

ESSAYS ON MICROECONOMIC THEORY AND ITS APPLICATIONS

by

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To Ersin, Gulnur, Osman, and Semiha

Dissertation Abstract

My thesis is comprised of three chapters. In the first chapter, coauthored with Stephen Spear, we study endogenous shocks driven by collective actions of managers. A good recent example of this is how the collective actions of bank managers engaging in securitization of loans ended up freezing the world financial markets in 2008. Motivated by examples like the 2008 crisis, we analyze how endogenous shocks driven by collective actions of managers impact social welfare by using a dynamic general equilibrium model. We first show that such endogenous shocks render competitive equilibrium allocations inefficient due to externalities. We establish that a socially optimal allocation can only be attained by paying managers the socially optimal wages, and this can be achieved by imposing wage taxes (or subsidies) on managers. Finally, we extend the model by allowing for information asymmetry, and show that it is not possible to attain a socially optimal (i.e., first-best) allocation. We instead examine second-best allocations.

In the second chapter, I study whether coalitions of consumers are beneficial to consumers when producers have market power. I refer to coalitions of consumers as *consumer unions* and the number of consumers in a union as *union size*. By constructing an imperfect competition model in a general equilibrium setting, I gauge how union size impacts consumer welfare. I establish, contrary to the literature on coalitions, that consumer welfare decreases with union size when the union size is above a threshold. I also prove that consumer unions discourage producers' investments, which may have repercussions for long-term consumer welfare. Finally, I show that depending on the production technology, having a higher number of producers can be more effective in promoting consumer welfare than consumer unions.

In the third chapter, coauthored with Stephen Spear, we study imperfectly competitive production economies in which technology exhibits arbitrary returns to scale including increasing returns. Increasing returns are well-documented empirically and widely recognized as the driving force of economic growth. Recognizing the significance of increasing returns, the general equilibrium literature has tried to incorporate it into the conventional general equilibrium framework. These attempts have usually been unsuccessful because of fundamental incompatibilities between increasing returns and the competitive paradigm. By using an imperfectly competitive model in a general equilibrium setting - in particular, the market game model, we prove the existence of equilibrium for arbitrary returns to scale in production including increasing returns. Via an extended example, we demonstrate the relationship between the number of increasing-returns firms and other parameters of the model.

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Chapter 1

1 Managerial Compensation with Systemic Risk: A Dynamic General Equilibrium Approach

1.1 Introduction

Economists have always been of two minds when it comes to modeling uncertainty. The earliest approach, which is called the *state-of-nature* (or *Savage*) approach, models the probabilities of possible outcomes as fixed and independent of agents' actions. Modeling shocks as exogenous and independent of agents' actions is reasonable in traditional models where agents take the states of the world as given. However, the state-of-nature approach may be inadequate when a model incorporates agents' actions that can affect the state of the economy, and hence the well-being or distress of firms and households in the economy.

A good recent example of how agents' actions affect the state of the economy is how the seizing up of the mortgage-backed securities market ended up freezing the world financial markets in 2008. In principle, bundling mortgages from different areas and different income profiles made a lot of sense as a way of diversifying the risk of a borrower defaulting on the mortgage. Packaging these up as securities to sell to large numbers of investors also further diversified the default risk. One drawback of the process, however, was the decoupling of loan origination from risk-bearing. Because of the moral hazard problem created by this, in a number of un- or under-regulated real estate markets, far too many so-called *sub-prime loans* were made to borrowers who clearly could not afford to carry the mortgage. This also generated an increase in housing values, which led a number of homeowners to increase consumption. When the inevitable defaults started, and the housing price bubble deflated, credit markets ended up in panic because no one knew what the various mortgage-backed securities were actually worth. Securitization of loans actually reduced an individual bank's risk exposure, but the collective actions of all bank managers engaging in securitization and attendant marketing of bad loans resulted in a spectacular failure.

Motivated by examples like the 2008 crisis, we study how endogenous shocks driven by collective actions of managers impact social welfare. In particular, we ask the following research questions: (Q1) Do markets deliver socially optimal allocations in the presence of endogenous

shocks? (Q2) If not, how can socially optimal allocations be implemented? (Q3) How should managers be compensated in the presence of endogenous shocks? Because managers' actions may be unobservable, we also analyze how endogenous shocks impact social welfare in the presence of information asymmetry. Specifically, we ask: (Q4) Is it possible to attain socially optimal (i.e., first-best) allocations in the presence of information asymmetry, and if not, how can second-best allocations be implemented?

To address these questions, we construct a dynamic general equilibrium model based on the overlapping generations framework. Our choice of the overlapping generations framework is based on the observation that for most established companies, the top tier managers are middle-aged or older.¹ This occurs because management activities are qualitatively different from even the most technically demanding production activities that firms engage in. Managers fundamentally work to minimize the risk of bad outcomes in their firms' production activities. This task necessitates a degree of comprehension of the overall structure and function of the firm that even very well-educated line workers typically do not have. Obtaining this knowledge requires a combination of early on-the-job training at entry level activities, typically followed by the attainment of an advanced degree (generally an MBA), and then another stint on the managerial career ladder learning the idiosyncrasies of the firm's overall performance. Because this all takes time to accomplish, we see a natural life-cycle division of labor across the age spectrum: young workers provide unskilled labor to production while old workers manage firms' production activities. To reflect this natural dichotomy, we work with an overlapping generations model in which young agents serve as line workers and old agents serve as managers.

To answer the research questions (Q1)-(Q4) listed above, we first show the existence of competitive equilibrium allocations (Proposition 1). We also show that endogenous shocks driven by collective actions of managers render competitive equilibrium allocations inefficient (Proposition 2) due to externalities. We establish that a socially optimal allocation can only be attained by paying managers the socially optimal wages, and this can be achieved by imposing wage taxes on managers (Theorem 1). Finally, we extend the model by incorporating information asymmetry,

¹According to Spencer Stuart, 87% of S&P 100 companies have CEOs older than 50 years old. For S&P 500 companies, median age of CEOs is 56, and average age of CEOs is 55.

and show that it is not possible to attain a socially optimal (i.e., first-best) allocation. We instead derive second-best allocations (Proposition 3).

Related Literature: Models in which agents' actions influence the probability distributions have been used in moral hazard literature since seminal papers of Holmstrom (1979) and Mirrlees (1999). Shorish and Spear (2005) apply this idea to the Lucas asset pricing model. Applying this idea to simple dynamic general equilibrium models is a more recent development. For example, Magill and Quinzii (2009) study neoclassical capital accumulation model in which firms' investment decisions control the probabilities of possible outcomes.

Another stream of related literature is overlapping generations studies that focus on the optimality of competitive equilibrium allocations. Economists have been curious about the optimality of competitive equilibria since the early models reveal the possibility of inefficient competitive equilibria. First, Cass (1972) and Gale (1973) provide ways to determine whether competitive equilibrium allocations are Pareto optimal. Then, Peled (1982) demonstrates Pareto optimality of competitive equilibria in a pure exchange model where agents live two periods. Aiyagari and Peled (1991) examine under which conditions competitive equilibria are Pareto optimal in a model with two-period lived agents. Chattopadhyay and Gottardi (1999) prove the optimality of competitive equilibria in a model where more than one good is traded at each period. Finally, Demange (2002) gives a comprehensive characterization of different optimality notions.

The remainder of the paper is organized as follows. In §3.2, we elaborate on model ingredients, and define competitive equilibria. In §1.3.1, we analyze the existence and optimality of competitive equilibria and managers' wages; in §1.3.2, we extend the model by allowing for information asymmetry. In §3.4, we conclude with a brief discussion; and we present all proofs in Appendix.

1.2 Model

We consider an infinite time horizon model that consists of a continuum of identical agents, a continuum of identical firms, a single consumption good, and an asset (i.e., equity). We work with an overlapping generations model in which agents become economically active at the age of 20, and live for two periods, each of which spans 30 years. Agents become young in the first periods of their lives and old in the second periods. At each period, new young agents are born, and

young agents of the previous period become old agents.² Firms produce the consumption good using labor inputs provided by agents, and pay wages in return. Moreover, agents purchase the consumption good, and hold ownership shares of firms (i.e., equities).

To reflect the natural life-cycle division of labor, young agents provide unskilled labor a_y whereas old agents provide skilled labor a_o . By the same reasoning, young agents serve as line workers while old agents serve as risk managers, and control firms' production processes.³ In exchange for their services, young agents earn (unit) wages ω_y , and old agents earn (unit) wages ω_o . Young agents experience no disutility from labor, and hence supply their labor inelastically. Old agents, on the other hand, experience disutility from labor, and their disutility function $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is increasing and strictly convex in their labor inputs a_o .

Agents' preferences are given by a von Neumann-Morgenstern utility function

$$E[U] = u(c_y) + \beta E[u(c_o) - \phi(a_o)],$$

where U is the utility function of an agent throughout his life; $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ are period utility functions, which are twice continuously differentiable, strictly increasing, strictly concave, and satisfy Inada conditions; c_y and c_o are consumption values of young and old agents, respectively; and $\beta \in (0, 1]$ is a discount factor.

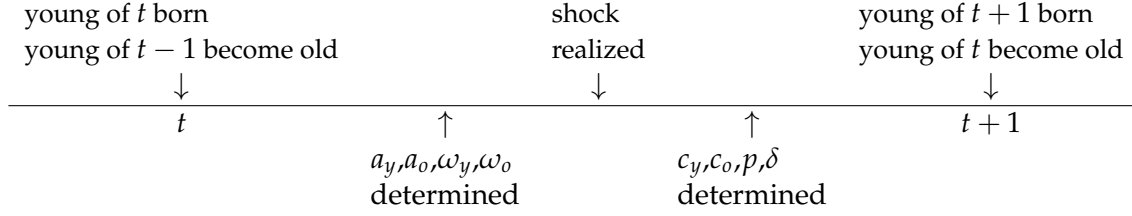
Firms engage in production processes determined by two factors: a deterministic and a stochastic component. The deterministic component is represented by a production function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which uses young agents' labor inputs a_y , and is increasing and concave in a_y . The stochastic component is represented by an individual output shock. For simplicity, output shock z takes on high (H) or low (L) value, i.e., $z \in \{z^H, z^L\}$, where $z^H > z^L$. Individual output shocks facing each firm are (perfectly) correlated, which in turn generate systemic risk.⁴ Systemic risk is controlled by the collective actions of managers (i.e., old agents) - in particular, probabilities of possible outcomes are influenced by old agents' labor inputs a_o . Probability function of high output shock $\pi : \mathbb{R}_+ \rightarrow [0, 1]$ is increasing and concave in a_o . The product of the deterministic and the stochas-

²We assume an initial old generation at period 0 to compare allocations at different periods.

³In the real world, managers have other duties but these duties are not essential for the problem we analyze, so our model focuses on their risk management activities.

⁴The assumption of perfectly correlated shocks is necessary for tractability, but it does not drive our results. As long as systemic risk is influenced by managerial actions, our results follow.

Table 1: Time Line



tic components is equal to total output γ , i.e., $\gamma = zf(a_y)$. Total output γ and production $f(a_y)$ are observable, so output shock z is also observable. Output shocks realized in different periods are independent, but need not be identically distributed because if a_o differs across periods, $\pi(a_o)$ differs, and hence the probability distribution across periods differs.

Agents hold ownership shares of firms, i.e., equities. Equity e is a productive asset, and the amount of equity is fixed and normalized to one. Initially, old agents possess equities (i.e., own the firms),⁵ but they sell these equities to young agents at a price p . At each period, equity holders earn dividend δ , which is equal to the remaining output after wages are paid, i.e., $\delta = \gamma - a_y\omega_y - a_o\omega_o$. Dividend δ takes on high (H) or low (L) value depending on the output shock, i.e., $\delta \in \{\delta^H, \delta^L\}$, where $\delta^H > \delta^L$.

As depicted in time line in Table 1, the sequence of events is as follows. First, new young agents are born, and young agents of the previous period become old. Second, labor inputs of young and old agents a_y and a_o , their wages ω_y and ω_o are determined, and hence labor markets clear. Third, output shock z is realized. Finally, depending on the output shock, consumption values of young and old agents c_y and c_o , equity price p , and dividend δ are determined. Thus, the consumption good and equity markets clear.

We next define competitive equilibria as follows. A *competitive equilibrium* allocation is a collection of choices for consumption values c_y and c_o , wages ω_y and ω_o , labor inputs a_y and a_o , equity holding e , equity price p , and dividend δ such that agents maximize their utilities, firms maximize their profits, and all markets clear. In what follows, we elaborate on components of a competitive equilibrium. First, agents face the following budget constraints in which the consumption good is

⁵Old agents are both managers and owners of the firms because if managers and owners were different agents, then managers would not necessarily act in the best interest of owners. This would create a moral hazard problem, which would distort the results of the paper.

numeraire (i.e., the price of the consumption good is normalized to one)

$$c_y = a_y \omega_y - pe \text{ and } c_o = a_o \omega_o + (p + \delta)e.$$

Agents solve the following problem by deciding on consumption values c_y and c_o , labor inputs a_o , and equities e

$$\begin{aligned} \max_{c_y, c_o, a_o, e} & u(c_y) + \beta E[u(c_o) - \phi(a_o)] \\ \text{s.t. } & c_y = a_y \omega_y - pe \text{ and } c_o = a_o \omega_o + (p + \delta)e. \end{aligned}$$

Second, firms solve the following problem by deciding on their demand for young and old agents' labor inputs

$$\max_{a_y, a_o} \left[\pi(a_o) z^H + (1 - \pi(a_o)) z^L \right] f(a_y) - a_y \omega_y - a_o \omega_o.$$

Third, the consumption good, equity, and labor markets clear. The market clearing condition for the consumption good (i.e., the overall resource constraint) is

$$c_y + c_o = \gamma = a_y \omega_y + a_o \omega_o + \delta.$$

Equity market clears when demand for equity is equal to the supply of equity. Because the amount of equity is fixed and normalized to one, the market clearing condition for equity is $e = 1$. Finally, labor markets clear where demand for labor is equal to the supply of labor. Because young agents supply their labor inelastically, the market clearing condition for young agents' labor is $a_y = \bar{a}_y$.

1.3 Analysis

This section proceeds as follows. In §1.3.1, we examine the existence and optimality of competitive equilibria, and managers' wages; in §1.3.2, we extend the model with information asymmetry.

1.3.1 Main Analysis

In our analysis, we restrict attention to *strongly stationary* equilibria, as is common in the literature (e.g., Peled 1982, Aiyagari and Peled 1991) because such equilibria allow for derivation of analytical results, and make the interpretation of these results easier. In the presence of strongly stationary competitive equilibria, endogenous variables depend only on the current realization of output shocks, i.e., endogenous variables do not depend on past realizations or other endoge-

nous variables (e.g., lagged variables). The following proposition shows the existence of strongly stationary competitive equilibria.

Proposition 1 *There exist strongly stationary competitive equilibria for overlapping generations economies.*

Given this existence result, we focus on strongly stationary equilibria for the rest of the paper.

We next shift our focus to efficiency issues, and investigate whether competitive equilibrium allocations are Pareto optimal. We define Pareto optimality as follows. A *Pareto optimal* allocation is a solution to the following planner's problem in which the social planner maximizes the weighted average of agents' utilities (where $\alpha \in [0, 1]$)

$$\max_{c_y^H, c_y^L, c_o^H, c_o^L, a_o} (1 - \alpha)E[u(c_y)] + \alpha(E[u(c_o)] - \phi(a_o)) \text{ subject to } c_y^S + c_o^S = \gamma^S. \quad (1)$$

The following proposition shows inefficiency of competitive equilibrium allocations.

Proposition 2 *Laissez-faire competitive equilibrium allocations are not Pareto optimal.*⁶

The intuition of Proposition 2 is as follows. Old agents work as risk managers, and earn wages equal to their marginal contributions to firms they work for, so there is no externality on the firm side. On the agent side, collective actions of old agents determine the probabilities of possible outcomes and hence the state of the economy. This situation creates two externalities. First, while solving his optimization problem, an individual old agent does not take into account the fact that old agents' collective actions affect the state of the economy and in turn his dividend.⁷ Second, old agents choose their actions without considering the effects of these actions on young agents although these actions influence young agents' wages through the state probabilities. These externalities render competitive equilibrium allocations inefficient. To restore efficiency, these externalities must be internalized.

The following theorem establishes how externalities can be internalized, and how Pareto optimal allocations can be implemented.

⁶This proposition also holds under *conditional* optimality notion, but we report *ex ante* optimality because it fits our model best. In our model, output shocks are driven by collective actions of old agents, and the resulting state probabilities affect both young and old agents, so we take unconditional expectation, and use *ex ante* optimality. However, in models wherein state probabilities are fixed and independent of agents' actions, conditional expectation is taken when the agent is young, and hence *conditional* optimality is used.

⁷Even if old agents consider the effect of their collective actions on their dividends, competitive equilibria still become Pareto suboptimal, and the other results also follow.

Theorem 1 *To attain a Pareto optimal competitive equilibrium, old agents must be paid optimal wages*

$$\omega_o^* = \frac{(1 - \alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) + \alpha\pi'(a_o)(u(c_o^H) - u(c_o^L))}{\alpha E[u'(c_o)]}.$$

Moreover, a Pareto optimal allocation $\{c_y^{H,*}, c_y^{L,*}, c_o^{H,*}, c_o^{L,*}, a_o^*\}$ can be implemented if the social planner imposes the following wage tax t_ω and equity tax t^s for $s = H, L$

$$t_\omega = \pi'(a_o^*)(z^H - z^L)f(\bar{a}_y) - \frac{\phi'(a_o^*)}{E[u'(c_o^*)]} \text{ and } t^s = \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{s,*})p^{s,*}}{u'(c_o^{s,*})}.$$

Theorem 1 implies that old agents' competitive wages are not socially optimal. Moreover, externalities created by old agents' collective risk management activities will be internalized if old agents are paid socially optimal wages. This occurs because old agents' optimal wages consider the effect of old agents' managerial actions on their dividends and young agents' wages.

To guarantee that old agents are paid optimal wages, the social planner (e.g., government) needs to impose wage taxes (or subsidies) on old agents, and the size of the wage tax must be equal to the competitive wage minus the optimal wage. By the help of the wage tax, externalities will be internalized. To support a Pareto optimal allocation as a competitive equilibrium, the social planner also needs to impose equity tax. Before equity tax is imposed, the first order condition with respect to equity is

$$E[u'(c_o)(p + \delta)] = u'(c_y^s)p^s. \quad (2)$$

Given that the agent purchases equity when young at a price p , sells it when old, and earns dividend δ , the left hand side of (2) is the marginal benefit of equity, and the right hand side is the marginal cost of equity. The marginal benefit of equity is equal to the marginal cost of equity at a competitive equilibrium. However, a Pareto optimal allocation does not necessarily satisfy this equilibrium condition. To support a Pareto optimal allocation as a competitive equilibrium, the social planner imposes equity tax t^s whose marginal cost is equal to the difference between the two terms in (2). Hence, we obtain

$$E[u'(c_o)(p + \delta)] - u'(c_y^s)p^s = t^s u'(c_o^s). \quad (3)$$

(3) is an Euler equation for equity because the marginal benefit of equity is equal to the (new) marginal cost of equity (initial marginal cost of equity plus the marginal cost of equity tax).

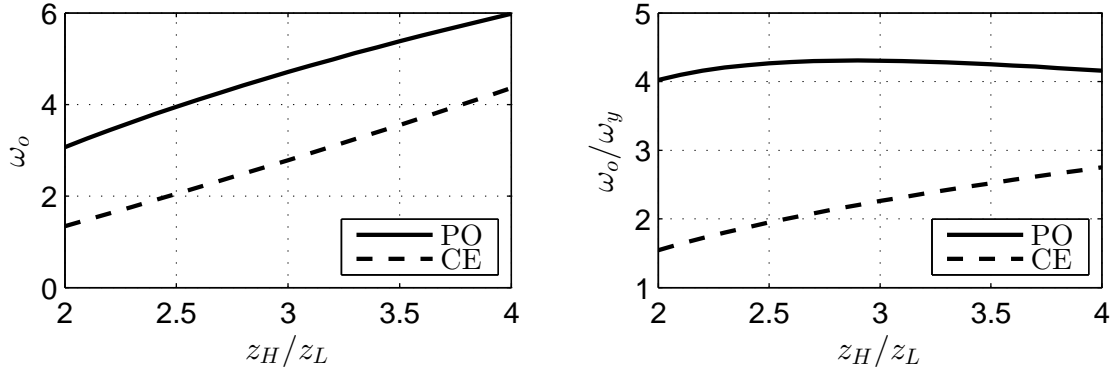
We next analyze how endogenous output shocks impact managers' (i.e., old agents') competitive and optimal wages.⁸ In particular, we examine numerically how the ratio of high output shock to low output shock z^H/z^L influences old agents' absolute wages ω_o and relative wages ω_o/ω_y . For the numerical analysis, we use the following functional forms. Agents' preferences are specified by logarithmic utility functions, i.e., $u(c_i) = \log(c_i)$ for $i = y, o$, and old agents' disutility functions are of the form $\phi(a_o) = a_o^d$, where $d > 1$. Production function exhibits constant returns to scale, and is of the form $f(\bar{a}_y) = \bar{a}_y$. Probability function of realizing high output shock is of the form $\pi(a_o) = 1 - \frac{1}{(1+a_o)^b}$.⁹

Figure 1(a) illustrates that when the ratio of high shock to low shock z^H/z^L increases, old agents' competitive and optimal wages increase by nearly the same ratio, so the gap between two wages stays almost the same. The intuition behind Figure 1(a) is as follows. Because endogenous shocks are influenced by old agents' actions (i.e., labor inputs), the higher the gap between high shock and low shock, the higher the impact of old agents' labor inputs on their firms and the economy. Higher impact on their firms raises competitive wages, and higher impact on the economy raises optimal wages.

Figure 1(b) depicts that when the ratio of high shock to low shock z^H/z^L rises, old agents' optimal relative wages remain almost the same. This figure has two implications. First, given that old agents' optimal absolute wages increase (Figure 1(a)), young agents' optimal wages also increase with z^H/z^L . This is because as z^H/z^L rises, total output rises, and higher total output yields higher wages to both young and old agents. Second, as z^H/z^L rises, young agents' optimal wages rise almost at the same rate as old agents' optimal wages, so the relative wage is nearly the same. This is because old agents' optimal wages consider the effect of old agents' managerial actions on young agents' wages. Figure 1(b) also demonstrates that old agents' competitive relative wages increase with z^H/z^L . This figure has two implications. First, because old agents' competitive relative wages increase slower than their absolute wages, young agents' competitive

⁸The expressions for competitive and optimal wages are given in (35) and (48), respectively in Appendix.

⁹ π satisfies assumptions made in §3.2, i.e., $\pi(0) = 0$, $\lim_{a_o \rightarrow \infty} \pi(a_o) = 1$, and π is increasing and concave in a_o .



(a) Competitive and optimal absolute wages.

(b) Competitive and optimal relative wages.

Figure 1: Comparison of competitive and optimal wages as a function of the ratio of high shock to low shock z^H/z^L , where young agents' labor inputs $\bar{a}_y = 10$, old agents' disutility exponent $d = 2$, probability function exponent $b = 2$, Pareto weight $\alpha = 0.5$, and discount factor $\beta = 1$.

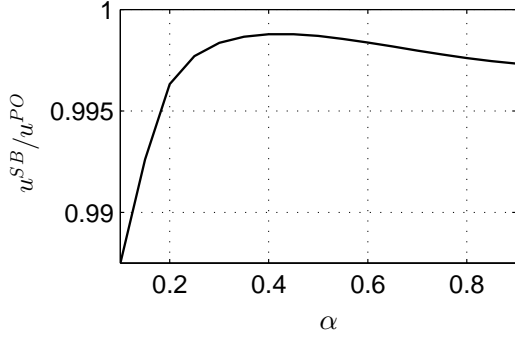
wages also increase, and this is because of externalities created by old agents' managerial actions. Second, young agents' competitive wages do not increase as much as their optimal wages do. This is because, unlike optimal wages, old agents' competitive wages do not consider the effect of old agents' managerial actions on young agents' wages.

