Marketing Implications of Shared Information Goods

by

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CHAPTER I

Introduction

Information goods are non-rivalrous, implying that multiple individuals or firms can access them without depriving others of such access. This property of information goods is different from physical goods and forms a very important component of our analyses both from an individual perspective as well as a from a strategic and competitive perspective. This characteristic feature of information goods allows them to be shared and for such sharing or shared situations to enhance both the availability of such goods as well as consequent consumer welfare or firm profitability. In this dissertation I examine the dynamic mechanisms of consumer and firm behavior, when faced with shared information goods. The first two essays examine sharing between consumers, which is mediated by social network relationships, involving both influence and strategic intent. The third essay examines sharing by firms in an imperfectly competitive marketplace.

My first essay examines why consumers (or users) make contributions that are necessary to keep social networks thriving. Several users post new user provided or 'user generated' content (photographs, videos, status messages) on online networking sites like Facebook and Twitter, and it is important for marketers to understand why they do so. I develop a dynamic game framework that can generally be applied to any setting where consumers contribute user generated content and apply it to data on the purchase of ringback tones and cellphone networks. In this framework, consumers compete for contribution status in their social network by providing conspicuous connected goods. I estimate the model using recently developed econometric approaches to tractably analyze dynamic games *Bajari et al.* (2007). I find that consumers can make contributions to increase their relative contribution status as well as to induce future contributions by others. I then discuss how the framework can be easily extended to include additional effects depending upon the context and availability of data.

In the second essay, I examine the dynamics of consumer behavior in response to actions by identifiable peer consumers. I focus on characterizing both time-invariant heterogeneity through a hierarchical Bayesian framework, as well as time-varying heterogeneity with a Hidden Markov Model (or HMM). I study the opinion of a consumer as a latent construct and use a HMM to understand how impressions received through the social network from peers can influence the decisions of individuals. Of specific interest is the degree and type of social relationship that can enable consumers to transition to a more susceptible or receptive state where they may be more inclined to take specific actions (like purchasing).

In the third essay on product strategy for commercial open source firms, I evaluate a market place where firms compete by building upon a truly available open source code that is also accessible to other firms or competing firms. I rationalize why we observe competitive firms freely contributing source code as open source, which is a shared information good that is public by definition. In the second essay, I develop a theory to understand why consumers contribute so much content in social network settings. These consumer contributions are the underpinnings of social network platforms like facebook, twitter and youtube and without these contributions by consumers or individuals these platforms might well not exist.

Overall, I study and disentangle individual and competitive factors that drive consumers to contribute connected goods. I develop a framework that can generally be applied to any setting where consumers contribute user generated content and apply it to data on the purchase of ringback tones and cellphone networks.

CHAPTER II

Sharing in a Social Network Context: Connected Goods

2.1 Introduction

Social networks are fast emerging in marketing as a novel phenomenon of interest. These include web-based platforms like Facebook, MySpace and Twitter, as well as networking platforms designed for mobile phones like GyPSii. Content-focused websites such as the New York Times, CNN and YouTube are adding their own social networking features, enabling their users to share or post content. For example CNN's iReport.com site allows users to post news reports, and also to connect to others users. In all the above cases, the common thread is the provision and consumption of *user-generated* and *user-contributed* content that is shared in a social setting. We take a structural approach to examining the strategic invenctives of individuals to contribute content, making it available for consumption by their social network friends or peers, specifying a structural model and recovering preferences from social interaction data.

Firms set up online networking sites with the goal that an active and thriving network can be monetized either through promotion, customer service or distribution activities. Managers understand that they need to target networks where consumers visit regularly, and where enough content is created by users to ensure a critical mass of responders (*Shih*, 2009; *Palmer*, 2009; *Goldenberg et al.*, 2009). Thus, an active and thriving community is essential for both network platforms like Facebook to be successful, as well as for marketers to leverage these new media to reach consumers. These factors make it very important to understand the drivers of activity in social networking on the web and mobile platforms. In fact, there is a recently released application on Facebook that determines how much each user is worth, based on social connections and user activity levels (*Taylor*, 2009).

As pointed out by *Trusov et al.* (2009), users visit online networks to either (a) create content, or (b) consume or experience content created by other users, or specifically their connected peers. At the basic form, social network platforms themselves do not focus on providing content, but provide tools that allow their users to create, manage and disseminate content more effectively. For example, Facebook enables a user to share a posted video, but the company itself rarely creates any videos. The main value addition of social networking sites is that they provide a social space for users that does not require synchronous or coordinated communication or presence. Rather, these sites provide a virtual location, where users can connect at their convenience and still receive access to network content and be able to participate in social conversations with multiple friends at the same time.

The type of content varies across different social network platforms. For example, Facebook users post videos, status messages, comments, and applications; Myspace users post music, photographs, lyrics and discussions; Twitter users post 'tweets' of their thoughts to their 'followers' that can be retweeted; CNN's iReport users post user-recorded videos of current events. These content goods include photographs, videos, 'tweets': information targeted to their friends (i.e. connected network peers) , rather than unaffiliated users or the general public. We abstract away the specific contexts to evaluate such contributions as "connected goods", because they exist only in the context of a social network of connected consumers: if there were no social connections or relationships between consumers, no one would experience connected goods. These goods have several novel aspects that are not captured by prior models of goods considered in economics and marketing.

We define a connected good as a *conspicuous information good contribution made by an individual, available for experiencing by the individual's peers in a social network setting.* Connected goods are an idealized abstraction that have the following properties:

1. The contributions are observable by the contributor's peers.

2. The contributor of the good incurs a cost for making a contribution (monetary, time,

effort etc).

3. The contributor need not get a *direct or immediate consumption* benefit from the good, i.e. the contributor either does not experience the product or service, or already has access to consume the good without making it a connected good.

4. The contributor's friends or peers obtain a benefit from experiencing or consuming the good, hence we refer to them as *experiencers*.

The definition and characteristics include both user-generated content and user-contributed content that is created by others. An example would be a music video created by the record publisher, but is shared by a user with her friends. Each unique information good posted to the network can be considered to be a connected good. Thus, a photograph, a video or a movie review from which consumers obtain experience utility can be considered to be a discrete information good.

It must be noted that we do not think of contributors and experiencers as different sets of individuals. Rather, these are roles played by individual consumers in a social network, and each person can be considered to be both a contributor and an experiencer of connected goods.

Connected goods differ from several well-established concepts of goods in the economics literature that incorporate some idea of shared consumption, including public goods and club goods. A connected good contribution will ordinarily not be made in the absence of a social network, i.e. an isolated individual will not derive utility from making a contribution to connected goods. This generalizes the notion of a public good, which can be specified as a connected good with all agents symmetrically connected to all others. Consumption of public goods is not restricted and non-rivalrous, in the sense that no one is denied the right to consume the good (*Cornes and Sandler*, 1996) and one person's consumption does not diminish another's, and this property carries over to connected goods. For example, blog posts would be considered a public good, rather than a connected good since consumption is rarely restricted. However, a private blog set up for friends and family would be closer to a connected good than a public good. Another point of comparison involves club goods that are shared goods and non-rivalrous, but where consumption is restricted to a set of members, e.g.

cable TV or golf clubs. These usually involve the appropriation of rents by an organization or company, and can be considered to be mediated by the market rather than through social connections. In the club goods case, it is possible to form a voluntary association of users who share costs, and can exclude others from consumption. Club goods are primarily market-mediated transactions, comparing with connected goods that are mediated by a social network. A physical world counterpart to a connected good that shares certain properties is the practice of gift-giving in social contexts (*Carmichael and MacLeod*, 1997). The act of giving a gift can be interpreted as a contribution to the donor's friend, but the donor would incur marginal costs for each person that a contribution is made.

When the contribution is an information good, the contributor does not incur marginal costs per peer for providing the connected good (*Shapiro and Varian*, 1998). This factor distinguishes the connected good setting from a physical world gift-giving example. In the case of gift-giving, or donations to charity or altruism, the motivation of the individual or group providing the good or service has been modeled to include 'warm-glow' altruism effects, signaling of wealth etc.

Consuming or experiencing connected goods is a primary motivation for users to visit social networking sites, whether on the mobile phone or via the web. However, it is puzzling to observe large quantities of connected goods being contributed to social media, when contributors do not obtain immediate consumption utility. The motivations applied to public goods, club goods, and gift giving seem inadequate in explaining why consumers contribute and whether and how their contribution activities are inter-dependent among the connected individuals in social networks (*Trusov et al.*, 2009; *Iyengar et al.*, 2009).

In this paper, we seek to understand the mechanisms that drive consumers to make connected goods contributions to social networks that are consumed by their peers.[1] Specifically, we address the following research questions: (i) What factors drive consumer contribution decisions? (ii) Are contribution decisions related across connected peer consumers and across time? (iii) How do the contributions differ depending on observable individual characteristics and network positions? (iv) What types of consumers are more influential in encouraging contributions? (v) If a consumer makes a contribution, how does it encourage

[1]We use the term individuals, users and consumers interchangeably when there is no possibility of confusion.

peers to contribute in a dynamic way?

To answer these questions, we begin with the properties of connected goods, specifically that they are conspicuous contributions and that when a consumer contributes, peers derive experience or consumption benefits. We translate theories of consumer behavior that apply under these circumstances to our setting of a social network. We therefore propose a theory-based model in which a forward-looking consumer plays a non-cooperative game with his peers to compete for contribution status and takes actions after considering how current actions can influence the future decisions of peers, which in turn can affect the future utility of the focal consumer. This makes the interactions between consumers strategic; the realized utilities will then depend on the strategies of all consumers. We use contribution recency serving as a state variable that is endogenously determined by the sequence of consumer contribution decisions. We then construct positional local status of each consumer in her ego network and model it as affecting the consumer's per-period utility. The contribution state interlinks the decisions of different consumers, and positional status ensures that an increase in one consumer's status will require that other consumers decrease in status. Thus, consumer contribution decisions have the following effects: (i) a contribution made by the focal consumer changes the positional ranking of his peers and thus their utilities, and contribution decisions. (ii) a focal consumer derives consumption utility from (both current and future) contributions made by his peers. Hence, our model recognizes not only the possibility that consumer contribution decisions are inter-related across peers within the social network, but also that contribution decisions are inter-temporally related. This implies a dynamic and competitive consumer decision making process which characterizes consumers as making current contributions to obtain future consumption utility: when a focal consumer contributes a *connected good,* her status increases and the status of her peers diminishes, making the peers more likely to make a contribution to restore their status. Such contributions by the focal consumer's peers in turn results in increased (future) consumption utility for her. Because we explicitly model the process of conspicuous and inter-temporal consumer decision making, our approach illustrates the micro-foundations of contribution decisions in a social network setting, rather than merely testing whether interdependence exists.

We adopt *Bajari et al.* (2007)'s two-step approach to estimate dynamic games with large numbers of agents. We operationalize and empirically demonstrate our model on a unique panel data set, which includes purchases of ringback tones offered by a global mobile phone company with a network of interconnected customers.[2]

Our results show that contribution status or positional utility plays an important role in determining contribution decisions, demonstrating competitive behavior between consumers in the data. Consumers value contributions made by their peers, and understand that competition for positional status enhances the likelihood of peers making a contribution when a consumer contributes. Note that the cost of contribution prevents consumers from making contributions too frequently. Consistent with our intuition, older consumers are shown to be less competitive, but receive a higher utility from contributions made by others, and tend to be more sensitive to the cost of contribution. Male consumers are more competitive and sensitive to positional status, and have higher consumption utility for experiencing contributions made by their peers, but they are less sensitive to contribution cost. Interestingly, consumers with higher centrality are more competitive, enjoy higher consumption utility, and are less sensitive to contribution cost.

We also conduct simulations to investigate how important consumers are in affecting the utilities of their peers and how the decision of one consumer can motivate others to contribute. Our results show that consumers who are more competitive in making their own contributions may not be the most influential consumers in the social network. For example, consumers with higher centrality are more competitive in contributing to maintain a higher contribution status, they may not have as much impact as consumers with middle levels of centrality, whose contribution changes the ranking order or more peers. Seeding the more central consumers can encourage contributions from more peers in a faster fashion, with most of the incremental contributions made by more competitive peers.

We contribute to the emerging literature on consumer behavior in social networks along the following dimensions. First, we address a relatively new and puzzling question of why individuals incur the cost to contribute the connected goods in a social network when the

[2]This company operates primarily in Asia. We are unable to specify a more detailed location due to data confidentiality issues.

contributor is different from the person obtaining consumption utility from experiencing the good. Second, based on existing theories, we conceptually formulate consumer contribution decisions as competitive among connected peers and dynamic across time with the aim to understand the micro-foundations that rationalize the contributions made by contributors. Accordingly, we adopt a structural approach to investigate the decision process behind consumer contributions, which involve inter-temporally related contribution decisions. Second, for connected goods, the person obtaining consumption utility (experiencer) is different from the contributor, suggesting that such variation in contributor's utility cannot identify consumption or experience utility. Our approach inter-temporally links the consumption utility to the contribution status, permitting us to identify consumption utility for these goods. Third, methodologically, we are the first to incorporate the structure of the social network into a dynamic game with strategic consumers. This paper extends the application area of dynamic games, mostly used in the industrial organization literature where the competition is between firms in a market.

The rest of this paper is organized as follows. Section 2.2 reviews the current literature and situates our contribution. Section 2.3 develops the model. Section 2.4 discusses the computational challenges, identification and the two-step estimation. Section 2.5.1 details the institutional setting, describes the data, and report results of an application of the model. Section 2.6 presents our conclusion, along with substantive implications for researchers and practitioners, and discusses future research avenues.

2.2 Literature Review

There is a rapidly growing empirical and analytical literature focusing on the impact of social factors on consumer decisions in varied contexts, ranging from sequential influence processes like product reviews, word-of-mouth to concurrent social coordination processes in groups, like choosing television shows or ordering meals in restaurants. Although the issues and problems are distinct, the literature is broadly connected by the idea that one consumer's opinion, choice or decision can affect and influence others in their formation of opinion or determination of choice.

There is a considerable theoretical literature that models the endogenous formation of links between agents in a social network: a primary motivation is to understand why individuals or firms develop ties with others, and how the structure of the resulting social network may impact overall welfare. There have been recent developments in marketing trying to understand why stores may link to one another from an empirical viewpoint by *Stephen and Toubia* (2009), who find that increase in linking can draw more traffic and hence revenue. For a theoretical treatment, see *Katona and Sarvary* (2008), where the authors examine why web sites find it optimal to link to other sites. Overall, we recognize that linking behavior is a highly important issue, but one that is beyond the scope of the current paper, and refer the interested reader to the book by *Jackson* (2008).

We restrict our focus to the interactions between consumers in an *existing (exogenous)* social network. Much of the empirical research in marketing has focused on providing statistical evidence to show that a focal consumer's choice is influenced by the choices of the consumer's peers. Primarily because the availability of data reflecting the relationships between individuals is challenging to obtain, earlier studies on social influence have examined both independent and dependent variables as aggregate effects to study voting patterns in a presidential election (*Smith and LeSage*, 2004) and how consumers allocate their budgets (*Alessie and Kapteyn*, 1991).

Emerging research proposes theoretical models to examine the equilibrium properties of games between individuals situated in social settings. These theoretical papers attempt to explain social effects by focusing on micro-foundations. For example, *Brock and Durlauf* (2001) develop a static, game-theoretical discrete-choice model where the action taken by an individual affects the utility of everyone in the reference group, and where consumers value conformity. Using a random fields approach, they examine the equilibrium properties of the limiting case with a large number of agents. They evaluate the possibility of multiple equilibria, and characterize the conditions on strategic complementarity between the actions of agents. Empirical applications that have been examined using the random fields approach include the effect of peers on high-school performance and on unemployment (*Topa*, 2001). Another key issue that has received attention is how an individual's action in the network relates to network position, and how the interconnection structure of the network affects outcomes. *Ballester et al.* (2006) analytically model the equilibrium activity level of each individual when activities are strategic complements or substitutes, and find that individual agents contribute to a degree proportional to their Bonacich (or Eigenvector) centrality. *Soetevent and Kooreman* (2007) examine the pure strategy equilibrium of a static game with individual-level and dyadic social data on high-school students, and find that choices like smoking or owning cellphones are driven by peer effects.

Hartmann (2009) develops structural models of consumer choices as a static non-cooperative game, with the estimation methodology treating the data as resulting from equilibrium outcomes. In this framework, he studies the context of consumers deciding whether to play golf with a partner versus playing alone. He models this as a static coordination game between consumers, similar to the classic 'Battle of the Sexes' and builds upon the discrete choice static game framework (*Bresnahan and Reiss*, 1991). The author models endogenous social interactions and infers social relationships from the data, positing individuals to be partners if they purchase within a specified time window of each other multiple times. He finds that a segment of consumers receives significant benefits from playing with others, and he evaluates the extent of these benefits. He also investigates whether a targeted marketing strategy focusing on the differences within groups or across groups serves better to increase revenue. We can also think of this paper as studying the formation of (temporary) links between players (so that they are linked when they decide to play golf together).

Since our setup is dynamic with inter-temporal tradeoffs in the presence of strategic interaction, we base our model on the framework characterizing dynamic games with forward-looking agents by *Ericson and Pakes* (1995) (E-P), which was developed to examine the dynamic effects of competitive strategic interactions between oligopolistic firms in a marketplace.

Our research differs from existing empirical research on social networks in the following ways. First, we examine a seemingly puzzling consumer decision: contribution to connected goods, i.e. conspicuous goods where the contribution is ostensibly for the benefit of the contributor's peers. We seek to address why consumers are willing to contribute even when they may not obtain immediate consumption utility. To the best of our knowledge, with the exception of *Iyengar et al.* (2009), most studies involving social networks examine the degree of activity rather than contribution of content. Second, consumers in our context do not obtain immediate *consumption* utility from their contribution decision. Rather, they influence their peers to contribute and thus obtain future consumption utility. This is in contrast to existing research where consumers obtain immediate consumption or experience utility directly from their own purchases, as well as additional utility if they incorporate the choices of their peers. Third, most empirical papers adopt statistical approaches such as a count model or logistic regression with the goal of empirically establishing that consumers influence each other. Our goal is to understand and explicitly model the micro-foundations of consumer interactions as a game played between individuals in a social setting, which rationalizes why we observe interdependence. Fourth, our data allows us to reconstruct the social network ties from transactions between interconnected consumers. Thus, we are not required to infer social relationships and are able to capture strong social ties as opposed to static declarations of relationships or by a survey. In addition, we have a record of purchases, an economic activity not ordinarily used in social network studies. Finally, this is among the first papers that apply empirical industrial organization approach to a consumer problem and is also one of the first to exam the dynamic strategic behavior of forward-looking consumers in a social network setting.

2.3 Model

We index consumers by $i \in \{1, \ldots, I\}$, and time periods by $t \in \{1, \ldots, \infty\}$. In each period, every consumer in the network chooses whether or not to contribute a new connected good that their peers can observe and experience. We use the binary indicator d_{it} to denote the contribution decision of consumer i in period t:[3]

$$d_{it} = \begin{cases} 0 & \text{do not contribute new connected good} \\ 1 & \text{contribute new connected good} \end{cases} \tag{2.1}$$

[3] We can extend this binary decision to incorporate a continuous or discrete-valued contribution depending on the specific context.

2.3.1 Theoretical Foundation

We follow the theoretical foundation for positional utility built by *Frank* (1985), which in turn is based on ideas on positional goods from *Hirsch* (1978). There is much observational and experimental evidence that social comparisons between individuals play an important role in consumer decision making (*Heffetz and Frank*, 2009). The essential idea is that individuals compare themselves with a reference group of peers and have a higher utility when their relative position is better than that of their peers.

While the idea that consumer preferences might be interrelated via social groups had been posited by *Duesenberry* (1949), the first model incorporating a setting where consumers compete for status by acquiring conspicuous goods was characterized by *Frank* (1985). This was based on the insightful argument by *Frank* (1993) was that when individuals make conspicuous choices, or attain conspicuous things, they are often driven not by the absolute consumption utility derived from consumption utility, but rather from the positional utility, which inherently involves an implicit comparison with others.

Two primary requirements for positional concerns to be important for consumers are as follows: (a) a consumer's choice is observable by peer consumers, i.e. the choices are conspicuous, and (b) the choice must be made solely at the individual level, not as a dyadic construct or between consumers. The former constraint would not permit private consumptions, whereas latter constraint would not include settings like dyadic gift-giving, even if observed by others.

The positional effect is likely to be especially strong in pre-existing social groups, rather than in a group of previously unaffiliated individuals. Other settings where such positional concerns are likely to be significant include workplaces, where employers can use explicitly positional goods like corner offices or executive-only lunchrooms to enhance competition between employees (*Auriol and Renault*, 2008).

Hopkins and Kornienko (2009) have developed a game-theoretic model incorporating the positional or status considerations. When relative social position offers utility, they find that individuals are motivated to take actions to compete for higher positional status and thus their choices, decisions or achievements will impact others in the social group. The equilibrium strategies and outcomes resulting from positional considerations explain a wide variety of outcomes in labor markets as well as other settings.

We extend the positional utility framework to apply to a connected good, which is a conspicuous *contribution* compared with prior applications to *consumption* goods. The notion of consumers playing a game of positional status is more appropriate when the status is local, i.e. consumers know their peers and can observe their actions and respond to them. The setting of a social network is ideal to examine status interactions, and the availability of data on both the contribution decisions and the social relationships from web or mobile-phone based networks will enable us to explicitly consider the strategic interactions within a social network of consumers.

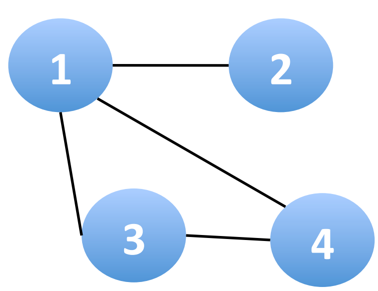
We model the status competition between consumers described above by characterizing a dynamic game with forward-looking consumers, whose decisions are interlinked due to the presence of a social network relationship. The positional status introduces strategic interactions between consumers, which we model as a game. Note that a consumer's current period contribution decision can impact her positional status, affecting future contribution decisions by peers, which in turn affect the consumer's future consumption utility. Hence, we must model consumers as forward-looking to capture the inherent inter-temporal tradeoff between current period cost of contribution and future expected consumption utility. We characterize consumers as playing pure Markov-perfect strategies in a dynamic game, where our focus is on examining the equilibrium strategy profile and outcome of the game. Following the Markovian logic, we posit state variables that encapsulate in a simple manner, the history of play in the game as it influences payoffs. Consumer utilities in any period depend on the current state and the law of motion, i.e. how states evolve over time as a result of the decisions made by consumers.

Our model parsimoniously captures the underlying dynamic consumer decision process: A consumer chooses to contribute whenever the current benefits from contributing due to self-expression, positional status as well as expected future consumption utility less the costs associated with contributing is greater than the current and expected future utilities obtained by not contributing. The expected benefits from contributing include a self-expression utility, higher positional status utility, higher future contribution levels by peer consumers and instrumental benefit interactions. These benefits have to be weighed not only against the cost of contributing as in traditional dynamic choice models,, but the expected change in the responses by peers.

The micro-foundations approach to this study requires several assumptions, some of which may be worth relaxing in specific problem settings, but remain beyond the scope of the current paper. First, we assume that social ties between individuals are exogenous and that strong social ties do not disappear over time (consumers who maintain a minimum level of communication are in fact in a social relationship). This allows us to focus on explaining why consumers contribute to connected goods. An approach that explicitly models how and why consumers form social relationships with specific peers, and how such relationships evolve would be very helpful (*Jackson*, 2008). A second assumption is that individuals perfectly observe the state of each of their friends in every period, which enables them to derive the status in their local social network. This assumption may be more reasonable in settings where consumers are in regular communication or are updated fairly frequently by the network platform regarding the actions of their peers.

2.3.2 Social Structure

We use Figure 2.1 to demonstrate the social structure with $I = 4$ individuals. It gives us the corresponding $I \times I$ social relationship matrix below. The (i, j)-th element of the symmetric social relationship matrix $\mathbf{R}$ is 0 if i and j do not have a social relationship, and 1 if they do.



$$\mathrm{R} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Figure 2.1: Example Network and Corresponding Social Relationship Matrix

A social network can be represented as a social relationship matrix $\mathbf{R}$, and since we characterize social ties as bidirectional, the social relationship matrix will be symmetric. Since the matrix $\mathbf{R}$ completely captures the structure of connections, and we can operationalize it in several different contexts, i.e. a work group relationship, a social relationship etc. The sum of row i (or column i) corresponds to the number of peers that consumer i is connected with. The relationship matrix is critical to our understanding of the interdependence in decisions, since a consumer can only observe the choices of peers with who she is connected, i.e. if element $R(i,j) = 0$, then i and j cannot have a direct influence on each other, but only through other consumers. Note that choosing a stronger measure of social relationship will result in fewer connections between consumers, but such networks are more likely to be stable over time.

