation probability increases faster with α, and the substitutability effect, which creates a negative incentive for firm 1, becomes more pronounced.

The Complementarity Effect

Next we consider the case in which only the complementarity effect exists by setting $b = 0$, while $m_H > m_L$. Following the same procedure as in the first case, we can obtain $du_1(x_1)/dx_1$

as follows:

$$\frac{du_1(x_1)}{dx_1} = \begin{cases} 1 & \text{if } x_1 < x^{(2)} - 2\epsilon; \\ 1 + \frac{(m_H - m_L)}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)^r & \text{if } x^{(2)} - 2\epsilon \leq x_1 < x^{(2)}; \\ 1 + \frac{(m_H - m_L)}{2\epsilon}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)^r\right\} & \text{if } x^{(2)} \leq x_1 \leq x^{(2)} + 2\epsilon; \\ 1 & \text{if } x_1 > x^{(2)} + 2\epsilon. \end{cases} \tag{2.6}$$

As in the first case, if $x_1 < x^{(2)} - 2\epsilon$ or $x_1 > x^{(2)} + 2\epsilon$, then $du_1(x_1)/dx_1 = 1$ because the capability index $\alpha = 0$ or $\alpha = 1$, respectively. In the two middle intervals of x_1 in (2.6), the second term of $du_1(x_1)/dx_1$ captures the marginal effect of a higher signal on complementarity, which decreases with ϵ. The intuition is similar to that in §4.2.1. Since $m_H > m_L$, it is easy to see that $du_1(x_1)/dx_1 > 0$ for any x_1. This means that as firm 1 observes a higher signal x_1, it anticipates that more firms will adopt the technology. This in turn will increase the likelihood of the new regulation being enforced under which firm 1 will have a higher payoff m_H than m_L under the current regulation. As a result, in equilibrium, firm 1 chooses a switching strategy around a threshold $x^{(1)}_{comp}$ as stated in the following proposition.

Proposition 2.2 *In the case of $b = 0$, the following results hold in equilibrium:*
(a) There exists threshold $x^{(1)}_{comp}$ such that firm 1 develops the green technology if and only if $x_1 \geq x^{(1)}_{comp}$.
(b) The threshold $x^{(1)}_{comp}$ is nondecreasing with r.

Contrary to Proposition 2.1(a), Proposition 2.2(a) shows that there exists a single threshold $x^{(1)}_{comp}$ that determines firm 1's strategy in equilibrium. Moreover, Proposition 2.2(b) suggests that the impact of r on the firm's incentive to develop the new green technology is opposite to that in Proposition 2.1(b): When the complementarity effect exists, a larger chance of the stricter standard being enforced incentivizes firm 1 to develop the new green technology, whereas it discourages firm 1 from doing so in the presence of the substitutability effect.

The Aggregate Effect

By combining the results stated in Propositions 2.1 and 2.2, we finally derive the following equilibrium for the general case in which both substitutability and complementarity effects are present (i.e., $b \geq 0$ and $m_H \geq m_L$).

Proposition 2.3 *(a) Proposition 2.1(a) continues to hold (with thresholds* $\underline{\lambda}_{aggr}$, $\overline{\lambda}_{aggr}$, $x^{(1)}_{aggr}$, $y^{(1)}_{aggr}$, *and* $z^{(1)}_{aggr}$ *replacing* $\underline{\lambda}_{subs}$, $\overline{\lambda}_{subs}$, $x^{(1)}_{subs}$, $y^{(1)}_{subs}$, *and* $z^{(1)}_{subs}$, *respectively) except that the condition* $\epsilon \geq \frac{b}{2}$ *is replaced with* $\epsilon \geq \epsilon^{(1)}$ *where* $\epsilon^{(1)}$ *is a real number lower than* $\frac{b}{2}$, *and* $\epsilon^{(1)}$ *is nondecreasing with b and nonincreasing with* $m_H - m_L$.
(b) There exists $b^{(x)}$ *(resp.,* $b^{(z)}$*) such that the thresholds* $x^{(1)}_{aggr}$ *(resp.,* $z^{(1)}_{aggr}$*) is nonincreasing with r if and only if* $b > b^{(x)}$ *(resp.,* $b > b^{(z)}$*). There exists* $b^{(y)}$ *such that the thresholds* $y^{(1)}_{aggr}$ *is nondecreasing with r if and only if* $b > b^{(y)}$.

When both complementarity and substitutability are present, a larger signal x_1 affects the expected value of the fundamental, the complementarity effect, and the substitutability effect. The marginal effect of a higher signal on the fundamental does not depend on the level of uncertainty, whereas its effect on substitutability and complementarity decreases with the level of uncertainty (see our discussion in §4.2.1 and §4.2.2). When the level of uncertainty is large, the former effect dominates the latter two, and the expected gain $u_1(x_1)$ increases with the signal x_1. In this case, firm 1's strategy is a switching strategy.

But when the level of uncertainty is small, the marginal effects of a higher signal on substitutability and complementarity are large. In this case, firm 1's signal is a very informative indicator of other firms' signals, and firm 1's belief on other firms' decisions changes significantly as firm 1's signal changes. In this situation, there are two cases to consider: First, when the marginal effect of a higher signal on complementarity dominates that on substitutability (which happens when $m_H - m_L$ is large and r is small), the expected gain $u_1(x_1)$ always increases with the signal x_1, resulting in $\epsilon^{(2)} = 0$. In this case, firm 1's strategy is again a switching strategy (similar to Proposition 2.2(a)). Second, when the marginal effect of a higher signal on substitutability dominates that on complementarity, the negative impact of substitutability can make firm 1's expected gain $u_1(x_1)$ change non-monotonically with x_1. In this case, similar to Proposition 2.1(a) (when complemenatrity was assumed to be zero), firm 1 may develop the technology if it observes a signal in two separate regions. However, the threshold $\epsilon^{(1)}$ is smaller than the threshold $b/2$ in Proposition 2.1(a) because the negative impact of substitutability is mitigated by the positive impact of complementarity in the aggregate model.

Proposition 2.3(b) combines the results of Proposition 2.1(b) and Proposition 2.2(b). As discussed earlier, a greater chance of the stricter standard being enforced (i.e., a lower r) discourages firm 1 from developing the new green technology under substitutability, whereas it encourages the firm to do so under complementarity. When both effects are present, Proposition 2.3(b) shows that when the competition intensity b is sufficiently large, the former substitutability effect outweighs the latter complementarity effect. In this case, the innovation of a new green technology can be encouraged if the government agency can make firms believe that the probability of regulation is small (r is large).

2.5 Comparisons with Two Benchmarks

In this section we compare our results derived in §4 under uncertain payoffs and regulation based on a capability index with two benchmarks: the case in which a stricter standard is enforced with a fixed probability, and the case in which the fundamental of the new green technology is common knowledge to every firm.

2.5.1 Comparison with Regulation Independent of a Capability Index

To study the impact of government consideration of industry capability on firms' decisions, we compare regulation that considers industry capability with regulation that ignores it. To model regulation that ignores industry capability, we assume the government agency enforces a stricter standard with a fixed probability p ($\in (0,1)$). In this case, there is a possibility that the stricter standard will be enforced even if no firms adopt the technology in period 2. All other assumptions remain the same as in the base model. Following a procedure similar to that in §4, we can characterize the equilibrium in this case as follows:

Lemma 2.2 *Suppose that the regulation probability is p. Then there exists a pure-strategy equilibrium if $\epsilon \geq b/2$. In equilibrium the following results hold:*
(a) Firm i ($\in [0,1)$) adopts the green technology in period 2 if and only if $x_i \geq \widehat{x}^{(2)}$, where $\widehat{x}^{(2)} = \frac{1}{2}b + f_L - (1-p)\, m_L - pf_H$.
(b) There exists threshold $\widehat{x}^{(1)}$ such that firm 1 develops the green technology if and only if $x_1 \geq \widehat{x}^{(1)}$.

Lemma 2.2(a) shows that firms' equilibrium strategies in period 2 take a similar form to those under regulation which considers industry capability, although the value of the threshold is different from that of the corresponding threshold in our base model. Also, in both cases, sufficiently large levels of uncertainty are required for the existence of an equilibrium, because the impact of strategic substitutability needs to be sufficiently small to ensure the existence of an equilibrium. We know from §4.1 that for our base model the impact of substitutability decreases with ϵ and it can be mitigated by strategic complementarity. But for the fixed probability model, due to the absence of complementarity under regulation ignoring industry capability, the threshold ($b/2$) for uncertainty is larger than that ($\epsilon^{(2)}$) under regulation which considers industry capability.

Lemma 2.2(b) shows that firm 1's equilibrium strategy is a switching strategy for the fixed probability model. Recall from part (a) that when $\epsilon \geq b/2$, a pure-strategy equilibrium exists in period 2. In this case, firm 1's expected gain always increases with the signal x_1. The intuition is similar to that of Proposition 2.3(a) when $\epsilon \geq \epsilon^{(1)}$.

We next compare a firm's incentive to develop the new green technology under regulation which considers industry capability with that under regulation which ignores industry capability. To this end, we compare the threshold $\widehat{x}^{(1)}$ in Lemma 2.2 with $x^{(1)}_{aggr}$ in Proposition 2.3, assuming that both regulation settings are equally good at motiving firms to adopt the new technology in period 2: Because the thresholds $\widehat{x}^{(2)}$ and $x^{(2)}$ in period 2 are functions of p and r, respectively, we choose values of p and r such that the two thresholds are the same. Given that both regulation schemes are equally effective in period 2, the following proposition establishes a condition under which one is more effective in period 1 than the other.

Proposition 2.4 *There exists threshold $\widehat{\lambda}$ such that $x^{(1)}_{aggr} \leq \widehat{x}^{(1)}$ if and only if $\lambda \leq \widehat{\lambda}$.*

Proposition 2.4 shows that when the first-mover advantage λ is small, regulation that considers industry capability is more effective in incentivizing firm 1 to develop the new green technology. When the first-mover advantage λ is small, firm 1 needs larger payoffs in periods 2 and 3 to earn positive expected gain in total. When firm 1 observes a large signal x_1, other firms are also likely to observe large signals. As a result, the capability index α is likely to be high, cutting firm 1's payoff in period 2. But under regulation which considers industry capability, the probability of regulation is also likely to be high because it increases with the capability index. In this case there is a high probability that firm 1's cost disadvantage in period 3 will be eliminated due to mandatory adoption. Therefore, regulation that considers industry capability works better when the first-mover advantage λ is small.

Proposition 2.4 bears important policy implications. A government agency's consideration of industry capability can effectively motivate a firm to develop a green technology only when the first-mover advantage is small. Such first-mover advantage is typically small, for example, if other firms can catch up with the technology quickly. For example, as mentioned in §1, BP

Amoco announced that it would lower the sulfur level in its gasoline in 40 cities in 1999. A few months later, Koch Petroleum followed BP by announcing that they would also sell gasoline with lower sulfur levels (Koch 1999). In this case the first-mover advantage was small, so our result suggests that regulation based on industry capability may work better for motivating green technology development in this case. A few months later, Koch Petroleum followed BP by announcing that they would also sell gasoline with lower sulfur level.

2.5.2 Comparison with the Case of Complete Information

Suppose the fundamental of the new green technology (θ) is common knowledge to every firm. In this case of complete information, we can show, similarly to Lemma 1, that if $b > f_H - m_L$ there exists no pure-strategy Nash equilibrium in period 2 for $\theta \in (f_L - m_L, b - f_H + f_L)$. In the remainder of this section, we thus focus on the case in which $b \leq f_H - m_L$.[7]

Lemma 2.3 *When θ is common knowledge to every firm, the following results hold in equilibrium:*
(a) In period 2, every firm adopts the new technology if and only if $\theta \geq \theta^{(2)} = f_L + \min\{-m_L, b - m_H\}$.
(b) If $b < m_H - m_L$ or $\lambda \in (-\infty, f_1 - f_L] \cup [f_1 - f_L + b - (m_H - m_L), \infty)$, then there exists $\theta^{(1)}$ such that firm 1 develops the green technology if and only if $\theta \geq \theta^{(1)}$. Otherwise, there exist $\theta^{(0)}$ and $\theta^{(1)}$ (where $\theta^{(1)} \leq \theta^{(2)} \leq \theta^{(0)}$) such that firm 1 develops the green technology if and only $\theta \in \left[\theta^{(1)}, \theta^{(2)}\right] \cup [\theta^{(0)}, \infty)$.

Having characterized the equilibrium under complete information, we now compare it with that of our base model under incomplete information. For brevity, we focus on comparing the threshold $\theta^{(1)}$ in Lemma 2.3 with $x^{(1)}_{aggr}$ in Proposition 2.3. The comparison between other thresholds can be done similarly.

Proposition 2.5 *The threshold for x_1 in the incomplete information case is strictly smaller than that in the complete information case (i.e., $x^{(1)}_{aggr} < \theta^{(1)}$) if $r < \max\{b/(m_H - m_L), 1\}$ and one of the following conditions is satisfied:*
(i) $b < m_H - m_L$ and $f_1 - f_L - b + m_H - m_L \leq \lambda < f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} + 2\epsilon$;

[7] We show in the proof of Lemma 2.3(a) that when $\theta \in [b - f_H + f_L, f_L - m_L]$, two pure-strategy symmetric equilibria exist. In this case, we follow the convention of the literature (e.g., Katz and Shapiro 1986) that firms choose the equilibrium that maximizes their payoffs.

(ii) $b > m_H - m_L$ *and* $f_1 - f_L + \frac{b}{2} - m_H + m_L + \frac{f_H - m_L}{r+1} - 2\epsilon < \lambda \leq f_1 - f_L$*;*

(iii) $b = m_H - m_L$ *and* $f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} - 2\epsilon < \lambda < f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} + 2\epsilon$*.*

Proposition 2.5 suggests that uncertainty surrounding the fundamental of the new green technology can incentivize firm 1 to develop the new green technology (i.e., $x_{aggr}^{(1)} < \theta^{(1)}$). This occurs under condition (i), (ii), or (iii) with sufficiently small r. Note that under complete information, a capability index is either 0 or 1; every firm observes the same fundamental θ, and hence follows the same strategy in period 2. By contrast, under incomplete information, a capability index can be any value between 0 and 1 because firms observe different signals about θ. This uncertainty can encourage firm 1 to develop the new technology.

To gain deeper understanding of Proposition 2.5, we examine firm 1's expected gain when it observes a signal $x_1 = \theta^{(1)}$ in our base model: If firm 1's expected gain is positive for the signal $x_1 = \theta^{(1)}$, then firm 1 develops the technology under incomplete information, whereas firm 1 is indifferent between developing and not developing the technology under complete information. In this case, uncertainty encourages firm 1 to develop the technology (i.e., $x_{aggr}^{(1)} < \theta^{(1)}$). Firm 1's expected gain depends on a capability index, which drives the substitutability and complementarity effects. When the capability index is 0, both effects are 0. When the capability index is 1, the substitutability effect can be smaller than, greater than, or equal to the complementarity effect. Those three scenarios correspond to conditions (i), (ii), and (iii) in Proposition 2.5, respectively.

Under the condition on λ in (i), when firm 1 observes $\theta = \theta^{(1)}$ under complete information, it expects that no firms will adopt the technology due to low benefits (hence, $\alpha = 0$). However, when it observes a signal $x_1 = \theta^{(1)}$ under incomplete information, it expects that some firms may observe sufficiently high signals and adopt the technology (hence, $\alpha > 0$). Under the condition $b < m_H - m_L$ in (i) and the condition $r < \max\{b/(m_H - m_L), 1\}$, the substitutability effect is smaller than the complementarity effect for any positive α. As a result, firm 1 expects higher expected gain under incomplete information because of a possibly positive capability index. This higher expected gain caused by uncertainty encourages firm 1 to develop the technology.

The intuition for condition (ii) is similar. Under the condition on λ in (ii), when firm 1 observes $\theta = \theta^{(1)}$ under complete information, it expects that all firms will adopt the technology (hence, $\alpha = 1$); but when it observes a signal $x_1 = \theta^{(1)}$ under incomplete information, it expects that some firms will observe sufficiently low signals, and they will not adopt the technology (hence, $\alpha < 1$). Under the condition $b > m_H - m_L$ in (ii) and the condition $r < \max\{b/(m_H - m_L), 1\}$, the substitutability effect is larger than the complementarity effect for high α, and the difference between the two effects is the largest at $\alpha = 1$ (which is the case under complete information). Since $\alpha < 1$ under incomplete information, firm 1 expects higher expected gain in this case.

The intuition for condition (iii) is slightly different. Under the condition on λ in (iii), firm 1 expects $0 < \alpha < 1$ when it observes a signal $x_1 = \theta^{(1)}$ under incomplete information. Under the condition $b = m_H - m_L$ in (iii) and the condition $r < \max\{b/(m_H - m_L), 1\}$, the substitutability effect is smaller than the complementarity effect for any $\alpha \in (0, 1)$ (which is the case under incomplete information), and these two effects are equal for $\alpha = 0$ or $\alpha = 1$ (which is the case under complete information). So firm 1 expects higher expected gain under incomplete information.

In summary, uncertainty is likely to encourage firm 1's development decision if the complementarity effect is strong and firm 1 observes low payoff signals for period 2 (conditions (i) and (iii)), or if the substitutability effect is strong and firm 1 observes high payoff signals (condition (ii)). Furthermore, in all the three conditions the ranges of λ become larger as ϵ becomes larger, indicating that a larger level of uncertainty is more likely to encourage the development of a green technology in these scenarios.

2.6 Extensions

This section examines two extensions of our base model. In §6.1, we consider a case in which a government agency might enforce a new regulation even if there is no technology available to meet the standard. In §6.2, we consider a case in which firm 1 licenses the technology to other firms.

2.6.1 Positive Regulation Probability in the Absence of Industry Capability

Our base model presented in §3 assumes that the regulation probability is α^r. This means that if firm 1 does not develop the new green technology, no firms will adopt the technology (hence $\alpha = 0$), and consequently the government agency will not enforce the new regulation. This captures the government's common reluctance to significantly disrupt industry.

However, there may be urgent environmental and health situations (such as forbidding toxic additives in food or replacing carcinogenic dyes in clothes) for which a government agency may not be willing to wait until a leading firm in an industry develops a new green technology (although the presence of such a technology is likely to encourage regulation). In such settings we can model the regulation probability as $p_0 + (1 - p_0)\,\alpha^r$ where p_0 ($\in [0, 1)$) represents the probability the government agency will enforce a stricter standard even when a new green technology has not been developed. With this change in the regulation probability, we can revise equations (2.1) and (2.2) as follows:

$$u_i(\theta, \alpha) = \theta - b\alpha + \alpha^r\,(1 - p_0)\,(f_H - m_L) + (1 - p_0)\,m_L - f_L + p_0 f_H; \tag{2.7}$$

$$u_1(\theta, \alpha) = \lambda + \theta + (1 - p_0)\,m_L - f_1 - b\alpha + \alpha^r\,(1 - p_0)\,(m_H - m_L)\,. \tag{2.8}$$

It is easy to see that (2.7) and (2.8) have the same functional forms as (2.1) and (2.2), respectively. As a result, although the specific values of thresholds now depend on the new parameter p_0, all the qualitative insights obtained in §4 and §5 continue to hold.

2.6.2 Licensing of the Technology

Our base model presented in §3 assumes that firm 1 does not receive additional revenue from licensing its green technology to other firms. However, if the technology were patented, it is plausible that firm 1 would receive additional revenue from licensing, proportional to the number of firms who adopt the technology. Thus, this can be modeled by adding $s\alpha$ to firm 1's revenue, where s is a positive constant, representing the maximum expected revenue from possible licensing fees. In this case, firm i's adoption cost f_L or f_H includes the possibility of paying a fee to firm 1. With this additional possibility, we can revise equation (2.2) as follows:

$$u_1(\theta, \alpha) = \lambda + \theta + m_L - f_1 - (b - s)\,\alpha + \alpha^r\,(m_H - m_L)\,. \tag{2.9}$$

It is easy to see that (2.9) has the same functional form as (2.2). Thus, although the specific values of thresholds depend on the new parameter s, our qualitative insights continue to hold.

2.7 Conclusion

A government agency's potential regulatory action is an important driving force for firms to develop and adopt a new green technology. Existing research assumes that a government agency's action is independent of industry capability, and that the benefit of the new technology is known. In practice, however, a government agency often takes into account industry capability, and firms face uncertainty in the technology's benefits. In this case firms' decisions exhibit both strategic substitutability (because the marketing benefit of a new green technology decreases as more firms adopt it) and complementarity (because the stricter standard is more likely to be enforced as more firms adopt it). We develop a novel model that captures these realistic features, and examines how they affect firms' development and adoption decisions.

Our analysis shows that regulation that considers an industry's capability index–compared with regulation that ignores it–can more effectively motivate development of a new green technology when the first-mover advantage from the technology is low. Therefore, for an industry in which firms can easily catch up with a new technology (thus reducing a firm's first-mover advantage), regulation that considers industry capability should be considered to encourage innovation. Our analysis further shows that in a setting in which industry capability affects regulation, uncertainty concerning the new technology's benefits can help motivate a firm to develop the new technology, because this uncertainty might soften competition or increase the probability of regulation. Finally, our findings bear important policy implications about the stringency of regulation (the probability of enforcing a stricter standard for a given capability index): More stringent regulation encourages more firms to adopt a green technology once it is invented, but may discourage a firm from developing it if the competition intensity is high. Therefore, for an industry with intense competition, a government agency may wish to enforce regulation with mild stringency to encourage innovation.

Chapter 3

Horizontal Mergers in Vertically-Differentiated Markets

3.1 Introduction

Mergers and acquisitions are a very important part in today's business world. Many companies expect to increase competitiveness through M&As and regard M&As as core strategies. As a result, mergers are extremely common in practice – 20,409 M&As occurred in 2013, worth 1.9 trillion dollars (WilmerHale 2014). Our focus in this paper is on studying the effects of a horizontal merger.

Despite their popularity among firms, mergers often raise concerns for antitrust agencies. Antitrust agencies are worried that a merger may reduce competition in a market and harm consumer welfare. Specifically, after a merger, reduced competition may raise prices, reduce qualities and varieties of products in that market. For example, in Horizontal Merger Guidelines, the U.S. Department of Justice and the Federal Trade Commission state the following: "Enhanced market power can also be manifested in non-price terms and conditions that adversely affect customers, including reduced product quality, reduced product variety...."

Indeed, merger-induced quality and variety changes are widely observed in practice. For example, Song (2015) shows that the quality level of Compaq's PC product line improved at a much slower speed than its competitors after the merger of HP and Compaq; i.e., the merged firm repositioned Compaq's product line to a lower quality level. Another example is about the merger of two CPU manufacturers. In late 1990s, VIA Technology, a Taiwanese manufacturer of integrated circuits, bought Centaur Technology and Cyrix, two low-end CPU manufacturers that competed with Intel and AMD. After the acquisitions, VIA only sold products based on designs from Centaur Technology.

Although it is quite common for mergers to cause quality and variety changes, there has been little research on evaluating their impacts on consumers. Most existing merger literature regards prices as the indicator of consumer welfare, and focuses solely on price changes. We aim to bridge this gap by studying firms' optimal product repositioning decisions after a merger, and analyzing how such decisions would affect consumer welfare.

Specifically, we study a merger of two firms in a vertically-differentiated market. In the market all customers prefer higher qualities, but their willingness to pay for higher qualities is different. Before the merger, each firm offers a single product with different quality. After the merger, all firm can change their qualities. Besides, the post-merge firm (the firm created by the merger of two firms) can either continue to offer two products to cover more customers, or offer a single product to achieve cost reduction through economies of scale.

Our analysis highlights the importance of considering firms' product repositioning when evaluating the impacts of a merger. As discussed above, most existing merger literature uses prices as the single indicator of whether a merger will harm consumers (e.g., Williamson 1968 and Whinston 2007). However, we find that a merger may decrease consumer welfare even if it induces the post-merger firm to reduce prices. This is because the post-merger firm reduces its product qualities to achieve lower costs. Although such lower costs reduce prices, the quality reduction harms consumer welfare and its negative impact outweigh the positive impact of lower prices. This finding is consistent with the empirical study in Song (2015). He shows that the price drop of Compaq products after the merger of HP and Compaq was primarily caused by quality reduction of Compaq products. In addition, it is a conventional wisdom that cost reduction generated by mergers can always benefit consumers. This result has been proven by many economists (e.g., see a comprehensive review by Whinston 2007), and has been regarded as the standard result in the theory of mergers. Indeed, many firms justify their proposed mergers by claiming that they can pass on cost savings to consumers. However, we find that cost reduction for the post-merge firm may not always benefit consumers when firms quality and variety decisions are considered. Although cost reduction can reduce the post-merger firm's price, it may also change the post-merger firm's decision from offering two products to offering a single product. Such product variety reduction can be harmful to consumers if the cost reduction is not sufficiently high.

The rest of this paper is organized as follows. In §2 we review the related literature. In §3 we describe our pre-merger model. In §4 we describe the post-merger model and analyze the equilibrium in the post-merger market. We conclude our paper in §5. Proofs are presented in the Appendix.

3.2 Literature Review

In this section, we first review economic literature on mergers, then review literature on vertical differentiation and product variety.

Mergers have always been a hot research topic for antitrust agencies and economists. The main focus is on whether a merger will increase consumer price and harm consumers. Stigler (1950) uses a Cournot model to analyze the formation of a cartel and shows that prices will increase due to such formation. Starting from Williamson (1968), economists have taken into account cost synergies generated mergers that may lower prices, and focus on the trade-off between reduced competition and lower costs (e.g., see a comprehensive review by Whinston 2007). Different from these papers based on Cournot competition, Deneckere and Davidson (1985) analyze a merger using a Bertrand competition model. The price competition among firms is more suitable for the analysis of retailer mergers because retailers are often price setters and the nature of their competition is very different from quantity competition in a Cournot model. For this reason, many researchers analyze mergers based on the model in Deneckere and Davidson (1985), such as Werden and Froeb (1994) and Davidson and Ferrett (2007). In these papers, prices are the only indicator of whether consumer welfare is harmed by mergers, whereas in our model, consumer welfare is affected by prices, qualities, and the number of products. We show that prices alone are not sufficient to determine the impact of a merger. It is possible that consumer welfare is harmed even if the post-merger firm reduces its prices after a merger.

There has been little research on product repositioning after a merger. The very few papers are about product repositioning in horizontally-differentiated markets based on the Hotelling model. Gandhi et al. (2008) study mergers between firms competing by choosing price and location analytically. They show that location repositioning greatly mitigates the anticompetitive effects of the merger. Berry and Walfogel (2001), Sweeting (2010), and Sweeting (2013) analyze mergers in the radio industry empirically and find evidence of product repositioning. These papers focus on mergers in markets where products are horizontally-differentiated. In such markets, consumers' preference differs based on their travel costs to different locations. So at the same prices, some consumers may prefer one product, while others may prefer another. Our paper is fundamentally different from these papers because we study mergers in markets where products are vertically-differentiated with different qualities. In our model, all consumers prefer products with higher qualities if the prices are the same. There is very little merger research in this area. We only find one empirical paper by Song (2015) which analyzes the merger of HP and Compaq in the consumer PC market. Song (2015) shows that the PC market is clearly vertically-differentitated, with HP in the middle-end and Compaq in the lower-middle end. However, due to lack of analytical research, Song (2015) uses Gandhi et al. (2008), which is based on horizontal differentiation, as his analytical benchmark. Our paper bridge the gap between merger literature and vertical differentiation literature and complements

Our model is based on vertical differentiation. The model of vertically differentiated model was first developed by Mussa and Rosen (1978). A number of papers extended this model (e.g., Shaked and Sutton 1982, Bonanno 1986). Moorthy (1988) studies a duopoly in which the marginal cost is a quadratic function of quality. We adopt this model as our benchmark to model the pre-merger market.

Our research is related to product variety because in our model the merging firms may stop producing a product from one merging firm. There are comprehensive reviews by Ho and Tang (1998) and Kok et al. (2008). The main difference between our paper and this stream of research is that we consider the impact of merger on product variety.

3.3 Pre-Merger Model

Our model is based on classical models of vertical differentiation. On the demand side, all consumers prefer more of a characteristic called "quality". However, their willingness to pay for the quality is different. We use θ to represent the consumer's willingness to pay. A type-θ consumer's utility of buying a product with quality q at price p is $\theta q - p$. If $\theta q - p$ is smaller than zero, the consumer will choose not to buy the product and get zero utility. We assume θ is uniformly distributed over $[0, \overline{\theta}]$ at unit density. Consumers can observe the product qualities and prices before they make purchase decisions. Each consumer will either buy one unit of product that maximize her utility, or choose not to buy any product if none of the products could provide her with positive utility.

On the supply side, we assume there are three firms in the market, indexed by i, $i = 1$, 2, and 3. Each firm offers one product. They try to maximize their profits. The competition occurs in two stages. In the first stage, firm i chooses a quality q_i ($q_i \geq 0$) simultaneously. In the second stage, after observing the qualities of other firms, firm decides its price p_i simultaneously with other firms. We assume firms choose prices after they choose quality level because usually prices are easier to change than qualities. For any chosen quality q_i, the marginal cost of producing one unit of this product is αq_i^2, where $\alpha > 0$ is the cost coefficient. The quadratic function form reflects the increasing marginal cost, and it has been widely used in literature (e.g., ...). We assume there is no fixed cost because fixed cost has no effect on the pre-merger equilibrium as long as each firm's profit is higher than the fixed cost.

Without loss of generality, we assume $q_1 \geq q_2 \geq q_3$. Denote by q_i^{pre}, p_i^{pre}, and π_i^{pre} the equilibrium quality, price, and profit of firm i, respectively. Denote by w^{pre} the equilibrium consumer welfare in equilibrium. We can show in equilibrium $q_1^{pre} > q_2^{pre} > q_3^{pre}$, $p_1^{pre} > p_2^{pre} > p_3^{pre}$, and $\pi_1^{pre} > \pi_2^{pre} > \pi_3^{pre}$ (see Lemma A1 in Appendix).

3.4 Post-Merger Model and Analysis

We focus our analysis on the case in which firm 2 and firm 3 merge to compete with firm 1. The analysis on cases with other merging firms can be obtained similarly. We provide such analysis in Online Appendix. We use firm m to refer to the new firm that is created by the merger, which we referred to as the post-merger firm. Firm m may either choose to continue producing two products with qualities q_2 and q_3, or choose only to produce a single product with quality q_m. If firm m only produces one product, it can achieve marginal cost reduction through economies of scale. Denote by s ($\in (0, 1]$) the percentage of cost reduction. Firm m's marginal cost of producing a product with quality q_m is $\alpha (1 - s) q_m$.

Next, we first investigate the case in which firm m produces two products. We then examine the case in which firm m produces only one product. We finally compare these two cases to analyze firm m's variety decision.

We compare the post-merger equilibrium in the case in which firm m produces two products with the pre-merger equilibrium in the following proposition. We use the super script (2) to denote the equilibrium in this case.

Proposition 3.1 *(Firm m produces two products) When firm m produces two products, there exists a unique Nash equilibrium in the post-merger market. In equilibrium:*

(a) The post-merger quality of any product is lower than its pre-merger quality; i.e., $q_i^{(2)} < q_i^{pre}$, $i = 1, 2, 3$.

(b) The post-merger price of product 1 *is higher than its pre-merger price, whereas the post-merger price of products* 2 *and* 3 *are lower than their pre-merger price, respectively; i.e.,* $p_1^{(2)} > p_1^{pre}$*, and* $p_i^{(2)} < p_i^{pre}$, $i = 2, 3$.

(c) Firm 1*'s post-merger profit is higher than its pre-merger profit; i.e.,* $\pi_1^{(2)} > \pi_1^{pre}$*. Firm* m*'s post-merger profit is higher than the sum of firm* 2*'s and firm* 3*'s pre-merger profits; i.e.,* $\pi_m^{(2)} > \pi_1^{pre} + \pi_2^{pre}$.

(d) The post-merger consumer welfare is lower than the pre-merger consumer welfare; i.e., $w^{(2)} < w^{pre}$.

Proposition 3.1(a) shows that after a merger, all firms reduce their qualities. In the pre-merger market, firm 2 competes with both firm 1 and firm 3. In the post-merger market, firm 2 and firm 3 become one firm and do not compete with each other. The post-merger firm only competes with firm 1. Therefore, it reduces its product qualities to further differentiate its products from firm 1's. Since its competitiors' qualities become lower, firm 1 also reduces its quality.

Proposition 3.1(b) shows that after a merger, the post-merger firm reduces its prices, but the nonparticipant firm (firm 1) increases its price. Since the post-merger firm reduces its qualities, it incurs lower marginal costs for products 2 and 3 (αs_i^2, $i = 2$, 3) which enable it to reduce its prices. Although the reduced competition intensity can cause firm m to increase it price, such effect is outweighed by the effect of lower marginal costs. However, the quality reduction is not very large for firm 1. So the effect of a lower marginal cost αs_1^2 cannot outweigh the effect of the reduced competition intensity. As a result, firm 1 increases its price after a merger.

Proposition 3.1(c) and Proposition 3.1(d) show that a merger increases firms' profits, but harms consumer welfare due to reduced competition.

When firm m only produces one product, clearly the equilibrium outcomes depend on the synergy level. We investigate how firms' decisions and other market outcomes change with the synergy level s in the following proposition. We use the super script (1) to denote the equilibrium in this case.

Proposition 3.2 *When firm m produces a single product in the post-merger market, there exists a unique Nash equilibrium. In equilibrium:*
(a) The qualities of both products ($q_1^{(1)}$ and $q_m^{(1)}$) are increasing with s.
(b) The prices of both products ($p_1^{(1)}$ and $p_m^{(1)}$) are increasing with s.
(c) Firm 1's post-merger profit ($\pi_1^{(1)}$) is decreasing with s, whereas firm m's post-merger profit ($\pi_m^{(1)}$) is increasing with s.
(d) The post-merger consumer welfare ($w^{(1)}$) is increasing with s.

When it becomes cheaper for the post-merger firm to produce higher-quality products, the post-merger firm increases its product qualities; facing the higher qualities from its competitior, firm 1 also increases its quality, as shown in Proposition 3.2(a). Proposition 3.2(b) shows that since all firms increases their qualities, they also increase their prices to match their qualities. Proposition 3.2(c) shows that cost reduction of the post-merger firm is beneficial to the post-merger firm, but the cost advantage of the post-merger firm is harmful to firm 1. Proposition 3.2(d) shows that cost reduction of the post-merger firm is beneficial to consumers because it induces all firms to increase their qualities. So when firm m only produces one product,

By combining the results above, we can analyze firm m's decision and its impact on consumer welfare in the following proposition.

Proposition 3.3 *There exists thresholds $s^{(\pi)} < s^{(w)}$ such that the following results hold:*
(a) Firm m produces one product if and only if $s \geq s^{(\pi)}$.
(b) The consumer welfare in the case in which firm m produces one product is greater than that in the case in which firm m produces two products (i.e., $w^{(1)} > w^{(2)}$) if and only if $s > s^{(w)}$.

