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Essays on Multiple Strategic Producers of Information

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To my mother, my wife, my son,

and in memory of

my father

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CHAPTER 1

Introduction

Information lies at the heart of the capital market: almost all activities in the capital market involve acquiring, analyzing, disseminating, or responding to information. Moving away from the capital market, firms devote significant resources to the production, disclosure, and use of information. In either setting, there are often multiple producers generating decision relevant information. For instance, several financial analysts issue research reports for a given company, and the principal groups several agents into a team and designs each agent's wage to depend on signals generated by all team members.

This dissertation presents analytical models where multiple information producers interact strategically. The guiding theme is that strategic interactions among information producers have important implications for the way information is produced, disseminated, and used, to the extent that models with multiple information producers generate qualitatively different results compared to models with only one information producer. The models presented in the dissertation intend to answer the following questions.

1.   When can herding behavior among financial analysts arise in a way that improves the information communicated to the market and therefore benefits investors?

2.   Is it in investors' best interest that financial analysts only report a subset of their information even though the report is forced to be truthful?

3.   When and why are managers compensated for their poor performance?

The results cast light on the observed accounting practices and institutions that conventional thinking and existing theories have difficulty explaining. Going beyond a positive theory, these results also have normative policy implications. When can regulators benefit investors by promoting herding behavior among financial analysts? When will policies aimed at protecting investors turn out to discourage information acquisition to the detriment of investors?

The key analytical tool used in the essays is the concept of *strategic complementarity* developed in simultaneous move games, and the broader notion that agents tend to act alike (herding) in sequential move games. Bulow et al. (1985) first use the term "strategic complementarity" to refer to games where each player's incentive to act in a certain way increases as other players act in that way as well. A bank run is an example of strategic complementarity: it is best for a depositor to withdraw her money if other depositors of the bank withdraw their money.

I will first discuss the central feature of strategic complementarity in a differentiable framework and then define it in a general setting.[1] Consider a two-person non-corporate simultaneous move game (A, U_i), where player $i \in \{1, 2\}$ chooses his action $a_i \in [0, A]$ and receives his utility U_i. $U_i = U_i(a_i | a_j; \theta)$ is player i's payoff if he takes the action a_i given the other player's action a_j and θ is a vector of payoff relevant parameters. U_i is assumed to be smooth (continuously differentiable), strictly increasing, and concave in a_i, i.e., $\frac{\partial U_i}{\partial a_i} > 0, \frac{\partial^2 U_i}{\partial a_i^2} < 0$. This game exhibits strategic complementarity if

$$(1.1) \qquad \frac{\partial^2 U_i}{\partial a_i \partial a_j} > 0$$

and condition (1.1) means that the marginal benefit of taking a high action increases in the level of the other players' action.

Since the definition of strategic complementarity is based on the players' payoffs without specifying the mechanism generating such payoff structures, it accommodates a large variety of games. For example, strategic complementarity can be driven by a complementary production technology (e.g., Bulow et al., 1985), complementary allocation rules in coordination games (e.g., Diamond and Dybvig, 1983), or the combination of the two (e.g., Baldenius and Glover, 2012).

In games with a strategic complementarity, players have incentives to act alike. This can be seen from the fact that each player's best response function is upward

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[1]Earlier work used similar examples to review the concept of strategic complementarity (e.g., Cooper, 1999; Vives, 2005).

sloping. Denote $B(a_j; \theta)$ as player i's best response function given a_j and θ. The strict concavity assumption suggests that $B(a_j; \theta)$ has a unique maximizer and satisfies the following condition (assuming the solution is interior):

$$\frac{\partial U_i(B(a_j; \theta)|a_j; \theta))}{\partial B(a_j; \theta))} = 0$$

Applying the implicit function theorem, one can show that the slope of the best response function is $\frac{d}{da_j} B(a_j; \theta) = -\frac{\partial^2 U_i / \partial a_i \partial a_j}{\partial^2 U_i / \partial a_i^2}$, which is positive if and only if the game exhibits strategic complementarity (i.e., condition 1.1 holds).

While the example above assumes convex action spaces and smooth payoff functions of the players, the idea of strategic complementarity is more general. Among others, Milgrom and Roberts (1990) and Vives (1990) study strategic complementarity in a general class of games called *supermodular games*,[2] which allows for non-smooth payoffs and complex strategy spaces. A complete discussion of supermodular games requires lattice-based theories developed by Topkis (1978) and is beyond the scope of the introduction. The purpose here is to highlight the counterpart of strategic complementarity in supermodular games.

Consider a 2 person simultaneous move game, where player i's action space $A_i, i \in \{1, 2\}$ contains finite, real-valued elements. In addition, I will not impose the smoothness assumption on players' payoff functions as I did earlier. The spirit

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[2]A game is a supermodular game if for each player i, (1) his action space is a complete lattice; (2) his payoff function U^i has increasing first differences (defined in the text); and (3) Given other players' action a_{-i}, U^i is supermodular in his own action a_i. Definition of part (1) and (3) can be found in Cooper (1999), Chapter 2.

of strategic complementarity in this general setting is captured by the concept of *increasing first differences* defined below.

Definition 1. *Let A_i be the action space of player $i \in \{1, 2\}$, $a, a' \in A_1$, and $b \in A_2$. The payoff U_i exhibits increasing first differences if for $a' > a, U_i(a', b) - U_i(a, b)$ increases in b.*

Clearly *increasing first differences* is a generalization of condition (1.1). Throughout the remainder, I do not differentiate the concept of increasing first differences from strategic complementarity, but instead use the term "complementarity" whenever agent i's incentive to choose a high action increases in his rival's action.

The idea that players tend to act alike is not unique to simultaneous games. Herding, which refers to players imitating their predecessor's action, carries the spirit of complementarity to sequential move games. To explore the connection between herding and strategic complementarity, consider a sequential move game where player 2 moves after player 1 taking a publicly observable action. Assuming that both players' action space is $\{0, 1\}$, we say that player 2 herds with player 1 if the followings hold.

$$
\begin{aligned}
(1.2) \qquad U_2(a_2 &= 1|a_1 = 1, I_2) > U_2(a_2 = 0|a_1 = 1, I_2), \forall I_2 \\
U_2(a_2 &= 0|a_1 = 0, I_2) > U_2(a_2 = 1|a_1 = 0, I_2), \forall I_2
\end{aligned}
$$

where I_2 is player 2's private information when he chooses his action.

Recall from Definition 2 that in the simultaneous move game with action space $\{0, 1\}$, player 2's payoff U_2 exhibits complementarity if[3]

$$(1.3) \qquad \begin{aligned} U_2(a_2 &= 1|a_1 = 1) - U_2(a_2 = 0|a_1 = 1) \\ &> U_2(a_2 = 1|a_1 = 0) - U_2(a_2 = 0|a_1 = 0) \end{aligned}$$

Abstracting from the context of two games, we can see that (1.2) implies (1.3). In other words, one can consider herding as the case where the degree of complementarity is so strong that the follower, once observing the predecessor's action, will ignore any private information he possess and choose to act alike. Previous research uses informational externality or reputation concerns to rationalize the strong complementarity between players' actions that leads to herding.

Banerjee (1992) and Bikhchandani et al. (1992) develop the earliest models where herding is due to an informational externality. Herding arises in their models because the information contained in the predecessor's action is strictly more informative about the underlying state than the follower's private signal and the follower wants to take an action as close to the underlying state as possible. Scharfstein and Stein (1990) and Trueman (1994) develop models where herding is driven by the agents' reputation concern. The model can be interpreted in a Principal-Agent setting, where the agents (players) herd in order to manipulate

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[3]Given the action space $\{0, 1\}$, a 2-person simultaneous game is a supermodular game if and only if U_i exhibits increasing first differences for both i (see Definition 2).

the (unmodeled) principal's perception about their ability (or type). Most herding models assume small action spaces of a players, which prevents agents with a slightly different posterior from choosing a slightly different action.

These herding models provide a good benchmark for understanding learning behavior, but they assume all players have the same incentives and do not behave strategically in the sense that they do not internalize the effect of their behavior on other players' choices. If information producers have heterogeneous incentives and interact strategically, which seems common in practice, the information cascade argument is questionable. For example, if the predecessor has an incentive to affect followers' actions, the predecessor may distort his action (that is to lie about his private signal). But the predecessor's distorted action lowers the informativeness of his action and therefore reduces the followers' incentive to herd. So once we allows for strategic interactions between players with heterogenous incentives, the predecessor's action and the followers' herding choice should be determined endogenously.

Another assumption adopted in the herding literature is that players' information is exogenously given. If players acquire their private information at a cost, one needs to reconsider the information cascade argument: if the player knows he or she is going to herd with the predecessors in the future, he or she will not acquire any information in the first place. So, as one endogenizes players' information acquisition, time consistency of the player's decision suggests that the effect

of herding depends critically on how it affects the players' incentive to acquire information in the first place.

Chapter 2 studies how an independent analyst interacts with an affiliated analyst when issuing stock recommendations, and how that interaction affects information acquired and communicated to the investor. This chapter builds on a herding model, but it endogenizes information acquisition and the sequence of actions, and introduces players with heterogeneous incentives. The model shows that the independent analyst disciplines/reduces the bias in the affiliated analyst's recommendation, but sometimes also herds with the affiliated analyst to improve recommendation accuracy. While casual intuition and existing research suggest that herding will jeopardize the ability to discipline, I show that in equilibrium herding and disciplining can be complements in the sense that the two roles coexist and reinforce each other.

In addition, the model shows that the independent analyst only herds with the affiliated analyst *conditionally* rather than perfectly (all the time), which implies that we will observe disagreement between the two analysts' recommendations from time to time. This finding is also associated with Welch's (2000) critique of existing herding models.

> This is because many herding theories are designed to explain a steady state in which all analysts herd perfectly, not to explain an ever-varying time-series of recommendations or a residual heterogeneity in opinion across analysts. (Welch, 2000, pp. 370)

Returning to the study of strategic complementarity in simultaneous move games, the literature has focused on identifying settings where strategic complementarity arises and studying the implication of strategic complementarity. Whether the game exhibits strategic complementarity has not been treated as a control variable yet.

Chapter 3 studies the optimal contract based on subjective/non-verifiable performance measures in a multi-period, principal-multi-agent game. The non-verifiable feature of the agents' performance measures limits the principal's ability to commit to reporting those measures truthfully. The repeated relationship enhances the principal's credibility as the agents can punish the principal if the latter fails to honor the contract. The repeated relationship also creates the room for implicit side contracts between the two agents. Whether the contract (in particular agents' payoffs induced by the contract) exhibits strategic complementarity or strategic substitutability has a subtle impact on the nature of the agent-agent side contract, and therefore is an important decision variable for the principal. For example, consider a game where both agents can either "shirk" or "work" and the principal wants to induce "work" from both agents as a collusion-proof equilibrium at a minimum cost. Baldenius and Glover (2012) shows that if the contract exhibits strategic complementarity, agents colluding on a (shirk, shirk) strategy is most expensive for the principal to break; while if the contract exhibit strategic substitutability, the two agents alternating between (work, shirk) and (shirk, work) is most expensive to break.

Investigating why and how the principal purposely designs the contract to exhibit strategic complementary, substitutability, or independence is the focus of Chapter 3. It shows that when the expected relationship horizon is long, the optimal contract exhibits strategic complementarity in order to motivate the agents to use implicit contracting and mutually monitor each other. When the expected horizon is short, the solution converges to a static bonus pool in the sense that the optimal contract rewards agents for (joint) bad performance in order to make the principal's promises to provide honest evaluations credible. For intermediate expected horizons, the optimal contract again rewards the agents for (joint) bad performance if the agents' credibility to collude with each other is relatively stronger than the principal's credibility to honor the contract. The reason is that paying for bad performance allows the principal to create a strategic independence in the agents' payoffs that reduces their incentives to collude. That is if the principal did not have to prevent tacit collusion between the agents in this case, she would not reward the agents for bad performance. She would instead use a relative performance evaluation scheme. The unappealing feature of relative performance evaluation is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has the agents taking turns making each other look good—they alternate between (work, shirk) and (shirk, work).

The remainder of the dissertation is organized as follows. Chapter 2 reconciles independent analysts' disciplining role over affiliated analysts' recommendation

bias and the observed herding behavior among financial analysts. Chapter 3 studies relational contracting based on subjective/non-verifiable performance measures as a foundation for bonus pools. Chapter 4 concludes with two extensions.

CHAPTER 2

Independent and Affiliated Analysts: Disciplining and Herding

ABSTRACT: The paper investigates strategic interactions between an independent analyst and an affiliated analyst when the analysts' information acquisition and the timing of their recommendations are endogenous. Compared to the independent analyst, the affiliated analyst has superior information but faces a conflict of interest. I show that the independent analyst's recommendation, albeit endogenously less informative than the affiliated analyst's, disciplines the affiliated analyst's biased forecasting behavior. Meanwhile, the independent analyst sometimes herds with the affiliated analyst in order to improve forecast accuracy. Contrary to conventional wisdom, I show that herding with the affiliated analyst may actually motivate the independent analyst to acquire more information upfront, reinforce his ability to discipline the affiliated analyst, and benefit investors.

2.1. Introduction

This paper investigates strategic interactions between an independent analyst and an affiliated analyst when the analysts' information acquisition and the timing

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of their recommendations are endogenous. In the paper, I differentiate affiliated analysts from independent analysts by two features: (a) the affiliated analyst faces a conflict of interest and (b) he has superior information compared to the independent analyst.

These two features have been widely noted by regulatory bodies, practitioners, and researchers. For example, the Global Analyst Research Settlement (Global Settlement) between the United States' regulators and the nation's top investment firms directly addresses conflicts of interest between research and investment banking businesses. An example of superior information affiliated analysts receive is confidential, non-public information they obtain in the due diligence process as an underwriter in an Initial Public Offering (IPO). The inappropriate release of such confidential information in a restricted period prior to Facebook's IPO was the focus of the Commonwealth of Massachusetts' case against Citigroup.[1] Empirically, Lin and McNichols (1998), Barber et al. (2007), and Mola and Guidolin (2009) document evidence suggesting that affiliated analysts face conflicts of interest when issuing stock recommendations, while Jacob et al. (2008) and Chen and Martin (2011) document evidence suggesting that analysts receive superior information because of their affiliations with the company.

On one hand, researchers argue that independent analysts' incentives are more aligned with investors and find the existence of independent analysts disciplines affiliated analysts' biased forecasting behavior (e.g., Gu and Xue, 2008). Consistent

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[1]http://www.sec.state.ma.us/sct/current/sctciti/Citi_Consent.pdf

with the disciplining argument, the Global Settlement requires investment banks to acquire and distribute three independent research reports along with their own reports for every company they cover. On the other hand, since independent analysts' information is inferior, it is reasonable to suspect they have incentives to herd with affiliated analysts, given the well-documented herding behavior among financial analysts (e.g., Welch, 2000; Hirshleifer and Teoh, 2003).

If independent analysts herd with affiliated analysts, to what extent is their disciplining role compromised? Casual intuition suggests that herding would jeopardize the ability to discipline, which is consistent with the prevailing view in academic research that analysts' herding behavior discourages information production and is undesirable from the investor's perspective. In the Abstract of *Herding Behavior among Financial Analysts: A Literature Review*, Van Campenhout and Verhestraeten (2010) write:

> Analysts' forecasts are often used as an information source by other investors, and therefore deviations from optimal forecasts are troublesome. Herding, which refers to imitation behavior as a consequence of individual considerations, can lead to such sub-optimal forecasts and is therefore widely studied.

Contrary to conventional wisdom, this paper shows that the independent analyst's disciplining role and herding behavior may reinforce each other. I show that if the independent analyst's informational disadvantage is large, herding with

the affiliated analyst actually motivates the independent analyst to acquire more information upfront, reinforces his disciplining role, and ultimately benefits the investor.

The model has three players: an affiliated analyst, an independent analyst, and an investor. Each analyst acquires a private signal about an underlying, risky asset (the firm) and publicly issues a stock recommendation at a time that is strategically chosen. When choosing the timing of their recommendations, both analysts face a trade-off between the accuracy and timeliness of their recommendations.[2] Compared to the independent analyst, the affiliated analyst is assumed to face a conflict of interest but has superior information. To model the affiliated analyst's conflict of interest, I assume he receives an additional reward (independent of the reward for timeliness and accuracy) if the investor is convinced to buy the stock. To model the independent analyst's informational disadvantage, I assume the signal he endogenously acquires is less precise than the affiliated analyst's signal due to exogenous higher information acquisition costs. The precision of the analysts' information is interpreted as a firm-wide choice (e.g., hiring a star analyst or devoting more resources to a specific industry) and is assumed to be publicly observed.

Due to his conflict of interest, the affiliated analyst has an incentive to over-report a bad signal in order to induce the investor to buy the stock. The model

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[2]See Schipper (1991) and Gul and Lundholm (1995) on the tradeoff between accuracy and timeliness of recommendations.

shows that the independent analyst disciplines the affiliated analyst's biased forecasting behavior in equilibrium. Intuitively, since the independent analyst's recommendation provides information to the investor, the extent the affiliated analyst can misreport his signal without being ignored by the investor is bounded by the quality of the independent analyst's recommendation.

The independent analyst's herding behavior also arises in equilibrium. Since the analysts' recommendations can be either favorable or unfavorable, the only reason for the independent analyst to delay his recommendation is to herd with the affiliated analyst. In equilibrium, the affiliated analyst's unfavorable recommendation is more informative than his favorable recommendation, so the independent analyst's expected benefit from waiting is higher if his private signal is good. The endogenous benefit of waiting, together with an exogenous cost of waiting, leads to a *conditional herding* equilibrium under which the independent analyst reports a bad signal immediately but waits and herds with the affiliated analyst upon observing a good signal.

Surprisingly, conditional herding causes the independent analyst to acquire more information and play a greater disciplining role than if he were prohibited from herding. The reason is that herding introduces an indirect benefit to information acquisition. By acquiring better information and reporting a bad signal right away, the independent analyst motivates the affiliated analyst to truthfully reveal a bad signal more often – this is the disciplining role. The affiliated analyst's more accurate reporting means that the independent analyst who receives a

good signal too will be more accurate, since he herds with the affiliated analyst. This indirect benefit of information acquisition derived from herding motivates the independent analyst to acquire better information upfront. That is, there is an induced complementarity between the independent analyst's ex-post herding and ex-ante information acquisition.

Empirical Implications

First, the model predicts a positive association between independent analysts' degree of herding[3] with affiliated analysts and the informativeness of affiliated analysts' recommendations for firms with high information acquisition costs. The predicted association is negative for firms with low information acquisition costs.

Second, the model predicts that the dispersion between affiliated and independent analysts' recommendations decreases over time. Moreover, the decrease of dispersion is driven by independent analysts' recommendations converging to affiliated analysts' recommendations but not vice versa. The prediction of a shrinking dispersion is consistent with O'Brien et al. (2005) and Bradshaw et al. (2006) who found that affiliated analysts' recommendations are more optimistic than independent analysts' only in the first several months surrounding public offerings, while there is no difference afterwards.[4]

Regulatory Implications

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[3]Welch (2000) proposes a methodology for estimating the degree of herding.
[4]O'Brien et al. (2005) write "we choose public offerings as a starting point because the financing event allows us to distinguish affiliated from unaffiliated analysts."

First, the model shows affiliated analysts can be disciplined by independent analysts even when the latter's recommendations are less informative and involve herding behavior. The result that the independent analyst herding with the affiliated analyst may actually benefit the investor is relevant in the light of the Jumpstart Our Business Startups Act (JOBS Act). The JOBS Act permits affiliated analysts to publish research reports with respect to an emerging growth company any time after its IPO.[5] The paper suggests that making it possible for independent analysts to herd with affiliated analysts right after the IPO may increase independent analysts' disciplining role and benefit investors.[6]

Second, the paper points out that regulations mitigating affiliated analysts' conflicts of interest such as the Global Settlement can hurt the investor in some cases. The reason is that such regulations may crowd out independent analysts' incentives to acquire information. The result offers a rationale for the evidence in Kadan et al. (2009) who found the overall informativeness of recommendations has declined following the Global Settlement and related regulations.

The paper proceeds as follows. Section 2.2 reviews the literature. Section 2.3 lays out the model, and Section 2.4 characterizes the equilibrium. Section

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[5]"Jumpstart Our Business Startups Act – Frequently Asked Questions About Research Analysts and Underwriters". http://www.sec.gov/divisions/marketreg/tmjobsact-researchanalystsfaq.htm

[6]Before the JOBS Act, affiliated analysts were restricted by the federal securities laws from issuing forward looking statements during the "quiet period," which extends from the time a company files a registration statement with the Securities and Exchange Commission until (for firms listing on a major market) 40 calendar days following an IPO's first day of public trading.

2.5 delivers the main point of the paper by illustrating how herding behavior motivates the independent analyst to acquire better information and enhances his disciplining role over the affiliated analyst in equilibrium. Section 2.6 develops more detailed empirical and regulatory implications. Section 2.7 discusses the robustness of the main results. Section 2.8 concludes the paper. Appendix A specifies results deferred from the main text, and Appendix B presents all proofs.

2.2. Related Literature

The paper is related to the herding literature. Banerjee (1992) and Bikhchandani, Hirshldifer and Welch (1992) are two seminal papers showing that agents may rationally ignore their own information and herd with their predecessor's action for statistical reasons. Scharfstein and Stein (1990) and Trueman (1994) develop models where herding is driven by the agents' reputation concern. Arya and Mittendorf (2005) show the manager may purposely disclose proprietary information in order to direct herding from outside information providers. While the classical herding literature assumes that agents act in an exogenous sequential order, the sequence of actions is endogenous in my model. Existing herding models show that herding behavior and the loss of information are inherently linked, while my paper finds a setting where herding behavior leads to more information acquisition ex-ante and more information being revealed ex-post.

The endogenous timing of analysts' actions was first studied by Gul and Lundholm (1995), who model the trade-off between the accuracy and the timeliness of

a forecast. Guttman (2010) gives conditions under which the time of the two analysts' forecasts cluster together or separate apart. In Gul and Lundholm (1995) and Guttman (2010), the analysts have homogeneous incentives. By modeling two analysts with heterogeneous incentives, my paper captures some institutional differences between affiliated and independent analysts and generates results that cannot be derived from earlier work.

Prior research has studied information acquisition in settings with a single analyst. Fischer and Stocken (2010) study a cheap-talk model and draw the conclusion that the analyst's information acquisition depends on the precision of public information. While the public information is provided by a non-strategic party in Fischer and Stocken (2010), both information providers behave strategically in my model. Langberg and Sivaramakrishnan (2010) endogenizes the analyst's information acquisition in a voluntary disclosure model similar to Dye (1985) and show the analyst's feedback can induce less voluntary disclosure from the manager.[7] My model contributes to this literature by developing an induced complementarity between the independent analyst's ex-post herding and ex-ante information acquisition.

The independent analyst's disciplining role in my model shares features of the disciplinary role of accounting information. Among others, Dye (1983), Liang (2000), and Arya et al. (2004) show that accounting information disciplines other

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[7]Taking the analyst's information acquisition as given, Arya and Mittendorf (2007) and Mittendorf and Zhang (2005) also model interactions between an analyst and a manager.

softer sources of information in principal-agent contracting settings. Like accounting information which is usually considered to be less informative and less timely than other information sources such as managers' voluntary disclosures, the independent analyst's recommendation in my model is also (endogenously) less informative and less timely than the affiliated analyst's recommendation.

Empirically, Gu and Xue (2008) document independent analysts' disciplining role: affiliated analysts' forecasts become more accurate and less biased when independent analysts are following the same firms than when they are not. They also document that independent analysts' forecasts are less accurate than affiliated analysts forecast ex-post. Both findings are consistent with predictions of my model. In addition, Gu and Xue (2008) argue their results suggest that independent analysts are better than affiliated analysts in representing ex-ante market expectations, which is in line with the model's assumption that the independent analyst's incentive is more aligned with the investor.

The model's assumption that analysts face a trade-off between accuracy and timeliness of their recommendations is motivated by empirical evidence. Schipper (1991) discusses the tradeoff between timeliness and accuracy of analysts' forecasts. Cooper et al. (2001) document that analysts forecasting earlier have greater impact on stock prices than following analysts, and Loh and Stulz (2011) find similar results in the context of analysts' recommendation revisions. Regarding the incentive to issue accurate forecasts, Mikhail et al. (1999), Hong and Kubik (2003), Jackson (2005), and Groysberg et al. (2011) document evidence that analysts are

rewarded for issuing accurate forecasts through higher payments, promising future careers, better reputations, and/or less turnover.

2.3. Model Setup

The model considers an economy consisting of an underlying, risky asset (the firm) and three players: an affiliated analyst, an independent analyst, and an investor. Whether an analyst is affiliated or independent is commonly known, and I will specify their differences later. The value of the firm is modeled as a random state variable ω whose prior distribution is also commonly known. Each analyst acquires a private signal about the value of the firm and then publicly issues a stock recommendation at a time that is strategically chosen. After observing both analysts' recommendations, the investor updates her belief about the value of the firm and makes an investment decision.

2.3.1. Endogenous Private Information Acquisition

The value of the firm is modeled as a state variable $\omega \in \{H, L\}$ with the common prior belief that both states are equally likely. At $t = 0$, the beginning of the game, the independent analyst (indicated by the superscript I) acquires his private signal $y^I \in Y^I = \{g, b\}$ about the underlying state ω at cost $c(p)$, where $p \in [\frac{1}{2}, 1]$ is the precision of y^I and is defined as follows

$$(2.1) \qquad p = \Pr(y^I = g | \omega = H) = \Pr(y^I = b | \omega = L)$$

The cost of information acquisition $c(p)$ increases in the precision p of the signal in a convex manner and is assumed to be

$$(2.2) \qquad c(p) = e \times (p - \frac{1}{2})^2$$

where e is a positive constant commonly known, and a greater e means acquiring information becomes more costly. The cost of not acquiring any information is zero, i.e., $c(p = \frac{1}{2}) = 0$.