1.3.2 Information Asymmetry

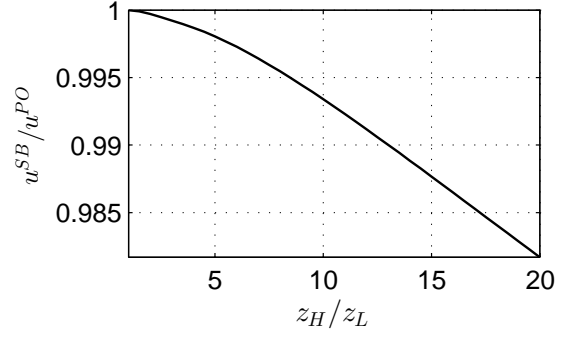
So far, we have analyzed the model in which managers' actions a_o and unit wages ω_o are observable by the social planner. However, in the real world, these variables may be unobservable because their calculations are difficult, and revelations are problematic due to incentives. We take this information asymmetry into account, and extend the model by allowing for unobservable actions (i.e., labor inputs) and unobservable wages for managers (i.e., old agents).

To implement a Pareto optimal allocation, the social planner needs to impose (unit) wage taxes on old agents. To do so, the social planner must be able to observe old agents' competitive wages ω_o or labor inputs a_o .¹⁰ When wages and labor inputs are unobservable, the social planner cannot impose wage tax, and hence cannot implement a Pareto optimal allocation. However, the social planner can make Pareto improvements by implementing a second-best allocation. In the second best, since a_o is unobservable, the social planner cannot decide on a_o . Instead, the social planner sets only consumption values c_y and c_o , and lets the market decide on a_o . In the market, labor input a_o is determined when labor supply is equal to labor demand. Labor supply equation (L^S)

¹⁰Because the social planner can observe labor income $a_o\omega_o$, observing a_o or ω_o suffices to calculate the other one.



(a) Welfare comparison where $z^H = 2, z^L = 0.5$.



(b) Welfare comparison where $\alpha = 0.5$.

Figure 2: The ratio of social welfare under second best to social welfare under Pareto optimal u^{SB}/u^{PO} as a function of Pareto weight α and the ratio of high shock to low shock z^H/z^L .

stemming from agents' optimization problems and labor demand equation (L^d) stemming from firms' optimization problems are

$$L^s : \left[\pi(a_o)u'(c_o^H) + (1 - \pi(a_o))u'(c_o^L) \right] \omega_o - \phi'(a_o) = 0 \quad (4)$$

$$L^d : \omega_o = \pi'(a_o)(z^H - z^L)f(\bar{a}_y). \quad (5)$$

Substituting (5) back into (4), and eliminating ω_o yields

$$\left[\pi(a_o)u'(c_o^H) + (1 - \pi(a_o))u'(c_o^L) \right] \pi'(a_o)(z^H - z^L)f(\bar{a}_y) - \phi'(a_o) = 0. \quad (6)$$

Because planner's problem (1) does not necessarily satisfy (6), we add (6) as a constraint to (1).

Thus, a second-best allocation is a solution to the following problem

$$\begin{aligned} \max_{c_y^H, c_y^L, c_o^H, c_o^L, a_o} & (1 - \alpha)E[u(c_y)] + \alpha(E[u(c_o)] - \phi(a_o)) \text{ subject to } c_y^s + c_o^s = \gamma^s \text{ and} \\ & \left[\pi(a_o)u'(c_o^H) + (1 - \pi(a_o))u'(c_o^L) \right] \pi'(a_o)(z^H - z^L)f(\bar{a}_y) - \phi'(a_o) = 0. \end{aligned}$$

The following proposition demonstrates how a second-best allocation can be implemented.

Proposition 3 *A second-best allocation $\{\widehat{c}_y^H, \widehat{c}_y^L, \widehat{c}_o^H, \widehat{c}_o^L, \widehat{a}_o\}$ can be implemented if the social planner imposes the following equity tax*

$$t^s = \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(\widehat{c}_y^s)\widehat{p}^s}{u'(\widehat{c}_o^s)}.$$

We next compare social welfare under a second-best allocation and under a Pareto optimal allocation as Pareto weight α and the ratio of high output shock to low output shock z^H/z^L change.

Figure 2(a) depicts that there is a gap between social welfare under a second-best allocation and under a Pareto optimal allocation (i.e., $u^{SB}/u^{PO} < 1$) for all values of α . As α increases, this gap reduces at first, but then it widens. Figure 2(b) shows that as z^H/z^L increases, social welfare under a second-best allocation diverges from social welfare under a Pareto optimal allocation. When high shock is equal to low shock (i.e., $z^H/z^L = 1$), old agents' actions a_o have no impact on the state of the economy. Then, failing to observe a_o and ω_o has no cost, and hence social welfare under second best reaches social welfare under Pareto optimal. When the gap between high shock and low shock widens, however, old agents' actions a_o have a sizeable impact on the state of the economy. Then, the cost of failing to observe a_o and ω_o is high because when these variables are unobservable, wage tax cannot be imposed, and hence optimal wages cannot be paid to old agents. This shows us that socially optimal allocations cannot be achieved without paying managers (i.e., old agents) the socially optimal wages.

1.4 Conclusion

In this paper, we have studied how endogenous output shocks driven by collective actions of managers impact social welfare. We show that these endogenous shocks render competitive equilibrium allocations inefficient. The intuition behind this inefficiency is that managerial actions create several externalities. We establish that a socially optimal allocation can only be attained by paying managers the socially optimal wages, which can be achieved by imposing wage taxes on managers. Finally, we extend the model by allowing for information asymmetry, and show that it is not possible to attain a socially optimal (i.e., first-best) allocation. We instead propose second-best allocations.

There are several future research avenues. First, in this paper, we analyze the impact of endogenous shocks (which are driven by managerial actions) on social welfare. It would be an interesting extension to analyze the impact of these shocks on business cycles - in particular, one can examine whether these endogenous shocks are important determinants of business cycle fluctuations. Second, our model suggests imposing wage taxes (or subsidies) on managers to restore efficiency. On the other hand, it is well-known that the drastic increase in managerial compensation over a couple of decades is an important determinant of the inequality of income distribution.

In this context, an interesting research avenue would be to investigate whether taxes suggested by our model alleviate or aggravate the inequality of income distribution. Finally, we examine the impact of endogenous shocks (which are driven by managerial actions) by using a model with no information asymmetry, and an important research avenue would be to examine the impact of these shocks by using a standard moral hazard model.

Chapter 2

2 Consumer Unions: Blessing or Curse?

2.1 Introduction

We examine whether and when coalitions of consumers are beneficial to consumers when producers have market power. Coalitions of consumers are formed when consumers cooperate or take joint actions. These coalitions countervail producers' market power, and hence they may benefit consumers. Thus, conventional wisdom and the literature on industrial organization (focusing on market power) suggest that when one side of the market possesses significant market power, enhancing the market power of the other side helps remedy problems arising from imperfect competition. By utilizing a comprehensive model, we identify a second effect, which has been overlooked before. Although coalitions of consumers improve consumers' market power, they may also induce producers to reduce production, so they may harm consumers.

Coalitions of consumers can be encountered in two forms. In addition to cases in which consumers literally act together (e.g., consumer cooperatives in Europe), other organizations can also be considered coalitions of consumers. In particular, numerous public and private organizations intermediate between consumers and producers by bargaining with producers on behalf of many consumers. We refer to such coalitions of consumers as *consumer unions*. We illustrate how consumer unions operate in public and private sectors in the following two examples:

- Established in each state of the United States, public utility commissions (PUCs) regulate capacity-constrained utilities such as electricity, natural gas, and telephone services. PUCs negotiate with utility producers on behalf of state residents over utility prices and allocations. In their mission statements, Texas and Florida PUCs define their goals for economic regulation as follows:
 "Provide fair and reasonable prices." (Florida PUC)
 "Protect consumers and act in the public interest." (Texas PUC)
- Online travel companies like Expedia and Priceline are intermediaries that book services

such as hotel rooms, airline seats, cruises, and rental cars for their customers. These intermediaries negotiate with service producers such as hotels and airlines to obtain lower prices and better services to their customers. These service producers are often constrained by a limited capacity, which is costly and time consuming to expand.

These online travel companies are regulated by the Antitrust Division of the Department of Justice. The Antitrust Division states its goals for regulation as follows: “Benefit consumers through lower prices ... promote consumer welfare through competition” (Antitrust Division).

The objective of this paper is to draw the borderline of when consumer unions are beneficial to consumers. Our criteria for being beneficial are the goals of regulatory agencies stated above. In particular, we ask the following research questions: (Q1) How do consumer unions impact consumer welfare? Because consumers’ long-term welfare depends on producers’ investments, we also analyze how consumer unions impact investments. Specifically, we ask: (Q2) How do consumer unions affect producers’ investments, and how do these investments in turn affect long-term consumer welfare? (Q3) How do consumer unions affect prices?

To address these questions, we model strategic interactions between consumer unions and producers in a general equilibrium environment with imperfect competition. For this purpose, we use a variant of the *market game* model, which is the natural extension of the Arrow-Debreu paradigm to accommodate small numbers of agents and the resulting strategic interactions among them. In the model, there are a finite number of producers and a finite number of consumer unions that bargain with producers on behalf of consumers over prices and allocations. We define the number of consumers in a union as *union size*. Given a fixed number of consumers, a larger union size represents a higher market power for consumers and a smaller number of consumer unions in the economy. Because the impact of consumer unions may depend on the production technology, we allow for arbitrary returns to scale in production.

To answer the research questions (Q1)-(Q3) listed above, we analyze the impact of a gradual change in union size. As our main result, we establish that consumer welfare decreases with union size when the union size is above a threshold (Theorem 2). Interestingly, this result is contrary to the literature on coalitions - in particular, its assumption of “superadditivity” (e.g., Shapley

1953), which means that coalitions are beneficial to their members.¹¹ In contrast, our main result shows that coalitions can be “subadditive,” i.e., they can be harmful to consumers. As a result, consumers may not benefit from the highest level of cooperation; they may be best off when there is some level of competition among consumer unions. The reason for reaching a different conclusion than the literature is that we incorporate production and consider both sides of the market. In particular, as consumers’ market power increases via larger union size, producers respond by reducing production.

We can summarize the other results that our analysis yields as follows. First, the threshold after which consumer welfare decreases with union size depends on the production technology and returns to scale (Corollary 2). Second, we prove that consumer unions discourage producers’ investments (Proposition 6), which in turn leads to a more dramatic fall in long-term consumer welfare. Third, we show that when the production technology exhibits increasing returns to scale, prices may increase with union size (Proposition 4). Finally, because consumer unions often fail to promote consumer welfare, we consider an alternative policy. In particular, we prove that a higher number of producers improves consumer welfare when the production technology exhibits decreasing or constant returns to scale (Proposition 7). This analysis proposes that having a higher number of producers via antitrust policy may be superior to consumer unions.

The implications of our findings are twofold. On the consumer side, consumers do not necessarily benefit from a higher level of cooperation; instead, they can benefit from some level of competition among consumer unions. For public organizations, this justifies why PUCs are at the state instead of the federal level.¹² For private intermediaries, this implies that fostering competition is crucial beyond classical antitrust concerns.¹³ Excessive market power of these private intermediaries may be detrimental to consumers not only because of the well-known “abuse of market power” phenomenon but also as a result of underproduction. On the producer side, when

¹¹Technically, superadditivity means that the value of the union of disjoint coalitions is higher than or equal to the sum of their individual values.

¹²Another advantage of smaller-scale regulatory agencies is that they are less prone to “regulatory capture,” which occurs when regulatory agencies end up acting in the best interest of producers instead of consumers.

¹³This finding may have significant implications given that these private intermediaries have been increasing their market power through mergers and acquisitions. For example, Expedia acquired Hotwire in 2003, CarRentals.com in 2008, and Trivago in 2013.

technology exhibits decreasing or constant returns to scale, policy makers should encourage the entry of new producers and foster competition among existing ones. For both goods and service producers, this implies that lower entry barriers and tighter antitrust policies may benefit consumers.

Related Literature: This paper is closely related to two streams of literature: the market game and the cooperative game theory literature.

Established by Shapley and Shubik (1977), the market game has been and will continue to be a prominent tool to model imperfect competition (e.g., Peck and Shell 1990; Peck and Shell 1991; Peck 2003). The market game has been applied to various topics (see, for example, Spear (2003) for price spikes in deregulated electricity markets, Goenka (2003) for information leakage, Koutsougeras (2003) for the violation of the law of one price). Our contribution to the market game literature is twofold. First, we blend cooperation and competition in a market game. Most of the market game literature focuses on noncooperative games, but one exception is Bloch and Ghosal (1994). Bloch and Ghosal (1994) study stable trading structures, and prove that the only strongly stable trading structure is the “grand coalition,” where all agents trade in the same market. In contrast, our paper shows that the grand coalition may not be formed because of the subadditive structure of coalitions. The reason for finding different results is that Bloch and Ghosal (1994) consider exchange economies, whereas we incorporate production, the reaction of the producer side of the market to coalitions. Second, we allow for arbitrary returns to scale in production that encompasses decreasing, constant, and increasing returns. Almost the entire literature restricts attention to pure exchange economies, and a few studies that consider production assume either decreasing returns (e.g., Dubey and Shubik 1977a) or constant returns (e.g., Spear 2003) for simplicity.¹⁴ We contribute to this literature by considering increasing returns and fully characterizing equilibrium under increasing returns.¹⁵

Another stream of related literature is the cooperative game theory literature. It is pioneered by von Neumann and Morgenstern’s monumental 1944 book and Shapley’s seminal 1953 paper (von Neumann and Morgenstern 1944; Shapley 1953), both of which assume that coalitions are

¹⁴Dubey and Shubik (1977a) assume convex production set, which implies nonincreasing returns to scale.

¹⁵For the full extension of the market game in the presence of increasing returns, see Korpeoglu and Spear (2014).

superadditive. Ever since Shapley (1953), most of the cooperative game theory literature assumes superadditivity (e.g., Krasa and Yannelis 1994; Clippel and Serrano 2008; Sun and Yang 2014). We contribute to this literature by showing a case under which this assumption is violated and by explaining why superadditivity may fail.

The rest of the paper is organized as follows. In §3.2, we elaborate on model ingredients, and define Nash equilibrium. In §2.3.1, we analyze how union size impacts prices and consumer welfare. In §2.3.2, we examine how union size affects producers' investments, and how these investments in turn affect long-term consumer welfare. In §2.3.3, we investigate how the number of producers via antitrust policy influences consumer welfare. In §3.4, we conclude and discuss the implications of our findings. We present all proofs in Appendix.

2.2 Model

To model strategic interactions between consumer unions and producers, we use the market game mechanism. In market games, agents trade goods at trading posts. There is a trading post for each good where agents can make bids to buy and make offers to sell the good. These bids are in terms of units of account and these offers are in terms of physical commodities. Agents make bids and offers based on their expectations of prices. Prices are formed by simultaneous actions (bids and offers) of all agents who buy or sell at the corresponding trading post. Equilibrium occurs when agents' price expectations come true.

We consider a static and deterministic model with multiple goods. To keep the production side of the economy simple, though, we interpret all but one of these goods as the production good available at different time periods, or equivalently, in different states of the world. Hence, there are T ($< \infty$) dated or stated production goods. For ease of illustration, we use electricity as the production good. Because electricity cannot be stored, it must be consumed as it is generated, and this considerably simplifies dealing with production decisions.¹⁶

Besides electricity, there is a single consumption good and two types of agents in the model. We can think of the consumption good as a composite good that is made up of all the goods in the economy except for electricity. Alternatively, we can interpret the consumption good as com-

¹⁶Electricity storage is negligible because according to the Electricity Information Administration, only 2% of electricity in USA in 2013 comes from storage.

modity money. Unlike electricity, the consumption good can be storable, so it is not a dated or stated good. The consumption good can be directly consumed by agents or used as an input to produce electricity. In addition to these two goods, there are two types of agents. There are P identical producers who are endowed with the technology to produce electricity and indexed by $j \in \{1, \dots, P\}$. Moreover, there are M identical consumers who are endowed with the consumption good and indexed by $h \in \{1, \dots, M\}$.

2.2.1 Agents

This section proceeds as follows. First, we elaborate on a producer's technology and actions. Second, we discuss a consumer's preferences. Finally, we explain how we model consumer unions and describe a union's actions.

First, each producer produces electricity by using the consumption good as an input with technology specified by a Cobb-Douglas production function. Producer j 's electricity output at period t is $q_j^t = \theta(\phi_j^t)^c$, where θ is the total factor productivity, ϕ_j^t is the consumption good input that producer j uses at period t , and c is returns to scale. The production technology exhibits decreasing, constant, and increasing returns to scale if $c < 1$, $c = 1$, and $c > 1$, respectively.¹⁷ Producer j faces a capacity constraint that restricts his output q_j^t to his capacity K , i.e., $q_j^t \leq K$ for all t . Producing beyond (i.e., expanding) the capacity requires substantial investment and time.¹⁸ Thus, each producer's capacity is fixed to \bar{K} in the short run, which is in line with the literature (e.g., Spear 2003). In the long run, each producer decides on his capacity by considering its cost. The unit cost of capacity is $\rho (> 0)$ units of the consumption good.

Each producer gets utility from consuming the consumption good, and he is not endowed with it. Thus, he purchases the consumption good from consumers. To do so, producer j makes a bid b_j^0 at the consumption good trading post. To finance his bid, producer j offers his electricity output q_j^t at the electricity trading post at each period t . Hence, producer j 's short-run set of actions is $A_j^{SR} = \{(b_j^0, q_j^1, \dots, q_j^T) \in \mathbb{R}_+^{T+1}\}$. In the long run, besides his bid and offers, each producer decides on his capacity as well. Thus, producer j 's long-run set of actions is

¹⁷Because production technologies may differ in different industries, we allow for arbitrary returns to scale. For example, in the electric power industry, technology exhibits increasing returns to scale up to a certain output level, and then exhibits constant returns to scale (see, for example, Christensen and Greene 1976b; Nelson 1985b).

¹⁸An example of capacity expansion is the construction or expansion of a new power plant.

$A_j^{LR} = \{(b_j^0, q_j^1, \dots, q_j^T, K) \in \mathbb{R}_+^{T+2}\}$. Each producer j 's preferences are specified by a logarithmic utility function, i.e., his utility is $\log(z_j^0)$, where z_j^0 is producer j 's consumption. However, logarithmic utility functions do not drive our results, and our results can be extended to more general utility functions (e.g., constant relative risk aversion).

Second, each consumer gets utility from consuming both electricity and the consumption good. Moreover, each consumer has time-varying preferences over electricity.¹⁹ Considering these two facts, we introduce exogenous weights $\alpha^0 (> 0)$, $\alpha^t (> 0)$ to represent the importance or significance of consuming the consumption good and electricity at period t , respectively. For example, if the consumption good demand is high, the consumption good weight α^0 is high; if electricity demand at period t is high, electricity weight α^t is high. As is in line with the literature (e.g., Bloch and Ghosal 1994; Spear 2003), each consumer h 's preferences are specified by a logarithmic utility function, i.e., $U_h = \sum_{t=1}^T \alpha^t \log(x_h^t) + \alpha^0 \log(x_h^0)$, where U_h is consumer h 's utility, x_h^t is his electricity allocation at period t , and x_h^0 is his consumption good allocation. Each consumer is endowed with ω units of the consumption good and does not have access to the technology to produce electricity.

Finally, consumers are represented by consumer unions in the sense that these unions make bids and offers on behalf of their members. We model consumer unions as exogenously formed coalitions of consumers to isolate the impact of union size. For analytical tractability, we focus on symmetric consumer unions, which is in line with the literature (e.g., Bloch and Ghosal 1994). There are R identical consumer unions that are indexed by $u \in \{1, \dots, R\}$ and N consumers in each union. The model captures all levels of cooperation, which is represented by union size N . If $N = 1$, each consumer acts on his own, i.e., there is no cooperation among consumers. If $1 < N < M$, consumers in the same union cooperate, and consumers in different unions compete. If $N = M$, all consumers are in the same union, i.e., there is full cooperation.²⁰ The level of cooperation is given in Table 2, where the level of cooperation among consumers increases if and

¹⁹For example, electricity demand is higher when the weather is very hot or very cold than when it is temperate. Another example is that daily electricity demand reaches its peak around 5:30 p.m. while it is very low after midnight.

²⁰Because $M = R \times N$, N values form a lattice. For analytical convenience and ease of illustration, we relax integer constraints of N , and allow for continuous N values, i.e., $N \in [1, M]$. This continuous relaxation (along with identical-unions assumption) allows us to completely characterize unions' actions, and it is a common practice in integer programming. However, our results hold without continuous relaxation as well.

Table 2: Level of Cooperation

$N = 1$	$1 < N < M$	$N = M$
no cooperation	some cooperation	full cooperation

only if union size N rises.

Union u 's endowment ω_u is equal to the total endowment of its members, i.e., $\omega_u = N\omega$. Consumer unions take actions on behalf of their members. Because each consumer gets utility from electricity but he is not endowed with it, union u purchases it from producers by making a bid b_u^t at the electricity trading post t . To finance this purchase, union u sells (some of) its consumption good by making a joint offer $q_u^0 (\leq \omega_u)$ at the consumption good trading post. Then, union u 's set of actions is $A_u = \{(b_u^1, \dots, b_u^T, q_u^0) \in \mathbb{R}_+^{T+1}\}$.

2.2.2 Prices and Allocations

This section explains how prices are formed and how goods are allocated. First, by using total bids and total offers, we calculate prices. Second, by using prices, we define allocation rules and budget constraints. Third, by using allocation rules, we verify that markets clear. Finally, we illustrate the market mechanism with a simple numerical example.

First, in the market game, the price of a good is equal to the ratio of the total bid (which represents the market demand) to the total offer (which represents the market supply).²¹ The total bid is equal to the sum of all individual bids, and the total offer is equal to the sum of all individual offers at the corresponding trading post. At the electricity trading post t , the total bid is $B^t = \sum_{u=1}^R b_u^t$, and the total offer is $Q^t = \sum_{j=1}^P q_j^t$. At the consumption good trading post, the total bid is $B^0 = \sum_{j=1}^P b_j^0$, and the total offer is $Q^0 = \sum_{u=1}^R q_u^0$. Then, electricity price at period t is $p^t = B^t / Q^t$, and the consumption good price is $p^0 = B^0 / Q^0$.

Second, in the market game, an agent's allocation is equal to the ratio of his own bid to the price of the good he purchases.²² Producer j 's (consumption good) allocation is $\frac{b_j^0}{p^0} = \frac{b_j^0}{B^0} Q^0$. Producer j uses ϕ_j^t units of the consumption good as production input at each period t . Because producer j 's output is $q_j^t = \theta(\phi_j^t)^c$, the total consumption good that he uses as production input is $\sum_{t=1}^T \phi_j^t =$

²¹ Adopting the convention of Shapley and Shubik (1977), if all offers at a trading post are zero, all bids are lost, and the price of the good is defined to be zero.

²² Following the lead of Shapley and Shubik (1977), if all bids at a trading post are zero, all offers are lost, and the allocation of the good is defined to be zero.

