Networks, Complementarities, and Transaction Costs: Applications in Finance and Economic Development

by

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To my family.

Abstract

Essay 1: Resource complementarity, alliances, and merger waves

The fact that mergers cluster in time is an important puzzle in finance. Recent explanations rely on over-valuation arguments or the existence of an economy-wide component in merger transaction costs. This essay proposes an alternative theory for merger waves. I develop a static game-theoretic model of network formation where firms combine complementary non-tradable resources, either by establishing an alliance or by merging. In a dynamic extension, I use the results from the static model and solve an option exercise game where distinct sets of firms may choose to merge. In both the static and dynamic cases, the existence of inter-industry alliances may propagate merger activity across sectors. The model is consistent with time-series data on the aggregate number of alliance and merger deals. In particular, the data seems to indicate that merger waves are preceded by an increase in alliance activity. The model also has implications beyond the topic of merger waves: (i) merger excess returns and/or merger frequency between firms in different industries should (a) display an inverted-U relationship with respect to complementarity, controlling for the level of coordination problems in alliances; and (b) controlling for complementarity, be higher in industries where alliances exhibit starker coordination problems; (ii) the model offers a rationale for the diversification discount, namely that it is optimal for diversifying acquirers to choose inexpensive targets, if the objective of the diversifying firm is to obtain only a subset of the target's resources; (iii) spin-offs take place optimally when the level of complementarity within the conglomerate is lower and/or the acquirer of the spun off division has a high firm-specific productivity, relative to the conglomerate; (iv) mergers may be socially inefficient, for reasons distinct from market power.

Essay 2: Social ties and economic development (with José Anchorena)

This paper develops a general equilibrium model where the set of goods includes ties between economic agents (e.g., friendships or acquaintances). We refer broadly to these as social ties. A tie between any two agents is produced according to a technology that uses time from both parties. The model also assumes that social ties contribute toward social capital, which economizes on transaction costs between members of the same community. Our theoretical approach yields the existence of multiple equilibria, which can be interpreted as rational outcomes in societies with different cultural be-

liefs, in the sense of Greif (1994). We calibrate this model to data on social ties and income per capita for a cross section of 27 countries. Our main quantitative findings are the following: (i) heterogeneity in the average number of social ties can account for a significant portion of the heterogeneity in income per capita across countries; (ii) the model can account for between 1/5 and 1/2 of the changes in use of time in the United States between 1900 and 2000; (iii) according to one measure, the calibrated model implies that without social capital countries would be between 1/2 and 3/4 their actual size in terms of income per capita. Theoretically we have the following additional results: (i) a preference for social ties may significantly mitigate an otherwise large underprovision of social capital; (ii) social capital is in some instances an important source of economic efficiency, very much complementary to labor and/or human capital; (iii) the elasticity of substitution between standard goods and social ties is an important determinant of the observed relationship between social capital and economic development; (iv) in some instances, an increase in productivity causes a decrease in welfare, via amplification of coordination failure in the production of social ties. Finally, in light of this model and our calibration, we conclude that social capital is mainly an externality of the consumption of social ties, and does not result from the agents' motivation to economize transaction costs.

Essay 3: Costly diversification and the diversification discount
I develop a stationary real options model with corporate restructuring costs that endogenously generates a diversification discount. This result requires that restructuring costs associated with spin-offs - i.e. refocusing moves - be significantly larger than those associated with acquisitions – i.e. diversifying moves. The discount is due to the fact that diversified firms performing poorly will still delay refocusing, given the high cost of implementing this strategy. Moreover, the model implies that a diversification discount is consistent with a cross-sectional distribution of firms where a significant portion of single-segment business units is diversified. Using data on US firms for the period 1984-2005 I analyze the differences in profitability and Tobin's Q between diversified and focused firms. The empirical findings are in line with previous literature, namely that diversified firms trade at a sizable discount. The data also reveals that diversified firms' Return on Assets (ROA) is on average 6% higher than that of focused firms, after controlling for industry effects. I calibrate the model to this sample and obtain a good quantitative fit to the data, namely the coexistence of a diversification discount and a higher profitability of diversified firms relative to focused firms.

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Contents

Introduction

As the title suggests, all three essays in my dissertation have three common themes: networks, complementarities, and transaction costs. While two of the chapters deal essentially with the topic of mergers and acquisitions, which perhaps one would readily associate with these three themes, one chapter is on social ties and economic development, where the connection is (maybe) less obvious. This introduction is not intended as a repetition of the abstract or the introductions to each essay. Rather, my objective is to attempt to establish the conceptual connection between the three essays.

First let us consider *networks*. What are networks and why are they interesting for economists and social scientists in general? In any setting a network refers to a structure in which agents (individuals, households, firms) are embedded. This structure can both condition the agents' behavior or, alternatively, be the result of such behavior. An example are social networks: agents may access job or investment opportunities via existing friends and acquaintances, or the fact that these friendships and acquaintances exist be itself a manifestation of the agents' motivation in seeking those same job or investment opportunities. This poses an empirical difficulty, namely to understand the direction of causality between network structure and agent behavior. This type of endogeneity issue is common in social sciences, and Carnegie Mellon has a reputation of addressing such issues using structural models. To a large extent that is what my co-author José Anchorena and I do in the chapter on social ties and economic development. A fully specified general equilibrium model with certain preferences and technology is calibrated to data, and we are able to make statements about which effects are quantitatively relevant. For example, while the model allows for two motives for agents building relationships – consumption of social ties and self-interested reduction of transaction costs – the calibration exercise suggests that it is the consumption motive that is prevalent.

The first step in constructing a model with networks is to specify the mechanism of link formation, and the payoff implications of the link structure. While the links in the social ties chapter refer to relationships between individuals, the same mechanism of link formation is used in the chapter on alliances and merger waves. The functional form for the directed link between agents i and j is the following:

$$a_{ij}^{\beta} a_{ji}^{1-\beta},$$

where a_{ij} is controlled by agent i. The functional form above has two fundamental characteristics, both of which matter for both chapters. The first is that it implies *voluntariness* in the process of link formation – any agent can shut off the link by setting $a_{ij} = 0$. The second is that there is an incentive to free ride in relationship building, which is gaged by the parameter β. If a_{ij} implies a cost (direct or indirect), and since a_{ji} is somewhat substitute to a_{ij} in the production of the link, then the agent may have an incentive to reduce a_{ij} from a first-best allocation. The coordination problems are consequential, for example for determining when mergers – achieving *ex post* first-best allocation of resources – are preferable to alliances. In the social ties chapter it is the existence of coordination problems that gives rise to the counter-intuitive result that an increase in productivity may decrease welfare. An obvious advantage of this functional form is its simplicity; whether it is empirically appropriate, strictly as a description of behavior, is still matter for further work.

So far I have discussed the mechanism underlying network formation. Perhaps more importantly, one needs to consider the motivation for the existence of such links in the first place. In all three chapters the motivation relates to *complementarities*. In the essay on alliances and merger waves firms establish alliances or merge in order to pool complementary resources (e.g., scale and know-how) in order to increase profit. In the essay on the diversification discount firms merge to take advantage of *synergies*. The synergies are modeled in reduced-form, but it is a notion of complementarity between firms' capabilities in different sectors that motivates this theoretical approach to diversification. In the essay on social ties and economic development, complementarity shows up in more than one form: (i) it is the fact that there is some complementarity between the consumption of standard commodities and social ties that rationalizes the co-existence of both; and (ii) social capital, in some instances, is complementary to other factors in fostering economic development, essentially because it enables efficient trade.

Finally we arrive at the topic of *transaction costs*. In the paper on social ties and economic development it is the fact that transaction costs are quantitatively important that creates an important role for social capital. In the paper on alliances and merger waves, it is only by exogenously assuming (merger) transaction costs that the economy is not fully integrated and alliances may be optimal. In the paper on the diversification discount it is the asymmetry between the transaction costs of a diversifying move relative to a spin-off that creates an average diversification discount. These are all *direct* effects of transaction costs. However, transaction costs may have important indirect effects. For instance, in the social ties essay, if transaction costs are high and there is no social capital to curb them, agents have little incentive to make the sacrifices (investment in human capital and labor) that will lead to an increased productivity in the future. In explaining merger waves, I show how a shock to the transaction costs in one location of the network of firms may *per se* trigger a cascade of merger activity.

It is also a tacit assumption about transaction costs that motivates the modeling approach to resource complementarity, in the chapter on alliances and merger waves. In this model there are no liquid markets for the exchange of production factors. It is the inability of trading separately each resource that gives rise to gains from two distinct firms pooling complementary resources. If these resources were traded in a competitive market, the rationale for alliances and mergers would disappear. This leads naturally to the discussion of why we observe certain markets but not others, which is outside the scope of my current work.

Chapter 1

Resource Complementarity, Alliances, and Merger Waves

1.1 Introduction

On October 7, 2007, software provider SAP announced the acquisition of Business Objects for an amount close to 7 billion USD. Business Objects is a leading provider of Business Intelligence, an important component of corporate software packages. A key aspect of the strategy of Business Objects is creating and maintaining a vast array of partners. One type of partnership relates to technological complementarities: Business Objects lists 59 companies as "complementary technology partners".[1]

In early November 2007 information services giant Dun & Bradstreet announces the acquisition of Purisma for 48 million USD. Purisma is a specialized software company in the area of Information Management, and it had established a partnership with Dun and Bradstreet one year earlier.[2] Purisma is also listed as one of the 11 top tier complementary technology partners by Business Objects.

The temporal closeness of these two mergers may be a coincidence. Nonetheless, it seems reasonable to assume that a big merger such as SAP's acquisition of Business Objects impacts the businesses of alliance partners. How far will this impact reach? Can it bear significantly in the restructuring decisions of other industries, such as Dun and Bradstreet's acquisition of Purisma? This paper addresses these questions from a theoretical perspective, by developing a model of merger and alliance activity. In particular, the model implies that the existence of inter-industry alliances may create a cascade of mergers.

The topic of mergers and acquisitions (M&A) has received significant attention in financial economics research. M&A activities are one important way in which resources are reallocated in the economy, and financial economists have been addressing questions such as (i) "does M&A create value?", (ii) "which firms gain from M&A?", and (iii) "what drives the decision to merge?". While there has been some consensus in terms of the answers to (i) and (ii), the answers to (iii) are still deemed incomplete and have motivated recent literature, such as Rhodes-Kropf and Robinson (2007), Lambrecht and Myers (2007), and Harford (2005).

In addressing the question of why firms merge, research has found that mergers cluster both in the cross section and in the time series; see Mitchell and Mulherin (1996) and Andrade, Mitchell, and Stafford (2001). The latter phenomenon is referred to as *merger waves*. Two main alternative explanations have been put forth for the

[1]Source: www.businessobjects.com

[2]Source: news site www.searchdatamanagement.com

existence of merger waves. The neoclassical hypothesis claims that mergers are the result of an adequate response of firms to industry and economy-wide shocks. Papers in this strand of research include Jovanovic and Rousseau (2002) and Harford (2005). The former uses a neoclassical investment model where large differences in firm-specific productivities trigger capital reallocation via takeovers. Harford (2005) relates time-varying financing constraints to restructuring activity. The competing theory to the neoclassical hypothesis argues that mergers are the result of financial market overvaluation. Shleifer and Vishny (2003) model privately informed managers that take advantage of misvaluations; Rhodes-Kropf and Viswanathan (2004) propose a rational model that supports this hypothesis.

The fact that periods of high market valuations coincide with a relaxing of firms' financing constraints creates difficulties in sorting out which of the competing explanations is a better description of reality. My model presents new testable implications that help overcome this ambiguity, namely in terms of the relationship between alliance and merger activity. In the model alliances and mergers are partially substitutable in terms of achieving synergies through the combination of non-tradable resources. I find support in the literature to the notion that alliances and mergers have a high degree of substitutability; see McConnell and Nantell (1985), Villalonga and McGahan (2005), and Wang and Zajac (2007), among others.

The key difference between the concept of an alliance (as employed in this paper) and a spot market transaction is that both parties commit part of their own non-tradable resources (skills) to a common project. The canonical example of an alliance is a joint venture. The pooling of resources may in practice assume other contractual forms, such as marketing and R&D agreements. An example is biotech-pharmaceutical alliances.[3] In these alliances pharmaceutical firms bring scale and marketing capabilities, while biotech firms bring scientific know-how. The reason why this form of organization is observed, *versus* for instance in-house biotech divisions at pharmaceutical companies, is beyond the scope of this paper. Robinson (2007) shows how in some instances strategic alliances may be a superior organizational form, relative to internal development.

The fact that firms cannot trade certain resources does not imply that they cannot combine them via alliances or mergers. In fact, empirically it has been shown that resource complementarity is a driver of both alliances and mergers; see for example

[3]See Nicholson, Danzon, and McCullough (2005) for a discussion of this type of partnership.

Wang and Zajac (2007) and Casciaro and Piskorski (2005).[4] In the model this is captured by allowing firms to establish bilateral links, through which resources are combined. In a merger this takes place under a hierarchical structure, while I model an alliance as a non-cooperative game using a reduced-form link formation mechanism. This approach follows from the the game-theoretic literature on network formation, e.g., Jackson and Wolinski (1996) and Bala and Goyal (2000).

In reality an alliance is usually a modular project, as in the case of joint ventures. I do not model these common projects explicitly (resource sharing impacts firms directly), but this is without loss of generality given the objective of the paper. The gains from any cooperative endeavor only matter to the extent that they create additional profits for the parent firms. These firms are the agents making decisions regarding the pooling of resources and bearing the ultimate consequences therefrom.

A recurrent theme in the alliances literature is the tension between cooperation and competition. The argument is that by forming alliances, one of the partners may improve its competitive edge over the other in the long run, by internalizing skills. This idea is explored in Hamel (1991), Kanna, Gulati, and Nohria (1998), and Habib and Mella-Barral (2007). My model somewhat captures this tension via the reduced-form mechanism of link formation.

Limitations to efficient coordination in an alliance could be overcome by a merger. However, mergers imply high transaction costs. These include costs of raising capital, lawyer and investment bank fees, and opportunity costs of managerial time. The fact that some mergers may face challenges by regulatory agencies can also be viewed as a source of transaction costs. In the model I take these costs as exogenous.

The key theoretical result of the paper is that *the existence of alliances propagates merger activity*, under certain conditions. The intuition for this result is as follows. In the model, when a merger takes place, it will imply a higher inward focus for the merging pair of firms. By inward focus I mean the degree to which the resources are maintained inside the firm (*versus* committed to alliances). In equilibrium this increased inward focus lowers the performance of adjacent firms, since they are now less able to obtain critical resources. This in turn implies under some conditions that there is an increased benefit for other pairs of firms to focus inwardly by merging. In other words, the merger decisions of one subset of firms carry important externalities, which

[4] Casciaro and Piskorski (2005) use the more sociological concept of mutual dependence, which for my purposes is equivalent to complementarity.

are in turn relevant for other firms' merger decisions. Without alliances the model has no channel through which one merger can affect the payoffs of other firms, so non-trivial merger waves are not possible without alliances. The model also accommodates merger waves being explained by a common component in merger costs, or shocks to resources that are important for a broad set of industries. I do not however explore these alternatives.

Data on the aggregate behavior of alliance and merger activity is supportive of my explanation for merger waves. In particular, data suggests that increases in merger activity are preceded by increases in alliance activity, which is consistent with the model. Beyond the connection between these data and the theory I am putting forth, considering alliances as somewhat substitutes for mergers helps to distinguish between the neoclassical and behavioral hypotheses for merger waves. Since the behavioral hypothesis relies on market over-valuation arguments, it does not apply to alliances. On the other hand, the fact that both mergers *and* alliances exhibit some degree of clustering is consistent with the notion that economy-wide and/or industry shocks are the fundamental cause of these aggregate movements.

I wish to point out that the mechanism behind merger propagation in this model does not rely on market power arguments. Although in practice it is possible that mergers in downstream (upstream) industries may take place as a reaction to horizontal mergers upstream (downstream), making this a potential factor behind merger waves, this is beyond the scope of the paper.[5] Merger propagation in my model is driven simply by the indirect effects of mergers on non-tradable resource allocation.

The paper has other results beyond the connection of alliances and merger waves. The first relates to the cross section of merger announcements. According to the model, merger excess returns and/or merger frequency between firms in different industries should (i) display an inverted-U relationship with respect to complementarity, controlling for the level of coordination problems in alliances; and (ii) controlling for complementarity, be higher in industries where alliances exhibit starker coordination problems. The model also predicts mergers that are not motivated by complementarities, with an argument very similar to Jovanovic and Rousseau (2002). As mentioned above, in Jovanovic and Rousseau (2002) mergers take place to channel tangible assets

[5] One would still have to explain why firms would merge for market power reasons if in the end they end up with the same (relative) market power, given downstream and/or upstream restructuring. Also, if mergers were motivated by collusive reasons, regulatory agencies would in principle intervene and these merges would not be observed.

to more productive firms. This is the so-called Q-theory of mergers and it implies that mergers should be driven by large gaps in Tobin's Q between merging firms. Other papers have linked the difference between acquirer and target Tobin's Q to merger gains, namely Lang, Stulz, and Walkling (1989) and Servaes (1991). Recently Rhodes-Kropf and Robinson (2007) challenge this notion, both theoretically and empirically. Inspecting the bivariate distribution of market-to-book ratios (a proxy for Tobin's Q) for acquirer and target, the authors find that merging pairs are usually similar in this characteristic. Moreover, most deals take place for high market-to-book pairs. The authors then present a rationalization for this finding based on asset complementarities and search costs. The model in my paper nests two types of mergers, which I term as *productivity mergers* (similar to Jovanovic and Rousseau (2002)) and *complementarity mergers* (similar to Rhodes-Kropf and Robinson (2007)). The latter take place as a substitute for alliances with high coordination problems, while the former channel certain resources to their most efficient use. Furthermore, the model relies on a combination of inter-firm asset complementarities and firm-specific productivity differentials in order to rationalize merger waves.

The model also offers a theoretical explanation for the diversification discount, namely that it is optimal for diversifying acquirers to choose a "cheap" target, relative to industry peers. The reason why this takes place in the model is that a diversifying firm seeks only a subset of the target's resources; the ones in which it is in short internal supply. This explanation for the diversification discount is consistent with the empirical findings of Graham, Lemmon, and Wolf (2002). This paper shows that approximately 50% of the discount is attributable to conglomerates buying firms that are already discounted. In the same extension where I analyze the diversification discount I provide a rationale for spin-offs. These take place when there is a firm in the economy with very high idiosyncratic productivity that acquires the division that was spun off and/or the complementarities within the conglomerate decrease. In principle these implications are empirically testable.

Finally, I illustrate with a numerical example that in the model mergers may be socially inefficient, for reasons distinct from market power. In particular, a merger may inadvertently destroy an equilibrium alliance configuration that was relatively efficient.

The paper is organized as follows. Section 1.2 develops the general theoretical framework. Section 1.3 analyzes what the model implies for a pair of firms. Section 1.4 shows why the theory implies the possibility of merger waves and contrasts this prediction with aggregate data. Section 1.5 discusses a simple dynamic version of the basic setup in continuous-time, as well as other applications of the theoretical framework. Section 1.6 concludes. The Appendix contains all proofs and a brief description of the numerical methods employed. The MATLAB code is available at `http://fernando.r.anjos.googlepages.com`

1.2 A model of bilateral resource sharing

A firm is comprised of a set of *business units* (BUs), which can be thought of as units of production. A business unit's *economic profit* is given by a function of that business unit's *resources*. These resources are taken as exogenous. A firm then maximizes the combined economic profit of its BUs, in a setting of certainty and perfect information.

In the model, different business units overlap in a subset of the types of resources that drive their economic profit. Also, BUs are allowed to establish bilateral links, through which they share these resources. This may happen whether or not two BUs belong to the same firm. To be precise, an *alliance* between firms A and B exists when at least one BU from firm A and one BU from firm B establish a link and share resources. A *merger* between firms A and B groups all respective BUs under centralized control. In contrast to an alliance link, which is the equilibrium outcome of a non-cooperative game, a merger link is set arbitrarily for BU pairs inside the same firm.

In this section, the boundaries of firms – in the sense of which BUs belong to which firm – are taken as given. I then define the equilibrium link formation process, conditional on these boundaries. In the next sections I will add the possibility of firms changing their boundaries by merging.

All proofs are contained in the Appendix.

1.2.1 Description of the economy

There are N business units (BUs) in the economy and M distinct resource types. BU i's economic profit is given by a mapping $Q_i : \Re_+^M \to \Re_+$, with $Q_i \in \mathcal{C}^2$. Each BU is endowed with a vector of resources, denoted by K_i. The $(M \times N)$ matrix of resource endowments is denoted as K. BUs establish links among them which allow for bilateral transfer of these resources. For any two BUs i and j, L_{ij} is the size of the link that gives i access to j's resources. The $(N \times N)$ matrix of links is denoted as L. The *effective* resources that BUs use to generate economic profit are computed as the inner product of links and resource endowments. Formally I define $\hat{K}_{mi}$ as the effective amount of resource m available to BU i for generation of economic profit, where $\hat{K}_{mi} = \sum_{j \in \{1,...,N\}} L_{ij} K_{mj}$ (in matrix notation $\hat{K} = KL^\top$); note that this also includes the BU's link with itself. I require that the following inequality be verified:

Condition 1 ("No free lunch")

$$\sum_{j \in \{1,...,N\}} L_{ji} \leq 1 \ , \ \forall i \in \{1, ..., N\}$$

This condition, which in equilibrium will hold in equality, implies that aggregate effective resources are constrained by aggregate endowments. It is analogous to a market-clearing condition, and it also excludes short selling. Note that the condition involves the summation of L_{ji} and not L_{ij}. There is nothing ruling out that some BU i uses all resources in the economy (in which case $\sum_j L_{ij} = N$). L_{ij} can be interpreted as the percentage of time that the resources of BU j are in use by BU i.[6]

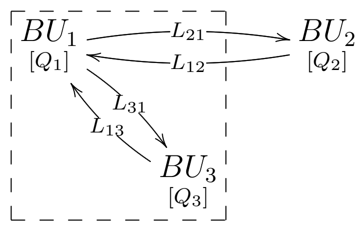
The key modeling assumptions made so far are the following: (i) BUs are unable to change their resource endowments through trade in markets; (ii) resources are scarce, in the sense that they cannot simultaneously be allocated to two different BUs, and (iii) the links BUs establish are not resource-specific, but rather imply that all resources flow.

The intuition behind (i) is that if resources were tradable, then their price would adjust such that profits tend to zero (I am ignoring the question of market power). Basic economic reasoning applies to assumption (ii) as well: if resources were not scarce, then they could not generate economic rents. Finally, the assumption that the links are not resource-specific is supported by at least three different arguments. First, this could be the case because contracts are incomplete. In turn, contracts may be incomplete either because (a) it is costly to exclude partners from accessing all resources; and/or (b) resources are difficult to measure. Second, it may be physically

[6]Under this interpretation, "knowledge" *per se* is not a resource. Rather, knowledge is assumed to be embodied in some resource, for instance a scientific team. In order for knowledge to create profit it needs to be put to use in one or more BUs. This means that codified knowledge (e.g., patents) does not qualify as a resource in this setting. However, once knowledge is codified, it is likely that it becomes tradable. In the model all resources are assumed to be non-tradable. It seems reasonable to assume that if a firm needs tradable knowledge, it acquires it in the market, thus foregoing the costs associated with either alliances or mergers.

impossible to unbundle resources; an example is a scientist with skills in two different fields. Third, this assumption is strictly necessary in the model for complementarity to drive both alliances and mergers, which is empirically supported.[7]

The size of the links is modeled as an equilibrium outcome and it will make a difference if two BUs belong to the same firm or not. Recall that the set of BUs is partitioned into firms, which are the units of control. Firm k is represented as F_k, for all $k \in \{1, ..., N_F\}$, where $N_F \leq N$, and it is a set of business units. An example of a three-BU, two-firm economy is depicted below:



In this example, BUs 1 and 3 belong to the same firm – represented by the dashed box – and there is no link between BUs 2 and 3. However, there is an alliance between the two firms, namely in the form of a link between BUs 1 and 2.

For the case where two BUs i and j belong to the same firm, it is straightforward to model the decision problem: the firm takes both L_{ij} and L_{ji} as controls. However, if i and j belong to different firms, there is no reasonable direct assignment of L_{ij} and L_{ji} as decision variables. This follows because BU i, if allowed, will set $L_{ij} = 1$ (obtaining all of BU j's resources) and $L_{ji} = 0$ (not giving any resources to BU j); while BU j prefers to do the exact opposite. In order to model the equilibrium link formation as a non-cooperative game, I resort to the following reduced-form approach, which is in the spirit of game-theoretic network formation literature – see Jackson and Wolinski (1996) or Bala and Goyal (2000). Bilateral link size is computed as a mapping $L : \Re^2_+ \rightarrow [0, 1]$, with $L \in \mathcal{C}^2$. In particular, defining BU i's control a_{ij} as the level of i's "commitment" to the relationship with j, I specify that

[7] See Wang and Zajac (2007) and Casciaro and Piskorski (2005).

Condition 2 (Joint effort and voluntariness)

$$
\begin{aligned}
&L_{ij} = L(a_{ij}, a_{ji}) \ , \ \forall (i,j) \in \{1, ..., N\}^2 \\
&\frac{\partial L_{ij}}{\partial a_{ij}}, \frac{\partial L_{ij}}{\partial a_{ji}} \geq 0 \ , \ \forall (i,j) \in \{1, ..., N\}^2 \\
&L_{ij}(x, 0) = L_{ij}(0, x) = 0 \ , \ \forall i, j \in \{1, ..., N\}^2 \ , \ \forall x \in \Re_+,
\end{aligned}
$$

which implies that (i) it takes a concerted action of both BUs in order for resource sharing to emerge, and (ii) no firm can be *forced* to share resources. The elements a_{ij} are collected in the $(N \times N)$ matrix A. I will refer to a_{ij} by the term *attention*. I point out that a_{ij} is not associated with some cost. Still, the model does generate constraints on A indirectly, via condition 1 (analogous to market clearing).

A nice property of this approach is the following. If two BUs belong to the same firm, then solving the problem in terms of the pair (L_{ij}, L_{ji}) or the pair (a_{ij}, a_{ji}) is identical since the relationship between the two is (generally) monotonic.[8] Thus we can focus on the problem of identifying $\{a_{ij}\}_{i,j=1,...,N}$, for any configuration of firms' boundaries, although still preserving the mechanism that a firm is in essence arbitrarily setting links between the BUs that it owns.

Firm k solves the following:

$$
\max_{\{a_{ij} | i \in F_k, j=1,...,N\}} \sum_{i \in F_k} Q_i \left(\hat{K}_i \right) \tag{1.1}
$$

s.t.

$$
\begin{aligned}
&A \backslash \{a_{ij} | i \in F_k, j = 1, ..., N\} \quad \text{given} && (1.2) \\
&\hat{K}_{mi} = \sum_{j \in \{1,...,N\}} L_{ij} K_{mj} \quad , \quad \forall i \in F_k, \ \forall m \in \{1, ..., M\} && (1.3) \\
&a_{ij} \geq 0 \quad (\lambda_{ij}) \quad , \quad \forall i \in F_k, \ \forall j \in \{1, ..., N\} && (1.4) \\
&\sum_{j \in \{1,...,N\}} L_{ji} \leq 1 \quad (\theta_i) \quad , \quad \forall i \in F_k && (1.5)
\end{aligned}
$$

Condition (1.2) states simply that the firm takes the strategies of other firms as given, i.e. it is best responding in a Nash sense. The notation $X \backslash Y$ has the standard set-theoretic interpretation and it corresponds to the set of elements in set X that are not elements of set Y. I am using the same label for a matrix and the corresponding

[8]Except when the first or second arguments of the link function are 0. This will not be relevant given the solution concept I use for the game (perfect equilibrium).

set of the matrix's elements (e.g., A), but from the context it is clear which one is meant. Condition (1.3) describes how effective resources are computed from links and endowments. Condition (1.4) is the non-negativity constraint on a_{ij} (necessary given the functional form of $L(a_{ij}, a_{ji})$). Inequality (1.5) is a restatement of the resource constraint. The Lagrangian for this problem is given by:

$$\mathcal{L}_k = \sum_{j \in F_k} \left\{ Q_i \left(\hat{K}_i(K, L) \right) + \sum_{n=1}^{N} \lambda_{ij} a_{ij} - \theta_i \left[\sum_{j=1}^{N} L(a_{ji}, a_{ij}) - 1 \right] \right\} \tag{1.6}$$

The necessary first-order conditions identify all a_{ij} such that $i \in F_k, j \in \{1, ..., N\}$ and are shown below:

$$\frac{\partial Q_i}{\partial a_{ij}} + \mathbb{1}_{\{j \in F_k\}} \frac{\partial Q_j}{\partial a_{ij}} + \lambda_{ij} = \theta_i \frac{\partial L(a_{ji}, a_{ij})}{\partial a_{ij}} + \mathbb{1}_{\{j \in F_k\}} \theta_j \frac{\partial L(a_{ij}, a_{ji})}{\partial a_{ij}} \quad, i \neq j \tag{1.7}$$

$$\frac{\partial Q_i}{\partial a_{ii}} + \lambda_{ii} = \theta_i \frac{\partial L(a_{ii}, a_{ii})}{a_{ii}} \tag{1.8}$$

I assume the second-order conditions for a maximum are verified (I explicitly check for this in the parametric version).

Taking resource endowments as fixed, and assuming that I can write the Lagrange multipliers as functions of parameters and variables, I will write the first-order conditions in shorthand as

$$f_{ij}(a_{ij}^*; A \backslash a_{ij}) = 0 \quad, \forall i, j \in \{1, ..., N\}. \tag{1.9}$$

This representation of the first-order conditions will be useful in the subsequent section, when considering existence and uniqueness issues (and corresponding proofs in the Appendix).

Note that it is an immediate consequence of condition 2 that the strategy pair ($a_{ij} = 0, a_{ji} = 0$) can be part of a pure-strategy Nash equilibrium (I do not consider mixed-strategy equilibria); when $a_{ji} = 0$ the strategy $a_{ij} = 0$ is weakly dominant. However, these equilibria are not always *perfect* (or stable), in which case they are excluded.

1.2.2 Existence and uniqueness results

I take firms' boundaries as exogenous, and solve for the equilibrium in terms of links. I am interested in perfect equilibria, so first I define an ϵ-equilibrium. A perfect equilibrium is the limit of an ϵ-equilibrium as $\epsilon \to 0$. The ϵ-economy is obtained by adding the term $\epsilon \sum_j \log a_{ij}$ to agent i's objective function. The perturbed first-order conditions to the problem are given by

$$f_{\epsilon,ij}(a_{ij}^*; A \backslash a_{ij}) \equiv f_{ij}(a_{ij}^*; A \backslash a_{ij}) + \frac{\epsilon}{a_{ij}^*} = 0, \tag{1.10}$$

which obviously rules out solutions that are not interior. The economic relevance of this construct is that it excludes unstable equilibria. The following condition is key in proving subsequent results.

Condition 3 (Extended solvability)

$$\begin{aligned} &\forall (i,j) \in \{1, ..., N\}^2, \epsilon > 0, A \backslash a_{ij} \in \Re_+^{N^2-1} \\ &\exists! a_{ij}^* : f_{\epsilon,ij}(a_{ij}^*, A \backslash a_{ij}) = 0, \end{aligned}$$

which means that the best-response correspondence is always a function.

Theorem 1 (Existence) *Under the assumptions stated above and the technical requirement $a_{ij} < \bar{a} < \infty, \forall (i,j)$, there exists an ϵ-equilibrium. Furthermore, this implies that there exists a perfect equilibrium.*

The proof of theorem 1 is a simple application of Schauder's fixed point theorem. The next theorem contains sufficient *bilateral* conditions for the global uniqueness of the perfect equilibrium.

Theorem 2 (Uniqueness) *If $\forall (i,j)$ with $i \in \{1, ..., N\}, j \in \{1, ..., N\} \backslash i, \exists!$ perfect equilibrium (a_{ij}^*, a_{ji}^*) in the 2-BU subgame defined by taking $A \backslash (a_{ij}, a_{ji}) = 0$, then the perfect equilibrium is unique.*

The important implication of this theorem is that one only has to check whether games between any two pair of BUs lead to unique equilibria. The proof of theorem 2 is a simple application of homotopy theory.[9]

[9] See Garcia and Zangwill (1982) for a textbook covering the application of homotopies to equilibrium problems.

In the economic networks literature there are different definitions of equilibrium (see Bloch and Jackson (2006)). One of them is *pairwise stability*, which implies that in equilibrium two conditions are met: (i) no agent wants to sever a link; and (ii) no pair of agents wants to form a new link. Obviously these conditions are met in the equilibrium of my model, although they are embedded in the right-continuity of the link formation function at 0. In this application, the pure-strategy Nash equilibrium is indeed a pairwise stable equilibrium too.

1.2.3 Parametrization

Throughout the remainder of the paper I use a specific parametrization, where Q and L are Cobb-Douglas:

$$Q_i = \prod_{m \in \{1,\ldots,M\}} \left(\hat{K}_{mi}\right)^{\alpha_{mi}} \tag{1.11}$$

$$L_{ij} = a_{ij}^{\beta} a_{ji}^{1-\beta} \quad , \quad \beta < 0.5 \tag{1.12}$$

The economic interpretation of the function Q is standard; it is similar to a production function with output elasticities given by the $(M \times N)$ matrix α with typical element α_{mj}.

By adopting the Cobb-Douglas form for the link function, the "joint effort and voluntariness" condition is satisfied. The requirement that the link function is constant-returns-to-scale is an innocuous normalization, since a_{ij} does not have a meaningful quantitative interpretation *per se*. Specifying that $\beta < 0.5$ implies that the level of partnership commitment of the BU *giving access* to resources is more important than the one of the BU *gaining access* thereto. More on the technical and economic content of condition $\beta < 0.5$ is provided below and in section 1.3.

When i and j belong to the same firm, the level of β is irrelevant, since links are set arbitrarily. In fact, the maximization problem can be stated in terms of controls (L_{ij}, L_{ji}), since they generally can be reversed back into the pair (a_{ij}, a_{ji}):

$$\begin{cases} a_{ij} = \left(\dfrac{L_{ji}^{1-\beta}}{L_{ij}^{\beta}}\right)^{\frac{1}{1-2\beta}} \\ a_{ji} = \left(\dfrac{L_{ij}^{1-\beta}}{L_{ji}^{\beta}}\right)^{\frac{1}{1-2\beta}} \end{cases} \tag{1.13}$$

The (non-perturbed) first-order conditions for a_{ij}, when i and j do *not* belong to the same firm, are the following:

$$\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right] = 0 \tag{1.14}$$

Assuming an interior equilibrium, (1.14) can be manipulated and the best-response function written as

$$a_{ij}^{*}(A, K, \alpha, \beta) = a_{ji} \left[\left(\frac{\beta}{1-\beta} \right) \times \frac{\sum_{m=1}^{M} \left(\frac{\alpha_{mi}}{\hat{K}_{mi}^{*}} \right) K_{mj}}{\sum_{m=1}^{M} \left(\frac{\alpha_{mi}}{\hat{K}_{mi}^{*}} \right) K_{mi}} \right]^{1/(1-2\beta)} . \tag{1.15}$$

The optimal a_{ij} depends on three factors: (i) how much attention BU j is investing in the partnership, measured by a_{ji}; (ii) the size of the coordination difficulties, measured by β; and (iii) the net technological benefit of the link, measured by the ratio of the summations. The net technological benefit of the link takes into account the fact that BU i both gains and looses by having a link with BU j. The gain component for resource type m is related to K_{mj} (numerator), while the loss component is related to K_{mi} (denominator). The term $\alpha_{mi}/\hat{K}_{mi}^{*}$ serves as a weight in evaluating the benefit of the link (in equilibrium) with respect to resource m. Note that if a resource is technologically irrelevant for BU i ($\alpha_{mi} = 0$) then this resource is not weighted. Also, since $\hat{K}_{mi}$ includes the amount of resource m that BU i is obtaining from all its links, then in equilibrium the importance of BU j as a partner depends on how exclusive a partner it is. BU j is important for BU i as long as some resource that i has in short own supply is also in short supply for i's other partners.

The intuition for why β gages the level of coordination failure is that it determines the incentive of one partner to free ride on the other partner's resources. Suppose for a moment that both partners "agreed" to the unconditionally optimal link, i.e. the unconditionally optimal level of resource sharing. If β is low, then the gains for one BU to deviate are simply that by reducing a_{ij}, the loss in accessing the partner's resources (via decrease in $L_{ij} = a_{ij}^{\beta} a_{ji}^{1-\beta}$) are disproportionately lower than the gain in retained own resources (via decrease in $L_{ji} = a_{ij}^{1-\beta} a_{ji}^{\beta}$). Both partners have an incentive to free ride if β is low, and that yields a coordination failure; as in the classic example of the prisoner's dilemma. The reason why the coordination failure is not always total is that for low levels of the link the marginal productivity of the accessed resources is very high. In fact, below some threshold for the link, the BU may want to increase a_{ij} even if a_{ji} is low. I further this discussion in section 1.3.

So far, alliance links show two sources of inefficiency: (i) a low β implies a high coordination failure: low (a_{ij}, a_{ji}) implies low (L_{ij}, L_{ji}); (ii) The relative size of non-overlapped resources is irrelevant for equilibrium outcomes (e.g., when $\alpha_{mi} = 0$). In the next section I discuss the conditions under which a non-hierarchical link reduces to the hierarchical case, i.e., alliances and mergers are equivalent.

The following theorem states that this parametrization satisfies the conditions for existence and uniqueness.

Theorem 3 *The chosen parametrization verifies both condition 3 (extended solvability) and the second-order conditions associated with the individual firm's maximization problem, hence there exists a perfect equilibrium in an economy populated by business units with this functional representation. Moreover, this parametrization satisfies the sufficient conditions required in theorem 2 for uniqueness.*

Next I define the key concept of BU-specific capital:

Definition 1 (BU-specific capital) *If BU i has a resource type $m(i)$ such that*

$$\alpha_{m(i)i}, K_{m(i)i} > 0$$
$$\alpha_{m(i)j} = K_{m(i)j} = 0 \quad , \forall j \neq i,$$

then $m(i)$ is BU-specific capital.

BU-specific capital is only useful for BU i, in the sense that it is a resource type that does not enter any other BU's profit function. The economic interpretation for this concept is that BUs may have organizational capital that is non-transferable. I will generally specify that there exists such an $m(i)$. BU-specific capital will play a similar role to the firm-specific productivities in Jovanovic and Rousseau (2002) in rationalizing merger activity.

The next three propositions present general results for this parametrization, which are invoked in subsequent sections.

Proposition 1 *In the chosen parametrization the amount of BU-specific resources is irrelevant for determining equilibrium links between BUs that belong to different firms.*

This result has the important implication that large inefficiencies will sometimes arise in voluntary alliances, since BU-specific resources are not taken into account in the equilibrium links, although they matter for economic profit. This follows from BU-specific capital not having any value as a medium of exchange in the network of firms. Another similar source of inefficiency is implied in the next proposition.

Proposition 2 *If BU i and BU j have only one resource type in common, and they do not belong to the same firm, then in equilibrium they will not establish an alliance.*

The intuition for this proposition is that in an alliance setting, BUs need to give something in exchange for the other partner to be interested. With an overlap in one type of resource only this is obviously not possible. Finally the following proposition is simply a formalization of a trivial effect.

Proposition 3 *If BU i and BU j have no resource types in common, then in equilibrium $L_{ij} = L_{ji} = 0$, i.e., there is no resource sharing, independently of whether the BUs belong to the same firm or not.*

In this model there is something to gain if resources can be shared in ways that improve economic profit generation. If two BUs have no resource type in common, then there is no possible benefit from establishing a link.

1.3 Alliances and mergers in a 2-BU economy

In this section I examine economies where there are only two business units, so that I can abstract from more systemic equilibrium effects. The analysis of the latter, as well as what the model implies in terms of merger waves, is postponed until section 1.4. In short, the immediate objective is to understand why a particular pair of BUs decides to merge, in the context of this model.

In order to examine mergers as endogenous objects, I now introduce the concept of *merger costs*. Throughout this section BUs have the option to merge, at a certain cost. In practice these costs may be direct, for instance lawyer and investment bank fees; or indirect, for example the opportunity cost of managerial time. These costs may also have fixed and proportional components. One important type of merger costs is related to the financing aspect of the deal; see Harford (2005). Finally, regulatory constraints can also be interpreted as merger costs.

The model predicts that there are two distinct types of mergers. The first kind are complementarity-driven, and take place if and only if alliances display strong coordination problems. I will term these mergers as *complementarity mergers*. The second kind are driven by asymmetric BU-specific resources. These can be interpreted as the firm-specific productivities in the Q-theory of mergers of Jovanovic and Rousseau (2002). These mergers do not take place as a substitute for inefficient alliances (i.e. alliances where the full gains from complementarity are not captured), but rather channel certain resources to the most efficient BU. I will term these mergers as *productivity mergers*.

I make the normalization throughout this section that technology is homogeneous of degree 1, i.e. $\sum_m \alpha_{mi} = 1$. All proofs are presented in the Appendix.

1.3.1 Complementarity mergers

This first case serves two purposes: (i) illustrate in a simple way the mechanics of the model; (ii) provide the key economic intuition behind this theory. In this case one also has the advantage of working with closed-form solutions. For now I shut down the channel of BU-specific capital. The technological and endowment matrices are the following:

$$\alpha = \begin{pmatrix} \frac{1}{1+\pi} & \frac{\pi}{1+\pi} \\ \frac{\pi}{1+\pi} & \frac{1}{1+\pi} \end{pmatrix} \quad K = \begin{pmatrix} 1+y & 1-y \\ 1-y & 1+y \end{pmatrix},$$

with $y \in [-1, 1]$. The resource size of both BUs is independent of y and constant at 2. The normalization of resource size is neutral in terms of results given the linearity of technology. The parameter π measures how important each resource type is for each BU. With $\pi = 0$ each BU only uses one resource type; with $\pi = 1$ both BUs use both resource types with equal output elasticity. Note that

$$\frac{\alpha_{21}}{\alpha_{11}} = \frac{\alpha_{12}}{\alpha_{22}} = \pi,$$

so π measures how (relatively) important resource type j is for BU i. Denote the relative ratio of resources by $x \equiv (1-y)/(1+y)$. Since

$$\frac{K_{21}}{K_{11}} = \frac{K_{12}}{K_{22}} = x,$$

the difference between π and x dictates how imbalanced each BU is, in terms of the matching between its endowment and its technology.

The exogenous imbalance between resources and technology will drive most of the implications of the model. In the real world, why would one observe resource imbalance? I present two arguments for this. One is the possibility of technological shocks which are not anticipated. This is in the spirit of Gort (1969) and Rhodes-Kropf and Robinson (2007). The second argument relates to the gains of specialization. In a dynamic setting, this could be modeled as BUs having heterogeneous costs with respect to the development of different resources (skills). Rational BUs anticipate that they will have partners for obtaining non-tradable resources where they have lower internal development capabilities and thus specialize. In other words, future anticipated complementarity may impact current investment decisions.

Next I derive the unconditional optimal allocation of resources for these BUs, which obtains when they are merged.

Proposition 4 *If BUs 1 and 2 are merged, then the optimal links are given by*

$$L_{12}^* = L_{21}^* = \max\left\{\min\left\{\frac{\pi - x}{(1-x)(1+\pi)}, 1\right\}, 0\right\} \tag{1.16}$$

$$L_{11}^* = L_{22}^* = 1 - \max\left\{\min\left\{\frac{\pi - x}{(1-x)(1+\pi)}, 1\right\}, 0\right\} \tag{1.17}$$

and the optimal relative share of effective resources in an interior optimum is

$$\frac{\hat{K}_{21}^*}{\hat{K}_{11}^*} = \frac{\hat{K}_{12}^*}{\hat{K}_{22}^*} = \pi. \tag{1.18}$$

The relative weight of the optimal effective resources in an interior optimum is the same *as if* the resources were traded in a competitive market (relative resource price in this simple example would be 1). Assuming that $x < 1$, then there will only be resource sharing if $\pi > x$, which means that each BU attaches a high output elasticity to the resource that is owned by the other BU. The same holds for $x > 1$ and $\pi < x$. Thus the bigger the wedge between π and x, the greater the benefits of a merger, relative to the no-link scenario. The same effect carries over partially to alliances.

The next proposition describes the equilibrium outcome in an alliance setting.

Proposition 5 *If BUs 1 and 2 do not belong to the same firm, equilibrium links are given by:*

$$L_{12}^* = L_{21}^* = \max\left\{\min\left\{\frac{\pi + x^2 - \left(\frac{1-\beta}{\beta}\right)x(1+\pi)}{(1-x)\left[\pi - x - \left(\frac{1-\beta}{\beta}\right)(\pi x - 1)\right]}, 1\right\}, 0\right\} \quad (1.19)$$

$$L_{11}^* = L_{22}^* = 1 - \max\left\{\min\left\{\frac{\pi + x^2 - \left(\frac{1-\beta}{\beta}\right)x(1+\pi)}{(1-x)\left[\pi - x - \left(\frac{1-\beta}{\beta}\right)(\pi x - 1)\right]}, 1\right\}, 0\right\} \quad (1.20)$$

Moreover, if $\beta < 0.5$ and there is an alliance, then the equilibrium link among the BUs is lower than that achieved in the merger case:

$$L_{ij}^*\big|_{alliance} - L_{ij}^*\big|_{merger} = \frac{\left[1 - \left(\frac{1-\beta}{\beta}\right)\right]\pi(1+x)^2}{(1+\pi)(1-x)\left[\pi - x - \left(\frac{1-\beta}{\beta}\right)(\pi x - 1)\right]} < 0 \quad , \quad i \neq j \quad (1.21)$$

Finally, if $\beta \to 0.5$, then the equilibrium outcome in an alliance converges to the outcome in a merger.

The last statement in proposition 5 has a simple economic intuition: if there are no coordination problems, then a merger adds nothing to an alliance. In the next sections alliances will be shown to carry other (endogenous) problems, but for now I want to address the question: *where do these reduced-form coordination problems come from, and what is the economic intuition behind this modeling approach?*

The way an equilibrium link is determined in an alliance is by identifying the crossing point of the BUs' best response functions. Figure 1.1 provides a graphical illustration of these functions.[10]

First let us look at the left panel of Figure 1.1, which depicts the best response of BU 1. Recall that β determines how much impact a_{12} bears on forming the link $L_{12} = a_{12}^{\beta} a_{21}^{1-\beta}$, which is giving access to BU 2's resources. It is thus intuitive that if own attention is more productive, it will optimally be set at a higher level. After some threshold for a_{21}, BU 1 prefers to cut back on attention. This can be viewed as an income/substitution trade-off. As BU 2 invests more in the link (a_{21} increases), the productivity of a_{12} increases (income effect). However, it is also true that a higher a_{21} allows the same link size with a lower a_{12} (substitution effect). And since the marginal

[10]Unfortunately there are no closed-form solutions for the best response functions.

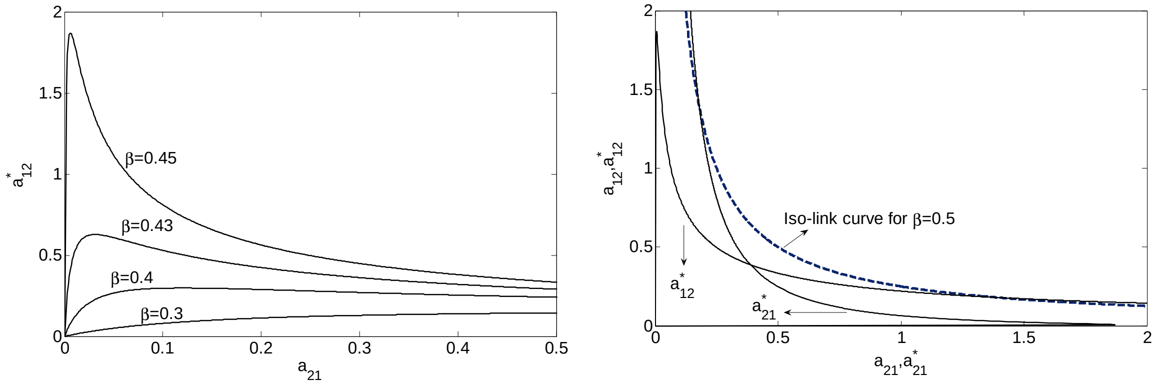


Figure 1.1: Left panel: each line represents the optimal attention that BU 1 dispenses to BU 2, for some given β, as a function of the attention that BU 2 dispenses to BU 1; $x = 0.2$. Right panel: the thicker and dashed curve depicts the possible combinations of a_{12} and a_{21} that yield the optimal unconditional link ($\beta = 0.5$); the other two curves represent the best response functions of the BUs for $\beta = 0.47$. In both panels $x = 0.2$ and $\pi = 1$.

value of the link is decreasing because of the decreasing marginal returns to effective resources, the substitution effect eventually dominates.

Why does the level of β drive the level of coordination failure in an alliance? For low β's, an increase in attention to the alliance partner generally implies that the focal BU is mostly letting go of own resources, in net terms. Looking at equation (1.12), this follows from the structure of the link function:

$$\left.\frac{\partial L_{ji}}{\partial a_{ij}}\right|_{a_{ij}=a_{ji}=a} = 1 - \beta > \beta = \left.\frac{\partial L_{ij}}{\partial a_{ij}}\right|_{a_{ij}=a_{ji}=a}$$

Since there is a substitution effect between own attention and the partner's attention, then this immediately implies the possibility of a free-rider problem. However, this problem is only made relevant by the fact that each BU is unable to unilaterally improve the (net) access to its partner's resources. The problem vanishes as $\beta \to 0.5$ because in this case increasing attention has a balanced outcome in terms of extra access gained (to the partner's resources) and extra access given (to own resources). To understand why the free-rider problem is not important *per se* consider the case $\beta = 0.5$.[11] Then there is a continuum of perfect equilibria comprised of all (a_{12}, a_{21}) such that $(a_{12}a_{21})^{0.5} = L^*$, where L^* is the optimal unconditional link.[12] This is de-

[11]If $\beta > 0.5$ there is no stable equilibrium in pure strategies.

[12]These equilibria are ruled out in the existence and uniqueness results by imposing the strict inequality $\beta < 0.5$, or equivalently condition 3.

picted in the right panel of Figure 1.1: as $\beta \rightarrow 0.5$ the best response functions will overlap exactly at the iso-link curve. These equilibria are stable because, at the margin, a change in a_{ij} yields the same effect in both L_{ij} and L_{ji}:

$$\left.\frac{\partial L_{ji}}{\partial a_{ij}}\right|_{\beta=0.5} = 0.5\left(\frac{a_{ji}}{a_{ij}}\right)^{0.5} = \left.\frac{\partial L_{ij}}{\partial a_{ij}}\right|_{\beta=0.5}$$

If $\beta < 0.5$ then it would compensate for BU i to slightly decrease a_{ij} from any equilibrium with $\beta = 0.5$, because this comes mostly at the expense of a decrease in L_{ji}. This process converges to an equilibrium because the (net) marginal value of the link increases when the link size is lowered.

To illustrate, consider a joint venture where each partner has private information over its own resources. Then there is the moral hazard problem of the incentive to bring in the poorer resources to the joint venture, given that gains are not fully appropriated by each of the partners.[13] However, there will still be a joint venture if the initial gains of this partnership are big enough to offset the moral hazard issues (and the resources are not too heterogeneous). The chosen mechanism of link formation also captures the spirit of the competition/cooperation tension in alliances, as long as resources are broadly interpreted. In this setting the resources that the focal BU is bringing to the partnership are more valuable than they "appear", because they will improve the partner's competitive edge over the focal BU in the long run. This is somewhat isomorphic to BUs giving more access to own resources than they ideally want to, as in the present case with $\beta < 0.5$.