At the same time, the affiliated analyst (indicated by the superscript A) is endowed with a private signal $y^A \in Y^A = \{g, b\}$ whose precision $p^A \in [\frac{1}{2}, 1]$ is defined analogously as in (1). Assuming the affiliated analyst costlessly receives his signal with a fixed precision p^A is a simplification not crucial to the model. It is enough to assume the cost of information acquisition is sufficiently lower for the affiliated analyst so that he acquires more precise information in equilibrium.[8]

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[8]I obtain qualitatively similar results by assuming both analysts simultaneously acquire information, and the affiliated analyst's information acquisition cost is $c(p^A) = \frac{e}{2} \times (p^A - \frac{1}{2})^2$. I am unable to obtain closed-form solutions for one of the key cutoff conditions under this alternative setup.

Conditional on the realization of the state ω, the signals received by the two analysts are independent. That is,

$$(2.3) \qquad \Pr(y^A, y^I | \omega) = \Pr(y^A | \omega) \Pr(y^I | \omega), \forall p$$

From each analyst's perspective, the conditional independence assumption says the other analyst's private signal is more likely to be the same as his own signal than to be different.

The paper assumes the precision (not the realization) of the analysts' signals, p^A and p, is observable. One can interpret the precision as the *firm-wide* research quality. In practice, it takes time and effort for the research firm to increase its information precision, such as setting up a larger research group for the industry, hiring a star analyst, or becoming part of the managers' network. These actions and investments have to be made up front and are, to a substantial extent, observable to the market.

2.3.2. Endogenous Timing of Public Recommendations

After observing their private signals at $t = 0$, both analysts simultaneously choose either to issue a stock recommendation immediately at $t = 1$ or to defer the recommendation to $t = 2$. While deferring a recommendation is costly (which will be made precise shortly), doing so may be worthwhile as recommendations issued

at $t = 1$ (if any) are observable and provide additional information to the analyst who waits until $t = 2$ to issue his recommendation.

Since each analyst issues only one recommendation in the model, a specific analyst can issue a recommendation at $t = 2$ if and only if he was silent earlier at $t = 1$. To be concrete, denote r_t^I as the recommendation issued by the independent analyst at time $t \in \{1, 2\}$ and R_t^I as his action space at t. Then we have

$$(2.4) \qquad\qquad r_1^I \in R_1^I = \{\widehat{H}, \widehat{L}, \emptyset\}$$

where $r_1^I = \emptyset$ means keeping silent at $t = 1$, and

$$(2.5) \qquad\qquad r_2^I \in R_2^I = \begin{cases} \{\widehat{H}, \widehat{L}\} \text{ if } r_1^I = \emptyset \\ \emptyset \text{ if } r_1^I \in \{\widehat{H}, \widehat{L}\} \end{cases}$$

The affiliated analyst's action space R_1^A (and R_2^A) is defined analogously as R_1^I (and R_2^I).

The analyst's small message space $\{\widehat{H}, \widehat{L}\}$ is less restrictive than might be thought initially: Kadan et al. (2009) document that most leading investment banks adopted a three-tier recommendation system similar to (Buy, Hold, Sell) after the Global Settlement and related regulations were implemented in 2002.[9]

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[9] A small message space is also assumed in most herding models (e.g., Scharfstein and Stein, 1990; Banerjee, 1992; Trueman 1994).

2.3.3. Analyst and Investor Payoffs

The independent analyst maximizes his payoff function U^I by choosing both what and when to recommend:

$$(2.6) \qquad U^I = Accurate + \delta \times Timely - c(p)$$

where $Accurate$ and $Timely$ have values of either zero or one and $c(p)$ is the cost of information acquisition defined in (2.2). $Accurate = 1$ if his recommendation r^A is consistent with the realization of the state ω, and 0 otherwise. $Timely = 1$ if the independent analyst makes a *non-null* recommendation ($r_1^I \in \{\widehat{H}, \widehat{L}\}$) early at $t = 1$, and $Timely = 0$ if he defers his recommendation to $t = 2$. The positive constant δ is the reward for issuing a timely recommendation and can be equivalently understood as the cost of deferring a recommendation to $t = 2$.

U^I captures the analyst's trade-off between the timeliness and accuracy of his recommendation, first discussed by Schipper (1991) and supported by subsequent empirical findings (e.g., Cooper et al., 2001; Loh and Stulz, 2011; Hong and Kubik, 2003; Jackson, 2005).[10]

The affiliated analyst maximizes his payoff function U^A by choosing both what and when to recommend:

$$(2.7) \qquad U^A = Accurate + \delta \times Timely + \alpha \times Buy$$

---

[10]The timeliness is also noted by practitioners. In an interview with the Wall Street Journal, an analyst said, "it is better to be first than to be out there saying something that looks like you're following everyone else." (*Small Time, in Big Demand.* The Wall Street Journal, June-05-2012.)

where *Accurate* and *Timely* are defined the same way as in the independent analyst's payoff (2.6). $Buy = 1$ if the investor eventually chooses to "Buy" after observing both recommendations, and 0 otherwise.

$\alpha \times Buy$ in U^A captures the affiliated analyst's *conflict of interest*, and the positive constant α measures the degree of the conflict of interest. Due to his conflict of interest, the affiliated analyst has an incentive to misreport his bad signal in order to induce the investor to buy. Among others, Dugar and Nathan (1995), Lin and McNichols (1998), Michaely and Womack (1999), and Mola and Guidolin (2009) document evidence suggesting affiliated analysts face conflicts of interest and tend to issue optimistic recommendations.

The investor makes her investment decision $d \in \{Buy, NotBuy\}$ at $t = 3$ after observing both analysts' recommendations, including the timing of the recommendations. The investor's payoff U^{Inv} is determined by her investment decision as well as the realization of the value of the firm.[11]

$$(2.8) \qquad U^{Inv} = \begin{cases} 1 & \text{if } d = Buy \text{ and } \omega = H \\ -1 & \text{if } d = Buy \text{ and } \omega = L \\ 0 & \text{if } d = NotBuy \end{cases}$$

Figure 1 summarizes the timeline of the game.

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[11]The paper does not model the market microstructure, specifically the supply of the share and the endogenous pricing function. Instead, the paper focuses on the strategic interactions between the two analysts and the information production in equilibrium.

Figure 1: Timeline
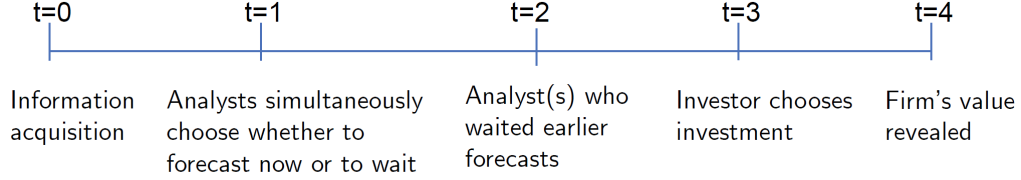
2.3.4. **Two Central Frictions: Incentives and Information**

Central to the model are strategic interactions caused by two frictions: (a) the affiliated analyst's conflict of interest and (b) the independent analyst's informational disadvantage. These two frictions differentiate the affiliated analyst from the independent analyst in the model.

To introduce the independent analyst's informational disadvantage, it is helpful to analyze a benchmark case in which the independent analyst is the only analyst in the economy. In the benchmark case, the independent analyst forecasts at $t = 1$ and independently in the sense that $r^I = \widehat{H}$ if and only if $y^I = g$. Denoting p^* as the optimal precision chosen by the independent analyst in the benchmark case, then p^* solves the following non-strategic optimization problem

$$(2.9) \qquad p^* = \arg\max_{p \in [\frac{1}{2}, 1]} p - e \times (p - \frac{1}{2})^2$$

Solving the program, we obtain $p^* = \frac{1+e}{2e}$. To capture the independent analyst's informational disadvantage, I assume $p^* < p^A$, which is equivalent to the following

assumption on the parameters of the model

(2.10)
$$e > \frac{1}{2p^A - 1}$$

As will be shown later, the assumption $e > \frac{1}{2p^A-1}$ is a sufficient condition under which the signal the independent analyst acquires is less precise than the affiliated analyst's signal in equilibrium. The assumption is supported by empirical evidence such as Jacob et al. (2008) who found affiliated analysts receive superior information compared to the information independent analysts receive.

The affiliated analyst's conflict of interest is captured by the term $\alpha \times Buy$ in his payoff function (2.7), and α measures the degree of the conflict of interest. To avoid trivial analyses, I assume the conflict of interest is neither too weak nor too strong, that is

(2.11)
$$2p^A - 1 + \delta = \underline{\alpha} \le \alpha \le \overline{\alpha} = \frac{2p^A - 1}{1 - p^A + p^*(2p^A - 1)}$$

If the affiliated analyst's conflict of interest is too weak $(\alpha < \underline{\alpha})$, he can perfectly reveal his private signal through his recommendation. If the affiliated analyst's conflict of interest is too strong $(\alpha > \overline{\alpha})$, he cannot credibly communicate his private signal at all. I characterize equilibria for $\alpha < \underline{\alpha}$ and $\alpha > \overline{\alpha}$ in Appendix A for completeness.

2.4. Equilibrium Analysis

This paper's equilibrium concept is Perfect Bayesian Equilibrium.[12] What makes the analysis challenging is the endogenous order of the analysts' actions as it complicates the possible history of the game and therefore players' strategies.[13] I present the analysis in two steps: I first analyze a benchmark case (in Subsection 4.1) where only the independent analyst can choose the timing of his recommendation and then allow both analysts to choose the timing of their recommendations (in Subsection 4.2). The reason to analyze the benchmark case is twofold. First, it is the simplest setting in which the independent analyst's disciplining role and herding behavior arise endogenously, and therefore represents a simpler model in which key tensions of the game can be illustrated. Second, the equilibrium characterized in the benchmark case carries over to the more general game both qualitatively and quantitatively.

2.4.1. Endogenous Timing of Independent Analyst's Recommendation

For the moment, suppose the affiliated analyst issues his recommendation at $t = 1$ and focus on the independent analyst's strategy. The analysis also illustrates the steps used in solving the more general game in Subsection 4.2.

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[12]A profile of strategies and system of beliefs (σ, μ) is a Perfect Bayesian Equilibrium of the extensive form game with incomplete information if it satisfies two properties: (i) the strategy profie σ is sequentially rational given the belief μ and (ii) the belief μ is derived from strategy profile σ by Bayes Rule for any information set H such that $\Pr(H|\sigma) > 0$.

[13]For example, when issuing a recommendation early at $t = 1$, the analyst is not sure whether it will be observed by the other analyst when making recommendations.

2.4.1.1. Properties simplifying the equilibrium analysis. Before solving the game using backward induction, I specify some properties (necessary conditions of the two analysts' strategies) of the equilibrium. These properties, which hold in the general game where both analysts can choose the timing of their recommendations, narrow the search for an equilibrium to a smaller family of strategies.

While the affiliated analyst can bias his recommendation in both directions, the following lemma tells us that focusing on over-reporting is without loss of generality.

Lemma 2. *The affiliated analyst never under-reports his good signal in equilibrium, i.e.,* $\Pr(r^A = \widehat{L}|y^A = g) = 0$.

Proof. All proofs are in Appendix B.

The following lemma narrows the search of the independent analyst's forecasting strategy in equilibrium.

Lemma 3. *If the independent analyst keeps silent at $t = 1$ in equilibrium, it must be that he herds with the affiliated analyst's recommendation r_1^A at $t = 2$ for any $r_1^A \neq \emptyset$.*

The lemma establishes a *perfect correlation* between waiting at $t = 1$ and herding behavior at $t = 2$ in equilibrium. The intuition is as follows: the independent analyst will not receive any informational gain from waiting (to observe r^A) unless his final recommendation is different from what he would have recommend if he did

not wait, i.e., $r^I(y^I, r^A) \neq r^I(y^I)$. In the language of voting theory, information about the affiliated analyst's signal is valuable to the independent analyst only when it is *pivotal*.[14] Two conditions are necessary for the independent analyst who receives y^I to benefit from waiting to observe the affiliated analyst's recommendation r^A: r^A disagrees with his own signal y^I, and the independent analyst herds with r^A in the sense that $r_2^I = r^A$. Since waiting is costly, it must be accompanied by a subsequent herding in equilibrium. This intuition leads to the following proposition.

Proposition 4. *(Endogenous Benefit of Waiting) In equilibrium, the independent analyst's expected gain from waiting to observe r^A is at least weakly higher if he receives a good signal than if he receives a bad signal.*

The proposition opens the gate for endogenous timing of the independent analyst's recommendation: since the independent analyst's benefit of waiting depends on the realization of his private signal while the cost of waiting δ is exogenous, independent analysts observing different signals may choose to forecast at different times in equilibrium.

The intuition for Proposition 4 is as follows. We know from Lemma 3 that the independent analyst does not benefit from waiting unless he subsequently herds

---

[14]The argument does not depend on the analyst's signal space being binary; it applies even if one introduces any continuous signal for the analysts. Instead, the analysts' small message space is critical to the argument. Herding would have not been in equilibrium if the analysts had a continuous message space.

with the affiliated analyst's recommendation indicating a *difference* in the two analysts' signals. Therefore upon observing $y^I = b$ (or $y^I = g$), the independent analyst's informational gain from waiting can be measured by the informativeness of the affiliated analyst's favorable recommendation $\widehat{H}$ (or unfavorable recommendation $\widehat{L}$). Given his incentive to over-report the bad signal, the affiliated analyst's unfavorable recommendation is more informative than his favorable recommendation in equilibrium, which implies the independent analyst's informational gain from waiting is higher if he observes a good signal than a bad signal.[15]

2.4.1.2. Equilibrium. The game is solved by backward induction. Taking the independent analyst's precision choice $p \geq \frac{1}{2}$ at $t = 0$ as given, the following lemma characterizes the unique subgame equilibrium.

Lemma 5. *When only the independent analyst can choose the timing of his recommendation, the unique subgame equilibrium following a given $p \geq \frac{1}{2}$ is*

*(i) **Independent Forecasting Equilibrium** if $\delta \geq \frac{(p^A - p)(2p - 1)}{p^A + p - 1}$, in which the independent analyst forecasts independently at $t = 1$, or*

*(ii) **Conditional Herding Equilibrium** if $\delta < \frac{(p^A - p)(2p - 1)}{p^A + p - 1}$, in which the independent analyst upon observing a bad signal forecasts $\widehat{L}$ at $t = 1$, but upon observing a good signal waits and subsequently herds with the affiliated analyst's recommendation at $t = 2$.*

---

[15]Rigorously, the probability that r^A disagrees with y^I is lower if $y^I = g$. However, as shown in the proof, the potential benefit of changing a recommendation upon disagreement more than offsets the lower probability of that disagreement.

In both cases, the affiliated analyst over-reports his bad signal with probability $\beta = \frac{p^A - p}{p^A + p - 1}$. *The investor bases her investment decision on the affiliated analyst's recommendation unless* $r^A = \widehat{H}$ *but* $r^I = \widehat{L}$, *in which case she does not buy with probability* $\frac{\alpha - (2p^A - 1)}{\alpha(1 - p^A - p + 2p^A p)}$.

The result is simple: given the initial precision choice p, the subgame equilibrium depends on the value of the exogenous cost of deferring recommendations to $t = 2$. If deferring his recommendation is extremely costly ($\delta \geq \frac{(p^A - p)(2p - 1)}{p^A + p - 1}$), the independent analyst forecasts early (and thus independently) regardless of the realization of his signal. If waiting becomes less expensive, the independent analyst waits and herds with the affiliated analyst's recommendation after observing a good signal, since the informational gain from waiting is higher in this case (Proposition 4).

It is worth noting that while Lemma 5 is derived as a mixed strategy equilibrium, the results do not hinge on the randomization of mixed strategies. I show in Section 7 that the main results of the paper are preserved in a richer game in which the equilibrium is in pure strategies.

The following proposition endogenizes the independent analyst's precision choice at $t = 0$ and specifies the overall equilibrium of the benchmark considered in this Subsection.

Proposition 6. *When only the independent analyst can choose the timing of his recommendation, the unique Perfect Bayesian Equilibrium is*

(i) Independent Forecasting Equilibrium if $\delta \geq \Pi$, in which the precision $p = p^*$.

(ii) Conditional Herding Equilibrium if $\delta < \Pi$, in which the precision $p = p^{ch}$.

The players' strategies in each equilibrium are specified in Lemma 5, $\Pi = \frac{4p^A p^{ch} - p^A - p^{ch}}{p^A + p^{ch} - 1} - \frac{e^2(1 - 2p^{ch})^2 + 2e + 1}{2e}$, $p^* = \frac{1+e}{2e}$, and $p^{ch} \in (\frac{1}{2}, p^A)$ is the unique real root to the cubic function[16]

$$(2.12) \qquad 2(p^A + p^{ch} - 1)^2(e - 2ep^{ch}) + (2p^A - 1)^2 = 0$$

The condition on δ in Proposition 6 ensures that (a) the precision p specified in the proposition is ex-ante optimal when the independent analyst chooses it, and (b) the equilibrium is sequentially rational (thus satisfies the conditions in Lemma 5) for the specified p.

2.4.2. Endogenous Timing of Both Analysts' Recommendations

Allowing both analysts to choose the timing of their recommendations substantially increases the possible history of the game and therefore leads to a much larger strategy space for each player. However as shown in the lemma below, the equilibrium characterized in the benchmark (studied in Subsection 4.1) continues to be an equilibrium of the general game.

Lemma 7. *For $\delta \geq \Pi$ ($\delta < \Pi$), the Independent Forecasting Equilibrium (Conditional Herding Equilibrium) characterized in Proposition 6 is an equilibrium of*

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[16]The cubic function has a unique real root and two non-real complex conjugate roots.

the general game in which the timing of both analysts' recommendations is endogenous.

The proof in Appendix B also shows that the equilibrium survives standard equilibrium refinements, particularly the Cho-Kreps's Intuitive Criterion and the (more demanding) Universal Divinity Criterion developed by Banks and Sobel.[17]
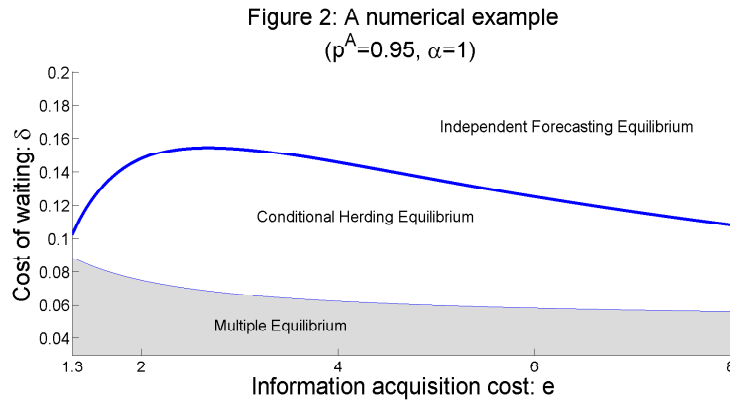
When the timing of both analysts' recommendations is endogenous, the possible history of the game increases and therefore the players' strategy spaces grow exponentially. To maintain tractability, I confine attention to equilibria where the affiliated analyst's waiting decision is in pure strategies. Equilibria with this property are summarized in the following lemma, and they are equilibria even if one allows for arbitrary mixed strategies.[18]

Lemma 8. *In addition to the equilibrium characterized in Lemma 7, another equilibrium emerges for small δ in which the affiliated analyst forecasts at $t = 2$ while the independent analyst forecasts at $t = 1$. Details of the additional equilibrium are specified in Appendix A.*

---

[17]I adopt the definition 11.6 in Fudenberg and Tirole (1991). Denote $D(t, T, m)$ as the set of the investor's mixed-strategy best responses to an out-of-equilibrium message m and beliefs concentrated on the support of the affiliated analyst's type space $T = \{g, b\}$ that makes a type-t affiliated analyst strictly prefer sending out m to his equilibrium message. Similarly denote $D^0(t, T, m)$ as the set of mixed best responses that makes type-t exactly indifferent. In my context, an equilibrium survives the Universal Divinity (or D2) criterion if and only if for *all* the out-of-equilibrium messages m, the equilibrium assigns zero probability to the type-message pair (t, m) if there exists another type t' such that $D(t, T, m) \cup D^0(t, T, m) \subset D(t', T, m)$.

[18]See Theorem 3.1 in Fudenberg and Tirole (1991): In a game of perfect recall, mixed strategies and behavior strategies (mixed strategies of extensive-form games) are equivalent. Then the claim is true by the definition of a Nash Equilibrium.

Figure 2 illustrates the lemma and shows the equilibrium (or equilibria) obtained for different values of the parameters. The shaded area in Figure 2 shows that both the Conditional Herding Equilibrium and the additional equilibrium characterized in Lemma 8 are equilibria of the game for small δ.

Figure 2: A numerical example
(p^A=0.95, α=1)

Independent Forecasting Equilibrium

Conditional Herding Equilibrium

Multiple Equilibrium

Cost of waiting: δ

Information acquisition cost: e

In the additional equilibrium characterized in Lemma 8, the affiliated analyst issues his recommendation later than the independent analyst. The independent analyst chooses his non-strategic precision p^* and forecasts early at $t = 1$ because he correctly conjectures that the affiliated analyst always forecasts at $t = 2$. The affiliated analyst issues $\widehat{H}$ if his own signal is good or the independent analyst issues $\widehat{H}$. If the affiliated analyst receives a bad signal and the independent analysts issues $\widehat{L}$, what the affiliated analyst issues depends on α: he issues $\widehat{L}$ if α is small, while he randomizes between $\widehat{L}$ and $\widehat{H}$ if α is large. The additional equilibrium fails the Universal Divinity Criterion for the small α case. In addition, while the paper assumes the cost of waiting δ is identical for both analysts, the affiliated

analyst with more precise information may have a higher cost of waiting than the independent analyst, which would also rule out the additional equilibrium in which the affiliated analyst forecasts later than the independent analyst.[19] Throughout the remainder, I confine attention to the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium (recall that one or the other of the two, but not both, exists for a given set of parameters).

The independent analyst's endogenous herding decisions in the Conditional Herding Equilibrium has a subtle effect on his ex-ante information acquisition p^{ch}. As will be shown in Section 5, herding with the affiliated analyst in equilibrium can motivate the independent analyst to acquire more information than he would acquire without herding ($p^{ch} > p^*$) and reinforce his ability to discipline the affiliated analyst's biased forecasting behavior.

2.5. Herding Reinforces Disciplining

This section delivers the main point of the paper. The independent analyst's disciplinary role over the affiliated analyst's forecasting strategy is important to understand the result and is formalized in the following lemma.

---

[19]Gul and Lundholm (1995) make a similar assumption and Zhang (1997) develops a model in which players with more precise signals choose to take actions earlier because of the information leakage. Nevertheless, the multiple equilibria problem can be seen as a limitation of this study and of signaling models in general. Using equilibrium refinements to narrow the set of equilibrium is itself controversial, because of the strong assumptions the refinements make. An alternative approach is to accept all equilibria as equally plausible. In my model, this would mean accepting that either the affiliated or the independent analyst might forecast first. Since the Conditional Herding Equilibrium is the one that best captures the notion of disciplining (which is the focus of the paper) and seems consistent with observed analyst behavior, I focus on that equilibrium.

Lemma 9. *(The Independent Analyst's Disciplinary Role) In equilibrium, the affiliated analyst will over-report his bad signal less often if the independent analyst acquires better information. Formally, we have $\frac{d}{dp}\beta < 0$ in equilibrium, where $\beta \doteq \Pr(r^A = \widehat{H}|y^A = b)$.*

Intuitively, as the independent analyst acquires more information, the investor puts more weight on the independent analyst's recommendation when making her investment decision, which means relatively less weight is given to the affiliated analyst's recommendation. Less attention from the investor makes the affiliated analyst *endogenously* care more about being accurate since the only reason he may over-report a bad signal is to convince the investor to buy the stock. In other words, the *endogenous weight* the affiliated analyst puts on the accuracy of his recommendation increases if the independent analyst acquires better information upfront. The independent analyst's disciplining effect is consistent with Gu and Xue (2008) who find that the affiliated analysts' recommendations become more accurate and less biased when independent analysts are following the same firms than when they are not.

Lemma 9 shows that the effectiveness of the independent analyst's disciplining role depends on how much information he acquires ex-ante, while the ex-post herding *per se* is irrelevant. Therefore instead of asking how the independent analyst's herding behavior affects his disciplining role, the real question is how the herding behavior affects the independent analyst's ex-ante information acquisition

(and thus the ability to discipline the affiliated analyst). The next Proposition shows that the independent analyst's ex-post herding behavior will motivate better information acquisition ex-ante (and therefore reinforces the disciplining role) if his informational disadvantage is large.

Proposition 10. *(Herding Motivates Better Information Acquisition) The independent analyst acquires more precise information in the Conditional Herding Equilibrium than in the Independent Forecasting Equilibrium if and only if his informational disadvantage is large. Formally, $p^{ch} > p^* \Leftrightarrow e > \frac{1}{(\sqrt{2}-1)(2p^A-1)}$.*

Why does the independent analyst spend more effort acquiring private information, knowing that he will discard that information ex-post half of the time (whenever $y^I = g$) and herds with the affiliated analyst? Analyzing the marginal benefit of information acquisition from the independent analyst's perspective provides the answer. In the Conditional Herding Equilibrium, the marginal benefit is

$$(2.13) \qquad \underbrace{\frac{1}{2} \times 1}_{\text{Direct benefit}} + \underbrace{\frac{1}{2}\{\beta(p) + \overbrace{(p^A - p)}^{\text{benefit of herding}} \overbrace{\frac{d}{dp}}^{\text{discipline}}[-\beta(p)]\}}_{\text{Indirect benefit associated with disciplining}}$$

where $\beta(p)$ is the equilibrium probability that the affiliated analyst over-reports his bad signal.