Endogenous Contribution States

We model the endogenous state s_{it} for consumer i in period t as representing the number of periods since the consumer has made a connected good contribution to the social network. The state of the consumer evolves deterministically depending on the consumer's choice in period t, indicated by d_{it}:

$$s_{i,t+1} = (s_{i,t} + 1)[1 - d_{i,t}] \tag{2.2}$$

This implies that the state increases by 1 when the consumer makes no contribution, and is reset to 0 when a contribution is made. The state represents the recency of contribution, which represents how active a consumer has been in the network. If a consumer contributes regularly to the connected good, then she is likely to have a low value of the state variable, whereas a consumer who has only made a contribution a long time ago has a high value. We use a $I \times 1$ vector to denote the states of the all the I individual consumers in the social network, $\mathbf{s_t} = (s_{1t}, \ldots, s_{It})$.

The vector of contribution states of consumers in the social network $\mathbf{s_t}$ is the primary variable that relates the decisions and utilities of the consumers, not only across consumers in a specific period, but over time and is critical to the underlying inter-temporal tradeoff. At each period of time, consumer derive utility based on the state of their own and the states of their connected peers. In turn, the periodical decisions made by consumers also affect their state. The state of the network $\mathbf{s_t}$ serves as endogenous state variables that drives all the dynamic effects in the model.[4]

We focus on strongly connected social networks where consumers know the states of their network peers with whom they are connected. When consumers are in contact with their peers fairly regularly, it is reasonable to assume that they are able to identify the contributions made by their peers. However, in a setting where consumers are in contact only rarely, such an assumption would be untenable, and we would expect knowledge about peers' states to be imperfect at best. If we do not observe the pattern of communications over time, inference may require such uncertainty to be built into the model.

2.3.3 Per-Period Utility Function

We assume consumer i's utility in period t to be affected by the consumer's self-state, the resulting positional status (or contribution status), consumption utility, the cost of contribution and an unobservable error term. More specifically, the utility function is defined as:

$$\begin{aligned} u(\mathbf{s_t}, d_{it}, \epsilon_{it}, \nu_{it}) = & \quad \theta_1 s_{it} + \psi(i, \mathbf{s_t}; \mathbf{R}, \theta_{\mathbf{2}}) \\ & + \theta_3 \chi(i, \mathbf{s_t}; \mathbf{R}) + \hat{u}(\mathbf{s_t}, \nu_{it}) + \theta_4 d_{it} p + \epsilon_{it}(d_{it}) \end{aligned} \tag{2.3}$$

In the per-period utility function, we allow the following factors to affect consumer contribution decisions: The first component is the utility derived from self-expression, where the contributor receives utility from active contributions to connected goods. The self-expression utility depends on the recency of their contribution, and captures an absolute effect that is independent of the decisions of peers. The second component is the utility contribution status, where an individual consumer's utility could be impacted if she makes more contributions or more frequent contributions relative to her peers. With a conspicuous good, each

[4]When we refer to 'state' without qualifiers, we mean the endogenous state vector. Since we consider contributions over time, our representation would involve frequency of contribution rather than the amount of contribution, which is more appropriate in a static setting. We would be unable to tease out these two effects further if we do not have information on consumers' motivations. Depending on the variety of connected goods available in specific settings, the definition of state may have to be expanded to allow different kinds of media (e.g. posting a video versus an album of photographs) to be weighted differently. Also, it would be more flexible to include more of history of the game in the state, but this would be less attractive from the twin viewpoints of behavioral requirements and analytical tractability.

individual can observe their friend's contributions, and following *Frank* (1993), we posit that the relative contribution or status affects utility. The third component is consumption utility, where the consumer obtains utility depending on the contribution to goods made by her peers. Whenever a focal consumer's peers make a contribution, the consumer receives utility from the good. The fourth component denotes monetary, cognitive and search costs to select an appropriate connected good that will be appreciated by peers, as well as opportunity costs and bandwidth costs if used in a mobile setting.

In the above utility function, all parameters are common knowledge[5], and the unobservables ϵ_{it} and ν_{it} are *private* shocks observed by each consumer and are *iid* across consumers and periods. The distribution of ϵ_{it} is type-I extreme value, whereas ν_{it} is distributed log-normally. Below, we explicate on the different terms in the consumers' period utility function.

Self-Expression

The state of the consumer i in period t, s_{it}, captures how the consumer's utility depends on the consumer's own state, and ignores the effect of other consumers. This can be interpreted as a consumer's need for self-expression by making a contribution when she deems a sufficient period of time to have passed since her previous contribution. The parameter θ_1 represents this overall effect, and a more negative value for θ_1 indicates that consumers more self-expression, whereas a positive value suggests inertia to make a contribution increases with the time passed since the previous contribution.

Positional Status

We describe below the factor of positional status on the utility of the consumer, as detailed in the theory overview. The second term in the utility function, the contribution status term $\psi(s_{it}, \mathbf{s_{-i,t}}; \boldsymbol{\theta_2})$ depends on the state of the consumer i as well as the state of all i's other peers in the network ($\mathbf{s_{-i,t}}$). The essential idea is that contributions to connected goods are made in a conspicuous fashion, and in such settings, consumers are known to value not just a higher level of absolute consumption but specifically value achieving higher levels than others belonging to a reference group (*Frank*, 1993). We refer to this factor as the contribution

[5]See *Fudenberg and Tirole* (1991) for a definition.

status or simply the status factor. In the web and mobile social networks, it would be appropriate to consider peers who belong to the network to be the reference group.

There are several research efforts that have examined the role of status as a motivating factor for consumers to post content online, e.g. reviews of products on Amazon.com or contributions to open source software (*Kollock*, 1999; *Roberts et al.*, 2006b). However, in several of these settings, the contributions are made anonymously, and even when identities are partially revealed (e.g. a unique login), the contributions are mostly public goods that are available in an unrestricted fashion. Prior studies on online communities have demarcated a clear role for status as a motivating factor in contributions (*Lampel and Bhalla*, 2007). Following the same logic, we conjecture status to have an even stronger effect on contribution decisions in social network settings, because individuals have clear social relationships with others in the network.

We consider the simplest possible ordinal measure of status, extending *Hopkins and Kornienko* (2009) (HK, hereafter). The status effect in period t is determined by the position of consumer i in the ordinal ranking of the states of all the consumers during the period t. who characterize consumers' status depending on the degree of consumption of a positional good. More specifically, the positional status of consumer i is determined by how many other peers of i have a lower status (or a higher state) than i, and is operationalized by the empirical CDF of the states in each period:

$$\Phi_{\mathbf{s_t}}(i, \mathbf{s_t}; \mathbf{R}) = \frac{|\{m: \quad r_{im} = 1 \wedge s_{mt} \leq s_{it}\}|}{|\{m: \quad r_{im} = 1\}|} \tag{2.4}$$

where r_{im} is the element in the ith row and m-th column of the relationship matrix $\mathbf{R}$ (see Figure 2.1). This empirical CDF construct represents the fraction in i's local network who have made a more recent contribution to the connected good than i. The positional status of consumer i in period t is the fraction of peers with a less recent contribution than i, or$[1 - \Phi_{\mathbf{s_t}}(i, \mathbf{s_t}; \mathbf{R})]$. Specifically, it is the fraction of i's peers who have a higher state than i in the period. Note that one's relative position can improve only when another person's position declines.

Therefore, we employ the following function form to capture the positional effects:

$$\psi(i, \mathbf{s_t}; \mathbf{R}, \theta_\mathbf{1}) = \theta_{21}\left[1 - \Phi_{\mathbf{s_t}}(i, \mathbf{s_t}; \mathbf{R})\right] + \theta_{22}\phi_{\mathbf{s_t}}(i, \mathbf{s_t}; \mathbf{R}) \tag{2.5}$$

The empirical PDF given by $\phi_{\mathbf{s_t}}(i, \mathbf{s_t}; \mathbf{R}) = \frac{|\{m: \quad r_{im}=1 \wedge s_{mt}=s_{it}\}|}{|\{m: \quad r_{im}=1\}|}$ indicates the fraction of i's peers who have exactly the same state as i in period t. It denotes positional ties in the state as well as contribution status. We treat positional ties separately to account for consumers' preference for equality of state (and hence status).

Using the empirical CDF and PDF allows us to characterize a measure that is invariant to network density, i.e. both dense networks where consumers have several connected peers, and sparse networks can be accommodated because of the implicit normalization. We model the effect of ties on the utility separately to allow the flexibility for consumer utility to be affected by the number of peers in the same state.

The coefficient θ_{21} captures the effect of status of consumer i, which is determined by how many other consumers have a lower status (or a higher state) than i. If θ_{21} is determined to be positive and significant, then consumers value having a higher status among their peers. If θ_{22} is significant and determined to have a lower magnitude than θ_{21}, then consumers place a lower value on being tied with their peers compared with being more current. We use parameter vector $\theta_\mathbf{2} = (\theta_{21}, \theta_{22})$ to denote all components associated with status-based utility.[6]

Experience Utility

In period t, consumer i has access to $\sum_{m \neq i} r_{im} d_{m,t-1}$ new connected goods, i.e. contributions made by her peers in the previous period, accounting for the presence of a social

[6]There are three significant modeling differences between HK's setup and ours. First, consumers in HK's model choose from a set of continuous levels, which makes it easy to rank them, whereas in our model consumers can only choose from two levels (contribute or don't contribute) in each period. We consider status to be based on the consumer's state (the number of periods since the consumer has made a contribution) to capture a more fine-grained notion of status, allowing the effects of contributions to persist beyond across periods. This characterization derives from the idea that a newer contribution is indicative of a higher level of contribution to the network. Second, HK's focus is on examining the comparative statics of the equilibrium of the static one-shot game, whereas our primary interest is in the dynamic effects of status-based competition between consumers. Third, we explicitly postulate the network structure in the status competition, whereas HK implicitly assume a *complete* network, in which each individual is connected to all others.

relationship. Unlike most marketing purchase decisions, where the person purchasing obtains consumption utility, with connected goods the person incurring the cost of contribution (contributor) is different from the person obtaining experience or consumption utility. Consumers value the amount of new connected goods available for consumption in each period, which can be viewed as an approximation that also captures declining utility for observing older contributions over time.[7]

Thus, the third term in the utility function represents the utility obtained in the current period from consuming these newconnected goods contributed by peers during the previous period:

$$\chi(i, \mathbf{s_t}; \mathbf{R}) = \sum_{m \neq i} r_{im} d_{m,t-1} = \sum_{m \neq i} r_{im} \mathbf{I}(s_{m,t} = 0) \tag{2.6}$$

Note that the consumption utility only depends on the contributions made by consumer i's peers, and not by any current decisions i makes in the current period. However, i's current period contribution ($d_{it} = 1$) may induce i's peers to contribute in future periods, e.g. $d_{m,t+1} = 1$, where i and m are peers. Consumer i can thus derive higher expected future consumption utility by making a contribution in the current period t. The consumption effect is represented by the parameter θ_3 and a larger positive value indicates that the consumer places a higher marginal consumption utility on contributions made by her peers.

Complementary Activity - Dyadic Social Communication

In social network settings, consumers receive utility from complementary activities, including communicating with their peers, i.e. resulting from some activity that is not *directly* related to the contributions to connected goods. The instrumental utility results from consumer communication activities even in the absence of contributions or consumptions of connected goods. Experimental evidence has demonstrated consumers receive additional utility from the complementary activity when they have a higher status (*Ball et al.*, 2001). Note that such an activity may not be important in several social networking contexts, and even when these factors exist, information on such factors may be difficult to obtain. The description here is specialized to social networks and to the specific application context we

[7]The consumption utility can then be viewed as approximation to the net present value of current and future consumption utilities.

examine in Section 2.5.

We model consumers deriving additional interaction utility from dyadic social interactions when they have a higher status. We posit that the amount of one-on-one time consumers spend communicating with their peers varies with contribution status. To capture this interaction between the contribution status and dyadic social communication, we model the utility for the latter flexibly with a simple non-linear formulation, with interaction effects between status and the utility of dyadic communication captured in a multiplicative manner. The decision on the aggregate amount of dyadic communication x is made every period, and consumers receive utility for dyadic communication given by:

$$\tilde{u}_i(x, s, \nu) = \left(\gamma_1 x + \gamma_2 x^2\right) \cdot \nu \cdot [1 + \gamma_3(1 - \Phi_s(s_i))] - c \cdot x, \qquad \log(\nu) \sim N(0, 1) \tag{2.7}$$

We choose a relatively simple and flexible functional form $(\gamma_1 x + \gamma_2 x^2)$ that allows for concave or convex utilities from dyadic social communication, i.e. γ_2 can be positive or negative. This could be regarded as an approximation to more a detailed specification of utility for dyadic communication. The additional factor $\gamma_3(1 - \Phi_s(s_i))$ represents the interaction between the dyadic communication and status.

Thus, in the above equation, the coefficients γ_1 and γ_2 capture the non-linear utility of dyadic communication, and ν is a log-normally distributed error term, and $1 - \Phi_s(s_i)$ represents the positional status. Its coefficient γ_3 measures whether having a higher status offers consumer additional instrumental utility from dyadic communication. Note that the consumer obtains utility through dyadic communication of amount x even when there is no interaction with status, i.e. when $\gamma_3 = 0$.

We can interpret the amount of time the consumer spends on communicating with peers as the amount for which marginal benefits of communicating further will equal the marginal costs. The marginal benefits are affected by the unobservable term ν. Thus, consumers may value dyadic communication more in some periods and less in others, and ν rationalizes the decision.

The optimal amount of dyadic communication and the corresponding utility are then denoted as:

$$
\begin{aligned}
x^*(\mathbf{s}, \nu) &= \arg\max_x \tilde{u}(x, \mathbf{s}, \nu) \\
\hat{u}(\mathbf{s_t}, \nu_{it}) &= \tilde{u}(x^*(\mathbf{s}, \nu), \mathbf{s_t}, \nu_{it})
\end{aligned}
\tag{2.8}
$$

Note that after choosing the optimal aggregate dyadic communication, x^*, the instrumental utility $\hat{u}$ only depends on the state and the current period realization of the random variable, indicated by ν_{it}. Note that the instrumental decision (dyadic communication) is affected by the state and status, but does not influence the dynamics of the model through the state. Thus, it is purely a 'static' decision, similar to the oligopoly market competition in prices among firms in *Ericson and Pakes* (1995). In principle, multiple instrumental factors can easily be incorporated in the model.

2.3.4 Heterogeneity

To incorporate heterogeneity that characterizes individuals into the structure of the dynamic game, we allow each parameter in the utility function (2.3) to depend on the observable time-invariant individual-level factors $Z_1, Z_2, \ldots, Z_l$ as follows:

$$
\theta_j = \theta_j^0 + \sum_{k=1}^{l} \theta_j^k Z_k \tag{2.9}
$$

where j indicates the effect included in the utility function. The variables $Z_1, Z_2, \ldots, Z_l$ may include both traditional demographic observables like age, gender as well as social network constructs like degree or eigenvector centrality that describe the individual's role. This structure represents a dyadic heterogeneity and includes network-based constructs of individual heterogeneity like degree centrality and eigenvector centrality. While most marketing studies have considered individual-level variables like age, gender to account for heterogeneity, it becomes really important in a network context to incorporate centrality, a measure of the importance of each consumer. This novel construct can help us understand whether we find support for sociology-inspired ideas which posit that more central individuals are more important to the network.

Incorporating the network structure permits us to focus on how consumers respond to choices made by their friends, rather than the aggregate level of contribution in the entire social network. Using the aggregate level implicitly assumes that consumers can observe and respond to all others in the network, irrespective of whether they actually communicate with them, which would be unrealistic in most social network contexts.[8]

In practice, since the focus is often on marginal effects, the variables may be mean-centered, so that θ_j^0 is interpreted as the effect for the average individual. θ_j^k denotes a vector of coefficients explaining whether consumers characterized by variable Z_k has higher or lower coefficient than an average consumer θ_j^0. The demographic characteristics serve as consumer-specific exogenous state variables in our models, so that even when the policy functions are identical, there is variation introduced by the exogenous variables. If we consider N_z individual characteristics to include in the state space, the exogenous state space will have $(N_Z \times I) + I^2$ elements, including I^2 elements for the relationship matrix, $\mathbf{R}$.

2.3.5 Consumers' Problem

Recall that the inter-temporal tradeoffs for consumers in making a contribution includes the current costs compared with future benefits from positional status, self-expression and consumption utility resulting from peers' contributions. We specify consumer i's problem as maximizing the sum of expected discounted future period utilities. This can be represented as:

$$\max_{(d_{i\tau})_{\tau=t}^{\infty}} \mathbf{E}\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(\mathbf{s_t}, d_{i\tau}, \nu_{i\tau}, \epsilon_{i\tau})|\mathbf{s_t}\right] \tag{2.10}$$

where β is the common discount factor for all consumers. The expectation is taken over the unobservable private per-period shocks for each consumer across future periods, and consumers have expectations over the shocks received by others. The flow utility or period utility payoff to the consumer, $u(\mathbf{s_t}, d_{i\tau}, \nu_{i\tau}, \epsilon_{i\tau})$, is defined in equation (2.3), and depends on the states of all consumers. The specific sequence of events in each period detailing the activity follows:

1. Consumers observe their own state and the state of their social network peers.

[8] Observing the aggregate or entire activity would be reasonable for public goods like product reviews posted by consumers on Amazon.com.

2. Consumers receive private draws of the unobservables for the contribution decision and instrumental decision.

3. Consumers make their decision on whether to make a contribution to the connected good and related instrumental decisions (if any).

4. Consumers receive utility from their self-expression (self-state), contribution status, consumption utility and incur the cost of contribution if any.

5. The states of consumers evolve as a result of their decisions.

We can transform the problem and characterize it by the Bellman equation that gives us the value function corresponding to a state, which in turn captures the expected present value of current and future discounted utilities that results from starting in that specific state. We write the Bellman equation as:

$$V_i(\mathbf{s}, \sigma; \theta) = \max_{\sigma_i(s) \in \{0,1\}} \left[u(\mathbf{s}, \sigma(\mathbf{s})) + \beta \int V_i \left(\mathbf{s}' | \mathbf{s}, \sigma_{-\mathbf{i}}\right) dP \left(\mathbf{s}' | \mathbf{s}, \sigma_{-\mathbf{i}}, \sigma_i\right) \right] \tag{2.11}$$

The value function reflects expected utility at the beginning of the period before the private shocks or unobservables are realized. In our setting, the state transitions are deterministic, so the expectation or integration is over the unobservables. We assume that consumers decide on a contribution strategy as a function of the current observable state, including both the endogenous state $\mathbf{s_t}$ in period t and the time-invariant exogenous characteristics of consumers in the social network.

Following the literature on dynamic games (*Ericson and Pakes*, 1995), we assume that consumers play a pure-strategy Markov-Perfect equilibrium. The strategies are constrained to depend only on the current state and current period unobservables, and map these quantities to an action set indicating the contribution decision. A Markov-perfect strategy for a consumer is represented formally as: $\sigma: \quad \mathbf{S} \times \mathbf{R}^{\mathbf{2}}_{+} \rightarrow \{0,1\}$. We denote consumer i's contribution strategy as $\sigma_i(\mathbf{s_t}, \epsilon_{\mathbf{it}})$, a deterministic function indicating a contribution or no-contribution decision made by consumer i. Note that the strategy σ_i represents the dynamic decision, and does not include the static instrumental decision. The strategy profile of consumers in the network is simply the vector of individual strategies and is indicated by:

$\sigma = (\sigma_{\mathbf{1}}, \ldots, \sigma_{\mathbf{I}})$. In a Markov-perfect Equilibrium (MPE), each consumer uses an equilibrium strategy to maximize the lifetime utility, and consumers expect their peers to use the equilibrium strategy as a function of the observable state and private period shocks. A strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_I^*)$ constitutes a Markov-Perfect equilibrium if the value function for each consumer corresponding to the equilibrium strategy is greater than the value function corresponding to any unilateral deviation from the strategy. Mathematically, this is represented as:

$$V_i(\mathbf{s}|\sigma_{\mathbf{i}}^*, \sigma_{-\mathbf{i}}^*) \geq V_i(\mathbf{s}|\sigma_{\mathbf{i}}^{'}, \sigma_{-\mathbf{i}}^*) \qquad \forall i, \mathbf{s}, \sigma^{'} \tag{2.12}$$

There are critical points of similarity and distinction when comparing the present model with studies on industrial settings based on the E-P framework (*Ericson and Pakes*, 1995; *Pakes and Mcguire*, 1994), which focus on the strategic competition between firms. The similarities include the behavioral assumptions made about the agents (consumers in our setting and firms in industrial organization studies), that they are forward-looking and maximize the sum of expected discounted future payoffs or utilities. The underlying mechanism in such models characterizes the data as representing the pure strategy Markov-perfect equilibrium played by agents in a dynamic game. The Markovian property requires the consideration of the strategy of a consumer only as it depends on the current state of the network, i.e. the consumer's own state, the state of peer consumers, and (potentially) state variables that are not assigned to a specific consumer.

2.3.6 Equilibria

We focus on one market or network of interconnected consumers, so the assumption that the data are generated from a single equilibrium is quite reasonable. While there may be multiple equilibria in dynamic games in general, as detailed in section 2.4, the advantage of the two-step estimation approach is that the equilibrium actually played will be used to consistently recover the parameters. If we do not use strategies different from those characterizing the equilibrium that corresponds to the data, we would not need to focus on other equilibria that may be consistent with the same parameter values as our estimates. To address the concern that a different equilibrium might result when choosing different policy functions, we run simulations that only require us to vary the states without recomputing the policy functions, as detailed in Section 2.5.4.

2.4 Identification and Estimation

We describe the empirical issues that naturally arise in the estimation of the model described in Section 2.3. We begin by specifying how individual-level information can be incorporated into the model by including it in the state space. We then proceed to discuss how the specified model is identified from variations in contribution patterns that we observe in the data. Finally, we discuss the estimation process for dynamic games in general and the two-step approach of *Bajari et al.* (2007) that we adopt.

2.4.1 Identification

The variation in contribution behavior by a consumer with respect to the states of peer consumers allows us to identify the status effect. The consumption effect is affected by the interaction between the status effect and the contribution decisions of peers. When a consumer faces a situation where several peers are making new contributions, the status of the focal consumer is reduced, whereas the consumption utility is increased. The inverse condition when contributions by peers happen infrequently allows a consumer to maintain a higher status even with less frequent contributions. For the instrumental utility of dyadic social communication, the variation in communication patterns over time and at different levels of contribution status allows us to identify the instrumental utility parameters.

Having detailed the sources of variation that help identification, we note that absent the data on both contributions and social connections, we would be unable to identify either the consumption (experience) utility or the status utility. In a model with myopic consumers ($\beta = 0$), the consumption utility coefficient θ_3 will not be identified. The reason is that consumers will not value expected future consumptions, and will not account for this effect when making their contribution decisions. Therefore, while consumers will experience consumption utility in the current period, their actions will not affect the future actions of their peers, and we will be unable to identify consumption utility.

We modeled the utility of altruism frequency as the consumer's utility from self-state in Section 2.3. If we alternatively consider altruism to imply that a consumer obtains utility only when a contribution is made, then that effect will not be separately identified from the cost of contribution if there is no variation in the cost. However, if there is a sufficient observable variation in the cost of contribution which can happen when the contributor pays a monetary price, then such an effect would be identified. The cost parameter c in the dyadic communication or instrumental benefit equation, given by (2.7) is not separately identified from γ_1 and γ_2, so we normalize it by setting $c = 1$.

2.4.2 Estimation

The overall goal is to estimate the model parameters, both the static parameters as well as the dynamic structural parameters. We denote the set of dynamic parameters by $\theta = (\theta_1^k, \theta_{21}^k, \theta_{22}^k, \theta_3^k, \theta_4^k)_{k=0}^{N_Z}$, so that we have $5 \times (1 + N_Z)$ parameters to estimate, where N_Z is the number of individual-level 'heterogeneity' variables. The static parameters are collected in the vector $\mathbf{\Gamma} = (\gamma_1, \gamma_2, \gamma_3)$. Note that the static variables are separately estimated, and used as an input to the estimation process for the dynamic parameters.

The estimation methods for dynamic games have evolved from the algorithms used to estimate single-agent dynamic discrete choice models. The basic approach to estimating single-agent settings, detailed in *Rust* (1987), is to compute the equilibrium given a set of parameter values using an 'inner loop', and search over the parameter space in an 'outer loop'. This approach is feasible in a dynamic game only with special structural properties when the number of players is very small, perhaps $N = 4$, depending on the state space. Despite advances in computing technology and developments in optimally reducing the computation like *Pakes et al.* (2004), a setting with even $N = 5$ agents (consumers or firms) can remain very much beyond the reach of these methods, especially with a large state space.

Due to the intractability of explicitly computing the Markov-Perfect equilibrium, which grows exponentially with the number of consumers, we adopt the multi-step approach advocated by *Bajari et al.* (2007) (BBL, henceforth). There are several recent advances proposed to alleviate the unique computational difficulties posed by dynamic games, and other algorithms that break the estimation down into steps include *Aguirregabiria and Mira* (2007) and *Pesendorfer et al.* (2008).

An alternative approach to estimating games with a large number of agents is the recently developed notion of Oblivious Equilibrium (OE), suggested by *Weintraub et al.* (2008). OE is a different equilibrium concept that characterizes the long-run states of the dynamic game, and is presented as an approximation to MPE, and converges to the MPE with a large number of agents. OE relies on the essential idea that agents may not react significantly to deviations by any one agent, since each agent's effect is small.[9] In an Oblivious Equilibrium, each agent is assumed to play the best response to the $long-run$ strategies of the other agents, and agents only keep track of the aggregate state of the other agents.

