

CROSS-SECTIONAL PHENOMENA AND  
NEW PERSPECTIVES ON MACRO-FINANCE PUZZLES

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*Dissertation Submitted in Partial Fulfillment  
of the Requirements for the Degree of*

DOCTOR OF PHILOSOPHY

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April 2016

*Dedicated to my lovely wife Anne-Sophie Perrin,  
and my dear father Michel Ehouarne.*

# Abstract

Traditional models of the business cycle rely on the assumption that the economy is populated by agents who have similar characteristics such as taste, income, or productivity. Despite offering simple and elegant macroeconomic models, this assumption masks the rich interplay between cross-sectional and aggregate cyclical movements. The goal of this dissertation is to show in two distinct contexts of heterogeneity – households and firms – how such interplay can shed new light on classic puzzles in the macro-finance literature such as the high volatility of unemployment or the large equity premium.

The first chapter, titled “The Macroeconomics of Consumer Finance”, studies the macroeconomic effects of consumer credit conditions in an incomplete-market, general equilibrium model where households hold unsecured debt, and firms use labor. I show that consumer finance disturbances can cause business cycle fluctuations through a rich interplay between credit and labor risks. As unemployment rises, households are more likely to default, translating into tighter credit conditions that reduce their consumption and cause further unemployment. Such feedback loop is reinforced by precautionary-saving motives among unconstrained households. Surprisingly, this mechanism can explain a large fraction of the volatility and persistence of U.S. unemployment even though it abstracts from traditional frictions like search or price stickiness.

In the second chapter, titled “Misallocation Cycles” and co-authored with Lars-Alexander Kuehn and David Schreindorfer, we estimate a general equilibrium model with firm heterogeneity and a representative household with Epstein-Zin preferences. Firms face investment frictions and permanent shocks, which feature time-variation in common idiosyncratic skewness. Quantitatively, the model replicates well the cyclical dynamics of the cross-sectional output growth and investment rate distributions. Economically, the model is able to generate business cycles through inefficiencies in the allocation of capital across firms. These cycles arise because (i) permanent Gaussian shocks give rise to a power law distribution in firm size and (ii) rare negative Poisson shocks cause time-variation in common idiosyncratic skewness.

# Acknowledgements

I am grateful to my advisor, co-author, and mentor Lars-Alexander Kuehn for his support and guidance all along this project. He has given me the freedom to think outside of the box and has guided me through my trials and errors with a lot of patience. His sense of humor and good mood made this journey extremely pleasant. I am also thankful to have David Schreindorfer as a co-author who is a bright and friendly scholar.

I am deeply in debt of my advisors Sevin Yeltekin and Ariel Zetlin-Jones for their incredible support and guidance. They shared with me invaluable insights into my job market paper and the whole job search process. I am also grateful for the many years I worked for Sevin as a Teaching Assistant for my favorite class “Global Economics”.

For helpful comments and suggestions on this dissertation and in particular on the first chapter (my job market paper), I would like to thank Bryan R. Routledge, Brent Glover, Burton Hollifield, Benjamin J. Tengelsen, and seminar participants at Carnegie Mellon University, HEC Montréal, Université du Québec à Montréal, Concordia University, Université Laval, Bank of Canada, and the Federal Reserve Board of Governors. I also would like to thank Daniele Coen-Pirani for serving on my committee. I also thank Lawrence Rapp for his help at the early stage of the job market process, and for setting such a good mood in the Ph.D. lounge.

For inspiring me to pursue a Ph.D., I would like to thank my master’s thesis advisor Stéphane Pallage and his wonderful colleagues at Université du Québec à Montréal.

Finally, I would like to thank my family and friends for supporting me during my ups and downs during this long journey. I dedicate this dissertation to my wife Anne-Sophie Perrin for giving me the strength to accomplish this great achievement, and my father Michel Ehouarne for having always believed in me.

# List of Figures

1.1	<b>Steady-State Policy Functions by Household Type</b> . . . . .	18
1.2	<b>Steady-State Excess Demand in the Goods Market.</b> . . . . .	21
1.3	<b>Model-based Stationary Distributions by Household Type</b> . . . . .	27
1.4	<b>Unemployment and Consumer Finance Statistics over the U.S. Business Cycle</b> . . . . .	29
1.5	<b>Cross sectional moments of credit balances and limits as % of annual income</b> . . . . .	30
1.6	<b>Employment Forecasting Rules in U.S. Data</b> . . . . .	33
1.7	<b>Excesses and Shortages of Goods in the Stochastic General Equilibrium.</b> . . . . .	34
1.8	<b>Impulse Response Functions by Consumption Quintile</b> . . . . .	37
2.1	Optimal capital and investment rate policies . . . . .	56
2.2	Optimal capital policies in the $\mu$ -distribution . . . . .	58
2.3	Robust cross-sectional moments of annual sales growth and investment rates . . . . .	66
2.4	Life Cycle of a Firm: Average Behavior and Quartiles. . . . .	75
2.5	Impulse response: Cross-Section . . . . .	76
2.6	Impulse response: Aggregates . . . . .	78
2.7	<b>Cross-Sectional Impulses by Firms' <math>\kappa</math>.</b> . . . . .	79
2.8	<b>Cross-Sectional Impulses by Firm Size.</b> . . . . .	81
2.9	Output Concentration . . . . .	82
2.10	Comparative statics for the exit rate . . . . .	83

# List of Tables

1.1	Model Statistics by Category of Borrowers . . . . .	17
1.2	Model Parameter Values . . . . .	22
1.3	Simulated and Actual Moments in Steady State . . . . .	24
1.4	Alternative Steady-State Estimation without Preference Heterogeneity . . . . .	26
1.5	OLS Coefficients of the Conditional Employment Forecasting Rules . . . . .	32
1.6	Unemployment and Consumer Finance Statistics Over the Business Cycle . . . . .	35
1.7	Household Economic Characteristics by Consumption Quintile in Steady State . . . . .	38
1.8	Consumption Insurance by Net Worth Quintile Over the Business Cycle . . . . .	39
2.1	Two stylized firm-level distributions . . . . .	60
2.2	Predefined and Estimated Parameter Values . . . . .	68
2.3	Moments Targeted in SMM Estimation . . . . .	70
2.4	Consumption Growth and Asset Prices . . . . .	73

# Contents

<b>1</b>	<b>The Macroeconomics of Consumer Finance</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.1.1	Literature . . . . .	6
1.2	Modeling Consumer Finance and Systemic Risk . . . . .	7
1.2.1	The Environment . . . . .	8
1.2.2	Household's Problem . . . . .	10
1.2.3	Bond Pricing . . . . .	11
1.2.4	Aggregate Resources . . . . .	11
1.2.5	General Equilibrium . . . . .	12
1.2.6	Steady State . . . . .	13
1.2.7	Solution Method . . . . .	13
1.3	Estimating the Model Steady State . . . . .	14
1.3.1	The Data . . . . .	15
1.3.2	Mapping the Model to U.S. Data . . . . .	16
1.3.3	Identifying Preference Heterogeneity in the Data . . . . .	18
1.3.4	Matching Moments . . . . .	19
1.3.5	Results . . . . .	20
1.3.6	Counterfactual Results without Preference Heterogeneity . . . . .	25
1.3.7	Understanding the Role of Preference Heterogeneity . . . . .	25
1.4	On the Cyclical Effects of Consumer Finance . . . . .	28
1.4.1	Stylized Facts of the Cyclical Nature of Consumer Finance . . . . .	28
1.4.2	Calibrating the Aggregate Financial Shock . . . . .	31
1.4.3	Approximate Aggregation in the Model and in U.S. Data . . . . .	32
1.4.4	The Joint Dynamics of Consumer Finance and Unemployment . . . . .	35
1.4.5	The Mechanism at Play: From Cross-Sectionial to Aggregate . . . . .	36
1.4.6	Welfare Implications: Procyclical Consumption Insurance . . . . .	39
1.4.7	Discussion . . . . .	40
1.5	Conclusion . . . . .	41

<b>2</b>	<b>Misallocation Cycles</b>	<b>43</b>
2.1	Introduction . . . . .	43
2.2	Model . . . . .	47
2.2.1	Production . . . . .	47
2.2.2	Firms . . . . .	50
2.2.3	Household . . . . .	51
2.2.4	Equilibrium . . . . .	52
2.3	Analysis . . . . .	52
2.3.1	Household Optimization . . . . .	52
2.3.2	Firm Optimization . . . . .	53
2.3.3	Aggregation . . . . .	55
2.3.4	Frictionless Economy . . . . .	59
2.3.5	Numerical Method . . . . .	61
2.4	Estimation . . . . .	63
2.4.1	Data . . . . .	64
2.4.2	Cyclical Properties of the Cross-Section of Firms . . . . .	65
2.4.3	Simulated Method of Moments . . . . .	67
2.4.4	Parameter Identification . . . . .	70
2.4.5	Baseline Estimates . . . . .	72
2.4.6	Alternative Specifications . . . . .	73
2.5	Model Implications . . . . .	74
2.5.1	Firms' Life Cycle . . . . .	74
2.5.2	Business Cycles . . . . .	74
2.5.3	Power Law and Consumption Dynamics . . . . .	81
2.6	Conclusion . . . . .	84
<b>A</b>	<b>Computational Appendix</b>	<b>85</b>
A.1	Value Function Iteration . . . . .	85
A.2	Non-Stochastic Simulation . . . . .	86
A.3	Steady State and Estimation . . . . .	86
A.4	Approximate-Aggregation Equilibrium . . . . .	87



# Chapter 1

## The Macroeconomics of Consumer Finance

### 1.1 Introduction

The Financial Crisis of 2007-09 sparked a large body of empirical research emphasizing a strong and significant relationship between household financial conditions, consumption, and unemployment (e.g. [Mian and Sufi \(2014a\)](#), [Mian et al. \(2013\)](#), [Melzer \(2013\)](#), [Dyan \(2012\)](#), [Bauer and Nash \(2012\)](#), [Haltenhof et al. \(2014\)](#)). At the trough of the recession in 2009, more than a million American households filed for bankruptcy while banks were charging off billions of dollars of consumer loans. The tumult was particularly acute in unsecured consumer credit markets where banks halved their originations of credit card loans, representing a fall of more than 50 billion dollars<sup>1</sup>. In the meantime, the unemployment rate more than doubled in less than two years and households cut back on their consumption by hundreds of billions of dollars. Despite the empirical evidence of a correlation between household finance and business cycle fluctuations, there has been limited progress in the theoretical and quantitative field toward understanding the key causal mechanism at play.<sup>2</sup>

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<sup>1</sup>Data on bankcard originations is from [Experian](#); other data are from U.S. Courts, BLS, BEA, and Board of Governors.

<sup>2</sup>For instance, [Justiniano et al. \(2015\)](#) explains that “the macroeconomic consequences of leveraging and deleveraging are relatively minor”, while [Nakajima and Ríos-Rull \(2014\)](#) studies models with various degrees of credit frictions to conclude that “the volatility of output is highly similar across all economies”. In opposite, in an influential empirical work, [Mian and Sufi \(2014b\)](#) use their geographic estimates based on the non-tradable goods sector to extrapolate that “a 9.5% reduction in housing net worth (which is what the economy experienced between 2007 and 2009) leads to a reduction in overall employment of 2.9% or 55% of the actual decline in total employment of 5.3%.”

The main contribution of this paper is to explain *how* disturbances in consumer finance can cause business cycle fluctuations through the rich interplay between credit and labor risks, and to quantify such effects in the U.S. economy. As unemployment rises, households are more likely to default, translating into tighter credit conditions in the form of higher interest rates and lower credit limits. Such tightening forces indebted households to decrease consumption, which reduces the firms' sales and thus causes further unemployment. The effects of this consumer credit channel are not limited to borrowers, however. Because of higher unemployment, unconstrained households also hold back on consumption to build up precautionary savings. Despite its simplicity, this theory is surprisingly powerful in explaining the high volatility and high first-order autocorrelation of U.S. unemployment, as well as key properties of unsecured credit markets: the weak correlation between bankruptcies and unemployment, the large volatility of bankruptcies, and the pro-cyclicality of revolving credit. My results highlight the role of consumer credit frictions (incomplete markets, intermediation, risk of default) in shaping the cyclical dynamics of employment and consumption.

The results are based on a standard dynamic stochastic general equilibrium model where production is based on labor, markets are incomplete, and households can hold unsecured credit accounts. Households face two types of idiosyncratic shocks: labor productivity and employment status. They can also differ in their preferences which remain fixed throughout time. There are risk-neutral intermediaries who are perfectly competitive and price individual credit loans according to the household's specific risk of default. The key novelty of this framework is the treatment of unemployment. In a typical bond economy, the funds market clears through the adjustment of a single interest rate on lending and borrowing, and so the goods market clears by Walras' law.<sup>3</sup> However, in my model, because of the presence of financial intermediaries, this adjustment process becomes inoperative. Instead, excesses and shortages of goods are eliminated by adjustments in the firm's production process through its choice of labor. For illustrative purposes, consider the following two polar cases. On one hand, if the

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<sup>3</sup>Walras' law states that, in general equilibrium, clearing all but one market ensures equilibrium in the remaining market.

unemployment rate were extremely high in the economy, production (quantity supplied) would be low because few households work, but consumption (quantity demanded) would be high because households would tend to dis-save or consume on credit to compensate for their unemployment spell. On the other hand, if unemployment were extremely low, production would be high, but consumption would not necessarily be as high because some households would be saving their income for precautionary reasons. Overall, the model admits an interior solution where the number of workers employed by the firms is consistent with the level of production required to satisfy all the goods demanded by the households. Therefore, my paper provides a Neoclassical theory of unemployment purely based on consumer credit frictions rather than traditional frictions like search or price stickiness.

To discipline the quantitative analysis, I use the triennial Survey of Consumer Finance over the sample period 1995-2013 to estimate the model parameters by matching a large set of aggregate and cross-sectional steady-state moments related to unsecured consumer credit markets and employment. At the aggregate level, the model reproduces well the unemployment rate, the credit card interest spread (average credit card interest rate minus fed fund rate), the bankruptcy rate (number of non-business bankruptcy filings under Chapter 7 as a fraction of civilian population), the total outstanding revolving credit expressed as a fraction of aggregate disposable income, and to some extent the charge-off rate on credit cards. At the cross-sectional level, the model matches the cross-sectional mean and higher order moments (variance, skewness, kurtosis) of the distribution of balance-to-income ratios (a measure of leverage in unsecured credit markets) and the distribution of credit card interest rates for non-convenience users (i.e. credit card users who do not fully repay their balance within a billing cycle). The model also reproduces the apparent disconnect between credit card interest rates and unsecured leverage observed in the data (cross-sectional correlation close to zero). The model is able to replicate all these aggregate and cross-sectional features because it allows for heterogeneity among households in their income shocks, subjective discount factors, and elasticities of intertemporal substitution. In a counterfactual estimation with only in-

come heterogeneity, I show the cross-sectional dispersion in unsecured leverage is almost three times lower than the data, and leverage is almost perfectly correlated with the interest rate. This stems from the fact that with only one dimension of heterogeneity, interest rates are directly a function of leverage. Therefore, as leverage rises, the cost of borrowing necessarily increases as well, which discourages households to lever up and yields a limited dispersion in leverage.

Turning to business cycle analysis, I solve the model by conjecturing an *approximate-aggregation* equilibrium where individuals only use information about current employment to form an expectation about future employment conditional on the realization of future shocks, rather than using the entire household distribution as a state variable.<sup>4</sup> In my model, business cycles are driven by exogenous shocks that affect the way intermediaries discount time. More precisely, intermediaries are modeled as more impatient when they lend money to the households compared to when they borrow from them (i.e. when they hold deposits). During bad times, the spread between these two subjective discount factors increases. Such time-varying spread captures all the different possible factors that could affect the intermediaries' lending decision (e.g. time-varying risk aversion, cost of processing loans, illiquidity of funds market, etc.). I infer the volatility and persistence of this financial shock by parameterizing the model such that it reproduces the persistence and volatility of credit card interest rate spread observed in U.S. data (defined as average credit card interest rate minus effective Fed funds rate). Under this parametrization, I show that the model matches well the unemployment dynamics as well as salient features of consumer finance. In what follows, I summarize the key predictions of the model.

First, unemployment is highly volatile and highly persistent, both in the model and in the data. The volatility of unemployment represents a quarter of its mean, and the coefficient of auto-correlation is close to unity. In the model, it takes more than ten years for the economy to recover from a one-quarter recession. Similarly, credit card interest spread is highly volatile and persistent, and displays a high positive correlation

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<sup>4</sup>See [Krusell and Smith \(1998\)](#). The method I use to solve the model is of independent interest and explained in details in the computational appendix.

with unemployment.

Second, bankruptcies and unemployment are weakly correlated. At first glance, this seems to suggest that consumer default does not affect the business cycle. However, this is not the case due to the subtle distinction between *expected* default and *realized* default. Realized default does not play a significant role because bankruptcies are extremely rare (less than half a percent of the population per year, in model and data). On the other hand, expected default is strongly counter-cyclical (proxied by credit spreads) and can have large macroeconomic effects because it affects *all* the borrowers through high interest rates and low credit limits.

Third, credit is pro-cyclical. This means that households borrow more during an expansion rather than during a recession. This result is counter-intuitive to most intertemporal-consumption-choice models which predict that households save during expansions and borrow during recessions in order to smooth their consumption path. This argument however misses a selection effect. When unemployment is low, households are less likely to default and thus benefit from higher credit limit and lower interest rates. During an expansion, there are more consumers that have access to credit markets, which leads to higher levels of debt overall although each household does not necessarily borrow more individually.

Fourth, in the model-based cross section, the biggest drop in consumption comes from the low-net-worth (high debt, low income) households who are no longer able to consume on credit as they did before the crisis. In particular, the average drop in consumption at the first quintile of the household distribution is twice as large as in the second and third quintiles, and three times larger than the fourth and fifth quintile. Such disparities across consumption groups also justifies the need to match the *right* amount of heterogeneity observed in the data.

Fifth, in the model, even the wealthiest households increase their savings and reduce their consumption during a recession. This result is due to an aggregation effect. Although it is true that the wealthy households who become unemployed *dis-save* in order to smooth consumption, it turns out that the ones who remain employed also save

more against the higher chances of being unemployed. Overall, the latter group is dominant and therefore aggregate savings increase among the wealthiest while consumption decreases.

The rich interplay between labor risk, credit risk, and the aggregate demand channel offers some new perspectives on public policies. Through the lens of the model, we can view consumption and unemployment fluctuations as a symptom of time-varying risk-sharing opportunities. Hence, to stimulate the economy, we could design policies aimed at reducing the idiosyncratic risk faced by households rather than simply spending more. Such policies could for example feature a household bailout or some other credit market regulations. Of course, the cost and benefit of each policy should be carefully analyzed in a structurally estimated model. This paper makes some progress toward this goal.

The remainder of the paper is organized as follows. After describing how my paper fits in the literature, I start Section 1.2 with a description of the model, and then estimate its parameters in steady state in Section 1.3. Section 1.4 presents and discusses the main business cycle results of the paper. Section 1.5 concludes with some remarks.

### 1.1.1 Literature

This paper belongs to the vast literature on incomplete-market economies (e.g. [Bewley \(1983\)](#), [Imrohoroglu \(1989\)](#), [Huggett \(1993\)](#), [Aiyagari \(1994\)](#), [Krusell and Smith \(1998\)](#), [Wang \(2003\)](#), [Wang \(2007\)](#), [Challe and Ragot \(2015\)](#)), with an emphasis on consumer credit default as in [Chatterjee et al. \(2007\)](#), [Livshits et al. \(2007\)](#), [Athreya et al. \(2009\)](#), and [Gordon \(2014\)](#). In a closely related paper, [Nakajima and Ríos-Rull \(2014\)](#) extends [Chatterjee et al. \(2007\)](#) to a dynamic setup to study the business cycle implications of consumer bankruptcy. The main difference with my paper is that the authors do not consider the interplay between credit risk and labor risk (in their model, there is a labor-leisure trade off but no unemployment). As a result, they find that “the volatility of output is highly similar across all economies [with different degrees of credit frictions]”.

My work contributes to the growing body of research that studies the macroeconomic effects of household financial conditions. With the exception of few papers like [Nakajima](#)

and [Ríos-Rull \(2014\)](#), it has been standard in this literature to drop either of the two main assumptions that my paper emphasizes: (i) aggregate uncertainty, (ii) incomplete markets.

Prominent papers of the former group are [Eggertsson and Krugman \(2012\)](#), [Guerrieri and Lorenzoni \(2011\)](#), and [Justiniano et al. \(2015\)](#). Such models prevent a formal business cycle analysis since they miss the effects of aggregate risk on households' borrowing, saving, and defaulting decisions, which are crucial in order to understand the aggregate effects of consumer finance on consumption and unemployment.

Important papers of the latter group are [Herkenhoff \(2013\)](#), [Bethune \(2014\)](#), [Midrigan and Philippon \(2011\)](#), and [Kehoe et al. \(2014\)](#). Such papers abstract from market incompleteness to focus on the role of various financial frictions like price stickiness and cash-in-advance constraints, or credit and labor search.

My work also complements the literature which focuses on the firm side (e.g. [Kiyotaki and Moore \(1997\)](#), [Bernanke et al. \(1999\)](#), [Jermann and Quadrini \(2012\)](#), [Brunnermeier and Sannikov \(2014\)](#), [Gomes and Schmid \(2010a\)](#), and [Shourideh and Zetlin-Jones \(2014\)](#)). Although there could be some overlaps between the two strands of literature, there are virtues in studying each side. For instance, when firms face credit frictions that impede their ability to invest or produce, they are eventually forced to exit or lose market shares to the benefit of bigger firms that do not face such frictions. In opposite, if households face credit frictions that hinder their ability to consume, they will neither “exit” the economy nor “lose market shares” to wealthier households. The lack of consumption from credit-constrained households is not necessarily offset by more consumption from unconstrained households as I show in my paper, thus having aggregate consequences on consumption and unemployment.

## 1.2 Modeling Consumer Finance and Systemic Risk

This section describes a parsimonious incomplete-market heterogeneous-agent bond economy in which households have the option to default. Production is stochastic and based on labor. Wages are set in perfectly competitive labor markets and reflect

household-specific labor productivities. The interest rate on saving and the cross section of household-specific interest rates on borrowing are set by perfectly competitive risk-neutral intermediaries who make zero profit in expectation. Since interest rates are pinned down by intermediaries, there is only one variable left to clear the goods market: employment. Except for market incompleteness and financial intermediation, the model is a standard Neoclassical framework: prices are perfectly flexible, goods and labor markets are frictionless, and firms are perfectly competitive.

### 1.2.1 The Environment

The economy is populated by a unit continuum of infinitely lived households indexed by  $i$ . Time is discrete, indexed by  $t$ , and goes forever. Households discount time at their own subjective rate  $\beta_i \in (0, 1)$ , and order stochastic streams of consumption goods  $(C_{it})_{t=0}^{\infty}$  according to [Epstein and Zin \(1989b\)](#) preferences:

$$U_{it} = \left\{ (1 - \beta_i) C_{it}^{1-1/\psi_i} + \beta_i \left( \mathbb{E}_t [U_{it+1}^{1-\gamma}] \right)^{\frac{1-1/\psi_i}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi_i}}, \quad (\text{Utility})$$

with  $\psi_i > 0$  their elasticity of intertemporal substitution (EIS), and  $\gamma > 0$  their coefficient of relative risk aversion (common across all households).<sup>5</sup> The amount of heterogeneity in preference parameters  $\theta_i \equiv (\beta_i, \psi_i) \in \Theta$  is fixed at time  $t = 0$  and characterized by the joint distribution  $F : \Theta \rightarrow (0, 1)$ .<sup>6</sup>

Households face two types of idiosyncratic shocks: labor productivity and employment status. Labor productivity is denoted by  $z_{it} \in \mathbb{R}_+$  and is drawn from a truncated Log-normal distribution with minimum value  $\underline{z}$ . On the other hand, employment status can take on two possible values  $\varepsilon_{it} \in \{0, 1\}$ . With probability  $\pi_t \in (0, 1)$ , household  $i$  is employed at time  $t$  ( $\varepsilon_{it} = 1$ ) and supplies inelastically one unit of labor to the firms in exchange of a wage payment  $W_{it} > 0$ . Otherwise, with probability  $1 - \pi_t$ , the household is unemployed ( $\varepsilon_{it} = 0$ ) and receives a compensation from the government

<sup>5</sup>For computational efficiency, whenever  $\psi_i > 1$ , I will use the compact form  $\mathcal{U}_{it} \equiv U_{it}^{1-1/\psi_i} / (1 - \beta_i)$ ,  $\mathcal{U}_{it} = C_{it}^{1-1/\psi_i} + \beta_i \left( \mathbb{E}_t [U_{it+1}^{\rho_i}] \right)^{\frac{1}{\rho_i}}$ , where  $\rho_i \equiv \frac{1-\gamma}{1-1/\psi_i}$ .

<sup>6</sup>I discuss the importance of allowing for preference heterogeneity when I estimate the model in [Section 3](#).



at the replacement rate  $\varrho \in (0, 1)$ .<sup>7</sup> The unemployment insurance program is funded by a proportional tax  $\tau \in (0, 1)$  on income, and any fiscal surplus is spent as public consumption  $G_t$ . Labor income net of taxes equals:

$$\Upsilon_{it} = (1 - \tau)(\varepsilon_{it} + (1 - \varepsilon_{it})\varrho)W_{it}. \quad (\text{Income})$$

The key feature of this model is that the employment probability  $\pi_t \in (0, 1)$  is simply pinned down by the firms. A representative firm operates the economy by *choosing* labor  $\pi_t \in (0, 1)$  and producing a single non-durable good  $Y_t \in \mathbb{R}_+$  with the linear technology:

$$Y_t = \int \varepsilon_{it} z_{it} di \Leftrightarrow Y_t = \pi_t, \quad (\text{Technology})$$

where the equivalence result holds because of the assumption that employment shocks  $\varepsilon_{it}$  are *iid* across households, and the mean of the idiosyncratic productivity draws  $z_{it}$  is normalized to unity. Under competitive pricing, workers are paid their marginal product, hence  $W_{it} = z_{it}$ .

Markets are incomplete in the sense that households are not allowed to trade consumption claims contingent on their idiosyncratic shocks  $(\varepsilon_{it}, z_{it})$ . This could be due to a moral hazard problem, though it is not explicitly modeled in this paper. Households only have access to a credit account with balance  $B_{it} \in \mathbb{R}$ , where  $B_{it} \leq 0$  is treated as savings with return  $r_t^f > 0$ . The price of \$1 borrowed today by household  $i$  is denoted by  $q_{it} \leq 1$  and reflects her specific risk of default. At any point in time, household  $i$  is free to default on her credit balance  $B_{it} > 0$ . By doing so, her outstanding debt is fully discharged ( $B_{it} \equiv 0$ ). Consequently, the household is not allowed to save or borrow in the current period (one-period autarky) and  $\Upsilon_{it} - \underline{\Upsilon}$  dollars are garnished from her income, where  $\underline{\Upsilon}$  is an exogenous limit on the amount of income the intermediaries can seize.<sup>8</sup>

<sup>7</sup>Notice that the modeling of the U.S. unemployment insurance is simplified along two dimensions to reduce the state space. First, benefits are computed in terms of the wage the household would have earned if employed, which corresponds to the wage of an actual employed worker who has the exact same productivity. In opposite, the U.S. benefits are computed with respect to the wage earned before the job was lost, hence requiring an extra state variable. Second, benefits are perpetual. Instead, in the U.S. they are limited to a certain number of periods, which would have to be tracked by an additional state variable.