$(\frac{1}{\theta})^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}}$. Moreover, because the unit cost of capacity is ρ units of the consumption good, producer j uses ρK units of the consumption good to hold capacity K . Producer j consumes the remaining allocation, so his consumption is

$$z_j^0 = \frac{b_j^0}{B^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K. \quad (7)$$

Union u 's electricity allocation at period t is $\frac{b_u^t}{p^t} = \frac{b_u^t}{B^t} Q^t$. Union u 's consumption good allocation is the remaining consumption good endowment after its offer, i.e., $\omega_u - q_u^0$. A union's allocation is distributed equally among its members. The electricity allocation of consumer h in union u is

$$x_h^t = \frac{1}{N} \frac{b_u^t}{B^t} Q^t, \forall t. \quad (8)$$

The consumption good allocation of consumer h in union u is

$$x_h^0 = \frac{\omega_u - q_u^0}{N}. \quad (9)$$

An agent faces a budget constraint that restricts his (total) bid to his (total) income. This income is the product of his offer and the price of the good he sells. In particular, union u 's and producer j 's budget constraints are (respectively)

$$\sum_{t=1}^T b_u^t \leq q_u^0 p^0 = \frac{q_u^0}{Q^0} B^0 \text{ and } b_j^0 \leq \sum_{t=1}^T q_j^t p^t = \sum_{t=1}^T \frac{q_j^t}{Q^t} B^t.$$

Third, in the market game, allocation rules are designed in a way that markets always clear, and generate feasible allocations. Thus, we verify that the total use (for consumption or as production input) of each good is equal to its total production or total endowment. The electricity market at period t clears as follows

$$\sum_{u=1}^R \frac{b_u^t}{B^t} Q^t = \frac{Q^t}{B^t} \sum_{u=1}^R b_u^t = Q^t = \sum_{j=1}^P q_j^t, \forall t.$$

The consumption good market clears as follows (where $\omega_u = N\omega$, and $RN = M$)

$$\sum_{j=1}^P \frac{b_j^0}{B^0} Q^0 + \sum_{u=1}^R (\omega_u - q_u^0) = \frac{Q^0}{B^0} \sum_{j=1}^P b_j^0 + \sum_{u=1}^R \omega_u - \sum_{u=1}^R q_u^0 = M\omega.$$

Finally, we illustrate the market mechanism with the following simple numerical example where consumers act noncooperatively, i.e., union size $N = 1$.

Example 1 Suppose that consumers 1, 2, 3, and 4 make bids for 60 units of electricity. If each consumer makes a \$1 bid, consumer 1's allocation is (where $B_{-1}^t = B^t - b_1^t$)

$$x_1^t = \frac{b_1^t}{b_1^t + B_{-1}^t} Q^t = \frac{1}{1 + 3 * 1} 60 = 15.$$

If consumers 2, 3, 4 still make \$1 bids, and consumer 1 makes a \$2 bid, his allocation becomes

$$x_1^t = \frac{b_1^t}{b_1^t + B_{-1}^t} Q^t = \frac{2}{2 + 3 * 1} 60 = 24.$$

When consumer 1 increases his bid from \$1 to \$2, his allocation increases from 15 to 24 units. Thus, doubling the bid leads to a less than double increase in allocation because agents' actions affect prices, i.e., prices are nonlinear. The summary of this example is given in Table 3.

Table 3: The Impact of Doubling a Single Union's Bid on Its Members' Individual Allocation When $N = 1$.

Bid of A	Bid of B,C,D	Allocation of A
1	1	$\frac{1}{1+3} \times 60 = 15$
2	1	$\frac{2}{2+3} \times 60 = 24$

2.2.3 Nash Equilibrium

This section proceeds as follows. First, we define Nash equilibrium using a best-response argument. Second, we define off-peak and peak periods and a producer's uncapacitated offer. Finally, we present a lemma that will be frequently used for the rest of the paper.

First, we adopt the standard definition of Nash equilibrium, that is, each agent makes a best response to other agents' actions. We denote other agents' actions as follows. The sum of the bids of unions other than u at period t is B_{-u}^t , the sum of the offers of unions other than u is Q_{-u}^0 , the sum of the bids of producers other than j is B_{-j}^0 , and the sum of the offers of producers other than j at period t is Q_{-j}^t . In this market game, unions make electricity bids based on their expectations of the electricity price and make consumption good offers based on their expectations of the consumption good price. Similarly, producers make consumption good bids based on their expectations of the consumption good price and make electricity offers based on their expectations of the electricity price. When expectations of unions and producers come true, Nash equilibrium occurs.

Before formally defining Nash equilibrium, we derive agents' best response functions stem-

ming from their optimization problems. In particular, union u solves the following problem

$$\max_{b_u^1, \dots, b_u^T, q_u^0} N \left[\sum_{t=1}^T \alpha^t \log \left(\frac{1}{N} \frac{b_u^t}{b_u^t + B_{-u}^t} Q^t \right) + \alpha^0 \log \left(\frac{\omega_u - q_u^0}{N} \right) \right] \quad (10)$$

$$\text{s.t.} \quad \sum_{t=1}^T b_u^t \leq \frac{q_u^0}{q_u^0 + Q_{-u}^0} B^0. \quad (11)$$

The objective of union u given in (10) is to choose $b_u^1, \dots, b_u^T, q_u^0$ that maximize the total utility of its members. Budget constraint (11) guarantees that union u 's bid does not exceed its income.

Producer j solves the following problem

$$\max_{b_j^0, q_j^1, \dots, q_j^T, K} \log \left(\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta} \right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K \right) \quad (12)$$

$$\text{s.t.} \quad b_j^0 \leq \sum_{t=1}^T \frac{q_j^t}{q_j^t + Q_{-j}^t} B^t, \quad (13)$$

$$q_j^t \leq K, \forall t. \quad (14)$$

The objective of producer j given in (12) is to choose $b_j^0, q_j^1, \dots, q_j^T, K$ that maximize his utility. Budget constraint (13) ensures that producer j 's bid does not exceed his income. At each period t , capacity constraint (14) guarantees that producer j 's output q_j^t does not exceed his capacity K .

We restrict attention to symmetric Nash equilibrium in which the same types of agents take the same actions. Next, we formally define symmetric Nash equilibrium where unions make bids \hat{b}^t and offers \hat{q}^0 , and producers make bids \hat{b}^0 , offers \hat{q}^t , and choose equilibrium capacity \hat{K} in the long-run equilibrium.

Definition 1 (i) The short-run Nash equilibrium $\{\hat{b}^1, \dots, \hat{b}^T, \hat{q}^1, \dots, \hat{q}^T, \hat{b}^0, \hat{q}^0\}$ is such that $\{\hat{b}^1, \dots, \hat{b}^T, \hat{q}^0\}$ solves (10) - (11) given $B_{-u}^t = (R-1)\hat{b}^t$, $Q_{-u}^0 = (R-1)\hat{q}^0$, $Q^t = P\hat{q}^t$, $B^0 = P\hat{b}^0$; $\{\hat{q}^1, \dots, \hat{q}^T, \hat{b}^0\}$ solves (12) - (14), and $K = \bar{K}$ given $B_{-j}^0 = (P-1)\hat{b}^0$, $Q_{-j}^t = (P-1)\hat{q}^t$, $B^t = R\hat{b}^t$, $Q^0 = R\hat{q}^0$.

(ii) The long-run Nash equilibrium $\{\hat{b}^1, \dots, \hat{b}^T, \hat{q}^1, \dots, \hat{q}^T, \hat{b}^0, \hat{q}^0, \hat{K}\}$ is such that $\{\hat{b}^1, \dots, \hat{b}^T, \hat{q}^0\}$ solves (10) - (11) given $B_{-u}^t = (R-1)\hat{b}^t$, $Q_{-u}^0 = (R-1)\hat{q}^0$, $Q^t = P\hat{q}^t$, $B^0 = P\hat{b}^0$; $\{\hat{q}^1, \dots, \hat{q}^T, \hat{b}^0, \hat{K}\}$ solves (12) - (14) given $B_{-j}^0 = (P-1)\hat{b}^0$, $Q_{-j}^t = (P-1)\hat{q}^t$, $B^t = R\hat{b}^t$, $Q^0 = R\hat{q}^0$.

As is common in the literature (e.g., Peck et al. 1992; Dubey and Shapley 1994), we focus on interior Nash equilibrium in which all bids and offers are positive. Moreover, Koutsougeras and Ziros (2008) show that trivial Nash equilibria in which all bids and offers are zero do not arise in

the presence of a cooperative structure in market games.

Second, a period t is either an off-peak or a peak period depending on electricity demand.²³ If electricity demand at period t (represented by the weight a^t) is low, t is likely to be off-peak. If electricity demand at period t is high, t is likely to be peak. We define off-peak and peak periods and the producer's uncapacitated offer as follows.

Definition 2 (i) A period t is off-peak if the Lagrange multiplier of (14), μ^t , is zero.

(ii) A period t is peak if the Lagrange multiplier of (14), μ^t , is positive.

(iii) Given capacity K , the producer's uncapacitated offer $q^{t,*}[K]$ is the solution to (12) - (13) given $b_j^0 = \widehat{b}^0$, $q_j^t = q^{t,*}[K]$, $B_{-j}^0 = (P-1)\widehat{b}^0$, $Q_{-j}^t = (P-1)q^{t,*}[K]$, $B^t = R\widehat{b}^t$, $Q^0 = R\widehat{q}^0$.

We define off-peak and peak periods by using the Lagrange multiplier μ^t to avoid ambiguities in the presence of degenerate cases wherein $q_j^t = K$ and $\mu^t = 0$. This definition makes economic sense as well because μ^t is the marginal benefit of capacity. The marginal benefit of capacity is zero at off-peak periods, and it is positive at peak periods. In particular, the set of off-peak periods is $\mathcal{L} = \{t \mid \mu^t = 0\}$, and the set of peak periods is $\mathcal{H} = \{t \mid \mu^t > 0\}$. The number of off-peak periods is $|\mathcal{L}|$, and the number of peak periods is $|\mathcal{H}|$, where $|\mathcal{L}| + |\mathcal{H}| = T$.

Finally, the producer's uncapacitated offer $q^{t,*}[K]$ is his utility-maximizing offer when capacity constraint (14) is relaxed. Whenever $q^{t,*}[K]$ is feasible, it is optimal for the producer to offer $q^{t,*}[K]$. In particular, if $q^{t,*}[K]$ is less than or equal to capacity K , the producer's equilibrium offer \widehat{q}^t is equal to $q^{t,*}[K]$. If $q^{t,*}[K]$ is higher than K , it is infeasible for the producer to offer $q^{t,*}[K]$, so \widehat{q}^t is equal to K . In Lemma 1, we prove that having $q^{t,*}[K]$ less than or equal to K is equivalent to having an off-peak period in equilibrium. We present all proofs in Appendix.

Lemma 1 The producer's uncapacitated offer $q^{t,*}[K] \leq K$ if and only if period t is off-peak in equilibrium.

At an off-peak period t , capacity constraint (14) is not binding, so the producer's equilibrium offer \widehat{q}^t is equal to $q^{t,*}[K]$. At a peak period t , capacity constraint (14) is binding, so \widehat{q}^t is equal to K . Thus, we have $\widehat{q}^t = q^{t,*}[K] \leq K$ for all $t \in \mathcal{L}$ and $\widehat{q}^t = K < q^{t,*}[K]$ for all $t \in \mathcal{H}$. Lemma 1 enables us to use " $q^{t,*}[K] \leq K$ " and "period t is off-peak" interchangeably for the rest of the paper.

²³Electricity and the consumption good demands are endogenous and derived from agents' optimization problems.

2.3 Analysis

This section is organized as follows. In §2.3.1, we analyze the impact of union size on prices and consumer welfare in the short run. In §2.3.2, we examine the impact of union size on the producer's capacity and the impact of this capacity on consumer welfare in the long run. In §2.3.3, we discuss the effect of a higher number of producers on consumer welfare.

Before presenting our results, we illustrate how unionization works by using Example 1. In Example 1, we show the effect of doubling the bid of a union involving one consumer ($N = 1$), whereas in Example 2, we show the same effect for a union involving two consumers ($N = 2$).

Example 2 *Suppose that union 1 consists of consumers 1 and 2, and union 2 consists of consumers 3 and 4. Unions 1 and 2 make bids for 60 units of electricity. If each union makes a \$2 bid (the bid share of consumer 1 is \$1 as in Example 1), consumer 1's allocation is*

$$x_1^t = \frac{1}{N} \frac{b_1^t}{b_1^t + b_2^t} Q^t = \frac{1}{2} \frac{2}{2+2} 60 = 15.$$

If union 2 continues to make a \$2 bid and union 1 makes a \$4 bid (the bid share of consumer 1 is \$2 as in Example 1), consumer 1's allocation becomes

$$x_1^t = \frac{1}{N} \frac{b_1^t}{b_1^t + b_2^t} Q^t = \frac{1}{2} \frac{4}{4+2} 60 = 20.$$

After consumers start to cooperate, when the bid share of consumer 1 is doubled, consumer 1's allocation increases from 15 to 20 units; as opposed to before cooperation, when consumer 1 doubles his bid, his allocation increases from 15 to 24 units. Hence, as union size N rises, the impact of doubling the bid on each consumer's electricity allocation falls. The summary of this example is given in Table 4.

Table 4: The Impact of Doubling a Single Union's Bid on Its Members' Individual Allocation When $N = 2$.

Bid of A&B	Bid of C&D	Allocation of A
2	2	$\frac{1}{2} \left(\frac{2}{2+2} \times 60 \right) = 15$
4	2	$\frac{1}{2} \left(\frac{4}{4+2} \times 60 \right) = 20$

2.3.1 The Short Run

In this section, our main question is how union size impacts consumer welfare in the short run when the producer's capacity is fixed to \bar{K} . Because the consumer gets utility from the consumption good and electricity (see §3.2.1), we examine how the consumer's equilibrium consumption

good allocation \hat{x}^0 and electricity allocation \hat{x}^t change with union size N . We also present how the relative electricity price \hat{p}^t / \hat{p}^0 changes with union size N .

We start the short-run analysis with Lemma 2 that characterizes the consumer's equilibrium consumption good allocation \hat{x}^0 , and shows how it changes with union size N .

Lemma 2 *The consumer's equilibrium consumption good allocation $\hat{x}^0 = \omega \left(1 - \sum_{t=1}^T W_N^t\right)$, where $W_N^t \equiv \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}$. Furthermore, \hat{x}^0 increases with union size N .*

We can interpret W_N^t as the relative weight of electricity at period t . Then, $\sum_{t=1}^T W_N^t$ is the relative weight of electricity, and $1 - \sum_{t=1}^T W_N^t$ is the relative weight of the consumption good. Lemma 2 is intuitive in the sense that the consumer's equilibrium consumption good allocation \hat{x}^0 is equal to the product of his endowment ω and the relative weight of the consumption good $1 - \sum_{t=1}^T W_N^t$. As union size N rises, \hat{x}^0 rises because ω does not change, W_N^t falls, and hence $1 - \sum_{t=1}^T W_N^t$ rises. We explain the intuition of Lemma 2 by taking the gist of Example 2 as our starting point. As union size N rises, the impact of increasing a union's bid b^t on the electricity allocation per consumer x^t decreases. In response, a union substitutes away from electricity towards the consumption good. In particular, a union reduces the consumption good offer per consumer \hat{q}^0 / N . Because the consumer's endowment ω does not change with N , the decrease in the offer per consumer leads to an increase in the consumption good allocation per consumer $\hat{x}^0 = \omega - \hat{q}^0 / N$.

The consumer's equilibrium electricity allocation \hat{x}^t is constrained by the total electricity offer. Then, to examine how \hat{x}^t changes with union size N , we first need to examine how the producer's equilibrium offer \hat{q}^t changes. As discussed in §3.2.3, \hat{q}^t depends on the producer's uncapacitated offer $q^{t,*}$. Thus, we start by presenting $q^{t,*}$, and demonstrating how it changes with N .

Lemma 3 *The producer's uncapacitated offer $q^{t,*} = \left(\frac{M\omega(P-1)^2 c \theta^{\frac{1}{c}}}{P^3}\right)^c (W_N^t)^c$ at period t , where $W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}$. Moreover, $q^{t,*}$ decreases with union size N .*

Notice that the producer's uncapacitated offer does not depend on his capacity K , so it is denoted by $q^{t,*}$ instead of $q^{t,*}[K]$ for the rest of the paper.

As union size N rises, $q^{t,*}$ falls because $\left(\frac{M\omega(P-1)^2 c \theta^{\frac{1}{c}}}{P^3}\right)^c$ does not change, and W_N^t falls. The intuition is as follows. As discussed in Lemma 2's intuition, the consumption good offer per con-

sumer \hat{q}^0/N falls with N . Because consumers are identical, total consumption good offer Q^0 also falls. This leads to a lower consumption good allocation $\frac{\hat{b}^0}{B^0}Q^0$ for the producer. When capacity constraint (14) is relaxed, the producer optimally splits his allocation between the input for production and his own consumption to maximize his utility (see §3.2.2). Since the producer's allocation is lower, it is optimal for him to reduce input for production $(\frac{1}{\theta})^{\frac{1}{c}} \sum_{t=1}^T (q^{t,*})^{\frac{1}{c}}$, and hence reduce his uncapacitated offer $q^{t,*}$.

As Lemma 3 shows, the producer's uncapacitated offer $q^{t,*}$ falls with union size N . Then, one of the following three cases occurs. First, if a period t is off-peak, i.e., $q^{t,*} \leq \bar{K}$, it remains off-peak after N rises. Second, if a period t is peak, i.e., $q^{t,*} > \bar{K}$, and if $q^{t,*}$ is still higher than \bar{K} after N rises, then t remains peak. Third, if a period t is peak, i.e., $q^{t,*} > \bar{K}$, and if $q^{t,*}$ falls below \bar{K} after N rises, then t turns into off-peak. Next, we address the first case in Proposition 4, the second case in Proposition 5, and the third case in Corollary 1.

First, Proposition 4 characterizes the consumer's equilibrium electricity allocation \hat{x}^t at an off-peak period t , and shows how it changes with union size N . Proposition 4 also presents the relative electricity price \hat{p}^t/\hat{p}^0 in equilibrium and how it changes with N .

Proposition 4 *At an off-peak period t , the consumer's electricity allocation $\hat{x}^t = \frac{\omega^c(P-1)^{2c}c^c\theta}{M^{1-c}P^{3c-1}}(W_N^t)^c$ in equilibrium; and \hat{x}^t decreases with N . Also, the relative electricity price $\frac{\hat{p}^t}{\hat{p}^0} = \frac{(M\omega)^{1-c}P^{3c-1}}{(P-1)^{2c}c^c\theta}(W_N^t)^{1-c}$ in equilibrium; and as N rises, \hat{p}^t/\hat{p}^0 falls, stays the same, and rises if $c < 1$, $c = 1$, and $c > 1$, respectively.*

As union size N rises, \hat{x}^t falls because $\frac{\omega^c(P-1)^{2c}c^c\theta}{M^{1-c}P^{3c-1}}$ does not change, and W_N^t falls. The intuition is as follows. At an off-peak period t , the producer's equilibrium offer \hat{q}^t is equal to his uncapacitated offer $q^{t,*}$ (see §3.2.3). As Lemma 3 presents, $q^{t,*}$ falls with N , so \hat{q}^t also falls. Because producers are identical, total electricity offer Q^t falls. This results in a lower electricity allocation $\hat{x}^t = \frac{\hat{b}^t}{B^t}Q^t$ for the consumer.

As union size N rises, the change in \hat{p}^t/\hat{p}^0 is determined by the change in $(W_N^t)^{1-c}$ because $\frac{(M\omega)^{1-c}P^{3c-1}}{(P-1)^{2c}c^c\theta}$ does not change. In particular, \hat{p}^t/\hat{p}^0 falls, stays the same, and rises if $c < 1$, $c = 1$, and $c > 1$, respectively. The intuition is as follows. Both total consumption good offer Q^0 and total electricity offer Q^t decrease with N but at different rates. The decrease in Q^0 is linear and the decrease in Q^t is in the order of c . When technology exhibits decreasing returns to scale ($c < 1$),

the former outweighs the latter, so \hat{p}^t / \hat{p}^0 falls. When technology exhibits constant returns to scale ($c = 1$), the former offsets the latter, so \hat{p}^t / \hat{p}^0 stays the same. When technology exhibits increasing returns to scale ($c > 1$), the latter outweighs the former, so \hat{p}^t / \hat{p}^0 rises.

Second, Proposition 5 presents the consumer's equilibrium electricity allocation \hat{x}^t at a peak period t , and shows how it changes if t remains peak after union size N rises. Proposition 5 also characterizes the relative electricity price \hat{p}^t / \hat{p}^0 in equilibrium, and demonstrates how it changes if t remains peak after N rises.

Proposition 5 *At a peak period t , the consumer's electricity allocation $\hat{x}^t = \frac{P\bar{K}}{M}$, and the relative electricity price $\frac{\hat{p}^t}{\hat{p}^0} = \frac{M\omega}{P\bar{K}} W_N^t$ in equilibrium. If period t remains peak as union size N rises, then \hat{x}^t does not change, and \hat{p}^t / \hat{p}^0 falls.*

If a period t remains peak after union size N rises, \hat{x}^t does not change because $P\bar{K}/M$ does not change. The intuition is as follows. If a period t remains peak after N rises, the producer's equilibrium offer \hat{q}^t is equal to his fixed capacity \bar{K} , so \hat{q}^t stays the same. Because producers are identical, total electricity offer Q^t stays the same. This leads to the same electricity allocation $\hat{x}^t = \frac{\hat{b}^t}{B^t} Q^t$ for the consumer. Moreover, if a period t remains peak after N rises, \hat{p}^t / \hat{p}^0 falls because $M\omega / P\bar{K}$ does not change, and W_N^t falls. This is because as N rises, total consumption good offer Q^0 falls and total electricity offer Q^t stays the same, so \hat{p}^t / \hat{p}^0 falls.

Third, Corollary 1 explains how the consumer's electricity allocation and the relative electricity price in equilibrium change if a peak period turns into off-peak after union size N rises.

Corollary 1 *At a peak period t (i.e., $q^{t,*} > \bar{K}$), as union size N rises, if $q^{t,*}$ falls below \bar{K} , period t turns into off-peak. When a peak period turns into off-peak, the consumer's electricity allocation \hat{x}^t and the relative electricity price \hat{p}^t / \hat{p}^0 in equilibrium change as in Proposition 4.*

As a consequence of Lemmas 2, 3, Propositions 4, 5, and Corollary 1, we establish in Theorem 2 that there exists a union size N^* after which consumer welfare decreases with union size.

Theorem 2 *There exists $N^* < M$ such that the consumer's utility decreases with N for all $N > N^*$.*

The intuition of Theorem 2 is as follows. A larger union size N impacts the consumer's utility in three ways. First, it raises the consumer's consumption good allocation \hat{x}^0 (Lemma 2). Second,

it reduces the consumer's electricity allocation \hat{x}^t at off-peak periods (Proposition 4), and it does not affect \hat{x}^t at peak periods (Proposition 5). Third, it lowers the producer's uncapacitated offer $q^{t,*}$ (Lemma 3), possibly turning some peak periods into off-peak periods. This implies that the set of off-peak periods \mathcal{L} after union size N rises is at least as large as the one before the rise in N (Corollary 1). Thus, the utility loss from electricity at off-peak periods is reinforced by having a weakly larger set of off-peak periods \mathcal{L} . Due to concavity of the consumer's utility function, reduction in electricity allocation leads to reduction in the consumer's utility for union sizes $N > N^*$. Thus, the consumer's utility decreases with union size when it is larger than N^* .

Theorem 2 establishes that the highest level of cooperation is not necessarily beneficial to consumers; instead, some level of competition among consumer unions can be more beneficial. This result is interesting because since Shapley (1953), virtually the entire literature on coalitions assumes superadditivity, which means that the value of the union of coalitions is higher than or equal to the sum of their individual values. In other words, the literature assumes that coalitions are beneficial to their members. In contrast, Theorem 2 proves that when the union size is above a certain threshold, coalitions are harmful to their members, i.e., coalitions are *subadditive*. Hence, contrary to a first intuition, when producers possess significant market power, enhancing consumers' market power to the fullest extent may not be beneficial to consumers. Instead, policy makers should establish or preserve a certain level of competition among consumer unions.

After showing the existence of a threshold for union size, we discuss a special case of this threshold. In particular, we establish in Corollary 2 that when there is constant or increasing returns to scale ($c \geq 1$) and the producer's capacity \bar{K} is sufficiently large, threshold N^* is so low (i.e., $N^* = 1$) that consumer welfare decreases with union size regardless of union size N .