Although this approach is very attractive for settings like the present one with a large number of agents, OE lacks the ability to incorporate the reflection effect and characterize consumption utility obtained when the contributor does not directly receive consumption utility, but only via an indirect process. The inability to specifically characterize the idea that a consumer may make a contribution in the current period to induce contributions by peers in future periods, from which the agent receives future consumption utility is a key contribution of our model and analysis. Therefore, we cannot use the OE method in our setting. Moreover, OE cannot easily incorporate arbitrary dependence structures that we require to model relationships in social network settings.

The BBL approach is prominent among the recent methodological advances that involve breaking down the estimation into multiple steps, and builds on the Conditional Choice Probability (CCP) developed by *Hotz and Miller* (1993) and the value function simulation method of *Hotz et al.* (1994). The primary benefit of this two-step method in estimating dynamic games is that we do not need to compute the equilibrium. Indeed, in our setting, computing the equilibrium even once is intractable due to the large number of agents. Another benefit specific to the *Bajari et al.* (2007) approach not shared by the other two-step estimators is the ability to handle a continuous state space without discretization. The trade-off with BBL and other two-step procedures when compared with explicit equilibrium computation

[9] OE relies on a light-tailed condition on the distribution of the agents' states to achieve this.

is that the two-step procedures do not incorporate all of the information, resulting in a loss of efficiency with small samples.[10]

In our context, the first step involves recovering the contribution policy of consumers as a function of the state of the social network. This is a reduced-form process of mapping the equilibrium strategies that are represented in the data to a policy function. The second step recovers the structural parameters of the model that affect the consumer's dynamic inter-temporal considerations.

Static Parameters

The static parameters corresponding to the instrumental benefit affect only the period utility functions, and do not affect the dynamics of the game between consumers. We estimate the static parameters by MLE, but alternative approaches like GMM can be used. The static parameter estimates are used as an input to the dynamic estimation procedure. The expected period utility for dyadic social communication can be mapped onto the state variable, and determined uniquely from the state of the consumer. We estimate the consumers' dyadic social communication by maximum likelihood. The utility for x amount of dyadic social communication is given by equation (2.7). Maximizing this utility as a function of the state variable $\mathbf{s}$ and the unobservable ν gives us the optimal amount of dyadic social communication chosen by the user. We use the data to perform an inverse mapping to the realized value of the random variable denoted as v_{it} and estimate the parameters $\mathbf{\Gamma} = (\gamma_1, \gamma_2, \gamma_3)$ by maximum likelihood. This mapping is derived from the FOC of (2.7) and is given by:

$$v_{it}(\mathbf{s_t}, x_{it}, \gamma) = \frac{c}{(\gamma_1 + 2\gamma_2 x_{it})\left[1 + \gamma_3\left(1 - \Phi_{\mathbf{s_t}}(s_{it})\right)\right]} \tag{2.13}$$

The likelihood for dyadic social communication is then specified as:

$$L_{DC}(\gamma) = \prod_{i=1}^{I}\prod_{t=1}^{T} f_V(v_{it}|\mathbf{s_t}, x_{it}, \gamma) \tag{2.14}$$

[10]See *Bajari et al.* (2007)for Monte Carlo evidence on the point of efficiency.

where f_V is the density function of the random variable V with log-normal distribution, $\log V \sim N(0,1)$. Since the current consumption of dyadic social communication does not affect the state variable in future periods, we first estimate $\mathbf{\Gamma}$ and treat it as known in the algorithm for recovering the structural parameters.

Dynamic Parameters: First-step Policy Function

The first step recovers the policy function from data and is used as an input to compute the value functions via simulation. We use a logit model to characterize the choices of consumers as a function of the state of the consumer, the positional status of the consumer among his peers, the number of peers the consumer is tied with for status, and the consumer's characteristics. If the amount of data were not an issue, we could consider a non-parametric approach to estimating the first step.

Dynamic Parameters: Second-step

In the second step, we utilize the recovered policy functions $\hat{\sigma}$ from the first stage to determine the estimates of the structural parameters. This second step consists of several stages, and we describe them in turn below.

In the first stage, the estimates of the policy functions along with the period utilities is used to obtain the value function by forward simulation as detailed in *Bajari et al.* (2007). This is designed to recover the value function for a specific consumer i beginning with an observed state $\mathbf{s_0}$, and is denoted by $V_i(\mathbf{s_0}|\sigma,\theta,\beta)$ for a policy σ. The value function thus depends upon the policy, the dynamic parameters represented by θ and the 'true' first-stage parameters, β. We linearize the value function, so that the computation of the inequalities can proceed independent of the parameter values. Thus, we represent the value function in the form of the dot product of the parameter-free vector W_i and the augmented parameter vector $[\theta,\, 1]$:

$$V_i(\mathbf{s}|\sigma,\theta,\beta) = W_i(\mathbf{s}|\sigma;\beta) \cdot [\theta,\, \mathbf{1}]$$

We can consider the parameter-free representation of the value function because the period utility function in (2.3) is linear in each of the parameters in Γ. For most of the estimation process, we are concerned only with the parameter-free value function $W_i(\mathbf{s}|\sigma)$.

The process involves the following computations performed for $t = 1, \ldots, T_S$ periods:

1. Draw a private vector of unobservables $\epsilon_{\mathbf{kt}}$ for each consumer $k \in \mathbf{I}$.

2. Determine the choices for each consumer according to the specified policy $d_{kt} = \sigma_k(s_t, \epsilon_{kt})$.

3. Obtain the state in period $t + 1$ using the transition rule in equation (2.2) for each consumer.

4. Calculate the period utility for consumer i.

This forward simulation procedure gives us the value function for a consumer i corresponding to any specified policy σ.

In the second stage, we perform the forward simulation to determine the value function under two different policy functions. The first policy $\hat{\sigma}$ is recovered from data in the first step and is treated as the solution to the consumers' problems, i.e. the equilibrium strategy profile. Note that since our dynamic decision is binary (contribute or do not contribute), we can interpret $\hat{\sigma}$ as corresponding to specifying a threshold error $\hat{\epsilon}_i(\mathbf{s})$ for each state, so that $\hat{\sigma}(\mathbf{s_t}, \epsilon_{it}) = 1 \Longleftrightarrow \epsilon_{it} > \hat{\epsilon}_i(\mathbf{s_t})$.

The alternative policy is a perturbation of $\hat{\sigma} = (\hat{\sigma}_{\mathbf{i}}, \hat{\sigma}_{-\mathbf{i}})$ for a focal consumer i that we denote as $\sigma' = \left(\sigma_i^{'}, \hat{\sigma}_{-i}\right)$. We construct the perturbed policy σ' from $\hat{\sigma}$ by adding a random disturbance to the threshold $\hat{\epsilon}_i(\mathbf{s})$ for consumer i. We determine the value function $W_i(\mathbf{s_0}|\sigma', \beta)$ corresponding to the alternative perturbed policy using the procedure detailed in the first stage above, using the same error draws as for the optimal value function, which reduces the variance introduced by simulation.

In the third stage, we draw from a set of inequalities $\mathcal{H}$, each element of which corresponds to a triple $(i, \mathbf{s}, \sigma')$, i.e. a specific consumer, a starting state for the network, $\mathbf{s}$ and an alternative (perturbed) policy σ'. The difference between the value function corresponding to the optimal policy and the value function corresponding to the perturbed policy, $W_i\left(\mathbf{s}|\hat{\sigma}, \beta\right) - W_i\left(\mathbf{s}|\sigma', \beta\right)$, must in theory always be positive since there are no profitable deviations from $\hat{\sigma}$. Therefore, whenever the difference is negative, the equilibrium conditions are violated, and we include the degree of violation captured by the difference between the value functions. The violation function is defined as:

$$g(i, \mathbf{s}, \sigma'; \beta) = \min\left[\left(W_i\left(\mathbf{s}|\hat{\sigma}, \beta\right) - W_i\left(\mathbf{s}|\sigma', \beta\right)\right)[\theta,\ \mathbf{1}], \quad 0\right]$$

The objective function is defined as:

$$Q(\theta, \beta) = \int g(i, \mathbf{s}, \sigma')^2 d\mathcal{H}$$

The value of the objective at the true parameters is zero, since there will be no violations of the equilibrium conditions when consumers play the equilibrium strategy.

To calculate the objective in practice, we obtain N_I inequalities from the set of inequalities, drawing $(i, \mathbf{s}, \sigma')$ and computing the associated value functions for consumer i_k. The sample analog of the inequalities is represented by the function $\tilde{Q}$ and used to form the objective for estimation:

$$\tilde{Q}(\theta; \hat{\beta}) = \frac{1}{N_I} \sum_{k=1}^{N_I} \min\left[\left(\tilde{W}_{i_k}\left(\mathbf{s^k}|\hat{\sigma}(\hat{\beta})\right) - \tilde{W}_{i_k}\left(\mathbf{s^k}|\sigma'^{(k)}(\hat{\beta})\right)\right)[\theta,\ \mathbf{1}], \quad 0\right] \tag{2.15}$$

where i_k represents the consumer chosen in the k-th inequality draw, and $\mathbf{s^k}$ is the starting state for that draw, and $\sigma'^{(k)}$ is the perturbed policy corresponding to that draw. The sample value functions $\tilde{W}$ correspond to the equilibrium value functions W and $\hat{\beta}$ represents the parameters for the first-step policy function estimation. The objective function is then defined as:

$$\hat{\theta} = \arg\min_{\theta} \tilde{Q}(\theta, \hat{\beta}) \tag{2.16}$$

The estimation procedure detailed above does not have a closed form for the asymptotic variance for the BBL estimator, because the variance depends on the inequality sampling procedure, and this makes the computation of standard errors difficult. To compute standard errors corresponding to the inequality estimator, *Bajari et al.* (2007) suggest the use of bootstrap or subsampling methods. Such resampling methods require the repeated estimation of hundreds, if not thousands of subsamples and are computationally intractable in our setting, since each estimation takes several hours. To overcome this hurdle, we therefore use the Laplace-type Estimator (LTE) proposed by *Chernozhukov and Hong* (2003), who prove that an MCMC approach can be effective in recovering the parameters of a generalized criterion function even in the absence of a likelihood function. The LTE essentially involves constructing a quasi-posterior density from the generalized sample criterion function (e.g., GMM), with the computation following an MCMC procedure.[11]

Since the BBL approach does not involve computing likelihood functions, but instead depends on the $Q(\theta, \beta)$ function defined in equation (2.15), the LTE is well-suited suited for estimation in conjunction with BBL. One additional advantage of using the LTE is that the standard errors do not need to estimated separately, and a single procedure can be used to obtain both the estimates and the standard error. In the LTE estimation process, we used $R = 10^6$ draws to obtain the quasi-posterior distribution. Following the guidelines suggested by *Chernozhukov and Hong* (2003), we set the candidate parameter to be a random walk from the current parameter value, with the variance of the random walk selected to ensure that the rejection rate falls in the [0.5, 0.8] range.

2.5 Empirical Application

The model described above can be applied to settings with an interconnected network of consumers, where we observe social relationships as well as contributions to connected goods. We apply the model to data from a mobile network, with data on social relationships derived from communication patterns. Subscribers purchase ringback tones that are a connected good, and make a contribution to their social network in doing so. Below, we describe the institutional details and the data we use in our application. We then report the parameter estimates from our BBL procedure which involves the use of a flexible first-stage reduced-form policy function estimation and a logit model to regress the purchase or contribution decision with the state.

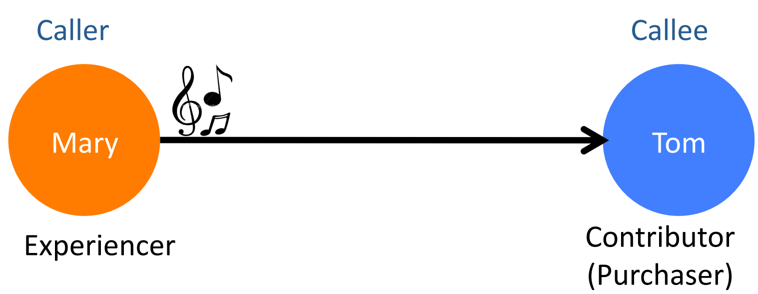
[11]Note that LTE is a classical estimator, not Bayesian, although the MCMC procedure for computing is commonly used in Bayesian analysis.

2.5.1 Institutional Setting

The data set is provided by a global mobile phone companies in a large Asian metropolis. Cellular service providers have relied primarily on revenue from voice calling services in the past decade. However, as this revenue source saturates, they are attempting to increase revenues from data services. Data services like ringtones and ringback tones, as well as video shows and TV-enabled content are expected to demonstrate double-digit increases over the next several years according to a recent research report by IDC, and are becoming primary drivers of growth for mobile phone carriers. For the mobile phone provider from whom we obtained our data, the revenue from the ringback tones represents the fastest-growing stream of data revenue (40 percent), followed closely by web services and ringtones.

Whereas ring tones have been available for a few years, ringback tones are more recent, and have become popular first in Asian countries before being introduced by carriers in the US. The increased adoption of data services by consumers has also been accompanied by the sales of smartphones that can access e-mail, the mobile web and other rich media content (*Lawton*, 2005).

Figure 2.2: Ringback Tone (Experienced by Mary, Purchased by Tom)



Ringback tones are purchased by a subscriber to replace the standard "ring-ring" sound with a musical tune that plays for for about 20 seconds, and often features popular contemporary music.[12] Subscribers purchase ringback tones by sending a text message requesting the tone, or by calling the customer service department of the mobile company. Ringback tones are heard by the purchaser's callers, and not by the purchaser whereas a ringtone is heard by the purchaser or callee. We demonstrate an example in Figure 2.2, where Tom is

[12]Note that ringback tone is activated at the network level, even before the call is transferred to the subscriber's phone.

the contributor or purchaser who has incurred a cost to contribute the new ringback tone, whereas Mary is the caller who experiences the tone when she calls Tom. Thus, Tom makes a conspicuous contribution to his social network peers, and seldom derives consumption utility from experiencing the tone.

The ringback tones are varied, and include popular musical tones as well as music from different eras, and instrumental tones. Most purchases we observe are of the popular music variety, but since we do not yet have access to explicit data characterizing the genre of other details of the tones, we are unable to further build upon this dimension. For the purpose of understanding why consumers contribute to connected goods, rather than focusing on specific song choice, we evaluate the purchase/no purchase (or contribute / no contribute) decisions.

We consider the ringback tone to be an ideal example of a connected good for two reasons. First, the social tie is over the phone and is likely to be stronger than designating someone as a friend in an online network. This setup provides us an offline context that may track social relationships and behavior more accurately than studies conducted with purely online data. Second, we can obtain not just communication ties between individuals but product purchase decisions that are similar to traditional marketing scenarios and unlike most online studies that track activities like updating photographs and videos, or installing freely available applications on Facebook. Our results can also be viewed as an illustration of how consumers use new data services that have a social component.

2.5.2 Data Description

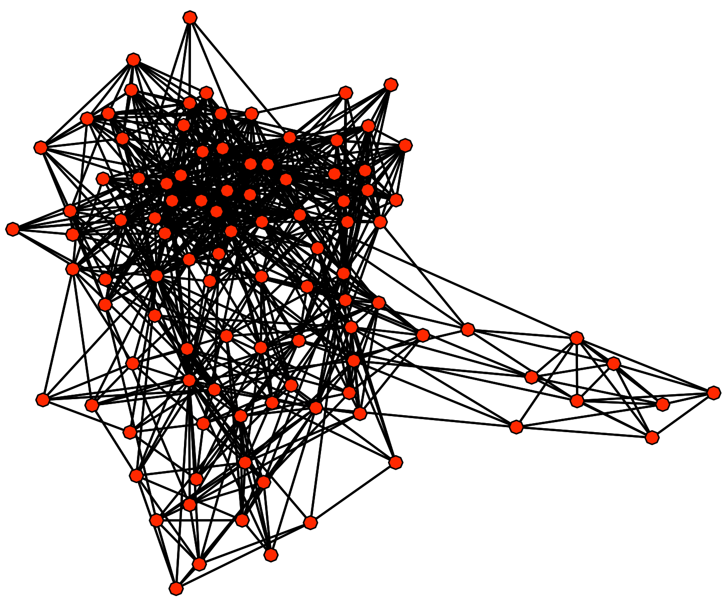
Our data is a panel data with the complete calling and purchase history of all the networked customers from a large cosmopolitan city over a nearly six-month period spanning Dec 2007 – May 2008. It has the complete call records indicating the phone numbers of the calling and called individuals, along with the date, time, and duration of each call. For each consumer, we observe their purchases of ringback tones with information on date, tone downloaded, and price paid. Moreover, we also have access to demographic variables such as age, gender, geographic code, etc.

We select the sample according the following rule: to ensure we focus on regular consumers, we begin by randomly selecting a seed from subsample where consumers contribute at least 3 times, i.e. the seed customer makes at least 3 purchases during the entire panel length. Then, from the calling history, we derive the social relationship matrix R for the seed customers and their peers by using the following rule: we specify a social relationship connection between two individuals whenever each in the pair makes at least 5 calls to the other. We do this to focus on stronger ties, ignoring the weak ties that might result from occasional calls. The social network graph is detailed in Figure 2.3 below. Each consumer is indicated as a node, and the edges connect consumers who have a social relationship. Beginning with the seed's connected peers (ego network), we recursively obtain the network. We repeat the recursive procedure for each consumer who is included until we span a depth of three levels. We determine which set of peers to include in the sample by maximizing the ratio of within-sample ties to total social ties for the sample, i.e. we choose a cohesive network where a person is included only when they have a high ratio of the number ties to consumers in the sample to the number of ties to those outside.

The sample data contains calling history of 109 networked customers. Table 2.1 below presents statistics for the social ties, network promotional exposures and purchases. Consumers make an average of around 15 calls per week to their peers, but there is a large variation in calling volume among consumers. The talk time per week, or amount of time consumers spend in voice conversations is a little more than an hour per week, and the variation in talk time is fairly large as well. Consumers purchase a ringback tone every 5 weeks on average, and again, we find a significant variation in purchase behavior across consumers. Our sample consumers have a majority of men and are mostly 20-45 years old. We note that ignoring the structure of the network connections, and assuming every consumer to be connected to everyone else is clearly likely to lead to biased results given the heterogeneity of the interconnection structure.

In the network connection depicted in Figure 2.3, notice that there is much heterogeneity with respect to the number of links (edges), and that some consumers are very heavily connected to other peers, whereas some have few such connections. The large concentration of consumers heavily interlinked in the top half of the graph may be more central to the network, whereas the consumers in the right half are more peripheral. It is important to take into account the position of each consumer in their social network. Although the social network literature has several centrality metrics, like degree and betweenness centrality, we compute the *eigenvector centrality* for each individual in the network, which is defined as the greatest eigenvalue of the social relationship matrix, $\mathbf{R}$. This variable has proven very appropriate because it captures not just the number of social network ties, but the importance of peers with whom the individual is connected. There is also theoretical support for this measure to be relevant when we consider the interaction between an individual and the network (*Ballester et al.*, 2006).[13] Google's PageRank algorithm was initially based on this measure to evaluate the importance of web pages and sites. Note that the graph does not indicate the call volume between individuals, merely the presence of a social relationship.

Figure 2.3: Social Network Derived from Call Patterns



2.5.3 Estimation

Dyadic Communication

The dyadic communication parameters are estimated using the likelihood function defined in equation (2.7). The parameter estimates are detailed in Table 2.2.

[13]See (*Wasserman and Faust*, 1994) for a definition.

We can see that the calling utility is positive in the linear term and negative in the quadratic term indicating decreasing marginal utility for calling time. We also observe that the status interaction term is positive and significant. This suggests that higher contribution status enhances value of dyadic voice communication and confirms our intuition that higher contribution status enhances the value of other social communication activities.

First Step – Policy Function

In the first step, we flexibly estimate the policy function as a parametric regression, specifically a logit model. These estimates must not be structurally interpreted, since they only represent the observed behavior in the data as recovered by the above specification of the reduced-form first stage. With large amounts of data, we would use a non-parametric approach to flexibly recover the policy function. However, we follow the literature in applying a parametric model with a small number of consumers and time periods (*Sweeting*, 2006; *Macieira*, 2007).

Second Stage – Structural Parameters

We next use the policy function recovered in the second step to estimate the structural parameters as described in Section 2.4.2. We list the structural parameter estimates below in Table 2.3. We describe the effects in the order they appear in the utility function described in (2.3). The coefficient of self-state can be interpreted as an self-expression, with consumers who have more negative of the coefficient θ_{11} as having a higher utility for relatively newer contributions. Surprisingly, we find that this coefficient is positive and significant. This implies that consumers are more likely to contribute when their contribution state is lower, i.e. they actually don't place a higher value on making a contribution when they have not contributed for a longer period of time. Older customers value having a lower state, which could be interpreted as a stronger preference for making a contribution due to self-expression. The gender of the individual does not have a statistically significant effect on the utility of self-expression. We find that consumers who are more centrally located in the social network value self-expression less.

Moving on to the contribution status effect, we find that the coefficient of positional status

θ_{21} is positive and significant, implying that consumers really care about their contribution status relative to their peers. They value have a higher contribution status, i.e. making more frequent or timely contributions relative to their peers and the competitive focus driving the dynamic game, is the effect of contribution status. Since contributions are effected whenever consumers update their ringback tones, consumers with tones replaced more recently than their peers will have a higher contribution status, if the behavior of their peers is constant.

The coefficient of the positional tie in contribution status on the utility of consumers is also positive and significant. It is also higher in magnitude than the coefficient of non-tied positional status. This confirms our conjecture that consumers also positively value being tied with their peers and that they value being tied with their peers differently compared with have a higher contribution. Relative to average consumers, older and male consumers place more value on positional ties. More interestingly, higher centrality consumers place a larger value on ties, and we expect that this may be because more central consumers have a larger number of connected peers, and competing intensely in situations with positional ties might be more costly for them.

The coefficient of consumption measures the marginal utility that consumers receive from contributions made by their peers, i.e. whenever their peers replace their ringback tones. As expected, consumers obtain a positive consumption utility from the contribution made by their peers. Consumption benefit is higher for older consumers, which is consistent with the observation that older consumers may contribute to induce contributions from peers, rather than to compete for positional status. In addition, more centrally located consumers in the network value consumption more.

The cost parameter characterizes a fixed cost of contribution rather than the price paid by the consumer, and must be interpreted with care since it is a measure of the fixed contribution cost incurred whenever the consumer makes a contribution. We find this coefficient to be negative, as expected, with older and male consumers having more negative values, and highly central consumers having less negative values of this cost effect.

2.5.4 Policy Simulation

Modeling the interdependence of consumers in a structural manner allows us to perform policy simulations to determine the answers to "what-if" type questions. Indeed, this has been long been recognized as one of the strengths of the structural modeling approach. We begin with the parameter estimates recovered in the previous section, and examine the effects of differences in the social network environment for the same set of consumers.

We do not focus on computing the MPE of our model due to computational constraints, since the large number of consumers in our sample does not permit that, and simulating different policy functions or varying parameters is therefore not the avenue for further analysis. Rather, the policy simulations we conduct allow for us to examine the implications of starting with different states of the network, varying the exogenous state. This method allows us to characterize the resulting dynamic effects that affect the mobile service provider.

Characterizing the Networked Value of a Consumer

Each consumer contributes to the social network, and thus creates value for peers in the network, but only captures part of the value. Since the product is an information good, the non-rivalrous property enables multiple peers to receive consumption benefits from a single contribution. We seek to answer the question: how much of the value created by a contribution is captured by the user? This is not as straightforward as it may initially seem: the contribution creates utility for peers, which in turn leads to the reflection effect for the contributing consumer. Our dynamic model permits the careful delineation of the value internalization by consumers in contributing to connected goods.

We find that on average consumers create experience or consumption utility for their peers by contributing to connected goods. There is a lot of heterogeneity in the sample of consumers with some creating little value for others. How is a consumer's local network affected when the consumer is removed from the network? This simulation will inform us of the value of each consumer, specifically it enables us to determine which consumers the network would suffer most from losing. We evaluate this factor by altering the network structure (relationship matrix), so as to remove all the social ties belonging to the focal consumer. We then study the effect on peers in the social network, paying specific attention to the utility effects and resulting contributions. As an example, consider removing consumer 18 from the network. In our setting, consumer 18 is connected to 7 other peers. It is not immediately clear whether 18's peers will lose utility from 18's absence. Obviously, they lose consumption utility because they do not have access to the contributions to connected goods made by 18 anymore. However, 18's absence may improve the status of the remaining peers, especially if 18 usually made frequent replacements, and therefore had a low state and high status. Thus, the effects of the status and consumption utility may be in opposite directions. Notice that in a densely connected network, the absence of a consumer may not impact status competition by much. However, in a setting where consumers have few peers, then the absence of a single consumer will significantly impact the status competition.