<sup>8</sup>This specification of wage garnishment allows me to express the household's recursive problem

There exists a large number of risk-neutral intermediaries with identical preferences. They discount time differently whether they lend resources to households or borrow from them (i.e. hold saving account). In particular, they use the discount factor  $\delta \in (0, 1)$  when making a borrowing decision, and  $\phi_t \delta$  otherwise when lending money to households. The spread between the two discount factors is the sole source of business cycle fluctuations and is modeled as a first-order Markov chain  $\phi_t \in \{\phi_1, \dots, \phi_N\}$  with transition probabilities  $\Pr(\phi_{t+1} = \phi_j | \phi_t = \phi_i) \equiv \pi_{ij}$ , such that  $\sum_{j=1}^N \pi_{ij} = 1$  for all  $i = 1, \dots, N$ . This financial shock captures different factors that could affect the intermediaries' lending decisions such as time-varying risk aversion, funds market illiquidity, costly processing loans, etc.

### 1.2.2 Household's Problem

At the beginning of each period  $t$ , household  $i$  observes her idiosyncratic shocks  $(\varepsilon_{it}, z_{it}) \in \mathbb{R}_+^2$ , her outstanding balance  $B_{it} \in \mathbb{R}$  as well as the aggregate state of the economy  $\omega_t \in \Omega_t$  (discussed below), and decides whether to default on her debt, if any:

$$V_{it} = \max\{V_{it}^{\text{def}}, V_{it}^{\text{pay}}\}. \quad (1.1)$$

The value of paying off her debt is

$$V_{it}^{\text{pay}} = \max_{B_{it+1} \leq \bar{B}} \left\{ C_{it}^{1-1/\psi_i} + \beta_i \left( \mathbb{E}_t \left[ V_{it+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi_i}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi_i}}, \quad (1.2)$$

subject to:

$$\Upsilon_{it} + q_{it} B_{it+1} \geq C_{it} + B_{it}, \quad (\text{Budget constraint})$$

where  $\bar{B} \in \mathbb{R}_+$  is an exogenous limit that prevents her from running Ponzi schemes<sup>9</sup>. As the household  $i$  is more likely to default, she faces a higher cost of rolling over her debt and therefore  $q_{it}$  decreases. Notice that in the limiting case that household  $i$  will almost surely default next period, her cost of borrowing becomes infinite. In that case, the household cannot borrow since  $q_{it} \rightarrow 0$  and thus  $q_{it} B_{it+1} \rightarrow 0$  regardless of the choice

solely in terms of her net worth  $N_{it} \equiv \Upsilon_{it} - B_{it}$  which greatly simplifies the computational method. For technical reasons, I restrict  $\underline{\Upsilon} \leq (1 - \tau)\varrho \underline{z}$  to ensure that  $\Upsilon_{it} - \underline{\Upsilon}_t \geq 0$  at any point in time (i.e. extremely poor households do not get a subsidy from the banks by defaulting on their debt).

<sup>9</sup>In practice,  $\bar{B}$  represents the upper limit of the grid for the state  $B_{it}$ , and is set to a value large enough so that the household's optimal choice of debt does not bind.

$B_{it+1} \leq \bar{B}$ . Upon default, household  $i$  contemplates one-period autarky: her debt is discharged, her income is garnished by the amount  $\Upsilon_{it} - \underline{\Upsilon}$ , and she cannot borrow or save at time  $t$ . Mathematically:

$$V_{it}^{\text{def}} = \left\{ C_{it}^{1-1/\psi_i} + \beta_i \left( \mathbb{E}_t \left[ V_{it+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi_i}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi_i}}, \quad (1.3)$$

where:

$$C_{it} \equiv \Upsilon_{it}, \quad \Upsilon_{it} \equiv \underline{\Upsilon}, \quad B_{it} \equiv 0, \quad B_{it+1} \equiv 0.$$

### 1.2.3 Bond Pricing

Credit balances are priced in a perfectly competitive environment. The intermediary's net present value of making a loan of size  $B_{it+1} \in \mathbb{R}_+$  at time  $t$  to consumer  $i$  is:

$$\Pi_{it} = -q_{it}B_{it+1} + \delta\phi_t \mathbb{E} \left[ \mathbb{1}_{\{V_{it+1}^{\text{def}} \geq V_{it+1}^{\text{pay}}\}} \underbrace{(\Upsilon_{it+1} - \underline{\Upsilon})}_{\text{Garnishment}} + \mathbb{1}_{\{V_{it+1}^{\text{def}} < V_{it+1}^{\text{pay}}\}} B_{it+1} \right],$$

where  $\mathbb{1}_{\{V_{it+1}^{\text{def}} \geq V_{it+1}^{\text{pay}}\}}$  is an indicator function that takes the value 1 if household  $i$  decides to default on her debt  $B_{it+1} \in \mathbb{R}_+$  next period, and 0 otherwise. Under perfect competition, expected profits are driven to zero  $\Pi_{it} = 0$ , and the bond price equals<sup>10</sup>

$$q_{it} = \delta\phi_t \mathbb{E} \left[ \mathbb{1}_{\{V_{it+1}^{\text{def}} \geq V_{it+1}^{\text{pay}}\}} \frac{\Upsilon_{it+1} - \underline{\Upsilon}}{B_{it+1}} + \mathbb{1}_{\{V_{it+1}^{\text{pay}} < V_{it+1}^{\text{def}}\}} \right]. \quad (1.4)$$

In a similar fashion, the intermediary's net present value of borrowing  $B_{it} < 0$  resources from household  $i$  is simply  $\Pi_{it} = -q_{it}B_{it+1} + \delta B_{it+1}$ , which does not involve risk. Profits are driven to zero in perfectly competitive markets, which trivially pins down the saving bond price (risk-free)  $q^f = \delta$ . Furthermore, the intermediaries' profits at the aggregate level are also driven to zero, so that all the loans are solely funded by deposits.

### 1.2.4 Aggregate Resources

At the aggregate level, output must equal private and public consumption since there is no capital accumulation in the model (no investment), nor imports and exports.

<sup>10</sup>This line of argument follows [Chatterjee et al. \(2007\)](#) and [Livshits et al. \(2007\)](#), and is commonly in the corporate finance literature: [Leland \(1994\)](#), [Hennessy and Whited \(2007\)](#), [Gomes and Schmid \(2010b\)](#), [Kuehn and Schmid \(2014\)](#)

Furthermore, since intermediaries' aggregate profits are driven to zero under pure and perfect competition, their consumption is null. Therefore,

$$Y_t = C_t + G_t. \quad (1.5)$$

Private consumption is simply the sum of households' individual consumptions  $C_t = \int C_{it} di$ . On the other hand, public consumption consists of the government expenditures financed by income taxes and net of transfers:

$$G_t = \underbrace{\int \tau z_{it} \varepsilon_{it} di}_{\text{Fiscal revenue}} - \underbrace{\int \varrho(1 - \tau) z_{it} (1 - \varepsilon_{it}) di}_{\text{Unemployment transfers}},$$

which is simply  $G_t = \tau \pi_t - \varrho(1 - \tau)(1 - \pi_t)$  since the employment shocks  $\varepsilon_{it} \in \{0, 1\}$  are *iid* across households, and the cross-sectional mean of the idiosyncratic productivity shocks is normalized to unity. For  $\tau \in (0, 1)$  large enough compared to  $\varrho \in (0, 1)$ ,  $G_t$  is always positive and co-moves with the employment rate (pro-cyclical).

### 1.2.5 General Equilibrium

Anticipating the definition of a recursive equilibrium, I will drop time indexes  $t$ , and mark next-period variables with a prime symbol ( $t'$ ). The household's state variables are her net worth defined as income minus credit balance,  $N \equiv Y - B \in \mathbb{R}$ , and her vector of preference characteristics  $\theta \in \Theta$ . The distribution of households over their individual states  $s \equiv (N, \theta) \in S \equiv \mathbb{R} \times \Theta$  is summarized by the probability measure  $\mu$  defined on the Borel algebra  $\mathcal{B}(S)$ . The aggregate state of the economy is the infinite dimensional object  $\omega \equiv (\mu, \phi) \in \Omega \equiv \mathcal{M} \times \mathbb{R}_+$ , where  $\mathcal{M}$  is the set of all possible measures on  $\mathcal{M} \equiv (S, \mathcal{B}(S))$ .

**Definition 1.** *A Recursive Competitive Equilibrium is a value function  $V : S \times \Omega \rightarrow \mathbb{R}$ , a pair of policies  $C : S \times \Omega \rightarrow \mathbb{R}_+$  and  $B' : S \times \Omega \rightarrow \mathbb{R}$ , a bond schedule  $q : \mathbb{R} \times \Omega \rightarrow [0, 1]$ , a time-varying employment rate  $\pi : \Omega \rightarrow [0, 1]$ , and a law of motion  $\mathcal{T} : \Omega \times \Phi \rightarrow \mathcal{M}$ ,  $\mu' = \mathcal{T}(\mu, \phi, \phi')$ , such that: (i) households solve their optimization problem characterized by (1.1)–(1.4), (ii) the goods market clears (1.5), and (iii) the evolution of  $\mu \in \mathcal{M}$  is consistent with households' optimal policies.*

Notice that if markets were complete and preferences identical, every household would consume the average private income of the economy  $\mathbb{E}_t[\Upsilon_{it}] = (1 - \tau)(\pi_t + (1 - \pi_t)\varrho)$ . Consequently, the goods market would clear regardless of the employment rate<sup>11</sup>. In particular, full employment is one of the equilibria in the complete-market case. Hence, the level of unemployment in the full model is an implicit measure of the degree of market incompleteness faced by consumers.

### 1.2.6 Steady State

It will be useful to define the model (non-stochastic) steady state as follows:<sup>12</sup>

**Definition 2.** *A Steady State Equilibrium is a value function  $v(s; \pi^*)$ , a pair of policies  $c(s; \pi^*)$  and  $b'(s; \pi^*)$ , a bond schedule  $q(b'; \pi^*)$ , a constant employment rate  $\pi^* \in [0, 1]$  and a time-invariant distribution  $\mu^* \in \mathcal{M}$  such that: (i) households solve their dynamic program problem stated in (1.1)–(1.4) but without aggregate uncertainty, (ii) the goods market clears (1.5), and (iii)  $\mu^* = T(\mu^*)$  where  $T$  corresponds to the operator  $\mathcal{T}$  in an environment without aggregate risk.*

In absence of aggregate uncertainty, the distribution  $\mu^*$  is time-invariant and hence not part of the household state space. Instead, I wrote the household's value and policies as explicit functions of  $\pi^*$  to highlight the fact that the employment rate is the only aggregate variable that the household cares about. This is an insight that will prove useful in the following section where I describe the solution method.

### 1.2.7 Solution Method

Since there are no known closed-form solution for this class of model, I will rely on numerical methods to solve the model. This task is however computationally challenging since the state  $\mu$  is an infinite dimensional object. Following [Krusell and Smith \(1998\)](#), I will remedy to this problem by conjecturing that the equilibrium features *approximate aggregation* in the sense that the entire households distribution can be summarized by

<sup>11</sup>To see this:  $\underbrace{\pi_t}_{\text{GDP}} = \underbrace{(1 - \tau)(\pi_t + (1 - \pi_t)\varrho)}_{\text{Consumption}} + \underbrace{\tau\pi_t - \varrho(1 - \tau)(1 - \pi_t)}_{\text{Government expenditure}} \Rightarrow \pi_t = \pi_t$ .

<sup>12</sup>Technically, an equilibrium with idiosyncratic shocks  $(\varepsilon, z)$  but no aggregate (financial) shocks  $(\phi)$

a finite set of moments. In their seminal paper, the authors used statistical moments of the idiosyncratic state variable (e.g. mean, variance) to proxy for the distribution. Subsequent research has explored the use of alternative moments such as prices (e.g. interest rate in [Krusell and Smith \(1997\)](#)).

Observe that in my model, in order to solve their optimization problem defined by (1.1)–(1.4), households only need to forecast the next-period employment rate  $\pi'$  in order to form expectations. Implicitly, this means that they do not require any specific information about the distribution  $\mu \in \mathcal{M}$  itself, but rather need some good predictors of  $\pi'$ . I conjecture and verify that current employment  $\pi$  is a good predictor of next-quarter employment  $\pi'$ , conditional on the realization of future shocks  $\phi'$ . Formally, I define:

**Definition 3.** *An Approximate-Aggregation Equilibrium is a value function  $V : S \times (0, 1) \rightarrow \mathbb{R}$ , a pair of policies  $C : S \times (0, 1) \rightarrow \mathbb{R}_+$  and  $B' : S \times (0, 1) \rightarrow \mathbb{R}$ , a bond schedule  $q : \mathbb{R} \times (0, 1) \rightarrow [0, 1]$ , and a forecasting rule  $\pi' = \Gamma(\pi, \phi') + \epsilon$ , such that (i) households solve their optimization problem characterized by (1.1)–(1.4), (ii) the goods market clears (1.5), and (iii) the forecasting errors  $\epsilon \in \mathcal{E}$  are near zero.*

### 1.3 Estimating the Model Steady State

The first test of the model is whether it can explain the unemployment rate and a large set of aggregate and cross-sectional consumer finance statistics in the steady state under plausible parameter values. I discipline this exercise in an over-identified moment matching procedure. Except for few parameters that have a clear counterpart in the data (e.g. income tax rate and unemployment insurance replacement rate), all the other model parameters are estimated by minimizing the squared relative distance between simulated and actual moments. The main result of this section is that under a plausible parametrization, the model can replicate the steady state unemployment rate while matching simultaneously a high credit card spread, a low bankruptcy rate, and a large cross-sectional dispersion in both unsecured leverage and credit card interest rates.

### 1.3.1 The Data

The main data that I use to estimate the model is the Survey of Consumer Finances (SCF) from the [Board of Governors](#). All the cross sectional moments are averaged across the seven triennial surveys available over the period 1995–2013. To assess the amount of heterogeneity in cost of borrowing and amounts borrowed in the unsecured credit markets, I look at two key variables: the balance-to-income ratio, and the credit card interest rate. The first variable that I consider is the balance-to-income ratio, which proxies for leverage in absence of collaterals. The balance refers to the amount of money the household still owes to the bank after her last payment. This means that the balance after payment excludes the typical “convenience” credit that does not entail interest fees if paid in full within a billing cycle. On the other hand, the income refers to the annual gross income before taxes. This includes all wages and salaries, unemployment compensations, pensions and other social security transfers, and income from interests, capital gains and business ownership. The second variable I consider is the interest rate paid on the most used credit card. This is a variable that started to be documented only in 1995 (except for the 1983 survey). The cross section of interest rates only reflects credit card holders who are revolvers and not simply convenience users.

I complement the SCF data with various aggregate statistics such as the credit card interest rate spread, the bankruptcy rate, the credit card charge-off rate, and the ratio of revolving credit to aggregate disposable income. I construct the credit card spread by taking the difference between the average credit card interest rate and the effective Federal Funds rate. The former is reported in [H.15 Selected Interest Rates \(Board of Governors\)](#) while the latter is documented in the [Report on the Profitability of Credit Card Operations \(Board of Governors\)](#). On the other hand, the bankruptcy rate is obtained by dividing non-business bankruptcy filings under Chapter 7 ([U.S. Bankruptcy Courts](#)) by the U.S. population (civilian, non-institutional, of age 16 and older).<sup>13</sup> To compute the ratio of outstanding revolving credit to disposable income,

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<sup>13</sup>Consumers can file for bankruptcy under three different chapters: 7, 11, and 13. Chapter 11 relates

I use the data on total outstanding revolving consumer credit from [G.19 Consumer Credit \(Board of Governors\)](#) and the aggregate disposable personal income reported in [Personal Income and Outlays, A067RC1 \(Bureau of Economic Analysis\)](#). The charge-off rate is documented in the [Report of Condition and Income \(Board of Governors\)](#). The charge-offs are the loans that are removed from the banks' books and charged as a loss. The charge-off rate is net of recoveries and computed as the ratio between charge-offs and the banks' outstanding debt. Lastly, the civilian unemployment rate is from the [U.S. Bureau of Labor Statistics \(BLS\)](#).

### 1.3.2 Mapping the Model to U.S. Data

A difficulty in mapping the model to the data is to distinguish between the credit card holders who carry a balance from one period to another (revolvers) and the ones who pay their balance in full within a billing cycle (convenience users). This stems from the fact that in the model there is no distinction between cash and debt. Thus, the household's explicit decision to pay-off a portion of her outstanding debt with cash instead of issuing new debt is indeterminate.<sup>14</sup> To tackle this issue, I split the consumers in my model as follows. Convenience users refer to the consumers who do not roll over their debt, that is, if there were a cash flow mismatch within a period  $t$ , they would be able to pay off their existing debt with their current income alone. Instead, the revolvers will refer to the consumers who *need* to borrow in order to pay off their past debt. In terms of state variables, this means that the revolvers are the households whose current income is smaller than current debt  $\Upsilon < B$ , which implies that they have a negative net worth  $N < 0$ . On the other hand, the convenience users are the ones who have enough income to cover their debt  $\Upsilon \geq B$ , or simply  $N \geq 0$ .

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to business owners and concerns less than 1% of all the filings. On the other hand, Chapter 11 represents 30% of all the filings but involves a repayment plan, which is something that my model does not account for. As a consequence, I only consider the filings under Chapter 7 (over 69% of the bankruptcy cases). There is a spike in bankruptcy filings around 2005 which reflects the Bankruptcy Abuse and Prevention and Consumer Protection Act (BAPCPA) that took effect on October 17, 2005.

<sup>14</sup>Mathematically, let  $X_{it}$  be cash and  $L_{it}$  the size of a new credit card loan, then the credit balance's law of motion is:

$$B_{it+1} = B_{it} - X_{it} + L_{it}.$$

In the model, only  $B_{it+1}$  and  $B_{it}$  are separately identified, but not  $X_{it}$  and  $L_{it}$ .

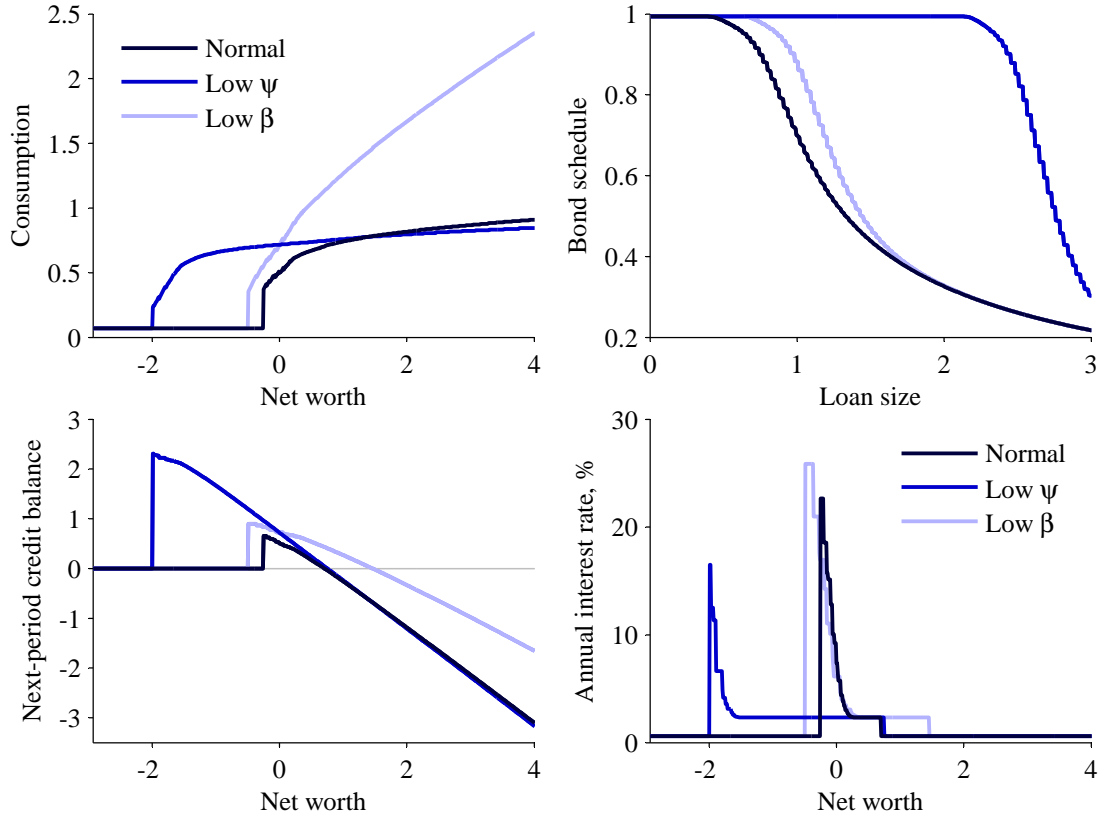


**Table 1.1: Model Statistics by Category of Borrowers**

This table compares the economic characteristics of the two different types of borrowers in the model: convenience users *versus* revolvers. Convenience users are defined as the borrowers who have enough income to repay fully their outstanding debt. Mathematically, this means  $\Upsilon \geq B$ , implying that their net worth is positive  $N \equiv \Upsilon - B \geq 0$ . On the other hand, revolvers are households who have too much debt compared to their current income, and do not have the choice but to roll over their debt.

Average characteristics	Type of borrower	
	Convenience user	Revolver
Outstanding debt	0.24	0.75
Current income	0.75	0.52
Consumption	0.68	0.57
Interest rate (annualized, %)	2.17	11.79
Unemployment rate (in %)	3.53	19.22
Default rate (annualized, %)	0	15.02
Subjective discount factor ( $\beta$ )	0.99	0.91
Elasticity of intertemporal substitution ( $\psi$ )	1.54	1.38
Fraction of borrowers by type (in %)	90.24	9.76

To check the validity of this categorization, Table 1.1 reports key model statistics about convenience users *versus* revolvers. Notice that more than 90% of the borrowers are classified as convenience users with this methodology. Their cost of borrowing is extremely low compared to revolvers (2.17% as opposed to 11.79%), which is consistent with the view that their borrowing activity does not entail a high credit risk.<sup>15</sup> Indeed, observe that the default rate among convenience users is exactly 0%, while it is 15% for revolvers. Furthermore, the convenience users hold low levels of debt, approximately three time less than the revolvers. It is interesting to remark that the revolvers in the model are particularly in distress: 20% of the revolvers are unemployed as opposed to 3.5% among convenience users. Overall, these statistics validate the model categorization: convenience users borrow small amounts, at a low cost and no associated risk of default, as opposed to revolvers who have high debt (larger than their income), pay a higher interest rate, and default frequently.



**Figure 1.1: Steady-State Policy Functions by Household Type**

This figure depicts the households' optimal consumption policy, their optimal next-period choice of credit balance (negative balances are savings), the optimal bond schedule offered by the intermediaries (given households' optimal behavior), and the equilibrium interest rate for each type of households (evaluating the bond schedule at the households' optimal borrowing policies). There are three types: normal, impatient (low  $\beta$ ), and high-incentive to smooth consumption (low  $\psi$  / low EIS). Notice that the default cutoffs are near zero for the normal households and the impatient ones, and near  $-2$  for the low-EIS households.

### 1.3.3 Identifying Preference Heterogeneity in the Data

To estimate the amount of preference heterogeneity required to match the large dispersion of balance-to-income ratios and credit card interest rates, I consider three different types of households in my model: high, medium, and low. High-type households are highly levered but their cost of borrowing is low. Low-type households are also highly levered *and* their cost of borrowing is high. Finally, medium-type households have low leverage and with moderate cost of borrowing. I identify these three types in the model

<sup>15</sup>In the model, the cost of borrowing for the convenience users is not exactly zero because they still need to compensate the intermediaries for their impatience. In reality, the convenience users do not have to pay an interest fee because banks get paid fees by the merchants who use credit card as a method of payment.

by looking at the household optimal policies in steady state (see Figure 1.1).

In particular, high-type households have a relatively low elasticity of intertemporal substitution ( $\psi$ ) compared to the other types, and low-type households have a relatively low subjective discount factor ( $\beta$ ). Notice in particular how households with low patience tend to borrow more while paying high interest rates. In the meantime, households with a low EIS are able to borrow more at lower net worth levels for a low cost. This stems from the fact that with a low elasticity of intertemporal substitution (EIS), such households have a strong desire to smooth consumption, and thus they want to avoid a default event. Since household types are observable<sup>16</sup>, the intermediaries recognize that such households have strong incentives not to default and hence they offer them a discount. For computational tractability, I consider the following distribution of types  $\{(\beta, \psi), (\beta, \underline{\psi}), (\underline{\beta}, \psi)\}$  which requires the estimation of only four parameters (high and low value for  $\beta$  and  $\psi$  respectively), and two associated probabilities (the third probability being linearly dependent).

### 1.3.4 Matching Moments

I proceed in two stages to estimate the values of the different parameters of the model. In the first stage, I directly assign values to the parameters that have a clear counterpart in the data. Three parameters are *calibrated* in this fashion: the income tax rate  $\tau$ , the Unemployment Insurance (UI) replacement rate  $\varrho$ , and the risk-free return on saving account  $r^f$  (which I interpret in the data as the effective Federal Funds rate). In the second stage, I obtain the rest of the parameter values (six out of nine) by minimizing the squared relative difference between a set of actual and simulated moments. Six parameters concern preference heterogeneity. Three other parameters are the steady state value of the financial shock (the intermediaries' discount spread)  $\mathbb{E}[\phi]$ , the volatility of the idiosyncratic labor productivity shock  $\sigma_z$ , and the coefficient of relative risk aversion  $\gamma$  (common across households).

To identify the parameter values, I select several cross-sectional and aggregate mo-

<sup>16</sup>The assumption that types are observable proxies for the fact that in reality creditors have access to credit scores, which keep track of the households' borrowing habits and hence provides useful information to infer their type.

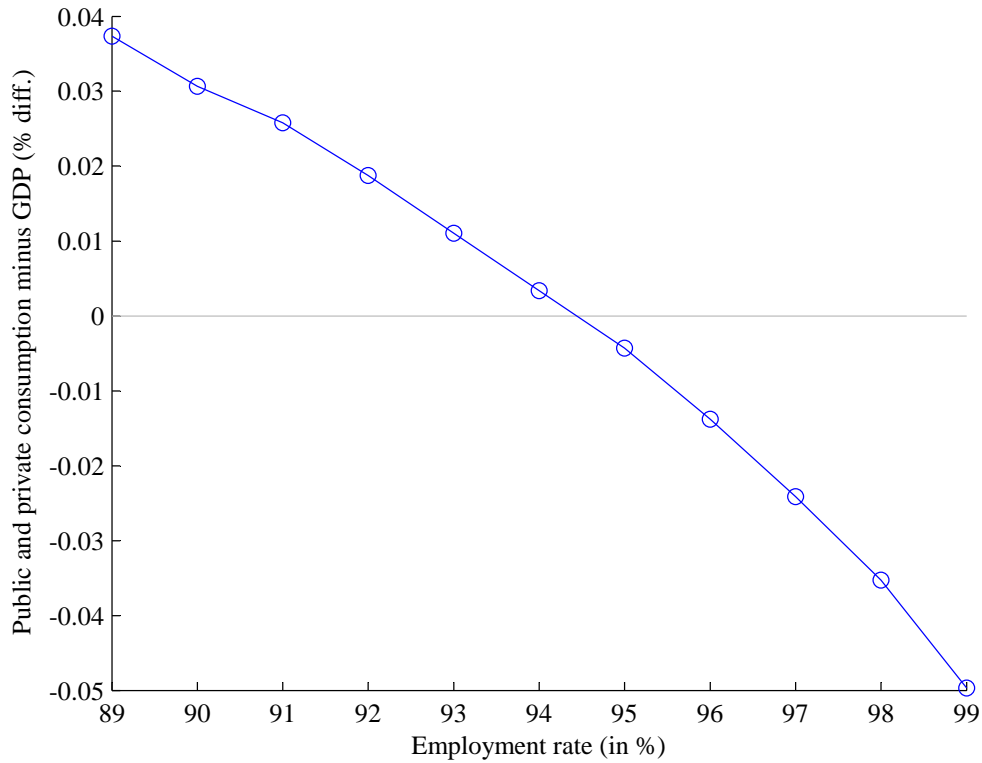
ments such as the employment rate, the bankruptcy rate, the credit card charge-off rate, the ratio of outstanding revolving credit to disposable income, the cross-sectional mean, variance, skewness and kurtosis of balance-to-income ratios, the cross-sectional mean, variance, skewness and kurtosis of credit card interest rates, the cross-sectional correlation between balance-to-income ratios and credit card interest rates, and the average recovery rate on defaulted loans.

