Financing Frictions and Firm Dynamics

by

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Abstract

My dissertation examines how financing frictions affect firms' dynamics. In particular, I incorporate financing constraints into a neoclassical investment framework augmented with aggregate and idiosyncratic uncertainty and exogenous stochastic discount factor. Financing constraints are defined as collateral and dividend constraints. Specifically, firms' only source of external finance is debt secured by their collateral assets.

The first essay, ”Effects of Financing Constraints on Investment-Cash Flow Sensitivities in a Dynamic Framework,” studies how financing frictions affect the relationship between corporate investment and internal funds. I demonstrate that the model is sufficient to replicate the major empirical evidence on the influence of financial constraints on corporate investment. On the one hand, supporting Fazzari, Hubbard and Peterson (1988)'s argument, given a cash flow innovation, firms that face tight financing constraints have high propensities to invest and low propensities to pay dividends, because they invest not only to utilize the profitable investment opportunities but also to relax financing constraints they face. On the other hand, as a result of dynamic feature of the model, firms that face tight financing constraints don't exhaust all their credit lines in the anticipation of future bad contingencies. Contrary to Kaplan and Zingales (1997)'s argument, the model illustrates that existence of unutilized credit lines is compatible with the presence of financing constraints. As a result, the model can reproduce quantitatively the empirical evidences of Fazzari, Hubbard and Peterson (1988) and Kaplan and Zingales (1997), and reconciles the controversy between them.

The second essay, ”Effects of Financing Constraints on Cross-Section of Stock Re-

turns," investigates the extent to which financing constraints explain documented cross-sectional variation in risk and expected returns. I show that, consistent with Fama and French (1995) and Chan and Chen (1991), the value and size premiums are rewards for systematic risk. Due to financing constraints, value firms and small firms are dominantly composed of financially distressed firms with low capital capacity and/or excessive leverage. Given that these financially distressed firms benefit during economic expansions and suffer during economic downturns, the high exposure to systematic risk leads to high risk premiums.

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Chapter 1

Effects of Financing Constraints on Investment-Cash Flow Sensitivities in a Dynamic Framework

1.1 Introduction

A growing body of research argues that changes in either net worth or internal funds will affect firms' investment behaviors if capital markets are imperfect. One of the most pronounced capital market imperfections is financing frictions. Fazzari, Hubbard and Peterson (1988) propose that because of external financing frictions, a firm's optimal investment level depends both on the availability of investment opportunities and on the availability of internal funds. Starting with Fazzari, Hubbard and Peterson (1988), several studies examine the influence of financing frictions on corporate investment by looking at the empirical sensitivity of investment to changes in cash flow, but they have conflicting results. Fazzari et al. document excess sensitivity of investment to cash flow for firms that pay low dividends, and suggest that investments undertaken by firms that are more likely to face financing constraints are more sensitive to their cash flows. Following Fazzari, Hubbard and Peterson (1988), Hoshi, Kashyap and Scharfstein (1991), Fazzari and Peterson (1993), Calomiris and Hubbard (1995), Gilchrist and Himmelberg (1996, 1998), and several others subsequently supported Fazzari, Hubbard and Peterson (1988)'s findings. However, Kaplan and Zingales (1997) find evidence that firms with unutilized credit lines exhibit higher investment-cash flow sensitivity than firms without any available credit line and interpret this result as evidence that firms that appear less financially constrained exhibit greater investment-cash flow sensitivities.

Despite the popularity and significance of this literature, little attention is given to the theoretical foundations. I develop a theory that is sufficient to replicate the empirical evidence on the influence of financial constraints on corporate investment. Specifically, I incorporate the financing constraints into a neoclassical investment model with aggregate and idiosyncratic uncertainty. Despite the common approach of formalizing financial constraint as cost constraints, in this thesis, it is assumed that financially constrained firms face quantity constraints instead of cost constraints. The quantity constraints come into play when borrowers restrict debt to be secured by collateral. In this setting, potential borrows cannot borrow more than their collateral value even if they are willing to pay more than the market interest rate.

The main reason for quantity constraint assumption is that there is an extensive literature supporting the relevance of quantity constraints empirically and theoretically. A large credit rationing literature, starting with Jaffee and Russel (1976), Greenwald, Stiglitz and Weiss (1984), and Stiglitz and Weiss (1981), supported by empirical literature by Kashyap, Stein, and Wilcox (1993), Kashyap, Lamont and Stein (1994) and Petersen and Rajan (1994, 1995), suggest that unavailability of external funds is more relevant than the higher cost of external funds in limiting firms' investments. Furthermore, as Kaplan and Zingales (1997) show, investment-cash flow implications of a model with costly external financing depend on the specific properties of production and cost functions since in equilibrium the slope of marginal cost of external funds should be equal to the slope of marginal productivity of investment. In a quantity constraint framework, however, capital demand and capital supply are not equal in equilibrium and therefore implications of the model do not depend on the sophisticated assumptions about marginal productivity and marginal cost functions. Another advantage of the quantity constraint is the endogeneity of the financing constraint. Since firms with higher collateral are able to borrow more, they can invest more into assets that will serve as collateral and as a result their ability to borrow will be even higher. In other words, the endogeneity of financing constraints can amplify the effect of a cash flow innovation on investment spending. The dynamic relationship between investment and borrowing constraint, which is called the ”credit multiplier effect” by Kiyotaki and Moore (1997), cannot be directly captured by a cost constraint framework. Finally, the secured debt assumption simplifies the numerical solution of the model considerably; it is not needed to solve for the interest rate since the secured debt assumption implies risk-free debt.

To investigate how financial frictions affect firms’ behaviors, I develop a partial equilibrium model where heterogeneous firms make rational financing and investment decisions in a dynamic framework under aggregate and idiosyncratic uncertainty. All firms have access to the same decreasing return to scale technology for a single homogeneous good and their capital structures change over time in response to idiosyncratic shocks. A firm’s investment policy is determined by the following trade off: a higher investment rate translates into higher expected profits and higher borrowing capacity, which can increase the investment rate even further, but it also indicates higher expected volatility of profits. In a dynamic framework with a concave value function, firms’ optimal debt policy and investment policy are determined by the trade off between higher profits and higher volatility.

Firm financing and investment decisions produced by the model have important implications for the cash flow sensitivity of investment under financial constraints. First of all, the model presented is able to reproduce the main empirical evidence on the influence of financial constraints on corporate investment. I find that firms with low cash flows take on more debt, invest more and pay fewer dividends. In general, low cash flow firms are small firms without any cash savings. Low cash flow firms borrow more than high cash flow firms and use the proceeds to increase their investment rate both to relax the financing constraints they face and to utilize the higher return associated with small size, independently of their future profitability. These findings support Fazzari, Hubbard and Peterson (1988)'s argument about the influence of financial constraints on corporate investment and they are consistent with the empirical evidence presented by Fazzari, Hubbard and Peterson (1988), Gilchrist and Himmelberg (1995) and Allayannis and Mozumdar (2001) on the excess sensitivity of highly constrained firms.

In addition to replicating the major empirical evidence, the model resolves the debate about financing constraints. The model suggests that for some firms the existence of unutilized lines of credit is compatible with the presence of financing constraints. The logic behind this result is as follows. As well as current constraints, future expected financing constraints have considerable impacts on investment and financing decisions. If forward looking firms expect to face financing constraints in the future, they will partially protect themselves with buffer stocks of cash or unused debt capacity. As a buffer against cash flow variation firms may find it cost effective, relative to asset liquidation, to maintain reserves of liquid assets. In this respect, the model is the consistent with the Myers and Mafluj (1984) for which financing constraints induce precautionary cash savings. The excess liquidity induced by precautionary motives implies that in a dynamic framework, a firm may be able to invest more at the margin at a moment in time and be financially constrained at the same time. This feature of the model reconciles the controversy between Fazzari, Hubbard and Peterson and Kaplan and Zingales by showing that unused debt capacity is not a good measure of degree of financial constraints.

Although the model supports the argument that cash flows have significant impact on firms' investments, its predictions are inconsistent with financial accelerator literature like Bernanke, Gertler, and Gilchrist (1996) and Oliner and Rudebusch (1996) that suggest stronger cash flow effects on investment after periods of tight money. Rather, the model predicts lower investment-cash flow sensitivity during economic downturns. While Bernanke, Gertler, and Gilchrist also modeled financing constraints as endogenous borrowing capacity, their one period model ignores the features that stem from the dynamic framework like precautionary motives and changes in continuation value of a firm due to counter-cyclical price of risk and persistency of productivity shocks. The main ideas of my model are as follows. Given that the borrowing capacity is inversely related to liquidation value, following an adverse aggregate shock, firms experience a reduced access to credit, which is consistent with the empirical evidence provided by Kashyap, Stein, Wilcox (1993) and Bernanke, Gertler, and Gilchrist (1996). In my analysis, tighter credit conditions and persistency of the productivity shocks force firms to maintain excess liquidity to meet future bad contingencies. In the meantime, adverse aggregate shocks reduce investment demand through lower productivity and expected continuation value through counter-cyclical discount rates. Besides, lower investment demand and lower continuation value increase the value of dividend payments. Overall, constrained firms borrow less, invest less and make more dividend payments in bad states of the economy and the lower productivity, lower continuation value and stronger precautionary motives dampen the cash flow sensitivity of investment during an economic downturn. In good states of the economy, however, financing and investment policies of the constrained firms are pretty different. Higher productivity, lower discount rates and relaxed financing constraints stimulate higher borrowing and investment rates and lower dividend rates. Given that firms use a greater portion of cash flows to finance investment and investment increases further through additional debt, the cash flow sensitivity of investment magnifies under a good aggregate shock.

Finally, the model has some implications on corporate liquidity. Using the approach adopted by Almeida, Campello and Weisbach (2005), I measure firm's propensity to save cash out of cash inflows. As Almeida, Campello and Weisbach suggest cash flow sensitivity of cash is an important measure of the significance of financial constraints. In my model, precautionary motives promote cash holdings as a buffer against future contingencies when a constrained firm starts a period with positive slack or zero debt. In overall, model entails positive cash flow sensitivity of cash. More than ever, during recession lower internal funds, lower propensity to invest and persistency of aggregate shocks induce stronger precautionary motives. The outcome of the model is consistent with the empirical evidence of Almeida et al., and provides additional support on the relationship between financial constraints and firm dynamics.

This chapter is largely an extension of the work of Almeida and Campello (2004) that use an endogenous collateral constraint in a simple one period framework, which does not capture dynamic behavior of firms' investment policies. To capture the dynamic effects, I integrate the same financing constraint to a neoclassical investment framework with time-varying price of risk as well as aggregate and idiosyncratic uncertainty.

Moyen (2005) also adopts a similar finance framework to examine the cash flow sensitivity controversy. In support of KZ, Moyen (2005) finds that high cash flow firms have investment policies that are more sensitive than low cash flow firms. The main difference of the theoretical model of Moyen (2005) is the formulation of financing constraints. In Moyen's model, constrained firms cannot change their debt policy; they cannot issue or retire new debt. Additionally, Moyen assumes that future discount rates are fixed. My model, on the other hand, illustrates that accounting for counter-cyclical discount rates and debt financing constrained by collateral value increase the cash flow sensitivity of a constrained firm substantially.

Gomes (2001) is another paper that studies theoretically the effect of financing constraints on cash-flow sensitivity of investment in an environment where heterogeneity and financial decisions are central to understanding firms' investment behavior. Using an industry equilibrium model Gomes shows that financing frictions are not sufficient to generate significant coefficients on cash flow. The key differences between my model and that of Gomes are that I: 1) formulated financial constraint as endogenous quantity constraint instead of cost constraint; 2) allow the corporation to save; and 3) incorporate aggregate uncertainty. In my model, the endogeneity of financing constraint gives rise to the credit multiplier effect that improves the investment-cash flow sensitivity of the constrained firm. On top, the aggregate uncertainty and its effect on discount rates increase the cash flow sensitivities even further. Contradicting Gomes's results, in the current model, financing frictions are sufficient to generate significant coefficients on cash flow. Gomes objects to the common practice of measuring investment-cash flow sensitivities and argues that cash flows contain information about the relationship between real investment demand and future investment opportunities. However, the inferences of this chapter is not likely to be driven by such a bias since the importance of financing constraints on firm dynamics are fortified by the cash flow sensitivity of cash test which use a financial variable as an endogenous variable as opposed to a real variable. For unconstrained firms, changes in cash holdings should not depend on current cash flows or on future investment opportunities so there is no reason to ascribe that significant coefficient of cash flow variable to its ability to forecast investment demand.

The remainder of the first chapter is organized as follows. Section 1.2 describes the model. Section 1.3 discusses the calibration and the simulation methodology. Section 1.4 presents the quantitative properties, and Section 1.5 presents the empirical implications of the model. Finally, Section 1.6 concludes. The description of the numerical method that is used to solve for the value function and the policy rules of the firm and tables and figures are attached in appendix.

1.2 Model

In this section, I describe my version of the neoclassical investment model studied. Time is discrete and the horizon is infinite. The industry is populated with continuum of competitive firms that produce a homogeneous product. I assume the price of capital is one and competitive firms take this price as given. Firms have access to the following production technology for converting capital, k_t into output y_t:

$$y_t = e^{x_t+z_t} k_t^{\alpha}, \tag{1.1}$$

where $0 < \alpha < 1$, and x_t and z_t are the aggregate and idiosyncratic productivity shocks at time t, respectively. Each firm experiences a different idiosyncratic shock and this results in heterogeneity between firms. Both x_t and z_t follow stationary autoregressive stochastic processes;

$$x_{t+1} = \overline{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \epsilon^x_{t+1}, \tag{1.2}$$

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon^z_{t+1}, \tag{1.3}$$

where ϵ^x_{t+1} and ϵ^z_{t+1} are IID standard normal shocks for all $t \geq 0$, ρ_x and ρ_z are persistence measures, σ^x and σ^z are conditional volatilities and $\overline{x}$ is average productivity level.

Since my focus is on the implications of collateral constraint on production side of the economy, following Berk, Green and Naik (1999) and Zhang (2005), I parameterize directly the pricing kernel that is necessary in the valuation of future cash flows, without explicitly modeling the consumer's problem. In this setting the pricing kernel can be considered as the marginal rate of substitution of a representative agent. Specifically, I assume

$$logM_{t+1} = log\beta + \gamma_t(x_t - x_{t+1}) \tag{1.4}$$

$$\gamma_t = \gamma_0 + \gamma_1(x_t - \overline{x}), \tag{1.5}$$

where M_{t+1} denotes the stochastic discount factor from time t to t+1 and $1 > \beta > 0$, $\gamma_0 > 0$, and $\gamma_1 < 0$ are constant parameters. The price of risk is time varying and negative γ_1 ensures that γ_t is decreasing with demeaned aggregate productivity shock. The counter-cyclical price of risk generated by the model can be motivated by the time-varying risk aversion implied by the external habit model of Campbell and Cochrane (1999).

At the beginning of period t, a typical firm decides whether to continue activity or to liquidate after observing productivity shocks x_t and z_t. If it chooses to continue, it commences production with the given level of capital, it pays a nonnegative fixed production cost, f, and it pays down its current debt. Next, it decides how much to invest, how to finance the investment (internal or external funds), and how large a dividend to pay to its shareholders. The funds required to obtain a stock of capital k_{t+1} for next period, given the current stock is k_t, will be represented by the investment function, i_t:

$$i_t = k_{t+1} - (1 - \delta)k_t, \qquad \delta \in [0, 1] \tag{1.6}$$

where δ is the depreciation rate of capital. When the firm is financially healthy, there is an adjustment cost, $h(i_t, k_t)$, which is a quadratic and symmetric function of the investment and capital level.

$$h(i_t, k_t) = \frac{\theta}{2}(\frac{i_t}{k_t})^2 k_t \tag{1.7}$$

Financial markets are imperfect in the model. Equity financing is not available in the model. There are only three potential sources of funds: current cash flow, internal savings and external borrowing. Firms are constrained by the amount of external funds available for borrowing. Lenders impose a collateral constraint requiring that the debt service is less than the liquidation value of the firm. As a result of this collateral constraint, any firm can borrow or lend only at the current market risk free rate r_t. Borrowing is one period debt but it can also be considered as long-term debt with a floating rate. Each period the firm decides whether to issue more debt $b_{t+1} > b_t$, to retire debt, $b_{t+1} < b_t$, or to roll over the existing debt $b_{t+1} = b_t$ at current interest rate. In the presence of financing constraints, firms may want to retain some of their internal funds as cash. To accommodate such policies, the state variable b is allowed to be negative. Since negative debt can be considered as lending in the model, when b_t and b_{t+1} are negative, the previous inequalities in absolute values imply increasing, decreasing and rolling over the existing cash savings, respectively.

In any given period, investment spending, i_t, debt service, $b_t(1 + r_{t-1})$, dividends paid to equity holders, d_t, as well as adjustment cost, $h(i_t, k_t)$, and fixed production cost, f, can be financed by internal cash flows, y_t, and one period debt, b_{t+1}. A firm is liquidated if it doesn't have enough funds to pay the fixed cost, f. Given the optimal decision rules in the continuation case, if the following inequality holds then the firm chooses to liquidate since it will fail to pay the fixed cost in case of production.

$$y_t - i_t - b_t(1 + r_{t-1}) + b_{t+1} - h(i_t, k_t) < f \tag{1.8}$$

When the firm is liquidated, its assets are sold, the debt payment is made and any residual left is distributed as dividends.

If the value of the firm at time t is denoted as $v(k_t, b_t, x_t, z_t)$ then the dynamic problem facing is :

$$\begin{aligned} v(k_t, b_t, x_t, z_t) = \max_{(k_{t+1}, b_{t+1})} & y_t + \Delta b_{t+1} - r_{t-1} b_t - 1_{(b_{t+1}<0)} b_{t+1}^2 \chi - i_t - h(i_t, k_t) - f + \\ & \int\int M_{t+1} v(k_{t+1}, b_{t+1}, x_{t+1}, z_{t+1}) \times Q(dx_{t+1}/x_t) \times Q(dz_{t+1}/z_t), \end{aligned} \tag{1.9}$$

subject to

$$d_t = y_t + \Delta b_{t+1} - r_{t-1} b_t - 1_{(b_{t+1}<0)} b_{t+1}^2 \chi - i_t - h(i_t, k_t) - f \tag{1.10}$$

$$\Delta b_{t+1} = b_{t+1} - b_t \tag{1.11}$$

$$(1 + r_t) b_{t+1} \leq k_{t+1}(1 - \delta) \tag{1.12}$$

$$d_t \geq 0 \tag{1.13}$$

where $Q(x_{t+1}/x_t)$, and $Q(z_{t+1}/z_t)$ are Markov transition matrixes of aggregate shock x and idiosyncratic shock z as defined above and χ is the cost of holding cash. The cost can be viewed as a tax disadvantage since corporate tax generally exceeds the tax on interest income or as an agency cost since managers may divert funds from value-maximizing activities. A convex cost structure ensures a well-defined firm problem.

The collateral constraint is modeled in the form of limited pledgeability of assets using Hart and Moore's (1994) inalienability of human capital assumption. The production technology requires not only physical investment, but also entrepreneur's human capital as inputs. Because of inalienability of human capital, the entrepreneur cannot credibly commit her input to the production process. Consequently, as Hart and Moore (1994) show, the entrepreneur faces credit constraints in this setting; he can only borrow up to the expected value of the firm in liquidation. This type of constraint is also studied by the influential papers Kiyotaki and Moore (1997) and Cooley and Quadrini (2001).

The dividend non-negativity constraint derives from the assumption that equity financing is not available in the model. I imposed a dividend non-negativity constraint like many recent studies (e.g., Moyen (2005), Sapriza and Zhang (2004), Whited (1992)) to be able to focus on the impacts of collateral constraint. Without the non-negativity constraint, when a firm needs additional funds it will pay a negative dividend, which is equivalent to costless equity financing and the collateral constraint would not have any impact on firm's problem.

Let ϕ_t and μ_t denote the Lagrange multipliers associated with the credit constraint, equation 1.12 and the dividend constraint, equation 1.13, respectively. It is possible to solve the constrained optimization problem above using the first-order conditions of equation 1.9 with respect to k_{t+1}, b_{t+1}, k_t and b_t and complementary slackness conditions on μ_t and ϕ_t. The first-order condition with respective to k_{t+1}, positive b_{t+1} and negative and b_{t+1} are:

$$E(M_{t+1}\frac{\partial V_{t+1}}{\partial k_{t+1}}) = (1+\mu_t)(g\frac{I_t}{k_t}+1) - \phi_t(1-\delta) \tag{1.14}$$

$$E(M_{t+1}\frac{\partial V_{t+1}}{\partial b_{t+1}}) = \phi_t(1+r_t) - (1+\mu_t) \tag{1.15}$$

$$E(M_{t+1}\frac{\partial V_{t+1}}{\partial b_{t+1}}) = (1+\mu_t)(1-2b_{t+1}\chi) - \phi_t(1+r_t) \tag{1.16}$$

,respectively.

Equation 1.14 states that the firm invests up to the point where the shadow cost of capital equals the marginal increase in next period's expected discounted value. Since an extra unit of investment relaxes the collateral constraint, the shadow cost of capital decreases in ϕ_t. If the firm chooses to borrow, equation 1.15 states that it borrows up to the point where the benefit of an extra unit of debt today equals the marginal loss in next period's expected discounted value. Similarly, equation 1.16 states that when holding cash is optimal, the firm saves up to the point where the cost of an extra unit of slack today equals the marginal benefit in next period's expected discounted value. The envelope conditions are:

$$\frac{\partial V_t}{\partial k_t} = (1 + \mu_t)\frac{\partial d_t}{\partial k_t} \tag{1.17}$$

$$\frac{\partial V_t}{\partial b_t} = -(1 + \mu_t)(1 + r_{t-1}) \tag{1.18}$$

The intuition behind equation 1.17 is that when the value generated by an additional unit of capital is high or when the internal funds generated by an additional unit of capital is low the demand for external funds will be high and as a result the dividend constraint is more likely to bind. Similarly, equation 1.18 says that when the decrease in value by an additional unit of debt is high or if the interest paid on debt is high then the dividend constraint is more likely to bind.

Using the first order conditions and envelope conditions it is easy to show that:

$$E(M_{t+1}\mu_{t+1}) = \frac{\mu_t}{1 + r_t} - \phi_t \tag{1.19}$$

We can define μ_t in terms of present and future risk free interest rates and credit constraint multipliers by iterating equation (19).

$$\mu_t = E_t[\sum_{k=1}^{\infty}(\prod_{j=0}^{k-1} R_{t+j})\phi_{t+k-1}] \tag{1.20}$$

where R_t is equal to $(1 + r_t)$. Equation (20) shows the relationship between dividend and collateral constraints. Collateral constraint binds whenever the firm is willing to borrow more money than its liquidation value. If the firm is likely to be bounded by the collateral constraint today or anytime in the future, then the demand for the external funds increases in the anticipation of future and dividend constraint also binds. If the model didn't have collateral constraint then the firm could have borrowed without any cost when it needs external fund and the dividend constraint would never bind.

1.3 Calibration

A closed form solution for the model does not exist so I calibrate the model and solve it using numerical methods. All parameters are calibrated at monthly frequency. Following Kydland and Prescott (1982) and Gomes (2001) the capital share α is set to be 0.3. The monthly depreciation rate of capital δ is set to be 0.01 which is consistent with the estimates of Cooper and Haltiwanger (2000).

The parameters driving the dynamics of productivity shocks are calibrated following Zhang (2005). The monthly persistence parameter of aggregate shock x, ρ_x, is set to be 0.983 and the monthly conditional volatility of x, σ_x, is set to be 0.0023. These estimates are consistent with the quarterly values used by Cooley and Prescott (1995). The monthly persistence parameter of aggregate shock z, ρ_z, is set to be 0.97 and the monthly conditional volatility of z, σ_z, is set to be 0.1. These parameters are consistent with the empirical evidence of Pastor and Veronesi (2003).