When the post-merger firm's profit produces two products, its profit and the consumer welfare do not depend on the synergy level s. When it produces one product, its profit and the consumer welfare increase with s, as shown in Proposition 3.2. So there exists thresholds $s^{(\pi)}$ and $s^{(w)}$ such that $\pi_m^{(1)} > \pi_m^{(2)}$ if and only if $s > s^{(\pi)}$, and $w^{(1)} > w^{(2)}$ if and only if $s > s^{(w)}$. What is interesting is that $s^{(\pi)} < s^{(w)}$. When $s \leq s^{(\pi)}$, the post-merger firm produces two products, and the consumer welfare is $w^{(2)}$. When $s^{(\pi)} < s < s^{(w)}$, the post-merger firm only produces one product, and the consumer welfare changes to $w^{(1)}$. Notice that in this interval of

s, consumers prefer two products from the post-merger firm because $w^{(1)} < w^{(2)}$. As a result, when s increases from the interval $\left[0, s^{(\pi)}\right]$ to the interval $\left(s^{(\pi)}, s^{(w)}\right)$, consumer welfare actually decreases. In other words, cost reduction generated by the merger actually harm consumers.

3.5 Conclusion

Although merger-induced quality and variety changes are widely observed in the business world, there has been little research on evaluating the impact of such changes on consumers. This paper bridges this gap in literature by analyzing mergers in vertically-differentiated markets. Our analysis highlights the importance of taking into account potential quality and variety changes when evaluating potential mergers. A merger may induce the post-merger firm to reduce prices, but still harm consumers because it can reduce product qualities. This analytical finding is consistent with the empirical finding in Song (2015). Furthermore, we find that cost reduction generated by mergers do not always benefit consumers. Cost reduction may cause the post-merger firm to reduce the number of products it offers. As a result, consumer welfare may be hurt because of the smaller number of available products.

Appendix A

Supplements to Chapter 1

A.1 Proofs of Main Results

We use superscript * to denote a firm's best response to the other firms' prices. For example, the joint best response price of all nonparticipant firms to the post-merger firm's price p_m is denoted by $p_3^*(p_m)$.

Lemma A1 *Suppose the following condition is satisfied:*

$$2b + \frac{n-2}{n}\gamma - \frac{\sigma w^2}{\phi(\Phi^{-1}\left(1 - \frac{w}{\underline{p}}\right))\underline{p}^3} > 0. \tag{A.1}$$

(a) $0 < dp_3^*/dp_m < 1$, $0 < dp_m^*/dp_3 < 1$ *and* $d\pi_3^*/dp_m > 0$.

(b) There exists a unique pure-strategy Nash equilibrium in the pre-merger market in which the symmetric equilibrium prices $p_1^{pre} = p_2^{pre} = \ldots = p_n^{pre}$ *are the unique solution of (1.3).*

(c) There exists a unique pure-strategy Nash equilibrium in the post-merger market in which $p_1^{post} = p_2^{post} = p_m^{post}$ and $p_3^{post} = p_4^{post} = \ldots = p_n^{post}$ *are the unique solutions of the following equations*:

$$a - \left(b + \gamma\frac{n-2}{n}\right)\left(2p_m^{post} - w_m\right) + \gamma\frac{n-2}{n}p_3^{post} - \frac{\sigma_m}{2}R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right) = 0; \tag{A.2}$$

$$a - \left(2b + \frac{n+1}{n}\gamma\right)p_3^{post} + \left(b + \frac{n-1}{n}\gamma\right)w + \gamma\frac{2}{n}p_m^{post} - \sigma R\left(\Phi^{-1}\left(1 - \frac{w}{p_3^{post}}\right)\right) = 0. \tag{A.3}$$

Proof of Proposition 1.1. (a) We compute $\frac{dp_m^{post}}{d\sigma_m}$ and $\frac{dp_3^{post}}{d\sigma_m}$ by applying the implicit function theorem to (A.2) and (A.3) as follows:

$$\begin{bmatrix} \frac{dp_m^{post}}{d\sigma_m} \\ \frac{dp_3^{post}}{d\sigma_m} \end{bmatrix} = -J^{-1} \times \begin{bmatrix} \frac{\partial}{\partial\sigma_m}\left(\frac{\partial\pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\Big|_{\mathbf{p}=\mathbf{p}^{post}}\right) \\ \frac{\partial}{\partial\sigma_m}\left(\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\big|_{\mathbf{p}=\mathbf{p}^{post}}\right) \end{bmatrix} = -J^{-1} \times \begin{bmatrix} \frac{\partial^2\pi_m(\mathbf{p},\sigma_m)}{\partial p_1\partial\sigma_m}\Big|_{\mathbf{p}=\mathbf{p}^{post}} \\ 0 \end{bmatrix},$$

where $J^{-1} = \dfrac{\begin{bmatrix} \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) & -\frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \\ -\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) & \frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \end{bmatrix}}{\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) - \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right)}$.

Substituting J^{-1} into above and using $\frac{dp_3^{post}}{dp_m^{post}} = -\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) / \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right)$, we can simplify $\frac{dp_m^{post}}{d\sigma_m}$ and $\frac{dp_3^{post}}{d\sigma_m}$ into

$$\frac{dp_m^{post}}{d\sigma_m} = \frac{-\left.\frac{\partial^2 \pi_m(\mathbf{p},\sigma_m)}{\partial p_1 \partial \sigma_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}}{\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) + \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \frac{dp_3^{post}}{dp_m^{post}}} \text{ and } \frac{dp_3^{post}}{d\sigma_m} = \frac{dp_3^{post}}{dp_m^{post}} \frac{dp_m^{post}}{d\sigma_m}. \tag{A.4}$$

We now show $\frac{dp_m^{post}}{d\sigma_m} < 0$ by examining its denominator and numerator, respectively. To compute the denominator of $\frac{dp_m^{post}}{d\sigma_m}$, we use (A.2) and $\frac{dR(t)}{dt} = \Phi(t) - 1$, getting $\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) = -2\left(b + \gamma \frac{n-2}{n}\right) + \frac{\sigma_m w_m^2}{2\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)(p_m^{post})^3} < 0$, and $\frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) = \frac{n-2}{n}\gamma > 0$. Using (A.1), we can show $\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) + \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) < 0$. Because $dp_3^{post}/dp_m < 1$ from Lemma A1(a), $\frac{\partial}{\partial p_m^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) + \frac{\partial}{\partial p_3^{post}} \left(\left.\frac{\partial \pi_m(\mathbf{p},\sigma_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} \right) \frac{dp_3^{post}}{dp_m^{post}} < 0$. To compute the numerator of $\frac{dp_m^{post}}{d\sigma_m}$, we first find the expression of $\pi_m(\mathbf{p}) = \pi_m^d(\mathbf{p}) - c_m(p_1, p_2)$. By substituting $y_m = \sigma_m \Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)$, $L_1(\mathbf{p})$ and $L_2(\mathbf{p})$ into $\pi_m^d(\mathbf{p})$ and $c_m(p_1, p_2, y_m)$ in (1.4) respectively, we have

$$\pi_m^d(\mathbf{p}) = \sum_{i=1}^{2} (p_i - w_m) \left\{ a - (b + \frac{n-1}{n}\gamma) p_i + \frac{\gamma}{n} \sum_{j \neq i}^{n} p_j \right\}, \text{ and} \tag{A.5}$$

$$c_m(p_1, p_2) = w_m \sigma_m \Phi^{-1}\left(1 - \frac{2w_m}{p_1 + p_2}\right) + \frac{p_1 + p_2}{2} \sigma_m R\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1 + p_2}\right)\right). \tag{A.6}$$

From (A.5), $\frac{\partial^2 \pi_m^d(\mathbf{p},\sigma_m)}{\partial \sigma_m \partial p_1} = 0$. From (A.6), by using $\frac{dR(t)}{dt} = \Phi(t) - 1$ we obtain $\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial \sigma_m \partial p_1}$ as follows:

$$\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial \sigma_m \partial p_1} = \frac{\partial}{\partial \sigma_m} \left\{ w_m \sigma_m \frac{\partial \Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)}{\partial p_1} - \frac{p_1+p_2}{2} \sigma_m \frac{2w_m}{p_1+p_2} \frac{\partial \Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)}{\partial p_1} + \frac{\sigma_m R\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)}{2} \right\}$$
$$= \frac{1}{2} R\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right) > 0.$$

Thus, $-\left.\frac{\partial^2 \pi_m(\mathbf{p},\sigma_m)}{\partial p_1 \partial \sigma_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = -\frac{\partial^2 \pi_m^d(\mathbf{p},\sigma_m)}{\partial \sigma_m \partial p_1} + \frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial \sigma_m \partial p_1} > 0$. Finally, because the denominator of $\frac{dp_m^{post}}{d\sigma_m}$ is negative and its numerator is positive, we get $\frac{dp_m^{post}}{d\sigma_m} < 0$, hence $\frac{dp_m^{post}}{d\rho} = \frac{\sigma}{\sqrt{2+2\rho}} \frac{dp_m^{post}}{d\sigma_m} < 0$.

Lastly, $\frac{dp_3^{post}}{d\sigma_m} = \frac{dp_3^{post}}{dp_m^{post}} \frac{dp_m^{post}}{d\sigma_m} < 0$ because $\frac{dp_m^{post}}{d\sigma_m} < 0$ from above and $dp_3^{post}/dp_m > 0$ from

Lemma A1(a). So $\frac{dp_3^{post}}{d\rho} = \frac{\sigma}{\sqrt{2+2\rho}} \frac{dp_3^{post}}{d\sigma_m} < 0$.

(b) We first prove $d\pi_m^{post}/d\rho < 0$. From (1.4), we can write

$$\frac{d\pi_m^{post}}{d\rho} = \left(\frac{\partial\pi_m^{post}}{\partial p_1} + \frac{\partial\pi_m^{post}}{\partial p_2}\right)\frac{dp_m^{post}}{d\rho} + \frac{\partial\pi_m^{post}}{\partial y_m}\frac{dy_m^{post}}{d\rho} + \frac{\partial\pi_m^{post}}{\partial\rho} + \frac{dp_3^{post}}{d\rho}\sum_{j=3}^{n}\frac{\partial\pi_m^{post}}{\partial p_j} = \frac{\partial\pi_m^{post}}{\partial\rho} + \frac{dp_3^{post}}{d\rho}\sum_{j=3}^{n}\frac{\partial\pi_m^{post}}{\partial p_j},$$

since $\frac{\partial\pi_m^{post}}{\partial p_1} = \frac{\partial\pi_m^{post}}{\partial p_2} = 0$ and $\frac{\partial\pi_m^{post}}{\partial y_m} = 0$ at $(p_1, p_2, y_m) = (p_m^{post}, p_m^{post}, y_m^{post})$ (by Envelop Theorem). From (1.4), using $R(t) = \phi(t) - t(1 - \Phi(t))$ and $\frac{dR(t)}{dt} = \Phi(t) - 1$, we have $\frac{\partial\pi_m^{post}}{\partial p_j} = \frac{2\gamma}{n}(p_m^{post} - w_m)$ and $\frac{\partial\pi_m^{post}}{\partial\rho} = -\frac{p_m^{post}\sigma}{\sqrt{2+2\rho}}R\left(\frac{y_m^{post}}{\sigma_m}\right) - p_m^{post}\sigma_m\frac{d}{d\rho}R\left(\frac{y_m^{post}}{\sigma_m}\right) = -\frac{p_m^{post}\sigma}{\sqrt{2+2\rho}}\phi(\frac{y_m^{post}}{\sigma_m})$. Finally, substituting $\frac{\partial\pi_m^{post}}{\partial p_j}$ and $\frac{\partial\pi_m^{post}}{\partial\rho}$ into $\frac{d\pi_m^{post}}{d\rho}$, we obtain $\frac{d\pi_m^{post}}{d\rho} = -\frac{p_m^{post}\sigma}{\sqrt{2+2\rho}}\phi(\frac{y_m^{post}}{\sigma_m}) + \frac{2\gamma}{n}(p_m^{post} - w_m)(n-2)\frac{dp_3^{post}}{d\rho} < 0$, where the inequality is due to $dp_3^{post}/d\rho < 0$ from part (a). Next, $d\pi_3^{post}/d\rho = (d\pi_3^{post}/dp_m)(dp_m^{post}/d\rho) < 0$ because $d\pi_3^{post}/dp_m > 0$ by Lemma A1(a) and $dp_m^{post}/d\rho < 0$ by part (a). □

Proof of Proposition 1.2. **(a)** We focus on proving that $dp_m^{post}/ds < 0$ if and only if $s > s^{(1)}$, and that $s^{(1)}$ is nondecreasing in σ_m with $s^{(1)} = 0$ at $\sigma_m = 0$. Then the result about p_3^{post} follows easily: $dp_3^{post}/ds = (dp_3^{post}/dp_m)(dp_m^{post}/ds) < 0$ if and only if $s > s^{(1)}$ because $dp_3^{post}/dp_m > 0$ from Lemma A1(a) and $dp_m^{post}/ds < 0$ if and only if $s > s^{(1)}$. The proof proceeds in three steps. In step 1, we get the expression of dp_m^{post}/dw_m. In step 2, we prove there exists a unique $w_m^{(1)}$ such that $dp_m^{post}/dw_m > 0$ if and only if $w_m < w_m^{(1)}$. In step 3, we use $w_m^{(1)}$ to compute $s^{(1)}$.

Step 1: Similar to Proposition 1.1(a), we apply the implicit function theorem to obtain

$$\frac{dp_m^{post}}{dw_m} = \frac{-\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}}{\frac{\partial}{\partial p_m^{post}}\left(\left.\frac{\partial\pi_m(\mathbf{p},w_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) + \frac{\partial}{\partial p_3^{post}}\left(\left.\frac{\partial\pi_m(\mathbf{p},w_m)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right)\frac{dp_3^{post}}{dp_m^{post}}}. \tag{A.7}$$

Since the denominator of (A.7) is negative (cf. the proof of Proposition 1.1(a)), $\frac{dp_m^{post}}{dw_m}$ has the same sign as $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = \left.\frac{\partial^2\pi_m^d(\mathbf{p},w_m)}{\partial w_m\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} - \left.\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial\sigma_m\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}$, where $\frac{\partial^2\pi_m^d(\mathbf{p},w_m)}{\partial w_m\partial p_1} = b + \frac{n-2}{n}\gamma$ from (A.5) and $\frac{\partial^2 c_m(p_1,p_2,\sigma_m)}{\partial\sigma_m\partial p_1} = \frac{\sigma_m w_m}{2\phi\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right)\{(p_1+p_2)/2\}^2}$ from (A.6) and $\frac{dR(t)}{dt} = \Phi(t) - 1$. Thus;

$$\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = b + \frac{n-2}{n}\gamma - \sigma_m\left\{2h\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)p_m^{post}\right\}^{-1}, \tag{A.8}$$

where $h(t) = \phi(t)/\{1 - \Phi(t)\}$ is the failure rate of a standard normal variable.

Step 2: We can show the existence and uniqueness of $w_m^{(1)}$ by proving the three statements: (i) when $w_m \to 0$, $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} > 0$, (ii) when w_m is sufficiently large, $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$, and (iii) $d\left\{\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right\}/dw_m < 0$ when $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$. Then from (i) and (ii), there exists at least one $w_m^{(1)}$, and the value of $w_m^{(1)}$ is determined by $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$. In addition, from (iii) $w_m^{(1)}$ is unique because $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ can cross zero only once as w_m increases. To show how $w_m^{(1)}$ changes with σ_m, we compute $d\sigma_m/dw_m^{(1)}$. By solving $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$ from (A.8), we have $\sigma_m = 2\left(b+\frac{n-2}{n}\gamma\right) h\left(\Phi^{-1}\left(1-\frac{w_m^{(1)}}{p_m^{post}}\right)\right)p_m^{post}$. Then,

$$\frac{d\sigma_m}{dw_m^{(1)}} = -\frac{2\left(b+\frac{n-2}{n}\gamma\right)h'\left(\Phi^{-1}\left(1-\frac{w_m^{(1)}}{p_m^{post}}\right)\right)}{\phi\left(\Phi^{-1}\left(1-\frac{w_m^{(1)}}{p_m^{post}}\right)\right)} + \frac{\partial\left\{2\left(b+\frac{n-2}{n}\gamma\right)h\left(\Phi^{-1}\left(1-\frac{w_m^{(1)}}{p_m^{post}}\right)\right)p_m^{post}\right\}}{\partial p_m^{post}} \left.\frac{dp_m^{post}}{dw_m}\right|_{w_m=w_m^{(1)}},$$

where the first term is negative due to $h'\left(\Phi^{-1}\left(1-\frac{w_m^{(1)}}{p_m^{post}}\right)\right) > 0$ and the second term is zero due to $\left.\frac{dp_m^{post}}{dw_m}\right|_{w_m=w_m^{(1)}} = 0$. Therefore, $dw_m^{(1)}/d\sigma_m < 0$.

Step 3: If $w < w_m^{(1)}$, then $dp_m^{post}/dw_m > 0$ and $dp_m^{post}/ds < 0$ for any $w_m \le w$. We define $s^{(1)} = 0$ in this case. If $w \ge w_m^{(1)}$, then we define $s^{(1)} = \left(w - w_m^{(1)}\right)/w$. In this case, $dp_m^{post}/ds < 0$ if and only if $s > s^{(1)}$. Since $dw_m^{(1)}/d\sigma_m < 0$, $ds^{(1)}/d\sigma_m > 0$ in this case. Since $w_m^{(1)}$ solves $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$, from (A.8), $s^{(1)}$ is the maximum of 0 and the unique s that solves $b + \frac{n-2}{n}\gamma - \sigma_m\left\{2h\left(\Phi^{-1}\left(1-\frac{w(1-s)}{p_m^{post}}\right)\right)p_m^{post}\right\}^{-1} = 0$.

(b) $\frac{d\pi_3^{post}}{ds} = \frac{dp_m^{post}}{ds}\frac{d\pi_3^{post}}{dp_m} < 0$ if and only if $s > s^{(1)}$ because $\frac{d\pi_3^{post}}{dp_m} > 0$ by Lemma A1(a) and $\frac{dp_m^{post}}{ds} < 0$ if and only if $s > s^{(1)}$ by part (a). In the rest of the proof, we prove $\frac{d\pi_m^{post}}{ds} > 0$ for all s by showing $\frac{d\pi_m^{post}}{dw_m} < 0$ for all w_m in each of the following two cases: (case I) $dp_m^{post}/dw_m \le 0$, and (case II) $dp_m^{post}/dw_m > 0$.

(Case I) From (1.4), $\frac{d\pi_m^{post}}{dw_m} = \left(\frac{\partial\pi_m^{post}}{\partial p_1} + \frac{\partial\pi_m^{post}}{\partial p_2}\right)\frac{dp_m^{post}}{dw_m} + \frac{\partial\pi_m^{post}}{\partial y_m}\frac{dy_m^{post}}{dw_m} + \frac{\partial\pi_m^{post}}{\partial w_m} + \frac{dp_3^{post}}{dw_m}\sum_{j=3}^{n}\frac{\partial\pi_m^{post}}{\partial p_j} = \frac{\partial\pi_m^{post}}{\partial w_m} + \frac{dp_3^{post}}{dp_m}\frac{dp_m^{post}}{dw_m}\sum_{j=3}^{n}\frac{\partial\pi_m^{post}}{\partial p_j}$, where $\frac{\partial\pi_m^{post}}{\partial p_1} = \frac{\partial\pi_m^{post}}{\partial p_2} = 0$ and $\frac{\partial\pi_m^{post}}{\partial y_m} = 0$ at $(p_1, p_2, y_m) = (p_m^{post}, p_m^{post}, y_m^{post})$ by Envelop Theorem. By computing $\frac{\partial\pi_m^{post}}{\partial p_j}$ and $\frac{\partial\pi_m^{post}}{\partial w_m}$ from (1.4) and substituting them into $\frac{d\pi_m^{post}}{dw_m}$, we get $\frac{d\pi_m^{post}}{dw_m} = -q_m^{post} + 2\gamma\frac{n-2}{n}(p_m^{post} - w_m)\frac{dp_3^{post}}{dp_m}\frac{dp_m^{post}}{dw_m} < 0$, where the inequality is due to $q_m^{post} > 0$, $\frac{dp_m^{post}}{dw_m} \le 0$ by the premise, and $\frac{dp_3^{post}}{dp_m} > 0$ by Lemma A1(a).

(Case II) Suppose that w_m is reduced by dw_m (> 0). Let $p_3^{post} - dp_3$, $p_m^{post} - dp_m$, $y_m^{post} - dy_m$ and $\pi_m^{post} - d\pi_m$ denote new equilibrium outcomes associated with the change of dw_m. Let π_m' denote the post-merger firm's expected profit at the new marginal cost $w_m - dw_m$ when $p_m = p_m^{post} - dp_m$, $p_3 = p_4 = ... = p_n = p_3^{post} - dp_3$, and $y_m = y_m^{post}$. Note that $\pi_m' \le \pi_m^{post} - d\pi_m$ because $y_m = y_m^{post}$ is chosen instead of $y_m = y_m^{post} - dy_m$. In order to prove $\frac{d\pi_m^{post}}{dw_m} < 0$, we will show $\pi_m' > \pi_m^{post}$, so that $\pi_m^{post} - d\pi_m \ge \pi_m' > \pi_m^{post}$. From (1.4), we can compute π_m^{post}, π_m' and then $\pi_m' - \pi_m^{post}$ as follows:

$\pi_m^{post} = (p_m^{post} - w_m)L_m^{post} - w_m y_m^{post} - p_m\sigma_m R\left(\frac{y_m^{post}}{\sigma_m}\right)$;

$\pi_m' = \left\{p_m^{post} - w_m + (dw_m - dp_m)\right\}\left\{L_m^{post} + 2bdp_m + \frac{n-2}{n}2\gamma\left(dp_m - dp_3\right)\right\} - (w_m - dw_m)\,y_m^{post}$

$$
\begin{aligned}
& -\left(p_m^{post}-dp_m\right)\sigma_m R\left(\tfrac{y_m^{post}}{\sigma_m}\right); \\
\pi_m' - \pi_m^{post} &= \left\{p_m^{post} - w_m + (dw_m - dp_m)\right\}\left\{2bdp_m + \tfrac{n-2}{n}2\gamma\left(dp_m - dp_3\right)\right\} \\
&\quad + (dw_m - dp_m)\, L_m^{post} + y_m^{post} dw_m + \sigma_m R\left(\tfrac{y_m^{post}}{\sigma_m}\right) dp_m \\
&> (dw_m - dp_m)\, L_m^{post} + y_m^{post} dw_m + \sigma_m R\left(\tfrac{y_m^{post}}{\sigma_m}\right) dp_m \\
&= \left(L_m^{post} + y_m^{post}\right) dw_m + \left\{\sigma_m R\left(\tfrac{y_m^{post}}{\sigma_m}\right) - L_m^{post}\right\} dp_m,
\end{aligned}
$$

where the inequality follows from $\frac{dp_m^{post}}{dw_m} > 0$ by the premise, $\frac{dp_m^{post}}{dw_m} < 1$ from step 2 of part (a), and $\frac{dp_3^{post}}{dp_m} < 1$ by Lemma A1(a). Using $dw_m > dp_m > 0$, we can simplify this inequality into $\pi_m' - \pi_m^{post} > \left\{y_m^{post} + \sigma_m R\left(\frac{y_m^{post}}{\sigma_m}\right)\right\} dp_m$. Finally, we complete the proof by showing that $y_m^{post} \geq -\sigma_m R\left(\frac{y_m^{post}}{\sigma_m}\right)$. Since $y_m - (y_m - \widetilde{\varepsilon}_m)^+ = \widetilde{\varepsilon}_m - (\widetilde{\varepsilon}_m - y_m)^+$, $y_m = E\left(y_m - \widetilde{\varepsilon}_m\right)^+ - E\left(\widetilde{\varepsilon}_m - y_m\right)^+$. Using $E\left(y_m - \widetilde{\varepsilon}_m\right)^+ \geq 0$ and $E\left(\widetilde{\varepsilon}_m - y_m\right)^+ = \sigma_m R(y_m/\sigma_m)$, $y_m^{post} \geq -\sigma_m R\left(\frac{y_m^{post}}{\sigma_m}\right)$. □

Proof of Proposition 1.4. **(a)** $\frac{dl_m^{post}}{d\rho} = \frac{\partial l_m^{post}}{\partial p_m^{post}} \frac{dp_m^{post}}{d\rho} = \frac{w_m}{\left(p_m^{post}\right)^2} \frac{dp_m^{post}}{d\rho}$. Since $\frac{dp_m^{post}}{d\rho} < 0$ from Proposition 1(a), we get $\frac{dl_m^{post}}{d\rho} < 0$. Similarly, we can prove $\frac{dl_3^{post}}{d\rho} < 0$.
(b) $\frac{dl_m^{post}}{dw_m} = -\frac{1}{p_m^{post}} + \frac{w_m}{\left(p_m^{post}\right)^2} \frac{dp_m^{post}}{dw_m} = \left(\frac{w_m}{p_m^{post}} \frac{dp_m^{post}}{dw_m} - 1\right) \frac{1}{p_m^{post}}$. Since $w_m < p_m^{post}$ and $\frac{dp_m^{post}}{dw_m} < 1$ from the proof of Proposition 2(a), $\frac{dl_m^{post}}{dw_m} < 0$. Therefore, $\frac{dl_m^{post}}{ds} = \frac{dl_m^{post}}{dw_m} \frac{dw_m}{ds} = \frac{dl_m^{post}}{dw_m}(-w) > 0$. Next, $\frac{dl_3^{post}}{ds} = \frac{w}{\left(p_3^{post}\right)^2} \frac{dp_3^{post}}{ds} > 0$ if and only if $s < s^{(1)}$ from Proposition 2(a).
(c) Since l_m^{post} decreases with ρ and increases with s, it suffice to show $l_m^{post} > l_1^{pre}$ at $\rho = 1$ and $s = 0$, which follows directly from $l_m^{post} = 1 - w/p_m^{post}$, $l_1^{pre} = 1 - w/p_1^{pre}$ and $p_m^{post} > p_1^{pre}$ at $\rho = 1$ and $s = 0$. Since $l_3^{post} = 1 - w/p_3^{post}$ increases with p_3^{post}, the result follows from Proposition 3(a).□

Proof of Proposition 1.5. A sketch of the proof is as follows. We first find a lower bound of $E[cs^{post}]$, denoted by $E[cs^{lbd}]$. We then show $E[cs^{lbd}] > E\left[cs^{pre}\right]$ at $s = s^{(2)}$ and that $E\left[cs^{lbd}\right]$ increases with s for $s \in [s^{(2)}, 1)$. Therefore, there exists a threshold $s^{(cs)} \in \left[0, s^{(2)}\right]$ such that $E[cs^{post}] > E\left[cs^{pre}\right]$ for any $s > s^{(cs)}$. □

Proof of Corollary 1.1. We provide a sketch of the proof for part (a). The proof of part (b) follows a similar procedure. We first get the n first-order conditions for the post-merger firm: $\frac{\partial \pi_m}{\partial p_1} = 0$, $\frac{\partial \pi_m}{\partial p_2} = 0$, and $\frac{\partial \pi_i}{\partial p_i} = 0$, $i = 3, 4, \ldots, n$. Using the implicit function theorem and the Cramer's rule, we obtain $\frac{dp_i^{post}}{d\rho} = -\left|J_i^{\rho}\right| / \left|J\right|$, where J is the Jacobian matrix of the n first-order conditions, and J_i^{ρ} is the matrix formed by replacing the ith column of J with the vector $\left(\left.\frac{\partial^2 \pi_m}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}, \left.\frac{\partial^2 \pi_m}{\partial p_2 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}, 0, ..., 0\right)^T$. We can show that the sign of $|J|$ is $(-1)^n$. In addition, we can show that $\left.\frac{\partial^2 \pi_m}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ and $\left.\frac{\partial^2 \pi_m}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$, and thus the sign of $|J_i^{\rho}|$ is also $(-1)^n$. So $\frac{dp_i^{post}}{d\rho} = -\left|J_i^{\rho}\right| / \left|J\right| < 0$. □

Proof of Corollary 1.2. We provide a sketch of the proof for part (a). We compare $\partial \pi_m / \partial p_1$ for the same price vector under two different demands $\widetilde{\varepsilon}_m$ and $\widetilde{\xi}_m$. We show that if the post-merger firm keeps its prices constant but chooses its safety stock optimally, then the expected lost sales is smaller for the less dispersive demand $\widetilde{\varepsilon}_m$. This results in a larger $\partial \pi_m / \partial p_1$ for the less dispersive demand. Consequently, if the prices are set at the equilibrium point for the more dispersive demand (resulting in $\partial \pi_m / \partial p_1 = 0$ for this demand), then $\partial \pi_m / \partial p_1 > 0$ for the less dispersive demand. The rest of the proof follows the procedure similar to the proof of Lemma 1.1(a). The proof of part (b) follows the procedure similar to the proof of Proposition 1.2(a). □

A.2 Parameter Values Used in Figures

The parameter values used in our numerical examples are motivated by the U.S. rental car market. In Figure 1-1, we used the following parameter values: $n = 3$, $a = 1$, $b = 0.6$, $\gamma = 0.5$, $w = 0.5$, and $\sigma = 0.3$. We use $n = 3$ to reflect the number of major rental car companies in the U.S. (i.e., Hertz, Avis and Enterprise). We used the normalized value of $a = 1$ for the demand intercept. The parameters b and γ are related to the price sensitivity of a firm's demand to its own price and other competitors' prices. According to McCarthy (1996), the own price elasticity for the US auto market is between -1.06 and -1.85, and the cross price elasticity is between 0.28 and 0.86. We chose the values of b and γ to yield the own price elasticity of -1.64 and the cross price elasticity of 0.59, which are consistent with McCarthy (1996). We chose $w = 0.5$ to yield a profit margin of 42% in the pre-merger market. From the quarterly financial reports of Hertz, we found that its gross profit margin was between 37% and 50% from 2006 to 2013. The profit margin in our numerical example is in the middle of this range. We used $\sigma = 0.3$ to add moderate uncertainty to demand.

Figure 1-2 is plotted over different levels of synergy s for a fixed value of ρ. For illustration, we provide three different figures for $\rho = 0$, 0.5 and 1, respectively. The case when $\rho = 1$ can be viewed as a benchmark case in which firm 1 and firm 2 maintain separate inventories after their merger. Other parameter values are the same as in Figure 1-1 except $\sigma = 0.5$. We used a larger σ to better illustrate the property that $s^{(1)}$ is nondecreasing with σ_m.

A.3 Supplemental Materials

Proof of Lemma A1. (a) We first prove $0 < dp_3^*/dp_m < 1$. Substituting the optimal safety stock $y_i = \sigma \Phi^{-1}\left(1 - \frac{w}{p_i}\right)$ to (1.2), computing the derivative with respect to p_i, and setting $p_3 = ... = p_n = p_3^*$ and $p_1 = p_2 = p_m^*$ yield:

$$-\left(2b + \tfrac{n+1}{n}\gamma\right) p_3^* - \sigma R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_3^*}\right)\right) + \tfrac{2\gamma}{n} p_m + a + \left(b + \tfrac{n-1}{n}\gamma\right) w = 0,$$

which implicitly defines $p_3^*(p_m)$. Using the implicit function theorem, we obtain

$$\frac{dp_3^*}{dp_m} = \frac{2\gamma}{n}\left\{\left(2b + \frac{n+1}{n}\gamma\right) - \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_3^*}\right)\right)(p_3^*)^3}\right\}^{-1}, \tag{A.9}$$

where we have used (A.12). To prove $0 < dp_3^*/dp_m < 1$, we finally show

$$\left(2b + \frac{n+1}{n}\gamma\right) - \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_3^*}\right)\right)(p_3^*)^3} > \frac{2\gamma}{n}. \tag{A.10}$$

To show that (A.10) holds, we use (A.1), and rewrite it as follows by adding $\frac{3\gamma}{n}$ to both sides of (A.1):

$$\left(2b + \frac{n+1}{n}\gamma\right) - \frac{\sigma w^2}{\phi(\Phi^{-1}\left(1 - \frac{w}{\underline{p}}\right))\underline{p}^3} > \frac{3\gamma}{n} \geq \frac{2\gamma}{n}.$$

Because $p_3^* \geq \underline{p}$, (A.10) holds if $\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^3}$ is decreasing with p. By using the failure rate of a standard normal variable $h(t) = \phi(t) / \{1 - \Phi(t)\}$, we can express $\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^3}$ as follows:

$$\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^3} = \frac{\sigma\left\{1-\Phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)\right\}w}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^2} = \frac{\sigma w}{h\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^2},$$

which is decreasing with p because $h(t)$ is increasing with t for the standard normal distribution.

Similarly, we can obtain the following equation that defines $p_m^*(p_3)$ implicitly:

$$-2\left(b+\gamma\tfrac{n-2}{n}\right)p_m - \tfrac{\sigma_m}{2}R\left(\Phi^{-1}\left(1-\tfrac{w_m}{p_m}\right)\right) + \gamma\tfrac{n-2}{n}p_3 + a + \left(b+\gamma\tfrac{n-2}{n}\right)w_m = 0,$$

and obtain $\frac{dp_m^*}{dp_3}$ as follows: $\frac{dp_m^*}{dp_3} = \frac{\gamma\frac{n-2}{n}}{2\left(b+\gamma\frac{n-2}{n}\right)-\sigma_m w_m^2\left\{2\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^*}\right)\right)(p_m^*)^3\right\}^{-1}}$. We prove $0 < dp_m^*/dp_3 < 1$ by using the following inequality that can be proven similarly to (A.10):

$$2\left(b + \gamma\frac{n-2}{n}\right) - \frac{\sigma_m w_m^2}{2\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^*}\right)\right)(p_m^*)^3} > \gamma\frac{n-2}{n}. \tag{A.11}$$

Lastly, we prove $d\pi_3^*/dp_m > 0$. Observe from (1.2) that π_3 depends on the prices of all firms, so that $\frac{d\pi_3}{dp_m} = \frac{\partial\pi_3}{\partial p_3}\frac{dp_3}{dp_m} + \frac{\partial\pi_3}{\partial y_3}\frac{dy_3}{dp_m} + \sum_{j=4}^{n}\frac{\partial\pi_3}{\partial p_j}\frac{\partial p_j}{\partial p_m} + \frac{\partial\pi_3}{\partial p_m}$. When (p_3, y_3) are chosen optimally to (p_3^*, y_3^*), the two terms in $\frac{d\pi_3}{dp_m}$ are zero (by Envelop Theorem). Since $p_3^* = ... = p_n^*$, we can rewrite $\frac{d\pi_3}{dp_m}$ as follows: $\frac{d\pi_3^*}{dp_m} = \sum_{j=4}^{n}\frac{\partial\pi_3^*}{\partial p_j}\frac{dp_j^*}{dp_m} + \frac{\partial\pi_3^*}{\partial p_m} = \frac{dp_3^*}{dp_m}\sum_{j=4}^{n}\frac{\partial\pi_3^*}{\partial p_j} + \frac{\partial\pi_3^*}{\partial p_m}$. Finally, using $\frac{\partial\pi_3}{\partial p_j} = (p_3 - w)\frac{\gamma}{n}$ (for $j = 4, ..., n$) and $\frac{\partial\pi_3}{\partial p_m} = (p_3 - w)\frac{2\gamma}{n}$ from (1.2), we can simplify $\frac{d\pi_3^*}{dp_m}$ into $\frac{d\pi_3^*}{dp_m} = \left(\frac{n-3}{n}\gamma\frac{dp_3^*}{dp_m} + \frac{2\gamma}{n}\right)(p_3^* - w)$, which is positive because $dp_3^*/dp_m > 0$ and $p_3^* > w$.