In total, I use 18 cross-sectional and aggregate moments to compute the method-of-moment objective function which maps a set of 9 parameter estimates to the sum of squared differences (percentage deviations) between actual and simulated moments. Each function evaluation requires solving the model steady state. In particular, given a set of parameters and a guessed steady state employment rate  $\pi^*$ , I solve the household's optimization problem and then use her optimal policies to simulate the economy until the household net worth distribution becomes stationary. Given this distribution, I then compute the steady-state aggregate consumption (public and private)  $C^*(\pi^*) + G^*(\pi^*)$  and compare it with aggregate production  $Y^* \equiv \pi^*$ . If they differ, I re-start the process with a new guess for  $\pi^*$ . The steady-state equilibrium can be viewed as the solution to the fixed point problem  $C^*(\pi^*) + G^*(\pi^*) - \pi^* = 0$  (an excess demand function set to zero) which I find with a simple bisection method. Figure 1.2 plots such function. The method-of-moment algorithm search for different sets of parameters in parallel with a genetic algorithm, and each set of parameters requires multiple rounds of value function iterations and non-stochastic simulations to find an associated steady-state equilibrium.

### 1.3.5 Results

Table 1.2 presents the estimated parameter values.

The first group of parameters  $\langle \varrho, \tau, \delta \rangle$  have direct counterparts in the data and thus set accordingly. In particular, the replacement ratio is set to  $\varrho = 40\%$  in accordance with the [UI Replacement Rates Report \(US Department of Labor\)](#). The proportional tax rate on output corresponds to the tax revenue of the government (at all levels) expressed as a percentage of GDP, which is approximately  $\tau = 25\%$  according to the [Revenue Statistics](#)



**Figure 1.2: Steady-State Excess Demand in the Goods Market.**

For each candidate solution  $\pi_s$ , I solve for the household's optimal policies which I then use to simulate the steady-state economy until I find a stationary household distribution  $\mu^*$ . Given  $\mu^*$ , I then compute the employment rate  $\pi_d$  that would be needed to satisfy all the public and private consumption implied by  $\pi_s$ . The excess demand is computed as  $\pi_d - \pi_s = f(\pi_s)$  for each candidate solution  $\pi_s$  (x-axis). The y-axis denotes the excess demand in relative terms  $100 * (\pi_d - \pi_s) / \pi_s$ .

(OECD database). The intermediary's subjective discount rate equals the inverse of the risk-free rate  $\delta = \frac{1}{1+r_f}$ . I interpret the return on deposit / saving accounts as the real effective Fed funds rate and I set  $\delta = 0.9985$  accordingly to match an annualized rate of  $\frac{1}{0.9985^4} - 1 = 0.60\%$  as observed over the period 1995–2013.<sup>17</sup>

The rest of the parameters are the solution to the over-identified moment matching procedure. The estimated parameters have all plausible values, consistent with various papers in the literature. The steady-state value of the intermediaries' discount spread  $\mathbb{E}[\phi]$  is 1.7% (annualized) which is small compared to the overall credit card spread (11.20%). If such a spread is interpreted as pure transaction costs, it is also consistent

<sup>17</sup>For consistency across data sets, I look at the same time interval as in the SCF, that is 1995–2013

**Table 1.2: Model Parameter Values**

This table summarizes the model parameter values. The first group of parameters has clear counterparts in the data and are set accordingly. The second and third group of parameters are estimated in steady state by minimizing the squared percentage deviation of 18 aggregate and cross-sectional simulated moments from their data counterparts. The model frequency is quarterly.

Parameters directly determined	Symbol	Value	Target	
Intermediaries' discount factor	$\delta$	0.9985	Fed funds rate	
Unemployment replacement ratio	$\varrho$	40%	Replacement rate	
Income tax rate	$\tau$	25%	Tax / GDP	
Parameters estimated unconditionally	Symbol	Value		
Coefficient of relative risk aversion	$\gamma$	2.7970		
Volatility of idio. productivity shock	$\sigma_z$	0.3127		
Intermediaries' spread (annual, %)	$\mathbb{E}[\phi]$	1.7495		
			Household type	
Parameters estimated by household type	Symbol	Normal	Low $\beta$	Low $\psi$
Household's subjective discount factor	$\beta$	0.9934	0.8387	0.9934
Elasticity of intertemporal substitution	$\psi$	1.5494	1.5494	0.6116
Household type's probability mass		0.9502	0.0351	0.0147

with the relatively small value of 4% used by [Livshits et al. \(2007\)](#). The implied annual idiosyncratic volatility of the labor productivity shock is 0.18 ( $\sigma_z = 0.31$ ), which is in the range of what [Storesletten et al. \(2004\)](#) estimates:  $\sqrt{0.0630} = 0.2510$ . The coefficient of relative risk aversion is 2.80 while the EIS is 1.55 (for medium-type households). These values are in the range of what is commonly used in the long-run asset pricing literature: around 5 for risk aversion and 1.5 for EIS (e.g. [Bansal and Yaron \(2004\)](#), [Kaltenbrunner and Lochstoer \(2010\)](#)). Furthermore, the subjective discount factor (for medium-type households) is estimated to be 0.9934, a value close what is typically used in macroeconomic models calibrated at the quarterly frequency. This also implies that households are slightly more impatient than the intermediaries ( $\delta = 0.9985$ ).

Turning to preference heterogeneity, the moment-matching procedure implies that the model economy is populated by a large group of medium-type households (95%), and two small groups of high- and low-type households (1.5% and 3.5% respectively). The high-type households have a low elasticity of intertemporal substitution (EIS), which is estimated to be lower than one as opposed to the other households whose EIS is greater

than one. The low-type households are substantially more impatient than the other households (0.8387 compared to 0.934).

Table 1.3 compares the simulated moments with the actual ones observed in the data.

Overall, the model does well in replicating the main aggregate and cross-sectional moments. In the cross section of balance-to-income ratios, the median is low while the mean is high, both in the model and in the data (4.11 and 8.48 compared to 5.53 and 8.12). Also, all the higher order moments are large in the model and in the data: standard deviation (8.87 *versus* 11.12), skewness (2.38 and 2.25), and kurtosis (9.67 and 8.60). The model is also successful in matching the cross-sectional moments of interest rates: a high mean and standard deviation (11.79 and 6.69 compared to 11.79 and 6.03), and a low skewness and kurtosis close to the Normal law (0.29 and 2.31 *versus* 0.10 and 2.35). The model also replicates the apparent disconnect between interest rates and balance-to-income ratios (cross-sectional correlation of 0 in the data, and  $-0.15$  in the model). At the aggregate level, the model matches well the credit card interest spread (11.20 in the model and 11.26 in the data), the unemployment rate (5.35 in the model and 6.01 in the data), and the bankruptcy rate (0.50 and 0.42), but tends to overstate the ratio of total outstanding revolving credit to aggregate disposable income (13.86 compared to 8.38).

The two main shortcomings of the model relate to the recovery rate on credit card loans which is too high, and the charge-off rate which is too low. The model performance along these dimensions is tied to the garnishment rule since a lower garnishment would decrease the recovery rate and increase the charge-offs. The garnishment rule was modeled as  $\Upsilon_{it} - \underline{\Upsilon}$ , with the restriction that  $\underline{\Upsilon} \leq (1 - \tau)\varrho\underline{z}$ , where  $\underline{z}$  is the lower bound of the truncated lognormal distribution of idiosyncratic labor productivity shocks. Such assumption made the model tractable by collapsing the household's state space into net worth only, rather than keeping track of income and debt separately. I conjecture that including  $\underline{z}$  in the set of parameters to be estimated would improve the model fit with this respect. Instead, in the current estimation procedure,  $\underline{z}$  is arbitrarily fixed at a

**Table 1.3: Simulated and Actual Moments in Steady State**

This table compares the moments generated by the model with the actual U.S. data. The cross-sectional moments are computed from the triennial Survey of Consumer Finances over the sample period 1995–2013. For consistency, the aggregate moments refer to the same sample period 1995–2013. The model statistics are averaged across 50 simulations of 30,000 households each (comparable to the average number of respondents in the Survey of Consumer Finance). The estimation procedure is over-identified: 9 parameters are pinned down by minimizing the squared percentage deviation of 18 aggregate and cross-sectional simulated moments from their U.S. data counterparts.

Moments (all annualized, expressed in %)	Data	Model
Credit card interest spread	11.26	11.20
Bankruptcy rate (non-business, Chapter 7)	0.42	0.50
Credit card charge-off rate	5.28	2.65
Revolving credit as % of disposable income	8.38	13.86
Unemployment rate	6.01	5.35
Cross-sectional moments of balance-to-income ratios <sup>a</sup>		
– median	4.11	5.53
– mean	8.48	8.12
– standard deviation	11.12	8.87
– skewness	2.25	2.38
– kurtosis	8.60	9.67
Total balances as % of aggregate annual income <sup>b</sup>	0.29	0.26
Cross-sectional moments of credit card interest rates <sup>c</sup>		
– mean		
– weighted by balances after last payment	11.53	10.84
– unweighted	11.79	11.79
– standard deviation	6.03	6.69
– skewness	0.10	0.29
– kurtosis	2.35	2.31
Cross-sectional correlation between balance & interest rate	0.00	–0.15
Recovery rate on defaulted loans <sup>d</sup>	≤ 5	24.46

<sup>a</sup>Balances after last payment computed as % of annual disposable income.

<sup>b</sup>Total sum of balances after last payment expressed as % of total sum of disposable income.

<sup>c</sup>Excludes convenience users who do not have balances left after having made their last payment.

<sup>d</sup>Furletti (2003) reports recovery rates on defaulted credit card loans of \$0.01 per \$100 debt for bankruptcy filings under chapter 7 and as much as \$5 per \$100 debt for non-bankrupt cases.



small value and  $\underline{Y}$  is set at its maximum possible value.

### 1.3.6 Counterfactual Results without Preference Heterogeneity

To check the robustness of all these estimation results, I re-estimate the model with no preference heterogeneity. Table 1.4 presents the estimated parameter values and the simulated moments obtained from this alternative estimation procedure. Notice in particular that in general all the newly estimated parameters have values comparable to their benchmark counterparts, except for the intermediaries' discount spread that is considerably understated (1.7495% in full estimation with preference heterogeneity compared to 0.0450% in alternative estimation without preference heterogeneity).

Overall, the estimation procedure that does not allow for preference heterogeneity tends to understate the cross-sectional dispersions in leverage and interest rates. It is particularly true for the former: the standard deviation is four times lower in the alternative estimation compared to the benchmark one, and the mean is two times lower. Furthermore, not allowing for preference heterogeneity has counter-factual implications for the relationship between credit card interest rates and balance-to-income ratios. While in the data and in the benchmark case they are weakly correlated (0.00 and  $-0.15$  respectively), their correlation is strong in the no-preference-heterogeneity case (0.96).

### 1.3.7 Understanding the Role of Preference Heterogeneity

A key result of the alternative estimation procedure is that heterogeneity in  $\beta$  and  $\psi$  plays an important role in explaining the large dispersion in balance-to-income ratios. Figure 1.3 illustrates this point by plotting the household distribution of net worth, balance-to-income ratios and non-convenience interest rates by preference type: “high” (high incentive to smooth; low  $\psi$ ), “medium” (normal), “low” (impatient; low  $\beta$ ).

What I want to emphasize here is that the  $\beta$ -heterogeneity and the  $\psi$ -heterogeneity have *different* implications at the cross-sectional level, and complement each other in generating the large dispersion in balance-to-income ratios *and* credit card interest rates. For instance, consider the case of heterogeneity in the elasticity of intertemporal

**Table 1.4: Alternative Steady-State Estimation without Preference Heterogeneity**

This table compares the moments generated by the model with the actual U.S. data. The cross-sectional moments are computed from the triennial Survey of Consumer Finances over the sample period 1995–2013. For consistency, the aggregate moments refer to the same sample period 1995–2013. The model statistics are averaged across 50 simulations of 30,000 households each (comparable to the average number of respondents in the Survey of Consumer Finance). The estimation procedure is over-identified: 5 parameters are pinned down by minimizing the squared percentage deviation of 17 aggregate and cross-sectional simulated moments from their U.S. data counterparts.

Estimated parameters	Symbol	Value
Subjective discount factor	$\beta$	0.9892
Elasticity of intertemporal substitution (EIS)	$\psi$	1.4377
Coefficient of relative risk aversion	$\gamma$	3.5530
Volatility of idiosyncratic productivity shock	$\sigma_z$	0.3022
Intermediaries' discount spread (annual, %)	$\mathbb{E}[\phi]$	0.0450
<hr/>		
Moments (all annualized, expressed in %)	Data	Model
Credit card interest spread	11.26	8.46
Bankruptcy rate (non-business, Chapter 7)	0.42	0.50
Credit card charge-off rate	5.28	1.33
Revolving credit as % of disposable income	8.38	5.88
Unemployment rate	6.01	5.06
Cross section of balance-to-income ratios <sup>a</sup>		
– median	4.11	3.27
– mean	8.48	4.10
– standard deviation	11.12	3.27
– skewness	2.25	0.86
– kurtosis	8.60	2.92
Total balances as % of aggregate annual income <sup>b</sup>	0.29	0.12
Cross section of credit card interest rates <sup>c</sup>		
– mean		
– weighted by balances after last payment	11.53	12.61
– unweighted	11.79	9.05
– standard deviation	6.03	4.59
– skewness	0.10	0.71
– kurtosis	2.35	2.39
Cross-sectional correlation between balance & interest rate	0.00	0.96

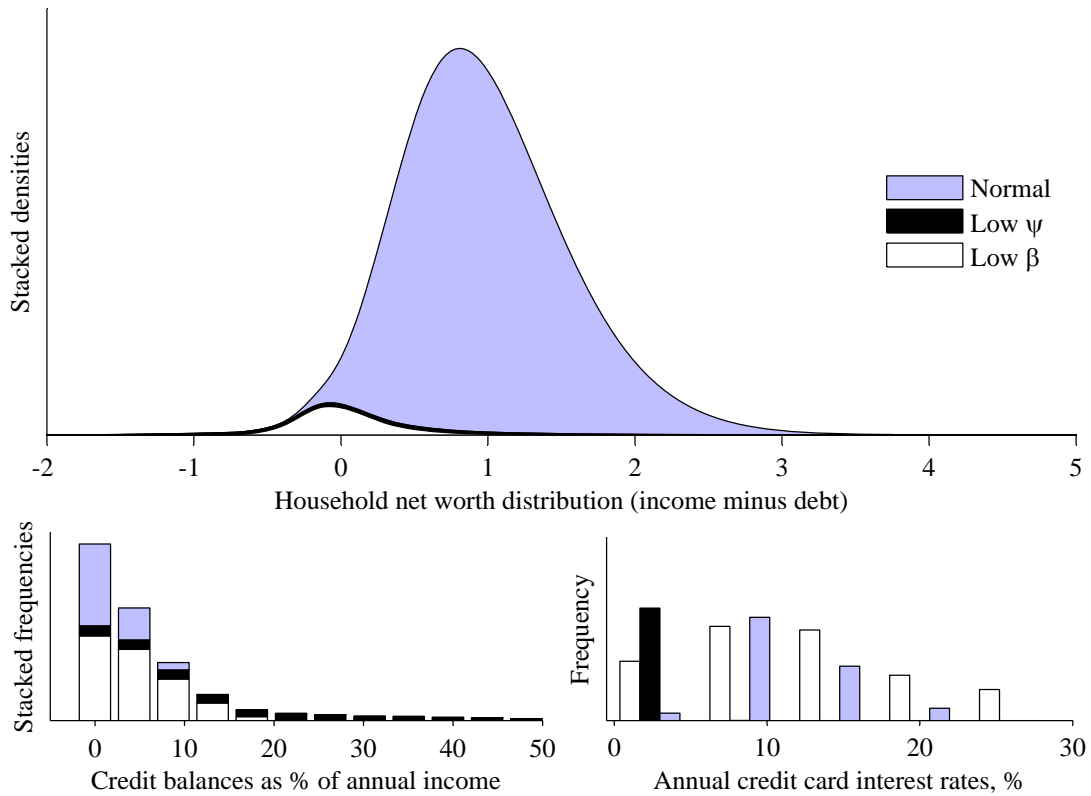
<sup>a</sup>Balances after last payment computed as % of annual disposable income.

<sup>b</sup>Total sum of balances after last payment expressed as % of total sum of disposable income.

<sup>c</sup>Excludes convenience users who do not have balances left after having made their last payment.

substitution  $\psi$ . By having low- $\psi$  households who dislike default, the model generates a group of “prime” borrowers who can borrow a lot at a discounted interest rate. If there were only one dimension of heterogeneity, interest rates would be directly a function of leverage and therefore as leverage rises, the cost of borrowing necessarily increases as well, which discourages households to lever up and yields a limited dispersion in leverage.

On the other hand, the impatient households contribute to the high mean of credit card interest rate. In the data, such mean is high because some households are eager to borrow even at high costs. This could be due to liquidity shocks or some other factors. A parsimonious way to capture this feature of the data is by considering that a small



**Figure 1.3: Model-based Stationary Distributions by Household Type**

The histograms of net worth and balance-to-income ratios represent cumulative frequencies. The height of a bar represents the total frequency among the three types “normal”, “low  $\psi$ ”, and “low  $\beta$ ”; not simply the frequency of the “normal” households. The third histogram represents the non-cumulative frequencies of interest rates by household type. The smooth net worth distribution is obtained by non-stochastic simulation, explained in details in the computational appendix. The other two distributions – balances and interest rates – are obtained by Monte Carlo simulation with one million households.

group of households are very impatient. Such group of households counter-balance the effects of the low- $\psi$  households who tend to decrease the average interest rate.

## 1.4 On the Cyclical Effects of Consumer Finance

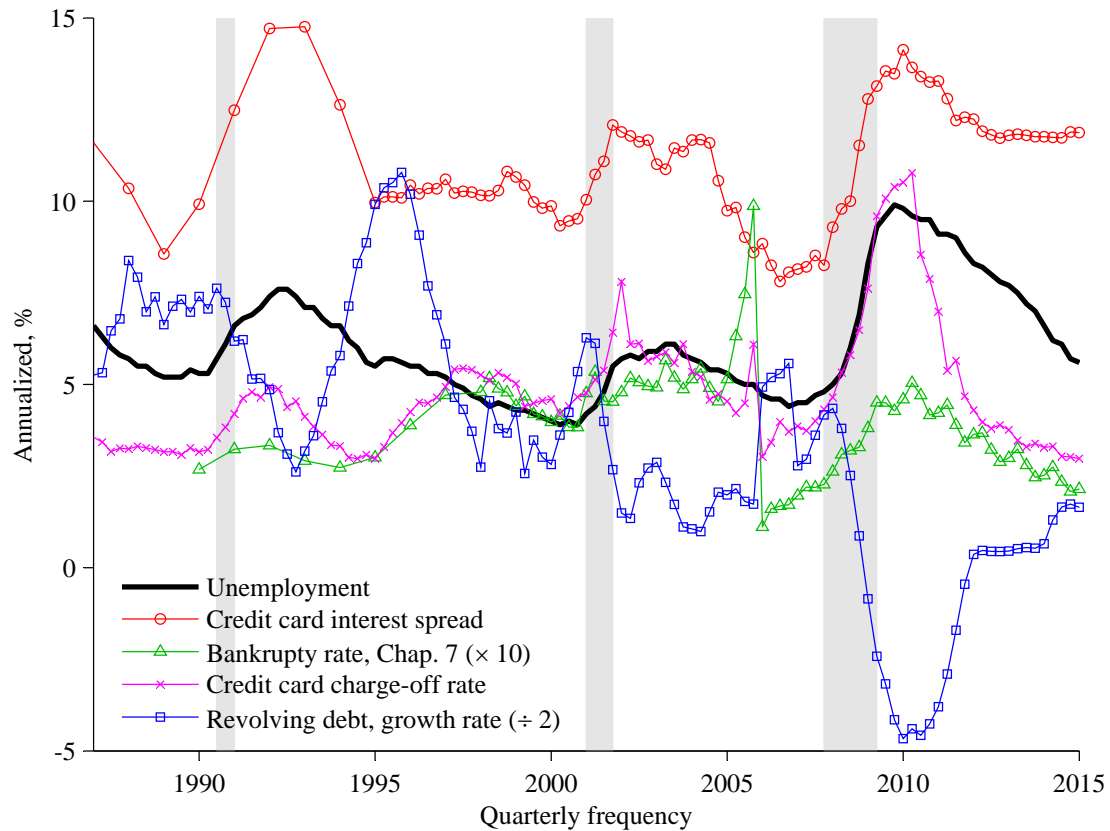
This section investigates the cyclical nature of consumer finance and its macroeconomic implications, both at the aggregate and cross-sectional levels. The main result of this section is that the model can explain the joint dynamics of unemployment and consumer finance observed over the business cycle since the early nineties.

### 1.4.1 Stylized Facts of the Cyclical Nature of Consumer Finance

I begin my analysis by documenting how some key financial variables related to unsecured consumer credit markets evolve over the business cycle. My focus is on the credit card interest spread, the bankruptcy rate (non-business Chapter 7), the growth rate of outstanding revolving credit, and the growth rate of its ratio to aggregate disposable income. Figure 1.4 plots the main variables of interest.

A striking pattern in the data is that consumer finance displays large movements over the business cycle, and such movements are not just specific to the recessionary episode associated with the Financial Crisis. Thus, a successful business cycle theory of consumer finance should not solely focus on specific factors of the Financial Crisis (e.g. banking and housing collapses) but also be general enough to encompass the cycles in the nineties and the early 2000s'. Notice in particular how movements in credit card spreads follow the unemployment rate, and how the onset of each recession (1990, 2001, 2007) is marked by an abrupt drop in the growth rate of revolving credit and a spike in both bankruptcy and charge-off rates. Interestingly, the last two variables display a similar cyclical pattern, suggesting that bankruptcies are a good proxy for losses in credit card markets.

An important feature of the data is that bankruptcies and charge-offs are less correlated with unemployment as opposed to credit card spread. A noticeable example is the spike in bankruptcy filings around 2005 which reflects the enactment of the Bankruptcy



**Figure 1.4: Unemployment and Consumer Finance Statistics over the U.S. Business Cycle**

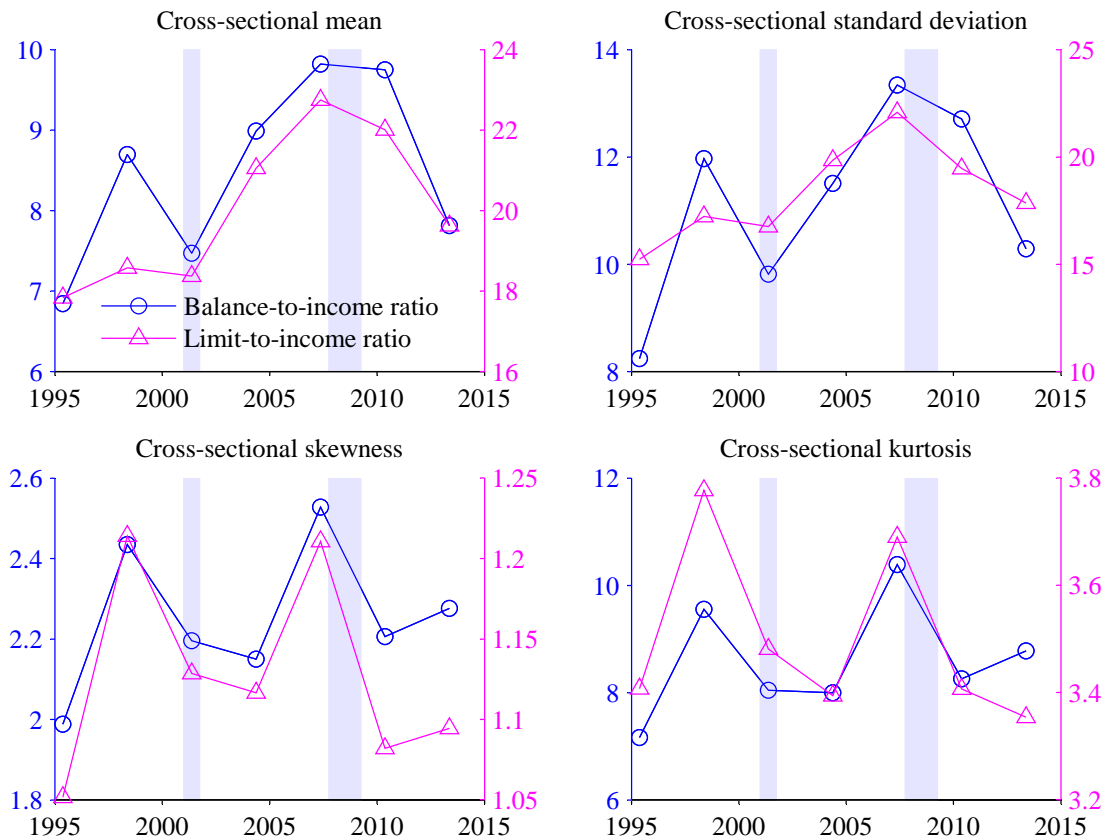
The unemployment rate time series is from the [BLS](#). The annualized credit card interest rate spread is the difference between the average credit card interest rate and the effective Federal Funds rate (both from Board of Governor under [Report on the Profitability of Credit Card Operations](#), and [H.15 Selected Interest Rates](#) respectively). The bankruptcy rate is total non-business bankruptcy filings under Chapter 7 ([U.S. Bankruptcy Courts](#)) over total U.S. population (civilian, non-institutional, of age 16 and older). The spike in bankruptcy filings around 2005 reflects the Bankruptcy Abuse and Prevention and Consumer Protection Act (BAPCPA) that took effect on October 17, 2005. Total outstanding revolving consumer credit is obtained from [G.19 Consumer Credit \(Board of Governors\)](#) and expressed in year-over-year growth rate. All data are quarterly, except for credit card spread prior to 1995 and bankruptcy rate prior to 1998 for which information was available only at annual frequency. Bankruptcy rates and revolving credit year-over-year growth rates are scaled for sake of comparison. Shaded areas correspond to [NBER](#)-dated recessions.

Abuse and Prevention and Consumer Protection Act (BAPCPA) on October 17, 2005. Such spike does not translate into higher unemployment or higher credit spread. An important distinction to make here is between *realized* default and *expected* default. In the data, consumer default is extremely frequent (less than half a percent of the population per year), and thus cannot have large quantitative effects by itself. However, credit risk affects *all* the borrowers through high interest rates and low credit limits.

Thus, unanticipated bankruptcy shocks are not likely to have large effects.

Another fascinating feature of the U.S. data is that revolving credit is procyclical.<sup>18</sup> This is counter-intuitive to the predictions of most standard models of inter-temporal consumption choices. In such models, households optimally borrow during bad times in order to smooth their consumption, thus credit is *counter*-cyclical. This feature of the data is robust to the level of aggregation. For instance, in Figure 1.5, I report the cross-sectional mean and higher moments of households' balance-to-income ratios from

<sup>18</sup>Additionally, remark that the growth rate in revolving credit displays a medium-term trend. Since the model is stationary and abstracts from medium- to long-term structural factors, I will filter this time series with a four-year centered moving average in both the U.S. data and the simulated time series when I will evaluate the business cycle performance of the model.



**Figure 1.5: Cross sectional moments of credit balances and limits as % of annual income** Survey of Consumer Finance, 1995–2013 (triennial). Balance-to-income ratios refer to balances after last payment as a fraction of annual disposable income (net of transfers). Balances after last payment refer to the amount of credit card debt the household still owes to the bank at the end of a billing cycle. The limit refers to the maximum amount the household can borrow on his most used credit card. Outliers at the 99<sup>th</sup> (90<sup>th</sup>) percentile are discarded to compute the cross-sectional moments of balance-to-income ratios (limit-to-income ratios). Shaded areas indicate NBER recessions.

the Survey of Consumer Finance over the sample period 1995–2013. The balance refers to credit card debt that is not fully repaid within a billing cycle, while the income refers to annual salary and transfer payments.