The pricing kernel parameters β, γ_0, and γ_1 are calibrated to match the average Sharpe ratio, the average real interest rate and the volatility of real interest rate where real interest rate and the maximum Sharpe ratio as:

$$R_t = \frac{1}{E_t[M_{t+1}]} = \frac{1}{\beta} e^{-\mu_m - \frac{1}{2}\sigma_m^2} \tag{1.21}$$

$$S_t = \frac{\sigma_t[M_{t+1}]}{E_t[M_{t+1}]} = \frac{\sqrt{e^{\sigma_m^2}(e^{\sigma_m^2} - 1)}}{e^{\frac{\sigma_m^2}{2}}} \tag{1.22}$$

where

$$\mu_m \equiv [\gamma_0 + \gamma_1(x_t - \overline{x})](1 - \rho_x)(x_t - \overline{x}) \tag{1.23}$$

$$\sigma_m = \sigma_x[\gamma_0 + \gamma_1((x_t - \overline{x})] \tag{1.24}$$

β, γ_0, and γ_1 are chosen to be 0.0994, 50 and -1000, respectively, which give an average Sharpe ratio of 0.42, an average annual real interest rate of 2.0 percent and an annual volatility of interest rate of 0.028. These moments are in line with the empirical literature (e.g., Campell and Cochrane (1999).

The adjustment cost parameter, θ is chosen to be 15 following Zhang (2005) and it is consistent with the empirical estimates of Whited (1992). Following Moyen (2005), the cost of cash holding χ is chosen to be 0.05. Finally, I calibrate the fixed cost of production f to be 0.025. Table 1.1 summarizes the key parameter values in the model.

Given these parameter values, I solved the model numerically via iteration on the Bellman equation. The solution produced the value function $V_c(k, b, x)$ and the policy functions $K_c'(k, b, x)$ and, $B_c'(k, b, x)$. Section 1.7.1 describes numerical procedure to solve the individual firm's maximization problem in more detail. To be able to study the effects of collateral constraint on the policy decision of the firm in more detail, I also solve the model for a firm that can raise unlimited external funds through negative dividends. The resulting value function and policy functions are denoted by $V_u(k, b, x)$, $K_u'(k, b, x)$, $B_u'(k, b, x)$. As discussed before without non-negativity constraints, when the firm needs additional funds it will pay negative dividends, which is equivalent to costless equity financing and the firm can act as an unconstrained firm. Comparing constrained and unconstrained firms gives us a better sense of direction of changes resulting from the financing constraints.

1.4 Quantative Properties

I start by analyzing the properties of the value functions and optimal policy functions of the constrained and unconstrained firms. Since my focus is on the behaviors of the constrained model, I will use unconstrained model results for comparison purposes. I first consider the behavior of investment function, which is defined by equation (6). I scale the investment by the book value of assets, k. Figure 1 plots the optimal investment rates I_u/k, I_c/k for different aggregate and idiosyncratic shocks. To study the effects of idiosyncratic productivity shock z, the aggregate productivity shock level x, is fixed at its long run average level $\overline{x}$ and similarly, to study the effects of x, z is fixed at its long run average level $\overline{\mathrm{z}}$. Several interesting patterns emerge from investment function analysis. In general, investment rates of both constrained and unconstrained firms are increasing in productivity level and decreasing in the firm size. Any firm, constrained or unconstrained, will invest more when it faces better productivity shocks. Lower investment rate with larger asset size is a direct outcome of the negative relationship between the marginal productivity of capital and the firm size.

Figure 1.1 plots I_u/k and I_c/k under different aggregate productivity shock levels. I_c/k is significantly lower than I_u/k when the firm size is small. Unlimited financing capability gives the unconstrained firm an edge in investment when capital level is low. When unconstrained firm's cash flow is low, it can still reach its first best investment level using costless, unlimited external financing. It pays negative dividend which is equivalent to equity financing. When the initial debt holding of the constrained firm, B_c, is zero, borrowing limit has an indirect effect on the low level of investment rate of the small constrained firm. Figure 1.2 plots B_c' for different level of aggregate productivity shocks. Although small firms cannot reach unconstrained levels of investment, they still do not borrow up to their limits. More borrowing translates into higher level of production, which implies higher expected profit and higher volatility of profits. Lower borrowing level is a direct consequence of the trade off between larger expected profits and higher volatility of the profits. The positive probability of a series of bad shocks prevents firms from extra borrowing because if they choose to borrow more today, in case of a series of bad shocks, the firm size will shrink in the future, the cash flow will decrease significantly and accordingly, the borrowing constraint will bind. Due to binding borrowing limit and insufficient internal cash flows to cover the fixed cost of production, the firm may get liquidated. In other words, more debt today increases the possibility of being constrained in the future and therefore, borrowing limit has indirect impacts on the low levels of borrowing and investment of the small constrained firm.

As the firm size gets bigger, the internal cash flow of the constrained firm become sufficient to reach to the desired rate of investment, and the constrained firm's investment rate catches up with the unconstrained rate. With bad aggregate productivity shocks, it even exceeds the unconstrained rate. A low aggregate shock reduces the investment rates of all the firms in the economy through two different channels. First, it reduces the investment rate directly by lowering the marginal productivity of investment. There is also an indirect effect associated with x, which stems from time-varying discount rates. When a bad aggregate productivity shock hits the economy, the discount rate increases and therefore for a given increase in the investment rate, the increase in expected continuation value of the firm will be lower. In other words, the contribution of investment to current firm value is lower so a bad shock reduces the propensity to invest through higher discount rates. As a result of the low marginal productivity and the high discount rate, the unconstrained firm chooses to decrease the firm size and give the proceeds to the shareholders as dividends. The large constrained firm prefers to disinvest as well but still I_c/k is significantly higher than I_u/k. Due to the positive probability of a series of bad shocks and probability of being constrained in the future, the constrained firm acts more conservative and tries to maintain a higher level of capital as a cushion for possible bad times. Due to adjustment and cash holding costs, it is cheaper to maintain a higher capital level relative to disinvest and save the proceeds as cash; therefore constrained firm prefers carrying unproductive capital over holding cash as a cushion.

As noted, financial constraints have indirect effects on unleveraged firms' investment decisions under aggregate shocks. Constrained firms avoid borrowing and they end up depressing their investment rates. The gap between unconstrained and constrained firms' investment rates is higher under good aggregate shocks relative to bad shocks. We can deduce that financing constraints bind unleveraged constrained firms more in economic booms compared to recessions. The reason behind this is the simultaneous impact of aggregate shocks on current productivity and continuation value. Given that current productivity is higher and future looks brighter, investment demand amplifies. Current cash flow increase as well but the increase in internal funds is not adequate to compensate the increase in investment demand. Since firms still pass up the borrowing option, the gap between internal funds and investment demand explain the significance of the financial constraints. This gap diminishes with firm size because while cash flow increases with capital level, investment demand decreases with it. In other words, financing constraints is not very restrictive for large firms. During bad times, the internal fund and investment demand diminish simultaneously but the gap between them is lower compared to that in good times.

In Figure 1.2, the investment rates follow a similar pattern when analyzed as a function of idiosyncratic shock z. The only exemption is the case when the firm size is very small and the idiosyncratic shock is negative. The conditional volatility of z is larger than the conditional volatility of x, therefore the cash flow of a firm is more sensitive to changes in z. When a negative z hits an extraordinarily small firm, firm's cash flow drops substantially. The borrowing constraint binds and the cash flow cannot cover the fixed cost. Consequently, the constrained firm can not survive and the firm gets liquidated. A firm that is close to the liquidation boundary borrows up to the limit and increases its investment rate as much as possible to move away from the boundary. Briefly, a small constrained firm is very vulnerable to negative idiosyncratic shocks and when it faces one, investment rate becomes very sensitive to internal cash flows and available external funds.

I also want to note that the significance of financing constraints is higher under negative z compared to positive z. The effect of z on internal funds dominates its effect on investment demand. This is not surprising given that idiosyncratic shock z is not related to future discount rate while the internal fund is vastly sensitive to changes in z due to its high conditional volatility.

Finally, I want to study the effect of stochastic discount rate assumption on investment rates. Figure 1.3 plots the optimal investment rates of a constraint firm for different aggregate shocks under constant discount rate and stochastic discount rate assumptions. Investment rates barely move with aggregate shock x under constant discount rate assumption. Due to low conditional volatility of x, the productivity and internal funds vary slightly with different aggregate shocks. Under stochastic discount rate assumption, however, x has an effect on both current productivity and expected continuation value; therefore the effect is amplified. In good times, stochastic discount rate model produces higher investment rates compared to constant discount rate model because not only today, but also future looks brighter. Similarly, in bad times, stochastic discount rate model produces lower investment rates compared to constant discount rate model because the decline in expected continuation value on the top of the drop in current cash flow depresses the investment demand significantly. Briefly, investment rates produced by stochastic discount rate model are more sensitive to aggregate productivity shocks compared to that produced by constant discount rate model.

Figures 1.4 and 1.5 depict the borrowing policies of a constrained firm with zero B_c and positive B_c, respectively, under different aggregate and idiosyncratic shocks. The most noticeable trend in Figure 1.4 is the tendency to save. The tendency to save stems from the dynamic feature of the model. In contrast to one period models, in a dynamic model, firms act strategically taking expected continuation value of the firm into account, as well as current profitability. A forward looking firm may want to save some cash as a buffer against the possibility of future financing constraints. While cash holdings require a reduction in current, valuable investments, the extra slack tomorrow will reduce the constrained firm's reliance on external financing and it will allow the firm to utilize more from the future profitable investment opportunities. Therefore, a constrained firm's cash policy is determined by a trade off between the profitability of current and future investments. Precautionary motives to hold cash that is derived from this trade off under financing constraints is important in explaining the excess cash holdings of the firms. In this sense, the chapter contributes to cash management literature by presenting a theory that will help explaining the empirical evidence.

An initially zero indebted constrained firm borrows in two cases. The first case is when the firm's size is small and it faces a negative idiosyncratic shock. Shocks are very persistent and the firm rationally anticipates another bad shock in the next period. As discussed earlier, if the firm size gets too small, the constrained firm can not survive after a negative z. Therefore, to avoid liquidation, the firm, whose size is close to the liquidation border, borrows up to the limits and enlarges the firm size. The second case is when the firm size is significantly large and the economy faces a bad aggregate shock. As mentioned before, under a bad aggregate shock the value maximizing firm discounts expected continuation value heavily and the dividend paid today is valued more by the shareholders. If the firm is far from the liquidation boundary, it chooses to borrow up to the limit and pay the proceeds as dividends.[1]

A noticeable pattern in Figures 1.4 and 1.5 is that the borrowing/saving level is higher/lower during good aggregate shock periods but lower/higher during positive idiosyncratic shock periods, and vice versa. This result stems from the trade off between the higher expected profits and higher volatility of profits that are attributable to higher borrowing and lower saving. Due to the counter-cyclical price of risk, a constrained firm becomes less risk averse under a good aggregate productivity shock and prefers to

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[1] Borrowing at the limit under bad aggregate shock and zero idiosyncratic shock occurs when the firm size is greater than 6. This part is not included in Figures 1.4 and 1.5 because due to small figure sizes it would be hard to identify the borrowing and saving patterns when the capital level is less than 6. Figures are available upon request.

borrow more/save less and uses the proceeds to increase investment and the expected profit level. If the firm is hit by a positive z, however, the risk averness of the firm does not change and the concern of higher volatility dominates the higher profitability expectations and accordingly, when the firm is large enough it prefers to borrow less/save more when it faces a more favorable idiosyncratic shock. The tendency of constrained firms to save less in good times and save more in recessions is also documented in empirical cash management literature by Almeida, Campello and Weisbach (2005).

Another rule evident from the figures is that the optimal level of debt depends not only on the capital level and productivity shocks but on the current level of debt as well. As the current debt level increases, holding everything else constant, the optimal borrowing level also increases. Finally, the debt policy is pretty persistent and decreasing with the firm size.

Figure 1.6 illustrates the optimal investment policy of a constrained firm with positive B_c under different productivity shocks. As the zero debt level case, the investment rate of an indebted firm is a non-decreasing function of both the aggregate and idiosyncratic shocks and also, the investment rate is more sensitive to idiosyncratic shocks when the firm is close to the liquidation border. Possibility of future bad shocks and the threat of liquidation forces small firms to borrow more and invest more. On the other hand, larger firms have larger cash flows and they don't rely on external borrowing to finance their investment. Large firms borrow only to pay more dividends when they face a bad aggregate shock.

Another way to see this is to look at the ratio $B'/(1-\delta)K'$ for different capital levels. By definition, this ratio is less than or equal to 1.[2] In general, the ratio is a decreasing function of the firm size and firm productivity. In the first panel of Figure 1.7, it seems like the borrowing constraint never binds under different aggregate productivity shocks at small capital levels because the constrained firm never reaches to the limiting point 1.

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[2] A shortcoming of the numerical procedure is that discretization of the state space limits the ability of the model to borrow at the boundary.

In fact, when the firm is extremely small, the collateral constraint binds. As discussed above, firm can not survive due to lack of internal cash flow and limiting borrowing constraint and hence, the equity value goes to zero. That is why the ratio is undefined, instead of 1, for very small capital levels even though the constraint binds. The same logic applies to idiosyncratic shocks. Besides, the small firm under negative productivity shock borrows on the limit point and the collateral constraint binds.

Gomes, Yaron and Zhang (2003) find that firms are more constrained in good times of the economy. Gomes, Yaron and Zhang (2003) define financing constraints as equity financing cost. As productivity increases, a firm's propensity to invest increases and higher investment requires more external financing which results in higher cost of external financing. In other words, firms are more financially constrained under good productivity shocks and financing premium is pro-cyclical. The pro-cyclical financing premium is consistent with the results presented for unleveraged firms. Since unleveraged firms prefer not to borrow unless they are small and unproductive in the firm-level, debt to equity ratio is not a good measure of financing constraints for them. As discussed earlier the gap between their internal funds and investment demand increases in good times and financing constraints bind them more. The outcome of my model differs from Gomes, Yaron and Zhang (2003)'s results when firm is initially leveraged. In my model, the significance of financing constraints is counter-cyclical for leveraged firms. The intuition behind this is simple. In a collateral constraint framework, financing constraint is endogenous in the liquidation value of the firm and as investment level increases, the borrowing capacity increases as well. Leveraged firms are highly risky and they need to invest in collateralized assets both to increase production and to relax the financing constraints. Under a good productivity shock, credit multiplier effect induces higher investment rates, and accordingly less binding financing constraints. On the other hand, under a bad productivity shock, the number of indebted firm liquidations increases substantially as a result of the limiting constraints. Firms that are close to liquidation want to over invest to move away from the liquidation boundary in anticipation of persistent bad shocks but they hardly service their current debt using external financing and they are bounded by the collateral limit. Lower investment rates result in lower borrowing capacity and binding financing constraints. Therefore, financing constraints bind more under bad productivity shocks.

We now focus our attention on the impact of financial slack. Figure 1.8 depicts the investment rates of constrained and unconstrained firms with slack for different productivity shocks. One interesting point is that investment rate of the unconstrained firm does not depend on the slack. This is not a surprising result considering that the unconstrained firm can effectively optimize its investment rate by using unlimited external financing. The constrained firm, on the other hand, sometimes cannot reach to the desired level of investment on account of financing constraints and limits its investment to the available funds. Financial slack helps constrained firm to relax the financial constraint it faces and reach to the unconstrained level of investment. Moreover, the liquidity reduces the likelihood of being constrained in the future. According to Figure 1.8, when the constrained firm does not have enough assets to generate the sufficient amount of internal funds for investment, it uses the initial cash stock it has to finance its investment. In view of that, investment rate increases and given the adequate amount of slack, it reaches to the unconstrained level. For a certain interval, it is even exceeds the unconstrained level. The difference is the precautionary investment against the possibility of future bad shocks and shrinking firm size. The decision between precautionary investment and precautionary savings is related to equations 1.14 through 1.18. As the equations state a constrained firm prefers production over saving in three cases: (1) When expected future internal funds generated by an additional unit of capital is high; (2) When next period interest rate is low; (3) When total cash holding cost is high. While positive idiosyncratic shock raises production significantly, a good aggregate shock simultaneously raises the current cash flow and reduces next period interest rate; therefore persistency of productivity shocks cause high expected value creation with an additional unit of capital. Accordingly, the constrained firms with good productivity shocks and/or high financial slack prefer extra production over cash holding.

1.5 Empirical Implications

1.5.1 Summary Statistics

I start by simulating the model for 2000 firms and 500 months. Only 1283 firms survived at the end of 500 months. The default rate is 0.29% per quarter, which is lower than the average exit rate, 1.2 % per quarter, in the sample of manufacturing firms analyzed by Evans (1987a). The default rate is lower than data because constrained model guarantees that firms never default on their debt payments. Default occurs only when a firm cannot pay the fixed productivity cost because collateral constraint prevents firms from choosing a debt level that is too burdensome to service in the following period after facing an unexpectedly low productivity shock.

I assume that all firms are initially identical. I drop the first 50 observations to minimize the effect of a possibly suboptimal starting point. The resulting firms' capital levels range between 0.07 and 2.2. Figure 1.9 plots the steady state distribution of the surviving firms' capital levels.

Table 1.2 reports a set of moments generated under the benchmark parameters in Table 1.1. The data source for moments of real interest rate is Campbell, Lo and MacKinlay (1997), for the debt-asset ratio is Hennessy and Whited (2006), for cash holding-asset ratio is Almeida, Campello and Weisbach (2005), for the rate of investment Livdan, Li and Zhang (2005) and for the rate of disinvestment is Abel and Eberly (2001).

Overall, Table 1.2 shows that the collateral constrained model does a reasonably good job in matching this set of basic moments. The close to perfect fit of the model to the first three moments is not surprising since the model is already calibrated to match the average annual Sharpe ratio, interest rate and volatility of interest rate. The investment and disinvestment rates are sensibly close to the data. While many studies are able to match the investment rate, they have difficulties in matching disinvestment rate, especially under quadratic adjustment costs. In this model, collateral constraints drive the high rates of costly disinvestment. Since the collateral constraints limit firms' access to external funds, when they are short of internal funds they have to disinvest to create the extra funds. Disinvestment generally occurs when firms face bad productivity shocks and they don't have sufficient internal funds to service their debt payments and to pay the fixed production cost, simultaneously. Finally, while both debt and cash holding rates are quite close to the real data moments, both of the ratios are a little lower than the real values. The reason behind this is the shortcoming of the model to account for simultaneous debt and cash holdings. The model does not allow borrowing for saving purposes, which reduces both borrowing and saving rates. I also want to note that even though the average debt to asset ratio is higher than average cash holding to asset ratio, the fraction of observations with cash holdings is 0.54. In other words, firms hold cash, for precautionary purposes, more often than they do borrow but the level of cash is less than the level of debt.

Table 1.3 displays some of the constrained firm's investment behavior in greater detail. Table 1.3 presents the means and correlation coefficients for capital stock $K_{i,t}$, beginning of period Tobin's q, $Q_{i,t-1}$, cash flow $CF_{i,t}$, investment $I_{i,t}$, dividends $D_{i,t}$, debt issues $B_{i,t}$, and finally the productivity shocks x_t and $z_{i,t}$. Specifically, I define cash flow, $CF_{i,t}$, as the beginning of the period funds and Tobin's q , $Q_{i,t}$, as the market to book value of assets.

$$CF_{i,t} = y_{i,t} - b_{i,t}r_{i,t-1} - \delta k_{i,t} - f \tag{1.25}$$

$$Q_{i,t} = (V_{i,t} + b_{i,t+1})/k_{i,t+1} \tag{1.26}$$

Cash flow, investment, dividend and debt variables are scaled by the capital level.

In this model aggregate shocks x_t, and idiosyncratic shocks $z_{i,t}$ are the variables that represent investment opportunities. Summer (81) motives a linear relationship between the investment capital ratio and Tobin's q using a quadratic adjustment cost framework.

According to the Table 1.3, the beginning of the period Tobin's q is a good proxy for the investment opportunities for this model as well because the correlation coefficient of $Q_{i,t-1}$ with x_t and $z_{i,t}$ are fairly high, 0.54 and 0.42 respectively. The reason of the higher correlation between Tobin's q and aggregate shock is the counter-cyclical discount rates. Tobin's q takes into account not only the value created today but also the expected value that will be created in the future. While both x_t and $z_{i,t}$ affect firm's productivity level, only x_t has an effect on discount rates. Since x_t changes the way a firm discounts expected continuation value as well as the productivity, Tobin's q responds more to variations in x_t compared to variations in $z_{i,t}$.

The correlation between the cash flow $CF_{i,t}$ and x_t and $z_{i,t}$ exhibits a different pattern. While the correlation coefficient between $CF_{i,t}$ and x_t is almost zero, the coefficient is pretty high for $CF_{i,t}$ and $z_{i,t}$. While both x_t and $z_{i,t}$ affect the productivity in the same direction their impact on the debt service is different. The aggregate shock doesn't have much impact on the debt level but the interest rate varies with it. Depending on the sign of $B_{i,t}$, which determines whether a firm is borrowing or saving, impact of the interest rate will differ. On the other hand, while $z_{i,t}$ and $B_{i,t}$ are negatively correlated, the interest rate remains constant. As a result, regardless of the sign of $B_{i,t}$, an increase/decrease in idiosyncratic shock level will also increase/decrease the cash flow.[3] In addition, higher volatility of $z_{i,t}$ results in higher sensitivity in cash flows.

Another interesting relation to explore is the relationship between dividend rate and productivity shocks. The dividend rate is negatively related to aggregate shocks but positively related to idiosyncratic shocks. The explanation is very similar to the previous cases. As a result of the counter-cyclical discount rates, in a good state of the economy the discount rate is low and thus the expected continuation value of the firm is high. This reduces the dividend ratio and increases the investment rate. In contrast, a higher $z_{i,t}$ promotes both investment rate and dividend rate since the productivity is

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[3] Table 1.3 records the correlation coefficients between $z_{i,t}$ and $B_{i,t+1}$, and x_t and $B_{i,t+1}$ but due to the persistence of the shocks, their correlation with $B_{i,t}$ is very close to the reported values.

high and the discount rate is constant.[4]

The rest of Table 1.3 is intuitive. The cash flow is positively correlated with the investment and dividend rates and negatively correlated with the borrowing rate. When there is an increase in the internal funds, firm uses these funds to raise capital, to make dividend payment and to retire some debt.

1.5.2 Investment-Cash Flow Sensitivities

In this section, I investigate the cross-sectional properties of the model. My intention is to find out whether the model presented with the collateral constraint can produce a cross section of investment-cash flow sensitivities that demonstrates similar pattern with the real data. The most popular approach to test investment-cash flow relationship is the Q model. According to Q model, under the assumption of perfect capital markets, a firm's investment decision is mainly determined by expectations of future profit opportunities, which is usually estimated by the ratio of the market value of assets to their replacement value.

In an influential paper, Fazzari, Hubbard and Peterson (1988) suggest that under financing frictions, in addition to the availability of profitable investment opportunities, the availability of internal funds is also an important determinant of a firm's optimal investment policy. To account for the financial frictions, the Q model is adjusted to include cash flow variable as a proxy for the availability of internal funds. A firm that faces financing constraints should exhibit positive investment-cash flow sensitivity. Since Fazzari et al., it has been a common practice to test the presence of financing constraints by checking whether investment-cash flow sensitivity coefficient that is obtained from adjusted Q model is significant for the firms that a priori are thought more likely to face financing frictions.

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[4] A higher $z_{i,t}$ promotes dividend rate if the firm is not close to liquidation boundary.

Specifically, the adjusted Q model of investment can be written as follows:

$$(\frac{I}{K})_{it} = a_i + a_1 Q_{i,t-1} + a_2(\frac{CF}{K})_{i,t} + u_{i,t} \tag{1.27}$$

where the fixed firm effect a_i, the investment q sensitivity a_1, and the investment-cash flow sensitivity a_2 are the coefficients to be estimated and $u_{i,t}$ is the error term.

There are several influential empirical studies that split the sample of firms according to different criteria that are used to identify the degree of financing constraints that a firm faces and then examine if the cash flow coefficient is different across the groups of firms. To explain the empirical evidence provided by these studies I split the data generated by the constrained model according to different firm characteristics identified by different criterion and report the regression coefficients obtained under each criteria. For comparison purposes, Table 1.5 reports the empirical findings of the related studies.