Merger gains

Mergers are exogenously costly. In order to analyze the propensity of the BU pair to merge I compute the absolute gains of a merger.[14] I set $\pi = 1$ in the subsequent analysis, i.e., both resources carry equal importance for both BUs. With $\pi = 1$ the variable y is measuring the degree of resource imbalance. The following proposition contains the implications of complementarity in terms of merger gains.

[13] I thank Robert Dammon for this insight.

[14] In this case relative gains display the same pattern as absolute gains.

Proposition 6 *For the special case of $\pi = 1$, the unconditionally optimal resource allocation obtained in a merger implies that the economic profit of both BUs is independent from y and β, and is given by:*

$$Q|_{merger} \equiv Q_1|_{merger} = Q_2|_{merger} = 1 \tag{1.22}$$

In an alliance setting, equilibrium economic profit is given by:

$$Q|_{alliance} \equiv Q_1|_{alliance} = Q_2|_{alliance} = \sqrt{1 - \left(\frac{1-2\beta}{y}\right)^2} \leq 1 \tag{1.23}$$

The economic profit in an alliance setting varies positively *with the degree of imbalance y (locally):*

$$\frac{\partial\, Q|_{alliance}}{\partial y} = \frac{(1-2\beta)^2}{y\sqrt{y^2-(1-2\beta)^2}} > 0 \tag{1.24}$$

The threshold for y below which BUs do not establish a link in equilibrium for the alliance setting is given by:

$$\bar{y} \equiv \sqrt{1-2\beta} \tag{1.25}$$

Finally, for $y < \bar{y}$ economic profit varies negatively *with the degree of imbalance y (locally):*

$$Q|_{no-link} \equiv Q_1|_{no-link} = Q_2|_{no-link} = \sqrt{1-y^2} \tag{1.26}$$

$$\frac{\partial Q|_{no-link}}{\partial y} = \frac{-y}{\sqrt{1-y^2}} < 0 \tag{1.27}$$

The first thing to note is the ambiguous effect of resource imbalance. For firms with alliances in place a higher resource imbalance actually *increases* economic profit, which is shown by equation (1.24). This is so because resource imbalance is at the same time a measure of complementarity between the BUs, and a higher complementarity offsets parts of the coordination problems generated by $\beta < 0.5$. In contrast, if resource imbalance is low ($y < \bar{y}$), the perfect equilibrium leads to a no-link scenario and resource imbalance actually hurts economic profits. This is shown in equation (1.27).

Naturally this ambiguity translates into the relationship between resource imbalance and merger gains, as depicted in the left panel of Figure 1.2. The kinks correspond to $\bar{y}$, for each level of β. Note that merger gains have a one-to-one relation with the economic profit of the no-merger scenario, since merger output does not depend either on y or β; see equation (1.22). A high imbalance corresponds to high merger gains as long as it is not *too* high and triggers a relatively efficient alliance. This is a novel look at the common concept of synergies, since usually the next best alternative is taken to be isolation. This perspective has both positive implications for the empirical analysis of merger frequency and announcement effects (see discussion below), and normative implications for the evaluation of merger decisions by companies.

Figure 1.2 also shows how merger gains vary with β, holding resource imbalance (equivalently complementarity) constant. Again the kinks represent the thresholds after which there is an alliance. Lower β's (i.e. better alliances) correspond to higher merger gains, which is intuitive.

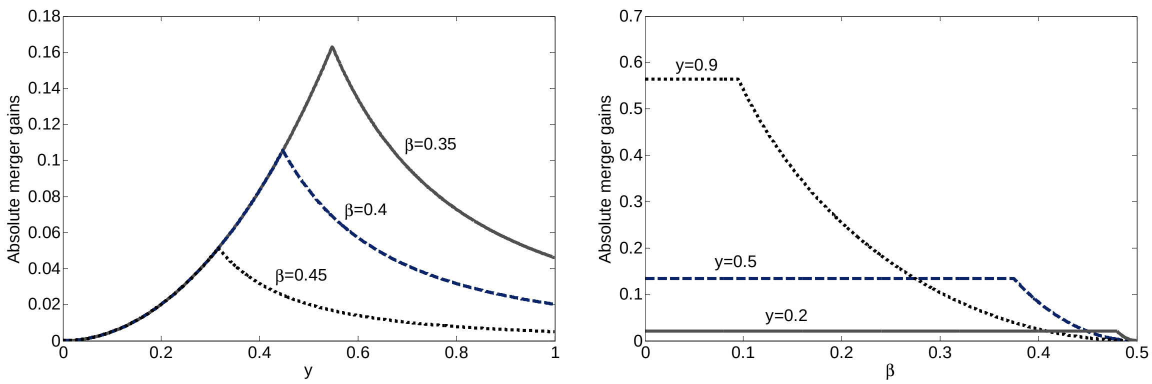


Figure 1.2: Left panel: shows absolute merger gains, for varying y. Right panel: shows absolute merger gains, for varying y. In both panels $\pi = 1$.

The relationships between merger gains, complementarity, and alliances are testable implications of the model for the cross section of merger announcement returns and/or merger frequency: (i) the excess returns associated with a merger and/or the merger frequency between firms in different industries should display an inverted-U relationship with respect to complementarity, controlling for the level of coordination problems in alliances; (ii) controlling for complementarity, the excess returns associated with a merger and/or the merger frequency between firms in different industries should be higher for industries where coordination problems are starker.

This particular case of the model is in line with the recent empirical findings of Rhodes-Kropf and Robinson (2007) about the bivariate distribution of market-to-book (MB) ratios of merging firms. The authors show that merging pairs of firms have similar MB ratios, although acquirers usually have a slightly higher MB ratio than their targets. This stands in opposition to the Q-theory of mergers in Jovanovic and Rousseau (2002), where it is the productivity gap between acquirers and targets that triggers capital reallocation. Rhodes-Kropf and Robinson (2007) offer a rationalization for this fact that is also partly based on asset complementarity. In my model the decision to merge for complementarity reasons is also unrelated to differences in productivity (productivities are equal by construction).

1.3.2 Productivity mergers

So far the underlying reason for firms to ally or merge has been resource complementarity. Mergers are preferable if transaction costs are low and alliances have strong coordination problems. In this section I explain why the model predicts a reason for mergers that is unrelated to complementarity and alliances. Consider the following technology and endowment matrices:

$$\alpha = \begin{pmatrix} \alpha_s & 0 \\ 0 & \alpha_s \\ 1-\alpha_s & 1-\alpha_s \end{pmatrix} \quad K = \begin{pmatrix} z & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

In this case, BUs overlap in one resource only, namely for $m = 3$. Resources $\{K_{ii}\}_{i=1,2}$ are BU-specific capital. The parameter α_s measures the importance of BU-specific capital for both BUs, and z measures the size differential in this resource. As stated generally in proposition 2 these BUs will not establish a link if they belong to different firms, independently of coordination difficulty. However, it is not true that a merger between these two BUs never adds value. The following proposition states under which conditions merger gains obtain.

Proposition 7 *Assume the two BUs decide to merge. For $z > 1$ optimality requires $L_{21} = 0$, i.e., the small BU does not receive resources from the big BU. The optimal flow of BU 2's resources to BU 1 is given by:*

$$L_{12}^* = \max\left\{\min\left\{(1-\alpha_s)^{\frac{1}{\alpha_s}} z - 1, 1\right\}, 0\right\} \tag{1.28}$$

The above implies that there is inertia in terms of merger activity, until z surpasses the following threshold $\bar{z}$:

$$\bar{z} \equiv \frac{1}{(1-\alpha_s)^{\frac{1}{\alpha_s}}} \tag{1.29}$$

Also, the small BU is fully emptied of resources ($L_{12}^* = 1$), and its economic profit is zero, for z larger than the threshold $\bar{\bar{z}}$ given below:

$$\bar{\bar{z}} \equiv \frac{2}{(1-\alpha_s)^{\frac{1}{\alpha_s}}} = 2\bar{z} \tag{1.30}$$

The intuition for why there is an inertial region is the following. It is never optimal to have both L_{12} and L_{21} being positive, since they imply *waste* of BU-specific capital. Recall that a positive link implies that all resources flow, and BU 1 has no use for BU 2's BU-specific capital. Thus, for $z > 1$ it can only be the case that the non-specific resources of the less profitable BU are somewhat transfered to the more profitable BU. This story is a Q-theory of mergers, as developed in Jovanovic and Rousseau (2002). However in my model, the waste of the less profitable BU's BU-specific capital is only offset if "productivities" have a big enough gap. This is reminiscent of the exogenous disassembling costs in Jovanovic and Rousseau (2002).

Next note that the more technologically important BU-specific capital is, the smaller the equilibrium transfer of resources L_{12}^*.[15] A caveat is in order: a higher α_s also implies a higher threshold $\bar{z}$ (see equation 1.29), so the effect described above is local.[16] This also implies that the inertial region is bigger when BU-specific capital is more important. The mechanism delivering this effect is an increase in the opportunity cost of wasting the small BU's BU-specific capital.

[15] The effect of α_s on L_{12}^* is given by:

$$\frac{\partial L_{12}^*}{\partial \alpha_s} = -z(1-\alpha_s)^{\frac{1}{\alpha_s}} \left[\frac{\log(1-\alpha_s)}{\alpha_s^2} + \frac{1}{\alpha_s(1-\alpha_s)} \right] \leq 0 \tag{1.31}$$

To prove that the expression above is negative it is only necessary to show that the bracketed term is positive, which is equivalent to showing that

$$G(\alpha_s) \equiv \log(1-\alpha_s) + \frac{\alpha_s}{1-\alpha_s} \geq 0$$

Computing the derivative of $G(\alpha_s)$ we obtain

$$G'(\alpha_s) = \frac{\alpha_s}{(1-\alpha_s)^2}$$

which is always non-negative. Since $G(0) = 0$ expression (1.31) is true.

[16] The effect of α_s on $\bar{z}$ is given by:

$$\frac{\partial \bar{z}}{\partial \alpha_s} = \frac{G(\alpha_s)}{\alpha_s^2(1-\alpha_s)^{1/\alpha_s}} > 0$$

where $G(.)$ is defined as in footnote 15.

The expression for absolute merger gains (AMG) is the following:

$$AMG = z^{\alpha_s} \left[(1 + L_{12}^*)^{(1-\alpha_s)} - 1 \right] - L_{12}^* \tag{1.32}$$

In an interior optimum, and making use of the envelope theorem, we have that

$$\frac{\partial AMG}{\partial z} = \alpha_s z^{\alpha_s - 1} \left[(1 + L_{12}^*)^{(1-\alpha_s)} - 1 \right] > 0, \tag{1.33}$$

which is readily verified after replacing L_{12}^* by the expression in equation (1.28). As in Jovanovic and Rousseau (2002), the gap in BU-specific productivity (measured by z) increases merger gains.

The fact that BU-specific (organizational) capital may induce an inertial region in terms of mergers has an interesting implication. In a dynamic setting, suppose that an economy-wide restructuring period takes place. In the aftermath, firms are now developing organizational capital related to their redrawn boundaries. Since organizational capital is a product of experience, z will evolve smoothly over time. Thus even if merger costs are low, the inertial region implies that there will still be a period of inactivity following a period of economy-wide restructuring. After enough time has passed, some firms will have a higher organizational capital and take over the low-organizational capital firms. This would restart the cycle.

1.4 Aggregate effects of alliance and merger activity

The analysis of a 2-BU economy allowed me to abstract from less trivial equilibrium effects. In this section I study how alliances, mergers and merger waves are related, for distinct pairs of *firms*. I start by modeling a 4-BU economy. Next I study the N-BU case. I also contrast the model's predictions with data on the worldwide aggregate number of alliance and merger deals, for the period 1988-2005.

In an extension to the main model, presented in section 1.5.1, I show in a simple continuous-time setting that the basic intuition carries over to the dynamic case.

1.4.1 4-BU economy

The workhorse of this section is a symmetric 4-BU economy. This is the minimal number of BUs that is necessary to analyze merger propagation. Each BU is endowed with specific and non-specific resources, and all output elasticities have the same magnitude $\bar{\alpha}$. With respect to the technology and endowment matrices, recall that the rows stand for resource types, while columns represent BUs. The (partitioned) technology matrix is given by:

$$\alpha_s = \begin{pmatrix} \bar{\alpha} & 0 & 0 & 0 \\ 0 & \bar{\alpha} & 0 & 0 \\ 0 & 0 & \bar{\alpha} & 0 \\ 0 & 0 & 0 & \bar{\alpha} \end{pmatrix} \quad \alpha_{ns} = \begin{pmatrix} \bar{\alpha} & \bar{\alpha} & 0 & 0 \\ \bar{\alpha} & \bar{\alpha} & 0 & 0 \\ 0 & \bar{\alpha} & \bar{\alpha} & 0 \\ 0 & \bar{\alpha} & \bar{\alpha} & 0 \\ 0 & 0 & \bar{\alpha} & \bar{\alpha} \\ 0 & 0 & \bar{\alpha} & \bar{\alpha} \end{pmatrix},$$

where α_s contains the output elasticities of BU-specific resources, and α_{ns} the output elasticities of non-specific resources. I interpret the degree of overlap in terms of (positive) technological parameters as a measure of industry relatedness. The row concatenation of α_s and α_{ns} yields the global α matrix. Analogously, the partitioned resource endowment matrix K is given by:

$$K_s = \begin{pmatrix} z & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & z \end{pmatrix} \quad K_{ns} = \begin{pmatrix} 1-y_{out} & 1+y_{out} & 0 & 0 \\ 1+y_{out} & 1-y_{out} & 0 & 0 \\ 0 & 1-y_{in} & 1+y_{in} & 0 \\ 0 & 1+y_{in} & 1-y_{in} & 0 \\ 0 & 0 & 1-y_{out} & 1+y_{out} \\ 0 & 0 & 1+y_{out} & 1-y_{out} \end{pmatrix}$$

Although it may at first look complex, the modeling approach is still parsimonious. There are only four parameters/variables, namely $\bar{\alpha}$, z, y_{in} and y_{out}. I set $\bar{\alpha} = 0.2$, which makes the technology of the inner BUs (2 and 3) linear.[17] The variable z, as in section

[17] This in turn implies that the outer BUs (1 and 4) have decreasing returns to scale. I also ran a version of this model where I set the BU-specific output elasticities of the outer BUs at a higher level (such that linear technology obtains), and there is no important qualitative change in results. The fact that this is not important in the model relates to the fact that the *amount* of BU-specific capital and its output elasticity are somewhat substitute.

1.3.2, denotes the size differential in BU-specific capital between the outer BUs (1 and 4) and the inner BUs (2 and 3). The variable y_{in} controls for the complementarity degree between inner BUs. The variable y_{out} controls for the complementarity degree between the outer BU and the adjacent inner BU. Inspecting the K_{ns} matrix one verifies that the outer BUs have no resource type in common, so they will never find it optimal to establish a link (see proposition 3). This is also true for BU pairs $(1, 3)$ and $(2, 4)$. I interpret this as the BUs belonging to unrelated industries. Note that the resource size of all BUs is unaffected by changes in y_{in} and y_{out}.

The importance of assuming a sparse structure for the technology and endowment matrices is that although one can make a clear theoretical distinction between "industries", the *connectedness* of the economy is still present. Without this connectedness it would not be possible to have propagation effects in this model.

I consider two merger options in this economy. One is between BUs 1 and 2, and the second between BUs 3 and 4. Thus there are four possible merger scenarios. This is without much loss of generality as long as β is high enough, since this implies that there is very little incentive for BUs 2 and 3 to merge; BUs 2 and 3 have an equal amount of BU-specific capital, so a productivity merger would never take place. Similarly BU pairs $(1, 4)$, $(1, 3)$, and $(2, 4)$ have no incentive to merge either since they do not overlap in any resource types. The only relevant merger option that I am excluding is full integration, which may be interpreted as an exogenous institutional feature of the economy.

There are four outcomes in terms of the economic profit vector Q, each associated with one of the four different merger scenarios. Since BUs are symmetric, this gives rise to low dimensionality in terms of the individual outcomes for each BU's economic profit (only 8). I denote these by $\{EP_w\}_{w=1,\ldots,8}$:

No mergers	(BU_1, BU_2)	(BU_3, BU_4)	$(BU_1, BU_2), (BU_3, BU_4)$
$Q = \begin{pmatrix} EP_1 \\ EP_2 \\ EP_2 \\ EP_1 \end{pmatrix}$	$Q = \begin{pmatrix} EP_3 \\ EP_4 \\ EP_5 \\ EP_6 \end{pmatrix}$	$Q = \begin{pmatrix} EP_6 \\ EP_5 \\ EP_4 \\ EP_3 \end{pmatrix}$	$Q = \begin{pmatrix} EP_7 \\ EP_8 \\ EP_8 \\ EP_7 \end{pmatrix}$

For my purposes it is useful to represent the economic problem as a 2-player, 2-strategies game in normal form. BUs 1 and 2 are one player, while BUs 3 and 4 are the other player. I denote merger costs for each BU pair by $c_{1,2}$ and $c_{3,4}$. Game payoffs correspond to the sums of the economic profits of the merging BUs, net of the merger costs. The pure strategies available for the players are M (stands for "merge") and NM (stands for "not merge"); I do not consider mixed strategies. The game matrix is then:

		(BU_3, BU_4)	
		NM	M
(BU_1, BU_2)	NM	$EP_1 + EP_2, EP_1 + EP_2$	$EP_5 + EP_6, EP_3 + EP_4 - c_{3,4}$
	M	$EP_3 + EP_4 - c_{1,2}, EP_5 + EP_6$	$EP_7 + EP_8 - c_{1,2}, EP_7 + EP_8 - c_{3,4}$

Without loss of generality let us focus on the choices of player (BU_1, BU_2). There are two types of mergers for this player. The first I term *type-1* and it is a merger where the other player is choosing NM. *Type-2* mergers, on the other hand, take place when player (BU_3, BU_4) chooses M. Formally I define (gross) absolute merger gains (AMG) for each type of merger as

$$AMG|_{type-1} \equiv \underbrace{(EP_3 + EP_4)}_{post-merger} - \underbrace{(EP_1 + EP_2)}_{pre-merger} \quad (1.34)$$

$$AMG|_{type-2} \equiv \underbrace{(EP_7 + EP_8)}_{post-merger} - \underbrace{(EP_5 + EP_6)}_{pre-merger}, \quad (1.35)$$

and correspond to the combined economic profit of the merged BUs, net of the combined payoffs for the NM strategy. This may be viewed as the opportunity costs of the merger, as opposed to costs $c_{1,2}$ and $c_{3,4}$. The difference between the (gross) advantage in merging for the two different merger types is then given by:

$$\Delta AMG \equiv AMG|_{type-2} - AMG|_{type-1} = \underbrace{(EP_7 + EP_8) - (EP_3 + EP_4)}_{\Delta post-merger} + \underbrace{(EP_1 + EP_2) - (EP_5 + EP_6)}_{\Delta pre-merger} \quad (1.36)$$

The difference in gains between the two different types of mergers reflects the possibility that the first merger may change the opportunity costs or final payoff of the second merger. In fact, the possibility that one player *may gain more from merging when the other player also merges* is this model's rationale for merger waves. A merger by

(BU_3, BU_4) as a "response" to lower costs $c_{3,4}$ may "trigger" a merger by (BU_1, BU_2), even if $c_{1,2}$ stays constant.[18]

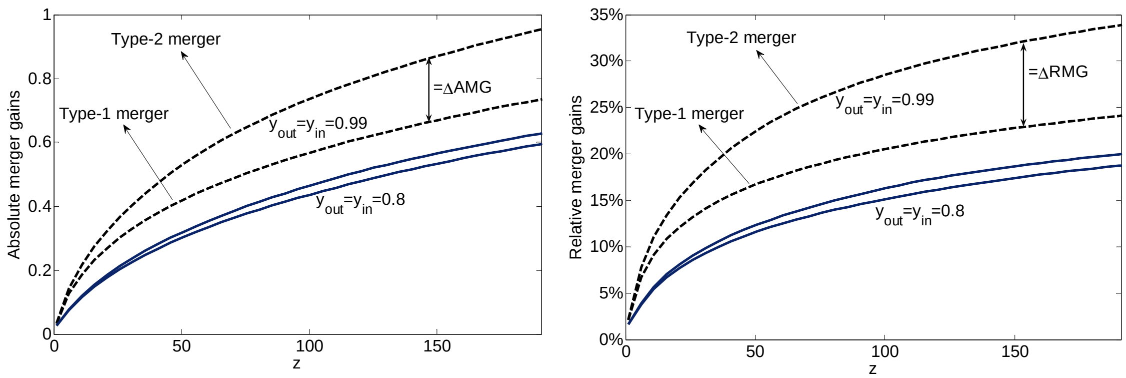


Figure 1.3: Left panel: shows absolute merger gains, for varying size differential in BU-specific capital; the thicker dashed curves represent the gains of a type-1 merger and a type-2 merger (higher values) for $y_{in} = y_{out} = 0.99$; the thinner solid curves represent the gains of a type-1 merger and a type-2 merger (higher values) for $y_{in} = y_{out} = 0.8$. Right panel: shows merger gains as a percentage of pre-merger economic profit, for varying size differential in BU-specific capital; the thicker dashed curves represent the gains of a type-1 merger and a type-2 merger (higher values) for $y_{in} = y_{out} = 0.99$; the thinner solid curves represent the gains of a type-1 merger and a type-2 merger (higher values) for $y_{in} = y_{out} = 0.8$. In both panels $\beta = 0.4$ and $\bar{\alpha} = 0.2$.

Figure 1.3 shows how merger gains vary with the degree of complementarity and the BU-specific resource differential. Both panels show that both merger gains and difference in gains between merger types increase with both size differential z and complementarities/imbalances y_{in}, y_{out}. Although not shown in the figure, both complementarities are associated with higher merger gains and higher difference in merger gains between merger types. The following proposition makes the discussion more formally precise:

Proposition 8 *The following three statements are true for the depicted 4-BU economy:*

1. *For any (β, y_{out}), there exists a threshold $\bar{y}_{in}$ such that*

$$y_{in} < \bar{y}_{in} \Rightarrow \Delta AMG = 0,$$

which means that there are no merger waves for $y_{in} < \bar{y}_{in}$.

[18] The quotations used in this sentence are justified since this is a static model, so in rigor there are no reactions to shocks.

2. *For any* (y_{in}, y_{out}) *such that in the no-merger case the equilibrium links between the inner BUs are positive, i.e.,* $L_{23} = L_{32} > 0$*, and* $EP_1 > EP_6$*, there exists a threshold* $\bar{z}$ *such that*

$$z > \bar{z} \Rightarrow \frac{\partial AMG}{\partial z} > 0, \frac{\partial \Delta AMG}{\partial z} > 0,$$

which implies that for $z > \bar{z}$*,* z *is a driver of both mergers and merger waves.*

3. *If the no-merger equilibrium is interior and* $EP_1 > EP_6$*, then the following holds:*

$$\frac{\partial AMG}{\partial y_{out}} > 0, \frac{\partial \bar{z}}{\partial y_{out}} < 0$$

This means that y_{out} *is also a driver of both mergers and merger waves.*

Point 1 in proposition 8 makes clear that in the context of this model *there cannot be merger waves without alliances.* For low y_{in} the inner BUs have low complementarity and so they do not establish a link in the no-merger scenario. If this is so, and given the symmetry of the economy, merger gains are equal and independent of one another. The inner alliance link is the way merger activity is propagated.

As in section 1.3.2, size differential in BU-specific capital is an important driver of merger activity. However, point 2 in the proposition shows that z also widens the gap between the gains of the two types of mergers. This happens via two distinct channels. The first is that after a type-1 merger, the bigger the z, the more resources are diverted from the inner BU to the outer BU. Recall that z is neutral with respect to equilibrium links in an alliance setting (proposition 1), so z only affects post-merger allocation. It is intuitive that for a higher difference in BU-specific capital more resources are transferred (exactly as in section 1.3.2). A higher transfer of resources from the inner BU to the outer BU after the first merger implies that the relationship between the two inner BUs is weakened. This naturally implies that the non-merged inner BU has a smaller economic profit, which reduces the opportunity costs of type-2 mergers.

The second channel through which z drives merger waves is that the first merger generates lower equilibrium marginal productivities of the resources that the non-merged inner BU shares with the non-merged outer BU. This implies that a smaller link between the non-merged pair is established and thus the economic profit of the non-merged outer BU is also hampered. This further reduces the opportunity costs of the second merger. Point 3 in proposition 8 shows that this second effect is amplified in proportion to y_{out}, which measures the dependence of the outer BUs on the inner BUs.

So far this has been a discussion about changes in opportunity costs. For z big enough there will however be no change in the post-merger payoffs for any type of merger. The reason is that for high z, inner BUs are simply emptied of resources, either in type-1 or type-2 mergers.

In short, the model predicts that merger waves should be more likely in situations of (i) high average Tobin's Q and Tobin's Q dispersion (via z), and (ii) high complementarity (via y_{out} and y_{in}). Interestingly, in the model the factors that trigger mergers are the same that trigger merger waves. Prediction (i) is clearly in line with the empirical findings that merger waves happen in times of high valuations and high Tobin's Q dispersion. How can one test prediction (ii)? If complementarity evolves smoothly enough, then before mergers take place one should observe an increase in alliance activity. According to the model, merger waves would be preceded by alliance waves.

In addition, since mergers imply a high inward focus, the model implies that they negatively impact the level of alliance activity (actually this is what is triggering the merger waves). For high enough z there are no alliances in this economy after the two mergers take place.

1.4.2 N-BU economy

In this section I extend the sparse structure used in the 4-BU case to an economy with N BUs. The technological matrices are now given by:

$$\underset{(N\times N)}{\alpha_s} = \begin{pmatrix} \bar{\alpha} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \bar{\alpha} \end{pmatrix} \quad \underset{(2N\times N)}{\alpha_{ns}} = \begin{pmatrix} \bar{\alpha} & 0 & \dots & 0 & \bar{\alpha} \\ \bar{\alpha} & 0 & \dots & 0 & \bar{\alpha} \\ \bar{\alpha} & \bar{\alpha} & \dots & 0 & 0 \\ \bar{\alpha} & \bar{\alpha} & \dots & 0 & 0 \\ 0 & \bar{\alpha} & \dots & 0 & 0 \\ 0 & \bar{\alpha} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \bar{\alpha} & 0 \\ 0 & 0 & \dots & \bar{\alpha} & 0 \\ 0 & 0 & \dots & \bar{\alpha} & \bar{\alpha} \\ 0 & 0 & \dots & \bar{\alpha} & \bar{\alpha} \end{pmatrix},$$

where as before α_s contains the output elasticities of BU-specific resources, and α_{ns} the output elasticities of non-specific resources. The degree of overlap in terms of (positive) technological parameters is interpreted as a measure of industry relatedness. The row concatenation of α_s and α_{ns} yields the global α matrix. I set $\bar{\alpha} = 0.2$ which implies the BUs have a linear production technology. The partitioned resource endowment matrix K is given by:

$$\underset{(N\times N)}{K_s} = \begin{pmatrix} z_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & z_N \end{pmatrix} \quad \underset{(2N\times N)}{K_{ns}} = \begin{pmatrix} 1-y & 0 & \dots & 0 & 1+y \\ 1+y & 0 & \dots & 0 & 1-y \\ 1+y & 1-y & \dots & 0 & 0 \\ 1-y & 1+y & \dots & 0 & 0 \\ 0 & 1+y & \dots & 0 & 0 \\ 0 & 1-y & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1-y & 0 \\ 0 & 0 & \dots & 1+y & 0 \\ 0 & 0 & \dots & 1+y & 1-y \\ 0 & 0 & \dots & 1-y & 1+y \end{pmatrix},$$

where $y \in [0,1]$ measures the degree of complementarity between BUs i and $i+1$ (label convention: if $i = 1$, then $i-1 \equiv N$; if $i = N$, $i+1 \equiv 1$).

Since the economy is composed of BUs that are similar in terms of non-specific resources and $\{z_i\}$ does not influence equilibrium alliance links (see proposition 1), if each BU is a separate firm then there is a closed-form expression for links, as shown in the next proposition.

Proposition 9 *If each BU corresponds to an isolated firm, then the following is true:*

1. *The equilibrium links L_{ij}, for all $i \neq j$, are scale-free (do not depend on N), and are given by L^*, according to the ensuing expression:*

$$L^* = \frac{C_1 + \sqrt{C_2}}{C_3}, \tag{1.37}$$

where

$$\begin{aligned} C_1 &= 14 - 20\beta - \left[y^2(26 - 16\beta)\right] && (1.38) \\ C_2 &= y^4 4(\beta - 2)^2 + y^2 8(13\beta^2 - 28\beta + 14) + 4(3\beta - 2)^2 && (1.39) \\ C_3 &= 2\left[y^2(21\beta - 33) + 9 - 13\beta\right]. && (1.40) \end{aligned}$$

2. *If $y = 1$, equilibrium links simplify to*

$$L^*|_{y=1} = \frac{\beta}{3 - \beta}. \tag{1.41}$$

3. *If $y = 1$, the equilibrium profit of BU i is given by*

$$Q_i^*|_{y=1} = \frac{\left[432 z_i \beta^2 (1 - \beta)^3\right]^{0.2}}{3 - \beta}. \tag{1.42}$$

From expressions (1.41) and (1.42) it is easy to check that both links and profit depend positively on β. This is intuitive since a lower β implies a higher coordination failure in alliances, as discussed in previous sections.

In terms of merger options I consider that for any BU triplet in the set

$$\{(1, 2, 3), (4, 5, 6), ...\},$$

with N a multiple of 3, BUs can choose to merge, either pairwise or fully. Without this assumption of merger-option exogeneity one would have to model competing bids. However, if $z_3 \approx z_4$, $z_6 \approx z_7$, ... and β is not too low, this assumption is not binding, since the corresponding BU pairs would never merge for positive merger costs. The following proposition shows how merger waves may obtain in this economy. I term BUs in the set $\{2, 5, 8, ...\}$ as *central* BUs.

Proposition 10 *Denote by* $\underline{c}(\{z_i\})$ *the* minimum *level of merger costs such that it is not optimal for any pair or triplet of adjacent BUs to merge, given the exogenous BU-specific capitals* $\{z_i\}$*, and conditional on all other triplets not merging (equivalently, if merger costs are set at* $\underline{c}(\{z_i\})$*, these costs are just high enough such that there exists a pure-strategy Nash equilibrium with no mergers). Then there exists* $(\{z_i\}, \beta, y, \delta)$ *such that if* $c_{k,k-1} = 0$ *for some central BU* k*, and* $\{c_{ij} = \underline{c}(\{z_i\}) + \delta\}_{i \neq j; i,j \neq k}$*, the only merger equilibrium is a dominating-strategy equilibrium where all BUs merge into* 3*-BU firms.*

Corollary 1 *Assume that the conditions for a dominating-strategy equilibrium in proposition 10 are met. Then solving the merger game sequentially, where each BU triplet decides whether or not to merge pairwise or fully, yields a subgame-perfect Nash equilibrium that coincides with the dominating-strategy equilibrium of proposition 10, where all BU triplets merge.*

What is the mechanism that delivers the result in proposition 10? Note that, for BUs k and $k-1$, merging is a dominating strategy, since this pair does not face merger costs. The fact that this merger takes place, as long as z_k is high enough relative to z_{k-1} and β and δ are low enough,[19] implies that BU $k+1$ is *also* optimally merged with BU pair $(k-1, k)$. Under the assumptions that $z_k >> z_{k-1}$ and $z_k >> z_{k+1}$, the resources of BUs $k-1$ and $k+1$ are diverted to BU k. Combined with a high enough complementarity degree y, this means that BUs $k+2$ and $k-2$ become worthless, since they no longer have access to critical (complementary) resources. This lowers the opportunity costs of the mergers for BU pairs $(k+2, k+3)$ and $(k-3, k-2)$, which as in section 1.4.1 means that merger gains are increased. If these mergers take place, then there is a higher incentive for BU $k-4$ to be merged with the pair $(k-3, k-2)$, and for BU $k+4$ with the pair $(k+2, k+3)$. In case these mergers also imply a massive allocation of resources to the central BU, which happens if these BUs have high enough BU-specific capital, the same mechanism will lead to triplets $(k-7, k-6, k-5)$ and $(k+5, k+6, k+7)$ merging. Induction implies that *all triplets merge, as long as central BUs have high enough BU-specific capital.* As in section 1.4.1, merger waves require high complementarity and high heterogeneity in BU-specific capital.

[19]See the proof for details.

1.4.3 Aggregate data analysis

In this section I analyze the time-series of alliance and merger activity. My proxy for the level of activity is the number of deals.[20] Data is summarized in Figure 1.4.

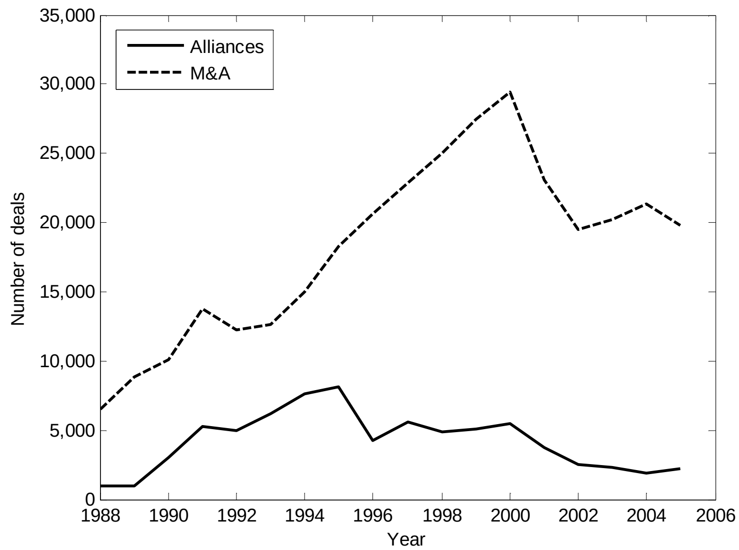


Figure 1.4: Source: The Boston Consulting Group (2005) (used with permission). The data refers to worldwide deals, and 2005 is only an estimate.

As may be visible in Figure 1.4, both series display high autocorrelation. Table 1.1 shows the estimates of first-order autocorrelation.

Alliances		M&A	
$\hat{\rho}$	0.79	$\hat{\rho}$	0.94
95%-CI	$[0.52, 0.92]$	95%-CI	$[0.85, 0.98]$

Table 1.1: Displays the first-order autocorrelation for the log-standardized time series of alliance and merger deals.

I interpret the high degree of autocorrelation as an alternative measure for merger and alliance waves. In this sample alliances have a lower autocorrelation than mergers, which is consistent with the model. Alliances are not associated with discrete events of major resource reallocation, as in the case for mergers. The fact that alliances display

[20] Alliance deals include joint ventures, strategic alliances, licensing and exclusive licensing agreements, research and development agreements, marketing agreements, manufacturing agreements and supply agreements.

a high autocorrelation in absolute terms is however not being explained by this model. A possible rationalization for this fact is that alliances play a role in the development of organizational capital. Another alternative is that alliances bear less coordination costs across time, as partners know each other better. This could be seen as a progressive increase of β in the model.

The contemporaneous correlation between the two (log-standardized) series is 0.49, with a 95%-confidence interval of $[0.19, 0.70]$. The model does predict that an increase in complementarity increases both the propensity for alliances and mergers to take place. This could be a possible explanation, especially if the cross section of merger costs is highly heterogeneous (so some firms for instance never merge). However, this could easily just be a business cycle effect. What is more striking is that the correlation between merger activity and alliance activity increases when we lag the former; this is shown in Table 1.2.

M&A lag	1	2	3	4	5
$\hat{\rho}$	0.54	0.57	0.69	0.86	0.90
95%-CI	$[0.24, 0.74]$	$[0.27, 0.77]$	$[0.44, 0.84]$	$[0.72, 0.94]$	$[0.80, 0.96]$

Table 1.2: displays the correlation between the log-standardized time series of alliance and lagged M&A deals.

In this sample alliance waves do precede merger waves, which is in line with the model. The correlation between series is highest for a 5-year lag (higher lags than 5 years are not reported). I performed the same analysis for lagged alliance series; the results are shown in Table 1.3.

M&A lead	1	2	3	4	5
$\hat{\rho}$	0.20	-0.32	-0.61	-0.73	-0.89
95%-CI	$[-0.15, 0.50]$	$[-0.60, 0.03]$	$[-0.80, -0.32]$	$[-0.87, -0.50]$	$[-0.95, -0.76]$

Table 1.3: displays the correlation between the log-standardized time series of lagged alliance and M&A deals.

The results are now the exact opposite. High merger activity in the present is correlated with low alliance activity in the future. I believe this too is consistent with the theory, since in the model mergers imply a higher inward focus, leaving less room for alliances.

There are alternative interpretations of this data. One is the argument put forth by Harford (2005) that merger waves are triggered by a generalized lowering of financing constraints. This is also consistent with the theory presented and does not require any mechanism of propagation. It is simply the fact that when merger costs are high, mergers do not take place. If mergers and alliances are indeed somewhat substitutes, one should expect to see a higher degree of alliance activity when merger costs are high. This however does not explain the positive contemporaneous correlation between merger and alliance activity.

A set of theories that have also been offered to rationalize merger waves rely on market over-valuation; see Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004). These theories cannot explain the apparent relation between alliance and merger activity, much less the existence of alliance waves. This is so because they require control transactions to happen for their argument to go through.

1.5 Extensions

1.5.1 Dynamic 4-BU economy

In this section I investigate how merger propagation takes place in a simple continuous-time setting. I show that the results obtained in section 1.4.1 (static setting) carry over to the dynamic case. BU-pairs make decisions about mergers and alliances, at each moment in time. Mergers are irreversible and alliance links are changed without frictions (like labor in the neoclassical investment model). Thus, cash flow *rates* ϕ for BU-pair $i \in \{1, 2\}$ are taken to be the combined equilibrium profits from the static model:[21]

$$\phi(t, \tau_i, \tau_j) = \begin{cases} EP_1 + EP_2 \equiv \phi_1 & \tau_i, \tau_j \geq t & (1.43) \\ EP_3 + EP_4 \equiv \phi_2 & \tau_i < t, \tau_j \geq t & (1.44) \\ EP_5 + EP_6 \equiv \phi_3 & \tau_i \geq t, \tau_j < t & (1.45) \\ EP_7 + EP_8 \equiv \phi_4 & \tau_i, \tau_j < t & (1.46) \end{cases}$$

[21]This static-dynamic approach is used in the dynamic industrial organization literature – see Ericson and Pakes (1995), among others.

where τ_i is the stopping time associated with the exercise of the merger option by BU-pair i. For simplicity of exposition I abstract from the primitives $y_{in}, y_{out}, z, \beta$ and focus on the relations between different ϕ's. I make the following assumptions:

$$AMG|_{type-1} = \phi_2 - \phi_1 \geq 0 \tag{1.47}$$

$$AMG|_{type-2} = \phi_4 - \phi_3 \geq 0 \tag{1.48}$$

$$\Delta AMG = (\phi_4 - \phi_3) - (\phi_2 - \phi_1) \geq 0 \tag{1.49}$$

AMG should be now interpreted as the gain in cash flow rate associated with a merger. The assumptions above are not arbitrary; rather, they are justifiable given the equilibrium outcomes derived in the previous section. In addition I also assume

$$\phi_2 \geq \phi_4, \tag{1.50}$$

which means that BU-pair i's combined cash flow in a post-merger setting is hit negatively when BU-pair j merges. In a quantitative exploration of the static model from the previous section I found that this condition always holds (not reported). Also, note that (1.49) and (1.50) taken together imply

$$\phi_1 \geq \phi_3, \tag{1.51}$$

which means that BU-pair i's combined cash flow in a pre-merger setting is hit negatively when BU-pair j merges. Recall that this effect (in the static model) is mainly a result of the non-merged inner BU loosing a valuable alliance partner (i.e., access to critical resources) after the first merger.

There is a single risk-neutral continuously-compounded discount rate r for all firms. The only stochastic elements in the economy are merger costs. Merger costs for pair $i \in \{1, 2\}$ at time t are denoted by C_{it}. I assume the logarithm of the merger costs (denoted by c_{it}) follows a scaled Brownian motion:

$$dc_{1t} = \sigma dB_{1t} \tag{1.52}$$

$$dc_{2t} = \sigma dB_{2t}, \tag{1.53}$$

where B_{1t} and B_{2t} are uncorrelated. The value for pair i, denoted by V_i, is given by:

$$V_i(t, c_{1t}, c_{2t}) = \begin{cases} \sup_{\tau_i} \left\{ \mathbb{E}\left[\int_t^\infty e^{-r(u-t)} \phi(u, \tau_i, \tau_j)\, du - e^{-r(\tau_i - t)} C_{i\tau_i} \right] \right\} & \tau_i \geq t \quad (1.54) \\ \mathbb{E}\left[\int_t^\infty e^{-r(u-t)} \phi(u, \tau_i, \tau_j)\, du \right] & \tau_i < t \quad (1.55) \end{cases}$$

τ_i is the stopping time associated with the exercise of the merger option by pair i. V_i depends on c_{jt} because it impacts the decision of pair j to merge, which in turn impacts i's cash flow rate. This is an option exercise game. The solution concept I employ is a symmetric Markov-perfect equilibrium. The following proposition contains the key results.

Proposition 11 *The value function for BU-pair $i \in \{1, 2\}$ is given by:*

$$V_i(t, c_{1t}, c_{2t}) = \begin{cases} \frac{\phi_1}{r} + \lambda_1 e^{-\frac{\sqrt{2r}}{\sigma} c_{it}} - \lambda_2 e^{-\frac{\sqrt{2r}}{\sigma} c_{jt}} + \\ \quad + \lambda_3 e^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4} c_i + \sqrt{1-\lambda_4} c_j\right)} & \tau_i, \tau_j \geq t \quad (1.56) \\ \frac{\phi_2}{r} - \lambda_5 e^{-\frac{\sqrt{2r}}{\sigma} c_{jt}} & \tau_i < t, \tau_j \geq t \quad (1.57) \\ \frac{\phi_3}{r} + \lambda_6 e^{-\frac{\sqrt{2r}}{\sigma} c_{it}} & \tau_i \geq t, \tau_j < t \quad (1.58) \\ \frac{\phi_4}{r} & \tau_i, \tau_j < t, \quad (1.59) \end{cases}$$

where $\lambda_1 - \lambda_6$ are non-negative. The optimal log-cost threshold c^* for the exercise of a type-2 merger option is

$$c^*|_{type-2} = \log\left(\frac{\phi_4 - \phi_3}{r + \sigma\sqrt{r/2}} \right). \tag{1.60}$$

The optimal threshold for a type-1 merger for BU-pair i is a function of c_j (the other pair's cost) and is determined implicitly. $c^*|_{type-1}$ has the following upper and lower bounds:

$$\log\left(\frac{\phi_2 - \phi_1}{r + \sigma\sqrt{r/2}} \right) \leq c^*|_{type-1} \leq \log\left(\frac{\phi_4 - \phi_3}{r + \sigma\sqrt{r/2}} \right) \tag{1.61}$$

Finally, as long as $r < \bar{r}$ (defined in the Appendix), the optimal threshold for a type-1 merger for BU-pair i decreases monotonically with c_j.

The intuition for the results in proposition 11 follows. The value function in equation (1.56) can be decomposed into three terms: (i) the value of a perpetuity paying ($\phi_1\, dt$) forever – this happens if $c_i, c_j \to \infty$, which means there is a zero probability that mergers take place within a finite time horizon; (ii) the negative impact of j merging to i's profits – this is captured by the term pre-multiplied by λ_2; (iii) the value of the option to merge, which amounts to

$$\lambda_1 e^{-\frac{\sqrt{2r}}{\sigma} c_{it}} + \lambda_3 e^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4} c_i + \sqrt{1-\lambda_4} c_j\right)}.$$

Note that this option value approaches zero as $c_i \to \infty$, which is economically intuitive: merger costs are so high that it is as if a merger option did not exist. An interesting feature of this option value is that it varies negatively with c_j. This means that the option is more valuable when the possibility of BU-pair j merging becomes more imminent. By merging, BU-pair i is able to somewhat offset the negative effect of BU-pair j's merger decision. Thus c_j affects the value function both negatively (via the term pre-multiplied by λ_2) and positively (via the increase in option value). The intuition for the other branches of the value function is similar, so I will skip this discussion.

How does this dynamic model speak to the topic of merger waves? Expressions (1.60) and (1.61) show that the threshold for a type-1 merger is at or below the threshold for a type-2 merger. As c_j decreases, which makes a merger for BU-pair j more likely, the threshold that justifies a merger by i increases, thus making this merger also more likely. Note that this is independent of the level of the merger costs c_i. Naturally the fact that i is more likely to merge also feeds back into j's policy. The equilibrium thresholds contain a component that can be seen as an amplification of the "natural" merger threshold (i.e., if $\Delta AMG = 0$).

Finally I want to discuss the role played by volatility and time-preferences in terms of merger activity. Expressions (1.60) and (1.61) show that thresholds are lower for a higher σ. The intuition for this is that the more volatile merger costs are, the more likely it is that they will take on low values. Hence the opportunity cost of waiting for low-cost states of nature in the future is lowered, which translates into lower optimal thresholds. The discount rate r has a qualitatively similar effect. If r is high, then the NPV gains of a merger are lower. Thus lower merger costs are required in order to make the merger choice economically feasible.

1.5.2 Spin-offs and diversification

This model can be applied to a setting where mergers are interpreted as diversifying or refocusing strategic moves. Consider an economy with three BUs, with the technology and endowment matrices given by

$$\alpha = \begin{pmatrix} \alpha_s & 0 & 0 \\ 0 & \alpha_s & 0 \\ 0 & 0 & \alpha_s \\ (1-\alpha_s)/2 & 0 & 0 \\ (1-\alpha_s)/2 & (1-\alpha_s) & (1-\alpha_s) \end{pmatrix} \quad K = \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \\ 1 & 0 & 0 \\ 1-y & 1 & 1 \end{pmatrix}.$$

BUs 2 and 3 are interpreted to belong to the same industry, although with different levels of BU-specific capital: z_2 and z_3, respectively. BU 1 belongs to a different industry, but also uses one of the resources required by BUs 2 and 3. The degree to which BU 1 is "short" with respect to this resource is gaged by y. In this economy there is no scope for alliances, since no pair of BUs shares more than one type of capital in common (see proposition 2). Nonetheless, as in section 1.3.2, resources may be more efficiently allocated via a merger. Let us start by considering the option for BU 1 to acquire one of the other two BUs. The results are shown in proposition 12.

Proposition 12 *For $j \in \{2,3\}$, and assuming $z_1 > z_j$, the post-merger allocation implies $L^*_{j1} = 0$ and*

$$L^*_{1j} = \max\left\{\min\left\{\left[\frac{(1-\alpha_s)z_1^{\alpha_s}}{2z_j^{\alpha_s}}\right]^{\frac{2}{(1+\alpha_s)}} - 1 + y, 1\right\}, 0\right\}. \tag{1.62}$$

The minimum threshold for z_1 such that a merger takes place is given by

$$z_1 \geq z_j \left[\frac{2(1-y)^{\frac{(1+\alpha_s)}{2}}}{(1-\alpha_s)}\right]^{\frac{1}{\alpha_s}} \equiv \bar{z}_1. \tag{1.63}$$

The threshold for z_1 after which BU j is emptied of its resources ($L_{1j} = 1$) is given by

$$z_1 \geq z_j \left[\frac{2(2-y)^{\frac{(1+\alpha_s)}{2}}}{(1-\alpha_s)}\right]^{\frac{1}{\alpha_s}} \equiv \bar{\bar{z}}_1. \tag{1.64}$$

The absolute merger gains are the following:

$$AMG = \begin{cases} \left(\frac{z_1^{2\alpha_s}}{z_j^{\alpha_s(1-\alpha_s)}}\right)^{\frac{1}{(1+\alpha_s)}} \left[\left(\frac{1-\alpha_s}{2}\right)^{\left(\frac{1-\alpha_s}{1+\alpha_s}\right)} \right. \\ \left. \quad -\left(\frac{1-\alpha_s}{2}\right)^{\frac{2}{(1+\alpha_s)}}\right] + z_j^{\alpha_s}(1-y) - z_1^{\alpha_s}(1-y)^{\frac{(1-\alpha_s)}{2}} & z_1 < \bar{\bar{z}}_1 \quad (1.65) \\ z_1^{\alpha_s}\left[(2-y)^{\frac{(1-\alpha_s)}{2}} - (1-y)^{\frac{(1-\alpha_s)}{2}}\right] - z_j^{\alpha_s} & z_1 \geq \bar{\bar{z}}_1 \quad (1.66) \end{cases}$$

The absolute merger gains vary negatively with z_j, i.e.

$$\frac{\partial AMG}{\partial z_j} \leq 0. \tag{1.67}$$

The key implication of proposition 12 is that the model predicts that it is the *low-value* BU (the one with lowest z) which is going to be acquired by BU 1. This is true both from the perspective of a merger threshold and absolute merger gains. This implication provides a rationalization for the *diversification discount* that is in line with the findings of Graham, Lemmon, and Wolf (2002). This paper shows that approximately 50% of the discount is attributable to conglomerates buying firms that are already discounted.

Equations (1.63) and (1.64) also reveal that mergers are facilitated (lower z_1 thresholds) for high y, that is, when the resource required by BU 1 is in very short own supply, which is intuitive. This supports the notion that complementarity is the driver of some diversification activity, as put forth in the literature on related diversification – see, e.g., Rumelt (1974), Bettis (1981), Amit and Livnat (1988), and Villalonga (2004).

In addition to a merger by two BUs in different industries, it may be efficient to join BUs 2 and 3. This is simply the case of productivity mergers analyzed in section 1.3.2. In the example at hand, I now consider that BUs 1 and 2 are merged (the "conglomerate") and BU 3 considers the acquisition of BU 2. In case this is economically efficient this would amount to a spin-off. Proposition 13 contains the results.

Proposition 13 *Assume for simplicity that $z_1 >> z_2$ and also $z_3 >> z_2$, such that if BUs 1 and 2 are merged the optimal L_{21} equals zero, and if BUs 2 and 3 are merged the optimal L_{23} equals zero. A necessary condition (sufficient for zero merger costs)* *for the spin-off to take place is the following:*

$$\left(\frac{z_1}{z_3}\right)^{\alpha_s} < \frac{2^{(1-\alpha_s)} - 1}{(2-y)^{\frac{(1-\alpha_s)}{2}} - (1-y)^{\frac{(1-\alpha_s)}{2}}} \tag{1.68}$$

The spin-off is facilitated with a lower y, since

$$\frac{\partial}{\partial y}\left[\frac{2^{(1-\alpha_s)} - 1}{(2-y)^{\frac{(1-\alpha_s)}{2}} - (1-y)^{\frac{(1-\alpha_s)}{2}}}\right] < 0. \tag{1.69}$$

Not surprisingly, what determines the spin-off is that the acquiring specialized firm has a relatively high productivity (high z_3), and BU 1 does not have a too short supply of the resource that it gets from BU 2. A lowering of the degree of resource complementarity within the conglomerate (lower y) will *ceteris paribus* increase the propensity of spin-offs.