Although the data is triennial, the general patterns are suggestive of a pro-cyclical behavior. In particular, notice how the cross-sectional mean falls in 2001 and during the years 2010 and 2013. A similar pattern can be observed in the dispersion of balance-to-income ratios: volatility, skewness, and kurtosis. Such cross-sectional movements are closely mirrored by cross-sectional movements in limit-to-income ratios, suggesting the key role of credit constraints in shaping debt dynamics.

### 1.4.2 Calibrating the Aggregate Financial Shock

I consider financial shocks as the main source of business cycle fluctuations. In particular, recall that in the model, intermediaries are more impatient when lending money to households compared to when borrowing from them (i.e. holding deposits), and such spread between their discount factors is subject to stochastic perturbations. To infer the dynamics of this spread, I discipline the model to reproduce the persistence and volatility of the credit card interest rate spread observed in U.S. data. To do so, I model the financial shock as a two-state Markov chain  $\phi \in \{\phi_g, \phi_b\}$  corresponding to good times and bad times (alternatively, expansions and recessions). This Markov chain has transition probabilities defined as:  $\mathbf{P} = [p_{gg}, p_{gb}; p_{bg}, p_{bb}]$ ,  $p_{gg} = 1 - p_{gb}$  and  $p_{bg} = 1 - p_{bb}$ . I choose  $p_{gb} = (p_{bg}p_b)/(1 - p_b)$  so that the unconditional probability of a recession in the model is consistent with the frequency of NBER recessions over the sample period 1950–2015, that is  $p_b = 0.0474$ . Then, I choose  $p_{bb} = 0.50$  to reproduce the high first-order auto-correlation observed in credit card interest rate spread. Finally, I set the annual discount spread to be 10% above its steady state value during bad times ( $\phi_b$ ), and solve for  $\phi_g$  such that the stochastic steady state of  $\phi$  equals the estimated value in steady state,  $\mathbb{E}[\phi] = 0.9957$  (or equivalently an annualized spread of 1.7495%). This procedure gives  $\phi_b = 0.9953$  in bad times, and  $\phi_g = 0.9958$  in good times (or equivalently annual spreads of 1.9245% and 1.7140% respectively).

**Table 1.5: OLS Coefficients of the Conditional Employment Forecasting Rules**

The OLS regression is  $\log \pi' = \alpha_0 + \alpha_1 d' + \alpha_2 \log \pi + \alpha_3 d' \log \pi + \epsilon$ . In the model, the dummy  $d$  takes the value 1 if  $\phi = \phi_b$  and 0 otherwise ( $\phi = \phi_g$ ). In the U.S. data, the dummy  $d$  takes the value 1 if the quarter is a recession as defined by the [NBER](#), and 0 otherwise (quarter is an expansion). The U.S. data covers the sample 1948:Q1–2015:Q2. Unemployment data is from [BLS](#). In the model, the sample covers 1,100 quarters (the first 100 quarters are discarded).

	Coefficient estimates			
	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$
Data	−0.000546	−0.001789	0.968837	0.097310
Model	−0.001114	−0.002667	0.960753	0.079831

### 1.4.3 Approximate Aggregation in the Model and in U.S. Data

Following [Krusell and Smith \(1998\)](#), I solve for an *approximate-aggregation* equilibrium of the model by guessing that households only use current employment rate ( $\pi$ ) to forecast next-quarter employment ( $\pi'$ ) rather than the infinite-dimensional object  $\mu \in \mathcal{M}$  (household distribution). More formally, let  $d \in \{0, 1\}$  be a recession dummy variable that takes the value 1 if  $\phi = \phi_b$  and 0 otherwise. Then, consider log-linear forecasting rules of the form:

$$\log \pi' = \alpha_0 + \alpha_1 d' + \alpha_2 \log \pi + \alpha_3 d' \log \pi + \epsilon, \quad (1.6)$$

where  $\epsilon$  are the smallest possible forecasting errors made by households in equilibrium.

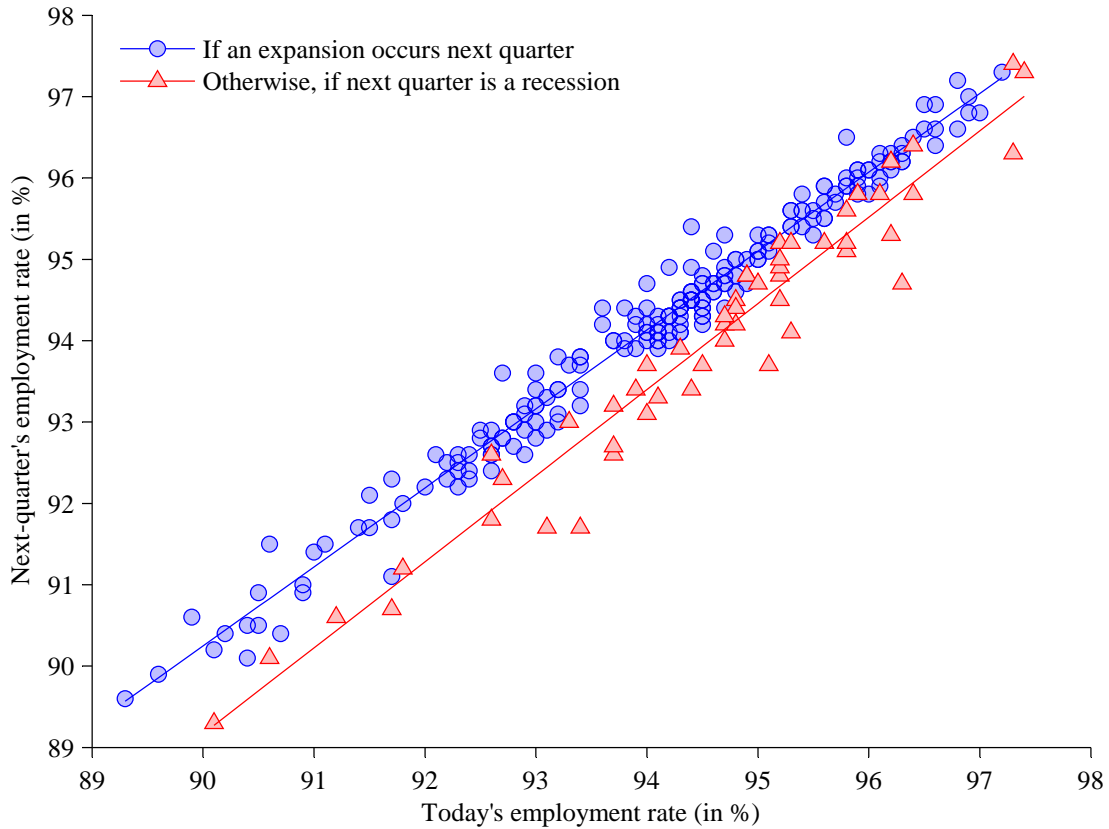
Since households in the model use this forecasting rule to predict employment, it is a pre-requisite of the model's success that a similar rule-of-thumb could be used in reality as well. The U.S. data-based forecasting rules are visualized in [Figure 1.6](#) (using postwar 1948:Q1–2015:Q1). Additionally, [Table 1.5](#) compares the OLS parameter estimates of [\(1.6\)](#) in the model and in U.S. data. It is interesting to notice that the model-based regression coefficients are in line with the ones based on the data, and thus suggests that approximate aggregation is a reasonable feature of the data.

To find the model equilibrium forecasting rules, I could solve for the unknown coefficients  $\{\alpha_0, \dots, \alpha_3\}$  in a guess-and-verify fashion. In particular, I would simulate an economy with a given set  $\{\alpha_0^*, \dots, \alpha_3^*\}$  and make sure that markets clear at each point in time. Then, I would run an OLS regression and check whether the forecasting errors



$\epsilon$  are small according to some criteria. If not, I would re-start the process with a new set of coefficients  $\{\alpha_0^*, \dots, \alpha_3^*\}$  obtained from OLS. Though standard in the literature, this process does not converge in my model because of the high non linearities that default and unemployment generate.

Instead, I consider a dual approach. Instead of imposing market clearing at each point in time, and checking whether the forecast errors are small, I assume that the economy evolves exactly as predicted by households (thus, there are no forecast errors), and check whether the goods market clear (the magnitude of shortages and excesses). Thus, I do not impose market-clearing ex-interim but only check it ex-post. With this approach, an equilibrium is a set of guessed coefficients  $\{\alpha_0^*, \dots, \alpha_3^*\}$  such that the

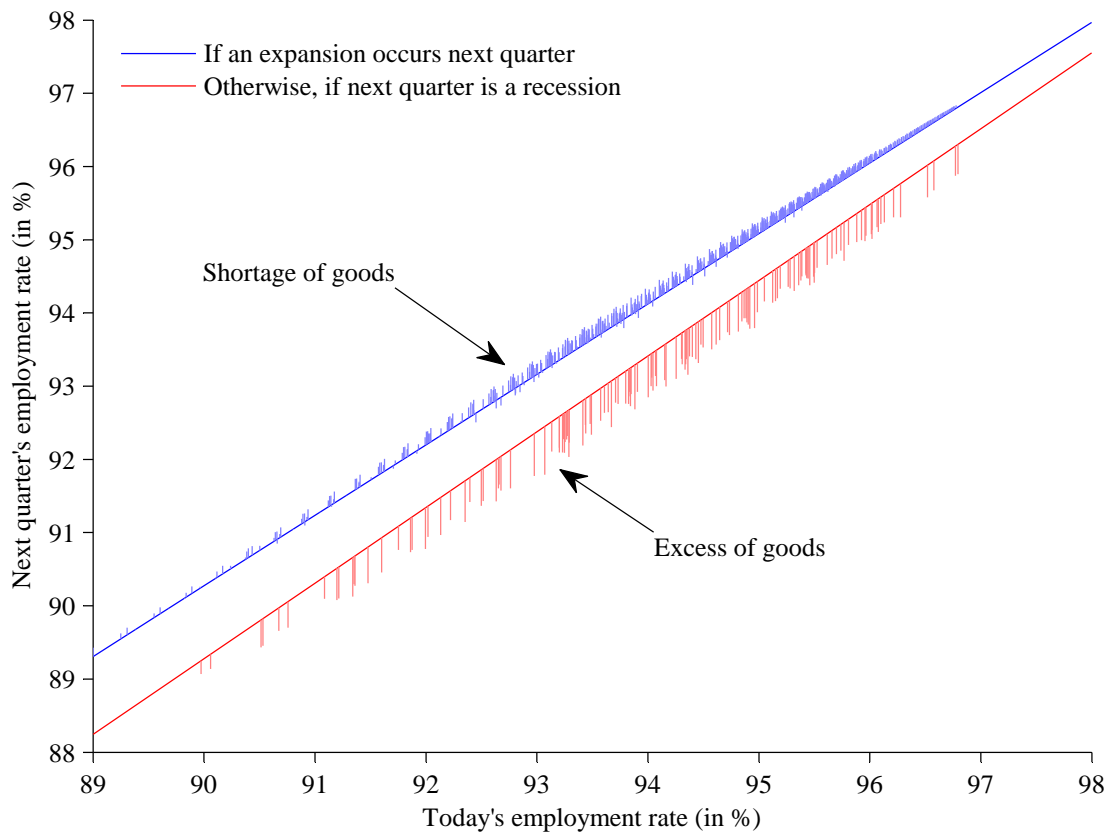


**Figure 1.6: Employment Forecasting Rules in U.S. Data**

U.S. data, 1948:Q1–2015:Q2 (from [BLS](#)). The data is sorted in two groups “next quarter is an expansion” and “next quarter is a recession” according to the [NBER](#) dates. The forecasting rules are from an Ordinary Least Squares log-linear model:  $\log \pi' = -0.000546 + -0.001789d' + 0.968837 \log \pi + 0.097310d' \log \pi + \epsilon$ , where  $d \in \{0, 1\}$  are recession dummies, and  $\pi$  corresponds to the employment rate.

excesses and shortages in the goods market are the *smallest* possible. Such excesses and shortages are simply given by the difference between the guessed employment rate  $\pi$  (production-based) and the one that would have been required to exactly satisfy all the consumption needs of the households and the government (consumption-based).

Figure 1.7 plots the employment forecasting rules (production path) against total actual consumption (expressed in employment units). Each vertical segment represents the magnitude of either a shortage (if above the production path) or an excess (if below). In equilibrium, the average excesses (shortages) represent only 0.1673% of the total quantities produced, while the maximum difference is 0.7135% and occurs mostly during recessions. Such errors are well within the range of the noise observed in the data,



**Figure 1.7: Excesses and Shortages of Goods in the Stochastic General Equilibrium.**

The two diagonal lines are the equilibrium production paths,  $\pi'_s = \Gamma(\pi_s, \phi')$ :  $\log \pi'_s = -0.001114 + -0.002667d' + 0.960753 \log \pi_s + 0.079831d' \log \pi_s + \epsilon$ . Each vertical line represent the distance between what is produced ( $\pi_s$ ) and the employment rate that would be required to satisfy all the public and private consumption ( $\pi_d$ ). The shorter the vertical lines, the small the excesses or shortages in the goods market, and hence the closer is the economy to market clearing.

**Table 1.6: Unemployment and Consumer Finance Statistics Over the Business Cycle**

U.S. data, 1990:Q1–2015:Q2. Model statistics refer to a sample of one million households simulated over 1,100 quarters, and the first 100 quarters are discarded. The model frequency is quarterly. The growth rates of revolving credit and unsecured leverage display a medium-term cycle that is filtered with a four-year centered moving average. For consistency, a similar filtering is applied to the model-generated time series of revolving credit growth and unsecured leverage growth. As expected, such filtering on stationary time series has virtually no quantitative effect. The bankruptcy rate (Chapter 7 filings divided by civilian population) and the credit card interest rate spread (average credit card interest rate minus effective Fed funds rate) are annualized.

Variables (all in %)	Mean		Volatility		First-order auto-corr.		Corr. with unemployment	
	Data	Model	Data	Model	Data	Model	Data	Model
Unemployment rate	6.11	5.82	1.57	1.63	0.98	0.99	1.00	1.00
Credit card spread	11.28	10.61	1.69	1.12	0.96	0.91	0.78	0.95
Bankruptcy rate	0.38	0.52	0.13	0.07	0.71	0.31	−0.04	0.34
Charge-off rate	4.87	2.78	1.69	0.38	0.93	0.33	0.51	0.47
Revolving credit <sup>a</sup>	−0.01	0.00	1.09	1.01	0.35	0.24	−0.31	−0.13
Leverage <sup>a</sup>	0.01	0.00	1.32	0.96	0.05	0.10	−0.21	−0.13

<sup>a</sup>Detrended growth rate.

and thus should not dramatically affect the main conclusions of the paper. As models of incomplete markets add more frictions and non-linearities, this approach offers an interesting and simple alternative to the standard methods.

#### 1.4.4 The Joint Dynamics of Consumer Finance and Unemployment

Table 1.6 presents the main business cycle results of the paper.

The credit spread dynamics are relatively well matched. In particular, it is highly volatile though it understates the data (1.12 compared to 1.69). It is also very persistent, the first-order auto-correlation coefficient is 0.96 in the data and 0.91 in the model. Lastly, the credit card spread co-moves strongly with the cycle, especially more in the model than in the data: the correlation between unemployment and the spread is 0.95 and 0.78 respectively.

Under this calibration, the model replicates salient features of the *joint* dynamics of employment and other key financial variables that pertain to unsecured consumer credit markets: growth rate of revolving credit, growth rate of the ratio of revolving credit to aggregate disposable income (which I will refer as unsecured leverage), bankruptcy rate

under Chapter 7, and to some extent the charge-off rate on credit cards.

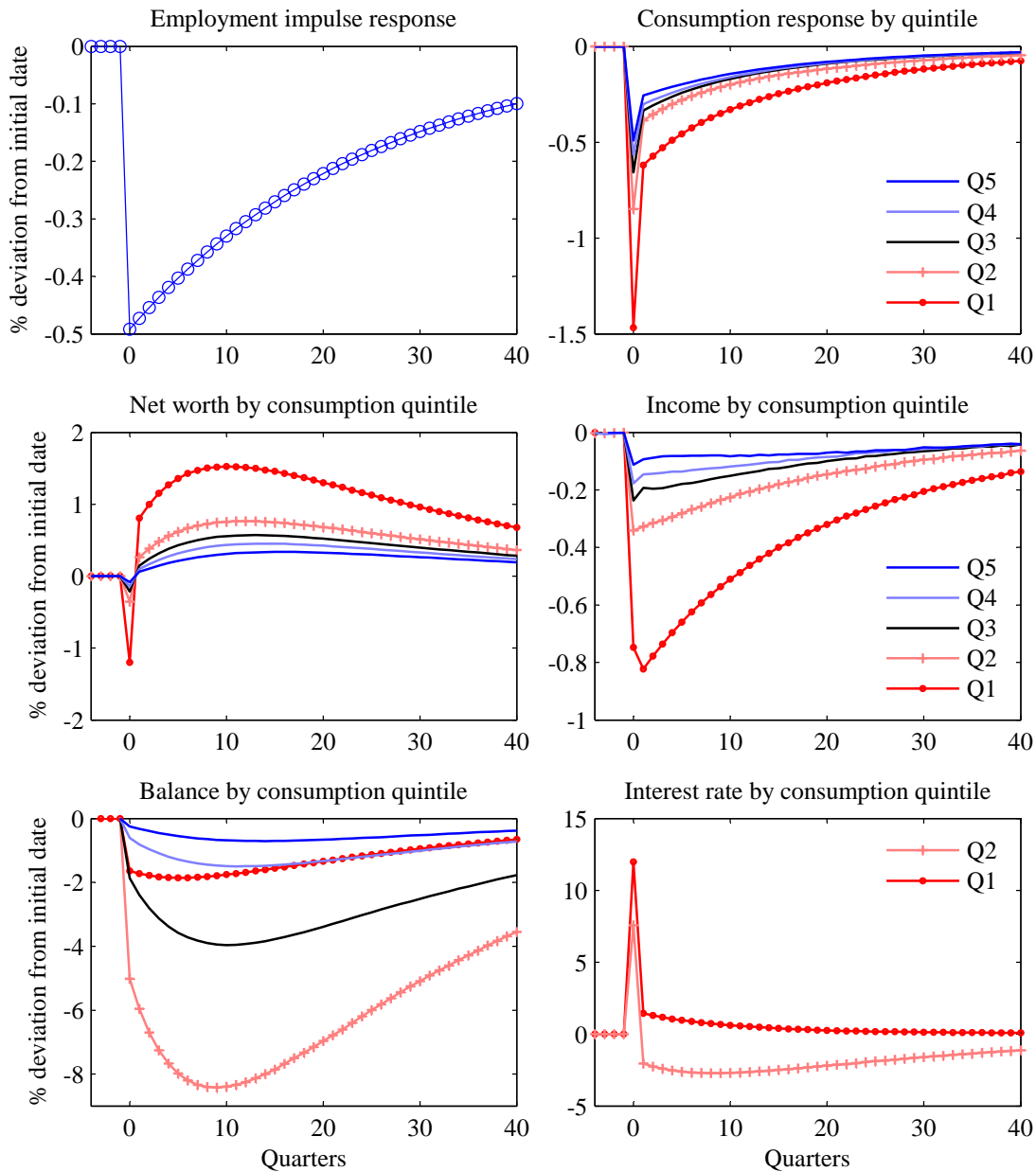
The model reproduces the weak correlation between the charge-off rate and unemployment (0.51 in data, 0.47 in model), and to some extent between the bankruptcy rate and unemployment ( $-0.04$  in data, 0.34 in model). However, the model falls short in explaining the large persistence of such time series (0.71 and 0.93 *versus* 0.31 and 0.33). As a consequence, the model also tends to understate the volatility of bankruptcies and charge-offs (0.07 compared to 0.13, and 0.38 compared 1.69). The low volatility of charge-offs is also due to the fact that in steady state their mean is halved. Adjusting for the mean, the model-based coefficient of variation is 13% compared to 35% in the data. A way of improving the model fit would be to focus on elements that increase the persistence of bankruptcies. For example, I would conjecture that modeling persistent labor income shocks (as opposed to purely transitory in the current model) could help along this dimension, but at the expense of a more demanding numerical method.

Turning to growth rates of revolving credit and unsecured leverage, the model accounts well for all of their dynamic properties. For instance, their volatilities are 1.09 and 1.01 respectively in the data, compared to 1.01 and 0.96 in the model. They are also weakly persistent, especially unsecured leverage more than revolving credit: 0.05 and 0.35 respectively in the data, and 0.10 and 0.24 in the model. Additionally, the model can rationalize the *pro-cyclicality* of revolving credit and unsecured leverage: the contemporaneous correlations between unemployment and these two series are  $-0.31$  and  $-0.21$  in the data, compared to  $-0.13$  and  $-0.13$  in the model.

Overall, the model provides a fair *quantitative* account of the *joint* dynamics of employment and household financial variables.

#### 1.4.5 The Mechanism at Play: From Cross-Sectional to Aggregate

Since in the model, unemployment fluctuations mirror movements in consumption, it is interesting to examine what part of the household distribution drives the business cycle dynamics. To understand the mechanism at play, I simulate an economy that recovers from a one-quarter recession. At each point in time, I sort households by consumption



**Figure 1.8: Impulse Response Functions by Consumption Quintile**

Economic characteristics are obtained by averaging across all the households who belong to the same quintile. The first quintile corresponds to households whose consumption is below the 20<sup>th</sup> percentile, the second quintile to households between the 20<sup>th</sup> and the 40<sup>th</sup> percentile, and so on. The numbers refer to Monte Carlo averages of 5 million households over 500 trials. The simulated economy experiences a one-quarter recession at time  $t = 0$  and then remains on an expansionary path for the remaining periods. At each point in time, households are sorted by consumption quintile and several average economic characteristics are computed.

**Table 1.7: Household Economic Characteristics by Consumption Quintile in Steady State**

Economic characteristics are obtained by averaging across all households who belong to the same quintile. The first quintile correspond to households whose consumption is below the 20<sup>th</sup> percentile, the second quintile to households between the 20<sup>th</sup> and the 40<sup>th</sup> percentile, and so on. The numbers refer to Monte Carlo averages of 5 million households over 500 trials.

Average characteristics	Sorted by consumption quintile				
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
Consumption	0.64	0.71	0.75	0.77	0.82
Net worth	0.28	0.64	0.91	1.21	1.80
Disposable income	0.57	0.68	0.74	0.80	0.89
Old balance	0.29	0.04	-0.16	-0.41	-0.91
New balance	0.36	0.07	-0.16	-0.44	-0.99
Interest rate (annual, %)	2.90	1.95	0.59	0.59	0.59
Default rate (annual, %)	0.88	0.00	0.00	0.00	0.00

quintile and record their economic characteristics such as income, net worth, debt, interest paid, and default rate.

Table 1.7 reports the averaged household characteristics by consumption quintile before the economy enters a recession, while Figure 1.8 shows how the different household groups by consumption quintile fare after the economy experienced a one-quarter recession.

The households in the first two consumption quintiles correspond to borrowers with low income. The borrowers in the first quintiles are particularly indebted, likely to default and face substantial interest rates. On the other hand, households in the third, fourth, and fifth consumption quintile are savers with high income.

The key insight of these *cross-sectional* impulse response functions is that the biggest drop in consumption comes from the first quintile, that is the group of households with low income and high levels of debt. In particular, the drop is almost as twice as large in the first quintile as in any other quintiles.

The second key insight is that even the wealthiest households, those in the first consumption quintile group, slowly build up precautionary savings and consequently hold back on consumption. Notice in particular how their credit balance impulse response (savings) is hump shaped. In opposite, the credit balance impulse response of the most

**Table 1.8: Consumption Insurance by Net Worth Quintile Over the Business Cycle**

I simulate a sample of one million households for 1,100 quarters (the first 100 are discarded). At each point in time, I sort households by net worth quintiles  $j = 1, \dots, 5$ . For each quintile group, I then run the OLS regression:  $\Delta \log C_{ijt} = \lambda_{jt} \Delta \log Y_{ijt} + \epsilon_{ijt}$ , where  $\Delta \log C_{ijt}$  and  $\Delta \log Y_{ijt}$  are log-growth rates of consumption and disposable income respectively, and  $\epsilon_{ijt}$  is an error term. The table below presents the average value of this elasticity across net worth groups over the business cycle, as well as their volatility and correlation with recession dummies  $d_t$ . The recession dummy takes the value 1 if the financial shock is  $\phi = \phi_b$  (bad times), and 0 otherwise.

	Net worth quintile, $j =$				
	1	2	3	4	5
$\mathbb{E}[\lambda_{jt} j]$	0.1303	0.0801	0.0634	0.0514	0.0400
$\text{std}(\lambda_{jt} j)$	0.0047	0.0020	0.0012	0.0008	0.0005
$\text{corr}(\lambda_{jt}, d_t j)$	0.6039	0.3430	0.2442	0.1249	-0.0054

credit-constrained households is monotonic in the aftermath of a recession.

#### 1.4.6 Welfare Implications: Procyclical Consumption Insurance

I complete my analysis of the cross-sectional effects of consumer finance disturbances by measuring the degree of consumption insurance across time periods and households.

There are two key implications of the model that I want to highlight. First, regardless of business cycle fluctuations, there is a large cross-sectional heterogeneity in risk-sharing opportunities. This stems from the fact that the lower the net worth of the household, the costlier it is to obtain new loans. Therefore, households with low or negative net worth fare worse than wealthy households with high net worth. Second, the cross-sectional dispersion in consumption insurance is *time-varying*. This is because during a recession, not only unemployment increases but it is also more difficult to borrow for poor households because of higher credit risk. Thus, households have a lower ability to smooth their income shocks exactly at the same time when shocks are larger and more frequent. In opposite, wealthy households do significantly suffer less during a recession since they smooth consumption by using precautionary savings.

To test these predictions, I simulate an economy subject to aggregate shocks. At each point in time, I sort households by net worth quintile  $j = 1, \dots, 5$ . Then, for each group  $j$ , I run the regression:

$$\Delta \log C_{ijt} = \lambda_{jt} \Delta \log Y_{ijt} + \epsilon_{ijt},$$

where  $\Delta \log C_{ijt}$  and  $\Delta \log Y_{ijt}$  are log-growth rates of consumption and disposable income respectively, and  $\epsilon_{ijt}$  is an error term. The parameter  $\lambda_{jt}$  has the interpretation of the income elasticity of consumption, and measures the degree of consumption insurance. As  $\lambda_{jt} \rightarrow 0$ , the households who belong to the  $j$ -th net worth quintile group at time  $t$  are close to full insurance. Table 1.8 presents the results of this simulation.

The first row of the table reports the average income elasticity of consumption by net worth quintile over the business cycle. The cross-sectional dispersion in consumption is *large*: the top 80% has an elasticity more than three times smaller than the bottom 20% (0.04 compared to 0.13). This means that a 10% fall in income results in only 0.4% consumption fall for the wealthiest households. This stems from the fact that they have large stocks of savings that they can use to smooth consumption. In opposite, the poorest households have low or negative net worth and thus can only borrow to smooth income shocks. A 10% fall in income translates in a 1.3% fall in consumption.

The second row of the table reports the volatility of the income elasticity of consumption across time periods. Notice that not only the degree of consumption insurance is low among the poorest but it is also the most volatile and countercyclical. Indeed, the third row of the table reports the correlation between  $\lambda_{jt}$  and the recession dummy  $d_t$ . The recession dummy takes the value 1 if the financial shock is  $\phi = \phi_b$  (bad times), and 0 otherwise. The volatility of the income elasticity of consumption is almost 10 times smaller among the top 80% compared to the bottom 20%. Furthermore, the fluctuations are acyclical among the richest households: the degree of consumption insurance among the richest households is not correlated with the amount of labor risk they face. In opposite, the degree of consumption insurance is strongly correlated with the cycle among the poorest households (correlation of 0.60 with the recession dummy  $d_t$ ). Thus, the model generates *pro-cyclical* consumption insurance among the poorest households.