First, following Fazzari et al., I separate simulated sample into groups based on dividends paid out to shareholders, $D_{i,t}/K_{i,t}$. The sample consists 450 time period and 1283 firms that survived over this period, which are 577350 observations in total. I compute the average dividend over this sample and the observations that have a dividend payout higher than the average is called the low retention group and similarly the observations that have a dividend payout lower than the average is called the high retention group. Given that Tobin's q included in the regression accounts for the investment opportunity set of the firm, in Table 1.4, Panels A and B, the cash flow coefficient estimate of the high retention group is higher than the coefficient estimate of the low retention group, $(0.22 > 0.03)$. According to the model, firms that are more constrained by financing constraints pay fewer dividends and retain more of their income to be able to finance their investment, and accordingly, their investments are more sensitive to the fluctuations of their cash flows. The estimates are consistent with the findings of Fazzari et al., where the cash flow coefficient of most constraint firm, 0.46, is higher than the coefficient estimate of the less constraint group, 0.023.

Second, in the spirit of Allayannis and Mozumdar (2001), the sample split is based on the beginning of period cash flow $CF_{i,t}/K_{i,t}$. The advantage of using cash flow as proxy for financing constraints is that it omits the current period's decisions. I compute the average cash flow and categorize the observations that have cash flows higher than the average as high cash flow group and the rest is categorized as the low cash flow group. According to Table 1.4, low cash flow group has higher cash flow-investment sensitivity than high cash flow group, (0.20 > 0.04), similar to the evidence provided by Allayannis and Mozumdar, (0.355 > 0.151). Since the correlation coefficient between cash flow and dividend is high, (0.55), it is not surprising that both criteria provide similar outcomes.

Allayannis and Mozumdar further argue that negative cash flow observations need additional attention because they represent abnormal cases where levels of investments hit bottom and can not respond to additional cash flow declines. Since these outliers does not add any value to the objective of measuring the impact of financing constraints on investment in normal situations, I run the same regression in equation 1.27 using only the positive cash flow observations, $CF^{+}_{i,t}/K_{i,t}$. The results are reported in Table 1.6. Consistent with Allayannis and Mozumdar results the cash flow sensitivity coefficients of the low cash flow group is amplified and it is still higher than the high cash flow group, (0.3986 > 0.063). Similarly, Panel A and B of Table 1.6 show that the cash flow sensitivity of the high retention group is amplified and it is still higher than the cash flow sensitivity of the low retention group, (0.4183 > 0.09).

Figure 1.10 explains these results further. It graphs the average investment and dividend rates at representative cash flow realizations. The simulated sample from the constrained model is sorted with respect to cash flow realizations and then divided into 20 groups. For each group of the average cash flow realizations, average investment rate and dividend rate are computed. In panel A, for negative cash flow realizations, the plot is flatter compared to low but positive cash flow realizations. Since the slope of the plot is reflected in the cash flow sensitivities, the estimated sensitivity coefficient of low cash flow group amplifies when negative cash flow observations are dropped. As cash flow increases further the plot becomes flatter and accordingly, the cash flow sensitivity coefficient of high cash flow group is lower. The high cash flow group consists of firms with large capital size and/or firms with slack. Since both increasing firm size and cash savings relax the financing constraints that firms face, the cash flow sensitivities drop as cash flow increases. Panel B shows the relationship between dividend and cash flow realizations. Since some of the low dividend observations correspond to negative cash flow realizations the cash flow sensitivity of high retention group is lower in Table 1.4 compared to Table 1.5. When negative cash flow observations are dropped, the correlation between cash flow and dividend increases significantly and high retention group and low cash flow group provide very similar results.

Next classification criterion is the firm size. Gertler and Hubbard (1988) argue that small firms are more likely to face information and incentive problems so they are more likely to face financing frictions. After Gertler and Hubbard (1988), Gilchrist and Himmelberg (1995) check the presence of financing constraints by splitting firms according to their sizes and they find that cash flow sensitivity of small firms is higher than the sensitivity of large firms, ($0.203 > 0.124$). Following Gilchrist and Himmelberg (1995), the sample is split according to the criterion that is based on firm size. Firms that are smaller than the average size are called small firm size group, and the rest is called the large firm size group. For the constrained model provided here, the degree of financial constraint depends both on the borrowing capacity and on the size of the firm's cash flows relative to its investment opportunities. Firm size is a great proxy for the degree of financial constraint for firms without slack because both the output level and the borrowing capacity is positively related to firm size. However, slack relaxes the financing constraints by increasing the size of a firm's cash flow keeping the investment demand constant and it reduces the cash flow sensitivity of investment. Since small firm size group consists of small firms both with slack and without slack, the cash flow sensitivity coefficient of small firm group is lower than the coefficients of the previous groups but it still is significantly higher than that of the large firm group, $(0.1559 > 0.02)$. The results provided by Gilchrist and Himmelberg (1995) are inline with the results produced by the model.

Fourth, the sample is divided into two groups to capture Kaplan and Zingales (1997)'s firm classification. This classification is based on the amount of unutilized credit available to a firm. Firms which borrow close to their credit limits are called low unutilized credit group and the rest is called high unutilized credit group. Specifically, the sample split is between firms with $(B_{i,t+1}/((1-\delta)K_{i,t+1}) \geq 0.9$ and firms with $(B_{i,t+1}/((1-\delta)K_{i,t+1}) < 0.9$. Consistent with the regression results of Kaplan and Zingales (1997), firms that exhaust their available credit exhibit lower cash flow sensitivity than firms that do not utilize all the credit available to them. There are several reasons behind this result. The first one is the fact that constrained firms do not always borrow at the margin. Considering the possibility of being constrained in the future, forward looking firms will partially protect themselves with cash savings or unused debt capacity. The precautionary motives that stems from this dynamic perspective distinguish the model from the static view of financing constraints employed by Kaplan and Zingales (1997). In a dynamic framework the amount of unutilized credit is not a good proxy for the degree of financing constraint a firm faces. The second reason is that more than half of the firms that borrow close to their margin have negative cash flows. As discussed above, investment levels of firms with negative cash flows have already hit bottom and cannot respond to additional cash flow declines so negative cash flows observations result in low investment-cash flow sensitivities. Finally, a small number of observations in the low unutilized credit group are coming from large firms who borrow at the margin to make dividend payments to shareholders after facing bad productivity shocks. Since changes in their cash flows will have a major impact on their dividend payments instead of their investments, these observations reduce the overall cash flow investment sensitivity.

Allayannis and Mozumdar (2001) showed that when the negative cash flow observations are excluded from Kaplan and Zingales's sample, the estimated sensitivity of low unutilized credit group is actually higher than that of high unutilized credit group. In the spirit of Allayannis and Mozumdar, when I re-estimate the sensitivity coefficients excluding the negative cash flow observations from the simulated data, the results in Table 1.6, Panel G and H, are supporting Allayannis and Mozumdar (2001)'s findings. The low unutilized credit group's sensitivity coefficient is significantly higher than the high unutilized credit group's sensitivity coefficient ($0.56 > 0.1057$). What is more, it is significantly higher than the sensitivity coefficients of all the previous classifications. This result is interesting but it is not surprising. Excluding negative cash flow observations from data implies leaving financially distressed firms out of the sample. A firm that issues debt when it is financially healthy typically uses part of the external funds to invest, which will in turn increase its borrowing and investment capacities. As discussed before, this so called ”credit multiplier effect” stems from the endogenous financing constraint and it increases the sensitivity of investment to internal funds. In other words, a change in the availability of internal funds when a firm is financially healthy has dual effects on investment, and hence the investment-cash flow sensitivity is amplified.

Finally, following Moyen (2004), I compare the cash flow-investment sensitivities of the data simulated from the constrained model and the unconstrained model. In contrast with the findings of Moyen, investment-cash flow sensitivities of constrained firms are higher than those of unconstrained firms, ($0.14 > 0.02$). The difference in findings is stem from the different definitions of financing constraints in our models. In Moyen (2004), unconstrained firms do issue risky debt when they need external financing and they face bankruptcy risk. Therefore, availability of internal funds is an important determinant of their investment policy and the unconstrained group's investment-cash flow sensitivity coefficient is significantly greater than zero. Constrained firms, on the other hand, had access to external capital markets at some point in the past but they no longer have access. They don't have any control over their debt policy so they only choose between investing and paying dividends. Under a positive cash flow innovation, since unconstrained firms can issue debt to finance additional investment, their investment rates increase more rapidly compared to constrained firms' investment rates and therefore, investment-cash flow sensitivity of unconstrained group is higher than cash flow sensitivity of the constrained group, (1.221 > 0.592).

In my model, unconstrained firms have riskless, unlimited access to external markets. Since unconstrained firms don't face any type of financing constraints, the availability of internal funds is irrelevant for their investment policy and their investment cash flow sensitivity is insignificant. Constrained firms, on the other hand, do have access to the external capital markets but their access is limited to their liquidation values. Constrained firms may issue some debt in periods of high cash flows to finance investment or they may choose to stock up some cash in bad times to finance future investment opportunities with internal funds. Therefore, the availability of external financing in the constrained model gives rise to significant investment-cash flow sensitivity coefficients. Besides, the endogeneity of financing constraint enhances the investment-cash flow sensitivities even further because firms invest both to exploit the profitable financing opportunities and to relax the borrowing constraints. In other words, credit multiplier effect gives more incentive to constrained firms to invest. Furthermore, countercyclical discount rate feature of the model, which doesn't exist in Moyen (2004), also contributes to the significant cash flow sensitivities of constrained firms. After facing a good/bad aggregate productivity shock, continuation value of a firm increases/decreases so propensity to invest also increases/decreases. Firms with slack, however, decrease the investment-cash flow sensitivity of the constrained group because slack relaxes the financing constraint they face and constrained firms with slack acts more like unconstrained firms. Therefore, the stochastic discount rates, the availability of limited borrowing capacity, the possibility of cash holding, and the endogeneity of borrowing capacity simultaneously contribute to the cash flow sensitivity coefficients produced by the constrained model.

1.5.3 Cash Flow Sensitivity of Cash

The model has significant implications on corporate liquidity and its relationship to the firms' internal funds, as well. The relation between financial constraints and a firm's propensity to save provide us one more way to identify whether financial constraints are an important determinant of a firm behavior. As discussed in detail above, if forward looking firms expect to face financing constraints in the future, they will partially protect themselves with buffer stocks of cash today. A constrained firm cash policy is determined by a trade off between profitability of current and future investments. While cash holdings as a buffer against future cash flow variations may be cost effective relative to asset liquidation, higher cash savings require a reduction in current, valuable investments. Therefore, financial constraints are related to a firm's propensity to save cash out of internal funds, which Almeida, Campello and Weisbach (2006) refer as the "cash flow sensitivity of cash". Unconstrained firms in this model have unlimited access to external markets so they have no need for cash. The analysis suggests that while constrained firms should display significant cash flow sensitivity of cash, unconstrained firms' cash holdings should bear no significant relation to their internal funds since they don't face any financing constraints.

To test this hypothesis, I use the regression specification from Almeida, Campello and Weisbach and estimate the change of cash holdings for constrained and unconstrained model, controlling for firm fixed effects. Specifically, I estimate the regression equation

$$(\frac{\Delta CashHoldings}{K})_{i,t} = a_i + a_1 Q_{i,t} + a_2(\frac{CF}{K})_{i,t} + a_3 K_{i,t} + u_{i,t} \tag{1.28}$$

Size is included to the cash holding equation as an explanatory variable because of economies of scale arguments in cash management literature (Opler et al (1999)). a_2 represents the cash flow sensitivity of cash.

The results of estimating equation 1.28 are summarized in Table 1.7. For constrained firms, the cash flow sensitivity of cash is positive and significant. The sensitivity estimate of cash is 0.111. In other words, a constrained firm saves around 11.1 cents for each additional dollar of internal funds. These results are consistent with the hypothesis that corporate liquidity is positively related to internal funds under financial constraints. The sensitivity estimate of cash is small, negative and insignificant for unconstrained firm. Since unconstrained firms have no use of cash there is no significant relation between cash holding and internal funds. The estimates are in line with the empirical evidence provided by Almeida, Campello and Weisbach (2006) that is summarized in Table 1.7 as well. Using annual data, Almeida, Campello and Weisbach test the presence of financing frictions using the regression equation 1.28 shows that for each dollar of cash flow, a constrained firm will save around $5 - 6$ cents, while unconstrained firms do nothing.

Macroeconomic Dynamics

Another important issue to consider is the role that aggregate shocks play in a constrained world in creating business cycle asymmetries. In this section, I explore how investment policies change in response to an aggregate shock that affects firms' ability to generate cash flows.

According to the model presented, a firm's collateralizable net worth is pro-cyclical. During booms, constrained investment increases and accordingly, the collateralized net worth and external financing capacity also increase. Besides, interest rates are countercyclical so interest paid for a loan is lower during booms. In other words, it is easier and cheaper to borrow during economic upturns. Conversely, in recessions, it is more difficult and expensive to borrow.

The analysis suggests that constrained firms' investment fluctuations may exhibit asymmetries in business cycles. Investment upswings in booms may be sharper than downswings during recessions. In upturns, higher productivity, lower discount rates and higher financing capacity stimulate investment and borrowing, and dampen dividend payments. Since a good aggregate shock enhances external financing capacity and higher external financing capacity increases the investment further, investment rates may be very sensitive to movements in borrower's cash flow in booms. In contrast, the lower productivity, lower continuation value and stronger precautionary motives reduce firms' propensity to invest and increase their propensity to save and pay dividends during an economic downturn. As a result, cash flow sensitivity of investment may be lower in recessions.

To test this hypothesis, I estimate an investment equation with two dummy variables, D_t^r ,that equals unity during recessions and equals zero otherwise and D_t^b that equals unity during booms and equals zero otherwise. Specifically, I estimate the equation

$$(\frac{I}{K})_{it} = a_i + a_1 Q_{i,t-1} + a_2(\frac{CF}{K})_{i,t} + a_3(D_t^r * \frac{CF}{K})_{i,t} + a_4(D_t^b * \frac{CF}{K})_{i,t} + u_{i,t} \tag{1.29}$$

The results of estimating equation 1.29 are summarized in Table 1.9. The third column reports the coefficient on the recession dummy times the cash flow variable. Recessions are defined as a large increase in the risk free rate, which is related to the changes in aggregate shock, x. Specifically, I consider a period in which risk free interest rate raised at least 15 basis points to be the date of monetary contraction and consider eight periods following the contraction as recession. For constrained firms, there is a reduction in the cash flow coefficient during recessions although it is not statistically significant. The reduction in the cash flow investment sensitivity is about 13 percent. The fourth column reports the coefficient on the boom dummy times the cash flow variable. I consider a period in which risk free interest rate decreased at least 15 basis points to be the date of monetary increase and consider eight periods following as economic boom. For constrained firms, there is a significant increase in the cash flow coefficient during booms. The increase in the cash flow investment sensitivity is about 19 percent. High productivity, high continuation value and cheap external funds magnify cash flow sensitivity of investment during booms. As explained above, these asymmetric sensitivity coefficients in business cycles stem from two distinct features of the model, which are credit multiplier effect of endogenous collateral constraint and stochastic discount rates. Higher cash flow sensitivity of a constrained firm during booms is consistent with the empirical evidence presented by Almeida and Campello (2004). Almeida and Campello (2004) finds that investment-cash flow sensitivities of constrained firms increase with asset tangibility and asset tangibility has a higher (positive) impact on constrained firms' investment-cash flow sensitivities during booms.

1.6 Conclusion

In this chapter, I have developed a partial equilibrium model to study the relationship between corporate investment and internal funds in a dynamic framework under aggregate and idiosyncratic uncertainty, counter-cyclical price of risk, and endogenous external financing constraint. Specifically, financing constraint is defined as collateral constraint, which is a function of firm's liquidation value. According to the model, a firm's investment and financing policy is determined by a trade off between higher profits and higher volatility.

The solution of the model demonstrates that financial constraints play a crucial role in investment and financing decisions of firms and their responses to aggregate shocks. Normally, small firms have high profitability but they have low cash flows, and low access to credit lines. To utilize the higher return associated with small size and to relax the financial constraints they face, small firms without financial slack, would like to borrow more, invest more and pay less dividends relative to large firms. As a result, supporting the evidence presented by Fazzari, Hubbard and Peterson (1988), firms that face tight financing constraints have high investment cash flow sensitivities.

Even though small firms without any cash reserves have a high propensity to borrow and use these extra funds for investment, in general, they don't exhaust all their credit lines in the anticipation of future bad contingencies. In other words, precautionary motives reduce investment and borrowing, and stimulate excess liquidity. Contrary to Kaplan and Zingales (1997)'s argument, the model illustrates that existence of unutilized credit lines is compatible with presence of financing constraints. As a result, the model produces results that are inline with the empirical evidence of both Fazzari, Hubbard and Peterson (1988) and Kaplan and Zingales (1997), and reconciles the controversy between them.

The model has important business cycle implications as well. Counter-cyclical interest rates, persistent aggregate shocks and endogenous collateral constraint imply higher investment-cash flow sensitivity during booms and lower investment-cash flow sensitivity during recessions. Business cycle behavior of cash flow sensitivity of investment provides additional support to the argument that financial constraints are an important determinant of firm dynamics.

Overall, I propose a model that is sufficient to replicate the major empirical evidence on the influence of financial constraints on corporate investment. There are several promising directions in which this model can be extended. Allowing borrowing and financial slack simultaneously, one can study the cash management implications of financing constraints in more detail. Finally, it is important to find out whether the qualitative results of this chapter go through when firms are allowed to have costly equity financing and/or risky debt.

1.7 Appendix

1.7.1 Numerical Solution Method

I use value function iteration procedure to solve the individual firm's problem. Optimal value function and policy function are solved on a grid where the state space for (k,b,x,z) is discrete. Piecewise linear interpolation is used to calculate the optimal investment, debt values and firm value that do not lie on the grid. The grid for capital stock is created following the method used by McGrattan (1999). The grid is defined by the formula, $k_i = k_{i-1} + a(exp(b(i-2)))$ where i is the index of grid points. Coefficients a and b are selected to provide 50 grid points between the interval $[0, \bar{k}]$. As in Gomes (2001), $\bar{k}$ is defined as $\pi(\bar{k}, \bar{x}, \bar{z}) - \delta\bar{k} \equiv 0$. Given the form of the profit function, $\bar{k}$ is well defined. Any k greater than $\bar{k}$ is not economically profitable. The benefit of the recursive construction of the capital stock grid points is having more grid points closer to the lower bound of k where the production function has the highest curvature.

The state space for b is more complicated, because debt issuance is restricted by the collateral constraint, which in turn depends on the level of capital stock. The state space for b is specified by choosing feasible points that satisfy equation (12) for each element of the state space k. Since next period capital stock k' is chosen from a compact set defined above, debt level is bounded above also. I set the lower bound of borrowing to be -2 and on account of the cost of holding cash; it appears to be sufficient given that there is no optimal debt rule chosen at the lower boundary.

The state variables x and z are transformed into discrete state with the method described in Rouwenhorst (1995), which can calibrate an extremely persistent AR(1) process that Tauchen and Hussey (1991) cannot handle. I used 5 grid points for x and 3 grid points for z. All the results are robust to finer grid points.

1.7.2 Tables and Figures

Table 1.1: Parameter Values

This table lists the key benchmark parameter values used to solve and simulate the model. The parameters are either estimates from empirical studies or calibrated to match a set of key moments in the model to the U.S. data.

Parameter		*Value*
Capital Share	α	0.3
Depreciation Rate	δ	0.01
Adjustment Cost	θ	15
Persistence of Aggregate Productivity Shock	ρ_x	0.983
Conditional Volatility of Aggregate Productivity Shock	σ_x	0.0023
Persistence of Idiosyncratic Productivity Shock	ρ_z	0.96
Conditional Volatility of Idiosyncratic Productivity Shock	σ_z	0.1
Pricing Kernel Parameters	β	0.994
	γ_0	50
	γ_1	-1000
Cost of holding cash	χ	0.05
Fixed Cost of Production	f	0.025

Table 1.2: Simulated Moments

This table reports a set of moments generated under the benchmark parameters in Table 1.1. The data source for moments of real interest rate is Campbell, Lo and MacKinlay (1997), for the debt-asset ratio is Hennessy and Whited (2006), for cash holding-asset ratio is Almeida, Campello and Weisbach (2005), for the rate of investment Livdan, Li and Zhang (2005) and for the rate of disinvestment is Abel and Eberly (2001).

Unconditional Moment	*Constrained Model*	*Data*
Average annual Sharpe ratio	0.42	0.43
Average annual interest rate	0.0174	0.018
Annual volatility of real interest rate	0.027	0.03
Annual average rate of investment	0.15	0.13
Annual average rate of disinvestment	0.04	0.02
Annual volatility of investment rate	0.096	0.07
Average debt-asset ratio (net of cash)	0.26	0.3
Average cash holding-asset ratio	0.12	0.15

Table 1.3: Means and Correlations

This table reports the means and correlation coefficients for capital stock $K_{i,t}$, beginning of period Tobin's q, $Q_{i,t-1}$, cash flows $CF_{i,t}$, investments $I_{i,t}$, dividends $D_{i,t}$, debt issues $B_{i,t}$, and finally the productivity shocks x_t and $z_{i,t}$. Cash flow, investment, dividend and debt variables are scaled by the capital level.

	$K_{i,t}$	$Q_{i,t-1}$	$\frac{CF_{i,t}}{K_{i,t}}$	$\frac{I_{i,t}}{K_{i,t}}$	$\frac{D_{i,t}}{K_{i,t}}$	$\frac{B_{i,t+1}}{K_{i,t}}$	x_t	$z_{i,t}$
Means	0.96	2.88	0.36	0.15	0.29	0.16	−2.90	0.04
Correlations:								
$K_{i,t}$	1.00							
$Q_{i,t-1}$	−0.1	1.00						
$\frac{CF_{i,t}}{K_{i,t}}$	0.15	0.49	1.00					
$\frac{I_{i,t}}{K_{i,t}}$	−0.22	0.41	0.31	1.00				
$\frac{D_{i,t}}{K_{i,t}}$	0.02	0.08	0.55	−0.08	1.00			
$\frac{B_{i,t+1}}{K_{i,t}}$	−0.30	−0.18	−0.46	0.13	−0.36	1.00		
x_t	0.34	0.54	−0.02	0.23	−0.32	0.01	1.00	
z_t	0.30	0.42	0.84	0.25	0.34	−0.34	−0.01	1.00

Table 1.4: Simulated Investment-Cash Flow Sensitivities

This table reports the results of the cross sectional regressions of the monthly investment rates, $(I_{i,t})/(K_{i,t})$, on the firm fixed effects, beginning of period Tobin's q, $Q_{i,t-1}$, and the cash flow scaled by the capital level, $(CF_{i,t})/(K_{i,t})$. Panels A through J report the coefficient estimates and t-stats of the regressions that use different criteria to identify the degree of financing constraints.

$Q_{i,t-1}$	$(\frac{CF}{K})_{i,t}$	$Q_{i,t-1}$	$(\frac{CF}{K})_{i,t}$
Panel A : High Retention Group		Panel B : Low Retention Group	
0.0012	0.2261	0.0091	0.0361
(61.66)	(67.95)	(215.28)	(25.28)
Panel C : Low Cash Flow Group		Panel D : High Cash Flow Group	
0.0035	0.2034	0.0084	0.0409
(56.52)	(51.64)	(216.89)	(23.15)
Panel E : Small Firm Size Group		Panel F : Large Firm Size Group	
0.0039	0.1559	0.0083	0.0287
(59.38)	(52.11)	(212.27)	(18.00)
Panel G : Low Unutilized Credit Group		Panel H :High Unutilized Credit Group	
0.0272	0.0399	0.0048	0.1395
(26.79)	(22.32)	(142.10)	(75.00)
Panel I : Constrained Model Group		Panel J :UnConstrained Model Group	
0.0041	0.13	0.03	0.04
(120.12)	(54.21)	(210.32)	(51.54)

Table 1.5: Empirical Investment-Cash Flow Sensitivities

This table reports the coefficient estimates and t-stats of the investment regressions presented by Fazzari, Hubbard, and Peterson (1988), Allayannis and Mozumdar (2001), Gertler, and Gilchrist and Himmelberg (1995), Kaplan and Zingales (1997), and Moyen(2004). $(I_{i,t}/(K_{i,t})$ represents investment rate, $Q_{i,t-1}$ the beginning of period Tobin's q, and, $(CF_{i,t})/(K_{i,t})$ the cash flow scaled by the capital level. Panels A through J report the coefficient estimates and standard errors of the regressions that use different criteria to identify the degree of financing constraints.

