

DISSERTATION DEFENSE

Sercan Yildiz

Valid Inequalities for Mixed-Integer Linear and Mixed-Integer Conic Programs

Friday, April 29, 2016
10:00 am
322 GSIA (West Wing)

Mixed-integer programming provides a natural framework for modeling optimization problems which require discrete decisions. Valid inequalities, used as cutting-planes and cutting-surfaces in integer programming solvers, are an essential part of today's integer programming technology. They enable the solution of mixed-integer programs of greater scale and complexity by providing tighter mathematical formulations of the feasible solution set. This dissertation presents new structural results on general-purpose valid inequalities for mixed-integer linear and mixed-integer conic programs.

Cut-generating functions are a priori formulas for generating a cut from the data of a mixed-integer linear program. This concept has its roots in the work of Gomory and Johnson from the 1970s. It has received renewed attention in the past few years. Gomory and Johnson studied cut-generating functions for the corner relaxation, which is obtained by ignoring the nonnegativity constraints on the basic variables in a tableau formulation. In our first three chapters, we consider models where these constraints are not ignored. In our first chapter, we generalize a classical result of Gomory and Johnson characterizing minimal cut-generating functions in terms of subadditivity, symmetry, and periodicity. Our analysis also exposes shortcomings in the usual definition of minimality in our general setting. To remedy this, we consider stronger notions of minimality and show that these impose additional structure on cut-generating functions. A stronger notion than the minimality of a cut-generating function is its extremality. While extreme cut-generating functions produce powerful cutting-planes, their structure can be very complicated. Gomory and Johnson identified a "simple" class of extreme cut-generating functions for the corner relaxation of a one-row integer linear program by showing that continuous, piecewise linear, minimal cut-generating functions with only two distinct slope values are extreme. In our second chapter, we establish a similar result for a one-row problem which takes the nonnegativity constraint on the basic variable into account. In our third chapter, we consider a related model where only nonbasic continuous variables are present. Conforti, Cornuejols, Daniilidis, Lemarechal, and Malick recently showed that not all cutting-planes can be obtained from cut-generating functions in this framework. They also conjectured a natural condition under which cut-generating functions might be sufficient. In our third chapter, we prove that this conjecture is true. This justifies the recent research interest in cut-generating functions for this model.

Despite the power of mixed-integer linear programming, many optimization problems of practical and theoretical interest cannot be modeled using a linear objective function and constraints alone. In the next four chapters, we turn to a natural generalization of mixed-integer linear programming which allows nonlinear convex constraints: mixed-integer conic programming. Disjunctive inequalities, introduced by Balas in the context of mixed-integer linear programming in the 1970s, have been a principal ingredient to the practical success of integer programming in the last two decades. In order to extend our understanding of disjunctive inequalities to mixed-integer conic programming, we pursue a principled study of two-term disjunctions on conic sets. In our fourth chapter, we consider two-term disjunctions on a general regular cone. A result of Kilinc-Karzan indicates that conic minimal valid linear inequalities are all that is needed for a closed convex hull description of such sets. First we characterize the structure of conic minimal and tight valid linear inequalities for the disjunction. Then we develop structured nonlinear valid inequalities for the disjunction by grouping subsets of valid linear inequalities. We analyze the

structure of these inequalities and identify conditions which guarantee that a single such inequality characterizes the closed convex hull of the disjunction. In our fifth and sixth chapters, we specialize our earlier results to the cases where the regular cone under consideration is a direct product of second order cones and nonnegative rays and where it is the positive semidefinite cone. These cases deserve attention because of their importance for mixed-integer second-order cone and mixed-integer semidefinite programming. We identify conditions under which our valid convex inequalities can be expressed in computationally tractable forms and present techniques to generate low-complexity relaxations when these conditions are not satisfied. In our final chapter, we provide closed convex hull descriptions for homogeneous two-term disjunctions on the second-order cone and general two-term disjunctions on affine cross-sections of the second-order cone, extending the results of the fifth chapter in two directions. Our results yield strong convex disjunctive inequalities which can be used as cutting-surfaces in generic mixed-integer conic programming solvers.