Corollary 2 *When $c \geq 1$, there exists \bar{K}_0 such that the consumer's utility is monotonically decreasing in union size N (i.e., $N^* = 1$) for all $\bar{K} > \bar{K}_0$.*

The intuition is as follows. A larger union size impacts the consumer's utility by raising his consumption good allocation \hat{x}^0 , and by reducing his electricity allocation \hat{x}^t at off-peak periods. Returns to scale c and capacity \bar{K} do not affect \hat{x}^0 , but they significantly affect \hat{x}^t . In particular, c influences how fast \hat{x}^t falls at off-peak periods, and \bar{K} influences the set of off-peak periods \mathcal{L} .

First, when a larger union size reduces the producer's allocation, the producer decreases input for production $\hat{\phi}^t$, thereby decreasing his offer $\hat{q}^t = \theta(\hat{\phi}^t)^c$ (see discussion of Lemma 3). How much offer \hat{q}^t decreases with a decrease in input $\hat{\phi}^t$ is measured by the *input elasticity of offer*. The input elasticity of offer is $\frac{\partial \log(\hat{q}^t)}{\partial \log(\hat{\phi}^t)} = \frac{\theta c(\hat{\phi}^t)^{c-1} \hat{\phi}^t}{\theta(\hat{\phi}^t)^c} = c$. When offer \hat{q}^t is elastic ($c \geq 1$), a decrease in $\hat{\phi}^t$ significantly reduces \hat{q}^t , which leads to a considerable fall in electricity allocation $\hat{x}^t = \frac{p\hat{b}^t}{B^t} \hat{q}^t$ for the consumer at an off-peak period t . Second, because $\hat{q}^t \leq \bar{K}$ for all $t \in \mathcal{L}$ (see §3.2.3), the set \mathcal{L} enlarges with \bar{K} . Thus, when $c \geq 1$ and \bar{K} is sufficiently large, the consumer's utility monotonically decreases with N .

Corollary 2 shows that when technology exhibits constant or increasing returns to scale ($c \geq 1$) and the producer's capacity \bar{K} is large enough, consumers do not benefit from any level of cooperation. In other words, when $c \geq 1$ and \bar{K} is sufficiently large, consumer welfare is maximized when consumers act noncooperatively (i.e., $N^* = 1$).

2.3.2 The Long Run

In this section, our main question is how union size affects the producer's capacity, and how this capacity in turn affects consumer welfare in the long run. In the long run, the producer chooses his equilibrium capacity \hat{K} by considering that the unit cost of capacity is ρ units of the consumption good. We establish in Proposition 6 that a larger union size induces the producer to reduce his equilibrium capacity in the long run.

Proposition 6 *In the long run, the producer's equilibrium capacity \hat{K} decreases with union size N .*

To build intuition for Proposition 6, we present the equation that determines \hat{K} ((90) in Appendix)

$$\frac{1}{c\hat{K}} \left(\frac{1}{\theta} \right)^{\frac{1}{c}} \sum_{t=1}^T \max \left\{ (q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}}, 0 \right\} = \rho.$$

The left-hand side is the marginal benefit of capacity, and the right-hand side is the marginal cost of capacity. Moreover, for all $\rho > 0$, there must be at least one peak period, i.e., $q^{t,*} > \hat{K}$ because otherwise, $\max \left\{ (q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}}, 0 \right\} = 0$ for all t . Lemma 3, which shows that the producer's uncapacitated offer $q^{t,*}$ falls with union size N , holds in the long run as well. The fall in $q^{t,*}$ reduces the marginal benefit of capacity. Because the marginal cost of capacity ρ does not change with N , the producer's equilibrium capacity \hat{K} decreases with N .

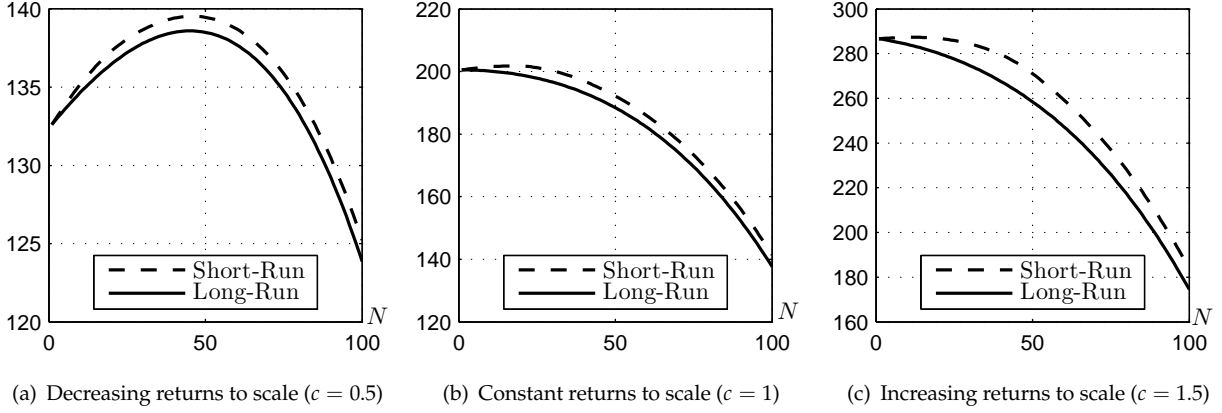


Figure 3: Comparison of consumer welfare in the short run and long run as a function of union size N , where the number of consumers $M = 200$, the number of producers $P = 10$, the consumer's endowment $\omega = 10$, productivity $\theta = 1$, and the unit cost of capacity $\rho = 2$.

We next examine how a decrease in the producer's equilibrium capacity impacts consumer welfare in the long run. Figure 3 demonstrates how the consumer's utility changes with union size N in the short run and long run. Figures 3(a), 3(b), and 3(c) have two features in common. First, there is a threshold N^* after which the consumer's utility decreases with N . Second, N^* in the long run is smaller than N^* in the short run. For instance, $N^* = 1$ (consumer welfare is maximized when consumers act noncooperatively) in the long run in Figures 3(b) and 3(c), whereas $N^* = 17$ and $N^* = 12$ in the short run in Figures 3(b) and 3(c), respectively.

The intuition of Figure 3 is as follows. As in Theorem 2's intuition, a larger union size impacts the consumer's utility via his consumption good allocation \hat{x}^0 (Lemma 2), electricity allocation \hat{x}^t at off-peak periods (Proposition 4), and \hat{x}^t at peak periods (Proposition 5). We prove in Lemma 6 of Appendix that the long-run Nash equilibrium is the same as the short-run Nash equilibrium if the short-run capacity \bar{K} is replaced with the long-run equilibrium capacity \hat{K} . Lemma 6 implies that Lemma 2 and Proposition 4 hold in the long run as well, but Proposition 5 alters as follows. At a peak period t , $\hat{x}^t = \frac{P\hat{K}}{M}$; and \hat{x}^t decreases with N because \hat{K} decreases with N (Proposition 6) in the long run. Thus, like the short run, a larger union size raises \hat{x}^0 , and reduces \hat{x}^t at off-peak periods; and unlike the short run, it reduces \hat{x}^t at peak periods as well. Hence, there is a threshold N^* in the long run, and it is smaller than N^* in the short run.

2.3.3 An Alternative Policy

As §2.3.1 and §2.3.2 demonstrate, consumer unions often fails to promote consumer welfare. This leads us to consider antitrust policy as an alternative. This section analyzes how a higher number of producers via antitrust policy affects consumer welfare. We first present and discuss Lemma 4 that summarizes interim results leading to the main result, Proposition 7.

Lemma 4 *As the number of producers P rises,*

- (i) *the consumer's consumption good allocation \hat{x}^0 does not change,*
- (ii) *the producer's uncapacitated offer $q^{t,*}$ falls; and if $c \leq 1$, total uncapacitated offer $Pq^{t,*}$ rises,*
- (iii) *if a period t remains or becomes off-peak, and $c \leq 1$, the consumer's electricity allocation \hat{x}^t rises,*
- (iv) *if a peak period t remains peak, \hat{x}^t rises in the short run; and*
- (v) *if $c \leq 1$, total equilibrium capacity $P\hat{K}$ rises in the long run.*

First, the consumer's consumption good allocation $\hat{x}^0 = \omega \left(1 - \sum_{t=1}^T W_N^t\right)$, where $W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}$ (see Lemma 2); and \hat{x}^0 does not change with the number of producers P . This is because when P changes, a union's offer \hat{q}^0 remains the same, which allocates the same amount of consumption good $\omega - \hat{q}^0/N$ to each member.

Second, the producer's uncapacitated offer $q^{t,*} = \frac{(P-1)^{2c}}{p^{3c}} \left(M\omega c \theta^{\frac{1}{c}} W_N^t\right)^c$ (see Lemma 3). As P rises, $q^{t,*}$ falls because $\frac{(P-1)^{2c}}{p^{3c}}$ falls, and $\left(M\omega c \theta^{\frac{1}{c}} W_N^t\right)^c$ does not change. However, when $c \leq 1$, a higher P raises total uncapacitated offer $Pq^{t,*} = \frac{(P-1)^{2c}}{p^{3c-1}} \left(M\omega c \theta^{\frac{1}{c}} W_N^t\right)^c$ because it raises $\frac{(P-1)^{2c}}{p^{3c-1}}$. The intuition is as follows. When the number of producers P rises, each producer reduces his uncapacitated offer $q^{t,*}$. However, when there is decreasing or constant returns to scale ($c \leq 1$), the rise in P outweighs the fall in $q^{t,*}$, which leads to a higher $Pq^{t,*}$.

As Lemma 4(ii) shows, the producer's uncapacitated offer $q^{t,*}$ falls with the number of producers P . When $q^{t,*}$ falls, an off-peak period (i.e., $q^{t,*} \leq \bar{K}$) remains off-peak, and this case is examined in Lemma 4(iii). When $q^{t,*}$ falls, a peak period (i.e., $q^{t,*} > \bar{K}$) either remains peak or turns into off-peak. The former case is discussed in Lemma 4(iv), and the latter case is equivalent to having an off-peak period before P rises, so it is covered in Lemma 4(iii).

Third, if a period remains or becomes off-peak after P rises, the consumer's electricity allocation $\hat{x}^t = \frac{(P-1)^{2c}}{p^{3c-1}} \frac{(\omega c W_N^t)^c \theta}{M^{1-c}}$ (see Proposition 4). When $c \leq 1$, as P rises, \hat{x}^t rises because $\frac{(P-1)^{2c}}{p^{3c-1}}$ rises,

and $\frac{(\omega c W_N^t)^c \theta}{M^{1-c}}$ does not change. The intuition is as follows. At an off-peak period t , the producer's equilibrium offer \hat{q}^t is equal to his uncapacitated offer $q^{t,*}$ (see §3.2.3). Because producers are identical, total electricity offer $Q^t = P\hat{q}^t = Pq^{t,*}$. As Lemma 4(ii) shows, when technology exhibits decreasing or constant returns to scale ($c \leq 1$), $Pq^{t,*}$ increases with P . Thus, when $c \leq 1$, a higher P raises Q^t , which leads to a higher electricity allocation $\hat{x}^t = \frac{\hat{b}^t}{B^t} Q^t$ for the consumer.

Fourth, if a peak period remains peak after P rises, the consumer's electricity allocation $\hat{x}^t = \frac{P\bar{K}}{M}$ (see Proposition 5) rises. This is because the producer's equilibrium offer \hat{q}^t is equal to his capacity \bar{K} at a peak period t (see §3.2.3). Because producers are identical, total electricity offer $Q^t = P\hat{q}^t = P\bar{K}$ increases with P , which leads to a higher electricity allocation $\hat{x}^t = \frac{\hat{b}^t}{B^t} Q^t$ for the consumer.

Finally, when $c \leq 1$, total equilibrium capacity $P\hat{K}$ increases with the number of producers P in the long run. The intuition is as follows. As P rises, the producer's uncapacitated offer $q^{t,*}$ falls (Lemma 4(ii)), which leads to a lower equilibrium capacity \hat{K} . When technology exhibits decreasing or constant returns to scale ($c \leq 1$), the rise in P outweighs the fall in $q^{t,*}$, which results in a higher $Pq^{t,*}$ (Lemma 4(ii)). A higher $Pq^{t,*}$ raises total equilibrium capacity $P\hat{K}$.

As a result of Lemma 4, we establish in Proposition 7 that when technology exhibits decreasing or constant returns to scale ($c \leq 1$), a higher number of producers improves consumer welfare in the short run and long run.

Proposition 7 *When $c \leq 1$, the consumer's utility increases with P in the short run and long run.*

The intuition of Proposition 7 differs in the short run and long run. In the short run, when $c \leq 1$, a higher number of producers does not affect the consumer's consumption good allocation \hat{x}^0 (Lemma 4(i)); and it raises the consumer's electricity allocation \hat{x}^t at off-peak (Lemma 4(iii)) and peak periods (Lemma 4(iv)). Thus, when technology exhibits decreasing or constant returns to scale ($c \leq 1$), the consumer's utility increases with the number of producers P in the short run. To build intuition for Proposition 7 in the long run, we use Lemma 6 of Appendix. We prove in Lemma 6 that the long-run Nash equilibrium is the same as the short-run Nash equilibrium if the short-run capacity \bar{K} is replaced with the long-run equilibrium capacity \hat{K} . Lemma 6 implies that Lemma 4(i) and 4(iii) hold in the long run as well, but Lemma 4(iv) alters as follows. At a peak

period t , the consumer's electricity allocation $\hat{x}^t = \frac{P\hat{K}}{M}$. When $c \leq 1$, as P rises, \hat{x}^t rises because $P\hat{K}$ rises (Lemma 4(v)). Thus, when $c \leq 1$, a higher P does not affect \hat{x}^0 , and it raises \hat{x}^t at off-peak and peak periods. Hence, when technology exhibits decreasing or constant returns to scale ($c \leq 1$), the consumer's utility increases with the number of producers P in the long run.

Proposition 7 establishes that a higher number of producers via antitrust policy promotes consumer welfare when technology exhibits decreasing or constant returns to scale. The implication of Proposition 7 is as follows. When there is decreasing or constant returns to scale, policy makers should encourage the entry of new producers and foster competition among existing ones. Proposition 7, in conjunction with Theorem 2, suggests that in industries wherein technology exhibits decreasing or constant returns to scale, having a higher number of producers is more effective in promoting consumer welfare than consumer unions. In industries wherein technology exhibits increasing returns to scale, however, antitrust policy is ineffective because only a few producers can survive, i.e., make nonnegative profits. For a more comprehensive discussion of this issue, see Korpoglu and Spear (2014).

2.4 Conclusion

In this paper, we have studied the impact of consumer unions, which bargain with producers on behalf of consumers over prices and allocations. When producers possess significant market power, consumer unions may benefit consumers by countervailing producers' market power. On the other hand, consumer unions may harm consumers by inducing producers to reduce production. This paper provides insights for policy makers about how union size impacts relative prices, consumer welfare, and production capacity.

A change in union size creates a ripple effect on markets, inducing a complex set of adjustments by both consumers and producers. By utilizing an imperfect competition model in a general equilibrium environment, we are able to capture various effects of a change in union size, and we obtain the following novel insights:

- While a larger union size successfully reduces relative prices under tight production capacity or under decreasing returns to scale, it fails to do so under constant or increasing returns to scale.

- Consumer welfare decreases with union size when the union size gets larger than a threshold, which depends on production capacity and returns to scale. This suggests that consumers may not benefit from the highest level of cooperation; they are best off when there is a certain level of competition among consumer unions.
- A larger union size discourages production capacity expansion, which in turn leads to a more dramatic fall in long-term consumer welfare.
- Under decreasing or constant returns to scale, consumer welfare can be promoted by a higher number of producers via antitrust policy. This suggests that consumers may benefit from the entry of new producers and competition among existing ones.

Although our results can be applied to both public and private organizations, their interpretation is slightly different. First, our findings justify why PUCs are at the state or even at the county level (e.g., San Francisco PUC) instead of being at the federal level. Moreover, our results can be potentially applied to other regulatory agencies. Second, our findings are robust to transfers between private intermediaries and consumers (e.g., Expedia and Priceline charge a certain fee for their services). In particular, a tighter antitrust policy is advisable because excessive market power of these intermediaries may not only increase the likelihood of excessive fees but also reduce service production.

There are several interesting avenues for future research. First, in this paper, we consider identical consumers and identical producers, but it would be an interesting extension to incorporate heterogeneity. For example, some consumers may assign higher weights to electricity than others. In this case, one can analyze whether consumer unions create a welfare shift across consumer types by benefiting some consumers but harming others. Second, while our model allows flexibility in union size N , one may consider a different case in which N is determined endogenously. This model allows for gauging the number of consumer unions that consumers form in equilibrium. This analysis may help assess whether private intermediaries like Expedia pose a threat of monopolization. Note that this model of endogenous N does not isolate the impact of union size N on consumer welfare, nor does it allow the evaluation of legally formed public unions like PUCs. Thus, models of exogenous and endogenous N are complementary to each other. Finally,

a special case of our model in which there is only one production good (i.e., $t = 1$) can be used to examine warehouse clubs such as Costco and Sam's Club or retailers who intermediate between manufacturers and consumers. An interesting research avenue would be to examine such warehouse clubs and retailers by using a model with multiple production goods (i.e., $t = T$) and by incorporating inventory decisions of producers.

Chapter 3

3 The Production Market Game and Arbitrary Returns to Scale

3.1 Introduction

Modern general equilibrium theory has been developed under the assumption of decreasing or constant returns to scale in production for analytical tractability, yet economic theory has long recognized the presence and significance of increasing returns. Since Adam Smith's pin factory example, increasing returns have been considered essential for explaining economic efficiency and possibilities for economic growth. Economists going as far back as Alfred Marshall have examined the relation between increasing returns and economic growth in general equilibrium frameworks. Furthermore, numerous empirical studies present evidence for the presence of increasing returns. In particular, we observe increasing returns in conventional industries such as manufacturing (e.g., Diewert and Wales 1987 and Ramey 1989), transportation, and public utilities (Christensen and Greene 1976a and Nelson 1985a) as well as new industries such as education, software, and internet-related business (e.g., Amazon and Google).

Recognizing the significance of increasing returns, the general equilibrium literature has attempted to incorporate it into conventional general equilibrium frameworks such as dynamic growth models and static Walrasian models. These attempts have usually been unsuccessful because of the fundamental incompatibilities between increasing returns and the competitive paradigm. Specifically, in dynamic growth models, an equilibrium may not exist or production may be unbounded; in static Walrasian models, firms may make long-run losses.

In dynamic growth models, the existence problem is solved if factors that drive economic growth (e.g., research and development) are allowed to be external to individual firms (Arrow 1962). However, in competitive models with externalities, if internal returns to scale is decreasing or constant, then it is indeterminate how agents are compensated for engaging in costly research necessary to drive economic growth (Shell 1966). An alternative that avoids this problem is to view such factors as public goods, and hence require the government to levy taxes to pay for these activities. However, this immediately implies that the internal factors will not be paid their

full marginal products. The unbounded-production problem can be avoided only by assuming that the marginal product of capital is diminishing given a fixed supply of labor (Arrow 1962). These problems disappear when increasing returns are incorporated into an imperfectly competitive model (Romer 1987). On the other hand, in the presence of internal increasing returns, fixed costs are large (implicitly, if not explicitly) but marginal costs are small (even zero) in comparison. Under the competitive analysis of static Walrasian models, marginal-cost pricing cannot be sustained without firms making long-run losses, which can be solved by requiring the government to subsidize these losses (Suzuki 2009). However, to subsidize these losses, the government will need to levy taxes, and hence productive agents again will not be paid their full marginal products.

In this paper, we study imperfectly competitive production economies in which technology exhibits arbitrary returns to scale including increasing returns. We incorporate increasing returns in the form of fixed costs, in the form of decreasing marginal costs, and their combination. We use the *market game* model introduced by Shapley and Shubik (1977). The market game model is the natural extension of the Walrasian model to accommodate small numbers of agents and the resulting strategic interactions among them. We extend the pure exchange version of the market game to production economies. We first show that the market game pricing and allocation mechanism generates quasi-concave profit functions so that all firms' objective functions are well-defined and have bounded solutions. We then prove, under fairly mild assumptions, that an equilibrium exists, although this result is weak in the sense that we cannot guarantee increasing-returns firms are actually active in equilibrium. This weakness stems from the fact that increasing-returns firms can make losses at low levels of production. For an arbitrary number of such firms, there is no general guarantee of non-negative profits. Hence, any improvements on this existence result would necessarily depend on details like the number of increasing-returns firms and the nature of their technology. Finally, we demonstrate the relationship between the number of increasing-returns firms and the input elasticity of production.

Related Literature: This paper is closely related to two streams of literature: increasing returns in general equilibrium frameworks (both dynamic growth and static Walrasian models) and the market game literature.

Alfred Marshall is one of the earlier economists to consider the effects of increasing returns, giving a general equilibrium interpretation to the relation between increasing returns and economic growth by introducing the distinction between internal and external economies of scale.²⁴ The interest of studying increasing returns in more contemporary economics has revived following the publication of Arrow (1962)'s paper on knowledge spill-overs and learning by doing. Arrow (1962) proposes that as investment and production take place, new knowledge is discovered, which improves the productivity of primary factors of production, and hence gives rise to increasing returns. Because this knowledge becomes publicly known, increasing returns are external to individual firms. In follow-up work, Frankel (1962) incorporates a new factor of production - what economists now call human capital - as a purely external input, in addition to the conventional labor and capital inputs, in an otherwise conventional neoclassical growth model. Frankel (1962)'s model with external human capital can generate globally increasing returns, even as firms operate competitively with respect to the conventional inputs. Shell (1966) uses a similar framework to provide an early model of endogenous growth, and he makes an early observation about the problem of compensating agents for engaging in costly research necessary to drive human capital accumulation. Specifically, when internal returns to scale is decreasing or constant, there will be nothing left to compensate agents involved in research and development. Romer (1987) also recognizes the problem of paying for the accumulation of the external factors of production. To tackle these problems, Romer (1987) develops an endogenous growth model by using a production version of the Dixit and Stiglitz (1977) model of monopolistic competition in which increasing returns arise because of specialization.

Besides trying to incorporate increasing returns into endogenous growth models, economic theorists have also tried to incorporate it directly into Walrasian models. It is easily shown, however, that increasing returns is fundamentally incompatible with the competitive paradigm. In particular, because marginal costs are small (even zero) compared to fixed costs in the presence of increasing returns, marginal-cost pricing cannot be sustained without firms making long-run

²⁴Marshall suggests that an increase in "trade-knowledge" cannot be kept secret, and this represents a sort of "external economy." This external economy justifies the use of a decentralized, price-taking equilibrium in the presence of aggregate increasing returns.

losses. If the government levies taxes to subsidize these losses, productive agents will not be paid their full marginal products. Hence, this literature has focused on the question of finding pricing rules which will implement Pareto optimal allocations in the presence of increasing returns (e.g., Suzuki 2009 and references therein). This literature is silent on the question of modeling increasing returns in general equilibrium environments with strategic interactions.

Another stream of literature to which this paper is related is the market game literature. Established by Shapley and Shubik (1977), the market game has been and will continue to be a prominent tool to model imperfect competition (e.g., Peck and Shell 1991, Koutsougeras 2003, Peck 2003). Almost the entire market game literature restricts attention to pure exchange economies. A few studies that consider production assume either convex production set, which implies non-increasing returns (Dubey and Shubik 1977b) or constant returns (Spear 2003) for simplicity. We contribute to this literature by fully extending the market game model to production economies in which technology exhibits arbitrary returns to scale including increasing returns.

3.2 The Model

To study imperfectly competitive production economies, we use the market game model. In market games, agents trade goods at trading posts. There is a trading post for each good where agents can make bids to buy and make offers to sell the good. These bids are in terms of units of account and offers are in terms of physical commodities. Agents make bids and offers based on their expectations of prices. Prices are formed by simultaneous actions (bids and offers) of all agents who buy or sell at the corresponding trading post. Equilibrium occurs when agents' price expectations come true.