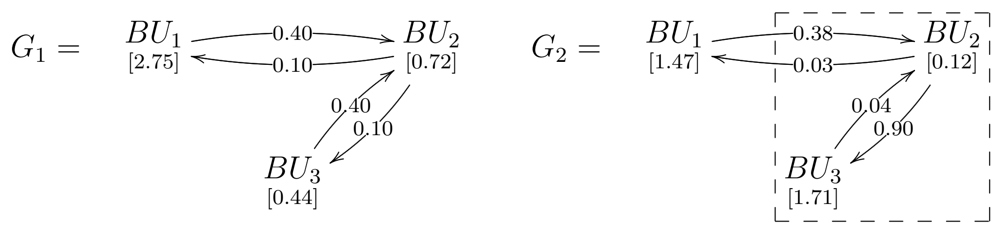
1.5.3 Socially inefficient mergers

In this section I work with an economy comprised of three business units, described by the following technology and endowment matrices:

$$\alpha = \begin{pmatrix} 0.4 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.4 \\ 0.5 & 0.4 & 0.5 \\ 0.1 & 0.4 & 0.1 \end{pmatrix} \quad K = \begin{pmatrix} 1000 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \\ 0.001 & 0.5 & 0.001 \\ 0.999 & 0.5 & 0.999 \end{pmatrix}$$

In terms of link formation I set $\beta = 0.4$. BUs 1 and 3 are very identical, in that they only differ in the amount of BU-specific capital they own, which is much higher for BU 1. BU 2 is structurally different, and much smaller, where I am measuring size as the summation of resource endowments across capital types. The situation I aim to depict in this example is the existence of a player, in this case BU 2, who owns a very valuable (i.e. relatively scarce) resource, in this case capital of type 4 (4-th row in matrix K). Not only is it the most important resource for both BUs 1 and 2 (since $\alpha_{41} = \alpha_{43} = 0.5$), but also both these BUs have a very short own supply of this capital (since $K_{41} = K_{43} = 0.001$). Furthermore, the fact that these two BUs demand this same resource from only one supplier implies additional demand pressure.

I complete the description of this economy by introducing an (exogenous) option for BUs 2 and 3 to merge. Below are the equilibria in these two scenarios; G_2 is the graph where BUs 2 and 3 exercise their option to merge (indicated by the dashed box); the bracketed numbers are the equilibrium profits of each BU; the numbers on the link arrows are the proportion of resources that flow from one BU to another.



First note how in the no-merger scenario BU 2 is enjoying a favorable position in respect of the terms of exchange with its partners, receiving 40% of each partner's resources, in exchange for 10% of its own endowment. Although it is a small business unit, it consumes what a naive observer (this is, someone not observing endowments, but observing link and cash flow size) would consider an unreasonable proportion of the partners' resources. This follows from BUs 1 and 3 being highly dependent on resource 4.

Next let us compare G_1 and G_2. Summing the cash flows in the merger scenario we obtain 3.3; the same computation for the no-merger scenario yields 3.91. It follows that an increase in the concentration of control in the economy is socially inefficient, although obviously if we were to merge all three firms the unconditional Pareto solution would obtain. This result somewhat echoes Cass and Citana (1998), who show that in gradually completing an incomplete market, sometimes there are decreases in social welfare (although fully complete markets Pareto dominate). A caveat is in order. In this model there are no people in the economy, so it is not evident whether business units are improving at the expense of consumers (here tacitly a resource).

The reason behind the social inefficiency lies in the disparities between BU-specific capital. While in the no-merger scenario a significant part of BU 2's resources were allocated to BU 1, although still at a suboptimal level, the situation is quite different with the merger, since BU 3 is able to transfer much more resources from BU 2 (90% *versus* 10%).

1.5.4 Asymmetries in a 2-BU economy

In this section I extend the model from section 1.3 to encompass two kinds of asymmetries between BUs. The objective is to conduct a robustness check of the results obtained in that section, which I show do not change importantly when asymmetries are considered. The first kind is *size asymmetry*, which means that one BU is bigger than the other, but they have the same degree of imbalance in resources with respect to technology. The second kind is *imbalance asymmetry*, where the BUs have the same size but different goodness of fit between resource mix and technology.

Size asymmetry

The technology and endowment matrices are given below:

$$\alpha = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \quad K = \begin{pmatrix} z & x \\ z \times x & 1 \end{pmatrix}$$

The variable z is gaging the size differential between BUs. Setting $z = 1$ reduces this model to the one in the section 1.3, with $\pi = 1$. Figure 1.5 shows how absolute merger gains vary with size differential. Merger gains increase in size, which is a direct consequence of there being more resources for the combined BU pair. Empirically, Rhodes-Kropf and Robinson (2007) find that size differential in a pair of firms increases the likelihood of observing a merger, which is thus consistent with the model. Not surprisingly merger gains are lower for higher β's. As in the symmetric case of section 1.3, as $\beta \rightarrow 0.5$ there are no merger gains, independently of the size differential.

Imbalance asymmetry

Now I consider that both BUs have the same size, but one is more imbalanced than the other. The technology and endowment matrices are given below:

$$\alpha = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \quad K = \begin{pmatrix} 1-y & 1 \\ 1+y & 1 \end{pmatrix}$$

The variable y measures how imbalanced BU 1 is with respect to the optimum given the technology (i.e. $y = 0$). The implications of varying y in terms of merger gains are depicted in figure 1.6. Merger gains display a non-monotonic relationship with variation

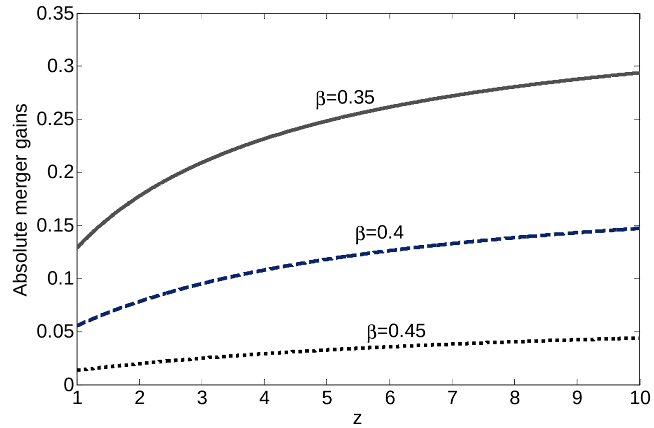


Figure 1.5: Shows how merger gains vary with size differential, for three levels of β; $x = 0.2$.

in relative imbalance. The peak (conditional on β) coincides with the threshold of y that triggers an alliance. For high β's this threshold is naturally lower; in fact, as in the previous cases of 2-BU economies, there are no merger gains for $\beta = 0.5$.

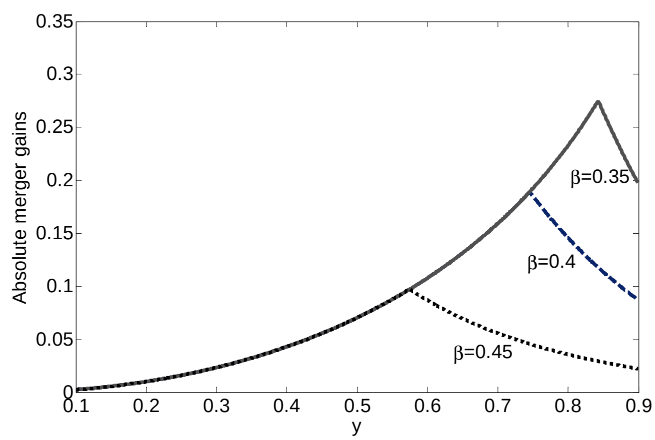


Figure 1.6: Shows how merger gains vary with the imbalance degree of BU 1, for three levels of β; $x = 0.2$.

1.5.5 Connection to resource dependence theory

Resource dependence theory is pervasive in organizational science, having been first proposed in Pfeffer and Salancik (1978). The theory posits that the success of organizations hinges on accessing critical resources from the (social) environment. Organizations pursue a range of different tactics in order to access these resources, and inter-organizational cooperation is one of these tactics. Another alternative would be a sequence of actions undertaken in order to diminish the intrinsic dependence on a certain resource. When the focal organization requires a resource that is controlled by another social actor, the organization is said to be constrained. A key notion in resource dependence theory is *constraint absorption*, which means that the control of the critical resource is transferred to the dependent organization. M&A activity is thus a natural setting for the test of the theory (full constraint absorption), and that is the empirical test carried out by Casciaro and Piskorski (2005).

It is only fair to point out that Casciaro and Piskorski (2005) develop a sharper version of resource dependence theory, more amenable to empirical verification. They start by reviewing the conceptual underpinnings of resource dependence, namely Emerson's theory of power-dependence relations (Emerson (1962)). For Emerson, *power* is a property of dyadic relations, and it is the inverse of bilateral *dependence*. Two dimensions pertain to the power-dependence view of the bilateral relationship: (i) *mutual dependence*, which measures the average degree of dependence of both parties; and (ii) *power imbalance*, which captures the level of asymmetry in dependence. Building on this decomposition of power-dependence relations, Casciaro and Piskorski (2005) argue that while mutual dependence should imply a higher frequency of constraint absorption, the opposite is true of power imbalance. The rationale for this is that with power imbalance, and holding everything else constant, one of the parties could have little incentive to facilitate constraint absorption. In fact, the constraint probably corresponds to a surplus that the high-power party is extracting from the relationship.

Using a sample from 468 industries in the period 1985-2000, Casciaro and Piskorski (2005) test whether the frequency of inter-industry merger activity can be explained by mutual dependence and power imbalance. They construct proxies for these variables by looking at the input-output patterns of transactions across economic sectors.

In my model, business units are formalized as functions that map resources to output. Firms overlap in certain resources and are also allowed to share them, so the model is in fact a mathematical formalization of resource dependence theory. And in effect, mutual dependence – or complementarity – is shown to be a driver of alliance and merger activity, albeit not the only one. The model is also somewhat related to Marsden (1983), where an equilibrium model of relationships is developed. However, the focus of that paper is to study the implications of *restricted access* in terms of equilibrium relationships, i.e. the network structure is to a large extent given. Since in my model I assume no frictions in link formation, access is in effect fully unrestricted.

1.6 Conclusion

This paper develops a model that attempts to explain merger waves by modeling mergers and alliances as mutually exclusive alternatives for the combination of complementary resources. The model is in line with previous findings and aggregate data. The key theoretical result is that the gains of a merger for a set of BUs is conditional on the merger decisions of other firms. The main mechanism that makes mergers more attractive when other mergers are taking place relates to changes in inward focus. If a set of BUs merges and spends less resources in alliances (i.e., lower outward focus), then some stand-alone businesses that depended heavily on these alliances are negatively impacted. In turn, this means that the opportunity cost of these businesses being acquired by some other firm is lowered.

Two natural extensions of the model are: (i) to use a dynamic equilibrium framework with endogenous firm-specific capital (organizational capital); this would help understand how alliances, mergers, and the evolution of firm-specific productivity are inter-related; and (ii) considering traded resources, alliances, and mergers simultaneously; this would deliver testable implications about the relation between traded production factors (prices and quantities) and the structure of the network of firms.

1.A Appendix

1.A.1 Proofs

The following lemma, which is an application of the Schauder fixed point theorem, will be necessary in showing existence and uniqueness results:[22] As in section 1.2.2, X simultaneously denotes a matrix and the corresponding set of the matrix's elements.

Lemma 1 *Partition the set of BUs into two sets Z and $\tilde{Z}$, and create two matrices $A_Z, A_{\tilde{Z}}$ where $a_{ij} \in A_Z \Leftrightarrow i \in Z$, and $a_{ij} \in A_{\tilde{Z}} \Leftrightarrow i \in \tilde{Z}$. If we take $A_{\tilde{Z}}$ as fixed, say $A_{\tilde{Z}} = X$, where $X \in \Re_+^{\#(\tilde{Z})}$ is exogenous, then under condition 3 and the technical requirement $a_{ij} < \bar{a}, \forall i, j$, there exists A^*_Z such that for all $i \in Z$ and $j \in \{1, ..., N\}$ the following holds*

$$f_{\epsilon,ij}(a^*_{ij}; A^*_Z \backslash a^*_{ij}, A_{\tilde{Z}} = X) = 0,$$

for any $\epsilon > 0$.

Proof. From condition 3, there is a set of maps $a^*_{\epsilon,ij} : \Re_+^{(\#(Z)N-1)} \rightarrow \Re_+$ such that, for all $i \in Z, j \in \{1, ..., N\}$

$$a^*_{\epsilon,ij}(A_Z \backslash a_{ij}; A_{\tilde{Z}} = X)$$

yields

$$f_{\epsilon,ij}(a^*_{\epsilon,ij}; A_Z \backslash a_{ij}, A_{\tilde{Z}} = X) = 0.$$

Now construct the mapping $A^*_{\epsilon,Z} : \Re_+^{\#(Z)^2-1} \rightarrow \Re_+^{\#(Z)^2-1}$ which is just a vector containing each map $a^*_{\epsilon,ij}$, for all $i \in Z$. From our differentiability assumptions on Q_i and L, plus the technical requirement $A < \bar{A}$, the mapping $A^*_{\epsilon,Z}$ is a bounded continuous function. It then follows by the Schauder fixed point theorem that this function has a fixed point, which completes the proof. ■

Proof of Theorem 1. Setting $Z = \{1, ..., N\}$ and $\tilde{Z} = \varnothing$, an application of lemma 1 guarantees existence of an ϵ-equilibrium. Next consider any sequence of equilibria such that $\epsilon_n \rightarrow 0$. Since the closure of the counter-domain of $A^*_{\epsilon,Z}$ is by assumption a compact set, then the limit of such a sequence exists and is contained in that set, as long as the sequence is Cauchy. This last property follows from the continuity of $A^*_{\epsilon,Z}$.■

It is now necessary to prove the following lemma as an intermediate step to obtain the uniqueness results.

Lemma 2 (Induction step) *Using the same notation as in lemma 1, define the Z-subgame by constraining $A_{\tilde{Z}} = X$, where $X \in \Re_+^{\#(\tilde{Z})}$ is exogenous. If there is a unique*

[22]For a version of the Schauder fixed point theorem see Başar and Olsder (1999).

ϵ-equilibrium to this subgame (denoted by $A^*_{\epsilon,Z}$), then by extending the subgame to any other BU, i.e. by not fixing some row vector in $A_{\tilde{Z}}$, the equilibrium of the extended subgame is also unique.

Proof. By lemma 1 there exists a solution to the $(Z+1)$-subgame, for any BU $n \in \tilde{Z}$ to which the subgame is extended; denote by $A^*_{\epsilon,(Z+1)}$ this equilibrium allocation. Also, let a^*_n stand for the $(1\times N)$-vector of controls of BU n in this allocation. Next construct the *homotopy function* $H_\epsilon : \Re_+^{\#(Z)N} \times [0,1] \rightarrow \Re^{\#(Z)N}$ with

$$H_\epsilon(A_Z, t; a^*_n) = \left\{ f_{\epsilon,ij}(a_{ij}; a_{nj} = t \times a^*_{nj}, A_{\tilde{Z}} \backslash a_n = X \backslash a_n) | i \in Z, j \in \{1, ..., N\} \right\}. \tag{1.A.1}$$

It is true by construction that

$$H_\epsilon(A^*_{\epsilon,Z}, 0; a^*_n) = H_\epsilon(A^*_{\epsilon,Z}, 1; a^*_n) = 0.$$

Also by lemma 1 there is a solution to the equation $H_\epsilon(A_Z, t; a^*_n) = 0$, for any $t \in [0,1]$; note that $t \times a^*_n$ is a possible strategy for BU n. From condition 3 this solution is locally unique. H_ϵ is continuously differentiable from our assumptions on Q_i and L, which means that the solution is a continuously differentiable function of t. The final step is to show, by contradiction, that this implies that the lemma is true. Assume that the $(Z+1)$-subgame has more than one equilibrium; pick any two of them. Independently of whether a^*_n is distinct in each of these two equilibria, there exists a continuously differentiable path from the Z-subgame allocation $A^*_{\epsilon,Z}$ thereto, defined by varying t from 0 to 1 in equation (1.A.1). Since these two equilibria are locally unique, such a path would have to bifurcate at some $t \in [0,1]$. But this is impossible for a continuously differentiable path.[23] ■

Proof of Theorem 2. Choosing a small enough ϵ, the conditions stated in the theorem about the 2-player subgames obviously hold for the perturbed subgames. Next, by induction using lemma 2 and these conditions, it follows that the full game has a unique ϵ-equilibrium.■

Proof of Theorem 3. The first step is to obtain an expression for the Lagrange multipliers θ_i. In an ϵ-equilibrium all a_{ij}'s are positive, hence $\lambda_{ij} = 0$, for all (i,j). Adding the perturbation term to equation (1.8) one obtains the following expression for θ_i:

$$\theta_i = Q_i \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} K_{mi} + \frac{\epsilon}{a_{ii}}, \tag{1.A.2}$$

which implies that $a_{ii} > 0$ in equilibrium. I now compute a_{ii} indirectly, using the resource constraint:

$$a_{ii} = L_{ii} = 1 - \sum_{j \neq i} L_{ji} \tag{1.A.3}$$

[23] An alternative proof of this lemma can be obtained through the application of the fixed point index theorem in Granas and Dugundji (2003), page 307; the proof of this theorem also involves homotopies.

So far we have transformed the constrained mathematical program into an unconstrained one (it is important to keep in mind that a_{ii} is now a function of the other controls).
Case 1: BUs i and j do not belong to the same firm. The perturbed first-order condition in equation (1.10) for this parametrization is given by:

$$\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right] + g_{\epsilon,ij} = 0, \tag{1.A.4}$$

where

$$g_{\epsilon,ij} \equiv \frac{\epsilon}{a_{ij}} \left[1 - \frac{(1-\beta)L_{ji}}{a_{ii}} \right]. \tag{1.A.5}$$

Setting $a_{ij} = 0$ makes the LHS of equation (1.A.4) equal to $+\infty$. Also, there is for sure an upper bound $\tilde{a}$ such that $a_{ij} = \tilde{a}$ implies $a_{ii} = 0$, which implies that the LHS of equation (1.A.4) goes to $-\infty$. By the intermediate value theorem, and the continuity of the LHS of equation (1.A.4) in a_{ij}, there exists a solution to the equation in the interval $(0, \tilde{a})$. Next I show that the second-order conditions for a maximum, evaluated at a_{ij} such that (1.A.4) holds (this eliminates one of the terms in the mechanical derivation), are verified; formally:

$$\sum_{m=1}^{M} \alpha_{mi} \left\{ \left(-\frac{1}{\hat{K}_{mi}^2} \right) Q_i x_{m1}(a_{ij})^2 + \frac{1}{\hat{K}_{mi}} Q_i x_{m2}(a_{ij}) \right\} + g'_{\epsilon,ij} < 0, \tag{1.A.6}$$

where

$$x_{m1}(a_{ij}) \equiv K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \tag{1.A.7}$$

$$x_{m2}(a_{ij}) \equiv K_{mj} \beta(\beta-1) a_{ij}^{(\beta-2)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta)(-\beta) a_{ji}^{\beta} a_{ij}^{-(1+\beta)} \tag{1.A.8}$$

$$g'_{\epsilon,ij} \equiv \frac{\partial g_{\epsilon,ij}}{\partial a_{ij}} = -\frac{\epsilon}{a_{ij}} \left[1 - \frac{(1-\beta)L_{ji}}{a_{ii}} + \frac{(1-\beta)^2 L_{ji}}{a_{ii}^2} \right] \tag{1.A.9}$$

Clearly the first term inside the summation in the LHS of (1.A.6) is always negative. Next I show that

$$\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i x_{m1}(a_{ij}) + g_{\epsilon,ij} = 0 \Rightarrow \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i x_{m2}(a_{ij}) + g'_{\epsilon,ij} < 0,$$

which implies that the second-order conditions hold. Note that the first part of the statement above is true because it is simply the first-order condition. Next multiply the second expression in the statement by $a_{ij}/(\beta-1)$, which changes the sign of the inequality, since $\beta < 1$. We can then write this equation as

$$\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i x_{m1}(a_{ij}) - g_{\epsilon,ij} + \frac{g'_{\epsilon,ij} a_{ij}}{(\beta-1)} +$$
$$+ \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \left(1 - \frac{(-\beta)}{(\beta-1)} \right) \right] =$$
$$= -\frac{\epsilon}{a_{ij}} \left[1 - \frac{(1-\beta)L_{ji}}{a_{ii}} \right] - \frac{\epsilon}{a_{ij}(\beta-1)} \left[1 - \frac{(1-\beta)L_{ji}}{a_{ii}} + \frac{(1-\beta)^2 L_{ji}}{a_{ii}^2} \right] +$$
$$+ \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \left(\frac{1-2\beta}{1-\beta} \right) \right] =$$
$$= \underbrace{\frac{\epsilon(1-\beta)L_{ji}}{a_{ij}a_{ii}} \left\{ \left(\frac{\beta}{1-\beta} \right) + 1 - \frac{1}{(1-\beta)} + \frac{1}{a_{ii}} \right\}}_{component\ 1} +$$
$$+ \underbrace{\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \left(\frac{1-2\beta}{1-\beta} \right) \right]}_{component\ 2}, \qquad (1.A.10)$$

where I made use of the first-order condition in the simplification. In equation (1.A.10) component 1 is clearly positive (since $a_{ii} < 1$ and $\beta < 0.5$), while the fact that $(1 - 2\beta) > 0$ makes all the terms inside the summation in component 2 also positive. This completes the claim that the second-order conditions are verified, whenever the first-order conditions are. Incidentally, the fact that the second-order conditions always have the same sign for any zero of the (continuously differentiable) first-order condition implies that the zero is unique. And this completes the proof for case 1.

Case 2: BUs i and j belong to the same firm. After a few algebra steps the first-order condition in this case is given by:

$$\sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right] + g_{\epsilon,ij} +$$
$$+ \sum_{m=1}^{M} \frac{\alpha_{mj}}{\hat{K}_{mj}} Q_j \left[K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} - K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} \right] + g_{\epsilon,ji} = 0 \qquad (1.A.11)$$

As in case 1, setting $a_{ij} = 0$ makes the LHS of equation (1.A.11) equal to $+\infty$. And again there is an upper bound $\tilde{a}$ such that $a_{ij} = \tilde{a}$ implies either $(a_{ii} = 0, a_{jj} \geq 0)$, or $(a_{jj} = 0, a_{ii} \geq 0)$, or both. In any case this implies that the LHS of the equation goes to $-\infty$. This completes proving existence of at least one zero for the first-order condition. Next construct the following homotopy function:

$$H(a_{ij},t) \equiv \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mj}\beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right] + g_{\epsilon,ij} + \\ +t \left\{ \sum_{m=1}^{M} \frac{\alpha_{mj}}{\hat{K}_{mj}} Q_j \left[K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} - K_{mj}\beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} \right] + g_{\epsilon,ji} \right\} \quad (1.A.12)$$

Setting $t = 0$ reduces to case 1, so a solution to $\{H(a_{ij}, t = 0) = 0\}$ exists and is unique. By a reasoning similar to the one outlined in lemma 2, and the fact that we already proved existence of a solution to $\{H(a_{ij}, t = 1) = 0\}$, this solution is also unique. This follows from H being continuously differentiable in t, which in turn is true because there is a solution for the equation $\{H(a_{ij}, t) = 0\}$, for any t; the argument is the same as the one for the existence when $t = 0$ (case 1) or $t = 1$ (case 2). The fact that this unique solution corresponds to a maximum follows from the positivity of the first-order condition from $a_{ij} = 0$ up to the solution; recall that the LHS of equation (1.A.11) is $+\infty$ when $a_{ij} = 0$.
To finalize the proof of the theorem I only have to show that the conditions required by theorem 2 for uniqueness are verified. This amounts to showing that for a 2-BU subgame there is exactly one perfect equilibrium. For this parametrization the special case where all BUs only use BU-specific capital implies that the unique equilibrium is one where $a_{ij} = 0$, for all $(i, j) \in \{1, ..., N\}$ (see proposition 3). Hence this is the unique perfect equilibrium in this special case. Given the continuity of the first-order conditions, and by the same kind of homotopic arguments made above, this implies that any 2-BU subgame has a unique equilibrium too (here the homotopy is easily constructed on the parameters α_{mi}).■

Proof of Proposition 1. First note that for $m = m(j)$ it is true by construction that $\alpha_{m(j)i} = 0$. Thus the LHS of the first-order condition (1.A.4) is unchanged with variations in $K_{m(j)j}$. Second, for $m = m(i)$, by construction we have $\hat{K}_{m(i)i} = K_{m(i)i}$ and $K_{m(i)j} = 0$. Since when $\epsilon \to 0$ we can simplify (1.A.4) by dividing both sides by Q_i, the term inside the summation when $m = m(i)$ equals to

$$-\alpha_{m(i)i} \frac{K_{m(i)i}}{\hat{K}_{m(i)i}} (1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} = -\alpha_{m(i)i}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta},$$

which does not depend on $K_{m(i)i}$.■

Proof of Proposition 2. Denote as m^* the type of capital that the BUs have in common. Define $\delta \equiv K_{m^*j}/K_{m^*i}$. The term associated with m^* in the LHS of the first-order condition (1.A.4) for BU i with respect to a_{ij} is

$$\frac{\alpha_{m^*i}}{\hat{K}_{m^*i}} Q_i \left[K_{m^*j}\beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{m^*i}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right]. \quad (1.A.13)$$

For BU j the LHS of the first-order condition with respect to a_{ji} is similarly given by

$$\frac{\alpha_{m^*j}}{\hat{K}_{m^*j}}Q_j\left[K_{m^*i}\beta a_{ji}^{(\beta-1)}a_{ij}^{(1-\beta)} - K_{m^*j}(1-\beta)a_{ij}^{\beta}a_{ji}^{-\beta}\right]. \tag{1.A.14}$$

After some manipulations, the sign of equation (1.A.13) is equal to the sign of

$$\delta - \left(\frac{1-\beta}{\beta}\right)\left(\frac{a_{ij}}{a_{ji}}\right)^{(1-2\beta)}.$$

And the sign of equation (1.A.14) is equal to the sign of

$$\begin{aligned}\left(\frac{\beta}{1-\beta}\right)\left(\frac{a_{ij}}{a_{ji}}\right)^{(1-2\beta)} - \delta = &-\left[\delta - \left(\frac{1-\beta}{\beta}\right)\left(\frac{a_{ij}}{a_{ji}}\right)^{(1-2\beta)}\right] \\ &-\left[\left(\frac{1-\beta}{\beta}\right) - \left(\frac{\beta}{1-\beta}\right)\right]\left(\frac{a_{ij}}{a_{ji}}\right)^{(1-2\beta)}.\end{aligned}$$

Since the last term in the second expression above is negative for $(a_{ij}, a_{ji}) > 0$ (recall: $\beta < 0.5$), then if the LHS of the first-order condition for BU i is positive, then necessarily the whole expression is negative too. Since for all $m \neq m^*$ the terms inside the summation in the LHS of the BUs' first-order conditions are negative or zero by construction, this implies that it is never possible to satisfy the first-order conditions of both BUs at the same time with $(a_{ij}, a_{ji}) > 0$ when $\epsilon \to 0$, which completes the proof.■

Proof of Proposition 3. From the assumption that BU i and BU j have no types of capital in common it follows that setting $\epsilon = 0$ the LHS of the first-order conditions for both i and j is negative whenever $a_{ij} > 0$ and $a_{ji} > 0$; note that by construction when $\alpha_{mi} > 0$ we have $K_{mj} = 0$ and the same holds for BU j. Inspecting equations (1.A.4) and (1.A.11), one verifies that this is true independently of whether the BUs belong to the same firm or not. Hence, in a perfect equilibrium (attained as $\epsilon \to 0$) it must be that $(a_{ij}, a_{ji}) = (0, 0)$, which implies $L_{ij} = L_{ji} = 0$.■

Proof of Proposition 4. First note that the endowment matrix can be written as

$$K = \frac{1}{1+y}\begin{pmatrix} 1 & x \\ x & 1 \end{pmatrix}.$$

The scalar multiplying the matrix may be ignored given the linearity of technology. Since the equilibrium is unique, then in this case it must be symmetric; this allows us to set $a_{12} = a_{21}$ in the the set of first-order conditions, using equation (1.A.11) with $\epsilon = 0$. After some manipulations this reduces to (the interior of) equation (1.16). Plugging the optimal links in the expressions for effective capital yields equation equation (1.18).■

Proof of Proposition 5. The endowment matrix can also be simplified as in the proof of proposition 4. The unique perfect equilibrium is also symmetric, which allows us to set $a_{12} = a_{21}$. Now we use the first-order conditions given by equation (1.A.4) with $\epsilon = 0$. After some algebra steps this yields equation (1.19).
To show that the statement in expression (1.21) is true, I show that the optimal link in the alliance case (equation (1.19)) is smaller than the optimal link in the merger case (equation (1.16)).

$$\frac{\pi + x^2 - \left(\frac{1-\beta}{\beta}\right) x(1+\pi)}{(1-x)[\pi - x - \left(\frac{1-\beta}{\beta}\right)(\pi x - 1)]} < \frac{\pi - x}{(1-x)(1+\pi)} \Leftrightarrow$$
$$\pi \left[x^2 - \left(\frac{1-\beta}{\beta}\right) x(1+\pi) + x + \left(\frac{1-\beta}{\beta}\right)(\pi x - 1)\right] <$$
$$-x\pi + x\left(\frac{1-\beta}{\beta}\right)(\pi x - 1) - \pi + \left(\frac{1-\beta}{\beta}\right) x(1+\pi) \Leftrightarrow$$
$$x^2 - \left(\frac{1-\beta}{\beta}\right) x + x - \left(\frac{1-\beta}{\beta}\right) < \left(\frac{1-\beta}{\beta}\right) x - x + x^2 \left(\frac{1-\beta}{\beta}\right) - 1$$
$$\Leftrightarrow (x+1)^2 > 0,$$

which is always true.■

Proof of Proposition 6. Since in equilibrium $L_{12} = L_{21} \equiv L^*$, in any scenario (merger or alliance) one can write economic profit as

$$Q = \sqrt{1 - y^2(1 - 2L^*)^2}. \tag{1.A.15}$$

Using the expressions from equations (1.16) and (1.19) one obtains equations (1.22) and (1.23), respectively. To obtain $\bar{y}$ in equation (1.25) I set the expression for the optimal link in an alliance setting equal to 0. Finally expression (1.26) is obtained by simply setting $L^* = 0$ in (1.A.15).■

Proof of Proposition 7. Given that in an optimum $L_{21} = 0$, one only needs the first-order condition for L_{12}, which after some manipulations is given by

$$\frac{\alpha_s Q_2}{1 - L_{12}} = \frac{\alpha_c Q_1}{1 + L_{12}} - \frac{\alpha_c Q_2}{1 - L_{12}}. \tag{1.A.16}$$

Further simplification yields (the interior of) expression (1.28). The upper bound of the inertia region in (1.29) is obtained by setting the interior of expression (1.28) to zero. Finally the threshold in (1.30) is obtained by setting the interior of expression (1.28) to 1.■

Proof of Proposition 8. To prove point 1 in the proposition, first note that for $y_{in} = 0$ the inner BUs do not establish a link, in any scenario. It follows that $EP_3 = EP_7$, $EP_4 = EP_8$, $EP_1 = EP_6$, $EP_2 = EP_5$, which plugged into equation (1.36) yields

$\Delta AMG = 0$. From the continuity of the first-order conditions there exists then a threshold $\bar{y}_{in} \geq 0$ such that $\Delta AMG = 0$.
Next I show that the statement in point 2 is true. Define $\bar{z} >> 1$ as the threshold for z such that in *any* merger we have $L_{outer,inner} \approx 1$ and $L_{inner,outer} \approx 0$. After such a threshold we have $EP_4 = EP_8 = 0$ and $EP_3 = EP_7 = (4z)^{\bar{\alpha}}$, which implies

$$AMG|_{type-1} = (4z)^{\bar{\alpha}} - EP_1 - EP_2 \quad (1.A.17)$$

$$AMG|_{type-2} = (4z)^{\bar{\alpha}} - EP_6 - EP_5 \quad (1.A.18)$$

$$\Delta AMG = EP_1 + EP_2 - EP_5 - EP_6. \quad (1.A.19)$$

Since EP_2 and EP_5 do not depend on z because z is neutral for alliance links (see proposition 1), then we have

$$\frac{\partial\ AMG|_{type-1}}{\partial z} = 4\bar{\alpha}(4z)^{(\bar{\alpha}-1)} - \frac{\partial EP_1}{\partial z} \quad (1.A.20)$$

$$\frac{\partial\ AMG|_{type-2}}{\partial z} = 4\bar{\alpha}(4z)^{(\bar{\alpha}-1)} - \frac{\partial EP_6}{\partial z} \quad (1.A.21)$$

$$\frac{\partial \Delta AMG}{\partial z} = \frac{\partial EP_1}{\partial z} - \frac{\partial EP_6}{\partial z}. \quad (1.A.22)$$

A few steps of algebra show that

$$\frac{\partial EP_1}{\partial z} = \bar{\alpha}\frac{EP_1}{z} \quad (1.A.23)$$

$$\frac{\partial EP_6}{\partial z} = \bar{\alpha}\frac{EP_6}{z}. \quad (1.A.24)$$

Combining equations (1.A.23)-(1.A.24) with equation (1.A.22) yields

$$\frac{\partial \Delta AMG}{\partial z} \Leftrightarrow EP_1 > EP_6, \quad (1.A.25)$$

which is assumed in the proposition. Next I show that for any type of merger,

$$\frac{\partial AMG}{\partial z} > 0. \quad (1.A.26)$$

After a few steps of algebra this is equivalent to showing that

$$4 > (1 - L_{inner,outer})\hat{K}_{51}\hat{K}_{61} \Leftrightarrow \quad (1.A.27)$$

$$4 > (1 - L_{inner,outer})\left[(1 + L_{outer,inner} - L_{outer,inner})^2 - y_{out}^2(1 - L_{inner,outer} - L_{outer,inner})^2\right]. \quad (1.A.28)$$

The RHS of the above expression is maximized for $L_{inner,outer} = 0$, $L_{outer,inner} = 1$ and $y_{out} = 0$, yielding 4. However, in an alliance equilibrium $L_{inner,outer} = 0$ implies

$L_{outer,inner} = 0$, so this concludes proving point 2.
Finally I show that point 3 in the proposition holds. Note that for $z \geq \bar{z}$ the value of y_{out} is irrelevant for the post-merger allocations, in any type of merger. Also, the equilibrium links between outer and inner BUs in the alliance settings are independent of z. Using the envelope theorem and after some manipulations we have that

$$\frac{\partial\, Q^*_{outer}|_{alliance}}{\partial y_{out}} < 0 \Leftrightarrow 1 > L_{inner,outer} + L_{outer,inner}, \tag{1.A.29}$$

which is always true because any alliance link in this setting must be smaller than 0.5 (otherwise there would be over-investment in links, which is impossible for $\beta < 0.5$). Similarly, it is also true that

$$\frac{\partial\, Q^*_{inner}|_{alliance}}{\partial y_{out}} < 0 \Leftrightarrow 1 > L_{outer,inner} + L_{inner,outer} + L_{inner,inner}. \tag{1.A.30}$$

It thus follows that for the same z, merger gains increase when y_{out} increases (lower opportunity costs). This also directly implies that $\bar{z}$ is lower.■

Proof of Proposition 9. Start by guessing that the all-alliance equilibrium implies a scale-free solution L^*. For some BU i this implies that the relevant effective resources are

$$\begin{aligned}
\hat{K}_{ii} &= (1 - 2L^*)z_i \\
\hat{K}_{2i-1,i} &= \hat{K}_{2i+2,i} = (1 - y) + L^*(3y - 1) \\
\hat{K}_{2i,i} &= \hat{K}_{2i+1,i} = (1 + y) - L^*(3y + 1).
\end{aligned}$$

Combining the above with the first-order conditions (1.A.4) for the optimal $a_{i,i+1}$ yields

$$\frac{[\beta(1 + y) - 2(1 - \beta)(1 - y)]}{[(1 - y) + L^*(3y - 1)]} + \frac{[\beta(1 - y) - 2(1 - \beta)(1 + y)]}{[(1 + y) - L^*(3y + 1)]} = \frac{(1 - \beta)}{1 - 2L^*},$$

which simplified yields equation (1.37). Setting $y = 1$ yields (1.41), which plugged into the profit function gives (1.42).■

Proof of Proposition 10. Without loss of generality set $k = 2$. The first step is to show that if $z_2 >> z_1$ (such that $L_{12} \approx 0$ and $L_{21} \approx 1$), $y = 1$, and BU 3 is *not* merged with $(1, 2)$, the following two statements are true:

$$L_{32} = \frac{\beta}{3 - 2\beta} \tag{1.A.31}$$

$$L_{23} \leq \frac{\beta}{3 - 2\beta} \tag{1.A.32}$$

To show that (1.A.31) holds, let us first write an expression for the profit of BU 2 after the merger with 1:

$$Q_2 = \left[16z_2(1-L_{32})^3 L_{23}\right]^{0.2}$$

The first-order condition of the above with respect to a_{23} (recall L_{23} and L_{32} are both functions of a_{23}) yields

$$\frac{0.2(-3)Q_2}{(1-L_{32})}(1-\beta)a_{32}^{\beta}a_{23}^{-\beta} + \frac{0.2Q_2}{L_{23}}\beta a_{23}^{\beta-1}a_{32}^{1-\beta} = 0,$$

which simplifies into (1.A.31). Next, to show that (1.A.32) holds, let us write the expression for the economic profit of BU 3:

$$Q_3 = \left[16z_3(1-L_{23}-L_{43})^3 L_{32}L_{34}\right]^{0.2}$$

The first-order condition with respect to a_{32} yields

$$\frac{0.2(-3)Q_3}{(1-L_{23}-L_{43})})(1-\beta)a_{23}^{\beta}a_{32}^{-\beta} + \frac{0.2Q_3}{L_{32}}\beta a_{32}^{\beta-1}a_{23}^{1-\beta} = 0 \Leftrightarrow$$
$$\Leftrightarrow L_{23} = \frac{\beta(1-L_{43})}{3-2\beta}.$$

Since $L_{43} \geq 0$ is always true, (1.A.32) follows. With (1.A.31) and (1.A.32) we can write the following upper bounds for Q_2 and Q_3:

$$Q_2 \leq \left[432z_2\frac{\beta(1-\beta)}{(3-2\beta)^4}\right]^{0.2} \equiv \bar{Q}_2 \tag{1.A.33}$$

$$Q_3 \leq \left[16z_3\frac{\beta^2}{(3-2\beta)^2}\right]^{0.2} \equiv \bar{Q}_3 \tag{1.A.34}$$

The next step is to analyze how merger gains change with the merger by BUs 1 and 2. Assuming that $z_2 >> z_3$ (such that $L_{23} \approx 1$ and $L_{32} \approx 0$ if BUs 2 and 3 merge), then the absolute merger gains of BUs 2 and 3 merging, relative to the full-alliance scenario ($c_{ij} = \underline{c}$ for all BUs), are bounded from above:

$$AMG|_{BUs\ 2\ and\ 3\ merge} \leq \left\{432z_2\frac{\beta(1-\beta)}{(3-2\beta)^4}\right\}^{0.2} - \frac{\left[432\beta^2(1-\beta)^3\right]^{0.2}}{(3-\beta)}\left(z_2^{0.2}+z_3^{0.2}\right), \tag{1.A.35}$$

where I have made use of equation (1.42) and am still assuming $y = 1$. Thus, under the assumptions that $z_2 >> z_1$ and $z_2 >> z_3$, the absolute merger gains associated with the merged BU pair $(1, 2)$ acquiring BU 3 has the following *lower* bound:

$$AMG|_{BU\ pair\ (1,2)\ buys\ BU\ 3} \geq (16z_2)^{0.2} - \left[432z_2\frac{\beta(1-\beta)}{(3-2\beta)^4}\right]^{0.2} - \left\{16z_3\frac{\beta^2}{(3-2\beta)^2}\right\}^{0.2} \tag{1.A.36}$$

Combining inequalities (1.A.35) and (1.A.36), one can derive a lower bound for the variation in absolute merger gains from joining BUs 2 and 3, after and before 1 has merged with 2:

$$\begin{aligned}\Delta AMG \quad \geq \quad & z_2^{0.2}\left\{16^{0.2} - 2\left[\frac{432\beta(1-\beta)}{(3-2\beta)^4}\right]^{0.2} + \frac{[432\beta^2(1-\beta)^3]^{0.2}}{(3-\beta)}\right\} + \\ & + z_3^{0.2}\left\{\frac{[432\beta^2(1-\beta)^3]^{0.2}}{(3-\beta)} - \left[16\frac{\beta^2}{(3-2\beta)^2}\right]^{0.2}\right\} \equiv \underline{\Delta AMG}\end{aligned} \tag{1.A.37}$$

It is easy to check from equation (1.A.37) that if $z_3 \approx 0$ (consistent with the assumption $z_2 >> z_3$) and $\beta < 0.27$, then $\underline{\Delta AMG} > 0$. This implies that it is a *dominating strategy* for BU 3 to be acquired by BU pair $(1, 2)$ as long as $\delta \leq \underline{\Delta AMG}$ (i.e. if merger costs are not too high), since I did not make any assumptions about L_{43} or L_{34}. The final step in the proof is to show that the merger of BUs 1, 2, and 3 makes it a dominating strategy for other BU triplets to merge. If $L_{21} = L_{23} \approx 1$, then BUs 1 and 3 have no longer resources to share with their other adjacent BUs; N and 4, respectively. If $y = 1$ this implies that $Q_N = Q_4 = 0$. This means that the merger gains of merging 4 with 5, and $N - 1$ with N, increase. As long as z_5 and z_N are high enough this also implies that the acquisitions of BUs $N-2$ and 6 are more profitable (just as the merger gains of acquiring BU 3 increase after 1 and 2 are merged). Applying induction and assuming that the BU-specific capitals of central BUs are high enough, this means that for low enough β and high enough y there is a dominating-strategy equilibrium where all BU triplets merge. Since BUs are playing dominating strategies this equilibrium is unique.■

Proof of Proposition 11. The financial-markets equilibrium conditions for the value function of BU-pair $i \in \{1, 2\}$ are given by the following expressions:[24]

$$rV_i(t, c_i, c_j)\, dt = \phi_t\, dt + \mathbb{E}[dV_i(t, c_i, c_j)] \tag{1.A.38}$$

where $j \in \{1, 2\}, j \neq i$ and $\phi_t \in \{\phi_1, \phi_2, \phi_3, \phi_4\}$; see equations (1.43)-(1.46). Applying Itô's lemma to $\mathbb{E}[dV_i]$, one can write (1.A.38) as the following partial differential equation (PDE):[25]

$$rV_i(t, c_i, c_j) = \phi_t + 0.5\sigma^2\left[\frac{\partial^2 V_i}{\partial c_i^2} + \frac{\partial^2 V_i}{\partial c_j^2}\right] \tag{1.A.39}$$

[24] See Dixit and Pyndick (1994) or Shreve (2004).
[25] Just for reference, equation (1.A.39) is the Klein-Gordon PDE from quantum field theory.

where I have made use of equations (1.52)-(1.53) and the fact that B_{1t} and B_{2t} are uncorrelated. V_i does not vary locally with t ($\phi's$ are constants), hence the non-appearance of a term $\partial V_i/\partial t$ in (1.A.39). To pin down V_i one needs to consider the optimal exercise of merger options, as well as the economically relevant boundary conditions. I start by determining V_i for the case where both mergers have already taken place; it is simple to check that $V_i = \phi_4/r$ verifies PDE 1.A.39, plus condition $\partial V_i/\partial c_i = \partial V_i/\partial c_j = 0$ (merger costs are now irrelevant). The next step is to determine the value function for the case where j is already merged, but i is not $(\tau_i \geq t, \tau_j < t)$. In this case the PDE reduces to an ODE:

$$rV_i = \phi_3 + 0.5\sigma^2 \frac{\partial^2 V_i}{\partial c_i^2} \tag{1.A.40}$$

The general solution that satisfies this equation, plus the boundary condition $\lim_{c_i \to \infty} V_i = \phi_3/r$ (i.e., c_i is so high that the option to merge has no value) is given by equation (1.58). I conjecture that there exists some threshold c^* such that for $c_i < c^*$ BU-pair i merges. The smooth-pasting condition is given by:[26]

$$\left(\left.\frac{\partial V_i|_{\tau_i \geq t, \tau_j < t}}{\partial c_i}\right|_{c_i = c^*} = -\lambda_6 \frac{\sqrt{2r}}{\sigma} e^{-\frac{\sqrt{2r}}{\sigma}c^*} \right) = \left(-e^{c^*} = \left.\frac{\partial \left[V_i|_{\tau_i, \tau_j < t} - e^{c_i} \right]}{\partial c_i} \right|_{c_i = c^*} \right) \tag{1.A.41}$$

Recall that c_i is the logarithm of the merger costs, hence the exponential in the RHS of the equation above. The value-matching condition is

$$V_i|_{\tau_i \geq t, \tau_j < t}(c^*) = \frac{\phi_4}{r} - e^{c^*}. \tag{1.A.42}$$

Combining the smooth-pasting and value-matching equations I obtain the expression for $c^*|_{type-2}$ in equation (1.60) and λ_6:

$$\lambda_6 = \frac{\sigma}{\sqrt{2r}} \left[\frac{\phi_4 - \phi_3}{r + \sigma\sqrt{r/2}} \right]^{\left(1 + \frac{\sqrt{2r}}{\sigma}\right)} \geq 0 \tag{1.A.43}$$

Given that I am solving for a symmetric equilibrium, the optimal exercise of j when i is already merged will also follow the same policy $c^*|_{type-2}$. The PDE for $(\tau_i < t, \tau_j \geq t)$ also reduces to an ODE. Taking into account the boundary condition $\lim_{c_j \to \infty} V_i = \phi_2/r$ (i.e., c_j is so high that it is as if BU-pair j will never merge), the solution is expressed as equation (1.57). The value-matching condition pins down λ_5:

$$V_i|_{\tau_i < t, \tau_j \geq t}(c_j = c^*|_{type-2}) = \frac{\phi_4}{r} \Leftrightarrow$$

[26] See Dixit and Pyndick (1994) for a presentation of smooth-pasting conditions in the context of real options methodology.

$$\lambda_5 = \frac{(\phi_2 - \phi_4)}{r}\left[\frac{\phi_4 - \phi_3}{r + \sigma\sqrt{r/2}}\right]^{\frac{\sqrt{2r}}{\sigma}} \geq 0 \tag{1.A.44}$$

The final step is to determine V_i for $(\tau_i, \tau_j \geq t)$. This amounts to finding a solution to (1.A.39) that verifies certain boundary conditions. I start by imposing

$$\lim_{c_j \to \infty} V_i(c_i, c_j) = V_i(c_i) \tag{1.A.45}$$

$$\lim_{c_i \to \infty} V_i(c_i, c_j) = V_i(c_j) \tag{1.A.46}$$

$$\lim_{c_i, c_j \to \infty} V_i(c_i, c_j) = \phi_1 / r, \tag{1.A.47}$$

which is economically intuitive. The functional form in equation (1.56) verifies these three conditions. The smooth-pasting and value-matching conditions are now, respectively:

$$\left.\frac{\partial V_i|_{\tau_i,\tau_j \geq t}}{\partial c_i}\right|_{c_i = c^*} = \left.\frac{\partial\left[V_i|_{\tau_i < t, \tau_j \geq t} - e^{c_i}\right]}{\partial c_i}\right|_{c_i = c^*} \tag{1.A.48}$$

$$V_i|_{\tau_i,\tau_j \geq t}(c_i = c^*, c_j) = V_i|_{\tau_i < t, \tau_j \geq t}(c_j) - e^{c^*} \tag{1.A.49}$$

Combining the two expressions I obtain an implicit function for $c^*|_{type-1}$, the threshold for the exercise of a type-1 merger option (for BU-pair i):

$$F(c^*|_{type-1}, c_j) \equiv \lambda_1 e^{-\frac{\sqrt{2r}}{\sigma}c^*}\left[1 + \frac{\sqrt{2r}}{\sigma}\right] + (\lambda_5 - \lambda_2)e^{-\frac{\sqrt{2r}}{\sigma}c_j} +$$
$$+\lambda_3 e^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4}c^* + \sqrt{1-\lambda_4}c_j\right)}\left[1 + \frac{\sqrt{2\lambda_4 r}}{\sigma}\right] - \frac{(\phi_2 - \phi_1)}{r} = 0 \tag{1.A.50}$$

The terms c^* in (1.A.50) refer to $c^*|_{type-1}$; the same applies to the expressions below. The following additional conditions pin down λ_1 and λ_2:

$$V_i|_{\tau_i,\tau_j \geq t}(c_i = c^*(c_j \to \infty), c_j \to \infty) = \frac{\phi_2}{r} - e^{c^*(c_j \to \infty)} \tag{1.A.51}$$

$$V_i|_{\tau_i,\tau_j \geq t}(c_i \to \infty, c_j = c^*(c_i \to \infty)) = \frac{\phi_3}{r}, \tag{1.A.52}$$

yielding

$$\lambda_1 = \frac{\sigma}{\sqrt{2r}}\left[\frac{\phi_2 - \phi_1}{r + \sigma\sqrt{r/2}}\right]^{\left(1 + \frac{\sqrt{2r}}{\sigma}\right)} \geq 0 \tag{1.A.53}$$

$$\lambda_2 = \frac{(\phi_1 - \phi_3)}{\sqrt{2r}} \left[\frac{\phi_2 - \phi_1}{r + \sigma\sqrt{r/2}}\right]^{\frac{\sqrt{2r}}{\sigma}} \geq 0, \tag{1.A.54}$$

and an expression for $c^*(\infty)$ which is the lower bound for $c^*|_{type-1}$ in (1.61). Since the equilibrium is symmetric, we can use the following smooth-pasting condition to pin down λ_3:

$$\left.\frac{\partial V_i|_{\tau_i,\tau_j \geq t}}{\partial c_i}\right|_{c_i=c_j=c^*|_{type-2}} = \left.\frac{\partial [V_i|_{\tau_i,\tau_j<t} - e^{c_i}]}{\partial c_i}\right|_{c_i=c_j=c^*|_{type-2}}, \tag{1.A.55}$$

where I made use of the fact that in equilibrium (and given continuity) the upper bound of $c^*|_{type-1}$ must be $c^*|_{type-2}$. If both c_i and c_j hit this threshold simultaneously then we go from a no-merger scenario directly to a two-mergers scenario. A few steps of algebra yield

$$\lambda_3 = \frac{\frac{(\phi_4-\phi_3)}{\sqrt{2r^3/\sigma + r}} - \lambda_1 \left[\frac{\phi_4 - \phi_3}{r+\sigma\sqrt{r/2}}\right]^{-\frac{\sqrt{2r}}{\sigma}}}{\sqrt{\lambda_4}\left[\frac{\phi_4-\phi_3}{r+\sigma\sqrt{r/2}}\right]^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4}+\sqrt{1-\lambda_4}\right)}}. \tag{1.A.56}$$

It is easy to show that under the assumptions (on ϕ) λ_3 is non-negative:

$$\begin{aligned}
&\lambda_3 \geq 0 \Leftrightarrow \\
&\frac{\phi_4 - \phi_3}{\sqrt{2r^3}/\sigma + r} \geq \lambda_1 \left[\frac{\phi_4 - \phi_3}{r + \sigma\sqrt{r/2}}\right]^{-\frac{\sqrt{2r}}{\sigma}} \Leftrightarrow \\
&\frac{\phi_4 - \phi_3}{\phi_2 - \phi_1} \geq \left(\frac{\phi_2 - \phi_1}{\phi_4 - \phi_3}\right)^{\frac{\sqrt{2r}}{\sigma}} \Leftrightarrow \phi_4 - \phi_3 \geq \phi_2 - \phi_1
\end{aligned}$$

The next step is to determine λ_4 (numerically) using the following value-matching condition:

$$V_i|_{\tau_i,\tau_j \geq t}(c_i = c^*|_{type-2}, c_j = c^*|_{type-2}) = \frac{\phi_4}{r} - e^{c^*|_{type-2}} \tag{1.A.57}$$