The first term corresponds to the independent analyst observing a bad signal, in which case he will forecast $r^I = \widehat{L}$ immediately. In this case, acquiring better

information mechanically increases the likelihood of his recommendation being accurate at a marginal rate of 1.

The second term corresponds to the independent analyst observing a good signal, in which case he will wait and herd with the affiliated analyst at $t = 2$. In this case, information acquisition has an indirect benefit. As the independent analyst acquires more information, the affiliated analyst faces more stringent discipline and his best response is to truthfully report an unfavorable recommendation $r^A = \widehat{L}$ more often (i.e., $\frac{d}{dp}[-\beta(p)] > 0$). The response by the affiliated analyst in turn implies that the independent analyst observing a good signal is more likely to enjoy a precision jump of $(p^A - p)$ by herding with the affiliated analyst's (more precise) unfavorable recommendation $\widehat{L}$. It is the very ex-post herding behavior that allows the independent analyst to benefit from the discipline effect he provides and motivates him to acquire more information than he would have acquired were he forced to forecast independently.

In the Independent Forecasting Equilibrium, the marginal benefit of information acquisition comes solely from the direct benefit, discussed in the first term of equation (2.13). Therefore, the independent analyst acquires more information in the Conditional Herding Equilibrium if and only if the indirect benefit via herding dominates the direct benefit. As the precision choice p decreases in the information acquisition cost e, the condition $e > \frac{1}{(\sqrt{2}-1)(2p^A-1)}$ in Proposition 10 simply puts a lower bound on the potential precision jump $p^A - p$, above which the indirect benefit outweighs the direct benefit. To illustrate Proposition 10, Figure 3 compares

the independent analyst's information acquisition p^* and p^{ch} as a function of the information acquisition cost parameter e, in which $p^A = 0.95$.

Figure 3: Herding motivates better information acquisition if e>e*
(p^A=0.95)
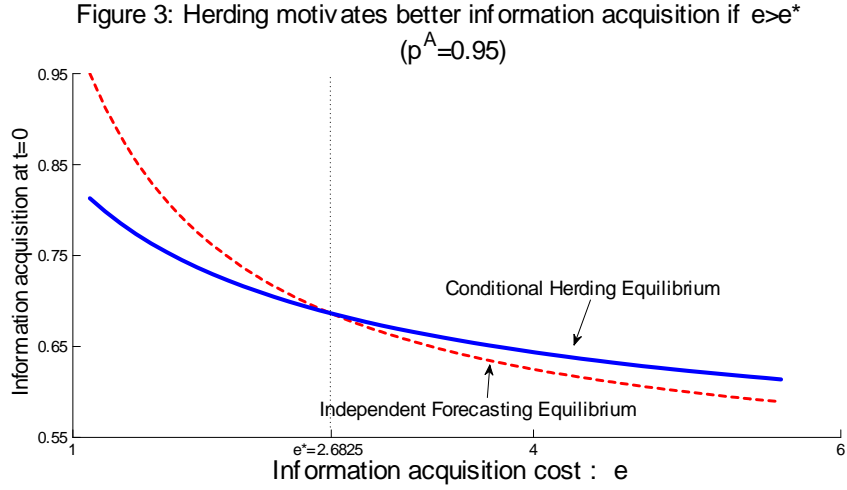
What is the effect of the independent analyst's herding behavior on the investor's payoff? The answer is not clear at this point: while the independent analyst may acquire better information in the Conditional Herding Equilibrium (Proposition 10), he sometimes discards that information and herds with the affiliated analyst who by assumption faces a conflict of interest. The following proposition summarizes the result.

Proposition 11. *(Herding Benefits the Investor) Forcing the independent analyst to forecast independently would make the investor weakly worse-off if and only if the independent analyst's informational disadvantage is large, i.e., $e >$*

$$\frac{1}{(\sqrt{2}-1)(2p^A-1)}.$$

The result confirms the idea that herding per se does not affect the independent analyst's disciplining role. Given the affiliated analyst's incentive to over-report a bad signal, the independent analyst's recommendation disciplines the affiliated analyst only when it is unfavorable ($r^I = \widehat{L}$). In equilibrium, the independent analyst reports his bad signal immediately while he herds only if his private signal is good, which does not compromise his ability to discipline the affiliated analyst. As shown in Proposition 10, if the independent analyst's informational disadvantage is large, his herding strategy motivates better information acquisition and, therefore, reinforces the disciplining benefit enjoyed by the investor.

Figure 4 compares the investor's utility in the Independent Forecasting Equilibrium (the dotted line) and the Conditional Herding Equilibrium as a function of e. Forcing the independent analyst to forecast independently implements the Independent Forecasting Equilibrium, however doing so weakly decreases the investor's payoff for $e > 2.6825$ as otherwise the equilibrium would be the Conditional Herding Equilibrium if the cost of waiting is not too large.

Figure 4: Herding benefits the investor if e>e*
(p^A=0.95)
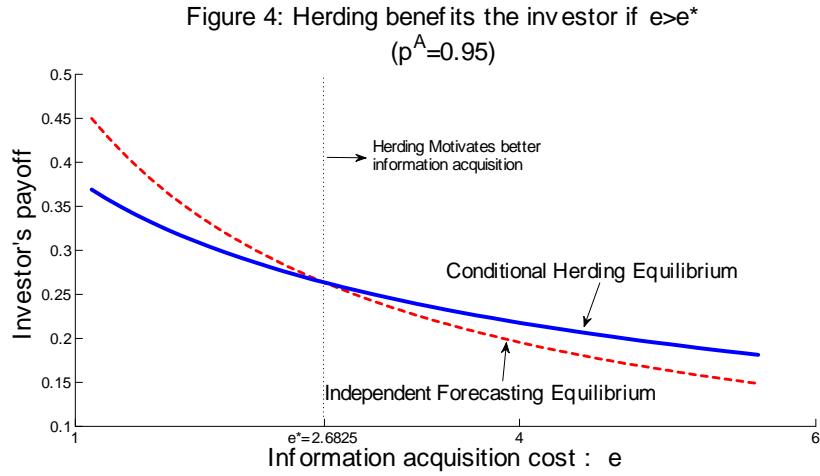
2.6. Empirical and Regulatory Implications

From Lemma 9 and Proposition 11, we know that the affiliated analyst's recommendation will become more informative if the independent analyst acquires better information, which is in line with the disciplining story documented by Gu and Xue (2008). Since the model derives the necessary and sufficient condition for the independent analyst's herding behavior to motivate better information acquisition (Proposition 10), it generates predictions about the association between the independent analyst's herding behavior and the informativeness of the affiliated analyst's recommendations. Moreover, depending on the characteristics of the firm, the sign of the association is different.

Corollary 12. *The model predicts a positive association between independent analysts' degree of herding and the informativeness of affiliated analysts' recommendations for firms with high information acquisition costs, while the predicted association is negative for firms with low information acquisition costs.*

This is a sharp prediction that can be used to test my model. Welch (2000) proposes a methodology for estimating the degree of herding and proxies for other variables are common in the existing literature.

The following prediction is about the dynamics of the dispersion of the analysts' recommendations over time.

Corollary 13. *The model predicts that the dispersion between independent and affiliated analysts' recommendations decreases over time even if no new information is released.*

Traditional wisdom attributes the decrease in the dispersion of analysts' recommendations to the arrival of *new* information, which decreases the uncertainty analysts face and leads to similar opinions. The model offers an alternative and more endogenous explanation. Instead of relying on exogenous "*new*" information available from outside, the decrease of dispersion in my model is caused by how "*old*" information is gradually comprehended and used over time inside the analyst market.

The corollary explains O'Brien et al. (2005) and Bradshaw et al. (2006) who find that affiliated analysts' recommendations are more optimistic than independent analysts' recommendations only in the first several months surrounding IPOs and SEOs, while there is no difference between the two recommendations made later. According to the model, only the independent analysts who observe bad signals choose to issue recommendations early, which explains the affiliated analysts' optimism at the beginning. We do not expect any difference later on because independent analysts who choose to wait will herd with affiliated analysts' recommendations. In addition, since Proposition 11 shows the optimality of the independent analyst's herding behavior from the investor's point of view, my model suggests that the empirical evidence documented above may actually come from the equilibrium (conditional herding equilibrium) that is favorable to investors. To the best of my knowledge, this prediction has not been tested outside public offering settings.

The next corollary addresses a potential, undesirable consequence of regulations mitigating the affiliated analyst's conflict of interest.

Corollary 14. *Regulations mitigating the affiliated analyst's conflict of interest such as the Global Settlement do not necessarily benefit the investor.*

In the model, a smaller α captures the effect of regulations mitigating the affiliated analysts's conflict of interest. While it is easy to show $\frac{d}{d\alpha}U^{Inv} = 0$ in equilibrium (which is driven by the mixed strategies), the idea that lowering the

affiliated analyst's bias does not necessarily benefit the investor is more general. In Section 7, I modify the base model so that only pure strategy equilibria exist and show that lowering the affiliated analyst's conflict of interest could *strictly decrease* the investor's payoff. The reason is that the independent analyst tends to put more trust in the affiliated analyst as the latter's conflict of interest becomes less severe. It could be that a smaller α completely wipes out the independent analyst's incentive to acquire information ex-ante and therefore the affiliated analyst faces no disciplining, in which case the investor is worse off.[20]

2.7. Robustness of Main Results

Due to simplifications made for tractability, the affiliated analyst and the investor play mixed strategies in the base model (see Lemma 5). I show in this section that the main results of the base model are preserved in a game where only pure strategy equilibria exist. To ease exposition, I restrict the affiliated analyst to issuing his recommendation at $t = 1$ (as in Subsection 4.1). The goal is to demonstrate that results of the paper are not driven by mixed strategies.

2.7.1. Modified setup

Three modifications are made to the base model. While the investor is assumed to be risk neutral with a binary action space $\{Buy, NotBuy\}$ in the base model, she is

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[20]The reasoning for the second part of Corollary 14 is similar to Fischer and Stocken (2010) who find more precise public information may completely crowd out an analyst's information acquisition.

now assumed to be risk averse (CARA utility) and has a continuous action space. The investor is endowed with e amount of "dollars" which can be invested between a risk-free asset and a risky asset (the firm). The return of the risk-free asset is normalized to be zero and the return of the risky asset is $\omega \in \{H = 1, L = -1\}$ with the common prior belief that both states are equally likely. Both assets pay out at the end of the game, and the time value of money is ignored for clean notation. A portfolio consisting of A units of the risk-free asset and B units of the risky asset costs the investor $A + B * m$ dollars to form and will generate wealth w to the investor at the end of the game

$$(2.14) \qquad\qquad w = A + B * m * (1 + \omega)$$

where m is the price of the risky asset when the portfolio is formed.[21] The investor maximizes the following utility function

$$(2.15) \qquad\qquad U^{INV} = -e^{-\rho * w}$$

where $\rho > 0$ is the coefficient of absolute risk aversion. The model does not allow short selling of the risky asset and therefore $B \geq 0$.

The reward the affiliated analyst receives for inducing the investor's buy action is modified to be proportional to the units of the risky asset the investor buys. If

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[21]The paper does not model the supply of the share and therefore m is taken as given.

the investor buys B units of the risky asset, the affiliated analyst's payoff function is

$$(2.16) \qquad\qquad U^A = Accurate + \alpha \times B$$

which is a natural extension of $U^A = Accurate + \alpha \times Buy$ used in the base model as (2.16) incorporates the fact that the risk averse investor will buy different numbers of shares of the risky asset in response to different recommendations.

Finally, instead of having a binary support $\{b, g\}$ in the base model, the affiliated analyst's private signal y^A is now assumed to have a continuous support:

$$(2.17) \qquad\qquad y^A = \omega + \tau$$

where $\omega \in \{H = 1, L = -1\}$ is the return of the risky asset and the noise term τ is normally distributed

$$(2.18) \qquad\qquad \tau \sim N(0, 1)$$

and the variance of τ is normalized to 1 without loss of generality.[22]

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[22] The probability density function $\varphi(y^A|\omega)$ satisfies the monotonic likelihood ratio property (MLRP) in the sense that $\frac{\varphi(y^A|\omega=H)}{\varphi(y^A|\omega=L)}$ increases in y^A.

2.7.2. Equilibrium analysis

As the investor is risk-averse, her holdings of the firm vary continuously with her posterior assessment of the firm. Intuitively, the investor will hold more of the risky asset if her posterior assessment is more optimistic, which is verified by the following lemma.

Lemma 15. *In equilibrium, the investor buys B units of the risky asset at a given price m:*

$$(2.19) \qquad B = \begin{cases} \dfrac{\log(\frac{q_H}{1-q_H})}{2*\rho*m} & \text{if } q_H \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

and $\frac{dB}{dq_H} \geq 0$, where $q_H = \Pr(\omega = H | r^A, r^I)$ is derived using Bayesian Rule given the prior distribution of ω and both analysts' equilibrium strategies.

The following lemma states the properties of the independent analyst's strategy in equilibrium. As in the base model (see Proposition 4), the independent analyst observing a good signal is more likely to wait and herd with the affiliated analyst, which opens the gate for the endogenous timing of the independent analyst's recommendation.

Lemma 16. *In equilibrium, the independent analyst will herd with r^A if he keeps silent at $t = 1$, and the gain from waiting is higher if he observes a good signal than if he observes a bad signal.*

The following lemma shows that the affiliated analyst follows an intuitive switching strategy in equilibrium.

Lemma 17. *In equilibrium, the affiliated analyst's strategy is characterized by a unique cut-off point $s < 0$ such that he forecasts $\widehat{H}$ if and only if the realization of his signal is greater than s. Formally, $r^A = \widehat{H} \Leftrightarrow y^A > s$.*

With all players' equilibrium strategies in place, we are ready to present the equilibrium.

Proposition 18. *The modified game only has pure strategy equilibria, and the equilibrium takes one of the following forms*

*(1) **Independent Forecasting Equilibrium** where the independent analyst forecasts independently at $t = 1$.*

*(2) **Conditional Herding Equilibrium** where the independent analyst forecasts independently at $t = 1$ if and only if his signal is bad while otherwise he waits and herds with the affiliated analyst at $t = 2$.*

*(3) **No Information Acquisition Equilibrium** where the independent analyst does not acquire private information and always herds with the affiliated analyst's recommendation at $t = 2$.*

In any equilibrium, the investor's investment strategy is defined in Lemma 15 and the affiliated analyst follows a switching strategy described in Lemma 17.

As in the base model, the endogenous benefit of waiting leads to a conditional herding equilibrium, under which the independent analyst reports his bad signal immediately while he waits and herds with the affiliated analyst otherwise.

The main result of the base model, that herding with the affiliated analyst motivates the independent analyst to acquire more information (Proposition 10) and ultimately benefits the investor (Proposition 11), arises in the modified game as well. Figure 5 plots the precision chosen by the independent analyst in the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium (characterized in Proposition 18) as a function of the information acquisition cost parameter e, in which $\alpha = 2$, $\delta = 0.05$, and $\rho = 0.2$. In this example, the unique equilibrium of the game is the Conditional Herding Equilibrium for all values of e. It is clear that the independent analyst acquires better information in the conditional herding equilibrium than in the Independent Forecasting Equilibrium if his informational disadvantage is large ($e > 9.465$ in Figure 5), which is consistent with Proposition 10. One can also check that the investor is strictly better-off in the Conditional Herding Equilibrium for $e > 9.465$, which is consistent with Proposition 11.

Figure 5: Herding motivates better information acquisition
($\alpha=2, \delta=0.05, \rho=0.2$)
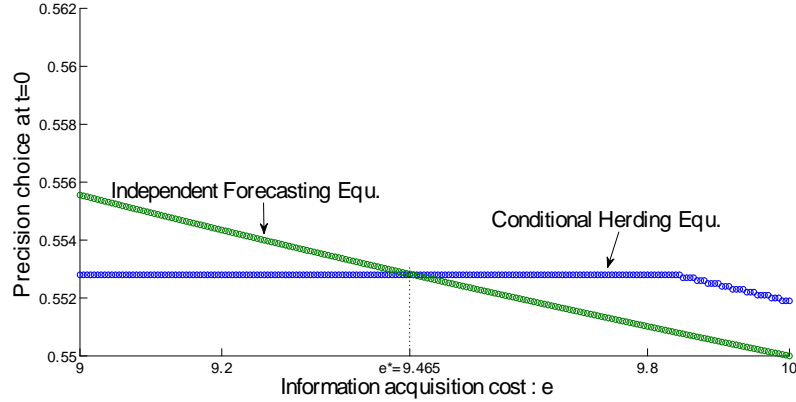
2.8. Concluding Remarks

The paper studies how an independent analyst interacts with an affiliated analyst. Inspired by features noted by practitioners and academic researchers, the paper assumes that, compared to the independent analyst, the affiliated analyst faces a conflict of interest but has superior information. Consistent with our intuition and empirical findings, the paper shows that the independent analyst both disciplines and herds with the affiliated analyst. On one hand, the independent analyst's incentive is more aligned with the investor and therefore he disciplines the affiliated analyst's biased forecasting behavior. On the other hand, the independent analyst sometimes defers his recommendation and herds with the affiliated analyst as the latter has more precise information.

While traditional wisdom suggests that disciplining and herding are in conflict with each other, I show that the independent analyst's disciplining role and herding

behavior may actually be complements in equilibrium. In particular, if the independent analyst's informational disadvantage is large, herding with the affiliated analyst actually motivates the independent analyst to acquire more information upfront, reinforces his disciplining role, and ultimately benefits the investor. This point and other findings of the paper are intended to improve our understanding of independent analysts' role and offer a rationale for some empirical observations.

The main point that herding can motivate better information acquisition and reinforce disciplining seems likely to apply to settings other than affiliated and independent analysts. For example, mutual fund managers base their portfolio choices on both buy-side and sell-side analysts' forecasts. While sell-side analysts potentially face conflicts of interest such as trade-generating incentives, it has been documented that their forecasts are more precise than buy-side analysts (e.g., Chapman et al., 2008). The paper suggests that buy-side analysts may serve a disciplinary role. Moreover, in order to induce buy-side analysts to acquire more information, fund managers may purposely allow buy-side analysts to herd with sell-side analysts by passing along the latter's forecast to buy-side analysts.

CHAPTER 3

A Multi-period Foundation for Bonus Pools

ABSTRACT:[1] This paper explores optimal discretionary rewards based on subjective/non-verifiable performance measures in a multi-period, principal-multi-agent model. The multi-period relationship creates the possibility of trust between the principal and the agents. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, creates opportunities for both cooperation/mutual monitoring (good implicit side-contracting) and collusion (bad implicit side-contracting). When the expected relationship horizon is long, the optimal contract emphasizes joint performance, which incentivizes the agents to use implicit contracting and mutual monitoring to motivate each other. When the expected horizon is short, the solution converges to a static bonus pool. A standard feature of a static bonus pool is that it rewards agents for (joint) bad performance in order to make the evaluator's promises to provide honest evaluations credible. For intermediate expected horizons, the optimal contract allows for more discretion in determining total rewards, which is typical in practice, but also sometimes rewards the agents for bad performance. The reason for rewarding bad performance is different than in the static setting—paying for bad performance

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[1]This essay is a joint work with Jonathan Glover.

allows the principal to create a strategic independence in the agents' payoffs that reduces their incentives to collude. That is if the principal did not have to prevent tacit collusion between the agents in this case, she would not reward the agents for bad performance. She would instead use a relative performance evaluation scheme. The unappealing feature of relative performance evaluation is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has the agents taking turns making each other look good—they alternate between (*work, shirk*) and (*shirk, work*).

3.1. Introduction

Discretion in awarding bonuses and other rewards is pervasive. Evaluators use discretion in determining individual rewards, the total reward to be paid out to all (or a subset of the) employees, and even in deviating from explicit bonus formulas (Murphy and Oyer, 2001; Gibbs et al., 2004). A common concern about discretionary rewards is that the evaluator must be trusted by evaluatees (Anthony and Govindarajan, 1998).

In a single-period model, bonus pools are a natural economic solution to the "trust" problem (Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009). When all rewards are discretionary (based on subjective assessments of individual performance), a single-period bonus pool rewards bad performance, since the total size of the bonus pool must be a constant in order to make the evaluator's promises credible.

The relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on subjective/non-verifiable performance measures (e.g., Baker, Gibbons, and Murphy, 1994), but this literature has mostly confined attention to single-agent settings.[2] This paper explores optimal discretionary rewards based on subjective/non-verifiable individual performance measures in a multi-period, principal-multi-agent model, which leads to discretionary rewards. The multi-period relationship creates the possibility of trust between the principal and the agents, since the agents can punish the principal for bad behavior. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, creates opportunities for both cooperation/mutual monitoring (good implicit side-contracting) and collusion (bad implicit side-contracting) between the agents.

When the expected relationship horizon is long, the optimal contract emphasizes joint performance, which incentivizes the agents to use implicit contracting and mutual monitoring to motivate each other to "*work*" rather than "*shirk*." The subjective measures set the stage for the managers to use implicit contracting and mutual monitoring to motivate each other, as in existing models with verifiable performance measures (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001).[3]

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[2]One exception is Levin (2002), who examines the role that trilateral contracting can have in bolstering the principal's ability to commit—if the principal's reneging on a promise to any one agent means she will loose the trust of both agents, relational contracting is bolstered.

[3]There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh (1993) models of explicit side-contracting

When the expected horizon is short, the solution converges to the static bonus pool. A standard feature of a static bonus pool is that it rewards agents for (joint) bad performance in order to make the evaluator's promises credible.

For intermediate expected horizons, the optimal contract allows for more discretion in determining total rewards, which is typical in practice, but also rewards the agents for bad performance. The reason for rewarding bad performance is different than in the static setting—paying for bad performance allows the principal to create a strategic independence in the agents' payoffs that reduces their incentives to collude. If the principal did not have to prevent tacit collusion between the agents, she would instead use a relative performance evaluation scheme. The unappealing feature of relative performance evaluation is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has the agents taking turns making each other look good—they alternate between *(work, shirk)* and *(shirk, work)*.[4] While it is natural to criticize discretionary rewards for bad performance (e.g., Bebchuk and Fried, 2006), our result provides a rationale for such rewards. In this light, individual performance evaluation can be seen as one of a class of incentive arrangements

can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

[4]Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

that create strategic independence—individual performance evaluation is the only such arrangement that does not involve rewarding poor performance.

In our model, all players share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important.

There is a puzzling (at least to us) aspect of observed bonus pools. Managers included in a particular bonus pool are being told that they are part of the same team and expected to cooperate with each other to generate a larger total bonus pool (Eccles and Crane, 1988). Those same managers are asked to compete with each other for a share of the total payout. We extend the model to include an objective/verifiable team-based performance measure. Productive complementarities in the objective team-based measure can make motivating cooperation among the agents optimal when it would not be in the absence of the objective measure. The productive complementarity also takes pressure off of the individual subjective measures, allowing for a greater degree of relative performance evaluation (and less pay-for-bad performance) than would otherwise be possible. Put differently, the combination of rewarding a team for good performance but also asking agents to compete with each other for a share of the total reward is not inconsistent with motivating cooperation and mutual monitoring. Instead, such commonly observed

schemes can be an optimal means of motivating cooperation and mutual monitoring when the principal's commitment is limited. The earlier theoretical literature on bonus pools did not develop this role for bonus pools because of their focus on static settings.

In contrast, productive substitutability in the objective team-based measure may preclude cooperation from being optimal and necessitate even greater pay-for-bad performance. In such cases, the productive substitutability also necessitates a strategic complementarity in the way the subjective measures are incorporated into the compensation arrangement (limiting the use of relative performance evaluation and resulting in more pay-for-bad performance). In particular, the subjective measures are used to create a payment complementarity that just offsets the productive substitutability in the objective measure, so that the net effect is an overall strategic payoff independence that is optimal in preventing collusion.

Murphy and Oyer (2001) report that 42% of the firms they study had discretion in determining the size of the bonus pool. Such discretion in determining total rewards is always optimal in our model but is much greater when cooperation and mutual monitoring is optimal than when they are not and is increasing in the degree of the complementarity of the agents' actions in the verifiable team-based measure. Murphy and Oyer (2001) also hypothesize and find that (positive) externalities across divisions are associated with greater use of discretion. Their reasoning is that such discretion can be used to reward such cooperation. Instead of productive complementarities creating the demand for discretion, productive

complementarities facilitate discretion in our model (reversing the direction of causality).

Like our paper, Kvaloy and Olsen (2006) study a multi-period, multi-agent model in which all performance measures are subjective. They exogenously rule out pay for bad performance, which is an important focus of our paper. Our extension is closely related to Baldenius and Glover (2012) on dynamic bonus pools. They take the form of the bonus pool as given, studying the impact of using a bonus pool with the features of a static one in a multi-period horizon. In particular, all of their bonus pools have the feature that the total payout does not depend on the subjective performance measures. In contrast, the focus of this paper is on optimal contracts, which incorporate discretion in determining the size of the bonus pool.

Baiman and Baldenius (2009) study the role of non-financial performance measures can have in encouraging cooperation by resolving hold-up problems. The empirical literature also provides evidence consistent with discretion being used to reward cooperation (e.g., Murphy and Oyer, 2001; Gibbs et al., 2004). Our model is consistent with this view in that the discretionary rewards are used to motivate cooperation when possible. Our analysis points out the importance of both the evaluator's and the evaluatees' reputation in sustaining cooperation through mutual monitoring.