We consider a static and deterministic model populated with output goods, input goods, firms, and households. There are $J < \infty$ sectors that produce different types of output goods. There are $N < \infty$ input goods used to produce output goods, and input goods are indexed by $n \in \{1, \dots, N\}$. There are $K_j < \infty$ firms that are endowed with the technology to produce output good $j \in \{1, \dots, J\}$, and firm k in sector j is indexed by k_j where $k \in \{1, \dots, K_j\}$. There are also H households who are endowed with input goods (and ownership shares of firms), and they are indexed by $h \in \{1, \dots, H\}$.

We assume that firms sell all of output goods they produce and households sell all of their endowments of input goods. In the absence of this assumption, i.e., when agents can make bids and offers simultaneously at the same trading posts, a well-known coordination indeterminacy arises. To avoid this indeterminacy, we assume that households are endowed only with input goods (from which they receive no utility), and receive utility only from output goods (which they are not endowed with). This is a common assumption in the international trade literature (e.g., Ohlin 1967) and see Peck et al. 1992 for further discussion of the importance of this assumption for determinacy of equilibrium in market games. There is also an economic rationale behind the Hecksher-Ohlin assumption. In competitive markets, it is well-known that Walrasian tatonnement process may not converge to the competitive equilibrium (Scarf 1960). However, the Hecksher-Ohlin assumption guarantees that the Walrasian tatonnement process will converge to the competitive equilibrium because it eliminates endowment-induced income effects in product markets. In market games, Kumar and Shubik (2004) provide an example in which tatonnement-like process may not converge to the equilibrium (analogous to Scarf (1960)'s counter example in competitive markets). However, we conjecture that the Hecksher-Ohlin assumption will induce the tatonnement-like process to converge to the equilibrium in market games as well.

3.2.1 Agents

This section proceeds as follows. First, we discuss a firm's endowments and actions. Second, we discuss a household's endowments and actions. Third, we elaborate on production technology.

First, firm k_j (firm k in sector j) produces output good j by using input goods, and his production technology is specified by $q_{k_j} = f^j(\phi_{k_j})$, where q_{k_j} is the output, $f^j : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$ is the production function, $\phi_{k_j} \in \mathbb{R}_+^N$ is the vector of input goods of firm k_j , and $f(0) = 0$. Because firms need input goods for production but they are not endowed with these goods, firms purchase input goods from households. To purchase input good n , firm k_j makes a bid $w_{k_j}^n$ at input trading post n , where $w_{k_j} = (w_{k_j}^1, \dots, w_{k_j}^N)$ is the vector of firm k_j 's bids. To finance these bids, firms sell (all of) output goods they produce. In particular, firm k_j sells his output q_{k_j} at output trading post j , and as a result of this sale, makes profit π_{k_j} .

Second, household h is endowed with input goods, and his vector of endowments is $g_h =$

$(g_h^1, \dots, g_h^N) \in \mathbb{R}_+^N$. Households do not have access to the technology to produce output goods, but they get utility from consuming output goods. Household h 's utility function u_h is at least twice continuously differentiable, strictly increasing, strictly concave, and his vector of (output goods') consumption is $x_h \in \mathbb{R}_+^J$. Because households get utility from consuming output goods but they are not endowed with these goods, households purchase output goods from firms. To purchase output good j , household h makes a bid b_h^j at output trading post j , where $b_h = (b_h^1, \dots, b_h^J)$ is the vector of household h 's bids. Because households need to finance these bids, they sell their endowments of input goods. Moreover, households are owners of firms - in particular, household h is endowed with ownership shares $\theta_h^{k_j}$ of firm k in sector j .

Third, because households own the firms, they pay for the fixed costs of the firms they own. The fixed cost that household h pays is $0 \leq \delta \leq 1$ of his endowment g_h^n for all $n \in \{1, \dots, N\}$. Because households do not get utility from input goods (i.e., primary factors of production), they sell all of their endowments of input goods after paying for the fixed costs of the firms they own. In particular, household h sells $e_h^n = g_h^n(1 - \delta)$ at input trading post n , and $e_h = (e_h^1, \dots, e_h^N) \in \mathbb{R}_+^N$. When fixed costs are too high ($\delta = 1$), no endowments are left to use as input for production. Then, firms make zero production, and hence households make zero bids on outputs. Therefore, we have a trivial equilibrium in which all agents make zero bids, and they do not engage in trade. When fixed costs are zero (i.e., $\delta = 0$), increasing returns appear in the form of decreasing marginal costs. When fixed costs are positive (i.e., $0 < \delta < 1$) and when technology exhibits decreasing (or constant) returns, decreasing (or constant) returns approximate increasing returns, that is, increasing returns appear in the form of fixed costs. When fixed costs are positive (i.e., $0 < \delta < 1$) and when technology exhibits increasing returns, increasing returns appear in the form of both fixed costs and decreasing marginal costs.

3.2.2 Prices and Allocations

This section explains how prices are formed and how goods are allocated. First, by using total bids and total offers, we calculate prices. Second, by using prices, we define allocation rules and budget constraints. Third, by using allocation rules, we verify that markets clear.

First, in the market game, the price of each good is equal to the ratio of the total bid (which

represents the market demand) to the total offer (which represents the market supply).²⁵ The total bid is equal to the sum of all individual bids, and the total offer is equal to the sum of all individual offers at the corresponding trading post. At input trading post n , the total bid W^n and the total offer E^n are

$$W^n = \sum_{j=1}^J \sum_{k=1}^{K_j} w_{k_j}^n \text{ and } E^n = \sum_{h=1}^H e_h^n.$$

At output trading post j , the total bid B^j and the total offer Q^j are

$$B^j = \sum_{h=1}^H b_h^j \text{ and } Q^j = \sum_{k=1}^{K_j} q_{k_j}.$$

The price r^n of input good n and the price p^j of output good j are

$$r^n = \frac{W^n}{E^n} \text{ and } p^j = \frac{B^j}{Q^j},$$

where $r = (r^1, \dots, r^N)$ is the vector of input prices, and $p = (p^1, \dots, p^J)$ is the vector of output prices.

Second, in the market game, each agent's allocation is equal to the ratio of his own bid to the price of the good he purchases.²⁶ Firm k_j 's allocation of input good n is

$$\phi_{k_j}^n = \frac{w_{k_j}^n}{r^n} = w_{k_j}^n \frac{E^n}{W^n},$$

and $\phi_{k_j} = (\phi_{k_j}^1, \dots, \phi_{k_j}^N)$ is the vector of (input good) allocations of firm k_j . Household h 's allocation of output good j is

$$x_h^j = \frac{b_h^j}{p^j} = b_h^j \frac{Q^j}{B^j},$$

where $x_h = (x_h^1, \dots, x_h^J)$ is the vector of (output good) allocations of household h .

Each agent faces a budget constraint that restricts his total bid to his total income. This income is the product of his offer and the price of the good he sells. In particular, firm k_j 's budget constraint is

$$\sum_{n=1}^N w_{k_j}^n \leq q_{k_j} p^j = q_{k_j} \frac{B^j}{Q^j}.$$

²⁵ Adopting the convention of Shapley and Shubik (1977), if all offers at a trading post are zero, all bids are lost, and the price of the good is defined to be zero.

²⁶ Following the lead of Shapley and Shubik (1977), if all bids at a trading post are zero, all offers are lost, and the allocation of the good is defined to be zero.

As mentioned in §3.2.1, households are endowed with input goods and ownership shares of firms, so households receive income from sales of input goods and shares of firms' profits. Moreover, because households own the firms, households pay for the fixed costs of firms they own. Hence, household h 's budget constraint is

$$\sum_{j=1}^J b_h^j \leq \sum_{n=1}^N \frac{W^n}{E^n} e_h^n + \sum_{j=1}^J \sum_{k=1}^{K_j} \theta_h^{k_j} \pi_{k_j},$$

where π_{k_j} is firm k_j 's profit.

Third, in the market game, allocation rules are designed in a way that markets always clear, and generate feasible allocations. Thus, we verify that the total use (for consumption or as production input) of each good equals its total endowment or total production. The market for input good $n \in \{1, \dots, N\}$ clears as follows

$$\sum_{j=1}^J \sum_{k=1}^{K_j} \phi_{k_j}^n = \sum_{j=1}^J \sum_{k=1}^{K_j} w_{k_j}^n \frac{E^n}{W^n} = E^n = \sum_{h=1}^H e_h^n.$$

The market for output good $j \in \{1, \dots, J\}$ clears as follows

$$\sum_{h=1}^H x_h^j = \sum_{h=1}^H b_h^j \frac{Q^j}{B^j} = Q^j = \sum_{k=1}^{K_j} q_{k_j}.$$

3.2.3 Nash Equilibrium

We adopt the standard definition of Nash equilibrium, that is, each agent makes a best response to other agents' actions. We denote other agents' actions as follows. At input trading post n , the sum of bids of all firms other than firm k_j is $W_{-k_j}^n$, the sum of offers of households other than household h is E_{-h}^n . At output trading post j , the sum of bids of households other than household h is B_{-h}^j , the sum of offers of firms other than firm k_j in sector j (which is also the total offer of other firms for output good j) is $Q_{-k_j}^j$. Before formally defining Nash equilibrium, we derive agents' best response functions stemming from their optimization problems. In particular, firm k_j solves the

following problem

$$\max_{w_{k_j}^1, \dots, w_{k_j}^N, q_{k_j}} \frac{B^j}{Q_{-k_j}^j + q_{k_j}} q_{k_j} - \sum_{n=1}^N w_{k_j}^n \quad (15)$$

$$\text{s.t.} \quad \sum_{n=1}^N w_{k_j}^n \leq \frac{B^j}{Q_{-k_j}^j + q_{k_j}} q_{k_j}, \quad (16)$$

$$q_{k_j} = f^j(\phi_{k_j}), \quad (17)$$

where $\phi_{k_j} = (w_{k_j}^1 \frac{E^1}{W^1}, \dots, w_{k_j}^N \frac{E^N}{W^N})$. The objective function of firm k_j given in (15) is to choose $w_{k_j}^1, \dots, w_{k_j}^N$ that maximize firm k_j 's profit π_{k_j} . Budget constraint (16) guarantees that firm k_j 's total cost does not exceed its revenue, i.e., (16) ensures that firm k_j 's profit π_{k_j} is nonnegative. The constraint (17) specifies firm k_j 's production technology. If we substitute (17) back into (15), firm k_j 's optimization problem collapses to

$$\max_{w_{k_j}^1, \dots, w_{k_j}^N} \frac{B^j}{Q_{-k_j}^j + f^j(\phi_{k_j})} f^j(\phi_{k_j}) - \sum_{n=1}^N w_{k_j}^n. \quad (18)$$

On the other hand, household h solves the following problem

$$\max_{b_h^1, \dots, b_h^J} u_h(x_h^1, \dots, x_h^J) \quad (19)$$

$$\text{s.t.} \quad \sum_{j=1}^J b_h^j \leq \sum_{n=1}^N \frac{W^n}{E^n} e_h^n + \sum_{j=1}^J \sum_{k=1}^{K_j} \theta_h^{k_j} \left(\frac{b_h^j + B_{-h}^j}{Q_{-k_j}^j + f^j(\phi_{k_j})} f^j(\phi_{k_j}) - \sum_{n=1}^N w_{k_j}^n \right). \quad (20)$$

The objective function of household h given in (19) is to choose b_h^1, \dots, b_h^J that maximize household h 's utility. Because households sell all of their endowments, offers of input goods are not decision variables for households. Budget constraint (20) guarantees that household h 's total bid does not exceed his total income. We next define Nash equilibrium as follows.

Definition 3 The Nash equilibrium $\{\widehat{w}_{k_j}^n, \widehat{b}_h^j, j \in \{1, \dots, J\}, k \in \{1, \dots, K_j\}, n \in \{1, \dots, N\}, h \in \{1, \dots, H\}\}$ is such that for all $j \in \{1, \dots, J\}$ and $k \in \{1, \dots, K_j\}$, $\widehat{w}_{k_j}^n, n \in \{1, \dots, N\}$ solves (18) given $\widehat{w}_{-k_j}^n, n \in \{1, \dots, N\}$ and for all $h \in \{1, \dots, H\}$, $\widehat{b}_h^j, j \in \{1, \dots, J\}$ solves (19) - (20) given $\widehat{b}_{-h}^j, j \in \{1, \dots, J\}$.

3.3 Analysis

This section is organized as follows. In §3.3.1, we show the existence of interior solution; in §3.3.2, we present the main existence theorem; in §3.3.3, we characterize the equilibrium with an extended example.

Before presenting our results, we derive first-order conditions we will use. The first-order condition of (18) with respect to $w_{k_j}^n$ (noting that $\phi_{k_j} = (\phi_{k_j}^1, \dots, \phi_{k_j}^N)$ and $\phi_{k_j}^n = w_{k_j}^n \frac{E^n}{W^n}$) is

$$\frac{B^j Q_{-k_j}^j}{(Q^j)^2} \frac{\partial f^j(\phi_{k_j})}{\partial \phi_{k_j}^n} \left[\frac{W_{-k_j}^n E^n}{(W^n)^2} \right] - 1 = 0, \forall n,$$

which boils down to (noting that $r^n = \frac{W^n}{E^n}$ and $p^j = \frac{B^j}{Q^j}$)

$$\frac{p^j}{r^n} \frac{\partial f^j(\phi_{k_j})}{\partial \phi_{k_j}^n} \frac{Q_{-k_j}^j}{Q^j} \frac{W_{-k_j}^n}{W^n} - 1 = 0. \quad (21)$$

Note that if the market consists of a very large number of firms, (21) boils down to the statement that the value of the marginal product of the n^{th} input is equal to the price of the n^{th} input because $Q_{-k_j}^j / Q^j$ and $W_{-k_j}^n / W^n$ will be almost one. Letting λ be the Lagrange multiplier of (20), the first-order condition of (19) - (20) with respect to b_h^j (noting that $x_h = (x_h^1, \dots, x_h^J)$, $x_h^j = b_h^j \frac{Q^j}{B^j}$) is

$$\frac{\partial u_h(x_h)}{\partial x_h^j} \left[\frac{Q^j}{B^j} \frac{B_{-h}^j}{B^j} \right] + \lambda \left[\frac{\sum_{k=1}^{K_j} \theta_h^{k_j} f^j(\phi_{k_j})}{Q^j} - 1 \right] = 0. \quad (22)$$

Note that if the market consists of a very large number of firms and households, (22) collapses to the statement that the marginal utility divided by the price is equal to the Lagrange multiplier because B_{-h}^j / B^j will be almost one and $f^j(\phi_{k_j}) / Q^j$ will be almost zero.

3.3.1 Existence of Interior Solution

In this section, we show that as long as output prices are strictly positive, any firm's profit function is strictly quasi-concave. For this analysis, we suppress consideration of the specific production sector, and hence we will drop index j from the notation and keep the notation as is otherwise. However, the analysis can easily be extended to the case with multiple production sectors. On the revenue side, let $h_k \equiv D_{\phi_k}(pq_k) = D_{\phi_k} f(\phi_k) \left[\frac{d(pq_k)}{dq_k} \right]$, where

$$\frac{d(pq_k)}{dq_k} = \frac{BQ_{-k}}{Q^2} \geq 0.$$

Letting production function f be homogeneous of degree $\gamma \geq 0$, Euler's theorem implies that $\phi_k \cdot D_{\phi_k} f(\phi_k) = \gamma f(\phi_k)$. Now consider

$$\phi_k \cdot h_k = \phi_k \cdot D_{\phi_k} f(\phi_k) \left[\frac{d(pq_k)}{dq_k} \right] = \gamma q_k \frac{BQ_{-k}}{Q^2} \geq 0,$$

and as $q_k \rightarrow \infty$, $\phi_k \cdot h_k \rightarrow 0$. Next, differentiating $\frac{d(pq_k)}{dq_k}$ with respect to q_k , we get

$$\frac{d^2(pq_k)}{dq_k^2} = \frac{d}{dq_k} \left[\frac{BQ_{-k}}{Q^2} \right] = -2 \frac{BQ_{-k}}{Q^3} < 0.$$

Then, it follows that

$$\begin{aligned} D_{\phi_k} h_k &= \frac{BQ_{-k}}{Q^2} D_{\phi_k}^2 f(\phi_k) - 2D_{\phi_k} f(\phi_k) \frac{BQ_{-k}}{Q^3} D_{\phi_k} f(\phi_k)^T \\ &= \frac{BQ_{-k}}{Q^2} \left[D_{\phi_k}^2 f(\phi_k) - \frac{2}{Q} D_{\phi_k} f(\phi_k) D_{\phi_k} f(\phi_k)^T \right]. \end{aligned}$$

Using Euler's theorem yields

$$\phi_k \cdot D_{\phi_k} f(\phi_k) = \gamma f(\phi_k).$$

Differentiating both sides with respect to ϕ_k gives

$$D_{\phi_k}^2 f(\phi_k) \phi_k = (\gamma - 1) D_{\phi_k} f(\phi_k) \text{ or}$$

$$D_{\phi_k}^2 f(\phi_k) \phi_k D_{\phi_k} f(\phi_k)^T = (\gamma - 1) D_{\phi_k} f(\phi_k) D_{\phi_k} f(\phi_k)^T.$$

Hence, denoting the identity matrix by I , we have

$$\begin{aligned} D_{\phi_k}^2 f(\phi_k) - \frac{2}{Q} D_{\phi_k} f(\phi_k) D_{\phi_k} f(\phi_k)^T &= D_{\phi_k}^2 f(\phi_k) + \frac{2}{Q(1-\gamma)} D_{\phi_k}^2 f(\phi_k) \phi_k D_{\phi_k} f(\phi_k)^T \\ &= D_{\phi_k}^2 f(\phi_k) \left[I + \frac{2}{Q(1-\gamma)} \phi_k D_{\phi_k} f(\phi_k)^T \right]. \end{aligned}$$

Now let production function f be strictly quasi-concave.²⁷ Then, along any direction ρ orthogonal to $D_{\phi_k} f$, we obtain

$$\rho^T D_{\phi_k} h_k \rho = \frac{BQ_{-k}}{Q^2} \rho^T D_{\phi_k}^2 f(\phi_k) \rho < 0$$

because $BQ_{-k}/Q^2 > 0$ and f is strictly quasi-concave. Thus, the firm's revenue function is quasi-concave.

²⁷Note that a strictly quasi-concave production function can exhibit any returns to scale, encompassing decreasing, constant, and increasing returns.

On the cost side, firm k 's allocation of input good n , $\phi_k^n = w_k^n \frac{E^n}{W^n}$ implies $\frac{W_{-k}^n + w_k^n}{w_k^n} = \frac{E^n}{\phi_k^n}$, which leads to $\frac{W_{-k}^n}{w_k^n} + 1 = \frac{E^n}{\phi_k^n}$. Then, we get

$$w_k^n = \frac{\phi_k^n W_{-k}^n}{E^n - \phi_k^n}.$$

As a result, total cost of firm k is

$$C(\phi_k) = \sum_{n=1}^N w_k^n = \sum_{n=1}^N \frac{\phi_k^n W_{-k}^n}{E^n - \phi_k^n}.$$

Then, $D_{\phi}^2 C(\phi_k)$ is a diagonal matrix consisting of

$$\frac{\partial^2 C(\phi_k)}{\partial (\phi_k^n)^2} = \frac{2W_{-k}^n E^n}{(E^n - \phi_k^n)^3} \geq 0.$$

Because this matrix is positive definite, cost function C is strictly convex. Hence, when γ is sufficiently small (e.g., $\gamma < 1$), profit function π will be strictly quasi-concave. In particular, the upper contour sets for profit function π of any firm will be convex. When we impose the following mild assumption

$$\lim_{\phi \rightarrow \infty} \frac{1}{f(\phi)} D_{\phi} f < \infty,$$

this will guarantee that as ϕ gets large, profit π becomes negative since cost C is unbounded while revenue $R = \frac{B}{Q} f \leq B$ is bounded. Thus, $D_{\phi} \pi$ will be asymptotically negative. The restriction itself bounds the production function below something exponential, and is sufficient, though not necessary. In this case, when $D_{\phi_k} f(0)$ and E^n are sufficiently large, we obtain

$$D_{\phi_k} \pi_k(0) = \frac{B}{Q_{-k}} D_{\phi_k} f(0) - \frac{W_{-k}^n}{E^n} > 0. \quad (23)$$

The following lemma shows the conditions under which a firm seeks an interior profit maximum.

Lemma 5 *In sectors with sufficiently small γ and sufficiently large $D_{\phi} f(0)$ and E^n , firms will seek interior profit maximum.*

3.3.2 Existence of Equilibrium

In this section, we present the main existence theorem.

Theorem 3 *Suppose that there exist at least two firms that use all input goods and satisfy condition (23). Then there exists an equilibrium in which all households make positive bids and some firms make positive*

bids (i.e., produce).

Proof. We will prove the existence of equilibrium by using Kakutani's fixed-point theorem. We first define a best-response mapping from a set into itself. When households make positive bids on output goods of two firms that use all inputs, the best response for these two firms, say \widehat{k}_j and \widetilde{k}_j , is to make positive production, and hence make positive bids on inputs goods. Similarly, when these two firms make positive bids on input goods, the best response for households is to make positive bids on output goods of these two firms. When households and these two firms start with positive bids, the best response for households and these two firms is to continue to make positive bids. Then, there exists sufficiently small $\epsilon > 0$ such that the vector of household h 's bids on output goods $b_h \geq \epsilon \cdot \iota$ for all $h \in \{1, \dots, H\}$ and the vector of firm k_j 's bids on input goods $w_{k_j} \geq \epsilon \cdot \iota$ for all $k_j \in \{\widehat{k}_j, \widetilde{k}_j\}$, where ι is the vector of ones. Note that we have a free normalization on all bids because all bids on input and output goods appear in both households' and firms' budget constraints. Thus, we define the vector of all bids β as follows

$$\beta = (b_1, \dots, b_H, w_{1_1}, \dots, w_{1_J}, w_{2_1}, \dots, w_{2_J}, \dots, w_{K_1}, \dots, w_{K_J}) \in \Delta_\epsilon^{JH+NK_jJ-1},$$

where b_h is J -dimensional vector for all $h \in \{1, \dots, H\}$, w_{k_j} is N -dimensional vector for all $k \in \{1, \dots, K_j\}$ and for all $j \in \{1, \dots, J\}$, and $\Delta_\epsilon^{JH+NK_jJ-1}$ is ϵ -trimmed unit simplex. We define a mapping $\zeta : \Delta_\epsilon^{JH+NK_jJ-1} \rightarrow \Delta_\epsilon^{JH+NK_jJ-1}$, where

$$\zeta(\beta) = \frac{1}{\iota^T \cdot \widehat{\beta}(\beta)} \widehat{\beta}(\beta),$$

ι is the vector of ones, and $\widehat{\beta}(\beta)$ is the vector of best responses to β .

We then show that there exists an equilibrium via Kakutani's fixed-point theorem. First, ζ is upper hemicontinuous via the maximum theorem given the standard assumptions on utility and production functions. Moreover, because $\beta \in \Delta_\epsilon^{JH+NK_jJ-1}$, we have $\iota^T \cdot \beta = 1$ and $\iota \geq \beta \geq \epsilon \cdot \iota$. Second, $\Delta_\epsilon^{JH+NK_jJ-1}$ is non-empty because at least $\left(\frac{1}{JH+NK_jJ-1}, \dots, \frac{1}{JH+NK_jJ-1}\right)$ is in $\Delta_\epsilon^{JH+NK_jJ-1}$. Third, $\Delta_\epsilon^{JH+NK_jJ-1}$ is compact because it is closed and bounded over $[\epsilon, 1]^{JH+NK_jJ-1}$. Finally, $\Delta_\epsilon^{JH+NK_jJ-1}$ is convex as follows. Let $\beta \in \Delta_\epsilon^{JH+NK_jJ-1}$, $\widetilde{\beta} \in \Delta_\epsilon^{JH+NK_jJ-1}$, and $\eta \in [0, 1]$. By definition, we have $\iota^T \cdot \beta = 1$ and $\iota^T \cdot \widetilde{\beta} = 1$ and also $\beta \geq \epsilon \cdot \iota$ and $\widetilde{\beta} \geq \epsilon \cdot \iota$. Then, we obtain

$\eta \iota^T \cdot \beta + (1 - \eta) \iota^T \cdot \tilde{\beta} = \eta + (1 - \eta) = 1$ and $\eta \beta + (1 - \eta) \tilde{\beta} \geq \eta \epsilon \cdot \iota + (1 - \eta) \epsilon \cdot \iota = \epsilon \cdot \iota$. Thus, $\eta \beta + (1 - \eta) \tilde{\beta} \in \Delta_\epsilon^{JH+NK_jJ-1}$. So, $\zeta(\cdot)$ has a fixed point, i.e., there is $\beta \in \Delta_\epsilon^{JH+NK_jJ-1}$ such that $\beta \in \zeta(\beta)$ via Kakutani's fixed-point theorem, and it is a Nash equilibrium of this game. ■

In the proof of Theorem 3, satisfying condition (23) is necessary to guarantee non-negative profits for firms, and the presence of at least two firms that use all inputs is important to open markets for input goods. The proof can be generalized to the case in which any number of firms use subsets of inputs as long as each input is used by at least two firms, and these two firms satisfy condition (23). Besides two firms that use all inputs, other firms can also be active depending on initial endowments.