The last step in the proof is to show that $c^*|_{type-1}$ for i decreases monotonically with c_j. Using equation (1.A.50) and invoking the implicit function theorem we have

$$\begin{aligned}
&\frac{dc^*}{dc_j} = -\frac{\partial F/\partial c_j}{\partial F/\partial c^*} = \\
&\frac{(\lambda_5 - \lambda_2)e^{-\frac{\sqrt{2r}}{\sigma}c_j} + \sqrt{1-\lambda_4}\lambda_3 e^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4}c^* + \sqrt{1-\lambda_4}c_j\right)}\left[1 + \frac{\sqrt{2\lambda_4 r}}{\sigma}\right]}{\lambda_1 e^{-\frac{\sqrt{2r}}{\sigma}c^*}\left[1 + \frac{\sqrt{2r}}{\sigma}\right] + \sqrt{\lambda_4}\lambda_3 e^{-\frac{\sqrt{2r}}{\sigma}\left(\sqrt{\lambda_4}c^* + \sqrt{1-\lambda_4}c_j\right)}\left[1 + \frac{\sqrt{2\lambda_4 r}}{\sigma}\right]}
\end{aligned} \tag{1.A.58}$$

A sufficient condition for $dc^*/dc_j < 0$ is

$$\lambda_5 > \lambda_2 \Leftrightarrow$$
$$r < 0.5\left[\sigma\frac{\log\left(\frac{\phi_2-\phi_4}{\phi_1-\phi_3}\right)}{\log\left(\frac{\phi_2-\phi_1}{\phi_4-\phi_3}\right)}\right]^2 \equiv \bar{r}, \tag{1.A.59}$$

and this completes the proof.■

Proof of Proposition 12. As shown in section 1.3.2, it would be inefficient for any merger link between a pair of BUs in this economy to have both links positive. If BUs 1 and j merge, the optimal link is obtained by maximizing the following function with respect to L_{1j}:

$$Q_1(L_{1j}) + Q_j(L_{1j}) = z_1^{\alpha_s}(1-y+L_{1j})^{\frac{(1-\alpha_s)}{2}}$$

The first-order conditions are readily obtained:

$$z_1^{\alpha_s}(1-y+L_{1j})^{\frac{(1-\alpha_s)}{2}-1}\frac{(1-\alpha_s)}{2} - z_j^{\alpha_s} = 0,$$

which simplified yield expression 1.62. The thresholds $\bar{z}_1$ and $\bar{\bar{z}}_1$ are obtained by setting L_{1j} to 0 and 1, respectively. The expressions for the absolute merger gains are obtained by plugging the optimal link into the objective function. It is trivial to see that expression (1.67) holds for $z_1 > \bar{\bar{z}}_1$. For $z_1 \leq \bar{\bar{z}}_1$ we have

$$\frac{\partial AMG}{\partial z_j} \leq 0 \Leftrightarrow \left(\frac{z_1}{z_2}\right)^{\frac{2\alpha_s}{(1+\alpha_s)}}\left(\frac{1-\alpha_s}{1+\alpha_s}\right)\left[\left(\frac{1-\alpha_s}{2}\right)^{\left(\frac{1-\alpha_s}{1+\alpha_s}\right)} - \left(\frac{1-\alpha_s}{2}\right)^{\frac{2}{(1+\alpha_s)}}\right] \geq 1-y.$$

Since $z_1 > \bar{z}_1$ is necessary for a merger to take place, it is sufficient to show

$$\left[\frac{2(1-y)^{\frac{(1+\alpha_s)}{2}}}{(1-\alpha_s)}\right]^{\frac{2\alpha_s}{\alpha_s(1+\alpha_s)}}\left(\frac{1-\alpha_s}{1+\alpha_s}\right)\left[\left(\frac{1-\alpha_s}{2}\right)^{\left(\frac{1-\alpha_s}{1+\alpha_s}\right)} - \left(\frac{1-\alpha_s}{2}\right)^{\frac{2}{(1+\alpha_s)}}\right] \geq 1-y.$$
$$\Leftrightarrow \left[(1-\alpha_s)^{\left(\frac{1-\alpha_s}{1+\alpha_s}\right)+1-\frac{2}{(1+\alpha_s)}}\right]\left[2^{\frac{2}{(1+\alpha_s)}-\left(\frac{1-\alpha_s}{1+\alpha_s}\right)}\right] -$$
$$\left[(1-\alpha_s)^{\frac{2}{(1+\alpha_s)}+1-\frac{2}{(1+\alpha_s)}}\right]\left[2^{\frac{2}{(1+\alpha_s)}-\frac{2}{(1+\alpha_s)}}\right] \geq 1+\alpha_s \Leftrightarrow 1+\alpha_s \geq 1+\alpha_s,$$

which finalizes the proof.■

Proof of Proposition 13. The merger gains associated with each merger, relative to the stand-alone case, are

$$\begin{aligned} AMG|_{1\&2} &= z_1^{\alpha_s}\left[(2-y)^{\frac{(1-\alpha_s)}{2}} - (1-y)^{\frac{(1-\alpha_s)}{2}}\right] - z_2^{\alpha_s} \\ AMG|_{2\&3} &= z_3^{\alpha_s}\left[2^{(1-\alpha_s)} - 1\right] - z_2^{\alpha_s}. \end{aligned}$$

To obtain the conditions such that the spin-off is economically efficient, conditional on zero merger (transaction costs), the following needs to hold:

$$AMG|_{2\&3} > AMG|_{1\&2}\,,$$

which is given by expression (1.68). To finalize the proof (last expression in the proposition) note that

$$\begin{aligned} \frac{\partial}{\partial y}\left[(2-y)^{\frac{(1-\alpha_s)}{2}}-(1-y)^{\frac{(1-\alpha_s)}{2}}\right] > 0 \Leftrightarrow \\ (2-y)^{\frac{1-\alpha_s-2}{2}} < (1-y)^{\frac{1-\alpha_s-2}{2}} \Leftrightarrow 1<2. \end{aligned}$$

■

1.A.2 Numerical methods

The MATLAB code is available at `http://fernando.r.anjos.googlepages.com`. The algorithm I use to find the equilibrium links between BUs, conditional on firms' boundaries, makes use of the analytical expressions for first and second derivatives. These are derived in the following expressions. If i and j belong to the same firm I solve the optimization problem using the links L_{ij} and L_{ji} directly as controls. The first- and second-order derivatives for L_{ij} are, respectively

$$\frac{\partial\left(Q_i+Q_j\right)}{\partial L_{ij}} = \frac{\partial Q_i}{\partial L_{ij}}+\frac{\partial Q_j}{\partial L_{ij}} \tag{1.A.60}$$

$$\begin{aligned} \frac{\partial^2\left(Q_i+Q_j\right)}{\partial L_{ij}^2} = & \sum_{m=1}^{M} K_{mj}\left[\frac{\alpha_{mi}}{\hat{K}_{mi}}\left(\frac{\partial Q_i}{\partial L_{ij}}-\frac{Q_i}{\hat{K}_{mi}}K_{mj}\right)-\right. \\ & \left.\frac{\alpha_{mj}}{\hat{K}_{mj}}\left(\frac{\partial Q_j}{\partial L_{ij}}+\frac{Q_j}{\hat{K}_{mj}}K_{mj}\right)\right], \end{aligned} \tag{1.A.61}$$

where

$$\frac{\partial Q_i}{\partial L_{ij}} = \sum_{m=1}^{M} K_{mj}\left(\frac{\alpha_{mi}}{\hat{K}_{mi}}Q_i\right) \tag{1.A.62}$$

$$\frac{\partial Q_j}{\partial L_{ij}} = -\sum_{m=1}^{M} K_{mj}\left(\frac{\alpha_{mj}}{\hat{K}_{mj}}Q_j\right). \tag{1.A.63}$$

If i and j do not belong the same firm, then I use the derivatives of Q_i with respect to a_{ij}. Below I restate the first derivative and show the expression for the second derivative:

$$\frac{\partial Q_i}{\partial a_{ij}} = \sum_{m=1}^{M} \frac{\alpha_{mi}}{\hat{K}_{mi}} Q_i \left[K_{mj} \beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} - K_{mi}(1-\beta) a_{ji}^{\beta} a_{ij}^{-\beta} \right] \tag{1.A.64}$$

$$\begin{aligned} \frac{\partial^2 Q_i}{\partial a_{ij}^2} &= \left(\frac{\partial Q_i}{\partial a_{ij}} \right)^2 \frac{1}{Q_i} + Q_i \sum_{m=1}^{M} \alpha_{mi} \left\{ \frac{1}{\hat{K}_{mi}} \left[\beta(\beta-1) a_{ij}^{(\beta-2)} a_{ji}^{(1-\beta)} K_{mj} - \right. \right. \\ & \left. (1-\beta)(-\beta) a_{ij}^{-(1+\beta)} a_{ji}^{\beta} K_{mi} \right] - \left\{ \frac{1}{\hat{K}_{mi}} \left[\beta a_{ij}^{(\beta-1)} a_{ji}^{(1-\beta)} K_{mj} - \right. \right. \\ & \left. \left. \left. (1-\beta) a_{ij}^{-\beta} a_{ji}^{\beta} K_{mi} \right] \right\}^2 \right\} \end{aligned} \tag{1.A.65}$$

The algorithm evaluates each control at a time (L_{ij} or a_{ij}, whichever the case) and I generally determine the next step by making a quadratic approximation. In particular, consider the quadratic equation

$$y = ax^2 + bx.$$

As long as $a < 0$ such a function has a maximum, given by $x = -b/(2a)$, which will be considered as the next step. I set a and b such that the first- and second-derivatives of the parabola and those of the objective function are matched, given the current value of the control (a_{ij} or L_{ij}) and the objective function (Q_i or $(Q_i + Q_j)$). In some regions the second derivatives of the objective function with respect to the control will not be negative (but, concavity always holds around the solution for the first-order conditions); in this case I use the direction given by the first derivative. The management of step size was optimized heuristically. Finally, the minimum accepted level of a_{ij} is progressively lowered, so that equilibria that are not perfect are excluded. For details the reader is referred to the code.

Chapter 2

Social Ties and Economic Development

Note: this is joint work with José Anchorena.

2.1 Introduction

In his popular book "Bowling Alone", Robert Putnam claims that *social capital* in the United States has increased up to the 1960s and decreased afterwards. This erosion of social capital notwithstanding, the American economy has kept a steady pace of economic growth in the last 50 years. This apparent ambiguity begs the question: what, if any, is the relation between social capital and economic development?

Traditionally social capital has been understood to be a driver of economic growth. The first step is to define social capital. Putnam (2000) (pg. 19) does so in the following way.

> Whereas physical capital refers to physical objects and human capital refers to properties of individuals, social capital refers to connections among individuals – social networks and the norms of reciprocity and trustworthiness that arise from them.

In economic terms, the above definition of social capital implies that a dense network of *social ties* adds value by lowering costs of coordination. This concept of social capital is also similar to Coleman (1988), who argues that closure (or degree of connectedness) is the key structural property of a social network in increasing efficiency of economic interactions. This does not imply however, from a normative perspective, that social capital should always be maximized. Durlauf and Fafchamps (2005) and Routledge and von Amsberg (2003) have pointed out that social capital is not without costs. On one hand social capital is sometimes associated with unlawful activity; on the other, it may be efficient to incur high coordination costs if it allows a higher productivity. In addition to the ambiguous effect of social capital in economic outcomes, some literature has pointed out that the term itself is ambiguous; see e.g. Solow (2000) or Arrow (2000). In particular, it is not clear whether social capital results from a deliberate process of accumulation by economic agents.

Our objective in this paper is twofold: (i) to document and attempt to explain the relationship between social ties and economic development; and (ii) to construct a framework that sheds light on the concept of social capital.

Why do social ties exist? One motive is certainly the pleasure individuals derive from relationships with other human beings; another relates to the personal advantages of a good social network. A related question is how these social ties are built. Friendships and acquaintances require emotional and temporal commitment, and these resources are scarce. We construct a general equilibrium model where social ties are considered explicitly, both as consumption goods and as building blocks of social capital. By social ties we thus mean a particular class of commodities that agents are able to produce, using as inputs time from each of the individuals in a bilateral relationship. It is important to point out that time is the only primitive resource in the model. This indirectly puts a price on social ties, thus allowing us to analyze the trade-offs between these and standard commodities.

In our general setup there is a global economy comprised of different villages, which in turn are populated by individuals. There is also a variety of standard tradable goods which can be exchanged within villages and between villages. Trade within a village incurs transaction costs, in terms of time, which can be economized with social capital. Social capital is modeled in reduced-form as an average of bilateral social ties. Trade between villages incurs in both transaction and transport costs, but agents are not allowed to build ties across villages. This is similar to the unfriendly-trade case in Routledge and von Amsberg (2003). Transport costs are taken exogenously. Beyond making decisions on social ties and consumption of standard commodities, individuals also choose how much time to devote to production.

Our model relies on the existence of multiple equilibria to rationalize the heterogeneity of observed outcomes across history and regions. These multiple equilibria are seen as resulting from differences in *cultural beliefs*. This is in the spirit of Krugman (1991), Cole, Mailath, and Postlewaite (1992), and Greif (1994). This last paper frames the issue in the exact terms that are relevant for our paper. The excerpt below illustrates what is meant by cultural beliefs:

> Yet if each player expects others to play a self-enforcing and hence an equilibrium strategy, is there any analytical benefit from distinguishing between strategies and cultural beliefs? Unlike strategies, cultural beliefs are qualities of individuals in the sense that cultural beliefs that were crystallized with respect to a specific game affect decisions in historically subsequent strategic situations. Past cultural beliefs provide focal points and coordinate expectations, thereby influencing equilibrium selection and society's enforcement institutions.

In our model, the belief that one should have a certain *number* of social ties is self-fulfilling. This in turn affects economic decisions. We refer to this as the degree of *communitarianism*. We will use the term *individualism* for instances where the level of communitarianism is low. Empirically, we find a positive association between a proxy for the degree of communitarianism and income per capita, in a cross section of countries. This is depicted in the left panel of Figure 2.1. The same figure – in the right panel – shows that the relationship between the average *strength* of social ties and income per capita also displays a systematic pattern, in this case a negative association.

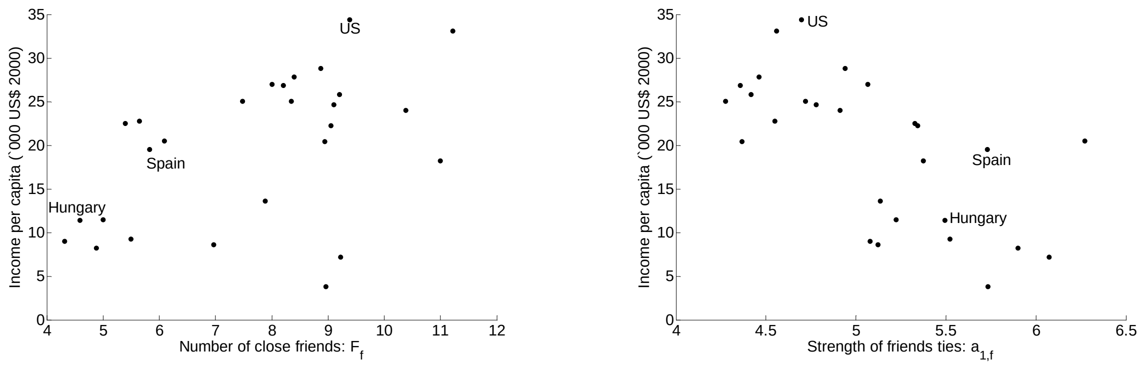


Figure 2.1: Left panel: plots the number of close friends vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is 0.49, and is statistically significant at the 1% level. Right panel: plots the average intensity of close friends' ties vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is −0.65, and it is statistically significant at the 0.1% level.

Our model presents a rationalization for the depicted relationships between social ties (number and strength) and income per capita. In particular, in the calibrated version of the model heterogeneity in number of ties can account for a significant share of the variability in current income per capita; around 65% measuring social ties as the number of close friends. In section 2.A.2 in the Appendix we conduct a more detailed empirical analysis of social ties – including individual and regional data –, and we find further evidence of a robust association between measures of economic development and characteristics of social ties.

Theoretically our framework helps understand the relationship between social ties, social capital, and other economic outcomes. The main theoretical findings are the following: (i) a preference for social ties may significantly mitigate an otherwise large underprovision of social capital; (ii) when social ties are a consumption good, it is theoretically possible to observe different qualitative associations between social ties and economic development, simply due to standard income and substitution effects – this implies that it may be naive to attribute a technological role to proxies of social capital such as social ties, without controlling for consumption motives; (iii) social capital is in some instances an important source of economic efficiency, very much complementary to labor and/or human capital; (iv) only in some societies – the more individualistic ones – is social capital the product of a deliberate process of accumulation; in others it is mostly an important positive externality from the consumption of social ties; and (v) we obtain the counter-intuitive effect that in highly communitarian societies, an increase in productivity may hinder welfare, via amplification of the level of coordination failure in the building of social relationships.

In the quantitative exercise, our main findings are: (i) heterogeneity in the number of social ties can account for a significant portion of the heterogeneity in income per capita across countries, as well as a large increase of this heterogeneity in the time series; (ii) the model can also account for between 1/5 and 1/2 of the changes in use of time in the United States between 1900 and 2000, and the changes in the share of the transaction services' sector for the Unites States, during the same period; (iii) the model is counter-factual with respect to: the time-series relationship between productivity, social capital, and leisure, in the period 1965-1985 in the United States; the evolution of the share of self-produced goods and external market goods; and in the cross section of countries, the relationship between labor and leisure with income per capita; (iv) according to one measure, our model implies that without social capital countries would be between 1/2 and 3/4 their actual size in terms of income per capita.

The paper is organized as follows. Section 2.2 develops the general theoretical setup and proposes a parametrization. Section 2.2.3 analyzes a simplified version of the static model, which captures the main intuitions behind the theory. Section 2.3 develops and calibrates a dynamic extension of the static setup, delivering the quantitative implications of the model for the cross section of countries, in terms of income per capita. Section 2.4 discusses our results and concludes. The Appendix contains all proofs, details on the construction of data for calibration, and a more detailed analysis of data on social ties.

2.2 Static model

2.2.1 General setup

The global economy is composed of V villages and N agents. Each agent $i \in \{1, ..., N\}$ belongs to a unique village v, denoted by $v(i)$. Any individual $i \in \{1, ..., N\}$ maximizes the following utility function

$$\mathcal{U}_i(c, s) = \{\phi\,[u_1(c_i)]^{\rho_{c,s}} + (1-\phi)\,[u_2(s_i)]^{\rho_{c,s}}\}^{\frac{1}{\rho_{c,s}}}\,, \tag{2.1}$$

subject to time and budget constraints, which will be defined shortly. The first component of utility refers to standard commodities, where c_i stands for the vector of different goods consumed by the agent. The second term in (2.1) is the utility that the agent derives from social ties with other members in her village. Agents cannot build social ties with members of other villages. $\rho_{c,s} \in (-\infty, 1]$ controls for the elasticity of substitution between tie utility and standard-commodities utility, given by the ratio $1/(1-\rho_{c,s})$.

Each agent produces one good only, denoted by $g(i)$. The budget constraint faced by i is given by

$$\sum_{g \in \cup G(v)} x_{ig}\, p_g \leq x^S_{ig(i)}\, p_{g(i)}, \tag{2.2}$$

where x_{ig} stands for the amount of good g demanded by i, above and beyond the amount consumed from own production, which we represent subsequently as $x^o_{ig(i)}$. $G(v)$ represents the set of goods produced in village v. The price of good g is denoted by p_g. Finally, the variable $x^S_{ig(i)}$ stands for the supply of good $g(i)$ by i.

For simplicity we assume that all goods are sold in the villages where they were produced, and buyers from other villages incur the associated iceberg transport cost. $\tau_{v_1 v_2}$ stands for the unitary transport cost incurred by an agent from village v_1 buying goods from an agent in village v_2. Transport costs are homogeneous across goods. Thus, consumption of good g by i, denoted by c_{ig}, is given by

$$c_{ig} = \begin{cases} x_{ig} & g \in G(v(i)) \\ x_{ig}\left(1 - \tau_{v(i)v}\right) & g \notin G(v(i)). \end{cases} \tag{2.3}$$

There are two technologies in the economy, one related to the production of standard commodities, the other related to the production of social ties:

$$y_{c,ig(i)} = f_c(r_{ig(i)}) \tag{2.4}$$

$$y_{s,ij} = f_s(a_{ij}, a_{ji}) \tag{2.5}$$

The variable $r_{ig(i)}$ stands for the time spent by i in the production of good $g(i)$. We refer to r as *transformational effort*, and it is intended to capture a combination of labor and investment in human capital. Physical capital is omitted from the production function. The variable a_{ij} represents the amount of time invested by i in the tie with j and is controlled by i. We refer to a_{ij} as *attention*. Social ties are thus produced using two inputs, namely the amount of time invested by each of the two agents in the tie.

Trading in this economy implies transaction costs (TC), which are fully borne by the buyer; we term this as *transactive effort*. Transaction costs correspond to time spent trading, and depend on the social capital (SC) and the amount of goods acquired:

$$TC_{ig} = f_{tc}(SC_{iv}, x_{ig}) \tag{2.6}$$

The amount of social capital usable by agent i is in turn a function of social ties:

$$SC_{iv} = \begin{cases} f_{sc}(s_i, \{s_j\}_{j \in v(i)}) & v = v(i) \\ 0 & v \neq v(i) \end{cases} \tag{2.7}$$

We let social capital be agent-specific (hence the dependence on s_i), but it also depends on the overall structure of ties, within the agent's village.

Time is the only scarce natural resource in this economy, and the following time constraint must be verified for any agent i:

$$r_{ig(i)} + \sum_{g \in \cup G(v)} TC_{ig} + \sum_{j \in v(i)} a_{ij} \leq 1, \tag{2.8}$$

where the total amount of time was normalized to 1. Time is thus put into three different uses: transformational effort, transactive effort, and producing ties. When using the term *effort* without qualification we mean transformational effort.

An equilibrium is characterized by solving all agents' (constrained) maximization problem, subject to the market-clearing condition

$$\sum_{i \in \{1,...,N\}} x^S_{ig(i)} = \sum_{i \in \{1,...,N\}} x_{ig} \ , \forall g \in \{1, ..., G\}. \tag{2.9}$$

2.2.2 Parametrization

To represent preferences for standard commodities we choose a utility function that is standard in the trade literature (see Dixit and Stiglitz (1977) and Krugman (1980)):

$$u_1(c_i) = \left(\sum_{g=1}^{G} c_{ig}^{\rho_c} \right)^{\frac{1}{\rho_c}} \tag{2.10}$$

This utility function implies a constant elasticity of substitution (CES) between different goods. In particular, the elasticity of substitution between any two goods equals $1/(1-\rho_c)$, with $\rho_c \in (-\infty, 1)$.[1] Having little to guide us in terms of a choice for u_2, we opted for a functional form that parallels u_1:

$$u_2(s_i) = \left(\sum_{j \in v(i)} s_{ij}^{\rho_s} \right)^{\frac{1}{\rho_s}} \quad , \tag{2.11}$$

which means that preferences for social ties also display a constant elasticity of substitution, with $\rho_s \in (-\infty, 1)$. Note that the tie of the individual with herself is part of the utility function too, and we interpret it as *leisure*. An important characteristic of these utility functions is a preference for variety, and this will bear significantly in our results.

The production functions for commodities and ties are defined by

$$f_c(r_{ig(i)}) \quad = \quad B_{v(i)} r_{ig(i)} \tag{2.12}$$

$$f_s(a_{ij}, a_{ji}) \quad = \quad a_{ij}^{\beta} a_{ji}^{1-\beta}. \tag{2.13}$$

Social ties are built using a constant-returns-to-scale technology, and B_v is the total productivity factor (of effort) for standard commodities in village v. The technology for building social ties in equation (2.13) implies that no agent can be forced into a

[1] We are ruling out the trivial case of perfect substitutability, which would reduce to the agent only consuming the good she produces.

relationship, since she can always set $a_{ij} = 0$. Note that this would not be possible if for instance this production function was CES (with an elasticity of substitution different than one). This property seems apt, given that we wish to depict social ties as a product of voluntary individual action. This mechanism of building ties is in the spirit of game-theoretic literature on network formation, e.g., Jackson and Wolinski (1996) and Bala and Goyal (2000). Our model borrows the approach of tie production from the chapter on alliances and merger waves.

The meaning of β in (2.13) relates to the degree of complementarity in the production of relationships. If $\beta < 0.5$, relationships – from the perspective of the consumer i – rely more heavily on attention being given by j, the other side of the tie. If $\beta > 0.5$, relationships depend mainly on the attention that the focal agent gives to the other agent. It is important to note that we are assuming that β is a non-cultural primitive. Although in equilibrium villages will display different degrees of "individualism", this will result from different equilibria being played, and not from different primitive preferences or technology for social ties. One can interpret the level of the inverse of β as the usual level of egotism displayed by an individual in building relationships with others, which we assume is determined biologically. Also note that the individual's tie with herself is given by a_{ii}, independently of β. Thus a_{ii} can be interpreted as leisure associated with spending time alone. This feature is a direct consequence of assuming a constant-returns-to-scale technology in the production of ties.

Transaction costs are borne by i if she acquires the goods in some marketplace and are given by

$$f_{tc}(SC_{iv}, x_{ig}) = \begin{cases} \frac{\alpha_0}{\left(1+\alpha_2 SC_{v(i)}\right)} x_{ig} & g \in G(v(i)) \\ \alpha_0 x_{ig} & g \notin G(v(i)). \end{cases} \tag{2.14}$$

In case the goods come from self production, transaction costs are assumed to be zero. The first branch of the transaction cost function shows that transaction costs within the village can be economized by social capital; the productivity of which is gaged by α_2. The function that maps social ties to social capital is given by the following expression:

$$f_{sc}(s_i, \{s_j\}_{j \in v(i)}) = \left(\sum_{j \in v(i), j \neq i} s_{ij} \right)^{\alpha_1} \left(\frac{\sum_{j \in v(i)} \sum_{k \in v(i), k \neq j} s_{jk}}{N_{v(i)}} \right)^{1-\alpha_1} \tag{2.15}$$

The first component of social capital, weighted by α_1, relates to the ties of individual

i with respect to other individuals in the village. We interpret this as a measure for how much the individual belongs to the community. The second component averages over all ties in the village. This is meant to capture the global effect of closure, in the sense of sociologist James Coleman. Social capital, as measured from the individual perspective of agent i, has thus a private and a public good component. It seems reasonable to assume that these two components are complementary, as in functional form (2.15). Global closure only matters as a way of solving individual moral hazard and informational problems to the extent that the individual is embedded enough in the network of community ties. This argument notwithstanding, social capital can be set to be a strictly public good, by imposing $\alpha_1 = 0$. The upper bound for social capital is 1, independently of the size of the village.

We can now write the Lagrangian associated with the maximization problem of some agent i.

$$
\begin{aligned}
\mathcal{L}_i \;=\; & \left[\phi \left\{ \left(x^o_{ig(i)} + x_{ig(i)}\right)^{\rho_c} + \sum_{\substack{g \neq g(i) \\ g \in G(v(i))}} x_{ig}^{\rho_c} + \sum_{v \neq v(i)} \sum_{g \in G(v)} \left[x_{ig}(1 - \tau_{v(i)v})\right]^{\rho_c} \right\}^{\frac{\rho_{c,s}}{\rho_c}} + \right. \\
& \left. (1-\phi) \left\{ \sum_{j \in v(i)} [f_s(a_{ij}, a_{ji})]^{\rho_s} \right\}^{\frac{\rho_{c,s}}{\rho_s}} \right]^{\frac{1}{\rho_{c,s}}} - \theta_i \left\{ r_{ig(i)} + \sum_{j \in v(i)} a_{ij} - 1 + \right. \\
& \left. + \sum_{g \in \cup G(v)} \alpha_0 \left[1 + \alpha_2 f_{sc} \left(\{f_s(a_{ij}, a_{ji})\}_{\substack{j \in v(i) \\ j \neq i}}, \{f_s(a_{jk}, a_{kj})\}_{j,k \in v(i)} \right) \right]^{-1} x_{ig} \right\} \\
& -\lambda_i \left\{ \sum_{g \in \cup G(v)} x_{ig} p_g - \left(B_{v(i)} r_{ig(i)} - x^o_{ig(i)}\right) p_{g(i)} \right\} + \sum_{j \in v(i)} \eta_{a,ij} a_{ij} + \eta_{r,i} r_{ig(i)} \\
& + \eta_{o,i} x^o_{ig(i)} + \sum_{g \in \cup G(v)} \eta_{x,ig} x_{ig}, \qquad (2.16)
\end{aligned}
$$

where we have made use of the substitutions

$$
\begin{aligned}
x^S_{ig(i)} &= f_c(r_{ig(i)}) - x^o_{ig(i)} = B_{v(i)} r_{ig(i)} - x^o_{ig(i)} \\
s_{ij} &= f_s(a_{ij}, a_{ji}).
\end{aligned}
$$

The Lagrange multiplier θ_i measures the shadow price of time and λ_i measures the shadow price of wealth. All η multipliers refer to non-negativity constraints. We assume that prices and attention from others are taken as given, i.e., we will be investigating Nash equilibria. Denoting by N_v the number of agents in village v, the price-taking behavior is consistent with $N_v/G_v >> 1$, which we assume.

The FOC with respect to self consumption x^o_{ig} yields the following relation:

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}}\,\phi\left\{\left(x^o_{ig(i)}+x_{ig(i)}\right)^{\rho_c}+\sum_{\substack{g\neq g(i)\\ g\in G(v(i))}}x_{ig}^{\rho_c}+\sum_{v\neq v(i)}\sum_{g\in G(v)}\left[x_{ig}(1-\tau_{v(i)v})\right]^{\rho_c}\right\}^{\frac{\rho_{c,s}}{\rho_c}-1}\times$$
$$\left(x^o_{ig(i)}+x_{ig(i)}\right)^{\rho_c-1}=\lambda_i p_{g(i)}+\eta_{o,i},\quad (2.17)$$

where $\mathcal{U}_i^*$ is the utility function evaluated at the optimum, i.e., the value function. The FOC with respect to the additional consumption of good $g(i)$ is

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}}\,\phi\left\{\left(x^o_{ig(i)}+x_{ig(i)}\right)^{\rho_c}+\sum_{\substack{g\neq g(i)\\ g\in G(v(i))}}x_{ig}^{\rho_c}+\sum_{v\neq v(i)}\sum_{g\in G(v)}\left[x_{ig}(1-\tau_{v(i)v})\right]^{\rho_c}\right\}^{\frac{\rho_{c,s}}{\rho_c}-1}\times$$
$$\times\left(x^o_{ig(i)}+x_{ig(i)}\right)^{\rho_c-1}=\theta_i\frac{\alpha_0}{(1+\alpha_2 SC_{v(i)})}+\lambda_i p_{g(i)}+\eta_{x,ig(i)}.\quad (2.18)$$

Suppose that i produces some good $g(i)$ and consumes part of it, such that $\eta_{o,ig(i)}=0$. Combining expressions (2.17) and (2.18), it is trivial to see that i will not acquire good $g(i)$. If this were the case, then $\eta_{x,ig(i)}=0$ and the following would have to be true:

$$\theta_i\frac{\alpha_0}{(1+\alpha_2 SC_{v(i)})}=0$$

This would generally not obtain, except if $\alpha_0=0$. The intuition for this is that as long as transaction costs are positive, then for any given price the agent would always be better off by marginally decreasing the supply of good $g(i)$ and marginally increasing self consumption. The FOC for goods $g\in v(i)$ different than $g(i)$ is

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}}\,\phi\left\{\left(x^o_{ig(i)}+x_{ig(i)}\right)^{\rho_c}+\sum_{\substack{g\neq g(i)\\ g\in G(v(i))}}x_{ig}^{\rho_c}+\sum_{v\neq v(i)}\sum_{g\in G(v)}\left[x_{ig}(1-\tau_{v(i)v})\right]^{\rho_c}\right\}^{\frac{\rho_{c,s}}{\rho_c}-1}\times$$

$$\times (x_{ig})^{\rho_c - 1} = \theta_i \frac{\alpha_0}{(1 + \alpha_2 SC_{v(i)})} + \lambda_i p_g + \eta_{x,ig}. \quad (2.19)$$

The FOC for consumption from other villages is

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}} \phi \left\{ \left(x^o_{ig(i)} + x_{ig(i)}\right)^{\rho_c} + \sum_{\substack{g \neq g(i) \\ g \in G(v(i))}} x_{ig}^{\rho_c} + \sum_{v \neq v(i)} \sum_{g \in G(v)} \left[x_{ig}(1 - \tau_{v(i)v})\right]^{\rho_c} \right\}^{\frac{\rho_{c,s}}{\rho_c} - 1} \times$$

$$\times \left[x_{ig}(1 - \tau_{v(i)v})\right]^{\rho_c - 1} (1 - \tau_{v(i)v}) = \theta_i \alpha_0 + \lambda_i p_g + \eta_{x,ig}. \quad (2.20)$$

Note that θ_i, the shadow price of time, enters equations (2.19) and (2.20). This means that if for some reason the agent is "poor" in terms of time (high θ_i), then *ceteris paribus* she will consume less traded goods, in order to economize in transaction costs.

Next we derive the decisions regarding social ties. Consider first the time spent in non-social leisure a_{ii}; the FOC with respect to this control yields

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}} (1 - \phi) \left\{ \sum_{j \in v(i)} \left(a_{ij}^{\beta} a_{ji}^{1-\beta}\right)^{\rho_s} \right\}^{\frac{\rho_{c,s}}{\rho_s} - 1} a_{ii}^{\rho_s - 1} = \theta_i - \eta_{a,ii}. \quad (2.21)$$

With respect to a_{ij} the FOC is slightly more complex:

$$(\mathcal{U}_i^*)^{1-\rho_{c,s}} (1 - \phi) \left\{ \sum_{j \in v(i)} \left(a_{ij}^{\beta} a_{ji}^{1-\beta}\right)^{\rho_s} \right\}^{\frac{\rho_{c,s}}{\rho_s} - 1} \left(a_{ij}^{\beta} a_{ji}^{1-\beta}\right)^{\rho_s - 1} \beta a_{ij}^{\beta - 1} a_{ji}^{1-\beta} =$$

$$= \theta_i \left\{ 1 - \frac{\left(\sum_{g=1}^{G} x_{iv(i)g}\right) \alpha_0 \alpha_1 \alpha_2 \beta a_{ij}^{\beta-1} a_{ji}^{1-\beta}}{\left[1 + \alpha_2 \left(\sum_{j \in v(i), j \neq i} a_{ij}^{\beta} a_{ji}^{1-\beta}\right)^{\alpha_1} \left(\frac{\sum_{j \in v(i)} \sum_{k \in v(i), k \neq j} a_{jk}^{\beta} a_{kj}^{1-\beta}}{N_{v(i)}}\right)^{1-\alpha_1}\right]^2} \times \right.$$

$$\left. \times \left[\left(\sum_{j \in v(i), j \neq i} a_{ij}^{\beta} a_{ji}^{1-\beta} \right)^{\alpha_1 - 1} \left(\frac{\sum_{j \in v(i)} \sum_{k \in v(i), k \neq j} a_{jk}^{\beta} a_{kj}^{1-\beta}}{N_{v(i)}} \right)^{1-\alpha_1} \right] \right\} - \eta_{a,ij}, \quad (2.22)$$

where we have made use of the assumption that $N_{v(i)}$ is large enough that agent i disregards the impact of a_{ij} in terms of the public good component of social capital. There is an intuitive relation between a_{ij} and a_{ii}, but given the complexity of the expression above we postpone this discussion until the next section. For the moment note that if $a_{ji} = 0$, then the LHS of equation (2.22) is also zero. Since the term pre-multiplied by θ_i becomes 1, an interior solution for a_{ij} does not exist. This particular feature of the model allows for multiple equilibria in terms of social ties, and it is the result of assuming the Cobb-Douglas functional form for the tie production function.[2]

Finally we derive the expression that gives the optimal effort for production of good g, namely r_{ig}.

$$\theta_i = \lambda_i B_{v(i)} p_{g(i)} + \eta_{r,i}, \tag{2.23}$$

where again we have made use of the assumption of large $N_{v(i)}$.

An interior solution with an in-village symmetric equilibrium is further characterized by the following conditions, for all $i \in v$:

$$x^o_{ig(i)} = c_{0v} \tag{2.24}$$

$$x_{ig}|_{g \in G(v), g \neq g(i)} = c_{1v} \tag{2.25}$$

$$x_{ig}|_{g \in G(v^*), v^* \neq v} = c_{2vv^*} \tag{2.26}$$

$$a_{ii} = a_{0v} \tag{2.27}$$

$$a_{ij}|_{j \in v, j \neq i} = a_{1v} \tag{2.28}$$

$$r_{ig(i)} = r_v \tag{2.29}$$

$$\#\{j | a_{ij} > 0, j \in v, j \neq i\} = F_v \tag{2.30}$$

$$p_g|_{\exists j \in v^* : g(j) = g} = p_{v^*} \tag{2.31}$$

$$\theta_i = \theta_v \tag{2.32}$$

$$\lambda_i = \lambda_v \tag{2.33}$$

Equation (2.30) refers to the *number of ties* of each agent. Any F_v is sustainable in equilibrium. The intuition behind this is that i will not pay attention to someone from whom she does not receive attention, and this is an equilibrium. It is important to emphasize that if agent j sets $a_{ji} = 0$, a_{ij} is *strictly* a better response than $a_{ij} = 0$ for agent i. This is so because time is valuable and $a_{ij} > 0$ *per se* translates into a zero increase to the utility of agent i. On the other hand, the marginal utility of a new tie is infinity at zero, so F_v does not have an upper bound. Also, note that the equality in equilibrium prices, stated in (2.31), is implied by the remaining conditions. The same

[2] Some equilibria with $(a_{ij}, a_{ji}) = (0, 0)$ are however not *pairwise stable*, an equilibrium definition that is important in the economic networks literature; see for example Jackson and Wolinski (1996). See proposition 15.

applies to the Lagrange multipliers.

Manipulating the system of first-order equations, constraints, and equilibrium conditions, an interior solution is given by the following system of equations:

$$\frac{c_{0v}}{c_{1v}} = \left[1+\frac{\alpha_0}{(1+\alpha_2 F_v a_{1v})}B_v\right]^{\frac{1}{1-\rho_c}} \tag{2.34}$$

$$\frac{c_{2vv^*}}{c_{1v}} = \left\{\frac{(1-\tau_{vv^*})^{\rho_c}\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F_v a_{1v})}\right]}{\alpha_0 B_v + p_{v^*}/p_v}\right\}^{\frac{1}{1-\rho_c}} \tag{2.35}$$

$$\frac{c_{0v}^{1-\rho_c}}{a_{0v}^{1-\rho_s}} = B_v \times \frac{\phi\left\{c_{0v}^{\rho_c}+(G_v-1)c_{1v}^{\rho_c}+\sum_{v^*\neq v}G_{v^*}\left[c_{2vv^*}(1-\tau_{vv^*})\right]^{\rho_c}\right\}^{\frac{\rho_{c,s}}{\rho_c}-1}}{(1-\phi)\left[a_{0v}^{\rho_s}+F_v a_{1v}^{\rho_s}\right]^{\frac{\rho_{c,s}}{\rho_s}-1}} \tag{2.36}$$

$$\frac{a_{0v}}{a_{1v}} = \left[\frac{1}{\beta}-(G_v-1)c_{1v}\frac{\alpha_0\alpha_1\alpha_2}{(1+\alpha_2 F a_1)^2}\right]^{\frac{1}{1-\rho_s}} \tag{2.37}$$

$$(BC) \qquad (G_v-1)c_{1v}+\sum_{v^*\neq v}G_{v^*}c_{2vv^*}\frac{p_{v^*}}{p_v} = B_v r_v - c_{0v} \tag{2.38}$$

$$(TC) \qquad r_v+(G_v-1)\frac{\alpha_0}{(1+\alpha_2 F_v a_{1v})}c_{1v}+\sum_{v^*\neq v}G_{v^*}\alpha_0 c_{2vv^*}+ +a_{0v}+F_v a_{1v} = 1 \tag{2.39}$$

$$(MC) \qquad \frac{N_v}{G_v}\left[B_v r_v - c_{0v}\right] = \left(N_v-\frac{N_v}{G_v}\right)c_{1v}+\sum_{v^*\neq v}N_{v^*}c_{2v^*v}, \tag{2.40}$$

where BC is the budget constraint, TC the time constraint, and MC the market-clearing condition. The latter is only necessary for relations between villages, since the in-village symmetry condition combined with the budget constraints imply in-village market clearing automatically.

Equation (2.34) shows that c_1 is (weakly) smaller than c_0. The agent adjusts her consumption bundle towards the good without transaction costs, i.e. self consumption. This bias is mitigated by social capital $F_v a_{1v}$; in societies with higher social capital, *ceteris paribus* one should observe a more balanced relation between self consumption and consumption of domestic goods. It is interesting to note that productivity B_v also affects the ratio c_0/c_1. The reason is that the agent can compensate for a higher marginal utility of c_1 for a greater quantity of c_0, if it is not costly to do so (which is the case with high B_v). Naturally the elasticity of substitution $1/(1-\rho_c)$ plays an important role in the relation between c_0 and c_1. In the extreme case of Leontieff preferences ($\rho_c = -\infty$), the ratio is always one. In the extreme case of perfect substitutability, $c_{1v} = 0$.

The relation between c_1 and c_2, stated in equation (2.35), is mediated by two channels: (i) the natural advantage of domestic consumption, via lower transaction costs (if social capital is positive) and zero transport costs; and (ii) the terms of trade, measured by the real exchange rate p_{v^*}/p_v.

The consumption of c_0 in relation to non-social leisure a_0, given in equation (2.36), is impacted mainly by three factors: (i) the elasticity of substitution between standard commodities and ties; (ii) the productivity of effort; and (iii) the relative preference for standard commodities, measured by ϕ.

Equation (2.37) shows that the agent will have higher non-social leisure a_0 relative to a_1 when β is small. This does not follow though from a lower primitive preference for social ties; it is just the case that with low β (high level of egotism), agents in any bilateral relation experience a coordination failure – for a detailed discussion of this mechanism see chapter 1. On the other hand, if transaction costs α_0 are high and the agent controls part of her social capital ($\alpha_1 > 0$), this implies *ceteris paribus* a higher a_1. This effect is reinforced if the agent is trading a lot of goods inside the village (high G_v) and/or consumes large quantities of these goods (high c_1).

2.2.3 Benchmark model

In this section we solve a benchmark case, where we simplify the model along some dimensions, but keep the most important features. The objective of this exercise is to understand the mechanics implied by our theoretical approach, especially with respect to the interaction of social ties (and capital) with other economic variables. The benchmark model is characterized by: (i) $\rho_{c,s} = \rho_c = \rho_s \equiv \rho$;[3] (ii) partial equilibrium with respect to the exterior and only one trade partner – terms of trade denoted by p^*, number of imported goods by G^*, and transport costs by τ. The benchmark case

[3]This assumption is relaxed in the calibration; see section 2.3.1.

can be reduced to the following system of equations (where a_1 is to be found):

$$a_1^*(a_1) = c_1^*(a_1)\left\{\frac{(1-\phi)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]}{\phi B\left[1/\beta-(G-1)\frac{\alpha_0\alpha_1\alpha_2}{(1+\alpha_2 F a_1)^2}c_1^*(a_1)\right]}\right\}^{\frac{1}{1-\rho}} \tag{2.41}$$

$$\begin{aligned} c_1^*(a_1) = {} & B(1-Fa_1)\left\{(G-1)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]+\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]^{\frac{1}{1-\rho}}\times\right. \\ & \left.\times\left[1+\left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}}+G^*\left(\frac{1-\tau}{p^*+\alpha_0 B}\right)^{\frac{\rho}{1-\rho}}\right]\right\}^{-1} \end{aligned} \tag{2.42}$$

$$c_0^*(a_1) = c_1^*(a_1)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]^{\frac{1}{1-\rho}} \tag{2.43}$$

$$c_2^*(a_1) = c_1^*(a_1)\left\{\frac{(1-\tau)^\rho\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]}{p^*+\alpha_0 B}\right\}^{\frac{1}{1-\rho}} \tag{2.44}$$

$$a_0^*(a_1) = c_1^*(a_1)\left\{\frac{(1-\phi)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]}{\phi B}\right\}^{\frac{1}{1-\rho}} \tag{2.45}$$

$$r^*(a_1) = 1-\left[(G-1)\frac{\alpha_0}{(1+\alpha_2 F a_1)}c_1^*(a_1)+G^*\alpha_0 c_2^*(a_1)+a_0^*(a_1)+Fa_1\right] \tag{2.46}$$

The following proposition establishes existence and uniqueness.

Proposition 14 *In the benchmark case with $\phi < 1$, for a given F there exists a unique interior symmetric equilibrium.*

Next we show that $F = N$ (maximal "communitarianism") must hold in any (symmetric) pairwise stable equilibrium, an important solution concept in the economic networks literature (see e.g. Jackson and Wolinski (1996)). Pairwise stability requires that no pair of connected agents prefers to sever their tie, and no pair of disconnected agents prefers to build a tie.

Proposition 15 *In the benchmark case with $\phi < 1$, any symmetric pairwise stable equilibrium requires $F = N$.*

The fact that a pairwise stable equilibrium requires $F = N$ follows from two key assumptions: (i) the preference for variety in social ties (recall $\rho < 1$); (ii) the absence of fixed costs in building or severing ties. With respect to the first assumption we are not sure if this behavioral representation is unreasonable. With respect to the second we believe it is somewhat unrealistic. If there were fixed costs of building and severing ties, there would probably exist pairwise stable equilibria for different F. The tractability of our framework would however be hampered if we introduced that sort of discontinuity. In this sense, we interpret our simple multiple Nash equilibria (for each F) as robust in light of these omitted fixed costs. Nonetheless, the model naturally favors "communitarianism" (high F), in the sense that it is more stable than individualism.

No transaction costs ($\alpha_0 = 0$)

Without transaction costs we are able to find a closed-form solution:

$$c_0 = c_1 = \frac{B}{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} \left(1 + F\beta^{\frac{1}{1-\rho}}\right)} \tag{2.47}$$

$$c_2 = \frac{B\left[\frac{(1-\tau)^\rho}{p^*}\right]^{\frac{1}{1-\rho}}}{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} \left(1 + F\beta^{\frac{1}{1-\rho}}\right)} \tag{2.48}$$

$$a_0 = \frac{\left[\frac{(1-\phi)}{\phi B^\rho}\right]^{\frac{1}{1-\rho}}}{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} \left(1 + F\beta^{\frac{1}{1-\rho}}\right)} \tag{2.49}$$

$$a_1 = \frac{\left[\frac{\beta(1-\phi)}{\phi B^\rho}\right]^{\frac{1}{1-\rho}}}{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} \left(1 + F\beta^{\frac{1}{1-\rho}}\right)} \tag{2.50}$$

$$r = \frac{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}}}{G + G^* \left(\frac{1-\tau}{p^*}\right)^{\frac{\rho}{1-\rho}} + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} \left(1 + F\beta^{\frac{1}{1-\rho}}\right)} \tag{2.51}$$

From equation (2.51) it is straightforward to see that a higher F implies a lower effort r. Also, an increase in F implies always a higher social capital, which is shown in the next proposition.

Proposition 16 *In the benchmark case with $\alpha_0 = 0$, a higher degree of communitarianism F implies higher social capital Fa_1.*

Thus, for a cross section of economies with different degrees of communitarianism, i.e. different F, in the case without transaction costs the model endogenously generates a negative relationship between social capital and effort. Note however that we are holding everything else constant. Also, as shown towards the end of this section, this negative relationship does not always obtain when the assumption of equal elasticities of substitutions (equal ρs) is relaxed. We will next discuss how variation in productivity, number of goods, and transport costs affects magnitudes of interest, since heterogeneity in these three dimensions is, we believe, typically associated with heterogeneity in economic development.

If $\rho > 0$, i.e. social ties and standard commodities are substitutes, increasing the productivity parameter B leads *ceteris paribus* to an increase in production effort and a decrease in social capital, which is trivially seen by looking at equations (2.51) and (2.50). The decrease in social capital is necessary to accommodate the increase in effort, which is optimal given the higher productivity. But this only takes place because standard commodities and ties are substitutes. In fact, if $\rho \leq 0$ an increase in B will generate a decrease in effort. The complementarity between the consumption of social ties and standard commodities implies that a higher level of the latter (from a higher B) is optimally accompanied by a higher level of the former. For this to take place the agent needs to reduce time spent in the production of the standard goods.

When $\rho > 0$ consumption increases with B – see equations (2.47) and (2.48) – and leisure a_0 decreases, since it is an activity that also requires time, now necessary for production. Our interpretation of this comparative statics exercise is that more developed economies, which display higher productivity, will naturally have both lower social capital and higher production effort, as long as transaction costs are low enough and the elasticities of substitution are relatively similar and high enough. Whether these qualifications are appropriate is ultimately an empirical question, which we discuss further in section 2.3.1.

Another dimension that one would naturally associate with different stages of development is the number of traded goods. In the case without transaction costs, an increase in G or in G^* has the same effect in effort and social capital as an increase in productivity when $\rho > 0$. A higher variety of goods coupled with a preference for variety makes the agents substitute away from leisure and social ties into standard commodities; this requires more production, so again time becomes more expensive. The effect in per-type-of-good consumption c_0, c_1, and c_2 is naturally a decrease, since the agent is trading off quantity of consumption for variety of consumption.

Next we consider how variation in transport costs, also in line with a notion of economic development, impacts both social capital and effort. Inspecting equations (2.51) and (2.50) again, we find that for $\rho > 0$ lower transport costs are associated with higher effort and lower social capital, so in essence the effect is the same as an increase in productivity.

Finally we wish to address the following question: is the negative relation between communitarianism F and effort r a more general feature of the model? The next proposition shows that this is *not* the case, and again that the elasticities of substitution play a determinant role.

Proposition 17 *Relaxing the assumption of a single ρ in the benchmark case without transaction costs, an increase in F leads to a decrease in r if and only if the following is true:*

$$\rho_s - \rho_{c,s} < \rho_s \left(1 - \rho_{c,s}\right) \left[\frac{F + \left(\frac{1}{\beta}\right)^{\frac{\rho_s}{1-\rho_s}}}{F + \left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho_s}}} \right] \tag{2.52}$$

The RHS of equation (2.52) is always positive as long as $\rho_s > 0$, so the assumption of equal ρs, which makes the LHS of said expression 0, implies that communitarianism F and effort r are negatively related. However, it is easy to see that, for instance if $\rho_{c,s} = 0$ and $\rho_s > 0$, expression (2.52) is false. In this case, as communitarianism increases agents dispense more effort in production. This is the result from social ties and standard commodities being (more) complementary.

So far the key message of this section is that even with the simplifying assumption that ties result primordially from preferences, as suggested by Arrow (2000), and do not play a technological role, whether or not one observes an increase in transformational effort with an increase in the degree of communitarianism is a function of whether standard goods and social ties are substitutes or complements.

Positive transaction costs ($\alpha_0 > 0$)

When positive transaction costs are considered, social ties are no longer a simple consumption good. Social ties are the building blocks of social capital, which economizes in-village transaction costs. To understand what the model implies when social ties have a technological role, we start by analyzing what happens when there are no preferences for social ties ($\phi = 1$). The results are presented in proposition 18 and corollary 2.

