Information Frictions and Real Rigidities in Production Networks*

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Abstract

This paper studies a production network model with information frictions and real rigidities, where (i) firms may be restricted in how effectively they can adjust (some or all of) their intermediate input quantities in response to changes in the economic environment and (ii) they need to choose their quantities under incomplete information about the realizations of shocks. Our characterization results show that these two frictions lead to a reduction in aggregate output, as firms may find it optimal to rely more heavily on less volatile supply chains, even if it comes at the cost of forgoing more efficient ones. We also find that the interaction between information frictions and real rigidities dampens the impact of productivity and aggregate demand shocks on aggregate output, while increasing the inflationary effects of positive shocks to nominal aggregate demand. The magnitudes of these effects depend on the distribution of the two frictions over the production network.

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1 Introduction

The production of goods and services in any modern economy is organized around complex, interlocking supply chains, as firms rely on a variety of different inputs for production. Due to the key role of intermediate goods in the production process, disruptions to the orderly flow of goods and services have been increasingly recognized by policy makers as a source of aggregate risk. Such concerns were the motivation behind the 2012 U.S. National Strategy for Global Supply Chain Security, which was based on the premise that "[i]ntegrated supply chains are fast and cost-efficient but also susceptible to shocks that can rapidly escalate from localized events into broader disruptions". The more recent experience during the Covid-19 pandemic illustrates how bottlenecks in the extensive network of production can have both significant micro and macroeconomic consequences.

While an extensive (and growing) body of work in macroeconomics studies how the economy's production network can serve as a mechanism for propagation and amplification of shocks,[1] this literature, for the most part, abstracts from two realistic frictions. First, the benchmark models of production networks assume that firms can adjust their input and output quantities frictionlessly in response to shocks. This is despite the fact that, in reality, many production processes, especially in manufacturing, require advance planning and setting up production lines that cannot be ramped up or down instantaneously. Similarly, while firms may be able to produce using different mixes of inputs, switching from one mix to another may require reorganizing the production process, a potentially costly and time-consuming endeavor.[2] Of course, such frictions may not be of first-order importance if firms can perfectly anticipate future shocks and can set up contingency plans accordingly. This brings us to the second friction missing from the benchmark models, which assume that firms make their production decisions under perfect information about all the shocks in the economy. But, this is not necessarily the case either, as it would require the firms to obtain and process a large volume of information about a wide range of shocks.

In this paper, we develop a model that incorporates these two frictions into an otherwise standard model of production networks. We consider an input-output model in which firms are subject to "information frictions" and "real rigidities," in the sense that (i) they may be restricted in how effectively they can adjust their input quantities in response to changes in the economic environment, and (ii) they may have to choose those quantities with only incomplete information about the realizations of shocks.

Specifically, we consider a multisector general equilibrium economy à la Long and Plosser (1983) and Acemoglu et al. (2012) in which firms are linked to one another via input-output linkages and are subject to industry-level productivity and aggregate demand shocks. Firms in each industry use Cobb-Douglas production technologies with constant returns to transform labor and intermediate

[1] Some recent examples include production network models with fairly general production functions (Baqaee and Farhi, 2019), endogenous technologies (Acemoglu and Azar, 2020), nominal rigidities (La'O and Tahbaz-Salehi, 2022; Rubbo, 2020), and extensive margin adjustments (Baqaee and Farhi, 2021; Acemoglu and Tahbaz-Salehi, 2020).

[2] Yet other reasons include capacity constraints and contracts with suppliers and customers, all of which may reduce the firm's short-term ability to adjust its production process in response to shocks.

inputs into output. Additionally, as in Baqaee and Farhi (2022), we allow for downward nominal wage rigidities. However, in a departure from the rest of the literature, we assume that firms make (some or all of) their intermediate input decisions in the presence of incomplete information about the realization of supply and demand shocks. This modeling approach has two key implications. First, it ensures that by the time the shocks are realized (or observed), certain input decisions made by the firms are sunk and hence cannot be adjusted. Second, while firms in our model are subject to quantity adjustment frictions, they nonetheless optimally plan their production in anticipation of future shocks (subject to their information sets).

As our main theoretical result, we provide a system of equations that (implicitly) characterizes equilibrium prices and quantities in terms of model primitives, namely, the realized supply and demand shocks, the economy's production network structure, each industry's set of rigid and flexible inputs, and the information sets of all firms in the economy. Despite being implicit, this characterization result captures the key economic forces that are active in the model. Specifically, it shows that, when deciding on their quantities under incomplete information, firms need to make forecasts about prices charged by their suppliers as well as the quantity demanded by their customers. As such, our characterization result highlights that equilibrium prices and quantities depend on firms' expectations of both upstream and downstream shocks.