3.3.3 Extended Example

In the previous section, we have shown the existence of equilibrium when condition (23) is satisfied for at least two firms. In this section, we will provide more complete characterization under the condition that all firms and households are identical, and under symmetric equilibrium. We will in turn relax the assumptions that production function f is homogeneous of degree γ and that condition (23) is satisfied. Because all firms are identical, there is a single sector in the economy, so we will drop index j . The following arguments can be generalized to multiple sectors but such generalization would complicate the analysis yet would not bring any new insights.

We first introduce best responses in this case. Under identical firms and households, each firm k 's profit maximization problem becomes

$$\max_{w_k^1, \dots, w_k^N} \frac{B}{Q_{-k} + f(\phi_k)} f(\phi_k) - \sum_{n=1}^N w_k^n, \quad (24)$$

and each household h 's utility maximization problem becomes

$$\max_{b_h} u_h(x_h) \quad (25)$$

$$\text{s.t. } b_h \leq \sum_{n=1}^N \frac{W^n}{E^n} e_h^n + \sum_{k=1}^K \theta_h^k \left(\frac{b_h + B_{-h}}{Q_{-k} + f(\phi_k)} f(\phi_k) - \sum_{n=1}^N w_k^n \right), \quad (26)$$

where $\phi_k = (w_k^1 \frac{E^1}{W^1}, \dots, w_k^N \frac{E^N}{W^N})$ and $x_h = b_h \frac{Q}{B}$. The first-order condition of (24) with respect to w_k^n

is

$$\frac{BQ_{-k}}{(Q_{-k} + f(\phi_k))^2} \frac{\partial f(\phi_k)}{\partial \phi_k^n} \frac{E^n W_{-k}^n}{(W^n)^2} - 1, \forall n. \quad (27)$$

Evaluating (27) at symmetric equilibrium $w_k^n = \widehat{w}^n$ for all $n \in \{1, \dots, N\}$ (noting that $\widehat{W}^n = K\widehat{w}^n$, $Q = Kf(\phi_k)$, and $\phi_k = (w_k^1 \frac{E^1}{W^1}, \dots, w_k^N \frac{E^N}{W^N})$) and letting $E = (E^1, \dots, E^N)$ gives

$$\begin{aligned} \frac{B}{f(\phi_k)} \frac{\partial f(\phi_k)}{\partial \phi_k^n} \frac{E^n (K-1)^2}{K^4 \widehat{w}^n} - 1 &= 0, \forall n \\ \frac{B}{f(\frac{E}{K})} \frac{\partial f(\frac{E}{K})}{\partial \phi_k^n} \frac{E^n (K-1)^2}{K^4 \widehat{w}^n} - 1 &= 0, \forall n. \end{aligned}$$

Letting λ be the Lagrange multiplier of (26), the first-order conditions of (25) - (26) are (noting that $E^n = He^n$ because of identical households)

$$u'(x_h) \frac{B_{-h}Q}{B^2} + \lambda \left(\sum_{k=1}^K \frac{\theta_h^k f(\phi_k)}{Q} - 1 \right) = 0 \quad (28)$$

$$\sum_{n=1}^N \frac{W^n}{H} + \sum_{k=1}^K \theta_h^k \left(\frac{b_h + B_{-h}}{Q} f(\phi_k) - \sum_{n=1}^N w_k^n \right) - b_h = 0. \quad (29)$$

Evaluating (28) and (29) at symmetric equilibrium $b_h = \widehat{b}$, $\theta_h^k = \frac{1}{H}$, $w_k^n = \widehat{w}^n$ (noting that $\widehat{B} = H\widehat{b}$, $Q = Kf(\phi_k)$, $\widehat{W}^n = K\widehat{w}^n$, and $x_h = b_h \frac{Q}{B}$), we obtain

$$\begin{aligned} u' \left(\frac{Q}{H} \right) \frac{(H-1)Q}{H^2 \widehat{b}} + \lambda \left(\frac{1}{H} - 1 \right) &= 0 \\ \sum_{n=1}^N \frac{K\widehat{w}^n}{H} + \frac{K}{H} \left(\frac{H\widehat{b}}{K} - \sum_{n=1}^N \widehat{w}^n \right) &= \widehat{b}. \end{aligned} \quad (30)$$

After simplifications, budget constraint (30) yields

$$\widehat{b} = \sum_{n=1}^N \frac{K\widehat{w}^n}{H} + \left(\widehat{b} - \frac{K}{H} \sum_{n=1}^N \widehat{w}^n \right) = \widehat{b}.$$

Thus, (30) yields an identity. Then, the following system of equations is necessary for equilibrium

$$\frac{H\widehat{b}}{f(\frac{E}{K})} \frac{\partial f(\frac{E}{K})}{\partial \phi_k^n} \frac{E^n (K-1)^2}{K^4} = \widehat{w}^n, \forall n \quad (31)$$

$$u' \left(\frac{Kf(\frac{E}{K})}{H} \right) \frac{Kf(\frac{E}{K})}{H\widehat{b}} = \lambda. \quad (32)$$

Finally, we need to check whether firms make nonnegative profits under any solution to (31) - (32) because only such a solution can be equilibrium. The profit of each firm under solutions to

(31) - (32) is

$$\pi = \frac{H \hat{b}}{K} - \sum_{n=1}^N \hat{w}^n.$$

Substituting (31) gives

$$\begin{aligned} \pi &= \frac{H \hat{b}}{K} \left[1 - \left(\frac{K-1}{K} \right)^2 \sum_{n=1}^N \frac{\partial f\left(\frac{E}{K}\right)}{\partial \phi_k^n} \frac{\frac{E^n}{K}}{f\left(\frac{E}{K}\right)} \right] \\ &= \frac{H \hat{b}}{K} \left[1 - \left(\frac{K-1}{K} \right)^2 \sum_{n=1}^N \mu^n(\phi_k) \right], \end{aligned}$$

where $\mu^n(\phi_k) = \frac{\partial f(\phi_k)}{\partial \phi_k^n} \frac{\phi_k^n}{f(\phi_k)}$. In fact, $\mu^n(\phi_k)$ is the input elasticity of production. Then, firms make nonnegative profits if and only if

$$\sum_{n=1}^N \mu^n\left(\frac{E}{K}\right) \leq \left(\frac{K}{K-1}\right)^2.$$

As a result, we reach the following proposition.

Proposition 8 *Given the number of firms K , input vector $E = (E^1, \dots, E^N)$, and production function f , a symmetric equilibrium exists if and only if*

$$\sum_{n=1}^N \mu^n\left(\frac{E}{K}\right) \leq \left(\frac{K}{K-1}\right)^2, \quad (33)$$

where $\mu^n(\phi_k) = \frac{\partial f(\phi_k)}{\partial \phi_k^n} \frac{\phi_k^n}{f(\phi_k)}$ for all $n \in \{1, \dots, N\}$, and $\phi_k = (\phi_k^1, \dots, \phi_k^N)$.

Corollary 3 shows the relation between the input elasticity of production, total endowments, and the number of firms.

Corollary 3 *Only a few firms can be active in equilibrium when i) the input elasticity of production is constant, or ii) the input elasticity of production is decreasing and total endowments are limited.*

Example 3 *Suppose that each firm uses input good ϕ , and its technology is specified by a Cobb-Douglas production function, i.e., $f(\phi) = \alpha (\phi)^c$, where $\alpha > 0$ and $c > 0$. The input elasticity of production is c , which is a constant. Then, (33) implies that the number of firms $K \leq 1 + \frac{1}{\sqrt{c-1}}$. For example, if $c = 1.44$, the number of firms $K \leq 6$.*

Note that in Example 3, the input elasticity of production c is in fact returns to scale. Thus,

when $c > 1$, technology exhibits increasing returns to scale. Therefore, in the presence of increasing returns, only a few firms can be active in equilibrium.

3.4 Conclusion

In this paper, we have studied imperfectly competitive production economies in which technology exhibits arbitrary returns to scale including increasing returns. We extend the pure exchange version of the market game to production economies. We first show the quasi-concavity of firms' profit functions and the existence of an interior solution. We then prove the existence of equilibrium under mild conditions. Finally, we demonstrate that only a few firms can be active in equilibrium in the presence of increasing returns.

As in pure exchange market games, Nash equilibrium allocations of production market games are suboptimal. However, unlike pure exchange market games, a large number of traders and a large number of firms may not approximate competitive outcomes. This is because only a limited number of firms can be active in the presence of increasing returns. On the other hand, the literature on Walrasian models with increasing returns requires centralized intervention to enforce pricing rules or to provide subsidies to cover firms' losses. The market game model, on the other hand, does not require such intervention, and hence the model is self-contained in this sense.

Appendix

A Managerial Compensation with Systemic Risk: A Dynamic General Equilibrium Approach

Proof of Proposition 1. We will outline the proof for the existence of strongly stationary equilibria. To do so, we first derive competitive equilibrium equations from agent's utility maximization and firm's profit maximization problems, and market clearing conditions. First, agents maximize their utilities subject to budget constraints by solving the following problem

$$\begin{aligned} \max_{c_y, c_o, a_o, e} \quad & u(c_y) + \beta E[u(c_o) - \phi(a_o)] \\ \text{s.t.} \quad & c_y = a_y \omega_y - pe \text{ and } c_o = a_o \omega_o + (p + \delta)e. \end{aligned}$$

The corresponding first order conditions are

$$\begin{aligned} \beta [E[u'(c_o)] \omega_o - \phi'(a_o)] &= 0, \\ -u'(c_y)p + \beta E[u'(c_o)(p + \delta)] &= 0 \\ c_y = a_y \omega_y - pe \text{ and } c_o &= a_o \omega_o + (p + \delta)e. \end{aligned}$$

Second, firms solve the following problem to maximize their profits

$$\max_{a_y, a_o} \left[\pi(a_o) z^H + (1 - \pi(a_o)) z^L \right] f(a_y) - a_y \omega_y - a_o \omega_o.$$

The corresponding first order conditions are

$$\omega_y = \left[\pi(a_o) z^H + (1 - \pi(a_o)) z^L \right] f'(a_y) \text{ and } \omega_o = \pi'(a_o) (z^H - z^L) f(a_y).$$

Third, the consumption good, equity, and labor market clearing conditions are

$$c_y + c_o = \gamma, e = 1, \text{ and } a_y = \bar{a}_y.$$

Finally, dividend is equal to the residual of total output after wages are paid, so we have

$$\delta = \gamma - a_y \omega_y - a_o \omega_o.$$

We then prove strong stationarity by construction. In any potential strongly stationary competitive equilibrium, labor input a_o cannot depend on past realizations; it can only depend on the current realization of the output shock. From the time line, we know that a_o is determined before

the output shock is realized when the agent is old, and hence a_o cannot depend on this shock. Furthermore, a_o cannot depend on the shock realized when the agent is young, either. Suppose not. From the time line, we know that a_o affects the output shock realized when the agent is old, and this shock affects other endogenous variables such as consumption values. If a_o depends on the shock realized when the agent is young, then other endogenous variables affected by a_o will depend on that previous shock. However, this contradicts with strong stationarity, which requires independence of endogenous variables from past realizations. Thus, a_o does not depend on the realization of any shock. As a result, labor inputs a_y and a_o , and wages ω_y and ω_o cannot depend on states because they are determined before the realization of the output shock. Consumption values, equity price, and dividend, on the other hand, may depend on states because they are determined after the realization of the output shock.

Before we attain competitive equilibrium equations, we make some simplifications. First, we take $\beta = 1$ for simplicity, and substitute $e = 1$ and $a_y = \bar{a}_y$ into all equations given above. Second, competitive equilibrium equations do not include the market clearing condition for the consumption good because budget constraints and dividend equation imply it as follows

$$c_y = \bar{a}_y \omega_y - p, c_o = a_o \omega_o + (p + \delta) \text{ and } \delta = \gamma - \bar{a}_y \omega_y - a_o \omega_o$$

$$c_y + c_o = \bar{a}_y \omega_y + a_o \omega_o + \delta \rightarrow c_y + c_o = \gamma.$$

Third, we obtain the following set of competitive equilibrium equations

$$\omega_y = \left[\pi(a_o)z^H + (1 - \pi(a_o))z^L \right] f'(\bar{a}_y) \quad (34)$$

$$\omega_o = \pi'(a_o)(z^H - z^L)f(\bar{a}_y) \quad (35)$$

$$\left[\pi(a_o)u'(c_o^H)(p^H + \delta^H) + (1 - \pi(a_o))u'(c_o^L)(p^L + \delta^L) \right] = u'(c_y^H)p^H \quad (36)$$

$$\left[\pi(a_o)u'(c_o^H)(p^H + \delta^H) + (1 - \pi(a_o))u'(c_o^L)(p^L + \delta^L) \right] = u'(c_y^L)p^L \quad (37)$$

$$\left[\pi(a_o)u'(c_o^H) + (1 - \pi(a_o))u'(c_o^L) \right] \omega_o - \phi'(a_o) = 0 \quad (38)$$

$$c_y^s = \bar{a}_y \omega_y - p^s \text{ for } s = H, L \quad (39)$$

$$c_o^{s'} = a_o \omega_o + (p^{s'} + \delta^{s'}) \text{ for } s' = H, L \quad (40)$$

$$\delta^s = \gamma^s - \bar{a}_y \omega_y - a_o \omega_o \text{ for } s = H, L. \quad (41)$$

This system of 11 independent equations and 11 variables $c_y^H, c_y^L, c_o^H, c_o^L, p^H, p^L, \delta^H, \delta^L, \omega_y, \omega_o, a_o$ has a solution based on the work of Kehoe and Levine (1984). ■

Proof of Proposition 2. We will outline the proof for inefficiency of competitive equilibrium allocations. To do so, we first derive the equations that a Pareto optimal allocation satisfies. A Pareto optimal allocation is a solution to the following planner's problem

$$\max_{c_y^H, c_y^L, c_o^H, c_o^L, a_o} (1 - \alpha)E[u(c_y)] + \alpha(E[u(c_o)] - \phi(a_o)) \text{ subject to } c_y^s + c_o^s = \gamma^s.$$

The corresponding first order conditions are

$$(1 - \alpha)\pi(a_o)u'(c_y^H) - \lambda^H = 0 \quad (42)$$

$$(1 - \alpha)(1 - \pi(a_o))u'(c_y^L) - \lambda^L = 0 \quad (43)$$

$$\alpha\pi(a_o)u'(c_o^H) - \lambda^H = 0 \quad (44)$$

$$\alpha(1 - \pi(a_o))u'(c_o^L) - \lambda^L = 0 \quad (45)$$

$$(1 - \alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) + \alpha(\pi'(a_o)(u(c_o^H) - u(c_o^L)) - \phi'(a_o)) = 0 \quad (46)$$

$$c_y^s + c_o^s = \gamma^s. \quad (47)$$

Let $\{c_y^s, c_o^s, p^s, \delta^s, \omega_y, \omega_o, a_o \mid s = H, L\}$ be a competitive equilibrium allocation. Then, by definition, $\{c_y^s, c_o^s, p^s, \delta^s, \omega_y, \omega_o, a_o \mid s = H, L\}$ satisfies the competitive equilibrium equations from (34) to (41). Suppose to the contrary that the competitive equilibrium allocation is Pareto optimal. Then, there must be Pareto weight α such that $\{c_y^s, c_o^s, p^s, \delta^s, \omega_y, \omega_o, a_o \mid s = H, L\}$ satisfies Pareto optimality equations from (42) to (47). In fact, (47) is already satisfied because (39), (40), and (41) imply it as follows

$$c_y^s + c_o^s = \bar{a}_y \omega_y + a_o \omega_o + \delta^s = \gamma^s.$$

Thus, $\{c_y^s, c_o^s, p^s, \delta^s, \omega_y, \omega_o, a_o \mid s = H, L\}$ should satisfy all competitive equilibrium and Pareto optimality equations from (34) to (46). We have a system of 16 independent equations and 14 variables $c_y^H, c_y^L, c_o^H, c_o^L, p^H, p^L, \delta^H, \delta^L, \omega_y, \omega_o, a_o, \alpha, \lambda^H, \lambda^L$. Since the number of equations exceeds the number of variables, it can be shown that this system has no solution by applying the procedures of Spear (1985) or Citanna and Siconolfi (2007). ■

Proof of Theorem 1. We first prove that paying optimal wages to old agents is a necessary con-

dition to attain a Pareto optimal competitive equilibrium. We then prove that a Pareto optimal allocation can be implemented by imposing wage tax and equity tax.

First, optimal wages $\omega_o^* = \frac{(1-\alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) + \alpha\pi'(a_o)(u(c_o^H) - u(c_o^L))}{\alpha E[u'(c_o)]}$ must be paid to old agents to achieve a Pareto optimal competitive equilibrium. As Proposition 2 shows, a competitive equilibrium allocation is not Pareto optimal because it fails to satisfy all competitive equilibrium and Pareto optimality equations from (34) to (46) simultaneously. Both competitive equilibrium and Pareto optimality equations involve first order conditions with respect to a_o ; and unless these two equations (i.e., (38) and (46)) are combined, a competitive equilibrium allocation cannot satisfy both of them. Thus, we combine them by substituting (38) into (46) as follows

$$E[u'(c_o)]\omega_o = \frac{(1-\alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) + \alpha\pi'(a_o)(u(c_o^H) - u(c_o^L))}{\alpha}.$$

Then, this equation yields optimal wage ω_o^* as follows

$$\omega_o^* = \frac{(1-\alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) + \alpha\pi'(a_o)(u(c_o^H) - u(c_o^L))}{\alpha E[u'(c_o)]}. \quad (48)$$

Therefore, paying optimal wages to old agents is a necessary condition for achieving a Pareto optimal competitive equilibrium.

Second, if the social planner imposes wage tax $t_\omega = \pi'(a_o^*)(z^H - z^L)f(\bar{a}_y) - \frac{\phi'(a_o^*)}{E[u'(c_o^*)]}$ and equity tax $t^s = \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{s,*})p^{s,*}}{u'(c_o^{s,*})}$, a Pareto optimal allocation can be implemented. Let $\{c_y^{H,*}, c_y^{L,*}, c_o^{H,*}, c_o^{L,*}, a_o^*\}$ be a Pareto optimal allocation. Then, by definition, it satisfies the Pareto optimality equations from (42) to (47). We will prove, by construction, that $\{c_y^{H,*}, c_y^{L,*}, c_o^{H,*}, c_o^{L,*}, a_o^*\}$ satisfies the competitive equilibrium equations from (34) to (41) after wage tax and equity tax are imposed.

One possible way to impose equity tax t^s is as follows

$$c_o^s = a_o\omega_o + (p^s - t^s + \delta^s)e + t^s.$$

At an equilibrium, equity market clearing condition implies that $e = 1$, and the equation above collapses to (40). Thus, equity tax does not affect budget constraints; it only affects the first order conditions with respect to equity (i.e., (36) and (37)). After equity tax is imposed, (36) and (37)

become the following equations, respectively

$$\pi(a_o)[u'(c_o^H)(p^H - t^H + \delta^H)] + (1 - \pi(a_o))[u'(c_o^L)(p^L - t^L + \delta^L)] = u'(c_y^H)p^H \quad (49)$$

$$\pi(a_o)[u'(c_o^H)(p^H - t^H + \delta^H)] + (1 - \pi(a_o))[u'(c_o^L)(p^L - t^L + \delta^L)] = u'(c_y^L)p^L. \quad (50)$$

On the other hand, wage tax only affects old agents' wage equation (35). After wage tax is imposed, (35) becomes the following equation

$$\omega_o = \pi'(a_o)(z^H - z^L)f(\bar{a}_y) - t_\omega. \quad (51)$$

Next, we will plug the Pareto optimal allocation $\{c_y^{H,*}, c_y^{L,*}, c_o^{H,*}, c_o^{L,*}, a_o^*\}$ into the updated competitive equilibrium equations (34), (38)-(41), and (49)-(51) one by one. First, substituting the Pareto optimal allocation into (38) gives old agents' optimal wages $\omega_o^* = \frac{\phi'(a_o^*)}{E[u'(c_o^*)]}$. Second, placing the Pareto optimal allocation into (34) yields young agents' optimal wages $\omega_y^* = [\pi(a_o^*)z^H + (1 - \pi(a_o^*))z^L]f'(\bar{a}_y)$. Third, substituting the Pareto optimal allocation, ω_y^* and ω_o^* into (41) provides dividend $\delta^{s,*} = \gamma^s - \bar{a}_y\omega_y^* - a_o^*\omega_o^*$, where $s = H, L$. Fourth, by placing the Pareto optimal allocation and ω_y^* into (39), we obtain prices $p^{s,*} = \bar{a}_y\omega_y^* - c_y^{s,*}$, where $s = H, L$. We now know the expressions for all 11 variables $c_y^{H,*}, c_y^{L,*}, c_o^{H,*}, c_o^{L,*}, p^{H,*}, p^{L,*}, \delta^{H,*}, \delta^{L,*}, \omega_y^*, \omega_o^*, a_o^*$. Finally, we verify the remaining equations. Plugging the Pareto optimal allocation along with $\omega_o^*, p^{s,*}$, and $\delta^{s,*}$ verifies (40) for $s = H, L$ as follows

$$c_o^{s,*} = a_o^*\omega_o^* + p^{s,*} + \delta^{s,*} = -c_y^{s,*} + \gamma^s = c_o^{s,*}.$$

Substituting the Pareto optimal allocation, ω_o^* and t_ω into (51) yields

$$\omega_o^* = \pi'(a_o^*)(z^H - z^L)f(\bar{a}_y) - t_\omega = \frac{\phi'(a_o^*)}{E[u'(c_o^*)]}.$$

The last step is to verify that (49) and (50) are satisfied. Placing the Pareto optimal allocation along

with $p^{s,*}$, $\delta^{s,*}$, and t^s into (49) gives

$$\begin{aligned}
u'(c_y^{H,*})p^{H,*} &= \pi(a_o^*)[u'(c_o^{H,*})(p^{H,*} - t^H + \delta^{H,*})] + (1 - \pi(a_o^*))[u'(c_o^{L,*})(p^{L,*} - t^L + \delta^{L,*})] \\
&= \pi(a_o^*) \left[u'(c_o^{H,*}) \left(p^{H,*} - \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{H,*})p^{H,*}}{u'(c_o^{H,*})} + \delta^{H,*} \right) \right] \\
&+ (1 - \pi(a_o^*)) \left[u'(c_o^{L,*}) \left(p^{L,*} - \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{L,*})p^{L,*}}{u'(c_o^{L,*})} + \delta^{L,*} \right) \right] \\
&= \pi(a_o^*)u'(c_o^{H,*})(p^{H,*} + \delta^{H,*}) + (1 - \pi(a_o^*))u'(c_o^{L,*})(p^{L,*} + \delta^{L,*}) - E[u'(c_o^*)(p^* + \delta^*)] + u'(c_y^{H,*})p^{H,*} \\
&= u'(c_y^{H,*})p^{H,*}.
\end{aligned}$$