### 1.4.7 Discussion

The take away from these results is that in presence of credit frictions (market incompleteness, financial intermediation, and risk of default), the actions of borrowers and



savers do no longer wash out at the aggregate level. The key reason put forth in my paper is that what is saved by some households is not one-to-one redistributed to other households in the form of loans. For example, think of the extreme case when some households save *but* credit risk is so high that not a single borrower can obtain a loan. This inefficiency in credit markets has real consequences. Since savers do not consume all their income, and credit-constrained borrowers cannot consume as much as they would want to, the overall consumption is lower than what would be required to reach full employment.

Hence, this paper provides an unemployment theory purely based on market incompleteness, financial intermediation, and risk of default, rather than traditional frictions like price stickiness or search.

This business cycle theory of consumer finance and unemployment offers new perspectives on the effects of government stimulus policies. Indeed, by viewing unemployment as a symptom of limited consumption insurance as evidenced in the section above, we can think of stimulus policies that aim at improving risk-sharing among households. Through the lens of the model, this means that for example an unemployment insurance program, though having negative effects on the labor supply because of search effects, has also positive effects through improving risk-sharing. Similarly, a household bailout policy could be beneficial since it would ease the pressure on poor households who have a low net worth. More broadly, any risk-related policy such as a medical insurance policy or regulations in the credit card markets could prove to be useful in the context of reducing unemployment. Of course, each policy would require a careful cost-benefit analysis in a well structurally estimated model. By proposing a simple model that links unemployment to the degree of consumer credit frictions, this paper makes a first step toward this goal.

## 1.5 Conclusion

I have developed a dynamic incomplete-market general equilibrium model where households can hold unsecured debt and production is based on labor. I estimated the model

parameters in steady state by matching a large set of aggregate and cross sectional moments. Turning to business cycle analysis, I have shown that the model matches well the joint dynamics of unemployment and some key consumer finance statistics. In future research, the model could be used to assess the costs and benefits of some stimulus policies such as a bailout of highly levered households.

The model could also be extended along several lines. In particular, intermediaries could be modeled as risk-averse households who solve their own optimization problem. Such extension would allow the model to shed light on the cyclical movements of the risk-free rate and risk premia. Also, permanent idiosyncratic shocks could be added to study the interaction between heterogeneity in leverage and wealth. Lastly, the model could be extended to account for capital accumulation in order to study the effects of consumer finance on a broader set of business cycle statistics.

## Chapter 2

# Misallocation Cycles

### 2.1 Introduction

A large body of research has shown that the cross-section of firms is characterized by a substantial degree of productivity and capital heterogeneity (e.g., [Eisfeldt and Rampini \(2006\)](#)). While the empirical facts about firm heterogeneity are well known, the aggregate consequences are not well understood. In this paper, we develop and estimate a simply general equilibrium model to illustrate how the dynamics of the cross-section of firms impact aggregate fluctuations and risk premia via the misallocation of capital resources. The key implication of our general equilibrium model is that idiosyncratic shocks do not integrate out at the aggregate level but instead generate cyclical movements in the higher moments of consumption growth and risk premia.

Our model is driven by a cross-section of heterogeneous firms, which face irreversible investment decisions, exit, and permanent idiosyncratic and aggregate productivity shocks. The representative household has recursive preferences and consumes aggregate dividends. To generate aggregate consequences from a continuum of idiosyncratic shocks via capital misallocation, our model mechanism requires both a *power law distribution* as well as *common idiosyncratic skewness* in productivity.

While most of the literature assumes a log-normal idiosyncratic productivity distribution arising from mean-reverting Gaussian shocks, idiosyncratic shocks are permanent and follow random walks in our model. With firm exit, distributions of random walks generate power laws as emphasized by [Gabaix \(1999\)](#) and [Luttmer \(2007\)](#). Quantita-

tively, the endogenous power law for firm size is consistent with the data, as reported in [Axtell \(2001\)](#), such that the largest 5% of firms generate more than 30% of consumption and output in our model.

In addition to the random walk assumption, we model innovations to idiosyncratic productivity not only with Gaussian but also with negative Poisson shocks, which induce common idiosyncratic skewness. These negative Poisson shocks do not capture rare aggregate disaster, as in [Gourio \(2012\)](#), because they wash out at the aggregate level in a frictionless model.<sup>1</sup> Instead, time variation in the size of common idiosyncratic skewness allows us to capture the cyclicity in the skewness of cross-sectional sales growth, consistent with the evidence in [Salgado et al. \(2015\)](#).

In the model, these features lead to large occasional inefficiencies in the allocation of capital across firms and it hinders the representative agent's ability to smooth consumption. Intuitively, in recessions aggregate productivity falls and the distribution of output growth becomes negatively skewed. Firms with negative idiosyncratic productivity draws are constrained because they cannot disinvest unproductive capital to raise dividends. At the same time, the representative household would like to reallocate capital to smooth consumption.

Because of the power law distribution in firm size, the share of output coming from large firms contributes disproportionately to aggregate consumption, so that negatively skewed shocks to their productivity are particularly painful. Consequently, the drop in dividends from the mass of constrained is large, given that they are large in size. While unconstrained firms increase dividends by reducing investment, they are smaller so that they are not able to offset the impact of large constrained firms on aggregate consumption. This effect implies that in recessions aggregate consumption falls by more than aggregate productivity, causing negative skewness and kurtosis, and it arises purely from the cross-sectional misallocation. In contrast, in models with log-normal productivity distributions the size difference between constrained and unconstrained firms is small so that the groups offset each other.

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<sup>1</sup>For disaster risk in consumption see [Barro \(2006\)](#), [Gabaix \(2012\)](#), and [Wachter \(2013\)](#).

While the impact of capital misallocation on output and consumption are short lived under temporary mean-reverting shocks, permanent Poisson shocks render misallocation distortions long lasting. Quantitatively, output and consumption growth become more volatile and persistent, even though the model is only driven i.i.d. innovations. Importantly, consumption growth is left skewed and leptokurtic, as in the data. Because the household cares about long lasting consumption distortions due to Epstein-Zin preferences, the welfare costs of capital misallocation and aggregate risk premia are large.

Our mechanism to generate aggregate fluctuations from idiosyncratic shocks obeying a power law is distinct from the granular hypothesis of [Gabaix \(2011\)](#). While Gabaix also argues that the dynamics of large firms matters for business cycles, he relies on the fact that the number of firms is finite in an economy so that a few very large firms dominate aggregate output. The impact of these very large firms does not wash at the aggregate level when firm size follows a power law. In contrast, we model a continuum of firms so such each individual firm has zero mass. In our model, the power law in firm size generates aggregate fluctuations based on capital misallocation, arising from the investment friction, and not because the economy is populated by a finite number of firms. In reality, both effects are at work to shape business cycles.<sup>2</sup>

Methodologically, we build on [Veracierto \(2002\)](#) and [Khan and Thomas \(2008\)](#), who find that microeconomic investment frictions are inconsequential for aggregate fluctuations in models with mean-reverting idiosyncratic productivity.<sup>3</sup> We show that a model with permanent shocks and a more realistic firm size distribution not only breaks this irrelevance result, but also produces risk premia that are closer to the data. We are not the first to model permanent idiosyncratic shocks, e.g., [Caballero and Engel \(1999\)](#) and [Bloom \(2009\)](#) do so, but these paper study investment dynamics in partial equilibrium frameworks.

Starting with the influential paper by [Bloom \(2009\)](#), a growing literature emphasizes time varying uncertainty in productivity as driver of business cycles, e.g. [Bloom](#)

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<sup>2</sup>Related to the granular notion, [Kelly et al. \(2013\)](#) derive firm volatility in sparse networks.

<sup>3</sup>[Bachmann and Bayer \(2014\)](#) show that the same irrelevance result holds with idiosyncratic volatility shocks.

[et al. \(2014\)](#), [Bachmann and Bayer \(2014\)](#), and [Kehrig \(2015\)](#). We differ from this literature by focusing on time varying skewness in the cross-section of sales growth. It is well-known that sales growth dispersion is strongly countercyclical. Less well-known is that this countercyclical dispersion is mainly driven by the left tail of the sales growth distribution. By just looking at the cyclicity of the IQR, one might conclude that in recessions, firms have more dispersed positive and negative productivity draws. But the cyclicity of Kelly skewness indicates that in recessions significantly more firms have extreme negative productivity draws. Our model is equipped to match this empirical fact because productivity is not only driven by Gaussian shocks but also by negative Poisson shocks.

This empirical fact is also reminiscent of [Guvenen et al. \(2014\)](#), who document that households' income shocks feature procyclical skewness. [Constantinides and Ghosh \(2015\)](#) and [Schmidt \(2015\)](#) show that procyclical skewness is quantitatively important for aggregate asset prices in incomplete market economies. Different from these papers, our paper focuses on heterogeneity on the productive side of the economy and analyzes the effect of skewed shocks on capital misallocation.

The first study to quantify capital misallocation is [Olley and Pakes \(1996\)](#). More recent contributions include [Hsieh and Klenow \(2009\)](#) and [Bartelsman et al. \(2013\)](#). We extend their measure of capital misallocation and derive a frictionless benchmark in a general equilibrium framework. The importance of capital misallocation for business cycles is illustrated by [Eisfeldt and Rampini \(2006\)](#).

Our study also relates to the literature on production-based asset pricing, including [Jermann \(1998\)](#), [Boldrin et al. \(2001\)](#), and [Kaltenbrunner and Lochstoer \(2010\)](#), which aims to make the real business cycle model consistent with properties of aggregate asset prices. While these models feature a representative firm, we incorporate a continuum of firms. This allows us to pay close attention to cross-sectional aspects of the data, thereby providing a more realistic micro foundation for the sources of aggregate risk premia. While [Kogan \(2001\)](#) and [Gomes et al. \(2003\)](#) also model firm heterogeneity, our model provides a tighter link to firm fundamentals such that we estimate model

parameters.

Our model mechanism is also related to the works of [Gabaix \(1999\)](#) and [Luttmer \(2007\)](#). [Gabaix \(1999\)](#) explains the power law of city sizes with random walks reflected at a lower bound. Using a similar mechanism, [Luttmer \(2007\)](#) generates a power law in firm size in a steady-state model. We extend this literature by studying the impact of a power law in firm size in a business cycle model with common idiosyncratic skewness shocks.

Starting with the influential paper by [Berk et al. \(1999\)](#), there exist a large literature, which studies the cross-section of returns in the neoclassical investment framework, e.g., [Carlson et al. \(2004\)](#), [Zhang \(2005\)](#), [Cooper \(2006\)](#), and [Gomes and Schmid \(2010b\)](#). For tractability, these papers assume an exogenous pricing kernel and link firm cash flows and the pricing kernel directly via aggregate shocks. In contrast, we provide a micro foundation for the link between investment frictions and aggregate consumption.

## 2.2 Model

Time is discrete and infinite. The economy is populated by a unit mass of firms. Firms own capital, produce output with a neoclassical technology subject to an investment irreversibility constraint, and face permanent idiosyncratic and aggregate shocks. The representative household has recursive preferences and consumes aggregate dividends. This section elaborates on these model elements and defines the recursive competitive equilibrium of the economy.

### 2.2.1 Production

Firms produce output  $Y$  with the neoclassical technology

$$Y = (X\mathcal{E})^{1-\alpha}K^\alpha, \quad (2.1)$$

where  $X$  is aggregate productivity,  $\mathcal{E}$  is idiosyncratic productivity,  $K$  is the firm's capital stock and  $\alpha < 1$  is a parameter that reflects diminishing returns to scale. Aggregate productivity  $X$  follows a geometric random walk

$$X' = \exp \{g_x - \sigma_x^2/2 + \sigma_x \eta'_x\} X, \quad (2.2)$$

where  $g_x$  denotes the average growth rate of the economy,  $\sigma_x$  the volatility of log aggregate productivity growth, and  $\eta_x$  an i.i.d. standard normal innovation.

Idiosyncratic productivity growth is a mixture of a normal and a Poisson distribution, allowing for rare but large negative productivity draws. These negative jumps capture, for instance, sudden drops in demand, increases in competition, the exit of key human capital, or changes in regulation. As we will see, they are also essential for allowing the model to replicate the cross-sectional distribution of firms' sales growth. Specifically, idiosyncratic productivity  $\mathcal{E}$  follows a geometric random walk modulated with idiosyncratic jumps

$$\mathcal{E}' = \exp \left\{ g_\varepsilon - \sigma_\varepsilon^2/2 + \sigma_\varepsilon \eta'_\varepsilon + \chi' J' - \lambda (e^{\chi'} - 1) \right\} \mathcal{E}, \quad (2.3)$$

where  $g_\varepsilon$  denotes the average firm-specific growth rate,  $\sigma_\varepsilon$  the volatility of the normal innovations in firm-specific productivity,  $\eta$  an i.i.d. idiosyncratic standard normal shock, and  $J$  an i.i.d. idiosyncratic Poisson shock with constant intensity  $\lambda$ . The jump size  $\chi$  varies with aggregate conditions  $\eta_x$ , which we capture with the exponential function

$$\chi(\eta_x) = -\chi_0 e^{-\chi_1 \eta_x} \quad (2.4)$$

with strictly positive coefficients  $\chi_0$  and  $\chi_1$ . This specification implies that jumps are negative and larger in worse aggregate times, i.e., for low values of  $\eta_x$ . Our specification for idiosyncratic productivity warrants a few comments.

First, [Bloom \(2009\)](#) structurally estimates the cyclical variation in the dispersion of idiosyncratic productivity, which is a symmetric measure of uncertainty. Our specification also leads to time variation in the higher moments of idiosyncratic productivity growth. In particular, equation (2.4) implies that firm-specific productivity shocks become more left skewed in recessions. Second, different from the uncertainty shocks in [Bloom \(2009\)](#) and [Bloom et al. \(2014\)](#), our assumptions imply that changes in idiosyncratic jump risk are neither known to firms ex ante nor persistent, and therefore do not cause wait-and-see effects. As we will show, however, they induce large changes in measured aggregate productivity via their effect on the efficiency of the cross-sectional capital distribution. Third, in contrast to the consumption based asset pricing literature with disaster risk in



consumption, for instance Barro (2006), Gabaix (2012), and Wachter (2013), we do not model time variation in the jump probability  $\lambda$ . If the jump probability were increasing in recessions, it would induce rising skewness in productivity and sales growth, while in the data it is falling.<sup>4</sup> Fourth, the idiosyncratic jump risk term  $\chi J$  is compensated by its mean  $\lambda(e^\chi - 1)$ , so that the cross-sectional mean of idiosyncratic productivity is constant (see equation (2.5) below). This normalization implies that aggregate productivity is determined solely by  $\eta_x$ -shocks, so that our model does not generate aggregate jumps in productivity as emphasized by, e.g., Gourio (2012). Because the size of the jump risk is common across firms, we refer to it as *common idiosyncratic skewness* in productivity.

Given the geometric growth in idiosyncratic productivity, the cross-sectional mean of idiosyncratic productivity is unbounded unless firms exit. We therefore assume that at the beginning of a period – before production takes place and investment decisions are made – each firm exits the economy with probability  $\pi \in (0, 1)$ . Exiting firms are replaced by an identical mass of entrants who draws their initial productivity level from a log-normal distribution with location parameter  $g_0 - \sigma_0^2/2$  and scale parameter  $\sigma_0$ . Whenever firms exit, their capital stock is scrapped and entrants start with zero initial capital.

Since the idiosyncratic productivity distribution is a mixture of Gaussian and Poisson innovations, it cannot be characterized by a known distribution.<sup>5</sup> But two features are noteworthy. First, due to random growth and exit, the idiosyncratic productivity distribution and thus firm size features a power law, as shown by Gabaix (2009). A power law holds when the upper tail of the firm size distribution obeys a Pareto distribution such that the probability of size  $S$  greater than  $x$  is proportional to  $1/x^\zeta$  with tail (power law) coefficient  $\zeta$ .<sup>6</sup>

Second, even though the distribution is unknown, we can compute its higher mo-

<sup>4</sup>Note that skewness of Poisson jumps  $J$  equals  $\lambda^{-1/2}$ .

<sup>5</sup>Dixit and Pindyck (1994) assume a similar process without Poisson jumps in continuous time and solve for the shape of the cross-sectional density numerically; see their chapter 8.4.

<sup>6</sup>In our model, the tail coefficient solves the nonlinear equation  $1 = (1 - \pi)Z(\zeta)$ , where  $Z(\zeta) = \exp\{\zeta g_\varepsilon - \zeta \sigma_\varepsilon^2/2 + \zeta^2 \sigma_\varepsilon^2/2 + \lambda(e^{\zeta \chi} - 1) - \zeta \lambda(e^\chi - 1)\}$ .

ments. Let  $\mathbb{M}_n$  denote the  $n$ -th cross-sectional raw moment of the idiosyncratic productivity distribution  $\mathcal{E}$ . It has the following recursive structure

$$\begin{aligned} \mathbb{M}'_n &= (1 - \pi) \exp\{ng_\varepsilon - n\sigma_\varepsilon^2/2 + n^2\sigma_\varepsilon^2/2 + \lambda(e^{n\chi'} - 1) - n\lambda(e^{\chi'} - 1)\}\mathbb{M}_n \\ &\quad + \pi \exp\{ng_0 - n\sigma_0^2/2 + n^2\sigma_0^2/2\}. \end{aligned} \quad (2.5)$$

The integral over idiosyncratic productivity and capital determines aggregate output. To ensure that aggregate output is finite, we require that the productivity distribution has a finite mean.<sup>7</sup> Equation (2.5) states that the mean evolves according to  $\mathbb{M}'_1 = (1 - \pi)e^{g_\varepsilon}\mathbb{M}_1 + \pi e^{g_0}$ , which is finite if

$$g_\varepsilon < -\ln(1 - \pi) \approx \pi. \quad (2.6)$$

In words, the firm-specific productivity growth rate has to be smaller than the exit rate. In this case, the first moment is constant and, for convenience, we normalize it to one by setting

$$g_0 = \ln(1 - e^{g_\varepsilon}(1 - \pi)) - \ln(\pi). \quad (2.7)$$

### 2.2.2 Firms

To take advantage of higher productivity, firms make optimal investment decisions. Capital evolves according to

$$K' = (1 - \delta)K + I, \quad (2.8)$$

where  $\delta$  is the depreciation rate and  $I$  is investment. As in [Khan and Thomas \(2013\)](#) and [Bloom et al. \(2014\)](#), we assume investment is partially irreversible, which generates spikes and positive autocorrelation in investment rates as observed in firm level data. Quadratic adjustment costs can achieve the latter only at the expense of the former, since they imply an increasing marginal cost of adjustment. Partial irreversibility means that firms recover only a fraction  $\xi$  of the book value of capital when they choose to disinvest. These costs arise from resale losses due to transactions costs, asset specificity, and the physical costs of resale.

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<sup>7</sup>[Luttmer \(2007\)](#) makes a related assumption (Assumption 4), which states that “a firm is not expected to grow faster than the population growth rate” to ensure that the firm size distribution has finite mean.

We show in Section 2.3 that partial irreversibility yields an (S,s) investment policy such that firms have nonzero investment only when their capital falls outside an (S,s) inactivity band.<sup>8</sup> A firm with an unacceptably high capital stock relative to its current productivity will reduce its stock only to the upper bound of its inactivity range. Similarly, a firm with too little capital invests only to the lower bound of its inactivity range to reduce the linear penalty it will incur if it later chooses to shed capital. Thus, partial irreversibility can deliver persistence in firms investment rates by encouraging repeated small investments at the edges of inactivity bands.

We summarize the distribution of firms over the idiosyncratic states  $(K, \mathcal{E})$  using the probability measure  $\mu$  and note that the aggregate state of the economy is given by  $(X, \mu)$ . The distribution of firms evolves according to a mapping  $\Gamma$ , which we derive in Section 2.3. Intuitively, the dynamics of  $\mu$  are shaped by the exogenous dynamics of  $\mathcal{E}$  and  $X$ , the endogenous dynamics of  $K$  resulting from firms' investment decisions, and firm entry and exit.

Firms maximize the present value of their dividend payments to shareholders by solving

$$V(K, \mathcal{E}, X, \mu) = \max_I \{D + (1 - \pi)\mathbb{E}[M'V(K', \mathcal{E}', X', \mu')]\}, \quad (2.9)$$

where

$$D = Y - I 1_{\{I \geq 0\}} - \xi I 1_{\{I < 0\}} \quad (2.10)$$

denotes the firm's dividends and  $M$  is the equilibrium pricing kernel based on aggregate consumption and the household's preferences, which we derive in Section 2.3.1.

### 2.2.3 Household

The representative household of the economy maximizes recursive utility  $U$  over consumption  $C$  as in Epstein and Zin (1989a):

$$U(X, \mu) = \max_C \left\{ (1 - \beta)C^{1 - \frac{1}{\psi}} + \beta \left( \mathbb{E} [U(X', \mu')^{1 - \gamma}] \right)^{(1 - \frac{1}{\psi}) / (1 - \gamma)} \right\}^{1 / (1 - \frac{1}{\psi})} \quad (2.11)$$

where  $\psi > 0$  denotes the elasticity of intertemporal substitution (EIS),  $\beta \in (0, 1)$  the subjective discount factor, and  $\gamma > 0$  the coefficient of relative risk aversion. In the

<sup>8</sup>See Andrew B. Abel (1996) for a continuous time model of partial irreversibility.

special case when risk aversion equals the inverse of EIS, the preferences reduce to the common power utility specification. The household's resource constraint is

$$C = \int D \, d\mu. \quad (2.12)$$

### 2.2.4 Equilibrium

A recursive competitive equilibrium for this economy is a set of functions  $(C, U, V, K, \Gamma)$  such that:

- (i) Firm optimality: Taking  $M$  and  $\Gamma$  as given, firms maximize firm value (2.9) with policy function  $K$  subject to (2.8) and (2.10).
- (ii) Household optimality: Taking  $V$  as given, household maximize utility (2.11) subject to (2.12) with policy function  $C$ .
- (iii) The good market clears according to (2.12).
- (iv) Model consistency: The transition function  $\Gamma$  is induced by  $K$ , aggregate productivity  $X$ , equation (2.2), idiosyncratic productivity  $\mathcal{E}$ , equation (2.3), and entry and exit.

## 2.3 Analysis

In this section, we characterize firms' optimal investment policy and the transition dynamics of the cross-sectional distribution of firms. We also derive closed-form solutions for a frictionless version of the model, which serves as a benchmark for quantifying the degree of capital misallocation and the wedge between actual and measured aggregate productivity. Because aggregate productivity contains a unit root, we solve the model in detrended units, such that detrended consumption  $c$  and wealth  $w$  are given by

$$c = C/X \quad w = W/X.$$

### 2.3.1 Household Optimization

The household's first order condition with respect to the optimal asset allocation implies the usual Euler equation

$$\mathbb{E}[M'R'] = 1 \quad (2.13)$$

where  $M'$  is the pricing kernel and  $R'$  is the return on equity, defined by  $V'/(V - D)$ . The pricing kernel is given by

$$M' = \beta^\theta (x')^{-\gamma} \left(\frac{c'}{c}\right)^{-\theta/\psi} \left(\frac{w'}{w - c}\right)^{\theta-1}, \quad (2.14)$$

where  $\theta = \frac{1-\gamma}{1-1/\psi}$  is a preference parameter and  $x' = X'/X$  is i.i.d. log-normal distributed. In the case of power utility,  $\theta$  equals one and wealth drops out of the pricing kernel. With Epstein-Zin preferences, the dynamics of both consumption and wealth evolve endogenously and are part of the equilibrium solution.

Consistent with the Euler equation (2.13), wealth is defined recursively as the present value of future aggregate consumption:

$$w = c + \beta \mathbb{E} \left[ (x')^{1-\gamma} (w')^\theta \left(\frac{c'}{c}\right)^{-\theta/\psi} \right]^{1/\theta}. \quad (2.15)$$

Firm exit introduces a wedge between wealth and the aggregate market value of firms. This stems from the fact that wealth captures the present value of both incumbents and entrants, whereas aggregate firm value relates to the present value of dividends of incumbent firms only.

### 2.3.2 Firm Optimization

Having solved for the functional form of the pricing kernel, we can characterize firms' optimal investment policy. The homogeneity of the value function and the linearity of the constraints imply that we can detrend the firm problem by the product of both permanent shocks  $X\mathcal{E}$ , as for instance in Bloom (2009). We define the firm-specific capital to productivity ratio  $\kappa = K/(X\mathcal{E})$ , the capital target to productivity ratio  $\tau = K'/(X\mathcal{E})$ , and the firm value to productivity ratio  $v = V/(X\mathcal{E})$ .

Given the linear cost structure, one can divide the value function into three regions. In the investing region ( $(1 - \delta)\kappa \leq \tau$ ), firms increase their capital to productivity ratio and the optimal firm value solves  $v_u$ ; in the disinvesting region ( $\tau \leq (1 - \delta)\kappa$ ), firms decrease their capital to productivity ratio and the optimal firm value solves  $v_d$ ; otherwise, firms are inactive. Firm value  $v$  is thus the maximum of the value of investing

$v_u$ , disinvesting  $v_d$ , or inactivity:

$$v_u(\kappa, \mu) = \max_{(1-\delta)\kappa \leq \tau} \left\{ \kappa^\alpha - (\tau - (1-\delta)\kappa) + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\kappa', \mu')\right] \right\}, \quad (2.16)$$

$$v_d(\kappa, \mu) = \max_{\tau \leq (1-\delta)\kappa} \left\{ \kappa^\alpha - \xi(\tau - (1-\delta)\kappa) + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\kappa', \mu')\right] \right\}, \quad (2.17)$$

$$v(\kappa, \mu) = \max \left\{ v_u(\kappa, \mu), v_d(\kappa, \mu), \kappa^\alpha + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v((1-\delta)\kappa/(x'\varepsilon'), \mu')\right] \right\} \quad (2.18)$$

where  $\varepsilon' = \mathcal{E}'/\mathcal{E}$ . Because both growth rates  $\varepsilon'$  and  $x'$  are i.i.d., the state space of the detrended firm problem reduces to  $(\kappa, \mu)$ . Importantly, for adjusting firms next period's capital to productivity ratio  $\kappa' = \tau/(x'\varepsilon')$  is independent of the current capital to productivity ratio. This fact implies that firms share a common time-varying capital target  $\tau$ , which is independent of their own characteristic  $\kappa$ . The optimal capital targets for the investing and disinvesting regions is given by  $\mathcal{T}_u(\mu)$  and  $\mathcal{T}_d(\mu)$ , respectively, and solves

$$\begin{aligned} \mathcal{T}_u(\mu) &= \arg \max_{\tau} \left\{ -\tau + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\tau/(x'\varepsilon'), \mu')\right] \right\}, \\ \mathcal{T}_d(\mu) &= \arg \max_{\tau} \left\{ -\xi\tau + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\tau/(x'\varepsilon'), \mu')\right] \right\}. \end{aligned}$$

Given these capital targets, the optimal policy of the firm-specific capital to productivity ratio can be characterized by an  $(S, s)$  policy and is given by

$$\kappa' = \max\{\mathcal{T}_u(\mu), \min\{\mathcal{T}_d(\mu), (1-\delta)\kappa\}\}/(x'\varepsilon') \quad (2.19)$$

where the max operator characterizes the investing region and the min operator the disinvesting one. Conditional on adjusting, the capital to productivity ratio of every firm is either  $\mathcal{T}_u$  or  $\mathcal{T}_d$ , independent of their own characteristic  $\kappa$  but dependent on the aggregate firm distribution  $\mu$ .