$Q_{i,t-1}$	$(\frac{CF}{K})_{i,t}$	$Q_{i,t-1}$	$(\frac{CF}{K})_{i,t}$
Fazzari, Hubbard, and Peterson (1998):			
Panel A : Most Constrained		Panel B : Least Constrained	
0.0008	0.461	0.0020	0.0230
(0.0004)	(0.027)	(0.0003)	(0.010)
Allayannis and Mozumdar (2001):			
Panel C : Financially Constrained		Panel D : Not Financially Constrained	
0.058	0.355	0.054	0.151
(0.009)	(0.015)	(0.008)	(0.013)
Gilchrist and Himmelberg (1995):			
Panel E : Small Firm		Panel F : Large Firm	
0.0056	0.203	0.027	0.124
(0.02)	(0.045)	(0.013)	(0.043)
Kaplan and Zinglades (1997):			
Panel G : Likely Constrained		Panel H :Never Constrained	
0.070	0.340	0.009	0.702
(0.018)	(0.042)	(0.006)	(0.041)
Moyen(2004):			
Panel I : Financing Constraints		Panel J :No Constraints	
-0.042	0.592	-0.031	1.221
(0.001)	(0.005)	(0.023)	(0.023)

Table 1.6: Simulated Investment-Cash Flow Sensitivities Excluding Negative Cash Flow Observations

This table reports the results of the cross sectional regressions of the monthly investment rates, $(I_{i,t}/(K_{i,t})$, on the firm fixed effects, beginning of period Tobin's q, $Q_{i,t-1}$, and the cash flow scaled by the capital level, $(CF^{+}_{i,t})/(K_{i,t})$. Panels A through J reports the coefficient estimates and t-stats of the regressions that use different criteria to identify the degree of financing constraints.

$Q_{i,t-1}$	$(\frac{CF^{+}}{K})_{i,t}$	$Q_{i,t-1}$	$(\frac{CF^{+}}{K})_{i,t}$
Panel A : High Retention Group		Panel B : Low Retention Group	
0.0029	0.4183	0.0047	0.09
(38.86)	(79.46)	(155.28)	(60.90)
Panel C : Low Cash Flow Group		Panel D : High Cash Flow Group	
0.0039	0.3986	0.0051	0.063
(50.41)	(50.83)	(116.68)	(22.98)
Panel E : Small Firm Size Group		Panel F : Large Firm Size Group	
0.0037	0.1945	0.0085	0.019
(56.55)	(54.64)	(223.21)	(4.09)
Panel G : Low Unutilized Credit Group		Panel H :High Unutilized Credit Group	
0.0212	0.56	0.004	0.1057
(13.34)	(34.16)	(162.75)	(73.77)
Panel I : Constrained Model Group		Panel J :UnConstrained Model Group	
0.0048	0.15	0.01	0.025
(128.12)	(64.73)	(280.95)	(58.64)

Table 1.7: Simulated Cash Flow Sensitivity of Cash

This table reports the results of the cross sectional regressions of the monthly change in cash holdings, $(\frac{\Delta CashHoldings}{K})_{i,t}$, on the firm fixed effects, Tobin's q, $Q_{i,t}$, the cash flow scaled by the capital level, and firm size, $K_{i,t}$. Top panel reports the simulated data results. The bottom panel reports results from Almeida, Campello and Weisbach (2006).

Simulated Data

$Q_{i,t}$	$(\frac{CF}{K})_{i,t}$	$K_{i,t}$
Constrained Firm		
-0.003	0.094	0.0027
(-73.61)	(114.62)	(43.91)
UnConstrained Firm		
0.0002	-0.006	-0.0012
(45.71)	(-2.1)	(-1.22)

Empirical Data

$Q_{i,t}$	$(\frac{CF}{K})_{i,t}$	$K_{i,t}$
Constrained Firm		
0.0029	0.0593	0.0019
(2.41)	(4.53)	(0.61)
UnConstrained Firm		
0.0001	-0.0074	0.0001
(0.01)	(-0.28)	(0.05)

Table 1.8: Macroeconomic Dynamcis

This table reports the results of the cross sectional regressions of the monthly investment rates, $I_{i,t}/K_{i,t}$, on the firm fixed effects, beginning of period Tobin's q, $Q_{i,t-1}$, the cash flow scaled by the capital level, $CF_{i,t}/K_{i,t}$, the cash flow interacted with a dummy variable that equals one during recessions,$(D_t^r * CF_{i,t})/(K_{i,t})$ and the cash flow interacted with a dummy variable that equals one during booms, $(D_t^b * CF_{i,t})/(K_{i,t})$.

$Q_{i,t-1}$	$(\frac{CF}{K})_{i,t}$	$D_t^r * (\frac{CF}{K})_{i,t}$	$D_t^b * (\frac{CF}{K})_{i,t}$
0.0048	0.1303	-0.018	0.0244
(113.92)	(51.18)	(-6.5910)	(15.04)

Figure 1.1: Investment Capital Ratio and Aggregate Shocks

This figure plots optimal investment capital ratios of constrained and unconstrained firms as a function of capital stock under different aggregate shocks.

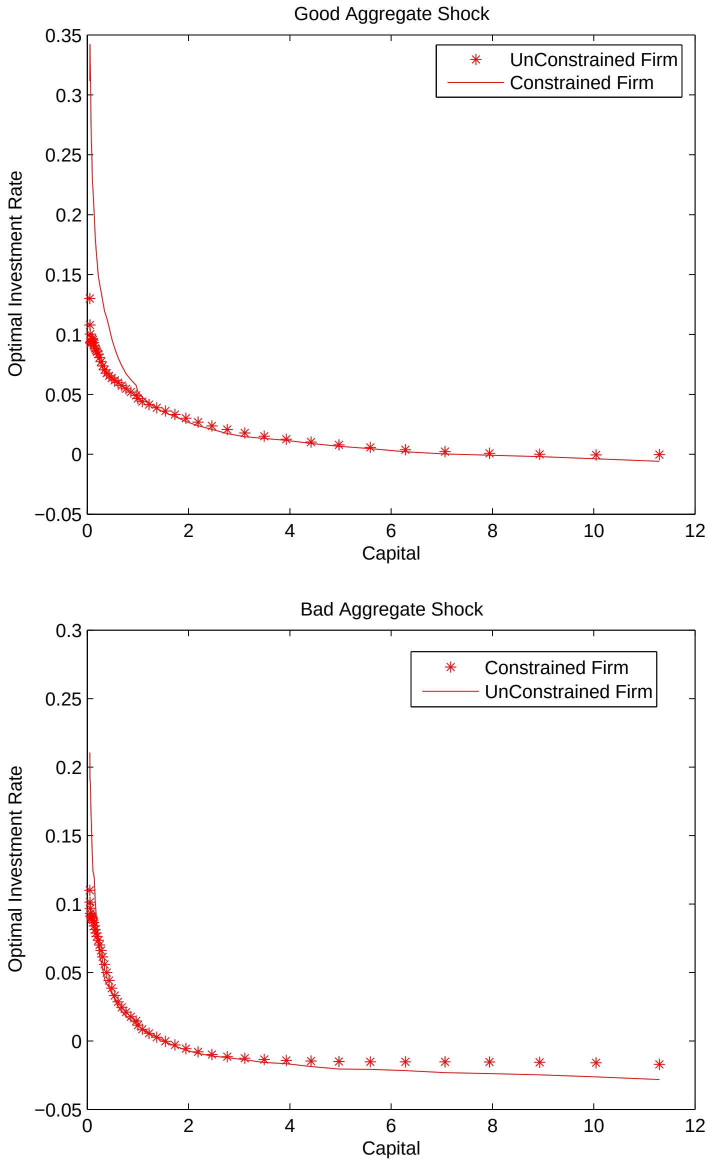


Figure 1.2: Investment Capital Ratio and Idiosyncratic Shocks

This figure plots optimal investment capital ratios of constrained and unconstrained firms as a function of capital stock under different idiosyncratic shocks.

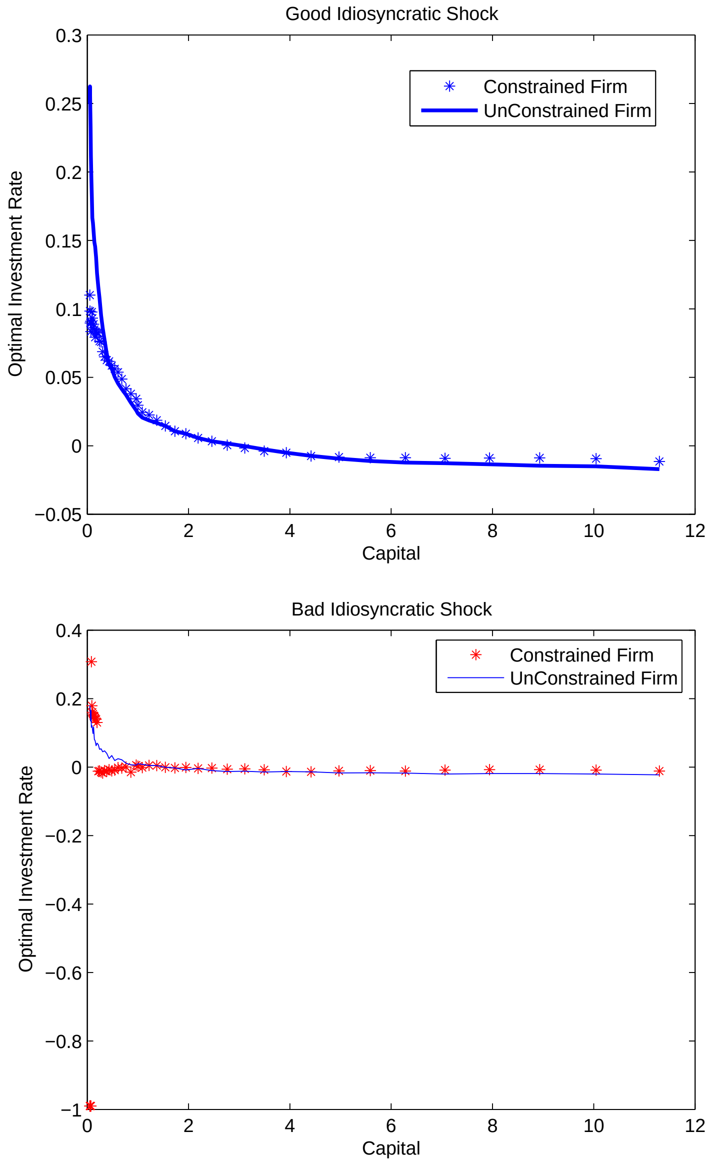


Figure 1.3: Investment Capital Ratios Under Stochastic Versus Constant Discount Rate Assumptions

This figure plots investment capital ratios produced by stochastic discount rate model and constant discount rate model as a function of capital stock under different aggregate and idiosyncratic shocks.

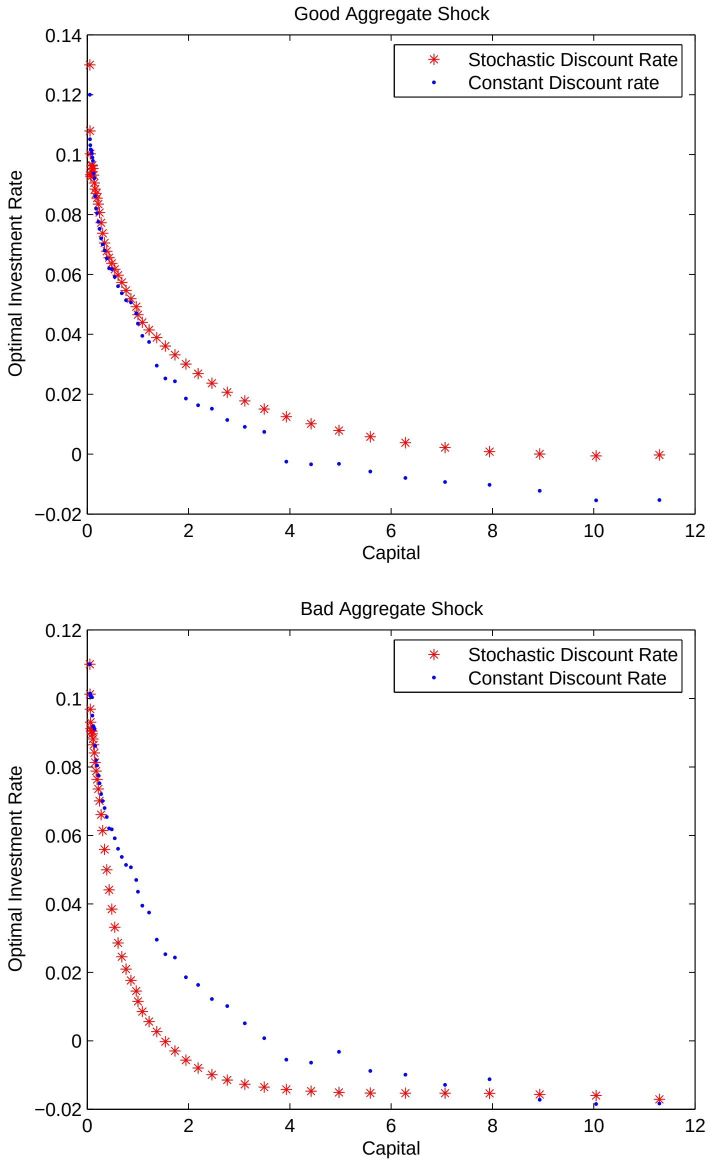


Figure 1.4: Borrowing Policy of an Unleveraged Firm

This figure plots optimal borrowing policy of an unleveraged firm as a function of capital stock under different aggregate and idiosyncratic shocks.

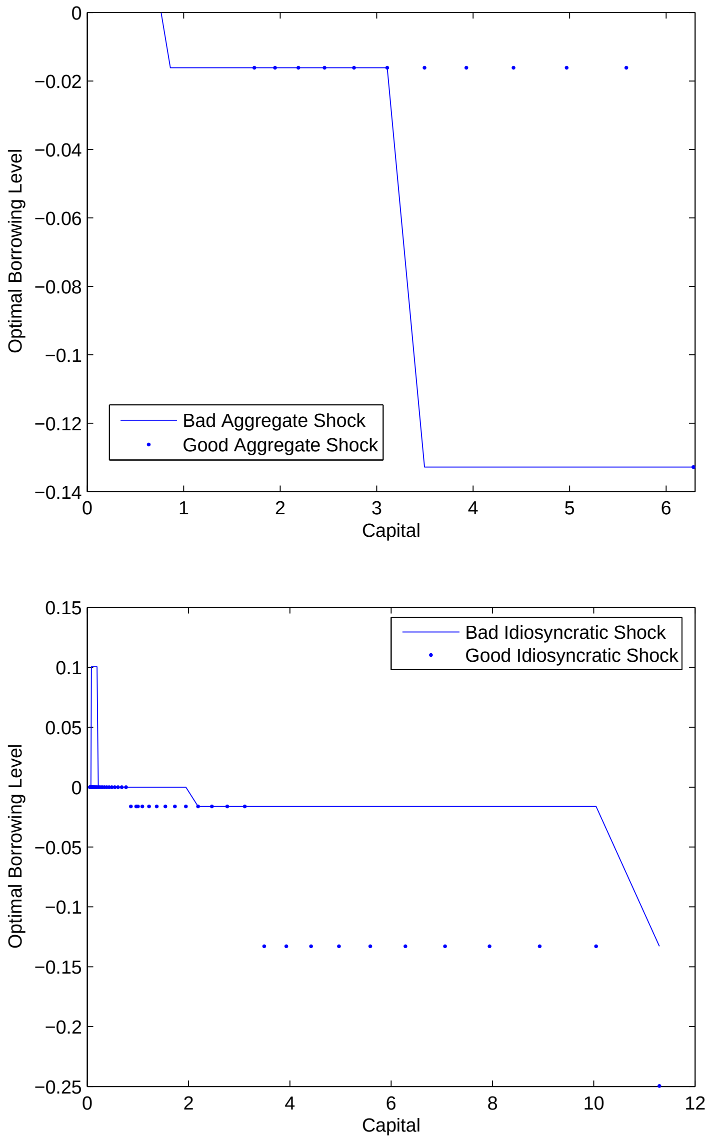


Figure 1.5: Borrowing Policy of a Leveraged Firm B=0.34

This figure plots optimal borrowing policy of a leveraged firm as a function of capital stock under different aggregate and idiosyncratic shocks.

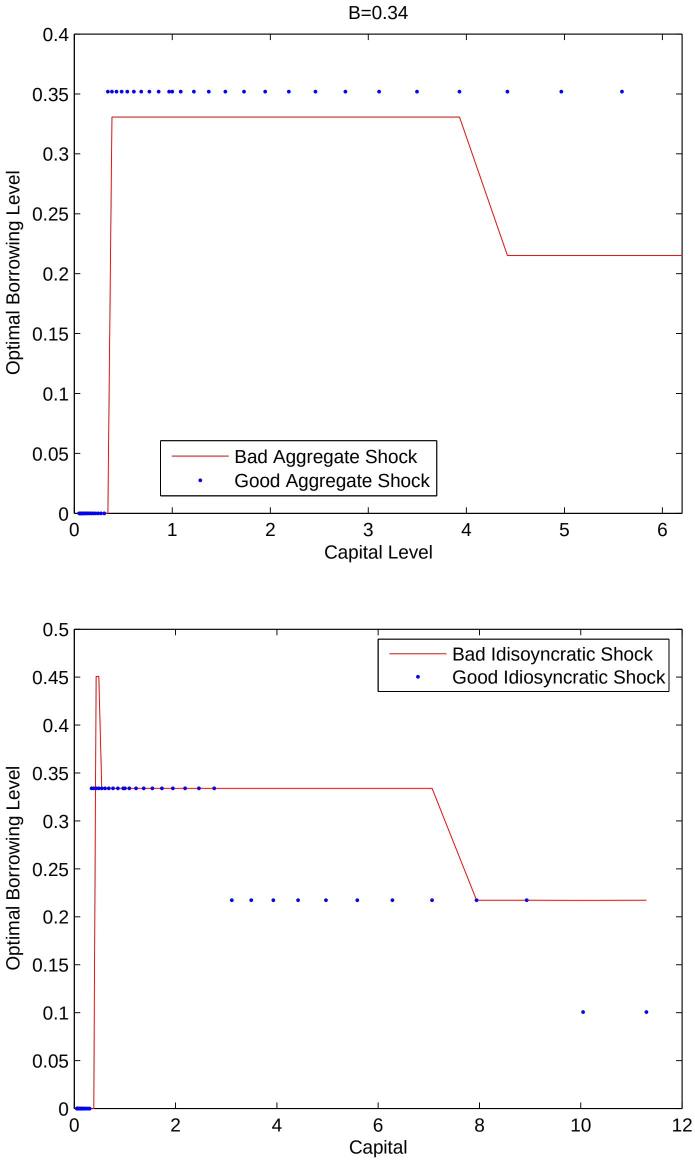


Figure 1.6: Investment Capital Ratio of a Leveraged Firm B=0.34

This figure plots investment capital ratio of a leveraged firm as a function of capital stock under different aggregate and idiosyncratic shocks.

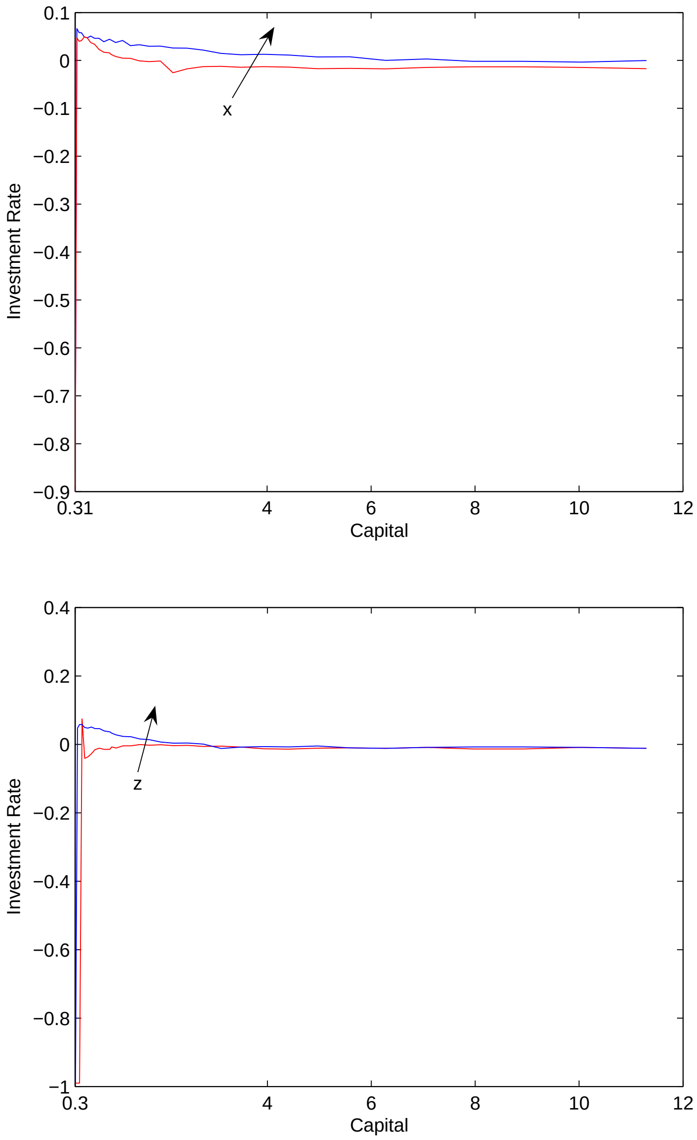


Figure 1.7: Borrowing Limit

This figure plots $B_{t+1}/((1-\delta)K_{t+1})$ as a function of capital stock under different aggregate and idiosyncratic shocks.

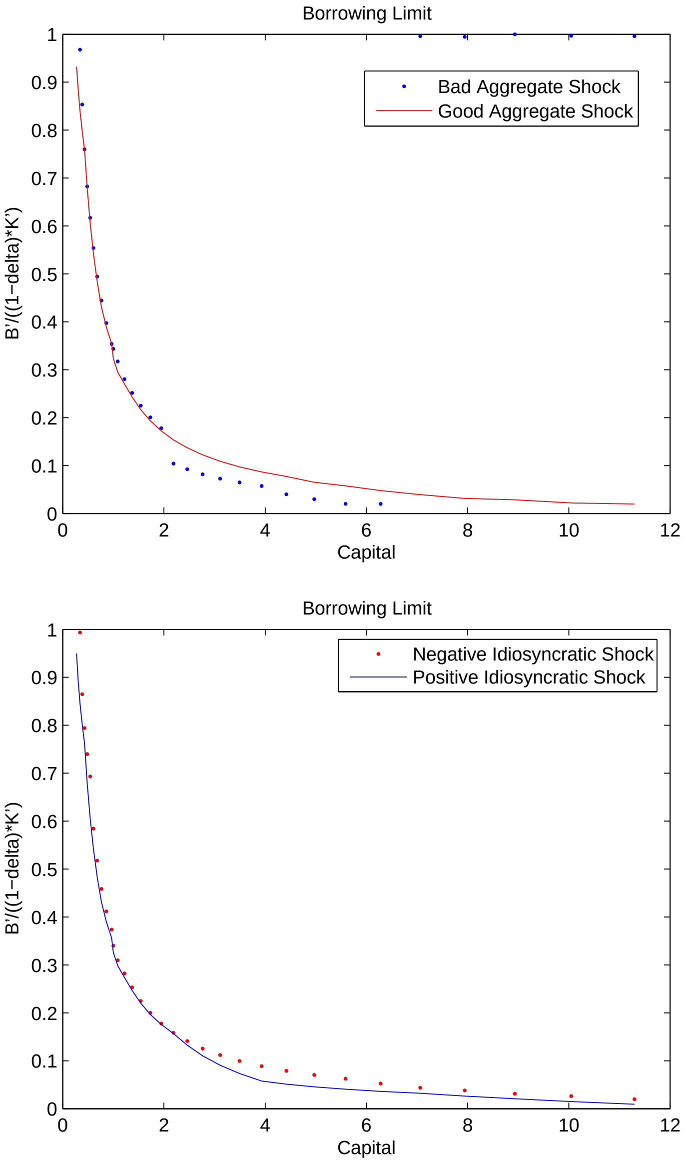


Figure 1.8: Investment Capital Ratio with Slack

The impact of aggregate and idiosyncratic shocks on the optimal investment rates of the constrained firm and the unconstrained firm as a function of financial slack when the capital level is low (k= 0.09).