Plugging the Pareto optimal allocation as well as $p^{s,*}$, $\delta^{s,*}$, and t^s verifies (50) as follows

$$\begin{aligned}
u'(c_y^{L,*})p^{L,*} &= \pi(a_o^*)[u'(c_o^{H,*})(p^{H,*} - t^H + \delta^{H,*})] + (1 - \pi(a_o^*))[u'(c_o^{L,*})(p^{L,*} - t^L + \delta^{L,*})] \\
&= \pi(a_o^*) \left[u'(c_o^{H,*}) \left(p^{H,*} - \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{H,*})p^{H,*}}{u'(c_o^{H,*})} + \delta^{H,*} \right) \right] \\
&+ (1 - \pi(a_o^*)) \left[u'(c_o^{L,*}) \left(p^{L,*} - \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{L,*})p^{L,*}}{u'(c_o^{L,*})} + \delta^{L,*} \right) \right] \\
&= \pi(a_o^*)u'(c_o^{H,*})(p^{H,*} + \delta^{H,*}) + (1 - \pi(a_o^*))u'(c_o^{L,*})(p^{L,*} + \delta^{L,*}) - E[u'(c_o^*)(p^* + \delta^*)] + u'(c_y^{L,*})p^{L,*} \\
&= u'(c_y^{L,*})p^{L,*}.
\end{aligned}$$

Thus, a Pareto optimal allocation is implemented when the social planner imposes wage tax

$$t_\omega = \pi'(a_o^*)(z^H - z^L)f(\bar{a}_y) - \frac{\phi'(a_o^*)}{E[u'(c_o^*)]} \text{ and equity tax } t^s = \frac{E[u'(c_o^*)(p^* + \delta^*)] - u'(c_y^{s,*})p^{s,*}}{u'(c_o^{s,*})}. \blacksquare$$

Proof of Proposition 3. We will show that if the social planner imposes equity tax $t^s = \frac{E[u'(c_o^*)(\hat{p} + \hat{\delta})] - u'(c_y^s)\hat{p}^s}{u'(c_o^s)}$,

a second-best allocation is implemented. We first derive the equations that a second-best allocation satisfies. A second-best allocation is a solution to the following planner's problem

$$\begin{aligned}
&\max_{c_y^H, c_y^L, c_o^H, c_o^L, a_o} (1 - \alpha)E[u(c_y)] + \alpha(E[u(c_o)] - \phi(a_o)) \text{ subject to } c_y^s + c_o^s = \gamma^s \text{ and} \\
&\left[\pi(a_o)u'(c_o^H) + (1 - \pi(a_o))u'(c_o^L) \right] \pi'(a_o)(z^H - z^L)f(\bar{a}_y) - \phi'(a_o) = 0.
\end{aligned}$$

The corresponding first order conditions are

$$\pi(a_o)[(1 - \alpha)u'(c_y^H) - \alpha u'(\gamma^H - c_y^H) - \mu u''(\gamma^H - c_y^H)\pi'(a_o)(z^H - z^L)f(\bar{a}_y)] = 0 \quad (52)$$

$$(1 - \pi(a_o))[(1 - \alpha)u'(c_y^L) - \alpha u'(\gamma^L - c_y^L) - \mu u''(\gamma^L - c_y^L)\pi'(a_o)(z^H - z^L)f(\bar{a}_y)] = 0 \quad (53)$$

$$(1 - \alpha)\pi'(a_o)(u(c_y^H) - u(c_y^L)) - \mu\phi''(a_o) + \mu E[u'(\gamma - c_y)]\pi''(a_o)(z^H - z^L)f(\bar{a}_y) \\ + (\alpha + \mu\pi'(a_o)(z^H - z^L)f(a_y))\pi'(a_o)(u(\gamma^H - c_y^H) - u(\gamma^L - c_y^L)) - \alpha\pi'(a_o)\phi'(a_o) = 0 \quad (54)$$

$$E[u'(c_o)]\pi'(a_o)(z^H - z^L)f(\bar{a}_y) - \phi'(a_o) = 0 \quad (55)$$

$$c_y^s + c_o^s = \gamma^s. \quad (56)$$

Let $\{\widehat{c}_y^H, \widehat{c}_y^L, \widehat{c}_o^H, \widehat{c}_o^L, \widehat{a}_o\}$ be a second-best allocation. Then, by definition, it satisfies the second-best equations from (52) to (56). We will prove, by construction, that $\{\widehat{c}_y^H, \widehat{c}_y^L, \widehat{c}_o^H, \widehat{c}_o^L, \widehat{a}_o\}$ satisfies the competitive equilibrium equations from (34) to (41) after equity tax t^s is imposed. The equity tax here is very similar to the one used in the proof of Theorem 1, so it only affects the first order conditions with respect to equity (i.e., (36) and (37)). After the equity tax is imposed, (36) and (37) become (49) and (50), and all other competitive equilibrium equations remain the same.

We will show that $\{\widehat{c}_y^H, \widehat{c}_y^L, \widehat{c}_o^H, \widehat{c}_o^L, \widehat{a}_o\}$ satisfies the updated competitive equilibrium equations (34), (35), (38) - (41), (49), and (50). First, (55) is obtained by substituting (35) into (38). Because the second-best allocation satisfies (55), it directly satisfies (35) and (38) as well. Hence, we have old agents' wages $\widehat{\omega}_o = \pi'(\widehat{a}_o)(z^H - z^L)f(\bar{a}_y)$. Second, plugging the second-best allocation into (34) yields young agents' wages $\widehat{\omega}_y = [\pi(\widehat{a}_o)z^H + (1 - \pi(\widehat{a}_o))z^L]f'(\bar{a}_y)$. Third, placing the second-best allocation, $\widehat{\omega}_y$ and $\widehat{\omega}_o$ into (41) gives dividend $\widehat{\delta}^s = \gamma^s - \bar{a}_y\widehat{\omega}_y - \widehat{a}_o\widehat{\omega}_o$ for $s = H, L$. Fourth, substituting the second-best allocation and $\widehat{\omega}_y$ into (39) provides prices $\widehat{p}^s = \bar{a}_y\widehat{\omega}_y - \widehat{c}_y^s$ for $s = H, L$. Now we know the expressions for all 11 variables $\widehat{c}_y^H, \widehat{c}_y^L, \widehat{c}_o^H, \widehat{c}_o^L, \widehat{p}^H, \widehat{p}^L, \widehat{\delta}^H, \widehat{\delta}^L, \widehat{\omega}_y, \widehat{\omega}_o, \widehat{a}_o$. Finally, we verify the remaining equations. Placing the second-best allocation along with $\widehat{\omega}_o, \widehat{p}^s$, and $\widehat{\delta}^s$ verifies (40) for $s = H, L$ as follows

$$\widehat{c}_o^s = \widehat{a}_o\widehat{\omega}_o + \widehat{p}^s + \widehat{\delta}^s = -\widehat{c}_y^s + \gamma^s = \widehat{c}_o^s.$$

Plugging the second-best allocation along with \widehat{p}^s , $\widehat{\delta}^s$, and t^s verifies (49) as follows

$$\begin{aligned}
u'(c_y^H)\widehat{p}^H &= \pi(\widehat{a}_o)[u'(c_o^H)(\widehat{p}^H - t^H + \widehat{\delta}^H)] + (1 - \pi(\widehat{a}_o))[u'(c_o^L)(\widehat{p}^L - t^L + \widehat{\delta}^L)] \\
&= \pi(\widehat{a}_o) \left[u'(c_o^H) \left(\widehat{p}^H - \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(c_y^H)\widehat{p}^H}{u'(c_o^H)} + \widehat{\delta}^H \right) \right] \\
&+ (1 - \pi(\widehat{a}_o)) \left[u'(c_o^L) \left(\widehat{p}^L - \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(c_y^L)\widehat{p}^L}{u'(c_o^L)} + \widehat{\delta}^L \right) \right] \\
&= \pi(\widehat{a}_o)u'(c_o^H)(\widehat{p}^H + \widehat{\delta}^H) + (1 - \pi(\widehat{a}_o))u'(c_o^L)(\widehat{p}^L + \widehat{\delta}^L) - E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] + u'(c_y^H)\widehat{p}^H = u'(c_y^H)\widehat{p}^H.
\end{aligned}$$

Substituting the second-best allocation along with \widehat{p}^s , $\widehat{\delta}^s$, and t^s verifies (50) as follows

$$\begin{aligned}
u'(c_y^L)\widehat{p}^L &= \pi(\widehat{a}_o)[u'(c_o^H)(\widehat{p}^H - t^H + \widehat{\delta}^H)] + (1 - \pi(\widehat{a}_o))[u'(c_o^L)(\widehat{p}^L - t^L + \widehat{\delta}^L)] \\
&= \pi(\widehat{a}_o) \left[u'(c_o^H) \left(\widehat{p}^H - \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(c_y^H)(\widehat{p}^H)}{u'(c_o^H)} + \widehat{\delta}^H \right) \right] \\
&+ (1 - \pi(\widehat{a}_o)) \left[u'(c_o^L) \left(\widehat{p}^L - \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(c_y^L)(\widehat{p}^L)}{u'(c_o^L)} + \widehat{\delta}^L \right) \right] \\
&= \pi(\widehat{a}_o)u'(c_o^H)(\widehat{p}^H + \widehat{\delta}^H) + (1 - \pi(\widehat{a}_o))u'(c_o^L)(\widehat{p}^L + \widehat{\delta}^L) - E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] + u'(c_y^L)\widehat{p}^L = u'(c_y^L)\widehat{p}^L.
\end{aligned}$$

Therefore, a second-best allocation will be implemented when the social planner imposes equity

$$\text{tax } t^s = \frac{E[u'(\widehat{c}_o)(\widehat{p} + \widehat{\delta})] - u'(c_y^s)\widehat{p}^s}{u'(c_o^s)}. \blacksquare$$

B Consumer Unions: Blessing or Curse?

Proof of Lemma 1. In four steps, we prove that a period t is off-peak (i.e., $\mu^t = 0$) in equilibrium if and only if $q^{t,*}[K] \leq K$. First, we derive the set of equations that the producer's uncapacitated offer $q^{t,*}[K]$ satisfies. Second, we derive the set of equations that a Nash equilibrium satisfies. Third, we show that if $q^{t,*}[K] \leq K$, then t is off-peak in equilibrium. Finally, we show that if t is off-peak in equilibrium, then $q^{t,*}[K] \leq K$.

First, as Definition 2 states, $q^{t,*}[K]$ is the solution to (12) - (13) given

$$b_j^0 = \widehat{b}^0, q_j^t = q^{t,*}[K], B_{-j}^0 = (P-1)\widehat{b}^0, Q_{-j}^t = (P-1)q^{t,*}[K], B^t = R\widehat{b}^t, Q^0 = R\widehat{q}^0. \quad (57)$$

Letting $\eta \geq 0$ be the Lagrange multiplier of (13), the Kuhn-Tucker conditions are

$$\frac{1}{\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K} Q^0 \frac{B_{-j}^0}{(b_j^0 + B_{-j}^0)^2} - \eta = 0 \quad (58)$$

$$-\frac{1}{\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K} \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q_j^t)^{\frac{1}{c}-1} + \eta B^t \frac{Q_{-j}^t}{(q_j^t + Q_{-j}^t)^2} = 0, \forall t \quad (59)$$

$$\sum_{t=1}^T \frac{q_j^t}{q_j^t + Q_{-j}^t} B^t - b_j^0 = 0. \quad (60)$$

Substituting (57) and (58) into (59) gives

$$\frac{R^2(P-1)^2 \widehat{b}^t \widehat{q}^0}{P^4 \widehat{b}^0} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q^{t,*}[K])^{\frac{1}{c}} = 0. \quad (61)$$

Second, as Definition 1 presents, a Nash equilibrium is the solution to (10) - (14) given

$$b_j^0 = \widehat{b}^0, q_j^t = \widehat{q}^t, B_{-j}^0 = (P-1)\widehat{b}^0, Q_{-j}^t = (P-1)\widehat{q}^t, B^t = R\widehat{b}^t, Q^0 = R\widehat{q}^0, \quad (62)$$

$$b_u^t = \widehat{b}^t, q_u^0 = \widehat{q}^0, B_{-u}^t = (R-1)\widehat{b}^t, Q_{-u}^0 = (R-1)\widehat{q}^0, Q^t = P\widehat{q}^t, B^0 = P\widehat{b}^0. \quad (63)$$

Letting $\lambda \geq 0$, $\eta \geq 0$, and $\mu^t \geq 0$ be Lagrange multipliers of (11), (13), and (14) respectively, the

Kuhn-Tucker conditions in the long run are (58), (60), and the following set of equations

$$\frac{N\alpha^t(b_u^t + B_{-u}^t)}{b_u^t} \frac{B_{-u}^t}{(b_u^t + B_{-u}^t)^2} - \lambda = 0, \forall t \quad (64)$$

$$\frac{-N\alpha^0}{\omega_u - q_u^0} + \lambda B^0 \frac{Q_{-u}^0}{(q_u^0 + Q_{-u}^0)^2} = 0 \quad (65)$$

$$\frac{q_u^0}{q_u^0 + Q_{-u}^0} B^0 - \sum_{t=1}^T b_u^t = 0 \quad (66)$$

$$-\frac{1}{\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K} \left(\left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q_j^t)^{\frac{1}{c}-1} + \eta B^t \left(\frac{Q_{-j}^t}{(q_j^t + Q_{-j}^t)^2} \right) \right) - \mu^t = 0, \forall t \quad (67)$$

$$-\frac{\rho}{\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K} + \sum_{t=1}^T \mu^t = 0 \quad (68)$$

$$q_j^t \leq K \text{ and } \mu^t(K - q_j^t) = 0, \forall t. \quad (69)$$

In the short run, (68) is replaced with $K = \bar{K}$. For these Kuhn-Tucker conditions to be necessary for Nash equilibrium, utility functions must satisfy Inada conditions.²⁸ An Inada condition requires that the producer's equilibrium consumption $\hat{z}^0 > 0$. Substituting (62) into (7), we get

$$\hat{z}^0 = \frac{R\hat{q}^0}{P} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (\hat{q}^t)^{\frac{1}{c}} - \rho K > 0. \quad (70)$$

Besides (70), two more conditions are used for the rest of the proof. Plugging (62) into (69) leads to one of them: the complementary slackness condition in equilibrium

$$\hat{q}^t \leq K \text{ and } \mu^t(K - \hat{q}^t) = 0, \forall t. \quad (71)$$

Substituting (58) and (62) into (67) yields the other one: the counterpart of (61) in equilibrium

$$\frac{1}{\hat{q}^t \left(\frac{R\hat{q}^0}{P} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (\hat{q}^t)^{\frac{1}{c}} - \rho K \right)} \left(\frac{R^2(P-1)^2 \hat{b}^t \hat{q}^0}{P^4 \hat{b}^0} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (\hat{q}^t)^{\frac{1}{c}} \right) = \mu^t. \quad (72)$$

Third, we prove that if $q^{t,*}[K] \leq K$, then t is off-peak, i.e., $\mu^t = 0$ in equilibrium. Suppose to the contrary that t is peak, i.e., $\mu^t > 0$ in equilibrium. Given that $\mu^t > 0$, $\hat{q}^t > 0$, and $\frac{R\hat{q}^0}{P} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (\hat{q}^t)^{\frac{1}{c}} - \rho K > 0$ (from (70)), we must have

$$\frac{R^2(P-1)^2 \hat{b}^t \hat{q}^0}{P^4 \hat{b}^0} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (\hat{q}^t)^{\frac{1}{c}} > 0, \quad (73)$$

to satisfy (72). We know from (61) that $\frac{R^2(P-1)^2 \hat{b}^t \hat{q}^0}{P^4 \hat{b}^0} - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q^{t,*}[K])^{\frac{1}{c}} = 0$, so we need $q^{t,*}[K] > \hat{q}^t$

²⁸Note that logarithmic utility functions given in §3.2.1 satisfy Inada conditions.

to satisfy (73). Since $\mu^t > 0$, $\hat{q}^t = K$ must hold to satisfy (71). Thus, $q^{t,*}[K] > \hat{q}^t = K$, which is a contradiction with the initial assumption that $q^{t,*}[K] \leq K$.

Finally, we prove that if t is off-peak, i.e., $\mu^t = 0$ in equilibrium, then $q^{t,*}[K] \leq K$. Suppose to the contrary that $q^{t,*}[K] > K$. We know from (71) that $K \geq \hat{q}^t$, which leads to $q^{t,*}[K] > K \geq \hat{q}^t$. Given that $q^{t,*}[K] > \hat{q}^t$ and $\frac{R^2(P-1)^2\hat{b}^t\hat{q}^0}{P^4\hat{b}^0} - (\frac{1}{\theta})^{\frac{1}{c}} \frac{1}{c} (q^{t,*}[K])^{\frac{1}{c}} = 0$ (from (61)), we need (73) to hold. Given (70), (73), and $\hat{q}^t > 0$, we must have $\mu^t > 0$ to satisfy (72). However, $\mu^t > 0$ contradicts the initial assumption that $\mu^t = 0$. ■

Lemma 6 *The long-run Nash equilibrium is the same as the short-run Nash equilibrium if the short-run capacity \bar{K} is replaced with the long-run equilibrium capacity \hat{K} .*

Proof. A Nash equilibrium is the solution to (10) - (14) given (62) and (63). The long-run Nash equilibrium is determined by the Kuhn-Tucker conditions (58), (60), (64) - (69). The short-run Nash equilibrium is determined by the Kuhn-Tucker conditions (58), (60), (64) - (67), (69), and $K = \bar{K}$. Given that the long-run equilibrium capacity \hat{K} is determined by (68), the long-run Nash equilibrium is the same as the short-run Nash equilibrium if \bar{K} is replaced with \hat{K} .

Proof of Lemma 2. We show that the consumer's equilibrium consumption good allocation is $\hat{x}^0 = \omega \left(1 - \sum_{t=1}^T W_N^t\right)$, where $W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}$; and \hat{x}^0 rises with N . Union u solves (10) - (11) given (63). The Kuhn-Tucker conditions are (64) - (66). Substituting (63), (64) into (65) gives

$$\hat{b}^t = \frac{P(R-1)^2(\omega_u - \hat{q}^0)\alpha^t\hat{b}^0}{R^3\hat{q}^0\alpha^0}. \quad (74)$$

Plugging (63) and (74) into (66), we obtain a union's equilibrium offer

$$\hat{q}^0 = \frac{N\omega \sum_{t=1}^T \alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}. \quad (75)$$

Substituting (63), (75) into (9), we get the consumer's equilibrium consumption good allocation

$$\hat{x}^0 = \frac{\omega_u - \hat{q}^0}{N} = \omega \left(1 - \sum_{t=1}^T W_N^t\right), \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T \alpha^t}. \quad (76)$$

As N rises, \hat{x}^0 rises because ω does not change, W_N^t falls, and hence $1 - \sum_{t=1}^T W_N^t$ rises. ■

Lemma 7 *A union's equilibrium bid is $\hat{b}^t = \frac{NP\alpha^t}{M \sum_{t=1}^T \alpha^t}$.*

Proof. Substituting (75) into (74) yields

$$\widehat{b}^t = \frac{NP\widehat{b}^0\alpha^t}{M\sum_{t=1}^T\alpha^t}. \quad (77)$$

Plugging (62) and (77) into (60), we have

$$\widehat{b}^0 = \sum_{t=1}^T \frac{\widehat{q}^t}{\widehat{q}^t + (P-1)\widehat{q}^t} \widehat{B}^t = \frac{R}{P} \sum_{t=1}^T \widehat{b}^t = \widehat{b}^0.$$

Since (60) collapses to $\widehat{b}^0 = \widehat{b}^0$, we normalize \widehat{b}^0 to 1.²⁹ Substituting $\widehat{b}^0 = 1$ into (77) gives $\widehat{b}^t = NP\alpha^t / M\sum_{t=1}^T\alpha^t$. ■

Proof of Lemma 3. We show that in the short run and long run, the producer's uncapacitated offer is $q^{t,*} = \left(\frac{M\omega(P-1)^2c\theta^{\frac{1}{c}}}{P^3}\right)^c (W_N^t)^c$, where $W_N^t = \frac{\alpha^t}{\alpha^0\left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T\alpha^t}$; and $q^{t,*}$ decreases with N . As Definition 2 states, $q^{t,*}[K]$ is the solution to (12) - (13) given (57). The Kuhn-Tucker conditions are (58) - (60). Substituting (57) into (60) gives $\sum_{t=1}^T \frac{R\widehat{b}^t}{P} = \widehat{b}^0$. Plugging (57) and (58) into (59), we get

$$q^{t,*}[K] = \left(\frac{R^2(P-1)^2\widehat{b}^t\widehat{q}^0c\theta^{\frac{1}{c}}}{P^4\widehat{b}^0}\right)^c. \quad (78)$$

As Lemma 6 shows, the long-run Nash equilibrium is the same as the short-run Nash equilibrium if the short-run capacity \bar{K} is replaced with the long-run equilibrium capacity \widehat{K} . Then, offer \widehat{q}^0 given in (75) and bid \widehat{b}^t given in (77) are a union's equilibrium actions in the short run and long run. Substituting (77) into $\sum_{t=1}^T \frac{R\widehat{b}^t}{P} = \widehat{b}^0$ yields an identity $\widehat{b}^0 = \widehat{b}^0$ as in Lemma 7. Plugging (75) and (77) into (78), we obtain the producer's uncapacitated offer as follows

$$q^{t,*} = \left(\frac{M\omega(P-1)^2c\theta^{\frac{1}{c}}}{P^3}\right)^c (W_N^t)^c, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0\left(\frac{M}{M-N}\right)^2 + \sum_{t=1}^T\alpha^t}. \quad (79)$$

Since the producer's uncapacitated offer does not depend on his capacity K , it is denoted by $q^{t,*}$ instead of $q^{t,*}[K]$. As N rises, $q^{t,*}$ falls because $\left(\frac{M\omega(P-1)^2c\theta^{\frac{1}{c}}}{P^3}\right)^c$ does not change, and W_N^t falls. ■

Proof of Proposition 4. The proof proceeds as follows. First, we show that at an off-peak period, the consumer's equilibrium electricity allocation is $\widehat{x}^t = \frac{\omega^c(P-1)^{2c}c^c\theta}{M^{1-c}P^{3c-1}}(W_N^t)^c$; and \widehat{x}^t decreases with union size N . Second, we show that the relative electricity price in equilibrium is $\frac{\widehat{p}^t}{\widehat{p}^0} = \frac{(M\omega)^{1-c}P^{3c-1}}{(P-1)^{2c}c^c\theta}(W_N^t)^{1-c}$; and as N rises, $\widehat{p}^t/\widehat{p}^0$ decreases, stays the same, and increases if $c < 1$, $c = 1$, and $c > 1$, respectively.

²⁹There is also an economic rationale behind this normalization. Because bids are in terms of units of account, until we specify how much unit of account is available in the model, bids and prices are undetermined.

First, at off-peak period t , the producer's equilibrium offer \hat{q}^t is equal to his uncapacitated offer $q^{t,*}$ given in (79). Hence, the producer's equilibrium offer is

$$\hat{q}^t = \left(\frac{M\omega(P-1)^{2c}\theta^{\frac{1}{c}}}{P^3} \right)^c (W_N^t)^c, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}. \quad (80)$$

Substituting (63) and (80) into (8), we obtain the consumer's equilibrium electricity allocation

$$\hat{x}^t = \frac{1}{N} \frac{P\hat{q}^t}{R} = \frac{\omega^c(P-1)^{2c}c^c\theta}{M^{1-c}P^{3c-1}} (W_N^t)^c, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}. \quad (81)$$

As N rises, \hat{x}^t falls because $\frac{\omega^c(P-1)^{2c}c^c\theta}{M^{1-c}P^{3c-1}}$ does not change, and W_N^t falls.