(b) For a given p_i, from (1.2) we can compute a unique optimal safety stock $y_i^* = \sigma\Phi^{-1}(1 - w/p_i)$. Given this optimal safety stock, firm i only needs to make a price decision; i.e., firm i's strategy space can be reduced to $[\underline{p}, \overline{p}]$ with price as the single decision variable. The original game can be reduced to a game with price as the only decision variable. By using this new game, we complete the rest of the proof in three steps. In step 1, we show there exists a Nash equilibrium in this new game. In step 2, we show that this Nash equilibrium is unique with p_1^{pre} as the equilibrium price for each firm. In step 3, we show the original game also has a unique equilibrium with p_1^{pre} as the equilibrium price and $y_1^{pre} = \sigma\Phi^{-1}(1 - w/p_1^{pre})$ as the equilibrium safety stock for each firm.

Step 1: To show the existence, we need to show that $\pi_i(\mathbf{p})$ is concave in p_i. Substituting the optimal safety stock $y_i^* = \sigma\Phi^{-1}(1 - w/p_i)$ back into (1.2), we obtain $\pi_i(\mathbf{p}) = (p_i - w)L_i(\mathbf{p}) - w\sigma\Phi^{-1}(1 - w/p_i) - p_i\sigma R\left(\Phi^{-1}(1 - w/p_i)\right)$. By using (A.12), we compute its second order derivative as follows: $\frac{\partial^2 \pi_i}{\partial p_i^2} = -2\left(b + \frac{n-1}{n}\gamma\right) + \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p_i}\right)\right)p_i^3}$. Comparing this equation with (A.1), we obtain that $2\left(b + \frac{n-1}{n}\gamma\right) \geq 2b + \frac{n-2}{n}\gamma$ and $\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p_i}\right)\right)p_i^3} < \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{\underline{p}}\right)\right)\underline{p}^3}$ (since $\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p_i}\right)\right)p_i^3}$ is decreasing with p_i as shown in the proof of Lemma A1(a)), and thus $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$. Therefore, $\pi_i(\mathbf{p})$ is concave in p_i. In addition, the strategy space $[\underline{p}, \overline{p}]$ is compact. So there exists a pure-strategy Nash equilibrium. Note from (1.2) that $p_i = \underline{p}$ or $p_i = \overline{p}$ yields non-positive expected profits for sufficiently small $\underline{p}$ or sufficiently large $\overline{p}$, respectively, and thus it cannot be optimal. So the optimal price must be a interior solution of the first-order condition.

Step 2: We first use the contraction mapping theorem to prove the uniqueness. We then prove p_1^{pre} is the equilibrium price. By using the contraction mapping theorem, Cachon and Netessine (2004) show that if $\left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right| > \sum_{j \neq i}\left|\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|$, then there exists a unique Nash equilibrium. From the expression of $\pi_i(\mathbf{p})$, we obtain $\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{\gamma}{n}$ and $\frac{\partial^2 \pi_i}{\partial p_i^2} = -2\left(b + \frac{n-1}{n}\gamma\right) + \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p_i}\right)\right)p_i^3} < 0$, where the inequality is due to concavity proved in step 1. So we need to show $\left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right| - \sum_{j \neq i}\left|\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right| = \left(2b + \frac{n-1}{n}\gamma\right) - \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p_i}\right)\right)p_i^3} > 0$. Following a procedure similar to step 1, we can show that (A.1) ensures this inequality holds, hence the uniqueness of the Nash equilibrium. Therefore, equilibrium prices is the solution of (1.3), which is obtained by computing $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ from the expression of $\pi_i(\mathbf{p})$, and then setting $p_1^{pre} = p_2^{pre} = ... = p_n^{pre}$. To prove the solution of (1.3) is unique, it suffices to show that the left side of (1.3), denoted by $f_1(p_1^{pre})$, is decreasing in p_1^{pre}. We can compute $f_1'(p) = -(2b + \frac{n-1}{n}\gamma) + \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^3} < 0$, where the inequality follows from (A.1).

Step 3: It is easy to verify that $p_i^{pre} = p_1^{pre}$ and $y_i^{pre} = \sigma\Phi^{-1}(1 - w/p_1^{pre})$ is a Nash equilibrium for the original game. In order to prove the uniqueness of the Nash equilibrium, from the index theory (page 48 in Vives 1999), it suffices to show that the determinant of the

Jacobian matrix of $\frac{\partial \pi_i}{\partial p_i}$ and $\frac{\partial \pi_i}{\partial y_i}$ has the sign of $(-1)^{2n}$ whenever $\frac{\partial \pi_i}{\partial p_i} = 0$ and $\frac{\partial \pi_i}{\partial y_i} = 0$. By using (1.2), we can calculate the elements of the Jacobian matrix J (which is a $2n$ by $2n$ matrix) as follows: for $i, j \leq n$ and $i \neq j$, $J_{i,i} = \frac{\partial^2 \pi_i}{\partial p_i^2} = -2(b + \frac{n-1}{n}\gamma)$, $J_{i,j} = \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{1}{n}\gamma$, $J_{i,i+n} = \frac{\partial^2 \pi_i}{\partial p_i \partial y_i} = 1 - \Phi\left(\frac{y_i}{\sigma}\right)$, $J_{i,j+n} = \frac{\partial^2 \pi_i}{\partial p_i \partial y_j} = 0$, $J_{i+n,i} = \frac{\partial^2 \pi_i}{\partial y_i \partial p_i} = 1 - \Phi\left(\frac{y_i}{\sigma}\right)$, $J_{i+n,j} = \frac{\partial^2 \pi_i}{\partial y_i \partial p_j} = 0$, $J_{i+n,i+n} = \frac{\partial^2 \pi_i}{\partial y_i^2} = -\frac{p_i}{\sigma}\phi\left(\frac{y_i}{\sigma}\right)$, and $J_{i+n,j+n} = \frac{\partial^2 \pi_i}{\partial y_i \partial y_j} = 0$. By using $y_i^* = \sigma\Phi^{-1}(1 - w/p_i)$ at $\frac{\partial \pi_i}{\partial y_i} = 0$, we can simplify the Jacobian matrix J as follows: $J = \begin{bmatrix} J_1 & J_3 \\ J_3 & J_2 \end{bmatrix}$, where the elements of J_1 are given as $(J_1)_{i,i} = -2(b + \frac{n-1}{n}\gamma)$ and $(J_1)_{i,j} = \frac{1}{n}\gamma$ for $i \neq j$ and $i, j \leq n$, J_2 is an n by n diagonal matrix with the ith diagonal element equal to $-\frac{p_i}{\sigma}\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)$, and J_3 is an n by n diagonal matrix with the ith diagonal element equal to w/p_i. Using row and column operations, we can compute $|J|$ as follows: $|J| = \begin{vmatrix} J_1 - J_3 J_2^{-1} J_3 & 0 \\ 0 & J_2 \end{vmatrix} = \left|J_1 - J_3 J_2^{-1} J_3\right| \cdot |J_2|$. Since J_2 is an n by n diagonal matrix with the ith diagonal element equal to $-\frac{p_i}{\sigma}\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)$, the sign of $|J_2|$ is $(-1)^n$. From the expression of J_1, J_2 and J_3, we obtain $\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,i} = -2(b + \frac{n-1}{n}\gamma) + \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)p_i^3} < 0$ (the inequality has been shown in step 1), and $\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,j} = \frac{1}{n}\gamma > 0$ for $i \neq j$. In addition, we compute the following: $\left|\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,i}\right| - \sum_{j \neq i}^{n} \left|\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,j}\right| = (2b + \frac{n-1}{n}\gamma) - \frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)p_i^3}$. Following the same procedure as in step 1, we can show that from (A.1), $\left|\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,i}\right| - \sum_{j \neq i}^{n} \left|\left(J_1 - J_3 J_2^{-1} J_3\right)_{i,j}\right| > 0$. Therefore, $J_1 - J_3 J_2^{-1} J_3$ is a diagonally dominant matrix with negative diagonal values and its sign is $(-1)^n$. So the sign of $|J| = \left|J_1 - J_3 J_2^{-1} J_3\right| \cdot |J_2|$ is $(-1)^{2n}$.

(c) The proof proceeds similarly to the proof of part (b) as follows.

Step 1. The proof for firm 3 to firm n is similar to part (b). To prove $\pi_m(\mathbf{p}) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - w_m\sigma_m\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right) - (p_1 + p_2)\frac{\sigma_m}{2}R\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)$ (which is obtained by substituting $y_m = \sigma_m\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)$ to (1.4)) is jointly-concave in p_1 and p_2, we need to show that $\nabla^2_{p_1,p_2}\pi_m(\mathbf{p})$ is negative definite. We can compute

$$\nabla^2_{p_1,p_2}\pi_m(\mathbf{p}) = \begin{bmatrix} -2\left(b + \frac{n-1}{n}\gamma\right) + \frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} & \frac{2\gamma}{n} + \frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} \\ \frac{2\gamma}{n} + \frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} & -2\left(b + \frac{n-1}{n}\gamma\right) + \frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} \end{bmatrix}.$$

For this symmetric matrix, to ensure that it is negative definite, we only need to show that it is diagonally dominant; i.e., $2\left(b + \frac{n-2}{n}\gamma\right) - \frac{\sigma_m w_m^2}{2\phi\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} > 0$. Similar to step 1 in part (b), this inequality follows directly from (A.1).

Step 2. The proof for firm 3 to firm n is similar to part (b). For the post-merger firm, we need to show $\left|\frac{\partial^2\pi_m}{\partial p_1^2}\right| > \sum\limits_{j\neq 1}\left|\frac{\partial^2\pi_m}{\partial p_1\partial p_j}\right|$. We compute $\frac{\partial^2\pi_m}{\partial p_1^2}$ and $\frac{\partial^2\pi_m}{\partial p_1\partial p_j}$ as follows: $\frac{\partial^2\pi_m}{\partial p_1^2} = -2\left(b+\frac{n-1}{n}\gamma\right)+\frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3}$, $\frac{\partial^2\pi_m}{\partial p_1\partial p_2} = \frac{2\gamma}{n}+\frac{\sigma_m w_m^2}{4\phi\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3}$, and $\frac{\partial^2\pi_m}{\partial p_1\partial p_j} = \frac{\gamma}{n}$ for $j\neq 1,2$. So we obtain a sufficient condition for $\left|\frac{\partial^2\pi_m}{\partial p_1^2}\right| > \sum\limits_{j\neq 1}\left|\frac{\partial^2\pi_m}{\partial p_1\partial p_j}\right|$: $\left(2b+\frac{n-2}{n}\gamma\right)-\frac{\sigma_m w_m^2}{2\phi\left(\Phi^{-1}\left(1-\frac{2w_m}{p_1+p_2}\right)\right)\left(\frac{p_1+p_2}{2}\right)^3} > 0$. Following a procedure similar to the proof of Lemma A1(a), we can show that the left-hand side of this inequality is increasing in p_1 and p_2, and decreasing in w_m and σ_m. Since p_1, $p_2 > \underline{p}$, $w_m \leq w$ and $\sigma_m/2 \leq \sigma$, we can use the bounds of w_m, σ_m, p_1 and p_2 to get the sufficient condition for this inequality: $2b+\frac{n-2}{n}\gamma-\frac{\sigma w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{\underline{p}}\right)\right)\underline{p}^3} > 0$, which is exactly (A.1).

Step 3. The proof follows the same procedure as in the proof of part (b). $\square$

Proof of Lemma 1.1. **(a)** The proof proceeds in two steps. First, we show $\frac{\partial\pi_m(\mathbf{p})}{\partial p_1}\big|_{p_1=p_2=\ldots=p_n=p_1^{pre}} > 0$, implying that if all firms charge the pre-merger equilibrium price p_1^{pre} in the post-merger market, then the post-merger firm has an incentive to raise its price. Second, we prove $p_1^{pre} < p_3^{post} < p_m^{post}$.

First, substituting $y_m = \sigma_m\Phi^{-1}\left(1-\frac{2w}{p_1+p_2}\right)$, $L_1(\mathbf{p})$ and $L_2(\mathbf{p})$ in (1.1) into $\pi_m(\mathbf{p}, y_m)$ in (1.4), we obtain the expression for $\pi_m(\mathbf{p})$. Then, using (1.3) and the following result:

$$\frac{dR\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)}{dp} = \left\{\Phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)-1\right\}\frac{d\Phi^{-1}\left(1-\frac{w}{p}\right)}{dp} = -\frac{w^2}{\phi\left(\Phi^{-1}\left(1-\frac{w}{p}\right)\right)p^3}, \tag{A.12}$$

we obtain $\frac{\partial\pi_m(\mathbf{p})}{\partial p_1}$ after simplification as follows: $\frac{\partial\pi_m(\mathbf{p})}{\partial p_1} = \left(2b+\frac{n-1}{n}\gamma\right)\left(p_1^{pre}-p_1\right)+\frac{\gamma}{n}\sum\limits_{j=2}^{n}\left(p_j-p_1\right)+\frac{\gamma}{n}\left(p_2-w\right)+\sigma R\left(\Phi^{-1}\left(1-\frac{w}{p_1^{pre}}\right)\right)-\sigma R\left(\Phi^{-1}\left(1-\frac{2w}{p_1+p_2}\right)\right)$. Substituting $p_1 = p_2 = p_3 = \ldots = p_n = p_1^{pre}$ into $\frac{\partial\pi_m(\mathbf{p})}{\partial p_1}$ yields $\frac{\partial\pi_m(\mathbf{p})}{\partial p_1} = \frac{1}{n}\gamma\left(p_1^{pre}-w\right) > 0$.

Next, suppose the post-merger firm raises its price to a new price $p_m^{(1)} = p_1^{pre}+\Delta p_m^{(1)}$ with $\Delta p_m^{(1)} > 0$ such that $\partial\pi_m(\mathbf{p})/\partial p_1|_{p_1=p_2=p_m^{(1)},p_3=p_4=\ldots=p_n=p_1^{pre}} = 0$. (Note that such $p_m^{(1)}$ exists, since $\pi_m(\mathbf{p})$ is jointly-concave in p_1 and p_2.) Since $0 < dp_3^*/dp_m < 1$ by Lemma A1(a), all nonparticipant firms raise their prices to $p_3^{(1)} = p_3^{pre}+\Delta p_3^{(1)}$ with $0 < \Delta p_3^{(1)} < \Delta p_m^{(1)}$. In response to $p_3^{(1)}$, the post-merger firm will raise its price again to $p_m^{(2)} = p_m^{(1)}+\Delta p_m^{(2)}$, where $0 < \Delta p_m^{(2)} < \Delta p_3^{(1)}$ (since $0 < dp_m^*/dp_3 < 1$ by Lemma A1(a)). This process continues until p_m and p_3 converge to p_m^{post} and p_3^{post} respectively in equilibrium. Since $0 < \Delta p_3^{(j)} < \Delta p_m^{(j)}$ in every stage j, equilibrium prices will satisfy $p_1^{pre} < p_3^{post} < p_m^{post}$.

(b) We first prove $\pi_m^{post}/2 > \pi_1^{pre}$. Let $\pi_m|_{(p_1,p_2,y_m)=(p_1^{pre},p_1^{pre},2y_1^{pre})}$ denote the post-merger firm's expected profit when $(p_1,p_2,y_m) = (p_1^{pre},p_1^{pre},2y_1^{pre})$ and $p_i = p_i^{post}$ for $i = 3,4,\ldots,n$. From π_m in (1.4) and π_i in (1.2), we can express $\pi_m|_{(p_1,p_2,y_m)=(p_1^{pre},p_1^{pre},2y_1^{pre})}$ and π_1^{pre} respectively as follows:

$\pi_m|_{(p_1,p_2,y_m)=\left(p_1^{pre},p_1^{pre},2y_1^{pre}\right)} = 2(p_1^{pre}-w)\left\{a - bp_1^{pre} + \gamma\frac{n-2}{n}\left(p_3^{post} - p_1^{pre}\right)\right\} - 2wy_1^{pre} - 2p_1^{pre}\sigma R\left(\frac{y_1^{pre}}{\sigma}\right)$;
$\pi_1^{pre} = (p_1^{pre} - w)\,(a - bp_1^{pre}) - wy_1^{pre} - p_1^{pre}\sigma R\left(\frac{y_1^{pre}}{\sigma}\right)$.
Then, we can compute $\frac{1}{2}\pi_m|_{(p_1,p_2,y_m)=\left(p_1^{pre},p_1^{pre},2y_1^{pre}\right)} - \pi_1^{pre} = (p_1^{pre} - w)\gamma\frac{n-2}{n}\left(p_3^{post} - p_1^{pre}\right) > 0$, where the inequality is due to $p_1^{pre} > w$ and $p_3^{post} > p_1^{pre}$ from part (a). Since $\pi_m^{post} \geq \pi_m|_{(p_m,y_m)=\left(p_1^{pre},2y^{pre}\right)}$ (because (p_m^{post}, y_m^{post}) maximize π_m given that $p_i = p_i^{post}$ for $i = 3, 4, ..., n$), $\pi_m^{post}/2 > \pi_1^{pre}$.

Next, we prove $\pi_3^{post} > \pi_1^{pre}$. Let $\pi_3|_{(p_3,y_3)=\left(p_1^{pre},y_1^{pre}\right)}$ denote firm 3's post-merger expected profit when $(p_3, y_3) = (p_1^{pre}, y_1^{pre})$ and $p_i = p_i^{post}$ for $i = m, 4, 5, ..., n$. We can then show

$$\pi_3|_{(p3,y_3)=\left(p_1^{pre},y_1^{pre}\right)} - \pi_1^{pre} = (p_1^{pre} - w)\left\{\gamma\frac{n-3}{n}\left(p_3^{post} - p_1^{pre}\right) + \gamma\frac{2}{n}\left(p_m^{post} - p_1^{pre}\right)\right\} > 0,$$

where the inequality is due to $p_1^{pre} > w$, $p_3^{post} > p_1^{pre}$ and $p_m^{post} > p_1^{pre}$ from part (a). Since $\pi_3^{post} \geq \pi_3|_{(p_3,y_3)=\left(p_1^{pre},y_1^{pre}\right)}$, $\pi_3^{post} > \pi_1^{pre}$.

Finally, we prove $\pi_3^{post} > \pi_m^{post}/2$. Let $\pi_3|_{(p_3,y_3)=\left(p_m^{post},y_m^{post}/2\right)}$ denote firm 3's post-merger expected profit when $(p_3, y_3) = (p_m^{post}, y_m^{post}/2)$ and $p_i = p_i^{post}$ for $i = m, 4, 5, ..., n$. Then, we can compute $\pi_3|_{(p_3,y_3)=\left(p_m^{post},y_m^{post}/2\right)} - \frac{1}{2}\pi_m^{post} = (p_m^{post} - w)\frac{1}{n}\gamma\left(p_m^{post} - p_3^{post}\right) > 0$, where the inequality is due to $p_m^{post} > p_3^{post}$ by part (a). Since $\pi_3^{post} \geq \pi_3|_{(p_3,y_3)=\left(p_m^{post},\frac{1}{2}y_m^{post}\right)}$, $\pi_3^{post} > \pi_m^{post}/2$. $\square$

Detailed Proof of Proposition 1.2. **(a)** The proofs of statements (i), (ii) and (iii) are presented below. To prove (i), we note from (A.8) that as $w_m \to 0$, $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} \to b + \frac{n-2}{n}\gamma > 0$. To prove (ii), we will show that $dp_m^{post}/dw_m < 1$ such that when w_m is sufficiently large, $(1 - \frac{w_m}{p_m^{post}}) \to 0$ and $h\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right) \to 0$. Thus, from (A.8), $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ when w_m is sufficiently large. We can compute the expression of dp_m^{post}/dw_m in (A.7) as follows:

$$\frac{dp_m^{post}}{dw_m} = \frac{(b+\frac{n-2}{n}\gamma)-\sigma_m\left\{2h\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)p_m^{post}\right\}^{-1}}{(2b+\frac{n-2}{n}\gamma)-\sigma_m\left\{2h\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)p_m^{post}\right\}^{-1}\frac{w_m}{p_m^{post}}+\frac{n-2}{n}\gamma\left(1-\frac{dp_3^{post}}{dp_m}\right)},$$

which is smaller than 1 because $b > 0$, $w_m < p_m^{post}$ and $dp_3^{post}/dp_m < 1$ from Lemma A1(a). To prove (iii), we compute $d\left\{\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right\}/dw_m$ as follows:

$$\frac{d\left\{\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right\}}{dw_m} = \frac{\partial\left\{\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right\}}{\partial w_m} + \frac{\partial\left\{\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right\}}{\partial p_m^{post}}\frac{dp_m^{post}}{dw_m}.$$

From (A.7), when $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$, $dp_m^{post}/dw_m = 0$. Substituting $dp_m^{post}/dw_m = 0$ and the expression in (A.8) to the above equation, we obtain the following after simplification:

$$\frac{d\left\{ \frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\Big|_{\mathbf{p}=\mathbf{p}^{post}} \right\}}{dw_m} = -\frac{\sigma_m h'\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)}{2\left(p_m^{post}\right)^2 h^2\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)} < 0,$$

where the inequality is due to $h'\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right) > 0$ because the normal distribution has an increasing failure rate. $\square$

Proof of Proposition 1.3. **(a)** The proof proceeds in three steps. First, we show $p_m^{post} > p_3^{post} > p_1^{pre}$ when $s = 0$ for all ρ. Second, we prove the existence of $s^{(2)}$. Finally, we prove that $s^{(2)}$ is nonincreasing with ρ.

First, by Lemma 1.1(a), $p_m^{post} > p_3^{post} > p_1^{pre}$ when $s = 0$ and $\rho = 1$. By Proposition 1.1(a), $dp_m^{post}/d\rho < 0$ and $dp_3^{post}/d\rho < 0$. In addition, since $dp_3^{post}/d\rho = (dp_3^{post}/dp_m)dp_m^{post}/d\rho$ and $0 < dp_3^{post}/dp_m < 1$ by Lemma A1(a), $\left|dp_3^{post}/d\rho\right| < \left|dp_m^{post}/d\rho\right|$. So $p_m^{post} > p_3^{post} > p_1^{pre}$ when $s = 0$ and $\rho \leq 1$.

Second, we consider the following two different cases: (Case I) $\frac{\partial \pi_m}{\partial p_1}|_{p_1=p_2=p_3=...=p_n=p_1^{pre}} \geq 0$ when $w_m \to 0$, and (Case II) $\frac{\partial \pi_m}{\partial p_1}|_{p_1=p_2=p_3=...=p_n=p_1^{pre}} < 0$ when $w_m \to 0$.
(Case I) Following the same procedure as in the proof of Lemma 1.1(a), we can show that when $s \to 1$ ($w_m \to 0$), $p_m^{post} \geq p_3^{post} \geq p_1^{pre}$ if $\frac{\partial \pi_m}{\partial p_m}|_{p_m=p_3=...=p_n=p_1^{pre}} \geq 0$. Since $p_m^{post} > p_3^{post} > p_1^{pre}$ when $s = 0$, and both p_m^{post} and p_3^{post} either decrease with s or first increase and then decrease with s by Proposition 1.2(a), $p_m^{post} > p_1^{pre}$ and $p_3^{post} > p_1^{pre}$ for any $0 \leq s < 1$. In addition, $dp_3^{post}/ds = (dp_3^{post}/dp_m)(dp_m^{post}/ds)$, where $0 < dp_3^{post}/dp_m < 1$ by Lemma A1(a); consequently $\left|\frac{dp_m^{post}}{ds}\right| > \left|\frac{dp_3^{post}}{ds}\right|$. Therefore, $p_m^{post} > p_3^{post}$ for any s. If we set $s^{(2)} = 1$, the result stated is satisfied.
(Case II) Following the same procedure as in the proof of Lemma 1.1(a), we can show that when $s \to 1$ ($w_m \to 0$), $p_m^{post} < p_3^{post} < p_1^{pre}$ if $\frac{\partial \pi_m}{\partial p_m}|_{p_m=p_3=...=p_n=p_1^{pre}} < 0$. Since $p_m^{post} > p_3^{post} > p_1^{pre}$ when $s = 0$ and $p_m^{post} < p_3^{post} < p_1^{pre}$ when $s \to 1$, there exists $s^{(2)} \in (0, 1)$ such that $p_m^{post} = p_1^{pre}$ at $s = s^{(2)}$. In addition, by substituting $p_m^{post} = p_1^{pre}$ into (A.3), we can verify $p_3^{post} = p_1^{pre}$ at $s = s^{(2)}$. By Proposition 1.2(a), if $s^{(1)} = 0$, p_m^{post} and p_3^{post} decrease with s, and otherwise they first increase with s when $s < s^{(1)}$ and then decrease with s when $s > s^{(1)}$. Furthermore, $\left|\frac{dp_m^{post}}{ds}\right| > \left|\frac{dp_3^{post}}{ds}\right|$ as shown in case I. Therefore, $p_m^{post} > p_1^{pre}$, $p_3^{post} > p_1^{pre}$ and $p_m^{post} > p_3^{post}$ if and only if $s < s^{(2)}$, where $s^{(2)}$ ($> s^{(1)}$) is unique.

Finally, we prove that $s^{(2)}$ is nonincreasing with ρ for each of the two cases considered above. *(Case I)* As shown above, if $\frac{\partial \pi_m}{\partial p_m}|_{p_m=p_3=...=p_n=p_1^{pre}} \geq 0$ when $w_m \to 0$, then $s^{(2)} = 1$. After substituting the optimal safety stock $y_m = \sigma_m \Phi^{-1}\left(1 - 2w/\left(p_1 + p_2\right)\right)$ into (1.4) and differentiating it with respect to p_1, we obtain

$$\frac{\partial \pi_m(\mathbf{p})}{\partial p_1}|_{p_1=p_2=p_3=...=p_n=p_1^{pre}} = a - bp_1^{pre} - \left(b + \gamma\frac{n-2}{n}\right)(p_1^{pre} - w_m) - \frac{1}{2}\sigma_m R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_1^{pre}}\right)\right). \quad \text{(A.13)}$$

Since $\lim_{w_m \to 0} \frac{\partial \pi_m(\mathbf{p})}{\partial p_m}|_{p_m=p_3=\ldots=p_n=p_1^{pre}} = -\left(2b + \gamma\frac{n-2}{n}\right)p_1^{pre} + a \geq 0$ is independent of ρ, $s^{(2)} = 1$ for any ρ in this case.

(Case II) Define $w_m^{(2)}$ such that $s^{(2)} = (w - w_m^{(2)})/w$. We prove $dw_m^{(2)}/d\sigma_m > 0$, which implies $ds^{(2)}/d\rho < 0$ because $\sigma_m = \sigma\sqrt{2+2\rho}$. Since $p_m^{post} = p_3^{post} = p_1^{pre}$ at $s = s^{(2)}$, $\frac{\partial \pi_m(\mathbf{p})}{\partial p_m}|_{p_m=p_3=\ldots=p_n=p_1^{pre}} = 0$ at $s = s^{(2)}$. Differentiating both sides of $\frac{\partial \pi_m(\mathbf{p})}{\partial p_m}|_{p_m=p_3=\ldots=p_n=p_1^{pre}} = 0$ (see (A.13)) with respect to σ_m and setting $w_m = w_m^{(2)}$ result in

$$\frac{dw_m^{(2)}}{d\sigma_m} = \frac{1}{2}R\left(\Phi^{-1}\left(1 - \frac{w_m^{(2)}}{p_1^{pre}}\right)\right)\left\{b + \frac{n-2}{n}\gamma - \frac{\sigma_m w_m^{(2)}}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m^{(2)}}{p_1^{pre}}\right)\right)\left(p_1^{pre}\right)^2}\right\}^{-1}.$$

To complete the proof by showing $dw_m^{(2)}/d\sigma_m > 0$, it suffices to show $b+\frac{n-2}{n}\gamma > \frac{\sigma_m w_m^{(2)}}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m^{(2)}}{p_1^{pre}}\right)\right)\left(p_1^{pre}\right)^2}$. Since $s^{(2)} > s^{(1)}$ (see above), by Proposition 1.2(a), $dp_m^{post}/dw_m > 0$ at $s = s^{(2)}$. From (A.7) and (A.8), $dp_m^{post}/dw_m > 0$ implies $b + \frac{n-2}{n}\gamma > \frac{\sigma_m w_m^{(2)}}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m^{(2)}}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^2}$. Since $p_m^{post} = p_1^{pre}$ at $s = s^{(2)}$, the result follows.

Taken in sum, $s^{(2)}$ is determined as follows. From (A.13), if $\lim_{w_m \to 0} \frac{\partial \pi_m(\mathbf{p})}{\partial p_m}|_{p_m=p_3=\ldots=p_n=p_1^{pre}} = -\left(2b + \gamma\frac{n-2}{n}\right)p_1^{pre} + a \geq 0$ (i.e., case I), then $s^{(2)} = 1$. Otherwise $s^{(2)}$ is the unique s that solves the following equation:

$$a - bp_1^{pre} - \left(b + \gamma\frac{n-2}{n}\right)\left\{p_1^{pre} - w\left(1-s\right)\right\} - \frac{1}{2}\sigma_m R\left(\Phi^{-1}\left(1 - \frac{w(1-s)}{p_1^{pre}}\right)\right) = 0.$$

(b) We first compare $\pi_m^{post}/2$ and π_1^{pre}. Since $\pi_m^{post}/2 > \pi_1^{pre}$ when $s = 0$ and $\rho = 1$ by Lemma 1.1(b), π_m^{post} increases with s by Proposition 1.2(b), and π_m^{post} decreases with ρ by Proposition 1.1(b), $\pi_m^{post}/2 > \pi_1^{pre}$ for any $0 \leq s < 1$ and $\rho \leq 1$.

Next, we compare π_3^{post} and π_1^{pre}. Since $p_m^{post} = p_3^{post} = p_1^{pre}$ at $s = s^{(2)}$ by part (a), $\pi_3^{post} = \pi_1^{pre}$ at $s = s^{(2)}$ by (1.2). Since $\pi_3^{post} > \pi_1^{pre}$ when $s = 0$, π_3^{post} increases (resp., decreases) with s when $s < s^{(1)}$ (resp., $s > s^{(1)}$) and $s^{(1)} < s^{(2)}$, $\pi_3^{post} < \pi_1^{pre}$ if and only if $s > s^{(2)}$.

Finally, let us compare $\pi_m^{post}/2$ and π_3^{post}. We consider the following two cases: (case I) There does not exist any s in $[0, 1)$ such that $\pi_m^{post}/2 = \pi_3^{post}$, and (case II) There exists at least one $s \in [0, 1)$ such that $\pi_m^{post}/2 = \pi_3^{post}$.

(Case I) In this case, either $\pi_m^{post}/2 > \pi_3^{post}$ for all $s \in [0, 1)$, or $\pi_m^{post}/2 < \pi_3^{post}$ for all $s \in [0, 1)$. We set $s^{(3)} = 0$ for the first case, and $s^{(3)} = 1$ for the second case. The second case is only possible if $s^{(2)} = 1$. This is because $\pi_m^{post}/2 > \pi_1^{pre} = \pi_3^{post}$ at $s = s^{(2)}$ if $s^{(2)} < 1$.

(Case II) Define $s^{(3)}$ as the largest s that satisfies $\pi_m^{post}/2 = \pi_3^{post}$. (Note that multiple s's may satisfy $\pi_m^{post}/2 = \pi_3^{post}$ because π_3^{post} may first increase with s and then decrease with s from Proposition 1.2; see our discussion in the main body.) The result that $\pi_m^{post}/2 > \pi_3^{post}$ if $s > s^{(3)}$

(where $s^{(3)} < s^{(2)}$) follows from the following earlier results: (i) $\pi_m^{post}/2 > \pi_1^{pre} = \pi_3^{post}$ at $s = s^{(2)}$; and (ii) π_m^{post} is increasing with $s \in \left(s^{(2)}, 1\right)$, and π_3^{post} is decreasing with $s \in \left(s^{(2)}, 1\right)$ because of Proposition 1.2(b) and $s^{(1)} < s^{(2)}$ from part (a).