The optimal investment rate policy, implied by (2.19), can be summarized by the same three regions of investment, inactivity, and disinvestment:

$$\frac{I}{K} = \begin{cases} \frac{\mathcal{T}_u(\mu) - \kappa}{\kappa} + \delta & (1-\delta)\kappa < \mathcal{T}_u & \text{investing,} \\ 0 & \mathcal{T}_u \leq (1-\delta)\kappa \leq \mathcal{T}_d & \text{inactive,} \\ \frac{\mathcal{T}_d(\mu) - \kappa}{\kappa} + \delta & \mathcal{T}_d < (1-\delta)\kappa & \text{disinvesting.} \end{cases}$$

In Figure 2.1, we plot both the optimal capital to productivity and investment rate policies for two arbitrary capital targets. Intuitively, when a firm receives a positive idiosyncratic productivity draw, its capital to productivity ratio  $\kappa$  falls. If the shock is large enough and depreciated  $\kappa$  is less than  $\mathcal{T}_u$ , it will choose a positive investment rate, which reflects the relative difference between target and current capital to productivity ratio as well as the depreciation rate. As a result, next period's capital to productivity ratio will reach  $\mathcal{T}_u$  in the investment region.

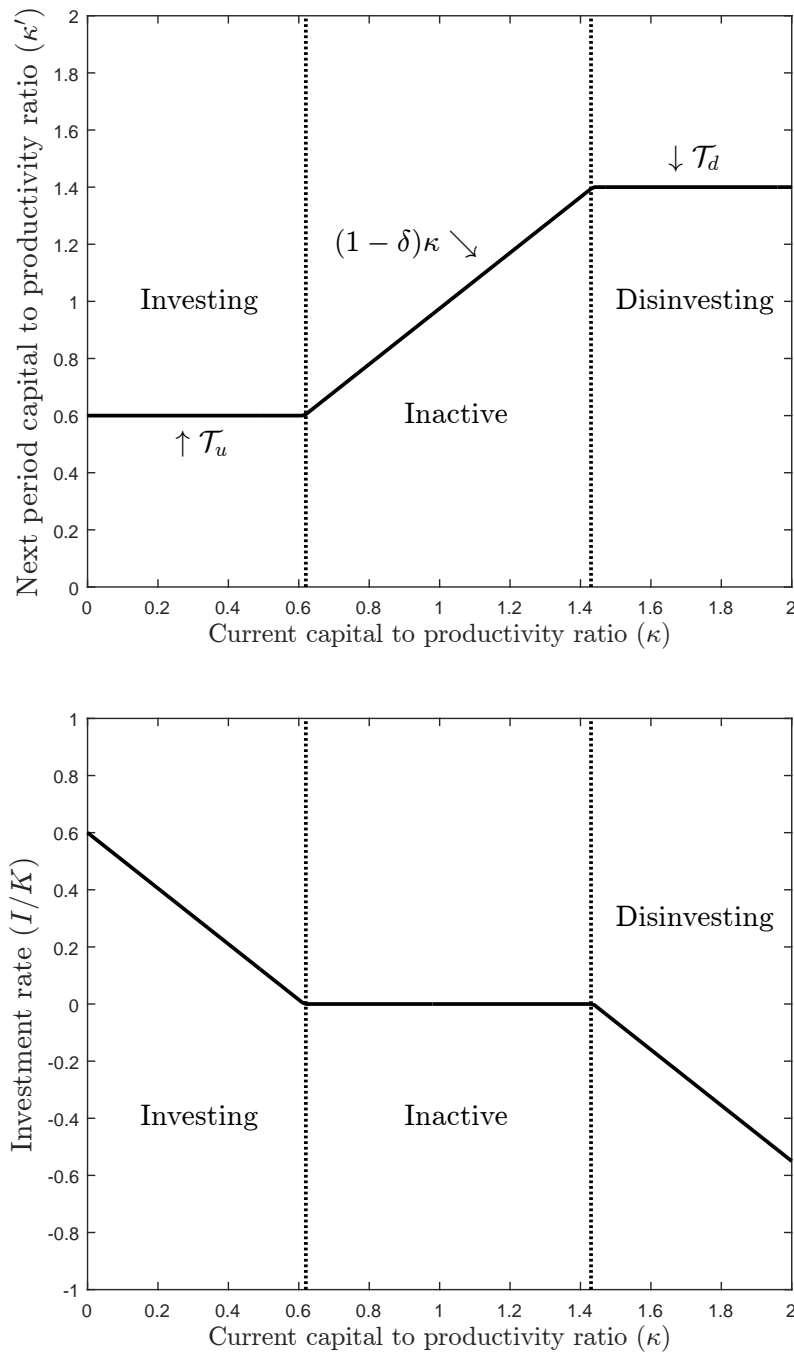
When a firm experiences an adverse idiosyncratic productivity draw, its capital to productivity ratio  $\kappa$  increases and it owns excess capital. If the shock is severe enough and depreciated  $\kappa$  is greater than  $\mathcal{T}_d$ , it will choose a negative investment rate, which reflects the relative difference between target and current capital to productivity ratio as well as the depreciation rate. As a result, next period's capital to productivity ratio will fall to  $\mathcal{T}_d$  in the disinvestment region. For small enough innovations, the depreciated capital to productivity ratio remains within  $\mathcal{T}_u$  and  $\mathcal{T}_d$ . In this region, firms are inactive and have a zero investment rate.

An important feature of our model is that there is heterogeneity in the duration of disinvestment constraintness. This feature arises because adverse idiosyncratic productivity shocks can arise either from a normal distribution or from a Poisson distribution. While adverse normal distributed shocks are short lasting, Poisson shocks are rare and large and therefore long lasting. As a result of Poisson shocks, the capital to productivity ratio rises dramatically, indicating a long duration of disinvestment constraintness.

### 2.3.3 Aggregation

In the previous section, we have shown that the firm-specific state space of the firm's problem reduces to the univariate capital to productivity ratio  $\kappa$ . One might therefore conjecture that the household only cares about the distribution of the capital to productivity ratio across firms. Yet the univariate distribution of the capital to productivity ratio is not sufficient to solve for equilibrium consumption.

Aggregate output is the integral over the product of capital and idiosyncratic pro-



**Figure 2.1:** Optimal capital and investment rate policies

ductivity and thus the correlation between capital and idiosyncratic productivity matters. While capital and idiosyncratic productivity are perfectly correlated in the frictionless economy, the investment friction renders capital and idiosyncratic productivity



imperfectly correlated. As a result, the joint distribution of capital and idiosyncratic productivity matters for aggregate consumption because it captures the degree of capital misallocation across firms, whereas in the frictionless model aggregate capital is a sufficient variable for the firm-level distribution.

Instead of normalizing capital by both permanent shocks  $X\mathcal{E}$ , we define detrended capital  $k$  by  $k = K/X$ . We can then summarize the distribution of firms over the idiosyncratic states by  $(k, \mathcal{E})$  using the probability measure  $\mu$ , which is defined on the Borel algebra  $\mathcal{S}$  for the product space  $\mathbf{S} = \mathbb{R}_0^+ \times \mathbb{R}^+$ .<sup>9</sup> The distribution of firms evolves according to a mapping  $\Gamma$ , which is derived from the dynamics of idiosyncratic productivity  $\mathcal{E}$  in equation (2.3) and capital. The law of motion for detrended capital can be obtained by multiplying firms' optimal policies (2.19) with idiosyncratic productivity  $\mathcal{E}$  and is given by

$$k' = \max\{\mathcal{E}\mathcal{T}_u(\mu), \min\{\mathcal{E}\mathcal{T}_d(\mu), (1 - \delta)k\}\}/x'. \quad (2.20)$$

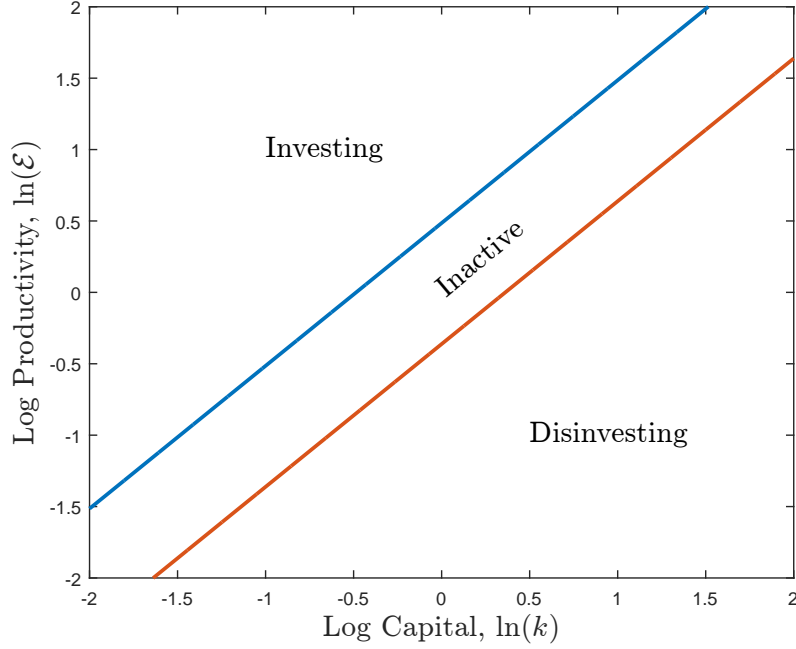
In Figure 2.2, we illustrate the three regions of  $\mu$  implied by the optimal capital policy (2.20). For the majority of firms, capital and idiosyncratic productivity are closely aligned such that these firms are optimally inactive. When idiosyncratic productivity exceeds capital, firms optimally invest such that next period's capital lies on the (blue) boundary to inactivity. Similarly, when capital exceeds idiosyncratic productivity, firms optimally disinvest such that next period's capital lies on the (red) boundary to inactivity.

Given the capital policy (2.20), the aggregate resource constraint detrended by  $X$  yields detrended consumption

$$c = \int \mathcal{E}^{1-\alpha} k^\alpha d\mu + (1+\xi)(1-\delta)\bar{k} - \int \max\{\mathcal{E}\mathcal{T}_u, (1-\delta)k\} d\mu - \xi \int \min\{\mathcal{E}\mathcal{T}_d, (1-\delta)k\} d\mu, \quad (2.21)$$

where  $\bar{k} = \int k d\mu$  denotes the aggregate capital stock. The first term is aggregate output, the second one is the book value of depreciated capital, the third one captures aggregate investment, and the fourth one aggregate disinvestment.

<sup>9</sup>Note that, with a slight abuse of notation, we continue to use the symbols  $\mu$  and  $\Gamma$  to denote the distribution of firms and its transition in the detrended economy.



**Figure 2.2:** Optimal capital policies in the  $\mu$ -distribution

The distribution  $\mu$  evolves over time according to the mapping  $\Gamma : (\mu, \eta'_x) \mapsto \mu'$ . To derive this mapping, note that the capital policy  $k'$  in equation (2.20) is predetermined with respect to the firm-level productivity shocks  $(\eta', J')$ . This implies that, conditional on current information and next period's aggregate shock  $\eta'_x$ , next period's characteristics  $(k', \mathcal{E}')$  are cross-sectionally independent of one another. Therefore, for any  $(\mathbf{K}, \mathbf{E}) \in \mathbf{S}$ ,

$$\mu'(\mathbf{K}, \mathbf{E}|\eta'_x) = \mu'_k(\mathbf{K}|\eta'_x) \times \mu'_\mathcal{E}(\mathbf{E}|\eta'_x), \quad (2.22)$$

where  $\mu_k$  and  $\mu_\mathcal{E}$  are the marginal distributions of capital and productivity, respectively.

The measure of firms with a capital stock of  $k' \in \mathbf{K}$  next period is simply the integral over the measure of firms who choose  $k'$  as their optimal policy this period and survive, plus the mass of entrants in the case  $0 \in \mathbf{K}$ .

$$\mu'_k(\mathbf{K}|\eta'_x) = (1 - \pi) \int 1_{\{k' \in \mathbf{K}\}} d\mu + \pi 1_{\{0 \in \mathbf{K}\}} \quad (2.23)$$

The measure of firms with an idiosyncratic productivity of  $\mathcal{E}' \in \mathbf{E}$  next period follows from the fact that, conditional on  $(\mathcal{E}, J', \eta'_x)$ ,  $\mathcal{E}'$  is log-normally distributed for continuing

firms. For entrants, idiosyncratic productivity is log-normally distributed as well. The distribution of  $\mathcal{E}'$  conditional on  $\eta'_x$  can therefore be computed as follows

$$\begin{aligned} \mu'_{\mathcal{E}}(\mathbf{E}|\eta'_x) &= \int_{\mathcal{E}' \in \mathbf{E}} \left\{ (1 - \pi) \int \sum_{j=0}^{\infty} p_j \phi \left( \frac{\ln(\mathcal{E}') - \left( \ln(\mathcal{E}) + g_{\varepsilon} - \frac{\sigma_{\varepsilon}^2}{2} + \chi' j - \lambda (e^{\mathcal{E}'} - 1) \right)}{\sigma_{\varepsilon}} \right) d\mu_{\mathcal{E}} \right. \\ &\quad \left. + \pi \phi \left( \frac{\ln(\mathcal{E}') - (g_0 - \sigma_{\varepsilon}^2/2)}{\sigma_{\varepsilon}} \right) \right\} d\mathcal{E}' \end{aligned} \quad (2.24)$$

where  $p_j = \lambda^j e^{-\lambda}/j!$  is the Poisson probability of receiving  $j$  jumps and  $\phi$  the standard normal density. Equations (2.22)–(2.24) define the transition function  $\Gamma$ .

### 2.3.4 Frictionless Economy

To understand the impact of irreversible investment on aggregate consumption and output, we also solve the frictionless economy as a benchmark. This benchmark allows us to quantify the degree of capital misallocation and to compute the resulting distortion in output and measured total factor productivity resulting from partial irreversible investment.

Without investment friction, the optimal firms' investment target can be solved for analytically

$$\mathcal{T}(\mu) = \left( \frac{(1 - \pi)\alpha \mathbb{E}[M'(x'\varepsilon')^{1-\alpha}]}{1 - (1 - \pi)(1 - \delta)\mathbb{E}[M']} \right)^{1/(1-\alpha)}$$

and optimal capital policy (2.20) simplifies to

$$k' = \mathcal{E}\mathcal{T}(\mu)/x'. \quad (2.25)$$

Intuitively, without the irreversibility constraint, firms are at their optimal capital target in every period.

In the frictionless case, it is feasible to derive a closed-form expression for the law of motion of aggregate capital. Specifically, by aggregating the optimal capital policy (2.25) across firms, it follows that

$$\bar{k}' = (1 - \pi)\mathcal{T}/x'.$$

Intuitively, aggregate capital is a weighted average of the investment target of incumbents and average capital of entrants. This aggregation result fails in the full model

		<b>Case I</b>		<b>Case II</b>	
$k_{high}$		0	0.5		$k_{high}$
$k_{low}$		0.5	0		$k_{low}$
		$\varepsilon_{low}$	$\varepsilon_{high}$		

**Table 2.1:** Two stylized firm-level distributions

because the optimal capital policy under partial irreversible investment (2.20) implies that future capital is a function of past shocks, rendering capital and idiosyncratic shocks correlated.

Similarly, the detrended aggregate resource constraint (2.21) in the frictionless economy simplifies to  $c = y + (1 - \delta)\bar{k} - \mathcal{T}$ . In contrast to the full model, aggregate output in the frictionless economy collapses to a function of aggregate capital  $\bar{k}$  and is given by

$$y = A(1 - \pi)^{1-\alpha}\bar{k}^\alpha, \quad (2.26)$$

where  $A$  is a Jensen term coming from the curvature in the production function.

In the full model, capital is misallocated across firms because unproductive firms find it costly to disinvest. We propose three measure of resource misallocation: a capital misallocation measure  $\mathcal{M}$ , an output distortion measure  $\mathcal{D}$ , and measured total factor productivity  $\mathcal{Z}$ .

The first misallocation measure is the correlation between capital and productivity

$$\mathcal{M} = 1 - \text{Corr}(\ln K', \ln \mathcal{E}). \quad (2.27)$$

In the frictionless case, capital is never misallocated and  $\mathcal{M} = 0$  in each period. In the full model, the more capital is misallocated across firms the larger is  $\mathcal{M}$ .

To illustrate capital misallocation in the context of our model, we present two stylized firm-level distributions in Table 2.1, where both idiosyncratic productivity and capital can only take on two values. The table entries are the probability mass for each point in the support of  $\mu$ , in line with the intuition of Figure 2.2. In Case I, productive firms hold a high capital stock, while unproductive firms hold a low capital stock. Consequently, there is no capital misallocation and  $\mathcal{M} = 1$ . In Case II, the scenario is reversed and capital completely misallocated with  $\mathcal{M} = -1$ .

Our capital misallocation measure is similar to the one in [Olley and Pakes \(1996\)](#), who suggest the covariance between size and productivity. This covariance measure has been used more recently by [Hsieh and Klenow \(2009\)](#) and [Bartelsman et al. \(2013\)](#). While covariances are unbounded, the upper bound of our correlation measure has a simple economic interpretation.

The second measure is the output loss due to capital misallocation. The output distortion measure  $\mathcal{D}$  is defined as the ratio of output in the full model relative to output on the frictionless economy and is given by

$$\mathcal{D} = \frac{\int \mathcal{E}^{1-\alpha} k^\alpha d\mu}{A(1-\pi)^{1-\alpha} \bar{k}^\alpha}. \quad (2.28)$$

The output distortion measure is tightly linked to our third measure of misallocation, namely, measured TFP. It is defined as TFP backed out from the full model, assuming that aggregate production can be described as a function of aggregate capital as in equation (2.26), and given by  $\mathcal{Z} = X\mathcal{D}$ . Interestingly, while the log growth rate of true aggregate TFP is i.i.d. and normally distributed, measured TFP can feature persistence in growth rates arising from misallocation

$$\Delta \ln \mathcal{Z}' = g_x - \sigma_x^2/2 + \sigma_x \eta'_x + \Delta \ln \mathcal{D}'. \quad (2.29)$$

### 2.3.5 Numerical Method

As in [Krusell and Smith \(1998\)](#), we approximate the firm-level distribution  $\mu$  with an aggregate state variable to make the model solution computable. Krusell and Smith and most of the subsequent literature used aggregate capital,  $\bar{k} = \int k d\mu$ , and higher cross-sectional moments of capital to summarize the policy-relevant information in  $\mu$ . Instead, we use detrended aggregate consumption  $c$ , and we argue that this approach is better suited for models with quantitatively important degrees of capital misallocation.

While aggregate capital (and higher moments of capital) depends only on the marginal distribution of capital, consumption depends on the joint distribution of capital and productivity. To illustrate the importance of this feature, consider again the stylized example in Table 2.1. In both cases, the aggregate capital stock equals  $(k_{low} + k_{high})/2$ .

Consumption, however, will be much higher in Case I, where capital is not misallocated and productive firms hold a high capital stock. In this case, aggregate output is higher and likely to remain so for the foreseeable future due to the permanent nature of shocks and the investment friction. Therefore, consumption is better suited than aggregate capital for summarizing the economically relevant aspects of  $\mu$  in our model. We suspect that this advantage carries over to other cross-sectional models that feature substantial amounts of capital misallocation.

Methodologically, the main difference between aggregate capital compared to consumption as state variable arises when specifying their law of motions. Tomorrow's capital stock for each firm is contained in the current information set, which implies that tomorrow's aggregate capital stock is contained in the current information set as well. Consequently, it is possible to approximate the law of motion for aggregate capital with a deterministic function. On the contrary, tomorrow's consumption is not known today but depends on tomorrow's realization of the aggregate shock  $\eta'_x$ . We approximate the law of motion for consumption with an affine function in log consumption

$$\ln c' = \zeta_0(\eta'_x) + \zeta_1(\eta'_x) \ln c. \quad (2.30)$$

These forecasting functions imply intercepts and slope coefficients, which depend on the future shock to aggregate productivity, i.e., they yield forecasts conditional on  $\eta'_x$ . As we illustrate quantitatively in Section 2.5, this functional form for aggregate consumption is not very restrictive as it allows for time variation in conditional moments of consumption growth.

In a model based on a representative household with power utility, the consumption rule (2.30) is sufficient to close the model. Because we model a representative household with recursive utility, we also have to solve for the wealth dynamics to be able to compute the pricing kernel (2.14). Khan and Thomas (2008) assume that marginal utility of consumption is a log linear function in aggregate capital and estimate the coefficients based on simulated data of the model. Instead, given consumption dynamics (2.30), we use the Euler equation for the return on wealth (2.15) to determine log wealth as a function log consumption, i.e.,  $w(c)$ . To this end, we minimize the Euler equation

error by iterating on the Euler equation. As a result, wealth dynamics and optimal consumption satisfy the equilibrium Euler equation and the model does not allow for arbitrage opportunities.

To summarize, our algorithm works as follows. Starting with a guess for the coefficients of the equilibrium consumption rule (2.30), we first solve for the wealth rule and then the firm's problem (2.16)–(2.18) by value function iteration. To update the coefficients in the equilibrium rule (2.30), we simulate a continuum of firms. Following [Khan and Thomas \(2008\)](#), we impose market clearing in the simulation, meaning that firm policies have to satisfy the aggregate resource constraint (2.21). The simulation allows us to update the consumption dynamics and we iterate on the procedure until the consumption dynamics have converged.

## 2.4 Estimation

The main goal of our paper is to relate aggregate fluctuations and risk premia to time variation in the efficiency of factor allocations at the firm level. Because such variation results from the interplay of idiosyncratic risk and frictions, it is crucial for our model to capture the cyclicity in the shocks that individual firms face. We therefore estimate productivity parameters based on a set of moments that reflects both the shape and cyclicity of the cross-sectional distribution. In particular, our simulated method of moments (SMM) estimation targets the cross-sectional distribution of firms' sales growth and investment rates, along with a set of aggregate quantity moments. Our paper is the first to estimate a general equilibrium model with substantial heterogeneity based on such a set of endogenous moments. This is made feasible largely due to modeling shocks as permanent, which allows us to reduce the dimensionality of the state space relative to earlier studies such as [Khan and Thomas \(2008\)](#), [Bachmann and Bayer \(2014\)](#), or [Bloom et al. \(2014\)](#).

### 2.4.1 Data

Our estimation relies on both aggregate and firm-level data over the period from 1976 to 2014. We use quarterly data but the moments are annual (based on four quarters). This allows us to make use of the higher information content of quarterly relative to annual data, while avoiding the seasonal variation of quarterly moments.

We define aggregate output as gross value added of the nonfinancial corporate sector, aggregate investment as private nonresidential fixed investment, and aggregate consumption as the difference between output and investment. All series are per capita and deflated with their respective price indices. Aggregate moments are based on four quarter log growth rates.

Firm-level data is taken from the merged CRSP-Compustat database. We eliminate financial firms (SIC codes 6000-6999) and utilities (SIC codes 4900-4999), because our model is inappropriate for these firms. Additionally, we only consider firms with at least 10 years of data. While this filter induces a sample selection bias, it also ensures that the time-variation in cross-sectional statistics is mostly driven by shocks to existing firms as opposed to changes in the composition of firms. In reality, such changes are driven by firms' endogenous entry and exit decisions, but this channel is outside of our model.

We estimate the model based on cross-sectional moments of sales growth and investment rates. Sales growth is defined as the four quarter change in log SALEQ, deflated by the implicit price deflator for GDP. The investment rate is defined as the sum of four quarterly investment observations divided by the beginning capital stock. We compute quarterly investment as the difference in net property, plant and equipment (PPENTQ), deflated by the implicit price deflator for private nonresidential fixed investment.<sup>10</sup> Capital is computed using a perpetual inventory method.<sup>11</sup> The cross-sectional dimension

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<sup>10</sup>This assumes that economic depreciation is equal to accounting depreciation. A preferable approach would be to define investment as the difference in gross PPE and subtract economic depreciation. Yet individual firms' economic depreciation is not observable.

<sup>11</sup>The perpetual inventory method assumes that  $K_{i,t} = (1 - \delta)K_{i,t-1} + I_{i,t}$ , initialized using PPENTQ deflated by the implicit price deflator for private nonresidential fixed investment. As in our calibration, we assume a quarterly depreciation rate of  $\delta = 2.5\%$ .



of our final sample grows from 915 firms in the first quarter of 1977 to 1501 firms in the last quarter of 2014.

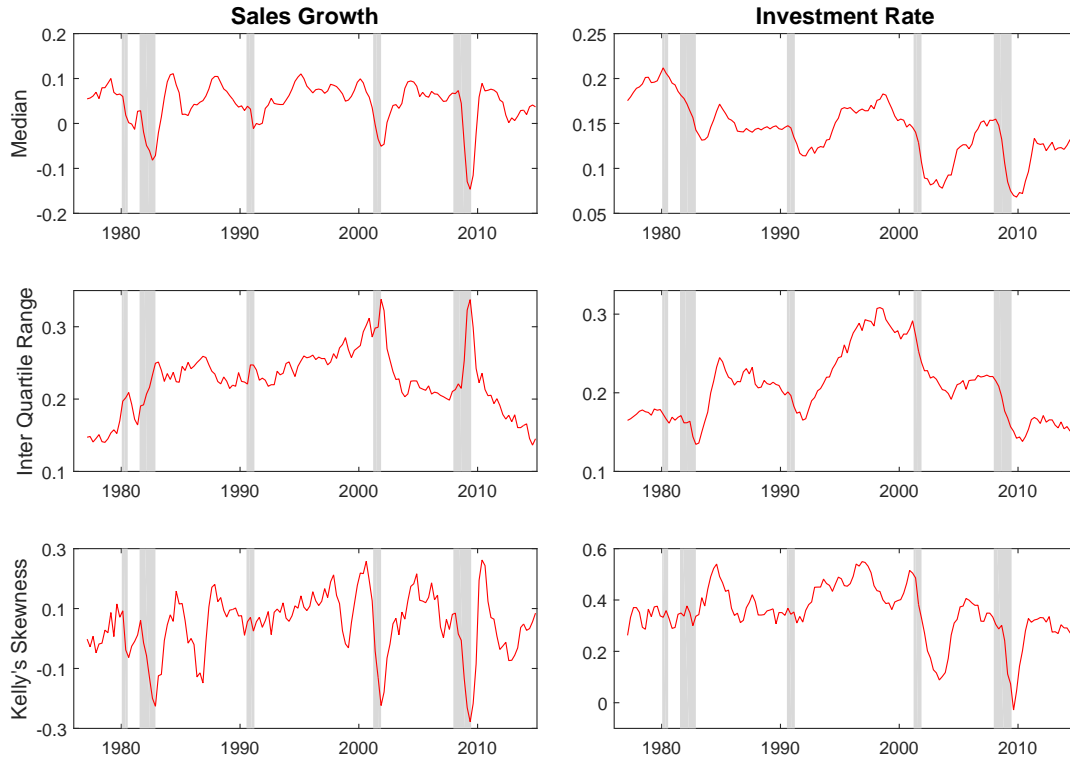
### 2.4.2 Cyclical Properties of the Cross-Section of Firms

In this section, we document how the cross-section of firms moves over the business cycle, and we discuss implication of the associated empirical facts. Figure 2.3 shows the evolution of the cross-sectional distributions of firms' sales growth (left column) and investment rates (right column) over time. We summarize both distributions with robust versions of their first three moments, i.e., we measure centrality with the median, dispersion with the inter quartile range (IQR), and asymmetry with Kelly skewness.<sup>12</sup> The two top panels of the figure show that recessions are characterized by sizable declines in sales growth and investment rates for the median firm. This observation is unsurprising. However, recessions are further characterized by pronounced changes in the *shape* of the cross-sectional distributions.

Sales growth becomes more disperse during recessions and its skewness switches sign from positive to negative. This evidence suggests that recessions coincide with an increase in idiosyncratic risk. Bloom (2009) and Bloom et al. (2014) provide ample additional evidence for the increase in dispersion and model it as an increase in the volatility of firms' Gaussian productivity shocks. However, the pronounced change in the skewness of sales growth shows that the countercyclicality of idiosyncratic risk is better described as resulting from an expansion of the left tail of the shock distribution as opposed to a symmetric widening of the whole distribution. Intuitively, recessions are times where a subset of firms receives very negative shocks, but it is not the case that an equal proportion of firms receives very positive shocks.