Proposition 18 *If In the benchmark case with $\phi = 1$, the following is true:*

1. *Social capital has the following upper bound $\overline{SC}$:*

$$\overline{SC} \equiv \frac{1}{\alpha_2} \left\{ \frac{\alpha_0 B(\alpha_1 \beta \alpha_2 - 1) - 1 - \frac{(1+\alpha_0 B)^{\frac{1}{1-\rho}}}{(G-1)} \left[1 + G^* \left(\frac{1-\tau}{p^* + \alpha_0 B} \right)^{\frac{\rho}{1-\rho}} \right]}{(1 + \alpha_0 B \alpha_1 \beta) + \frac{(1+\alpha_0 B)^{\frac{1}{1-\rho}}}{(G-1)} \left[1 + G^* \left(\frac{1-\tau}{p^* + \alpha_0 B} \right)^{\frac{\rho}{1-\rho}} \right]} \right\} \tag{2.53}$$

2. *For log utility ($\rho = 0$), the following condition is necessary and sufficient for the existence of an equilibrium with $a_1 > 0$:*

$$\alpha_2 \geq \frac{(1 + \alpha_0 B)(G + G^*)}{\alpha_1 \beta \alpha_0 B(G - 1)} \tag{2.54}$$

Corollary 2 *The results of proposition 18 imply that in the benchmark case with $\phi = 1$, the following statements are true:*

1. *If $\alpha_2 \leq \frac{1}{\alpha_1 \beta} \left(1 + \frac{1}{\alpha_0 B} \right)$, there is no interior symmetric equilibrium, i.e. $a_1 = 0$.*
2. *If $\tau < 1$ and $\rho > -\infty$, there always exists a* high enough *G^* such that there exists no symmetric equilibrium with $a_1 > 0$.*
3. *For log utility ($\rho = 0$), an increase in G^* implies a monotonic increase of the minimum value of α_2 that sustains an equilibrium with positive a_1.*
4. *A decrease in transport costs τ leads to a decrease in the upper bound for social capital if and only if $\rho > 0$.*

Point 1 in corollary 2 states that social capital is only observed in equilibrium if the productivity of social capital is high enough. In particular note that if either social capital is a pure public good ($\alpha_1 = 0$) or agents are completely egotistic ($\beta = 0$), social capital is zero even if its productivity (α_2) is arbitrarily high (but finite). The (symmetric) socially optimal amount of social capital does not depend on β nor α_1, given the way ties and social capital were constructed. The degree to which the observed social capital departs from this optimum is a function of the product $\alpha_1 \beta$. Interestingly this means that the two sources of coordination failure – free-riding in relationship building and in social capital – compound in this model. This implies that even if β or α_1 are not excessively low, social capital may be severely underprovided because the product $\beta\alpha_1$ is small. Other determinants of non existence of social capital are the number of traded goods with the exterior, transport costs, transaction costs, and productivity; as shown in points 1-4 in corollary 3. However, if these factors *per se* determine that social capital is not observed, this is not the result of a coordination failure. Rather, social capital may extinguish its usefulness, for instance if agents have to spend a significant amount of time trading with the exterior (high G^*). It is also interesting to note, both in point 1 in the corollary and equation 2.54 in proposition 18, that the effect of transaction costs α_0 always appears multiplied by productivity B: social capital only exists if the ratio $\alpha_0 B$ is high enough. The reason for this complementarity is that the level of transaction costs only matters to the extent that society is productive enough to generate a significant amount of trade. Otherwise self consumption is the main source of utility for standard commodities and there is little use for social capital.

The fact that agents may indeed derive consumption utility from social ties may help solve the problem of underprovision of social capital. The next proposition shows how a preference for social ties delivers a lower bound for social capital $\underline{SC}$, even when it is a pure public good. This implies that if α_2 is high, there is an important positive externality to the transaction technology that is associated with preferences.

Proposition 19 *In the benchmark case with $\phi < 1$, there is a lower bound for social capital, given by:*

$$\underline{SC} \equiv \frac{\beta F}{\left[G + G^* \left(\frac{1-\tau}{p^*+\alpha_0 B}\right)^{\frac{1}{1-\rho}}\right] \left(\frac{\phi B^\rho}{1-\phi}\right)^{\frac{1}{1-\rho}} + 1 + \beta F} \tag{2.55}$$

Also, for the log-utility case and $\alpha_1 = 0$ (social capital is a pure public good), the lower bound given by (2.55) coincides with the actual social capital.

Corollary 3 *The result of proposition 19 implies that the following statements are true:*

1. *The lower bound for social capital given by equation (2.55) is* increasing *in F, β, τ, and α_0.*

2. *The lower bound for social capital given by equation (2.55) is* decreasing *in G and G^*.*

3. *Both ρ and B have an ambiguous effect in the lower bound for social capital given by equation (2.55).*

As in the case with $\phi = 1$, the lower bound for social capital may become relatively low if the economic environment changes. This would for instance happen with an increase in the number of imported goods G^*. The level of coordination failure in building relationships, gaged by β, may also drive the lower bound for social capital to an arbitrarily low level. Naturally, in this model it is impossible to create social capital in a society of fully egotistical individuals ($\beta = 0$), even when these individuals enjoy social ties highly (low ϕ) and social capital is very productive (high α_2).

A natural question to ask about this model is whether the fact that social capital plays a relevant technological role helps reduce coordination problems with respect to the consumption of social ties. The answer is "not significantly", and the explanation ensues. The level of coordination failure in relationship building is a function of how low β is. But if β is low, then $\beta\alpha_1$ is low too, and as argued above this is what underlies a technological underprovision of social capital. On the other hand, even if $\alpha_1 = 0$, if agents have a strong preference for social ties and β is relatively high, they have a private incentive to build ties. This then translates into a positive externality, namely the reduction in transaction costs.

The remaining of our exploration of the static model with positive transaction costs is numerical, given that expressions either do not exist in closed-form or are too cumbersome. We followed a strategy of setting the parameters at values guided by the calibration exercise of section 2.3.1, and then doing selective comparative statics. Our main interest is in the equilibrium outcomes for social capital and transformational effort. Figure 2.2 depicts how changing unitary transaction costs affects both these variables.

The right panel in figure 2.2 shows how lower transaction costs α_0 are associated with lower social capital, for any F. This happens for two reasons. The first is that when α_0 is very high there is almost no trading, i.e. c_1 and c_2 are small. Since the agent does not spend a significant amount of time trading, she can produce and consume social ties, and consume leisure. This last effect is shown in the right panel of figure 2.3. Obviously the agent could spend this extra time in transformational effort, but with a decreasing marginal utility for c_0 this does not happen. The second reason why social capital increases with α_0 is that with higher transaction costs the technological importance of social capital increases (this is true only because $\alpha_1 > 0$).

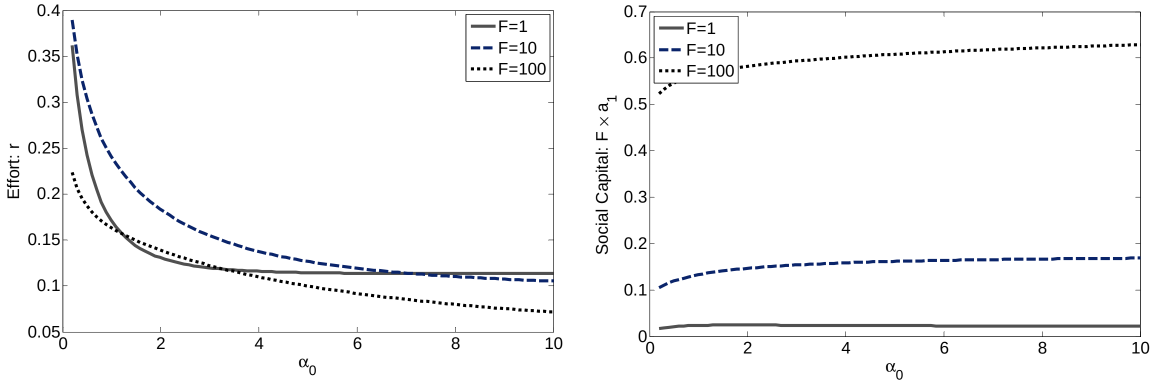


Figure 2.2: Left panel: shows how effort varies with transaction costs α_0. Right panel: shows how social capital varies with transaction costs α_0. In both panels the remaining parameters are set at: $G = G^* = 6$, $B = 3$, $\phi = 0.18$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

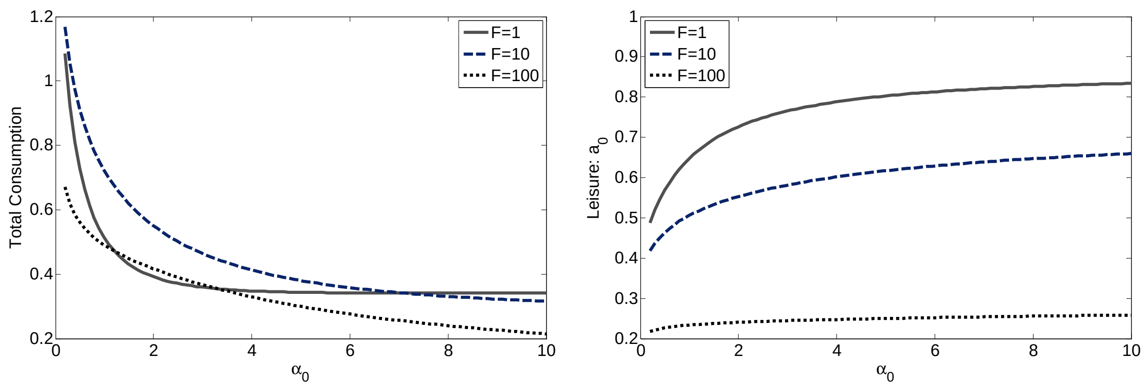


Figure 2.3: Left panel: shows how total consumption $[c_0 + (G-1)c_1 + G^* c_2 p^*]$ varies with transaction costs α_0. Right panel: shows how leisure varies with transaction costs α_0. In both panels the remaining parameters are set at: $G = G^* = 6$, $B = 3$, $\phi = 0.18$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

It is interesting to note, in the left panel of figure 2.2, that the relationship between communitarianism and effort is not monotonic, fixing the level of transaction costs α_0. Let us compare the extreme cases of $F = 1$ and $F = 100$. For high enough transaction costs, both societies simply do not consume c_1 or c_2 (not shown). The effort of low-F societies is higher just because when there are few social ties, this leaves more time for transformational activities. As transaction costs initially decrease, the effort of the high-F economy is higher (relative to the low-F case), for an intermediate region of α_0 (e.g. $\alpha_0 = 2$). This obtains because trading is still so costly for the low-F society that the incentive to produce for obtaining goods beyond self production is relatively low. If we were to model a reduced-form production function for this economy, then for intermediate transaction costs social capital and effort are complementary production factors. As α_0 further decreases, the technological role of social ties becomes less important; in fact it disappears when $\alpha_0 = 0$. We are now back in the case where the effort of low-F societies is higher simply because agents do not spend time in ties.

With positive transaction costs there may now be a technological motive to build ties, namely saving transaction costs. However, from an individual agent's perspective, this is only a valid motive as long as social capital is not a pure public good. It turns out that an increase in α_1 translates into an increase in a_1, which means that part of the observed social ties in equilibrium result from a *deliberate* intent of reducing transaction costs. This is shown in the left panel of figure 2.4.

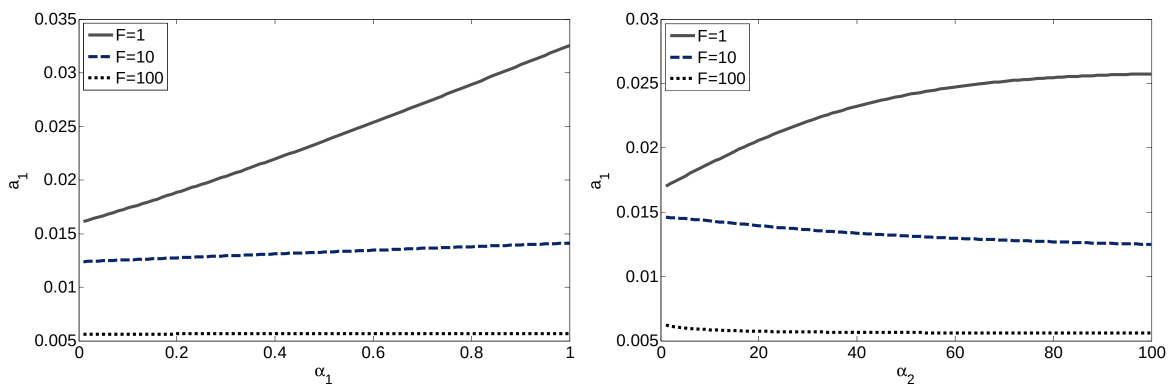


Figure 2.4: Left panel: shows how attention a_1 varies with control over agent-specific social capital α_1; remaining parameters set at: $G = G^* = 6$, $B = 3$, $\phi = 0.18$, $\alpha_0 = 1$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$. Right panel: shows how attention a_1 varies with social capital productivity α_2; remaining parameters set at: $G = G^* = 6$, $B = 3$, $\phi = 0.18$, $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

A question that arises is why changing α_1 only impacts a_1 if F is low (individualistic societies). The intuition for this effect is the following. Agents value variety in ties, and so with a high F they end up with high social capital Fa_1 just for consumption motives. This means that for communitarian societies in this model the marginal benefit of further increasing social ties for a technological motive is low. On the other hand, when F is low, Fa_1 will also tend to be low for consumption reasons, given the decreasing marginal utility of each tie. This means that the marginal benefit of strengthening the existing ties for technological reasons is higher, and therefore we observe a stronger response of a_1 to α_1. Ironically, Fa_1 is more aptly interpreted as social capital (in a deliberate technological sense) in individualistic societies. In communitarian societies, Fa_1 is more a measure for a particular class of consumption goods, namely social ties, which creates an important positive technological *externality* in transaction activities. The qualitative effect of changes in α_2 (the productivity of social capital) also depends on the level of communitarianism, as shown in the right panel of figure 2.4. In fact, if F is high, an increase in this productivity leads to a substitution away from social ties. The income effect dominates for low F, and social capital increases with α_2.

The introduction of transaction costs not only creates direct effects, but also mediates the relationship between other parameters and endogenous variables. Next we analyze how $\alpha_0 > 0$ changes the response of social capital and effort to variations in productivity, which is depicted in figure 2.5. For a given F, the effect of B is qualitatively the same as in the case without transaction costs: higher productivity implies higher transformational effort and lower social capital. The non-monotonic relation between the level of communitarianism and effort, for a given B, parallels the case where we vary transaction costs (figure 2.2). The incentive to produce is a function of relative trade efficiency; for low B, the ratio B/α_0 is low and so individualistic societies (with low social capital) follow predominantly a strategy of self consumption.

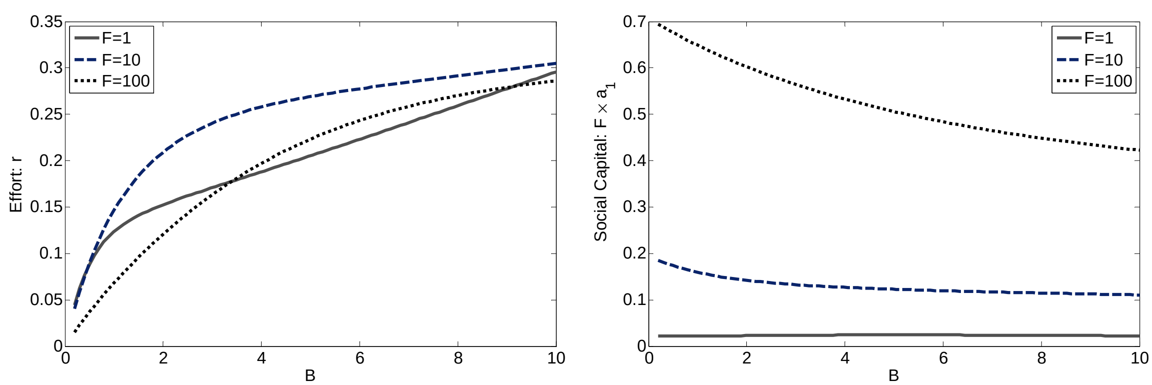


Figure 2.5: Left panel: shows how investment in human capital varies with productivity B. Right panel: shows how social capital varies with productivity B. In both panels the remaining parameters are set at: $G = G^* = 6$, $\phi = 0.18$, $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

As mentioned before, another dimension that we would naturally associate with development is a higher variety of goods. Next we investigate whether the introduction of transaction costs changes the relationship between variety, social capital, and effort. With high transaction costs ($\alpha_0 = 10$) the effect of a higher diversity of goods on effort may actually be the opposite of the case with $\alpha_0 = 0$, as shown in the left panel of figure 2.6. The intuition is that with high transaction costs and a high number of goods, agents spend a significant amount of time trading; this is so because diversity is preferred to quantity. Although agents do need to produce in order to be able to trade, and consequently would like to increase effort, with high α_0 this effect may be totally offset, by the fact that they need to allocate time to trade. Not surprisingly this is more of an issue for low-F societies, that cannot rely on social capital to obtain transactive efficiency. The effect on social capital is generally the same as with $\alpha_0 = 0$ (but slightly positive for $F = 1$); a higher diversity of goods is associated with lower social capital, holding F constant. This effect is the most pronounced for high-F societies.

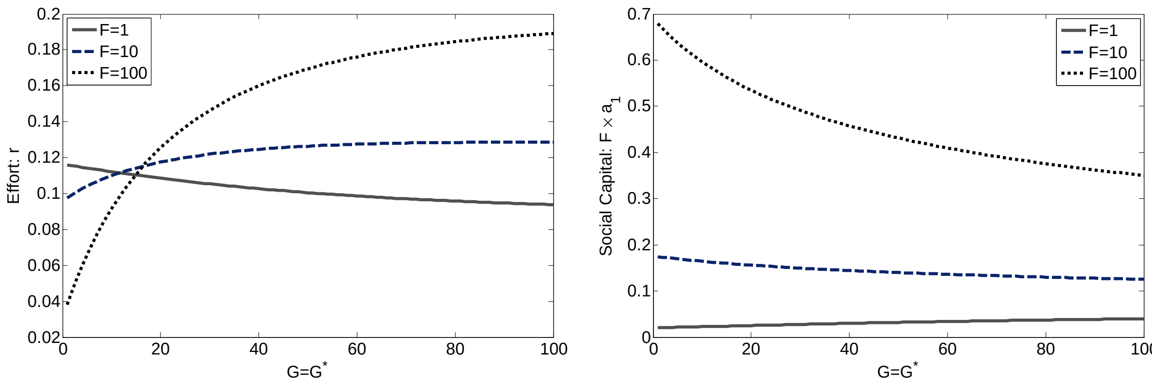


Figure 2.6: Left panel: shows how effort varies with number of goods $G = G^*$. Right panel: shows how social capital varies with number of goods $G = G^*$. In both panels the remaining parameters are set at: $B = 3$, $\phi = 0.18$, $\alpha_0 = 10$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

For lower transaction costs ($\alpha_0 = 1$), the response of effort and social capital to variety is qualitatively the same as without transaction costs (not shown). We also found that a decrease in transport costs delivers similar effects as an increase in the number of goods (not shown). This is intuitive, since high transport costs implies a relatively low consumption of foreign goods, which is the case with a lower number of goods.

Finally we do a simple welfare analysis, shown in figure 2.7. Not surprisingly the communitarian societies have a higher utility. This follows from a preference for diversity in ties and a higher transactive efficiency.

Both panels in figure 2.7 depict a counter-intuitive effect: for communitarian societies (high F) an increase in productivity or an increase in the number of goods leads to a decrease in welfare. The reason why this obtains is the following. Since $\beta < 1$, there is a coordination failure in building bilateral relationships. In other words, agents

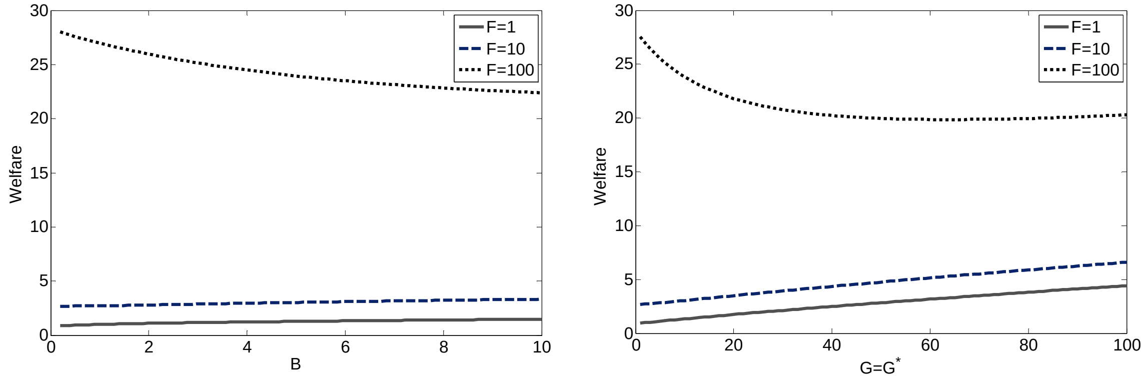


Figure 2.7: Left panel: shows how welfare varies with productivity B; remaining parameters set at: $G = G^* = 6$, $\phi = 0.18$, $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$. Right panel: shows how welfare varies with number of goods $G = G^*$; remaining parameters set at: $B = 3$, $\phi = 0.18$, $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\alpha_2 = 44.2$, $\rho = 0.54$, $p^* = 1$, $\tau = 0.1$, $\beta = 0.18$.

are consuming social ties below the first-best. An endogenous determinant of the coordination failure is how much the agent has to gain individually from deviating from a first-best allocation of attention. This gain is actually in part a function of the level of productivity and the number of goods. In particular, note that if $B = 0$ and/or $G = G^* = 0$, agents spend all their time in social ties and leisure, thus coordination failure is significantly mitigated. The incentive to deviate from a strategy with high a_1 has to do with the opportunity cost of time for the individual agent, which is higher if there are more goods to trade and/or production is more efficient.

In this section we took a first cut into some predictions of the model. Those depend importantly on: (i) the degree of communitarianism (F); (ii) the elasticity of substitution between ties and standard goods; and (iii) the levels of some key parameters, namely the number of goods, the unitary transaction costs, the productivity of social capital, and the productivity of transformational effort. In section 2.3 we will tighten these predictions by calibrating the model and comparing with data. Before proceeding with this, we relax the "small open economy" assumption.

2.2.4 Equilibrium exchange rate

So far we have been analyzing a "small open economy", where the terms of trade are given. In this section we explore how trade between villages with distinct degrees of communitarianism (F) and productivity (B) affects the equilibrium (real) exchange rate. The global economy is comprised of two villages with otherwise identical parameters. The equilibrium exchange rate $p^* \equiv p_{v_2}/p_{v_1}$ is given by

$$p^* = \frac{c_{2,v_2}}{c_{2,v_1}}, \tag{2.56}$$

where p_{v_i} is the price index in village i, and c_{2,v_i} is the quantity of imported goods consumed in village i. This is readily checked by combining the budget constraints and market-clearing conditions for each village. Equation 2.56 represents the equilibration of the trade balance between the two villages.

We chose four numerical scenarios to illustrate how distinct B and F affects prices and quantities. In order to not make this exercise cumbersome, all parameters besides B and F were set at their calibrated levels from section 2.3.1. In terms of each village's F, we set $F_{v_1} = 20$ (high communitarianism) and $F_{v_2} = 1$ (low communitarianism). Table 2.1 shows the results.

Scenario	$p^* = \frac{c_{2,v_2}}{c_{1,v_1}}$	r_{v_1}	r_{v_2}	a_{1,v_1}	a_{1,v_2}
(1) $B_{v_1} = B_{v_2} = 5$	0.140	0.301	0.262	0.009	0.026
		(0.327)	(0.237)	(0.009)	(0.023)
(2) $B_{v_1} = 5, B_{v_2} = 1$	2.545	0.343	0.149	0.009	0.020
		(0.327)	(0.155)	(0.009)	(0.021)
(3) $B_{v_1} = B_{v_2} = 1$	0.812	0.151	0.154	0.013	0.021
		(0.150)	(0.155)	(0.013)	(0.021)
(4) $B_{v_1} = 1, B_{v_2} = 5$	0.151	0.129	0.263	0.012	0.026
		(0.150)	(0.237)	(0.013)	(0.023)

Table 2.1: This table shows the effects of requiring that the real exchange rate equilibrates the trade balance. The numbers in brackets represent the outcomes for each scenario when $p^* = 1$ is assumed.

Scenarios (1) and (3), where the productivity of both villages is similar, show how the equilibrium exchange rate is affected by a difference in F only. In both these scenarios the (relative) price of the goods of the low-F village is low. The intuition for this is the following. Since the high-F village has a significant amount of social capital, foreign goods need to be competitively priced, since the agents naturally bias consumption towards the goods with lower transaction costs, i.e. c_1. The same effect would obtain if there was a subsidy for the consumption of domestic goods.

A difference in B is an important determinant of exchange rates. Inspecting scenario (2) reveals that now village 2 has relatively expensive goods. This happens because village 1 is able to produce goods at a much lower cost (high B), and so these goods are cheap. In scenario (4) this same mechanism leads to a lower exchange rate compared to scenario (3). The goods of village 2 have a low production cost (high B), so they are traded at a relatively lower price.

Finally, note that the introduction of general equilibrium considerations did not lead to significant changes in quantities of effort and ties, as shown by the last 4 columns of table 2.1.

2.3 Dynamic model and quantitative implications

Our objective in this section is to understand whether the model can explain *quantitatively* the heterogeneity in income per capita across time and countries. To generate a time path for each country, we devise a simple dynamic extension of the static model. Each agent lives one period – say from t to $t+1$ – and maximizes a combination of its own lifetime utility plus the discounted utility of her progeny. Agent i's total indirect utility at t is denoted by $\mathcal{J}_i$ (we assume all decisions are made at t):

$$\mathcal{J}_i(B_t) = \max\left\{\mathcal{U}_i(c_i, s_i; B_t) + \gamma \mathcal{J}_{i'}(B_{t+1})\right\}, \tag{2.57}$$

where for notational economy we omitted the village subscript and the dependence of utility on other parameters besides productivity B. We assume these parameters are fixed across time. The label i' denotes agent i's progeny; γ is the discount factor.

The link between the two time periods is the transformational effort. In particular, we assume that, contemporaneously, productivity is the same for all agents, and its law of motion is given by:

$$B_{t+1} = B_t\left(1 - \delta + \sum_i \frac{r_{ig(i)}}{N}\right), \tag{2.58}$$

where the parameter δ is the depreciation rate of productivity. An interpretation for our law of motion for productivity is that agents learn by doing (hence the dependence on effort), and cannot be excluded from knowledge (hence productivity not being agent-specific). Naturally this implies that for large N agents cannot materially influence the utility of their progeny (note that the upper bound for r is 1), and our dynamic model reduces to a repetition of the static optimization and equilibrium problem, only with an endogenous evolution of productivity B. In particular, we still solve for a symmetric equilibrium at each stage, so the law of motion reduces to

$$B_{t+1} = B_t \left(1 - \delta + r_t\right). \tag{2.59}$$

We make the strong assumption that the only difference in primitives between countries at $t = 0$ (the year 1700 in data) is the degree of communitarianism (F), which we are able to proxy for using sociometric data. It is likely the case that certain aspects of the economic environment that we parametrize also changed between 1700 an 2000 (e.g., unitary transaction costs). However, abstracting from these variations helps us understand the quantitative importance of the degree of communitarianism *per se* in terms of economic development. It is also not clear whether the concomitant variation of some other parameters besides F would hinder or strengthen our argument. We detail this discussion towards the end of the paper, in section 2.4.

2.3.1 Calibration

We calibrate the model to data of the United States for the year 1985. We then assume that the parameters are equal for all countries, and simulate how economies with different "cultures" of ties (different F's), which are considered fixed, evolve over time. We set the length of a period to 20 years, and interpret each period's decisions as the decisions of individuals in a generation. For the calibration we use mainly the following pieces of data: (i) output per capita and its components; and (ii) average distribution of use of time.

In the context of our model, total output is defined by the following

$$GDP = Br + B \times TC = c_0 + (G-1)c_1 + G^* c_2 + B \times TC, \tag{2.60}$$

where the last three components are measured output, and the first three components are associated with transformational production. The product $B \times TC$ is the value of transactive effort (in terms of goods).[4]

From the Penn World Table we obtain the measured output per capita for the United States in 1985, slightly higher than 24 thousand dollars (of the year 2000, see table 2.2). We adjusted this amount, using estimates by Devereux and Locay (1992), to account for officially unmeasured home production, and obtained a value for total output per capita slightly higher than 29 thousand dollars. This amount was

[4]The relative price of time in terms of goods is given by the ratio $\theta/(\lambda p)$, which is simply B – see equation (2.23).

distributed into four components (which correspond to the four types of goods in our model): home production (c_0, 22%), production for the local market ($(G-1)c_1$, 35%), production for the foreign market (G^*c_2, 4%), and transaction services production (value of TC, 39%). In Appendix 2.A.3 we describe in detail the sources and steps for the construction of these estimates.

	1985	1965
Measured GDP (2000 US\$, '000)	24.39	15.49
Adjustment GDP for home production	1.20	1.29
Total GDP (2000 US\$, '000)	29.27	19.94
Share home production $\left(\frac{c_0}{GDP}\right)$	0.22	0.27
Share internal market $\left(\frac{(G-1)c_1}{GDP}\right)$	0.35	0.34
Share external market $\left(\frac{G^*c_2}{GDP}\right)$	0.04	0.02
Share transaction services $\left(\frac{B\times TC}{GDP}\right)$	0.39	0.37

Table 2.2: Shares of output, United States, 1965-1985.

From Ramey and Francis (2008) we obtain data on use of time for the average person of age 10 or more for the year 1985. Apart from sleeping and personal care, which are assumed constant, table 2.3 shows the distribution among market work, home work, schooling, and leisure, of which we provide the components due to watching television and participating in social activities. We identify *total effort* in our model to the sum of the first three components. But, in our model, effort can be used in two sectors: one associated to the production of consumption goods, the other associated to transaction services. We use an estimate of market labor share in each sector (transformation vs. transaction) to separate effort in each of them. The results are that time used in transformation production (r) was in 1985 slightly higher than 30%, time used in transaction services (TC) was equal to 20%, time in social activities (Fa_1) was almost 10%, and time in (non-social) leisure (a_0) was slightly less than 40%. Again, in Appendix 2.A.3 we detail the construction of this classification.

Given these magnitudes, we proceed now to the calibration of the parameters, using first-order conditions and constraints. Table 2.4 shows the values of the parameters. We consider two cases. In the first one we set the elasticity of substitution between standard goods and social ties, $\rho_{c,s}$, equal to the calibrated elasticity of substitution within standard goods ρ_c, and also assume $\rho_c = \rho_{c,s} = \rho_s$. This is the benchmark

	1985		1965	
	Hours per week	Share	Hours per week	Share
Total effort	**45.4**	**0.51**	**47.5**	**0.53**
Market work plus travel	22.0	0.25	20.7	0.23
Home work	19.5	0.22	22.2	0.25
Schooling	3.8	0.04	4.6	0.05
Total leisure	**43.4**	**0.49**	**42.8**	**0.47**
Social activities	8.6	0.10	11.8	0.13
Non-social leisure	34.8	0.39	31.0	0.34
Television	17.5	0.20	13.5	0.15
Labor share in market transaction services	-	0.39	-	0.36
Effort in transformation	27.6	0.31	30.6	0.34
Effort in transaction	17.8	0.20	16.9	0.19

Table 2.3: Components of use of time (other than sleeping and personal care), United States, 1965-1985.

model described in section 2.2.3. In the second case we calibrate ρ_c and $\rho_{c,s}$ separately, which is a generalization of the benchmark model.

We set $G = G^*$ to 6, the number of broad consumption goods (food and alcoholic beverages, housing, apparel, transportation, health care, and entertainment). Hummels (2007) reports that ocean freight and air freight as a percentage of value shipped were (in 1985) in the range of 0.09 to 0.11; so we set τ to 0.1. We normalize the terms of trade, p^*, to 1 in 1985. Given consumption values and time used in production, we solve for B_{1985} in the budget constraint (2.38), and obtain a value of 5.72.

We used simultaneously equations (2.34), (2.35), and (2.39) to obtain values for ρ_c, α_0, and α_2. These are 0.54, 0.64, and 44.2, respectively. The first one implies an elasticity of substitution between standard goods slightly higher than 2.[5] We will consider three possible measures of the number of ties, which we describe below. The one that we mainly use is the number of close friends, the one shown in table 2.4. We set F_f^{US} to 8, the median quantity in 2001. We assume α_1 equal to 0.5 and ρ_s equal to ρ_c,[6] and use equation (2.37) to obtain a value for β equal to 0.18.

[5]It is interesting to note that an increase in the number of goods, $G = G^*$, would imply a higher elasticity of substitution, a relation that is supported by the data; e.g. Broda and Weinstein (2006).

[6]We tried different values for α_1, holding the calibration for the other parameters constant, and did not find significant quantitative differences.

At this point both cases, the benchmark and the generalized benchmark, differ. In the first one we just assume $\rho_{c,s} = \rho_c$, and calibrate ϕ, using equation (2.36), to a value of 0.18. In the second case, we use equation (2.36) for the years 1985 *and* 1965 to calibrate both values ϕ and $\rho_{c,s}$. These are 0.46 and -0.20, similar to the ones used commonly in the business cycle literature (e.g., Cooley and Prescott (1995), who offer values of 0.36 and 0 respectively). The slight negative elasticity of substitution between standard goods and (total) leisure implies that, with increasing productivity of transformational effort, total leisure will increase, as a result of the income effect slightly dominating the substitution effect.[7]

Finally, we have to determine values for the depreciation rate of productivity, δ, and the productivity level in 1700, B_{1700}. Given all the parameters calibrated above, we use a "shooting" algorithm to obtain these values, targeting the ratio between measured output in 1990 and 1750, informed by Lucas (2002), table 5.2, a ratio equal to 21.6.

Parameter	Case		Target
	$\rho_{c,s} = \rho_c$	$\rho_{c,s}$ calibrated	
G	6	6	# of broad sectors
G^*	6	6	# of broad sectors
τ	0.1	0.1	Hummels (2007)
B_{1985}^{US}	5.72	5.72	equation (2.38)
α_0	0.64	0.64	eqs. (2.34), (2.35), (2.39)
α_2	44.2	44.2	eqs. (2.34), (2.35), (2.39)
ρ_c	0.54	0.54	eqs. (2.34), (2.35), (2.39)
F_f^{US}	8	8	observation ISSP
α_1	0.5	0.5	assumed
ρ_s	0.54	0.54	assumed equal to ρ_c
$\rho_{c,s}$	0.54	-0.20	see text
β	0.18	0.18	eq. (2.37)
ϕ	0.18	0.46	eq. (2.36)
δ	0.03	0.10	ratio GDP_{1990}^{US} to GDP_{1750}^{US}
B_{1700}	0.61	0.069	"shooting" to obtain B_{1985}

Table 2.4: Parameter values

[7] To compute the equation for 1965, we need to calculate the productivity in 1965, B_{1965}, which we find to be equal to 3.72. The implied (average) annual rate of effort productivity growth between 1965 and 1985 is 2.2%, not an unreasonable estimate.

2.3.2 Simulation

We measure the distribution of the average number of ties across 27 countries, using the International Social Survey Programme 2001 (ISSP 2001).[8] We have three measures for number of ties: number of members of nuclear family (F_{nf}), number of close friends (F_f), and number of ties in secondary associations (F_{sa}). While the first two measures are straightforward, given the information of the ISSP, the third one needs some elaboration. There are 7 types of groups or associations informed: (i) political parties; (ii) unions; (iii) religious organizations; (iv) sports groups; (v) charitable organizations; (vi) neighborhood groups; and (vii) other associations. If a person belongs to a group, we considered that the person would have ties with 10 members of the group. Then we summed ties across groups and took country means. With this methodology we found that the average person from the United States would have around 23 ties, while the average person from France would have 12 ties, and the average person from Brazil 7.[9] Table 2.14 in Appendix 2.A.3 presents the means (medians in the case of nuclear family and close friends) for each country for each of the measures of number of ties. At the bottom, it shows that the number of close friends and the number of ties in secondary associations are highly correlated between them, and not correlated with the number of nuclear family ties. It also shows that the number of nuclear family ties is negatively associated with income per capita, while the number of close friends and the number of secondary association ties are positively associated with income per capita.

The values for the United States are respectively $F^{US}_{nf} = 6$, $F^{US}_{f} = 8$, and $F^{US}_{sa} = 23$. We simulate six cases, using one of the three measures for number of ties at a time, and a high or low $\rho_{c,s}$, as explained in the calibration. Different values for F^{US} and $\rho_{c,s}$ imply different values calibrated for β, ϕ, and δ. We simulate 15 periods (of 20 years each), that we map to the period between 1700 and 2000. We assume that there are no differences in parameter values across countries in 1700, including an identical effort productivity, B_{1700}. In that year, all differences in economic outcomes are assumed to

[8] See section 2.A.2 in the Appendix for details.

[9] The number of ten ties per person per group is rather arbitrary. But we do not have at the moment information about the number of this kind of ties, or on time allocated to them. We think of these ties as weaker than family and close friends' ties, in terms of average time allocated per tie. In fact, secondary associations' ties could be interpreted as a proxy for loose friends and acquaintances. The fact that we are using the same assumption for all countries for the average number of ties per secondary association does however restrict the model's degrees of freedom. In fact, setting this number to 10 can be interpreted as calibrating the model with low $\rho_{c,s}$ and F_{sa} to the coefficient of variation in data – see table 2.5.

be due to differences in number of ties, which will be considered fixed over time. But, with time, differences in the allocation of time to effort imply differences in generated productivities, which, by the year 2000, may account for a large share of the differences in economic outcomes.

Given this initial productivity value, we generate a path for each country for the variables of interest, inputing the differences in number of ties across countries. The results are shown in table 2.5. Using a high $\rho_{c,s}$, equal to the calibrated ρ_c, we observe in row (6) that the model can account for only 5% of the observed coefficient of variation of income per capita in 2000, if we use nuclear family ties as the measure of number of ties. By contrast, the model can account for a larger share of the variation in 2000, 39%, if we use instead secondary association ties, and it can account for 18% of the variation if we use instead close friends ties. Row (7) presents the cross-country correlation for the income per capita generated by the model and data. The three measures order the countries fairly well, with a higher correlation for the secondary associations ties. If we use a low $\rho_{c,s}$, calibrated as explained above, the differences in outcomes for different measures of number of ties are starker. In this case, differences in family ties generate a higher variation in income per capita by the year 2000 (around 39%), but the correlation between model and data is negative, implying that the model predicts incorrectly which countries will develop faster. By contrast, using close friends and secondary associations measures of number of ties imply both a greater variation in income per capita by the year 2000, and a positive and strong correlation in income per capita between model and data.

Lucas (2002) reported income per capita across regions of the world (for 21 broad regions) for the years 1750 and 1990. Using these data, in row (8) we compute the ratio of the coefficient of variation of income per capita in 1990 to the coefficient of variation in 1750, and conclude that the coefficient was multiplied by 4 in that period. We compare that value to the ratio generated by the model, informed in row (9). The model with a high elasticity of substitution between standard commodities and social ties generates too much or too low variation in income per capita in 1990, given the variation in 1750. The model with a low $\rho_{c,s}$ does better (with values around 2.5), though it still does not capture completely the magnification in variation that happened between 1750 and 1990.

The main differences in the simulated series between a high and a low elasticity of substitution between standard commodities and social ties (high vs. low $\rho_{c,s}$) are

		$\rho_{c,s}$ high			$\rho_{c,s}$ low		
		$n.f.$	$fr.$	$s.a.$	$n.f.$	$fr.$	$s.a.$
F^{US}	(1)	6	8	23	6	8	23
$CV\ F$ (Data)	(2)	0.22	0.35	0.47	0.22	0.35	0.47
$CV\ IPC$ 2000 (Data)	(3)	0.43	0.43	0.43	0.43	0.43	0.43
$CV\ IPC$ 1700 (Model)	(4)	0.02	0.03	0.03	0.03	0.05	0.10
$CV\ IPC$ 2000 (Model)	(5)	0.02	0.08	0.17	0.17	0.28	0.45
% accounted	(6)=(5)/(3)	5	18	39	39	65	104
Correlation IPC 2000	(7)	0.43	0.53	0.68	−0.35	0.58	0.69
$\frac{CVIPC1990}{CVIPC1750}$ (Data)	(8)	4.0	4.0	4.0	4.0	4.0	4.0
$\frac{CVIPC1990}{CVIPC1750}$ (Model)	(9)	0.8	1.8	5.8	2.9	2.6	2.3

Table 2.5: The effects of number of ties on distribution of income per capita. $n.f.$ indicates nuclear family; $fr.$, friends; and $s.a.$, secondary associations. CV indicates coefficient of variation. IPC is income per capita. Row (7) presents the correlation between model and data for income per capita in the year 2000.

the time trends in social capital and effort in transformation of goods. With a low elasticity, the time trend for social capital is increasing, while the trend for effort in transformation is decreasing, as can be seen in figure 2.8. On the contrary, with a high elasticity, the time trend for social capital is decreasing, while the trend for effort in transformation is increasing, as can be seen in figure 2.9. In the first case, the income effect of an increase in productivity dominates the substitution effect, and therefore the consumption of social ties (and leisure) increase with productivity. In the second case, the substitution effect is dominant and therefore an increase in productivity causes an increase in effort and a decrease in social ties.

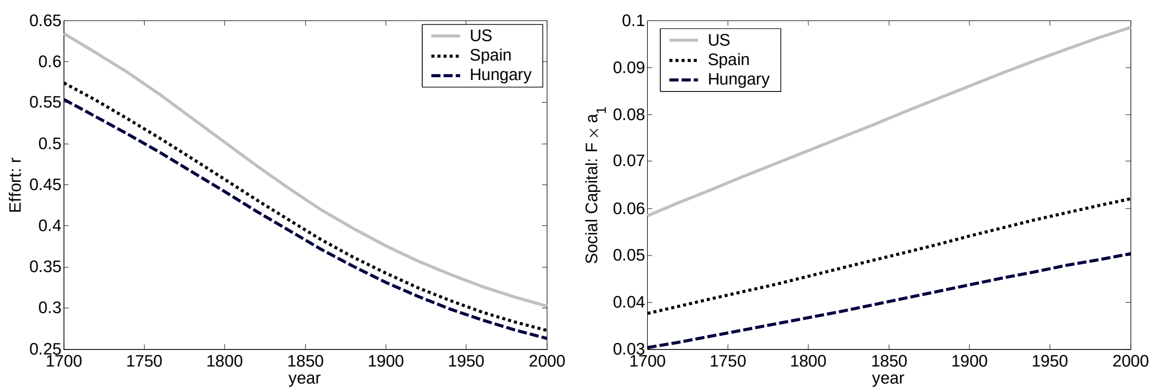


Figure 2.8: Low $\rho_{c,s}$. Left panel: shows changes in effort time between years 1700 and 2000. Right panel: shows changes in time use in social ties in the same period.

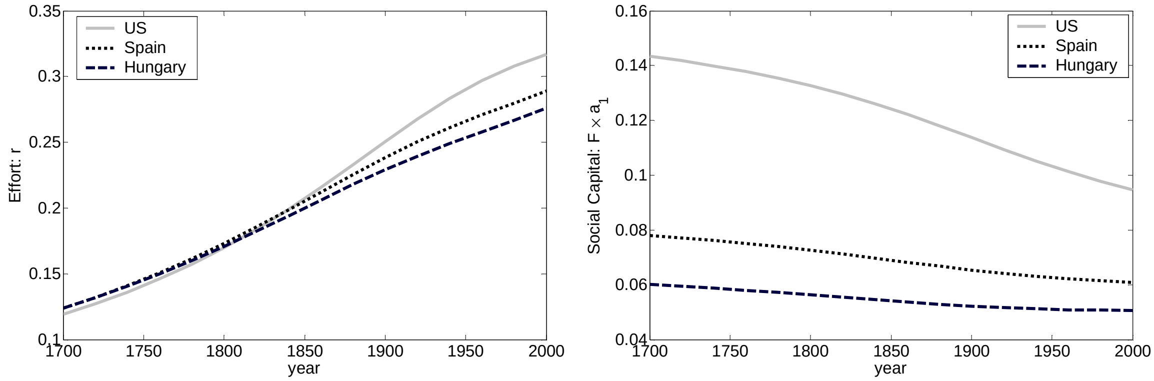


Figure 2.9: High $\rho_{c,s}$. Left panel: shows changes in effort time between years 1700 and 2000. Right panel: shows changes in time use in social ties in the same period.

Interestingly, one could associate the first case with observations for the United States in the earlier part of the 20th century, when effort in transformation of goods was diminishing but social capital was increasing (at least by Putnam's account), and associate the second case with observations in the later part of the 20th century, when social capital was decreasing (again by Putnam's account) while the decline in effort was slowed down or even reversed. At this point we will not take a position with respect to which is the most plausible parameter value, a high or a low elasticity of substitution between standard goods and social ties. We would just emphasize that increasing and decreasing social capital in time are both, in principle, consistent with equilibrium behavior and with increasing overall welfare.

From these results, we conclude that close friends and secondary associations are better measures for number of ties, given the purpose of our model. Next we will consider how the model fares in other dimensions, considering first time series for the United States and later country cross-sectional issues. We use number of close friends as our measure for number of ties, and a low elasticity of substitution between standard goods and social ties. Most of the observations that follow will not change if instead we use secondary associations' ties. Some observations will change if instead we consider a high elasticity of substitution, and we comment when that is the case.

Time series - United States

Table 2.6 presents data on time use between 1900 and 2005. We have bundled together market work, home work, and schooling into what we call effort. Then we have separated into two groups of effort, transformational production and transaction services (see Appendix 2.A.3 for details). It is clear that time in transformational production has decreased (32%), while time in transaction services has increased drastically (145%). Time in total leisure has increased by a smaller amount (7%). The bottom part shows the changes provided by the simulated model in the last 100 years. Qualitatively they move in the right direction, but quantitatively they are smaller, in the order of 1/5 to 1/2. This is due mainly to the fact that, in the model, the inter-generational growth rates in productivity are decreasing in time (due to decrease effort in time), while they are increasing in the data. The change in these magnitudes would be better accounted for if labor and schooling were separated in the model, and most of the increase in inter-generational productivities attributed to increase in schooling.

We have calibrated δ such that we obtain an increase of around 22 times in income per capita between 1750 and 1990. Between 1900 and 2000 total production in the model (GDP) increases by 197% while in the data it increased by 481%. By contrast, between 1750 and 1850 the United States grew 81%, while the model shows growth by more than 465%. Again, this is due to the fact that the model, as calibrated, presents higher growth rates earlier than later. With a separation of labor and schooling periods, increasing schooling in time, and productivity change depending on schooling we would obtain increasing growth rates, and this counter-factual feature of the model would be eliminated.[10]

Table 2.6 also presents data for the division of total leisure between social activities (socializing, religion, and organizations in Robinson and Godbey (1997) classification) and non-social leisure for the years 1965 and 1985. Between those years we observe a decrease in social activities and an increase in non-social leisure (most of it can be accounted for by an increase in time watching television). By contrast, our model presents increases in both time used in social activities and time in non-social leisure, with a higher increase in the first component. It would be interesting to obtain data on these components for other years, and check if the predictions of the model accord better with the data in the first 2/3 of the 20th century.

This anomaly can be understood better if we recall equation (2.37). It is clear that, for a fixed F, an increase in internal market consumption, c_1, with an increase in (non-social) leisure, a_0, must imply an increase in strength of ties, a_1. In other

[10]This feature of the model is also eliminated if we use a high $\rho_{c,s}$, because, as we have seen (figure 2.9), in that case effort increases with productivity, and therefore the growth rates of productivity are larger later than earlier.

	Data			Model		
	Share		% change	Share		% change
	1900	2000		1900	2000	
Total effort	**0.56**	**0.52**	**-14**	**0.53**	**0.51**	**-5**
Transformational effort	0.47	0.32	−32	0.38	0.30	−20
Transaction services effort	0.08	0.20	145	0.16	0.21	30
Total leisure	**0.44**	**0.48**	**7**	**0.47**	**0.49**	**6**
Social activities	na	na	na	0.09	0.10	14
Non-social leisure	na	na	na	0.38	0.40	4

Table 2.6: Comparison of model and data: use of time, United States, 1900-2000. na indicates not available.

words, our model does not allow for a decrease in social capital with increasing leisure and local consumption. Alternatively, equation (2.37) indicates that an increase in internal market consumption and a decrease in strength of ties must be accompanied by a stronger decrease in leisure than in strength of ties. In other words, in light of this model, it is not the decrease in social activities *per se* that is surprising, but that decrease accompanied by economic growth and higher leisure. In this context, the decrease in social capital that allegedly happened in the United States in the last part of the 20th century can only be understood with a change in the equilibrium number of ties, F.

What about the shares of home (c_0), internal market ($(G-1)c_1$), external market ($G^* c_2$), and transaction services ($B \times TC$) in total output? The share of transaction services in the model increases from 0.30 in 1900 to 0.40 in 2000, an increase of 36%. Using Wallis and North (1986) estimates, we calculated an increase from around 0.19 in 1900 to 0.40 in 2000, an increase of 106%. Again, the model predicts changes in the right direction but the magnitudes are smaller than in the available data.

Table 2.7 presents the predicted changes in shares of these components in GDP between 1700 and 2000. The most striking features are the increasing share of home production and the drastically decreasing share of external market output. The available data with respect to those shares is scarce, but all indications are that home production diminished steadily, while the share of external market output does not have a clear trend (but increasing between 1900 and 2000). Why does the share of home production increase in the model? An increase in productivity increases the opportunity cost of time, and therefore the cost of incurring in transaction costs. Therefore there is a stronger incentive to consume more of the self-produced good, *vis à vis* the marketed good. Interestingly though, imports suffer the greater loss in share, given the inability of the agents to economize in transaction costs of those goods. By contrast, the share of internal market goods consumption diminish mildly at first and more strongly only later in time. The reallocation of consumption of goods from the market to self-produced is a strong and highly counter-factual feature of the model. Of course, other things might be changing too: lower unitary transportation costs (a decrease in τ) might be increasing the incentives to import rather than self-consume (see, e.g., Hummels (2007)); lower unitary transaction costs (a decrease in α_0) might be increasing the incentives to consume marketed products (see, e.g., North (1994), p. 363); the invention of new goods (an increase in either G or G^* or both) might be increasing the incentives to consume those goods instead of consuming the self-produced good. Equally or more important, we are not modeling here the decision of the agents to specialize in production. The increase in productivity might be conditional on the execution of a narrower set of tasks. Those tasks will hardly be enough to produce any consumable good, which creates an incentive to reduce self-produced consumption and increase marketed consumption. All these possibilities are material for future research.

	Self produced c_0	Internal market $(G-1)c_1$	External market G^*c_2	Transaction services $B \times TC$
Model				
1700	0.09	0.44	0.44	0.03
1800	0.11	0.45	0.31	0.13
1900	0.15	0.43	0.12	0.30
2000	0.23	0.33	0.03	0.40
Change 1900/2000, %	53	−24	−71	36
Data				
1900	0.42	0.36	0.03	0.19
2000	0.22	0.32	0.06	0.40
Change 1900/2000, %	−48	−10	115	106

Table 2.7: Comparison of model and data: production composition (shares of total output), United States, 1900-2000.

Country cross section

What about the cross-sectional predictions of the model? A higher income per capita (associated with a higher number of ties) is predicted to be negatively correlated with the average strength of ties. In the model a 1% increase in average strength of ties is associated with a 2.6% decrease in income per capita, while in the data it is associated with a 3.2% decrease in income per capita. Figure 2.1 in section 2.1 is borne out by the model. In this respect the model is quite accurate.

Table 2.8 presents use of time data for four countries around the year 2000: United States, United Kingdom, Italy and Mexico. We include Mexico, despite the fact that it is not part of the sample in the ISSP, because it is a relatively low income per capita country with use of time data. The means are taken over different groups of population, given the way information is provided, ranging from age 12 and above for Mexico to 16 and above for the United Kingdom. This fact makes the comparisons for schooling unclear, but when we aggregate market work, home work, and schooling into overall effort the comparisons are clear. We observe that, with increase in income per capita, total effort diminishes (around −25% between Mexico and the United States), while total leisure increases (slightly more than 45%). The model predicts just the opposite: countries with higher income per capita exhibit higher effort time (23% between the range extremes) and lower leisure time (−11% between the range extremes). This is another counter-factual feature of the model that could be addressed by separating schooling from labor. In that case, higher schooling could be associated with higher income per capita, lower labor, and higher total leisure. With respect to the use of leisure, the data shows an increase of more than 50% in time dedicated to social activities (between Mexico and the United States). The model predicts a substantial increase with income per capita. In this respect, the model fares better.