Taken in sum, $s^{(3)}$ is the maximum of 0 and the largest s that solves the following equation:

$$\left\{p_m^{post} - w\left(1-s\right)\right\}\left\{a - \left(b + \tfrac{n-2}{n}\gamma\right)p_m^{post} + \tfrac{n-2}{n}\gamma p_3^{post}\right\}$$
$$-w\left(1-s\right)\tfrac{\sigma_m}{2}\Phi^{-1}\left(1 - \tfrac{w(1-s)}{p_m^{post}}\right) - p_m^{post}\tfrac{\sigma_m}{2}R\left(\Phi^{-1}\left(1 - \tfrac{w(1-s)}{p_m^{post}}\right)\right)$$
$$= \left\{p_3^{post} - w\right\}\left\{a - \left(b + \tfrac{2}{n}\gamma\right)p_3^{post} + \tfrac{2}{n}\gamma p_m^{post}\right\} - w\sigma\Phi^{-1}\left(1 - \tfrac{w}{p_3^{post}}\right) - p_3^{post}\sigma R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_3^{post}}\right)\right),$$

where the left-hand side is the expression of $\pi_m^{post}/2$ obtained by substituting $w_m = w\left(1-s\right)$ and $y_m^{post} = \sigma_m\Phi^{-1}\left(1 - w\left(1-s\right)/p_m^{post}\right)$ to (1.4), and the right-hand side is the expression of π_3^{post} obtained by substituting $y_3^{post} = \sigma\Phi^{-1}\left(1 - w/p_3^{post}\right)$ to (1.2). $\square$

Derivation of the demand from the utility function. We derive the demand function $D_i = L_i\left(\mathbf{p}\right) + \widetilde{\varepsilon}_i$ (where $L_i\left(\mathbf{p}\right)$ is given in (1)) from the utility function of a representative consumer given in (1.7). The representative consumer determines her consumption bundle $\mathbf{D}$ that maximizes her utility gain: $u\left(\mathbf{D}\right) - \sum_{i=1}^{n} p_i D_i$. The first order conditions for this maximization problem are $\partial u\left(\mathbf{D}\right)/\partial D_i - p_i = 0$ for $i = 1, 2, \ldots, n$, which can be obtained from (1.7) as follows:

$$\frac{\partial u\left(\mathbf{D}\right)}{\partial D_i} - p_i = \frac{1 + \frac{\gamma}{nb}}{b+\gamma}\left(a + \widetilde{\varepsilon}_i - D_i\right) + \frac{\gamma}{nb\left(b+\gamma\right)}\sum_{j\neq i}^{n}\left(a + \widetilde{\varepsilon}_j - D_j\right) - p_i = 0. \tag{A.14}$$

Now we solve for n unknown D_i $(i = 1, 2, \ldots, n)$ from n first order conditions. By adding all first order conditions and rearranging terms, we obtain $\sum_{j=1}^{n}\left(a + \widetilde{\varepsilon}_j - D_j\right) = b\sum_{j=1}^{n} p_j$. Substituting this expression into (A.14) yields

$$\frac{1}{b+\gamma}\left(a + \widetilde{\varepsilon}_i - D_i\right) + \frac{\gamma}{n\left(b+\gamma\right)}\sum_{j=1}^{n} p_j - p_i = 0.$$

From this equation, we can solve for D_i as follows:

$$D_i(\mathbf{p},\widetilde{\varepsilon}_i) = a + \widetilde{\varepsilon}_i + \frac{\gamma}{n}\sum_{j=1}^{n} p_j - \left(b+\gamma\right)p_i = a - bp_i + \gamma\left(\frac{1}{n}\sum_{j=1}^{n} p_j - p_i\right) + \widetilde{\varepsilon}_i. \quad \square \tag{A.15}$$

Derivation of the consumer surplus function. Total consumer surplus is the utility the representative consumer derives from the consumption bundle less the money spent on the consumption, so $cs(\mathbf{p}, \mathbf{D}) = u\left(\mathbf{D}\right) - \sum_{i=1}^{n} p_i D_i$. Substituting the expression of $u\left(\mathbf{D}\right)$ given in

(1.7) into $cs(\mathbf{p},\mathbf{D})$ yields

$$\begin{aligned} cs(\mathbf{p},\mathbf{D}) &= \sum_{i=1}^{n}\left[\frac{1+\frac{\gamma}{nb}}{b+\gamma}\left(a+\widetilde{\varepsilon}_i-\frac{1}{2}D_i\right)+\frac{\gamma}{nb\,(b+\gamma)}\sum_{j\neq i}^{n}\left(a+\widetilde{\varepsilon}_j-\frac{1}{2}D_j\right)\right]D_i-\sum_{i=1}^{n}p_iD_i \\ &= \sum_{i=1}^{n}\left[\frac{1+\frac{\gamma}{nb}}{b+\gamma}\left(a+\widetilde{\varepsilon}_i\right)+\frac{\gamma}{nb\,(b+\gamma)}\sum_{j\neq i}^{n}\left(a+\widetilde{\varepsilon}_j\right)-\frac{1}{2\,(b+\gamma)}\left(D_i+\frac{\gamma}{nb}\sum_{j=1}^{n}D_j\right)-p_i\right]D_i. \end{aligned} \tag{A.16}$$

To simplify (A.16), we compute $D_i+\frac{\gamma}{nb}\sum_{j=1}^{n}D_j$ using the expression of D_i in (A.15) as follows:

$$\begin{aligned} & D_i+\frac{\gamma}{nb}\sum_{j=1}^{n}D_j \\ = \; & a+\widetilde{\varepsilon}_i-bp_i+\gamma\left(\frac{1}{n}\sum_{j=1}^{n}p_j-p_i\right)+\frac{\gamma}{nb}\sum_{j=1}^{n}\left\{a+\widetilde{\varepsilon}_j-bp_j+\gamma\left(\frac{1}{n}\sum_{k=1}^{n}p_j-p_j\right)\right\} \\ = \; & \left(1+\frac{\gamma}{nb}\right)\left(a+\widetilde{\varepsilon}_i\right)+\frac{\gamma}{nb}\sum_{j\neq i}^{n}\left(a+\widetilde{\varepsilon}_j\right)-\left(b+\gamma\right)p_i. \end{aligned}$$

Finally, after substituting the above expression of $D_i+\frac{\gamma}{nb}\sum_{j=1}^{n}D_j$ and the expression of D_i in (A.15) into (A.16), we obtain the following after simplification:

$$cs\left(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}\right)=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{1+\frac{\gamma}{nb}}{b+\gamma}\left(a+\widetilde{\varepsilon}_i\right)+\frac{\gamma}{nb\,(b+\gamma)}\sum_{j\neq i}^{n}\left(a+\widetilde{\varepsilon}_j\right)-p_i\right]\left\{L_i\left(\mathbf{p}\right)+\widetilde{\varepsilon}_i\right\}. \quad \square$$

Detailed Proof of Proposition 1.5. When $s^{(2)}=1$, it is possible $E[cs^{post}]<E[cs^{pre}]$ for all $s<1$. In this case, we define $s^{(cs)}=1$. When $s^{(2)}<1$, we prove the existence of $s^{(cs)}\in[0,s^{(2)})$ in the following three steps. In step 1, we obtain a lower bound of $E[cs^{post}]$, denoted by $E\left[cs^{lbd}\right]$. In step 2, we show $E[cs^{lbd}]>E[cs^{pre}]$ at $s=s^{(2)}$. In step 3, we show that $E\left[cs^{lbd}\right]$ increases with s in $[s^{(2)},1)$.

Step 1. Since $E\left[cs^{post}\right]=(n-2)\,E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]+E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$, we first find the expressions of $E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$ and $E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$ to find a lower bound of $E\left[cs^{post}\right]$. To find the expression of $E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$, we simplify $cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)$ into $cs_3\left(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}\right)=v_3\left(p_3,\widetilde{\varepsilon}\right)\left\{L_3\left(\mathbf{p}\right)+\widetilde{\varepsilon}_3\right\},$

where $v_3(p_3, \widetilde{\boldsymbol{\varepsilon}})$ and its expected value are defined from (1.8) as follows:

$$v_3(p_3, \widetilde{\boldsymbol{\varepsilon}}) = \frac{1}{2(b+\gamma)}\left\{\left(1+\frac{\gamma}{nb}\right)(a+\widetilde{\varepsilon}_3) + \frac{\gamma}{nb}\sum_{j\neq 3}^{n}(a+\widetilde{\varepsilon}_j) - (b+\gamma)p_3\right\}, \quad (A.17)$$

$$E[v_3(p_3, \widetilde{\boldsymbol{\varepsilon}})] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3(p_3, \boldsymbol{\varepsilon}) f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon} = \frac{a-bp_3}{2b}. \quad (A.18)$$

Let $z_3 = \Phi^{-1}\left(l_3^{post}\right)$. Using (1.9), (A.17) and (A.18), we can write $E\left[cs_3\left(\mathbf{p}^{post}, \widetilde{\boldsymbol{\varepsilon}}\right)\right]$ as follows:

$$\begin{aligned} E\left[cs_3\left(\mathbf{p}^{post}, \widetilde{\boldsymbol{\varepsilon}}\right)\right] &= \int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3\left(p_3^{post}, \boldsymbol{\varepsilon}\right)\left\{L_3\left(\mathbf{p}^{post}\right)+\varepsilon_3\right\} f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon}_{-3} d\varepsilon_3 \\ &\quad + \int_{\sigma z_3}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3\left(p_3^{post}, \boldsymbol{\varepsilon}\right)\left\{L_3\left(\mathbf{p}^{post}\right)+\sigma z_3\right\} f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon}_{-3} d\varepsilon_3. \\ &= \frac{a-bp_3^{post}}{2b} L_3\left(\mathbf{p}^{post}\right) + \int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3\left(p_3^{post}, \boldsymbol{\varepsilon}\right)\varepsilon_3 f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon}_{-3} d\varepsilon_3 \\ &\quad + \int_{\sigma z_3}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3\left(p_3^{post}, \boldsymbol{\varepsilon}\right)\sigma z_3 f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon}_{-3} d\varepsilon_3, \quad (A.19) \end{aligned}$$

where $\boldsymbol{\varepsilon}_{-3} = (\varepsilon_1, \varepsilon_2, \varepsilon_4, \ldots \varepsilon_n)$. We next simplify the second and third terms in (A.19). Let $f_3(\varepsilon_3)$ and $f_{3,j}(\varepsilon_3, \varepsilon_j)$ denote the marginal density of ε_3 and the joint density of $(\varepsilon_3, \varepsilon_j)$, respectively. Then substituting (A.17) into the second term in (A.19) yields

$$\begin{aligned} &\int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} v_3\left(p_3^{post}, \boldsymbol{\varepsilon}\right)\varepsilon_3 f(\boldsymbol{\varepsilon})\, d\boldsymbol{\varepsilon}_{-3} d\varepsilon_3 \quad (A.20) \\ &= \frac{1}{2}\int_{-\infty}^{\sigma z_3}\left\{\left(\frac{a}{b}-p_3\right)\varepsilon_3 + \frac{\left(1+\frac{\gamma}{nb}\right)}{(b+\gamma)}\varepsilon_3^2\right\} f_3(\varepsilon_3)\, d\varepsilon_3 + \frac{\gamma}{2nb(b+\gamma)}\sum_{j\neq 3}^{n}\int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\varepsilon_3\varepsilon_j f_{3,j}(\varepsilon_3, \varepsilon_j)\, d\varepsilon_j d\varepsilon_3 \\ &= \frac{1+\frac{\gamma}{nb}}{2(b+\gamma)}\sigma^2\left\{\Phi(z_3) - z_3\phi(z_3)\right\} - \frac{a-bp_3}{2b}\sigma\phi(z_3) + \frac{\gamma}{2nb(b+\gamma)}\sum_{j\neq 3}^{n}\int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\varepsilon_3\varepsilon_j f_{3,j}(\varepsilon_3, \varepsilon_j)\, d\varepsilon_j d\varepsilon_3, \end{aligned}$$

where the last equality is due to $\int_{-\infty}^{\sigma z_3}\varepsilon_3 f_3(\varepsilon_3)\, d\varepsilon_3 = \sigma\phi(z_3)$ and $\int_{-\infty}^{\sigma z_3}\varepsilon_3^2 f_3(\varepsilon_3)\, d\varepsilon_3 = \sigma^2\left\{\Phi(z_3) - z_3\phi(z_3)\right\}$. To calculate the second term in (A.20), we let $\widetilde{u}_3 = \widetilde{\varepsilon}_3/\sigma$ and $\widetilde{u}_j = (\widetilde{\varepsilon}_j - \rho\widetilde{\varepsilon}_3)/\sigma$. We have $E[\widetilde{u}_3] = E[\widetilde{u}_j] = 0$, $Var[\widetilde{u}_3] = 1$, and $Var[\widetilde{u}_j] = 1-\rho^2$. In addition, since $Cov(\widetilde{u}_3, \widetilde{u}_j) = Cov(\widetilde{\varepsilon}_3, \widetilde{\varepsilon}_j)/\sigma^2 - \rho Cov(\widetilde{\varepsilon}_3, \widetilde{\varepsilon}_3)/\sigma^2 = 0$, $\widetilde{u}_3$ and $\widetilde{u}_j$ are independent. So we can calculate the integral in the second term of (A.20) as follows:

$$\int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\varepsilon_3\varepsilon_j f_{3,j}\left(\varepsilon_3,\varepsilon_j\right)d\varepsilon_j d\varepsilon_3=\sigma^2\int_{-\infty}^{z_3}\int_{-\infty}^{\infty}u_3\left(u_j+\rho u_3\right)\phi\left(u_3\right)\frac{\phi\left(\frac{u_j}{\sqrt{1-\rho^2}}\right)}{\sqrt{1-\rho^2}}du_j du_3$$

$$=\sigma^2\rho\int_{-\infty}^{z_3}u_3^2\phi\left(u_3\right)du_3\int_{-\infty}^{\infty}\frac{\phi\left(\frac{u_j}{\sqrt{1-\rho^2}}\right)}{\sqrt{1-\rho^2}}du_j=\sigma^2\rho\left\{\Phi\left(z_3\right)-z_3\phi\left(u_3\right)\right\},$$

where the equality is due to $\int_{-\infty}^{\infty}u_j\frac{\phi\left(\frac{u_j}{\sqrt{1-\rho^2}}\right)}{\sqrt{1-\rho^2}}du_j=E\left[\widetilde{u}_j\right]=0$. By substituting this into (A.20), we can simplify the second term in (A.19) as follows:

$$\int_{-\infty}^{\sigma z_3}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}v_3\left(p_3^{post},\boldsymbol{\varepsilon}\right)\varepsilon_3 f\left(\boldsymbol{\varepsilon}\right)d\boldsymbol{\varepsilon}_{-3}d\varepsilon_3$$

$$=-\frac{a-bp_3^{post}}{2b}\sigma\phi\left(z_3\right)+\frac{\left(1+\frac{\gamma}{nb}\right)}{2\left(b+\gamma\right)}\sigma^2\left\{\Phi\left(z_3\right)-z_3\phi\left(z_3\right)\right\}+\frac{\left(n-1\right)\gamma\sigma^2\rho\left\{\Phi\left(z_3\right)-z_3\phi\left(z_3\right)\right\}}{2nb\left(b+\gamma\right)}.$$

Similarly, we can simplify the third term in (A.19) as follows:

$$\int_{\sigma z_3}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}v_3\left(p_3^{post},\boldsymbol{\varepsilon}\right)\sigma z_3 f\left(\boldsymbol{\varepsilon}\right)d\boldsymbol{\varepsilon}_{-3}d\varepsilon_3$$

$$=\frac{a-bp_3^{post}}{2}\sigma z_3\left\{1-\Phi\left(z_3\right)\right\}+\frac{\left(1+\frac{\gamma}{nb}\right)}{2\left(b+\gamma\right)}\sigma^2 z_3\phi\left(z_3\right)+\frac{\left(n-1\right)\gamma\sigma^2\rho z_3\phi\left(z_3\right)}{2nb\left(b+\gamma\right)}.$$

So we can rewrite (A.19) as follows after combining terms:

$$\begin{aligned}E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]&=\frac{a-bp_3^{post}}{2b}\left[L_3\left(\mathbf{p}^{post}\right)-\sigma\phi\left(z_3\right)+\sigma z_3\left\{1-\Phi\left(z_3\right)\right\}\right]+\frac{\sigma^2\Phi\left(z_3\right)}{2\left(b+\gamma\right)}\left\{1+\frac{\gamma}{nb}+\frac{\left(n-1\right)\gamma\rho}{nb}\right\}\\&=\frac{a-bp_3^{post}}{2b}\left\{L_3\left(\mathbf{p}^{post}\right)-\sigma R\left(z_3\right)\right\}+\frac{\sigma^2\Phi\left(z_3\right)}{2\left(b+\gamma\right)}\left\{1+\frac{\gamma}{nb}+\frac{\gamma}{nb}\left(n-1\right)\rho\right\},\end{aligned}\tag{A.21}$$

where the last equality is due to $R\left(z_3\right)=\phi\left(z_3\right)-z_3\left\{1-\Phi\left(z_3\right)\right\}$.

We next find the expression of $E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$. Letting $p_1=p_2=p_m$, we can write $cs_m\left(\mathbf{p},\boldsymbol{\varepsilon}\right)$ as

$$\begin{aligned}cs_m\left(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}\right)&=cs_1\left(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}\right)+cs_1\left(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}\right)\\&=\frac{1}{2\left(b+\gamma\right)}\sum_{i=1}^{2}\widetilde{\varepsilon}_i\left\{L_i\left(\mathbf{p}\right)+\widetilde{\varepsilon}_i\right\}+\frac{1}{2}\left\{\frac{a}{b}+\frac{\gamma}{nb\left(b+\gamma\right)}\sum_{j=1}^{n}\widetilde{\varepsilon}_j-p_m\right\}\sum_{i=1}^{2}\left\{L_i\left(\mathbf{p}\right)+\widetilde{\varepsilon}_i\right\}\\&=\frac{\widetilde{\varepsilon}_1^2+\widetilde{\varepsilon}_2^2}{2\left(b+\gamma\right)}+\frac{\widetilde{\varepsilon}_1+\widetilde{\varepsilon}_2}{4\left(b+\gamma\right)}L_m\left(\mathbf{p}\right)+\frac{1}{2}\left\{\frac{a}{b}+\frac{\gamma}{nb\left(b+\gamma\right)}\sum_{j=1}^{n}\widetilde{\varepsilon}_j-p_m\right\}\left\{L_m\left(\mathbf{p}\right)+\widetilde{\varepsilon}_1+\widetilde{\varepsilon}_2\right\},\end{aligned}$$

where the last equality is due to $L_1(\mathbf{p}) = L_2(\mathbf{p}) = L_m(\mathbf{p})/2$. Let $\widetilde{\varepsilon}_m = \widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2$ and $\widetilde{\zeta}_m = \widetilde{\varepsilon}_1 - \widetilde{\varepsilon}_2$. We can show that $\widetilde{\varepsilon}_m$ and $\widetilde{\zeta}_m$ are independent, and that the correlation coefficient between $\widetilde{\varepsilon}_m$ and $\widetilde{\varepsilon}_j$ $(j \geq 3)$ is $\frac{2\rho}{\sqrt{2+2\rho}}$. Substituting these new variables into the above equation and noting that $\widetilde{\varepsilon}_1^2 + \widetilde{\varepsilon}_2^2 = \frac{1}{2}\widetilde{\varepsilon}_m^2 + \frac{1}{2}\widetilde{\zeta}_m^2$, we obtain

$$cs_m(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}}) = \frac{\widetilde{\zeta}_m^2}{4(b+\gamma)} + \frac{1}{2}\left\{\frac{a}{b} - p_m + \frac{\widetilde{\varepsilon}_m}{2(b+\gamma)}\left(1 + \frac{2\gamma}{nb}\right) + \sum_{j=3}^{n} \frac{\gamma\widetilde{\varepsilon}_j}{nb(b+\gamma)}\right\}\{L_m(\mathbf{p}) + \widetilde{\varepsilon}_m\}. \tag{A.22}$$

Let $z_m = \Phi^{-1}\left(l_m^{post}\right)$. Since $\widetilde{\zeta}_m$ and $\widetilde{\varepsilon}_m$ are independent, the expected value of the first term in (A.22) can be expressed as

$$\begin{aligned} &\tfrac{1}{4(b+\gamma)}\int_{-\infty}^{\infty} \zeta_m^2 \frac{\phi\left(\frac{\zeta_m}{\sigma\sqrt{2-2\rho}}\right)}{\sigma\sqrt{2-2\rho}} d\zeta_m \left\{\int_{-\infty}^{\sigma_m z_m} \frac{\phi(\varepsilon_m)}{\sigma_m} d\varepsilon_m + \int_{\sigma_m z_m}^{\infty} \frac{L_m + \sigma_m z_m}{L_m + \varepsilon_m} \frac{\phi\left(\frac{\varepsilon_m}{\sigma_m}\right)}{\sigma_m} d\varepsilon_m\right\} \\ &= \tfrac{\sigma^2(2-2\rho)}{4(b+\gamma)}\left\{\Phi(z_m) + \int_{z_m}^{\infty} \tfrac{L_m+\sigma_m z_m}{L_m+\sigma_m t}\phi(t)\,dt\right\}. \end{aligned}$$

The second term in (A.22) is similar to $cs_i(\mathbf{p},\widetilde{\boldsymbol{\varepsilon}})$ in (1.8), so we can obtain its expected value similarly to $E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$. Putting them together, we can write $E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$ as follows (where we use the correlation coefficient of $\frac{2\rho}{\sqrt{2+2\rho}}$ between $\widetilde{\varepsilon}_m$ and $\widetilde{\varepsilon}_j$):

$$\begin{aligned} E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right] &= \frac{\sigma^2(2-2\rho)}{4(b+\gamma)}\left\{\Phi(z_m) + \int_{z_m}^{\infty} \frac{L_m+\sigma_m z_m}{L_m+\sigma_m t}\phi(t)\,dt\right\} + \frac{\sigma^2(2+2\rho)\Phi(z_m)\left(1+\frac{\gamma}{nb}\right)}{2(b+\gamma)} \\ &\quad + \frac{\sigma\Phi(z_m)}{2(b+\gamma)}\frac{(n-2)\gamma\sigma\sqrt{2+2\rho}\frac{2\rho}{\sqrt{2+2\rho}}}{nb} + \frac{a-bp_m^{post}}{2b}\left\{L_m\left(\mathbf{p}^{post}\right) - \sigma_m R(z_m)\right\} \\ &= \frac{\sigma^2(1-\rho)}{2(b+\gamma)}\int_{z_m}^{\infty} \frac{L_m+\sigma_m z_m}{L_m+\sigma_m t}\phi(t)\,dt + \frac{a-bp_m^{post}}{2b}\left\{L_m\left(\mathbf{p}^{post}\right) - \sigma_m R(z_m)\right\} \\ &\quad + \frac{\sigma^2\Phi(z_m)}{b+\gamma}\left\{1 + \frac{\gamma}{nb} + \frac{\gamma}{nb}(n-1)\rho\right\}. \end{aligned} \tag{A.23}$$

Define $E\left[cs_m^{lbd}\right] = E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right] - \frac{\sigma^2(1-\rho)}{2(b+\gamma)}\int_{z_m}^{\infty} \frac{L_m+\sigma_m z_m}{L_m+\sigma_m t}\phi(t)\,dt$ $\left(< E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]\right)$. From (A.23) we get

$$E\left[cs_m^{lbd}\right] = \frac{a-bp_m^{post}}{2b}\left\{L_m\left(\mathbf{p}^{post}\right) - \sigma_m R(z_m)\right\} + \frac{\sigma^2\Phi(z_m)}{b+\gamma}\left\{1 + \frac{\gamma}{nb} + \frac{\gamma}{nb}(n-1)\rho\right\}. \tag{A.24}$$

Finally, we let $E\left[cs^{lbd}\right] = E\left[cs_m^{lbd}\right] + (n-2)E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$. Then $E\left[cs^{lbd}\right] \leq E\left[cs^{post}\right] = E\left[cs_m\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right] + (n-2)E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$.

Step 2. Following the same procedure as in the calculation of $E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$, we can express $E\left[cs_1\left(\mathbf{p}^{pre},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$ as follows:

$$E\left[cs_1\left(\mathbf{p}^{pre},\widetilde{\boldsymbol{\varepsilon}}\right)\right]=\frac{a-bp_1^{pre}}{2b}\left\{L_1\left(\mathbf{p}^{pre}\right)-\sigma R\left(\Phi^{-1}\left(l_1^{pre}\right)\right)\right\}+\frac{\sigma^2 l_1^{pre}}{2\left(b+\gamma\right)}\left\{1+\frac{\gamma}{nb}+\frac{\gamma}{nb}\left(n-1\right)\rho\right\}. \tag{A.25}$$

Note that $E\left[cs^{pre}\right]=nE\left[cs_1\left(\mathbf{p}^{pre},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$. Using $\mathbf{p}^{post}=\mathbf{p}^{pre}$ and $l_3^{post}=l_3^{pre}$ at $s=s^{(2)}$, we can compute $E\left[cs^{lbd}\right]-E\left[cs^{pre}\right]$ from (A.21), (A.24) and (A.25) as

$$\begin{aligned}E\left[cs^{lbd}\right]-E\left[cs^{pre}\right] &= \frac{\sigma^2\left(l_m^{post}-l_1^{pre}\right)}{b+\gamma}\left\{1+\frac{\gamma}{nb}+\frac{\gamma}{nb}\left(n-1\right)\rho\right\}\\ &\quad+\frac{a-bp_1^{pre}}{2b}\left\{2\sigma R\left(\Phi^{-1}\left(l_1^{pre}\right)\right)-\sigma_m R\left(\Phi^{-1}\left(l_m^{post}\right)\right)\right\}\\ &\geq 0,\end{aligned}$$

where the inequality is due to $l_m^{post}>l_1^{pre}$ at $s=s^{(2)}$ and $2\sigma\geq\sigma_m$.

Step 3. We can express $\frac{dE\left[cs^{lbd}\right]}{ds}$ as follows: $\frac{dE\left[cs^{lbd}\right]}{ds}=\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}\frac{dp_m^{post}}{ds}+\frac{\partial E\left[cs^{lbd}\right]}{\partial l_m^{post}}\frac{\partial l_m^{post}}{\partial s}$. Since $\frac{dp_m^{post}}{ds}<0$ and $\frac{\partial l_m^{post}}{\partial s}>0$ for $s\geq s^{(2)}$, if $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}<0$ and $\frac{\partial E\left[cs^{lbd}\right]}{\partial l_m^{post}}>0$, then $\frac{dE\left[cs^{lbd}\right]}{ds}>0$ for $s\geq s^{(2)}$. Below we first prove that $\frac{\partial E\left[cs^{lbd}\right]}{\partial l_m^{post}}>0$ and then derive a condition for $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}<0$.

Since $\frac{\partial E\left[cs^{lbd}\right]}{\partial l_m^{post}}=\frac{\partial E\left[cs^{lbd}\right]}{\partial z_m}\frac{dz_m}{dl_m^{post}}$ and $\frac{dz_m}{dl_m^{post}}=\frac{1}{\phi(z_m)}>0$, it suffices to show $\frac{\partial E\left[cs^{lbd}\right]}{\partial z_m}>0$ for $\frac{\partial E\left[cs^{lbd}\right]}{\partial l_m^{post}}>0$. From $E\left[cs^{lbd}\right]=E\left[cs_m^{lbd}\right]+\left(n-2\right)E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$ and (A.24), we get

$$\frac{\partial E\left[cs^{lbd}\right]}{\partial z_m}=\frac{\partial E\left[cs_m^{lbd}\right]}{\partial z_m}=\frac{a-bp_m^{post}}{2b}\sigma_m\{1-\Phi\left(z_m\right)\}+\frac{\sigma^2\phi(z_m)}{b+\gamma}\left\{1+\frac{\gamma}{nb}+\frac{\gamma}{nb}\left(n-1\right)\rho\right\}>0.$$

We next derive a condition for $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}<0$. Since $E\left[cs^{lbd}\right]=E\left[cs_m^{lbd}\right]+\left(n-2\right)E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]$, we can express $dE\left[cs^{lbd}\right]/dp_m^{post}$ as follows:

$$\begin{aligned}\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}} &= \frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}}+\left(n-2\right)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}}+\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_3^{post}}\frac{dp_3^{post}}{dp_m^{post}} \qquad \text{(A.26)}\\ &\quad+\left(n-2\right)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_3^{post}}\frac{dp_3^{post}}{dp_m^{post}}+\left(n-2\right)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial l_3^{post}}\frac{\partial l_3^{post}}{\partial p_3^{post}}\frac{dp_3^{post}}{dp_m^{post}}.\end{aligned}$$

To simplify $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}$, we next derive $\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}}$, $\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_3^{post}}$, $\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_3^{post}}$ and $\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}}$. From (A.24), we get

$$\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} = -\frac{1}{2}\left\{L_m\left(\mathbf{p}^{post}\right) - \sigma_m R\left(z_m\right)\right\} - \frac{a - bp_m^{post}}{b}\left(b + \frac{n-2}{n}\gamma\right); \tag{A.27}$$

$$\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_3^{post}} = \frac{a - bp_m^{post}}{b}\frac{n-2}{n}\gamma. \tag{A.28}$$

Similarly, from (A.21) we can compute

$$\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_3^{post}} = -\frac{1}{2}\left\{L_3\left(\mathbf{p}^{post}\right) - \sigma R\left(z_3\right)\right\} - \frac{a - bp_3^{post}}{2b}\left(b + \frac{2}{n}\gamma\right); \tag{A.29}$$

$$\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}} = \frac{\gamma}{nb}\left(a - bp_3^{post}\right). \tag{A.30}$$

Using (A.27) and (A.30), we can show that the first two terms in (A.26) are negative because

$$\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} + \frac{(n-2)\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}} =$$
$$-\tfrac{1}{2}\left\{L_m\left(\mathbf{p}^{post}\right) - \sigma_m R\left(z_m\right)\right\} - \left(a - bp_m^{post}\right) - \tfrac{n-2}{n}\gamma\left(p_3^{post} - p_m^{post}\right) < 0,$$

where the inequality is due to $p_m^{post} \leq p_3^{post}$ for $s \geq s^{(2)}$. Furthermore, since $0 < \frac{dp_3^{post}}{dp_m^{post}} < 1$,

$$\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} + (n-2)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}} \leq \left\{\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} + (n-2)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}}\right\}\frac{dp_3^{post}}{dp_m^{post}} < 0.$$

Using this inequality, we can get an upper bound of $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}}$ from (A.26):

$$\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}} \leq$$
$$\left[\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} + \frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_3^{post}} + (n-2)\left\{\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}} + \frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_3^{post}} + \frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial l_3^{post}}\frac{\partial l_3^{post}}{\partial p_3^{post}}\right\}\right]\frac{dp_3^{post}}{dp_m^{post}}.$$

Thus, $\frac{dE\left[cs^{lbd}\right]}{dp_m^{post}} < 0$ if the following condition holds:

$$\frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_m^{post}} + \frac{\partial E\left[cs_m^{lbd}\right]}{\partial p_3^{post}} + (n-2)\left\{\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_m^{post}} + \frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial p_3^{post}} + \frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial l_3^{post}}\frac{\partial l_3^{post}}{\partial p_3^{post}}\right\} < 0.$$

By substituting (A.27), (A.28), (A.29), and (A.30), this condition can be further simplified into

$$2\left(a - bp_m^{post}\right) - \frac{1}{2}\sigma_m R\left(z_m\right) + (n-2)\left\{\left(a - bp_3^{post}\right) - \frac{\sigma}{2}R\left(z_3\right)\right\} > (n-2)\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\boldsymbol{\varepsilon}}\right)\right]}{\partial l_3^{post}}\frac{\partial l_3^{post}}{\partial p_3^{post}}. \tag{A.31}$$

The left-hand side of (A.31) satisfies the following:

$$2\left(a-bp_m^{post}\right)-\tfrac{1}{2}\sigma_m R\left(z_m\right)+(n-2)\left\{\left(a-bp_3^{post}\right)-\tfrac{\sigma}{2}R\left(z_3\right)\right\}>n\left(a-bp_3^{post}\right)-\tfrac{n\sigma}{2}R\left(z_3\right)$$
$$\geq \tfrac{n}{2}\left(a-bp_3^{post}\right),$$

where the first inequality is due to $p_3^{post}\geq p_m^{post}$ and $z_m>z_3$ (because $l_m^{post}>l_3^{post}$) for $s\geq s^{(2)}$, and the second inequality is due to $L_3\left(\mathbf{p}^{post}\right)-\sigma R\left(z_3\right)>0$ and $L_3\left(\mathbf{p}^{post}\right)=a-bp_3^{post}-\frac{2\gamma}{n}\left(p_3^{post}-p_m^{post}\right)\leq a-bp_3^{post}$. The right-hand side of (A.31) is smaller than $\frac{(n-2)\sigma\left(a-bp_3^{post}\right)w}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)\left(p_3^{post}\right)^2}$ because from (A.21)

$$\begin{aligned}
\frac{\partial E\left[cs_3\left(\mathbf{p}^{post},\widetilde{\varepsilon}\right)\right]}{\partial l_3^{post}} &= \frac{a-bp_3^{post}}{2b}\frac{\sigma w}{\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)p_3^{post}}+\frac{\sigma^2}{2\left(b+\gamma\right)}\left\{1+\frac{\gamma}{nb}+\frac{\gamma}{nb}\left(n-1\right)\rho\right\}\\
&\leq \frac{\left(a-bp_3^{post}\right)\sigma}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)}-\frac{\left(a-bp_3^{post}\right)\sigma\left(1-\frac{w}{p_3^{post}}\right)}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)}+\frac{\sigma^2}{2\left(b+\gamma\right)}\left(1+\frac{\gamma}{b}\right)\\
&= \frac{\left(a-bp_3^{post}\right)\sigma}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)}-\frac{\sigma\left\{\left(a-bp_3^{post}\right)\left(1-\frac{w}{p_3^{post}}\right)-\sigma\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)\right\}}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)}\\
&\leq \frac{\left(a-bp_3^{post}\right)\sigma}{2b\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)},
\end{aligned}$$

where the first inequality is due to $\rho\leq 1$, and the second inequality is due to $a-bp_3^{post}\geq L_3\left(\mathbf{p}^{post}\right)$ for $s\geq s^{(2)}$ and

$$\begin{aligned}
\pi_3^{post} &= L_3\left(\mathbf{p}^{post}\right)\left(p_3^{post}-w\right)-w\sigma\Phi^{-1}\left(l_3^{post}\right)-p_3^{post}\sigma R\left(\Phi^{-1}\left(l_3^{post}\right)\right)\\
&= p_3^{post}\left\{L_3\left(\mathbf{p}^{post}\right)\left(1-\tfrac{w}{p_3^{post}}\right)-\tfrac{w}{p_3^{post}}\sigma\Phi^{-1}\left(l_3^{post}\right)-\sigma R\left(\Phi^{-1}\left(l_3^{post}\right)\right)\right\}\\
&= p_3^{post}\left\{L_3\left(\mathbf{p}^{post}\right)\left(1-\tfrac{w}{p_3^{post}}\right)-\sigma\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)\right\}\\
&> 0.
\end{aligned}$$

By using the bounds of the left- and right-hand sides of (A.31), we finally have the following sufficient condition for (A.31): $b\geq\frac{(n-2)\sigma w}{n\phi\left(\Phi^{-1}\left(l_3^{post}\right)\right)\left(p_3^{post}\right)^2}$. □

Detailed Proof of Corollary 1.1. Before we prove (a) and (b), we first derive the condition for the existence of a unique Nash equilibrium in the post-merger market. To do so, we first obtain n first-order conditions on prices. We then compute the Jacobian matrix J of the left-hand side of the first-order conditions. We finally use J to obtain the conditions for the existence of a unique Nash equilibrium.