Another characteristic of recessions is the fact – first documented by Bachmann and Bayer (2014) – that the dispersion in firms' investment rates *declines*. This procyclicality is suggestive of nonconvexities in firms' capital adjustment cost because in the absence

<sup>12</sup>Kelly skewness is defined as  $KSK = \frac{(p_{90} - p_{50}) - (p_{50} - p_{10})}{p_{90} - p_{10}}$ , where  $p_x$  denotes the  $x$ -th percentile of the distribution. It measures asymmetry in the center of the distribution as opposed to skewness that can result from tail observations. Similar to the median and IQR, Kelly skewness is thus robust to outliers.



**Figure 2.3:** Robust cross-sectional moments of annual sales growth and investment rates

of such frictions, the increase in the dispersion of firms' productivity would lead to a *larger* dispersion in investment rates.

Bachmann and Bayer (2014) argue that the same fact is informative about the cyclicity of idiosyncratic risk. In particular, they show that a model with uncertainty shocks and wait-and-see effects in the spirit of Bloom et al. (2014) counterfactually produces a procyclical investment rate dispersion when uncertainty shocks tend to be large – as calibrated by Bloom et al. The intuition for this result is that when Gaussian volatility increases during recessions, the subset of firms receiving large positive shocks will undertake large positive investments, which leads to an increase in the cross-sectional dispersion of investment rates. When changes in uncertainty are large enough, this effect dominates the real options effect that causes firms to delay their investments in the face of increased uncertainty, which all else equal reduces the dispersion in investment rates. On the other hand, when uncertainty shocks are more moderately sized, the model becomes consistent with the procyclical investment rate

dispersion, but uncertainty shocks no longer induce serious business cycles. [Bachmann and Bayer \(2014\)](#) therefore argue that the countercyclical dispersion of investment rates places a robust and tight upper bound on degree of countercyclicality of idiosyncratic risk, and they challenge the view advocated by [Bloom \(2009\)](#) and [Bloom et al. \(2014\)](#) that firm level risk shocks are an important driver of business cycles. However, we note that this conclusion relies on a model with (a) real option effects and (2) time-variation in idiosyncratic risk that results from a *symmetric* change in the dispersion of shocks. Neither of these features are present in our model.

### 2.4.3 Simulated Method of Moments

This section explains how we estimate the model parameters. The full set of model parameters includes preference  $(\beta, \gamma, \psi)$ , technology  $(\delta, \alpha)$ , entry and exit  $(\pi, \sigma_0)$ , and productivity  $\theta \equiv (\chi_0, \chi_1, \lambda, g_\varepsilon, \sigma_\varepsilon, g_X, \sigma_X)$  parameters. Since it is not feasible computationally to estimate the full set of parameters, we focus on estimating the vector of productivity parameters  $\theta$ .

Values for the remaining parameters are taken from previous literature and are shown in Panel A of [Table 2.2](#). Following [Bansal and Yaron \(2004\)](#), we assume that the representative agent is fairly risk averse,  $\gamma = 10$ , and has a large EIS,  $\psi = 2$ . The time discount rate of  $\beta = 0.995$  is chosen to achieve a low average risk-free rate. Capital depreciates at a rate of 2.5% and the curvature of the production function equals 0.65, similar to [Cooper and Haltiwanger \(2006\)](#). Firms exit the economy with a rate of 2%, similar to the value reported in [Dunne et al. \(1988\)](#). The productivity draws of entrants has a mean pinned down by condition [\(2.7\)](#) and a volatility of 10%. As estimated by [Bloom \(2009\)](#), we assume partial irreversibility costs of  $\xi = 0.7$ .

Productivity parameters are estimated with the SMM, which minimizes a distance metric between key moments from actual data,  $\Psi^D$ , and moments from simulated model data,  $\Psi^M(\theta)$ . Given an arbitrary parameter vector  $\theta$ , the model is solved numerically as outlined in [Section 2.3.5](#). In solving the model, we use an equilibrium simulation of length 1820 quarters, which equals ten times the time dimension of the actual data

Table 2.2: Predefined and Estimated Parameter Values

Parameter	Spec-1	Spec-2	Spec-3	Description
<b>A: Predefined Parameters</b>				
$\delta$	0.025	0.025	0.025	Depreciation rate
$\alpha$	0.65	0.65	0.65	Curvature in production function
$\sigma_0$	0.1	0.1	0.1	Volatility of productivity of new entrants
$\pi$	0.02	0.02	0.03	Exit probability
$\beta$	0.995	0.995	0.995	Time discount factor
$\xi$	0.7	0.7	0.7	Proportional resale value of capital
$\gamma$	10	2	10	Risk aversion
$\psi$	2	0.5	2	Elasticity of intertemporal substitution
<b>B: Estimated Parameters</b>				
$\chi_0$	0.2384 [0.0000]	0.2915	0.2421	Parameter of jump size function
$\chi_1$	0.7027 [0.0000]	0.4189	0.7300	Parameter of jump size function
$\lambda$	0.0941 [0.0000]	0.0896	0.0900	Parameter of jump intensity function
$g_\varepsilon$	0.0146 [0.0003]	0.0149	0.0139	Mean idio. productivity growth rate
$\sigma_\varepsilon$	0.0496 [0.0001]	0.0416	0.0541	Parameter of idiosyncratic volatility function
$g_X$	0.0030 [0.0012]	0.0011	0.0029	Mean aggregate productivity growth rate
$\sigma_X$	0.0356 [0.0001]	0.0369	0.0334	Volatility of aggregate productivity growth rate

Notes: Panel A shows calibrated parameters and Panel B shows parameters estimated via SMM with standard errors in brackets. The model is solved at a quarterly frequency. Spec-1 equals the benchmark specification. Spec-2 replaces the recursive utility function with a time-separable power utility function with a low value for the relative risk aversion parameter. Spec-3 allows for time-variation not only in the jump size, but also in the jump intensity and the volatility of Gaussian idiosyncratic shocks.

plus an initial 300 quarters that we discard so as to start from the ergodic distribution. We then fix the equilibrium path of consumption that results from the equilibrium simulation and simulate a finite panel of firms for the same path of the economy.<sup>13</sup>

<sup>13</sup>While the simulation step of the model solution is based on a continuum of firms that are tracked

Based on the simulated data panel, we calculate the model moments  $\Psi^M(\theta)$  as well as the objective function  $Q(\theta) = [\Psi^D - \Psi^M(\theta)]'W[\Psi^D - \Psi^M(\theta)]$ . The parameter estimate  $\hat{\theta}$  is found by searching globally over the parameter space to find the minimizing parameter vector. We use an identity weighting matrix and implement the global minimization via a genetic algorithm with wide parameter bounds. Computing standard errors for the parameter estimate requires the Jacobian of the moment vector, which we find numerically via a finite difference method.

To identify the parameter vector  $\theta$ , we rely on a combination of aggregate and cross-sectional moments. First, we include the time series means of the six cross-sectional moments depicted in Figure 2.3. Doing so ensures that we capture the average shape of the conditional distributions of sales growth and investment rates, and therefore also the shape of their long run distributions. Second, we include the time series standard deviations of the same six cross-sectional moments to capture the amount of time-variation in the conditional cross-sectional distributions. Third, we rely on time series correlations between three cross-sectional moments and aggregate output growth to capture the cyclicity of the cross section. In particular, we include the cyclicity of the dispersion in sales growth documented by Bloom (2009), the cyclicity of the skewness in sales growth documented by Salgado et al. (2015), and the cyclicity in the dispersion of investment rates documented by Bachmann and Bayer (2014). Relying on three measures of cyclicity jointly ensures that we capture various aspects of how the cross section co-moves with the cycle. Lastly, we include the mean growth rate of aggregate output, and the standard deviations of aggregate output, consumption, and investment to ensure that productivity parameters reflect not only the cross-section but also remain consistent with macro aggregates. In total, we estimate 7 productivity parameters based on the 19 moments shown in the data column of Table 2.3.

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using a histogram (a so-called nonstochastic simulation), this approach is not feasible for determining cross-sectional moments that span multiple quarters. The reason is that multi-period transition functions become too high dimensional to be manageable computationally. However, the fact that the simulation of a finite panel of firms is based on the same path for aggregate shocks (and aggregate consumption) as the model solution implies that the Monte Carlo sample can be interpreted as subsample of the continuum of firms. We choose the number of simulated firms high enough to ensure that the simulated cross-sectional moments are not affected by the Monte Carlo noise stemming from the finiteness of the sample.

Table 2.3: Moments Targeted in SMM Estimation

		Data	Spec- 1	Spec- 2	Spec- 3	
<b>A: Cross-Sectional Sales Growth Moments</b>						
Median	<i>mean</i>	0.045	0.035	0.030	0.034	
	<i>std</i>	0.046	0.031	0.030	0.029	
IQR	<i>mean</i>	0.225	0.186	0.183	0.184	
	<i>std</i>	0.042	0.016	0.011	0.016	
	<i>corr</i> $[\cdot, g\bar{Y}]$	-0.332	-0.332	-0.351	-0.331	
Kelly	<i>mean</i>	0.046	0.075	0.077	0.070	
	<i>std</i>	0.104	0.128	0.082	0.133	
	<i>corr</i> $[\cdot, g\bar{Y}]$	0.586	0.588	0.597	0.594	
<b>B: Cross-Sectional Investment Rate Moments</b>						
Median	<i>mean</i>	0.142	0.126	0.125	0.125	
	<i>std</i>	0.032	0.043	0.029	0.041	
IQR	<i>mean</i>	0.207	0.256	0.255	0.253	
	<i>std</i>	0.043	0.024	0.019	0.023	
	<i>corr</i> $[\cdot, g\bar{Y}]$	0.244	0.249	0.266	0.244	
Kelly	<i>mean</i>	0.352	0.337	0.310	0.335	
	<i>std</i>	0.104	0.200	0.135	0.191	
<b>C: Aggregate Quantity Moments</b>						
Output Growth	<i>mean</i>	0.015		0.009	0.002	0.009
	<i>std</i>	0.030	0.033	0.031	0.032	
Consumption Growth	<i>std</i>	0.026	0.023	0.039	0.023	
Investment Growth	<i>std</i>	0.066	0.046	0.027	0.047	

Notes: The table summarizes the moments used in the SMM estimation. Panels A and B contain time series statistics of cross-sectional moments. For example, the row for IQR *mean* in Panel A contains the time series mean of the cross-sectional sales growth IQR. Panel C contains time series moments of aggregate quantity growth rates. All statistics refer to annual moments, i.e. annual sales growth rates, annual investment rates, as well as annual aggregate quantity growth rates. The model parameters related to each specification are shown in Table 2.2.

#### 2.4.4 Parameter Identification

In what follows, we discuss the main sources of identification for each estimated parameter. The drift and volatility parameters in aggregate productivity,  $g_X$  and  $\sigma_X$ , are pinned down by the mean and volatility of aggregate output growth. An increase in the

volatility of Gaussian idiosyncratic shocks,  $\sigma_\varepsilon$ , increases the cross-sectional skewness of investment rates because costly reversibility implies that positive investments are easier to undertake than negative investments. This parameter is therefore identified by the mean Kelly's skewness of investment rates. Given a value for the drift in aggregate productivity, the drift of idiosyncratic productivity,  $g_\varepsilon$ , can be identified from the average of the cross-sectional median of sales growth. While the four parameters discussed thus far also have small effects on other moments, their main source of identification is fairly clear.

On the other hand, the parameters that govern Poisson jumps in idiosyncratic productivity have a strong effect on a large number of moments. This implies both that they are very well identified, and that the large number of moments that we include in the estimation allows for a very good test of whether including such jumps is a reasonable assumption. The main effects are as follows.

1. **Cyclicalit y of the investment rate dispersion.** Increasing  $\chi_0$  increases the magnitude of idiosyncratic productivity jumps. Because such jumps occur more frequently in bad aggregate times while the volatility of Gaussian shocks is time-invariant, increasing  $\chi_0$  also leads to more time-variation in idiosyncratic risk. According to [Bachmann and Bayer \(2014\)](#), such an increase should lower the procyclicality of the investment rate dispersion. In our model, however, it has the opposite effect.
2. **Cyclicalit y of the sales growth dispersion.** Increasing  $\chi_1$  implies that the jump size increases more during in bad aggregate times, making productivity shocks more disperse.

On the other hand, increasing  $\chi_1$  has a negligible effect on the cyclicalit y of the investment rate dispersion.

As a consequence, the cross sectional productivity and sales growth distributions become more disperse during recessions, and the countercyclicality of the dispersion in sales growth increases.

This is reflected in the correlation between the IQR of sales growth and aggregate

output becoming more negative.

The same effect arises from an increase in the frequency of jumps,  $\lambda$ , because it makes the aforementioned effect quantitatively stronger.

3. **Time-variation of the cross-sectional distributions.** Making jumps larger, more cyclical, or more frequent by increasing  $\chi_0$ ,  $\chi_1$ , or  $\lambda$  increases the volatility of all cross-sectional moments. This occurs because jumps induce time-variation in the cross-sectional productivity distribution due to their cyclicity. Changes in the shape of the productivity distribution coincide with changes in the shape of both outcome variables (sales and investment).
4. **Consumption smoothing.** Making jumps larger, more cyclical, or more frequent by increasing  $(\chi_0, \chi_1, \lambda)$  increases the volatility of aggregate consumption growth. As we will show in Section 2.5, this arises from the fact that jumps in idiosyncratic productivity induce capital misallocation across firms that hinders the representative agent's ability to smooth consumption. At the same time, the three jump parameters leave the volatility of aggregate output growth nearly unaffected.

In summary, jumps are well-identified as they significantly effect many of the moments included in the estimation, and their presence has a number of interesting economic consequences that are directly in line with the data.

### 2.4.5 Baseline Estimates

SMM parameter estimates are shown in Table 2.2, whereas data and model moments are shown in Table 2.3. Our benchmark specification is shown in the columns labeled Spec-1 (we will return to the alternative specifications below). As Table 2.2 shows, all estimated parameters are well-identified as indicated by the very small standard errors. The estimated jump intensity of  $\hat{\lambda} = 0.0941$  implies that firms receive negative jumps in productivity about once every 11 quarters, whereas the estimated parameters of the jump size function ( $\hat{\chi}_0 = 0.2384$  and  $\hat{\chi}_1 = 0.7027$ ) imply that the average jump size is about  $-31\%$ . The log growth rate of idiosyncratic productivity has a Gaussian volatility of  $\hat{\sigma}_\varepsilon = 5.39\%$  and drift parameter of  $\hat{g}_\varepsilon = 1.46\%$  per quarter, well below the



Table 2.4: Consumption Growth and Asset Prices

	Spec-1	Spec-2	Spec-3
<b>Panel A: Consumption Growth</b>			
Autocorrelation	0.330	0.058	0.462
Skewness	-0.600	-0.440	-0.395
Kurtosis	3.541	3.323	3.411
<b>Panel B: Returns</b>			
Excess return on wealth	1.79%	0.07%	1.60%
Risk-free Rate	1.48%	2.06%	1.62%
Sharpe Ratio	0.573	0.062	0.543

Notes: The table summarizes moments related to consumption risks and risk premia. These moments were not targeted in the SMM estimation. The model parameters related to each specification are shown in Table 2.2.

threshold of  $\pi = 2\%$  that is required to ensure finiteness of the cross-sectional mean of productivity – see Equation 2.6. Lastly, the log growth rate of aggregate productivity has a drift parameter of  $\hat{g}_X = 0.30\%$  and a volatility of  $\hat{\sigma}_X = 3.56\%$  per quarter.

## 2.4.6 Alternative Specifications

In this section, we illustrate estimation results for a number of alternative model specifications in order to highlight the role of pre-specified parameters. The results of these experiments are contained in the additional columns of Tables 2.2-2.4.

Columns labeled "Spec-2" show results for a preference parameter calibration that implies time-separable utility. In particular, we calibrate the EIS to a low value of  $\psi = 0.5$  as typically assumed in the macroeconomics literature. Table 2.2 shows that the estimated productivity parameters are very similar to those in our benchmark specification, with the exception of  $\chi_1$  and  $g_X$ . The estimated parameter values imply that the size of productivity jumps is less cyclical than in the benchmark, whereas aggregate quantities grow at a lower rate. Table 2.3 shows that, while cross-sectional moments are matched similarly well as in the benchmark specification, the volatilities of output, consumption, and investment are very different from their data counterparts. Therefore, the low EIS leads to a tension between matching cross-sectional and aggregate

facts. While one could certainly match quantity volatilities by assigning a larger weight to these moments in the criterion function, this would come at the expense of no longer matching cross-sectional facts.

The next alternative specification labeled "Spec-3" changes the benchmark specification by assuming a higher exit rate of  $\pi = 3\%$  as opposed to 2% in the benchmark. While this only leads to small differences in estimated productivity parameters and fit of the targeted moments, the higher exit rate implies a lower power law coefficient, i.e. a lower concentration of large firms. As shown in Table 2.4 and Figure 2.10 (both of which we discuss below), this results in an improved ability to smooth consumption, lower risk premia, and less misallocation relative to the benchmark.

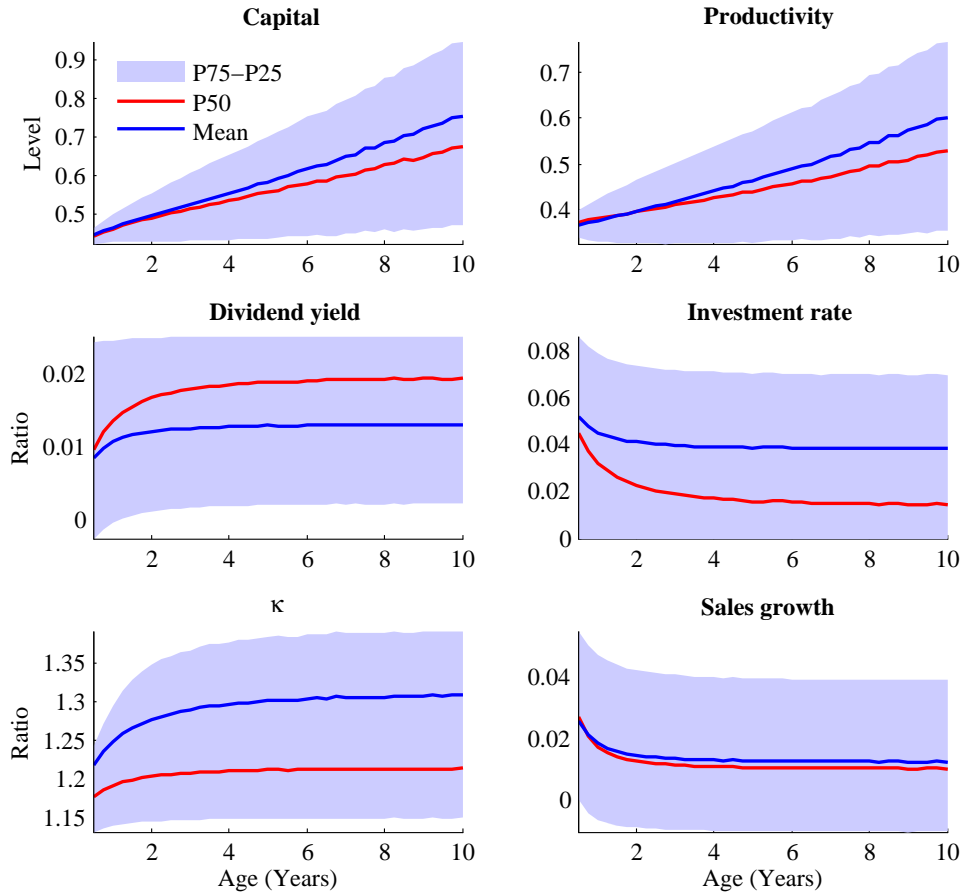
## 2.5 Model Implications

### 2.5.1 Firms' Life Cycle

To understand the nature of aggregate fluctuations in our model, it is useful to first characterize the behavior of a typical firm over its life cycle. Figure 2.4 shows how various firm characteristics change as firms age. Due to the geometric growth in firm-specific productivity shown in the top-right plot, firms average capital stock increases approximately linearly with age (top-left). In addition, older firms are more likely to hold excess capital, as indicated by their higher average capital-to-productivity ratio  $\kappa$  (bottom-left). The reason for this effect is that older firms are more likely to have received a negative Poisson shock to productivity during their lifetime, which tends to be followed by an extended period of inactivity. Because they are less likely to be constrained, young firms have higher investment rates (middle-right) and lower payout rates (middle-left), and as a result higher sales growth rates (bottom-right).

### 2.5.2 Business Cycles

This section illustrates via an impulse response analysis how the cross-section of firms and macroeconomic aggregates respond to adverse aggregate shocks. We choose to model a recession as two consecutive  $\eta_x$ -shocks of -1.36 standard deviations. This choice

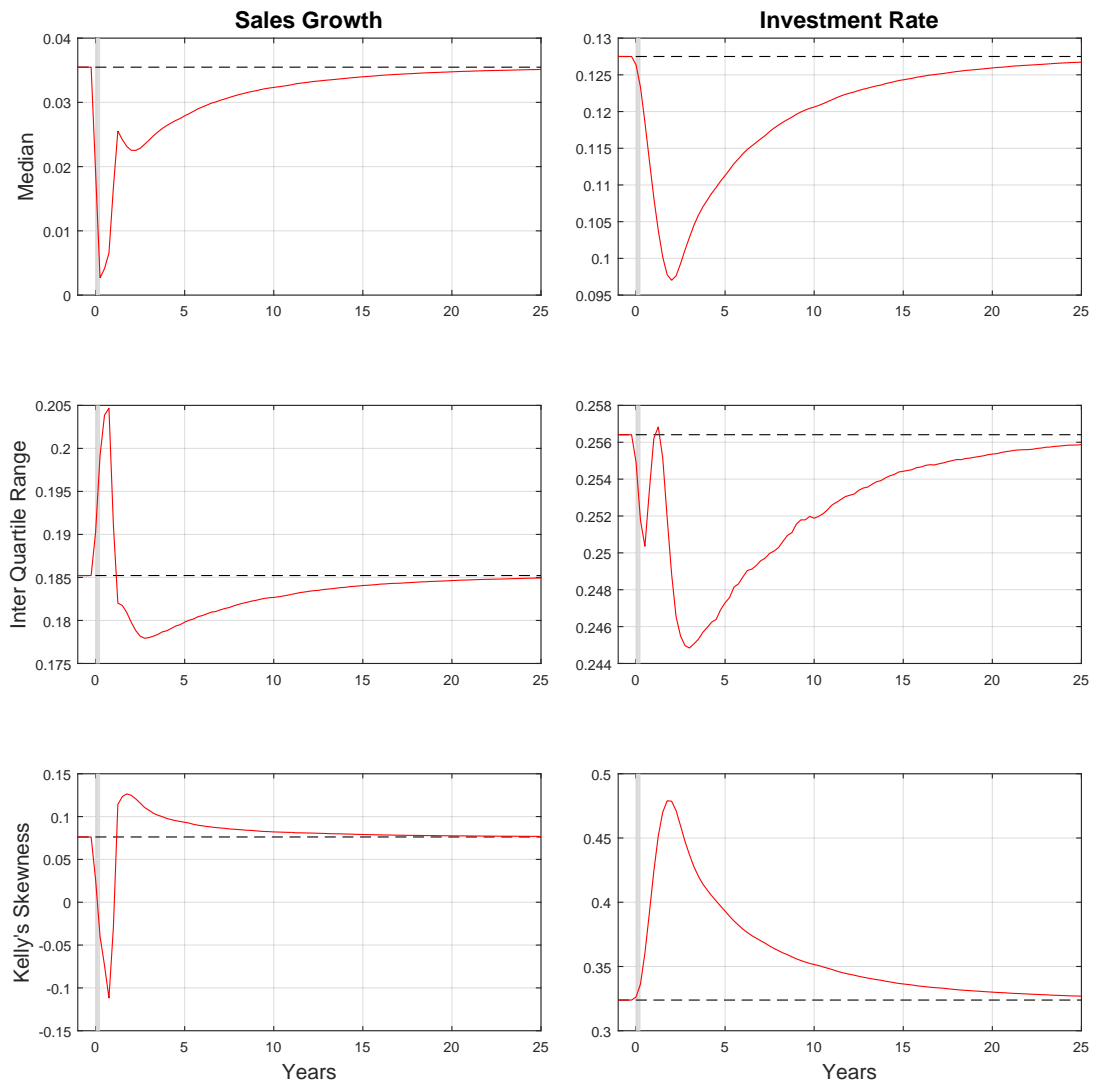


**Figure 2.4:** Life Cycle of a Firm: Average Behavior and Quartiles.

results in a reduction in aggregate output of about 4% over the course of the recession, similar to the U.S. experience during the Great Recession.<sup>14</sup> To compute the impulse response functions, we simulate 1,000 equilibrium paths of the economy by imposing market clearing in each period, and we compute the average of a given statistic across all paths. We then use the identical shock sequences once more, change the shock realizations in periods 500 and 501 to  $\eta_x = -1.36$ , and compute the average of the statistic across all paths again. In our figures, we then report the percent deviation of the shocked economy from the unconditional one. The initial 500 periods are discarded.

**Cross-sectional firm dynamics.** Figure 2.5 shows how the cross-sectional sales

<sup>14</sup>The choice for the exact value of -1.36 standard deviations resulted from the discretization of the  $\eta_x$ -shocks used for our numerical model solution.



**Figure 2.5:** Impulse response: Cross-Section

growth and investment rate distributions respond to a recession. On impact, annual sales growth for the median firm falls from 3.5% to about zero, and then immediately rebounds to 2.5% four quarters after the last recession quarter. The reason why it remains below trend is that annual sales growth depends positively on investment rates, which remain below trend for an extended period following the recession. The fast recovery of sales growth and the much slower recovery of investment rates is consistent with the data shown in Figure 2.3.