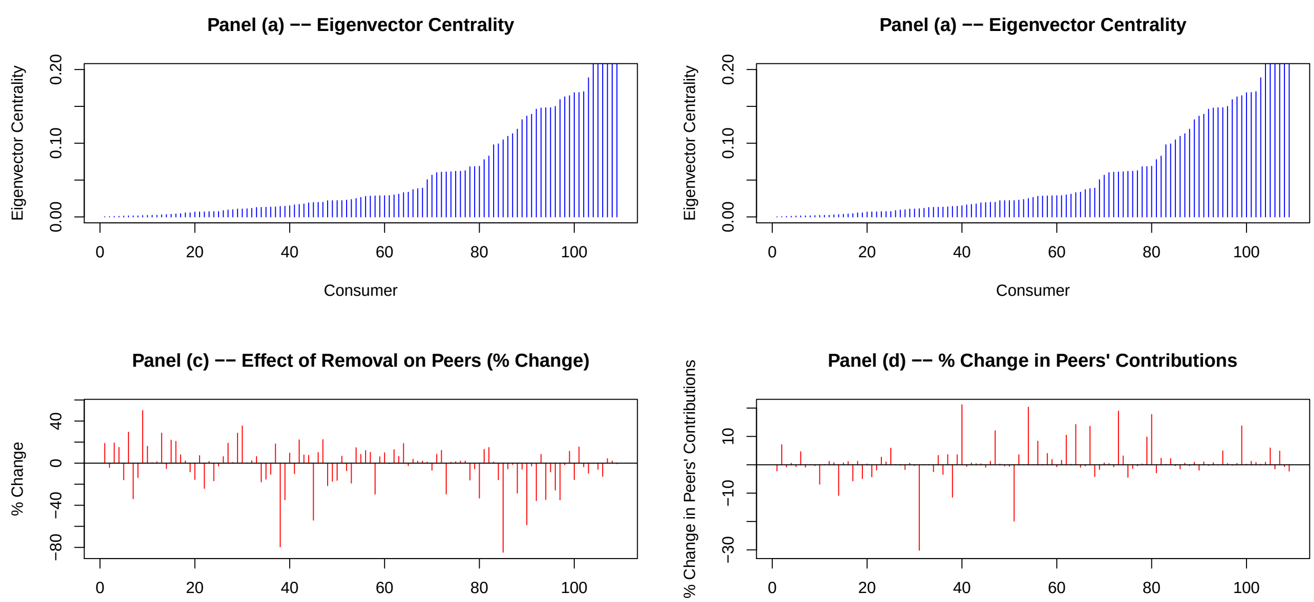
The simulation details are as follows: we begin the network with the same endogenous state for the original full network and the altered network with one consumer's social ties removed. In both cases, we simulate the system $N_{SIM} = 1000$ times, with each simulation proceeding for T_{MAX} periods, chosen so that the degree of discounting is less than 1%. We compute the current value of the expected discounted stream of utilities in all cases. We simply aggregate over the utilities of peers to obtain the overall utility in the presence and absence of a specific consumer.

Broadly, we simulate the model to determine the effects of absence of a consumer on peers by modifying the exogenous state. We obtain the difference in utilities for consumers who are ordered by their centrality scores. The results of the simulation are detailed in Figure 2.4 below. The horizontal axis represents consumers after ordering them based on eigenvector centrality, with the least central consumers on the left and with consumers becoming more central as we move rightward along the x-axis. The vertical axis in panel (c) captures the fractional change in peer utilities when the consumer is removed from the network, i.e. when consumer X is removed, what is the percentage change in the utilities of X's peers?

More importantly, much of the literature in social network analysis posits that centrality is key to identifying important players in the network. We measure importance by how much the absence of a consumer affects connected peers. There are several points that stand out from examining Panel (c).

First, we observe that not all consumers have a negative effect on their peers when removed, i.e. their peers benefit when they are not present. From the factors discussed above, i.e. contribution status and consumption utility, we know that when a consumer is absent, the loss of consumption utility experienced by peers is always positive. We expect that when certain consumers leave, they diminish the high intensity of status-based competition, so that their absence leads to less intense competitive pressures, and the peers benefit from this effect. Second, the range of difference in utility is approximately -84 % to + 50 %, and we find that 58% of consumers have a positive effect, so that their absence results in a loss of utility to their connected peers. Third, we find a higher concentration of negative impacts on the right end of the centrality distribution. This finding is counterintuitive, implying that when more central consumers leave, we might actually have a positive utility impact on that consumer's peers.

Figure 2.4: Policy Simulation



We find an interesting inverted-U shaped curve depicting the percentage change in contributions by peers following the removal of a consumer, detailed in Panel (d). Specifically, consumers with low or high levels of centrality have a lower effect on their peers, as compared to consumers with middling levels of centrality. We expect that for low centrality consumers, their peers are not very central either, and from the previous section, we know that low centrality consumers value consumption less and are therefore less likely to be influenced by making a contribution. Hence, the absence of such a consumer is not likely to lead to much loss due to lower contributions and the impact of absence may be small. For highly central consumers, we expect that the marginal value added due to higher contributions to peers to induce consumption may not have a large marginal impact, since the impact of any one consumer on the status of peers will likely be small.

Suppose we choose the top 10% (11 consumers) as indicated by the eigenvector centrality measure and evaluate whether these consumers are the most valuable to the network. We find that only one of these consumers would rank in the top 10 % as chosen by the actual contribution difference due to the absence of that consumer, as indicated in Panel (d).

2.6 Conclusion, Limitations and Future Work

Enabled by technology-mediated web and mobile social networks, we find a world of shared content available on multiple devices that is primarily generated or contributed by consumers. These connected goods are contributed by consumers with the intended experiencers being network peers of the contributor. Consumers visit these network platforms primarily because they value content contributed by their peers. It is therefore important for companies to understand the drivers of the social networking phenomenon: what motivates consumers to contribute to connected goods, adding content to the social network even though they do not obtain immediate consumption utility? Why do some consumers contribute a lot and others little? How does leadership in network contribution vary with network position and network characteristics? What are the substantive implications of understanding the drivers of contributions?

In this paper, we recognize that consumers contribute to connected goods for the following inter-related reasons: (a) consumers obtain a utility from having a higher relative contribution status compared to their peers, (b) consumers have (future) consumption utility from the connected goods provided by their peers. The combination of these two factors imply that consumer contribution decisions are dynamically inter-related in the sense that it can induce peers to contribute due to the change of relative contribution status. We formulate consumers' decisions on whether to actively contribute content to the social network as a dynamic game in the tradition of *Ericson and Pakes* (1995), and focus on Markov Perfect Equilibrium as the solution concept. In this game, forward-looking consumers forward-looking consumers strategically manage their relative network contribution status in order to maximize long-term utility. They evaluate the inter-temporal tradeoff between contributing at the current time period without receiving any immediate consumption benefit but with the intention of influencing the contribution decision of other consumers so as to enjoy future consumption benefits delivered through peers in the social network.

Based on a panel data with purchase history of ringback tones of consumers as well as phone calls between pairs of consumers, we construct the social network and estimate the parameters of our model using the two-step estimation approach proposed by *Bajari et al.* (2007). We demonstrate that positional contribution status plays a significant role in explaining consumer contribution decisions in a social network setting. This can encourage a "race" among consumers, who make strategic contributions involving inter-temporal tradeoffs to maximize their current and expected future utilities. We conduct policy simulations to investigate the influence of consumers in the network and evaluate the evolution of network contribution decisions when a fraction of consumers are recruited as seeds to encourage their peers to contribute.

In summary, we establish the microfoundations explaining why consumers are willing to contribute to connected goods: they contribute to burnish their network contribution status among their peers, and they receive consumption or experience utility when their friends contribute. This study is among the first to apply the empirical analysis of dynamic games, originally developed to study industrial organization, in the context of investigating interdependence in the decisions of consumers connected by a social network. Our results suggest several directions for managers who face challenges in attracting more voluntary contributions of user generated content when designing an online social network platform. Since competing for status and consumption benefit from contribution made by peers are strong drivers of contribution, designers can leverage their platform design to take advantage of these factors. More specifically, our results suggest the following guidelines:

1. Social network platforms can be designed to better encourage competition among peers

by frequently announcing recent contributions made by each consumer and by providing a clear ranking order of contributions. For example, companies can explicitly display the ranking of contribution status of each individual relative to her peers. Such a message might say "Alex is the #1 contributor of videos in January. Her total views are 232." Such an explicit ordering would likely lead consumers to implicitly compete and strengthen the status effect.

2. Incorporate an instrumental benefit, like a prize for some of the higher contributors, especially when the network has a lower level of activity. This will make the implicit competition more explicit.

3. When selecting seeding consumers, the firm should differentiate consumers with high centrality and with high influence. Even though central consumers have many more connections, the impact of their high ranking status may be diluted and thus less influential in encouraging peers to contribute. It is more effective for firms to target consumers considering the impact of the current state vector of the network.

Our research is subject to assumptions and limitations which we view as open avenues for further exploration. First, future research can relax the assumption that social ties between individuals are exogenous. An approach that explicitly models how and why consumers form social relationships with specific peers, and how such relationships evolve would be very helpful (*Jackson*, 2008). Second, for computational feasibility, we assume individuals perfectly observe the state of each of their friends in every period. It will be interesting to consider web network platforms, where there is a large amount of message and multimedia traffic among consumers, where it might be difficult to form an accurate impression of the state of each peer. As the number of connected peers increases, and consumers have hundreds of friends, we can instead allow them to have an imperfect perception of the contributions made by their peers. Third, a particularly promising direction would be to consider different types of contributions, extending the one-dimensional nature of connected goods in our setup. When consumers in the platform have access to different media, consumers face trade-offs between contributing multiple types of connected goods, each with different measure of status. It will be instructive to study consumer choice of contribution types. Fourth, another avenue would be to investigate how consumers may be interdependent in their responses to marketing activities by firms, especially in the context of mobile and web social networks. Finally, the overall framework with status competition in the dynamic competitive setting developed in this paper can be extended to other settings. One such context is the contributions to open source software, where developers make contributions to publicly available open source to signal their ability and attain status in the project team (*Lerner and Tirole*, 2002b). Another setting where status-based competition may prove particularly useful is in designing salesforce compensation schemes including sales contests.

Table 2.1: Sample Statistics

Call and Purchase Statistics	***Mean***	***SD***
Weekly incoming call volume	14.70	21.96
Weekly outgoing call volume	14.70	19.84
Weekly ringback tone purchases	0.18	0.21
Weekly voice talk time (in minutes)	68.11	111.52
Demographics		
Age (in years)	37.01	11.42
Gender (1 = Male, 0 = Female)	0.88	0.22
Centrality	0.06	0.07

Table 2.2: Dyadic Communication (Instrumental Benefit) Estimates

Symbol	**Parameter**	**Estimate**	**SE**
γ_1	Linear Calling Utility	2.31	1.12×10^{-1}
γ_2	Quadratic Calling Utility	-6.20×10^{-6}	3.15×10^{-6}
γ_3	Status Interaction	9.56×10^{-1}	2.04×10^{-1}

Table 2.3: Structural Estimates of Dynamic Model

Parameter	Symbol	Estimate	Low CI	High CI
Self-state	θ_1	0.012	0.009	0.015
	Age	-0.050	-0.058	-0.042
	Gender	0.001	-0.003	0.006
	Centrality	0.067	0.055	0.078
Positional Status	θ_{21}	0.327	0.295	0.361
	Age	-0.786	-0.824	-0.732
	Gender	0.085	0.049	0.123
	Centrality	1.352	1.333	1.371
Positional Ties	θ_{22}	1.099	1.053	1.173
	Age	-0.076	-0.105	-0.044
	Gender	1.207	1.173	1.256
	Centrality	1.085	1.061	1.104
Consumption	θ_3	0.877	0.830	0.939
	Age	1.041	1.020	1.060
	Gender	1.077	1.052	1.120
	Centrality	0.959	0.921	1.023
Cost	θ_4	-2.186	-2.227	-2.150
	Age	-2.140	-2.193	-2.111
	Gender	-0.256	-0.347	-0.208
	Centrality	1.675	1.609	1.735

CHAPTER III

A Hidden Markov Model of Consumer Susceptibility to Network Influence

The explosive growth of the social networking phenomenon has been well recognized by the media and trade press over the past few years. Several kinds of social networks like Facebook, Myspace and Twitter that serve to connect people online, as well as smartphone-enabled phone networks are becoming increasingly important as social media. Users sign up with microblogging services like Twitter to follow the activities and messages of their friends and acquaintances and to post their own updates, which are similar to blog posts but limited in length. Whenever a user posts an update, called a tweet, the users followers are notified. One major advantage of twitter is that it is designed to be accessible across multiple networks i.e, on the web as well as via text messages on the cell phone, and it has reported a growth of more than 1000% over the past year (2008-2009). Marketing managers at companies like Dell and HP are already leveraging these social media as an effective promotional vehicle. When users follow Dell, the company provides updates on its products, special deals, coupons and real-time feedback on user concerns, and the consumers friends and followers can observe these interactions.

In addition to general purpose networks like Facebook and Twitter, there are several other online sites that have added a social network feature as a complement to their core function. For example, Netflix primarily rents movies, but users can connect to their friends and observe what movies their friends have watched as well as the star rating given by them. Another example is Amazons Shelfari.com, which allows users to list the books they have read, their opinions and has a social networking feature to discover the reading habits of their friends. In addition to books and movies, social networks have sprung up around fashion groups, like StyleHive.com and FashMatch.com where individuals post their latest looks, and discuss opinion and evaluate fashion trends as well as discover who has similar tastes. Even newspapers like the New York Times with its online TimesPeople service have begun to offer content read by friends in the users Facebook network.

In all these settings including Netflix, Shelfari, Facebook, StyleHive examples, consumers receive promotions regarding adoption, purchase or consumption choices made by their friends. These promotions are not directed by the consumer at a specific person, rather they are observable by anyone who is connected to the consumer. More importantly, they are more likely to be perceived as credible by the receiver, since the promotion comes from a friend. The study of consumer-to-consumer social influence in product purchase has become important as consumers turn to technologically driven social media to obtain information and opinion on products and services, in addition to the social interaction that such media were designed for. Social media have dramatically lowered the costs of interactions and allow consumers to communicate with a subset of her social network in a very customized manner. The current and potential uses of social media in marketing has made it critical to examine and characterize interpersonal influence in such settings.

We examine the impact of a consumers exposure to the product used by another person in her social network. These exposures thus provide a network promotion effect that is the focus of this study. The consumers exposure to this network promotion can trigger a purchase, and can further result in additional purchases or adoptions by others in the social network. The social network promotion can be leveraged by viral marketing strategies that provide the product to a few carefully selected seed consumers, and rely on these consumers to promote the product to others, with the expectation that this propagation of influence can continue further. This strategy of carefully selecting seeds has recently been used by firms as varied as Google, for its GMail web-based e-mail product, and by Ford for its new Fiesta car aimed at the youth market (Barry, 2009). Several product firms choose prominent bloggers to promote their products, sometimes offering them a first chance at using a novel product.

A major objective of our analysis is to evaluate the effectiveness of network promotion, where consumers can be influenced by others with whom they share a social relationship. We incorporate and measure asymmetric influence as well as susceptibility to influence among consumers connected by dyadic social ties. We are broadly interested in examining the factors underlying the success of voluntary consumer-to-consumer promotions in a social network setting.

To better understand the consumer-to-consumer promotion through the social network, we seek to answer the following questions:

1. When are consumers most and least susceptible to network influence? Is network position or status a good predictor of susceptibility?

2. What is the long-term impact of promotional messages received from peers?

3. How important is the network promotion compared with other factors that may drive a purchase?

3.1 Related Literature

Early studies established the importance of evaluating the interdependence between consumer choices or preferences *Duesenberry* (1949) and researchers have long recognized social effects at least at the class or group level (*Leibenstein*, 1950), though not necessarily at the disaggregate dyadic level . This issue received little empirical attention in the intervening years, but has been gaining increasing prominence recently. To keep the literature review tractable, we focus on studies that use micro-level data to evaluate the adoption, purchase or consumption decisions for consumers with interdependence in preferences. Diffusion processes at the micro-level are also motivated by implicit communication between individual consumers, although such studies rarely use data on the individual-level social ties or networks.

Marketing researchers have empirically evaluated how individuals make simultaneous choices in a group setting, where individuals determine how to coordinate their decisions, and how their choice is different when the individual is not part of a larger group. Examples of these settings include choosing a movie in a theater, evaluating restaurants for dinner and several other group activities.

Much of the recent work has focused on the temporal influence effects, where consumers make decision sequentially, and the current paper belongs in this stream as well. Recent work examines both online and offline settings, and relies on individual-level observations. However, most studies use survey data to measure social ties, and the well-known recall biases apply in those settings. *Nair et al.* (2009) examine the influence of opinion leaders with regard to drug prescriptions - they posit a Poisson model of the number of prescriptions, and evaluate the influence of the social network on doctors prescriptions. They find that opinion leaders can strongly influence the amount of specific drugs prescribed by doctors. In their analysis, they control for unobserved correlation driving both prescriptions and social ties via individual-specific fixed effects. This eliminates the confounding effect of time-invariant homophily, that would imply that individuals are more likely to have ties with those similar to themselves.

Evaluating online and mobile communication settings which may differ from face-to-face interactions, perhaps the closest related studies are *Trusov et al.* (2009) and *Iyengar et al.* (2009). To the best of our knowledge, these are the only papers that actually use non-survey recorded measures of social relationships in the context of studying individual choices, and with the exception of *Iyengar et al.* (2009) and *Hartmann* (2009), the variable of interest is consumption of activities, not purchase decisions. *Trusov et al.* (2009) investigate how influential users updating their profile pages, status messages and content on Facebook may induce other users to increase their activity as well. They use a Poisson count model for login activity as their primary variable of interest, and evaluate the heterogeneity in how other users activities can be influenced by the focal user. *Iyengar et al.* (2009) evaluate how the purchase behavior of users on a Korean social networking site is influenced by the purchase activity of their peers, and model two decisions: binary purchase decision using a utility model, and quantity of purchase using a log-linear regression. They include lagged expressions of a users connections to others, separating indegree and outdegree effects to understand whether more connected consumers purchase more, as well as the social influence effect of observing when a friend has made a purchase.

3.2 Data and Institutional Setting

The institutional setting and data are identical to that described in Chapter II.

3.3 Hidden Markov Model

Hidden Markov models have found major use in the macroeconomics literature and these applications of hidden Markov model primarily deal with aggregate data. From a microeconomics and marketing point of view, the studies that have primarily used Hidden Markov models to construct customer-relationship management models include *Netzer et al.* (2008) and *Montgomery et al.* (2004) among others. Netzer models alumni giving to education institutions using hidden Markov model where the relationship between the individual and the institution is captured as the hidden state. Individual donors transition between states when they attend reunions or volunteer for events at their Alma mater thus the transitions between states is itself dependent on covariates which makes it a non homogeneous hidden Markov model. Structural models at the individual level have captured the hidden state as representing the degree of product and quality uncertainty, where consumers learn overtime about the true quality with each consumption experience. Now the hidden Markov models don't impose such specific structures rather they let the data determine what the hidden states are and how many there are. In the current model, we allow individual consumers to transition between states and the transitions are influenced not just by the consumer's actions but whether other peer consumers have taken similar actions or whether the focal consumer has engaged in social interactions with his or her peers.

The states of the hidden Markov model are not observable (by definition), but affect the decisions and choices of individual consumer. Only by observing the choices can we infer the state that the consumer is given at any given point of time.

The main components of the hidden Markov model are:

1. The initial state

2. The transitions between states

3. The choices as functions of state and other covariates.

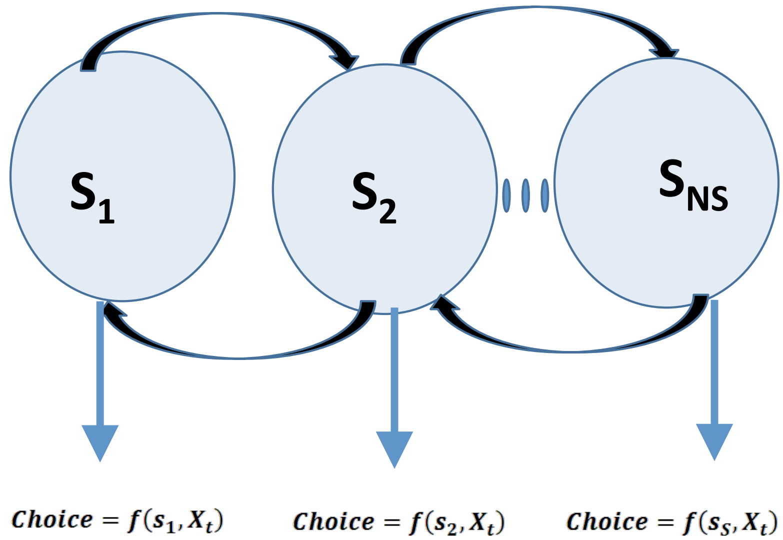


Figure 3.1: Hidden Markov Model with S states

An example of the general Hidden Markov model is given in Figure reffig:hmm.

The initial state distribution is denoted by $\pi_i = (\pi_i(s_1), \ldots, \pi_i(s_S))$ which is different for each consumer and $\pi_i(s_k)$ denotes the probability of the consumer being in the state k initially at $t = 0$. The transition matrix is denoted by $\mathbf{\Gamma_{it}}$ and depends on both the individual and covariates corresponding to the individual at time t.

We estimate the hidden Markov model by embedding it in a hierarchical Bayseian framework which allows us to model individual specific heterogeneity. The parameters of coefficient corresponding to each individual are modeled to depend on the individual's demographic characteristics which are time invariant, which complements the hidden states which are time varying.

3.3.1 Consumer Model

Consumers are indexed by $i \in \{1, \ldots, I\}$ and time periods by $t \in \{1, \ldots, T\}$. Each consumer i can be in any one of a set of latent (hidden) states, denoted by $s_{it} \in \mathbf{S} = \{1, \ldots, S\}$.

The consumer's purchase decision in time t is represented as R_{it} defined as follows:

$$R_{it} = \begin{cases} 0, \text{ if consumer } i \text{ makes no purchase in period } t \\ 1, \text{ if consumer } i \text{ purchases a new CRBT in period } t \end{cases}$$

We use a multinomial logit specification for the choice process undertaken by consumers. The utility the consumer receives from a purchase is indicated by u_{it} given as:

$$\begin{aligned} u_{it} = \; & \beta_{0i}(s) + \beta_{1i}(s) \textstyle\sum_{k \neq i} E_{ikt} I(\sum_{\sigma > \tau_{ikt}}^{t} R_{k\sigma} > 0) + \beta_{2i}(s) \sum_{k \neq i} E_{ikt} I(\sum_{\sigma > \tau_{ikt}}^{t} R_{k\sigma} = 0) \\ & + \beta_{3i}(s) \textstyle\sum_{k \in \mathbf{T_i^Z}} I(\sum_{\sigma > \tau_{ikt}}^{t} R_{k\sigma} > 0) + \beta_{4i}(s) \sum_{k \in \mathbf{T_i^Z}} E_{ikt} I(\sum_{\sigma > \tau_{ikt}}^{t} R_{k\sigma} = 0) + \epsilon_{it} \end{aligned}$$

The variable E_{ikt} represents the number of calls made by consumer i to network peer k in period t, whereas R_{kt} represents the purchase decision of the latter. We denote τ_{ikt} as the period prior to t when consumer i had called peer k, so it can be defined as: $\tau_{ikt} = \min \tau \{\tau < t : \sum_{tau}^{t-1} E_{ik\tau} = 0\}$.

Thus, the construct $\sum_{k \neq i} E_{ikt} I(\sum_{\sigma > \tau_{ikt}}^{t} R_{k\sigma} > 0)$ captures the number of impressions that consumer i receives from all her peers who have made a purchase decision in the same period, and we call this the new impressions. The next term in the utility function, $\sum_{k \neq i} E_{ikt} \mathbf{I} \left(\sum_{\tau=0}^{t-1} R_{k\tau} \right)$ captures the impressions that i has received prior to period t from her peers, and we term these re-impressions.

The next two terms are similar in the sense that they capture new impressions and re-impressions. However, we do not include all peers, but only peers who have a specific relationship with consumer i: Simmelian peers. There has been strong support for the idea that Simmelian peers have a fundamentally different relationship than a simple dyadic interaction that transcends the strength of a dyadic interaction. Simmelian peers are defined as individuals embedded in a triadic (or larger) group, and the presence of a third peer can alter the effects significantly. A pair of consumers i and k have a Simmelian relationship if in addition to having a bi-directional dyadic tie, there is a third m who has dyadic ties with both i and k. The set $\mathbf{T_i^Z}$ is therefore defined as $\mathbf{T_i^Z} = \{k : T_{ik}^{Z} = 1\}$ where T_{ik}^{Z} indicates the existence of a Simmelian relationship between i and k.

$$T_{ik}^{Z} = 1 \iff E_{ik}E_{ki} = 1 \bigwedge \exists m \neq i, k \text{ s.t. } E_{im}E_{mi}E_{km}E_{mk} = 1, \text{ where}$$
$$E_{ik} = \mathbf{I}\left(\sum_{t=1}^{T} E_{ikt} > 0\right)$$

In sum, the purchase utility in any state s is determined by i's impressions and re-impressions received from peers and Simmelian peers.