We can express the expected profits of a nonparticipant firm i ($i=3,4,...,n$) and the post-merger firm respectively as follows:

$$\pi_i(\mathbf{p}, y_i) = (p_i - w_i)\left\{a_i - \left(b_i + \tfrac{n-1}{n}\gamma\right)p_i + \tfrac{\gamma}{n}\sum_{j\neq i} p_j\right\} - w_i y_i - p_i\sigma_i R\left(\tfrac{y_i}{\sigma_i}\right), \text{ and}$$

$$\pi_m(\mathbf{p}, y_m) = \sum_{j=1}^{2}(p_i - w_m)\left\{a_i - \left(b_i + \tfrac{n-1}{n}\gamma\right)p_i + \tfrac{\gamma}{n}\sum_{j\neq i} p_j\right\} - w_m y_m - (p_1k_1 + p_2k_2)\,\sigma_m R\left(\tfrac{y_m}{\sigma_m}\right),$$

where k_1 (> 0) (resp., k_2) represents a portion of lost sales at location 1 (resp., location 2) with $k_1 + k_2 = 1$. From the expression above, we can compute the first-order conditions on y_m and y_i, and obtain the optimal safety stock for a given price vector $\mathbf{p}$: $y_m^* = \sigma_m\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)$ and $y_i^* = \sigma_i\Phi^{-1}\left(1 - \frac{w_i}{p_i}\right)$, where $\overline{p}_m = p_1k_1 + p_2k_2$. Substituting y_m^* and y_i^* into $\pi_m(\mathbf{p}, y_m)$ and $\pi_3(\mathbf{p}, y_3)$ above, we obtain

$$\begin{aligned}\pi_m(\mathbf{p}) = &\sum_{j=1}^{2}(p_i - w_m)\left\{a_i - \left(b_i + \frac{n-1}{n}\gamma\right)p_i + \frac{\gamma}{n}\sum_{j\neq i} p_j\right\} \\ &- \sigma_m w_m\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right) - \overline{p}_m\sigma_m R\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right), \text{ and}\end{aligned} \tag{A.32}$$

$$\pi_i(\mathbf{p}) = (p_i - w_i)\left\{a_i - \left(b_i + \frac{n-1}{n}\gamma\right)p_i + \frac{\gamma}{n}\sum_{j\neq i} p_j\right\} - w_i\Phi^{-1}\left(1 - \frac{w_i}{p_i}\right) - p_i\sigma_i R\left(\Phi^{-1}\left(1 - \frac{w_i}{p_i}\right)\right). \tag{A.33}$$

We compute the first-order conditions for p_1, p_2, and p_i ($i > 2$) respectively as:

$$a_1 - \left(b_1 + \frac{n-1}{n}\gamma\right)(2p_1 - w_m) + \frac{2\gamma}{n}p_2 + \frac{\gamma}{n}\sum_{j\neq 1,2} p_j - \frac{\gamma}{n}w_m - k_1\sigma_m R\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right) = 0; \tag{A.34}$$

$$a_2 - \left(b_2 + \frac{n-1}{n}\gamma\right)(2p_2 - w_m) + \frac{2\gamma}{n}p_1 + \frac{\gamma}{n}\sum_{j\neq 1,2} p_j - \frac{\gamma}{n}w_m - k_2\sigma_m R\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right) = 0; \tag{A.35}$$

$$a_i - 2\left(b_i + \frac{n-1}{n}\gamma\right)p_i + \frac{\gamma}{n}\sum_{j\neq i} p_j + \left(b_i + \frac{n-1}{n}\gamma\right)w_i - \sigma_i R\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right) = 0. \tag{A.36}$$

We then compute the Jacobian matrix J for these first-order conditions and get the following expressions for $J_{i,j}$:

$$J_{1,1} = \frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_1^2} = -2\left(b_1 + \frac{n-1}{n}\gamma\right) + k_1^2 \frac{\sigma_m w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right)\overline{p}_m^3}; \tag{A.37}$$

$$J_{2,1} = J_{1,2} = \frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_1 \partial p_2} = \frac{2\gamma}{n} + k_1 k_2 \frac{\sigma_m w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right)\overline{p}_m^3}; \tag{A.38}$$

$$J_{2,2} = \frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_2^2} = -2\left(b_2 + \frac{n-1}{n}\gamma\right) + k_2^2 \frac{\sigma_m w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right)\overline{p}_m^3}; \tag{A.39}$$

$$J_{i,i} = \frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} = -2\left(b_i + \frac{n-1}{n}\gamma\right) + \frac{\sigma_i w_i^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_i}{p_i}\right)\right)p_i^3}, \ i > 2; \tag{A.40}$$

$$J_{i,j} = \frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} = \frac{1}{n}\gamma, \ i > 2 \text{ and } j \neq i; \tag{A.41}$$

$$J_{i,j} = \frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i \partial p_j} = \frac{1}{n}\gamma, \ i \leq 2 \text{ and } j > 2. \tag{A.42}$$

To ensure the expected profit of firm i is concave in p_i, we require $J_{ii} < 0$. In addition, to ensure there exists a unique Nash equilibrium, we require that J is diagonally dominant; i.e., $|J_{i,i}| > \sum_{j \neq i} |J_{i,j}|$ (Cachon and Netessine 2004). We can verify $J_{i,j} > 0$ for $i \neq j$. So the condition simplifies to $\sum_{j=1}^{n} J_{i,j} < 0$. Substituting the expression for $J_{i,j}$, we get the following conditions:

$$\begin{aligned}
-\left(2b_1 + \tfrac{n-2}{n}\gamma\right) + k_1 \frac{\sigma_m w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right)\overline{p}_m^3} &< 0, \\
-\left(2b_2 + \tfrac{n-2}{n}\gamma\right) + k_2 \frac{\sigma_m w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m}\right)\right)\overline{p}_m^3} &< 0, \\
-\left(2b_i + \tfrac{n-1}{n}\gamma\right) + \frac{\sigma_i w_i^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_i}{p_3}\right)\right)p_i^3} &< 0 \ \text{ for} i > 2.
\end{aligned}$$

(a) We first prove $dp_i^{post}/d\rho < 0$ $(i = 1, 2, \ldots, n)$. We then prove $d\pi_i^{post}/d\rho < 0$ $(i = 3, 4, \ldots, n)$.

Using the implicit function theorem and the Cramer's rule, we obtain $dp_i^{post}/d\rho = -\left|J_i^\rho\right| / |J|$, where J_i^ρ is the matrix formed by replacing the ith column of J with the vector $\left(\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\Big|_{\mathbf{p}=\mathbf{p}^{post}}, \frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_2 \partial \rho}\Big|_{\mathbf{p}=\mathbf{p}^{post}}, 0, ..., 0\right)^T$. This vector is obtained by differentiating the first-order conditions with respect to ρ. We next show that the signs of $|J|$ and $|J_i^\rho|$ are both $(-1)^n$ such that $dp_i^{post}/d\rho < 0$. Since J is symmetric and strictly diagonally dominant with negative diagonal elements, the sign of $|J|$ is $(-1)^n$. To obtain the sign of $|J_i^\rho|$, we use column expansion and obtain

$$|J_i^\rho| = \left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} (-1)^{i+1} |M_{1,i}| + \left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_2 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} (-1)^{i} |M_{2,i}|, \tag{A.43}$$

where $M_{j,i}$ is a matrix formed by deleting the jth row and ith column of J. We then use (A.43) to obtain the signs of $|J_1^\rho|$, $|J_2^\rho|$ and $|J_i^\rho|$, $i = 3, ...n$, respectively.

To show that the sign of $|J_1^\rho|$ is $(-1)^n$, from (A.43) we will show $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_2 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$, and the signs of $|M_{1,1}|$ and $|M_{2,1}|$ are $(-1)^{n-1}$ and $(-1)^n$, respectively. From (A.34) and (A.35), we obtain $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = -k_1 \frac{\sigma_1 \sigma_2}{\sigma_m} R\left(\Phi^{-1}\left(1 - \frac{w_m}{\bar{p}_m^{post}}\right)\right) < 0$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_2 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = -k_2 \frac{\sigma_1 \sigma_2}{\sigma_m} R\left(\Phi^{-1}\left(1 - \frac{w_m}{\bar{p}_m^{post}}\right)\right) < 0$. Since $M_{1,1}$ is a matrix formed by deleting the first row and first column of J, $M_{1,1}$ is symmetric and diagonally dominant with negative diagonal values. So the sign of $|M_{1,1}|$ is $(-1)^{n-1}$. Finally, we compute $|M_{2,1}| = \begin{vmatrix} J_{1,2} & J_{1,3} & \cdots & J_{1,n} \\ J_{3,2} & J_{3,3} & \cdots & J_{3,n} \\ \vdots & \vdots & \ddots & \vdots \\ J_{n,2} & J_{n,3} & \cdots & J_{n,n} \end{vmatrix}$. We substitute $J_{i,j} = \frac{1}{n}\gamma$ for $i \neq j$, $i > 2$ from (A.41), and $J_{i,j} = \frac{1}{n}\gamma$ for $i \leq 2$, $j > 2$ from (A.42) into the expression of $|M_{2,1}|$ and get the following:

$$\begin{aligned}
|M_{2,1}| &= \begin{vmatrix} J_{1,2} & \frac{\gamma}{n} & \cdots & \frac{\gamma}{n} \\ \frac{\gamma}{n} & J_{3,3} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{\gamma}{n} \\ \frac{\gamma}{n} & \ddots & \frac{\gamma}{n} & J_{n,n} \end{vmatrix} \\
&= \begin{vmatrix} J_{1,2} & \frac{\gamma}{n} & \frac{\gamma}{n} & \cdots & \frac{\gamma}{n} \\ 0 & J_{3,3} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ \vdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ 0 & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & J_{n,n} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} \end{vmatrix} \\
&= J_{1,2} \begin{vmatrix} J_{3,3} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & J_{n,n} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} \end{vmatrix},
\end{aligned}$$

where the second equality is obtained by multiplying the first row with $-\frac{\gamma}{nJ_{1,2}}$ and adding the product to the other rows, and the last equality is obtained from column expansion. We obtain $J_{1,2} > 0$ from (A.38). In order to obtain the sign of $|M_{2,1}|$ from the above expression, we will show that the following symmetric matrix is diagonally dominant with negative diagonal values such that the sign of its determinant is $(-1)^{n-2}$:

$$\begin{bmatrix} J_{3,3} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) \\ \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & \cdots & \frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) & J_{n,n} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} \end{bmatrix}.$$

To do so, we will show $J_{i,i} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} + (n-3)\frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) < 0$ and $\frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) > 0$ such that $\left|J_{3,3} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n}\right| > (n-3)\frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right)$. Since $\frac{\gamma}{nJ_{1,2}} < 1$ from (A.38), we obtain $\frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) > 0$. Since J is diagonally dominant with negative diagonal values, $J_{i,i} + (n-1)\frac{\gamma}{n} = J_{i,i} + \sum_{j \neq i} J_{i,j} < 0$ for $i \geq 3$, where the equality is from (A.41). Adding both sides $-\frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} - 2\frac{\gamma}{n} - (n-3)\frac{\gamma}{n}\frac{\gamma}{nJ_{1,2}}$ results in $J_{i,i} - \frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} + (n-3)\frac{\gamma}{n}\left(1 - \frac{\gamma}{nJ_{1,2}}\right) < -\frac{\gamma}{nJ_{1,2}}\frac{\gamma}{n} - 2\frac{\gamma}{n} - (n-3)\frac{\gamma}{n}\frac{\gamma}{nJ_{1,2}} < 0$ because $J_{1,2} > 0$ from (A.38).

Following the same procedure as in the case of $|J_1^\rho|$, we can show that the sign of $|J_2^\rho|$ is also $(-1)^n$.

Finally, from (A.43) we show that the sign of $|J_i^\rho|$, $i \geq 3$, is $(-1)^n$ by proving that the signs of $|M_{1,i}|$ and $|M_{2,i}|$ are $(-1)^{n+i}$ and $(-1)^{n+i-1}$, respectively. We compute $M_{1,i}$ as follows:

$$M_{1,i} = \begin{bmatrix} J_{2,1} & J_{2,2} & \cdots & J_{2,i-1} & J_{2,i+1} & \cdots & J_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ J_{i-1,1} & J_{i-1,2} & \cdots & J_{i-1,i-1} & J_{i-1,i+1} & \cdots & J_{i-1,n} \\ J_{i,1} & J_{i,2} & \cdots & J_{i,i-1} & J_{i,i+1} & \cdots & J_{i,n} \\ J_{i+1,1} & J_{i+1,2} & \cdots & J_{i+1,i-1} & J_{i+1,i+1} & \cdots & J_{i+1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ J_{n,1} & J_{n,2} & \cdots & J_{n,i-1} & J_{n,i+1} & \cdots & J_{n,n} \end{bmatrix}.$$

By switching the $(i-1)$th row with the row above it $(i-2)$ times, we obtain

$$|M_{1,i}| = (-1)^{i-2} \begin{vmatrix} J_{i,1} & J_{i,2} & \cdots & J_{i,i-1} & J_{i,i+1} & \cdots & J_{i,n} \\ J_{2,1} & J_{2,2} & \cdots & J_{2,i-1} & J_{2,i+1} & \cdots & J_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ J_{i-1,1} & J_{i-1,2} & \cdots & J_{i-1,i-1} & J_{i-1,i+1} & \cdots & J_{i-1,n} \\ J_{i+1,1} & J_{i+1,2} & \cdots & J_{i+1,i-1} & J_{i+1,i+1} & \cdots & J_{i+1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ J_{n,1} & J_{n,2} & \cdots & J_{n,i-1} & J_{n,i+1} & \cdots & J_{n,n} \end{vmatrix}$$

$$
= (-1)^i \begin{vmatrix}
\frac{\gamma}{n} & \frac{\gamma}{n} & \cdots & \cdots & \cdots & \cdots & \frac{\gamma}{n} \\
J_{2,1} & J_{2,2} & \frac{\gamma}{n} & \ddots & \ddots & \cdots & \vdots \\
\frac{\gamma}{n} & \frac{\gamma}{n} & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & J_{i-1,i-1} & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & J_{i+1,i+1} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \frac{\gamma}{n} \\
\frac{\gamma}{n} & \frac{\gamma}{n} & \cdots & \cdots & \cdots & \frac{\gamma}{n} & J_{n,n}
\end{vmatrix},
$$

where the last equality is obtained by substituting $J_{k,j} = \frac{1}{n}\gamma$ for $k \neq j$, $k > 2$ from (A.41), and $J_{k,j} = \frac{1}{n}\gamma$ for $k \leq 2$, $j > 2$ from (A.42). By multiplying the first row by -1 and adding this product to the other rows, we can simplify the above equation to the following equation:

$$
|M_{1,i}| = (-1)^i \begin{vmatrix}
\frac{\gamma}{n} & \frac{\gamma}{n} & \cdots & \cdots & \cdots & \cdots & \frac{\gamma}{n} \\
J_{2,1} - \frac{\gamma}{n} & J_{2,2} - \frac{\gamma}{n} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & J_{i-1,i-1} - \frac{\gamma}{n} & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & J_{i+1,i+1} - \frac{\gamma}{n} & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & 0 & \cdots & \cdots & \cdots & 0 & J_{n,n} - \frac{\gamma}{n}
\end{vmatrix}
$$

$$
= (-1)^i \frac{\gamma}{n} \begin{vmatrix}
J_{2,2} - \frac{\gamma}{n} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & J_{i-1,i-1} - \frac{\gamma}{n} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & J_{i+1,i+1} - \frac{\gamma}{n} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & J_{n,n} - \frac{\gamma}{n}
\end{vmatrix}
$$

$$
- (-1)^i \left(J_{2,1} - \frac{\gamma}{n}\right) \begin{vmatrix}
\frac{\gamma}{n} & \frac{\gamma}{n} & \cdots & \cdots & \cdots & \cdots & \frac{\gamma}{n} \\
0 & J_{3,3} - \frac{\gamma}{n} & 0 & \cdots & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & J_{i-1,i-1} - \frac{\gamma}{n} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & J_{i+1,i+1} - \frac{\gamma}{n} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & J_{n,n} - \frac{\gamma}{n}
\end{vmatrix}
$$

$$
= (-1)^i \left\{ \frac{\gamma}{n} \prod_{k=2, k \neq i}^{n} \left(J_{k,k} - \frac{\gamma}{n}\right) - \frac{\gamma}{n} \left(J_{2,1} - \frac{\gamma}{n}\right) \prod_{k=3, k \neq i}^{n} \left(J_{k,k} - \frac{\gamma}{n}\right) \right\},
$$

where the second equality is from column expansion. Since $J_{k,k} - \frac{\gamma}{n} < J_{k,k} < 0$ and $J_{2,1} - \frac{\gamma}{n} > 0$ from (A.38), the sign of $|M_{1,i}|$ is $(-1)^{n+i-2}$ from the above expression. Similarly, we can show that the sign of $|M_{2,i}|$ is $(-1)^{n+i-1}$.

(b) We prove $dp_i^{post}/ds > 0$ if $s < s_{asym}^{(1)}$ in three steps. First, following the same procedure as in the proof of part (a), we can show that if $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},w)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ are both negative, then $dp_i^{post}/dw_m < 0$. Second, we show that there exists a $\underline{w}_m^{(1)}$ such that $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},w)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ are both negative if $w_m > \underline{w}_m^{(1)}$. To do so, we obtain the expressions of $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},w)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ by differentiating (A.34) and (A.35) with respect to w_m:

$$\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = \left(b_1 + \frac{n-2}{n}\gamma\right) - \frac{k_1 \sigma_m w_m}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m^{post}}\right)\right)\left(\overline{p}_m^{post}\right)^2}, \text{ and} \quad \text{(A.44)}$$

$$\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} = \left(b_2 + \frac{n-2}{n}\gamma\right) - \frac{k_2 \sigma_m w_m}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{\overline{p}_m^{post}}\right)\right)\left(\overline{p}_m^{post}\right)^2}. \quad \text{(A.45)}$$

Following the same procedure as in the proof of Proposition 1.2(a), we can show that when w_m is sufficiently large, $\frac{\sigma_m w_m}{\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)p_m^2}$ becomes sufficiently large such that $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ from (A.44) and (A.45). Let $\underline{w}_m^{(1)}$ be the largest w_m that satisfies $max\{\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}, \left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\} = 0$. For any $w_m > \underline{w}_m^{(1)}$, $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$ and $\left.\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_2 \partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}} < 0$, and thus $dp_i^{post}/dw_m < 0$. Finally, we use $\underline{w}_m^{(1)}$ to calculate $s_{asym}^{(1)}$. If $\underline{w}_m^{(1)} \geq \min\{w_1, w_2\}$, we define $s_{asym}^{(1)} = 0$. If $\underline{w}_m^{(1)} < \min\{w_1, w_2\}$, we define $s_{asym}^{(1)} = (\min\{w_1, w_2\} - \underline{w}_m^{(1)})/\min\{w_1, w_2\}$. So $dp_i^{post}/ds > 0$ if $s < s_{asym}^{(1)}$.

Finally, we prove $d\pi_i^{post}/ds > 0$ $(i = 3, 4, \ldots, n)$ if $s < s_{asym}^{(1)}$. We can write $d\pi_i^{post}/ds$ as follows: $\frac{d\pi_i^{post}}{ds} = \sum_{j=1}^{n} \frac{\partial \pi_i^{post}}{\partial p_j^{post}} \frac{dp_j^{post}}{ds}$. Since p_i^{post} is chosen optimally, $\partial \pi_i^{post}/\partial p_i^{post} = 0$. In addition, we get $\partial \pi_i^{post}/\partial p_j^{post} = \frac{\gamma}{n}(p_i^{post} - w_i)$ $(j \neq i)$ from (A.33). Substituting the expressions of $\partial \pi_i^{post}/\partial p_i^{post}$ and $\partial \pi_i^{post}/\partial p_j^{post}$ $(j \neq i)$ into the expression of $d\pi_i^{post}/ds$, we get the following:

$$\frac{d\pi_i^{post}}{ds} = \frac{\gamma}{n}(p_i^{post} - w_i) \sum_{j \neq i}^{n} \frac{dp_j^{post}}{ds} < 0,$$

where the inequality is due to $p_i^{post} > w_i$ and $dp_j^{post}/ds < 0$ if $s < s_{asym}^{(1)}$ from above. □

Detailed Proof of Corollary 1.2. (a) Denote by $f_{\widetilde{\varepsilon}_m}(\cdot)$ and $f_{\widetilde{\xi}_m}(\cdot)$ the density functions of $\widetilde{\varepsilon}_m$ and $\widetilde{\xi}_m$, respectively. Denote by $F_{\widetilde{\varepsilon}_m}(\cdot)$ and $F_{\widetilde{\xi}_m}(\cdot)$ the distribution functions of $\widetilde{\varepsilon}_m$ and $\widetilde{\xi}_m$, respectively. Denote by $\pi_{\widetilde{\varepsilon}_m}$ and $\pi_{\widetilde{\xi}_m}$ the expected profits of the post-merger firm when facing $\widetilde{\varepsilon}_m$ and $\widetilde{\xi}_m$, respectively. The proof proceeds in the following two steps. First, we obtain the expression for $\frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}}$, where $\widehat{\mathbf{p}}^{post} = \left(\widehat{p}_m^{post}, \widehat{p}_m^{post}, \widehat{p}_3^{post}, ..., \widehat{p}_n^{post}\right)$ is the equilibrium price vector when the post-merger firm faces $\widetilde{\xi}_m$. Second, we prove $\frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} \geq 0$ if $\widetilde{\varepsilon}_m \preceq_{disp} \widetilde{\xi}_m$. Therefore, if the post-merger firm faces random demand $\widetilde{\varepsilon}_m$, but all firms charge the equilibrium prices for random demand $\widetilde{\xi}_m$ (i.e., $\widehat{p}_i^{post}$, $i = m, 3, 4, ..., n$), then the post-merger firm has an incentive to raise its price. Then following the same procedure as in the proof of Lemma 1.1(a), we obtain $p_i^{post} \geq \widehat{p}_i^{post}$.

We first get the expression of $\frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}}$. Following the same procedure as in the proof of Lemma 1.1(a), we get

$$\frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1} - \frac{\partial \pi_{\widetilde{\xi}_m}(\mathbf{p})}{\partial p_1} = \frac{1}{2} R_{\widetilde{\xi}_m}\left(F^{-1}_{R_{\widetilde{\xi}_m}}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right) - \frac{1}{2} R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{R_{\widetilde{\varepsilon}_m}}\left(1 - \frac{2w_m}{p_1+p_2}\right)\right),$$

where $R_{\widetilde{\varepsilon}_m}(y_m) = \int_{y_m}^{\infty} (t - y_m) f_{\widetilde{\varepsilon}_m}(t)\, dt$ and $R_{\widetilde{\xi}_m}(y_m) = \int_{y_m}^{\infty} (t - y_m) f_{\widetilde{\xi}_m}(t)\, dt$. Since $\widehat{p}^{post}$ is the equilibrium price vector when the post-merger firm faces the demand $\widetilde{\xi}_m$, $\frac{\partial \pi_{\widetilde{\xi}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} = 0$. So from the above equation we obtain

$$\begin{aligned} \frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} &= \frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} - \frac{\partial \pi_{\widetilde{\xi}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} \\ &= \frac{1}{2} R_{\widetilde{\xi}_m}\left(F^{-1}_{R_{\widetilde{\xi}_m}}\left(1 - \frac{w_m}{\widehat{p}_m^{post}}\right)\right) - \frac{1}{2} R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{R_{\widetilde{\varepsilon}_m}}\left(1 - \frac{w_m}{\widehat{p}_m^{post}}\right)\right). \end{aligned}$$

We next show that $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) \leq R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right)$ for any $0 < z < 1$ such that $\frac{\partial \pi_{\widetilde{\varepsilon}_m}(\mathbf{p})}{\partial p_1}|_{\mathbf{p}=\widehat{\mathbf{p}}^{post}} \geq 0$ from the above equation. Note that from the definition of lost sales $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) = \int_{F^{-1}_{\widetilde{\varepsilon}_m}(z)}^{\infty} \left\{t - F^{-1}_{\widetilde{\varepsilon}_m}(z)\right\} f_{\widetilde{\varepsilon}_m}(t)\, dt$ and $R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right) = \int_{F^{-1}_{\widetilde{\xi}_m}(z)}^{\infty} \left\{t - F^{-1}_{\widetilde{\xi}_m}(z)\right\} f_{\widetilde{\xi}_m}(t)\, dt$. Let $X = \widetilde{\varepsilon}_m - F^{-1}_{\widetilde{\varepsilon}_m}(z)$ and $Y = \widetilde{\xi}_m - F^{-1}_{\widetilde{\xi}_m}(z)$. Denote by $F_X(\cdot)$ and $F_Y(\cdot)$ the distribution functions of X and Y, respectively. Denote by $f_X(\cdot)$ and $f_Y(\cdot)$ the density functions of X and Y, respectively. Then $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) = \int_0^{\infty} t f_X(t)\, dt$ and $R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right) = \int_0^{\infty} t f_Y(t)\, dt$. Using integration by parts, we can rewrite $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right)$ and $R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right)$ as $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) = \int_0^{\infty} \{1 - F_X(t)\}\, dt$ and $R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right) = \int_0^{\infty} \{1 - F_Y(t)\}\, dt$. So we obtain $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) - R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right) = \int_0^{\infty} \{F_Y(t) - F_X(t)\} dt$. We next show $F_Y(t) \leq F_X(t)$ for $t \geq 0$ so that $R_{\widetilde{\varepsilon}_m}\left(F^{-1}_{\widetilde{\varepsilon}_m}(z)\right) - R_{\widetilde{\xi}_m}\left(F^{-1}_{\widetilde{\xi}_m}(z)\right) \leq 0$. From the definition of X and Y, we obtain $F_X(0) = F_Y(0) = z$. Let $u = F_Y(t)$. Then $F_Y(t) \leq F_X(t)$ for $t \geq 0$ is equivalent to $u \leq F_X\left(F_Y^{-1}(u)\right)$ for $u \in [z, 1)$. This inequality is equivalent to $F_X^{-1}(u) \leq F_Y^{-1}(u)$ because $F_X^{-1}(\cdot)$ is an increasing function. Since $F_X(0) = F_Y(0) = z$, we obtain $F_X^{-1}(z) = F_Y^{-1}(z) = 0$. Then $F_X^{-1}(u) \leq F_Y^{-1}(u) \Leftrightarrow F_X^{-1}(u) - F_X^{-1}(z) \leq F_Y^{-1}(u) - F_Y^{-1}(z)$. From the definition of dispersive ordering, this inequality holds if $X \preceq_{disp} Y$. Since the dispersive order is location-invariant, $\widetilde{\varepsilon}_m \preceq_{disp} \widetilde{\xi}_m$ implies $X \preceq_{disp} Y$ from the definitions of X and Y.

(b) Following the same procedure as in the proof of Proposition 1.2(a), we can show that $\frac{dp_m^{post}}{dw_m}$ has the same sign as $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}|_{\mathbf{p}=\mathbf{p}^{post}}$ and

$$\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}|_{\mathbf{p}=\mathbf{p}^{post}} = b + \gamma \frac{n-2}{n} - \frac{1}{2p_m^{post} h_m \left(F_m^{-1}\left(1 - w_m/p_m^{post}\right)\right)}, \tag{A.46}$$

where $h_m(\cdot)$ is the failure rate for $\widetilde{\varepsilon}_m$. We consider two cases: (i) $1/2h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)p_m^{post} > b + \gamma\frac{n-2}{n}$ at $w_m = w$, and (ii) $1/2h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)p_m^{post} \leq b + \gamma\frac{n-2}{n}$ at $w_m = w$.

In case (i), at $w_m = w$, from (A.46) $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m} < 0$ and thus $dp_m^{post}/dw_m < 0$. As w_m decreases, p_m^{post} increases and $h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)$ also increases because $\widetilde{\varepsilon}_m$ has an increasing failure rate. If the upper bound of the failure rate is not sufficiently large, then the last term in (A.46), $1/2p_m^{post}h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)$, is smaller than $b+\gamma\frac{n-2}{n}$, so that $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}$ remains negative. In this case, p_m^{post} increases with s for all $s \in [0,1]$, so we set $s_{non}^{(1)} = 1$. If the failure rate has a sufficiently large upper bound, then as w_m becomes sufficiently small, $h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)$ becomes sufficiently large such that $1/2p_m^{post}h_m\left(F_m^{-1}\left(1-w_m/p_m^{post}\right)\right)$ $< b+\gamma\frac{n-2}{n}$ and $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}|_{\mathbf{p}=\mathbf{p}^{post}} > 0$ from (A.46). There exists a $w_m^{(1)}$ such that $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_1 \partial w_m}|_{\mathbf{p}=\mathbf{p}^{post}} =$ 0 and $dp_m^{post}/dw_m = 0$ at $w_m = w_m^{(1)}$. Following the same procedure as in the proof of Proposition 1.2(a), we can show that this $w_m^{(1)}$ is unique. We define $s_{non}^{(1)} = \left(w - w_m^{(1)}\right)/w$, and $dp_m^{post}/ds > 0$ if and only if $s < s_{non}^{(1)}$.

In case (ii), when $w_m = w$, $\frac{\partial^2 \pi_m(\mathbf{p},w_m)}{\partial p_m \partial w_m} > 0$ and $dp_m^{post}/dw_m > 0$. Following the same procedure as in the proof of Proposition 1.2(a), we can show that dp_m^{post}/dw_m can cross zero only once from negative to positive as w_m decreases from $w_m = w$. Since $dp_m^{post}/dw_m > 0$ at $w_m = w$, $dp_m^{post}/dw_m > 0$ for all $w_m \leq w$. Therefore, $s_{non}^{(1)} = 0$ and $dp_m^{post}/ds < 0$ for any $s > s_{non}^{(1)} = 0$.

Following the same procedure as in the proof of $d\pi_i^{post}/ds > 0$ $(i = 3, 4, \ldots, n)$ in Corollary 1.1(b), we can prove $d\pi_i^{post}/ds > 0$ if and only if $s < s_{non}^{(1)}$. □

Proof of Corollary 1.3. Before we prove part (a), we derive conditions for the existence and uniqueness of a pure-strategy equilibrium in the post-merger market. Following the same procedure as in the proof of Lemma A1(c), we need the following conditions for nonparticipant firms: $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} < 0$, $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} > 0$, and $\left|\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2}\right| > \sum_{j \neq i}\left|\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j}\right|$, $i = 3, \ldots, n$ and $j \neq i$, and the following conditions for the post-merger firm $\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i^2} < 0$, $\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i \partial p_j} > 0$, and $\left|\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i^2}\right| >$ $\sum_{j \neq i}\left|\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i \partial p_j}\right|$, $i = 1, 2$ and $j \neq i$. We first analyze the conditions for nonparticipant firms, and then analyze the conditions for the post-merger firm.

For nonparticipant firm i, let its inventory $q_i = L_i(\mathbf{p}) + \delta_i(\mathbf{p})y_i$. Then $\pi_i(\mathbf{p}, y_i) = (p_i - w)L_i(\mathbf{p}) - w\delta_i(\mathbf{p})y_i - p_i\delta_i(\mathbf{p})R(y_i)$. Using the first-order condition on y_i, it is easy to get the optimal $y_i^* = \Phi^{-1}(1 - w/p_i)$. Substituting this into $\pi_i(\mathbf{p}, y_i)$ results in

$$\pi_i(\mathbf{p}) = (p_i - w)L_i(\mathbf{p}) - \delta_i(\mathbf{p})\left\{w\Phi^{-1}\left(1 - \frac{w}{p_i}\right) + p_i R\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)\right\}. \qquad \text{(A.47)}$$

We can compute $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ as follows:

$$\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} = L_i(\mathbf{p}) - \left(b + \tfrac{n-1}{n}\gamma\right)(p_i - w) + \left(\beta + \tfrac{n-1}{n}\theta\right)\left\{w\Phi^{-1}\left(1 - \tfrac{w}{p_i}\right) + p_i R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_i}\right)\right)\right\}$$
$$-\delta_i(\mathbf{p})R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_i}\right)\right).$$

From the expression of $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ above, we can compute $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2}$ and $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j}$ as follows:

$$\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} = -2\left(b + \tfrac{n-1}{n}\gamma\right) + 2\left(\beta + \tfrac{n-1}{n}\theta\right)R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_i}\right)\right) + \frac{\delta_i(\mathbf{p})w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)p_i^3}, \text{ and}$$

$$\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i p_j} = \tfrac{\gamma}{n} - \tfrac{\theta}{n}R\left(\Phi^{-1}\left(1 - \tfrac{w}{p_i}\right)\right).$$

Using the above expressions, we obtain the following sufficient conditions for $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} < 0$, $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} > 0$ for $j \neq i$, and $\left|\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2}\right| > \sum_{j \neq i}\left|\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j}\right|$:

$$\left(2b + \frac{n-1}{n}\gamma\right) - \left(2\beta + \frac{n-1}{n}\theta\right)R\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right) - \frac{\delta_i(\mathbf{p})w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right)p_i^3} > 0, \text{ and} \qquad \text{(A.48)}$$

$$\gamma - \theta R\left(\Phi^{-1}\left(1 - \frac{w}{p_i}\right)\right) > 0. \qquad \text{(A.49)}$$

For the post-merger firm, we can express its expected profit as

$$\pi_m(\mathbf{p}) = (p_1 - w_m)L_1(\mathbf{p}) + (p_2 - w_m)L_2(\mathbf{p}) - \sqrt{\delta_1^2(\mathbf{p}) + \delta_2^2(\mathbf{p}) + 2\rho\delta_1(\mathbf{p})\delta_2(\mathbf{p})}$$
$$\left\{w_m\Phi^{-1}\left(1 - \frac{2w_m}{p_1 + p_2}\right) + \frac{p_1 + p_2}{2}R\left(\Phi^{-1}\left(1 - \frac{2w_m}{p_1 + p_2}\right)\right)\right\}. \qquad \text{(A.50)}$$

Using the above expression, we can obtain the conditions for $\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i^2} < 0$, $\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i \partial p_j} > 0$, and $\left|\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i^2}\right| > \sum_{j \neq i}\left|\frac{\partial^2 \pi_m(\mathbf{p})}{\partial p_i \partial p_j}\right|$, $i = 1, 2$ and $j \neq i$. By setting $p_1 = p_2 = p_m$ in equilibrium, we obtain the following conditions:

$$\left(2b+\frac{n-2}{n}\gamma\right)-\left(2\beta+\frac{n-2}{n}\theta\right)\frac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)-\frac{\sqrt{2+2\rho}\delta_1(\mathbf{p})w_m^2}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)p_m^3}>0; \tag{A.51}$$

$$\gamma-\frac{1}{2}\sqrt{2+2\rho}\theta R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)>0. \tag{A.52}$$

Following the same procedure as in the proof of Lemma A1(a), we can show that Lemma A1(a) still holds: $0<dp_3^*/dp_m<1$ and $0<dp_m^*/dp_3<1$.

(a) We prove the following two statements before we prove our main result: (i) $\frac{dp_m^{post}}{d\rho}$ has the same sign for all $\rho\in[-1,1]$, and (ii) p_m^{post} is increasing with β at $\rho=1$.

To prove (i), it suffices to show the following statement (iii): if $\frac{dp_m^{post}}{d\rho}=0$ for some $\rho\in[-1,1]$, then $\frac{dp_m^{post}}{d\rho}=0$ for all $\rho\in[-1,1]$. The reason is as follows. Suppose the sign of $\frac{dp_m^{post}}{d\rho}$ changes with ρ. Then there exist ρ_1 and ρ_2 ($\rho_1,\rho_2\in[-1,1]$ and $\rho_1\neq\rho_2$) such that $\frac{dp_m^{post}}{d\rho}>0$ for ρ_1 and $\frac{dp_m^{post}}{d\rho}\leq 0$ for ρ_2. By continuity, there must exists $\rho_0\in[-1,1]$ such that $\frac{dp_m^{post}}{d\rho}=0$. By statement (iii), this implies that $\frac{dp_m^{post}}{d\rho}=0$ for all $\rho\in[-1,1]$. This contradicts the premise that $\frac{dp_m^{post}}{d\rho}>0$ for ρ_1. Therefore, the sign of $\frac{dp_m^{post}}{d\rho}$ cannot change with ρ.