Second, we calculate equilibrium electricity price \hat{p}^t and equilibrium consumption good price \hat{p}^0 by using (62), (63), (75), (77), and (80) as follows

$$\hat{p}^t = \frac{R\hat{b}^t}{P\hat{q}^t} = \frac{\hat{b}^0\alpha^t}{\sum_{t=1}^T \alpha^t} \left(\frac{M\omega(P-1)^{2c}\theta^{\frac{1}{c}}\alpha^t}{P^3 \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right)} \right)^{-c}, \hat{p}^0 = \frac{P\hat{b}^0}{R\hat{q}^0} = \frac{P\hat{b}^0 \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right)}{M\omega \sum_{t=1}^T \alpha^t}.$$

Then, the relative electricity price in equilibrium \hat{p}^t/\hat{p}^0 is

$$\frac{\hat{p}^t}{\hat{p}^0} = \frac{(M\omega)^{1-c}P^{3c-1}}{(P-1)^{2c}c^c\theta} (W_N^t)^{1-c}, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}. \quad (82)$$

As N rises, the change in \hat{p}^t/\hat{p}^0 is determined by the change in $(W_N^t)^{1-c}$ because $\frac{(M\omega)^{1-c}P^{3c-1}}{(P-1)^{2c}c^c\theta}$ does not change, and W_N^t falls. Then, \hat{p}^t/\hat{p}^0 decreases, stays the same, and increases if $c < 1$, $c = 1$, and $c > 1$, respectively. ■

Proof of Proposition 5. The proof has two steps. First, we prove that at a peak period t , the consumer's equilibrium electricity allocation is $\hat{x}^t = \frac{P\bar{K}}{M}$; and show how it changes with union size N . Second, we prove that the relative electricity price in equilibrium is $\frac{\hat{p}^t}{\hat{p}^0} = \frac{M\omega}{P\bar{K}} W_N^t$; and show how it changes with N .

First, at a peak period t , the producer's equilibrium offer \hat{q}^t is equal to his capacity \bar{K} and strictly less than his uncapacitated offer $q^{t,*}$. Substituting (63) and $\hat{q}^t = \bar{K}$ into (8), we get the consumer's equilibrium electricity allocation

$$\hat{x}^t = \frac{1}{N} \frac{P\hat{q}^t}{R} = \frac{P\bar{K}}{M}. \quad (83)$$

As N rises, $q^{t,*}$ decreases (Lemma 3). If $q^{t,*}$ is still higher than \bar{K} , t remains peak, and \hat{x}^t is given in (83). As N increases, \hat{x}^t does not change because $P\bar{K}/M$ does not change. If $q^{t,*}$ falls below \bar{K} after

N rises, t turns into off-peak, and \hat{x}^t is given in (81), which is decreasing in N .

Second, we calculate equilibrium electricity price \hat{p}^t and equilibrium consumption good price \hat{p}^0 by using (62), (63), (75), (77), and $\hat{q}^t = \bar{K}$ as follows

$$\hat{p}^t = \frac{R\hat{b}^t}{P\hat{q}^t} = \frac{\hat{b}^0\alpha^t}{\bar{K}\sum_{t=1}^T\alpha^t}, \hat{p}^0 = \frac{P\hat{b}^0}{R\hat{q}^0} = \frac{P\hat{b}^0}{M\omega\sum_{t=1}^T\alpha^t} \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right).$$

Then, the relative electricity price in equilibrium \hat{p}^t/\hat{p}^0 is

$$\frac{\hat{p}^t}{\hat{p}^0} = \frac{M\omega}{P\bar{K}} W_N^t, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}. \quad (84)$$

If t remains peak as N rises, \hat{p}^t/\hat{p}^0 is given in (84). As N increases, \hat{p}^t/\hat{p}^0 falls because $M\omega/P\bar{K}$ does not change, and W_N^t falls. If t turns into off-peak as N rises, \hat{p}^t/\hat{p}^0 is given in (82), which decreases, stays the same, and increases if $c < 1$, $c = 1$, and $c > 1$, respectively. ■

Lemma 8 *There exists $N_0 < M$ such that the set of off-peak periods $\mathcal{L} \neq \emptyset$ for all $N > N_0$.*

Proof. By Lemma 3, the producer's uncapacitated offer is $q^{t,*}[N] = \left(\frac{M\omega(P-1)^2 c \theta^{\frac{1}{c}} \alpha^t}{P^3 \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right)} \right)^c \geq 0$.

As N approaches M , we have the following limit

$$\lim_{N \rightarrow M} q^{t,*}[N] = \lim_{N \rightarrow M} \left(\frac{M\omega(P-1)^2 c \theta^{\frac{1}{c}} \alpha^t}{P^3 \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right)} \right)^c = 0.$$

By the definition of limit, for all capacity $K > 0$, there exists N_0 such that $q^{t,*}[N] < K$ for all $N > N_0$. By Lemma 1, $q^{t,*}[N] < K$ implies that t is an off-peak period, i.e., $\mathcal{L} \neq \emptyset$. ■

Proof of Theorem 2. In three steps, we prove that there exists $N^* < M$ such that the consumer's utility decreases with N for all $N > N^*$. The proof proceeds as follows. First, we derive a sufficient condition for the consumer's utility to decrease with N . Second, we prove that such an N^* exists. Third, we show that this sufficient condition is satisfied for all $N > N^*$.

First, suppose that the initial union size is N_1 , and it increases to N_2 . Under N_1 (resp., N_2), the producer's equilibrium offer is \hat{q}_1^t (resp., \hat{q}_2^t), the consumer's equilibrium electricity allocation is \hat{x}_1^t (resp., \hat{x}_2^t), his consumption good allocation is \hat{x}_1^0 (resp., \hat{x}_2^0), his utility is \hat{U}_1 (resp., \hat{U}_2), the set of off-peak periods is \mathcal{L}_1 (resp., \mathcal{L}_2), and the set of peak periods is \mathcal{H}_1 (resp., \mathcal{H}_2). Also, $\hat{q}^t = q^{t,*}$ for all $t \in \mathcal{L}$, and $\hat{q}^t = \bar{K}$ for all $t \in \mathcal{H}$ (§3.2.3). The producer's uncapacitated offer $q^{t,*}$ decreases with N (Lemma 3), so we have $\hat{q}_2^t \leq \hat{q}_1^t$ for all t and $\hat{q}_2^t < \hat{q}_1^t$ for all $t \in \mathcal{L}_1$. Note that $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and

$\mathcal{H}_2 \subseteq \mathcal{H}_1$. As union size rises from N_1 to N_2 , the change in the consumer's utility is

$$\widehat{U}_2 - \widehat{U}_1 = \left(\sum_{t=1}^T \alpha^t \log(\widehat{x}_2^t) + \alpha^0 \log(\widehat{x}_2^0) \right) - \left(\sum_{t=1}^T \alpha^t \log(\widehat{x}_1^t) + \alpha^0 \log(\widehat{x}_1^0) \right). \quad (85)$$

Substituting (63) into (8) gives $\widehat{x}^t = P\widehat{q}^t/M$. Plugging $\widehat{x}^t = P\widehat{q}^t/M$ into 85, we get

$$\begin{aligned} \widehat{U}_2 - \widehat{U}_1 &= \left(\sum_{t=1}^T \alpha^t \log\left(\frac{P}{M}\widehat{q}_2^t\right) + \alpha^0 \log(\widehat{x}_2^0) \right) - \left(\sum_{t=1}^T \alpha^t \log\left(\frac{P}{M}\widehat{q}_1^t\right) + \alpha^0 \log(\widehat{x}_1^0) \right) \\ &= \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}\widehat{q}_2^t\right) + \alpha^0 \log(\widehat{x}_2^0) \right) - \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}\widehat{q}_1^t\right) + \alpha^0 \log(\widehat{x}_1^0) \right) + \sum_{t \in \mathcal{H}_1} \alpha^t \left(\log\left(\frac{P}{M}\widehat{q}_2^t\right) - \log\left(\frac{P}{M}\widehat{q}_1^t\right) \right) \\ &\leq \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}\widehat{q}_2^t\right) + \alpha^0 \log(\widehat{x}_2^0) \right) - \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}\widehat{q}_1^t\right) + \alpha^0 \log(\widehat{x}_1^0) \right) \\ &= \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}q_2^{t,*}\right) + \alpha^0 \log(\widehat{x}_2^0) \right) - \left(\sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}q_1^{t,*}\right) + \alpha^0 \log(\widehat{x}_1^0) \right). \end{aligned} \quad (86)$$

The inequality follows because $\widehat{q}_2^t \leq \widehat{q}_1^t$ for all t , and hence $\sum_{t \in \mathcal{H}_1} \alpha^t (\log(\frac{P}{M}\widehat{q}_2^t) - \log(\frac{P}{M}\widehat{q}_1^t)) \leq 0$. The last equality follows from $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and $\widehat{q}^t = q^{t,*}$ for all $t \in \mathcal{L}$. Note that by Lemma 8, the set of off-peak periods $\mathcal{L}_1 \neq \emptyset$ for sufficiently large union size $N_1 < M$.

Next, we prove that (86) is negative, so $\widehat{U}_2 < \widehat{U}_1$ for sufficiently large N_1 . Let $\widehat{U}_{\mathcal{L}_1}[N] = \sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}q^{t,*}[N]\right) + \alpha^0 \log(\widehat{x}^0[N])$, then (86) is equal to $\int_{N_1}^{N_2} \frac{\partial \widehat{U}_{\mathcal{L}_1}[N]}{\partial N} dN$. To show that (86) is negative (i.e., $\int_{N_1}^{N_2} \frac{\partial \widehat{U}_{\mathcal{L}_1}[N]}{\partial N} dN < 0$), it suffices to show that $\frac{\partial \widehat{U}_{\mathcal{L}_1}[N]}{\partial N} < 0$ for all $N > N_1$ for sufficiently large N_1 . Note that $\widehat{U}_{\mathcal{L}_1}[N]$ is continuously differentiable in N because $q^{t,*}[N]$ (given in (79)) and $\widehat{x}^0[N]$ (given in (76)) are continuous and differentiable functions of N . Before calculating $\partial \widehat{U}_{\mathcal{L}_1}[N]/\partial N$, we calculate $\widehat{U}_{\mathcal{L}_1}[N]$ by substituting (76) and (79) as follows

$$\begin{aligned} \widehat{U}_{\mathcal{L}_1}[N] &= \sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{P}{M}q^{t,*}[N]\right) + \alpha^0 \log(\widehat{x}^0[N]) \\ &= \sum_{t \in \mathcal{L}_1} \alpha^t \log\left(\frac{\omega^c (P-1)^{2c} c^c \theta}{M^{1-c} P^{3c-1}} (W_N^t)^c\right) + \alpha^0 \log\left(\omega \left(1 - \sum_{t=1}^T W_N^t\right)\right). \end{aligned}$$

Then, $\partial \widehat{U}_{\mathcal{L}_1}[N]/\partial N$ is

$$\begin{aligned} \frac{\partial \widehat{U}_{\mathcal{L}_1}[N]}{\partial N} &= c \sum_{t \in \mathcal{L}_1} \frac{\alpha^t}{W_N^t} \frac{\partial W_N^t}{\partial N} + \frac{\alpha^0}{1 - \sum_{t=1}^T W_N^t} \frac{\partial (1 - \sum_{t=1}^T W_N^t)}{\partial N} \\ &= \frac{2\alpha^0}{\alpha^0 + \left(\frac{M-N}{M}\right)^2 \sum_{t=1}^T \alpha^t} \left[\sum_{t=1}^T \alpha^t \frac{M-N}{M^2} - \sum_{t \in \mathcal{L}_1} \alpha^t \frac{c}{M-N} \right]. \end{aligned}$$

We have $\partial \widehat{U}_{\mathcal{L}_1}[N]/\partial N < 0$ if

$$N > M - M \left(\frac{c \sum_{t \in \mathcal{L}_1} \alpha^t}{\sum_{t=1}^T \alpha^t} \right)^{\frac{1}{2}}. \quad (87)$$

Second, we prove the existence of $N^* < M$ such that (87) is satisfied for all $N > N^*$. In particular, we show the existence of

$$N^* = \min_{N \in [1, M]} \left\{ N \mid N \geq M - M \left(\frac{c \sum_{t \in \mathcal{L}_1} \alpha^t}{\sum_{t=1}^T \alpha^t} \right)^{\frac{1}{2}} \right\}. \quad (88)$$

To do so, we apply the Weierstrass Theorem. Because N is continuous and the constraint set is compact (closed and bounded), we just need to verify that the constraint set is nonempty. By Lemma 8, $\mathcal{L}_1 \neq \emptyset$ for sufficiently large $N_1 < M$. Moreover, electricity weight $\alpha^t > 0$ for all t , so we have $\sum_{t \in \mathcal{L}_1} \alpha^t > 0$. Given that $c > 0$, plugging $N = M$ into (87) yields $M > M - M \left(\frac{c \sum_{t \in \mathcal{L}_1} \alpha^t}{\sum_{t=1}^T \alpha^t} \right)^{\frac{1}{2}}$, which implies that the constraint set is nonempty. It also implies that $N^* < M$ because there must exist $N \in (N_1, M)$ such that $N > M - M \left(\frac{c \sum_{t \in \mathcal{L}_1} \alpha^t}{\sum_{t=1}^T \alpha^t} \right)^{\frac{1}{2}}$.

Third, we show that (87) is satisfied for all $N > N^*$. By definition, N^* satisfies (87) with weak inequality. As N rises, the left-hand side of (87) rises. Moreover, the producer's uncapacitated offer $q^{t,*}$ falls by Lemma 3, which may turn some peak periods into off-peak periods. Then, the new set of off-peak periods is $\mathcal{L}_2 \supseteq \mathcal{L}_1$, and hence the right-hand side of (87) is nonincreasing in N . Thus, (87) holds for all $N > N^*$. ■

Proof of Corollary 2. In three steps, we show that when $c \geq 1$, there exists \bar{K}_0 such that the consumer's utility monotonically decreases with union size N for all $\bar{K} > \bar{K}_0$. First, we find a \bar{K}_0 such that all periods are off-peak. Second, we derive a condition under which the consumer's utility decreases with N . Third, we show that this condition is satisfied for all N when $c \geq 1$.

First, let $q^{1,*}, \dots, q^{T,*}$ be the producer's uncapacitated offers at periods $t = 1, \dots, T$. Moreover, let $\bar{K}_0 = \max\{q^{1,*}, \dots, q^{T,*}\} + \epsilon$, where $\epsilon > 0$. Note that $q^{t,*} < \infty$ for all t because $q^{t,*} = \theta(\phi^{t,*})^c$ and $\phi^{t,*} < \omega$ for all t . By definition of \bar{K}_0 , $q^{t,*} < \bar{K}_0$ for all t . By Lemma 1, having $q^{t,*} < \bar{K}_0$ for all t is equivalent to having off-peak periods for all t .

Second, the consumer's utility $U[N]$ decreases with union size N for all N if $\frac{\partial U[N]}{\partial N} < 0$. Before calculating $\partial U[N]/\partial N$, we first calculate $U[N]$ by using the fact that all periods are off-peak and

by plugging (76) and (81) as follows

$$\begin{aligned} U[N] &= \sum_{t=1}^T \alpha^t \log(\hat{x}^t[N]) + \alpha^0 \log(\hat{x}^0[N]) \\ &= \sum_{t=1}^T \alpha^t \log\left(\frac{\omega^c (P-1)^{2c} c^c \theta}{M^{1-c} P^{3c-1}} (W_N^t)^c\right) + \alpha^0 \log\left(\omega \left(1 - \sum_{t=1}^T W_N^t\right)\right). \end{aligned}$$

Then, $\partial U[N]/\partial N$ is

$$\frac{\partial U[N]}{\partial N} = \frac{2\alpha^0 \sum_{t=1}^T \alpha^t}{\alpha^0 + \left(\frac{M-N}{M}\right)^2 \sum_{t=1}^T \alpha^t} \left[\frac{(M-N)^2 - cM^2}{M^2(M-N)} \right].$$

Because $\alpha^0 > 0$ and $\alpha^t > 0$, we have $\partial U[N]/\partial N < 0$ for all $N > M - Mc^{\frac{1}{2}}$.

Third, when $c \geq 1$, we have $M - Mc^{\frac{1}{2}} \leq 0$. Because $N \in [1, M]$, we have $N > M - Mc^{\frac{1}{2}}$ for all N . Therefore, when $c \geq 1$, there exists \bar{K}_0 such that the consumer's utility monotonically decreases with N for all $\bar{K} > \bar{K}_0$. ■

Proof of Proposition 6. In two steps, we prove that the producer's equilibrium capacity \hat{K} decreases with union size N . First, we derive a necessary condition for the long-run Nash equilibrium. Second, by using this condition, we show that \hat{K} decreases with N .

First, as Definition 1 presents, the long-run Nash equilibrium is the solution to (10) - (14) given (62) and (63). The Kuhn-Tucker conditions are (58), (60), and (64) - (69). Plugging (58) into (67), and summing over t , we have

$$\frac{1}{\frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K} \left[\sum_{t=1}^T Q^0 \frac{B_{-j}^0}{(b_j^0 + B_{-j}^0)^2} B^t \frac{Q_{-j}^t}{(q_j^t + Q_{-j}^t)^2} - \sum_{t=1}^T \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q_j^t)^{\frac{1}{c}-1} \right] = \sum_{t=1}^T \mu^t.$$

Because we have $z_j^0 = \frac{b_j^0}{b_j^0 + B_{-j}^0} Q^0 - \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \sum_{t=1}^T (q_j^t)^{\frac{1}{c}} - \rho K > 0$ (see §3.2.3), substituting (68) yields

$$\sum_{t=1}^T Q^0 \frac{B_{-j}^0}{(b_j^0 + B_{-j}^0)^2} B^t \frac{Q_{-j}^t}{(q_j^t + Q_{-j}^t)^2} - \sum_{t=1}^T \left(\frac{1}{\theta}\right)^{\frac{1}{c}} \frac{1}{c} (q_j^t)^{\frac{1}{c}-1} = \rho. \quad (89)$$

To derive a necessary condition for the long-run equilibrium, we need to substitute the long-run Nash equilibrium into (89). As Lemma 6 shows, the long-run Nash equilibrium is the same as the short-run Nash equilibrium if the short-run capacity \bar{K} is replaced with the long-run equilibrium capacity \hat{K} . Then, producers make bids \hat{b}^0 and offers \hat{q}^t , where $\hat{q}^t = \hat{K}$ for all $t \in \mathcal{H}$, and $\hat{q}^t = q^{t,*}$ given in (79) for all $t \in \mathcal{L}$. Unions make bids \hat{b}^t given in Lemma 7 and offers \hat{q}^0 given in

(75). Then, plugging (62), (75), and (77) into (89) gives

$$\sum_{t=1}^T \frac{M\omega(P-1)^2}{P^3} W_N^t \left(\frac{1}{\hat{q}^t} \right) - \sum_{t=1}^T \left(\frac{1}{\theta} \right)^{\frac{1}{c}} \frac{1}{c} (\hat{q}^t)^{\frac{1}{c}-1} = \rho, \text{ where } W_N^t = \frac{\alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}.$$

Substituting $\hat{q}^t = \hat{K}$ for all $t \in \mathcal{H}$ and $\hat{q}^t = q^{t,*}$ for all $t \in \mathcal{L}$, we get

$$\sum_{t \in \mathcal{H}} \frac{M\omega(P-1)^2}{P^3 \hat{K}} W_N^t + \sum_{t \in \mathcal{L}} \frac{M\omega(P-1)^2}{P^3} \frac{W_N^t}{q^{t,*}} - \sum_{t \in \mathcal{H}} \left(\frac{1}{\theta} \right)^{\frac{1}{c}} \frac{1}{c} (\hat{K})^{\frac{1}{c}-1} - \sum_{t \in \mathcal{L}} \left(\frac{1}{\theta} \right)^{\frac{1}{c}} \frac{1}{c} (q^{t,*})^{\frac{1}{c}-1} = \rho.$$

Plugging (79), we get

$$\sum_{t \in \mathcal{H}} \left[(q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}} \right] = \rho c \theta^{\frac{1}{c}} \hat{K}.$$

We know from Lemma 1 that $q^{t,*} > \hat{K}$ for all $t \in \mathcal{H}$ and $q^{t,*} \leq \hat{K}$ for all $t \in \mathcal{L}$. It implies that $(q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}} > 0$ for all $t \in \mathcal{H}$ and $(q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}} \leq 0$ for all $t \in \mathcal{L}$. Thus, we replace $\sum_{t \in \mathcal{H}} \left[(q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}} \right]$ with $\sum_{t=1}^T \max \left\{ (q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}}, 0 \right\}$, and obtain the following necessary condition for the long-run Nash equilibrium

$$\sum_{t=1}^T \max \left\{ (q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}}, 0 \right\} = \rho c \theta^{\frac{1}{c}} \hat{K}. \quad (90)$$

Note that for all $\rho > 0$, there must be at least one peak period, i.e., $\mathcal{H} \neq \emptyset$ because otherwise, $\max \left\{ (q^{t,*})^{\frac{1}{c}} - (\hat{K})^{\frac{1}{c}}, 0 \right\} = 0$ for all t .

Second, we prove that the producer's equilibrium capacity \hat{K} is decreasing in union size N . Suppose to the contrary that \hat{K} is nondecreasing in N . Because ρ , c , and θ do not change with N , the right-hand side of (90) is nondecreasing in N . To satisfy (90), the left-hand side must also be nondecreasing in N . By Lemma 3, $q^{t,*}$ decreases with N ; and since $c > 0$, so does $(q^{t,*})^{\frac{1}{c}}$. Given that the left-hand side of (90) is nondecreasing in N , we need \hat{K} to be decreasing in N . However, this contradicts the initial assumption that \hat{K} is nondecreasing in N . Therefore, the producer's equilibrium capacity \hat{K} is decreasing in union size N . ■

Proof of Lemma 4. Because cases (i)- (iv) are straightforward, we only prove the case (v) that is $P\hat{K}$ is increasing in P when $c \leq 1$. To do so, we use (90), which is a necessary condition for the long-run Nash equilibrium. Multiplying both sides of (90) with P gives

$$\sum_{t=1}^T \max \left\{ (P^c q^{t,*})^{\frac{1}{c}} - (P^c \hat{K})^{\frac{1}{c}}, 0 \right\} = \rho c \theta^{\frac{1}{c}} P \hat{K}. \quad (91)$$

Note that for all $\rho > 0$, there must be at least one period t such that $(P^c q^{t,*})^{\frac{1}{c}} - (P^c \hat{K})^{\frac{1}{c}} > 0$

(i.e., t is a peak period) because otherwise, $\max \left\{ (P^c q^{t,*})^{\frac{1}{c}} - (P^c \widehat{K})^{\frac{1}{c}}, 0 \right\} = 0$ for all t . Suppose to the contrary that $P\widehat{K}$ is nonincreasing in P when $c \leq 1$. Because ρ , c , and θ do not change with P , the right-hand side of (91) is nonincreasing in P . To satisfy (91), the left-hand side must also be nonincreasing in P . As Lemma 3 shows, $q^{t,*} = \left(\frac{M\omega(P-1)^2 c \theta^{\frac{1}{c}} \alpha^t}{P^3 \left(\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t \right)} \right)^c$. Then, we have $(P^c q^{t,*})^{\frac{1}{c}} = \left(\frac{P-1}{P} \right)^2 \frac{M\omega c \theta^{\frac{1}{c}} \alpha^t}{\alpha^0 \left(\frac{M}{M-N} \right)^2 + \sum_{t=1}^T \alpha^t}$, which is increasing in P . Given that the left-hand side of (91) is nonincreasing in P , we need $P^c \widehat{K}$ to be increasing in P . Moreover, since $P\widehat{K}$ is nonincreasing in P , \widehat{K} must be decreasing in P . Given that $P\widehat{K}$ is nonincreasing in P , and $P^c \widehat{K}$ is increasing in P , we must have $c > 1$. However, $c > 1$ contradicts the initial assumption that $c \leq 1$. Thus, $P\widehat{K}$ increases with P when $c \leq 1$. ■

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