In period t, for consumer i the option of not purchasing is denoted $u_{it}^{0} = \epsilon_{it}^{0}$. Both ϵ_{it} and ϵ_{it}^{0} are assumed to be *iid* distributed as Type I EV, which makes the purchase probabilities the familiar logit case.

3.4 State Transitions

We model the state transition process for each consumer to follow a non-homgeneous process that depends on several variables that vary over time periods.

The state transition process in period t is denoted $\mathbf{\Gamma_t}$, and is defined as follows:

$$\mathbf{\Gamma_t} = \begin{pmatrix} \gamma_{11t} & \gamma_{12t} & \cdots & \gamma_{1St} \\ \gamma_{21t} & \gamma_{22t} & \cdots & \gamma_{2St} \\ \vdots & \vdots & & \vdots \\ \gamma_{S1t} & \gamma_{S2t} & \cdots & \gamma_{SSt} \end{pmatrix}$$

We do not restrict the state transition probabilities to be constant over time, and instead allow for a flexible non-homogeneous HMM. The elements of the state transition matrix are modeled as functions of co-variates, $\mathbf{Z_{it}}$.

To ensure tractability, we restrict the state transition process to move between adjacent states. Such an assumption is commonly made in applied settings in the marketing literature (*Netzer et al.*, 2008).

We model the transition probabilities at state s to follow an ordered logit process, where consumers can either move to the next higher state, stay at the current state or move to the next lower state. Thus, the probability of each of these events is given by:

$$\begin{aligned}
\gamma_{s,(s-1),t} &= \frac{exp(\eta_{s,L})}{exp(\eta_{s,L})+exp(\phi_{\mathbf{i}}\mathbf{Z_{it}})} \\
\gamma_{s,(s),t} &= \frac{exp(\eta_{s,H})}{exp(\eta_{s,H})+exp(\phi_{\mathbf{i}}\mathbf{Z_{it}})} - \frac{exp(\eta_{s,L})}{exp(\eta_{s,L})+exp(\phi_{\mathbf{i}}\mathbf{Z_{it}}} \\
\gamma_{s,(s+1),t} &= 1 - \frac{exp(\eta_{s,H})}{exp(\eta_{s,H})+exp(\phi_{\mathbf{i}}\mathbf{Z_{it}})}
\end{aligned}$$

The above ordered logit specification ensures that the transition probabilities sum to one, i.e. $\sum_{\hat{s}=(s-1)}^{(s+1)} \gamma_{s,\hat{s},t} = 1, \forall t$.

In the present specification, we include the following variables in $\mathbf{Z_{it}}$:

(1) Network Centrality, $Z_{i,1,t}$: We expect the state transition to be influenced by whether a consumer is more central and in contact with not just a larger number of peers, but more centrally located peers.

(2) Novelty of Current Tone, $Z_{i,2,t}$: If consumers purchase tones for the experience of their peers, then the fraction of peers who have heard the existing tone will impact the new purchase decision.

(3) Incoming Call Volume, $Z_{i,3,t}$: The incoming call volume will let consumers know that repeated exposure to the same ringback tone may decrease its desirability. In addition, this could also indicate social engagement by peers with the focal consumer.

3.5 Results

The preliminary results of the estimation are reported in Table **??**.

	Parameter	Posterior Mean	Posterior SD
Transition Parameters			
1	Constant	16.10	10.2
2	Centrality	85.13	37.7
3	Novelty	-114.16	62.1
4	IncomingCalls	63.29	35.1
5	ExpThreshold(1)	-59.49	33.8
6	ExpThreshold(2)	-46.32	22.4
Choice Parameters			
7	Constant(1)	91.22	39.5
8	New Impressions(1)	-62.40	32.6
9	Simmelian New Impressions(1)	14.10	9.7
10	Old Impressions(1)	-0.38	23.3
11	Simmelian Old Impressions(1)	-45.71	19.3
12	Constant(2)	-6.67	8.1
13	New Impressions(2)	84.15	45.2
14	Simmelian New Impressions(2)	-6.85	5.7
15	Old Impressions(2)	-9.41	18.3
16	Simmelian Old Impressions(2)	16.31	17.2

Table 3.1: Hidden Markov Estimation Results with S=2

CHAPTER IV

Sharing in a Market Context: Product Strategy for Open Source Software

Open source software is built through public collaboration and is distributed for free. The source code is published openly allowing others to modify and enhance it, in contrast to the traditional model of software where firms keep their source code private.[1] The free and open model has led to the increasing adoption of open source among consumers and firms, and has fundamentally altered the landscape of competition in the software market.[2]

A growing number of firms build commercial products based on open source software (*Economist*, 2009). These commercial open source software (COSS) firms enhance the features and usability of the existing open source software, resulting in a product that contains both publicly and privately developed components. A variety of open source licenses exist that dictate how modified versions may be distributed.[3] Certain licenses require COSS firms to release feature improvements to the public, where competing firms can incorporate them into their own products. Thus, some firms are able to free-ride on the contributions of others, a practice which Microsoft CEO Steve Ballmer referred to as "a cancer that attaches itself

[1]Open source software should be distinguished from two other forms of freely available software. Some firms make their software available for free ("freeware") but do not make the source code available (e.g. Adobe's Acrobat Reader). Another form is voluntary open source, where a firm releases the source code but with strong restrictions on its use and redistribution. We do not consider these cases because the strategic issues involved differ significantly from those faced by COSS firms.

[2]Popular examples of open source software include the Linux operating system, Firefox browser, OpenOffice, Apache Web Server, SugarCRM, and MySQL, among others. See http://www.sourceforge.net for a web-based repository of open source applications.

[3]We focus on the two types of licenses most common and relevant to the COSS industry: the GNU General Public License (`http://www.gnu.org`) and the Berkeley Software Distribution License (`http://www.opensource.org/licenses/bsd-license.php`). See *Laurent* (2004) for more discussion.

in an intellectual property sense to everything it touches," (*New York Times*, 2003).

The unique institutional arrangements discussed above raise a number of puzzling issues. First, why should a firm develop additional features for its product if competitors can freely appropriate these features for their products? Second, technology experts have pointed to cases where COSS products are of comparable or even higher quality than similar products produced by traditional, closed source software firms (*Dedeke*, 2009). How does a market where firms face a strong incentive to free-ride develop high quality products? Third, does the mandatory sharing of features always result in the provision of more open source? Fourth, when are firms and consumers truly hurt by the "cancer" of free-riding?

Despite the growing importance of the COSS industry, no research examines firms' marketing strategies in this novel setting. To address these questions, we incorporate the unique aspects of this industry into a stylized model, and analyze the competitive outcomes to shed light on these empirical puzzles. Our contributions are as follows:

(1) To understand competition between firms involved in the open source market, we develop a two-sided model in which firms interact strategically in both a product market and a developer market. Modeling the interaction between these markets allows us to endogenize the level of open source software available to the firms, which influences their product design decisions. We show that ignoring the developer market leads to a different ordering of product quality and surplus outcomes across licensing regimes.

(2) We show that free-riding is sustainable in equilibrium, that a COSS market can produce products of higher quality than a closed source market, and that open source contributions can be *lower* when firms are required to make their improvements public. We derive conditions under which different open source licenses maximize consumer surplus, and demonstrate that free-riding under shared features can actually *increase* both industry profits and consumer surplus under certain conditions.

(3) We contribute to the literature on individuals signaling on the job market. Whereas traditional signaling models (e.g., *Spence* (1973)) abstract from product market competition, we examine an imperfectly competitive market where developers' signals—as contributions to open source—are substitutes to a firm's own investments in product

quality. The signals are freely appropriable by all firms in the market, and thus impact firms' competitive interactions and industry outcomes.

We now provide a brief overview of the model and then discuss our key results in more detail. Our model has two interacting markets: a product market consisting of COSS firms who sell software products to consumers, and a developer market in which firms hire developers to create software products.

The product market is a vertically differentiated duopoly of ex-ante identical software firms who choose product quality and prices. A product's quality is a function of its feature and usability components. Depending on the open source license, the firm may or may not be required to publicly release the features it develops. In a *private features market*, a product's feature component is the sum of the open source features and any additional features the firm develops, which are not made public. In a *shared features market*, a product's feature component is the sum of the open source features and any additional features that *both* firms develop, which must be contributed to open source. A shared features market can enable either firm to free-ride on the contributions of the other firm. In contrast to features, usability improvements are always kept private; firms do not have access to a competitor's usability components regardless of the license.

Firms hire developers, whose skill-levels are heterogeneous and unobserved, to create additional features.[4] Consistent with empirical findings (*Roberts et al.*, 2006a), developers contribute to open source software to signal their skill-level (or type) to firms. As its popularity has risen, more firms rely on open source contributions to evaluate developers' skills.[5] Moreover, a developer's open source contributions are more transparent than those made to proprietary code that is unavailable for review (*LinuxWorld*, 2007). In our model firms therefore make wage offers based on developers' open source contributions. The interaction between the product and developer markets determines equilibrium wages, the provision of open source software, and product qualities and prices. In particular, the level of open

[4]Open source features contributed by the developers and features created by the firm are considered substitutes. Software developer labor is the primary input to producing commercial or independent open source software (*Boehm*, 1981; *Lakhani and von Hippel*, 2003; **?**)

[5]A recent report estimates that up to 15% of all available information technology jobs call for open source software skills (*InfoWorld*, 2008), and the August 2009 Elance Work Index, a measure of firms' hiring of online contractors, placed three open source skills in the top 10 of 100 in-demand skills (*Reuters*, 2009).

source software available for firms to incorporate into their own products is determined by developers' efforts to signal their type.

Developers form expectations of the wage offered by firms conditional on market type (shared or private features). These expectations determine developers' incentives to contribute to open source. A higher expected wage tends to increase the initial level of open source, which in turn affects firms' decisions to develop more features. If the initial level of open source is high, then firms have less incentive to further improve their product's features because consumer preferences are concave. However, this puts downward pressure on wages because firms are willing to pay less for an additional unit of quality. Equilibrium product quality results from a balance between developers' incentives to contribute to open source and firms' willingness to pay for marginal product improvements. Thus, endogenizing the provision of open source software is critical to understanding COSS firms' product and pricing strategies. One important contribution of our paper is that we explicitly model the link between the provision of open source software and COSS products derived from open source.

We characterize the firms' optimal product design and pricing decisions and demonstrate how the issues raised earlier result from the competitive strategies of COSS firms. We compare the models' equilibrium outcomes in three cases: a shared features, a private features market, and a closed source market. We include the traditional closed source market in order to contrast our results to a market where firms do not base their products on open source. Our key results, mirroring the puzzles, are the following.

First, when the open source license requires firms to share feature contributions, free-riding exists in equilibrium: the (ex-post) high quality firm creates additional open source features and the low-quality firm does not. However, both firms develop positive levels of usability. The intuition is that the high-quality firm contributes to open source because the complementary nature of features and usability increases the value of differentiating on usability, and firms can privately appropriate the benefits from usability. The low-quality firm has less of an incentive to contribute features for two reasons: it can free-ride on the high-quality firm and its marginal value of additional features is lower because it develops less usability. This free-riding outcome is consistent with empirical findings that the high-quality Red Hat Inc. contributes significantly more code to Linux than any other vendor

(*Pal and Madanmohan*, 2002; *SD-Times*, 2008).

Second, comparing product quality in the private and shared features markets, we find quality can be higher with free-riding in the shared features market because of reduced competition between firms in the developer market, which lowers the cost of quality for the firms. This outcome is particularly strong when the market size is large and signaling is costly for developers. In the private features case, a large market causes firms to compete intensely for developers, raising their wages and the cost of quality to firms. In the shared features market, the effect of higher wages is muted because only the high-quality firm hires developers, resulting in a lower wage and hence a lower cost of quality.

Third, open source contributions can be *lower* when firms are required to make public their improvements to the open source software. Intuition might suggest that a mandate to share contributions produces more open source. Instead, we show that when the market size is large enough and signaling is inexpensive, the shared features market results in fewer open source contributions than the private features market.

Fourth, consumers and the low-quality firm are unambiguously better off in the shared features market with free-riding compared to the private features market. Under certain conditions, the high quality firm may also be better off in the shared features market. The intuition for consumer surplus is that free-riding between firms in the product market reduces product differentiation and increases the price competition between firms, which in turn benefits consumers. Traditional closed source markets are worse for consumers than either licensing regime unless signaling costs are excessive.

Fifth, the model's predictions change if we ignore the developer market and make wages and initial open source exogenous. Such a "one-sided" model predicts: (a) that the private features market always produces a better product than the shared features market, (b) that the high-quality firm always earns higher profits in the private features market than in the shared features market, and (c) that consumers are better off in the private features market than in the shared features market under certain conditions. These results contrast with the findings from our two-sided model in which both developer wages and open source quality are endogenous. In addition, our model incorporates a mechanism for developers to contribute to open source without resorting to a behavioral motivation, such as altruism, to rationalize developer contributions. This comparison highlights the importance of jointly modeling the developer market and product market.

Our paper relates to two distinct streams of literature that have largely remained disparate. The first stream includes the literature on strategic product design that focuses on functionality, product line variety, and pricing in a competitive setting.[6] Our model is broadly related to models of vertical differentiation with endogenous product quality decisions, such as *Shaked and Sutton* (1982), *Moorthy* (1988), *Desai* (2001), and *Kuksov* (2004). We contribute to this literature by modeling a setting where firms compete with products that include public and private components. Several recent papers study firms' strategies in the context of competition between open source and closed source software. *Leppamaki and Mustonen* (2003) examine the strategy of a monopolist firm that hires developers to create a competing open source product. The model assumes a perfect market for developers, ignores the multi-dimensional aspect of quality, and does not consider strategic interaction between the firms and developers. *Economides and Katsamakas* (2006) model two-sided pricing of operating systems and applications and evaluate the effects of competition between the platform providers. *Casadesus-Masanell and Ghemawat* (2006) focus on the dynamic pricing strategy of a profit-maximizing firm (Microsoft) facing a competitor (Linux) who prices at zero in the presence of network effects. However, the quality of the open source product in these papers is exogenous and neither paper explores the strategic interaction between firms or product design choices. Our work is complementary to these as we examine COSS products that do not compete with open source but are instead based on open source.

The second stream of literature examines the motivation of developers to contribute open source projects. Although a number of explanations have been considered, we focus on the one with the most empirical support: economic signaling incentives related to higher wages or career concerns (*Roberts et al.*, 2006a; *Hertel et al.*, 2003).[7] Contributing to open source projects can be a strong job market signal because potential employers can review a developer's contributions (*Leppamaki and Mustonen*, 2003).[8] *Lerner and Tirole* (2002a)

[6] *Krishnan and Ulrich* (2001) provide a comprehensive survey of product design.

[7] Other explanations include "ego gratification" due to peer recognition for technically challenging tasks (*Hars*, 2002) and altruism (*Andreoni*, 1990).

[8] An apt example of this can be found at `http://blogs.techrepublic.com.com/opensource/?p=821`.

conclude in a review of the literature that "a considerable amount of evidence is consistent with an economic perspective."

Thus, our paper integrates the above streams by jointly modeling the interaction between the open source developer market and the game of strategic interaction between COSS firms. We also contribute to the literature on individuals signaling on the job market to demonstrate their types. In the canonical model of *Spence* (1973), workers signal through investments in their education to a market of perfectly competitive firms who earn zero profits in equilibrium. Our model builds on this work, where developer contributions to open source features serve as non-dissipative signals to COSS firms seeking to hire high-skilled developers. In contrast, our model consists of an imperfectly competitive, vertically differentiated market where firms earn positive profits in equilibrium. Integrating the developer market is critical for our results because the relevant market outcomes depend on the feedback loop between developers' signaling incentives and firms' product development decisions.

Our findings have several managerial implications for firms involved in the open source industry. First, the type of license determines the structure of competition and has important effects in the product market as well as the developer market. These interactions must be understood in order to accurately determine the optimal product strategy. Second, when faced with asymmetric information or an imperfect market for product developers, firms can leverage it to reduce the intensity of competition and increase profitability. Managers in related industries who need to understand the COSS industry can recognize several competitive incentives that drive product strategy.

The rest of the paper is organized as follows. In the next section, we briefly discuss the history of the COSS industry and provide details on the distinction between features and usability. Section 4.2 presents the model for the private features market, first describing the product market and then the developer market. Section 4.3 modifies the basic model to accommodate a shared features market (Section 4.3.1) and a closed source software market (Section 4.3.2). Section 4.3.3 compares the models' outcomes in terms of equilibrium product quality, firm profits, and consumer surplus, and relates these findings to the observed industry puzzles. Section 4.4 concludes with a discussion of the managerially relevant aspects of competitive strategy in COSS markets and some directions for further research.

4.1 The Commercial Open Source Industry

This section provides some background on the unique institutional setting of the commercial open source industry, in which firms develop and sell software products based on freely available open source software.

The open source movement gained prominence in the 1990s as a small community of expert developers who made the source code to their programs freely available for anyone to use and modify. The growth of open source software has been rapid: market researcher *IDC* (2007) estimates that the value of this market will grow to $5.8 billion in 2011. A more recent *IDC* (2009) report estimates that revenue from open source software will grow at a 22% compound annual growth rate, mainly due to the fact that "the open source software market has seen a strong boost from the current economic crisis." Large technology firms such as IBM, Sun Microsystems, and Hewlett Packard have launched multi-billion dollar open source initiatives, in a trend that is likely to continue.[9]

Early adopters of open source software were primarily computer enthusiasts. Most consumers did not find the freely available open source software to be user-friendly and required significant technical expertise to use effectively (*Lakhani and von Hippel*, 2003). To meet this hand-holding need, COSS firms add value to open source software by improving the product quality in two distinct dimensions:

1. **Features:** provide additional functionality, in the form of new or modified source code, that extends the basic operations of the software. These changes could take the form of an installation program, a graphical user interface, or administrative tools. For example, Sun's StarOffice suite has additional features providing compatibility and support for different document formats than the open source OpenOffice.

2. **Usability:** enhance the user's ability to effectively use the product's available features. Usability enhancements often take the form of non-software services, such as online help, technical assistance, documentation, packaging, and other support services.[10]

A prime example of a COSS firm is Red Hat Inc. The company's commercial version

[9] Infoworld, "Open Source Platforms: IBM Invests $1 Billion in Linux," December 18, 2000.

[10] Usability can also include software code or additional programs that make the interface more accessible.

of the freely available Linux open source software is designed to simplify and extend the management and administration of the Linux operating system. Red Hat Linux operates under the GNU General Public License, which implies that Red Hat must make publicly available any feature contributions it makes to Linux, but can keep any usability enhancements private. Red Hat makes significant contributions to the Linux kernel, the Linux X Windows System, the GNU Compiler Collection (GCC), and others, all of which are made public under Linux's license (Pal and Madanmohan, 2002; Software Development Times, 2008). Red Hat provides customers with additional services, such as extensive documentation, installation and maintenance, and support programs that are available to customers who purchase their commercial product.

Numerous firms have, or continue to, adopt the COSS model, and COSS products now span a wide range of applications from productivity suites to business intelligence to customer-relationship management.[11] Industry professionals clearly recognize that building upon open source is a novel business strategy (*Goldman and Gabriel*, 2005; *Riehle*, 2007). Despite significant commercial interest in the COSS market, the academic literature lacks a model that jointly captures the incentives of developers to contribute to open source and the strategies of firms to build COSS products. We present such a model in the next section.

4.2 The Private Features Market

We model a duopoly of *ex-ante* identical firms competing in two separate but interconnected markets: the first is a *product market* in which consumers purchase software produced by the firms, and the second is a *developer market* in which firms hire developers. Our model of the product market builds on earlier work of vertical differentiation (*Shaked and Sutton*, 1982; *Moorthy*, 1988) in which firms choose product quality, and we incorporate the unique aspects of the COSS industry that alter firms' competitive strategies.

We begin with the private features model, and extend it later to the shared features and closed source cases in Section 4.3. The sequence of stages in our model, common to all the specifications, is as follows:

[11] An exhaustive list of commercial applications based on open source can be found at Wikipedia, http://en.wikipedia.org/wiki/Commercial_open_source_applications.

Table 4.1: Notation for Main Constructs in the Model

Symbol	Definition
Market Types	
P, S, C	Superscripts for private features, shared features and closed source markets
Exogenous Parameters	
M	Market size
c_L, c_H	Cost of contributing one feature to open source for low- and high-type developers
c_s	Cost of developing one unit of usability
η	Productivity of a high-type developer
$\hat{w}$	Minimum wage guaranteed in the alternative market for high-type developers
Equilibrium Strategies and Outcomes	
p_j, q_j	Price and quality of firm j's product
f_j, s_j	Number of developers hired and usability investment by firm j
f_0	Number of open source features due to developer contributions
e_L, e_H	Contribution to open source by low- and high-type developers
$w(e)$	Wage schedule in the developer market, with $e \in \{e_L, e_H\}$

Stage 1: Developers contribute to open source to signal their skill-level.

Stage 2: Firms simultaneously make wage offers to developers, who decide whether to accept. Firms set product features and usability, resulting in an overall product quality.

Stage 3: Firms simultaneously set prices in the product market.

Stage 4: Consumers make their purchase decisions.

In Stage 4, consumers purchase software after observing all qualities and prices. Firms set prices in Stage 3 after observing both firms' product qualities. In Stage 2, firms observe developer signals and simultaneously make wage offers and product development decisions. In Stage 1, developers choose whether to contribute to open source features to signal their skill-level given their beliefs about the wage.

Table 4.1 above details the notation used in our model and equilibrium analysis, distinguishing between exogenous parameters and endogenous outcomes in the model.

4.2.1 Model

Product Market: Consumers and Firms

Consumers choose whether to purchase one unit of software from either of the COSS firms or not to purchase a product, in which case they receive a utility of zero.[12]Consumers are heterogeneous in their preferences for quality, and a consumer indexed by θ has utility for a software product of quality q at price p given by:

$$U(q;\theta) = \theta q - p$$

The marginal valuation for quality is distributed uniformly, $\theta \sim \mathbf{U}[0, M]$, and market coverage is determined endogenously.[13]

The quality q of a software product depends on its level of features F and its level of usability s. We follow industry practice and view these dimensions as mutually exclusive (*Boehm*, 1981). Quality is defined by the production function

$$q = Q(F, s).$$

A software product's features define the set of tasks that can be accomplished with the product, whereas usability refers to the ease with which a consumer can make use of the product's features. Consumers value having more features and greater usability. However, an abundance of features may create an overly complex product. Without a sufficient level of usability, consumers may not be able to take advantage of all the features. Conversely, a high level of usability is more beneficial in conjunction with a large number of features.[14] We therefore model these two dimensions of quality as (imperfect) complements, requiring $\frac{\partial^2 Q}{\partial s \partial F} > 0$. A simple functional form that captures this complementarity and is concave in

[12]Equivalently, the no-purchase option corresponds to the consumer choosing the free open source software because without any usability they receive a utility of zero.

[13]Expert consumers may value features but not usability. These customers would not purchase the software product but have value for the amount of open source features developed in equilibrium. We do not model such customers in the product market analysis but examine the effect of the equilibrium choice on expert consumers' surplus by determining the number of open source features.

[14]In general, consumers do not benefit from products with a significant imbalance between their level of features and usability. *Thompson et al.* (2005) shows that consumers who purchase overly complex products face "feature fatigue" and that improving usability can help consumers effectively utilize the features.

both features and usability is Cobb-Douglas: $Q(F, s) = (F \cdot s)^{\frac{1}{4}}$. The above formulation of consumer preferences, product quality and the sequence of events are common to all markets.

Figure 4.1 illustrates the structure of the product market when features are either private or shared. The upper panel depicts the private features market where both firms initially have access to f_0 features from the open source community, and the lower panel depicts the shared features market (discussed in Section 4.3.1) where firms contribute any features they develop to open source. In both cases, f_0 is determined in equilibrium through developer signaling in the developer market (discussed in the next subsection).

Firm $j \in \{1, 2\}$ determines its product quality by hiring f_j feature developers and choosing a level s_j of usability. Feature developers vary in their skill-level $\eta \in \{\eta_L, \eta_H\}$. Each developer produces η units of features when hired. In the private features market, firm j's product consists of $F_j = (f_0 + \eta f_j)$ features, where η is the average skill-level of hired developers. The overall quality of firm j's product is

$$q_j = Q(F_j, s_j) = [(f_0 + \eta f_j)\, s_j]^{\frac{1}{4}},$$

which depends on the existing open source features f_0, the additional features ηf_j the firm creates, and the usability s_j the firm develops.