We first find the expression of $\frac{dp_m^{post}}{d\rho}$. We can compute the first-order conditions $\left.\frac{\partial\pi_m(\mathbf{p})}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}=0$ and $\left.\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}}=0$ from (A.50) and (A.47) respectively as follows:

$$\begin{aligned}&a-\left(b+\frac{n-2}{n}\gamma\right)p_m^{post}+\frac{(n-2)\gamma}{n}p_3^{post}-\left(b+\frac{n-2}{n}\gamma\right)(p_m^{post}-w_m)\\&-\frac{\sqrt{2+2\rho}}{2}\left\{\alpha-\left(\beta+\frac{n-2}{n}\theta\right)p_m^{post}+\frac{\theta(n-2)}{n}p_3^{post}\right\}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\\&+\frac{\sqrt{2+2\rho}}{2}\left(\beta+\frac{n-2}{n}\theta\right)\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}=0;\end{aligned} \tag{A.53}$$

$$\begin{aligned}&-\left(b+\frac{n-1}{n}\gamma\right)(p_3^{post}-w)+\frac{2\gamma}{n}p_m^{post}-\left\{\alpha-\left(\beta+\frac{2}{n}\theta\right)p_3^{post}+\frac{2\theta}{n}p_m^{post}\right\}R\left(\Phi^{-1}\left(1-\frac{w}{p_3^{post}}\right)\right)\\&+a-\left(b+\frac{2}{n}\gamma\right)p_3^{post}+\left(\beta+\frac{n-1}{n}\theta\right)\left\{w\Phi^{-1}\left(1-\frac{w}{p_3^{post}}\right)+p_3^{post}R\left(\Phi^{-1}\left(1-\frac{w}{p_3^{post}}\right)\right)\right\}=0.\end{aligned} \tag{A.54}$$

Following the same procedure as in the proof of Proposition 1.1(a), we obtain the following:

$$\frac{dp_m^{post}}{d\rho} = \frac{-\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}}{\frac{\partial}{\partial p_m^{post}}\left(\left.\frac{\partial \pi_m(\mathbf{p},\rho)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) + \frac{\partial}{\partial p_3^{post}}\left(\left.\frac{\partial \pi_m(\mathbf{p},\rho)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right)\frac{dp_3^{post}}{dp_m^{post}}}. \tag{A.55}$$

By using (A.53) and (A.54), we can simplify the denominator of the above expression as follows:

$$-2\left(b+\frac{n-2}{n}\gamma\right)+2\left(\beta+\frac{n-2}{n}\theta\right)\frac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)+$$
$$\frac{\sqrt{2+2\rho}\,\delta_1(\mathbf{p}^{post})w_m^2}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^3}+\frac{n-2}{n}\left\{\gamma-\theta\frac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}\frac{dp_3^{post}}{dp_m^{post}} \tag{A.56}$$

To show that the expression in (A.56) is negative, we add $\frac{n-2}{n}\gamma - \frac{n-2}{n}\frac{\sqrt{2+2\rho}}{2}\theta R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)$ to both sides of (A.51) and obtain the following:

$$2\left(b+\tfrac{n-2}{n}\gamma\right)-2\left(\beta+\tfrac{n-2}{n}\theta\right)\tfrac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-\tfrac{w_m}{p_m}\right)\right)-\frac{\sqrt{2+2\rho}\,\delta_1(\mathbf{p}^{post})w_m^2}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)p_m^3}$$
$$> \tfrac{(n-2)\gamma}{n}-\tfrac{(n-2)\theta}{n}\tfrac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-w_m/p_m^{post}\right)\right)>0,$$

where the last inequality follows from (A.52). Since $\frac{dp_3^{post}}{dp_m^{post}}<1$ from Lemma A1(a), the above inequality can be rewritten as:

$$2\left(b+\tfrac{n-2}{n}\gamma\right)-2\left(\beta+\tfrac{n-2}{n}\theta\right)\tfrac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-\tfrac{w_m}{p_m}\right)\right)-\frac{\sqrt{2+2\rho}\,\delta_1(\mathbf{p}^{post})w_m^2}{2\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m}\right)\right)p_m^3}$$
$$> \left\{\tfrac{(n-2)\gamma}{n}-\tfrac{(n-2)\theta}{n}\tfrac{\sqrt{2+2\rho}}{2}R\left(\Phi^{-1}\left(1-w_m/p_m^{post}\right)\right)\right\}\frac{dp_3^{post}}{dp_m^{post}},$$

which implies the expression in (A.56), the denominator of $\frac{dp_m^{post}}{d\rho}$ in (A.55), is negative. Therefore, from (A.55), $\frac{dp_m^{post}}{d\rho}$ has the same sign as $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_m \partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}$, which can be computed from (A.53) as follows:

$$\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = -\frac{1}{2\sqrt{2+2\rho}}\left\{\alpha-2\left(\beta+\frac{n-2}{n}\theta\right)p_m^{post}+\frac{n-2}{n}\theta p_3^{post}\right\}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)$$
$$+\frac{1}{2\sqrt{2+2\rho}}\left(\beta+\frac{n-2}{n}\theta\right)w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right). \tag{A.57}$$

Next, using the expression of $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_m\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}$, which has the same sign as $\frac{dp_m^{post}}{d\rho}$, we show that if there exists $\rho_0 \in [-1,1]$ such that $\frac{dp_m^{post}}{d\rho}=0$ at $\rho=\rho_0$, then $\frac{dp_m^{post}}{d\rho}=0$ for all $\rho\in[-1,1]$. Note that p_m^{post}, p_3^{post}, and ρ_0 are the solution of first-order conditions (A.53) and (A.54), and

$\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1 \partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$. From (A.57), $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1 \partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$ can be written as

$$-\left\{\alpha - 2\left(\beta + \frac{n-2}{n}\theta\right)p_m^{post} + \frac{n-2}{n}\theta p_3^{post}\right\} R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)$$
$$+\left(\beta + \frac{n-2}{n}\theta\right) w_m \Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right) = 0. \tag{A.58}$$

Using (A.58), we can rewrite the first-order condition (A.53) as

$$a - 2\left(b + \frac{n-2}{n}\gamma\right)p_m^{post} + \frac{(n-2)\gamma}{n}p_3^{post} + \left(b + \frac{n-2}{n}\gamma\right)w_m + \frac{\sqrt{2+2\rho}}{2}\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}$$
$$= \quad a - 2\left(b + \frac{n-2}{n}\gamma\right)p_m^{post} + \frac{(n-2)\gamma}{n}p_3^{post} + \left(b + \frac{n-2}{n}\gamma\right)w_m = 0. \tag{A.59}$$

Notice that ρ does not appear in (A.54), (A.58) or (A.59). Therefore, p_m^{post} and p_3^{post} do not depend on ρ, while satisfying $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$ for any ρ. This implies that $\frac{dp_m^{post}}{d\rho} = 0$ and $\frac{dp_3^{post}}{d\rho} = \frac{dp_3^{post}}{dp_m^{post}}\frac{dp_m^{post}}{d\rho} = 0$ for all $\rho \in [-1, 1]$.

We prove (ii) by following the same procedure as in the proof of Proposition 1.1(a). First, we apply the implicit function theorem to the first-order conditions $\left.\frac{\partial\pi_m(\mathbf{p})}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$ and $\left.\frac{\partial\pi_3(\mathbf{p})}{\partial p_3}\right|_{\mathbf{p}=\mathbf{p}^{post}} = 0$, and get the following expressions:

$$\begin{bmatrix} \frac{dp_m^{post}}{d\beta} \\ \frac{dp_3^{post}}{d\beta} \end{bmatrix} = -J^{-1} \times \begin{bmatrix} \frac{\partial}{\partial\beta}\left(\left.\frac{\partial\pi_m(\mathbf{p},\beta)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) \\ \frac{\partial}{\partial\beta}\left(\frac{\partial\pi_3(\mathbf{p},\beta)}{\partial p_3}\big|_{\mathbf{p}=\mathbf{p}^{post}}\right) \end{bmatrix},$$

where J is defined in Proposition 1(a). Next, we show that $J^{-1} < 0$, $\frac{\partial}{\partial\beta}\left(\left.\frac{\partial\pi_m(\mathbf{p},\beta)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) > 0$ and $\frac{\partial}{\partial\beta}\left(\frac{\partial\pi_3(\mathbf{p},\beta)}{\partial p_3}\big|_{\mathbf{p}=\mathbf{p}^{post}}\right) > 0$ such that $\frac{dp_m^{post}}{d\beta} > 0$. From the first-order conditions in (A.53) and (A.54), we compute all the elements of J at $\rho = 1$ as follows:

$J_{1,1} = -2\left(b + \frac{n-2}{n}\gamma\right) + 2\left(\beta + \frac{n-2}{n}\theta\right)R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right) + \frac{\left\{\alpha - \left(\beta + \frac{n-2}{n}\theta\right)p_m^{post} + \frac{\theta(n-2)}{n}p_3^{post}\right\}w_m^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^3}$,

$J_{2,2} = -\left(2b + \frac{n+1}{n}\gamma\right) + \left(2\beta + \frac{n+1}{n}\theta\right)R\left(\Phi^{-1}\left(1 - \frac{w}{p_3^{post}}\right)\right) + \frac{\left\{\alpha - \left(\beta + \frac{2}{n}\theta\right)p_3^{post} + \frac{2\theta}{n}p_m\right\}w^2}{\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_3^{post}}\right)\right)\left(p_3^{post}\right)^3}$,

$J_{1,2} = \frac{(n-2)\gamma}{n} - \frac{(n-2)\theta}{n}R\left(\Phi^{-1}\left(1 - \frac{w_m}{p_m^{post}}\right)\right)$, and

$J_{2,1} = \frac{2\gamma}{n} - \frac{2\theta}{n}R\left(\Phi^{-1}\left(1 - \frac{w}{p_3^{post}}\right)\right)$.

By following the same procedure as in the analysis of the sign of $\frac{dp_m^{post}}{d\rho}$ in the proof of statement (i), from (A.48), (A.49), (A.51) and (A.52), we obtain $-J_{2,2} > J_{2,1} > 0$ and $-J_{1,1} > J_{1,2} >$

0. So $J^{-1} = \frac{1}{J_{2,2}J_{1,1}-J_{1,2}J_{2,1}} \begin{bmatrix} J_{2,2} & -J_{1,2} \\ -J_{2,1} & J_{1,1} \end{bmatrix} < 0$. We compute $\frac{\partial}{\partial \beta}\left(\left.\frac{\partial \pi_m(\mathbf{p},\beta)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right)$ and $\frac{\partial}{\partial \beta}\left(\frac{\partial \pi_3(\mathbf{p},\beta)}{\partial p_3}|_{\mathbf{p}=\mathbf{p}^{post}}\right)$ from the first-order conditions in (A.53) and (A.54) as follows:

$\frac{\partial}{\partial \beta}\left(\left.\frac{\partial \pi_m(\mathbf{p},\beta)}{\partial p_1}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) = w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right) + 2p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)$, and

$\frac{\partial}{\partial \beta}\left(\frac{\partial \pi_3(\mathbf{p},\beta)}{\partial p_3}|_{\mathbf{p}=\mathbf{p}^{post}}\right) = w\Phi^{-1}\left(1-\frac{w}{p_3^{post}}\right) + 2p_3^{post}R\left(\Phi^{-1}\left(1-\frac{w}{p_3^{post}}\right)\right)$.

It is easy to verify that $\frac{\partial}{\partial \beta}\left(\left.\frac{\partial \pi_m(\mathbf{p},\beta)}{\partial p_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\right) > 0$ and $\frac{\partial}{\partial \beta}\left(\frac{\partial \pi_3(\mathbf{p},\beta)}{\partial p_3}|_{\mathbf{p}=\mathbf{p}^{post}}\right) > 0$.

Using statements (i) and (ii), we next prove that there exists a threshold $\widehat{\beta}$ for a fixed θ such that $\frac{dp_m^{post}}{d\rho} < 0$ if $\beta < \widehat{\beta}$. The result for $\frac{dp_3^{post}}{d\rho}$ follows because $\frac{dp_3^{post}}{d\rho} = \frac{dp_3^{post}}{dp_m^{post}}\frac{dp_m^{post}}{d\rho}$ where $\frac{dp_3^{post}}{dp_m^{post}} > 0$ (as Lemma A1(a) still holds). Since we have shown that $\frac{dp_m^{post}}{d\rho}$ has the same sign as $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_m \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ in the proof of (i), and the sign of $\frac{dp_m^{post}}{d\rho}$ is independent of ρ from statement (i), we only need to show that there exists a threshold $\widehat{\beta}$ such that if $\beta < \widehat{\beta}$, $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} < 0$. If $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} \geq 0$ at $\beta = 0$ (which happens when θ is sufficiently large), the result holds when setting $\widehat{\beta} = 0$. In the rest of the proof, we focus on the case when $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} < 0$ at $\beta = 0$. For this case, it suffices to show that if β is sufficiently large, then $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} > 0$. From the first-order condition in (A.53), we can compute $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1}$ as

$$\begin{aligned}\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} &= -\frac{1}{4}\left\{\alpha - \left(\beta + \frac{n-2}{n}\theta\right)p_m^{post} + \frac{n-2}{n}\theta p_3^{post}\right\} R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right) \\ &\quad + \frac{1}{4}\left(\beta + \frac{n-2}{n}\theta\right)\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right) + p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}.\end{aligned} \tag{A.60}$$

We will show that $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} > 0$ for a sufficiently large β in each of the two cases: (*Case I*) $p_m^{post} \geq p_3^{post}$, and (*Case II*) $p_m^{post} < p_3^{post}$.

(*Case I*) Define the following function:

$$\begin{aligned}g_1\left(\mathbf{p}^{post},\beta\right) &= \left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} - \frac{1}{4}\frac{n-2}{n}\theta\left(p_m^{post} - p_3^{post}\right) R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right) \\ &= -\frac{1}{4}\left(\alpha - \beta p_m^{post}\right) R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right) \\ &\quad +\frac{1}{4}\left(\beta + \frac{n-2}{n}\theta\right)\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right) + p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}.\end{aligned}$$

Since $p_m^{post} \geq p_3^{post}$, $\left.\frac{\partial^2 \pi_m(\mathbf{p},\rho)}{\partial p_1 \partial \rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1} \geq g_1\left(\mathbf{p}^{post},\beta\right)$. In the remainder, we show that $\frac{dg_1}{d\beta}$ is greater than a positive constant, so that $g_1\left(\mathbf{p}^{post},\beta\right) > 0$ for a sufficiently large β. From the expression of $g_1\left(\mathbf{p}^{post},\beta\right)$ we obtain

$$\frac{dg_1}{d\beta}=\frac{1}{4}p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)+\frac{1}{4}\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}$$
$$+\frac{1}{4}\left\{\beta R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)+\frac{\left(\alpha-\beta p_m^{post}\right)w_m^2}{\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^3}+\left(\beta+\frac{n-2}{n}\theta\right)R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}\frac{dp_m^{post}}{d\beta}.$$

Since $\frac{dp_m^{post}}{d\beta}>0$ from statement (ii), every term in $\frac{dg_1}{d\beta}$ is positive. In particular, as β increases, the second term increases because $\frac{d}{dp_m^{post}}\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\}=R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)>0$. So $\frac{dg_1}{d\beta}$ is greater than a positive constant. Therefore, $g_1\left(\mathbf{p}^{post},\beta\right)>0$ for a sufficiently large β.

(*Case II*) Since $L_3\left(\mathbf{p}^{post}\right)=a-bp_3^{post}+\frac{2}{n}\gamma\left(p_m^{post}-p_3^{post}\right)>0$ and $p_m^{post}-p_3^{post}<0$, we obtain $p_3^{post}<a/b$. Substituting this inequality to (A.60) results in

$$\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1}>-\frac{1}{4}\left\{\alpha-\left(\beta+\frac{n-2}{n}\theta\right)p_m^{post}+\frac{n-2}{n}\theta\frac{a}{b}\right\}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)$$
$$+\frac{1}{4}\left(\beta+\frac{n-2}{n}\theta\right)\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}.$$

Define the right-hand side of the above inequality as $g_2\left(\mathbf{p}^{post},\beta\right)$:

$$g_2\left(\mathbf{p}^{post},\beta\right)=-\frac{1}{4}\left\{\alpha-\left(\beta+\frac{n-2}{n}\theta\right)p_m^{post}+\frac{n-2}{n}\theta\frac{a}{b}\right\}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)$$
$$+\frac{1}{4}\left(\beta+\frac{n-2}{n}\theta\right)\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}.$$

Then $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1}>g_2\left(\mathbf{p}^{post},\beta\right)$. From the expression of $g_2\left(\mathbf{p}^{post},\beta\right)$, we obtain the following:

$$\frac{dg_2}{d\beta}=\frac{1}{4}p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)+\frac{1}{4}\left\{w_m\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)+p_m^{post}R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}$$
$$+\frac{1}{4}\left\{\beta R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)+\frac{\left(\alpha-\beta p_m^{post}\right)w_m^2}{\phi\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\left(p_m^{post}\right)^3}+\left(\beta+\frac{n-2}{n}\theta\right)R\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)\right\}\frac{dp_m^{post}}{d\beta}.$$

The expression of $\frac{dg_2}{d\beta}$ is the same as $\frac{dg_1}{d\beta}$ in (Case I). Thus, for a sufficiently large β, $\left.\frac{\partial^2\pi_m(\mathbf{p},\rho)}{\partial p_1\partial\rho}\right|_{\mathbf{p}=\mathbf{p}^{post},\rho=1}>g_2\left(\mathbf{p}^{post},\beta\right)>0$.

(b) Following the same procedure as in the proof of part (a), we can show that the sign of $\frac{dp_m^{post}}{dw_m}$ is the same as $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$. If $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_m\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}\geq 0$ at $w_m=w$, then we can show that $\frac{dp_m^{post}}{ds}<0$ for all $s>0$, hence $s_{gen}^{(1)}=0$. If $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}<0$ at $w_m=w$, then $\frac{dp_m^{post}}{dw_m}<0$ at this point. We compute the expression of $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}$ as follows:

$$\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}=$$
$$\left(b+\frac{n-2}{n}\gamma\right)-\frac{\sqrt{2+2\rho}}{2}\frac{\alpha-\left(\beta+\frac{n-2}{n}\theta\right)p_m^{post}+\frac{\theta(n-2)}{n}p_3^{post}}{h\left(\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right)\right)p_m^{post}}+\frac{\sqrt{2+2\rho}}{2}\left(\beta+\frac{n-2}{n}\theta\right)\Phi^{-1}\left(1-\frac{w_m}{p_m^{post}}\right).$$

As $w_m\to 0$, the second term approaches zero, and the first and third terms are positive, so $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}>0$. Thus, there exists $w_m^{(1)}$ such that $\left.\frac{\partial^2\pi_m(\mathbf{p},w_m)}{\partial p_1\partial w_m}\right|_{\mathbf{p}=\mathbf{p}^{post}}=0$ at $w_m=w_m^{(1)}$.

Following the same procedure as in the proof of Proposition 1.2(a), we can show that p_m^{post} is increasing with w_m if and only if $w_m < w_m^{(1)}$. Define $s_{gen}^{(1)} = (w - w_m^{(1)})/w$. Then p_m^{post} is increasing with s if and only if $s < s_{gen}^{(1)}$.

Following the same procedure as in the proof of $d\pi_i^{post}/ds < 0$ ($i = 3, 4, \ldots, n$) in Corollary 2.3(b), we can write $d\pi_i^{post}/ds$ as follows: $\frac{d\pi_i^{post}}{ds} = \sum_{j \neq i}^{n} \frac{\partial \pi_i^{post}}{\partial p_j^{post}} \frac{dp_j^{post}}{ds}$. Since we have shown $\frac{dp_j^{post}}{ds} > 0$ if $s < s_{gen}^{(1)}$, we need to show $\frac{\partial \pi_i^{post}}{\partial p_j^{post}} > 0$ ($j \neq i$) to prove $\frac{d\pi_i^{post}}{ds} < 0$. From (A.47) we can calculate $\frac{\partial \pi_i^{post}}{\partial p_j^{post}}$ as follows:

$$\frac{\partial \pi_i^{post}}{\partial p_j^{post}} = \frac{\gamma}{n}(p_i - w) - \frac{\theta}{n}\left\{w\Phi^{-1}\left(1 - \frac{w}{p_i^{post}}\right) + p_i^{post} R\left(\Phi^{-1}\left(1 - \frac{w}{p_i^{post}}\right)\right)\right\}.$$

By using $R(t) = \phi(t) - t\{1 - \Phi(t)\}$, we can simplify the above equation as follows:

$$\frac{\partial \pi_i^{post}}{\partial p_j^{post}} = \frac{\gamma}{n}(p_i - w) - \frac{\theta}{n} p_i^{post} \phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i^{post}}\right)\right) =$$
$$\frac{p_i^{post} - w}{n}\left\{\gamma - \theta\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i^{post}}\right)\right)(1 - \frac{w}{p_i^{post}})^{-1}\right\}.$$

So if $\gamma > \theta\phi\left(\Phi^{-1}\left(1 - \frac{w}{p_i^{post}}\right)\right)(1 - \frac{w}{p_i^{post}})^{-1}$, $\frac{\partial \pi_i^{post}}{\partial p_j^{post}} > 0$. $\square$

Appendix B

Supplements to Chapter 2

B.1 Proofs for Main Results

Proof of Lemma 2.1. We prove that a switching strategy around $x^{(2)}$ is an equilibrium by showing that if all firms but firm i follow this switching strategy, then firm i's best response is to follow this switching strategy. (In Lemma O1 of Online Appendix, we show how we derive this strategy using the argument of higher-order beliefs.)

We first derive firm i's expected gain $u_i(x_i)$ as a function of x_i, and then show that $u_i(x_i) \geq 0$ if and only if $x_i \geq x^{(2)}$. For notational convenience, define $\eta_i \equiv \frac{x_i - x^{(2)}}{2\epsilon}$. Given that every other firm follows a switching strategy around $x^{(2)}$, we can use the expressions of α in (2.3) and integrate $u_i(\theta, \alpha)$ in (2.1) to get the expression of $u_i(x_i)$. When $x_i < x^{(2)} - 2\epsilon$, we get $\alpha = 0$ and $u_i(x_i) = x_i - (f_L - m_L)$. When $x^{(2)} - 2\epsilon \leq x_i \leq x^{(2)}$, we obtain the following expression of $u_i(x_i)$:

$$\begin{aligned} u_i(x_i) &= \int_{x_i-\epsilon}^{x_i-\epsilon} \frac{\theta}{2\epsilon} d\theta - (f_L - m_L) + \frac{1}{2\epsilon} \int_{x^{(2)}-\epsilon}^{x_i+\epsilon} \left\{ (f_H - m_L) \left(\frac{\theta + \epsilon - x^{(2)}}{2\epsilon} \right)^r - b \frac{\theta + \epsilon - x^{(2)}}{2\epsilon} \right\} d\theta \\ &= x_i - (f_L - m_L) + \int_0^{\frac{x_1 + 2\epsilon - x^{(2)}}{2\epsilon}} \left\{ -b\alpha + (f_H - m_L) \alpha^r \right\} d\alpha, \qquad \text{(B.1)} \end{aligned}$$

where the equality is obtained by changing the integration variable from θ to $\alpha = \frac{\theta + \epsilon - x^{(2)}}{2\epsilon}$. By using $\eta_i = \frac{x_i - x^{(2)}}{2\epsilon}$ (which is between -1 and 0 because $x^{(2)} - 2\epsilon \leq x_i \leq x^{(2)}$), we rewrite $u_i(x_i)$ in (B.1) as:

$$u_i(\eta_i) = x^{(2)} + 2\epsilon\eta_i - (f_L - m_L) + \int_0^{1+\eta_i} \left\{ -b\alpha + (f_H - m_L) \alpha^r \right\} d\alpha.$$

Similarly, when $x^{(2)} < x_i \leq x^{(2)} + 2\epsilon$, we obtain

$$u_i(\eta_i) = x^{(2)} + 2\epsilon\eta_i - (f_L - m_L) + \int_{\eta_i}^{1}\{-b\alpha + (f_H - m_L)\alpha^r\}\,d\alpha + \eta_i(f_H - m_L - b).$$

When $x_i > x^{(2)} + 2\epsilon$, we obtain $u_i(x_i) = x_i - b$.

Next, we prove $u_i(x_i) \geq 0$ if and only if $x_i \geq x^{(2)}$ by showing: (i) $u_i\left(x^{(2)}\right) = 0$, and (ii) if $\epsilon \geq \epsilon^{(2)}$, then $u_i(x_i)$ is nondecreasing with x_i. To prove (i), from (B.1), we obtain $u_i\left(x^{(2)}\right) = x^{(2)} - (f_L - m_L) - \frac{1}{2}b + \frac{f_H - m_L}{r+1}$. By substituting $x^{(2)} = \frac{1}{2}b + f_L - \frac{rm_L + f_H}{r+1}$ into $u_i\left(x^{(2)}\right)$, we get $u_i\left(x^{(2)}\right) = 0$. To prove (ii), we derive conditions for $du_i(x_i)/dx_i \geq 0$ in each interval of x_i, and express those conditions as $\epsilon \geq \epsilon^{(2)}$. When $x_i < x^{(2)} - 2\epsilon$ or $x_i > x^{(2)} + 2\epsilon$, it is easy to show $du_i(x_i)/dx_i = 1 > 0$. When $x^{(2)} - 2\epsilon \leq x_i \leq x^{(2)}$, we analyze the sign of $du_i(\eta_i)/d\eta_i$ because $du_i(x_i)/dx_i = 2\epsilon du_i(\eta_i)/d\eta_i$ where $\epsilon > 0$. In preparation, we compute $\frac{du_i}{d\eta_i} = 2\epsilon - b(1+\eta_i) + (f_H - m_L)(1+\eta_i)^r$, $\frac{d^2u_i}{d\eta_i^2} = -b + r(f_H - m_L)(1+\eta_i)^{r-1}$, and $\frac{d^3u_i}{d\eta_i^3} = r(r-1)(f_H - m_L)(1+\eta_i)^{r-2}$. First, consider the case when $r \leq 1$. Then $\frac{d^3u_i}{d\eta_i^3} \leq 0$. So if $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=-1} \geq 0$ and $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=0} \geq 0$, then $\frac{du_i}{d\eta_i} \geq 0$ for any $-1 \leq \eta_i \leq 0$. Since $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=-1} = 2\epsilon$ and $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=0} = 2\epsilon + (f_H - m_L) - b$, $\frac{\partial u_i}{\partial \eta_i} \geq 0$ if $\epsilon \geq \frac{b-(f_H-m_L)}{2}$. Second, consider the case when $r > 1$. Then $\frac{d^3u_i}{d\eta_i^3} > 0$, and $\frac{du_i}{d\eta_i}$ achieves its minimum value of $2\epsilon + \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} b\left\{\frac{1}{r} - 1\right\}$ at $\eta_i = \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} - 1$. If $r \geq \frac{b}{f_H-m_L}$, then $0 \leq \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} \leq 1$ and $\frac{du_i}{d\eta_i} \geq 0$ if $\epsilon \geq \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right)$; otherwise, $\frac{du_i}{d\eta_i}$ is decreasing with $\eta_i \in [-1, 0]$, so $\frac{du_i}{d\eta_i} > 0$ if $\epsilon \geq \frac{b-(f_H-m_L)}{2}$. Following the same procedure as above, when $x^{(2)} < x_i \leq x^{(2)} + 2\epsilon$, we can obtain the condition for $\frac{du_i}{dx_i} \geq 0$ as follows: $\epsilon \geq \frac{b-(f_H-m_L)}{2}$ if $r \geq \min\left\{1, \frac{b}{f_H-m_L}\right\}$, and $\epsilon \geq \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left\{\frac{1}{r} - 1\right\} + \frac{b}{2} - \frac{1}{2}(f_H - m_L)$ if $r < \min\left\{1, \frac{b}{f_H-m_L}\right\}$. By combining the conditions for different intervals of x_i, we have $\frac{du_i}{dx_i} \geq 0$ for any x_i if $\epsilon \geq \epsilon^{(2)}$, where

$$\epsilon^{(2)} = \begin{cases} \frac{b-(f_H-m_L)}{2} - \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right) & \text{if } r < \min\left\{1, \frac{b}{f_H-m_L}\right\}; \\ \max\left\{0, \frac{b-(f_H-m_L)}{2}\right\} & \text{if } \min\left\{1, \frac{b}{f_H-m_L}\right\} \leq r \leq \max\left\{1, \frac{b}{f_H-m_L}\right\}; \\ \left\{\frac{b}{r(f_H-m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right) & \text{if } r > \max\left\{1, \frac{b}{f_H-m_L}\right\}. \end{cases} \tag{B.2}$$

We show that $\epsilon^{(2)}$ in (B.2) is nondecreasing with b and nonincreasing with $f_H - m_L$ in Online Appendix (Lemma O2). □

Proof of Proposition 2.1. Since $x_1 = \theta + \tilde{\varepsilon}_1$, for any given x_1, the posterior distribution of θ is a uniform distribution on $[x_1 - \epsilon, x_1 + \epsilon]$. Since $\tilde{\varepsilon}_i$ is independent of $\tilde{\varepsilon}_1$, it is also independent of θ. Thus, $x_i = \theta + \tilde{\varepsilon}_i$ is a sum of two uniformly distributed random variables that are independent. Using convolution, we can show that the posterior distribution of x_i for any given x_1 is a symmetric triangular distribution on $[x_1 - 2\epsilon, x_1 + 2\epsilon]$. Using this result, we next prove (a) and (b).

(a) Following the same procedure as in the proof of Lemma 2.1, we obtain the expression of $u_1(x_1)$ in (2.4), and the expression of $\frac{\partial u_1}{\partial x_1}$ in (2.5). We next consider two cases: $\epsilon \geq b/2$ and $\epsilon < b/2$.

For the case of $\epsilon \geq b/2$, from (2.5) $\frac{\partial u_1}{\partial x_1} \geq 0$. So u_1 crosses zero only once. Let $x_{subs}^{(1)}$ be the solution of $u_1(x_1)$. Then $u_1(x_1) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

For the case of $\epsilon < b/2$, we will first prove that u_1 increases with x_1, then decreases with x_1, and finally increases with x_1 again. We then compare the local maximum and minimum of u_1 with 0 to determine firm 1's decision. First, we show the shape of u_1 on $\left[x^{(2)} - 2\epsilon, x^{(2)}\right]$. From (2.5), $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)} - 2\epsilon} > 0$, $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)}} < 0$, and $\frac{\partial^2 u_1}{\partial x_1^2} = -\frac{b}{(2\epsilon)^2} < 0$ when $x^{(2)} - 2\epsilon \leq x_1 < x^{(2)}$. Therefore, There exists $x_1^M \in \left(x^{(2)} - 2\epsilon, x^{(2)}\right)$ such that at $x_1 = x_1^M$, $\frac{\partial u_1}{\partial x_1} = 0$ and $u_1(x_1)$ achieves its local maximum $u_1(x_1^M)$. Solving $\frac{\partial u_1}{\partial x_1} = 0$ in this case yields $x_1^M = x^{(2)} - 2\epsilon + \frac{(2\epsilon)^2}{b}$. Substituting this expression to (2.4) we obtain $u_1(x_1^M) = \lambda - f_1 + x^{(2)} - 2\epsilon + \frac{2\epsilon^2}{b}$. Similarly, from (2.5) we get $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)}} < 0$, $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)} + 2\epsilon} > 0$, and $\frac{\partial^2 u_1}{\partial x_1^2} = \frac{b}{(2\epsilon)^2} > 0$ when $x^{(2)} \leq x_1 \leq x^{(2)} + 2\epsilon$. So There exists $x_1^m \in \left(x^{(2)}, x^{(2)} + 2\epsilon\right)$ such that at $x_1 = x_1^m$, $\frac{\partial u_1}{\partial x_1} = 0$ and $u_1(x_1)$ achieves its local minimum value $u_1(x_1^m) = \lambda - f_1 + x^{(2)} + 2\epsilon - b - \frac{2\epsilon^2}{b}$.

We next analyze three cases: (Case I) $u_1(x_1^M) < 0$, (Case II) $u_1(x_1^m) > 0$, and (Case III) $u_1(x_1^m) \leq 0$ and $u_1(x_1^M) \geq 0$.

(Case I) From the expression of $u_1(x_1^M)$, $u_1(x_1^M) < 0$ when $\lambda < -x^{(2)} + 2\epsilon + f_1 - m_L$. Let $\underline{\lambda}_{subs} = -x^{(2)} + 2\epsilon + f_1 - m_L$. Then $u_1(x_1^M) < 0$ if and only if $\lambda < \underline{\lambda}_{subs}$. In this case, given that the local maximum $u_1(x_1^M) < 0$, $u_1(x_1)$ can cross zero once in the interval (x_1^m, ∞). Denote by $x_{subs}^{(1)}$ the solution of $u_1(x_1) = 0$. Then $u_1(x_1) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

(Case II) Similar to (Case I), we can show $u_1(x_1^m) > 0$ when $\lambda > \overline{\lambda}_{subs}$, where $\overline{\lambda}_{subs} = -x^{(2)} - 2\epsilon + f_L + b + \frac{2\epsilon^2}{b} - m_L$. There exists $x_{subs}^{(1)} < x_1^m$ such that $u_1(x_1) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

(Case III) Similarly, $u_1(x_1^m) \leq 0$ and $u_1(x_1^M) \geq 0$ when $\underline{\lambda}_{subs} \leq \lambda \leq \overline{\lambda}_{subs}$. We can prove that There exists $x_{subs}^{(1)}$, $y_{subs}^{(1)}$, and $z_{subs}^{(1)}$ ($x_{subs}^{(1)} \leq x_1^M \leq y_{subs}^{(1)} \leq x_1^m \leq z_{subs}^{(1)}$) such that $u_1(x_1) \geq 0$ if and only if $x_1 \in [x_{subs}^{(1)}, y_{subs}^{(1)}] \cup [z_{subs}^{(1)}, \infty)$.

(b) In order to compute $\frac{dx_{subs}^{(1)}}{dr}$, we apply the implicit function theorem to the equation $u_1(x_1) = 0$ and obtain the following: $\frac{dx_{sub}^{(1)}}{dr} = -\left.\frac{\partial u_1/\partial r}{\partial u_1/\partial x_1}\right|_{x_1 = x_{subs}^{(1)}}$. From the proof of part (a), $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1=x_{subs}^{(1)}} > 0$ and $\frac{dx^{(1)}}{dr}$ has the same sign as $-\left.\frac{\partial u_1}{\partial r}\right|_{x_1=x_{subs}^{(1)}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial r}\right|_{x_1=x_{subs}^{(1)}}$ in the following four cases: (Case I) $x_{subs}^{(1)} < x^{(2)} - 2\epsilon$, (Case II) $x^{(2)} - 2\epsilon \le x_{subs}^{(1)} \le x^{(2)}$, (Case III) $x^{(2)} < x_{subs}^{(1)} \le x^{(2)} + 2\epsilon$, and (Case IV) $x_{subs}^{(1)} > x^{(2)} + 2\epsilon$.

(Cases I and IV) From (2.4), we obtain $\frac{\partial u_1}{\partial r} = 0$. So $\frac{dx_{sub}^{(1)}}{dr} = 0$ and $x_{sub}^{(1)}$ is independent of r.