The remainder of the paper is organized as follows. Section 3.2 presents the basic model. Section 3.3 studies implicit side-contracting between the agents that is

harmful to the principal, while Section 3.4 studies implicit side-contracting between the agents that is beneficial to the principal. Section 3.5 characterizes the optimal overall contract. Section 3.6 studies an extension in which there are both individual subjective performance measures (as in the rest of the paper) and an objective team-based performance measure. Section 3.7 concludes.

3.2. Model

A principal contracts with two identical agents, $i \in \{A, B\}$, to perform two independent and ex ante identical projects (one for each agent) in each period. Each agent chooses a personally costly effort $e^i \in \{0, 1\}$ in each period t, i.e., the agent chooses either "*work*" ($e^i_t = 1$) or "*shirk*" ($e^i_t = 0$). Whenever it does not cause confusion, we drop sub- and superscripts. Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to be 1. The outcome from each project, denoted by x^i, is assumed to be either high ($x^i = H > 0$) or low ($x^i = L = 0$).

Agent i's effort choice stochastically affects the outcome of the project under his management, in particular $q_1 = \Pr(x^i = H | e^i = 1)$, $q_0 = \Pr(x^i = H | e^i = 0)$, and $0 < q_0 < q_1 < 1$. Note that each agent's effort choice does not affect the other agent's probability of producing a good outcome. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e^i = 1$) in every period. The principal's problem is to design

the contract that motives both agents to *work* in each and every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort choice in each period. Communication from the agents to the principal is blocked—the outcome pair (x^i, x^j) is the only signal on which the agents' wage contract can depend. Denote by w^i_{mn} the wage agent i receives if his outcome is m and his peer's outcome is n; $m, n \in \{H, L\}$. The wage contract provided to agent i is a vector $w^i = \{w^i_{HH}, w^i_{HL}, w^i_{LH}, w^i_{LL}\}$. Given wage scheme w^i and assuming that agents i and j choose efforts level $k \in \{1, 0\}$ and $l \in \{1, 0\}$ respectively, agent i's expected wage is:

$$\pi(k, l; w^i) = q_k q_l w^i_{HH} + q_k(1 - q_l)w^i_{HL} + (1 - q_k)q_l w^i_{LH} + (1 - q_k)(1 - q_l)w^i_{LL}.$$

All parties in the model are risk neutral and share a common discount rate r, capturing the time value of money or the probability the contract relationship will end at each period (the contracting horizon). The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

(Non-negativity) $$w_{mn} \geq 0, \forall m, n \in \{H, L\}$$

Unlike Che and Yoo (2001), we assume the outcome (m, n) is unverifiable. The principal, by assumption, can commit to a contract form but cannot commit to

reporting the unverifiable performance outcome (m, n) truthfully.[5] Therefore, the principal must rely on a self-enforcing implicit (relational) contract to motivate the agents. We consider the following trigger strategy played by the agents: both agents behave as if the principal will honor the implicit contract until the principal lies about one of the performance measures, after which the agents punish the principal by choosing (*shirk, shirk*) in all future periods. This punishment is the severest punishment the agents can impose on the principal. The principal will not renege if:

(Principal's IC) $\dfrac{2\left[q_1 H - \pi(1, 1; w)\right] - 2q_0 H}{r} \geq \max\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}$

This constraint guarantees the principal will not claim the output pair from the two agents as (m', n') if the true pair is (m, n). The left hand side is the cost of lying.[6] The agents choosing (*shirk,shirk*) and the principal paying zero to each agent is a stage-equilibrium. Therefore, the agents' threat is credible. The right hand side of this constraint is the principal's benefit of lying about the performance signal.

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[5]In contrast, Kvaloy and Olsen (2006) assume the principal cannot commit to the contract, which makes it optimal to set wLL = 0. Our assumption that the principal can commit to the contract is intended to capture the idea that the contract and the principal's subjective performance rating of the agents' performance can be verified. It is only the underlying performance that cannot be verified.

[6]If she reneges on her implicit promise to report truthfully, the principal knows the agents will retaliate with (*shirk, shirk*) in all future periods. In response, the principal will optimally choose to pay a fixed wage (zero in this case) to each agent.

3.3. Collusion

In this section of the paper, we derive the optimal contract while considering only the possibility of negative agent-agent implicit side-contracts (collusion). Assuming the principal truthfully reports the outcome, both agents choosing work is a static Nash equilibrium if:

(Static NE) $$\pi(1,1;w) - 1 \geq \pi(0,1;w)$$

The following two conditions make the contract collusion-proof. First, the contract has to satisfy the following condition to prevent joint shirking:

(No Joint Shirking) $$\pi(1,0;w) - 1 + \frac{\pi(1,1;w) - 1}{r} \geq \frac{1+r}{r}\pi(0,0;w)$$

The left hand side is the agent's expected payoff from unilaterally deviating from (*shirk, shirk*), or "*Joint Shirking,*" for one period by unilaterally choosing *work* and then being punished indefinitely by the other agent by playing the stage game equilibrium (*work, work*) in all future periods, while the right hand side is his expected payoff from sticking to *Joint Shirking* strategy.

Second, the following condition is needed to prevent agents from colluding by "*Cycling,*" i.e., alternating between (*shirk, work*) and (*work, shirk*):

(No Cycling) $$\frac{1+r}{r}[\pi(1,1;w) - 1] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;w) + \frac{(1+r)}{r(2+r)}[\pi(1,0;w) - 1]$$

The left hand side is the agent's expected payoff if he unilaterally deviates by choosing *work* when he is supposed to *shirk* and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff if the agent instead sticks to the *Cycling* strategy.

The reason that it suffices to consider only these two conditions is that these are these two forms of collusion have the agents colluding in the most symmetric way, which makes the incentives the principal needs to provide to upset collusion most costly. If the agents adopted a less symmetric collusion strategy, the principal could find a less costly contract that would ensure the agent who benefits the least from collusion would abandon the collusive agreement. The argument is the same as in Baldenius and Glover (2012, Lemma 1).

It is helpful to distinguish three classes of contracts and point out how they influence the two collusion strategies above. The wage contract creates a *strategic complementarity* (between the two agents' effort choice) if $\pi(1,1) - \pi(0,1) > \pi(1,0) - \pi(0,0)$, which is equivalent to a payment complementarity $w_{HH} - w_{LH} > w_{HL} - w_{LL}$. Similarly, the contract creates a strategic substitutability if $\pi(1,1) - \pi(0,1) < \pi(1,0) - \pi(0,0)$, or equivalently $w_{HH} - w_{LH} < w_{HL} - w_{LL}$. The contract creates strategic independence if $\pi(1,1) - \pi(0,1) = \pi(1,0) - \pi(0,0)$, or $w_{HH} - w_{LH} = w_{HL} - w_{LL}$. This classification of wage schemes determines the collusion strategy that is most profitable/attractive from the agents' point of view and, thus, the most costly collusion strategy from the principal's point of view. No

Cycling is the binding collusion constraint under a strategic payoff substitutability, while No Joint Shirking is the binding collusion constraint under a strategic complementarity.[7] Investigating when and why the principal purposely designs the contract to exhibits a strategic complementarity, substitutability, or independence is the focus of our analysis.

The basic problem faced by the principal is to design a minimum expected cost wage contract $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ that assures (*work, work*) in every period is the equilibrium-path behavior of some collusion-proof equilibrium. The contract also has to satisfy the principal's reneging constraint, so that she will report her assessment of performance honestly. The problem is summarized in the following linear program:

$$\min_{\{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}} \pi(1,1)$$

s.t.

Static NE

No Joint Shirking $(LP-1)$

No Cycling

Principal's IC

Non-negativity

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[7]Mathematically, the *No Joint Shirking* constraint implies the *No Cycling* constraint if the contract exhibits a strategic complementarity, and the reverse implication is true if the contract exhibits a strategic substitutability.

Since the two agents are symmetric, it is sufficient to minimize the expected payment made to one agent. The following lemma states that the optimal contract always satisfies $w_{LH} = 0$.

Lemma 19. *Setting $w_{LH} = 0$ is optimal.*

Proof. All proofs of this essay are in the Appendix C. □

The proof of Lemma 19 explores the symmetry between w_{HL} and w_{LH}. The principal can always provide better incentives to both agents by decreasing w_{LH} and increasing w_{HL}. Proposition 20 characterizes how the optimal contract changes as the discount rate increases (both parties become impatient).

Proposition 20. *Depending on the value of r, the solution to $LP - 1$ is ($w_{LH} = 0$ in all cases):*

(i) IPE: $w_{HH} = w_{HL} = \frac{1}{q_1 - q_0}, w_{LL} = 0$ for $r \in (0, \delta^C]$;

(ii) BP^I: $w_{HH} = \frac{(q_1 - q_0)^2 H - (1+r)(q_1 + r - 1)}{(q_1 - q_0)(1 - r(q_1 + r - 1))}$, $w_{HL} = w_{HH} + w_{LL}$, $w_{LL} = \frac{(q_1 - q_0)^2(1+r)H - (1+r)(q_1 + r)}{(q_1 - q_0)(1 - r(q_1 + r - 1))}$

for $r \in (\delta^C, \tau^1]$;

(iii) RPE: $w_{HH} = \frac{(1 - q_1)q_1(1+r) + (q_1 - q_0)^2(q_1(2+r) - 1 - r)H}{(q_1 - q_0)\left(q_1^2 - r(1+r) + q_1 r(2+r)\right)}$, $w_{HL} = \frac{(q_1 - q_0)q_1(2+r)H - \frac{(1+r)(q_1^2 + r)}{q_1 - q_0}}{q_1^2 - r(1+r) + q_1 r(2+r)}$,

$w_{LL} = 0$ for $r \in (\max(\delta^C, \tau^1), \delta^F]$;

(iv) BP^S: $w_{HH} = \frac{(1+r)(r - (1 - q_1)^2) - (q_1 - q_0)^2(1 - q_1)(2+r)H}{(q_1 - q_0)(2q_1 + (3 - q_1)q_1 r + r^2 - 2(1+r))}$, $w_{HL} = 2w_{HH}$, $w_{LL} = \frac{(1+r)((2 - q_1)q_1 + r) + (q_1 - q_0)^2(q_1(2+r) - 2(1+r))H}{(q_1 - q_0)(2q_1 + (3 - q_1)q_1 r + r^2 - 2(1+r))}$ for $r > \max(\tau^1, \delta^F)$;

where $\tau^1 = \frac{2q_1 - 1}{(1-q_1)^2}$ and δ^C, δ^F are increasing functions of H, which are specified in the Appendix C.

We use superscripts "C", "S", and "I" to denote a (payoff) strategic complementarity, strategic substitutability, and strategic independence induced by the contract. In Proposition 1, individual performance evaluation (IPE) is optimal if both parties are patient enough ($r \leq \delta^C$). IPE is the benchmark solution—it is also the optimal contract when the performance measures are verifiable, since the agents operate independent individual production technologies.

As r increases, IPE is no longer feasible, because the impatient principal has incentive to lie when the output pair is (H, H). To see this, note that given the IPE contract, the right hand side of the Principal's IC constraint is $\frac{2}{q_1 - q_0}$ while the left hand side of the constraint is strictly decreasing in r. As the principal becomes less patient, she eventually has incentives to misreport the output pair as (L, L) when it is actually (H, H). The Principal's IC constraint starts binding at $r = \delta^C$. As r increases further, the gap between w_{HH} and w_{LL} must be decreased in order to prevent the principal from misreporting.

The principal has two methods of decreasing the gap between w_{HH} and w_{LL}. First, she can decrease w_{HH} and increase w_{HL}—a form of relative performance evaluation (RPE). Second, she can increase w_{LL} so that the contract rewards bad performance in the sense that both agents are rewarded even though they both produce bad outcomes, corresponding to solution BP^I in Proposition 20.

This second approach compromises the agents' incentive to *work* because increasing w_{LL} makes *shirk* more attractive to both agents; as a result, w_{HH} or w_{HL} needs to be increased even more to provide enough effort incentive to the agents. One may think that BP^I will never be preferred to *RPE*. In fact, the only reason that BP^I is optimal is that it is an efficient way of combatting agent-agent collusion on the *Cycling* strategy. *RPE* creates a strategic substitutability in the agents' payoffs that makes Cycling particularly appealing. *RPE* relies on w_{HL} to provide incentives, creating a strategic substitutability in the agents' payoffs. Under *RPE*, each agent's high effort level has a negative externality on the other agent's effort choice, making the *Cycling* collusion more difficult (more expensive) to break up than the *Joint Shirking* collusion strategy. Since collusion is more costly to prevent for small r, the principal purposely designs the contract to create strategically independent payoffs. This intuition and the tradeoff between the benefit and cost of *RPE* (increasing w_{HL}) relative to BP^I (increasing w_{HH} and w_{LL}) is illustrated with a numerical example in Figure 6.

In the example, $q_0 = 0.5$, $q_1 = 0.7$, and $H = 100$. The principal uses *IPE* if she is sufficiently patient ($r \leq 3.33$), while she has to chooses between *RPE* and BP^I for $r > 3.33$ (the origin of Figure 6 at which both solutions are equally costly to the principal). The solid line represents the cost of *RPE* (Solution iii) relative to BP^I (Solution ii), calculated as $q_1(1-q_1)(w_{HL}^{iii} - w_{HL}^{ii})$, where the superscripts *iii* and *ii* refer the corresponding solutions in Proposition 20. The dotted line measures the relative benefit of *RPE*, or equivalently the cost of increasing w_{LL} and w_{HH}

under BP^I, calculated as $q_1^2(w_{HH}^{ii} - w_{HH}^{ii}) + (1 - q_1)^2 w_{LL}^{ii}$. The intersection of two lines determines the critical discount rate $\tau^1 = 4.4$ used to choose between the two solutions. In this example, bonus pool type contract BP^I (pay without performance) emerges sooner than one might expect, since RPE is feasible for $r < 4.4$ and does not make payments for poor performance.
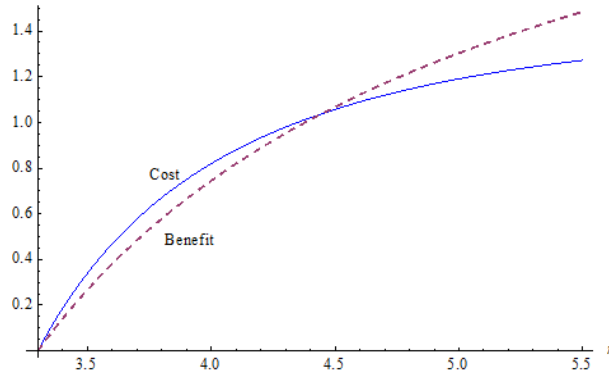

Figure 6: Cost and Benefit of RPE relative to BP^I

In general, BP^I emerges sooner than the RPE whenever the agents' credibility to collude is relatively stronger than the principal's credibility to honor her promises. We know from Proposition 20 that BP^I is optimal if and only if $\delta^C < \tau^1$, which is equivalent to restricting the high output of the project H is not too large, since δ^C increases in H while τ^1 is independent of H. Note that the principal's ability to commit to honoring the IPE contract is enhanced by a high value of H, since the punishment the agents can bring to bear on the principal is more severe. Therefore the region over which she can commit to the IPE contract becomes larger (δ^C becomes bigger) as H increases. For sufficiently large H, δ^C becomes so large that once the principal cannot commit to honoring the IPE ($r = \delta^C$),

it has already passed the point (τ^1) where the two agents can enforce their own collusion. Once the principal does not have to worry about the tacit collusion between the agents, she will instead use RPE and avoid the payment made to poor performance. Coming back to the same numerical example as in Figure 6, if one increases the value of H from 100 to 128.61, δ^C will exceed τ^1 and the region that used to be BP^I with $H = 100$ ($3.3 < r < 4.4$) now becomes IPE since the principal has a greater ability to commit with a higher H. Moreover, when the principal loses her commitment at $r = \delta^C = 4.4$, the agents are already so impatient that their *cycling* collusion is of no concern, and therefore the principal will offer RPE (instead of BP^I).

Solution BP^S emerges as r eventually becomes large enough (both parties become extremely impatient) and BP^S is similar to the static bonus pool. As pointed out in Levin (2003), "the variation in contingent payments is limited by the future gains from the relationship." The variation of wage payment is extremely limited under BP^S, because both parties are sufficiently impatient ($r > \max(\tau^1, \delta^F)$) and (thus) the future gains from the relationship are negligible. As a result, the principal has to set $w_{HL} = 2w_{HH}$ and also increase w_{LL} to make the contract self-enforcing. This coincides with the traditional view that bonus pools will eventually come into play because they are the only self-enforcing compensation form in such cases.

3.4. Cooperation and Mutual Monitoring

Agent-agent side contracting can also be beneficial to the principal: the fact that the agents observe each other's effort choice gives rise to the possibility that they could be motivated to mutually monitor each other to work as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001), as long as playing (*work, work*) Pareto-dominates all other possible action combinations (including *Cyclying*). Consider the following a trigger strategy used to enforce (*work, work*): both agents play work until one agent i deviates by choosing *shirk*; thereafter, agent j punishes i by choosing *shirk*:

(Mutual Monitoring)
$$\frac{1+r}{r}\left[\pi(1,1;w) - 1\right] \geq \pi(0,1;w) + \frac{1}{r}\pi(0,0;w)$$

Two conditions are needed for agents' mutual monitoring. First, each agent's expected payoff from playing (*work, work*) must be at least as high as from by playing the punishment strategy (*shirk, shirk*). In other words, (*work, work*) must Pareto dominate the punishment strategy from the agents' point of view in the stage game. Otherwise, (*shirk, shirk*) will not be a punishment at all:

(Pareto Dominance)
$$\pi(1,1;w) - 1 \geq \pi(0,0;w)$$

Second, the punishment (*shirk, shirk*) must be self-enforcing. The following constraint ensures (*shirk, shirk*) will be a stage game Nash equilibrium:

(Self-Enforcing Shirk) $\pi(0,0;w) \geq \pi(1,0;w) - 1$

The following linear program formalizes the principal's problem:

$$\min_{\{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}} \pi(1,1)$$

$s.t.$

Mutual Monitoring

Pareto Dominance

Self-Enforcing Shirk $(LP-2)$

No Cycling

Principal's IC

Non-negativity

Two points are worth noting. First, since (*shirk, shirk*) is now Pareto dominated by (*work, work*) from the agents' point of view, the collusion strategy *Joint Shirking* considered in the previous section can be dropped and the contract is collusion-proof as long as it satisfies the *No Cycling* constraint. Second, since the first three conditions sustain (*work, work*) in all periods as equilibrium play, the

trigger strategy considered in the *No Cycling* constraint, i.e. playing (*work, work*) indefinitely, is self-enforcing. Proposition 21 characterizes the optimal solution.

Proposition 21. *Depending on the value of r, the solution to LP-2 is (with* $w_{LH} = 0$ *in all cases):*

(i) JPE1: $w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$, $w_{HL} = w_{LL} = 0$ *for* $r \in (0, \delta^A]$;

(ii) BP1^C: $w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1-(1-q_1)r)H-(1+r)(q_1^2-1+r)}{(q_1-q_0)(q_0+q_1+(1-q_1)q_1 r-r^2)}$, $w_{HL} = 2w_{LL}$, $w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1 r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1+(1-q_1)q_1 r-r^2)}$ *for* $r \in (\delta^A, \min\{\tau^0, \delta^D\}]$;

(iii) JPE2: $w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0-(1-q_1)(1+r))H}{(q_1-q_0)((q_1-1)r(1+r)+q_0(q_1+r))}$, $w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_1+q_0+q_1 r)H}{(1-q_1)r(1+r)-q_0(q_1+r)}$, $w_{LL} = 0$ *for* $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$;

(iv) BP2^C: $w_{HH} = \frac{q_1-q_0+q_1 q_0-q_1^2+q_0 r-(q_1-q_0)^2(1-q_1)H}{(q_1-q_0)((q_1-1)r+q_0(q_1+r-1))}$, $w_{HL} = \frac{q_0-2q_1 q_0+q_1^2+r-2q_0 r-(q_1-q_0)^3 H}{(q_1-q_0)((1-q_1)r-q_0(q_1+r-1))}$, $w_{LL} = \frac{q_0(q_1-q_0)^2 H-q_0(q_1+r)}{(q_1-q_0)((1-q_1)r-q_0(q_1+r-1))}$ *for* $r \in (\max\{\tau^0, \delta^C\}, \delta^D]$;

where $\tau^0 = \frac{q_1+q_0-1}{(1-q_1)^2}$, δ^C *is same as in Proposition 20.* δ^A δ^D *are increasing functions of H and are specified in the Appendix C.*

It is easy to check that all the solutions create a strategy complementarity between the two agents' effort choice (denoted by the C superscript). When r is small, the optimal contract same as the "Joint Performance Evaluation" (*JPE1*) contract studied in Che and Yoo (2001), i.e., the agents are rewarded only if the outcomes from both agents are high. Starting from *JPE1*, as r increases, the principal has to increase w_{HH} because it becomes more difficult to motive the impatient agents to mutually monitor each other. However, the principal is also becoming less patient, and increasing w_{HH} will eventually become too expensive

for the principal to truthfully report the output (H, H). The principal will have to choose between *JPE2* and $BP1^C$ if she cannot credibly commit to honoring the *JPE1* contract. According to Proposition 21, $BP1^C$ follows the full commitment contract *JPE1* if and only if $\delta^A < \tau^0$. This condition is equivalent to H being not too large, since δ^A increases in H while τ^0 is independent of H. Using a similar argument to the one given just after Proposition 20, we know a higher value of H enhances the principal's ability to commit to the *JPE* solution. If H is large enough, δ^A will be so large that agent-agent side-contracting becomes fragile even before the principal's weakened commitment comes into play. Although increasing w_{HH} is most efficient in exploiting mutual monitoring between the agents, it comes at the cost that w_{LL} must also be increased. The principal uses $BP1^C$ only if the agents are still somewhat patient so that the efficiency of w_{HH} in exploiting mutual monitoring among the agents dominates its cost.

We illustrate the intuition for why and when the $BP1^C$ emerges sooner (relative to *JPE2*) using another numerical example: $q_0 = 0.5$, $q_1 = 0.9$, and $H = 100$. In this example, *JPE1* violates the Principal's IC constraint whenever $r > \delta^A = 14.12$ (the origin of Figure 7) and she has to either increase w_{HL} (*JPE2*) or increase w_{LL} so that she can increase w_{HH} ($BP1^C$). Similar to Figure 6, the solid line in Figure 7 represents the relative cost of *JPE2* compared to $BP1^C$ calculated as $q_1(1 - q_1)(w_{HL}^{iii} - w_{HL}^{ii})$ while the dotted line is the relative benefit calculated as $q_1^2(w_{HH}^{ii} - w_{HH}^{ii}) + (1 - q_1)^2 w_{LL}^{ii}$. One can see the principal chooses $BP1^C$ over *JPE2* if the two agents are somewhat patient ($r < \tau^0 = 40$) when it is efficient

to use w_{HH} to exploit mutual monitoring. If one sets $H \geq 279.61$ instead, δ^A will exceed τ^0 and the region that used to be $BP1^C$ ($14.12 < r < 40$) is now replaced by $JPE2$ as the principal's ability to commit is greater. Therefore when the principal loses her commitment, the agents are already so impatient that it is not worth increasing w_{HH} to induce mutual monitoring at the cost of increasing w_{LL}.
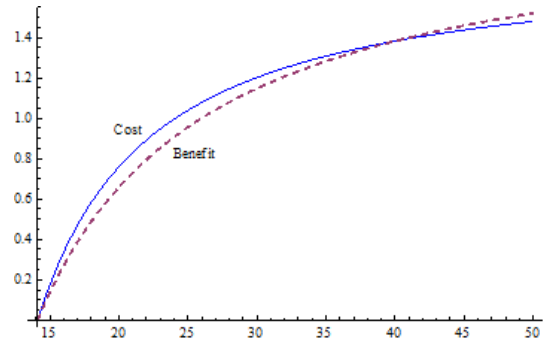

Figure 7: Cost and Benefit of $JPE2$ relative to $BP1^C$

A feature of Proposition 21 is that no feasible solution exists when r is large enough. There is a conflict between principal's desire to exploit the agents' mutual monitoring and her ability to commit to truthful reporting. Once r is sufficiently large, the intersection between the Mutual Monitoring constraint and the Principal's IC constraint is an empty set.

3.5. The Optimal Contract

In this section of the paper, we compare the solutions to $LP-1$ and $LP-2$ and characterize the overall optimal contract. The following proposition summarizes the result.

Proposition 22. *The principal prefers the solution to $LP-2$ (cooperation) to the solution from $LP-1$ (collusion) if and only if (i) $\tau^0 < \delta^C$ and $r < \delta^C$ or (ii) $\tau^0 \geq \delta^C$ and $r < \min\{\tau^0, \delta^D\}$.*

Not surprisingly, the key determinant of whether motivating cooperation is optimal is the discount rate. A longer expected contracting horizon gives the principal more flexibility in the promises she can offer and enables the agents to make credible promises to each other to motivate cooperation. As a result, there is a solution to $LP-1$ and a solution to $LP-2$ that are never optimal in the overall solution. Of the solutions to $LP-1$, IPE is never optimal overall. When the principal's ability to commit is strong enough that IPE is feasible, there is always a preferred solution involving cooperation under $LP-2$. Similarly, whenever the principal's and the agents' ability to commit is so limited that only BP2^C can be used to motivate cooperation, there is always a solution from $LP-1$ that dominates it.