Table 2.9 presents the model's shares of output for three countries, with different number of ties. With respect to the data, there is some presumption that less developed countries would have a higher share of self-produced consumption (see Devereux and Locay (1992)), and a lower share of transaction services. Neither presumption is borne by the model.

	United States	United Kingdom	Italy	Mexico
Year of survey	2000	2005	2002-2003	2002
Population	Age 14+	Age 16+	Age 15+	Age 12+
Total effort	**0.29**	**0.31**	**0.32**	**0.39**
Market work	0.15	0.18	0.17	0.16
Home work	0.13	0.12	0.14	0.18
Schooling	0.01	0.01	0.02	0.05
Leisure	**0.25**	**0.23**	**0.18**	**0.17**
Social activities	0.05	0.06	na	0.03
Television	0.10	0.11	na	0.07
Other leisure	0.10	0.06	na	0.07
Personal care	0.46	0.46	0.50	0.45
Income per capita, '000	34.4	24.7	22.5	8.1
# of close friends (2001)	8	9	4	na
# of ties in associations (2001)	23	14	10	na

Table 2.8: Shares of use of time for four countries, around year 2000.

	Self produced c_0	Internal market $(G-1)c_1$	External market G^*c_2	Transaction services $B \times TC$
High F (e.g. US)	0.23	0.33	0.03	0.40
Medium F (e.g. Spain)	0.21	0.32	0.07	0.40
Low F (e.g. Hungary)	0.20	0.31	0.08	0.40

Table 2.9: Production composition, country cross section.

Social capital

In our model there are two motives for building social ties. One is a preference motive; individuals enjoy spending time with the "usual suspects" from time to time. The other motive is technological; individuals trade more efficiently if they have ties with other members of their community. Usually, a social network that economizes in production or transaction is called social capital. In this section we try to measure the importance of social capital for observed outcomes. In particular, we ask: by how much do the predictions of the model change if social ties are ineffective in economizing transaction costs? In other words, we shut down the technological channel, by setting the productivity of social capital, α_2, to zero, and compare with previous results.

Table 2.10 shows the results. In the baseline case, with productive social capital

($\alpha_2 = 44$), the amount of time dedicated to social ties increases between 23 and 26% from 1700 to 2000. By contrast, with zero productivity of social capital ($\alpha_2 = 0$), the time dedicated to building social ties only increases between 5 and 8% in the same period of time. The fact that agents cannot economize in transaction costs has an important impact on growth. Indeed, the measured income per capita of the country with the larger number of ties is multiplied by 83 if social capital is productive, but only by 44 if it is not. Likewise, the income per capita for the country with the fewer number of ties is multiplied by 35 and 26, respectively. This is due, of course, to changes in endogenous productivities, which, in turn, are affected by the reallocation of time away from effort, given the complementarity between effort and social capital.

	Baseline case $\alpha_2 = 44.2$		No use for social capital $\alpha_2 = 0$	
	Higher F	Lower F	Higher F	Lower F
Social capital, $Fa_1, \%$	53	65	22	27
Measured GDP	$\sim \times 83$	$\sim \times 35$	$\sim \times 44$	$\sim \times 26$
Total GDP	$\sim \times 100$	$\sim \times 40$	$\sim \times 60$	$\sim \times 30$
Productivity (B)	$\sim \times 120$	$\sim \times 51$	$\sim \times 68$	$\sim \times 38$
Share of internal market, $\frac{(G-1)c_1}{GDP}$, %	−23	−28	−81	−67
% of variation accounted	65		45	
Correlation IPC 2000	0.58		0.58	

Table 2.10: The effects of social capital. Changes between 1700 and 2000.

It is interesting to note how all this translates into changes in the share of internal market goods in total output. In the baseline case, this share is reduced by around 25% between 1700 and 2000. This is due to the fact that, with higher productivity, the opportunity cost of time increases, and therefore these goods become costlier (*vis à vis* self-produced consumption) given that they incur in transaction costs. With zero productivity of social capital, though, the share of internal market goods in total output is reduced by a much larger amount, in the order of 67 to 81%. Figure 2.10 summarizes these effects for the United States. In the year 2000, with no productivity of social capital, transformational productivities are reduced by almost 40%, and internal consumption is reduced by more than 80%.

The case with zero productivity of social capital can account, in the year 2000, for 45% of the cross-country variation in income per capita (as opposed to 65% in the

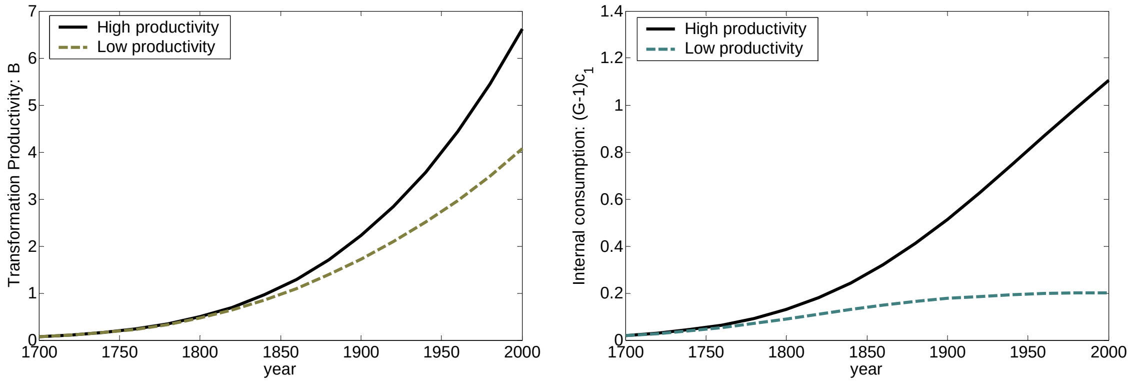


Figure 2.10: High ($\alpha_2 = 44$) and low ($\alpha_2 = 0$) productivity of social capital. Left panel: shows changes in transformational productivities between 1700 and 2000. Right panel: shows changes in consumption directed to the internal market in same period.

baseline case), and presents a similar correlation of income per capita between model and data (around 0.58).

The main motive for allocating time to social ties is the preference motive. In the year 2000, for the country with higher number of ties, the technological motive accounts for only 15% of the time allocated to social activities. But this exercise shows that social capital *per se* can be an important force in shaping world economic outcomes: without it, and according to the model, the income per capita of any country today would be between 1/4 and 1/2 smaller, the internal markets would be much smaller, and the standard deviation in income per capita would be around 20% smaller.

2.4 Discussion and conclusion

In this section we have three objectives. First, to summarize what we have learned, both analytically and quantitatively. Second, to expand on how this model helps us to think about social capital. Third, to discuss the model and its limitations, and outline possible courses for future research.

2.4.1 Summary of results

In the analytical section, we have found the following main results:

1. With no preference for ties, it is possible for social capital to be zero ($a_1 = 0$), even if the productivity of social capital is highly positive (proposition 18). This

may obtain for two reasons: (i) if there is a severe (combined) coordination failure (egotism in relationships and/or public good nature of social capital); and (ii) an intense trading activity with the exterior, which reduces the importance of in-village transaction costs. The problem of underprovision of social capital is mitigated when agents have a preference for social ties.

2. The level and relation between the different elasticities of substitution are a key determinant of the observed co-movements between social capital and other economic variables, even when social ties do not play a technological role.

3. With positive transaction costs social capital may play an important technological role, very much complementary to transformational effort.

4. The technological motive for investing in social ties is mostly relevant for societies with a low number ties.

5. For highly communitarian societies an increase in productivity may lead to a decrease in welfare, which obtains because of an amplification of the coordination failure in building relationships.

In the dynamic quantitative section we found the following main results:

1. Differences in number of ties can account for a fairly large share of the worldwide variation in income per capita in 2000.

2. More communitarian societies, measured by the number of close friends or the participation in civic associations, present faster economic growth. Indeed, more communitarian societies present higher social capital and higher transformational effort. In this sense, social structure can determine a country's path of economic growth.

3. In the time series, the model can account for between 1/5 and 1/2 of the changes in the United States components of use of time.

4. In the time series, the model cannot account for diminishing social capital with increasing (non-social) leisure.

5. In the time series, the model can account reasonably well for changes in transaction services and internal market shares; it cannot account for changes in self-produced and external market shares.

6. In the country cross section, the model can account well for the negative relationship between income per capita and average strength of ties, as well as for the positive relationship between income per capita and social capital. It does not do well in accounting for the relationships of effort and/or leisure and income per capita.

7. The technological role of social ties is quantitatively important: if the productivity of social capital were zero, the model predicts that all national economies would be between 1/2 and 3/4 their actual size. This effect is mainly attributable to a positive externality of the consumption of social ties, and not the result of deliberate accumulation of ties for purely technological reasons.

2.4.2 Social capital

A key link between social structure and economic development, emphasized by both sociologists and economists, is *social capital*. It is a key link because economic development is always associated to the spread of trade, and it is argued that social capital can diminish significantly the costs of trade, commonly known as transaction costs. How does social capital affect transaction costs? The idea is that social capital both facilitates the flow of *information* and creates *trust* among potential traders. Increased information and trust diminishes transaction costs, by eliminating uncertainty about the potential actions of the trading partners at different contingencies. In other words, social capital reduces transaction costs by assuring coordination and reducing malfeasance.

The concept of social capital has been criticized. One critique is why to call it "capital". Both Arrow (2000) and Solow (2000) ask for a change of name. The former says that social capital does not imply an essential characteristic of "capital", a "deliberate sacrifice in the present for future benefit". The latter asks "what is social capital a stock of?", and suggests to call it instead "patterns of behavior". In this paper we argue that social capital is sometimes a pure by-product of social interactions, sometimes the outcome of a deliberate accumulation process, requiring the sacrifice of transformational effort and/or trading time and/or leisure. In light of this natural ambiguity, social capital is more aptly interpreted as "capital" in some instances than in others. In any case, our model and calibration suggest that the amount of transaction time saved by social capital is quite significant.

Another related usual critique is associated to the fact that the originator of the concept (Coleman (1988)) and its main popularizer (Putnam (1993)) gave examples that portray the increase in social capital as unequivocally good. By now, there is quite an agreement that different kinds of social capital can increase or decrease welfare. A decrease in welfare can be due to different mechanisms. First, some associations can be created for unproductive or destructive activities; e.g. the Mafia or crime gangs. Second, as shown by Routledge and von Amsberg (2003), frequent migration can destroy social capital but increase welfare by a more efficient allocation of labor. Third, more generally, there might be more efficient solutions than social capital accumulation to the market failures of incomplete information and lack of enforcement, as emphasized by Durlauf and Fafchamps (2005). For example, an impersonal third-party court and police can create *generalized* trust, which is considered to have a low marginal cost, as opposed to the *personal* trust associated to social capital. In this paper we offer one more reason why the increase in social capital might not be welfare improving: it requires investment in time, which might be better used as investment in human capital, labor, or leisure.[11]

Social capital is called "social" for two reasons. One is that it might imply externalities. The benefits of an association might accrue to non-members. This is the public good character of social capital. The other, more fundamental, reason for calling it "social" is that it is a product of social ties, the "connections among individuals" in Putnam's definition. But why do social ties emerge? Arrow (2000) states: "there is considerable consensus also that much of the reward for social interactions is intrinsic – that is, the interaction is the reward – or at least that the motives for interaction are not economic. People may get jobs through networks of friendship or acquaintance, but they do not, in many cases, join the networks for that purpose. This is not to deny that networks and other social links may also form for economic reasons." In our model we follow this statement, and include two motives for the construction of social ties, an intrinsic, preference motive, and a technological, social capital motive. However, in the model, we found that for highly communitarian societies (high F), the technological motive (from an individual agent's perspective), becomes redundant.

When referring to social ties, there are two main concepts in which we are interested: the number of ties, known in the network literature as network size; and the average

[11]Kumar and Matsusaka (2006), who deal with personal networks, and Hoff and Sen (2005), who deal with the extended family, offer models where crystallized social interactions are obstacles to the spread of trade.

strength of ties, known as network density (Marsden (1990)). Additionally there are many types of social ties. Outside of the market work, we would consider at least three different types of ties: family, friends, and secondary associations. Do all of them contribute to social capital? The empirical literature (see empirical studies in Dasgupta and Serageldin (2000)) has mostly focused on the secondary associations, unions, churches, political parties, clubs, the so-called civic society (but see chapter 6 in Putnam (2000)). But, if the main attributes of social capital are the presence of trust and the flow of information between members, then it is clear that family and friends ties should be considered too. In our model we distinguish between number and strength of ties, but we do not model explicitly different types of ties. In the calibration and simulation, though, we consider the three types separately and ask the question: which type of ties is most consistent with both the increase in inequality across countries and with certain countries growing faster than others? The answer is that close friends and secondary associations ties are more consistent than family ties.

We have said that our model provides another possible reason for welfare to increase as social capital diminishes. Interestingly, though, our setting also can explain why there might be a feeling of "malaise" when social capital diminishes, despite the fact that productive efficiency might be increasing (as it is argued to have happened in the US in the second half of the 20th century). An increase in productivity creates a stronger incentive for individual agents to free ride in their relationships; in equilibrium this may yield a relatively low consumption of social ties, relative to the first-best. This seems however to be an issue only for highly communitarian societies (see figure 2.7).

In our model ties can have a high or low strength (a high or low a_1). We would like to mention that a weak tie in our model is quite different from Granovetter (1973)'s weak ties. While in his work weak ties are important because they connect ("bridge") isolated groups with strong within group ties, in our model we have weak ties with closure. Of course, this characteristic can be modified if we allow for the individual to have weak and strong ties simultaneously (say, having a family *and* acquaintances).

Another aspect of social capital we are not dealing with is production social capital, of which an example follows. Two researchers have offices next to each other, and they do small talk from time to time. One of them introduces his colleague into some results in a new area of research. This last person finds that those results are useful for his work in progress. His productivity has been increased by the social connection. We want to be clear we are not considering this aspect of social capital. By contrast, social capital in our model is related to transaction efficiency. In this we follow Putnam's comment, when he relates social capital to reciprocity, and states: “A society characterized by generalized reciprocity is more efficient than a distrustful society, for the same reason that money is more efficient than barter. If we don't have to balance every exchange instantly, we can get a lot more accomplished.” (p.21). The distinction is important because the effects of productivity increase considered in the simulation can be very different with production instead of transaction social capital.

2.4.3 Limitations and further research

We have seen that the model does well in accounting for some dimensions of the data, but not for others. In this section we discuss which we consider limitations of the model, and what extensions and explorations we consider worthwhile.

First we would like to consider the utility functional form, as the simulation relies heavily on it. The “love of variety” functional form for standard goods is quite plausible in this context. Some modifications might be reasonable; e.g., a different elasticity of substitution between domestic and foreign goods, and within domestic goods. But, overall, it seems a reasonable approximation to reality. More controversial is the “love of variety” functional form for social ties. The essential question here is: do people prefer to have many ties with low intensity (in terms of time), or a few ties with high intensity. As we model it, our answer has been the former. But we consider this a very much open empirical question. That is the reason why, in our calibrated version, communitarian societies present higher welfare than individualistic societies. Given that a higher number of ties implies (in most cases analyzed) higher social ties *and* higher effort, we observe that both components of welfare, the component due to consumption of standard goods and the one due to social ties, are higher in these societies (see figure 2.11). If, on the contrary, individuals would prefer a lower number of intense social ties, then we might have an interesting trade-off: for technological reasons, to economize in transaction costs, people would prefer to have a high number of ties; for preference reasons, they would prefer a low number of ties. With economic development there could be some tension, as one component of welfare might be increasing while the other one decreases. We think it would be important that more empirical work informs utility functions for social ties, in a similar way that empirical work has informed utility functions for standard commodities.

Another aspect of the utility functional form that deserves scrutiny is how (non-social) leisure enters into it. We have modeled it in such a way that leisure is func-

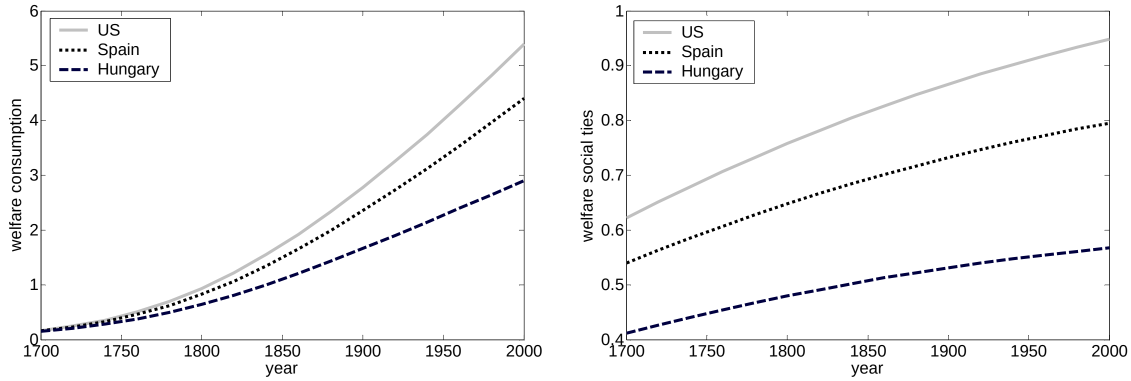


Figure 2.11: Left panel: it shows changes utility due to changes in consumption between years 1700 and 2000. Right panel: it shows changes in utility due to changes in social ties in the same period.

tionally equivalent to a tie with oneself. But this, of course, does not have to be the case. For example, one can think that (non-social) leisure is indeed complementary to consumption of standard goods, but that the composite standard goods-leisure is a substitute for social ties. As an example, the past 40 years of increase in time watching television, with increase in income per capita and decrease in socializing time, could be better accounted for by such a function.[12] An alternative view comes from asking the question: is watching television (or "surfing" the Internet, or playing video games) part of non-social leisure? Or is it an alternative way of socializing, a way in which, in terms of our model, the number of (asymmetric) ties is very large and the strength small? We started this paragraph by considering that it may be necessary to modify the model's utility functional form, and finished it by saying that it might be necessary to redefine what it means "social activities" in the data. We consider that further data investigation, as well as analytical inputs from sociology and social psychology, could clarify this issue.

In our calibration we fixed the number of ties of each country over time, and explore what were the consequences in the time series and cross-sectionally of differences in that variable. Each number of tie is an equilibrium, and we consider that culture

[12]For example, one such function is

$$U(c,l,s) = \left\{ \phi \left[(1-a)c^{\rho_{c,l}} + a^{\rho_{c,l}} \right]^{\frac{\rho_{(c,l),s}}{\rho_{c,l}}} + (1-\phi) s^{\rho_{(c,l),s}} \right\}^{\frac{1}{\rho_{(c,l),s}}}, \tag{2.61}$$

with $\rho_{c,l} < 0$, and $\rho_{(c,l),s} > 0$, where c and s are composites of standard goods and social ties respectively, and l is non-social leisure.

determines it. We found that the effects of "culture" on income per capita and welfare were important. A natural question arises: does culture change? If so, when, how, and how fast? It would seem natural that culture changes when the welfare costs of maintaining a specific culture are high. For example, one can consider that Japan changed its cultural beliefs after loosing World War II. This led to the adoption of new institutions; the militaristic tradition was abandoned, and a more democratic society evolved. Hayashi and Prescott (2006) have analyzed the case of the structural change between agriculture and industry in Japan before and after the war. Before the war a constraint in migration for the eldest son of a farmer was in place. After the war, the code was modified to permit his migration. The result was an increase in productivity, by reallocation of labor towards city industry, that the authors associate with the extraordinary income per capita growth rates after the war. We can imagine that the Japanese concluded that their (cultural) beliefs were constraining their development, changed those beliefs toward a less paternalistic society, and consequently changed their institutions (the code). On the other hand, if a culture has served well a country, it is reasonable to expect their citizens not to push for a change. An example, more in line with the cultural beliefs we analyzed, is the case of the United States. We have seen that, when measured as participation in secondary associations, the United States leads the pack in number of ties. But this was already observed by Tocqueville (1899) in 1835: "in no country in the world has the principle of association been more successfully used, or more unsparingly applied to a multitude of different objects, than in America." We conclude that culture can be fairly stable if it is associated with success. On the other hand, cultural change seems to be associated with negative shocks to the national self-esteem. In this context, a natural extension to our analysis would be to measure the distribution of number of ties at different moments in history, and relate changes in the distribution with changes in income per capita.

In the empirical literature on culture there is no practical distinction between different preferences across societies and different beliefs which imply different equilibria. Culture encompasses both. For example, Fernández and Fogli (2007) empirically assess that a national culture (with respect to the role of women in society) can influence women's labor force participation, while Tabellini (2006) considers the effect of regional culture (with respect to trust, respect of others, and self-determination) on income per capita, but it is not identified if individuals in different societies have different preferences or values (say, different values of the role of women in society, or more or less respect for others) or if they just have different beliefs of what the other individuals will do (say, condemn or not a woman that works at the market, or the level of respect for others). In our model we take a stance in which a particular culture is identified with a particular equilibrium. Nonetheless, in further research it would be interesting to study what is the relationship between a particular equilibrium and future preferences.

Another relevant aspect of reality that we are leaving aside is diffusion of technology. In the same way that we are postulating inter-generational externalities, we could postulate inter-societal externalities; that is, diffusion. With diffusion, the different productivities generated along time will be very much diminished (see Eaton and Kortum (1999)).

An important extension to our model would be to consider separately labor and schooling. In this case, future productivity might depend on both these types of effort (depending on the relevance of learning by doing versus formal schooling). We consider that this extended model might account for (i) an increase over time in income per capita growth rates (for the leaders); (ii) in the time series, a better quantitative account of use of time; (iii) in the country cross section, a better qualitative (and quantitative) account of the relationship between income per capita and both labor and leisure.

A substantial modification of our model would be to consider an additional endogenous choice: the possibility of specialization in a more or less narrow set of tasks of production. If productivity gains require specialization, there would be less incentive for individuals with higher productivities to self produce more. This would probably align the model with data better, with respect to the shares of production, both in the time series and cross-sectionally. It would also give a still more relevant role to trade, and therefore, to the technological motive for social capital.

Finally we want to refer that although we applied our theoretical framework to a country setting, other settings could be investigated. The model could be calibrated to regional or city data, or even to "social villages" within a region, e.g. ethnic communities.

2.A Appendix

2.A.1 Proofs

Proof of proposition 14. First we show that equation (2.41) has a solution. Start by setting $a_1 = 0$ in the RHS of (2.41) and (2.42). This yields $c_1^* > 0$ and $a_1^* > 0$. Now set a_1 close to $1/F$, such that $c_1^* = \epsilon$, with $\epsilon > 0$ arbitrarily small. This yields a_1^* arbitrarily close to zero. If (2.41) and (2.42) are continuous in a_1, then by the intermediate value theorem, there exists $a_1^s \in (0, 1/F)$ such that $a_1^*(a_1^s) = a_1^s$. It is straightforward to see that c_1^* is continuous in $a_1 \in [0, 1/F]$. To prove continuity of equation (2.41) we have to show that the denominator of the fraction is always non-zero. We claim that for $\phi < 1$ this is the case, in particular

$$\frac{1}{\beta} > (G-1)\frac{\alpha_0\alpha_1\alpha_2}{(1+\alpha_2 F a_1)^2} c_1^*(a_1). \tag{2.A.1}$$

Manipulating equation (2.41) we can write

$$(G-1)\frac{\alpha_0\alpha_1\alpha_2}{(1+\alpha_2 F a_1)^2} c_1 = \frac{1}{\beta} - \left(\frac{c_1}{a_1}\right)^{1-\rho} \frac{(1-\phi)(1+\alpha_2 F a_1 + \alpha_0 B)(1+\alpha_2 F a_1)}{\phi B (1+\alpha_2 F a_1)^2}.$$

Any interior solution implies $a_1 > 0$ and $c_1 > 0$, thus statement (2.A.1) is true for $\phi < 1$. Since c_1^*, c_2^*, a_0^* are positive continuous functions of a_1 for $a_1 < 1/F$, the last step in proving existence is to show $r^*(a_1^s) > 0$. Manipulating (2.41)-(2.46) and denoting

$$\begin{aligned} L_1(a_1) &\equiv \frac{\alpha_0 B}{(1+\alpha_2 F a_1)} \geq 0 \\ L_2 &\equiv \left(\frac{1-\phi}{\phi\epsilon_0^\rho}\right)^{\frac{1}{1-\rho}} \geq 0 \\ L_3 &\equiv \left[\frac{(1-\tau)^\rho}{p^* + \alpha_0\epsilon_0}\right]^{\frac{1}{1-rho}} \geq 0, \end{aligned}$$

we obtain

$$r^*(a_1) = (1 - F a_1) \times \\ \left\{1 - \frac{(G-1)L_1(a_1) + [1+L_1(a_1)]^{\frac{1}{1-\rho}}\left(L_2 + L_3 G^* \alpha_0 \epsilon_0\right)}{(G-1)\left[1+L_1(a_1)\right] + [1+L_1(a_1)]^{\frac{1}{1-\rho}}\left[1 + L_2 + L_3 G^*(p^* + \alpha_0\epsilon_0)\right]}\right\},$$

which is positive for any $a_1 \in (0, 1/F)$. Next we show that the equilibrium is unique, which amounts to showing that equation (2.41) has a unique solution. Construct the homotopy function $H : [0, 1/F] \times [0, 1] \rightarrow \Re$ with

$$H(a_1, t) = a_1^*(a_1|\alpha_0 = t \times \alpha_0^*) - a_1,$$

where now α_0^* denotes the exogenously specified transaction costs. Setting $t = 0$ reduces to the case without transaction costs, which has a unique solution, as shown in equations (2.47)-(2.51). Also, we have shown that there exists a solution to (2.41), which implies there is a solution for $H(a_1, t) = 0$, for any $t \in (0, 1]$. H is continuously differentiable in a_1 and t, which means that the solution to $H(a_1, t) = 0$ is a continuously differentiable function of t. Combining this with the uniqueness of the solution at $t = 0$ implies that the solution for any $t \in (0, 1]$ is also unique.[13]■

Proof of proposition 15. Suppose a pairwise stable equilibrium exists with $F < N$. Now pair up two disconnected agents i and j. Reducing the time each of these agents invests in r by a small amount ϵ corresponds to a finite decrease in utility of approximately $\theta\epsilon$ (envelope theorem). The increase in utility from the tie corresponds however to ∞ when $\epsilon \rightarrow 0$, as long as agents have a preference for ties ($\phi < 1$). Thus $F < N$ cannot correspond to a pairwise stable equilibrium. Now consider that $F = N$. Since in this model any agent could unilaterally sever a tie at zero cost (but does not, as implied by the existence result for any F, including $F = N$), then this equilibrium is (trivially) pairwise stable.■

Proof of proposition 16. It is sufficient to show that $\partial(Fa_1)/\partial F > 0$. Using equation (2.50), we obtain

$$\begin{aligned} &\frac{\partial(Fa_1)}{\partial F} > 0 \Leftrightarrow a_1 + F\frac{\partial a_1^*}{\partial F} > 0 \Leftrightarrow \\ &a_1 - Fa_1^2 > 0 \Leftrightarrow a_1 < 1/F, \end{aligned}$$

which is always true.■

Proof of proposition 17. Using equations (2.34)-(2.40), combined with the assumption of one trade partner with given terms of trade p^*, effort r is given in closed-form by

$$r = 1 - \frac{B}{B + \frac{H_1}{H_2}},$$

where

$$\begin{aligned} H_1 &= \left(\frac{\beta B\phi}{1-\phi}\right)^{\frac{1}{1-\rho_{c,s}}} \left[G + G^*\left(\frac{1-\tau}{p^*}\right)^{\frac{\rho_c}{1-\rho_c}}\right] \times \\ &\times \left[G + G^*(1-\tau)^{\frac{1}{1-\rho_c}}\left(\frac{1}{p^*}\right)^{\frac{\rho_c}{1-\rho_c}}\right]^{\frac{(\rho_{c,s}-\rho_c)}{\rho_c(1-\rho_{c,s})}} \end{aligned}$$

[13] See Garcia and Zangwill (1982) for a textbook introduction to the use of homotopies in solving equilibrium problems.

$$H_2 = \left[F + \left(\frac{1}{\beta}\right)^{\frac{\rho_s}{1-\rho_s}}\right]^{\frac{(\rho_{c,s}-\rho_s)}{\rho_s(1-\rho_{c,s})}} \left[F + \left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho_s}}\right].$$

It is true that

$$\text{sign}\left\{\frac{\partial r}{\partial F}\right\} = -\text{sign}\left\{\frac{\partial H_2}{\partial F}\right\}.$$

Computing the derivative of H_2 with respect to F we obtain

$$\begin{aligned}\frac{\partial H_2}{\partial F} &= \frac{(\rho_{c,s}-\rho_s)}{\rho_s(1-\rho_{c,s})}\left[F + \left(\frac{1}{\beta}\right)^{\frac{\rho_s}{1-\rho_s}}\right]^{\frac{(\rho_{c,s}-\rho_s)}{\rho_s(1-\rho_{c,s})}-1} \left[F + \left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho_s}}\right] + \\ &+ \left[F + \left(\frac{1}{\beta}\right)^{\frac{\rho_s}{1-\rho_s}}\right]^{\frac{(\rho_{c,s}-\rho_s)}{\rho_s(1-\rho_{c,s})}},\end{aligned}$$

which is negative if and only if expression (2.52) in the proposition holds.■

Proof of proposition 18. We begin by proving point 1. Setting $\phi = 1$ in equation (2.41), the equality can only be verified if

$$c_1 = \frac{(1+\alpha_2 F a_1)^2}{(G-1)\alpha_0\alpha_1\alpha_2\beta}.$$

Combining this with equation (2.42), we obtain:

$$\begin{aligned}\frac{(1+\alpha_2 F a_1)^2}{(G-1)\alpha_0\alpha_1\alpha_2\beta} = B(1-Fa_1)\Bigg\{(G-1)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right] + \\ + \left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]^{\frac{1}{1-\rho}}\left\{1+G^*\left(\frac{1-\tau}{p^*+\alpha_0 B}\right)^{\frac{\rho}{1-\rho}}\right\}\Bigg\}^{-1}\end{aligned}$$

The above expression allows us to write the following inequality:

$$(1+\alpha_2 F a_1) \leq \frac{(G-1)(1-Fa_1)\alpha_0 B\alpha_1\beta\alpha_2}{(G-1)\left[1+\frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right] + (1+\alpha_0 B)^{\frac{1}{1-\rho}}\left[1+G^*\left(\frac{1-\tau}{p^*+\alpha_0 B}\right)^{\frac{\rho}{1-\rho}}\right]},$$

which simplified with respect to Fa_1 yields expression (2.53) for $\overline{SC}$. Next we prove point 2. If $\rho = 0$ and $\phi = 1$, we can manipulate (2.41)-(2.42) such that an interior solution for a_1 can be written as the solution to the following quadratic equation:

$$\begin{aligned}&a_1^2(\alpha_2 F)^2(G+G^*) + a_1(\alpha_2 F)\left[(2+\alpha_0 B)(G+G^*) + \alpha_1\beta\alpha_0 B(G-1)\right] + \\ &(1+\alpha_0 B)(G+G^*) - \alpha_1\beta\alpha_2\alpha_0 B(G-1) = 0 \qquad (2.A.2)\end{aligned}$$

Since the terms in a_1^2 and a_1 in the LHS of (2.A.2) are both positive (implying a positive slope for the $a_1 > 0$ region), then a necessary condition for $a_1 > 0$ to be a solution is that the constant term be negative, i.e.

$$(1 + \alpha_0 B)(G + G^*) - \alpha_1 \beta \alpha_2 \alpha_0 B(G - 1) \leq 0,$$

which simplified with respect to α_2 yields expression (2.54). This condition is sufficient for the existence of an interior a_1 as long as $a_1 < 1/F$. Setting $a_1 = 1/F$ the LHS of (2.A.2) simplifies to

$$(G + G^*)(1 + \alpha_2)(1 + \alpha_0 B + \alpha_2),$$

which is always positive. This implies that $a_1 < 1/F$.■

Proof of proposition 19. Using equations (2.41) and (2.42), we can write

$$a_1 = \frac{B(1 - Fa_1)}{\left\{(G-1)\left[1 + \frac{\alpha_0 B}{(1+\alpha_2 F a_1)}\right]^{\frac{-\rho}{1-\rho}} + D\right\}} \left[\frac{\left(\frac{1-\phi}{\phi B}\right)}{1/\beta - (G-1)\frac{\alpha_0\alpha_1\alpha_2}{(1+Fa_1)^2}c_1}\right]^{\frac{1}{1-\rho}}, \qquad (2.A.3)$$

where

$$D \equiv 1 + \left(\frac{1-\phi}{\phi B^\rho}\right)^{\frac{1}{1-\rho}} + G^* \left(\frac{1-\tau}{p^* + \alpha_0 B}\right)^{\frac{1}{1-\rho}}.$$

It follows that the following inequality holds:

$$a_1 \geq \left(\frac{1-\phi}{\phi B}\right)^{\frac{1}{1-\rho}} \frac{B(1 - Fa_1)\beta}{(G - 1 + D)}$$

Simplifying the above with respect to a_1 and multiplying both sides of the inequality by F yields (2.55). To show that for the log-utility case with $\alpha_1 = 0$ the lower bound coincides with the actual social capital, we use equation (2.A.3) to obtain a closed-form expression for a_1:

$$a_1 = \frac{B(1 - Fa_1)}{\left[(G-1) + 1 + \left(\frac{1-\phi}{\phi}\right) + G^*\left(\frac{1-\tau}{p^*+\alpha_0 B}\right)\right]} \left[\frac{\left(\frac{1-\phi}{\phi B}\right)}{\frac{1}{\beta}}\right],$$

which simplifies to

$$a_1 = \frac{\beta}{\left[G + G^*\left(\frac{1-\tau}{p^*+\alpha_0 B}\right)\right]\left(\frac{\phi}{1-\phi}\right) + 1 + \beta F}.$$

Multiplying both sides of the expression above by F yields the expression for the lower bound.■

2.A.2 Summary and analysis of data on social ties

The purpose of this Appendix is to provide more disaggregated information on the association between social ties and economic development. We would prefer to work with income as our measure of economic development at all levels of aggregation. However, data on income at the individual level is not consistent across countries, and therefore we use degree of education as our measure of economic development at the individual and regional levels (except in the case in which we analyze individual data within the United States). The findings are the following. At the individual level, (i) there is a strong negative association between number of family ties and economic development; (ii) there is a strong negative association between average visits to nuclear family members and economic development; (iii) there is a positive association between number of close friends and economic development; (iv) there is a strong negative association between average visits to close friends and economic development; (v) there is a strong positive association between belonging to a secondary association and economic development; (vi) there is a weak positive association between frequency of participation in secondary associations and economic development. At the regional level, observations (i) to (v) are confirmed; observation (vi) is no longer statistically significant. At the national level, observations (i) to (v) are confirmed if the dependent variable is income per capita; observation (vi) is no longer statistically significant. Two additional findings are (vii) in general, country fixed effects diminish the size of the coefficients, which indicates the presence of a national factor on individual observables; and (viii) the degree of association between measures of social ties and of development are stronger (economically) with more aggregate data.

In our view, these results provide support to the way we introduced social ties into the traditional microeconomic framework. In particular, there is a consumption component (ties enter the utility function) and a cultural component (degree of communitarianism selects which equilibrium is played). In data it also seems to be the case that there are two relatively independent components in social ties one at a regional or national level, the other at an individual level.

The best systematic evidence of the relationship between social ties and economic development we were able to obtain is provided by the International Social Survey Programme (ISSP).[14] This program includes two chapters on social networks, one for the year 1986 and another for the year 2001. Individuals in many countries were asked, among other questions, about number and frequency of contacts with different types of agents: family members, friends and civic institutions. Using the dataset for the year 2001, which includes 30 countries, more than 350 regions, and around 37,000 individuals, we explored the degree of association between social ties and economic development. We regressed measures of economic development (education or income per capita) on measures of social ties (number of members in group, frequency of visits, frequency of other type of contact, and belonging and frequency of participation in secondary associations). This exploration can be done at different levels of analysis:

[14]Webpage: www.issp.org

individual, regional and national, where the last two imply taking averages of the relevant variables across geographical units. Tables 2.11, 2.12 and 2.13 present the results of this analysis.[15]

The first column in table 2.11 records the association between degree of education (from incomplete primary to complete university) and social ties for the whole sample of individuals. In this regression we controlled for sex, age, and marital status of the individual. The first measure of social ties is the frequency of visits to their mother. A higher value for this variable implies a higher frequency of visits. We observe that a higher degree of education is negatively associated with frequency of visits, with a coefficient value of -0.064. The number below the coefficient indicates the t-statistic, while the third value indicates the number of observations on which the regression is run. In this case, the coefficient is statistically highly significant. Economically, a decrease in visits from "daily" to "less often than several times a year" (from 2 to 7) is associated with around one third of an upward change in the degree of education (say, from complete primary to incomplete secondary).

The second measure of social ties is frequency of "other contacts with mother, other than visiting", which includes contacts by telephone, letter, or e-mail. The coefficient is 0.086 and it is highly significant, which indicates that the frequency of other contacts is positively associated with degree of education, with a decrease in visits from "daily" to "less often than several times a year" (from 2 to 7) associated with around 0.4 of an upward change in the degree of education (say, from complete primary to incomplete secondary).

Putting the two associations in the preceding two paragraphs together, we observe that, with development, individuals shift their technology of building ties with their mother, from explicit visiting to long-distance contact. The next six variables show that this shift occurs for contacts with all other members of the nuclear family: father, brother or sister, and son or daughter.

The bottom part of the table shows that the number of members in nuclear family (F_{nf}) and the average (across members) strength of ties ($a_{1,nf}$) are both negatively associated with degree of education.

The second column of table 2.11 shows the degree of association between social ties and degree of education (as in column one) but including country fixed effects. In general, the effect of including country dummy variables is to keep the sign of the coefficient, but to diminish the degree of association between the variables. For example, the frequency of visits to mother is less strongly related to degree of education with country fixed effects than without them (the coefficient changes from -0.064 to -0.047). This might indicate that there are two components in the degree of association between social ties

[15] In this section we take an agnostic view on causal relationships; we think all of the following coefficients as partial correlation coefficients. In previous sections, where we presented a model and its solution, we took a position, considering effects in both directions: the number of ties effects economic development through an allocation of time, and economic development effects the average strength of ties by changing the relative productivity of effort. A reader who does not agree with that argument might profit nevertheless from this section.

Level of analysis	Individual				Regional		National	
Dependent variable	Education			Income	Education		Education	Income
Countries	All	All	US	US	All	All	All	All
Visits mother	-0.064*	-0.047*	-0.075*	-878	-0.372*	-0.218*	-0.138	-5415**
	-11.86	-9.54	-3.56	-1.78	-6.40	-5.11	-0.49	-1.93
	19199	19199	737	486	248	248	29	27
Other contacts mother	0.086*	0.073*	0.043	264	0.195*	0.16*	.275	7850*
	13.01	12.13	1.67	0.46	3.24	3.99	1.28	4.15
	14564	14564	673	453	248	248	29	27
Visits father	-0.042*	-0.029*	-0.018	-573	-0.346*	-0.160*	-0.075	-3614
	-7.28	-5.62	-0.76	-1.08	-6.47	-3.92	-0.30	-1.39
	13988	13988	570	382	248	248	29	27
Other contacts father	0.066*	0.072*	0.074*	395	0.049	0.10*	0.149	7050*
	9.20	11.33	3.12	0.75	0.77	2.62	0.62	2.88
	10785	10785	536	364	248	248	29	27
Visits brother or sister	-0.133*	-0.073*	-0.115*	-510	-0.586*	-0.370*	-0.489	-7322*
	-24.47	-14.59	-5.23	-0.96	-8.51	-5.21	-1.61	-2.19
	25443	25443	944	553	248	248	29	27
Other contacts brother or sister	0.034*	0.045*	-0.023	-489	0.050	0.270*	0.342	6573**
	5.31	7.73	-0.92	-0.80	0.58	4.68	1.12	2.02
	23221	23221	918	539	248	248	29	27
Visits son or daughter	-0.057*	-0.016*	-0.062**	153	-0.302*	-0.005	-0.361	-8704*
	-8.28	-2.48	-1.97	0.21	-4.89	-0.10	-1.31	-3.41
	14189	14189	435	196	248	248	29	27
Other contacts son or daughter	0.093*	0.079*	-0.060	-489	0.163**	0.190*	0.393	8213*
	9.23	8.49	-1.28	-0.48	2.41	5.12	1.48	3.07
	9367	9367	356	152	248	248	29	27
F_{nf}	-0.105	-0.086	-0.197	-258	-0.138	-0.194	-0.145	-2729
	-38.48*	-33.05*	-8.93*	-0.95	-3.72*	-7.59*	-1.22	-1.96**
	31964	31964	1138	647	248	248	29	27
$a_{1,nf}$	-0.119*	-0.072*	-0.110*	-936	-0.479*	-0.321*	-0.251	-6815*
	-23.24	-15.03	-4.90	-1.74	-7.28	-4.94	-0.85	-2.35
	31040	31040	1111	636	248	248	29	27
Country dummies	no	yes	-	-	no	yes	-	-

Table 2.11: Education, income and nuclear family ties. For each variable, the upper number is the coefficient of the regressor, the second number is the t-statistic and the third number is the number of observations used in the regression. All regressions controlled for age and sex. The regression at the individual level controlled for marital status. The regression at the individual level, with income as the dependent variable, controlled for number of hours worked. * means the variable is statistically significant at the 0.01 level. ** means the variable is statistically significant at the 0.05 level. Education is measured by the degree obtained. A higher number implies a higher degree. Income is measured in US$ 2000.

and economic development: (i) an intra-country component and (ii) and inter-country component, with the second one reinforcing the first one. The second component might be associated to national cultures.

The third column shows the degree of association between measures of social ties and degree of education for individuals of the United States. The sign of associations as well as the point estimates are similar to the ones of the whole sample with country dummies, but the statistical significance is smaller as the number of observations is much smaller.

The fourth column shows the degree of association between measures of social ties and income of the respondent for individuals of the United States. (At the individual level, for the world-wide sample, we did not conduct regressions with income of the respondent as the dependent variable because, as income was reported in national currency, and it seemed to be related to different periods of time (monthly, annually), it was not clear we could compare it across countries). The point estimates have, in general, the sign of previous columns, but they are not statistically significant.

The fifth and sixth columns present associations between social ties measures and degree of education at the regional level. The differences in economic development and social ties can be large between regions of the same country; think of the northeast and south of the US, or north and south of Italy. For most countries the survey provides a variable that indicates the region of the country where the individual lives. The regional variable is one of a few that is decided by the national agency that conducts the survey; therefore the number of regions varies greatly across countries. For example, Hungary includes twenty regions while Brazil includes five. We obtained more than 350 regions, though only 248 presented information on degree of education.

The fifth column presents the associations at the regional level without country fixed effects. The signs of the coefficients are all equal to the signs of the coefficients at the individual level analysis. The point estimates of the coefficients are, in general, larger than the ones at the individual level analysis. For example, a decrease in the frequency of visits to mother from "daily" to "less often than several times a year" (from 2 to 7) is associated with almost two notches up in degree of education (say, from complete primary to complete secondary). Again, these coefficients are, in general, reduced by the inclusion of country dummies (sixth column).

The seventh and eighth columns present associations between social ties measures and economic development at the national level. The signs of the coefficients, either in column seventh where the dependent variable is degree of education, or in column eighth where the dependent variable is income per capita, are equal to the signs in both the individual and regional level analysis. The sizes of the point estimates in the seventh column do not differ much from the ones for the regional analysis, but they are mostly not statistically significant, due to the smaller number of observations. The point estimates in the eighth column are, in general, both large and statistically significant. For example, a decrease in number of visits to mother from "daily" to "less often than several times a year" (from 2 to 7) is associated with an increase in income per capita of more than 25 thousands dollars. (The country with the highest income per capita is the US with almost 35 thousand dollars (of the year 2000), while the country with the lowest income per capita is the Philippines with less than 4 thousand dollars (of the year 2000)).

The negative associations between number of family ties and average strength of family ties and income per capita can be observed, at the national level, in figure 2.12.

Table 2.12 shows the association between measures of friends' ties with economic development. In the first column we observe, first, that the number of close friends, either at

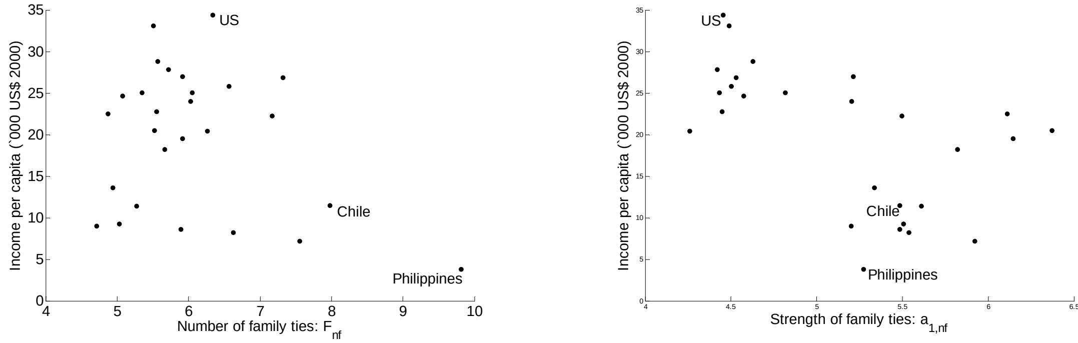


Figure 2.12: Left panel: plots the number of nuclear family ties vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is −0.28, and it is statistically significant at the 15% level. Right panel: plots the average intensity of nuclear family ties vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is −0.56, and it is statistically significant at the 1% level.

work or outside of work, reported by the individuals, is positively associated with the degree of education. Second, the frequency of visits to the closest friend is negatively associated to the degree of education, where a change from "daily" to "less often than several times a year" (from 2 to 7) implies almost one higher degree of education (say, from complete primary to incomplete secondary). Third, other type of contact with the closest friend (by telephone, letter or e-mail) is positively associated with the degree of education. All of these associations are statistically significant. Again, with friends, as with family, there is a shift from visiting to long-distance contact as the education level increases.

With friends, as with family, we obtain smaller size of coefficients if we control for country fixed effects, indicating the possibility of a cultural component, and we obtain higher degree of association if the analysis is done at the regional level. In this case, for example, a decrease in the frequency of visits to the closest friend from "daily" to "less often than several times a year" is associated with an increase of almost three notches up in the degree of education (say, from complete primary to university incomplete). And, again, the point estimates of the degree of association between these ties and income per capita at the national level are large. For example, a decrease in number of visits to friends from "daily" to "less often than several times a year" is associated with an increase in income per capita of more than 50 thousands dollars.

Level of analysis	Individual				Regional		National	
Dependent variable	Education			Income	Education		Education	Income
Countries	All	All	US	US	All	All	All	All
No. close friends F_f	0.024*	0.013*	0.042*	0.8	0.073*	0.005	0.050	2075*
	16.31	9.94	6.92	0.01	3.64	0.26	0.65	2.64
	30361	30361	1124	640	248	248	29	27
No. close friends at work	0.037*	0.006	0.007	389	0.120**	-0.050	0.078	568
	9.65	1.64	0.51	1.38	2.49	-1.38	0.37	0.26
	20858	20858	750	644	248	248	29	27
Visits close friend $a_{1,f}$	-0.185*	-0.119*	-0.100*	-1216**	-0.538*	-0.230*	-0.656**	-10325*
	-32.34	-22.53	-4.77	-2.47	-6.91	-3.28	-2.09	-2.99
	27278	27278	1074	622	248	248	29	27
Other contacts close friend	0.087*	0.076*	0.002	323	0.345*	0.295*	0.386	9414*
	17.16	16.61	0.10	0.63	5.27	6.40	1.63	4.57
	26273	26273	1048	606	248	248	29	27
Country dummies	no	yes	-	-	no	yes	-	-

Table 2.12: Education, income and close friends ties. See notes in table 2.11.

Figure 2.1 in the introduction to the paper shows the association between number of close friends and strength of friends ties (in terms of visits) with income per capita at the national level.

Table 2.13 shows the associations between secondary associations' ties (also known as civic society) and economic development. For each type of association (political party, union, clubs, etc.) we have a variable indicating belonging to the association, and another indicating frequency of participation in the association (conditional on belonging). At the individual level, belonging is generally positively associated with degree of education. The only case hat goes in the opposite direction is for religious organizations. These effects are statistically and economically strong. For example, belonging to a club is associated to having almost one half higher notch in degree of education. The association between frequency of participation and degree of education is less clear. It is positive and statistically significant for political parties, unions, sport clubs, and other associations, but statistically insignificant for the rest.

Again, in the second column, we find that the size of the coefficients are, in general (but not always), smaller with the inclusion of country fixed effects. Interestingly, the only coefficients that change sign between column 1 and column 2 are the ones related to belonging in religious organizations and attendance to religious services: while without country dummies the association between belonging to religious organizations and degree of education is negative, with country dummies this association turns positive. In other words, within countries, on average, there is a positive association being part of religious organizations and degree of education, but individuals in countries with lower formal education participate more actively in religious organizations than individuals in countries with higher formal education. In this unique case the aggregate component (possibly the national culture) does not reinforce, but undermines the individual component.

In the bottom part of the table we find a summary of the relationship between secondary associations ties and economic development. We compute the overall participation in associations (F_{sa}) and the average frequency of participation ($a_{1,sa}$). It is clear that there is a strong and positive association at all levels of aggregation between membership and economic development. By contrast, there is little association between frequency of participation and level of economic development as most coefficients in that last row are statistically insignificant. Figure 2.13 shows the association between belonging to secondary associations and strength of those ties with income per capita, at the national level.

Level of analysis	Individual				Regional		National	
Dependent variable	Education			Income	Education		Education	Income
Countries	All	All	US	US	All	All	All	All
Political party								
Belong	0.136*	0.248*	0.395*	3547**	1.11*	-0.464	1.83	41213**
	5.44	11.08	5.21	2.05	2.71	-1.28	1.30	2.26
	30328	30328	1131	64	248	248	29	27
Frequency	0.179*	0.105*	0.042	-3625**	-0.177**	-0.021	-0.729	5264
	6.84	4.26	0.49	-2.11	-2.01	-0.47	-1.41	0.86
	3709	3709	284	165	234	234	29	27
Union								
Belong	0.641*	0.515*	0.453*	8767*	1.02*	0.619**	1.60	34939*
	32.3	28.5	5.85	5.30	4.30	2.17	0.96	3.63
	30373	30373	1131	643	248	248	29	27
Frequency	0.097*	0.112*	-0.077	-742	-0.205**	-0.065	-0.768	-1693
	4.65	5.89	-0.98	-0.38	-1.93	-1.12	-1.39	-0.26
	6542	6542	276	183	247	247	29	27
Religious org.								
Belong	-0.143*	0.096*	0.243*	1395	-0.06	-0.355**	-0.688	15853**
	-8.12	5.69	3.49	0.87	-0.32	-2.04	-0.94	2.05
	30500	30500	1132	643	248	248	29	27
Frequency	0.016	0.125*	0.203*	-543	-0.084	0.048	-0.438	-2969
	0.87	7.69	3.62	-0.43	-0.83	0.79	-0.99	-0.55
	9414	9414	716	413	248	248	29	27
Sports/hobby club								
Belong	0.415*	0.313*	0.502*	2950**	1.06*	0.252	0.961	35742**
	23.55	29.22	7.44	1.92	4.89	0.99	1.11	4.46
	30750	30750	1132	643	248	248	29	27
Frequency	0.153*	0.085*	0.101	2052	0.286**	-0.050	0.456	13835**
	7.32	4.46	1.16	0.90	1.88	-0.60	0.72	2.11
	9359	9359	439	267	248	248	29	27
Charitable org.								
Belong	0.517*	0.415*	0.481*	3960**	2.15*	0.573	1.81	53514*
	21.45	19.33	6.98	2.50	6.17	1.68	1.36	3.33
	29070	29070	1131	642	233	233	27	26
Frequency	0.042	0.032	0.07	1689	0.003	-0.006	-0.24	3511
	1.50	1.21	0.92	0.79	0.06	-0.07	-0.49	0.61
	3998	3998	382	222	229	229	27	26

Table 2.13: Education, income and secondary associations ties. See notes in table 2.11.