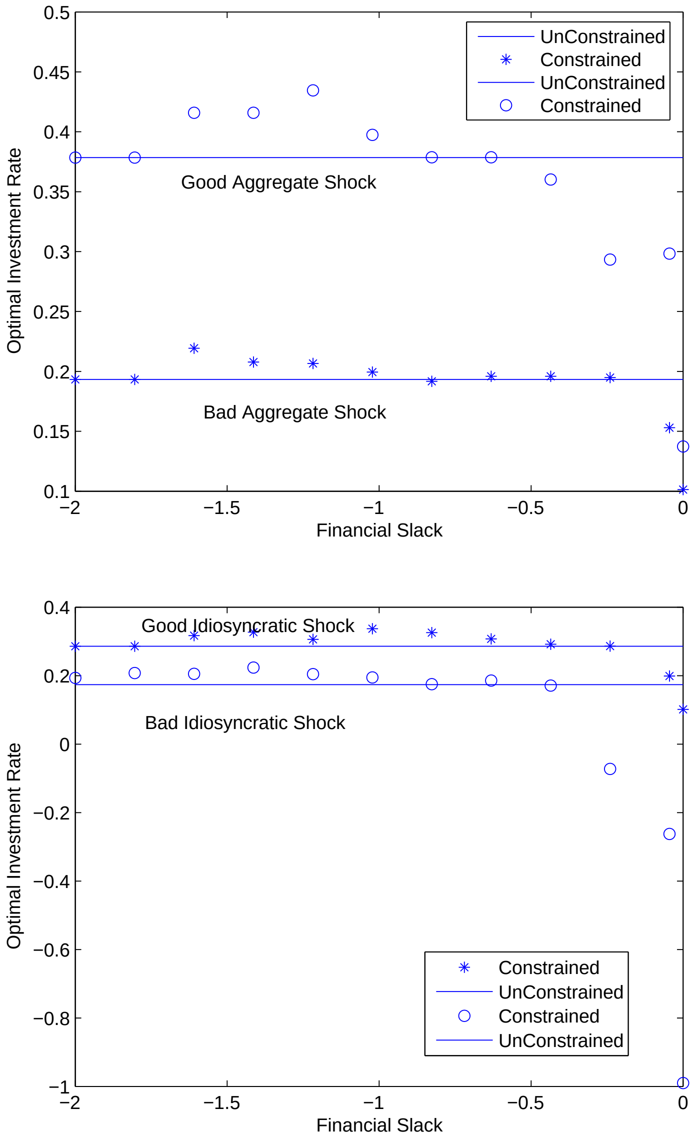


Figure 1.9: Size distribution of the simulated firms.

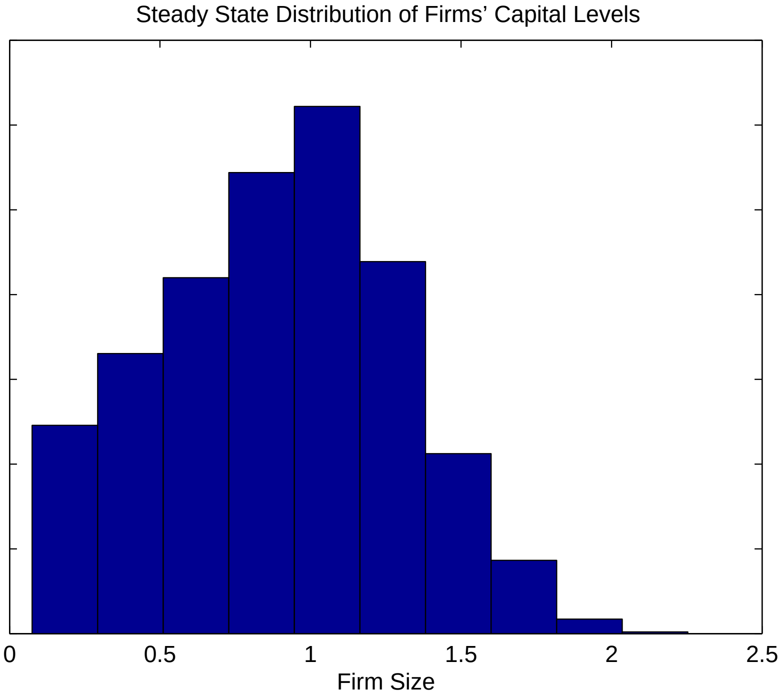
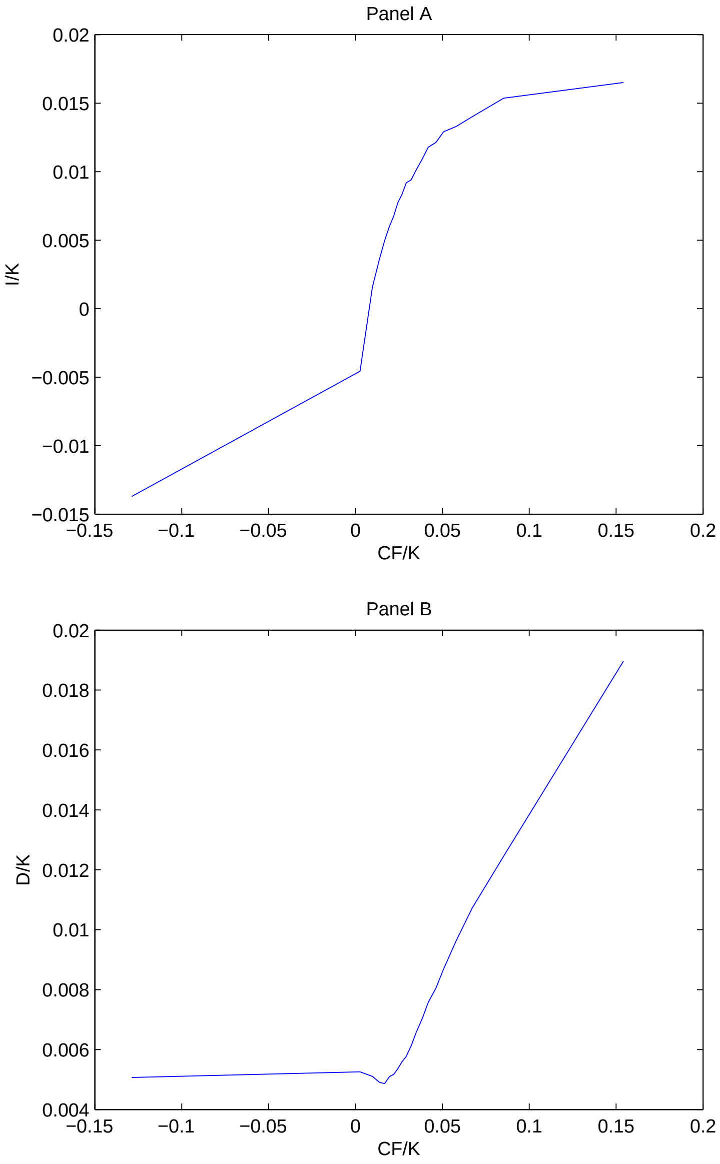


Figure 1.10: Investment-Cash Flow and Dividend-Cash Flow Relations

This figure graphs the average investment and dividend rates at representative cash flow realizations. The simulated sample from the constrained model is sorted with respect to cash flow realizations and then divided into 20 groups. For each group of the average cash flow realizations, average investment rate and dividend rate are computed.



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Chapter 2

Effects of Financing Constraints on Cross-Section of Stock Returns

2.1 Introduction

In the last two decades, financial economists have documented mounting evidence on the size premium for smaller companies and value premium for high book-to-market equity companies. Starting with Banz (1981) and Reinganum (1981), a large body of literature on size premium reported a negative relation between a firm's market capitalization and its stock performance. The notion of value premium is emerged from Fama and French (1992) and Lakonishok, Schleifer and Vishny (1994) who showed that firms with typical value characteristics such as low market-to-book equity have a tendency to outperform growth stocks, with high market-to-book equity. Some other studies that report return premia for value firms are DeBondt and Thaler (1985, 1987), Fama and French (1995, 1996, 1998), Jaffe Keim and Westerfield (1989), and Davis, Fama and French (2000). Based on the empirical evidence, it can be claimed that a zero net investment strategy such as short-selling growth stocks and buying value stocks will produce positive returns.

Although the existences of the value and size premiums are considered to be an empirical fact, there is an ongoing debate on their sources. Fama and French (1992) regard the value premium as a reward for systematic risk in the context of a linear multifactor model. In addition, Fama and French (1995) show that there is a book-to-market factor in fundamentals such as earnings and sales. Fama and French (1996) argue that the value premium is related to firm leverage, which can be considered as a proxy for financial distress. On the size premium side, Chan and Chen (1991) show that small stocks with a high book-to-market ratios are firms that recently have performed poorly and are vulnerable to financial distress.

Alternatively, some researchers have attributed the observed size and value effect to investor overreaction, lack of efficient pricing ability and data-snooping biases; see for example Lakonishok, Schleifer and Vishny (1994), Haugen and Baker (1996), and Lo and Mackinlay (1990).

This chapter develops a structural model of firm behavior to address the issues regarding the risk premia in the cross-section. I analyze theoretically firms' market equity, book-to-market, beta and expected return characteristics as functions of state variables such as capital stock, current debt and productivity shocks using the neoclassical framework of optimal investment (Abel and Eberly (1994, 1996)). Specifically, I develop a partial equilibrium model where heterogeneous firms make rational financing and investment decisions under aggregate and firm-specific uncertainty. The model has two distinctive features. First, firms' only source of external finance is debt secured by their collateral assets. Second, discount rate is specified as stochastic discount rate with time-varying price of risk.

I solve the dynamic investment problem and then study the effects of the financing constraints on risk and expected returns using calibration and simulation exercises. The model has several important implications: (i) Financing constraints and financial leverage reduce firms' values and enhance book-to-market ratios; (ii) A firm that is more likely to be bounded with financing constraints, i.e. a firm with low capital stock and/or high financial leverage and/or low firm-level productivity, earns higher equity return.

Intuitively, under limited financing opportunities, constrained firms' capacity to invest and future prospects are weaker than those of unconstrained firms. Hence, financing constraints lead to lower firm values and higher book-to-market equity ratios. The extent of these effects depends on the likelihood of future binding constraints. Due to the lower firm value and inferior future prospects, a firm that is more likely to be constrained in the future carries a higher risk of liquidation and earns higher returns.

Financial constraints are more restrictive for a firm with (i) low capital stock (ii) relatively high current debt, (iii) low firm-specific productivity due to the following reasons. Collateral limit is an increasing function of capital stock; therefore lower capital stock leads to lower debt capacity. Given that small firms are very productive, their investment demands are high but their internal funds and debt capacities are low. As a result, small firms are more financially constrained. Leveraged firms, on the other hand, cannot scale down without increasing the likelihood of liquidation under low productivity shocks. While a risk-averse firm tries to avoid volatility, higher current debt elevates the expected volatility of profits, raises the likelihood of getting constrained in the future, and accordingly, boosts the liquidation pressure. Finally, current cash flow and investment demand are both functions of firm-specific productivity but cash flow is more sensitive to firm-specific productivity relative to investment demand so the gap between internal funds and investment demand increases under low firm-level productivity. Hence, firms with low firm-level productivity are more likely to be constrained.

The link between financial constraints and aggregate productivity is more complicated and crucial for the rest of the analysis. The stochastic discount rate is a negative function of aggregate productivity. As future looks gloomier, the price of risk increases and firms discount future value more heavily. As a result, both cash flow and investment demand diminish. The main findings show that unless a small firm experiences a bad firm-level productivity shock, the impact of lower investment demand dominates the impact of lower internal funds and the firm ends up being less financially constrained and less risky. However, financial constraints are more likely to be binding for small firms that experience low aggregate shock on the top of low firm-level productivity. The underlying reason is the persistency of productivity shocks and liquidation risk. The severe decline in current cash flow pushes the firm down to the liquidation boundary, where internal fund is inadequate to cover the fixed cost even after exhausting the debt capacity. To prevent liquidation, small firms borrow and invest at the limits to move away from liquidation boundary even though the capital is very unproductive. Briefly, likelihood of binding financing constraints and risk are related negatively with aggregate shocks for small, unproductive firms but they are positively related to aggregate shocks for the rest of the firms.

In the view of these quantative properties of the model, I investigate the extent to which the simulated data can explain documented cross-sectional asset price anomalies. The main findings are (i) Small firm and value firm produced returns are superior to those generated by large firms and growth firms, respectively; (ii) Both book-to-market equity and size are associated with persistent differences in profitability, such that small (value) stocks earn persistently lower returns on equity than big (growth) stocks; (iii) Small (value) stocks persistently have a tendency to hold debt whereas big (growth) stocks have a tendency to hold cash; (iv) Size and value premiums are counter-cyclical. Basically, the results of my theoretical model are inline with the previously discussed empirical findings and they support the argument that size and value premiums are rewards for holding stocks of firms under relative distress.

The intuition behind cross-sectional properties is straightforward given the quantative properties discussed above. Firms with low productivity, high leverage and/or large capital stock have high book-to-market ratios in the model. The first two types of value firms are more likely to be constrained so they are relatively distressed. As noted before, firms that are more likely to be constrained are more risky and earn higher returns. Although the last type of value firms, the large firm, is not very risky, the first two types of firms dominate the value group in the simulations. On average, the high book-to-market firms are riskier than low book-to-market firms and they earn a positive return premia for the risk spread.

Another implication of the model is the persistent spread between the profitability of value firms and growth firms. In my model, precautionary motives promote cash holding as a buffer against future contingencies if the constrained firm starts a period with zero debt or positive slack. An initially unleveraged firm chooses to borrow if and only if it is an unproductive small firm. In addition, the debt policy implied by the model is pretty persistent. In other words, unproductive small value firms, whose book-to-market ratios are already high, choose to borrow and this financing policy enhances their book-to-market ratio even further. For 11 years around the portfolio formation period, on average, value firms hold positive debt whereas growth firms hold cash. The persistency of value and growth firms' debt policies lead to the persistent dispersion between their profitabilities.

Similarly, firms with low capital stocks, low productivities and relatively high financial leverages have low firm values. With the same intuition described above, small firms are relatively distressed and earn a risk premia over big firms. Also, they are persistently leveraged on average and have lower profitability compared to big firms.

Financing constraints are the main driving forces of the value premium in the model. Both small firm effect and book-to-market effect are amplified under the constrained model because liquidation risk is higher when there is limited financing. Incorporating the financing constraints into a neoclassical framework of optimal investment model to analyze the theoretical effects of financing constraints on risk and expected returns is the main contribution of this thesis. The dynamic feature of the model, in addition to financing constraints, brings about the precautionary motives of the forward looking firms and impacts constrained firms' financing decisions and their risk and return structures even further. For example, although some financially healthy, less distressed firms cannot reach to the unconstrained investment level, they still avoid debt in anticipation of future contingencies and they end up earning lower equity returns than their unconstrained counterparts.

The stochastic discount rate with time-varying price of risk assumption facilitates studying the cyclical behavior of the risk and expected returns. For example, the risk spread between value and growth firms are counter-cyclical in the model. For a distressed value firm, current cash flow is vital for survival. Due to higher productivity and lower interest rates, cash flow improves under better economic shocks and risk level drops. A growth firm, on the other hand, is more risky under better conditions because, as noted above, the gap between investment demand and internal fund increases with aggregate shocks. Therefore, the risk and return spreads between value and growth firms are counter-cyclical.

Moreover, under time-varying price of risk assumption, the model produces higher value and size premiums relative to constant price of risk assumption. The model with counter-cyclical price of risk discounts future more heavily in bad times compared to a model with constant price of risk. With lower continuation value, a firm's propensity to disinvest will be higher in bad times. However, firms with relatively high financial leverage cannot scale down without increasing the likelihood of liquidation and consequently they became more risky and earn higher returns leading to higher value and size premiums in bad times under counter-cyclical price of risk assumption compared to the constant price of risk assumption.

The findings of the chapter are consistent with the empirical evidence provided by Fama and French (1992, 1995) and Chan and Chen (1991). Due to financing constraints, value firms and small firms are relatively more financially distressed and the financial distress risk expands during economic recessions and contracts during economic booms. As a result, financial distress is a systematic risk factor, and value and small firms that face financial distress risk earn higher returns. These results show that the so called cross-sectional anomalies are indeed consistent with rational expectations.

This chapter sits at the intersection of two lines of research. The first line is a new and growing line of research, pioneered by Berk, Green and Naik (1999), that provides rational explanations for value premium based on optimal firm-level investment decisions. Berk et al. (1999) show that the relative weight of growth options versus assets-in-place captures the size effect in expected returns while the systematic risk in assets-in-place is linked to the book-to-market effect in a dynamic real options model. Building on the work of Berk et al. (1999), a series of papers such as Gomes, Kogan and Zhang (2003), Carlson, Fisher and Giammarino (2004), and Zhang (2005) relate stock return dynamics to firms' real investment decisions. Gomes, Kogan, and Zhang (2003) extend the work of Berk et al. (1999) to a general equilibrium model and show that expected returns and firm characteristics are related through beta. Carlson, Fisher, and Giammarino (2004) argue that a monopolistic firm's dynamic investment model with operating leverage and finite growth opportunities can produce asset betas containing time-varying size and book-to-market components. Finally, Zhang (2005) incorporates asymmetric adjustment costs into industry equilibrium model and shows that value firms with excessive unproductive capital capacity have more difficulty than growth firms in scaling down their capital stocks especially during economic downturns and hence earn higher risk premiums.

Although, all these studies provide an economic rationale for the relationship between expected returns and firm characteristics, they only focus on all-equity financed firms and none of them explicitly model financial leverage. My thesis is substantially different from all those above because I introduce financial leverage into the production model to capture the effect of financial distress on cross-section of stock returns. The significance of financial leverage is also validated by some recent empirical research. The studies such as Griffin and Lemmon (2002) and Vassalou and Xing (2004) show that cross-sectional features of stock returns, such as the size and book-to-market effects, are much stronger for firms with high financial leverage and hence with low credit quality. Basically, Griffin and Lemmon (2002) provide evidence that the value premium is most significant among firms with high probabilities of financial distress, and Vassalou and Xing (2004) demonstrate that both size and book-to-market effects are concentrated in high default risk firms. This chapter tries to provide a theoretical foundation to the documented effect of financial distress on cross-sectional risk and return by incorporating financial constraints into a neoclassical framework of optimal investment with debt financing.

This chapter is also related to a second line of research that deals with credit markets and frictions. Some theoretical studies such as Stiglitz and Weiss (1981), Myers and Majluf (1984), and Hart and Moore (1998) suggest that informational asymmetries, costly monitoring and contract enforcement problems lead to imperfect substitutability between internal and external funds and result in financing frictions that limit firms' access to external financing. These studies claim that unavailability of external funds is more relevant than higher cost of external funds in limiting firms' investments. The theoretical claim is also supported by empirical literature by Kashyap, Stein, and Wilcox (1993), Kashyap, Lamont, and Stein (1994) and Peterson and Rajan (1994, 1995). Some recent papers like Kiyotaki and Moore (1997), Gomes (2001), Cooley and Quadrini (2001) and Cooley, Marimon and Quadrini (2003) study models of industry equilibrium where a fraction of heterogeneous firms are bounded with quantity constraints. These papers show that financing imperfections can explain some stylized facts about aggregate output and industry dynamics like growth and investment. This chapter takes the results of these studies as evidence that financing imperfections like credit constraints can facilitate understanding asset pricing dynamics as well since investment policies and other firm dynamics will be reflected in stock prices. Accordingly, I focus on the effects of financing imperfections on cross-section of returns rather in a partial equilibrium model with time-varying price of risk.

As a final point, Gomes, Yaron and Zhang (2006) adopt a similar approach to this chapter, and investigate whether financing constraints are quantatively important for the cross-section of returns. Using GMM to estimate the stochastic investment Euler equation imposed on stock returns, they argue that the role of financing frictions in explaining the cross-sectional returns is insignificant unless the effect of financing constraints is pro-cyclical. The main difference between two models is the nature of financing constraints. While Gomes et al. (2006) formulize financing constraint as cost constraint; in this thesis, following the extensive credit rationing literature, it is assumed that firms face quantity constraints. According to the results implied by quantity constraint, financing frictions are significant in explaining cross-section of stock returns. Besides, the cyclicality of financing constraints is endogenously determined in the model and depends on firm-specific variables. For example, for bigger firms, the total effect of financing constraints is indeed pro-cyclical but for smaller unproductive firms and/or leveraged firms the effect is counter-cyclical.

The chapter is organized as follows. Section 2.2 illustrates the theoretical model. Section 2.3 describes the calibration methodology. Section 2.4 presents the quantitative properties of the model. Section 2.5 presents the empirical implication of the model. Section 2.6 concludes.

2.2 Model

In this chapter, I use the same neoclassical investment model that is presented in section 1.2. For the sake of completeness, I briefly outline the firm's dynamic maximixation problem in this section.

If the value of the firm at time t is denoted as $v(k_t, b_t, x_t, z_t)$ then the dynamic problem the firm facing is :

$$\begin{aligned} v(k_t, b_t, x_t, z_t) = \max_{(k_{t+1}, b_{t+1})} & \; y_t + \Delta b_{t+1} - r_{t-1} b_t - 1_{(b_{t+1}<0)} b_{t+1}^2 \chi - i_t - h(i_t, k_t) - f + \\ & \int\int M_{t+1} v(k_{t+1}, b_{t+1}, x_{t+1}, z_{t+1}) \times Q(dx_{t+1}/x_t) \times Q(dz_{t+1}/z_t), \end{aligned} \tag{2.1}$$

subject to

$$d_t = y_t + \Delta b_{t+1} - r_{t-1} b_t - 1_{(b_{t+1}<0)} b_{t+1}^2 \chi - i_t - h(i_t, k_t) - f \tag{2.2}$$

$$y_t = e^{x_t + z_t} k_t^{\alpha}, \tag{2.3}$$

$$\Delta b_{t+1} = b_{t+1} - b_t \tag{2.4}$$

$$i_t = k_{t+1} - (1 - \delta)k_t \tag{2.5}$$

$$h(i_t, k_t) = \frac{\theta}{2}(\frac{i_t}{k_t})^2 k_t \tag{2.6}$$

$$(1 + r_t)b_{t+1} \leq k_{t+1}(1 - \delta) \tag{2.7}$$

$$d_t \geq 0 \tag{2.8}$$

where k is capital stock, b is debt holding, f is fixed production cost, x and z are aggregate and idisyncratic productivity shocks, respectively, α is the capital share, δ is the depreciation rate, χ is the cost of holding cash and θ is the adjustment cost parameter.

Both x_t and z_t follow stationary autoregressive stochastic processes with ;

$$x_{t+1} = \overline{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \epsilon^x_{t+1}, \tag{2.9}$$

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon^z_{t+1}, \tag{2.10}$$

where ϵ^x_{t+1} and ϵ^z_{t+1} are IID standard normal shocks for all t≥ 0, ρ_x and ρ_z are persistence measures, σ^x and σ^z are conditional volatilities and $\overline{x}$ is average productivity level. $Q(x_{t+1}/x_t)$, and $Q(z_{t+1}/z_t)$ are Markov transition matrixes of x_t and z_t, respectively.