The principal's and agents' relative credibility is also key, as the following numerical examples illustrate. In the first example, the agents' ability to commit is always limited relative to that of the principal, and collusion is never the driving determinant of the form of the optimal compensation arrangement. Consider the example: $q_0 = 0.53$, $q_1 = 0.75$, and $H = 200$, which corresponds to Case (i) in Proposition 22 ($\tau^0 < \delta^C$). When $LP-1$ is optimal, RPE is used immediately and BP^I is never optimal.
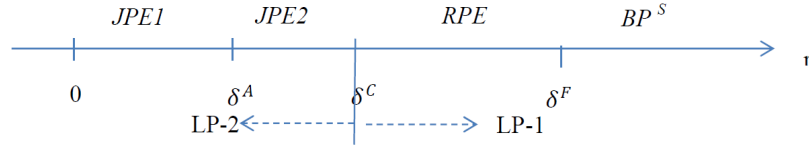
The second example also falls under Case (i) of Proposition 22: $q_0 = 0.47$, $q_1 = 0.72$, and $H = 100$. The principal's ability to commit is high relative to the agents', but the relative comparison is not as extreme. In this case, we move from BP^I to RPE, since the principal still has enough ability to commit to make RPE feasible after the discount rate is so large that the agents' collusion is not the key determinant of the form of the compensation contract (BP^I).
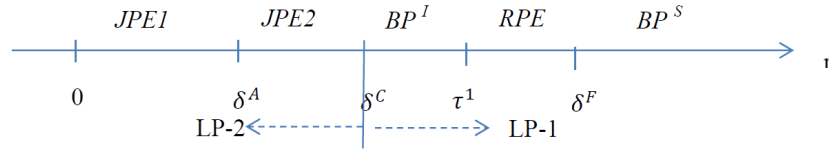
Next, consider a numerical example corresponding to Case (ii) in Proposition 22 ($\tau^0 \geq \delta^C$). $q_0 = 0.53$, $q_1 = 0.75$, and $H = 100$. In this case, the principal's ability to commit is low relative to the agents'. Once the discount rate is large enough that the agents' collusion is not the key determinant of the form of the compensation contract, the principal's ability to commit is also quite limited and BP^S is the only feasible solution. The principal's low relative credibility also leads to $BP1^C$ being optimal in this example when $JPE2$ was in the previous two examples.
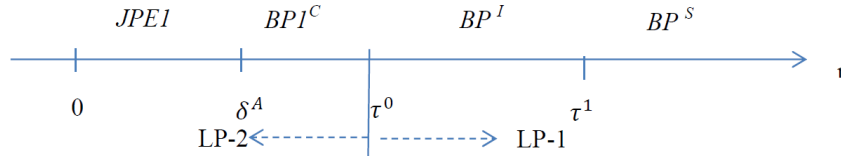
3.6. Incorporating an Objective Team-based Performance Measure

In a typical bonus pool arrangement, the size of the bonus pool is based, at least in part, on an objective team-based performance measure such as group or divisional earnings (Eccles and Crane, 1988). Suppose that such an objective measure y exists and define $p_1 = \Pr(y = H | e^A = e^B = 1)$, $p = \Pr(y = H | e^A = 1, e^B = 0) = \Pr(y = H | e^A = 0, e^B = 1)$, and $p_0 = \Pr(y = H | e^A = e^B = 0)$.

Consider the following numerical example: $q_0 = 0.47$, $q_1 = 0.72$, $p_0 = 0.1$, $p = 0.8$, $p_1 = 0.9$, $r = 5$, and $H = 27$. In this example, the team-based performance measure y exhibits a large production substitutability[8] and cooperation is not feasible. If only subjective measures are used in contracting, the optimal wage scheme is $w = (w_{LL}, w_{LH}, w_{HL}, w_{HH}) = (4.27, 0, 8.97, 4.70)$, or BP^I. Once objective measure is incorporated into the contract, use the first subscript on the wage payment to denote the realization of the objective measure. For example, w_{Hmn} is the payment made to agent i when y is H, x_i is m, and x_j is n; $m, n = L, H$. The optimal wage scheme is $w = (w_{LLL}, w_{LLH}, w_{LHL}, w_{LHH}, w_{HLL}, w_{HLH}, w_{HHL}, w_{HHH}) = (0, 0, 0, 0, 2.922, 0, 5.843, 3.675)$, which creates a strategic substitutability between the two agents' effort choice. There is a relatively small improvement in expected wages by introducing y: 9.19 without y and 5.96 with y. The objective measures are valuable because of their informativeness (Holmstrom, 1979) and because its verifiability nature compared to x, but the productive substitutability still precludes corporation from being optimal.

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[8]y exhibits a production substitutability if and only if $p_1 - p < p - p_0$.

Continue with the same example, except now assume $p = 0.2$, which exhibits a large production complementarity $(p_1 - p > p - p_0)$. In this case, productive complementarity in the objective team-based measure can make motivating cooperation among the agents optimal when it would not be in the absence of the objective measure. The optimal wage scheme is $w = (0, 0, 0, 0, 0.26, 0, 2.76, 1.38)$ and the expected total wages are 2.33, compared to 9.19 without incorporating the objective measure and 5.96 when the objective measure is incorporated but has a productive substitutability $(p = 0.8)$. The productive complementarity also takes pressure off of the individual subjective measures, allowing for a greater degree of relative performance evaluation than would otherwise be possible. The greater degree of substitutability in the subjectively determined wages reduces the amount of pay for bad performance (w_{HLL}) to 0.26 from 2.92 when the objective measure has a productive substitutability $(p = 0.8)$. In this example, agents' payment is higher if the team performance is good: $w_{Hmn} > w_{Lmn}, \forall m, n$, while each agent's reward is higher if the other agent's individual performance measure is poor: $w_{HmL} > w_{HmH}, \forall m$. Put differently, the combination of rewarding a team for good performance but also asking agents to compete with each other for a share of the total reward endogenously arises from a contract that motivates cooperation and mutual monitoring when the principal's commitment is limited. The example can be viewed as suggesting a new rationale for having employees whose actions are productive complements grouped into a single bonus pool.

3.7. Conclusion

A natural next step is to test some of the paper's empirical predictions. In particular, the model predicts that the form of the wage scheme will depend on (i) the expected contracting horizon, (ii) the relative ability of the principal and the agents to honor their promises, and (iii) the productive complementarity or substitutability of a team-based objective measure. A particularly strong prediction is that we should see bonus pool type incentive schemes that create strategic independence in the agents' payoffs in order to optimally prevent collusion. These particular bonus pool type arrangements should be observed when the agents' ability to collude is strong relative to the principal's ability to make credible promises and both are limited enough to be binding constraints. When the principal's credibility and the agents' credibility are both severely limited, we should instead observe bonus pool type arrangements that create a strategic substitutability in the agents' payoffs, since these allow for greater relative performance evaluation which is efficient absent collusion concerns. When instead the arrangement is used to motivate cooperation and mutual monitoring (which we suspect is more common), we should see productive and incentive arrangements that, when combined, create strategic complementarities.

CHAPTER 4

Conclusion

This section concludes with two extensions, reinforcing the idea that investigating strategic interactions of multiple information producers can cast light on observed accounting practices and institutions. Part one presents an extension of the model developed in Chapter 2, and the analysis highlights the possibility that regulations aimed at facilitating information acquisition can actually distort analysts' incentive to acquire information to the detriment of investors. Part two presents a new model as an attempt to rationalize the empirical evidence that financial analysts only release a subset of their information by showing that this practice is in investors' best interest. I show that there is a (endogenous) strategic complementarity between the two analysts' information acquisition and that the increase in the quantity of the information might be outweighed by the decrease in the quality of the information.

Lowering information acquisition cost can discourage information acquisition

The conventional wisdom is that, *caeteris paribus*, making it cheaper to acquire information leads to a higher level of information acquisition. While the conventional wisdom is true in a decision problem, it is not immediately clear once we consider strategic interactions between multiple information producers.

Consider the model developed in Chapter 2 (see Figure 1 in Chapter 2 for the time line). Let us now allow the independent analyst to defer his information acquisition from $t = 0$ to $t = 2$ when the recommendation issued at $t = 1$ is observed. For the purpose of highlighting the main point, I will fix the affiliated analyst's recommendation at $t = 1$. The following proposition summarizes the equilibrium of this modified game.

Proposition 23. *The unique Perfect Bayesian Equilibrium of the modified game is*[1]

(i) Independent Forecasting Equilibrium if the cost of waiting $\delta > \bar{\delta}$.

(ii) Conditional Herding Equilibrium if $\delta \in [\underline{\delta}, \bar{\delta}]$.

(iii) Random Revising Equilibrium if $\delta \leq \underline{\delta}$.

Proof. Similar to the proof of Proposition 1 in Chapter 2. □

Chapter 2 discusses all results except for the Random Revising Equilibrium. In this equilibrium, the independent analyst does not acquire information upfront.

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[1]The proposition assumes the affiliated analyst's conflict of interest $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. For $\alpha > \bar{\alpha}$ and $\alpha < \underline{\alpha}$, the game has trivial equilibria summarized in the Appendix A of Chapter 2. $\underline{\delta}$ and $\bar{\delta}$ are constant. As proved in Chapter 2, the equilibrium continues to be an equilibrium of the general game where both analysts can choose the timing of their recommendations.

He will not acquire any information if the affiliated analyst issued $r^A = \widehat{L}$ at $t = 1$; if $r^A = \widehat{H}$, he will randomizes between herding with r^A and acquiring his own information with precision $p^* = \frac{1+e}{2e}$. The independent analyst overturns $r^A = \widehat{H}$ whenever he observes a bad signal himself. The strategic interaction between the two analysts in this equilibrium resembles in many ways the interaction between managers and auditors discussed in the strategic auditing literature: while the affiliated analyst (manager) has the incentive to over report a bad signal, the independent analyst (auditor) has the technology to costly verify the reported good news.

An interesting finding is that lowering the information acquisition cost can discourage the independent analyst's ex-ante information acquisition and hurt investors.

Claim 24. *Lowering information acquisition cost decreases the investor's utility discontinuously whenever the equilibrium changes to the Random Revising Equilibrium in which the independent analyst does not acquire information upfront.*

In the Random Revising Equilibrium, the independent analyst has the incentive to save the ex-post information acquisition cost after observing the affiliated analyst's high recommendation. Such incentive jeopardizes the independent analyst's ability to discipline the affiliated analyst's biased behavior, and therefore is undesirable from the investor's perspective. If we consider the independent analyst's payoff in the Random Revising Equilibrium as his fallback position of not

acquiring information upfront, one can show that lowering information acquisition cost makes this fallback position more attractive. Therefore when the cost of waiting is small, lowering information acquisition cost can sometimes completely wipe out the independent analyst's incentive to acquire information upfront and hurt investors.

More information can be bad: tradeoffs between quantity and quality

It has been noticed that financial analysts only issue a subset of their information (e.g., Beyer et al., 2010). For example, analysts forecast earnings per share (EPS) for a company but normally do not issue the revenue forecast even though they use the latter in calculating EPS. One can argue this practice is preferred by financial analysts (the supply side) because releasing too many details has proprietary cost (such as the risk of revealing the core pricing technology analysts are using). I propose a model to show that this practice can be in the best interest of investors (the demand side). That is, even if the investor has the ability to force analysts to tell all the information they know truthfully, the investor will rationally choose to let analysts withhold some of their information.

Consider a model where two ex-ante identical analysts sell information to a continuum of risk-averse investors who subsequently involve in a speculative trading round in a noisy rational expectation equilibrium. The following time-line summarizes the sequence of actions.

t=0   t=1   t=2

1. Analysts acquire information;   Investor forms $\check{v}$ realized and
                                        her portfolio   investor consumes
2. Analysts set a price for signal(s);
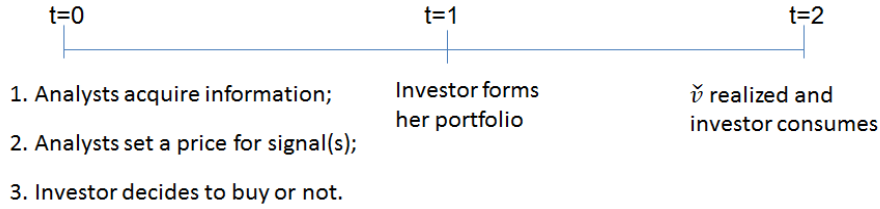
3. Investor decides to buy or not.

Figure 8: Time line

Admati and Pfleiderer (1986) study a similar setting where a monopolist sells information. While Admati and Pfleiderer (1986) focus on deriving the information monopolist's optimal selling strategy, I introduce multiple analysts, and the focus is the strategic interaction between the two analysts and how that affects their endogenous information acquisition.

The economy has one risk-free asset whose value is normalized to one and one risky asset whose value $\widetilde{v}$ is normally distributed $\widetilde{v} \sim N(v, 1)$. The economy has a continuum of identical investors. Each investor is endowed with W_0 units of risk-free asset and is characterized by the following utility function

$$U = -\exp(-\rho W_2)$$

where ρ is the risk aversion parameter and W_2 is the value of the investor's portfolio at $t = 2$. Noisy traders (or liquidity traders) provide the random supply of the per-capita risky asset $z \sim N(0, \sigma_Z^2)$.

Two ex-ante identical analysts acquire private signal about $\widetilde{v}$ and sell their information to all investors simultaneously at $t = 0$. Each analyst $i \in \{1, 2\}$

observes two signals (x_i, y_i) whose structure is as follows.

$$x_i = \widetilde{v} + \epsilon_i \ , \ y_i = \widetilde{v} + \delta_i$$

where

$$\epsilon_i \sim N(0, \sigma_\epsilon^2)$$

(4.1) $$\delta_i \sim N(0, \sigma_\delta^2(e_i)), e_i \in \{L, H\}$$

$$\epsilon_i, \epsilon_j, \delta_i, \delta_j \text{ independent}$$

The variance of the signal x_i is exogenous while the variance of y_i depends on the analyst's information acquisition effort $e_i \in \{L, H\}$. The analyst's personal cost of choosing $e_i = H$ (high information acquisition effort) is e, and the cost of $e_i = L$ is normalized to zero. The effort choice is publicly observed and choosing high information acquisition effort improves the precision (or lowers the variance) of the signal of y_i as follows

$$\sigma_{Good} = \sigma_\delta(e_i = H) < \sigma_\delta(e_i = L) = \sigma_{Bad}$$

One interpretation of this information structure is that skills required to observe signal x_i are mostly prerequisite for financial analysts, while acquiring a precise y_i requires additional work from the analyst. For example, one may think that x_i is the industry knowledge that analysts have to know before following a firm in that

industry, and y_i is the firm specific knowledge (such as its marketing strategy) that analysts need to devote additional effort to acquire.

Analysts wants to maximize the total revenue from selling their information to investors. I compare the two analysts' information acquisition and investors' payoffs in two regimes. The first regime is a *full disclosure regime* where each analyst sells both x_i and y_i to investors, and the second is a *partial disclosure regime* where each analyst only sells y_i (the one with endogenous variance) to investors. To focus on the effect of the number of signals sold on endogenous information acquisition, I assume that analysts truthfully sell his signals to the investors.[2] Denote S_i as the signal(s) sold by analyst $i \in \{1, 2\}$, then we have $S_i = \{x_i, y_i\}$ in the full disclosure regime and $S_i = \{y_i\}$ in the partial disclosure regime. Given forecasts are truthful, the analyst's objective is to find the maximum price he can charge for his signal(s). To break ties, I assume that investors will buy the analyst's information whenever indifferent.

For a given level of information acquisition, the subgame from $t = 1$ is a standard noisy rational expectation equilibrium setting (see Admati, 1985 for example). It is then a well known result that there exists a unique linear rational expectation equilibrium price function $\widetilde{p}$ for the risky asset:

$$\widetilde{p} = \alpha_0 + \alpha_1 \widetilde{v} - \alpha_2 \widetilde{z}$$

---

[2]This assumption is similar to the assumption in the voluntary disclosure literature that disclosure, once made, is truthful.

where $\alpha_0, \alpha_1, \alpha_2$ are constant.

Restricting to equilibrium with linear price function indicated above, one can show that there is a unique equilibrium for each pair of information acquisition. In equilibrium, all investors buy signals from both analysts and the price c_i charged by analyst $i \in \{1, 2\}$ is

$$
\begin{aligned}
c_1 &= \frac{1}{2\rho} \log \left[\frac{var(\widetilde{v}|\widetilde{p}, S_2)}{var(\widetilde{v}|\widetilde{p}, S_1, S_2)} \right] \\
c_2 &= \frac{1}{2\rho} \log \left[\frac{var(\widetilde{v}|\widetilde{p}, S_1)}{var(\widetilde{v}|\widetilde{p}, S_1, S_2)} \right]
\end{aligned}
$$
(4.2)

where $\widetilde{p}$ is the market price and S_i is the signal(s) sold by analyst $i \in \{1, 2\}$. Expression (4.2) is intuitive: the maximum price the analyst i can charge depends on how much uncertainty his information S_i can resolve *on top of* the uncertainty already resolved by the information contained in the market price $\widetilde{p}$ and the other analyst's signals S_j.

Knowing how their information is valued by the investor, the two analysts simultaneously choose how much information to acquire the beginning of the game. The numerical example below lists the two analysts' payoffs in a 2×2 matrix, and it highlights the trade-off between the quantity and the quality of analysts' information.

Numerical Example

$e = 0.1, \rho = 0.1, \sigma_Z = 7, \sigma_\varepsilon = 2, \sigma_{Good} = 1, \sigma_{Bad} = 2$

	H	L
H	0.37, 0.37	0.92, 0.24
L	0.24, 0.92	0.66, 0.66

Sell y only

	H	L
H	0.3, 0.3	0.65, 0.29
L	0.29, 0.65	0.66, 0.66

Sell both y and x

Compared to the partial disclosure regime (the payoff matrix on the left), the full disclosure regime (on the right) changes the informativeness of the market price in the way that generates strategic complementarity between two analysts' information acquisition. The consequence is that the game has two Pareto-ordered equilibria: both analysts acquiring low quality information (i.e., L, L) Pareto dominates the equilibrium where they both acquire high quality information from the analysts' perspective. One can check that the investor's expected equilibrium payoff is higher in the partial disclosure regime than in the full disclosure regime (assuming that analysts choose the Pareto dominant equilibrium between the two equilibria). In this example, investors are strictly better-off in the partial disclosure regime where analysts withhold some of their private information, as the increase in the quantity of the signal is outweighed by the decrease in the quality of the signal.

One limitation of this result is that analysts' information selling process is restricted to be truthful. This restriction makes the informativeness of market

price too high, which means that the value of analysts' information decays too fast. It seems to be natural that allowing analysts to add noisy to their signals before selling them to investors encourages analysts' information acquisition because the value of their information decays at a slower pace. Characterizing the analysts' optimal way of adding noise and verifying the conjecture that adding noise to signals benefit both analysts and investors are left for future research.

References

[1] Admati, A, 1985. A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica*, 53: 629-658.

[2] Admati, A., and Pfleiderer, P., 1986. A monopolistic market for information. *Journal of Economic Theory* 39: 400-438.

[3] Anthony, R. and V. Govindarajan, 1998. Management Control Systems, Mc-Graw Hill.

[4] Arya, A., J. Fellingham, and J. Glover, 1997. Teams, Repeated Tasks, and Implicit Incentives. *Journal of Accounting and Economics*, 23: 7-30.

[5] Arya, A., Glover, J., Mittendorf, B. and Zhang, L., 2004. The disciplining role of accounting in the long-run. *Review of Accounting Studies*, 9: 399-417.

[6] Arya, A. and B. Mittendorf, 2004. Using Job Rotation to Extract Employee Information. *Journal of Law, Economics, and Organization*, 20: 400-414.

[7] Arya, A. and Mittendorf, B., 2005. Using disclosure to influence herd behavior and alter competition. *Journal of Accounting and Economics*, 40: 231–246.

[8] Arya, A. and Mittendorf, B., 2007. The interaction among disclosure, competition between firms, and analyst following. *Journal of Accounting and Economics*, 43: 321–339.

[9] Baiman, S. and T. Baldenius, 2009. Nonfinancial Performance Measures as Coordination Devices. The Accounting Review, 84: 299-330.

[10] Baiman, S. and M. Rajan, 1995. The Informational Advantages of Discretionary Bonus Schemes. *The Accounting Review*, 70: 557-579.

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[11] Baker, G., R. Gibbons, and K. Murphy, 1994. Subjective Performance Measures in Optimal Incentive Contracts. *Quarterly Journal of Economics*, 109: 1125-1156.

[12] Baldenius, T. and J. Glover, 2012. Relational Contracts With and Between Agents. Working Paper, Carnegie Mellon University and New York University.

[13] Banerjee, A. V., 1992. A Simple Model of Herd Behavior. *Quarterly Journal of Economics*, 107: 797–817.

[14] Barber, B., Lehavy, R., and Trueman, B., 2007. Comparing the Stock Recommendation Performance of Investment Banks and Independent Research Firms. *Journal of Financial Economics*, 85: 490-517.

[15] Bebchuk, L. and J Fried, 2006. Pay Without Performance, Harvard University Press.

[16] Bikhchandani, S., Hirshleifer, D. and Welch, I., 1992. A Theory of Fads, Fashion, Custom, and cultural Change as Informational Cascades. *Journal of Political Economy*, 100: 992–1026.

[17] Bradshaw, T. M, Richardson, A. S., and Sloan, G. R., 2006. The relation between corporate financing activities, analysts' forecasts and stock returns. *Journal of Accounting and Economics*, 42: 53-85.

[18] Bulow, J. I., Geanakoplos, J. D. and Klemperer, P. D., 1985. Multimarket Oligopoly: Strategic Substitutes and Complements. *Journal of Political Economy*, 93: 488-511.

[19] Chapman, C., Healy, P. and Groysberg, B., 2008. Buy-Side vs. Sell-Side Analysts' Earnings Forecasts. *Financial Analysts Journal*, 64: 25-39.

[20] Che, Y. and S. Yoo, 2001. Optimal Incentives for Teams. *American Economic Review*, June: 525-541.

[21] Chen, T. and Martin, X., 2011. Do bank-affiliated analysts benefit from lending relationships? *Journal of Accounting Research*, 49: 633-674.

[22] Cho, I. and Kreps, D. M., 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102: 179-221.

[23] Cooper, J. W., 1999. Coordination games: Complementarities and macroeconomics. Cambridge University Press.

[24] Cooper, R. A., Day, T. E., and Lewis, C. M., 2001. Following the leader: A study of individual analysts' earnings forecasts. *Journal of Financial Economics*, 61: 383-416.

[25] Demski, J. and D. Sappington, 1984. Optimal Incentive Contracts with Multiple Agents. *Journal of Economic Theory*, 33: 152-171.

[26] Diamond, D. W. and P. H. Dybvig, 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91: 401-419.

[27] Dugar, A. and Nathan, S., 1995. The effect of investment banking relationships on financial analysts' earnings forecasts and investment recommendations. *Contemporary Accounting Research*, 12: 131–160.

[28] Dye, R. A., 1983. Communication and post-decision information. *Journal of Accounting Research*, 21: 514-533.

[29] Dye, R. A., 1985. Disclosure of nonproprietary information. *Journal of Accounting Research*, 23: 123-145.

[30] Eccles, R.G. and D.B. Crane, 1988. Doing Deals: Investment Banks at Work. Harvard Business School Press.

[31] Ederhof, M., 2010. Discretion in Bonus Plans. *The Accounting Review*, 85: 1921–1949.

[32] Ederhof, M., M. Rajan, and S. Reichelstein, 2011. Discretion in Managerial Bonus Pools. *Foundations and Trends in Accounting*, 54: 243-316.

[33] Fischer, P. E. and Stocken, P. C., 2010. Analyst information acquisition and communication. *The Accounting Review*, 85: 1985-2009.

[34] Fudenberg, D. and Tirole, J., 1991. Game Theory. The MIT Press.

[35] Gibbs, M., K. Merchant, W. Van der Stede, and M. Vargus, 2004. Determinants and Effects of Subjectivity in Incentives. T*he Accounting Review*, 79: 409-436.

[36] Groysberg, B., Healy, P. M. and Maber, D. A., 2011. What drives sell-side analyst compensation at high-status investment banks? *Journal of Accounting Research*, 49: 969-1000.

[37] Glover, J., 2012. Explicit and Implicit Incentives for Multiple Agents. *Foundations and Trends in Accounting*, 7: 1-71.

[38] Gu, Z. and Xue, J., 2008. The superiority and disciplining role of independent analysts. *Journal of Accounting and Economics*, 45: 289-316.

[39] Gul, F. and Lundholm,R., 1995. Endogenous timing and the clustering of agents' decisions. *Journal of Political Economy,* 103: 1039–1066.

[40] Guttman, I., 2010. The timing of analysts' earnings forecasts. *The Accounting Review*, 85: 513–545.

[41] Hirshleifer, D. and Teoh, S. H., 2003. Herding behavior and cascading in capital markets: a review and synthesis. *European Financial Management*, 9: 25-66.

[42] Holmstrom, B., 1979. Moral Hazard and Observability. *Bell Journal of Economics*, 10: 74-91.

[43] Hong, H. and Kubik, J., 2003. Analyzing the analysts: career concerns and biased earnings forecasts. *The Journal of Finance*, 58: 313–351.

[44] Irving, Ronald S., 2003. Integers, Polynomials, and Rings: A Course in Algebra. Springer.

[45] Jackson, A., 2005. Trade generation, reputation, and sell-side analysts. *The Journal of Finance*, 60: 673–717.

[46] Jacob, J., Rock, S. and Weber, D., 2008. Do non-investment bank analysts make better earnings forecasts? *Journal of Accounting, Auditing and Finance*, 23: 23–60.

[47] Kadan, O., Madureira, L., Wang, R. and Zach, T., 2009. Conflict of interest and stock recommendations: the effects of the global settlement and related regulations. *Review of Financial Studies*, 22: 4189-4217.