Level of analysis	Individual				Regional		National	
Dependent variable	Education			Income	Education		Education	Income
Countries	All	All	US	US	All	All	All	All
Neighborhood org.								
Belong	0.214*	0.180*	0.326*	3186	0.619**	0.615**	0.658	24365**
	9.40	8.69	3.92	1.61	1.94	1.99	0.56	1.94
	30343	30343	1132	643	248	248	29	27
Frequency	-0.006	0.02	-0.126	-4995	-0.201**	-0.073	-0.136	2097
	-0.21	0.83	-1.24	-1.71	-1.96	-1.39	-0.26	0.36
	4671	4671	228	115	240	240	29	27
Other associations								
Belong	0.573*	0.387*	0.566*	6666*	2.02*	1.05*	2.1	59265*
	25.97	19.36	7.96	4.09	6.50	3.30	1.59	5.08
	30112	30112	1132	643	248	248	29	27
Frequency	0.109*	0.069*	0.123	2921	0.025	0.078	0.015	6644
	4.15	2.80	1.55	1.45	0.47	0.76	0.03	1.18
	4851	4851	335	190	245	245	29	27
Religious services								
Frequency	-0.058*	0.013*	0.055*	-513	-0.14*	-0.083	-0.204**	-3470
	-12.22	2.78	3.14	-1.25	-2.81	-1.57	-1.93	-1.65
	28722	28722	1133	645	239	239	28	26
Union membership								
Belong	0.592*	0.344*	0.013	5703**	-0.175	0.713*	0.72	-1894
	27.60	16.63	0.11	2.43	-0.76	4.99	1.34	-0.29
	22115	22115	757	441	247	247	28	26
F_{sa}	1.04*	1.09*	1.18*	12235*	1.953*	1.006**	1.76	63375*
	28.30	31.48	9.95	4.35	4.81	1.93	1.24	4.28
	31964	31964	1138	647	248	248	29	27
$a_{1,sa}$	0.030*	0.051*	0.118**	-1681	-0.098	-0.095	-0.508	3558
	2.08	3.99	1.99	-1.17	-0.65	-0.96	-.84	0.53
	19682	19682	939	539	248	248	27	27
Country dummies	no	yes	-	-	no	yes	-	-

Table 2.13: Cont.: education, income and secondary associations ties. See notes in table 2.11.

The point estimates of the degree of association between belonging to secondary association and income per capita, at the national level, are again large. For example, an increase from "never" to "more than twice in a year" participation in activities of a political party is associated with an increase of almost 40 thousands dollars (of the year 2000). In economic terms, these associations are highly significant.

We will now summarize the results of this analysis. With economic development, we observe (i) a decrease in number of nuclear family members; (ii) a decrease in frequency of visits to members of nuclear family; (iii) an increase in other, long-distance contacts with members of nuclear family and friends; (iv) an increase in number of close friends; (v) a decrease in frequency of visits to closest friend; (vi) an increase in memberships to secondary associations; (vii) a weak increase in frequency of participation in secondary organizations. An additional result of the analysis is, (viii) the degree of association between social ties and economic development is stronger when we pass from the individual level to a regional level analysis; this might indicate a regional (or national)

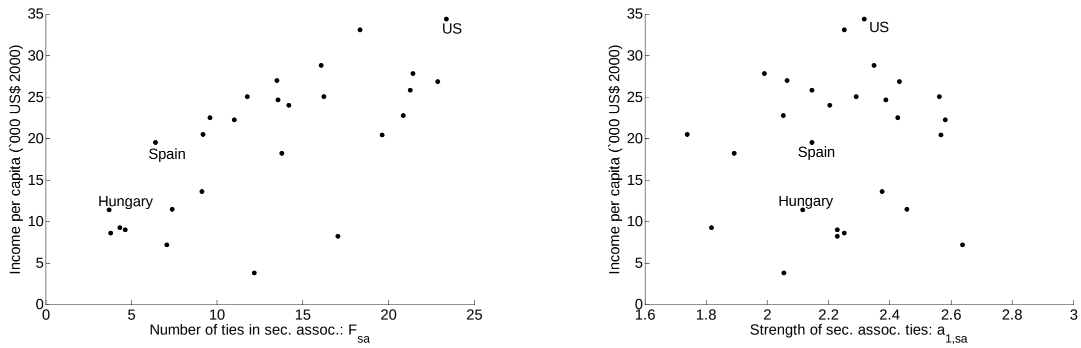


Figure 2.13: Left panel: plots the number of secondary associations' ties vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is 0.69, and it is statistically significant at the 0.1% level. Right panel: plots the average intensity of secondary associations' ties vs. income per capita in 2000, for a group of 27 countries; the correlation coefficient is 0.09, and it is not statistically significant.

cultural component.[16]

[16] We can set these stylized facts next to other pieces of evidence to obtain a clearer picture. These other pieces of evidence are: (i) a decrease in extended family size with economic development (Goode (1966)); (ii) a decrease in time-intensive participation in secondary associations at high levels of economic development (Putnam (2000): "From the point of view of social connectedness, however, the new organizations are sufficiently different from classic "secondary associations" that we need to invent a new label – perhaps "tertiary associations". For the vast majority of their members, the only act of membership consists in writing a check for dues or perhaps occasionally reading a newsletter. Few ever attend any meetings of such organizations –many never have meetings at all– and most members are unlikely ever knowingly to encounter any other member", p. 52); (iii) an increase in the number of these "tertiary associations" at high levels of economic development (Putnam (2000), p.49). Let's now divide the social ties in five group types: (i) nuclear family (nf), (ii) extended family (ef), (iii) friends (f), (iv) secondary associations (sa), (v) and tertiary associations (ta). Let's assign the variable F_i to the network size of group type i, and the variable a_i to the network density (mean strength of ties) of group type i (where $i = nf, ef, f, sa$ or ta). Let's associate strength of tie in a bilateral relationship with time invested in that relationship. Then the total time dedicated to group i will be: $F_i a_i$. Let's assume plausibly that $a_{nf} > a_{ef} > a_f > a_{sa} > a_{ta}$, at any point in time or level of economic development. The observations above indicate that, with economic development: (i) the network sizes of groups with strong ties (F_{nf} and F_{ef}) diminish, and the network sizes of groups with weak ties (F_f, F_{sa} and F_{ta}) increase; (ii) the network densities of family and friends (a_{nf}, a_f) diminish, while the network density of secondary associations (a_{sa}) seems roughly constant. If this is the case, the average strength of tie ($\bar{a} = \frac{\sum_i a_i F_i}{\sum_i F_i}$) will be declining with economic development for two reasons: (i) the diminishing time invested in ties in each group; (ii) a composition effect due to the increase of network sizes of groups with weak ties and decrease in network sizes of groups with strong ties.

2.A.3 Data construction for calibration

The objective of this Appendix is to describe in detail the sources and construction of the data that is offered in tables 2.2 to 2.8.

Table 2.2. Measured GDP: Penn World Table (real GDP per capita; constant prices: chain series). Adjustment GDP for home production: using Table I of Devereux and Locay (1992). Share home production: Table I of Devereux and Locay (1992). Share transaction services: 1965: table 3.13, Wallis and North (1986), averages between estimation I and II, average values for 1960 and 1970; 1985: *idem*, assumed value equal to 1970; both values have to be multiplied by (1 - share home production). Share external market: Penn World Table, openness in current prices, divided by two and multiplied by (1 - share home production) × (1 - share transaction services). Share internal market: residual.

Table 2.3. Market work plus travel, home work, schooling and total leisure: Ramey and Francis (2008); population: age 10 and above. Social activities and television based on percentages of free time informed by Robinson and Godbey (1997), tables 14 and 9 respectively. Labor share in market transaction: 1965: Wallis and North (1986), table 3.1, average 1960 and 1970; 1985: assumed average between *idem* for 1970 and own calculation for 2001, based on ISSP 2001, using Wallis and North (1986) methodology.

Table 2.5. F: own calculations, using ISSP 2001. F_{nf}: sum of variables v4r (number of brothers and sisters), v8r (number of adult sons and daughters), v66r (number of children aged 18 or less) plus 2 for father and mother. F_f: total numbers of friends; sum of variables v23r, v24r and v25r. $F_s a$: if a person participates in a secondary association we assume he has 10 ties in that association; we sum then across 7 types of associations. Income per capita (IPC 2000) for 27 countries: Penn World Table (real GDP per capita; constant prices: chain series). Ratio coefficient of variation in 1990 to 1750: using Lucas (2002), table 5.2. Table 2.14 below offers the means (secondary associations) or medians (nuclear family and close friends) per country.

Table 2.6. Same sources as table 2.3: Ramey and Francis (2008) for years 1900 and 2000. Share of labor in transaction services: Wallis and North (1986) for 1900, table 3.1; ISSP for 2001.

Table 2.7. Constructed with same methodology as table 2.2. Share home production: 1900: assumed to be equal to the one in 1930, informed by Devereux and Locay (1992), table I; 2000: assumed to be equal to the one in 1985, informed by *idem*. Transaction services share: 1900: Wallis and North (1986), table 3.13, average between I and II, multiplied by (1 - share home production); 2000: *idem*, assumed to be equal to 1970, average between I and II. External sector share: 2000: Penn World Table, openness in current prices, divided by two and multiplied by (1 - share home production) × (1 - share transaction services); 1900: Irwin (1996), table I, share of exports in GDP, assumed equal to value in 1913.

Table 2.8. United States, 2000: Ramey and Francis (2008); population: 14 and above. United Kingdom, 2005: ONS (2006), table 2.1, population: 16 and above. Italy, 2002-2003: ISTAT (2007), table 1.1, population: 15 and above. Mexico, 2002:

INEGI (2005), tables 2.2 and 2.17, population: 12 and above.
A table with information on average number of ties per country, used in the simulation, follows.

Country	Type of tie			Income
	Nuclear family	Close friends	Secondary associations	per capita (2000 US$ '000)
United States	6	8	23	34.36
Norway	5	11	18	33.09
Switzerland	5	8	16	28.83
Denmark	5	8	21	27.83
Austria	5	7	14	27.00
Canada	7	7	23	26.82
Australia	6	9	21	25.83
Germany	5	7	16	25.06
France	6	6	12	25.04
United Kingdom	5	9	14	24.67
Japan	6	9	14	23.97
Finland	5	5	21	22.74
Italy	4	4	10	22.49
Israel	6	8	11	22.24
Cyprus	6	6	9	20.46
New Zealand	6	8	20	20.42
Spain	5	4	6	19.54
Slovenia	5	10	14	18.21
Czech Republic	5	6	9	13.62
Chile	8	3	7	11.43
Hungary	5	3	4	11.38
Russia	5	4	4	9.26
Latvia	4	3	5	9.00
Poland	6	5	4	8.61
South Africa	6	3	17	8.23
Brazil	7	8	7	7.19
Philippines	10	7	12	3.83
Correlations				
Friends	0.07	1	-	-
Sec. associations	0.06	0.56	1	-
Income per capita	-0.35	0.58	0.69	1

Table 2.14: Number of ties and income per capita.

Chapter 3

Costly Refocusing and the Diversification Discount

3.1 Introduction

The empirical literature pioneered by Lang and Stulz (1994) and Berger and Ofek (1995) reports that on average, stand-alone firms are worth more than their diversified counterparts. Moreover, the proportion of diversified firms in the economy is high. Coupled together, these two facts suggest an economic puzzle: is this evidence that firms are being inefficient in a systematic way? I present a real options approach that offers a novel rationalization for this regularity. I model an economy which is built up of single-segment *business units*. These modular components are organized either as specialized or diversified firms, according to value-maximizing decisions. The key ingredient in the model is the existence of asymmetric corporate restructuring costs associated with spin-offs (specialization moves) and acquisitions (diversification moves). These costs are the *strike prices* associated with the real options available to the firm. The source and nature of these costs are outside the scope of the paper. My objective is to tentatively provide a useful abstraction with which to frame the diversification discount quantitatively, rather than hypothesize about detailed mechanisms of causation. The argument of the paper is that the source of the diversification discount is simply that the choice to diversify is more costly to reverse that the choice to (re)focus. While this may seem tautological at first, I consider other implications of the asymmetric reversibility, namely in terms of the proportion of diversified firms in the economy, and the average difference in profitability between diversified and focused firms. The model is able to match data in these other dimensions, which I interpret as being supportive of my explanation for the discount. The success of the model is however limited in capturing some time-series characteristics of the data, such as a strong positive contemporaneous correlation between the proportion of diversified firms in the economy and the cross-sectional average excess profitability of diversified firms.

The diversification discount has been the source of a large body of literature.[1] On the empirical side many studies have complemented the results of Lang and Stulz (1994) and Berger and Ofek (1995). Some of these papers use Tobin's Q as a normalized measure for firm value, while others compare assets and sales multipliers. Table 1 in Villalonga (2003) summarizes the estimates of the diversification discount. For authors using Tobin's Q, the (uncorrected) estimates of the diversification value differential range from a minimum of -0.59 (in Servaes (1996)) to a maximum of -0.08

[1] For a survey see Villalonga (2003).

(in Villalonga (2004)). For authors comparing assets and sales multipliers the same estimates range from -15% (in Berger and Ofek (1995)) to 0% (in Lins and Servaes (1999), for Germany). Just for reference, Lang and Stulz (1994) report a discount of -0.54 to -0.27 in terms of Tobin's Q.

On the theoretical side several approaches have taken place. Some papers use agency costs or market inefficiencies to rationalize the discount – see, amongst others, Jensen (1986) and Scharfstein and Stein (2000). More recently some authors have approached this empirical regularity as the result of value-maximizing behavior. Bernardo and Chowdry (2002) use real options combined with learning, while Maksimovic and Phillips (2002) and Gomes and Livdan (2004) link diversification decisions to industry-specific shocks. In Matsusaka (2001), firms diversify in order to leverage organizational capabilities that are not industry-specific. Although my paper is clearly an extension of this strand of the literature, the approach is novel, to the extent that I am using corporate restructuring costs as the key explanatory variable. There are also some recent theoretical papers on mergers and acquisitions that relate to the type of event that is modeled in the paper, however where the diversification discount is not the key research issue. These include Jovanovic and Rousseau (2002), Lambrecht and Myers (2007) and Hackbarth and Morellec (2006). The last two papers also employ real options as a modeling tool.

One of the motivations for this paper is the fact that previous theoretical literature pays little attention to *the weight of diversification in the economy*. If empirical studies were to show a diversification discount but only 1% of firms were diversified, then there would be no puzzle. In fact, what makes the diversification discount a relevant topic of research in financial economics is the possibility that firms are being inefficient on a large scale. The fraction of diversified firms e.g. in Lang and Stulz (1994) is an impressive 60%.

The paper has two theoretical results, namely: (i) in order for the economy to display a diversification discount, then spin-offs need to be more costly than acquisitions, and (ii) this asymmetry necessarily implies that a significant portion of single-segment business units be diversified. Furthermore, a comparative statics exercise yields that the higher the discount, the higher the proportion of diversified firms. This is counter-intuitive at first, but the reason why the model delivers this last effect will later become clear. A direct theoretical link between these two economic aggregates, namely the diversification discount and the proportion of diversified firms, seems relevant and is novel, at least to the best of my knowledge. In Gomes and Livdan (2004), the model also generates a high proportion of diversified firms in the economy, but the link between this magnitude and the diversification discount is left unexplained, since this is not the focus of the paper. Furthermore the authors rely on decreasing returns to scale to generate the discount, which I do not consider in my model.

An intuitive explanation for the theoretical results is that restructuring costs may be interpreted as the inverse of the firm's *propensity to restructure*. High spin-off costs imply that firms have a high propensity to diversify, *relative* to specialize. Thus on average there are more diversified firms. Simultaneously, high spin-off costs make it worthwhile for diversified firms to stay diversified, even when that strategy impacts cash flow negatively. Under appropriate conditions, this effect makes diversified firms less valuable, on average. Note that this is not incompatible with value maximization, since in the model, at some point in time, it was optimal to diversify. However, when the benefits of diversification disappear, it may still be suboptimal to specialize, given the spin-off costs.

I conduct a more detailed calibration exercise, using data on US firms for the period 1984-2005. The sample delivers an average discount that is in line with previous literature, namely 0.34 in terms of Tobin's Q. A striking feature of this sample is that the profitability of diversified firms – measured by the ratio of EBIT to Assets – is on average 6% higher than that of focused firms. This measure was constructed in the same way as the measure for the diversification discount, which accounts for industry effects.[2] While the coexistence of a diversification discount is hard to reconcile with diversified firms being more profitable, the model is able to fit these two dimensions of the data. The finding that the profitability of diversified firms is higher is in line with the findings in Schoar (2002), where diversified firms are shown to be on average more *productive* (+7% than comparable single-segment firms). The fact that diversified firms are on average more profitable seems at odds with the findings of Berger and Ofek (1995). However, the authors use the 2-digit SIC classification to determine whether a firm is diversified or not, while I use the 4-digit SIC classification. This means that some firms classified as focused in Berger and Ofek (1995) are classified as diversified in my sample. Moreover, these firms are the ones conducting a strategy of *related diversification*, which has been shown in the empirical strategy literature to increase profitability – see Rumelt (1974), Bettis (1981), and Amit and Livnat (1988).

[2] This measure is constructed using as a benchmark portfolios of single-segment firms.

Finally I extend the model in two ways: (i) allowing for uncertain strategy implementation (or uncertain real option exercise); and (ii) size heterogeneity. In (i) I investigate how a model where *attempted* diversification moves that are more likely to be implemented than attempted refocusing moves can deliver a diversification discount. The model developed in this extension is somewhat isomorphic to the main model, but the predictions in terms of announcement returns are highly counter-factual. In (ii) I consider the implications of (exogenous) size heterogeneity in terms of (a) the mean size of focused vs. average firms; (b) the proportion of diversified firms conditional on size; and (c) the average discount, also conditional on size. One of the implications of this extension is that bigger diversified firms should trade at a higher discount, which I find to be supported by the data using reduced-form multivariate regressions.

The paper is organized as follows: (i) a stationary model of optimal investment with real options is constructed; (ii) I present a particular case with closed-form solutions; (iii) I solve the model for the general case and conduct a simple calibration / comparative statics exercise; (iv) I conduct a more detailed empirical analysis and calibration, using data from 1984 to 2005; (v) I present two extensions of the main version of the model; and finally (vi) a brief conclusion ends. Some technical aspects of the model are contained in a small appendix.

3.2 Model

The economy is populated by infinitely-lived, industry-specific business units (BUs) that *produce* cash flow. BUs can be organized in two alternative ways: (i) as stand-alone, focused firms, or (ii) as part of a multi-industry, i.e. diversified, corporation. I refer to these two states as the BU's *corporate status*. For simplicity I consider only the possibility of a firm operating in two distinct industries. No intra-industry restructuring options are considered, which implies that restructuring decisions correspond to diversification strategies. Although this may be a partial representation of how firms become diversified, since this may happen via internal growth, it certainly seems to capture the essence of how firms refocus – see Ravenscraft and Scherer (1987), Kaplan and Weisbach (1992), and Campa and Kedia (2002). Graham, Lemmon, and Wolf (2002) also show that acquisitions and diversification are closely related. In their sample almost two thirds of the firms that increase the number of segments implement this strategy via acquisition.

Each BU is paired with a possible diversification partner from another industry. If these two BUs decide to merge, they receive an additional source of cash flow. Specifically, if BUs i and j are paired, their cash flows are given in continuous time, according to the following time rates π:

$$\begin{cases} \pi_{i,t} = K_i w_{i,t} + (K_i + K_j) m_{ij,t} x_{ij,t} \eta_i \\ \pi_{j,t} = K_j w_{j,t} + (K_i + K_j) m_{ij,t} x_{ij,t} (1 - \eta_i) \end{cases} \tag{3.1}$$

K_i is the constant capital stock of BU i. The variable $m_{ij,t}$ is endogenous and denotes an indicator function, with $m = 1$ if the BUs are merged and $m = 0$ otherwise. The variables $w_{i,t}$ and $x_{ij,t}$ are two exogenous components of BU i's profitability (similarly for j). The component $x_{ij,t}$ of profitability in equation (3.1) is only active if the BUs are diversified ($m = 1$). I will refer to x as the source of *synergies* associated with the diversification option. The total amount of synergy cash flow seized by the BU pair if merged at time t is $(K_i + K_j)x_{ij,t}\,dt$. The parameter η_i in equation (3.1) denotes the fraction captured by BU i. These synergies can for instance be related to an increase in cost efficiency or to additional revenues from cross selling. A good example for the latter are mergers involving banks and insurance companies: it seems reasonable to "push" home insurance to a customer entering into a mortgage loan.[3] Other authors rely on the construct of synergy, like Matsusaka (2001) or Gomes and Livdan (2004), to motivate gains from diversification. The real options problem can also be framed as one of switching technologies, which is common in this literature. Here x represents the additional cash flow associated with a different *organizational technology*. A model with this switching feature is in line with what is observed in data: many unrelated acquisitions are later divested.[4]

A cost will be incurred whenever the BU pair changes its corporate status, i.e. when m changes. I assume that these costs are proportional to the combined capital stock and possibly asymmetric: $(K_i + K_j)\theta_u$ if m changes from 0 to 1, and $(K_i + K_j)\theta_d$ in the opposite case. The restructuring costs are borne instantaneously by the current shareholders of the firm; this can be seen as a negative dividend. The magnitude $(K_i + K_j)\theta_u$ could represent the costs of replacing a board of directors, or the investment required to create a new joint brand. In this model $(K_i + K_j)\theta_u$ does not represent the control premium paid to target shareholders. In fact, a control premium is simply

[3] I prefer not to discuss in detail why such a synergy is not't attainable through a contract; informational and strategic aspects play certainly a role here.

[4] See Ravenscraft and Scherer (1987) and Kaplan and Weisbach (1992).

the counterpart of the value created by synergies, and so is embedded in firm value.[5] An illustration for $(K_i + K_j)\theta_d$ would be the investment required to split a common back-office, or the conglomerate loosing control over important strategic information by spinning off the BU. In addition, restructuring costs could possibly represent less direct mechanisms that determine a firm's propensity to diversify. An example would be the classical empire-building theory, suggested in Jensen (1986). There is even literature that is specific enough to state that managers are biased towards diversification.[6] If this is the case and one assumes that all other restructuring costs are approximately symmetric, then the model's prediction of costly spin-offs is reasonable. Finally $(K_i + K_j)\theta_u$ and $(K_i + K_j)\theta_d$ may capture search costs associated with merger activity, in which target and acquirer need to be matched.

In this model, restructuring costs are thus best seen as a latent variable about which one learns indirectly by inspecting magnitudes such as the diversification discount. Naturally it would be interesting to be able to measure these costs *a priori*, but such task is outside the scope of the paper.

For now I normalize K to 1 for all BUs. This means that henceforth, when discussing firm values, this actually refers to the firm's Tobin's Q. A large fraction of the empirical literature on the diversification discount focuses on this magnitude. I also abstract from game-theoretic considerations and focus on first-best outcomes only. With the further assumption that BUs split synergy cash flows evenly ($\eta_i = 0.5$), the initial problem is isomorphic to finding the optimal exercise policies of a switch option for one BU, with cash flow given by

$$\pi_t = w_t + m_t x_t, \tag{3.2}$$

where for notational economy I drop the BU subscript. With symmetric allocation of restructuring costs to each BU in the BU pair, then the switch costs for the isomorphic model with one BU are simply θ_u (θ_d) when m changes from 0 to 1 (1 to 0).

On the demand side of the economy all agents are assumed to be risk-neutral, so it follows that first-best firm value corresponds to maximizing the sum of discounted future expected cash flows. The instantaneous risk-free rate is constant and denoted

[5] The distinction between a control premium and $(K_i + K_j)\theta_u$ stems from the fact that the former is a wealth transfer amongst shareholders, while the latter represents a cost incurred by the firm. Target shareholders will cash in the control premium, but not the amount $(K_i + K_j)\theta_u$. This distinction, although maybe subtle, is material.

[6] See e.g. Denis, Denis, and Sarin (1997).

by r. From the assumptions so far it follows that BU value V_t has two separable components, i.e. $V_t \equiv S_t + J_t$, with

$$S_t = \mathbb{E}_t \left[\int_t^\infty e^{-r(u-t)} w_u du \right] \tag{3.3}$$

$$J_t = \sup_{\{m\}} \left\{ \mathbb{E}_t \left[\int_t^\infty e^{-r(u-t)} m_u x_u \, du - \sum_{\{\tau_u\}} e^{-r(\tau_u - t)} \theta_u - \sum_{\{\tau_d\}} e^{-r(\tau_d - t)} \theta_d \right] \right\}, \tag{3.4}$$

where $\{\tau_u\}$ and $\{\tau_d\}$ represent the set of random stopping times at which a merger or a de-merger takes place, respectively.

The first component S_t is the strict stand-alone value of the firm, while J_t is the value of synergies. Note that J_t captures the value of both the (realized) additional cash flows as well as the value of corporate restructuring flexibility. The endogenous variable $\{m\}$ is the whole sequence of restructuring decisions that the firm will make across time. On a more technical note, $\{m\}$ is a stochastic process adapted to the filtration generated by x.

It has been established empirically that both productivity and profitability are mean-reverting (see Moyen (1999) or Fama and French (2000)). Hence I assume that both w_t and x_t are mean-reverting, which implies that S_t and J_t are also mean-reverting. I assume that both w_t and x_t follow (arithmetic) Ornstein-Uhlenbeck (OU) processes:

$$dw = \kappa_w(\mu_w - w)dt + \sigma_w d\bar{B}_{w,t} \tag{3.5}$$

$$dx = \kappa_x(\mu_x - x)dt + \sigma_x d\bar{B}_{x,t}, \tag{3.6}$$

where $\bar{B}_{w,t}$ and $\bar{B}_{w,t}$ are Brownian motions with correlation ρ.

This is a continuous-time model, hence in equilibrium the following equalities must hold:[7]

$$rS_t dt = w_t dt + \mathbb{E}_t [dS_t] \tag{3.7}$$

$$rJ_t dt = m_t x_t dt + \mathbb{E}_t [dJ_t] \tag{3.8}$$

Applying Itô's lemma to $\mathbb{E}_t [dS_t]$ and $\mathbb{E}_t [dJ_t]$, and using (3.5)-(3.6), one can write (3.7)

[7] See e.g. Dixit and Pyndick (1994) or Shreve (2004).

and (3.8) as ordinary differential equations (ODEs):

$$rS(w) = S_w(w)\kappa_w(\mu_w - w) + \frac{1}{2}S_{ww}(w)\sigma_w^2 + w \tag{3.9}$$

$$rJ(x;m) = J_x(x;m)\kappa_x(\mu_x - x) + \frac{1}{2}J_{xx}(x;m)\sigma_x^2 + mx \tag{3.10}$$

Since there is no optionality associated with w and S, then the particular solution to ODE (3.9) gives us immediately an expression for $S(w)$:

$$S(w) = \frac{\mu_w}{r} + \frac{w - \mu_w}{r + \kappa_w} \tag{3.11}$$

It is straightforward to verify that the functional form above verifies ODE (3.9). Also, the economic intuition for this valuation formula is simple: on average the value of $S(w)$ corresponds to a perpetuity, with fluctuations associated with mean reversion. If the state variable w is above the mean μ_w, then there is an addition to mean value; the opposite holds when $w < \mu_w$. The speed of mean reversion κ_w determines the weight of the random component in $S(w)$.

To obtain J one needs to determine the optimal policy for the exercise of the restructuring options. I start by guessing that there exist boundaries $\overline{x}$ and $\underline{x}$, for which it is optimal to merge (diversify) and de-merge (specialize), respectively. Value-matching, smooth-pasting and transversality conditions applied to the general solutions of (3.10) identify J. In this model, the real options are *infinitely compound*, since for instance the option to merge is the option to acquire additional cash flows plus an option to de-merge; which itself is an option on the option to merge.

The agent homogeneity and stationarity of this simple two-state model echoes the methodological approach of structural macroeconomics. As in that literature, a link to real data is easily achieved by comparing conditional and unconditional magnitudes with the predictions of the model.

The reader who is familiar with this type of model and/or is only interested in the quantitative implications of the theory should skip ahead to section 3.2.2. In what immediately follows I explain the valuation mechanics of the model, and briefly discuss the methodology employed to solve ODE (3.10).

Throughout the remainder of this section I assume $\rho = 0$, which implies S_t and J_t are independent. With this assumption an average difference in valuation between diversified and focused firms can only arise from J_t. I denote the mean of $S(w)$ by μ_S.

3.2.1 A particular case with closed-form solution

Taking $\kappa_x = 0$ yields an analytic solution of ODE (3.10), more amenable to understanding the valuation mechanics of the model. It can easily be shown that the general solution to the simplified ODE is:

$$\begin{aligned} J(x;m=0) &= A_1 e^{\frac{\sqrt{2r}}{\sigma_x}x} + A_2 e^{-\frac{\sqrt{2r}}{\sigma_x}x} && (3.12)\\ J(x;m=1) &= B_1 e^{\frac{\sqrt{2r}}{\sigma_x}x} + B_2 e^{-\frac{\sqrt{2r}}{\sigma_x}x} + \frac{x}{r}, && (3.13)\end{aligned}$$

for any constants A_1, A_2, B_1 and B_2. The component x/r in (3.13) is the particular solution of the (non-homogeneous) simplified ODE and stands for the value of an infinite stream of cash flows when x_t is a zero-drift Brownian motion with starting value x.

From an economic standpoint it is natural to impose the following two boundary conditions:

$$\lim_{x\to-\infty} J(x;m=0) = 0 \qquad (3.14)$$

$$\lim_{x\to+\infty} J(x;m=1) - \frac{x}{r} = 0, \qquad (3.15)$$

where the first means that the option to *switch on* the synergies becomes worthless as x becomes small enough, while the second means that the option to *switch off* the synergies becomes worthless as x becomes big enough. With these two conditions we are now looking for solutions of the form:

$$\begin{aligned} J(x;m=0) &= A e^{\frac{\sqrt{2r}}{\sigma_x}x} && (3.16)\\ J(x;m=1) &= B e^{-\frac{\sqrt{2r}}{\sigma_x}x} + \frac{x}{r}, && (3.17)\end{aligned}$$

for (positive) constants A and B. The value of the option to merge, given by equation (3.16), is increasing in x, while the opposite holds for the option to de-merge, given by the first term in (3.17).

The two value-matching conditions pin down A and B as functions of $\overline{x}$ and $\underline{x}$:

$$J(\overline{x};m=0) = J(\overline{x};m=1) - \theta_u \qquad (3.18)$$

$$J(\underline{x}; m=1) \quad = \quad J(\underline{x}; m=0) - \theta_d \tag{3.19}$$

Only $\overline{x}$ and $\underline{x}$ are left to determine. For this I use the smooth-pasting (first-order) conditions:[8]

$$\left.\frac{\partial}{\partial x}\right|_{x=\overline{x}} [J(x; m=0) - J(x; m=1)] \quad = \quad 0 \tag{3.20}$$

$$\left.\frac{\partial}{\partial x}\right|_{x=\underline{x}} [J(x; m=0) - J(x; m=1)] \quad = \quad 0 \tag{3.21}$$

The set of equations (3.18)-(3.21) identifies A, B, $\overline{x}$ and $\underline{x}$ uniquely. The simplified system of equations (solvable numerically) is presented in the Appendix.

Figure 3.1 illustrates how a diversification spread can be generated within the context of this model. Both panels show that for a sufficiently high x the merged firm (i.e. the firm with $m = 1$) is worth more, on average, in the interval $[\underline{x}, \overline{x}]$. Recall that S_t is assumed to be independent of J_t, so on average a higher synergy value J_t implies a higher total firm value $V_t = S_t + J_t$.

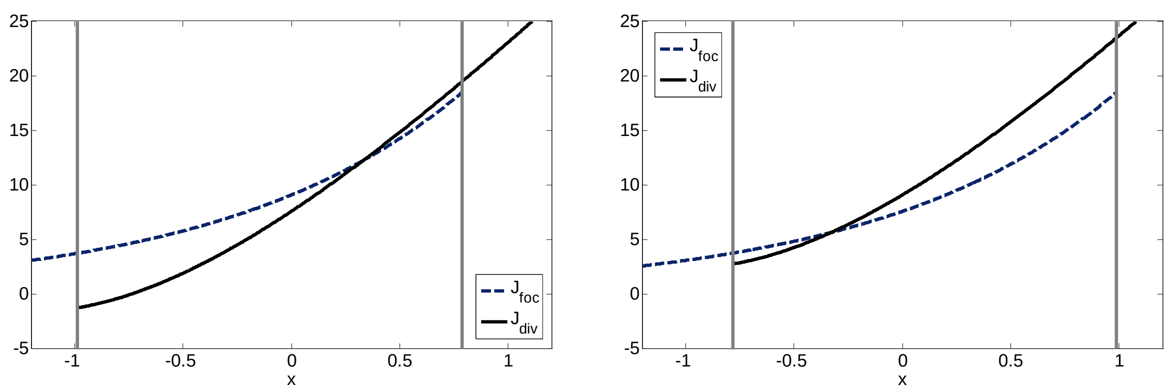


Figure 3.1: Example. The left panel displays a case with $\theta_u = 1$ and $\theta_d = 5$, i.e. spin-offs are more costly than acquisitions. The right panel displays the opposite case with $\theta_u = 5$ and $\theta_d = 1$. The remaining parameters were calibrated equally in both panels: $r = 0.05$ and $\sigma_x = 0.35$. The vertical lines mark the optimal exercise policies, $\underline{x}$ and $\overline{x}$.

For the case where spin-offs are more costly than acquisitions (left panel of Fig. 3.1), the two branches of the value function intercept at a higher x, relative to the

[8] See Dixit and Pyndick (1994) for the logic of smooth-pasting to determine optimal exercise of real options. The second-order conditions are always verified.

opposite case (right panel of Fig. 3.1). Thus the size and relative weight of the restructuring costs drive the pricing differential between specialized vs. diversified BUs, in the interval $[\underline{x}, \overline{x}]$. Note that if there were no restructuring costs, i.e. if $\theta_u = \theta_d = 0$, then $\overline{x}$ and $\underline{x}$ would collapse, which implies that the two branches of the value function would not overlap, but join continuously at the optimal exercise point. I will henceforth refer to the interval $[\underline{x}, \overline{x}]$ as the *overlapping region*. This is the region for which it makes sense to make a *ceteris paribus* comparison between the price of BUs of distinct corporate status.

The economic intuition for this price differential is simply that, in some scenarios, it would be preferable for the BU to have a different corporate status, in the sense that restructuring costs would be circumvented. However, this does not mean that *ex ante* the BU was not maximizing value, rather it means that *ex post* it ran into a less fortunate event. And even an *ex post* judgment on how unfortunate the value-maximizing choice actually was should be made with caution. One has to bear in mind that the dividends already paid out to shareholders are no longer part of firm value, however they indeed accrued to someone's wealth. This is actually a general conceptual problem associated with a methodology where cross-sectional empirical findings underly statements regarding efficiency: cumulative (intermediate) gains are not taken into account.[9]

Although the strict mechanics of firm value are quite evident in this particular case, it is still not a suitable approach to the research issue at hand. Considering $\kappa_x = 0$ not only contradicts the empirical characteristic of productivity, but also makes x not have a stationary distribution. Without the latter it is impossible to discuss an (average) diversification discount.

3.2.2 General case

Solving the model

For ease of exposition I restate equation (3.10) below:

$$rJ(x;m) = J_x(x;m)\kappa_x(\mu_x - x) + \frac{1}{2}J_{xx}(x;m)\sigma_x^2 + mx$$

The above equation actually denotes two distinct ODE's, one for each value of m.

[9]This advocates the use of event studies focusing on realized returns. However, this methodology also has downsides, since returns are very noisy and also one needs to control for risk.

This section briefly presents the method used to obtain a solution for these equations, which constitute the general case of the specified model.

The homogeneous ODE associated with (3.10) is given in its classical notation by (replacing J by y):

$$y'' + y' 2\kappa_x \left(\frac{\mu_x - x}{\sigma_x^2} \right) + y \left(\frac{-2r}{\sigma_x^2} \right) = 0 \tag{3.22}$$

It is easier to make the substitution $z = (x - \mu_x)$ and work with the ODE

$$y'' + y' z \left(\frac{-2\kappa_x}{\sigma_x^2} \right) + y \left(\frac{-2r}{\sigma_x^2} \right) = 0. \tag{3.23}$$

The particular solution of (3.10), denoted as y_p, is easily obtained:

$$y_p(z) = \frac{\mu_x}{r} + \frac{z}{r + \kappa_x} \tag{3.24}$$

Naturally y_p refers to the case $m = 1$, otherwise equation (3.10) is already an homogeneous equation. As in the $\kappa_x = 0$ case, the particular solution to the ODE corresponds to the value of an infinite stream of cash flows when x_t follows the specified OU process with starting value x. Note that setting $\mu_x = \kappa_x = 0$ one obtains $y_p = x/r$. Considering two linearly independent solutions to (3.23), $f_1(z)$ and $f_2(z)$, then it follows that the general solution to (3.10) is:[10]

$$J(x; m = 0) = A_1 f_1(z(x)) + A_2 f_2(z(x)) \tag{3.25}$$

$$J(x; m = 1) = B_1 f_1(z(x)) + B_2 f_2(z(x)) + y_p(z(x)) \tag{3.26}$$

To obtain f_1 and f_2 using a power series is straightforward, since the variable coefficients in ODE (3.23) are polynomial. In fact, the solution can be represented as a Kummer series.[11] But an issue arises. According to the specified process, x can take on negative values. But it does not make sense to consider negative f_1 and f_2, since we are dealing with option values, bounded from below at zero. In this sense I preferred to work with a finite-difference method (FDM), since it allows to identify strictly positive f_1 and f_2 easily. Moreover, one also wishes to apply transversality conditions as before:

$$\lim_{x \to -\infty} J(x; m = 0) = 0 \tag{3.27}$$

[10]See Zill (2001) for an introduction to differential equations.
[11]See e.g. http://eqworld.ipmnet.ru/en/solutions/ode/ode-toc2.pdf

$$\lim_{x\to+\infty} J(x;m=1) - \left[\frac{\mu_x}{r} + \frac{x-\mu_x}{r+\kappa_x}\right] = 0 \tag{3.28}$$

The above transversality conditions look pretty much the same as in the Brownian-motion case, because although the OU process is mean-reverting, the following is true:

$$\mathbb{E}[x_T|x_0] = x_0 e^{-\kappa_x T} + (1 - e^{-\kappa_x T})\mu_x, \tag{3.29}$$

which implies $\lim_{x_0\to+\infty} \mathbb{E}[x_T|x_0] = +\infty$ and $\lim_{x_0\to-\infty} \mathbb{E}[x_T|x_0] = -\infty$ for any finite T. In words, when x drifts towards large absolute values, it will need a very large amount of time (on average) to return to $\underline{x}$ or $\overline{x}$, whichever the case. This renders the optionality value irrelevant, i.e. it tends to zero. In addition, it is also natural to impose that f_1 be monotone increasing and f_2 monotone decreasing. In the case with $\kappa_x = 0$ this was already embedded in the analytic functional forms, which were exponentials. After having f_1 it is easy to obtain f_2, by simply setting $f_2(z) = f_1(-z)$. It is easily checked that $f_1(-z)$ verifies ODE (3.23) as long as f_1 does. Moreover, if f_1 is positive, monotone increasing and $\lim_{z\to-\infty} f_1(z) = 0$, then $f_2(z)$ will be positive, monotone decreasing and $\lim_{z\to\infty} f_2(z) = 0$. The method for finding f_1 is described in the Appendix.

With these specifications of f_1 and f_2 the solution to look for is of the form

$$J(x;m=0) = Af_1(z(x)) \tag{3.30}$$

$$J(x;m=1) = Bf_2(z(x)) + \frac{\mu_x}{r} + \frac{x-\mu_x}{r+\kappa_x}, \tag{3.31}$$

for positive A, B. Again I use the value-matching and smooth-pasting conditions (numerically) to pin down A, B, and the optimal policies $\underline{x}$ and $\overline{x}$. This procedure is detailed in the Appendix.

Comparative statics

The objective of this section is to understand how key parameters, e.g. θ_u and θ_d, drive the model. In order to reduce the degrees of freedom of the model, I calibrate some parameters using results from previous literature. A more detailed structural calibration, as well as analysis of recent data, is conducted in section 3.3.

One of the advantages of working with OU processes is that they have a stationary

Gaussian distribution. The main features of the OU process for synergies x_t are:[12]

1. $\mathbb{E}[x] = \mu_x$
2. $Var(x) = \frac{\sigma_x^2}{2\kappa_x}$
3. $Cov(x_t, x_s) = e^{-\kappa_x|t-s|}\frac{\sigma_x^2}{2\kappa_x}$

However this is not enough for my analysis. A bivariate stationary distribution for x and m needs to be found. This distribution is readily obtained by combining the univariate (unconditional) distribution of x with (i) knowledge of $\underline{x}$ and $\overline{x}$, and (ii) the dynamics of the OU process. The methodology employed is detailed in the Appendix.

The model has seven free parameters, namely μ_S, κ_x, r, σ_x, μ_x, θ_u, and θ_d. I set r at 0.05, following Kydland and Prescott (1982). Several other authors using optimal investment models choose this magnitude for calibration.[13] The (annual) autocorrelation coefficient of x is given by $e^{-\kappa_x}$. Following Moyen (1999) I set this magnitude at 0.6, which implies $\kappa_x = 0.5108$. I am using the assumption that the autocorrelation of x is similar to that of productivity, which seems reasonable; recall that x is one of the components of firm productivity/profitability in the model (see equation (3.2)).

To pin down the remaining parameters I will take magnitudes implied by the model and compare them to data. First I want the model to give a sensible magnitude for the unconditional average of the value of synergies J. To this end I use the following conditions:

$$\mathbb{E}[V] = \mu_S + \mathbb{E}[J] = 2 \tag{3.32}$$

$$\mathbb{E}[J]/\mathbb{E}[V] = \mathbb{E}[J]/(\mu_S + \mathbb{E}[J]) = 0.15 \tag{3.33}$$

Condition (3.32) above links the unconditional average of Tobin's Q in the economy to its empirical counterpart. I set this magnitude at 2, since the median market-to-book ratio for NYSE stocks in 2005 was approximately this size.[14] The value of the RHS of equation (3.33) is less straightforward. Nonetheless I believe that the assumption that on average J is a *marginal* fraction of firm value makes sense. If not, then

[12]For the properties of OU processes see Karatzas and Shreve (1991) or Tapiero (1998) or http://www.puc-rio.br/marco.ind/revers.html

[13]See e.g. Berk, Green, and Naik (1999) and Alti (2003).

[14]Source: Kenneth R. French website.

the conceptual frontier between industries would probably be ill-defined. A range of $10\% - 20\%$ for this magnitude is not shocking, so I set it at 0.15. The system of equations (3.32)-(3.33) implies an unconditional average for J:

$$\mathbb{E}[J] = 2 \cdot 0.15 = 0.3 \tag{3.34}$$

Following the same logic of synergies not being a "structural component" of the firm's business, I set $\mu_x = 0$. Three parameters are left: σ_x, θ_u and θ_d. Considering that equation (3.34) pins down one of these parameters, we require only two additional conditions to fully identify the model. The two magnitudes I will be looking at are: (i) the diversification discount, and (ii) the unconditional average weight of diversified BUs in the economy.

Instead of just looking at the parameters implied by the model for a specific level of the diversification discount and the proportion of diversified BUs, I constructed scenarios varying σ_x and the restructuring costs θ. This exercise of comparative statics will help better understand the mechanics and intuition of the model, and is also justified by the fact that I have little *a priori* guidance for the parameters σ_x, θ_u and θ_d. In addition, although it seems reasonable to state that the sign of the diversification pricing spread is negative, some authors have found diversification premia, after employing more sophisticated econometric techniques (see Villalonga (1999), Villalonga (2004), and Campa and Kedia (2002)). Hence it is also interesting to analyze under which conditions the model implies a diversification premium. The scenario analysis presented below meets this objective too.

It seems intuitive to consider that it is the relation $\theta_u - \theta_d$ that is relevant, and not the parameters' absolute values.[15] Hence, and to simplify the analysis at this point, I consider only three possible variations of corporate restructuring costs, namely: (i) $\theta_u = 0$, i.e. only spin-offs are costly, (ii) $\theta_u = \theta_d$, i.e. spin-offs and acquisitions have symmetric costs, and (iii) $\theta_d = 0$, i.e only acquisitions are costly. In terms of σ_x I used four different magnitudes: 0.05, 0.1, 0.2 and 0.3. This choice is less arbitrary than it seems, as will become apparent later.

Results

[15]In other words, what is assumed to be relevant is the cost of spin-offs *relative* to the cost of acquisitions. In section 3.3 I have to abandon this assumption to match the desired dimensions of data.

Table 3.1 shows, for each scenario, the diversification spread and the proportion of diversified firms in the economy. The former is given by $\mathbb{E}[J|m=1] - \mathbb{E}[J|m=0]$, while the latter corresponds simply to $\Pr(m=1)$.

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	0.0522	0.0635	0.0747
0.10	**−0.0138**	0.0948	0.2058
0.20	**−0.2045**	0.1320	0.4972
0.30	**−0.3956**	0.1544	0.7881

a) Diversification spread

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	0.5088	0.5031	0.4959
0.10	**0.5675**	0.5029	0.4397
0.20	**0.6896**	0.5041	0.3158
0.30	**0.7693**	0.5063	0.2339

b) Proportion of diversified BUs

Table 3.1: Panel a) shows the size of the diversification discount given by $\mathbb{E}[J|m=1] - \mathbb{E}[J|m=0]$. Panel b) shows the proportion of diversified BUs implied by the model, which corresponds to the unconditional probability $\Pr(m=1)$.

It is interesting to note in Table 3.1a) that a diversification discount only obtains for *a high enough volatility combined with costly spin-offs*. Moreover, by inspecting Table 3.1b), the same scenarios necessarily imply *a high proportion of diversified BUs in the economy*, as measured by the unconditional probability of $m = 1$. These two results taken together are counter-intuitive: the higher the discount, the higher the proportion of diversified firms. In what follows I provide the economic intuition for why this obtains.

As a first step it is useful to analyze what is implied in terms of more primitive variables. Table 3.2 presents the level of restructuring costs and the probabilistic mass of the overlapping region $[\underline{x}, \overline{x}]$. The latter measures the intensity of corporate restructuring activity and is a key link between restructuring costs, the diversification discount, and the weight of diversification.

Table 3.2a) shows for each scenario how high the restructuring costs need to be in order to maintain the chosen average value of synergies, i.e. having $\mathbb{E}[J] = 0.3$ (see equation (3.34)). It is clear that for any of the three restructuring cases (columns), a higher σ_x implies higher restructuring costs. This stems from the fact that J has an optionality component, hence a higher volatility leads to a higher value. In order to keep the average option value constant, the restructuring costs must go up. One also observes that the *total* size of restructuring costs is very similar for each column (note that the middle column only represents "half" of the *total* restructuring costs $\theta_u + \theta_d$).

The right panel of Table 3.2 shows the unconditional probability that x is in the overlapping region, the interval $[\underline{x}, \overline{x}]$. The main dispersion in values is due to variation

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	0.0237	0.0118	0.0236
0.10	**0.2390**	0.1215	0.2400
0.20	**0.8140**	0.4380	0.8180
0.30	**1.4359**	0.8100	1.4420

a) Level of restructuring costs

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	0.5448	0.5409	0.5429
0.10	**0.8381**	0.8439	0.8384
0.20	**0.9190**	0.9397	0.9198
0.30	**0.9350**	0.9651	0.9359

b) Size of overlapping region

Table 3.2: Panel a) shows for each value of σ_x what level of θ gives $\mathbb{E}[J] = 0.3$. Panel b) corresponds to the probability of x being in the overlapping region, and is given by the formula $\Phi\left(\frac{\overline{x}}{(\sigma_x/\sqrt{2\kappa_x})}\right) - \Phi\left(\frac{\underline{x}}{(\sigma_x/\sqrt{2\kappa_x})}\right)$, where Φ stands for the cumulative standardized normal distribution.

in volatility. The mechanics for this effect are as follows. A higher volatility implies higher restructuring costs. The dual for the value-maximization problem as framed in the paper is the minimization of these costs. More specifically, a value-maximizing firm minimizes the frequency with which a corporate status shift takes place. For higher restructuring costs, the firm lowers this frequency by lowering $\underline{x}$, increasing $\overline{x}$, or both. This means that corporate status shifts become less probable, which amounts to have a larger probability mass in the overlapping region.

From the discussion so far, the basic intuition for why a discount obtains is clear. For higher spin-off costs there is less incentive of the diversified firm to specialize, although it may be experiencing significantly negative cash flows – this drives value down. There is however a small caveat to this reasoning that will be detailed later on.

The other dimension for which it is relevant to characterize the optimal policies is their degree of asymmetry. This is shown in Table 3.3. Note that high spin-off

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	1.0450	1.0089	0.9652
0.10	**1.1837**	1.0047	0.8528
0.20	**1.5105**	1.0035	0.7281
0.30	**1.5691**	1.0031	0.6667

a) Asymmetry ratio $\underline{x}/\overline{x}$

σ_x	$\theta_u = 0$	$\theta_u = \theta_d$	$\theta_d = 0$
0.05	1.0446	1.0087	0.9657
0.10	**1.5542**	1.0124	0.6594
0.20	**3.6015**	1.0150	0.2813
0.30	**6.5530**	1.0164	0.1519

b) Probabilistic asymmetry ratio

Table 3.3: Panel a) shows the degree of asymmetry in terms of the optimal restructuring policies. Panel b) shows the degree of asymmetry using the ratio of probabilities $\left[1 - \Phi\left(\frac{\overline{x}}{(\sigma_x/\sqrt{2\kappa_x})}\right)\right] / \Phi\left(\frac{\underline{x}}{(\sigma_x/\sqrt{2\kappa_x})}\right)$, where Φ stands for the cumulative standardized normal distribution.

costs imply a bias towards diversification. The intuition is that the firm not only minimizes the frequency of corporate status shifts, but also distorts the symmetry of optimal policies in favor of the less costly restructuring event. If spin-offs are costly but acquisitions are for free, then there will be a tendency towards the latter.

So far I have explained that spin-offs being costly yields (i) a big overlapping region, and (ii) a tendency towards diversification moves. It is clear that the asymmetry in restructuring policies leads to a higher unconditional probability that a BU is diversified, since this event takes place more often. The mechanics that deliver the diversification discount are however less straightforward. Figure 3.2 plots the two branches of the value function of synergies for two alternative scenarios.