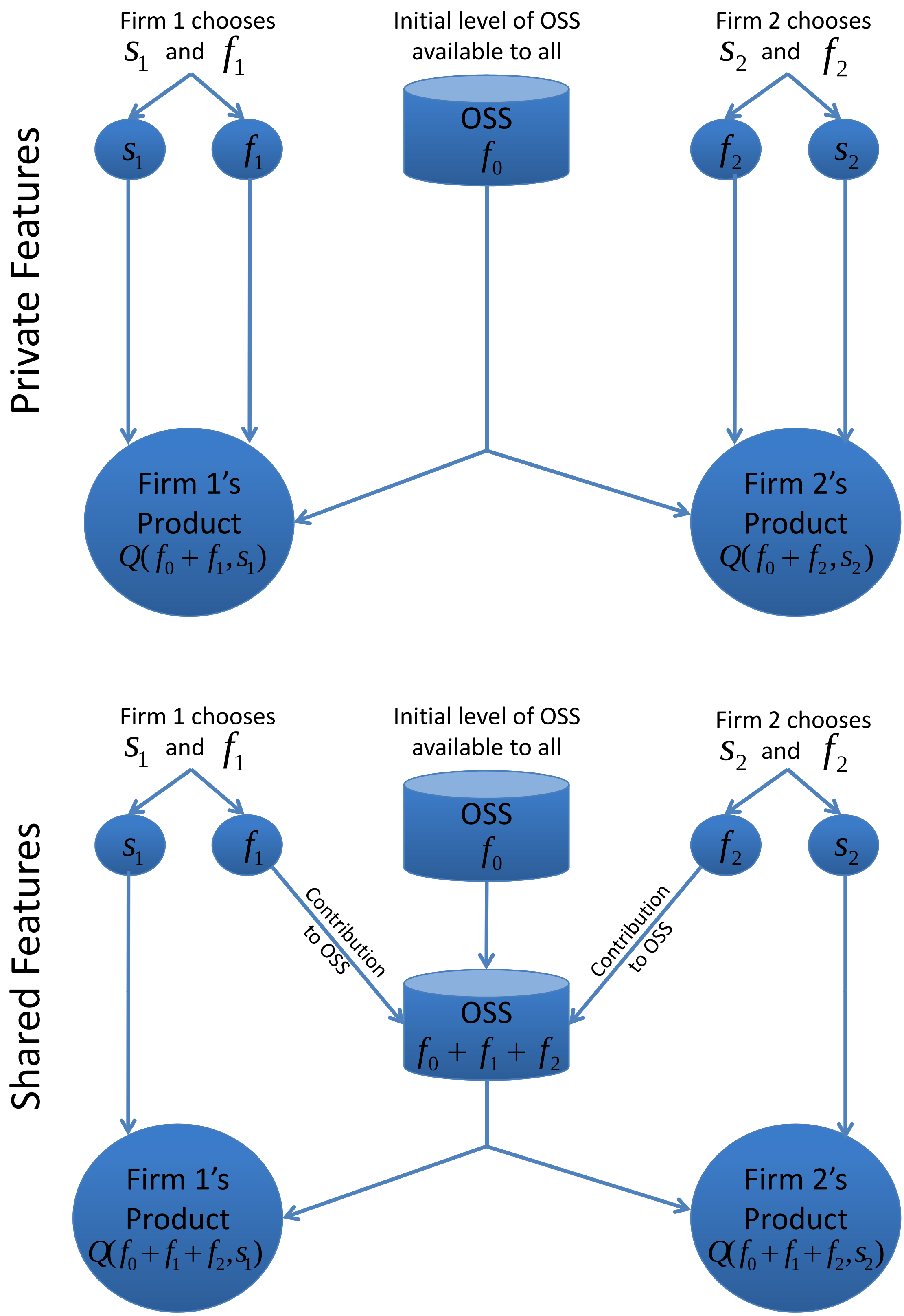
Each developer receives a wage of w and the marginal cost of creating usability is c_s.[15] The resulting total production cost $C(f, s)$ is simply the sum of the cost developing features and of making the product more usable, and is given as:

$$C(f, s) = w \cdot f + c_s \cdot s$$

Letting $q_1 > q_2$ without loss of generality, define consumer $\hat{\theta}_{12}$ to be indifferent between firm 1 and firm 2's products, such that $\hat{\theta}_{12} \cdot q_1 - p_1 = \hat{\theta}_{12} \cdot q_2 - p_2$. Consumer $\hat{\theta}_{20}$ is indifferent between purchasing firm 2's product and not purchasing either firm's product, such that

[15]We focus on the labor market for feature developers because creating new functionality is a more specialized and challenging skill, and thus a more credible signal of skill-level (*Roberts et al.*, 2006a; *Hars*, 2002). Equivalently, we could assume there is a perfectly competitive market for usability developers, who are inelastically supplied at some exogenous wage. Our analysis and results would be identical in this case.

Figure 4.1: Product Market: Comparison of Private and Shared Features



$\hat{\theta}_{20} = p_2/q_2$. The market size for the firms are:

$$m_1 = \frac{1}{M}\left(M - \hat{\theta}_{12}\right) = \frac{1}{M}\left(M - \frac{p_1 - p_2}{q_1 - q_2}\right) \text{ and } m_2 = \frac{1}{M}\left(\hat{\theta}_{12} - \hat{\theta}_{20}\right) = \frac{1}{M}\left(\frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}\right)$$

Marginal costs of production are zero because software is an information good. Firms set prices to maximize revenues $R_j = p_j m_j(p_j, p_{-j}, q_j, q_{-j})$, which are a function of the product prices and qualities and the market size:

$$R_1 = \max_{p_1}\left(M - \frac{p_1 - p_2}{q_1 - q_2}\right) p_1 \text{ and } R_2 = \max_{p_2}\left(\frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}\right) p_2 \tag{4.1}$$

These revenue functions are sufficient for us to derive the optimal prices in Stage 3 given product quality choices in Stage 2. Next we consider how the developer market influences firms' quality choices in the product market.

Market for Developers

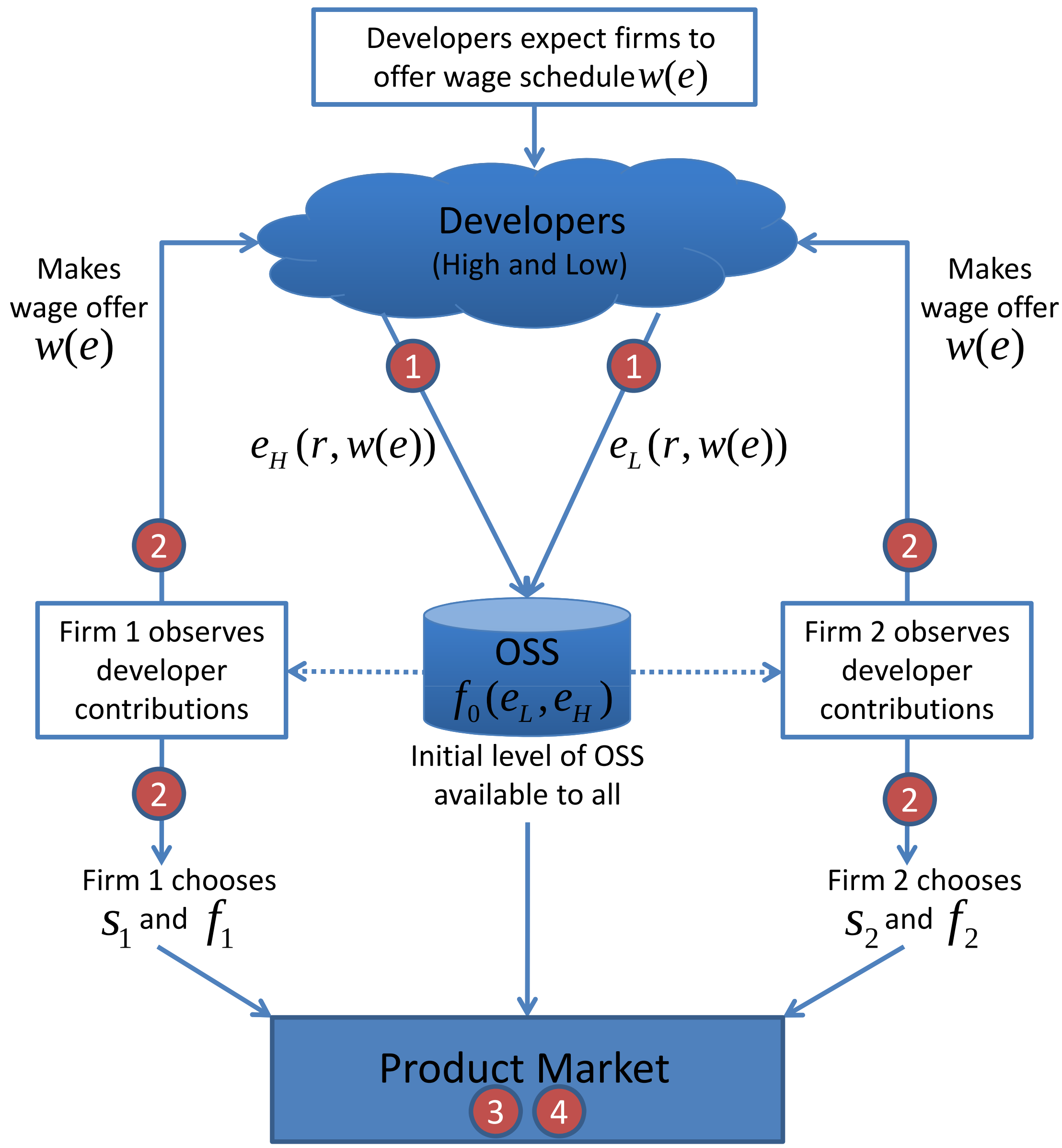
Figure 4.2 displays the developer market and its link to the product market. The numbers correspond to the stages in the game outlined at the beginning of Section 3. First, developers form expectations about wages based on the market, and contribute to open source to signal their skill level to firms. These contributions are publicly observable to the firms, which use them to evaluate developers' skills. Second, firms observe the contributions and make wage offers $w(e)$.[16] Firms simultaneously hire developers and choose their product's level of features and usability. Third, firms compete in the product market.

Developers are heterogeneously distributed between high-skilled (high-type) and low-skilled (low-type) developers. The cost of contributing to open source differs according to the developer's type. To contribute e features, high-types incur a cost of $c_H \cdot e$ and low-types incur $c_L \cdot e$, with $c_H < c_L$ indicating high-types face a lower cost. High-type developers have a reservation option $r \sim \psi(\cdot)$, with CDF $\Psi(\cdot)$ and support in the range $[\underline{R}, \bar{R}]$ where $0 \leq \underline{R} < \bar{R}$.[17] This option represents the utility a developer derives from her current job,

[16]Firms' wage offers are consistent with developers' beliefs in equilibrium.

[17]No further assumptions on the functional form of Ψ are required for our results. We use the general form in our description since it helps in presenting and interpreting different effects.

Figure 4.2: Developer Market



Note: The expected and actual wage schedules are the same due to rational expectations in equilibrium. The numbers in the diagram correspond to the stages of the game, as outlined at the beginning of Section 3.

and she will accept a new wage offer only if it exceeds r by the cost of signaling.

Our focus is on market level outcomes rather than the marginal effects of developer skills; our results primarily depend on the overall impact of signaling rather than the relative mix of types who signal. Low-type developers may masquerade as high-type developers so firms mistakenly hire them at a higher wage. Open source projects have "gatekeepers" who screen for low-quality code and decide which submissions to include in the project (*Bagozzi and Dholakia*, 2006). Gatekeepers ensure that high-type developers have an incentive to contribute more to open source to separate themselves from the low-types. To simplify our exposition, we make two assumptions. First, low-type developers have a fixed reservation option $R_L = 0$, and second, we normalize the low-type developer's skill-level to $\eta_L = 0$. These assumptions make it unprofitable for firms to hire the low-types, but both conditions could be relaxed and yield similar results.[18] We drop the subscript on η_H, and refer to it as η, because only the skill-level of high-type developers influences the equilibrium outcomes.

If a developer signals and accepts an offer from either firm, her wage schedule, $w(e)$, depends on her contribution to open source, e. A developer of type $t \in \{L, H\}$ contributing e features receives utility $u_t(w, e) = w(e) - c_t e$ and her optimal contribution level is

$$e_t = \arg\max_{e} w(e) - c_t e \tag{4.2}$$

subject to the incentive compatibility (IC) and individual rationality (IR) conditions:

$$\begin{aligned} (IC_H) \quad & w(e_H) - c_H e_H \geq w(e_L) - c_H e_L \\ (IR_H) \quad & w(e_H) - c_H e_H \geq r \\ (IC_L) \quad & w(e_L) - c_L e_L \geq w(e_H) - c_L e_H. \end{aligned} \tag{4.3}$$

The IR and IC constraints must hold for a developer in all separating equilibria.[19] To ensure

[18] We only require that at the equilibrium wage, the marginal increase in a firm's profits from hiring an incremental low-type developer is sufficiently low. This condition would be met if $\eta_L > 0$ but still low enough such that firms would not find it profitable to offer them a positive wage. Similar intuition would allow us to set $R_L > 0$. Our results are applicable when both of these values are generalized, but doing so complicates the notation without adding any depth to the results.

[19] We focus only on separating equilibria, and not on pooling equilibria, in order to convey the main insights in the simplest manner.

separation, high-type developers must find it incentive compatible to exert a high level of effort (IC_H) and must be sufficiently compensated to work for the COSS firms given her reservation option (IR_H). Low-type developers must find it prohibitively expensive to imitate the high-type developers (IC_L). Firms do not find it optimal to hire low-type developers at any positive wage after their type is revealed, leading them to set $w(e_L) = 0$. Given that the wage for low-type developers never exceeds their reservation wages (in any separating equilibrium), they do not contribute to open source, implying that $e_L = 0$. Therefore, the individual rationality condition for low-type developers, IR_L, does not need to be satisfied. The condition in IC_L reduces to $0 \geq w(e_H) - c_L e_H$. Developers with reservation values below a threshold, $r \leq w(e_H) - c_H e_H$, choose to signal and developers with $r > w(e_H) - c_H e_H$ choose their reservation option and do not signal. Thus the number of developers who signal is

$$N_H = \Psi(w(e_H) - c_H e_H). \tag{4.4}$$

This condition characterizes the supply of developers available for hire. Given an arbitrary wage w, the initial level of open source features available in Stage 1, $f_0(w)$, is a simple function of the number of developers and their contributions:

$$f_0(w) = N_H e_H = \Psi(w(e_H) - c_H e_H)e_H. \tag{4.5}$$

Note that the IR and IC conditions are necessary but not sufficient to determine the equilibrium wage. We must know the demand for developers from the firms to calculate the market-clearing wage. The wage influences developer contributions and yields different levels of open source features. A higher wage raises initial developer contributions, which also raises the initial quality available to the COSS firms. This alters the firms' incentives to further invest in product quality, and reduces their need to hire additional developers. The returns to quality also depends on the structure of the product market, i.e., private features versus shared features. Thus, we must account for competition between the firms for developers and the nature of the product market.

Discussion and Comparison

We briefly discuss and compare the role of signaling in our model to prior work. In the canonical model of *Spence* (1973), workers signal through investments in education to a market of perfectly competitive firms. Our model builds on this work, where developer contributions to open source features serve as signals to COSS firms seeking to hire high-skilled developers. In contrast, our model considers a vertically-differentiated duopoly where firms earn positive profits because developer heterogeneity within types allows firms to extract a surplus for each developer they hire who is not at the margin. An imperfectly competitive market is applicable to a broader range of industries and institutional settings.

Most signaling papers in marketing focus on a firm (the principal) who signals in order to convey information to consumers (the agents) about the firm's product quality (*Milgrom and Roberts*, 1986; *Balachander and Srinivasan*, 1994; *Moorthy and Srinivasan*, 1995; **?**).[20] Our setting differs from these earlier contexts in the following ways. First, the principals are the developers who signal and the agents are the COSS firms who observe the signals and make job offers. Second, the signaling contributions themselves serve as a substitute to a firm's investment in product features, i.e. the more developers contribute to open source, the less firms need to invest in building new features. Third, signals have a "spillover" effect: each contribution is freely available to *any* firm for its product, not just to the employer. In this sense, the signals are not privately appropriable by individual agents, compared to Spence's case where an individual's educational investment only benefits the employer. Signaling by a developer not only affects that developer's utility and the COSS firm that hires him, but it also changes the strategic interaction between the COSS firms in the market. These conflicting incentives create a critical and unique link between the product and developer markets. Thus, we generalize the basic job market signaling model to an imperfectly competitive market with publicly appropriable signals.

These distinctions enrich our model and help us more accurately capture the nature of competition between and for developers in the COSS market. In the next subsection, we characterize the equilibrium interaction between the product and developer markets, starting

[20] *Desai and Srinivasan* (1995) and **?** consider franchisee and retailer agents, respectively, who are uncertain about the potential demand for a firm's product.

with the last stage and working backwards.

4.2.2 Equilibrium Analysis

In this section, we analyze the subgame-perfect equilibrium of the above model. We begin with the last stage and solve the model using backward induction. We examine the firms' product design decisions, explicitly characterizing how many open source features are produced in equilibrium. The importance of modeling the developer market lies in the interaction between developers' contributions to open source features, f_0, and the quality levels of products subsequently offered by the COSS firms. We place intermediate results that are not central to our analysis and discussion in the Appendix.

Pricing Equilibrium

The pricing subgame in Stage 3 applies to all cases of the product market. The optimal price for each firm is a best response of the price set by the other firm, and depends on the quality levels of both products. Firm revenues, prices, and the consumer surplus are represented as functions of product quality levels in Proposition A1. This establishes the equilibrium outcomes of the pricing sub-game conditional on quality decisions made in Stage 2.

Factors such as wages, costs, and market characteristics determine revenue and prices only in the sense that they affect the quality levels chosen by the firms in the previous stage, and the pricing subgame does not directly depend on these factors. We proceed to characterize the equilibrium of Stage 2 of the game, how firms make product design choices of features and usability, and evaluate the degree of product differentiation in equilibrium.

Product Quality Equilibrium

The solution of the sub-game in Stage 2 requires the firms to strategically determine the optimal mix of features and usability depending on the license regime. Both firms simultaneously determine their product features and usability, and hence product quality. The profit functions are derived by substituting the feature and usability levels into the revenue functions in Proposition A1 and accounting for the costs of development. This provides the partial-equilibrium responses by the COSS firms given (1) an arbitrary wage in the developer market and (2) an arbitrary level of open source features.

We collapse the Stage 2 sub-game involving two strategic quality components to one overall quality level for each firm. If a firm's best response quality only depends on the overall quality of its competitor's product and not the specific mix of usability and features, then the firm will minimize the cost of attaining a given quality level through an optimal mix of usability and features. Proposition A2 in the Appendix establishes such a result, allowing us to solve for the equilibrium levels of features and usability, detailed below.

Proposition 1. ***[Private Features Product Market]*** *In the private features market with firms vertically differentiated in (partial) equilibrium, each firm develops both usability and features. Specifically, given f_0 open source features initially available and wage w in the developer market, firm j will choose quality and its components to be:*

$$q_j = \frac{M\phi_j}{2}\sqrt{\frac{\eta}{c_s w}}, \quad f_j = \frac{M^2\phi_j^2}{4}\sqrt{\frac{\eta}{c_s w^3}} - \frac{f_0}{\eta}, \quad s_j = \frac{M^2\phi_j^2}{4}\sqrt{\frac{\eta}{{c_s}^3 w}}$$

for $j \in \{1, 2\}$, where the constants ϕ_1 and ϕ_2 are defined in the Appendix. No symmetric equilibrium exists in which firms choose the same quality, $q_1 = q_2$.

Both firms create features in addition to the freely available open source code f_0. Firms reduce the number of features they develop when there are more open source features, since these are substitutes. We find that firms differentiate their products more on the less expensive dimension of quality, implying that if features are less expensive to produce, the firms will differentiate more on features ($f_1 - f_2 > s_1 - s_2$). The intuitive explanation is that firms differentiate their products more on the dimension that yields a greater return to such differentiation. This result is a partial equilibrium treatment, and it is necessary to understand how the wage w is set in equilibrium to evaluate the overall impact. We therefore turn to examining the forces determining the wage in the developer market.

Developer Market Equilibrium

The previous section derived the product market equilibrium taking the developer wage w as fixed or exogenous. The wage level influences both the firms' demand for high-type developers and the number of developers willing to signal through contributions to open source. Here we determine the equilibrium wage by equating the firms' demand for developers with the supply of developers who are willing to contribute to open source. We focus on separating equilibria in the developers' signaling game where the high-type and low-type developers choose to make different contribution levels to open source software.

We start with the supply side, establishing the conditions required for the high-type and low-type developers to separate. Recall that the skill-level of low-type developers implies that firms do not offer them a positive wage if their type is known, prompting low-type developers to make no contribution to open source, $e_L = 0$. For a separating equilibrium in the signaling game, we must identify conditions on the wage schedule that lead to positive contributions by some high-type developers and that ensures that low-type developers do not find it rational to imitate them. The following result provides the necessary conditions for a least-cost separating (LCS) equilibria in terms of the wage and open source contributions.

Proposition 2. ***[Separating Equilibrium]*** *The separating equilibria for the signaling game between the developers and firms is characterized by the following conditions where r is the reservation utility of a developer and w is the market wage for a developer who is known to be high-type:*

(i) The necessary conditions for contributions for each developer type are:

$$e_L(r,w) = 0, \quad e_H(r,w) \in \begin{cases} \{0\}, & r > w\left(1 - \frac{c_H}{c_L}\right) \\ \left[\frac{w}{c_L}, \frac{w-r}{c_H}\right], & r \leq w\left(1 - \frac{c_H}{c_L}\right) \end{cases}$$

(ii) In a least-cost separating (LCS) equilibrium, the high-type developers contribute $e_H^{LCS}(r,w)$ and the common out-of-equilibrium beliefs $\mu(H|e,w)$ for firms are

$$e_H^{LCS}(r,w) = \begin{cases} 0, & r > w\left(1 - \frac{c_H}{c_L}\right) \\ \frac{w}{c_L}, & r \leq w\left(1 - \frac{c_H}{c_L}\right) \end{cases} \quad \text{and } \mu(H|e,w) = \begin{cases} 0, & e < e^{LCS}(w) \\ 1, & e \geq e^{LCS}(w) \end{cases}$$

This LCS equilibrium is the only equilibrium that satisfies the Intuitive Criterion.

(iii) In the least-cost separating equilibrium, the number of high-type developers who contribute to open source features is $N_H^{LCS}(w) = \Psi(w - c_H e^{LCS}(w))$ and the number of features developed in Stage 1 is $f_0^{LCS}(w) = \Psi(w - c_H e^{LCS}(w)) \cdot e^{LCS}(w)$.

Part (i) identifies the necessary conditions that any separating equilibrium must satisfy. These conditions arise from the individual rationality and incentive compatibility constraints in (4.3). Since a separating equilibrium allows firms to perfectly distinguish developer types, low-types do not contribute to open source, $e_L = 0$. The high-type developers must contribute a sufficient number of features to ensure low-types do not find imitation profitable, captured in IC_L, and using which we obtain $e_H(w) \geq \frac{w}{c_L}$. However, some high-type developers must also find signaling to be preferred to their reservation utility, i.e. whenever $r \leq w - c_H e_H = w\left(1 - \frac{c_H}{c_L}\right)$, the developer will choose to signal. Note that an infinite number of separating equilibria exist in which a high-type developer with reservation utility $r \leq w\left(1 - \frac{c_H}{c_L}\right)$ makes a contribution in the continuum $\left[\frac{w}{c_L}, \frac{w-r}{c_H}\right]$. Each equilibrium is equally valid without imposing further restrictions on out-of-equilibrium beliefs.

Part (ii) establishes the out-of-equilibrium beliefs that satisfy the Intuitive Criterion (*Cho and Kreps*, 1987) to determine a unique LCS equilibrium. Signaling models often admit a multiplicity of equilibria, and we use the Intuitive Criterion to refine 'unreasonable' equilibria. In our context, least-cost refers to the minimum separation required at each prevailing wage. This purification of out-of-equilibrium beliefs requires that any observed deviation from the equilibrium path will more likely be from the type that could profit the most from the deviation. This refinement gives us the minimum contribution to open source the high-type developers must make to sustain a separating equilibrium, and the corresponding minimum wage firms must pay to guarantee separation.

Part (iii) focuses on the minimum amount of separation given by $e^{LCS}(w)$. Developers with reservation option $r < r^{LCS}(w) = w - c_H e^{LCS}(w) = w\left(1 - \frac{c_H}{c_L}\right)$ choose to signal, implying that the number of high-type developers who signal is $N_H^{LCS} = \Psi(r^{LCS}) = \Psi(w - c_H e^{LCS}(w))$. The number of open source features produced in Stage 1 is the product of the number of developers who signal and each developer's contribution, $f_0^{LCS}(w) = N_H^{LCS} e^{LCS}(w)$. This result allows us to focus on the (expected) wage level that determines f_0.

Having established the conditions for a LCS equilibrium, we restrict attention only to such outcomes in the remainder of the paper and drop the LCS superscript.

Proposition 2 characterizes the supply of developers. Examining the demand for developers, we observe that the number of high-type developers that firm j wishes to hire at wage w is $f_j\left(f_0(w), w\right)$. The equilibrium wage w^P in the private features market equates the firms' aggregate demand of high-type developers to the aggregate supply of developers who prefer the wage to their reservation option:

$$f_1\left(f_0(w^P), w^P\right) + f_2\left(f_0(w^P), w^P\right) = N_H(w^P). \tag{4.6}$$

Below this wage level, fewer high-type developers will contribute to open source than firms are willing to hire, and above this wage level more developers will be inclined to contribute than firms desire to hire. Thus, the equilibrium wage serves to balance the signaling incentives of developers and firms, and explicitly links the product and developer markets.

Combining the market clearing condition in Equation (4.6) with the demand results in Proposition 1, the equilibrium wage in the private features market is given below.