[48] Kvaloy, Ola and Trond E. Olsen, 2006. Team Incentives in Relational Employment Contracts. *Journal of Labor Economics*, 24: 139-169.

[49] Langberg, N. and Sivaramakrishnan, K., 2010. Voluntary disclosures and analyst feedback. *Journal of Accounting Research*, 48: 603-646.

[50] Levin, J., 2002. Multilateral Contracting and the Employment Relationship. *Quarterly Journal of Economics*, 117: 1075-1103.

[51] Levin, J., 2003. Relational Incentive Contracts. *American Economic Review*, 93: 835-847.

[52] Liang, P., 2000. Accounting recognition, moral hazard, and communication. *Contemporary Accounting Research*, 17: 457-490.

[53] Lin, H. and McNichols, M., 1998. Underwriting relationships, analysts' earnings forecasts and investment recommendations. *Journal of Accounting and Economics*, 25: 101–127.

[54] Loh, R. K., and Stulz, R. M., 2011. When are analyst recommendation changes influential? *Review of Financial Studies*, 24: 593-627.

[55] Michaely, D., and Womack, K., 1999. Conflict of interest and the credibility of underwriter analyst recommendations. *Review of Financial Studies*, 12: 653–686.

[56] Mikhail, B. M., Walther, B. R. and Willis, R. H., 1999. Does Forecast Accuracy Matter to Security Analysts?. *The Accounting Review*, 74: 185–200.

[57] Milgrom, P. and Roberts, J., 1990. Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities. *Econometrica*, 58: 1255-1277.

[58] Mittendorf, B. and Zhang, Y., 2005. The role of biased earnings guidance in creating a healthy tension between managers and analysts. *The Accounting Review*, 80: 1193–1209.

[59] Mola, S., and Guidolin, M., 2009. Affiliated mutual funds and analyst optimism. *Journal of Financial Economics*, 93: 108-137.

[60] Mookherjee, D., 1984. Optimal Incentive Schemes with Many Agents. *Review of Economic Studies*, 51: 433–446.

[61] Murphy, K., and P. Oyer, 2001. Discretion in Executive Incentive Contracts: Theory and Evidence. Working Paper, Stanford University.

[62] O'Brien, P., McNichols, M. and Lin, H., 2005. Analyst impartiality and investment banking relationships. *Journal of Accounting Research*, 43: 623–650.

[63] Rajan, M. and S. Reichelstein, 2006. Subjective Performance Indicators and Discretionary Bonus Pools. *Journal of Accounting Research*, 44: 525-541.

[64] Rajan, M. and S. Reichelstein, 2009. Objective versus Subjective Indicators of Managerial Performance. *The Accounting Review*, 84: 209-237.

[65] Scharfstein, D. S. and Stein, J. C., 1990. Herd behavior and Investment. *American Economic Review*, 80: 465-479.

[66] Schipper, K., 1991. Analysts' forecasts. *Accounting Horizon*, 5: 105-131.

[67] Tirole, J., 1986. Hierarchies and Bureaucracies: On the Role of Collusion in Organizations. *Journal of Law, Economics and Organization*, 2: 181-214.

[68] Tirole, J., 1992. Collusion and the theory of organizations. In J.-J. Laffont, ed., Advances in economic theory: Invited papers for the sixth world congress of the Econometric Society 2. Cambridge: Cambridge University Press.

[69] Topkis, D. M., 1978. Minimizing a submordular function on a lattice. *Operations Research*, 26: 305-321.

[70] Trueman, B., 1994. Analyst forecasts and herding behavior. *Review of Financial Studies*, 7: 97-124.

[71] Van Campenhout, G. and Verhestraeten, J. 2010. Herding behavior among financial analysts: a literature review. Working paper.

[72] Vives, X, 1990. Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, 19: 305-321.

[73] Vives, X, 2005. Complementarities and Games: New Developments. *Journal of Economic Literature*, 43: 437-479.

[74] Welch, I., 2000. Herding among security analysts. *Journal of Financial Economics*, 58: 369-396.

[75] Zhang, J., 1997. Strategic delay and the onset of investment cascades. *Rand Journal of Economics*, 28: 188-205.

Appendix

1. Appendix A

Equilibrium for $\alpha > \overline{\alpha}$ and $\alpha < \underline{\alpha}$

(i) For $\alpha > \overline{\alpha}$, the game has an equilibrium in which the affiliated analyst issues a fixed recommendation at $t = 1$, that is $r^A \equiv \widehat{L}$ or $r^A \equiv \widehat{H}$. The independent analyst chooses $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$, that is $r^I = \widehat{H} \Leftrightarrow y^A = g$. The investor bases her investment decision on r^I alone unless the affiliated analyst makes an out-of-equilibrium recommendation, in which case the investor does not buy.

(ii) For $\alpha < 2p^A - 1$, the game has an equilibrium in which the affiliated analyst truthfully reports his signal at $t = 1$, i.e., $r^A = \widehat{H} \Leftrightarrow y^A = g$. For $\delta \leq \alpha \leq \underline{\alpha} = 2p^A - 1 + \delta$, the game has an equilibrium in which the affiliated analyst perfectly signals his signal by the timing of his recommendation: he issues $\widehat{H}$ at $t = 2$ upon observing a good signal while otherwise he issues $\widehat{L}$ at $t = 1$. The independent analyst chooses precision $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$ if $\delta > p^A - (p^* - c(p^*))$ while otherwise he acquires no information and herds with r^A at $t = 2$. The investor bases her investment on r^A alone.

Details of the additional equilibria summarized in Lemma 8

For $\delta < \widehat{\delta}$, the affiliated analyst issues his recommendation at $t = 2$ while the independent analyst chooses precision $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$. Particularly,

(i) If $\alpha \leq \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$ and $\delta < \widehat{\delta}$, the affiliated analyst issues $\widehat{L}$ if and only if both $y^A = b$ and $r_1^I = \widehat{L}$, and the investor buys if and only if the affiliated analyst issues $\widehat{H}$ at $t = 2$.

(ii) If $\alpha > \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$ and $\delta < \widehat{\delta}$, the affiliated issues $r^A = \widehat{H}$ unless both his signal is bad and the independent analysts issues $\widehat{L}$, in which case the affiliated analyst issues $\widehat{H}$ with probability $\beta = \frac{p_H - p^*}{p_H + p^* - 1}$. The investor bases her investment decision on r^A unless $r^A = \widehat{H}$ but $r^I = \widehat{L}$, in which case she does not buy with probability $1 - \frac{1 - p^* - p_H}{\alpha(p_H + p^* - 1 - 2p_H p^*)}$.

In both cases, the investor will not buy if the affiliated analyst forecasts at $t = 1$ (the out-of-equilibrium path). Constants $\widehat{\delta} = p^* + p^* \alpha + p^A(\alpha - 2p^* \alpha - 1)$.

2. Appendix B

Notation: p^A (p) is the precision of the affiliated (independent) analyst's signal y^A (y^I); δ is the cost of deferring a recommendation, e is the information acquisition cost parameter, and α measures the affiliated analyst's degree of conflict of interest.

Proof of Lemma 2. Denote $\beta(p) \doteq \Pr(r^A = \widehat{H} | y^A = b, p)$ and $\gamma(p) \doteq \Pr(r^A = \widehat{L} | y^A = g, p)$, I will show that in equilibrium $\gamma(p) = 0$. The argument holds for all p and therefore I will write β and γ for simplicity. Denote $I_{\widehat{H}} = \Pr(\omega = H | r^A = \widehat{H})$ and $I_{\widehat{L}} = \Pr(\omega = L | r^A = \widehat{L})$ as the informativeness of $r^A = \widehat{H}$ and $r^A = \widehat{L}$

respectively. Notice $I_{\widehat{H}}, I_{\widehat{L}} \in [1 - p^A, p^A]$ are well-defined as $\alpha \leq \overline{\alpha}$ guarantees both $r^A = \widehat{L}$ and $r^A = \widehat{H}$ can be observed in equilibrium. It is an important observation that

$$(I_{\widehat{H}} - \frac{1}{2})(I_{\widehat{L}} - \frac{1}{2}) = \frac{(2p^A - 1)^2(\beta + \gamma - 1)^2}{4\left(1 - (\gamma - \beta)^2\right)} \geq 0$$

and that $I_{\widehat{H}} = \frac{1}{2} \Leftrightarrow I_{\widehat{L}} = \frac{1}{2} \Leftrightarrow \beta + \gamma = 1$.

First, I claim that $I_{\widehat{H}} = \frac{1}{2}$ (thus $I_{\widehat{L}} = \frac{1}{2}$) cannot hold in equilibrium. Suppose the opposite is true, then both $r^A = \widehat{H}$ and $r^A = \widehat{H}$ are ignored by the investor, which means that the affiliated analyst is strictly better off by forecasting truthfully, i.e., $r^A = \widehat{H}$ if and only if $y^A = g$. However the truthful reporting strategy implies $I_{\widehat{H}} = I_{\widehat{L}} = p^A$ and contradicts $I_{\widehat{H}} = I_{\widehat{L}} = \frac{1}{2}$. Since $I_{\widehat{H}} = \frac{1}{2}$ cannot be part of an equilibrium, we are left with two possible scenarios: $I_{\widehat{H}}, I_{\widehat{L}} < \frac{1}{2}$ or $I_{\widehat{H}}, I_{\widehat{L}} > \frac{1}{2}$.

Next, I claim that $I_{\widehat{H}}, I_{\widehat{L}} < \frac{1}{2}$ cannot hold in equilibrium. Suppose by contradiction that in equilibrium $I_{\widehat{L}} < \frac{1}{2}$ and $I_{\widehat{H}} < \frac{1}{2}$. Given $y^A = b$, the affiliated analyst's payoff is $p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{L}\right]$ if he forecasts $r^A = \widehat{L}$, and is $1 - p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{H}\right]$ if $r^A = \widehat{H}$. The expectation operator $E[\cdot|y^A, r^A]$ is taken over y^I, taking the independent analyst's strategy and the investor's strategy as given. The independent analyst's payoff function (7) guarantees that in equilibrium his strategy must satisfy $I_{\widehat{H}} < \frac{1}{2} \Rightarrow \Pr(r^I = \widehat{H}|y^I; Time) \leq \Pr(r^I = \widehat{H}|y^I; Time) \forall y^I$. The $Time$ parameter reflects that whether the independent analyst's information set contains r^A depends on the $timing$ of his recommendation (or waiting strategy), which is measurable only

with respect to y^I and cannot depend on r^A. In addition, the investor's pay-off function (8) guarantees that in equilibrium her strategy must be such that $I_{\widehat{H}} < \frac{1}{2} \Rightarrow \Pr(Buy|r^I, r^A = \widehat{H}) \leq \Pr(Buy|r^I, r^A = \widehat{L}) \forall r^I$. Given such properties of the independent analyst's strategy and the investor's strategy, we have $I_{\widehat{H}} < \frac{1}{2} \Rightarrow E\left[Buy|y^A = b, r^A = \widehat{H}\right] \leq E\left[Buy|y^A = b, r^A = \widehat{L}\right]$ in equilibrium, and

$$I_{\widehat{H}} < \frac{1}{2} \Rightarrow p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{L}\right] > 1 - p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{H}\right].$$

Therefore $I_{\widehat{H}} < \frac{1}{2}$ implies that forecasting $\widehat{L}$ must be a dominant strategy in equilibrium for the affiliated analyst if $y^A = b$. This implies $\beta(p) = 0$ for $\forall p$ and thus $I_{\widehat{L}} \geq \frac{1}{2}$, a contradiction to the assumption that $I_{\widehat{L}} < \frac{1}{2}$.

Finally we are left with $I_{\widehat{H}}, I_{\widehat{L}} > \frac{1}{2}$ and I claim that $\gamma(p) = 0$ in equilibrium. Following the similar argument developed above, one can show $I_{\widehat{H}} > \frac{1}{2} \Rightarrow E\left[Buy|y^A = g, r^A = \widehat{H}\right] \geq E\left[Buy|y^A = g, r^A = \widehat{L}\right]$ and

$$I_{\widehat{H}} > \frac{1}{2} \Rightarrow p^A + \alpha * E\left[Buy|y^A = g, r^A = \widehat{H}\right] > 1 - p^A + \alpha * E\left[Buy|y^A = g, r^A = \widehat{L}\right].$$

Therefore, in any equilibrium with $I_{\widehat{H}} > \frac{1}{2}$, $r^A = \widehat{H}$ is the affiliated analyst's strict best response upon observing $y^A = g$, and this proves the claim $\gamma(p) = 0$. $\qquad\square$

Proof of Lemma 3. The lemma is surely true if the independent analyst's private signal y^I and the affiliated analyst's recommendation r^A imply the same recommendation, so what is left is the case when y^I and r^A imply different recommendations. Suppose by contradiction that the independent analyst will stick to y^I if r^A implies differently, which means after all he forecasts independently in the sense that $r^I = \widehat{L}$ if and only if $y^I = b$. But there is a profitable deviation for the the independent analyst by simply forecasting independently at $t = 1$ and avoiding the waiting cost δ, a contradiction. □

Proof of Proposition 4. Consider the case in which both $r^A = \widehat{H}$ and $r^A = \widehat{L}$ are on the equilibrium path. After observing $y^I \in \{g, b\}$ with precision p, the independent analyst will obtain expected utility $p + \delta - c(p)$ if he forecasts immediately. On the other hand, if he defers his recommendation to $t = 2$ in equilibrium, we know by Lemma 3 that he will herd with r^A at $t = 2$. Let $E\left[Gain|y^I\right]$ be the expected informational gain from deferring a recommendation given y^I, we have

$$E\left[Gain|y^I = b\right] = p(1 - \beta)p^A + (1 - p)\left[p^A + (1 - p^A)\beta\right] - (p + \delta)$$

$$E\left[Gain|y^I = g\right] = p\left[p^A + (1 - p^A)\beta\right] + (1 - p)(1 - \beta)p^A - (p + \delta).$$

where $\beta = \Pr(r^A = \widehat{H}|y^A = b)$ is the probability that the affiliated analyst over-reports a bad signal, and we know from Lemma 2 that we can ignore the

under-reporting strategy as long as both $r^A = \widehat{H}$ and $r^A = \widehat{L}$ can be observed in equilibrium. It is easy to check that

$$E\left[Gain|y^I = g\right] - E\left[Gain|y^I = b\right] = (2p-1)\beta \geq 0.$$

If only $r^A = \widehat{H}$ (or $r^A = \widehat{L}$) is reported on the equilibrium path, then r^A is uninformative and the gain from observing it is zero for the independent analyst regardless of his signal y^I. $\qquad\qquad\square$

Proof of Lemma 5. First, I show that, in equilibrium, the independent analyst will not defer his recommendation upon observing $y^I = b$. Suppose by contradiction this is not the case. Then the independent analyst will also defer his recommendation upon observing $y^I = g$. The result is the independent analyst will unconditionally defer his recommendation, and (by Lemma 3) herd with the affiliated analyst's recommendation r^A. Knowing this, the affiliated analyst will forecast $r^A = \widehat{H}$ for all y^A, which makes his recommendation completely uninformative and contradicts the assumption that the independent analyst chooses to herd in the first place.

Upon observing $y^I = g$, the independent analyst will either forecast $\widehat{H}$ if he chooses to forecast at $t = 1$, or herd with r^A at $t = 2$ if he chooses to defer (by Lemma 3).

Given the independent analyst's forecasting strategy and $p > \frac{1}{2}$, the affiliated analyst sets $\beta = \Pr(r^A = \widehat{H}|y^A = b) = \frac{p^A - p}{p^A + p - 1}$ so that the investor is indifferent from "Buy" and "Not Buy" upon observing $r^A = \widehat{H}$ but $r^I = \widehat{L}$. The investor, upon observing $r^A = \widehat{H}$ but $r^I = \widehat{L}$, chooses not to buy with probability $\rho = \frac{\alpha - (2p^A - 1)}{\alpha(1 - p^A - p + 2p^A p)}$ so that the affiliated analyst is indifferent between reporting $\widehat{H}$ and $\widehat{L}$ upon observing a bad signal $y^A = b$. One can see that $0 \leq \rho \leq 1$ is guaranteed in the non-trivial region $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$.

Given $y^I = g$, the independent analyst obtains an expected payoff $p + \delta - c(p)$ if he forecasts early, while his expected payoff from waiting (and herding with r^A) is $p\left[p^A + (1 - p^A)\beta\right] + (1 - p)(1 - \beta)p^A - c(p)$. Substituting β from above, simple algebra shows that the independent analyst will forecast his good signal at $t = 1$ if and only if

$$\delta \geq \frac{(p^A - p)(2p - 1)}{p^A + p - 1}.$$

Collecting conditions completes the proof. □

Proof of Proposition 6. Denote p^* (p^{ch}) as the optimal precision the independent analyst chooses if the equilibrium of the overall game is the Independent Forecasting Equilibrium (and the Conditionally herd equilibrium). Then p^* satisfies

(.1) $$p^* = \arg\max_{p \in [\frac{1}{2}, 1]} p + \delta - e \times \left(p - \frac{1}{2}\right)^2,$$

which gives $p^* = \frac{1+e}{2e}$. p^{ch} satisfies

$$p^{ch} \in \arg\max_p \frac{1}{2}(p+\delta) + \frac{1}{2}\left[p(p^A + (1-p^A)\beta) + (1-p)(1-\beta)p^A\right] - e(p - \frac{1}{2})^2.$$

p^{ch} is solved from the following f.o.c

(.2)
$$e - 2e \times p^{ch} + \frac{(2p^A - 1)^2}{2(p^A + p^{ch} - 1)^2} = 0.$$

Since $p^A + p^{ch} - 1 > 0$, p^{ch} can be solved equivalently from the following cubic function

$$2(p^A + p^{ch} - 1)^2(e - 2e \times p^{ch}) + (2p^A - 1)^2 = 0.$$

This equation has a unique real root and two complex conjugate roots for any $p^A > \frac{1}{2}$ (Chapter 10 in Irving, 2003), and it is easy to check the second-order condition is $-2e - \frac{(2p^A - 1)^2}{(p^A + p - 1)^3} < 0$.

The independent analyst's expected payoff at $t = 0$, after plugging in p^* and p^{ch}, is denoted as U^I_{IF} in the Independent Forecasting Equilibrium and U^I_{CH} in the Conditional Herding Equilibrium. Simple algebra shows that

$$U^I_{CH} > U^I_{IF} \Leftrightarrow \delta < \Pi,$$

where
$$\Pi = \frac{4p^A p^{ch} - p^A - p^{ch}}{p^A + p^{ch} - 1} - \frac{e^2(1 - 2p^{ch})^2 + 2e + 1}{2e}.$$

Together with the self-fulfilling conditions characterized in Lemma 5, one can see the overall equilibrium is the Conditional Herding Equilibrium if δ is in the following set

$$\left\{ \delta : \delta < \min\left(\Pi \, , \frac{(p^A - p^{ch})(2p^{ch} - 1)}{p^A + p^{ch} - 1} \right) \right\} = \left\{ \delta : \delta < \Pi \right\},$$

where the equality is by straightforward algebra $\frac{(p^A - p^{ch})(2p^{ch} - 1)}{p^A + p^{ch} - 1} - \Pi = \frac{(2ep^{ch} - 1)^2}{2e} > 0$.

Analogously, the overall equilibrium is the Independent Forecasting Equilibrium if δ is in the following set

$$\left\{ \delta : \delta > \max\left(\Pi \, , \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1} \right) \right\},$$

and the remainder of the proof is to show $\Pi \geq \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1}$, which verifies the proposition.

Proving $\Pi \geq \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1}$ follows the graphic investigation of three claims. Claim 1: Both Π and $\delta^* \doteq \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1}$ strictly increase in e for $e < e^*$ while strictly decreases in e for $e > e^*$ (where $e^* = \frac{1}{(\sqrt{2} - 1)(2p^A - 1)}$). Claim 2: Π and δ^* achieve the *same* global maximum value at $e = e^*$, i.e., $\max \Pi = \Pi(e = e^*) = \delta^*(e = e^*) = \max \delta^*$. Claim 3: $\left| \frac{d\delta^*}{de} \right| > \left| \frac{d\Pi}{de} \right|$ holds for $e < e^*$ and $e > e^*$.

Proof of Claim 1: For the δ^* part, first notice that $\delta(p) = \frac{(p^A - p)(2p - 1)}{p^A + p - 1}$ is strictly concave in p and $\frac{d\delta}{dp} > 0$ if and only if $p < 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}$. As $p^* = \frac{1 + e}{2e}$ and $\frac{dp^*}{de} < 0$, we know $p^* < 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}$ if and only if $e > e^* = \frac{1}{(\sqrt{2} - 1)(2p^A - 1)}$.

Finally, since $\delta^* = \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1} = \delta(p = p^*)$, the claim for δ^* is true by applying the Chain Rule $\frac{d\delta^*}{de} = \frac{d\delta^*}{dp^*}\frac{dp^*}{de}$ and the fact that $\frac{dp^*}{de} < 0$.

For the Π part, rewrite Π as $\Pi(e, p^{ch}(e))$ to emphasize that its second argument p^{ch} is also a function of e. Differentiating $\Pi(e, p^{ch}(e))$ with respect to e,

$$\frac{d}{de}\Pi(e, p^{ch}(e)) = \frac{\partial\Pi(e, p^{ch}(e))}{\partial e} + \frac{\partial\Pi(e, p^{ch}(e))}{\partial p^{ch}}\frac{dp^{ch}}{de}.$$

Notice that

$$\frac{\partial\Pi(e, p^{ch}(e))}{\partial p^{ch}} = 2(e - 2ep^{ch}) + \frac{(2p^A - 1)^2}{(p^A + p^{ch} - 1)^2} = 0.$$

The last equality comes from the fact that p^{ch} is the optimal precision chosen in the conditional herding equilibrium, and by (.2) that p^{ch} satisfies

$$e - 2ep^{ch} + \frac{(2p^A - 1)^2}{2(p^A + p^{ch} - 1)^2} = 0.$$

Therefore, $\frac{d}{de}\Pi(e, p^{ch}(e))$ can be simplified as

$$\frac{d}{de}\Pi(e, p^{ch}(e)) = \frac{\partial\Pi(e, p^{ch}(e))}{\partial e}$$
$$= \frac{1}{2}\left(\frac{1}{e^2} - (2p^{ch} - 1)^2\right).$$

Straightforward calculus shows

$$\frac{d\Pi}{de} > 0 \Leftrightarrow p^{ch} < \frac{1 + e}{2e}.$$

We know from Proposition 10 that $p^{ch} < \frac{1+e}{2e} = p^*$ if and only if $e < e^*$. Monotonic transformation gives

$$\frac{d\Pi}{de} > 0 \Leftrightarrow e < e^*.$$

which verifies Claim 1.

Proof of Claim 2: As both Π and δ^* are continuous functions, the fact that they achieve their global maximum value at $e = e^*$ is a direct consequence of Claim 1. We know by Proposition 10 that $p^* = p^{ch} = 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}$ at $e = e^*$. Substituting p^* and p^{ch} verifies the claim.

Proof of Claim 3: We know from Claim 1 that $\frac{d\Pi}{de}$ can be simplified as $\frac{1}{2}(\frac{1}{e^2} - (2p^{ch} - 1)^2)$, and algebra shows that $\frac{d\delta^*}{de} = \frac{1 - e(2 + e(2p^A - 1)^2 - 4p^A)}{e^2(e - 1 - 2ep^A)^2}$. Simple algebra shows

$$
\begin{aligned}
dif &= \frac{d\Pi}{de} - \frac{d\delta^*}{de} \\
&= -2p^{ch^2} + 2p^{ch} - \frac{1}{2} + \frac{\frac{3}{2} + \frac{2}{(-1+e-2ep^A)^2} - \frac{4}{1-e+2ep^A}}{e^2}.
\end{aligned}
$$

Evaluating dif at $e = e^*$ we know $dif(e = e^*) = 0$, where the last equation uses the result from Proposition 10 that $p^{ch} = p^* = 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}$ at $e = e^*$. Since dif is quadratic in p^{ch} which is positive by definition, it is easy to verify that

$$dif \leq 0 \Leftrightarrow p^{ch} \geq 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}.$$

Applying the Implicit Function Theorem to the f.o.c defining p^{ch}, we have

$$\frac{dp^{ch}}{de} = \frac{1 - 2p^{ch}}{2e + \frac{(2p^A - 1)^2}{(p^A + p^{ch} - 1)^3}} < 0.$$

Monotonic transition gives

$$dif \leq 0 \Leftrightarrow e \leq e^*.$$

which, together with Claim 1, verifies the claim.

Finally $p^{ch} \in (\frac{1}{2}, p^A)$ is easy to show by combining the fact $\frac{dp^{ch}}{de} < 0$ and the results of Proposition 10. □

Proof of Lemma 7. One can verify the lemma by replicating the proof of Proposition 6 (players assigning probability one to $y^A = b$ on the out-of-equilibrium path supports the equilibrium). The remainder shows that the specified out-of-equilibrium belief satisfies both the Intuitive Criterion and the Universal Divinity Criterion.

For the purpose of equilibrium refinement, it is sufficient to check beliefs assigned to the affiliated analyst's out-of-equilibrium messages only.[1] As the affiliated analyst forecasts at $t = 1$ in equilibrium, the game has two out-of-equilibrium messages: $r^A = \widehat{H}$ at $t = 2$ and $r^A = \widehat{L}$ at $t = 2$, which are denoted as $r_2^A = \widehat{H}$ and $r_2^A = \widehat{L}$.

---

[1]Since the independent analyst's payoff does not depend on the investor's action, one can assign arbitrary beliefs to his out-of-equilibrium actions and those beliefs will survive both equilibrium refinement criteria used in the paper.