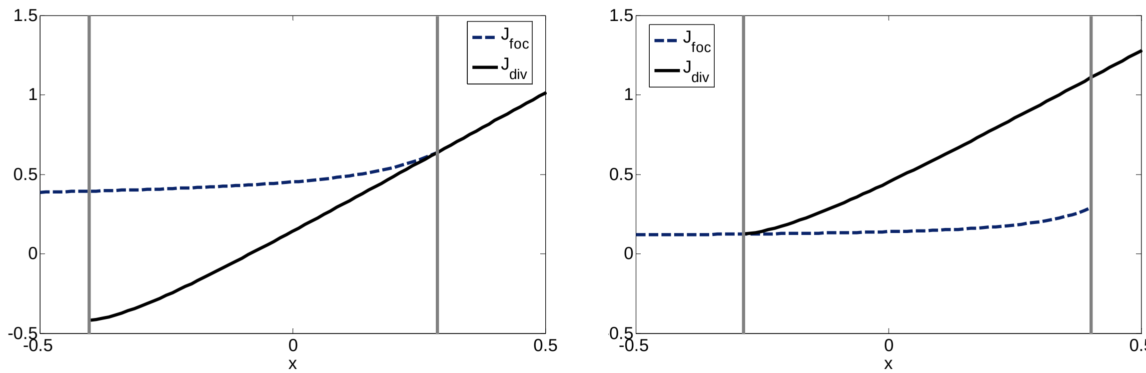


Figure 3.2: The left panel depicts the two branches of the value function for the scenario with $\sigma_x = 0.20$ and $\theta_u = 0$. The left panel depicts the two branches of the value function for the scenario with $\sigma_x = 0.20$ and $\theta_d = 0$. The vertical lines mark the optimal exercise policies, $\underline{x}$ and $\overline{x}$.

Both panels show the value of synergies for different levels of x and m. Note that the graph has similar properties to those presented in Fig. 3.1, the case of $\kappa_x = 0$. In the left panel the two branches cross exactly at $\overline{x}$, since the scenario depicted has $\theta_u = 0$, i.e. no acquisition costs. At $\underline{x}$ (the left vertical line), the difference between the two branches is exactly θ_d. For this scenario, as for any other scenario with $\theta_u = 0$, the stand-alone BU is always more valuable than the diversified BU in the overlapping region $[\underline{x}, \overline{x}]$. Having an average negative spread in the overlapping region is a necessary condition to obtain a diversification discount. In fact, I should point out that the model has a *natural bias* towards showing a premium, since outside the overlapping region, a diversified firm is obviously more valuable on average. Note that the optionality value associated with a merger, given by $J(x; m = 0)$, can never exceed the value of $J(x; m = 1)$, for $x \geq \overline{x}$. It is exactly the probability of receiving $J(x; m = 1) - \theta_u$ that makes $J(x; m = 0)$ valuable. This natural bias towards a diversification premium is the caveat in the intuitive explanation given for the discount in a previous paragraph. Since the discount is computed through averaging the vertical distance between the value function's branches, this difference needs to be in favor of specialized firms most of the time. This happens only if both the size of the overlapping region and the distance between branches are big enough. Note in the right panel of Figure 3.2, the case for costly diversification moves, that a diversified firm is always more valuable. No computations are required to conclude that a diversification premium ensues.

From the discussion above, it should also be clear why the counter-intuitive relation between the level of the discount and the proportion of diversified firms arises. *Ceteris paribus*, higher spin-off costs imply both a bigger overlapping region (lower frequency of restructuring) and a higher bias towards diversification (i.e. bias towards the less costly move). The former drives the discount up by giving more weight to outcomes where specialized firms are worth less; the latter makes specialization less likely, which implies a higher probability of finding diversified firms.

In this model a diversifying move (merger) takes place when the BU pair has a relatively high value. Since firms follow value-maximizing behavior, mergers are related to states of nature where synergy cash flows are relatively high. The assumption of value maximization is in line with some other papers in the diversification literature – Gomes and Livdan (2004), Bernardo and Chowdry (2002), Maksimovic and Phillips (2002) – but certainly seems hard to reconcile with the empirical findings of Graham, Lemmon, and Wolf (2002). In this paper the authors show that approximately half of the discount can be attributed to conglomerates acquiring *already discounted* firms. Although my model implies the opposite, I am not modeling in detail the merger process: how targets and acquirers are matched, the issue of competition for targets, and where exactly the synergies come from. One could conjecture that conglomerates are able to achieve the same level of synergies by acquiring the "cheapest" firm in a certain industry. This depends on how these synergies are achieved, which is beyond the scope of this paper.

3.3 Empirical analysis

In this section I conduct an empirical analysis of a sample of US firms for the 1984-2005 period, and use this data to calibrate the model and test its predictions.

3.3.1 Diversification in the 1984-2005 period

I use a panel of firms from the 1984-2005 period to analyze the dynamics of the diversification discount. This is the data in Custódio (2008), a paper where the connection between the *degree* of diversification (according to a relatedness measure) and the size of the discount is studied. The sample consists of a panel with 99,805 firm-years from COMPUSTAT Industrial Annual and COMPUSTAT Segment files, including stand-alone and diversified firms that are present in both. Following the previous literature, the following firms are excluded: (i) firms with segments in the financial sector (SIC codes 6000 to 6999); (ii) agriculture (SIC code $<$ 1000); (iii) government (SIC 9000); (iv) other non-economic activities (SIC 8600 and 8800); and (v) unclassified services (SIC 8900).

The diversification discount is computed using an *excess value* measure, following Lang and Stulz (1994), Servaes (1996), and Villalonga (2004). Excess value is computed as the difference between the firm's Tobin's Q and its imputed Q. This last measure is constructed using an asset-weighted average of single-segment firms in the same industry. The average discount for the whole sample is 0.34, which is line with previous empirical studies. In terms of Tobin's Q, a diversification discount between 0.08 and 0.59 has been reported (see section 3.1 for details.).

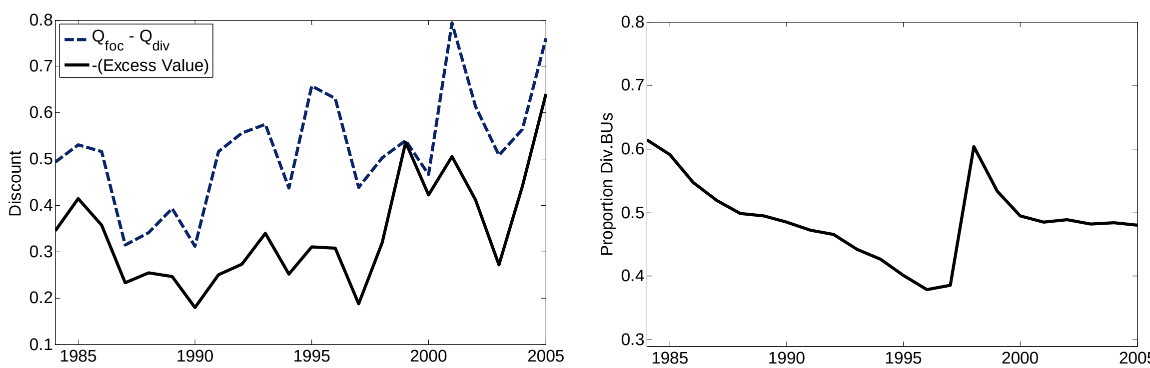


Figure 3.3: Left panel: shows the evolution of the diversification premium, for the period 1984-2005. Right panel: shows the evolution of the proportion of diversified BUs , for the same period.

The discount was also computed separately for each year in the sample, as shown in the left panel of figure 3.3. It is clear from the inspection of this time series that the discount fluctuates significantly, even with an average sample size of more than 1,100 diversified firms per year. Thus it seems there is an aggregate component in the diversification discount. I also show the simple difference in Tobin's Q between diversified and focused firms, which is strongly correlated with excess value (correlation of 0.72). This simple measure overstates the discount (full-sample mean of 0.52), which means that on average diversified firms are relatively more active in low-Q industries.

The right panel in figure 3.3 shows a proxy for the proportion of diversified BUs in the economy. I computed this by considering each segment a BU. Having the total number of segments for the sample, I divided the number of segments associated with diversified firms by the total number of segments. In this sample, the mean segment size of a diversified firm is 30% bigger than a focused firm. Also, there is little fluctuation in the cross-sectional average number of segments of diversified firms over time: the minimum is 2.7 and the maximum is 3. The average proportion of diversified BUs in the sample is 50%.

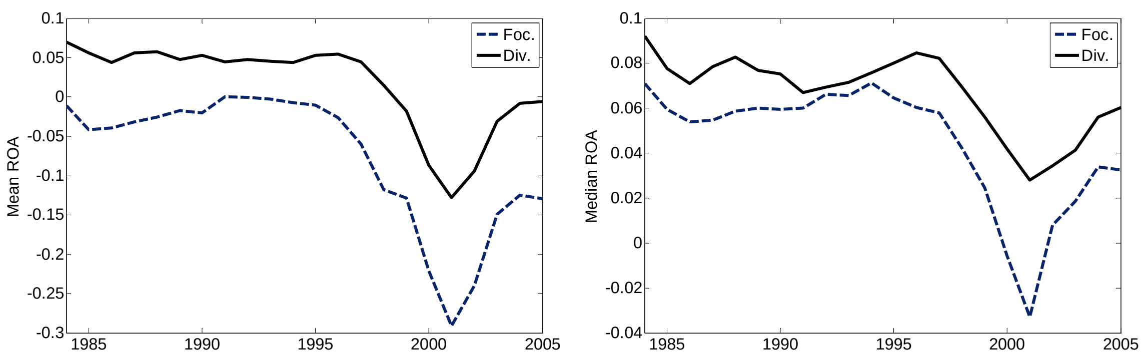


Figure 3.4: Left panel: shows the evolution of the cross-sectional mean profitabilities, for the period 1984-2005. Right panel: shows the evolution of the cross-sectional median profitabilities, for the same period.

Next I inspect differences in profitability between diversified and focused firms. The left panel of figure 3.4 shows how the cross-sectional average of ROA evolved for the sample period. Diversified and focused firms' profitability is strongly correlated, with a sample coefficient of 0.97. Two striking features in this sample are that the mean ROA of focused firms was always negative, and diversified firms were on average more profitable every year. In terms of medians this effect is dampened, as shown in the right panel of 3.4. Also, the median profitability of focused firms is in general positive.

As I explained in section 3.1, the finding that the profitability of diversified firms is higher is not inconsistent with the (opposite) findings of Berger and Ofek (1995), and is in line with Schoar (2002). In order to control for industry effects, I constructed a measure for *excess profitability*. This was done in the same way as excess value, i.e. using an imputed value for the profitability of diversified firms. Figure 3.5 compares the raw difference in profitabilities to the measure of excess profitability. The raw difference is always above the industry-adjusted measure, which means that diversified firms are on average more active in more profitable industries.

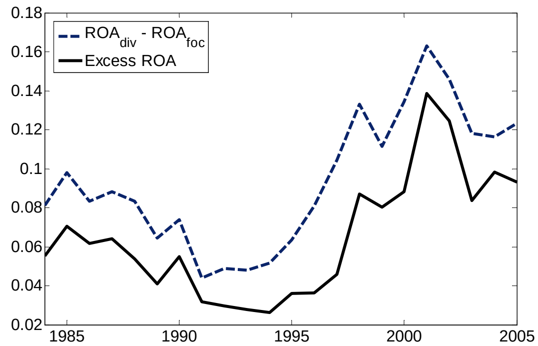


Figure 3.5: Shows the evolution of the difference in mean profitabilities between diversified and focused firms, for the period 1984-2005.

In section 3.3.2 I will use moments computed from this sample to calibrate the model. Finally, I defer discussing some other aspects of the sample until then.

3.3.2 Structural calibration

In what follows I conduct a structural calibration of the model, using the data for the 1984-2005 period. In particular I pin down the parameters for the processes driving profitability, which reduces the model's degrees of freedom. Nonetheless I still have to indirectly infer about the values of some parameters, namely the size of restructuring costs. Although I am able to compute the moments required to identify the model's parameters using analytical methods (numerical integration), I still need to simulate an artificial panel of data to investigate other testable implications of the model, for example the correlation between the discount and the proportion of diversified BUs. For the purpose of simulation I pair up N BUs in a simple way: BU i is paired up with BU $(N/2+i)$. Below I restate the process for each BU i's profitability:

$$\begin{aligned}
\pi_{i,t} &= w_{i,t} + m_{i,t} x_t \\
dw_{i,t} &= \kappa_w(\mu_w - w_{i,t})\, dt + \sigma_w\, d\bar{B}^w_{i,t} \\
dx_{i,t} &= \kappa_x(\mu_x - x_{i,t})\, dt + \sigma_x\, d\bar{B}^x_{i,t}
\end{aligned}$$

Unlike section 3.2.2 I allow for a mean of synergies μ_x different than zero. Without this additional degree of freedom I am unable to simultaneously match the discount, mean excess profitability, and the mean proportion of diversified BUs.

Parametrization of stochastic processes

I consider two aggregate shocks, one to baseline productivity, the second to synergies. I allow these shocks to be correlated, with coefficient ρ_{xw}. The Brownian motions driving the profitability components of BU i are constructed in the following way

$$d\bar{B}^w_{i,t} = \sqrt{\rho_w}\, dB^w_{G,t} + \sqrt{1-\rho_w}\, dB^w_{i,t} \;, \rho_w \geq 0 \tag{3.35}$$

$$d\bar{B}^x_{i,t} = \sqrt{\rho_x}\, d\tilde{B}^x_{G,t} + \sqrt{1-\rho_x}\, dB^x_{i,t} \;, \rho_x \geq 0, \tag{3.36}$$

with $d\tilde{B}^x_{G,t}$ computed as

$$d\tilde{B}^x_{G,t} = \rho_{xw}\, dB^w_{G,t} + \sqrt{1-\rho^2_{xw}}\, dB^x_{G,t}, \tag{3.37}$$

where $B^w_{G,t}$ and $B^x_{G,t}$ are independent Brownian motions. The correlation structure yields

$$\mathbb{E}\left[d\bar{B}^w_{i,t} d\bar{B}^w_{j,t}\right] = \rho_w\, dt \;, i \neq j \tag{3.38}$$

$$\mathbb{E}\left[d\bar{B}^x_{i,t} d\bar{B}^x_{j,t}\right] = \rho_x\, dt \;, i \neq j; j \neq N/2 \pm i \tag{3.39}$$

$$\mathbb{E}\left[d\bar{B}^w_{i,t} d\bar{B}^x_{i,t}\right] = \rho_{xw}\sqrt{\rho_x \rho_w}\, dt. \tag{3.40}$$

Also, for large N we have

$$\mathbb{E}[w|m=0] = \mathbb{E}[w|m=1]$$
$$\mathbb{E}[S|m=0] = \mathbb{E}[S|m=1],$$

so as in section 3.2.2 the only source of the diversification discount is J.

Calibration

In the model, it is easy to show that the processes for the average $w_{i,t}$ and $x_{i,t}$ are also Ornstein-Uhlenbeck, with the same speed of mean-reversion as the individual processes and instantaneous volatilities of $\sqrt{\rho_w}$ and $\sqrt{\rho_x}$, respectively. The correlation coefficient between these mean processes converges to ρ_{xw}, for large N. The cross-sectional standard deviations for $w_{i,t}$ and $x_{i,t}$ also have a simple expression for large N. I thus calibrate the w and x processes using the data on (i) profitability of focused firms, and (ii) difference in profitability between diversified and focused firms, respectively:[16]

$$\begin{aligned}
&auto\,corr|_{annual}\left(\frac{w_{i,t}}{N}\right) = e^{-\kappa_w} = 0.9 \text{ (}auto\,corr \text{ of mean } ROA_{foc}\text{)} \\
&auto\,corr|_{annual}\left(\frac{x_{i,t}}{N}\right) = e^{-\kappa_x} = 0.8 \text{ (}auto\,corr \text{ of mean } Excess\,ROA\text{)} \\
&SD\left(\sum_i \frac{w_{i,t}}{N}\right) = \sqrt{\frac{\rho_w}{2\kappa_w}}\sigma_w = 0.09 \text{ (}SD \text{ of mean } ROA_{foc}\text{)} \\
&SD\left(\sum_i \frac{x_{i,t}}{N}\right) = \sqrt{\frac{\rho_x}{2\kappa_x}}\sigma_x = 0.03 \text{ (}SD \text{ of mean } Excess\,ROA\text{)} \\
&SD_{cs}(w_{i,t}) = \sqrt{\frac{1-\rho_w}{2\kappa_w}}\sigma_w = 0.54 \text{ (}SD_{cs} \text{ of } ROA_{foc}\text{)} \\
&SD_{cs}(x_{i,t}) = \sqrt{\frac{1-\rho_x}{2\kappa_x}}\sigma_x = 0.28 \text{ (}SD_{cs} \text{ of } Excess\,ROA\text{)} \\
&corr\left(\sum_i \frac{w_{i,t}}{N}, \sum_i \frac{x_{i,t}}{N}\right) = \rho_{xw} = \\
&\quad = -0.93 \text{ (}corr \text{ between mean } ROA_{foc} \text{ and mean } Excess\,ROA\text{)},
\end{aligned}$$

where $corr$ means correlation coefficient, SD means standard deviation, and SD_{cs} cross-sectional standard deviation. It is worth pointing out that the correlation between excess profitability and profitability of focused firms is extremely negative (-0.93).

Four parameters are still not identified, namely μ_w, μ_x, θ_u, and θ_d. I use the following moment conditions to pin them down as well as possible:

$$\begin{aligned}
&\mathbb{E}[V|m=0] = 2.26 \text{ (mean of } Q_{foc}\text{)} \\
&\mathbb{E}[x|m=1] = 0.06 \text{ (mean of } Excess\,ROA\text{)} \\
&\mathbb{E}[m=1] = 0.5 \text{ (mean of } Prop.Div.\text{)} \\
&\mathbb{E}[V|m=0] - \mathbb{E}[V|m=1] = 0.34 \text{ (mean of } Discount\text{)}
\end{aligned}$$

[16] This is not entirely accurate, since the process of *observed* synergies, according to the model, is truncated. Nonetheless I am only calibrating volatility in this way, and the conditional instantaneous volatility is the same for the truncated process. Computing this moment in a simulation environment, I did not find a significant quantitative difference between the volatility of the primitive process, i.e. σ_x, and the volatility of observed synergies.

It would seem natural to calibrate μ_w to the mean ROA_{foc}, but this would yield negative firm values, so I preferred to match Q_{foc}. From a numerical methods perspective I can use the same methodology as in section 3.3 to compute these moments (using numerical integration).

Table 3.4 summarizes the parameter values in the calibration.

Parameter	Calibration
r	0.05
κ_w	0.11
μ_w	0.10
σ_w	0.24
ρ_w	0.03
κ_x	0.22
μ_x	-0.03
σ_x	0.20
ρ_x	0.01
ρ_{xw}	-0.93
θ_u	1.1
θ_d	2.4

Table 3.4: Calibrated parameters.

It is interesting to note that the calibration implies a negative average value of synergies μ_x. This squares with the economic intuition that different industries should have little to gain systematically with collaboration. Otherwise this would mean that industry frontiers were ill-defined.

Results

As for the moment conditions used in the identification process, we have a good fit to data, as shown in table 3.5, in particular a discount is obtained, even when diversified firms are 5% more profitable than focused firms, on average.

I computed other moments in the data and the model,[17] which I report in table 3.6. The results are mixed. In terms of a strict quantitative match the calibration is in line with data for one moment only, namely the correlation between excess profitability

[17] I generated an artificial panel of data with $N = 1,000$ BUs, for a period of 20 years. The time step used in the discretization was 1/12 (one month).

Moment	Data	Model
Q_{foc}	2.26	2.26
$Excess\,ROA$	0.06	0.05
$Prop.Div.$	0.50	0.53
$Discount$	0.34	0.31

Table 3.5: Comparison between the data and the model, for moments used in the identification of parameters μ_w, μ_x, θ_u, and θ_d.

and the proportion of diversified BUs, at approximately 0.35. Economically the sign of this magnitude is reasonable: in times where the aggregate shock to synergies is higher, more BUs choose to become (or stay) diversified. This evidence certainly argues against an explanation for the discount based on agency arguments or irrationality.

Moment	Data	Model
$SD(Discount)$	0.12	0.24
$Auto\,Corr(Discount)$	0.51	0.84
$SD(Prop.Div.)$	0.06	0.03
$Auto\,Corr(Prop.Div.)$	0.61	0.98
$Corr(Discount, Prop.Div.)$	0.25	−0.26
$Corr(\text{mean } ROA_{foc}, Discount)$	−0.61	0.48
$Corr(\text{mean } ROA_{foc}, Prop.Div)$	−0.12	−0.54
$Corr(\text{mean } Excess\,ROA, Discount)$	0.65	−0.95
$Corr(\text{mean } Excess\,ROA, Prop.Div.)$	0.35	0.34

Table 3.6: Comparison between the data and the model, for moments not used in the identification of parameters. Implementation with two aggregate shocks.

While the model overshoots the volatility of *Discount* and the auto correlation of both *Discount* and *Prop.Div.*; and understates the volatility of *Prop.Div.*, the relation between these variables is preserved. The volatility of *Discount* is higher than the volatility of *Prop.Div.*, and the auto correlation of *Prop.Div.* is higher than that of *Discount*. Also, the signs of the auto correlations are correct, which also happens with the correlation between mean ROA_{foc} and *Prop.Div.* This last correlation is negative, which means there are more diversified firms when focused firms are doing poorly. This is not surprising, given the negative correlation between mean $Excess\,ROA$ and mean ROA_{foc}.

Next I discuss the dimensions not captured by the model. The correlation between *Excess ROA* and *Discount* in the model is naturally extremely negative; in times where synergies are high, diversified firms are worth more. In data this correlation is surprisingly positive, at 0.65. The mismatch between the model and the data in terms of the correlation between *Discount* and *Prop.Div.* seems to be another manifestation of this same disagreement: the model implies that there are less diversified firms when the discount is high (i.e. excess profitability low). Interpreting this in light of our comparative statics result, namely that by increasing spin-off costs one observes both a higher discount and a higher proportion of diversified BUs, these mismatches may be indicative that restructuring costs fluctuate over time. Finally, the mismatch in correlations between the model and the data in terms of the series mean ROA_{foc} and *Discount* is simply a repetition of the same effect, since mean ROA_{foc} and mean *Excess ROA* are extremely negatively correlated.

In section 3.2.2 I showed that a diversification discount can be obtained by setting $\theta_u = 0$ and $\mu_x = 0$. However, in the calibration I set $\theta_u > 0$ and $\mu_x < 0$. In fact, without using θ_u and μ_x it is not possible to simultaneously match the discount, the mean excess profitability, and the proportion of diversified firms. Table 3.7 presents a sensitivity analysis, with θ_u set at 0. Inspection of this table shows that it would be possible to match the discount and the proportion of diversified firms with θ_d between 1 and 2, and μ_x at around -0.03. However, this would imply too high excess profitabilities for diversified firms, as shown in the corresponding cells in panel b) of table 3.7.

	a) Discount μ_x			b) Excess ROA μ_x			c) Prop.Div. μ_x		
θ_d	-0.06	-0.03	0.00	-0.06	-0.03	0.00	-0.06	-0.03	0.00
1	-0.10	-0.18	-0.27	0.16	0.16	0.17	0.42	0.48	0.56
2	0.49	0.37	0.24	0.10	0.10	0.10	0.46	0.58	0.68
3	1.02	0.82	0.59	0.03	0.03	0.03	0.59	0.75	0.87

Table 3.7: Panel a) shows the diversification discount, for each pair (θ_d, μ_x). Panel b) shows the mean excess profitability of diversified BUs, for each pair (θ_d, μ_x). Panel c) shows the proportion of diversified BUs, for each pair (θ_d, μ_x). In all cases $\theta_u = 0$.

The reason why $\theta_u > 0$ allows the matching of excess profitability is the following. For some pair (θ_d, μ_x), increasing θ_u leads to two effects: (i) decrease in the discount; (ii) decrease in the proportion of diversified firms (to see why this is the case see section

3.2.2). This allows me to choose a high θ_d to match the excess profitability; and then correct the discount and the proportion of diversified firms by increasing θ_u. Note that setting θ_d between 2 and 3 with $\theta_u = 0$ delivers a match to excess profitability (panel b) in table 3.7), however either the discount or the proportion of diversified firms would be too high (see panels a) and b) in table 3.7).

3.4 Extensions

3.4.1 Uncertain strategy implementation

The model, as defined so far, implies that whenever a BU pair hits the threshold for option exercise (be it a merger or a spin-off), this exercise is implemented with certainty. This also means that there are no announcement effects in the model, given that the market correctly anticipates the change in strategy, and state variables follow continuous stochastic processes. In what follows I study an extension of the model where strategy implementation (diversifying or refocusing) is uncertain. This uncertainty in strategy implementation aims to capture costs that may not be financial (no cash outflow) but still impact optimal decisions and prices. An example for this would be a search process for a diversification partner, or a conglomerate looking for a potential buyer in a spin-off operation.

Formally I define the following: (i) if the BU pair is not merged between times t and T ($\{m = 0\}_t^T$), and continuously attempts to merge between t and T, merging occurs in this period according to an independent Poisson process with intensity $\lambda_u(T-t)$; (ii) if the BU pair is merged between times t and T ($\{m = 1\}_t^T$), and continuously attempts to split between t and T, splitting occurs in this period according to an independent Poisson process with intensity $\lambda_d(T-t)$. In this extension, the real options available to the firm are options to *take a gamble* on the implementation of a certain strategy. Denoting by m_t^* the endogenous indicator function that assumes a value of 1 if the firm is attempting to merge if $m_t = 0$, and the value of 0 in the opposite case, the value of synergies is given by:

$$J_t = \sup_{\{m^*\}} \left\{ \mathbb{E}_t \left[\int_t^\infty e^{-r(u-t)} m_u x_u \, du - \sum_{\{\tau_u\}} e^{-r(\tau_u - t)} \theta_u - \sum_{\{\tau_d\}} e^{-r(\tau_d - t)} \theta_d \right] \right\}, \quad (3.41)$$

where as before $\{\tau_u\}$ and $\{\tau_d\}$ represent the set of random stopping times at which a merger or a de-merger takes place, respectively. As in the case with certain strategy implementation, equilibrium requires

$$rJ_t dt = m_t x_t dt + \mathbb{E}_t \left[dJ_t \right].$$

Applying Itô's lemma to $\mathbb{E}_t [dJ_t]$, the value functions are described by the following system of four differential equations:[18]

$$\begin{aligned} rJ(x; m=0, m^*=0) &= J_x(x; m=0, m^*=0)\kappa_x(\mu_x - x) + \\ &\quad + J_{xx}(x; m=0, m^*=0)\frac{\sigma_x^2}{2} \qquad (3.42) \\ (r+\lambda_u)J(x; m=0; m^*=1) &= \lambda_u \left[J(x; m=1, m^*=1) - \theta_u \right] + \\ &\quad + J_x(x; m=0, m^*=1)\kappa_x(\mu_x - x) + J_{xx}(x; m=0, m^*=1)\frac{\sigma_x^2}{2} \qquad (3.43) \\ rJ(x; m=1, m^*=1) &= J_x(x; m=1, m^*=1)\kappa_x(\mu_x - x) + \\ &\quad + J_{xx}(x; m=1, m^*=1)\frac{\sigma_x^2}{2} + x \qquad (3.44) \\ (r+\lambda_d)J(x; m=1; m^*=0) &= \lambda_d \left[J(x; m=0, m^*=0) - \theta_d \right] + \\ &\quad + J_x(x; m=1, m^*=0)\kappa_x(\mu_x - x) + J_{xx}(x; m=1, m^*=0)\frac{\sigma_x^2}{2} + x \qquad (3.45) \end{aligned}$$

The value of the firm when it is in gamble mode is a weighted average of the payoffs of each corporate status; the weights are a function of the Poisson intensities λ_u and λ_d. As before, value-matching and smooth-pasting conditions pin down the thresholds where the firm steps into a gamble mode (attempting to merge or to split). I assume that the transition into gamble mode takes place without cost, so the value-matching conditions are given by

$$J(\overline{x}; m=0, m^*=0) = J(\overline{x}; m=0, m^*=1) \qquad (3.46)$$

$$J(\underline{x}; m=1, m^*=1) = J(\underline{x}; m=1, m^*=0). \qquad (3.47)$$

The smooth-pasting conditions are

[18]See Dixit and Pyndick (1994) as a reference for real option valuation and optimal exercise with Poisson processes.

$$\frac{\partial}{x}\bigg|_{x=\overline{x}} [J(x; m = 0, m^* = 0) - J(x; m = 0, m^* = 1)] = 0 \quad (3.48)$$

$$\frac{\partial}{x}\bigg|_{x=\underline{x}} [J(x; m = 1, m^* = 1) - J(x; m = 1, m^* = 0)] = 0. \quad (3.49)$$

To match this model to data it is also necessary to derive the new stationary distribution for (x, m). This and the algorithm to find the value functions are described in the Appendix. I investigate whether asymmetry in λs could deliver the same effect as the previous asymmetry in θs. To this end, and to reduce the model's degree of freedom, I set $\theta_u = \theta_d = 0.1$. Since the median market-to-book ratio is approximately 2, then θ would correspond to 5% of the market value of a median firm. The quantitative implications of the model are practically unchanged for θ between 0.01 and 0.5. As before I used the mean and volatility of *Excess ROA*, the mean of *Discount*, the mean of Q_{foc} and the mean of *Prop.Div.* to calibrate the parameters σ_x, μ_w, μ_x, λ_u, and λ_d. The parameters were set at the magnitudes described in table 3.8 (I do not analyze correlations in this section). This model is also able to match data, as shown in table 3.9.

Parameter	Calibration
r	0.0500
κ_w	0.1100
μ_w	0.0860
σ_w	0.2400
κ_x	0.2200
μ_x	−0.0350
σ_x	0.1830
θ_u	0.100
θ_d	0.100
λ_u	1.5400
λ_d	0.0572

Table 3.8: Calibrated parameters.

This model is somewhat isomorphic to the model with certain strategy implementation, as can be seen in figure 3.6. The intuition is simple: if the intensity λ_d is set at 0, the diversifying move becomes irreversible, which happens in the previous version

Moment	Data	Model
Q_{foc}	2.26	2.26
$Excess\,ROA$	0.06	0.04
$Prop.Div.$	0.50	0.52
$Discount$	0.34	0.33

Table 3.9: Comparison between the data and the model, for moments used in the identification of parameters μ_w, μ_x, λ_u, and λ_d.

of the model for $\theta_d = \infty$. Since the unconditional mean of synergies is negative, the more irreversible the diversifying move, the higher the unconditional average of the discount. The same logic applies to the proportion of diversified BUs. Curiously, in terms of policies the effect is distinct. In the calibrated version of the model at hand, the value of $\overline{x}$ (0.3663) is well above the absolute value of $\underline{x}$ (-0.0176), the opposite of what happened when strategy implementation was certain.

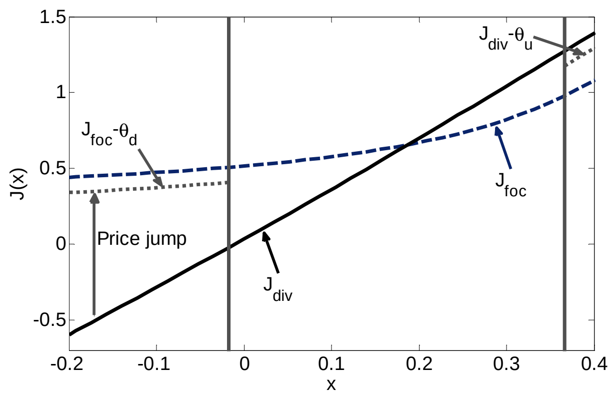


Figure 3.6: Shows the two branches of the value function, as a function of the state variable x. The vertical lines mark the optimal exercise policies, $\underline{x}$ and $\overline{x}$.

A distinct feature of the uncertain strategy implementation is that the model predicts jumps. Whenever a firm is able to implement the desired strategy (merger or spin-off), the price increases. This is just the market response to the successful undertaking of a positive NPV project. In figure 3.6 I highlighted the price jump that takes place when a spin-off (refocusing) is successful, but qualitatively the same effect obtains with a successful merger (diversification). Naturally this is a model that predicts positive announcement effects for *both* acquisitions and divestitures, given that the firm is value-maximizing. It is also the case that if the discount is merely a product of asymmetry in λs, then price jumps would have to be significantly higher for spin-offs than for acquisitions. Empirically this does not seem to be the true. Mulherin and Boone (2000), in a simultaneous study of acquisitions and divestitures, find that the combined wealth effects for acquisitions (mean=3.56%, median=1.99%) are higher than for divestitures (mean=3.04%, median=3.64%).

3.4.2 Size heterogeneity

One of the dimensions in terms of how diversified firms and focused firms differ is that the segments of diversified firms are, in my sample, 30% bigger. So far the model is unable to accommodate this, since I assume an equal K for all BUs in the economy. Next I consider an exogenous distribution of BU size. For simplicity I assume that for each BU pair, both BUs have the same capital size. The distribution of $\log(K)$ follows a Normal distribution (independent of the other random variables in the model), with mean μ_k and standard deviation σ_k. I assume the following in terms of per-unit-of-K merger costs for BU i: (i) $\theta_u|_i = \theta_u^* K_i$, and (ii) $\theta_d|_i = \theta_d^* K_i$. This means that the ratio between merger and spin-off costs is the same for any firm, independently of size; and bigger firms have disproportionately higher restructuring costs. It will soon become apparent why the opposite assumption would not work. Next I calibrate the model using the same moments as before, plus the requirement that the average size of a focused firm is 1, while the average size of a segment of a diversified firm is 1.3; these two additional conditions identify μ_k and σ_k. The chosen values are shown in table 3.10. I am able to obtain a reasonable match to data, as shown in table 3.11.

Where does the size difference come from? Different levels of restructuring costs will imply different proportions of diversified BUs.[19] Figure 3.7 shows how the discount and the proportion of diversified BUs vary with size.

Given our assumptions about the relation between size and restructuring costs, a higher K corresponds to higher θ_u and θ_d. It is the case that for most of the range of K this implies a higher proportion of diversified BUs, as shown in the left panel of figure 3.7. This is the reason why the opposite assumption (decreasing costs with scale) would yield the counter-factual prediction of larger focused firms. The assumptions made also imply that larger firms exhibit a bigger discount, as shown in the right panel of figure

[19] Also note that this implies that assuming proportional restructuring costs simply yields that focused and diversified BUs have the same average size.

Parameter	Calibration
r	0.05
κ_w	0.11
μ_w	0.10
σ_w	0.24
κ_x	0.22
μ_x	−0.05
σ_x	0.20
θ_u^*	1.0
θ_d^*	2.7
μ_k	−0.30
σ_k	0.95

Table 3.10: Calibrated parameters.

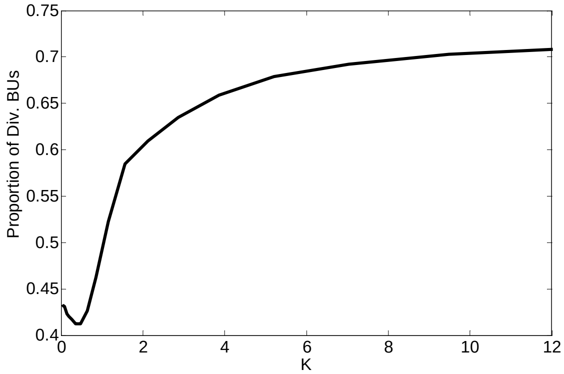
Moment	Data	Model
Q_{foc}	2.26	2.26
$Excess\, ROA$	0.06	0.08
$Prop.Div.$	0.50	0.48
$Discount$	0.34	0.26
$\mathbb{E}[K\|m = 0]$	1.00	0.97
$\mathbb{E}[K\|m = 1]$	1.30	1.34

Table 3.11: Comparison between the data and the model, for moments used in the identification of parameters μ_w, μ_x, λ_u, and λ_d.

3.7. I tested this implication by regressing excess value (subsample of diversified firms) on firm size and other controls. The results are shown in table 3.12. The coefficient for size is negative and statistically significant in all 6 models, which is consistent with the model's (qualitative) predictions.

3.5 Conclusion

This paper proposes a stationary real options model with restructuring costs to rationalize the diversification discount. The model is parsimonious, and although only solvable numerically, computation time is not an issue. The model can deliver a diver-



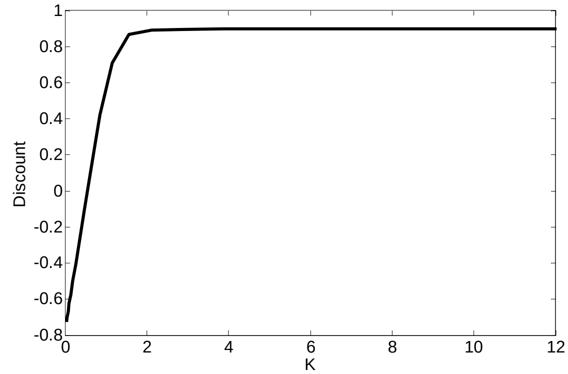


Figure 3.7: Left panel: shows how for each size class the implied proportion of diversified BUs. Right panel: shows for each size class the implied discount.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
$Log(Assets)$	-0.283	-0.287	-0.287	-0.067	-0.069	-0.052
	(-11.1)	(-11.2)	(-11.1)	(-12.6)	(-13.1)	(-10.2)
$Capex/Sales$		0.34	0.342		0.722	0.501
		(2.9)	(2.9)		(9.9)	(6.8)
$EBIT/Sales$			0.004			-0.238
			(0.07)			(-6.20)
Fixed Effects	Y	Y	Y	N	N	N
R^2	0.015	0.017	0.016	0.015	0.023	0.041
N	$25,102$	$25,102$	$25,102$	$25,102$	$25,102$	$25,102$

Table 3.12: This table shows the estimated coefficients in multivariate regressions where the dependent variable is *excess value*, for the subsample of diversified firms. No year dummies are included. T-statistics are reported in parenthesis and standard errors are corrected for heteroskedasticity.

sification discount under the condition that spin-offs are significantly more costly than acquisitions. This additionally implies that the proportion of diversified business units in the economy is relatively high. The direct driver for these results is the structure of the optimal restructuring policies. If spin-offs are costly, then they take place less often than acquisitions, which induces a high (unconditional) probability that a business unit is diversified. In terms of the diversification discount, two opposite effects need to be taken into account. First, diversified firms potentially earn additional positive cash flow from synergies. Second, these firms face the possibility that these additional cash flows become negative at some point in time. Moreover, if a spin-off is very costly, it may be still suboptimal to specialize, despite current negative negative cash flows. A diversification discount obtains when the second effect more than offsets the first, or in other words, for high enough spin-off costs.

I conducted an empirical analysis using data on US firms for the period 1984-2005, having found an average discount that is in line with previous studies. The discount coexists with the fact that on average, diversified firms' ROA is on average 6% higher than that of focused firms. I calibrate the model to this sample and am able to simultaneously capture these two features of the data.

One aspect that I abstract from is asymmetries of information. If managers are better informed than the market about the value of synergies, and if they act in a value-maximizing way, then in equilibrium they will diversify too soon relative to the first-best (full-information) policies.[20] This could complement the rationale for the discount put forth in this paper, since an early exercise of the diversification option naturally leads to an increase in the discount.

---

[20] Suppose the option to diversify has positive value. Then managers will only exercise the option if the "static NPV" exceeds the option value. Suppose this happens at some threshold $\overline{x}$. Denoting the cash flow state variable by x as before, at $x = \overline{x}$ we have NPV=option value (at market prices). Also, the option value is lower for any $x < \overline{x}$, since the option value naturally increases with x increasing. But this means that all managers who did not diversify have $x < \overline{x}$, thus the unconditional average option value must be lower or equal than the option has zero value. It follows that the only equilibrium is for managers to exercise with $\overline{x}$ such that NPV=0 and the option looses all value.

3.A Appendix

3.A.1 Obtaining the value functions

Detail of the solution for $\kappa_x = 0$

The system of equations below identifies A, B, $\overline{x}$ and $\underline{x}$ uniquely, for the case with $\kappa_x = 0$.

$$
\begin{aligned}
A &= \frac{\underline{G}\overline{G}}{\overline{G}^2 - \underline{G}^2}\left\{\frac{1}{\underline{G}}\left[\frac{\overline{x}}{r} - \theta_u\right] - \frac{1}{\overline{G}}\left[\frac{\underline{x}}{r} + \theta_d\right]\right\} && (3.A.1)\\
B &= \frac{\underline{G}\overline{G}}{\overline{G}^2 - \underline{G}^2}\left\{\underline{G}\left[\frac{\overline{x}}{r} - \theta_u\right] - \overline{G}\left[\frac{\underline{x}}{r} + \theta_d\right]\right\} && (3.A.2)\\
\overline{x} &= \frac{\sigma_x}{\sqrt{2r}}\ln\left[\frac{1/r + \sqrt{(1/r)^2 - 8ABr/\sigma_x^2}}{2A\sqrt{2r}/\sigma_x}\right] && (3.A.3)\\
\underline{x} &= \frac{\sigma_x}{\sqrt{2r}}\ln\left[\frac{1/r - \sqrt{(1/r)^2 - 8ABr/\sigma_x^2}}{2A\sqrt{2r}/\sigma_x}\right], && (3.A.4)
\end{aligned}
$$

where $\overline{G} \equiv e^{\frac{\sqrt{2r}}{\sigma_x}\overline{x}}$ and $\underline{G} \equiv e^{\frac{\sqrt{2r}}{\sigma_x}\underline{x}}$.

Solving ODE (3.10)

For theory and numerical methods on how to solve continuous-time control problems the reader is referred to Fleming and Soner (2005) or Kushner and Dupuis (2000). The problem at hand is rather simple given that the usual partial differential equation reduces to an ODE. I discretize the state space for x using a step of δ, from x_{min} to x_{max}. Denote the number of states by N (in my computation I used $N = 1,000$). The time step is denoted by h. I define a random walk process for x with probability of an arithmetic up move of size δ given by $p(x)$; the probability of a down move is $1 - p(x)$. Next I match the per-unit-of-time drift and variance of the random walk with those implied by the OU process, i.e:

$$
\begin{aligned}
p(x)\delta + (1 - p(x))(-\delta) &= -\kappa_x(x - \mu_x)h && (3.A.5)\\
p(x)\delta^2 + (1 - p(x))\delta^2 &= \sigma_x^2 h && (3.A.6)
\end{aligned}
$$

which simplifies into:

$$
\begin{aligned}
h &= \frac{\delta^2}{\sigma_x^2} && \text{(3.A.7)} \\
p(x) &= 0.5\left[1-(x-\mu_x)\frac{\kappa_x}{\sigma_x}\sqrt{h}\right] && \text{(3.A.8)}
\end{aligned}
$$

I will henceforth refer to $p(x)$ as $p_n \equiv p(x_n)$, where x_n is the n-th point in our discrete grid. The following bounds for x_{min} and x_{max} need to be observed in order to keep $p(x)$ in the unit interval:

$$
\begin{aligned}
x_{max} &\leq \frac{\sigma_x}{\kappa_x\sqrt{h}} && \text{(3.A.9)} \\
x_{min} &\geq -\frac{\sigma_x}{\kappa_x\sqrt{h}} && \text{(3.A.10)}
\end{aligned}
$$

Recall that to solve ODE (3.10) we only need to find f_1. It is economically intuitive to represent f_1 in the following recursive way:

$$
f_{1,n} = (1-rh)\left[p_n f_{1,n+1} + (1-p_n)f_{1,n-1}\right], \qquad \text{(3.A.11)}
$$

which means that the current value of f_1 is its discounted future expected value (recall that there are no dividends for f_1, since it is only an option value). Manipulating the equation one easily writes down a recursion for f_1:

$$
f_{1,n+1} = \frac{1}{p_n}\left[\frac{f_{1,n}}{1-rh} - (1-p_n)f_{1,n-1}\right] \qquad \text{(3.A.12)}
$$

To find an f_1 that is positive, converges toward 0 as x goes to $-\infty$ and is monotone increasing, I use the following process. I start by choosing a large negative x_{min} (i.e. I use a very coarse grid) and set $f_{1,0} = f_{1,1} \geq 0$ such that $f_1(z_n = 0) = 1$ (recall that $z = x - \mu_x$). This implies the required properties for all $x \geq x_{min}$. The approximation error relates to the fact that I am assuming that the first derivative of f_1 is zero at x_{min}, which actually is only true at $-\infty$. Next I identify $x_0 = x_{min}$ and x_1 such that 99.9% of the stationary distribution of x is in the grid (recall that this distribution is known *a priori*). I will refer to this second grid as the *fine* grid. The final step is to obtain $f_{1,0}$ and $f_{1,1}$ for the fine grid, which I do by linearly interpolating the values of f_1 determined in the first step, i.e. using the coarse grid. Also note that since I am covering 99.9% of the distribution of x, the discretization is accurate enough for the paper's objectives. Although specified in an economically intuitive way, the method I am employing is indeed an FDM. Next I show this. Define the following difference approximations:

$$y' = \frac{y_{n+1} - y_{n-1}}{2\delta} \quad (3.A.13)$$
$$y'' = \frac{y_{n+1} - 2y_n + y_{n-1}}{\delta^2} \quad (3.A.14)$$

The above system can be rewritten as:

$$y_{n-1} = 0.5y''\delta^2 - \delta y' + y \quad (3.A.15)$$
$$y_{n+1} = 0.5y''\delta^2 + \delta y' + y \quad (3.A.16)$$

Defining $r^* : (1 - rh) = 1/(1 + r^*h)$ and plugging equations (3.A.15) and (3.A.16) into equation (3.A.11) (I also replace f_1 by y), plus making use of the fact that $\delta^2 = \sigma_x^2 h$, one can write:

$$y''0.5\sigma_x^2 + y'\frac{\sigma_x(2p_n - 1)}{\sqrt{h}} - yr^* = 0 \quad (3.A.17)$$

Since $\sigma_x(2p_n - 1)/\sqrt{h} = -\kappa_x x$ by construction, I have shown that I am actually employing an FDM, i.e. a valid numerical approximation to solving ODE (3.10). Finally the algorithm for pinning down A, B, $\underline{x}$ and $\overline{x}$ numerically follows. (i) Given $\underline{x}$ and $\overline{x}$, choose A and B such that the value-matching conditions hold. This can be done by an iteration where A is set to match the value-matching condition (3.18) and then B is set to match the value-matching condition (3.19). (ii) Choose $\overline{x}$ such that the smooth-pasting condition (3.20) holds. (iii) Choose $\underline{x}$ such that the smooth-pasting condition (3.21) holds. (iv) Iterate until the numerical error associated with the smooth-pasting conditions is at its minimum.

Uncertain strategy implementation

As before p_n denotes the probability of an up move at x_n. The system of ODEs (3.42)-(3.45) can be written in the following economically intuitive way:

$$J_{n,1} = (1 - rh)\left[p_n J_{n+1,1} + (1 - p_n)J_{n-1,1}\right] \quad (3.A.18)$$
$$J_{n,2} = \lambda_u h\left[J_{n,3} - \theta_u\right] + (1 - rh)(1 - \lambda_u h)\left[p_n J_{n+1,2} + (1 - p_n)J_{n-1,2}\right] \quad (3.A.19)$$
$$J_{n,3} = (1 - rh)\left[p_n J_{n+1,3} + (1 - p_n)J_{n-1,3}\right] + x_n h \quad (3.A.20)$$
$$J_{n,4} = \lambda_d h\left[J_{n,1} - \theta_d\right] + (1 - \lambda_d h)\left\{(1 - rh)\left[p_n J_{n+1,4} + (1 - p_n)J_{n-1,4}\right] + x_n h\right\}, \quad (3.A.21)$$
$$(3.A.22)$$

where

$$
\begin{aligned}
J_{n,1} &\equiv J(x_n; m=0, m^*=0) && (3.A.23)\\
J_{n,2} &\equiv J(x_n; m=0, m^*=1) && (3.A.24)\\
J_{n,3} &\equiv J(x_n; m=1, m^*=1) && (3.A.25)\\
J_{n,4} &\equiv J(x_n; m=1, m^*=0). && (3.A.26)
\end{aligned}
$$

For given policies $\underline{x}$ and $\overline{x}$ I start with a guess for all branches of the value function and then iterate until obtaining convergence. To pin down the optimal policies I compute numerically the derivatives of each branch of the value function and change the policies until the smooth-pasting conditions are satisfied.

3.A.2 Obtaining a stationary distribution for (x, m)

Certain strategy implementation

Start by discretizing the normal density function for x using the same grid as in the previous section. Denote by β_n the probability mass associated with bin n. Next define α_n as the conditional probability of finding a firm with $m = 1$ (i.e. diversified) in bin n. Given our random walk discretization of the OU process (meaning that x always goes either up or down), then it follows that in the stationary bivariate distribution for (x, m) the following must hold:

$$\alpha_{\underline{n}}\beta_{\underline{n}} = \alpha_{\underline{n}+1}\beta_{\underline{n}+1} \cdot [1 - p_{\underline{n}+1}], \tag{3.A.27}$$

where $\underline{n}$ is the grid index associated with $\underline{x}$. The intuition for the above equation is as follows. We know from the model that below $\underline{x}$, i.e for n below $\underline{n}$, there are no diversified BUs (it is outside the overlapping region). From time step t to time step $t + h$, "all" diversified BUs that were there at t disappear from bin $\underline{n}$. This follows because of the random walk characteristic of x. Hence, to have an equilibrium, the "number" of diversified firms that ends up in bin $\underline{n}$ at time $t + h$ must correspond to the "outflow" of diversified firms that were there at t. Since diversified firms can only come from "above", then equation (3.A.27) follows. For intermediate classes the equilibrium equation is almost as simple:

$$\alpha_n\beta_n = \alpha_{n+1}\beta_{n+1} \cdot [1 - p_{n+1}] + \alpha_{n-1}\beta_{n-1}p_{n-1}, \tag{3.A.28}$$

with the difference that for $n \geq \underline{n}$ we need to account for the firms that come from "below". Since the $\beta's$ are known, then we only have to use the following recursion to pin down the $\alpha's$:

$$\alpha_{n+1} = \frac{\beta_n\alpha_n - \beta_{n-1}\alpha_{n-1}p_{n-1}}{\beta_{n+1}(1 - p_{n+1})}, \tag{3.A.29}$$

where the value of $\alpha_{\underline{n}}$ is set such that $\alpha_{\overline{n}} = 1$, since at this point all firms are diversified.

Uncertain strategy implementation

Now I have to adjust the flows from and to neighboring bins taking into account the Poisson processes (in the discretization these are binomial processes). β_n and γ_n stand for the unconditional probability mass of BUs in bin n and the conditional probability that a BU in bin n is focused, respectively. The following system of equations describes the stationary distribution.
If $x_n < \underline{x}$,

$$\gamma_n = \frac{1}{\beta_n}\left\{\beta_{n+1}(1-p_{n+1})\left[\gamma_{n+1}+(1-\gamma_{n+1})\lambda_d h\right]+\beta_{n-1}p_{n-1}\left[\gamma_{n-1}+(1-\gamma_{n-1})\lambda_d h\right]\right\} \tag{3.A.30}$$

If $x_n = \underline{x}$,

$$\gamma_n = \frac{1}{\beta_n}\left\{\gamma_{n+1}\beta_{n+1}(1-p_{n+1})+\beta_{n-1}p_{n-1}\left[\gamma_{n-1}+(1-\gamma_{n-1})\lambda_d h\right]\right\} \tag{3.A.31}$$

If $\underline{x} < x_n < \overline{x}$,

$$\gamma_n = \frac{1}{\beta_n}\left\{\gamma_{n+1}\beta_{n+1}(1-p_{n+1})+\gamma_{n-1}\beta_{n-1}p_{n-1}\right\} \tag{3.A.32}$$

If $x_n = \overline{x}$,

$$\gamma_n = \frac{1}{\beta_n}\left\{\gamma_{n+1}\beta_{n+1}(1-p_{n+1})(1-\lambda_u h)+\gamma_{n-1}\beta_{n-1}p_{n-1}\right\} \tag{3.A.33}$$

If $x_n > \overline{x}$,

$$\gamma_n = \frac{1}{\beta_n}(1-\lambda_u h)\left[\gamma_{n+1}\beta_{n+1}(1-p_{n+1})+\gamma_{n-1}\beta_{n-1}p_{n-1}\right] \tag{3.A.34}$$

I obtain this distribution by guessing a starting value for $\{\gamma_n\}$ ($\{\beta_n\}$ is known) and iterating until I obtain convergence.

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