Corollary 1. *The wage level in the private features market w^P is implicitly described by:*

$$\frac{M^2\left(\phi_1^2+\phi_2^2\right)}{4}\sqrt{\frac{\eta}{c_s}} = (w^P)^{\frac{3}{2}}\Psi\left(w^P\left[1-\frac{c_H}{c_L}\right]\right)\left[1+2\frac{w^P}{c_L}\right] \tag{4.7}$$

and a subset of high-type developers each contribute $e_H = \left(w^P\right) = \frac{w^P}{c_L}$ features to open source.

Given the implicit relationship between the wage and model primitives, we examine the comparative statics of the wage with respect to the market size, the cost of signaling, and the cost of producing usability.

Proposition 3. *The least-cost separating equilibrium wage for high-type developers satisfies the following properties: It is increasing in market size ($\frac{\partial w^P}{\partial M} > 0$), signaling cost ($\frac{\partial w^P}{\partial c_H} > 0$), and skill-level ($\frac{\partial w^P}{\partial \eta} > 0$), and decreasing in the cost of usability, ($\frac{\partial w^P}{\partial c_s} < 0$).*

Both firms compete for high-type developers, and developers make contributions to open source anticipating the competition between firms and the resulting wage schedule. The wage increases with the market size M because a larger market causes firms to invest more in creating a higher quality product and competition between firms drives wages higher. The wage is higher when producing usability is less expensive because a low c_s permits firms to invest more in usability. A higher level of usability raises the value of features for consumers and firms due to complementarity, which in turns ensures that firms invest more in features, thus raising developer wages. When signaling is costly for the high-type developers, fewer signal and more choose their reservation option, resulting in fewer open source features. The marginal value of features is then higher for firms since the substitution effect with open source features is weaker, which increases the wage.

4.3 The Shared Features and Closed Source Markets

The last section presented the basic model of competition between COSS firms in a private features market. Next we alter the model to accommodate the shared features and the closed source cases. In the shared features market, the structure of the developer market stays the same but the product market changes to account for the fact that firms must share with competitors any features that they develop. In the closed source market, the product market is identical to the private features case, but firms no longer build their products using open source and developers no longer contribute open source. We establish the equilibrium and then compare the outcomes across the three market regimes.

4.3.1 Shared Features Market

We alter the product market model in Section 3.1 to accommodate shared features, and present the equilibrium outcomes that change as a result of this modification.

4.3.1.1 Model

The only difference between the shared features market and the private features market presented earlier is the formulation for product quality. To see this distinction most clearly, compare the two panels of Figure 4.1. In the upper panel, firms in the private features market incorporate open source features and only their own privately developed features into their products. In contrast, the shared features market in the bottom panel shows that firms must contribute any features they develop back to open source, which subsequently become part of both firms' products. Products in the shared features market are identical in terms of features and only differ in usability, and the product quality of firm $j \in \{1, 2\}$ is:

$$q_j = Q(F_j, s_j) = [(f_0 + \eta f_j + \eta f_{-j})\, s_j]^{\frac{1}{4}}$$

where firm j hires f_j features developers and chooses s_j usability. The difference between the quality here and in the private features market is the extra (ηf_{-j}) term due to the shared features. The rest of the product market in the shared features case is identical to the private features market. However, this extra term fundamentally alters the strategic interaction and equilibrium outcomes as we demonstrate below.

4.3.1.2 Equilibrium Analysis

Unlike the private features market, we cannot uniquely characterize a firm's best response using overall product quality. Proposition A2 does not apply because each firm's feature contribution directly affects the quality of the other firm's product. The potential for free-riding on features leads to decreased product differentiation and increased price competition, making it unclear whether *any* firm will contribute to features in equilibrium. The following result provides the outcome for a market where consumers place sufficient value on quality such that firms develop additional features beyond those supplied by open source.

Proposition 4. *In the shared features market, the ex-post high-quality firm will develop features but the low-quality firm will not do so. The features, usability, and overall quality levels in this partial equilibrium with arbitrary open source features f_0 and wage w are:*

$$f_1 = \frac{M^2\sigma_1^3\sqrt{\frac{\eta(\sigma_1-\sigma_2)^3}{w^3 c_s}}}{(4\sigma_1-\sigma_2)^3} - \frac{f_0}{\eta} \qquad f_2 = 0$$

$$s_1 = \frac{M^2\sigma_1^5\sqrt{\frac{\eta(\sigma_1-\sigma_2)}{w {c_s}^3}}}{4\sigma_1-\sigma_2} \qquad s_2 = \frac{M^2\sigma_1\sqrt{\frac{\eta(\sigma_1-\sigma_2)}{w {c_s}^3}}\sigma_2^4}{4\sigma_1-\sigma_2}$$

$$q_1 = \frac{M\sigma_1^2\sqrt{\frac{\eta(\sigma_1-\sigma_2)}{w c_s}}}{4\sigma_1-\sigma_2} \qquad q_2 = \frac{M\sigma_1\sigma_2\sqrt{\frac{\eta(\sigma_1-\sigma_2)}{w c_s}}}{4\sigma_1-\sigma_2}$$

where the constants σ_1 and σ_2 are defined in the Appendix.

We find that only the high-quality firm develops features beyond the open source features f_0 available due to developer contributions. The low-quality firm fully free-rides on the features provided by the high-quality firm. Why does the high-quality firm develop a positive level of features? The intuition comes from the fact that the quality dimensions are complements, implying that the marginal utility of usability for consumers is increasing in the level of features. Although features do not contribute directly to product quality differentiation, they do magnify the effect of usability differences between the firms. Note that the low-quality firm invests less in usability than the high-quality firm, and the high-quality firm develops features to enhance the degree of product differentiation, which results from having a higher usability product. Thus, the increase in the differentiation from usability makes it worth creating features that reduce the intensity of competition.

The product market outcome above has a critical implication for the developers' market: only the high-type firm hires developers because the low-type firm does not develop features ($f_2 = 0$). With the high-quality firm acting as a monopsonist in the developers' market, it only pays the minimum wage to the high-type developers. However, firms not directly competing with the two COSS firms often value high-type developers. For example, a company in the embedded Linux market may value a developer who has contributed features to the Linux kernel, but the company does not compete directly with Red Hat.

We formalize this notion by modeling an alternative market in which a high-type developer receives a fixed wage of $\hat{w}$ when her type is known. The alternative market provides a minimum value to signaling by developers who may otherwise choose not to signal when faced with a monopsonist COSS firm. How is the equilibrium wage w^S then determined?

The high-quality firm cannot commit to offer a higher wage than $\hat{w}$ before developers signal, since such a commitment would not be credible after the developers have signaled. The wage offer cannot be lower than $\hat{w}$ or the firm will not be able to hire any high-type developers. The high-quality firm cannot induce the high-type developers to contribute more to open source than the minimum amount $e_H(\hat{w})$ required to separate them from low-type developers. The firm makes wage offers after the signaling stage, and even if the developers made more contributions than required for signaling, the firm does not need to offer a wage higher than $\hat{w}$ in Stage 2. Therefore, the wage offered by the high-quality firm is exactly $\hat{w}$. The developers expect the firm to pay exactly $\hat{w}$ and to contribute the minimum level required for separation, i.e. $e_H(\hat{w}) = \frac{\hat{w}}{c_L}$. This occurs even though it may be Pareto–improving for firms to pay a higher wage and for developers to contribute a higher level to open source features. These considerations determine both the wage level and the contributions made by developers to open source software. Note that the private market scenario is not impacted by the presence of an alternative market, as long as the wage level of the alternative market $\hat{w}$ is not extremely high or low (see Assumption A1 in the Appendix).

Corollary 2. *If the alternative market wage $\hat{w}$ is neither too high nor too low (A1), the wage level in the shared features market $w^S = \hat{w}$. A subset of high-type developers each contribute $e_H\left(w^S\right) = \frac{w^S}{c_L}$ features to open source.*

4.3.2 Closed Source Market

The product market model and analysis for the closed source market is identical to the private features case, but the developer market is significantly different. In the closed source market, firms face no information asymmetry, and observe the skill-levels (types) of developers. This obviates the need for signaling by the high-type developers. Since the rationale for high-type developers contributing to open source was to reveal their superior skill-level, no open source software is produced in this market. We can interpret this case as traditional software firms such as Microsoft or Oracle competing in both the product market and in the developers market for skilled developers.

4.3.2.1 Model

We model the closed source market as a limiting case in which high-type developers face zero contribution cost ($c_H = 0$) and low-type developers face an infinite contribution cost ($c_L \to \infty$). These costs are consistent with the requirement that no open source features are created in Stage 1: using Proposition 2 and applying the new costs, $\lim_{c_L \to \infty} e_H(w) = \lim_{c_L \to \infty} \frac{w}{c_L} = 0$, which implies $f_0 = 0$. Developers retain their reservation option and may accept offers from firms, as in the private features market.

4.3.2.2 Equilibrium Analysis

The number of high-type developers willing to give up their reservation utility at a wage w is simply $\Psi(w)$. The implicit expression for the equilibrium wage in the closed source market can be derived from (4.7).

Corollary 3. *The wage level in the private features market w^C is implicitly described by:*

$$\frac{M^2\left(\phi_1^2+\phi_2^2\right)}{4}\sqrt{\frac{\eta}{c_s}}=\left(w^C\right)^{\frac{3}{2}}\Psi(w^C) \tag{4.8}$$

and a subset of high-type developers each contribute $e\left(w^C\right)=\frac{w^C}{c_L}$ features to open source.

How do the properties of the wage here differ from the private features market? We find the comparative statics effects derived for the private features wage in Proposition 3 apply to the closed source case, except for the effect of signaling cost, c_H.[21] Observe that Proposition 1, which applies to the product market, continues to hold in the closed source market because only the developer market determines the wage and the effects of signaling directly apply there. However, the wage level affects firms' product quality decisions, implying that the outcomes in the closed source market will differ from those in the private features market.

4.3.3 Comparison of Market Regimes

We have derived equilibria under three market regimes: private features, shared features and closed source. In each case, *ex-ante* identical firms compete in product and developer markets and are differentiated *ex-post*. We focus on how profits, product qualities, and price levels compare across these regimes. We also highlight potential insights for firms operating in industries related to open source software and discuss some policy implications.

Our first result below compares the equilibrium wage across the different markets.

Result 1. ***[Wages]*** *The equilibrium wage for high type developers in the developers market is ordered as follows when the cost of signaling is not excessive:*

$$w^S < w^P < w^C$$

[21] We do not list this finding as a formal result. It is clear from the proof of Proposition 3 that the effects after taking the limits as $c_H \to 0$ and $c_L \to \infty$ will not be altered.

The wage is highest in the closed source market and lowest in the shared features market. Competition between firms in the private features market drives wages higher, and $w^S < w^P$ as we expect. The intuition behind $w^P < w^C$ is less obvious, and depends on the relative magnitude of two competing effects. First, the *signaling effect* reduces the number of developers in the private features market because each must incur a signaling cost, which leads to higher wages. Second, the *substitutability effect* causes the marginal value of features to firms to be lower in the private features market due to the presence of open source features. We find that the substitutability effect dominates the signaling effect, and developer wages are lower in the private features market compared with the closed source market.

We next examine the equilibrium provision of open source features. This outcome should be of interest to firms who engage in open source, to consumers who use products based on open source, and to policy makers who view open source as an important driver of innovation in the software market. Further, the initial creator of an open source project, who chooses the specific license, might seek to maximize the total contributions to the project.

Result 2. *[**Contribution to Open Source**] The equilibrium contribution to open source features is higher under the private features market compared with the shared features market when the market size is large and signaling costs for high-type developers are low.*

In the shared features market, both firms have access not only to the contributions to open source made from high-type developers, but also to the features developed by the *ex-post* high-quality firm. In contrast, the private features market allows firms to keep their features private leaving only the signaling contributions to features as open source. Thus, we expect more open source features in the shared features market. However, when the consumer market is large, firms compete for developers more intensely in the private features market because there is a greater value in producing a higher quality product. This raises the incentive for high-type developers to separate themselves from low-type developers. When separation is relatively easy (c_H is small compared with c_L), more high-type developers enter the market and more developers contribute to open source features. As we later show, this result would not hold in an alternative model that treats open source as exogenous.

We proceed to evaluate how the characteristics of the product and developers markets determine the equilibrium levels of usability and features and the overall level of quality.

Result 3. *[**Product Quality**] Comparing the markets we find that:*

(i) The low-quality product always has a lower quality level in the private features market compared to the shared features market.

(ii) For small (large) market size and low (high) signaling costs, the high-quality software product in the private features market is characterized by higher (lower) quality levels compared to the shared features market.

(iii) When separation is easy, both firms in the private features market produce a higher quality product compared to the closed source market.

(iv) The usability ratio $\frac{s_1}{s_2}$ is always larger for the shared features market compared to the private features market, and the features ratio $\frac{f_1}{f_2}$ is larger under the private features market. The quality differentiation captured by the ratio $\frac{q_1}{q_2}$ is higher for the private features market compared to the shared features market.

The low-quality firm's quality is always higher in the shared features market since the firm is able to free-ride on the features provided by the high-quality firm. This effect holds even when the low-quality firm develops a lower level of usability than in the private features market, and is independent of the model parameters, such as market size or signaling costs. In fact, as the market size increases, the quality difference of the low-quality firm compared across the private features and shared features markets becomes larger.

The effect of market on the high-quality firm's quality level is more nuanced: it depends on the market size, signaling cost, and cost to develop usability. When the market size is large and usability costs are low, there is higher demand for developers and lower cost of developing a higher quality product, which increases the equilibrium wage high-type developers receive. These effects are collectively stronger for the shared features market than in the private features market; therefore, the quality level chosen by the high-quality firm is higher in the shared features market. This effect is stronger when the signaling cost is higher for the high-type developers, which leads to increased competition between the firms in the private features market. In contrast, when the conditions above are reversed, the high-quality product is better under the private features market.

Next, we compare the quality levels between the private features and the closed source markets. When the cost of signaling for high-type developers is not excessive (Assumption A2), product quality is higher in the private features market. The underlying reason is that competition between the firms for developers is diminished due to the presence of open source features, resulting in lower wages. If both developer types have similar signaling costs, the excessive distortion of open source contributions required for the separating equilibrium leads to diminished participation by high-type developers, lowering product quality in the private features market.

The marginal benefit of usability increases with the level of public contribution of features, which implies that more signaling by programmers increases the firms' incentives to develop usability. In the shared features market, firms differentiate more on usability, but this differentiation is insufficient to overcome the fact that both products have the same level of features because features are publicly available. The quality differentiation in the shared features market is therefore lower than in the private features case. Prices are proportional to quality levels by Proposition A1.

A comparison of the equilibrium levels of usability and features, which determine firms' profits in each market, is presented next.

Result 4. *[**Profits**] The low-quality firm makes a higher profit under the shared features market. When the market size is small (large) or signaling is easy (difficult), the high-quality firm makes a higher profit under the private features (shared features) market.*

The gains to free-riding for the low-quality firm in the shared features market are too significant to be affected by market parameters, and the low-quality firm always prefers the shared features market. When the market is not too large and signaling costs are low, the high-quality firm prefers the private features market because the competition for developers is less intense and the firm does not have to share features with the low-quality firm. However, when competition becomes too intense, the high-quality firm would prefer to let the low-quality firm free-ride in order to decrease the competition in the market for developers.

Next, we examine the effect of market on consumers, detailed in the following result.

Result 5. *[**Consumer Surplus**] Consumer surplus is higher in the shared features market* *compared to the private features or closed source markets under all market conditions.*

The above result is surprisingly general and does not depend on the parameters. From the consumers' perspective, the shared features market is preferred, and points out that socially-conscious contributors should consider licensing their software under a framework that makes features publicly available. This result is counter to what we might intuitively expect: lack of information by firms regarding the true types of developers increases the surplus to consumers. The reasoning is that both the utility of the signal (open source features) used in the products developed by the firms and decreased competition in the developers market due to free-riding on features by the low-quality firm serve to benefit consumers. However, the consumer surplus must be balanced against potentially lower profits made by firms and lower wages received by high-type developers under the shared features market.

4.3.4 Comparison of Results with Exogenous Developer Market

Our primary motivation for modeling signaling in the developer market is that it provides a mechanism to endogenize the level of open source. Consistent with prior work (*Lerner and Tirole*, 2002a), developers signal their skill-level through contributions to open source in order to receive an equilibrium wage from the firms. In this section, we consider an alternative model in which the wage for developers is fixed at an exogenous level w, as opposed to being determined in equilibrium through signaling contributions. We contrast the results from this model with the results from the complete two-sided market. This allows us to assess the importance of accounting for the endogeneity of wages and the initial level of open source.

Let q_1^k for $k \in \{S, P, C\}$ be the high-quality product in either the shared features market, private features market, or closed source market. Similarly, let Π_1^k be the profits (revenues) for the ex-post high-quality firm and CS^k be consumer surplus. Imposing an exogenous wage amounts to setting w^* to be equal across product market regimes. Substituting this fixed wage into past results, the following orderings for product quality, profits, and surplus hold:

Proposition 5. *Under Assumptions (A1) and (A2), when the developer market is exogenous and the wage is fixed at w, the following results hold under all conditions:*

1. *The private features market always produces the highest overall quality product with the most features compared to the shared features market*

$$q_1^S < q_1^P \quad and \quad f_1^S < f_1^P,$$

2. *The ex-post high quality firm in the private features market always makes higher profits than in the shared features market*

$$\Pi_1^S < \Pi_1^P,$$

3. *Consumer surplus is the same in the closed source and private features markets*

$$CS^C = CS^P.$$

Compared to the full two-sided model, an exogenous wage leads the shared features market to bias the quality of both firms' products lower in equilibrium. As a consequence, the private features market produces the highest quality product and the high-quality firm earns higher profits as a result. In contrast, when we model the developer market, we find that the shared features market can produce higher quality products and can raise both firms' profits and consumer surplus under certain conditions. These comparisons illustrate the significance of integrating the developer market into the model.

4.4 Discussion and Conclusion

We construct a two-sided model of the commercial open source market in which firms compete in both a product market and a developer market. Developers contribute to open source to signal their skill-level to firms, which determines the level of open source available to firms for their own products. Firms hire developers to build their products, and then compete in a vertically-differentiated market for consumers. We compare equilibrium product prices, qualities, profit, and surplus under two types of open source licenses and contrast these to the baseline of traditional closed source firms.

Our model helps rationalize several puzzles observed in the industry, such as why Red Hat contributes significantly more to Linux than any other firm and why a market with mandatory sharing can actually produce higher quality products than a proprietary market. We show that ignoring the signaling incentives of developers in the model would lead to different equilibrium predictions. For example, if the level of open source were exogenous, the model would predict that product quality in the private features market would always be higher, in contrast to what we observe in the real world.

First, we show that the mandatory sharing of feature contributions between firms can actually raise profits and consumer surplus compared to the case of closed source firms. The availability of open source contributions and their role as a signaling device allows firms to improve the quality of their products beyond what they would have obtained if allowed to keep their contributions private. Second, competition between COSS firms in the private features market can induce developers to contribute more to open source features compared with the shared features market. Competition between firms in the private features market leads to a high equilibrium developer wage, which creates a strong incentive for high-skilled developers to contribute to open source to ensure separation from low-skilled developers, resulting in an overall increase in the number of open source features. Third, asymmetric information causes the firms to distort their quality choices relative to traditional closed source firms. Firms in the private features market differentiate more on usability and firms in the shared features market develop less usability.

Our model also extends prior work on modeling signaling on the job market. Firms compete in an imperfectly competitive market, and developers' open source contributions are substitutes for a firm's development of product features. Since all firms benefit from these contributions, developers' signals affect firms' product design decisions. In contrast, in *Spence* (1973), workers' educational investments solely benefit the worker and single employer, and the model considers a perfectly competitive product market.

Our model could apply more generally than in the context of open source software alone. We use the terms "features" and "usability" for sake of expositional clarity, but one could more broadly think of a product composed of public and private components that are mutually exclusive and complementary. Firms integrate existing contributions with their own material to form the public dimension of the product, and firms exclusively produce the private component. For example, user-generated media web sites where firms post a mixture of free and exclusive content in addition to user-generated material may be one such example.

Our work lends itself to extension along several dimensions. First, firms face a choice in deciding whether to make their own software open source and which license to adopt. The choice of license is likely to affect subsequent develop of the product and could ultimately play a large role in determining the eventual success of the product. Second, many COSS firms draw significant revenue from services that are separate from their products but that leverage their expertise. These services can help firms subsidize the cost of developing their COSS product and provide an additional competitive angle. Third, our model could be extended to allow firms to further innovate on product quality after observing one period of market outcomes.

The market for software built upon open source is growing rapidly. The open source movement recently made the leap to mobile computing platforms with the release of the Google Android operating system (**?**). We expect the commercial open source market to attract significant attention from researchers in the future who want to examine the unique aspects of product design, pricing, and firm strategy in this new and important industry.

CHAPTER V

Conclusion

I have examined contexts where individual consumers and firms can contribute non-rivalrous information goods that can be used by other consumers or firms. I study the behavior of firms competing in an imperfectly competitive market, and demonstrate the conditions under which they would prefer to share information goods. I also examine the behavior of consumers who contribute user-generated and provided content freely in social network settings, which has implications for the newly developing UGC phenomenon.

Several novel aspects of contributions affect the incentives of the contributors, and taking a strategic perspective helps me analyze the drivers of contributions, and I determine empirically. Taking a strategic perspective helps analyze the drivers of contribution both at an individual (or firm) level, as well as the competitive implications of settings as diverse as friends' social networks and product markets.

The framework I have developed in this dissertation has normative implications for consumers, firms, as well as policy makers. It clearly lays out the opportunity for further study on how competitive incentives as well as social factors can be harnessed to provide more content, to charge or subsidize such content. In developing the framework, I have leveraged the fact that shared information goods have interesting properties that make them quite different from ordinary goods.

At both the consumers and firm level, the move towards openness of information has fundamentally new implications for marketing, from positive as well as normative viewpoints. Understanding and rationalizing seemingly puzzling behavior by individuals and firms is a key positive contribution, and has novel informative implications on rules of competition, degree of information provided, and platform design for consumers and participating firms. I hope that this dissertation marks a first step in creating a better understanding of the phenomenon of openness of content.

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APPENDICES

APPENDIX A

Proofs

Appendix A

Denote the uniform *pdf* and *cdf* of $\theta \sim U[0, M]$ by $g(\theta) = \frac{1}{M}$ and $G(\theta) = \frac{\theta}{M}$. Proposition A1 is similar, but not identical, to the corresponding case in *Shaked and Sutton* (1982). The firm with higher *ex-post* quality sets a higher price, and the ratio of prices increases with the quality ratio. The pricing power of both firms diminishes when the quality levels are closer than when they are dissimilar due to the reduced intensity of competition.

Proposition (A1)**.** *In a vertically differentiated duopoly with quality levels q_1 and q_2, with $q_1 > q_2$ The optimal prices are set at: $p_1(q_1, q_2) = \frac{2Mq_1(q_1-q_2)}{4q_1-q_2}$ and $p_2(q_1, q_2) = \frac{Mq_2(q_1-q_2)}{4q_1-q_2}$. The revenues of the firms are $R_1(q_1, q_2) = \frac{4Mq_1^2(q_1-q_2)}{(4q_1-q_2)^2}$ and $R_2(q_1, q_2) = \frac{Mq_1(q_1-q_2)q_2}{(4q_1-q_2)^2}$. The consumer surplus is $CS(q_1, q_2) = \frac{Mq_1^2(4q_1+5q_2)}{2(4q_1-q_2)^2}$.*

Proof of Proposition (A1)**.** *Firms focus only on revenue to set prices as quality choices are sunk in the prior stage. The FOCs with respect to price are:*

$$\left.\frac{\partial R_1}{\partial p_1}\right|_{p_1,p_2} = M - \frac{p_1}{q_1 - q_2} - \frac{p_1 - p_2}{q_1 - q_2} = 0, \left.\frac{\partial R_2}{\partial p_2}\right|_{p_1,p_2} = \frac{p_1 - p_2}{q_1 - q_2} + p_2\left(-\frac{1}{q_2} - \frac{1}{q_1 - q_2}\right) - \frac{p_2}{q_2} = 0$$

Solving these FOCs simultaneously, we obtain

$$p_1(q_1, q_2) = 2Mq_1\left(\frac{q_1 - q_2}{4q_1 - q_2}\right) \text{ and } p_2(q_1, q_2) = \frac{Mq_2(q_1 - q_2)}{4q_1 - q_2}$$

Substituting these prices in (4.1), we obtain the expression in the proposition. The consumer surplus with these quality levels given $g(\theta) = \frac{1}{M}$ is:

$$CS(q_1, q_2) = \int_{\theta_2}^{\theta_1} (\theta q_2 - p_2)\, g(\theta) d\theta + \int_{\theta_1}^{M} (\theta q_1 - p_1)\, g(\theta) d\theta = \frac{M}{2} \frac{q_1^2 \left(4q_1 + 5q_2\right)}{\left(4q_1 - q_2\right)^2}$$

Proposition (A2)**.** ***[Quality Decomposition]*** *The optimal level of features and usability to contribute to a quality target q_j for firm j when the first stage produces f_0 features due to developers' signaling actions is as follows:*

1. *Low-quality region: When $q_j^2 < f_0\sqrt{\frac{w}{\eta c_s}}$, the optimal quality decomposition is*

$$f_j = 0, \quad s_j = \frac{q_j^4}{f_0}, \quad \text{and } C(q_j) = c_s \frac{q_j^4}{f_0} \tag{A.1}$$

2. *High-quality region: When $q_j^2 \geq f_0\sqrt{\frac{w}{\eta c_s}}$, the optimal quality decomposition is*

$$f_j = \sqrt{\frac{c_s}{w\eta}} q_j^2 - \frac{f_0}{\eta}, \quad s = \sqrt{\frac{w}{c_s \eta}} q_j^2, \quad \text{and } C(q_j) = 2\sqrt{\frac{w c_s}{\eta}} q_j^2 - w f_0 \tag{A.2}$$

where f_j and s_j are the optimal levels of features and usability, and $C(q)$ is the minimum cost of obtaining quality q.