Universal Divinity Criterion: I illustrate the argument for the out-of-equilibrium message $r_2^A = \widehat{H}$ (referred as m for short), and the argument for $r_2^A = \widehat{L}$ is similar.

Some notation is necessary to apply the criterion. Let $BR(\mu, r^I, m)$ be the investor's *pure-strategy* best response to the out-of-equilibrium message m, given the belief μ over the affiliated analyst's type (his signal $y^A \in \{g, b\}$) and the independent analyst's recommendation r^I. Similarly let $MBR(\mu, r^I, m)$ be the set of *mixed-strategy* best response to m, given μ and r^I, that is the set of all probability distributions over $BR(\mu, r^I, m)$.

Then define $D(t, T, m)$ to be the set of the investor's mixed-strategy best responses to the out-of-equilibrium message m and beliefs concentrated on support of affiliated analyst's type space T that makes type $t \in \{g, b\}$ strictly prefer m to his equilibrium payoff.

$$(.3) \qquad D(t, T, m) = \bigcup_{\{\mu:\mu(T)=1\}} \{x \in MBR(\mu, r^I, m) \, s.t. u^*(t) < u(m, x, t)\}.$$

where $x \doteq \Pr(Buy|m)$ is the probability that the investor buys the stock upon observing the out-of-equilibrium message m, $u^*(t)$ is the type-t affiliated analyst's payoff on the equilibrium path, $u(m, x, t)$ is type-t's expected payoff by sending out m when the investor reacts to it with x, and $\mu = \Pr(y^A = g|m)$ is the investor's belief that the affiliated analyst is *good* type. Similarly let $D^0(t, T, m)$ be the set

of mixed best responses that make type t exactly indifferent. Finally, $D(t, T, m)$ and $D^0(t, T, m)$ are functions of r^I, which I will return to later.

Algebra shows that the *good* type affiliated analyst (with $y^A = g$) prefers the out-of-equilibrium message $r_2^A = \widehat{H}$ to his equilibrium action if

$$x > \frac{(2p^A - 1)\left[\alpha + e2p^A + e(2p^A + 2p(1 + \alpha - 2p^A) - \alpha)\right] + \delta}{(2p^A - 1 + e)\alpha} \doteq A$$

where $p \in \{p^*, p^{ch}\}$ is the independent analyst's precision choice in equilibrium. Likewise, the *bad* type affiliated analyst (with $y^A = b$) prefers sending out $r_1^A = \widehat{H}$ if $x > \frac{2p^A - 1 + \delta}{\alpha}$. Furthermore, the following is true under the maintained assumption $\alpha > \underline{\alpha}$ (see (2.11))

$$(.4) \qquad\qquad\qquad A > B.$$

Now calculate $MBR(\mu, r^I, m)$, the investor's *mixed-strategy* best response. Notice the investor's best response depends not only on her belief about the affiliated analyst's type, but also on the independent analyst's recommendation. Denote $\mu^*(r^I = \widehat{L})$ as the probability of $y^A = g$ such that the investor is indifferent about buying or not buying upon observing $r^I = \widehat{L}$; and similarly $\mu^*(r^I = \widehat{H})$ as the probability of $y^A = g$ such that the investor is indifferent about buying or not buying upon observing $r^I = \widehat{H}$. Clearly

$$\mu^*(r^I = \widehat{H}) < \mu^*(r^I = \widehat{L}).$$

The set of investor's mixed best response $MBR(\mu, r^I, m)$ is

$$
MBR(\mu, r^I, m) = \begin{cases}
x = 0 \text{ if } \mu < \mu^*(r^I = \widehat{H}) \\
x \in [0, 1] \text{ if } \mu = \mu^*(r^I = \widehat{H}) \text{ and } r^I = \widehat{H} \\
x = 0 \text{ if } \mu = \mu^*(r^I = \widehat{H}) \text{ and } r^I = \widehat{L} \\
x = 0 \text{ if } \mu^*(r^I = \widehat{H}) < \mu < \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{L} \\
x = 1 \text{ if } \mu^*(r^I = \widehat{H}) < \mu < \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{H} \\
x = 1 \text{ if } \mu = \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{H} \\
x \in [0, 1] \text{ if } \mu = \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{L} \\
x = 1 \text{ if } \mu > \mu^*(r^I = \widehat{L}).
\end{cases}
$$

Therefore

$$
\begin{aligned}
D(g, T, m) &= \underset{\{\mu : \mu(T) = 1\}}{\cup} \{x > A \cap MBR(\mu, r^I, m)\} \\
&= \begin{cases}
x > A \text{ if } r^I = \widehat{H} \\
x > A \text{ if } r^I = \widehat{L}
\end{cases} \\
&= x > A.
\end{aligned}
$$

To understand the second equality, note that set $MBR(\mu, r^I, m)$ inside the union operation depends on both μ and r^I while the union is taken only with respect to μ, and therefore the outcome is a function of r^I. The last equality shows that the set $D(good, T, m)$ degenerates to a deterministic set.

Similarly, one can show that $D(b, T, m)$ is as follows:

$$
\begin{aligned}
D(b, T, m) &= \underset{\{\mu : \mu(T) = 1\}}{\cup} \{x > B \cap MBR(\mu, m)\} \\
&= \begin{cases} x > B \text{ if } r^I = \widehat{H} \\ x > B \text{ if } r^I = \widehat{L} \end{cases} \\
&= x > B.
\end{aligned}
$$

As we know from (.4) $A > B$, we have

$$
(.5) \qquad\qquad D(good, T, m) \cup D^0(good, T, m) \subset D(bad, T, m).
$$

According to the Universal Divinity Criterion, (.5) means the equilibrium should assign probability zero to type $y^A = g$ upon observing the out-of-equilibrium message m, which is consistent with the strategy specified in Lemma 7.

Intuitive Criterion: On one hand, suppose players assign probability one to $y^A = g$ upon observing any of the two out-of-equilibrium messages $r_2^A = \widehat{H}$ and $r_2^A = \widehat{L}$. Given the proposed belief, it is easy to show that the affiliated analyst observing $y^A = b$ (*bad*-type) is strictly better off by sending out either of the two out-of-equilibrium messages than choosing his equilibrium action. This implies that neither of the out-of-equilibrium messages can be eliminated for the *bad*-type affiliated analyst by equilibrium dominance used in the Intuitive Criterion (Cho and Kreps, 1987 Page 199-202). On the other hand, even if any of the out-of-equilibrium messages can be eliminated for the *good*-type ($y^A = g$) affiliated

analyst, the *bad*-type affiliated analyst does not have incentive to send out that message and being identified. $\square$

Proof of Lemma 8. Recall that the details of the additional equilibrium are stated in Appendix A.

Part (i): $\alpha \leq \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$ ensures that it is a strict best response for the affiliated analyst to issue $\widehat{L}$ upon observing both $y^A = b$ and $r_1^I = \widehat{L}$. $\delta \leq \widehat{\delta}$ prevents the affiliated analyst from deviating the equilibrium by issuing recommendations at $t = 1$. It is then easy to verify the equilibrium.

Part (ii): $\beta = \frac{p^A - p^*}{p^A + p^* - 1}$ is chosen so that the investor is indifferent between "Buy" and "Not Buy" after observing $\{r^A = \widehat{H} \cap r^I = \widehat{L}\}$. Likewise, $\rho = 1 - \frac{1 - p^* - p^A}{\alpha(p^A + p^* - 1 - 2p^A p^*)}$ is chosen so that affiliated analyst is indifferent between issuing $\widehat{H}$ and $\widehat{L}$ when observing $\{y^A = b, r^I = \widehat{L}\}$, and $0 \leq \rho \leq 1$ requires $\alpha > \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$. Substituting β and ρ, one can show $\delta < \widehat{\delta}$ prevents the affiliated analyst from deviating the equilibrium by issuing recommendations at $t = 1$. $\square$

Proof of Lemma 9. Recall that $\beta = \frac{p^A - p}{p^A + p - 1}$ in both the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium. Simple algebra verifies the Lemma. $\square$

Proof of Proposition 10. From the Proof of Proposition 6 we know $p^* = \frac{1+e}{2e}$ while p^{ch} maximizes

$$U(p) = \frac{1}{2}p + \frac{1}{2}\left[p(p^A + (1-p^A)\beta) + (1-p)(1-\beta)p^A - \delta\right] - e(p - \frac{1}{2})^2$$

and $U'(p)|_{p=p^{ch}} = 0$. Also, it is easy to show

$$U''(p) = -2e - \frac{(2p^A - 1)^2}{(p^A + p - 1)^3} < 0.$$

Therefore, $U'(p)$ is strictly decreasing with respect to p. Evaluating $U'(p)$ at $p = p^*$, we obtain

$$U'(p)|_{p=p^*} = \frac{2(2ep^A - e)^2}{(2ep^A - e + 1)^2} - 1.$$

Algebra shows

$$U'(p)|_{p=p^*} > 0 = U'(p)|_{p=p^{ch}} \Leftrightarrow e > \frac{1}{(\sqrt{2}-1)(2p^A - 1)}.$$

Since $U''(p) < 0$, $U'(p)|_{p=p^*} > U'(p)|_{p=p^{ch}}$ implies $p^* < p^{ch}$. Therefore,

$$p^{ch} > p^* \Leftrightarrow e > \frac{1}{(\sqrt{2}-1)(2p^A - 1)}.$$

This completes the proof. □

Proof of Proposition 11. Denote p^{eq} as the precision acquired by the indepen-dent analyst in equilibrium, which is given in Proposition 6, one can show the

investor's equilibrium payoff is

$$U^{Inv}(p^{eq}) = \frac{(2p^A - 1)(2p^{eq} - 1)}{2(p^A + p^{eq} - 1)}$$

and

$$\frac{d}{dp^{eq}}U^{Inv}(p^{eq}) > 0.$$

This inequality, together with Proposition 10, completes the proof. $\square$

Proof of Corollary 12. Direct implication of Proposition 10 $\square$

Proof of Corollary 13. The dispersion of analysts' recommendation is the ex-ante percentage that two analysts' recommendation are different ($\widehat{H}$ versus $\widehat{L}$) among all the recommendations observed by the investor up to period t. In the Conditional Herding Equilibrium, one can show $Dispersion_1 = \frac{\Pr(r^A = \widehat{H}, y^I = b)}{\Pr(y^I = b)}$ and $Dispersion_2 = \Pr(r^A = \widehat{H}, y^I = b)$, where the subscript represents period t. It is clear that $Dispersion_1 > Dispersion_2$. $\square$

Proof of Corollary 14. In the text. $\square$

Proof of Lemma 15. Given the investor's endowment is e, holding y units of risky asset and x units of risk-free asset will generate wealth w

$$
\begin{aligned}
w &= (1+\omega)*m*y+x \\
&= (1+\omega)*m*y+e-m*y \\
&= \omega*m*y+e.
\end{aligned}
$$

The second equality makes uses of the budget constraint $e = m*y+x$. The optimal holding B is

$$
B = \arg\max_{y\geq 0} q_H * -e^{-\rho*(e+my)} + (1-q_H) - e^{-\rho*(e-my)},
$$

where $q_H = \Pr(\omega = H|r^A, r^I)$ is the posterior probability of $\omega = H$. Solving the program we obtain

$$
B = \begin{cases} \frac{\log(\frac{q_H}{1-q_H})}{2*\rho*m} & \text{if } q_H \geq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}
$$

Simple algebra verifies the lemma. $\qquad\square$

Proof of Lemma 16. The proof of Lemma 3 can be used to prove the first part of the lemma.

To show the second part of the lemma, let us first state a necessary condition for the *affiliated* analyst's strategy to be in equilibrium. Point-wise mappings $\beta(y^A, p) \doteq \Pr(r^A = \widehat{H}|y^A, p)$ and $\gamma(y^A, p) \doteq \Pr(r^A = \widehat{L}|y^A, p)$ for $\forall y^A, \forall p$ characterize the affiliated analyst's strategy, and I will write $\beta(y^A)$ and $\gamma(y^A)$ for short

as the argument below holds for all p. The informativeness of r^A is calculated as follows

$$
\begin{aligned}
I_{\widehat{H}} &= \Pr(\omega = H | r^A = \widehat{H}) = \frac{\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A}{\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A + \int_{-\infty}^{+\infty} \beta(y^A)\varphi_L dy^A} \\
I_{\widehat{L}} &= \Pr(\omega = L | r^A = \widehat{L}) = \frac{\int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A}{\int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A + \int_{-\infty}^{+\infty} \gamma(y^A)\varphi_H dy^A}.
\end{aligned}
$$

where φ_H and φ_L are the probability density function of y^A conditional on state $\omega = H$ and L. One can show that

$$
(I_{\widehat{H}} - \frac{1}{2})(I_{\widehat{L}} - \frac{1}{2}) \geq 0
$$

and $(I_{\widehat{H}} - \frac{1}{2})(I_{\widehat{L}} - \frac{1}{2}) = 0 \Leftrightarrow \int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A = \int_{-\infty}^{+\infty} \beta(y^A)\varphi_L dy^A \Leftrightarrow I_{\widehat{H}}, I_{\widehat{L}} = \frac{1}{2}$. Arguments developed in Lemma 2 can be used to show that (1) in equilibrium $I_{\widehat{H}}, I_{\widehat{L}} > \frac{1}{2}$, and (2) in equilibrium $r^A = \widehat{H}$ is the affiliated analyst's strict best response for any $y^A \geq 0$ and therefore

$$
(.6) \qquad\qquad \beta(y^A) = 1, \forall y^A \geq 0 \text{ in equilibrium.}
$$

Now turn to the independent analyst's expected payoff by deferring his forecast to $t = 2$ after observing a signal y^I with precision p, denoted as $U_{t2}^I(y^I, p)$. We

know that waiting implies subsequent herding in equilibrium. Therefore

$$U_{t2}^I(b,p) \;=\; p * \int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A + (1-p)\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A$$

$$U_{t2}^I(g,p) \;=\; p * \int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A + (1-p)\int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A,$$

and

$$
\begin{aligned}
U_{t2}^I(g,p) - U_{t2}^I(b,p) \;&=\; (2p-1)\left(\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A - \int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A\right)\\
&=\; (2p-1)\left(\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A + \int_{-\infty}^{+\infty} \beta(y^A)\varphi_L dy^A - 1\right)\\
&\geq\; (2p-1)\left(\int_{0}^{+\infty} \beta(y^A)\varphi_H dy^A + \int_{0}^{+\infty} \beta(y^A)\varphi_L dy^A - 1\right)\\
&=\; (2p-1)\left(\int_{0}^{+\infty} \varphi_H dy^A + \int_{0}^{+\infty} \varphi_L dy^A - 1\right)\\
&=\; 0.
\end{aligned}
$$

The inequality is by $\beta(y^A) \geq 0$, the second last equality is by $\beta(y^A) = 1, \forall y^A \geq 0$ (see (.6)), and the last equality uses the fact that $\int_0^{+\infty} \varphi_H dy^A + \int_0^{+\infty} \varphi_L dy^A = 1$ due to the symmetry of ω (i.e., $L = -H$). $\qquad \square$

Proof of Lemma 17. I claim that if (in equilibrium) the affiliated analyst chooses to forecast $\widehat{L}$ after observing his signal $y^A = a$, he will also forecast $\widehat{L}$ for any signal $y^A < a$. Similarly, if (in equilibrium) the affiliated analyst chooses to forecast $\widehat{H}$ after observing his signal $y^A = b$, then he will also forecast $\widehat{H}$ for any signal $y^A > b$.

Denote $U^A(r^A, y^A)$ as the affiliated analyst's expected utility when his private signal is y^A and he forecasts r^A. In particular,

$$U^A(r^A = \widehat{H}, y^A) = \Pr(\omega = H|y^A) + \alpha * E[shares(r^A = \widehat{H}, r^I(y^I; Time))|y^A]$$

$$U^A(r^A = \widehat{L}, y^A) = \Pr(\omega = L|y^A) + \alpha * E[shares(r^A = \widehat{L}, r^I(y^I; Time))|y^A].$$

where $shares(r^A, r^I)$ is the number of risky asset the investor buys after observing r^A and r^I and in equilibrium follows Lemma 15. The expectation operator $E[\cdot|y^A]$ is taken over y^I while taking the independent analyst's strategy $r^I(y^I; Time)$ as given. The $Time$ parameter in $r^I(y^I; Time)$ reflects that whether the independent analyst's information set contains r^A depends on the timing of his recommendation (or waiting strategy) which (for a given p) is measurable only with respect to y^I. Then define

$$\Delta_{\widehat{H} \succ \widehat{L}}(y^A) = U^A(r^A = \widehat{H}, y^A) - U^A(r^A = \widehat{L}, y^A).$$

Collecting terms, one can rewrite $\Delta_{\widehat{H} \succ \widehat{L}}(y^A)$ as

$$\Delta_{\widehat{H} \succ \widehat{L}}(y^A) = Benefit_{\widehat{H} \succ \widehat{L}}(y^A) - Cost_{\widehat{H} \succ \widehat{L}}(y^A),$$

in which

$$Benefit_{\widehat{H} \succ \widehat{L}}(y^A) = \alpha * E[shares(r^A = \widehat{H}, r^I(y^I; Time)) - shares(r^A = \widehat{L}, r^I(y^I; Time))|y^A]$$

and

$$Cost_{\widehat{H} \succ \widehat{L}}(y^A) = \Pr(\omega = L|y^A) - \Pr(\omega = H|y^A)$$

$$= \frac{\varphi_L}{\varphi_H + \varphi_L} - \frac{\varphi_H}{\varphi_H + \varphi_L},$$

where φ_H and φ_L are the probability density function of y^A conditional on state $\omega = H$ and L. From the Monotonic Likelihood Ration Property (MLRP), we know that $\frac{\varphi_H}{\varphi_L}$ increases in y^A and

$$\frac{d}{dy^A} Cost_{\widehat{H} \succ \widehat{L}}(y^A) < 0.$$

$Benefit_{\widehat{H} \succ \widehat{L}}(y^A)$ can be written as follows

$$\alpha \Pr(y^I = g|y^A) \left[shares(r^A = \widehat{H}, r^I(g; Time)) - shares(r^A = \widehat{L}, r^I(g; Time)) \right]$$

$$+\alpha \Pr(y^I = b|y^A) \left[shares(r^A = \widehat{H}, r^I(b; Time)) - shares(r^A = \widehat{L}, r^I(b; Time)) \right].$$

Notice that terms in both square brackets above are independent of the *realization* of y^A. Further, I claim that the following holds in equilibrium, regardless of the independent analyst's waiting strategy.

$$(.7) shares(r^A = \widehat{H}, r^I(g; Time)) - shares(r^A = \widehat{L}, r^I(g; Time))$$

$$\geq shares(r^A = \widehat{H}, r^I(b; Time)) - shares(r^A = \widehat{L}, r^I(b; Time)).$$

To see this, we know from Lemma 16 that the independent analyst's waiting strategy can only take three forms: (i) always wait for $\forall y^I$, (ii) wait if and only if $y^I = b$, and (iii) never wait for $\forall y^I$. Since waiting implies herding in equilibrium, it is easy to check the inequality above holds for case (i) and case (ii). In case (iii), the independent analyst forecasts independently and one can calculate in this case

$$\left[shares(\widehat{H}, r^I(g)) - shares(\widehat{L}, r^I(g))\right] - \left[shares(\widehat{H}, r^I(b)) - shares(\widehat{L}, r^I(b))\right]$$

$$= \log(\frac{p}{1-p}) \geq 0.$$

The equality makes uses of $shares(\widehat{L}, \widehat{L}) = 0$ and verifies (.7).

MLRP implies $\frac{d}{dy^A} \Pr(y^I = g|y^A) > 0$. Therefore we have

$$\frac{d}{dy^A} Benefit_{\widehat{H} \succ \widehat{L}}(y^A) > 0,$$

which makes uses $\frac{d}{dy^A} \Pr(y^I = b|y^A) = -\frac{d}{dy^A} \Pr(y^I = g|y^A)$.

Since $\Delta_{\widehat{H} \succ \widehat{L}}(y^A) = Benefit_{\widehat{H} \succ \widehat{L}}(y^A) - Cost_{\widehat{H} \succ \widehat{L}}(y^A)$, we have

$$\frac{d}{dy^A} \Delta_{\widehat{H} \succ \widehat{L}}(y^A) > 0.$$

Since $\Delta_{\widehat{H} \succ \widehat{L}}(y^A) > 0$ means $r^A = \widehat{H}$ while $\Delta_{\widehat{H} \succ \widehat{L}}(y^A) < 0$ means $r^A = \widehat{L}$, $\frac{d}{dy^A} \Delta_{\widehat{H} \succ \widehat{L}}(y^A) > 0$ verifies the claim. $\qquad \square$

Proof of Proposition 18. Direct implication of Lemmas 15, 16, and 17. $\qquad \square$

3. Appendix C

$$\delta^A = \frac{1}{2}\left[\frac{(q_0-q_1)^2 q_1 H - 1 - q_1^2 +}{\sqrt{((q_0-q_1)^2 q_1 H - 1 - q_1^2)^2 + 4(q_0-q_1)^2(q_0+q_1)H - 4q_1^2}}\right],$$

$$\delta^C = (q_0-q_1)^2 H - q_1,$$

$$\delta^D = (q_0-q_1)^2(q_0+q_1)H - q_0 - q_1^2,$$

$$\delta^F = \frac{1}{2}\left[\frac{(q_0-q_1)^2(2-q_1)H - q - 2q_1 + q_1^2 +}{\sqrt{((q_0-q_1)^2(2-q_1)H - q - 2q_1 + q_1^2)^2 + 4(q_1-2)q_1 - 8(q_0-q_1)^2(q_1-1)H}}\right],$$

$$\tau^0 = \frac{q_1 + q_0 - 1}{(1-q_1)^2},$$

$$\tau^1 = \frac{2q_1 - 1}{(1-q_1)^2}.$$

Note that $\delta^A, \delta^C, \delta^D,$ and δ^F are increasing in H, and we assume throughout the paper that H is larger enough to rank term by comparing the coefficient of the linear term of H. In particular, we obtain (i) $\delta^A < \delta^C < \delta^F$ and (ii) $\delta^C < \delta^D$ if and only if $q_0 + q_1 > 1$. The agent's effort is assumed to be valuable enough ($q_0 - q_1$ is not too small) and $(q_0 - q_1)^2 H \geqslant 1 + \frac{1}{q_1 - q_0}$.

Proof of Lemma 19. One can rewrite LP-1 as follows.

$$\min(1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

$s.t$

$$(1-q_1)w_{LL} + q_1 w_{LH} - (1-q_1)w_{HL} - q_1 w_{HH} \leqslant \frac{-1}{q_1 - q_0} \text{"Stage NE"}$$

$((2-q_1-q_0)+(1-q_0)r)w_{LL}+(q_1-1+q_0+q_0r)w_{LH}+((q_0-1)(1+r)+q_1)w_{HL}-$

$(rq_0+q_0+q_1)w_{HH} \leqslant \frac{-(1+r)}{q_1-q_0}$"No Joint Shirking"

$(1-q_1)(2+r)w_{LL}+(-1+q_1(2+r))w_{LH}+((q_1-1)(1+r)+q_1)w_{HL}-q_1(2+r)w_{HH} \leqslant$

$-\frac{1+r}{q_1-q_0}$"No Cycling"

$((1-q_1)^2-r)w_{LL}+(1-q_1)q_1w_{LH}+(1-q_1)q_1w_{HL}+(q_1^2+r)w_{HH} \leqslant (q_1-q_0)H$

$ICP_{HH \succ LL}$

$((1-q_1)^2-r)w_{LL}+(q_1-q_1^2+\frac{r}{2})w_{LH}+(q_1-q_1^2+\frac{r}{2})w_{HL}+q_1^2w_{HH} \leqslant (q_1-q_0)H$

$ICP_{HH \succ LL}$

$(1-q_1)^2w_{LL}+(q_1-q_1^2-\frac{r}{2})w_{LH}+(q_1-q_1^2-\frac{r}{2})w_{HL}+(q_1^2+r)w_{HH} \leqslant (q_1-q_0)H$

$ICP_{HH \succ HL}$

$(1-q_1)^2w_{LL}+(q_1-q_1^2+\frac{r}{2})w_{LH}+(q_1-q_1^2+\frac{r}{2})w_{HL}+(q_1^2-r)w_{HH} \leqslant (q_1-q_0)H$

$ICP_{HL \succ HH}$

$((1-q_1)^2+r)w_{LL}+(1-q_1)q_1w_{LH}+(1-q_1)q_1w_{HL}+(q_1^2-r)w_{HH} \leqslant (q_1-q_0)H$

$ICP_{LL \succ HH}$

$((1-q_1)^2+r)w_{LL}+(q_1-q_1^2-\frac{r}{2})w_{LH}+(q_1-q_1^2-\frac{r}{2})w_{HL}+q_1^2w_{HH} \leqslant (q_1-q_0)H$

$ICP_{LL \succ HH}$

$-w_{LL} \leqslant 0; \quad -w_{HL} \leqslant 0; \quad -w_{HH} \leqslant 0; \quad -w_{LH} \leqslant 0.$

$ICP_{mn \succ m'n'}$ is the family of IC constraints for the principal. $ICP_{mn \succ m'n'}$ ensures the principal prefers reporting outcomes mn truthfully rather than reporting $m'n'$ given the agents' threat to revert to the stage game equilibrium of Joint Shirking if the principal lies.