(Case II) From (2.4) and $x^{(2)} = \frac{1}{2}b + f_L - \frac{f_H}{r+1}$ in Lemma 2.1, we obtain $\frac{\partial u_1}{\partial r} = \frac{\partial u_1}{\partial x^{(2)}}\frac{\partial x^{(2)}}{\partial r} = \frac{b}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)\frac{f_H}{(r+1)^2} > 0$. So $\frac{dx_{sub}^{(1)}}{dr} < 0$.

(Case III) Similar to Case II, we obtain $\frac{\partial u_1}{\partial r} = \frac{b}{2\epsilon}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)\right\}\frac{f_H}{(r+1)^2} > 0$. So $\frac{dx_{sub}^{(1)}}{dr} < 0$.

Following the same procedure, we can prove the results for $y_{subs}^{(1)}$ and $z_{subs}^{(1)}$. $\square$

Proof of Proposition 2.2. Following the same procedure as in the proof of Lemma 2.1, we obtain the expression of $\frac{\partial u_1}{\partial x_1}$ in (2.6). We can verify that $\frac{\partial u_1}{\partial x_1} > 0$ in all intervals. So u_1 crosses zero only once. Let $x_{comp}^{(1)}$ be the solution of $u_1(x_1) = 0$. Then $u_1(x_1) > 0$ if and only if $x_1 > x_{comp}^{(1)}$.

(b) The proof follows the same procedure as in the proof of Proposition 2.1(b). $\square$

Proof of Proposition 2.3. (a) Similar to the proof of Proposition 2.1(a), we can show that there are two cases. In the first case, $u_1(x_1)$ is nondecreasing with x_1, and There exists threshold $x_{aggr}^{(1)}$ such that $u_1(x_1) > 0$ if and only if $x_1 > x_{aggr}^{(1)}$. In the second case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again. In this case, there exists $x_1^M < x_1^m$ such that $u_1(x_1)$ achieves a local maximum at x_1^M and a local minimum at x_1^m. If $u_1\left(x_1^M\right) < 0$ or $u_1(x_1^m) > 0$, then there exists threshold $x_{aggr}^{(1)}$ such that $u_1(x_1) > 0$ if and only if $x_1 > x_{aggr}^{(1)}$. If $u_1\left(x_1^M\right) \ge 0$ and $u_1(x_1^m) \le 0$, then there exists $x_{aggr}^{(1)} \le x_1^M \le y_{aggr}^{(1)} \le x_1^m \le z_{aggr}^{(1)}$ such that $u_1(x_1) \ge 0$ if and only if $x_1 \in [x_{aggr}^{(1)}, y_{aggr}^{(1)}] \cup [z_{aggr}^{(1)}, \infty)$. We let $\underline{\lambda}_{aggr}$ be the value of λ such that $u_1\left(x_1^M\right) = 0$ if $\lambda = \underline{\lambda}_{aggr}$, and $\overline{\lambda}_{aggr}$ be the value of λ such that $u_1(x_1^m) = 0$ if $\lambda = \overline{\lambda}_{aggr}$. Then $u_1\left(x_1^M\right) \ge 0$ and $u_1(x_1^m) \le 0$ if and only if $\underline{\lambda}_{aggr} \le \lambda \le \overline{\lambda}_{aggr}$.

Finally, we show in Online Appendix (Lemma O3) that the first case (respectively, the second case) occurs if $\epsilon \ge \epsilon^{(1)}$ (respectively, $\epsilon < \epsilon^{(1)}$), where $\epsilon^{(1)}$ is given as follows:

$$\epsilon^{(1)} = \begin{cases} \frac{1}{2}\{b - (m_H - m_L)\} - \left\{\frac{b}{r(m_H - m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right) & \text{if } r < \min\left\{1, \frac{b}{m_H - m_L}\right\}; \\ \max\left\{0, \frac{1}{2}\{b - (m_H - m_L)\}\right\} & \text{if } \min\left\{1, \frac{b}{m_H - m_L}\right\} \le r \le \max\left\{1, \frac{b}{m_H - m_L}\right\}; \\ \left\{\frac{b}{r(m_H - m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right) & \text{if } r > \max\left\{1, \frac{b}{m_H - m_L}\right\}. \end{cases}$$

Similar to Lemma 2.1, we can show that $\epsilon^{(1)}$ is nondecreasing with b and nonincreasing with $m_H - m_L$.

(b) As in the proof of Proposition 2.1(b), we can show that $\frac{dx_{aggr}^{(1)}}{dr}$ has the same sign as $-\left.\frac{\partial u_1}{\partial r}\right|_{x_1 = x_{aggr}^{(1)}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial r}\right|_{x_1 = x_{aggr}^{(1)}}$ in the following four cases: (Case I) $x_{aggr}^{(1)} < x^{(2)} - 2\epsilon$, (Case II) $x^{(2)} - 2\epsilon \le x_{aggr}^{(1)} \le x^{(2)}$, (Case III) $x^{(2)} < x_{aggr}^{(1)} \le x^{(2)} + 2\epsilon$, and (Case IV) $x_{aggr}^{(1)} > x^{(2)} + 2\epsilon$.

(Cases I and IV) From (B.8), $\frac{\partial u_1}{\partial r} = 0$. So $\frac{dx_{aggr}^{(1)}}{dr} = 0$ and $x_{aggr}^{(1)}$ is independent of r.

(Case II) From (B.10) and $x^{(2)} = \frac{1}{2}b + f_L - \frac{rm_L + f_H}{r+1}$ in Lemma 2.1, we obtain the following:

$$\begin{aligned} \frac{du_1}{dr} &= \frac{\partial u_1}{\partial r} + \frac{\partial u_1}{\partial x^{(2)}} \frac{\partial x^{(2)}}{\partial r} + \frac{du_1}{d\eta_1} \frac{\partial \eta_1}{\partial x^{(2)}} \frac{\partial x^{(2)}}{\partial r} \\ &= \frac{(1+\eta_1)}{r+1} \left[-(m_H - m_L)(1+\eta_1)^{r-1} \left\{ \frac{1+\eta_1}{r+1} - (1+\eta_1)\ln(1+\eta_1) + \frac{f_H - m_L}{2\epsilon(r+1)} \right\} + \frac{1}{2\epsilon} \frac{(f_H - m_L)b}{r+1} \right]. \end{aligned}$$

Let $b^{(x)} = (m_H - m_L)\left(1+\eta_{aggr}\right)^{r-1} \left\{ \frac{2\epsilon\left(1+\eta_{aggr}\right)}{f_H - m_L} - \frac{2\epsilon(r+1)\left(1+\eta_{aggr}\right)}{f_H - m_L} \ln\left(1+\eta_{aggr}\right) + 1 \right\}$, where $\eta_{aggr} = \frac{x_{aggr}^{(1)} - x^{(2)}}{2\epsilon}$. Finally, from the above equation, we get $\frac{du_1}{dr} > 0$, and thus $\frac{dx_{aggr}^{(1)}}{dr} < 0$ if and only if $b > b^{(x)}$.

(Case III) Similar to (Case II), we can show that $\frac{dx_{aggr}^{(1)}}{dr} < 0$ if and only if $b > b^{(x)}$, where $b^{(x)} = \frac{m_H - m_L}{1 - \eta_{aggr}} \left\{ \frac{2\epsilon\left(1 - \eta_{aggr}^{r+1}\right)}{f_H - m_L} + \frac{2\epsilon(r+1)\eta_{aggr}^{r+1}}{f_H - m_L} \ln \eta_{aggr} + 1 - \eta_{aggr}^{r} \right\}$.

Following the same procedure, we can prove the results for $y_{aggr}^{(1)}$ and $z_{aggr}^{(1)}$. □

Proof of Lemma 2.2. **(a)** Following the same procedure as in the proof of Lemma 2.1, we show that if all firms but firm i follow a switching strategy around $\widehat{x}^{(2)}$, then firm i's best response is to follow this switching strategy as well. Similar to Lemma 2.1, we first derive the expected gain $\widehat{u}_i(x_i)$ when other firms follow a switching strategy around $\widehat{x}^{(2)}$. We can show that if $\epsilon \ge b/2$, then $\widehat{u}_i\left(\widehat{x}^{(2)}\right) = 0$, and $\widehat{u}_i(x_i)$ is nondecreasing with x_i. Thus $\widehat{u}_i(x_i) > 0$ if and only if $x_i > \widehat{x}^{(2)}$.

(b) Following the same procedure as in the proof of Lemma 2.1, we obtain the expression of

$\widehat{u}_1(x_1)$ as follows:

$$\widehat{u}_1(x_1) = \begin{cases} \lambda - f_1 + m_L + p(m_H - m_L) + x_1 & \text{if } x_1 < \widehat{x}^{(2)} - 2\epsilon; \\ \lambda - f_1 + m_L + p(m_H - m_L) + x_1 - \frac{b}{2}\left(\frac{x_1 - \widehat{x}^{(2)} + 2\epsilon}{2\epsilon}\right)^2 & \text{if } \widehat{x}^{(2)} - 2\epsilon \le x_1 < \widehat{x}^{(2)}; \\ \begin{array}{l} \lambda - f_1 + m_L + p(m_H - m_L) + x_1 \\ -\frac{b}{2}\left\{1 - \left(\frac{x_1 - \widehat{x}^{(2)}}{2\epsilon}\right)^2\right\} - b\left(\frac{x_1 - \widehat{x}^{(2)}}{2\epsilon}\right) \end{array} & \text{if } \widehat{x}^{(2)} \le x_1 \le \widehat{x}^{(2)} + 2\epsilon; \\ \lambda - f_1 + m_L + p(m_H - m_L) + x_1 - b & \text{if } x_1 > \widehat{x}^{(2)} + 2\epsilon. \end{cases} \tag{B.3}$$

The rest of the proof follows the same procedure as in the proof of Proposition 2.1. □

Proof of Proposition 2.4. We first solve for the value of r that results in $x^{(2)} = \widehat{x}^{(2)}$. Noting that $x^{(2)} = \frac{1}{2}b + f_L - \frac{rm_L + f_H}{r+1}$ from Lemma 2.1 and $\widehat{x}^{(2)} = \frac{1}{2}b + f_L + (1-p)(-m_L) - pf_H$ from Lemma 2.2(a), we solve $x^{(2)} = \widehat{x}^{(2)}$ and obtain $r = 1/p - 1$.

To compare $x^{(1)}_{aggr}$ and $\widehat{x}^{(1)}$, we compare $u_1(x_1)$ in (B.8) and $\widehat{u}_1(x_1)$ in (B.3) for $x_1 = \widehat{x}^{(1)}$ given that $r = 1/p - 1$. If $u_1(\widehat{x}^{(1)}) \ge \widehat{u}_1(\widehat{x}^{(1)}) = 0$, then $x^{(1)}_{aggr} \le \widehat{x}^{(1)}$ because $u_1(x_1) \ge 0$ for $x_1 \ge x^{(1)}_{aggr}$. To compare $u_1(\widehat{x}^{(1)})$ and $\widehat{u}_1(\widehat{x}^{(1)})$, we compute $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)})$ using (B.3) and the expression of $u_1(x_1)$ in the proof of Proposition 2.3:

$$u_1\left(\widehat{x}^{(1)}\right) - \widehat{u}_1\left(\widehat{x}^{(1)}\right) = \begin{cases} -p(m_H - m_L) & \text{if } \widehat{x}^{(1)} < \widehat{x}^{(2)} - 2\epsilon; \\ p(m_H - m_L)\left\{\left(1 + \frac{\widehat{x}^{(1)} - \widehat{x}^{(2)}}{2\epsilon}\right)^{\frac{1}{p}} - 1\right\} & \text{if } \widehat{x}^{(2)} - 2\epsilon \le \widehat{x}^{(1)} < \widehat{x}^{(2)}; \\ (m_H - m_L)\left(\frac{\widehat{x}^{(1)} - \widehat{x}^{(2)}}{2\epsilon}\right)\left\{1 - p\left(\frac{\widehat{x}^{(1)} - \widehat{x}^{(2)}}{2\epsilon}\right)^{\frac{1}{p} - 1}\right\} & \text{if } \widehat{x}^{(2)} \le \widehat{x}^{(1)} \le \widehat{x}^{(2)} + 2\epsilon; \\ (1-p)(m_H - m_L) & \text{if } \widehat{x}^{(1)} > \widehat{x}^{(2)} + 2\epsilon. \end{cases} \tag{B.4}$$

From (B.4) $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)}) = 0$ if $m_H = m_L$. In this case $x^{(1)}_{aggr} = \widehat{x}^{(1)}$. We next examine the case in which $m_H > m_L$. From (B.4) $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)}) < 0$ if $\widehat{x}^{(1)} < \widehat{x}^{(2)} - 2\epsilon$, and $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)}) > 0$ if $\widehat{x}^{(1)} > \widehat{x}^{(2)} + 2\epsilon$. We next show that $d\left\{u_1\left(\widehat{x}^{(1)}\right) - \widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}/d\widehat{x}^{(1)} \ge 0$ such that there exists $x^{(p)}$ such that $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)}) \ge 0$ if and only if $\widehat{x}^{(1)} \ge x^{(p)}$. From (B.4) we get the following:

$$\frac{d\left\{u_1\left(\widehat{x}^{(1)}\right)-\widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}}{d\widehat{x}^{(1)}}=\begin{cases}0 & \text{if } \widehat{x}^{(1)}<\widehat{x}^{(2)}-2\epsilon;\\ \frac{m_H-m_L}{2\epsilon}\left(1+\frac{\widehat{x}^{(1)}-\widehat{x}}{2\epsilon}\right)^{\frac{1}{p}-1} & \text{if } \widehat{x}^{(2)}-2\epsilon\leq\widehat{x}^{(1)}<\widehat{x}^{(2)};\\ \frac{m_H-m_L}{2\epsilon}\left\{1-\left(\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)^{\frac{1}{p}-1}\right\} & \text{if } \widehat{x}^{(2)}\leq\widehat{x}^{(1)}\leq\widehat{x}^{(2)}+2\epsilon;\\ 0 & \text{if } \widehat{x}^{(1)}>\widehat{x}^{(2)}+2\epsilon.\end{cases}$$

We can verify $d\left\{u_1\left(\widehat{x}^{(1)}\right)-\widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}/d\widehat{x}^{(1)}\geq 0$ for any $\widehat{x}^{(1)}$.

We finally change the condition $\widehat{x}^{(1)}\geq x^{(p)}$ to a condition on λ using the implicit function theorem. Note that $\widehat{x}^{(1)}$ is the solution of $\widehat{u}_1\left(x_1\right)=0$ and $\widehat{u}_1\left(x_1\right)$ increases linearly with λ from (B.3). By the implicit function theorem, $\frac{d\widehat{x}_1}{d\lambda}=-\left.\frac{\partial\widehat{u}_1/\partial\lambda}{\partial\widehat{u}_1/\partial x_1}\right|_{x_1=\widehat{x}^{(1)}}<0$ because $\widehat{u}_1\left(x_1\right)$ increases with x_1 at $x_1=\widehat{x}^{(1)}$. So there exists $\widehat{\lambda}$ such that $\widehat{x}^{(1)}\geq x^{(p)}$ if and only if $\lambda\leq\widehat{\lambda}$. □

Proof of Lemma 2.3. **(a)** We provide a sketch of proof here, while providing the detailed proof in Online Appendix. First, by analyzing the case in which firm i chooses $a_i=0$ given that $a_j=0$ $(j\neq 1,i)$, we obtain the following: $a_i=0$ $(i\neq 1)$ is an equilibrium if $\theta<f_L-m_L$. Next, by analyzing the case in which firm i chooses $a_i=1$ given that $a_j=1$ $(j\neq 1,i)$, we obtain the following: $a_i=1$ $(i\neq 1)$ is an equilibrium if $\theta>b-f_H+f_L$. Finally, for $b-f_H+f_L\leq\theta\leq f_L-m_L$, since both equilibria are possible, we choose the equilibrium that maximizes firms' total payoffs and obtain the following: $a_i=1$ if $\theta\geq\theta^{(2)}$, where $\theta^{(2)}=f_L+\min\left\{-m_L,b-m_H\right\}$.
(b) We first examine firm 1's decision when $\alpha=1$ or $\alpha=0$, respectively. In the case of $\alpha=1$ (which occurs when $\theta\geq\theta^{(2)}$), from (2.2), $\pi_1\left(1;\theta,1\right)-\pi_1\left(0;\theta,1\right)=\lambda+\theta-f_1-b+m_H$. Define $\theta^{(a)}\equiv f_1-\lambda+b-m_H$ (the threshold of θ when all other firms adopt the technology). Then $\pi_1\left(1;\theta,1\right)-\pi_1\left(0;\theta,1\right)=\theta-\theta^{(a)}$. So $a_1=1$ if and only if $\theta\geq\theta^{(a)}$. Similarly, in the case of $\alpha=0$ when $\theta<\theta^{(2)}$, $a_1=1$ if and only if $\theta\geq\theta^{(n)}\equiv f_1-\lambda-m_L$ (the threshold of θ when no other firms adopt the technology).

We analyze two cases: (Case I) $b\geq m_H-m_L$ (such that $\theta^{(a)}\geq\theta^{(n)}$), and (Case II) $b<m_H-m_L$ (such that $\theta^{(a)}<\theta^{(n)}$). Combining the conditions in (Case I) and (Case II), we can get the conditions specified in Lemma 2.3.
(Case I) We plot the relative positions of $\theta^{(2)}$, $\theta^{(n)}$, and $\theta^{(a)}$ in Figure B-1. Figures B-1(a) corresponds to the scenario in which $\theta^{(2)}\leq\theta^{(n)}$. Note that $a_1=1$ if one of the following two conditions is met: (i) no other firms adopt the technology ($\alpha=0$) and $\theta\geq\theta^{(n)}$, or (ii) all other firms adopt the technology ($\alpha=1$) and $\theta\geq\theta^{(a)}$. Using the relative position of $\theta^{(a)}$ with respect to interval $[\theta^{(2)},\infty)$ (in which $\alpha=1$) shown in Figure B-1(a), we can obtain the

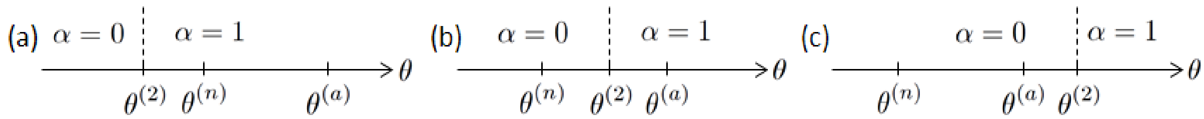

Figure B-1: The Relative Positions of $\theta^{(2)}$, $\theta^{(n)}$, and $\theta^{(a)}$ when: (a) $\theta^{(2)} \leq \theta^{(n)}$, (b) $\theta^{(n)} < \theta^{(2)} < \theta^{(a)}$, and (c) $\theta^{(2)} \geq \theta^{(a)}$.

following: $a_1 = 1$ if and only if $\theta \geq \theta^{(1)}$, where $\theta^{(1)} = \theta^{(a)}$. Similarly, for the scenario in which $\theta^{(n)} < \theta^{(2)} < \theta^{(a)}$, $a_1 = 1$ if and only if $\theta \in [\theta^{(1)}, \theta^{(2)}] \cup [\theta^{(0)}, \infty)$, where $\theta^{(1)} = \theta^{(n)}$ and $\theta^{(0)} = \theta^{(a)}$. Using the expressions of $\theta^{(2)}$, $\theta^{(n)}$, and $\theta^{(a)}$, we can simplify the condition $\theta^{(n)} < \theta^{(2)} < \theta^{(a)}$ as follows: $f_1 - f_L < \lambda < f_1 + b + m_L - m_H - f_L$. Similarly, for the scenario in which $\theta^{(2)} \geq \theta^{(a)}$, $a_1 = 1$ if and only if $\theta \geq \theta^{(1)}$, where $\theta^{(1)} = \theta^{(n)}$.

(Case II) Following the same procedure as in (Case I), we obtain the following: $a_1 = 1$ if and only if $\theta \geq \theta^{(1)}$, where $\theta^{(1)} = \theta^{(a)}$ if $\theta^{(2)} \leq \theta^{(a)}$; $\theta^{(1)} = \theta^{(2)}$ if $\theta^{(a)} < \theta^{(2)} < \theta^{(n)}$; and $\theta^{(1)} = \theta^{(n)}$ if $\theta^{(2)} \geq \theta^{(n)}$. $\square$

Proof of Proposition 2.5. We provide a sketch of proof here, while providing the detailed proof in Online Appendix. The proof focuses on the case in which $b < m_H - m_L$. The case in which $b = m_H - m_L$ or $b > m_H - m_L$ can be proved similarly. We examine three scenarios shown in Figure B-1. For each scenario, similar to the proof of Proposition 2.4, we analyze the condition for $u_1(\theta^{(1)}) \geq 0$ to get a sufficient condition for $x_{aggr}^{(1)} \leq \theta^{(1)}$ because $u(x_1) < 0$ for any $x_1 < x_{aggr}^{(1)}$. We first obtain the values of $u_1(\theta^{(1)})$ on boundary points $x^{(2)} - 2\epsilon$ and $x^{(2)} + 2\epsilon$, and then analyze the changing patterns of $u_1(\theta^{(1)})$ in the following intervals to get the condition for $u_1(\theta^{(1)}) \geq 0$: $(-\infty, x^{(2)} - 2\epsilon)$, $\left[x^{(2)} - 2\epsilon, x^{(2)} + 2\epsilon\right]$, and $(x^{(2)} + 2\epsilon, \infty)$. We finally combine the conditions in all scenarios to derive the sufficient condition in the Proposition 2.5.
$\square$

B.2 Supplemental Materials

This online appendix provides the detailed proofs that are omitted in Appendix.

Lemma O1 *If $r \leq 1$ and $b \leq r(f_H - m_L)$, then the equilibrium described in Lemma 2.1 is unique.*

Proof. Before we proceed to the proof, we first prove $u_i(\theta, \alpha)$ in (2.1) is increasing with θ and nondecreasing with α. From (2.1), $\partial u_i(\theta, \alpha)/\partial\theta = 1$ and thus $u_i(\theta, \alpha)$ is increasing with θ. In addition, we can get $\partial u_i(\theta, \alpha)/\partial\alpha = -b + r(f_H - m_L)\alpha^{r-1}$. Since $0 \leq \alpha \leq 1$, $r \leq 1$, and $b \leq r(f_H - m_L)$, we have $\partial u_i(\theta, \alpha)/\partial\alpha \geq 0$ and thus $u_i(\theta, \alpha)$ is nondecreasing with α.

We prove the argument in 3 steps. In step 1, we will prove by induction that a strategy survives n rounds of iterated deletion of strictly dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{x}^{(n)}; \\ 1 & \text{if } x_i > \overline{x}^{(n)}. \end{cases} \tag{B.5}$$

In step 2, we prove $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ and $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ such that as $n \to \infty$, $\underline{x}^{(n)} \to \underline{x}$ and $\overline{x}^{(n)} \to \overline{x}$. In step 3, we finally prove $\underline{x} = \overline{x} = x^{(2)}$ such that the switching strategy around $x^{(2)}$ is the only strategy that survives iterated deletion of strictly dominated strategy.

Step 1: We first prove the statement holds for $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$. Let $\overline{x}^{(0)} = b + (f_L - m_L) + \epsilon$. Since $x_i = \theta + \widetilde{\varepsilon}_i$ and $\widetilde{\varepsilon}_i$ is uniformly distributed on $[-\epsilon, \epsilon]$, for any $x_i > \overline{x}^{(0)}$, $\theta > b + (f_L - m_L)$ and from (2.1) $u_i(\theta, \alpha) > \alpha^r(f_H - m_L) \geq 0$. So $a_i = 1$ for $x_i > \overline{x}^{(0)}$. Similarly, we let $\underline{x}^{(0)} = f_L - f_H - \epsilon$ and get the following: $a_i = 0$ if $x_i < \underline{x}^{(0)}$.

We next prove that a strategy that survives $n+1$ rounds of iterated deletion of strictly dominated strategies is in the form of (B.5); i.e., if firm i knows that other firms follow a strategy in the form of (B.5) with thresholds $\underline{x}^{(n)}$ and $\overline{x}^{(n)}$, then firm i's best response should be in the form of (B.5) with thresholds $\underline{x}^{(n+1)}$ and $\overline{x}^{(n+1)}$. Note that firm i expects $a_j = 1$ $(j \neq i)$ for $x_j > \overline{x}^{(n)}$, and $a_j = 0$ $(j \neq i)$ for $x_j < \underline{x}^{(n)}$. Firm i is not sure firm j's strategy for $\underline{x}^{(n)} \leq x_j \leq \overline{x}^{(n)}$. So the lowest value of α is achieved when $a_j = 0$ for $\underline{x}^{(n)} \leq x_j \leq \overline{x}^{(n)}$. Since $u_i(\theta, \alpha)$ is nondecreasing with α, firm i expects the lowest gain in this case. It is easy to see that in this case other firms follow a switching strategy around $\overline{x}^{(n)}$. Define $u_i^*(x_i, x)$ the expected value of $u_i(\theta, \alpha)$ given that firm i observes x_i and any other firm will follow a switching strategy around x. Then $u_i^*\left(x_i, \overline{x}^{(n)}\right)$ is the lower bound of firm i's expected gain. If $u_i^*\left(x_i, \overline{x}^{(n)}\right) > 0$, then firm i chooses $a_i = 1$. Let $\overline{x}^{(n+1)}$ be the solution of $u^*\left(x_i, \overline{x}^{(n)}\right) = 0$. Following a similar procedure to that in the proof of Lemma 2.1, we can show that $u^*\left(x_i, \overline{x}^{(n)}\right)$ is increasing with x_i. So $a_i = 1$ for any $x_i > \overline{x}^{(n+1)}$. Similarly, $u_i^*\left(x_i, \underline{x}^{(n)}\right)$ is the upper bound of firm i's expected gain. Let $\underline{x}^{(n+1)}$ be the solution of $u^*\left(x_i, \underline{x}^{(n)}\right) = 0$. Then $a_i = 0$ for any $x_i < \underline{x}^{(n+1)}$.

Step 2: We prove $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ and $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ by induction. Since we have shown that $u^*(x_i, x)$ increases with x_i and $u^*\left(\overline{x}^{(1)}, \overline{x}^{(0)}\right) = 0$, we need to show $u^*\left(\overline{x}^{(0)}, \overline{x}^{(0)}\right) \geq 0$ to prove $\overline{x}^{(1)} \leq \overline{x}^{(0)}$. Following a similar procedure to that in the proof of Lemma 2.1, we get

$$u^*(x, x) = x - (f_L - m_L) - \frac{1}{2}b + \frac{f_H - m_L}{r+1}. \tag{B.6}$$

Using $\overline{x}^{(0)} = b + (f_L - m_L) + \epsilon$ we get $u^*\left(\overline{x}^{(0)}, \overline{x}^{(0)}\right) = \frac{1}{2}b + \frac{f_H - m_L}{r+1} + \epsilon > 0$. Similarly, we get $\underline{x}^{(1)} \geq \underline{x}^{(0)}$.

We next show $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ given that $\overline{x}^{(n)} \leq \overline{x}^{(n-1)}$. Following a similar procedure to that in the proof of Lemma 2.1, we can show that $u^*(x_i, x)$ is decreasing with x. Since $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n-1)}\right) = 0$ and $\overline{x}^{(n)} \leq \overline{x}^{(n-1)}$, we get $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n)}\right) \geq 0$. Since $u^*(x_i, x)$ increases with x_i, $u^*\left(\overline{x}^{(n+1)}, \overline{x}^{(n)}\right) = 0$, and $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n)}\right) \geq 0$, we get $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$. Similarly, we obtain the following: $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ given that $\underline{x}^{(n)} \geq \underline{x}^{(n-1)}$.

Step 3: As $n \to \infty$, $\underline{x}^{(n)} \to \underline{x}$ and $\overline{x}^{(n)} \to \overline{x}$, where $u^*(\underline{x}, \underline{x}) = 0$ and $u^*(\overline{x}, \overline{x}) = 0$. From (B.6) we can show that $x^{(2)}$ is the unique solution to $u^*(x, x) = 0$. Thus $\underline{x} = \overline{x} = x^{(2)}$. □

Remark. The proof of Lemma O1 uses an argument of iterated deletion of strictly dominated strategies. This argument was used in Morris and Shin (2003) to derive a unique equilibrium when only strategic complementarity among firms is present. This process can also be viewed as a process in which firm i considers its higher-order beliefs to eliminate possible strategies. First, firm i considers its own signal and gets a strategy specified by (B.5) with thresholds $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$. Then by considering its belief that all other firms follow a strategy specified by (B.5) with thresholds $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$, firm i refines its strategy and gets a strategy specified by (B.5) with thresholds $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$. Next by considering its belief about other firms' strategy with $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$, firm i gets a strategy with thresholds $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$. This process continues and finally firm i gets a switching strategy with a threshold $x^{(2)}$.

We use an argument of iterated deletion of strictly dominated strategies to derive the equilibrium strategy in Lemma 2.1 (a switching strategy around $x^{(2)}$) and prove the uniqueness of this equilibrium under the condition $r \leq 1$ and $b \leq r(f_H - m_L)$. We show in Lemma 2.1 that this switching strategy continues to be an equilibrium strategy for every firm under a more general condition. Similarly, Karp et al. (2007) proved that every firm following a switching strategy is an equilibrium in a one-period setting with both strategic complementarity and substitutability.

Lemma O2 *The threshold* $\epsilon^{(2)}$ *is nondecreasing with* b *and nonincreasing with* $(f_H - m_L)$.

Proof. In the proof we focus on proving that $\epsilon^{(2)}$ is nondecreasing with b. The statement $\epsilon^{(2)}$ is nonincreasing with $f_H - m_L$ can be proven similarly. To prove $\epsilon^{(2)}$ is nondecreasing with b, we first prove $d\epsilon^{(2)}/db \geq 0$ in all intervals specified in (B.2). We then prove $\epsilon^{(2)}$ at the boundary point $b = r\left(f_H - m_L\right)$.

First, from (B.2) we calculate $d\epsilon^{(2)}/db$ as follows:

$$\frac{d\epsilon^{(2)}}{db} = \begin{cases} \frac{1}{2} - \frac{1}{2}\left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} & \text{if } r < \min\left\{1, \frac{b}{f_H - m_L}\right\}; \\ 0 & \text{if } \frac{b}{f_H - m_L} \leq r \leq 1 \text{ and } b \leq f_H - m_L; \\ \frac{1}{2} & \text{if } 1 \leq r \leq \frac{b}{f_H - m_L} \text{ and } b > f_H - m_L; \\ \frac{1}{2}\left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} & \text{if } r > \max\left\{1, \frac{b}{f_H - m_L}\right\}. \end{cases} \tag{B.7}$$

When $r < \min\left\{1, \frac{b}{f_H - m_L}\right\}$, $\frac{b}{r(f_H - m_L)} > 1$ and $\frac{1}{r-1} < 0$; so $\left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} < 1$ and $\frac{d\epsilon^{(2)}}{db} > 0$ in (B.7). When $r \geq \min\left\{1, \frac{b}{f_H - m_L}\right\}$, from (B.7) $\frac{d\epsilon^{(2)}}{db} \geq 0$. Taken together, $d\epsilon^{(2)}/db \geq 0$ in all intervals.

Next, we prove that $\epsilon^{(2)}$ is continuous in b at $b = r\left(f_H - m_L\right)$. We consider three cases: (Case I) $r < 1$, (Case II) $r = 1$, and (Case III) $r > 1$.

(Case I) As b increases from $b \leq r\left(f_H - f_L\right)$ to $b > r\left(f_H - f_L\right)$, from (B.2) the expression of $\epsilon^{(2)}$ changes from 0 to $\frac{b - (f_H - m_L)}{2} - \left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right)$. We can show as $b \to r\left(f_H - f_L\right)$ from the right side, $\frac{b - (f_H - m_L)}{2} - \left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right) \to 0$. So $\epsilon^{(2)}$ is continuous in b.

(Case II) As b increases from $b \leq f_H - f_L$ to $b > f_H - f_L$, from (B.2) the expression of $\epsilon^{(2)}$ changes from 0 to $\frac{b - (f_H - m_L)}{2}$. It is easy to verify $\epsilon^{(2)}$ is continuous at $b = f_H - f_L$.

(Case III) As b increases from $b \leq r\left(f_H - f_L\right)$ to $b > r\left(f_H - f_L\right)$, from (B.2) the expression of $\epsilon^{(2)}$ changes from $\left\{\frac{b}{r(f_H - m_L)}\right\}^{\frac{1}{r-1}} \frac{b}{2}\left(1 - \frac{1}{r}\right)$ to $\frac{b - (f_H - m_L)}{2}$. We can verify $\epsilon^{(2)}$ is continuous at $b = r\left(f_H - f_L\right)$. □

Lemma O3 *If* $\epsilon \geq \epsilon^{(1)}$, *where* $\epsilon^{(1)}$ *is given in the proof of Proposition 2.3(a), then* $u_1\left(x_1\right)$ *increases with* x_1. *Otherwise* $u_1\left(x_1\right)$ *first increases with* x_1, *then decreases with* x_1, *and finally increases with* x_1 *again.*

Proof. Following the same procedure as in the proof of Lemma 2.1, we obtain the expression of $u_1(x_1)$ as follows:

$$u_1(x_1) = \begin{cases} \lambda - f_1 + x_1 + m_L & \text{if } x_1 < x^{(2)} - 2\epsilon; \\ \lambda - f_1 + x_1 + m_L - \frac{b}{2}\left(1 + \frac{x_1 - x^{(2)}}{2\epsilon}\right)^2 & \\ +\frac{(m_H - m_L)}{r+1}\left(1 + \frac{x_1 - x^{(2)}}{2\epsilon}\right)^{r+1} & \text{if } x^{(2)} - 2\epsilon \le x_1 \le x^{(2)}; \\ \lambda - f_1 + x_1 + m_L - \frac{b}{2}\left\{1 - \left(\frac{x - x^{(2)}}{2\epsilon}\right)^2\right\} & \\ +\frac{(m_H - m_L)}{r+1}\left\{1 - \left(\frac{x - x^{(2)}}{2\epsilon}\right)^{r+1}\right\} + (m_H - m_L - b)\left(\frac{x_1 - x^{(2)}}{2\epsilon}\right) & \text{if } x^{(2)} \le x_1 \le x^{(2)} + 2\epsilon; \\ \lambda - f_1 + x_1 + m_H - b, & \text{if } x_1 > x^{(2)} + 2\epsilon. \end{cases} \tag{B.8}$$

For the first and last intervals of x_1, it is easy to observe that $u_1(x_1)$ is increasing with x_1. We focus our analysis on the two middle intervals. Let $\eta_1 = \frac{x_1 - x^{(2)}}{2\epsilon}$. Then $\eta_1 \in [-1, 0]$ for $x^{(2)} - 2\epsilon \le x_1 \le x^{(2)}$, and $\eta_1 \in [0, 1]$ for $x^{(2)} \le x_1 \le x^{(2)} + 2\epsilon$. Let

$$u_1(\eta_1) = \begin{cases} \lambda + x^{(2)} + 2\epsilon\eta_1 + m_L - f_L - \frac{b}{2}(1 + \eta_1)^2 + \frac{(m_H - m_L)}{r+1}(1 + \eta_1)^{r+1} & \text{if } \eta_1 \le 0; \\ \lambda + x^{(2)} + 2\epsilon\eta_1 + m_L - f_L - \frac{b}{2}\left(1 - \eta_1^2\right) + \frac{m_H - m_L}{r+1}\left(1 - \eta_1^{r+1}\right) & \\ -\{b - (m_H - m_L)\}\eta_1^{r+1} & \text{if } \eta_1 > 0. \end{cases} \tag{B.9}$$

Then from (B.8) we get $u_1(x_1) = u_1(\eta_1)$ and $\frac{du_1}{dx_1} = \frac{1}{2\epsilon}\frac{du_1(\eta_1)}{d\eta_1}$ for $x^{(2)} - 2\epsilon \le x_1 \le x^{(2)} + 2\epsilon$ (corresponding to $-1 \le \eta_1 \le 1$). We calculate the first, second, and third order derivatives of $u_1(\eta_1)$ as follows:

$$\frac{du_1(\eta_1)}{d\eta_1} = \begin{cases} 2\epsilon - b(1 + \eta_1) + (m_H - m_L)(1 + \eta_1)^r & \text{if } \eta_1 \le 0; \\ 2\epsilon + b\eta_1 - (m_H - m_L)\eta_1^r + (m_H - m_L) - b & \text{if } \eta_1 > 0; \end{cases} \tag{B.10}$$

$$\frac{d^2u_1(\eta_1)}{d\eta_1^2} = \begin{cases} -b + r(m_H - m_L)(1 + \eta_1)^{r-1} & \text{if } \eta_1 \le 0; \\ b - r(m_H - m_L)\eta_1^{r-1} & \text{if } \eta_1 > 0; \end{cases} \tag{B.11}$$

$$\frac{d^3u_1(\eta_1)}{d\eta_1^3} = \begin{cases} r(r-1)(m_H - m_L)(1 + \eta_1)^{r-2} & \text{if } \eta_1 \le 0; \\ -r(r-1)(m_H - m_L)\eta_1^{r-2} & \text{if } \eta_1 > 0. \end{cases} \tag{B.12}$$

We next discuss three cases: (Case I) $r = 1$, (Case II) $r > 1$, and (Case III) $r < 1$.