Finally, the pricing kernel is formulazed as:

$$logM_{t+1} = log\beta + \gamma_t(x_t - x_{t+1}) \tag{2.11}$$

$$\gamma_t = \gamma_0 + \gamma_1(x_t - \overline{x}), \tag{2.12}$$

where M_{t+1} denotes the stochastic discount factor from time t to t+1 and $1 > \beta > 0$, $\gamma_0 > 0$, and $\gamma_1 < 0$ are constant parameters.

2.3 Calibration

As mentioned in Chapter 1, a closed form solution for the model does not exist so I calibrate the model and solve it using numerical methods. The calibration and numerical solution are based on the work presented in sections 1.3 and 1.7.1. The calibration parameters are presented in Table 1.1.

2.4 Qualitative Properties

I start by analyzing the properties of firm characteristics, such as firm value, book-to-market equity, beta and expected return. For comparison purposes, I will present the key properties of both the constrained and unconstrained firms. Understanding how model behaves without financing constraints serves as a natural starting point to evaluate the effects of financing constraints. The firm characteristics are analyzed as a function of three state variables: aggregate productivity shock x, idiosyncratic shock z, and firm capital level k. The fourth state variable, debt level b, is set to be 0, unless stated otherwise. Through out the analysis, to study the effects of two of the variables, the third one is set at its long-run average level.

2.4.1 Value Function

I first consider the behavior of value function. Figure 2.1 depicts the firm value, v, as a function of aggregate productivity shock x, idiosyncratic shock z, and firm capital level k.

First, notice that financing constraints reduce firm value significantly. Unlimited financing capability gives the unconstrained firm an edge in investment. Effectively, when unconstrained firm's cash flow is low, it can still reach its first best level of investment using costless, unlimited external financing. It pays negative dividend which is equivalent to equity financing. Financing constraints, on the other hand, have an indirect effect on small constrained firm's investment rate, borrowing rate and value. As noted in the first chapter, Figures 1.1 and 1.4 illustrate that, in general, an unconstrained firm's investment rate is higher than that of a constrained firm and yet, the constrained firm does not utilize its borrowing capacity. The indirect effect is attributable to the positive probability of a series of bad shocks in the future. The intuition behind this is the following. Although small unlevered firms cannot reach unconstrained levels of investment, as in Figure 1.4, they still do not borrow up to their limits to increase their investment rates. A higher borrowing level translates into higher level of production, which implies higher expected profits and higher volatility of profits. Due to the positive probability of a series of future bad shocks and the likelihood of being bounded by financing constraints in the future, the risk-averse forward looking constrained firm reduces the expected volatility of its profit via lower external financing and lower investment rates, which result in a lower firm value.

Figure 2.1 shows that the spread between constrained and unconstrained firm values is higher for small firms with lower idiosyncratic productivity shock z even though the constrained investment rate actually exceeds its unconstrained equivalent. This is again intuitive, because under the bad shock, the firm that is close to the liquidation boundary borrows up to the limit and increases its investment rate as much as possible to move away from the boundary. In that case, even though, the investment rate actually exceeds that of its unconstrained equivalent, it's firm value drops substantially. The persistency of idiosyncratic shocks and high probability of future liquidation dampen the expected future value of the constrained firm, and accordingly, current value of the constrained firm shrinks. To sum up, a constrained firm's value is lower than a similar unconstrained firm's value because the constrained firm either cannot reach the optimum investment level of its unconstrained counterpart or it has a lower future expected value.

Second, the value function v is increasing and concave in capital level k for both constrained and unconstrained firms. When idiosyncratic shock z is fixed at its long run average level $\bar{z}$, as a result of the decreasing return to scale feature of the technology, v has a steeper slope for lower values of capital. In other words, smaller firms have more growth opportunities. The same logic applies to unconstrained firms when aggregate productivity shock x is set to be $\overline{x}$ and v is analyzed as a function of k and z. A comparison of bottom plots of Figure 2.1 indicates that financial constraints work to reduce the slope of v for very low values of capital under bad idiosyncratic shocks. The reason is the following. The conditional volatility of z is larger than the conditional volatility of x, therefore cash flow of a firm is more sensitive to changes in z. When a negative z hits an extraordinarily small firm, firm's cash flow drops substantially and the borrowing constraint binds. As a result, cash flow cannot cover the fixed cost f and the firm gets liquidated. A firm that is close to the liquidation boundary faces a high probability of getting bounded by the financing constraints and getting liquidated. Hence, the value of a small constrained firm that is close to the liquidation boundary is very low and the slope of v is almost flat for extraordinarily small firms that face a negative idiosyncratic shock z.

Finally, the value function v is increasing both in aggregate and idiosyncratic productivity shocks. This pattern is also noticeable in Figure 2.2, which illustrates value function of constrained and unconstrained firms for different capital levels. In both Figures 2.1 and 2.2, sensitivity of unconstrained firm's value with respect to aggregate shock x is higher than that with respect to idiosyncratic shock z. This result stems from counter-cyclical feature of the discount factor. While both x and z affect firms' productivity levels through cash flows, only x has a direct effect on discount rates. Thus, a change in x has an impact on expected next period firm value $E_t[M_{t+1}v_{t+1}]$ through M_{t+1}. In particular, an increase in x_t increases $E[M_{t+1}]$, and hence raises the continuation value of the firm and vice versa. The sensitivity of a constrained large firm is close to the sensitivity of its unconstrained counterpart. The reason is the following. Diminishing marginal return assumption induces lower marginal productivity for higher levels of capital. In addition, the cash flow is positively related to the capital level. Due to lower investment demand and higher internal funds, the external financing demand of a large firm is low. Besides, the collateral limit defined in the constrained model is positively related to the capital level so larger firms are allowed to borrow more even though they need less debt financing. Consequently, constrained firms with large capital stocks can act more like unconstrained firms and their values are more sensitive to aggregate shock x compared to idiosyncratic shock z.

According to the lower plots in Figure 2.1, however, under the constrained model, the value of an extraordinarily small firm is more sensitive to idiosyncratic shock z compared to aggregate shock x. The same trend can be observed in Figure 2.2 as well. Since there is a fixed production cost in the model, such a firm is considered as financially distressed and the level of current cash flow is crucial for its survival. Consistent with the data, the volatility of idiosyncratic shock z is higher compared to aggregate shock x so sensitivity of cash flow with respect to idiosyncratic shock z is higher than that of aggregate shock x. To sum up, high sensitivity of financially distressed firms' values to idiosyncratic shock z can be attributed to the high conditional volatility of z.

2.4.2 Tobin's Q

Next, I consider Tobin's Q, which is the market value of equity dividend by the book value of equity. Tobin's Q has received much attention in the empirical literature as a popular proxy of growth opportunities. Figure 2.3 reveals the relationship between Tobin's Q and aggregate productivity shock x, idiosyncratic shock z, and firm capital level k.

Tobin's Q follows a similar pattern with the firm value function v. In general, firm-level Tobin's Q, TQ, is decreasing in capital level k and increasing in idiosyncratic shock z. However, the relationship between TQ and z for a constrained firm is reversed for lower capital levels under a negative idiosyncratic shock. As explained above, a distressed firm that is close to the liquidation boundary faces a high probability of getting bounded by the financing constraints and getting liquidated. Consistent with Figure

2.3, under a negative idiosyncratic shock, growth options are not valuable much for distressed firms. In addition, consistent with Figure 2.1, level of Tobin's Q of a constrained firm is significantly lower than that of an unconstrained firm. Since financing constraints dampen a firm value through direct and indirect effects described above, TQ is also lower for firms facing financing constraints.

Figure 2.4 plots Tobin's Q of a constrained firm that is financed with debt. According to the figure, debt financing reduces TQ considerably. This result stems from the relationship between debt financing b, and firm value v. As we have seen in equation 1.17, b_t and v_t are inversely related. Debt financing reduces firm value directly through current interest payments, and indirectly through higher expected profit volatility. Higher volatility of expected profits raises the probability of binding future financing constraints and liquidation, and therefore expected future value of the firm drops. In turn, growth opportunities and TQ are also reduced.

Overall, my model predicts that financing constraints reduce firm value, investment rate and Tobin's Q, and the magnitude of these effects decreases in the capital level k, idiosyncratic shock z and aggregate shock x, and increases in debt b.

2.4.3 Book-to-Market

Figure 2.5 depicts the constrained and unconstrained firms' book-to-market ratios with respect to aggregate productivity shock x, and idiosyncratic shock z for different capital levels. Book-to-market equity, the reciprocal of Tobin's Q, is commonly used in asset pricing literature to predict expected returns. One of the well-established features of financial data is the fact that firms with high book-to-market equity that are commonly referred as value firms tend to consistently deliver higher returns than firms with low book-to-market equity that are referred as growth firms. A number of interesting patterns emerge from plots.

First of all, the value spread, defined as the dispersion of book-to-market between value and growth firms is higher for lower levels of capital. As capital level rises, the probability of financial distress decreases and in return, the value spread shrinks.

Second, Figure 2.5 shows that idiosyncratic productivity shock z is the main source of the value spread, controlling for size. Book-to-market equity is a non-increasing function of idiosyncratic shock z, controlling for size and debt level. For a given point of time, among firms with the same capital and debt levels, firms that face positive idiosyncratic shocks have lower book-to-market equity compared to the ones that face negative idiosyncratic shocks.

Finally, the value spread is higher in recessions (low x) than that in expansions (high x) but the difference is less pronounced for small constrained firms. As discussed above for unconstrained firms and constrained big firms, the firm value v is more sensitive to aggregate shock x since x affects both the current cash flow and discount factor M_{t+1}. As a result, during an economic boom future prospects get brighter and current idiosyncratic shock z doesn't have a big impact on book-to-market ratio of an unconstrained firm. However, due to financial constraints, a small constrained firm relies more heavily on current cash flow and current cash flow is more sensitive to idiosyncratic shock z. In view of that, even during an economic boom a small constrained firm's book-to-market ratio, and accordingly the value spread, relies heavily on idiosyncratic shock z.

2.4.4 Expected Return and Beta

Last of all, I investigate how risk and expected return are related to firm characteristics, within the constrained model. Figures 2.6 through 2.9 depict excess return, $E_t[R^e_{t+1}]-r_t$, and risk, β_t, as functions of capital stock k, borrowing level b, aggregate productivity shock x, and idiosyncratic productivity shock z. These figures have several important indications for my inquiry.

To begin with, both the expected excess return and beta decrease in capital stock k and productivity shock z. In other words, in both constrained and unconstrained models, small firms and less productive firms are riskier and earn higher expected returns.

As mentioned in the first chapter, Figure 1.1 indicates that the investment rates of small firms respond to cash flow more strongly than those of large firms. In view of that, when small firms face different productivity shocks they face higher adjustment costs and their dividends covary more with both aggregate and idiosyncratic shocks. This means that firms with low book values are more risky and earn higher expected returns. The empirical evidence documented by Berk (1996) confirms the negative correlation between book value and average returns. In addition, Figure 2.7 shows that size effect is higher for firms with low z than for firms with high z, especially for constrained firms. As mentioned before, financial constraints are more likely to be binding for small firms with low z and hence small constrained firms with low idiosyncratic shock z earn very high returns. Finally, as shown in Figure 2.6, the size effect is counter-cyclical which is documented by Lettau and Ludvigson (2001).

Next, Figures 2.6 and 2.8 indicate that while β increases with aggregate shock x, expected return decreases with x. This is not a counter-intuitive result considering that price of risk is counter-cyclical. Expected return is the product of risk and price of risk. The model suggests that x and β are positively correlated. As x increases, both the internal funds and the expected future value of the firm increase. Higher expected future value results in higher investment demand. Since risk, β, is positively correlated with aggregate shock x, it can be deduced that the increase in investment demand dominates the increase in internal funds. While the risk is higher, future looks brighter after a good shock x and the price of risk shrinks. Although the firm faces a higher risk, the price of risk in the economy is lower and the firm ends up earning lower returns. In other words, the impact of price of risk on expected return dominates the impact of risk denoted by β and therefore, expected return is negatively correlated with aggregate shock x. The only exception to the positive relation between x and beta is the effect of aggregate shock on the beta of a small firm that faces negative idiosyncratic shock z. When a small firm faces a negative z, the liquidation concerns dominate the investment motives. Given that a good aggregate shock increases the current cash flow, it takes some of the liquidation pressure away and reduces the beta.

Figure 2.8 also shows that risk and return moves together under an idiosyncratic shock z and this is consistent with the previous argument given that z doesn't affect the price of risk.

Another important point that needs special attention is the discrepancy between constrained and unconstrained model returns. Both Figure 2.6 and Figure 2.7 show that in most cases, firms face lower risk and earn lower expected returns under constrained model compared to the unconstrained model. This result seems surprising but it is intuitive. As discussed earlier, due to the anticipation of bad shocks and possibility of being constrained in the future, risk-averse firms try to curtail the expected volatility of profits by reducing their external financing demands and investing less in the constrained model. Therefore, a constrained firm's future profitability is less volatile compared to that of an unconstrained firm. However, this pattern is broken when a constrained firm that is close to the liquidation boundary faces a negative idiosyncratic shock. Under the liquidation risk, survival motives overcome precautionary motives and the constrained firm borrows up to the limit and boosts its investment. As a corollary, constrained small firm's expected future volatility of profit, and accordingly expected return, exceed those of the unconstrained firm under a negative z. This result is consistent with the predictions of Chan and Chen (1991) and Perez-Quiros and Timmermann (2000) who claim that due to lower liquidity in tight credit market conditions, relatively unprofitable firms earn higher returns.

Finally, I want to point out the impact of financial leverage on constrained firm's expected return. A comparison of Figure 2.8 and Figure 2.9 illustrates that financial leverage increases a constrained firm's risk and expected return. As discussed earlier, financial leverage reduces firm value and Tobin's Q due to higher volatility of profits and higher likelihood of restricting future financing constraints. As a consequence of the same rationales, financial leverage boosts the riskiness of a constrained firm. The risk and expected return of the leveraged firm related negatively with productivity shocks x and z.

2.5 Empirical Implications

In this section, I study the time-series and cross-sectional implications of the model. I construct 50 samples of data consisting 2000 × 900 firm-month data points. I assume that all firms are initially identical. I drop the first 60 observations to minimize the effect of a possibly suboptimal starting point.

2.5.1 Time Series

Book-to-market ratio, the reciprocal of Tobin's Q, is commonly used in asset pricing literature to predict expected returns. In this section, I use simulated analysis to examine the implications of my model for the relation between returns and book-to-market ratio at the aggregate level both in monthly and annual frequency. Specifically, I investigate whether begging of period book-to-market ratio is a good estimator of end of period market return in a univariate regression. For ease of comparison, I replicate closely the empirical experiment in Pointiff and Schall (1999). Table 2.1 reports the estimates derived by regression performed on the simulated data and the corresponding empirical regressions of Pointiff and Schall (1999). The simulated data statistics are the time series averages that are averaged across samples.

Table 2.1 shows that the relation between beginning of period aggregate book-to-market and end of period value-weighted market return is significantly positive, both in monthly and annual frequency. In section 2.4.4, I argued that the expected return is a decreasing function of aggregate shock x, which is the main driving force of time-series fluctuations at aggregate level. The relation between book-to-market and x is not always monotone but Figure 2.5 illustrates that when idiosyncratic shock is at its long run average, 0, book-to-market is also a decreasing function of x. It follows that the model can generate the direction of the univariate linear relation between the realized equity returns and book-to-market ratio, yet it underestimates the magnitude of the slope coefficient. The sensitivity of book-to-market ratio to aggregate shock x is lower compared to its unconstrained model counterpart for small firms, which may partially explain the lower slope coefficient.

2.5.2 Cross-Section

This section establishes the key quantative results. I focus mainly on the cross-sectional relations between realized equity returns and firms' characteristics such as size and book-to-market equity.

The Size and Book-to-Market Effects

Panels A and B of Table 2.2 compare summary statistics of my model with those reported in Zhang (2005). Post-ranking average returns and unconditional betas for 10 portfolios constructed by one-dimensional sort of stocks on book-to-market and on size respectively are computed on the basis of the simulated panels. Every panel of data consist of 2000×840 firm-month data points. I repeat the entire simulation 50 times and report the average results of the sorting procedure across simulations. HML and SMB portfolios are constructed following Fama and French (1995).

Panel A of Table 2.2 reports the mean and unconditional beta statistics for 10 portfolios sorted on the book-to-market equity. As it can be noticed, the model does a good job in capturing the direction of the historical data. Both historical and simulated data panels exhibit a positive relation between average returns and book-to-market equity. HML return produced by the model appears to be lower than its empirical counterpart. The magnitude of risk and average returns are also lower in the model. The reason behind this is the general tendency to save and play safe in the model. As discussed earlier, borrowing constraints lead to precautionary motives, which give rise to lower investment and external financing rates and accordingly, borrowing constraints end up depressing the risk and average return levels. In addition to the precautionary motives, absence of inflation in the model drags down the returns as well. The average return levels catch up with the historical levels only for the highest two book-to-market portfolios. The high book-to-market portfolios consist of relatively large firms with high levels of capital stock and financially distressed firms with relatively low levels of capital stock and/or high levels of debt. As discussed in section 2.4, book-to-market, risk and average return are negatively correlated with capital stock and positively correlated with borrowing level. The high risk and return of distressed firms dominate the lower risk and average return levels of large, financially healthy firms in the high book-to-market portfolios and in the overall, their portfolio risk and returns match well with their empirical equivalents.

Similarly, Panel B of Table 2.2 reports the mean and unconditional beta statistics for 10 portfolios sorted on size, which is defined as the market value of the equity. The empirical findings show that average return and market value of equity exhibit a negative relation. Thus, small firms on average appear to earn more than large firms. In section 2.4.4, it is noted that the constrained model captures the size effect when size is measured as book value of equity. Lower plot in Figure 2.1 illustrates that for a given level of capital, market value of equity is positively related to idiosyncratic shock z. Provided that risk and return decreases with z, the size effect is actually stronger when size is measured as market value of equity relative to book value. As in the book-to-market case, the model captures the direction of historical data, yet the magnitudes of average portfolio risks and returns produced by the simulated data are lower than their historical counterparts. The risk and return of the smallest portfolio and SMB portfolio, however, are actually higher than their empirical counterparts. As a result of borrowing constraints, firms with the lower market values are mostly the distressed ones with few capital stocks and/or with high debt holdings and these distressed firms are very risky and earn high returns.

To analyze the cross-sectional properties of the model further, I replicate closely the cross-sectional regressions of stock returns on market beta, firm size and book-to-market in Fama and French (1992). The market betas are obtained using two-step procedure of Fama and French (1992). First, pre-ranking betas are estimated by regressing firm's excess stock returns on market excess returns over past 60 months. Then, post-ranking betas are estimated monthly for 100 portfolios constructed by two-dimensional sort of stocks on market value and pre-ranking betas. The post-ranking betas are then assigned to each stock of its portfolio.

Table 2.3 summarizes the cross-sectional regression estimates produced by the simulated data and empirical findings of Fama French (1992). The slope coefficients are the time-series averages of cross-sectional regressions and t-statistics are time-series averages divided by the time-series standard deviations.

The first univariate regression report the cross-sectional relation between realized equity return and market beta. The model does a poor job in replicating the direction of the relation. The slope coefficient of market beta produced by the model is negative, small in absolute value and insignificant. The empirical slope coefficient is insignificant as well but it is positive. The negative, insignificant slope produced by the model could be due to the fact that the relation between risk and return in response to an aggregate shock x is unclear. While for financially leveraged firms and/or small unproductive firms the risk and return are both positively related with x, for the rest of the firms in the economy the risk and return move in opposite directions in response to x. As discussed in Section 2.4.4, the negative relation between risk and return under x is attributable to the counter-cyclical price of risk. As beta increases with a higher aggregate shock x, the price of risk decreases and the decline in price of risk dominates the increase in risk. Consequently, some firms earn lower expected returns despite the fact that they carry more risk and vice versa. Given that price of risk is not a function of idiosyncratic shock z, risk and return move together under different idiosyncratic shocks. In accordance, theoretically, the direction of the relation between beta and return is unclear.

The other univariate regression estimates in the second and fourth rows of Panel

B indicate that there is a significant negative relationship between size and stock returns, and a significant positive relationship between book-to-market and stock returns. While the direction of the relation is inline with the empirical findings, the model overestimates the size effect and underestimates the book-to-market effect. The univariate regression estimates confirm the results in Table 2.2 which reports average returns of 10 portfolios sorted on book-to-market and size.

The analysis of joint regressions of beta, size and book-to-market equity on stock return suggest that size and book-to-market effects observed in the univariate cases survive and each regressor enters with a correct sign.

Overall, the results are consistent with Fama and French (1992), suggesting that both firm size and book-to-market equity have explanatory power in explaining average returns, but the estimated beta has little or no-cross-sectional explanatory power.

Value and Size Factors in Earnings

In this section, I investigate the economic mechanism behind the explanatory power of size and book-to-market equity in explaining average returns. I present simulation evidence that sheds light on the question about how size and book-to-market equity relate to economic fundamentals by analyzing the productivity differences between value and growth firms. I show that (i) both size and book-to-market equity are proxies for firm productivity, (ii) controlling for one factor, the other one is still associated with persistent differences in profitability, and (iii) book-to-market equity and size capture partly the cross-sectional variation in average returns that is related to financial distress.

Figure 2.10 depicts the average values of profitability and financial leverage for book-to-market portfolios and size portfolios in the simulated data, for 11 years around portfolio formation. Profitability is defined as the return on book equity and measured by $(\Delta k_{jt} + d_{jt})/(k_{jt-1} - b_{jt-1})$, where k denotes the capital stock, b denotes financial leverage and d is the dividend payout.

Consistent with the empirical findings of Fama and French (1995), book-to-market equity is associated with persistent differences in profitability. Low book-to-market stocks are, on average, more profitable than high book-to-market stocks for 5 years before and after portfolio formation. In the nutshell, book-to-market equity wears at least two heads: low values indicate sustained strong profitability; high values indicate persistent distress. In accordance, book to market equity largely captures cross-sectional variation in average returns that is related to financial distress and growth opportunities.

Similarly, top right plot in Figure 2.10 shows that size is also a proxy for profitability. Small stocks have persistently lower returns on equity than big stocks. Low market equity says that market judges the prospects of a firm to be poor relative to firms with higher market values and the size factor in earnings is the source of the corresponding risk factor in returns. Firm size captures partly the cross-sectional variation in average returns that is related to financial distress and asset-in-place.

In the constrained model, financial distress, growth opportunities and asset-in-place are closely related to financial constraints. To confirm that book-to-market equity and size subsume the effects of financial constraints on risk and expected returns, I study average values of financial leverage for book-to-market portfolios and size portfolios for 11 years around portfolio formation. Financial leverage is a good proxy for financial constraints in my constrained model since positive debt holdings increase the probability of financial distress via lower cash flow and higher volatility of future profits, whereas positive cash holding acts as a buffer against the possibility of future financial constraints. Figure 2.11 shows that while small firms and value firms hold positive debt on average, big firms and growth firms hold cash on average. Persistency of financial leverage policy explains the persistency in profitability. This result suggests that both size and book-to-market can largely capture the cross-sectional variation in equity returns due to financing constraints.

Finally, following Fama and French (1995), I study the earning behaviors of firms classified as low and high on book-to-market equity and small and big on size. As we have seen in Table 2.3, the explanatory powers of size and book-to-market equity survive under a joint regression. Figure 2.12 shows that controlling for size, growth firms earn persistently higher returns on equity than value firms for six years around the portfolio formation. Figure 2.12 also shows that controlling for book-to-market equity, big firms have higher profitability for five years around formation period. Briefly, size and book-to-market equity are collectively informative factors in profitability as they are in equity returns. This result supports the findings in Fama and French (1995).