Suppose the optimal solution is $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Consider the solution $w' = \{w'_{HH}, w'_{HL}, w'_{LH}, w'_{LL}\}$, where $w'_{LH} = 0$, $w'_{HL} = w_{HL} + w_{LH}$, $w'_{LL} = w_{LL}$ and $w'_{HH} = w_{HH}$. It is easy to see that w and w' generate the same objective function value. We show below that the constructed w' satisfies all the $ICP_{mn \succ m'n'}$ constraints and further relaxes the rest of the constraints (compared to the original contract w). Since w_{LH} and w_{HL} have the same coefficient in all the $ICP_{mn \succ m'n'}$ constraints, w' satisfies these constraints as long as w does. Denote the coefficient on w_{LH} as C_{LH} and the coefficient on w_{HL} as C_{HL} for the "Stage NE", "No Joint Shirking," and "No Cycling" constraints. We can show that $C_{LH} - C_{HL} = r > 0$ holds for each of the three constraints. Given $C_{LH} > C_{HL}$, it is easy to show that w' will relax the three constraints compared to the solution w. $\qquad\square$

Proof of Proposition 20. By Lemma 19, one can rewrite LP-1 as follows.

$\min (1 - q_1)^2 w_{LL} + (1 - q_1)q_1 w_{HL} + q_1^2 w_{HH}$

$s.t$

$(1 - q_1)w_{LL} - (1 - q_1)w_{HL} - q_1 w_{HH} \leqslant \frac{-1}{q_1 - q_0}$"Stage NE"$(\lambda_1)$

$((2 - q_1 - q_0) + (1 - q_0)r)w_{LL} + ((q_0 - 1)(1 + r) + q_1)w_{HL} - (rq_0 + q_0 + q_1)w_{HH} \leqslant \frac{-(1+r)}{q_1 - q_0}$"No Joint Shirking"$(\lambda_2)$

$(1 - q_1)(2 + r)w_{LL} + ((q_1 - 1)(1 + r) + q_1)w_{HL} - q_1(2 + r)w_{HH} \leqslant -\frac{1+r}{q_1 - q_0}$"No Cycling"$(\lambda_3)$

$((1 - q_1)^2 - r)w_{LL} + (1 - q_1)q_1 w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HH \succ LL}(\lambda_4)$

$$((1-q_1)^2 - r)w_{LL} + (q_1 - q_1^2 + \tfrac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HL \succ LL}(\lambda_5)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 - \tfrac{r}{2})w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HH \succ HL}(\lambda_6)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 + \tfrac{r}{2})w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HL \succ HH}(\lambda_7)$$

$$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{LL \succ HH}(\lambda_8)$$

$$((1-q_1)^2 + r)w_{LL} + (q_1 - q_1^2 - \tfrac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{LL \succ HL}(\lambda_9)$$

$$-w_{LL} \leqslant 0(\lambda_{10}); \quad -w_{HL} \leqslant 0(\lambda_{11}); \quad -w_{HH} \leqslant 0(\lambda_{12}).$$

Denote the objective function of (LP-1) by $f(w)$, the left-hand side less the right-hand side of the ith constraints by $g_i(w)$, and the Lagrangian Multiplier of the ith constraint by λ_i, then the Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. The first-order-conditions (FOCs) of the Lagrangian with respect to the three wage (w_{LL}, w_{HL}, w_{HH}) are as follows:

$$(\text{FOC} - w_{LL}) \quad 1 + \lambda_1 - \lambda_{10} + 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - \lambda_1 q_1$$

$$- 2(1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1 + (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2$$

$$+ (\lambda_3 - \lambda_4 - \lambda_5 + \lambda_8 + \lambda_9 - \lambda_3 q_1)r - \lambda_2(-2 + q_0 + q_1 + (-1 + q_0)r) = 0;$$

$$(\text{FOC} - w_{HL}) \quad \lambda_1(-1 + q_1) + q_1 + q_1(2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$$

$$- (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1) + \frac{1}{2}(\lambda_5 - \lambda_6 + \lambda_7 - \lambda_9$$

$$+ 2\lambda_3(-1 + q_1))r + \lambda_2(-1 + q_0 + q_1 + (-1 + q_0)r) - \lambda_{11} - \lambda_3 = 0;$$

$(\text{FOC}-w_{HH})$ $\quad (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2 + (\lambda_4 + \lambda_6 - \lambda_7 - \lambda_8)r$

$$- \lambda_3 q_1(2 + r) - \lambda_2(q_0 + q_1 + q_0 r) - \lambda_{12} - \lambda_1 q_1 = 0$$

The optimal solution is one that (i) satisfies all 12 constraints, (ii) satisfies the three FOC above, (iii) satisfies the 12 complementary slackness conditions $\lambda_i g_i(w) = 0$, and (iv) all the Lagrangian multipliers are non-negative, i.e. $\lambda_i \geqslant 0$. For $r \leqslant \delta^c$, the solution listed below satisfies $(i) - (iv)$ and thus is optimal. Under this solution, denoted as IPE, the wage payments are derived by solving the following three binding constraints in (LP-1): Stage NE, No Joint Shirking, and w_{LL}. (No Cycling is also binding, and the Lagrangian multipliers under this solution are not unique due to the degeneracy of the problem. However finding one set of λ satisfying $(ii) - (iv)$ is enough to show the optimality.)

The IPE solution is:

$w_{LL} = 0$, $w_{HL} = w_{HH} = \frac{1}{q_1 - q_0}$

$\lambda_1 = q_1$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$,

$\lambda_5 = 0$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$,

$\lambda_9 = 0$, $\lambda_{10} = 1 - q_1$, $\lambda_{11} = 0$, $\lambda_{12} = 0$.

Under IPE, the $ICP_{HH \succ LL}$ constraint imposes the upper bound δ^c on r. The optimal solution changes when $r \geqslant \delta^c$. For $\delta^c < r < \tau^1$, the solution is listed below. This solution, denoted as BP^I, is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, No Joint Shirking, and No Cycling.

The BP^I solution is:

$$w_{LL} = \frac{(q_1-q_0)^2(1+r)H-(1+r)(q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))}, \ w_{HL} = w_{HH} + w_{LL},$$

$$w_{HH} = \frac{(q_1-q_0)^2H-(1+r)(-1+q_1+r)}{(q_1-q_0)(q-r(-1+q_1+r))};$$

$$\lambda_1 = 0, \ \lambda_2 = \frac{r(1+r+q_1^2r-2q_1(q+r))}{(q_1-q_0)(1+r)(-1+(-1+q_1)r+r^2)}, \ \lambda_3 = \frac{r(-1+q_0+q_1-r+q_0r-q_1^2r)}{(q_0-q_1)(1+r)(-q+(-1+q_1)r+r^2)}, \ \lambda_4 =$$

$$\frac{1+r-q_1r}{-1-(1-q_1)r+r^2},$$

$$\lambda_5 = 0, \ \lambda_6 = 0, \ \lambda_7 = 0, \ \lambda_8 = 0,$$

$$\lambda_9 = 0, \ \lambda_{10} = 0, \ \lambda_{11} = 0, \ \lambda_{12} = 0.$$

Under BP^I, both the non-negativity of w_{LL} and the Stage NE constraints require $r > \delta^c$ and $\lambda_2 \geqslant 0$ requires $r < \tau^1$. The optimal solution changes if $r > \tau^1$. For $\max\{\tau^1, \delta^c\} < r \leqslant \delta^F$, the optimal solution is listed below. The solution, denoted as RPE, is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, No Cycling, and w_{LL}.

The RPE solution is:

$$w_{LL} = 0, \ w_{HL} = \frac{(q_1-q_0)q_1(2+r)H-\frac{(1+r)(q_1^2+r)}{q_1-q_0}}{q_1^2-r(1+r)+q_1r(2+r)},$$

$$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(-1-r+q_1r(2+r))H}{(q_1-q_0)(q_1^2-r(1+r)+q_1r(2+r))},$$

$$\lambda_1 = 0, \ \lambda_2 = 0, \ \lambda_3 = \frac{(1-q_1)q_1r}{r(1+r)-q_1^2-q_1r(2+r)}, \ \lambda_4 = \frac{q_1^2}{r(1+r)-q_1^2-q_1r(2+r)},$$

$$\lambda_5 = 0, \ \lambda_6 = 0, \ \lambda_7 = 0, \ \lambda_8 = 0,$$

$$\lambda_9 = 0, \ \lambda_{10} = \frac{r(1+r+q_1^2r-2q_1(1+r))}{r(1+r)-q_1^2-q_1r(2+r)}, \ \lambda_{11} = 0, \ \lambda_{12} = 0.$$

Under RPE, the Stage NE constraint and $\lambda_{10} \geqslant 0$ yields two lower bounds δ^C and τ^1 on r. $ICP_{HL \succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leqslant \delta^F$. $w_{HH} \geqslant 0$ also requires $r > s \equiv \frac{2q_1-1+\sqrt{(2q_1-1)^2+4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, \delta^C, \tau^1\}, \delta^F] = (\max\{\delta^C, \tau^1\}, \delta^F]$. Consider the case where $s > \delta^C$ (as the

claim is trivial if instead $s \leqslant \delta^C$). Since δ^C increases in H while s is independent of H, one can show $s > \delta^C$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $\delta^F < \delta^C$ for $H < H^*$. Therefore $s > \delta^C$ implies $\delta^F < \delta^C$, in which case both $(\max\{s, \delta^C, \tau^1\}, \delta^F]$ and $(\max\{\delta^C, \tau^1\}, \delta^F]$ are empty sets. For $r > \max\{\delta^F, \tau^1\}$, the optimal solution is listed below and denoted as BP^S. Under BP^s, the optimal payment is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, $ICP_{HL \succ LL}$, and No Cycle.

The BP^S solution is:

$$w_{LL} = \frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}, w_{HL} = 2w_{HH},$$

$$w_{HH} = \frac{(1+r)(-(1-q_1)^2+r)-(q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))},$$

$$\lambda_1 = q_1, \lambda_2 = 0, \lambda_3 = \frac{r}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_4 = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2r-r^2-q_1(2+3r)},$$

$$\lambda_5 = \frac{2(1+r+q_1^2r-2q_1(1+r))}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 1 - q_1, \lambda_{11} = 0, \lambda_{12} = 0.$$

Where the two lower bound δ^F and τ^1 on r are derived from the non-negativity constraint of w_{LL} and λ_5. Collecting conditions verifies the proposition. $\square$

Proof of Proposition 21. The program can be written as follows.

$$\min(1 - q_1)^2 w_{LL} + (1 - q_1)q_1 w_{LH} + (1 - q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

$s.t$

$$(2 - q_0 - q_1)w_{LL} + (q_0 + q_1 - 1)w_{LH} + (q_0 + q_1 - 1)w_{HL} - (q_0 + q_1)w_{HH} \leqslant \frac{-1}{q_1 - q_0}$$

"Pareto Dominance" (λ_1)

$$(2 - q_0 - q_1 + r(1 - q_1))w_{LL} + (q_0 + q_1 + q_1r - 1)w_{LH} + (q_0 + (q_1 - 1)(1+r))w_{HL} -$$

$(q_0 + q_1 + q_1 r)w_{HH} \leqslant \frac{-(1+r)}{q_1 - q_0}$ "Mutual Monitoring" (λ_2)

$(1-q_1)(2+r)w_{LL}+(-1+q_1(2+r))w_{LH}+((q_1-1)(1+r)+q_1)w_{HL}-q_1(2+r)w_{HH} \leqslant$

$-\frac{1+r}{q_1-q_0}$"No Cycling"(λ_3)

$((1-q_1)^2 - r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{HH \succ LL}(\lambda_4)$

$((1-q_1)^2 - r)w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{LH} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{HL \succ LL}(\lambda_5)$

$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{LH} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{HH \succ HL}(\lambda_6)$

$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{LH} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{HL \succ HH}(\lambda_7)$

$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{LL \succ HH}(\lambda_8)$

$((1-q_1)^2 + r)w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{LH} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H$

$ICP_{LL \succ HL}(\lambda_9)$

$-w_{LL} \leqslant 0(\lambda_{10}); \quad -w_{HL} \leqslant 0(\lambda_{11}); \quad -w_{HH} \leqslant 0(\lambda_{12}); \quad -w_{LH} \leqslant 0(\lambda_{13});$

$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \leqslant \frac{1}{q_1-q_0}$ "Self-Enforcing Shirk"(λ_{14})

We first solve a relaxed program without the "Self-Enforcing Shirk" constraint and then verify that solutions of the relaxed program satisfy the "Self-Enforcing Shirk" constraint.

Without the "Self-Enforcing Shirk" constraint, the same argument used in Lemma 19 can be used to show that setting $w_{LH} = 0$ is optimal. Denote the

objective function of (LP-2) by $f(w)$ the left-hand side less the right-hand side of the ith constraints by $g_i(w)$, and the Lagrangian Multiplier of the ith constraint by λ_i. The Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. After setting $w_{LH} = 0$ and simplifying the problem, FOCs for the three wage payments (w_{LL}, w_{HL}, w_{HH}) are as follows:

(FOC-w_{LL})

$$1 - \lambda_{10} + 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - 2(1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1$$

$$+ (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2 - \lambda_1(q_0 + q_1 - 2) +$$

$$(\lambda_3 - \lambda_4 - \lambda_5 + \lambda_8 + \lambda_9 - \lambda_3 q_1)r - \lambda_2(-2 + q_0 + q_1 + (-1 + q_1)r) = 0$$

(FOC-w_{HL})

$$q_1 + \lambda_1(q_0 + q_1 - 1) + q_1(2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1)$$

$$+ \frac{1}{2}(\lambda_5 - \lambda_6 + \lambda_7 - \lambda_9 + 2\lambda_3(-1 + q_1))r + \lambda_2(q_0 + (-1 + q_1)(1 + r)) - \lambda_{11} - \lambda_3 = 0$$

(FOC-w_{HH})

$$q_1^2 + \lambda_5 q_1^2 + \lambda_9 q_1^2 - \lambda_1(q_0 + q_1) + \lambda_7(q_1^2 - r) + \lambda_8(q_1^2 - r) - \lambda_3 q_1(2 + r) +$$

$$\lambda_4(q_1^2 + r) + \lambda_6(q_1^2 + r) - \lambda_2(q_0 + q_1 + q_1 r) - \lambda_{12} = 0$$

Again, the optimal solution is one that (i) satisfies all constraints in LP-2, (ii) satisfies the three FOCs above, (iii) satisfies the complementary slackness conditions $\lambda_i g_i(w) = 0$, and (iv) has non-negative Lagrangian multipliers. For $r < \delta^A$, the solution listed below satisfies $(i) - (iv)$ and thus is optimal. This solution, denoted as $JPE1$, is obtained by solving the three binding constraints: Mutual Monitoring, w_{HL}, and w_{LL}.

The $JPE1$ solution is:

$w_{LL} = 0, w_{HL} = 0, w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$;

$\lambda_1 = q_1, \lambda_2 = \frac{q_1^2}{q_0+q_1+q_1 r}, \lambda_3 = 0, \lambda_4 = 0,$

$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$

$\lambda_9 = 0, \lambda_{10} = \frac{q_0-2q_0 q_1+q_1(1+r-q_1 r)}{q_0+q_1+q_1 r}, \lambda_{11} = \frac{q_0 q_1}{q_0+q_1+q_1 r}, \lambda_{12} = 0.$

Under $JPE1$, the $ICP_{HH \succ LL}$ constraint yields the upper bound on r. The optimal solution changes when $r > \delta^A$. For $\delta^A < r \leqslant \min(\tau^0, \delta^D)$, the solution listed below satisfies $(i) - (iv)$ and becomes optimal. This solution, denoted as BPI^C, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{HH \succ LL}$, and $ICP_{HH \succ HL}$.

The $BP1^C$ solution is:

$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1 r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1 r-r^2)}, w_{HL} = 2 * w_{LL},$

$w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1+(-1+q_1)r)H-(1+r)(-1+q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1 r-r^2)};$

$\lambda_1 = 0, \lambda_2 = \frac{-r}{q_0+q_1+q_1 r-q_1^2 r-r^2}, \lambda_3 = 0, \lambda_4 = \frac{q_0-(q_1-2)((q_1-q)r-1)}{q_0+q_1+q_1 r-q_1^2 r-r^2},$

$\lambda_5 = 0, \lambda_6 = \frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1 r-q_1^2 r-r^2}, \lambda_7 = 0, \lambda_8 = 0,$

$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0$.

Under $BP1^C$, the non-negativity of w_{LL} and w_{HL} requires $r > \delta^A$, while the Pareto Dominant constraint and $\lambda_6 \geqslant 0$ impose upper bounds δ^D and τ^0 respectively. If $r > \tau^0$, the solution changes because otherwise $\lambda_6 < 0$. For $\min\{\tau^0, \delta^A\} < r \leqslant \delta^C$, the solution is listed below. This solution, denoted as $JPE2$, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{HH \succ LL}$, and w_{LL}.

The $JPE2$ solution is:

$$w_{LL} = 0, w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_0+q_1+q_1 r)H}{(1-q_1)r(1+r)-q_0(q_1+r)},$$

$$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0+(-1+q_1)(q+r))H}{(q_1-q_0)((-1+q_1)r(1+r)+q_0(q_1+r))};$$

$$\lambda_1 = 0, \lambda_2 = \frac{(q_1-1)q_1 r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_3 = 0, \lambda_4 = \frac{-q_0 q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = \frac{r(q_0-(q_1-1)((q_1-q)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under $JPE2$, the non-negativity of w_{HL} requires $r > \delta^A$ and $\lambda_{10} \geqslant 0$ yields another lower bound τ^0 on r. The non-negativity of w_{HH} and w_{HL} also requires $r > s' \equiv \frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)}$. In addition, both the Pareto Dominance and No Cycle constraints require $r < \delta^C$. We claim $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C]$. The claim is trivial if $s' \leqslant \delta^A$ and therefore consider the case where $s' > \delta^A$. Since δ^A increases in H while s' is independent of H, one can show $s' > \delta^A$ is equivalent to $H < H'$ for a unique positive H'. Meanwhile, algebra shows that $\delta^c < \delta^A$ for $H < H'$. Therefore $s' > \delta^A$ implies $\delta^c < \delta^A$, in which case $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C] = \varnothing$. For $\max\{\tau^0, \delta^C\} < r \leqslant \delta^D$ and $q_1 + q_0 \geqslant 1$, the optimal solution is as follows. This solution, denoted as $BP2^C$,

is obtained by solving the three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$.

The $BP2^C$ solution is:

$$w_{LL} = \frac{q_0(q_1-q_0)^2 H - q_0(q_1+r)}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))}, \quad w_{HL} = \frac{(q_1-q_0)^2 + r - 2q_0 r - (q_1-q_0)^3 H}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))},$$

$$w_{HH} = \frac{(q_0+q_1)(q_1-1) + q_0 r + (q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))};$$

$$\lambda_1 = \frac{-q_0 + (-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r + q_0(-1+q_1+r)}, \quad \lambda_2 = \frac{q_0+q_1-1}{(q_1-1)r + q_0(-1+q_1+r)}, \quad \lambda_3 = 0, \quad \lambda_4 = \frac{q_0(1-q_1)}{(q_1-1)r + q_0(-1+q_1+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0$$

Under $BP2^C$, the non-negativity of λ_1 requires $q_1 + q_0 \geqslant 1$. Given $q_1 + q_0 \geqslant 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > \delta^C$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0+q_1-1}$. The other lower bound τ^0 on r is generated by intersecting requirements for $\lambda \geqslant 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leqslant \delta^D$. We claim $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\tau^0, \delta^C\}, \delta^D]$. Subtracting q_1 from both sides of $\delta^C \leqslant \delta^D$ and collecting terms, one obtains $s'' \leqslant \delta^C$ which means $\delta^C \leqslant \delta^D$ if and only if $s'' \leqslant \delta^C$. Therefore $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\tau^0, \delta^C\}, \delta^D]$ is verified. As r becomes even larger, the problem (LP-2) becomes infeasible because the intersection of the Mutual Monitoring constraint and the Pricipal's IC constraint(s) is an empty set.

Finally, tedious algebra verifies that the solutions characterized above satisfy the "Self-Enforcing Shirk" constraint that we left out in solving the problem. Therefore adding this constraint back does not affect the optimal objective value. □

Proof of Proposition 22. The proposition is proved by showing a sequence of claims.

Claim 1: LP-1 is optimal for $r > \max\{\delta^C, \delta^D\}$.

Claim2: $BP2^C$ of LP-2 is never the overall optimal contract.

Claim 3: JPE^1 of LP2, if feasible, is the overall optimal contract.

Claim 4: JPE^2 of LP2, if feasible, is the overall optimal contract.

Claim 5: $BP1^C$ of LP2, if feasible, is the overall optimal contract.

Claim 6: $\min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$.

Using Claims 1 - 5, one can verify the following statement: when $\min\{\tau^0, \delta^D\} \leqslant \delta^C$, LP-2 is optimal if and only if $r < \delta^C$; otherwise for $\min\{\tau^0, \delta^D\} > \delta^C$, LP-2 is optimal if and only if $r < \min\{\tau^0, \delta^D\}$. Claim 6 shows that condition $\min\{\tau^0, \delta^D\} > \delta^C$ is equivalent to $\tau^0 > \delta^C$ and, thus, is equivalent to the statement in the proposition.

Proof of Claim 1: The claim is trivial as we know from Proposition 2 that LP-2 does not have feasible solution on the region.

Proof of Claim 2: Recall that $BP2^C$ of LP-2 is obtained by solving the following three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$. It is easy to see that $\pi(0, 0; w) = \pi(0, 1; w)$ when both Mutual Monitoring constraint and the Pareto Dominance constraint are binding, in which case the Mutual Monitoring constraint can be re-written as follows:

$$\frac{1+r}{r}[\pi(1, 1; w) - 1] \geqslant \pi(0, 1; w) + \frac{1}{r}\pi(0, 1; w)$$

Note this is same as the "Stage NE" constraint in LP-1 and therefore all the constraints in (LP-1) are implied by those in (LP-2) under the $BP2^C$ solution. In this case, (LP-2) has a smaller feasible set, so it can never do strictly better than (LP-1).

Proof of Claim 3: We know from Proposition 2 that $JPE1$ is the optimal solution of LP2 for $r \in (0, \delta^A]$, over which the optimal solution of LP1 is IPE (Proposition 1). Substituting the corresponding solution into the principal's objective function, we obtain $obj_{JPE1} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1 r)}$ and $obj_{JPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{JPE} - obj_{JPE1} = \frac{q_0 q_1}{(q_1-q_0)(q_0+q_1+q_1 r)} > 0$, which verifies the claim.

Proof of Claim 4: $JPE2$ is the solution of LP2 for $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$, over which IPE is the corresponding solution of LP1. Algebra shows that $obj_{JPE2} = \frac{(q_1-1)q_1 r(1+r)+q_0(q_0-q_1)^2 q_1 H}{(q_0-q_1)((q_1-1)r(1+r)+q_0(q_1+r))}$, $obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE2} - obj_{IPE} \leqslant 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)} \leqslant r \leqslant \delta^C$ (with equality on the boundary). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)}, \tau^0, \delta^A\} \leqslant r \leqslant \delta^C$, which was shown in the proof of Proposition 2 to be equivalent to $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$. Therefore, $JPE2$ is the overall optimal contract whenever it is feasible.

Proof of Claim 5: We know that $BP1^C$ is the solution of LP2 if $r \in (\delta^A, \min\{\tau^0, \delta^D\}]$. In this region, IPE and BP^I are potential solutions in LP1 because the other two solutions (RPE and BP^S) require $r \geqslant \tau^1 > \tau^0$. Let us compare first $BP1^C$ of LP2 and BP^I of LP1. It is easy to show $obj_{BP1C} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1 r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1 r-r^2)}$ and $obj_{BPI} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_0-q_1)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BP1C} < obj_{BPI}$ for $\delta^C < r \leqslant \min\{\tau^0, \delta^D\}$ where both solutions are feasible.

Showing $BP1^C$ is always more cost efficient than the IPE solution is more involved and is presented in two steps. We first derive the sufficient condition for this to be true and then show that the sufficient condition holds whenever both solutions are optimal in their corresponding program, namely $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Given obj_{BP1^C} and obj_{IPE} defined above, one can show that $obj_{BP1^C} < obj_{IPE} \Leftrightarrow r < \delta$, where

$$\delta = \frac{1}{2(1-q_1)}[((q_1 - q_0)^2 H - q_1)q_1(1 - q_1) - 1 +$$

$$\sqrt{(((q_1 - q_0)^2 H - q_1)q_1(1 - q_1) - 1)^2 + 4(1 - q_1)((q_1 - q_0)^2 H - q_1)(q_1 + q_0)}]$$

Note that if $\delta \geqslant \delta^C$, then $r < \delta$ (thus $obj_{BP1^C} < obj_{IPE}$) is satisfied trivially for $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Consider the opposite case in which $\delta < \delta^C$. For $q_0 \in [0, q_1)$, one can show that $\delta < \delta^C$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. Since the latter condition contradicts the maintained assumption that $(q_1 - q_0)^2 H > q_1$, we consider $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ only. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$, one can show $\delta > \tau^0$ for any $q_0 \in [0, q_1)$. Therefore, under the maintained assumption $(q_1 - q_0)^2 H > q_1$, $\delta < \delta^C$ implies $\tau^0 < \delta$. If the choice is between $BP1^C$ and IPE, $r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Then $\tau^0 < \delta$ implies $r < \delta$. $r < \delta$ implies $obj_{BP1^C} < obj_{IPE}$ whenever both are feasible (which is in the region $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$).

Proof of Claim 6: The "only if" direction is trivial. To show the "if" direction,

note that if $\tau^0 > \delta^C$, we know $q_1 + q_0 > 1$ as otherwise $\tau^0 < 0 < \delta^C$. Under the maintained assumption on H, $q_1 + q_0 > 1$ implies $\delta^D > \delta^C$. Therefore, $\min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$. $\qquad\square$