(Case I) From (B.10), $\frac{du_1(\eta_1)}{d\eta_1}$ is linear in η_1. In addition, $\frac{du_1(-1)}{d\eta_1} = \frac{du_1(1)}{d\eta_1} = 2\epsilon > 0$ and $\frac{du_1(0)}{d\eta_1} = 2\epsilon - b + (m_H - m_L)$. There are two cases regarding the sign of $\frac{du_1(0)}{d\eta_1}$. First, if $\epsilon \ge \frac{b - (m_H - m_L)}{2}$ (which implies $2\epsilon - b + (m_H - m_L) \ge 0$), then $\frac{du_1(\eta_1)}{d\eta_1} \ge 0$ for any $\eta_1 \in [-1, 1]$, and $\frac{du_1(x_1)}{dx_1} \geq 0$ in this case. Second, if $\epsilon < \frac{b-(m_H-m_L)}{2}$, then $\frac{du_1(0)}{d\eta_1} < 0$, and $\frac{du_1(\eta_1)}{d\eta_1}$ decreases from positive to negative, and then increases from negative to positive. In this case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again. The condition for the monotonic change of $u_1(x_1)$ is $\epsilon \geq \frac{b-(m_H-m_L)}{2}$.

(Case II) We need to compare the minimum value of $\frac{du_1(\eta_1)}{d\eta_1}$ with 0 to determine the shape of $u_1(x_1)$. From (B.12), $\frac{d^3u_1(\eta_1)}{d\eta_1^3} > 0$ for $\eta_1 \leq 0$ and $\frac{d^3u_1(\eta_1)}{d\eta_1^3} < 0$ for $\eta_1 > 0$. So $\frac{du_1(\eta_1)}{d\eta_1}$ is convex in η_1 for $\eta_1 \leq 0$ and concave in η_1 for $\eta_1 > 0$. We can show that there exists a minimum value of $\frac{du_1(\eta_1)}{d\eta_1}$ on $(-\infty, 0]$. To get this minimum value, we solve $\frac{d^2u_1^-(\eta_1)}{d\eta_1^2} = 0$ from (B.11) and get $\eta_1^{(1)} = \left\{\frac{b}{r(m_H-m_L)}\right\}^{\frac{1}{r-1}} - 1$.

If $\eta_1^{(1)} \leq 0$ (which happens when $\frac{b}{r(m_H-m_L)} \leq 1$), then $\frac{du_1(\eta_1)}{d\eta_1}$ achieves its minimum value $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} = 2\epsilon - \left\{\frac{b}{r(m_H-m_L)}\right\}^{\frac{1}{r-1}} b\left(1-\frac{1}{r}\right)$. If $\epsilon \geq \frac{1}{2}\left\{\frac{b}{r(m_H-m_L)}\right\}^{\frac{1}{r-1}} b\left(1-\frac{1}{r}\right)$ (which implies $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} \geq 0$), then $\frac{du_1(\eta_1)}{d\eta_1} \geq 0$ for any $\eta_1 \in [-1,1]$ and $u_1(x_1)$ increases with x_1. If $\epsilon < \frac{1}{2}\left\{\frac{b}{r(m_H-m_L)}\right\}^{\frac{1}{r-1}} b\left(1-\frac{1}{r}\right)$ (which implies $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} < 0$), we can show that $\frac{du_1(\eta_1)}{d\eta_1}$ changes from positive to negative, and then from negative to positive for $\eta_1 \in [-1,1]$ using $\frac{du_1(-1)}{d\eta_1} = \frac{du_1(1)}{d\eta_1} = 2\epsilon > 0$ and the fact that $\frac{du_1(\eta_1)}{d\eta_1}$ is convex in η_1 for $\eta_1 \leq 0$ and concave in η_1 for $\eta_1 > 0$. In this case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again.

If $\eta_1^{(1)} > 0$ (which happens when $\frac{b}{r(m_H-m_L)} > 1$), then $\frac{du_1(\eta_1)}{d\eta_1}$ achieves its minimum value $\frac{du_1(0)}{d\eta_1} = 2\epsilon - b + (m_H - m_L)$. So we get the condition for $\frac{du_1(\eta_1)}{d\eta_1} \geq 0$ is $\epsilon \geq \frac{b-(m_H-m_L)}{2}$. The rest of the proof is similar to case in which $\eta_1^{(1)} \leq 0$.

(Case III) Similar to (Case II), we get the conditions for the monotonic change of $u_1(x_1)$ as follows: $\frac{b}{r(m_H-m_L)} \geq 1$ and $\epsilon \geq \frac{b-(m_H-m_L)}{2} - \frac{1}{2}\left\{\frac{b}{r(m_H-m_L)}\right\}^{\frac{1}{r-1}} b\left(1-\frac{1}{r}\right)$, or $\frac{b}{r(m_H-m_L)} < 1$ and $\epsilon \geq \frac{b-(m_H-m_L)}{2}$.

Finally, by combining all the conditions in (Case I), (Case II), and (Case III), we can prove that if $\epsilon \geq \epsilon^{(1)}$, $u_1(x_1)$ increases with x_1, where the expression of $\epsilon^{(1)}$ is given in the proof of Proposition 2.3(a). □

Detailed Proof of Lemma 2.3(a). For the case in which firm i expects none of other firms to adopt the technology ($\alpha = 0$), by (2.1), $\pi_i(1;\theta,0) - \pi_i(0;\theta,0) = \theta - (f_L - m_L)$. If $\theta < f_L - m_L$, then $\pi_i(1;\theta,0) < \pi_i(0;\theta,0)$, so $a_i = 0$ for $i \neq 1$ in equilibrium. For the case in which firm i expects all other firms to adopt the technology ($\alpha = 1$), by (2.1), $\pi_i(1;\theta,1) - \pi_i(0;\theta,1) =$

$\theta-(b-f_H+f_L)$. Thus, if $\theta \geq b-f_H+f_L$, $a_i=1$ for all $i \neq 1$ in equilibrium. Putting the conditions together, we find that: (i) $a_i=0$ is the unique equilibrium when $\theta < b-f_H+f_L$; (ii) $a_i=1$ is the unique equilibrium when $\theta > f_L-m_L$; and (iii) both $a_i=1$ and $a_i=0$ can be an equilibrium when $b-f_H+f_L \leq \theta \leq f_L-m_L$.

We next find the equilibrium that maximizes firms' total payoffs when $b-f_H+f_L \leq \theta \leq f_L-m_L$. Similar to §3, we can obtain $\pi_i(1;\theta,1)=-f_L+\theta-b+m_H$, and $\pi_i(0;\theta,0)=0$. So $a_i=1$ if $\pi_i(1;\theta,1) \geq \pi_i(0;\theta,0)$, which can be simplified to $\theta \geq f_L+b-m_H$. Since $f_H \geq 0 \geq m_H$, $f_L+b-m_H \geq b-f_H+f_L$. But either $f_L-m_L \geq f_L+b-m_H$ or $f_L-m_L < f_L+b-m_H$ is possible. Define $\theta^{(2)} \equiv f_L+\min\{-m_L, b-m_H\}$. Then $a_i=1$ if and only if $\theta \geq \theta^{(2)}$. $\square$

Detailed Proof of Proposition 2.5. Similar to the procedure in the proof of Proposition 2.4, we analyze the condition for $u_1\left(\theta^{(1)}\right) \geq 0$ to get a sufficient condition for $x_{aggr}^{(1)} \leq \theta^{(1)}$ because $u(x_1) < 0$ for any $x_1 < x_{aggr}^{(1)}$. We further simplify the sufficient condition to the conditions in the Proposition 2.5.

We first get a sufficient condition for $u_1\left(\theta^{(1)}\right) \geq 0$. In this proof, we focus on the case in which $b < m_H-m_L$. Conditions when $b > m_H-m_L$ or $b=m_H-m_L$ can be derived similarly. For the case in which $b < m_H-m_L$, we consider the following cases: (Case I) $\theta^{(2)} \geq \theta^{(n)}$, (case II) $\theta^{(a)} < \theta^{(2)} < \theta^{(n)}$, and (case III) $\theta^{(2)} \leq \theta^{(a)}$.

(Case I) Let $\eta^{(\theta)}=\frac{\theta^{(1)}-x^{(2)}}{2\epsilon}$. Using $\theta^{(1)}=\theta^{(n)}$ from Lemma 2.3(b) and $u_1(x_1)$ in (B.8), we can express $u_1\left(\theta^{(1)}\right)$ as follows:

$$u_1\left(\eta^{(\theta)}\right)=\begin{cases} 0 & \eta^{(\theta)} < -1; \\ \frac{(m_H-m_L)}{r+1}\left(\eta^{(\theta)}\right)^{r+1}-\frac{b}{2}\left(\eta^{(\theta)}\right)^2 & -1 \leq \eta^{(\theta)} \leq 0; \\ \frac{m_H-m_L}{r+1}\left\{1-\left(\eta^{(\theta)}\right)^{r+1}\right\}-\frac{b}{2}\left\{1-\left(\eta^{(\theta)}\right)^2\right\} & \\ \quad +\left\{(m_H-m_L)-b\right\}\eta^{(\theta)} & 0 < \eta^{(\theta)} \leq 1; \\ m_H-m_L-b & \eta^{(\theta)} > 1. \end{cases} \tag{B.13}$$

When $\eta^{(\theta)} < -1$, $u_1\left(\eta^{(\theta)}\right)=0$; when $\eta^{(\theta)} > 1$, $u_1\left(\eta^{(\theta)}\right)=m_H-m_L-b > 0$. Following the same procedure as in the proof of Proposition 2.3(a), we can show that $u_1\left(\eta^{(\theta)}\right)$ changes with $\eta^{(\theta)}$ in one of the following ways: (i) When $r < b/(m_H-m_L)$, $u_1\left(\eta^{(\theta)}\right)$ first increases with $\eta^{(\theta)}$, and then decreases with $\eta^{(\theta)}$; (ii) When $b/(m_H-m_L) \leq r \leq 1$, $u_1\left(\eta^{(\theta)}\right)$ increases with $\eta^{(\theta)}$; and (iii) When $r > 1$, $u_1\left(\eta^{(\theta)}\right)$ first decreases with $\eta^{(\theta)}$, and then increases with $\eta^{(\theta)}$ (see Lemma O4 for the proof). So if $r \leq 1$ and $\eta^{(\theta)} > -1$, then $u_1\left(\eta^{(\theta)}\right) \geq 0$. Using

$\eta^{(\theta)} = \frac{\theta^{(1)} - x^{(2)}}{2\epsilon}$, $\theta^{(1)} = \theta^{(n)}$ and the expression of $x^{(2)}$ in Lemma 2.1, we rewrite $\eta^{(\theta)} > -1$ as follows: $\lambda < f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} + 2\epsilon$.
(Case II) Similar to (Case I), we can show a sufficient condition for $u_1\left(\eta^{(\theta)}\right) > 0$ is $\eta^{(\theta)} \geq 1$, which can be rewritten as $\frac{1}{2}b - m_H + \frac{rm_L + f_H}{r+1} \geq 2\epsilon$.
(Case III) Similar to (Case I), we can show that $u_1\left(\eta^{(\theta)}\right) \leq 0$ for any $\eta^{(\theta)}$ if $r \geq b/\left(m_H - m_L\right)$, and $u_1\left(\eta^{(\theta)}\right) \leq 0$ for any $\eta^{(\theta)} \leq -1$ and $\eta^{(\theta)} > 1$.

We get a sufficient condition on λ and r from (Case I): $\lambda < f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} + 2\epsilon$ and $r \leq 1$. In addition, using the expressions of $\theta^{(2)}$ and $\theta^{(n)}$ from the proof of Lemma 2.3, we rewrite the condition for this case, $\theta^{(2)} \geq \theta^{(n)}$, to the following: $\lambda \geq f_1 - f_L + m_H - m_L - b$. The final sufficient condition we get is $f_1 - f_L + m_H - m_L - b \leq \lambda < f_1 - f_L - \frac{1}{2}b + \frac{f_H - m_L}{r+1} + 2\epsilon$.
□

Lemma O4 *The function $u_1\left(\eta^{(\theta)}\right)$ changes with $\eta^{(\theta)} \in [-1, 1]$ in one of the following ways:*
(i) if $r < \min\{1, b/\left(m_H - m_L\right)\}$, then $u_1\left(\eta^{(\theta)}\right)$ first increases with $\eta^{(\theta)}$, and then decreases with $\eta^{(\theta)}$;
(ii) if $\min\{1, b/\left(m_H - m_L\right)\} \leq r \leq \max\{1, b/\left(m_H - m_L\right)\}$, then $u_1\left(\eta^{(\theta)}\right)$ changes monotonically with $\eta^{(\theta)}$;
(iii) if $r > \max\{1, b/\left(m_H - m_L\right)\}$, then $u_1\left(\eta^{(\theta)}\right)$ first decreases with $\eta^{(\theta)}$, and then increases with $\eta^{(\theta)}$.

Proof. In the proof we focus on the case in which $b < m_H - m_L$. The case in which $b = m_H - m_L$ or $b > m_H - m_L$ can be proved similarly.

We can calculate $\frac{du_1}{d\eta^{(\theta)}}$ and $\frac{d^2 u_1}{d\left(\eta^{(\theta)}\right)^2}$ from (B.13) as follows for $-1 \leq \eta^{(\theta)} \leq 1$:

$$\frac{du_1\left(\eta^{(\theta)}\right)}{d\eta^{(\theta)}} = \begin{cases} -b\left(1+\eta^{(\theta)}\right) + \left(m_H - m_L\right)\left(1+\eta^{(\theta)}\right)^r & -1 \leq \eta^{(\theta)} \leq 0; \\ b\eta^{(\theta)} - \left(m_H - m_L\right)\left(\eta^{(\theta)}\right)^r + \left(m_H - m_L - b\right) & 0 < \eta^{(\theta)} \leq 1; \end{cases}, \text{ and} \tag{B.14}$$

$$\frac{d^2 u_1\left(\eta^{(\theta)}\right)}{d\left(\eta^{(\theta)}\right)^2} = \begin{cases} -b + r\left(m_H - m_L\right)\left(1+\eta^{(\theta)}\right)^{r-1} & -1 \leq \eta^{(\theta)} \leq 0; \\ b - r\left(m_H - m_L\right)\left(\eta^{(\theta)}\right)^{r-1} & 0 < \eta^{(\theta)} \leq 1. \end{cases} \tag{B.15}$$

We next examine the following three cases: (Case I) $r < b/\left(m_H - m_L\right)$, (Case II) $b/\left(m_H - m_L\right) \leq r \leq 1$, and (Case III) $r > 1$.
(Case I) For $-1 \leq \eta^{(\theta)} \leq 0$, from (B.15) it is easy to show $\frac{d^2 u_1}{d\left(\eta^{(\theta)}\right)^2}$ changes from positive to negative when $\eta^{(\theta)}$ increases from -1 to 0. Since from (B.14) $\frac{du_1(-1)}{d\eta^{(\theta)}} = 0$ and $\frac{du_1(0)}{d\eta^{(\theta)}} =$

$-b+(m_H-m_L)>0$, $\frac{du_1(\eta^{(\theta)})}{d\eta^{(\theta)}} \geq 0$. For $0<\eta^{(\theta)} \leq 1$, from (B.15) it is easy to show that $\frac{d^2u_1}{d(\eta^{(\theta)})^2}$ changes from negative to positive when $\eta^{(\theta)}$ increases from 0 to 1. Since $\frac{du_1(0)}{d\eta^{(\theta)}} = -b+(m_H-m_L)>0$ and $\frac{du_1(1)}{d\eta^{(\theta)}}=0$, we get that $\frac{du_1}{d\eta^{(\theta)}}$ changes from positive to negative for $0<\eta^{(\theta)} \leq 1$. Taken together, $\frac{du_1}{d\eta^{(\theta)}}$ changes from positive to negative when $\eta^{(\theta)}$ increases from -1 to 1. So $u_1\left(\eta^{(\theta)}\right)$ first increases with $\eta^{(\theta)}$, and then decreases with $\eta^{(\theta)}$.

(Case II) For $-1 \leq \eta^{(\theta)} \leq 0$, from (B.15) it is easy to show $\frac{d^2u_1(\eta^{(\theta)})}{d(\eta^{(\theta)})^2} \geq 0$. Since $\frac{du_1(-1)}{d\eta^{(\theta)}}=0$ and $\frac{du_1(0)}{d\eta^{(\theta)}} = -b+(m_H-m_L)>0$, $\frac{du_1(\eta^{(\theta)})}{d\eta^{(\theta)}} \geq 0$. For $0<\eta^{(\theta)} \leq 1$, from (B.15) it is easy to show $\frac{d^2u_1(\eta^{(\theta)})}{d(\eta^{(\theta)})^2} \leq 0$. Since $\frac{du_1(0)}{d\eta^{(\theta)}} = -b+(m_H-m_L)>0$ and $\frac{du_1(1)}{d\eta^{(\theta)}}=0$, $\frac{du_1(\eta^{(\theta)})}{d\eta^{(\theta)}} \geq 0$. Taken together, $\frac{du_1}{d\eta^{(\theta)}} \geq 0$ for $-1 \leq \eta^{(\theta)} \leq 1$. So $u_1\left(\eta^{(\theta)}\right)$ increases with $\eta^{(\theta)}$.

(Case III) For $-1 \leq \eta^{(\theta)} \leq 0$, from (B.15) it is easy to show $\frac{d^2u_1}{d(\eta^{(\theta)})^2}$ changes from negative to positive when $\eta^{(\theta)}$ increases from -1 to 0. Since $\frac{du_1(-1)}{d\eta^{(\theta)}}=0$ and $\frac{du_1(0)}{d\eta^{(\theta)}} = -b+(m_H-m_L)>0$, $\frac{du_1}{d\eta^{(\theta)}}$ changes from negative to positive when $\eta^{(\theta)}$ increases from -1 to 0. For $0<\eta^{(\theta)} \leq 1$, from (B.15) it is easy to show that $\frac{d^2u_1(\eta^{(\theta)})}{d(\eta^{(\theta)})^2}$ changes from positive to negative when $\eta^{(\theta)}$ increases from 0 to 1. Since $\left.\frac{du_1}{d\eta^{(\theta)}}\right|_{\eta^{(\theta)}=0}>0$ and $\left.\frac{du_1}{d\eta^{(\theta)}}\right|_{\eta^{(\theta)}=1}=0$, we get that $\frac{du_1}{d\eta^{(\theta)}} \geq 0$. Taken together, $\frac{du_1}{d\eta^{(\theta)}}$ changes from negative to positive for $-1<\eta^{(\theta)} \leq 1$. So $u_1\left(\eta^{(\theta)}\right)$ first decreases with $\eta^{(\theta)}$, and then increases with $\eta^{(\theta)}$. $\square$

Appendix C

Supplements to Chapter 3

To simplify our analysis, we denote by $q_i = a_i\overline{\theta}/\alpha$, $i = 1$, 2, and 3.

Lemma A1 *there exists a unique Nash equilibrium in the post-merger market. In equilibrium:*
(a) $q_1^{pre} = 0.4324\overline{\theta}/\alpha$, $q_2^{pre} = 0.2803\overline{\theta}/\alpha$, $q_3^{pre} = 0.1602\overline{\theta}/\alpha$.
(b) $p_1^{pre} = 0.2191\overline{\theta}^2/\alpha$, $p_2^{pre} = 0.09912\overline{\theta}^2/\alpha$, $p_3^{pre} = 0.04115\overline{\theta}^2/\alpha$.
(c) $\pi_1^{pre} = 0.006785\overline{\theta}^3/\alpha$, $\pi_2^{pre} = 0.006292\overline{\theta}^3/\alpha$, $\pi_3^{pre} = 0.003496\overline{\theta}^3/\alpha$.
(d) $w^{pre} = 0.06370\overline{\theta}^3/\alpha$.

Proof. **(a)** We first obtain the expression of π_i. We then solve for the equilibrium backwards: we first solve for the equilibrium prices for given qualities; we then solve for the equilibrium qualities.

To get the expression of π_1, we need to derive the demand of product 1. A consumer buys product 1 other than product 2 if and only if $\theta q_1 - p_1 \geq \theta q_2 - p_2$, which simplifies to $\theta \geq \frac{p_2-p_2}{q_1-q_2}$.[1] So the demand of product 1 is $\overline{\theta} - \frac{p_2-p_2}{q_1-q_2}$ and the profit of firm 1 is given as $\pi_1 = \left(p_1 - \alpha q_1^2\right)\left(\overline{\theta} - \frac{p_1-p_2}{q_1-q_2}\right)$. Similarly, $\pi_2 = \left(p_2 - \alpha q_2^2\right)\left(\frac{p_1-p_2}{q_1-q_2} - \frac{p_2-p_3}{q_3-q_3}\right)$, and $\pi_3 = \left(p_3 - \alpha q_3^2\right)\frac{p_2-p_3}{q_3-q_3}$.

We then solve for equilibrium prices for given qualities. For given q_1 and q_2, we obtain the following: $\partial\pi_1^2/\partial p_1^2 = -2/(q_1 - q_2) < 0$. So the optimal p_1 must satisfy $\partial\pi_1/\partial p_1 = 0$. Similarly, the optimal p_2 and p_3 must satisfy $\partial\pi_2/\partial p_2 = 0$ and $\partial\pi_3/\partial p_3 = 0$, respectively. We calculate

[1] The case....

$\partial\pi_i/\partial p_i$ and write the three equations ($\partial\pi_i/\partial p_i = 0$, $i = 1, 2, 3$) as follows:

$$\begin{aligned}
\overline{\theta} - \frac{2p_1 - p_2 - \alpha q_1^2}{q_1 - q_2} &= 0; \\
\frac{p_1 - 2p_2 + \alpha q_2^2}{q_1 - q_2} - \frac{2p_2 - \alpha q_2^2 - p_3}{q_2 - q_3} &= 0; \\
\frac{p_2 + \alpha q_3^2 - 2p_3}{q_2 - q_3} - \frac{2p_3 - \alpha q_3^2}{q_3} &= 0.
\end{aligned}$$

By substituting $q_i = a_i\overline{\theta}/\alpha$ into the above equations, we can solve for p_i as follows:

$$p_1 = \frac{\overline{\theta}^2}{\alpha}\frac{a_1^3(4a_2 - a_3) - 3a_1^2a_2a_3 - a_2^2a_3(2a_2 + a_3) + a_1a_2(2a_2^2 + a_3^2) + (a_1 - a_2)(4a_1a_2 - a_1a_3 - 3a_2a_3)}{2\left(a_1(4a_2 - a_3) - a_2(a_2 + 2a_3)\right)} \tag{C.1}$$

$$p_2 = \frac{\overline{\theta}^2 a_2}{\alpha}\frac{a_1^2(a_2 - a_3) - a_2a_3(2a_2 + a_3) + a_1(2a_2^2 + a_3^2) + (a_1 - a_2)(a_2 - a_3)}{a_1(4a_2 - a_3) - a_2(a_2 + 2a_3)}, \tag{C.2}$$

$$p_3 = \frac{\overline{\theta}^2 a_3}{\alpha}\frac{a_1^2(a_2 - a_3) - 3a_2a_3(a_2 + a_3) + 2a_1a_2(a_2 + 2a_3) + (a_1 - a_2)(a_2 - a_3)}{a_1(4a_2 - a_3) - a_2(a_2 + 2a_3)}. \tag{C.3}$$

We next solve for equilibrium qualities. We substitute (C.1), (C.2), and (C.3) into the expression of π_i and obtain the following after simplification:

$$\pi_1 = \frac{\overline{\theta}^3}{\alpha}\frac{(a_1 - a_2)}{4(a_1(4a_2 - a_3) - a_2(a_2 + 2a_3))^2} \left[a_1^2(4a_2 - a_3) - a_2a_3(2a_2 + a_3 - 3) + a_1\left\{2a_2^2 + a_3 - 2a_2(2 + a_3)\right\}\right]^2 \tag{C.4}$$

$$\pi_2 = \frac{(a_1 - a_2)a_2^2(a_1 - a_3)(a_2 - a_3)(-1 - a_1 + a_2 + a_3)^2}{4(a_1(4a_2 - a_3) - a_2(a_2 + 2a_3))^2}\frac{\overline{\theta}^3}{\alpha}, \tag{C.5}$$

$$\pi_3 = \frac{a_2(a_2 - a_3)a_3(a_1^2 + a_1(1 + 2a_2 - 2a_3) - a_2(1 + a_3))^2}{4(a_1(4a_2 - a_3) - a_2(a_2 + 2a_3))^2}\frac{\overline{\theta}^3}{\alpha}. \tag{C.6}$$

Using above equations, we calculate $\partial\pi_i/\partial p_i$, solve $\partial\pi_i/\partial p_i = 0$, and get the only solution that results in positive π_i: $a_1 = 0.4324$, $a_2 = 0.2803$, and $a_3 = 0.1602$. We finally verify this solution is a Nash equilibrium. Using (C.1), we can verify that when $a_2 = 0.2803$ and $a_3 = 0.1602$, firm 1 achieves its maximum profit by choosing $a_1 = 0.4324$. Similarly, we can verify firm 2's best response to $a_1 = 0.4324$ and $a_3 = 0.1602$ is to choose $a_2 = 0.2803$, and firm 3's best response to $a_1 = 0.4324$ and $a_2 = 0.2803$ is to choose $a_3 = 0.1602$. So in equilibrium, $q_1^{pre} = 0.4324\overline{\theta}/\alpha$, $q_2^{pre} = 0.2803\overline{\theta}/\alpha$, $q_3^{pre} = 0.1602\overline{\theta}/\alpha$.

(b) We can prove the result using (C.1), (C.2), and (C.3) and $a_1 = 0.4324$, $a_2 = 0.2803$, and

$a_3 = 0.1602$.

(c) We can prove the result using (C.4), (C.5), and (C.6) and $a_1 = 0.4324$, $a_2 = 0.2803$, and $a_3 = 0.1602$.

(d) We can calculate the consumer welfare as follows:

$$\begin{aligned} w^{pre} &= \int_{p_3^{pre}/q_3^{pre}}^{(p_2^{pre}-p_3^{pre})/(q_2^{pre}-q_3^{pre})} \left(\theta q_3^{pre} - p_3^{pre}\right) d\theta + \int_{(p_2^{pre}-p_3^{pre})/(q_2^{pre}-q_3^{pre})}^{(p_1^{pre}-p_2^{pre})/(q_1^{pre}-q_2^{pre})} \left(\theta q_2^{pre} - p_2^{pre}\right) d\theta \\ &\quad + \int_{((p_1^{pre}-p_2^{pre})/(q_1^{pre}-q_2^{pre})}^{\overline{\theta}} \left(\theta q_1^{pre} - p_1^{pre}\right) d\theta \\ &= 0.06370\overline{\theta}^3/\alpha. \quad \square \end{aligned}$$

Proof of Proposition 3.1. In the post-merger market, firm m's profit is given as $\pi_m = \left(p_2 - \alpha q_2^2\right)\left(\frac{p_1-p_2}{q_1-q_2} - \frac{p_2-p_3}{q_3-q_3}\right) + \left(p_3 - \alpha q_3^2\right)\frac{p_2-p_3}{q_3-q_3}$. So the first order conditions for prices in the second stage are as follows: $\partial\pi_1/\partial p_1 = 0$, $\partial\pi_m/\partial p_2 = 0$, and $\partial\pi_m/\partial p_3 = 0$; the first order conditions for qualities in the first stage are as follows: $\partial\pi_1/\partial q_1 = 0$, $\partial\pi_m/\partial q_2 = 0$, and $\partial\pi_m/\partial q_3 = 0$. Following similar steps as in the proof of Lemma A1, we can obtain the following: $q_1^{(2)} = 0.4208\overline{\theta}/\alpha$, $q_2^{(2)} = 0.2198\overline{\theta}/\alpha$, $q_3^{(2)} = 0.1099\overline{\theta}/\alpha$, $p_1^{(2)} = 0.2313\overline{\theta}^2/\alpha$, $p_2^{(2)} = 0.08457\overline{\theta}^2/\alpha$, $p_3^{(2)} = 0.03625\overline{\theta}^2/\alpha$, $\pi_1^{(2)} = 0.01464\overline{\theta}^3/\alpha$, $\pi_m^{(2)} = 0.01319\overline{\theta}^3/\alpha$, and $w^{(2)} = 0.04925\overline{\theta}^3/\alpha$. By comparing the results with Lemma A1, we can prove Proposition 3.1. $\square$

Proof of Proposition 3.2. We first solve for the equilibrium prices for given qualities. Following similar steps as in the proof of Lemma A1, we solve $\partial\pi_1/\partial p_1 = 0$ and $\partial\pi_m/\partial p_2 = 0$, and obtain the following:

$$\begin{aligned} p_1 &= \frac{\alpha\left\{2q_1^2 + (1-s)\,q_2^2\right\} + 2\,(q_1 - q_2)\,\overline{\theta}}{4q_1 - q_2} q_1, \\ p_2 &= \frac{\alpha q_1\left\{q_1 + 2\,(1-s)\,q_2\right\} + (q_1 - q_2)\,\overline{\theta}}{4q_1 - q_2} q_2. \end{aligned}$$

We next substitute the above equations into the expression of π_1 and π_m and obtain the following:

$$\begin{aligned} \pi_1 &= \frac{\left[\alpha\left\{2q_1^2 - q_1q_2 - (1-s)q_2^2\right\} + 2(-q_1 + q_2)\overline{\theta}\right]^2}{(q_1 - q_2)(4q_1 - q_2)^2} q_1^2, \\ \pi_2 &= \frac{\left[\alpha\left\{q_1^2 - 2(1-s)q_1q_2 + (1-s)q_2^2\right\} + (q_1 - q_2)\overline{\theta}\right]^2}{(q_1 - q_2)(4q_1 - q_2)^2} q_1q_2. \end{aligned}$$

Unfortunately, there is no close-form solution for $\partial\pi_1/\partial q_1 = 0$ and $\partial\pi_m/\partial q_2 = 0$. Notice that usually the synergy level s is small, we first solve the equilibrium at $s = 0$, and then use the

Taylor expansion at $s = 0$ to approximate results.[2] To do so, we first solve $\partial\pi_1/\partial q_1 = 0$ and $\partial\pi_m/\partial q_2 = 0$ at $s = 0$ and obtain $q_1 = 0.4098\overline{\theta}/\alpha$ and $q_2 = 0.1994$. We then use the Taylor expansion to the third order and obtain the following:

$$q_1^{(1)} = \left(0.4098 + 0.01733s + 0.3728s^2 + 1.1411s^3\right)\overline{\theta}/\alpha, \tag{C.7}$$

$$q_2^{(1)} = (0.1994 + 0.2536s + 0.4624s^2 + 1.7415s^3)\overline{\theta}/\alpha. \tag{C.8}$$

Using the above equations, we can calculate prices, profits and consumer welfare as follows:

$$\begin{aligned}
p_1^{(1)} &= \left(0.2267 + 0.0793s + 0.2219s^2 + 0.9912s^3\right)\overline{\theta}^2/\alpha,\\
p_2^{(1)} &= \left(0.0750 + 0.0968s + 0.2014s^2 + 0.8852s^3\right)\overline{\theta}^2/\alpha,\\
\pi_1^{(1)} &= \left(0.0164 - 0.0285s - 0.0601s^2 - 0.0071s^3\right)\overline{\theta}^3/\alpha,\\
\pi_m^{(1)} &= (0.0121 + 0.0185s + 0.0487s^2 + 0.0061s^3)\overline{\theta}^3/\alpha,\\
w^{(1)} &= (0.0470 + 0.0339s + 0.0596s^2 + 0.0068s^3)\overline{\theta}^3/\alpha.
\end{aligned}$$

Using the above equations, we can prove Proposition 3.2. □

Proof of Proposition 3.3. **(a)** Firm m produces one products if and only if $\pi_m^{(2)} \leq \pi_m^{(1)}$. Using the expressions of $\pi_m^{(2)}$ and $\pi_m^{(1)}$ in Proposition 3.1 and Proposition 3.2, respectively, we obtain the following: $\pi_m^{(1)} \geq \pi_m^{(2)}$ if and only if $s \geq 5.2\%$. We define $s^{(\pi)} = 5.2\%$.

(b) Using $\pi_m^{(2)}$ and $\pi_m^{(1)}$ in Proposition 3.1 and Proposition 3.2, respectively, we obtain the following: $w^{(1)} \geq w^{(2)}$ if and only if $s \geq 6.0\%$. We define $s^{(w)} = 6.0\%$.

(c) Using $w^{(pre)}$ and $w^{(1)}$ in Lemma A1 and Proposition 3.2, we obtain the following: $w^{(1)} \geq w^{(2)}$ if and only if $s \geq 31.4\%$. We define $s^{(pre)} = 31.4\%$. □

[2] Our numerical...

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