Proof of Proposition (A2)**.** *When the product contains no open-source features, the firm's problem, $(f, s) = \min_{f,s} wf + c_s s$ subject to the constraint $(\eta f s)^{\frac{1}{4}} = q$ yields $f(q) = q^2 \frac{c_s}{\eta w}$ and $s(q) = q^2 \frac{w}{\eta c_s}$. The overall cost of providing quality q is then $C(q) = wf(q) + c_s s(q)$, so $C(q) = 2\sqrt{\frac{w c_s}{\eta}} q^2$.*

In the low-quality region, the firm only develops usability to a level of $\frac{q_j^4}{f_0}$. The firm does not add features beyond f_0 because either consumers do not value them sufficiently or the market is too small. We focus on the high-quality region, where the existing level of open source f_0 is inadequate for product market competition, because the low-quality region obviates the need for the developer market and does not explain why developers contribute to open source.[1]

[1]In the low quality region, firms only invest in usability, and do not hire developers. In our model, this

When COSS firms compete in the product market, firm j's quality is $q_j = [(f_0 + \eta f_1 + \eta f_2)\, s_j]^{\frac{1}{4}}$ *and depends on the feature levels contributed by both firms. Solving for an optimal value of* f_1 *requires the value of* f_2*, which determines* q_1*. Therefore, we cannot decompose the quality uniquely.* $\square$

Proof of Proposition 1. *The profit functions after incorporating the pricing sub-game are:*

$$\pi_1(f_0, f_1, s_1, f_2, s_2) = \frac{4M\,[(f_0+f_1)\,s_1]^{\frac{1}{2}}\left([(f_0+f_1)\,s_1]^{\frac{1}{4}} - [(f_0+f_2)\,s_2]^{\frac{1}{4}}\right)}{\left([(f_0+f_2)\,s_2]^{\frac{1}{4}} - 4\,[(f_0+f_1)\,s_1]^{\frac{1}{4}}\right)^2} - wf_1 - c_s s_1$$

$$\pi_2(f_0, f_1, s_1, f_2, s_2) = \frac{M\,[(f_0+f_1)\,(f_0+f_2)\,s_1 s_2]^{\frac{1}{4}}\left([(f_0+f_1)\,s_1]^{\frac{1}{4}} - [(f_0+f_2)\,s_2]^{\frac{1}{4}}\right)}{\left(4\,[(f_0+f_1)\,s_1]^{\frac{1}{4}} - [(f_0+f_2)\,s_2]^{\frac{1}{4}}\right)^2} - wf_2 - c_s s_2$$

The quality decomposition result from Proposition (A2) implies we can reduce the duopoly market competition in the private features market as represented by the following profits:

$$\Pi_1(q_1, q_2) = R_1(q_1, q_2) - C(q_1), \Pi_2(q_1, q_2) = R_2(q_1, q_2) - C(q_2)$$

where the revenue expressions are given in Proposition (A1) and the expression for minimum cost of achieving a quality level is obtained from Proposition (A2). The quality best responses for each firm is given by the solution to these FOCs:

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{2Mq_1\,(8q_1^2 - 6q_2q_1 + 4q_2^2)}{(4q_1 - q_2)^{\,3}} - 2cq_1 = 0 \text{ and } \frac{\partial \Pi_2}{\partial q_2} = \frac{Mq_1^2\,(4q_1 - 7q_2)}{(4q_1 - q_2)^{\,3}} - 2cq_2 = 0$$

where $c = 2\sqrt{\frac{wc_s}{\eta}}$*. Solving these FOCs, we obtain* $q_1 = \frac{M\phi_1}{2}\sqrt{\frac{\eta}{wc_s}}$ *and* $q_2 = \frac{M\phi_2}{2}\sqrt{\frac{\eta}{wc_s}}$*, where* ϕ_1 *and* ϕ_2 *are constants that are the positive real solutions to the polynomials below:*

$$(\phi_1) \quad -128 + 1168x - 31111x^2 + 235824x^3 = 0,\ (\phi_2) \quad -16 + 944x - 13057x^2 + 58956x^3 = 0$$

Using these and the optimal quality decomposition results from Proposition (A2) we obtain the

will not constitute an equilibrium since developers will only contribute when they expect to be hired. Thus, the low-quality outcome cannot occur in equilibrium and we do not consider it further.

stated results. Consider any potential symmetric equilibrium characterized by the equilibrium features and usability outcomes $f_1 = f_2 = f^*$ and $s_1 = s_2 = s^*$. With equal quality levels, the firms will charge equal prices (Proposition A1), and obtain half the market. If the firms charge different prices (say $p_1 > p_2$), all consumers will prefer firm 2's product since the qualities are equal. Further, we demonstrate that both firms charge zero prices. If either firm charges $p' > 0$, its competitor can obtain the entire market by offering a price of $p'' = p' - \epsilon$, where $\epsilon > 0$ is a small deviation. Recall that costs sunk in Stage 2 do not affect pricing in Stage 3. Therefore, firms earn zero revenue and have positive costs in any symmetric equilibrium. Consider a profitable deviation by firm 2, setting $s_2 = s' = s^* - \delta$, where $\delta > 0$ is a small deviation. Firm 2 can obtain higher revenues by part (i), and has lower development costs, thereby increasing profits beyond the symmetric equilibrium outcome. Thus, there is no equilibrium with symmetric strategies. □

Proof of Proposition 2. *For the shared features market, the FOCs for the firm profits with respect to both features f_j and usability s_j for $j = 1, 2$ are:*

$$\frac{\partial \Pi_1}{\partial f_1} = \frac{M\eta\sqrt{s_1}\left(\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)}{\left(f_0 + \eta\left(f_1 + f_2\right)\right)^{\frac{3}{4}}\left(\sqrt[4]{s_2} - 4\sqrt[4]{s_1}\right)^2} - w,$$

$$\frac{\partial \Pi_1}{\partial s_1} = \frac{\sqrt[4]{f_0 + \eta\left(f_1 + f_2\right)}M\left(\left(2\sqrt[4]{s_2} - 3\sqrt[4]{s_1}\right)\sqrt[4]{s_2} + 4\sqrt{s_1}\right)}{\sqrt{s_1}\left(4\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)^3} - c_s$$

$$\frac{\partial \Pi_2}{\partial s_2} = \frac{M\sqrt[4]{f_0 + \eta\left(f_1 + f_2\right)}\sqrt{s_1}\left(4\sqrt[4]{s_1} - 7\sqrt[4]{s_2}\right)}{4\left(4\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)^3 s_2^{3/4}} - c_s,$$

$$\frac{\partial \Pi_2}{\partial f_2} = \frac{M\eta\sqrt[4]{s_1}\left(\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)\sqrt[4]{s_2}}{4\left(f_0 + \eta\left(f_1 + f_2\right)\right)^{\frac{3}{4}}\left(\sqrt[4]{s_2} - 4\sqrt[4]{s_1}\right)^2} - w$$

We check whether an interior solution is possible in which both COSS firms can hire developers to develop features. Take the difference between the FOCs with respect to features:

$$\frac{\partial \Pi_1}{\partial f_1} - \frac{\partial \Pi_2}{\partial f_2} = \frac{M\eta\sqrt[4]{s_1}\left(\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)}{4\left(f_0 + \eta\left(f_1 + f_2\right)\right)^{3/4}\left(4\sqrt[4]{s_1} - \sqrt[4]{s_2}\right)}$$

We find that $\frac{\partial \Pi_1}{\partial f_1} - \frac{\partial \Pi_2}{\partial f_2} > 0$ at any wage level since $s_1 > s_2$ always holds. Therefore, both

FOCs $\frac{\partial \Pi_1}{\partial f_1} = 0$ *and* $\frac{\partial \Pi_2}{\partial f_2} = 0$ *cannot be simultaneously satisfied, which implies we cannot have an interior solution for both firms. Thus, either* $\frac{\partial \Pi_1}{\partial f_1} > 0$ *or* $\frac{\partial \Pi_2}{\partial f_2} < 0$ *or both must hold. If* $\frac{\partial \Pi_1}{\partial f_1} > 0$*, firm 1 will find it optimal to hire more developers and add features. Since firm 1 can improve its profits when* $\frac{\partial \Pi_1}{\partial f_1} > 0$*, we cannot have an equilibrium. Therefore,* $\frac{\partial \Pi_1}{\partial f_1} = 0$ *in equilibrium and* $f_1^* > 0$*. On the other hand,* $\frac{\partial \Pi_2}{\partial f_2} < 0$ *implies that firm 2 would find it optimal to hire fewer developers, and the inequality is consistent with* $f_2^* = 0$ *where it would still apply. Observe this result does not depend on* w *or any other parameters.*

Setting $f_2^* = 0$ *and using the FOCs for the others, after much algebraic manipulation we obtain the results in the proposition, with the constant* σ_2 *defined as the positive real root of the polynomial equation*

$$(\sigma_2): \qquad 203695116x^5 - 28488008x^4 + 1778324x^3 - 50117x^2 + 593x - 2 = 0, \text{ and}$$

$$\sigma_1 = \frac{1}{72612520768273} \times \left(219880056370426\, \sigma_2 - \frac{20684094246117}{\sigma_2^2} + \frac{890714199899}{\sigma_2^5} - \frac{15973663528}{\sigma_2^8} + \frac{63603928}{\sigma_2^{11}}\right)$$

□

Proof of Proposition 3. *For a separating equilibrium, we need to characterize the conditions on contributions to open source* e_L *and* e_H *for each type of developer. The market wage is* w *for the high-type developer and* 0 *for the low-type, where* w *is determined in equilibrium. Consider the binding constraints* IR_H *and* IC_L *specified in (4.3). We argued in §4.2.1 that* $e_L = 0$*. The* IR_H *condition implies* $w(e_H) - c_H e_H \geq r$ *for a high-type developer with reservation utility* r*, who will only signal up to a level* $e_H \leq \frac{w-r}{c_H}$*. Only high-type developers with low reservation option, i.e.* $r < \Psi(w - c_H e_H)$ *will choose to enter by signaling. These conditions correspond to part (i).*

The least-cost separation is achieved by the high type contributing just enough to open source that it deters the low-type from masquerading: $e_H^{LCS}(w) = \frac{w}{c_L}$ *and the corresponding belief by the firms to support that* $\mu(H|e, w) = \begin{cases} 0, e < \frac{w}{c_L} \\ 1, e > \frac{w}{c_L} \end{cases}$*. To prove this LCS contribution is an equilibrium, suppose the high type deviates from the least-cost contribution to* $e' =$

$e_H^{LCS}(w)+\epsilon$, where $\epsilon > 0$. The best possible belief for any type at this level e' is $\mu(H|e', w) = 1$. For the high type, this deviation to e' is not profitable since $w - c_H e' < w - c_H e_H^{LCS}(w)$. Therefore, no such deviation contributing beyond $e^{LCS}(w)$ is profitable, and any deviation below $e_H^{LCS}(w)$ will result in the firm believing the developer is low-type, implying $e_H^{LCS}(w)$ is an equilibrium strategy for the high-types. For the low-type developer, for any $e > e_H^{LCS}$, we have $w - c_L e < 0$ so the low-type will not deviate from $e_L^{LCS} = 0$.

We apply the intuitive criterion for part (ii) to eliminate non-LCS equilibria: Suppose another equilibrium exists where the high type contributes e'. This equilibrium requires that firms' beliefs on the equilibrium path are: $\mu(H|e, w) = \begin{cases} 0, e < e' \\ 1, e \geq e' \end{cases}$. Consider a deviation $\tilde{e}$ from the equilibrium path where $\frac{w}{c_L} < \tilde{e} < e'$. The best possible belief, $\mu(H|\tilde{e}, w) = 1$ is still not sufficient to induce the low-type developers to contribute $\tilde{e}$ since $w - c_L\tilde{e} < 0$. Therefore, only the high-type developer could have deviated to $\tilde{e}$, and the intuitive criterion requires the firms to assign beliefs $\mu(H|e', w) = 1$ after observing $\tilde{e}$. This reasoning leads to an inconsistent off-equilibrium-path belief and we can therefore eliminates this equilibrium. We can apply this criterion to filter any equilibrium with high type contributing $e' > e_H^{LCS}(w)$, and the only remaining equilibrium is the least-cost separating equilibrium.

In a least-cost equilibrium, the high-type developers who signal are those with reservation utilities $r < \tilde{r}^{LCS} = w - c_H e_H^{LCS}(w)$ so the number of entering developers is $\Psi(\tilde{r}^{LCS}) = \Psi(w - c_H e_H^{LCS}(w))$, which proves part (iii). □

Proof of Proposition 4. *We apply the implicit function theorem to Equation (4.7):*

$$\frac{dw^P}{dc_H} = -\frac{w^{\frac{3}{2}}\left(\frac{-w}{c_L}\right)\psi\left(w\left[1 - \frac{c_H}{c_L}\right]\right)}{\left(\frac{3}{2}w^{\frac{1}{2}} + \frac{5w^{\frac{3}{2}}}{c_L}\right)\Psi\left(w\left[1 - \frac{c_H}{c_L}\right]\right) + \left(w^{\frac{3}{2}} + \frac{2w^{\frac{5}{2}}}{c_L}\right)\left(1 - \frac{c_H}{c_L}\right)\psi\left(w\left[1 - \frac{c_H}{c_L}\right]\right)}\Bigg|_{w=w^P} > 0$$

We find the comparative statics of wages with respect to c_s, η, and M in a similar manner.

□

Proof for Corollary 2. For this corollary, we require the following assumption:

Assumption (A1)**.** *The wage in the alternative market satisfies the following condition:*

$$(\hat{w})^{\frac{3}{2}}\left[\hat{w}\left(1-\frac{c_H}{c_L}\right)\left(1+\frac{\hat{w}}{c_L}\right)\right] < \frac{M^2(\phi_1^2+\phi_2^2)}{4}\sqrt{\frac{\eta}{c_s}} < (\hat{w})^{\frac{3}{2}}\left[\hat{w}\left(1-\frac{c_H}{c_L}\right)\left(1+\frac{2\hat{w}}{c_L}\right)\right]$$

The first condition ensures that demand from the high-quality firm for developers at wage $\hat{w}$ in the shared features market does not exceed the number of high type developers signaling, whereas the second ensures that when both firms hire developers in the private features market, a higher wage induces developers to signal beyond $e_H^{LCS}(\hat{w}) = \frac{\hat{w}}{c_L}$. □

In order to ensure sufficient conditions for the following results, we assume that the signaling cost for high-type developers is not excessive:

Assumption (A2)**.** $c_H < \tilde{c}_H$ *where* $\Psi\left(w^C\left(1-\frac{\tilde{c}_H}{c_L}\right)\right)\left[1+\frac{2w^C}{c_L}\right] = \Psi(w^C)$

Proof of Result 1. *Denote* $\xi^P(w) = (w)^{\frac{3}{2}}\psi\left(w\left[1-\frac{c_H}{c_L}\right]\right)\left(1+2\frac{w}{c_L}\right)$ *and* $\xi^C(w) = (w)^{\frac{3}{2}}\Psi(w)$*. We observe that as* $c_H \to 0$*,we get* $\lim\limits_{c_H\to 0}\xi^P(w^P) = (w^P)^{\frac{3}{2}}\Psi(w^P)\left[1+2\frac{w^P}{c_L}\right]$ *and since* $\xi^C(w^C) = (w^C)^{\frac{3}{2}}\Psi(w^C)$ *we get* $(w^P)^{\frac{3}{2}}\Psi(w^P)\left[1+2\frac{w^P}{c_L}\right] = (w^C)^{\frac{3}{2}}\Psi(w^C)$*, implying* $w^P < w^C$*. For high values of* c_H *approaching* c_L*, we have* $\lim\limits_{c_H\to c_L}\xi^P(w^P) = \Psi(w^P\times 0)\left[1+2\frac{w^P}{c_L}\right] = \frac{M^2\left(\phi_1^2+\phi_2^2\right)}{4}\sqrt{\frac{\eta}{c_s}} \Rightarrow w^P \to \infty$*. First, we establish* w^P *is increasing in* c_H*:* $\frac{dw^P}{dc_H} = -\frac{\frac{\partial\xi^C}{\partial c_H}}{\frac{\partial\xi^C}{\partial w}} > 0$ *since* $\frac{\partial\xi^C}{\partial c_H} = -\frac{1}{c_L}(w)^{\frac{5}{2}}\psi\left(w\left[1-\frac{c_H}{c_L}\right]\right)\left(1+2\frac{w}{c_L}\right) < 0$ *and* $\frac{\partial\xi^C}{\partial w} > 0$.

Since w^P *is continuous and increasing in* c_H*, we know* $\exists\tilde{c}_H$ *so that when* $c_H < \tilde{c}_H$ *we get* $w^P < w^C$ *and when* $c_H > \tilde{c}_H$ *we have* $w^P > w^C$ *ceteris paribus. The threshold value* $\tilde{c}_H$ *is defined as solving* $\xi^P(w^C;\tilde{c}_H) = \frac{M^2\left(\phi_1^2+\phi_2^2\right)}{4}\sqrt{\frac{\eta}{c_s}}$*, which is the condition specified in Assumption (A2).* □

Proof of Result 2. *The contribution to open source in the private features market is* $F^P = \Psi\left(w^P\left[1-\frac{c_H}{c_L}\right]\right)\frac{w^P}{c_L}$ *whereas in the shared features market, contributions made by the firms and due to developers signaling is* $F^S = \frac{M^2\sigma_1^3(\sigma_1-\sigma_2)^{\frac{3}{2}}}{(4\sigma_1-\sigma_2)^3}\sqrt{\frac{\eta}{\hat{w}^3 c_s}}$*. Substituting from the implicit wage equation (4.7), we obtain* $F^P > F^S \iff \frac{1}{2+\frac{c_L}{w}} > \frac{4\sigma_1^3(\sigma_1-\sigma_2)^{\frac{3}{2}}}{\phi_1^2(4\sigma_1-\sigma_2)^3}\frac{1}{\hat{w}^{\frac{3}{2}}}$ *which holds when* $\hat{w}$ *is large,* c_L *is low, or when* w^P *is high, which in turn occurs when* M *is large or* c_s *is low.* □

Proof of Result 3. *For part (i), we obtain the quality levels from Propositions 1 and 4 to find:* $\frac{q_2^P(w^P)}{q_2^S(w^S)} = \frac{\phi_2(4\sigma_1-\sigma_2)}{2\sigma_1\sigma_2\sqrt{\sigma_1-\sigma_2}}\sqrt{\frac{\hat{w}}{w^P}} < \sqrt{\frac{\hat{w}}{w^P}}$*. We know the final fraction is less than 1* *from Proposition 1. Comparing the quality levels for the high-quality product in both the shared features and private features markets, we find* $\frac{q_1^P(w^P)}{q_1^S(w^S)} = \frac{\phi_1(4\sigma_1-\sigma_2)}{2\sigma_1^2\sqrt{\sigma_1-\sigma_2}}\sqrt{\frac{\hat{w}}{w^P}}$*. For* $q_1^P > q_1^S$*, we must have* $w^P < w^S\left(\frac{\phi_1(4\sigma_1-\sigma_2)}{2\sigma_1^2\sqrt{\sigma_1-\sigma_2}}\right)^2$*. This condition can only hold for small market sizes and low signaling costs and proves part (ii). Comparing the private features and closed source markets, we obtain from 1 and Assumption (A2) that* $w^P < w^C$*. The other parameters are identical across these two markets, which gives us part (iii) since quality depends inversely on wage levels. Part (iv) requires* $\frac{q_1^P}{q_2^P} = \frac{\phi_1}{\phi_2} > \frac{q_1^S}{q_2^S} = \frac{\sigma_1}{\sigma_2}$ *which we can numerically verify.* □

Proof of Result 4. *For the low-quality firms, the profits under different markets can be derived from the equilibrium quality levels as:*

$$\Pi_2^S = \gamma_2^S M^2\sqrt{\frac{\eta}{\hat{w}c_s}} \text{ and } \Pi_2^P = \gamma_2^P M^2\sqrt{\frac{\eta}{w^P c_s}} + \Psi\left(w^P\left[1-\frac{c_H}{c_L}\right]\right)\frac{(w^P)^2}{\eta c_L}$$

where $\gamma_2^S = \frac{\sigma_1\sigma_2(\sigma_2-\sigma_1)\left(\sigma_2^5+\sigma_1^2\left(16\sigma_2^3-1\right)+\sigma_1\left(\sigma_2-8\sigma_2^4\right)\right)}{\sqrt{\sigma_1-\sigma_2}(4\sigma_1-\sigma_2)^3}$ *and* $\gamma_2^P = \frac{\phi_2\left(\phi_1(8\phi_2-1)\phi_2-\phi_2^3-\phi_1^2(16\phi_2-1)\right)}{2(\phi_2-4\phi_1)^2}$*. The low-quality firm has a higher profit under shared features than the private features market with the same wage since* $\gamma_2^S > \gamma_2^P$*. Since* $w^P > w^S$ *from Proposition 1, the firm always has a higher profit in the shared features market. The profits of the high-quality firms under different market conditions are similarly derived to be:*

$$\Pi_1^S = \Psi\left(\hat{w}\left[1-\frac{c_H}{c_L}\right]\right)\frac{(\hat{w})^2}{\eta c_L}+\gamma_1^S M^2\sqrt{\frac{\eta}{\hat{w}c_s}}, \quad \Pi_1^P = \Psi\left(w^P\left[1-\frac{c_H}{c_L}\right]\right)\frac{(w^P)^2}{\eta c_L}+\gamma_1^P M^2\sqrt{\frac{\eta}{w^P c_s}}$$

where $\gamma_1^S = \frac{\sigma_1^3\sqrt{(\sigma_1-\sigma_2)}\left(\sigma_1\left(3-\sigma_1(\sigma_2-4\sigma_1)^2\right)-3\sigma_2\right)}{(4\sigma_1-\sigma_2)^3}$ *and* $\gamma_1^P = \frac{\phi_1^2\left(4(2\phi_2\phi_1+\phi_1)-16\phi_1^2-\phi_2(\phi_2+4)\right)}{2(\phi_2-4\phi_1)^2}$.

When $\hat{w} = w^P$*, then we find that* $\Pi_1^P > \Pi_1^S$ *since* $\gamma_1^S < \gamma_1^P$*. Observe that* Π_1^P *decreases with* w^P*, which is higher when the market size* M *is large, or when signaling becomes difficult (*c_H *is high). These conditions therefore result in lower profits for the high-quality firm in the private features setting.* □

Proof of Result 5. *From Proposition A1, we can rewrite the consumer surplus expression as:* $CS(q_1,q_2) = q_1\left[\left(\frac{q_1}{q_2}\right)\frac{4\frac{q_1}{q_2}+5}{\left(4\frac{q_1}{q_2}-1\right)^2}\right]$*. Observe that the term in square brackets only depends on the quality ratio, which is independent of the wage and other model primitives but depends on the market. We know that when the wages are identical* $CS^S(w) > CS^P(w)$ *and the surplus is increasing in the quality level, which in turn decreases with the equilibrium* *wage. Since $w^S < w^P$, the surplus inequality will continue to hold at the equilibrium wage. Comparing CS^C and CS^P, we observe that the quality ratios and therefore the term within square brackets is identical for these two markets. Since the consumer surplus is directly proportional to the quality level of the high-quality product and $w^C > w^P$, we know $q_1^C < q_1^P$ since q_1 decreases with wage level. This reasoning then implies $CS^S > CS^P > CS^C$.* □

Proof of Proposition 5. *The proof of part (i) is derived from the expressions in Propositions 1 and 4. When the wage is the same across markets,*

$$\frac{q_1^S}{q_1^P} = \frac{2\sigma_1^2\sqrt{\sigma_1 - \sigma_2}}{\phi_1} < 1$$

Similarly comparing the expression for features, we find that $f_1^S < f_1^P$. For part (ii), the proof of Result 4 shows that $\gamma_1^S < \gamma_1^P$. If $\hat{w} = w^P$, then $\Pi_1^S < \Pi_1^P$. This result is independent of any parameters. For part (iii), consider the proof of Result 5, where the consumer surplus depends on q_1 and the ratio $\frac{q_1}{q_2}$. The ratio is identical for both private features and closed source markets, and we find from Proposition 1 that the quality only depends on the wage. This implies product quality and consumer surplus will be equal in the closed source and private features markets. □