C. Cyclical Properties of Risk and Return

In this section, I analyze the cyclical properties of risk and return. In Figure 2.5, we see that a firm has a high book to market equity in three cases: (i) when firm's capital stock is low and it faces a negative idiosyncratic shock z, (ii) when firm carries significant amount of debt, and (iii) when firm's capital stock is high. A value firm with little capital stock that faces negative idiosyncratic shock z is close to the liquidation boundary. As we have discussed in Section 2.4.1, the current cash flow is crucial for firms under liquidation risk. Given that the current cash flow improves under good economic conditions, firm's risk level and expected return diminish with higher aggregate productivity. The second type of firms that are leveraged are also riskier than unleveraged firms and the gap enhances under bad productivity shocks owing to higher interest rates, bad future prospects, and higher distress risk. Besides, time-varying price of risk assumption drives up the leveraged firms' riskiness as well. According to Figure 2.13, the discount factor is lower on average in a counter-cyclical price of risk model compared to a constant price of risk model. Since future value is discounted more heavily in bad times of the economy, the continuation value is lower and propensity to disinvest is higher in recessions under time-varying price of risk. However, firms with relatively high leverage cannot scale down without increasing the likelihood of liquidation. Therefore, they became more risky and earn higher returns in bad times under counter-cyclical price of risk assumption relative to constant price of risk assumption. The effect of aggregate shocks on riskiness of the last type of value firms is pro-cyclical unless the firm carries a significant amount of debt. As noted in Section 2.4, the gap between investment demand and internal fund increases during good times since the effect of aggregate productivity on future value dominates its effect on current cash flow and investment demand of a financially healthy firm increases more than its internal fund.

The growth firms are financially healthier and carry more growth opportunities. Since they face a higher adjustment cost to scale up in economic booms compared to recessions, their risk level is increasing slightly with aggregate shock x.

A similar analysis can be done for small and big firms. Small firms are heavily leveraged on average and more likely to be bounded by financial constraints. Therefore, their risk and return is counter-cyclical. On the contrary, financially healthy big firms are less risky than small firms and their riskiness increase slightly with aggregate shock x. Consequently, the risk and return of small minus big portfolios are also counter-cyclical.

Table 2.4 reports the average conditional portfolio betas in good and bad times for both the historical and simulated data. Four states of the world are defined: (i) the months with the worst 10 % observations of the expected market risk premium; (ii) the remaining months with the expected market risk premium below its average; (iii) the months with the expected market risk premium above its average but other than the 10 % very best; and (iv) the months with the 10 % very best observations. Panels A and B illustrate the conditional betas averaged across 50 simulations. The data in Panel C and Panel D are from Petkova and Zhang (2002). HML stands for the conditional beta dispersion between value and growth firms and SMB stands for the conditional beta dispersion between small and big firms.

Consistent with the data, conditional beta spread between value and growth firms enhances during bad times, when expected risk premium is high, and contracts during good times of the economy, when expected risk premium is low. According to results, small unproductive firms and leveraged firms dominate the value group and the risk level of value group seems to be an increasing function of aggregate shock x. Although the direction of the relationship is right, the magnitude of the beta dispersion is lower than its empirical counterpart. A similar pattern is also observed in small and big portfolios.

D. Predictability of Value-minus-Growth Return

Lastly, I will analyze the predictability of HML return. Table 2.5 reports a predictive regression of the HML return on the value spread, the earning growth spread, and demeaned aggregate productivity. The HML is defined as the value-minus-growth return. Value spread is the log of the book-to-market spread between high and low portfolios, and the earning growth spread is the log of the return spread between low and high portfolios. Since it is a predictive regression, HML return is measured at the end of the period and all predictive variables are measured at the beginning of the period.

Earning growth and aggregate productivity are powerful predictors of HML return in the model. The slope of demeaned aggregate productivity is significant and negative in annual frequency so HML return is counter-cyclical. The slope of earning growth is positive and significant in both annual and monthly frequencies. The value spread is also a significant predictor in annual frequency but its predictive power seems to be less than the other two variables.

2.6 Conclusion

In recent years, the apparent abilities of value and size strategies to generate above-average returns in US stock market are well documented. In this chapter, I propose a simple theoretical framework that can produce simultaneously value premia and size premia in the cross section of equity returns and demonstrate the impact of financial distress on these patterns.

To account for the financial distress, I incorporate financing constraints into a neoclassical industry equilibrium model with stochastic discount rate as well as aggregate and idiosyncratic uncertainty. I investigate the extent to which financing constraints explain documented cross-sectional variation in risk and expected returns. I define financing frictions as collateral and dividend constraints. Specifically, firms' only source of external finance is debt secured by their collateral assets.

I show that due to financing constraints, value firms and small firms are dominantly financially distressed. The financially distressed firms with low capital capacity and/or excessive leverage benefit during economic expansions and suffer during economic downturns. In view of that, the high exposure to systematic risk leads to high risk premiums.

While the directions of the value and size premiums are in line with the evidence, the magnitude of the value premium is lower than its empirical counterpart. As described, financing constraints and counter-cyclical price of risk features of the model can partially explain the value premium but there is still room for improvement. Some other features of the economy that can potentially affect asset prices, like tax policies, irreversible investment or costly equity financing, can also be integrated in the neoclassical investment model. On the size premium side, it seems like the model does a better job in capturing the direction and the magnitude of the documented size premium.

2.7 Appendix

2.7.1 Tables and Figures

Table 2.1: Time Series Regressions

This table reports the time-series regressions of value-weighted market return on aggregate book-to-market ratio. Aggregate book-to-market ratio is defined as the sum of book values of all the firms in the market divided by the sum of market values. The regression is conducted at both monthly and annual frequencies. The first row of the panel reports the historical results, corresponding to Table II in Pointiff and Schall (1999). The second row reports simulated data estimates. The average slopes and adjusted R^2's are in percentage. t-statistics are adjusted for autocorrelated residuals using Newey-West (1987) method with 12 lags.

$$R^{vw}_{t+1} = a + b \times (k/v^e)_t + \epsilon_{t+1}$$

	Monthly			Annual		
	Slope	*t-stat*	*Adjusted R^2*	*Slope*	*t-stat*	*Adjusted R^2*
Data	3.02	-	1.00	42.18	-	16.00
Model	1.74	3.66	3.05	20.43	4.39	20.31

Table 2.2: Properties of Portfolio Sorted on Size and Book-to-Market

This table reports the summary statistics of 10 book-to-market portfolios (Panel A) and 10 size portfolios (Panel B), including annualized means, m, and market betas, β, both from historical data and model simulations. The average HML return (the value premium) and the average SMB return (the size premium) are in annualized percent. All the model moments are averaged across 50 artificial samples. All returns are simple returns.

Panel A: 10 B/M Portfolios *Panel B: 10 Size Portfolios*

	Model		*Data*			*Model*		*Data*	
	m	β	m	β		m	β	m	β
Low	0.09	0.83	0.11	1.01	Small	0.20	1.65	0.18	1.47
2	0.09	0.81	0.12	0.98	2	0.15	1.21	0.16	1.40
3	0.11	0.82	0.12	0.95	3	0.11	1.17	0.15	1.33
4	0.10	0.88	0.11	1.06	4	0.10	1.15	0.15	1.26
5	0.11	0.91	0.13	0.98	5	0.11	1.12	0.15	1.26
6	0.10	0.94	0.13	1.07	6	0.11	1.02	0.14	1.21
7	0.12	0.98	0.14	1.13	7	0.09	0.94	0.14	1.16
8	0.12	1.03	0.15	1.14	8	0.08	0.87	0.13	1.11
9	0.16	1.09	0.17	1.31	9	0.05	0.81	0.13	1.08
High	0.15	1.19	0.17	1.42	Big	0.06	0.81	0.11	0.93
HML	0.03	0.11	0.04	0.14	*SMB*	0.03	0.25	0.02	0.12

Table 2.3: Cross-sectional Regressions

This table reports the results of cross-sectional regressions of the monthly realized returns, R_{t+1}, on the market beta, β, the firm size, $logME$, the ratio of book equity to market equity, $log\frac{BE}{ME}$. Panel A presents the results from Fama and French (1992), Table III, while Panel B presents the results from simulated data. The slope and the t-Statistic coefficients are Fama-Macbeth (time-series) estimates.

β	$logME_t$	$log\frac{BE_t}{ME_t}$
	Panel A: Historical Data	
0.15		
(0.46)		
	-0.15	
	(2.58)	
-0.37	-0.17	
(1.21)	(3.41)	
		0.50
		(5.71)
	-0.11	0.35
	(1.99)	(4.44)
	Panel B: Simulated Data	
-0.09		
(0.68)		
	-0.31	
	(7.72)	
-0.12	-0.36	
(0.98)	(5.15)	
		0.21
		(3.01)
	-0.29	0.22
	(3.64)	(4.41)

Table 2.4: Cyclical Properties of Time-Varying Portfolio Betas

This table reports the average betas of 10 book-to-market portfolios, and 10 size portfolios, in good and bad times, defined by sorting on the expected market risk premium. Four states of the world are defined: ”Worst” is identified with the worst 10 % expected market premium months;”−” is the remaining below average risk premium months other than the 10 % worst; ”+” is the above average risk premium months other than the 10 % best; and ”Best” is the 10 % best months in the sample. HML denotes the risk spread between value and growth and SMB denotes the risk spread between small and big firm portfolios. Panels A and B illustrate simulated data results. Panels C and D illustrate empirical findings from Petkova and Zhang (2002).

10 B/M Portfolios					*10 Size Portfolios*				
	Panel A					*Panel B*			
	Worst	−	+	*Best*		*Worst*	−	+	*Best*
Low	1.02	1.03	0.93	0.92	Small	1.01	1.06	1.18	1.70
2	1.01	1.03	1.03	0.92	2	1.01	1.04	1.18	1.24
3	1.03	0.90	0.97	0.95	3	0.99	1.00	1.02	1.12
4	0.93	0.93	0.88	0.94	4	0.97	0.96	1.04	1.02
5	0.91	0.89	0.83	0.94	5	0.95	0.96	0.95	0.98
6	0.87	0.86	0.88	0.96	6	0.92	0.92	0.90	0.97
7	0.88	0.88	0.92	1.03	7	0.90	0.93	0.86	0.85
8	0.98	0.97	1.01	1.11	8	0.83	0.88	0.88	0.84
9	1.10	1.11	1.14	1.24	9	0.86	0.88	0.88	0.84
High	1.16	1.18	1.21	1.44	Big	0.81	0.80	0.83	0.81
HML	0.05	0.09	0.13	0.23	SMB	0.07	0.12	0.17	0.22

	Panel C					*Panel D*			
	Worst	−	+	*Best*		*Worst*	−	+	*Best*
Low	1.13	1.08	1.04	0.95	Small	1.08	1.18	1.27	1.60
2	1.00	1.02	1.03	0.98	2	1.16	1.25	1.29	1.46
3	1.01	1.01	1.00	0.92	3	1.13	1.17	1.22	1.37
4	0.94	0.93	0.97	1.09	4	1.10	1.15	1.17	1.28
5	0.89	0.88	0.90	0.99	5	1.08	1.13	1.17	1.29
6	0.86	0.90	0.97	1.15	6	1.08	1.11	1.14	1.25
7	0.79	0.88	1.02	1.28	7	1.09	1.11	1.12	1.17
8	0.75	0.89	1.06	1.33	8	1.04	1.06	1.08	1.13
9	0.77	0.98	1.20	1.58	9	0.98	1.01	1.05	1.12
High	0.87	1.09	01.31	1.70	Big	0.94	0.94	0.94	0.93
HML	−0.32	−0.15	0.05	0.40	SMB	0.22	0.21	0.16	0.15

Table 2.5: Predictability of HML Return

This table reports from predictive regressions of HML returns on, separately, the value spread, VP, earning growth spread, EG, aggregate productivity, x. The regression is conducted at both monthly and annual frequencies. t-statistics, reported in parenthesis, are adjusted for autocorrelated residuals using Newey West (1987) with 12 lags. The adjusted R^2's are in percentage.

Panel A: Monthly Predictive Regressions

Intercept	*VP*	*EG*	x-$\overline{x}$	*Adjusted* R^2
0.006	0.009			0.382
(1.99)	(2.03)			
0.010		0.016		0.484
(2.96)		(2.87)		
0.009			-0.295	0.842
(3.05)			(-1.48)	

Panel B: Annual Predictive Regressions

Intercept	*VP*	*EG*	x-$\overline{x}$	*Adjusted* R^2
0.064	0.060			9.161
(1.20)	(2.96)			
0.132		0.137		12.173
(1.39)		(3.59)		
0.113			-4.809	12.415
(5.71)			(-3.89)	

Figure 2.1: Value Function and Productivity Shocks

This figure plots firm value for constrained and unconstrained firms as a function of capital level k, aggregate productivity x, and firm-level productivity z.

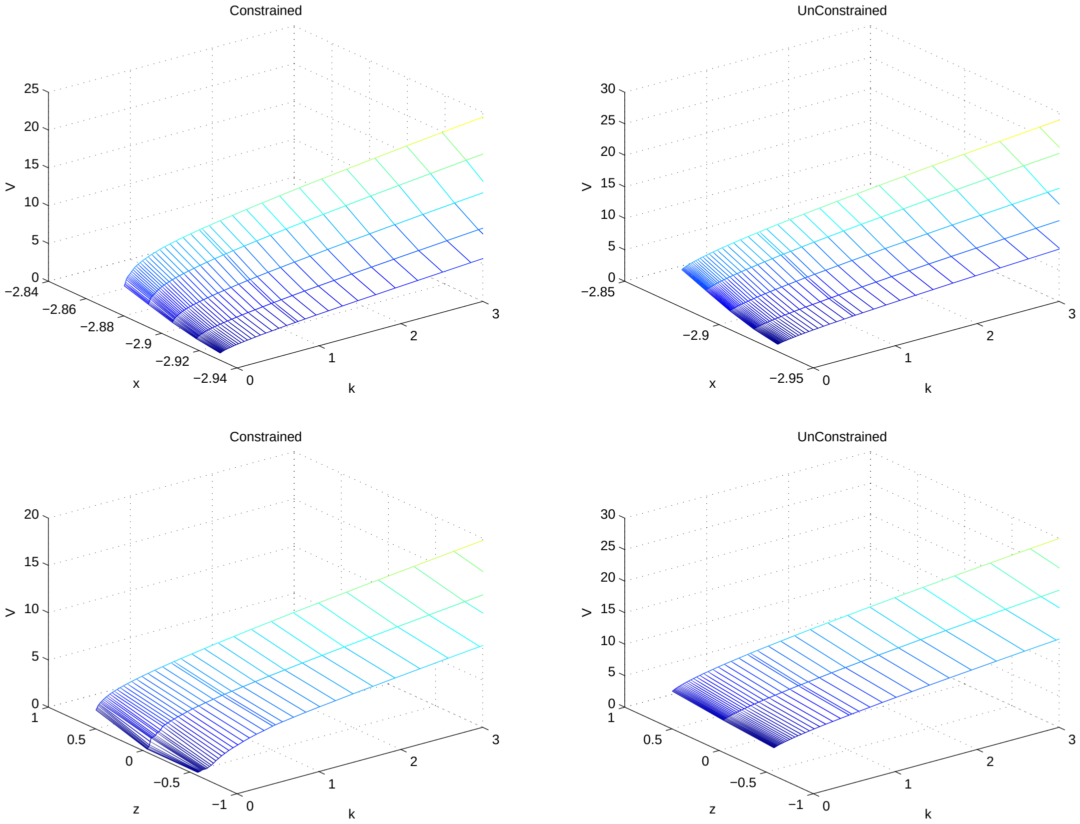
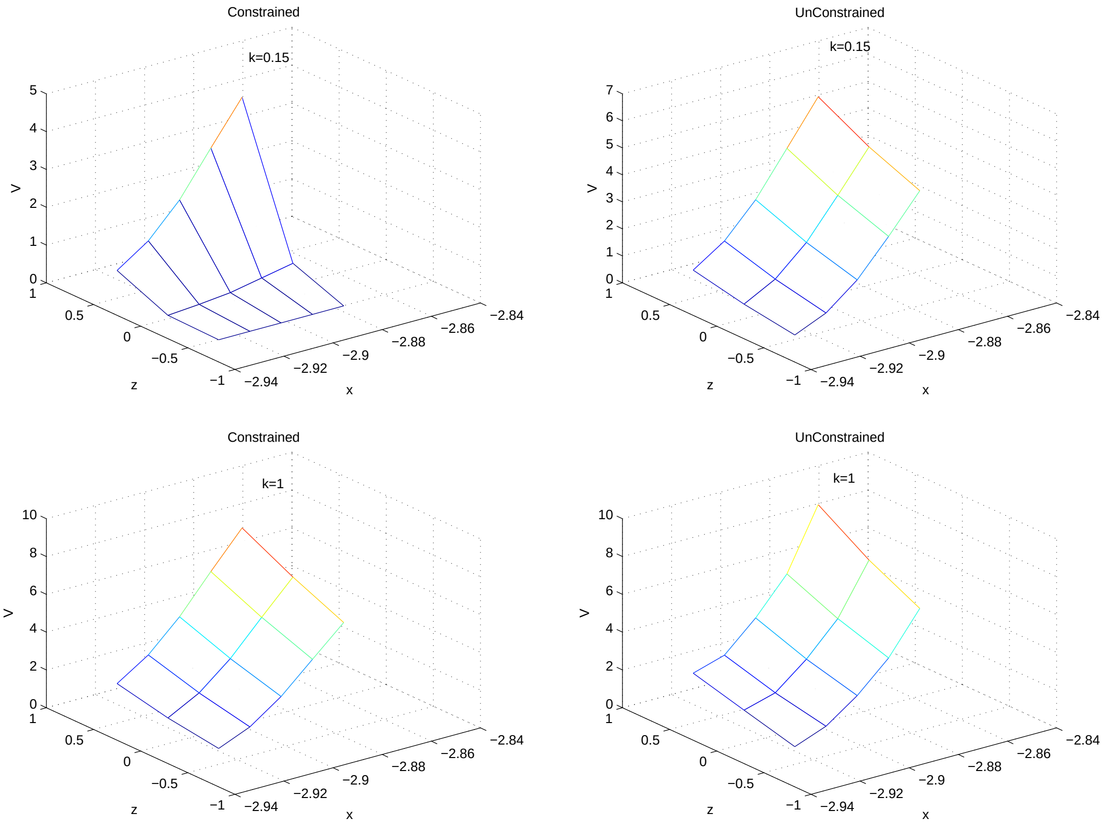


Figure 2.2: Value Function and Capital Level

This figure plots firm value for constrained and unconstrained firms as a function of capital level k, aggregate productivity x, and firm-level productivity z.



Constrained
k=0.15

UnConstrained
k=0.15

Constrained
k=1

UnConstrained
k=1

Figure 2.3: Tobin's Q and Productivity Shocks

This figure plots Tobin's Q for constrained and unconstrained firms as a function of capital level k, aggregate productivity x, and firm-level productivity z. Tobin's Q, $v/(k-b)$, is defined as the ratio of market equity and book equity.

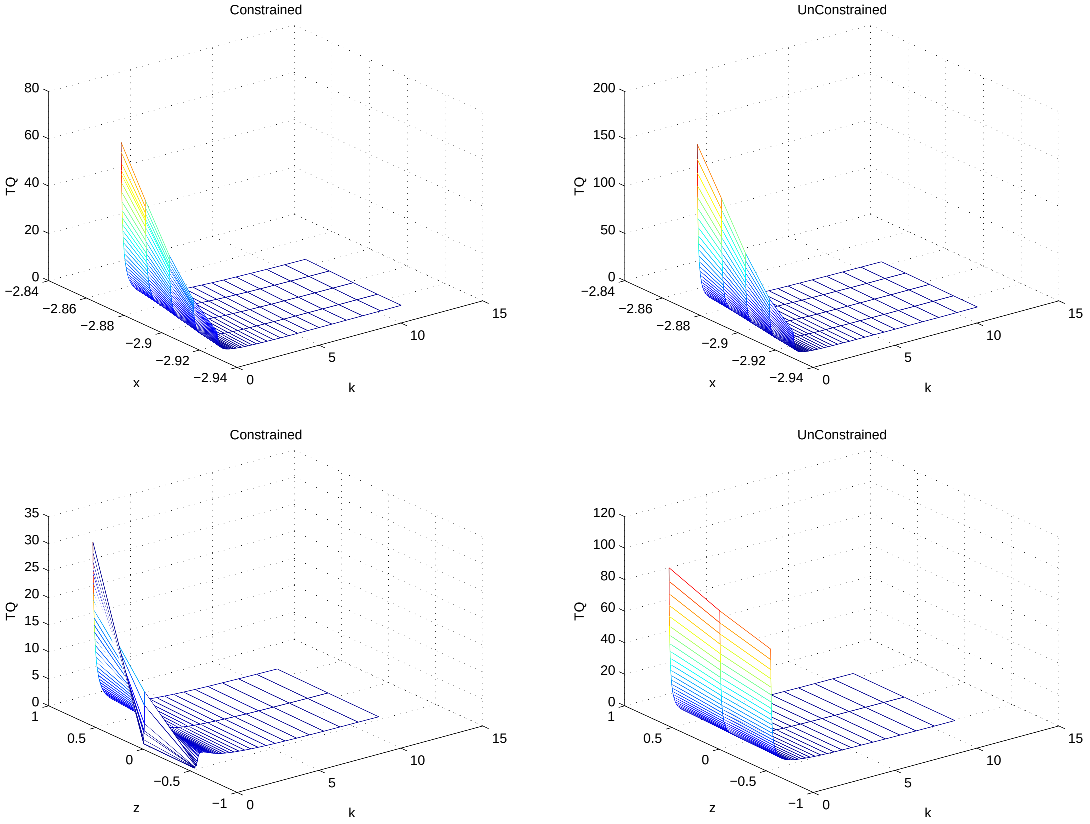


Figure 2.4: Tobin's Q and Financial Leverage

This figure plots Tobin's Q for an indebted constrained firm as a function of its capital level k, aggregate productivity x, and firm-level productivity z. Tobin's Q, $v/(k-b)$, is defined as the ratio of market equity and book equity. The borrowing level is held constant at 0.34.

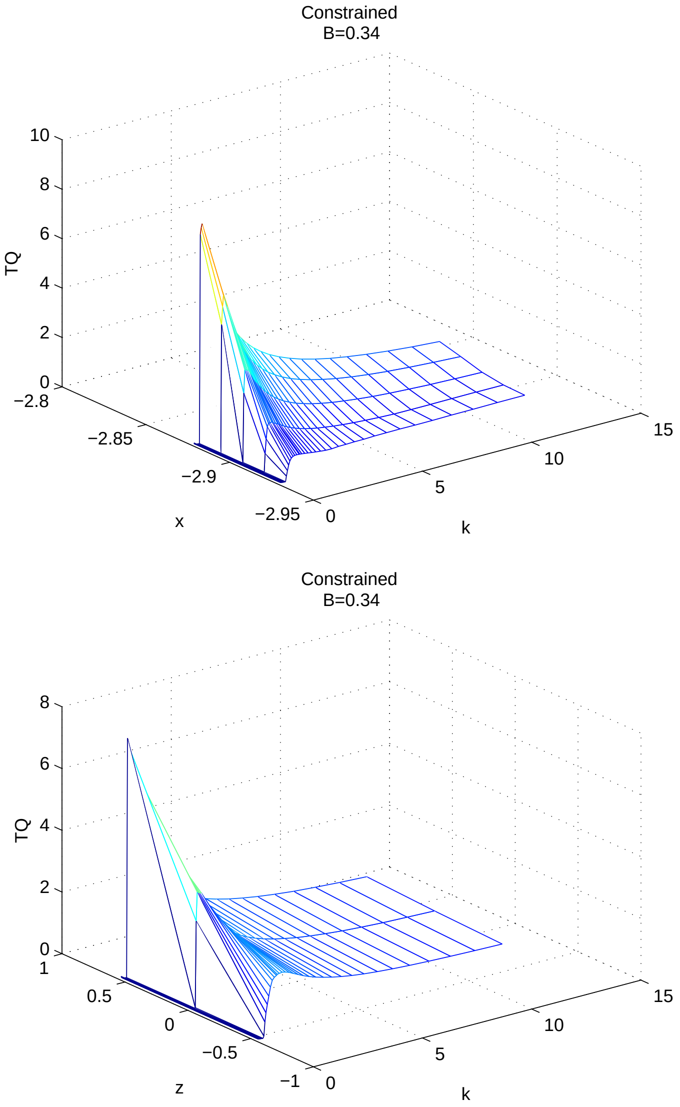


Figure 2.5: Book-To-Market and Capital Level

This figure plots firm value for constrained and unconstrained firms as a function of capital level k, aggregate productivity x, and firm-level productivity z.