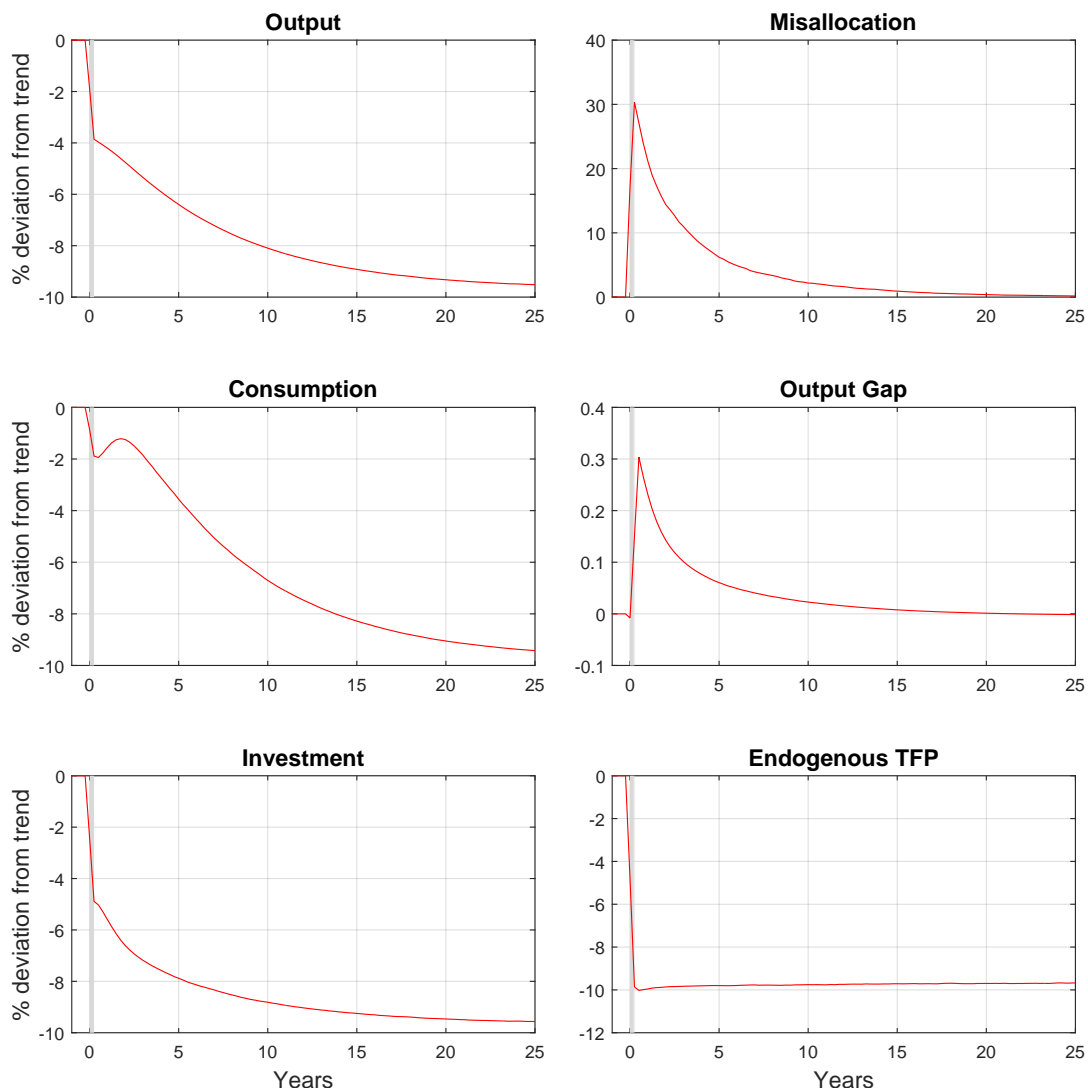
A similar difference in the speed of mean reversion can be observed for the dispersions

of both cross sections. The middle-left panel of Figure 2.5 shows that the IQR of sales growth increases from 18.5% to 20.5% upon impact of the recession, and then quickly returns to its pre-recession value with a slight undershooting. On the other hand, the IQR of investment rates falls on impact and, with the exception of a short-lived hump, continues to decline for 2-3 years following the recession. Once again, both of these effects mimic what we see in the data. In the model, the increased dispersion of sales growth results from the fact that negative jumps in productivity are larger during recessions. For the same reason, we observe a sharp but short-lived drop in the skewness of sales growth during recessions (bottom-left panel), once again mimicking the cross-sectional data.

Finally, the skewness of investment rates in our model changes very little during recessions, but increases strongly in its immediate aftermath (bottom-right panel). The effect in the data shown in Figure 2.3 is somewhat ambiguous for this moment. During the first three recessions in our sample the skewness stays fairly constant, whereas it drops sharply in the last two. On the other hand, it increases strongly following four of the five recessions in the data. While our model is consistent with the increase in skewness following most recessions, it doesn't produce the reduction in skewness observed for some recessions empirically.

**Aggregates.** Figure 2.6 shows how both aggregate quantities and misallocation measures respond to a recession. On impact of the shocks, the degree of capital misallocation creases by 30% (top-right plot), and the output gap relative to the frictionless benchmark increases. While all three quantities fall on impact, investment falls more than consumption due to the household's high EIS. After the initial sharp drop, quantities slowly adjust to their long-term paths.

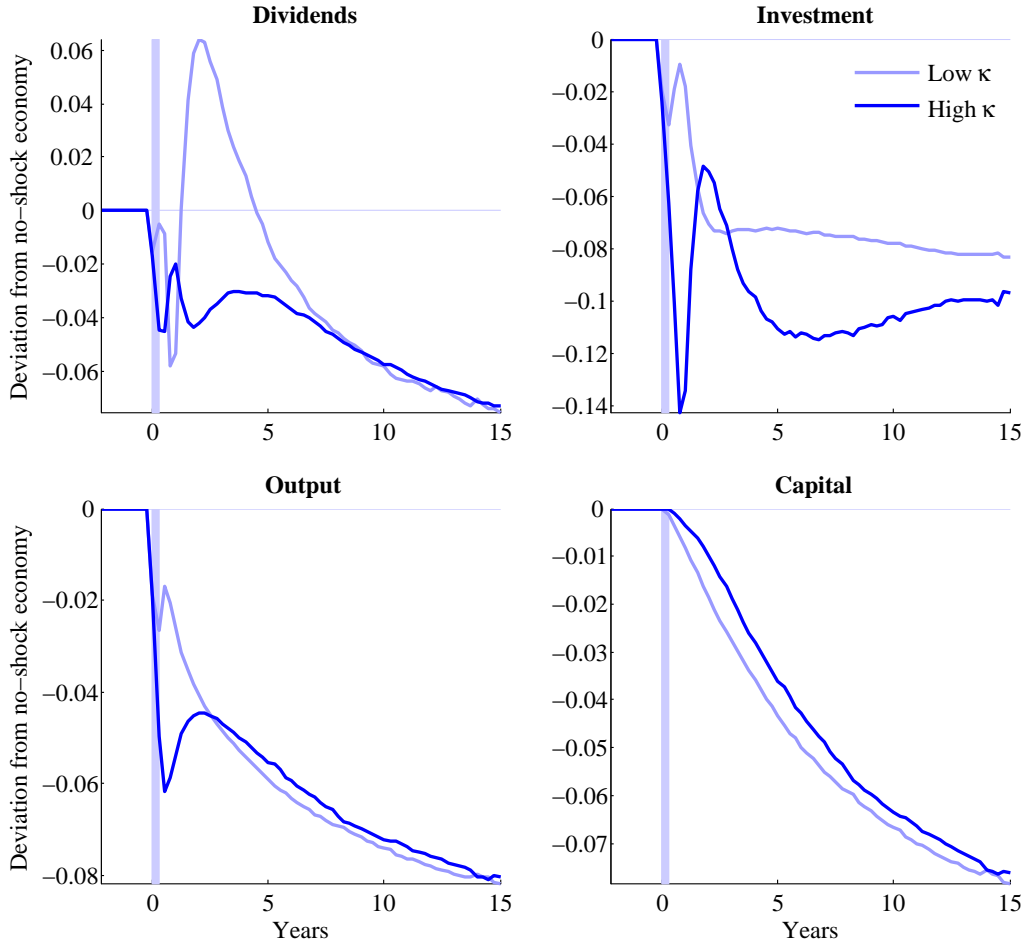
To understand these aggregate dynamics in more detail, we explore how firm policies and their distribution vary over the business cycle. As explained in the section 2.3, heterogeneity in firms policies arises solely due to differences in firms' capital to productivity ratio  $\kappa$ , but for the equilibrium the joint density over capital and productivity matters.



**Figure 2.6:** Impulse response: Aggregates

In Figure 2.7, we plot the response of several policy-related firm characteristics separately for low and high  $\kappa$  firms. In normal times, low  $\kappa$  firms have received positive idiosyncratic shocks and are investing firms, whereas high  $\kappa$  firms have received negative shocks and are inactive or disinvesting ones. The figure shows that, while output and investment of both types drop in recessions, output and investment of high  $\kappa$  firms drop dramatically and significantly more than low  $\kappa$  firms. This effect arises because high  $\kappa$  firms are often the ones hit by an adverse Poisson shock.

A key result that we show in Figure 2.7 is that firms' responses to an aggregate



**Figure 2.7: Cross-Sectional Impulses by Firms'  $\kappa$ .**

$\kappa$  is closely linked to the book-to-market ratio. Shaded areas denote a recession of length of two quarters. Firms are compared with their counterparts in a no-shock scenario. Units are relative deviations from no-shock scenario. Firms are sorted every  $t - 1$  periods and their behavior are recorded at time  $t$ .

shock are also unsynchronized. This comes from the fact that firms that are very close to inaction just before a recession cannot adjust as much as firms with a very low  $\kappa$ . Proportionally, firms with high  $\kappa$  tend to wait longer before adjusting because they are closer to the inaction region (need some time to depreciate back to optimal target). This unsynchronized response can also be observed in output, where the hump response for low- $\kappa$  firms precede the hump response for high  $\kappa$  firms.

There is an interesting complementarity between low and high  $\kappa$  firms in terms of generating persistence and amplification. Figure 2.7 shows that the output of low  $\kappa$  firms – which are the most efficient and have the lowest book-to-market ratio – responds

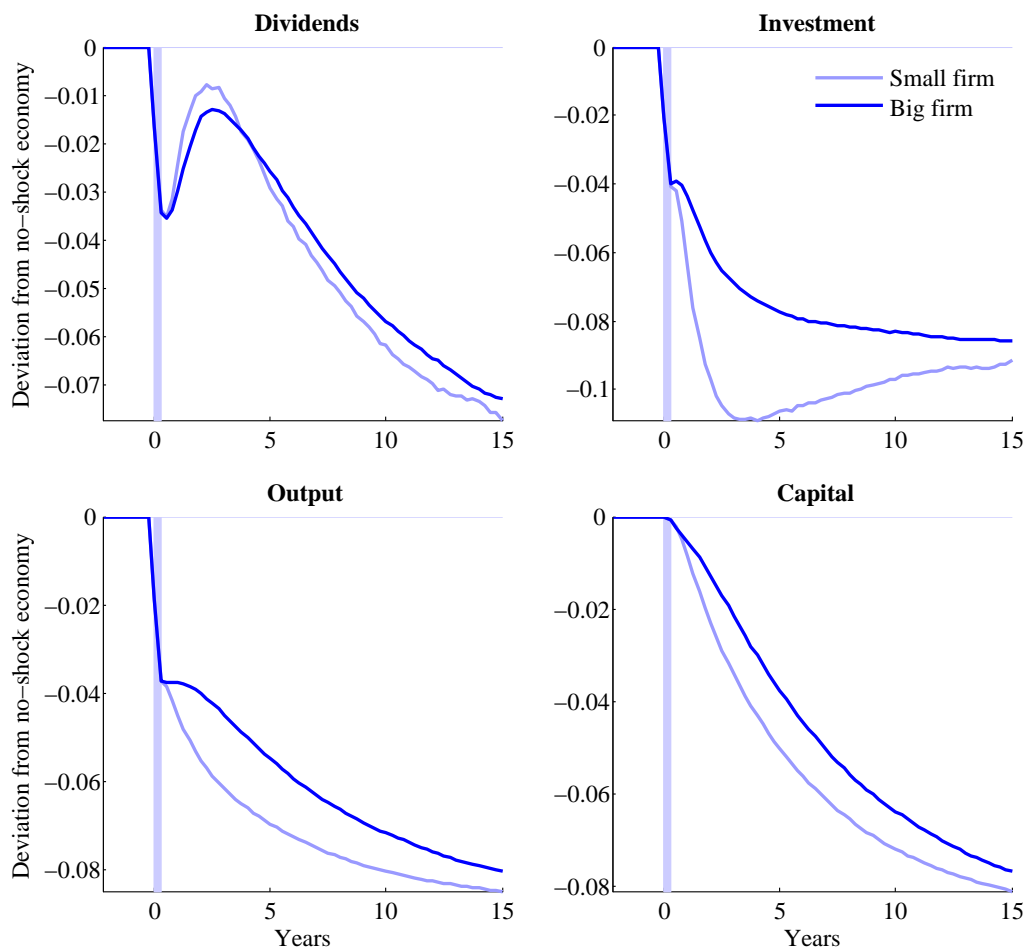
proportionally much less than that of high  $\kappa$  firms. As a consequence, inefficient firms are responsible for the biggest drop in output on impact. This stems from the fact that most of the inefficient firms are in the vicinity of the inaction region, and thus their behavior is very sensitive to the business cycle. On the other hand, the efficient firms are far from the inactive region and thus less sensitive to business cycle fluctuations.

In Figure 2.8, we plot the average response of small and big firms. The response of small and big firms differ in our model economy, even though optimal firm decisions are scale-free due to our assumption of random walk productivity. This stems from the fact that firm size correlates with its inefficiency  $\kappa$  (book-to-market ratio) because of life-cycle effects (see Figure 2.4).

The main implication of Figure 2.8 is that aggregate impulses mimic the behavior of big firms. This effect arises because the power law in firm size implies that a small fraction of large firms dominates the model economy. In recessions, both small and big firms reduce investment. Yet large firms are on average more constrained than small firms and thus their investment cannot fall much. In contrast, small firms are on average less constrained and their investment is very responsive to shocks. In case small and large firms had a similar impact on aggregate consumption, the investment behavior of small firms could offset the impact of large firms. But the power law in firm size implies that the share of dividends coming from small firms is too small to compensate for the loss in dividends generated by large firms.

Our model economy generates a hump-shaped aggregate consumption response to a skewness shock. The explanation for this behavior lies in our investment frictions and the different speeds of adjustment of investment compared to output. First of all, note that on impact investment decreases dramatically (investment targets fall as consumption decreases). At the aftermath of the recession, a large proportion of firms is located in the inaction region (their  $\kappa$  skyrockets when hit by a jump). In such region, firms are constrained and cannot disinvest, which translates into a plateau as observed during the first year following the recession. Meanwhile, output smoothly falls as capital slowly depreciates. However, in the later years, investment decreases much





**Figure 2.8: Cross-Sectional Impulses by Firm Size.**

Shaded areas denote a recession of length of two quarters. Firms are compared with their counterparts in a no-shock scenario. Units are relative deviations from no-shock scenario. Firms are sorted every  $t - 1$  periods and their behavior are recorded at time  $t$ .

faster than output as more firms become unconstrained. It is the difference of speed between investment (slow at the beginning and fast later on) and output (smooth) that gives rise to the hump shape of consumption (since consumption is constrained by output and investment).

### 2.5.3 Power Law and Consumption Dynamics

The combination of a unit root in idiosyncratic productivity and random exit in our model results in a firm size distribution whose right tail exhibits a power law. In particular, the log right tail probabilities (above the 50<sup>est</sup> percentile), with firm size

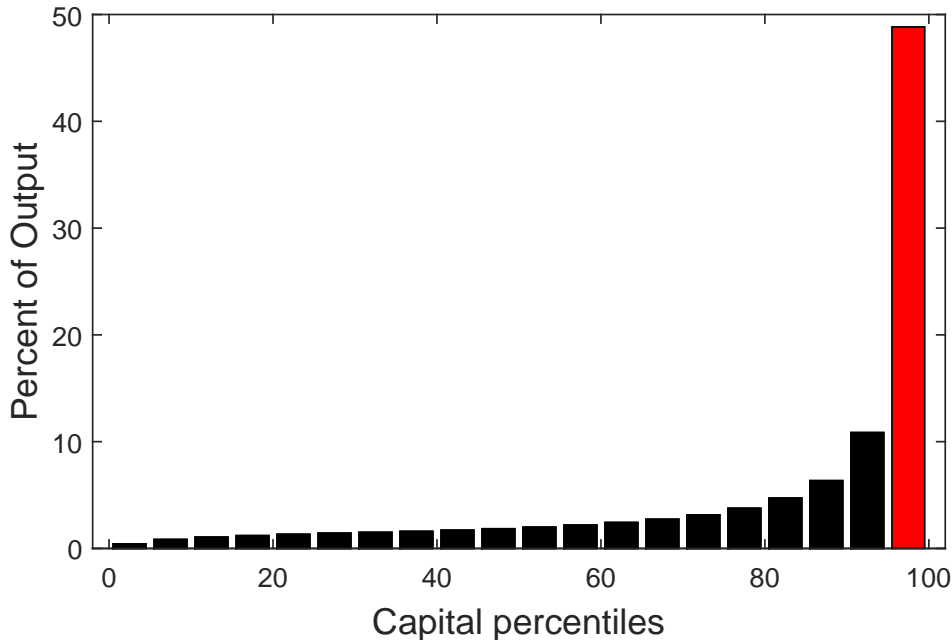
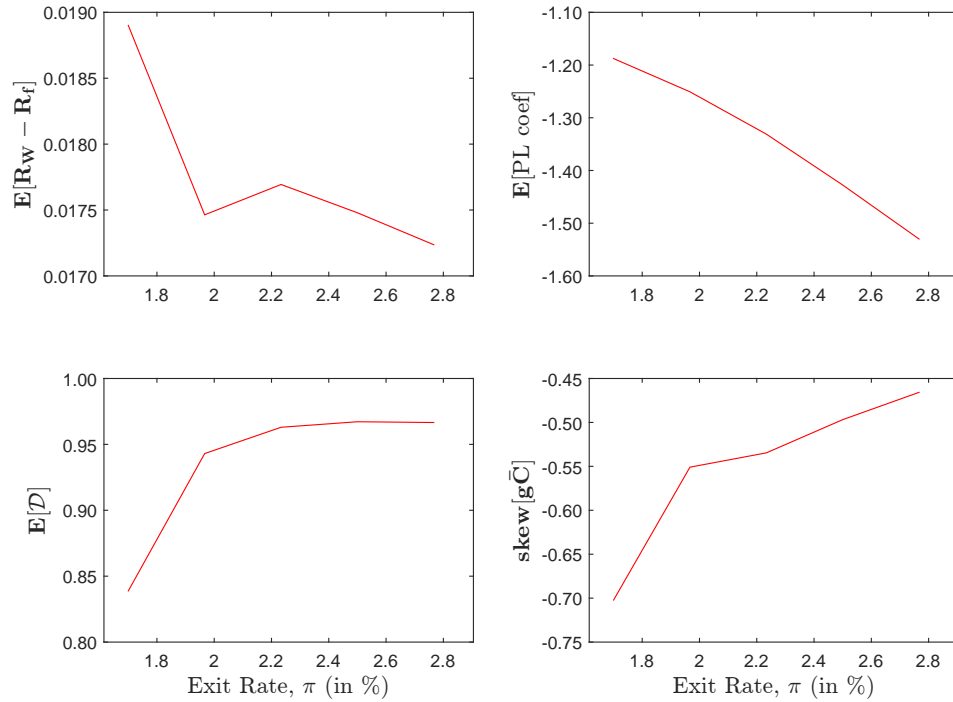


Figure 2.9: Output Concentration

measured by log capital, lie on a straight line with a slope of  $-1.25$  for our benchmark estimation results. This means that the firm size distribution is well approximated in its right tail by a Pareto distribution with right tail probabilities of the form  $1/S^\xi$ , with a tail index  $\xi$  of 1.25. To illustrate the economic effect of the power law, Figure 2.9 shows the degree of output concentration implied by our benchmark calibration. Specifically, it shows the fraction of aggregate output produced by firms in various percentiles of the capital distribution.<sup>15</sup> On average, the largest 5% of firms (in terms of capital) produce about 30% of aggregate output.

Due to the importance of large firms for output and consumption, permanent negative shocks to their productivity are particularly painful. Consequently, the drop in dividends from the mass of constrained firms is large, given that they are large in size (see the discussion in Section 2.5.1). While unconstrained firms increase dividends by reducing investment, they are smaller so that they are not able to offset the impact of large constrained firms on aggregate consumption. In other words, the firm size distri-

<sup>15</sup>To produce the figure, we simulate a continuum of firms and record the fraction of output produced by the 5% smallest firms (in terms of capital), firms between the 5th and 10th size percentiles, etc. in each period. We then average these fractions across all periods in the simulation.



**Figure 2.10:** Comparative statics for the exit rate

bution in combination with negative jumps in productivity implies that it is difficult for the representative household to smooth consumption during recessions. In contrast, in models with log-normal productivity distributions the size difference between constrained and unconstrained firms is small so that the groups offset each other.

Figure 2.10 illustrates this channel quantitatively via a comparative statics exercise that varies the exit probability  $\pi$ . A larger exit probability implies that firms survive shorter on average, which reduces the mass of large firms. The top-right panel shows that the power law coefficient ( $-\xi$ ) decreases as  $\pi$  increases, meaning that the right tail of the firm size distribution becomes thinner. This implies that it becomes easier to smooth consumption by offsetting the losses in consumption from large firms with the dividend payments of unconstrained (and relatively smaller) firms. As a consequence, the left skewness of consumption growth is reduced (bottom-right panel), the loss in output relative to the frictionless benchmark decreases (bottom-left panel), and risk premia decline (top-left panel).

## 2.6 Conclusion

We study the impact of capital misallocation on business cycle dynamics and risk premia in a dynamic, stochastic general equilibrium model with firm heterogeneity. In our model economy, firms face irreversible investment decisions, exit, and persistent idiosyncratic and aggregate productivity shocks. The representative household has Epstein-Zin preferences. We solve for the equilibrium dynamics of the Epstein-Zin pricing kernel by aggregating dividends and firm-values across heterogeneous firms to obtain consumption and wealth.

We differ from the existing literature by focusing on time varying skewness in the cross-section of sales growth. It is well-known that sales growth dispersion is strongly countercyclical. Less well-known is that this countercyclical dispersion is mainly driven by the left tail of the sales growth distribution. By just looking at the cyclicity of the IQR, one might conclude that in recessions, firms have more dispersed positive and negative productivity draws. But the cyclicity of Kelly skewness indicates that in recessions significantly more firms have extreme negative productivity draws. Our model is equipped to match this empirical fact because productivity is not only driven by Gaussian shocks but also by negative Poisson shocks.

Even though the model is only driven i.i.d. innovations, it replicates well the level, volatility, and persistence of capital misallocation in the US economy, which we define as correlation between size and productivity. In the frictionless benchmark model, capital and productivity is perfectly aligned across firms, implying a correlation of one, and output growth is not persistent. In the full model, investment is irreversible, implying that unproductive firms have excess capital. While the impact of capital misallocation on output and consumption are in the typical neoclassical model short lived, permanent Poisson shocks render misallocation distortions long lasting. Quantitatively, output and consumption growth become more volatile and output growth is more persistent because capital is sticky. Importantly, consumption growth is left skewed and leptokurtic, as in the data.

# Appendix A

## Computational Appendix

This appendix provides details about the algorithms used to solve and estimate the model in “The Macroeconomics of Consumer Finance”. Interested readers are encouraged to download the full commented code on my webpage.<sup>1</sup>

### A.1 Value Function Iteration

The household’s optimization problem is solved by iterating on the system (1.1)-(1.5) defined in Section 1.2. The household’s state variables are  $\{N, \theta, \pi\}$ . The net worth grid  $\vec{N}$  contains 901 points equally spaced from  $-3$  to  $6$ . The vector of preference parameters  $\theta$  is simply a grid of three indexes  $\{1,2,3\}$  corresponding to “high” (low  $\psi$ ), “medium”, “low” (low  $\beta$ ). The employment grid  $\vec{\pi}$  contains 10 points equally spaced by 1% increment from 89% to 98% (alternatively the unemployment grid goes from 2% to 11% by 1% increment). The next-period balance grid  $\vec{B}'$  (control variable) contains 1,801 points equally spaced from  $-6$  to  $3$ . Values outside of the grid are linearly interpolated. I iterate on policies to accelerate the convergence whenever possible.

To obtain a smooth bond schedule as shown in Figure 1.1 (top right panel), I compute the household’s expectation over labor productivity shocks  $z'$  with a fine probability grid of 100 points (I initially set up a grid of 9 quadrature nodes, then discard the 7 interior points and replace them with 98 equally spaced points).

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<sup>1</sup>The algorithms are implemented in Matlab R2014a. Webpage: [www.ehouarne.com](http://www.ehouarne.com).

## A.2 Non-Stochastic Simulation

To obtain a smooth net worth distribution as shown in Figure 1.3, I update the household distribution over  $S \equiv (N, \theta)$  with a non-stochastic method. This approach takes advantage of the fact that since  $\log z \sim \mathcal{N}\left(-\frac{\sigma_z^2}{2}, \sigma_z^2\right)$  (after properly adjusting for the truncation below  $\underline{z}$ ) and  $N' = [\varepsilon' + (1 - \varepsilon')\varrho](1 - \tau)z' - B'(S, \pi)$  is a linear transformation of  $z'$ , then we can write the probability density function of  $N'$  conditional on states  $(S, \pi)$  and future realized shocks  $(\varepsilon', \phi')$  as

$$f_{N'|S, \pi; \varepsilon', \phi'}(N') = \left[ \left( N' + \frac{B(S; \pi)}{x'} \right) \sigma_z \sqrt{2\varpi} \right]^{-1} \\ \times \exp \left\{ - \left[ \frac{1}{\sigma_z \sqrt{2}} \log \left( \frac{N' + B(S; \pi)}{[\varepsilon' + (1 - \varepsilon')\varrho](1 - \tau)} \right) + \frac{\sigma_z}{2\sqrt{2}} \right]^2 \right\},$$

where  $\varpi$  is the number pi. Notice that  $f_{N'|S, \pi; \varepsilon', \phi'}$  is a shifted log-Normal probability density function with support  $(-B'(S; \pi), +\infty)$ . It is then easy to get the distribution  $\mu'$  by doing the following steps: (1) evaluate the pdf at a 2-dimensional grid  $\langle \vec{N}, \vec{N}' \rangle$  and call this matrix  $\mathbf{W}_1$ , (2) multiply  $\mathbf{W}_1$  with proper probability weights associated with  $(N, \varepsilon')$  and call this matrix  $\mathbf{W}_2$ , (3) sum across rows of  $\mathbf{W}_2$  to obtain the new probability weights  $\mu'|\theta$ , (4) repeat the step for each preference type  $\theta \in \Theta$  and multiply with associated type probabilities to obtain the entire distribution  $\mu'$  (which is defined over a two-dimensional grid  $\langle \vec{N}, \vec{\theta} \rangle$ ).

## A.3 Steady State and Estimation

To complement the information provided in subsection 1.3.4, I explain in details how I solve the model steady state given a set of parameter estimates, and how I find these estimates.

Given a set of parameter estimates and a guessed steady state employment rate  $\pi^*$ , I solve the household's problem by value function iteration as exposed in subsection A.1. Given the resulting optimal policies, I then simulate the economy by updating the distribution  $\mu$  as described in subsection A.2 until reaching a stationary form. Once it has reached a stationary form, I use it to compute the aggregate consumption (public

and private)  $G(\pi^*) + C(\pi^*)$  (implicit function of  $\pi^*$ ). All these steps are wrapped in a routine that computes the excess demand  $G(\pi^*) + C(\pi^*) - \pi^*$  as a function of  $\pi^*$ . I use a bisection algorithm to find the (unique) root of this function in the interval  $\pi^* \in [0.89, 0.99]$ . Figure 1.2 plots such function.

To estimate the model, I search for the parameter values that minimize the method-of-moment objective function with a parallelized genetic algorithm. The algorithm works as follows: (1) set up an initial vector of random parameter guesses, (2) for each guess, find the steady state, compute all the moments of interest and compute the method-of-moment objective criterion, (3) create new guesses for the next round by combining and mutating only the current best guesses (the ones with the smallest objective criteria). Iterate on this process until the objective criterion does no longer improve.

## A.4 Approximate-Aggregation Equilibrium

To complement the information provided in subsection 1.4.3, I describe in the following steps how I solve for an approximate equilibrium: (1) guess a set of forecasting rule coefficients  $\{\alpha_0^*, \dots, \alpha_3^*\}$  and use it to solve the household's optimization problem, (2) use the forecasting rule to generate a production path and simulate the economy along this path, (3) at each point in time, compute the implied aggregate public and private consumption (measured in employment units), (4) compute the sup-norm (denoted by  $n^*$ ) of the difference between the guessed production path and the implied path of aggregate public and private consumption.

The steps (1)–(4) are wrapped in a routine that gives  $n^*$  as a function of  $\{\alpha_0^*, \dots, \alpha_3^*\}$ . The smallest sup-norm  $n^*$  can then be found by calling an optimizer such as one based on the derivative-free Nelder-Mead Simplex Method. As an initial guess for  $\{\alpha_0^*, \dots, \alpha_3^*\}$ , I assume that the forecasting rules are constant and solve the model with no aggregate shock. I then solve the full model with aggregate uncertainty by homotopy where I gradually increase the persistence and volatility of the aggregate financial shock  $\phi$  and use the solution of each increment  $i$  as a guess for the next increment  $i + 1$ .

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