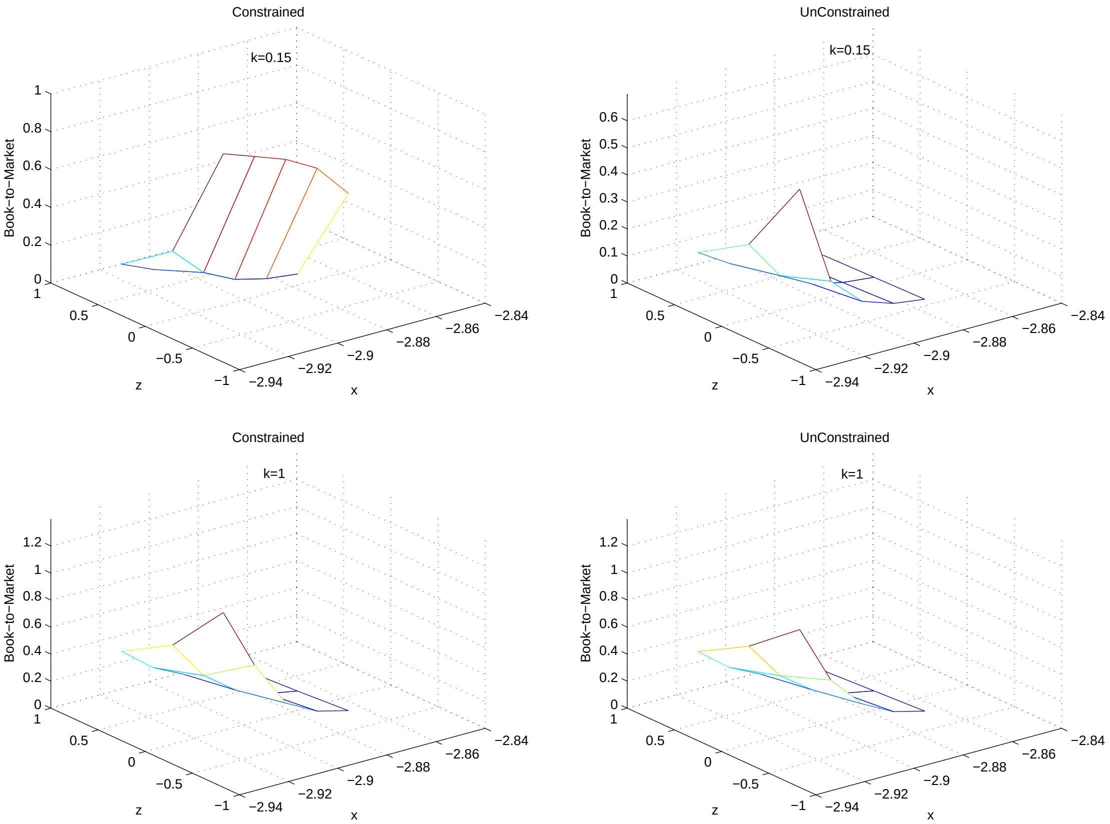


Figure 2.6: Beta and Expected Return for z= 0 and b=0

This figure plots expected return and beta for constrained and unconstrained firms as a function of capital level k, and aggregate productivity x.

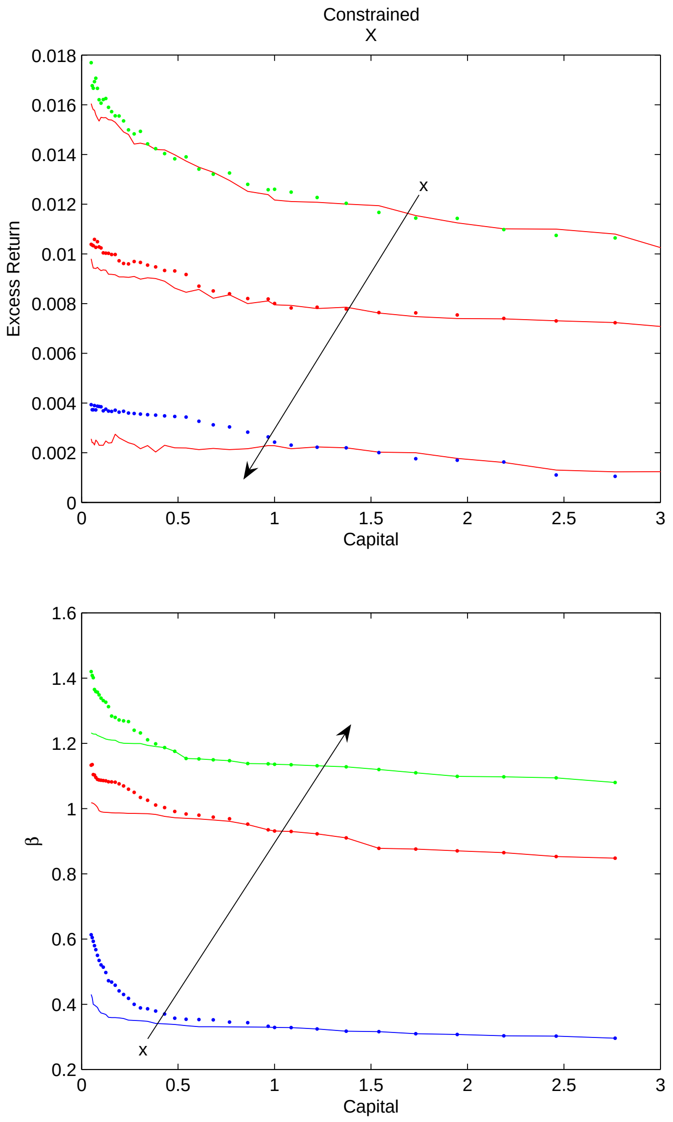


Figure 2.7: Beta and Expected Return for x=-2.9 and b=0

This figure plots expected return for constrained and unconstrained firms as a function of capital level k, and firm-level productivity z.

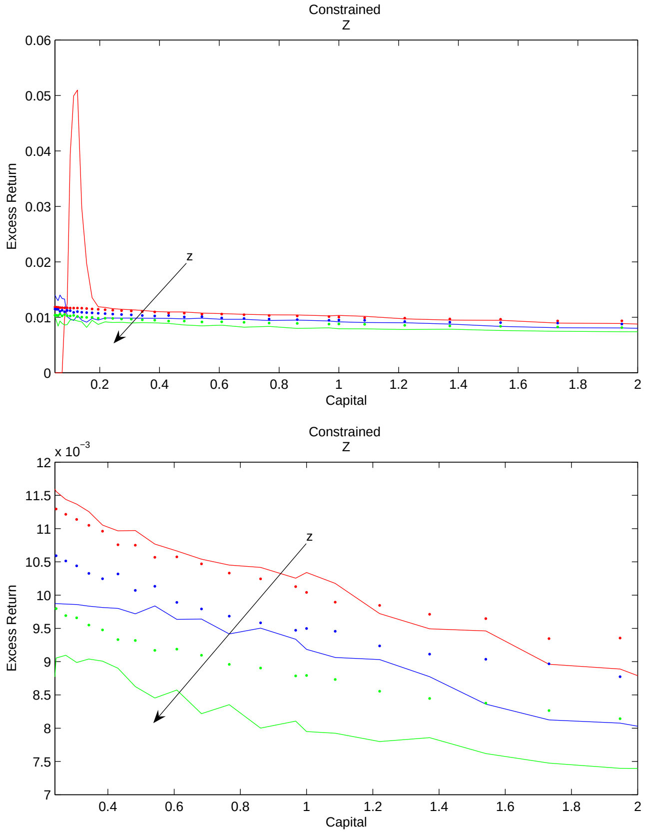


Figure 2.8: Beta and Expected Return for k=0.15 and b=0

This figure plots beta and expected return for constrained firm as a function of aggregate productivity x, and firm-level productivity z.

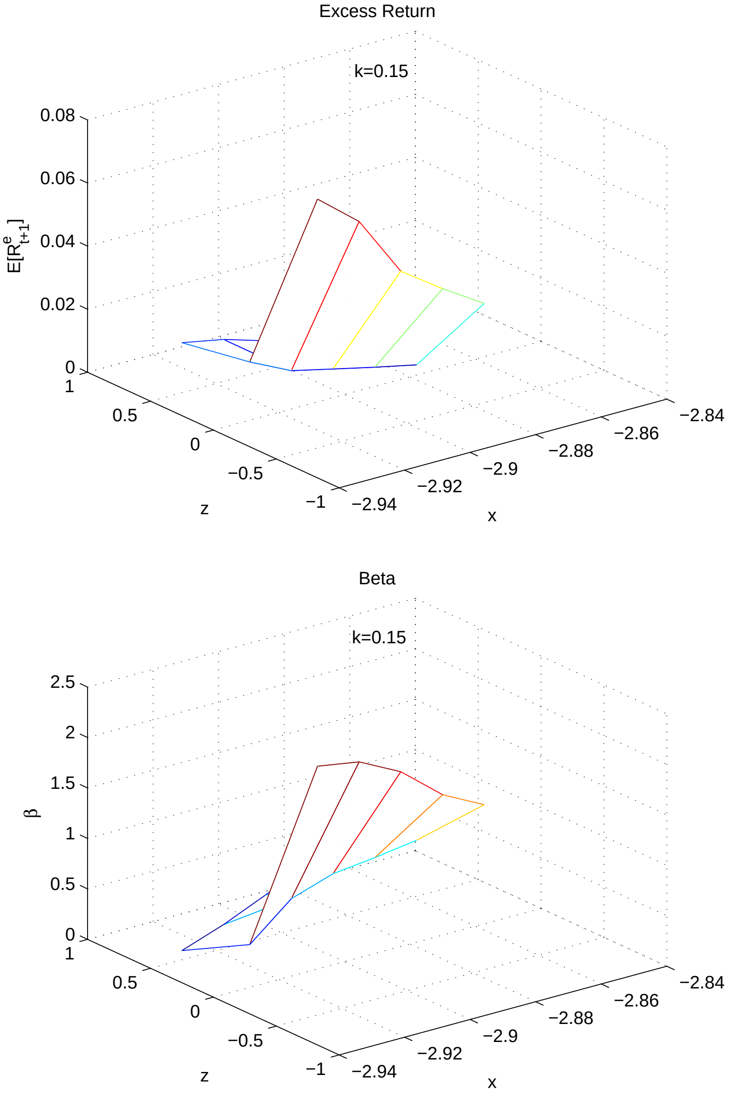


Figure 2.9: Beta and Expected Return for k=0.15 and b=0.04

This figure plots beta and expected return for constrained firm as a function of aggregate productivity x, and firm-level productivity z.

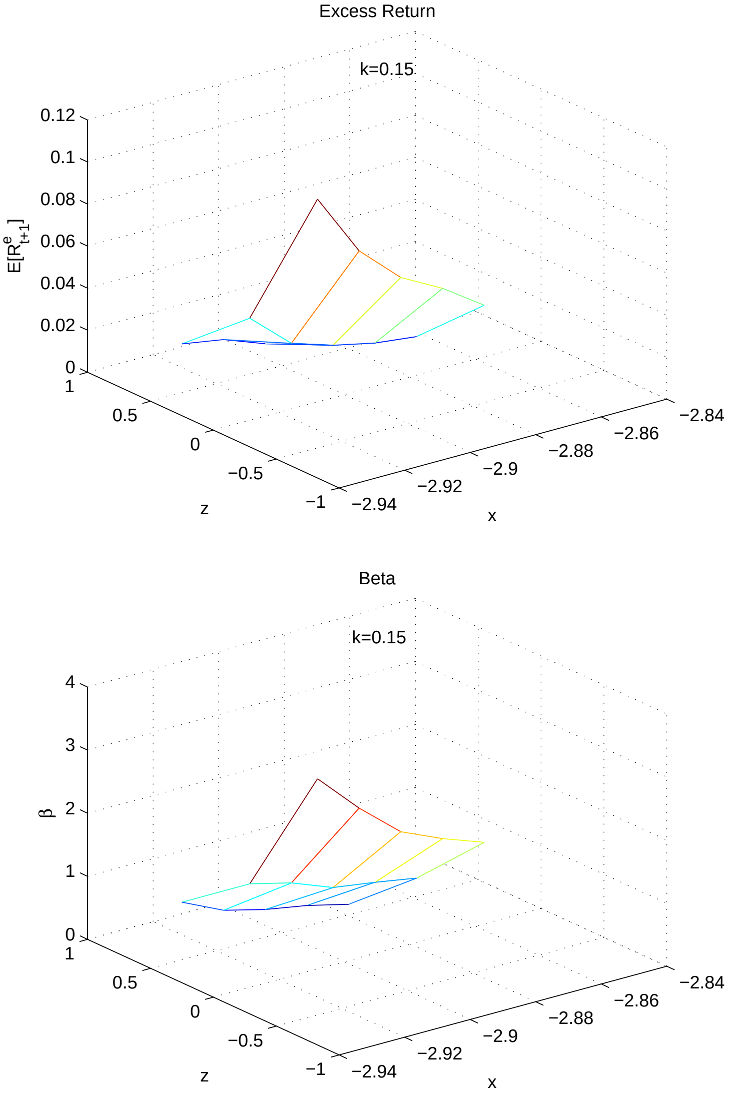


Figure 2.10: Value and Size Factor in Earnings - Seperately

This figure illustrates the relation between the book to market ratio and firm profitability in the simulated data. The firm profitability is measured by the return on equity, that is $(\Delta k_{jt} + d_{jt})/(k_{jt-1} - b_{jt-1})$, where k_t denotes the book value of equity, b_t denotes debt holding and d_t is the dividend payout. Panel A shows the 11-year evolution of earnings on book equity for value and growth portfolios. Panel B shows the 11-year evolution of earnings on book equity for small and big portfolios. Time 0 in the horizontal axis is the portfolio formation year.

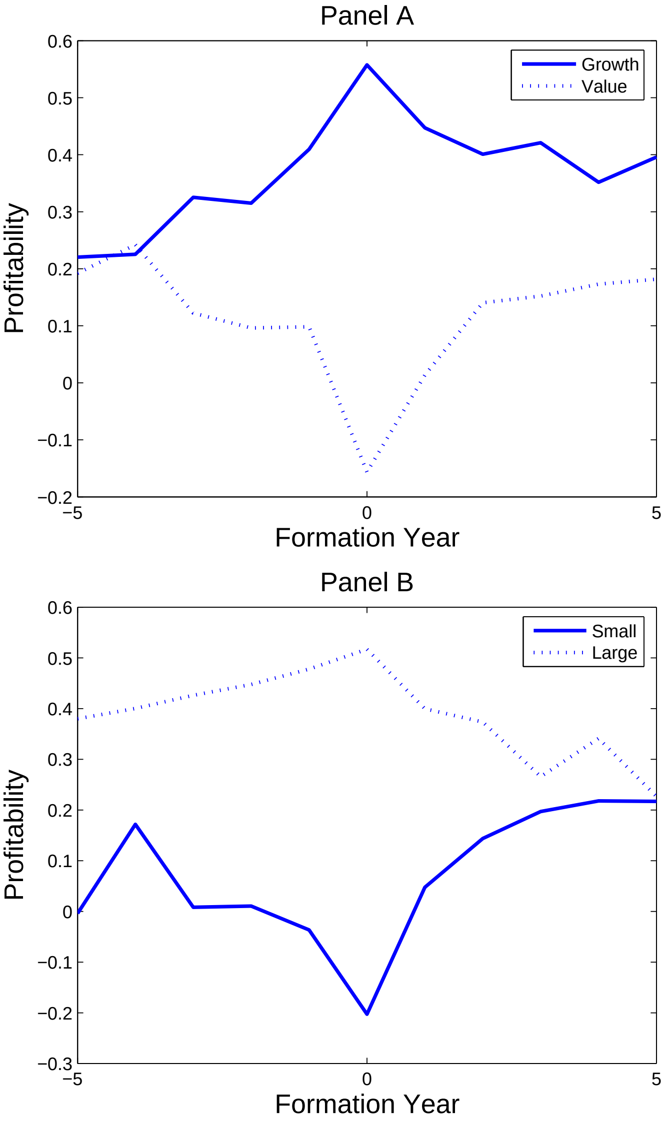


Figure 2.11: Value and Size Factor in Financial Leverage

Panel A illustrates the relation between the financial leverage and book-to-market equity in the simulated data. The plot shows the 11-year evolution of financial leverage for value and growth portfolios. Panel B illustrates the relation between the financial leverage and market equity in the simulated data. The plot shows the 11-year evolution of financial leverage for small and big portfolios. Time 0 in the horizontal axis is the portfolio formation year.

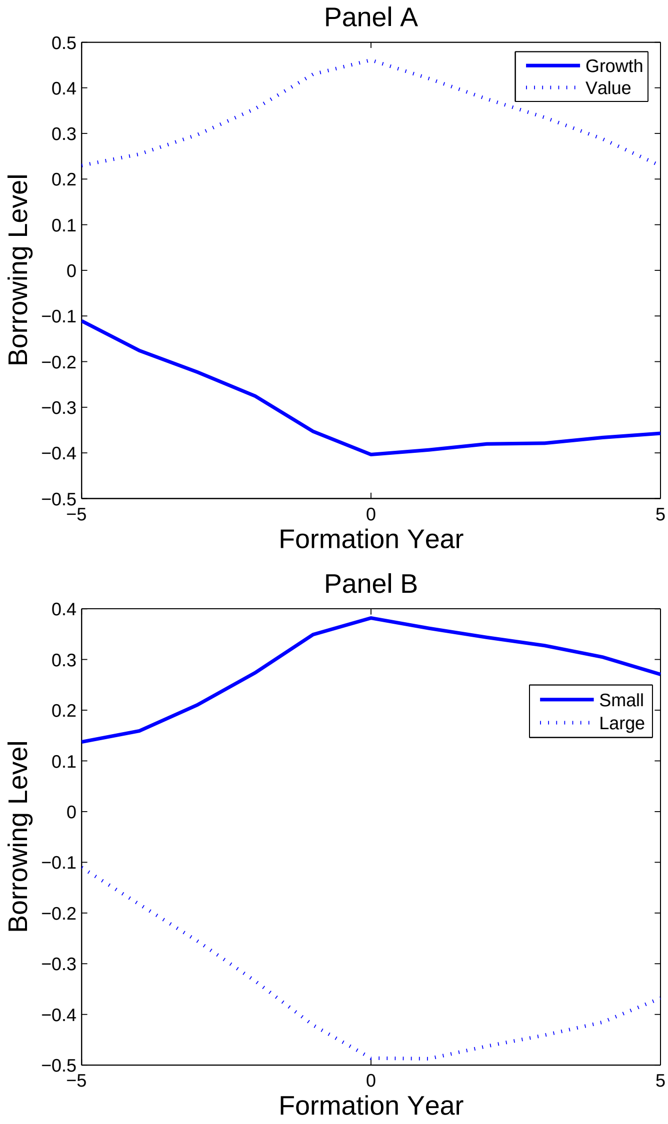


Figure 2.12: Value and Size Factor in Earnings - Together

This figure illustrates the relation between the book to market ratio, market equity and firm profitability in the simulated data. The firm profitability is measured by the return on equity, that is $(\Delta k_{jt}+d_{jt})/(k_{jt-1}-b_{jt-1})$, where k_t denotes the book value of equity, b_t denotes the financial leverage and d_t is the dividend payout. The figure shows the 11-year evolution of earnings on book equity for size/book-to-market portfolios. Time 0 in the horizontal axis is the portfolio formation year.

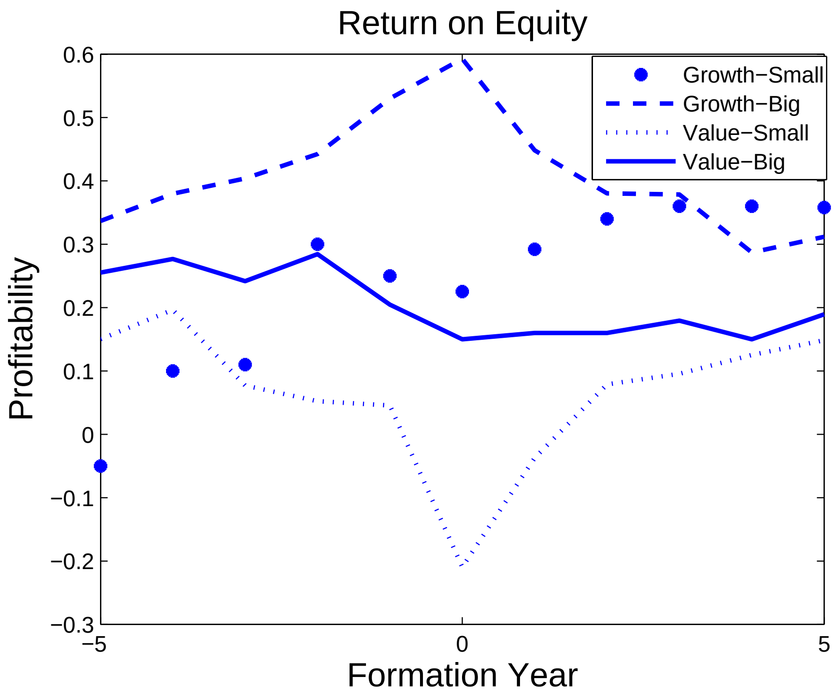
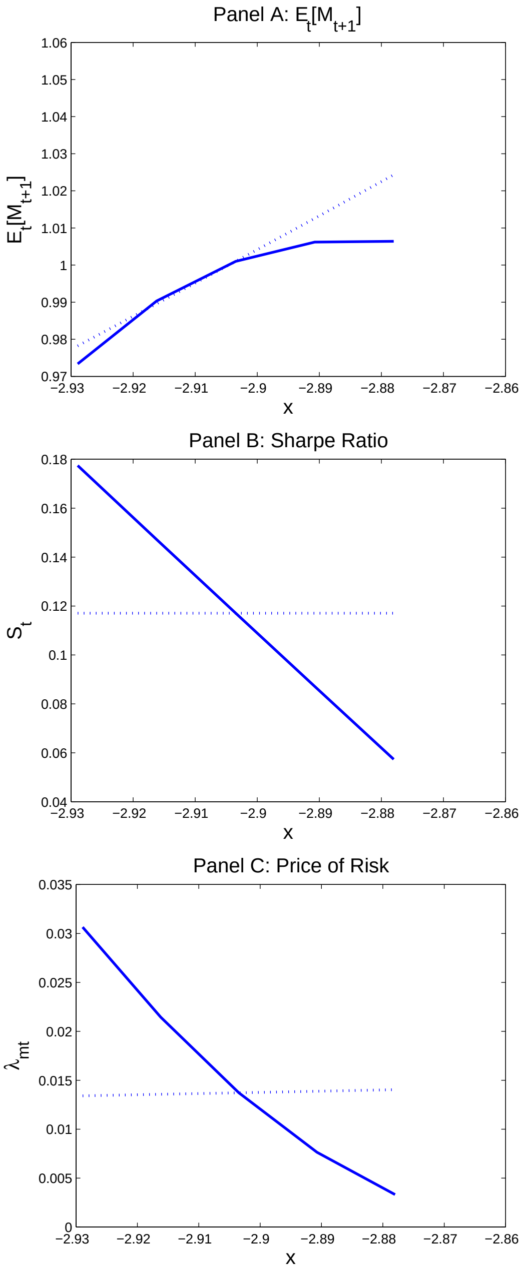


Figure 2.13: Properties of Pricing Kernal

This figure plots the key moments of the pricing kernel, $E_t[M_{t+1}]$, price of risk $\lambda_{mt} = \sigma_t^2[M_{t+1}]/Et[M_{t+1}]$, and Sharpe ratio $S_t = \sigma_t[M_{t+1}]/E_t[M_{t+1}]$, all in monthly frequency, against the aggregate productivity x_t. The solid lines are for the case with γ_1 = -1000 and the broken lines are for the case with $\gamma_1 = 0$.



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