We then apply our implicit characterization result to three specific environments that lend themselves to closed-form solutions. As our main case study, we consider an economy consisting of only a single rigid industry subject to information frictions, with the remaining industries capable of adjusting their quantities with no frictions. Focusing on such an environment allows us to identify the role played by real rigidities at each industry separately, and in particular, identify the industries that can act as production bottlenecks for the rest of the economy. We find that information frictions result in a reduction in aggregate output. This is because when firms make their intermediate input decisions under uncertainty about the realizations of (supply or demand) shocks, they find it optimal to rely more heavily on less volatile suppliers, even if this comes at the cost of forgoing more efficient ones. Additionally, our result indicates that, all else equal, a rigid industry functions as a tighter "production bottleneck" for the entire economy if it is simultaneously (i) an overall large supplier in the economy and (ii) an important direct or indirect customer of other firms in the economy.

We then apply our results to study how incomplete information and the resulting frictions in quantity adjustments change the mapping from supply and demand shocks to aggregate output and inflation. We find that, in the presence of real rigidities and information frictions, the first-order impact of productivity shocks is dampened compared to the fully flexible benchmark; that this dampening effect is stronger the higher the degree of real rigidities; and that the extent to which shocks to an industry propagate to aggregate outcomes depends on the size of the rigid industry and its exposure to the shock. As for shocks to aggregate demand, we find that, in contrast to a simple benchmark without information frictions, the real effect of aggregate demand shocks is dampened, a positive demand shock is inflationary, and that the magnitudes of both effects depend on the exact position of the rigid industry in the production network.

We follow up these results by focusing on two other information structures: one in which all firms in the economy observe the same public signal, and a more general case with an arbitrary information structure (albeit for a simplified production network structure). Focusing on these environments, we show how the effect of real rigidities and information frictions are amplified over the production network.

We then conclude the paper with a simple quantitative assessment of our model's implications.

Related Literature. As mentioned earlier, our paper belongs to the literature on production networks, which explores the implications of the disaggregated structure of the economy for aggregate, macroeconomic outcomes. In addition to the papers mentioned earlier, some of the more recent works in this literature include Bigio and La'O (2020), Liu and Tsyvinski (2021), and Baqaee and Farhi (2022). See Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2022) for recent surveys.[3] We contribute to this literature by relaxing two of the key standing assumptions in most production network models: that firms can make decisions under complete information and can frictionlessly adjust their intermediate input decisions in response to changes in the economic environment. We study how incomplete information together with frictions in quantity adjustments change the mapping from supply and demand shocks to aggregate output and inflation.

Our approach in using incomplete information in modeling frictions builds on earlier works, such as Mankiw and Reis (2002), Woodford (2003), and Mackowiak and Wiederholt (2009), among others. For the most part, this literature relies on incomplete information as a source of *nominal* rigidities, with the assumption that firms set their nominal prices without complete information about the economy's fundamentals. In contrast to the bulk of this literature, firms in our framework are subject to *real* rigidities and choose their input quantities in the presence of incomplete information. As such, our paper is more closely related to Angeletos, Iovino, and La'O (2016) and Angeletos and La'O (2020), who consider models in which firms' incomplete information about the shocks is a source of both real and nominal rigidities. Our point of departure from these two papers is our focus on how real rigidities arising from firms' incomplete information interact with the economy's production network structure.

More closely related to our work are two recent papers that also explore the role of incomplete information in the context of supply chains. Kopytov, Nimark, Mishra, and Taschereau-Dumouchel (2022) develop a model of endogenous network formation to investigate how uncertainty about the productivity of suppliers impacts firms' choice of technology. As in our paper, one of the key tradeoffs faced by firms in their framework is that supply chain uncertainty induces firms to rely more heavily on less volatile inputs, even if this comes at the cost of forgoing more efficient one.[4] They key distinction, however, is in the two paper's modeling approach: Kopytov et al. (2022) assume that firms choose their production technology under incomplete information, but can flexibly adjust their quantity demand in response to shocks. As such, and given the assumption of constant returns, firms in their framework only need to form forecasts about their marginal costs. In contrast, firms

[3]See Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019), and Carvalho et al. (2021) for some empirical studies of the role of production networks in propagation and amplification of shocks.

[4]Also see Grossman, Helpman, and Lhuillier (2022) for a related mechanism in the context of global supply chains in the presence of relationship-specific risk and country-wide supply disturbances.

in our framework are forced to make (some or all of) their quantity decisions prior to observing the shocks. As a result, they not only need to form forecasts about their upstream prices, but also about their downstream demand.

The second related paper is the recent work of Bui, Huo, Levchenko, and Pandalai-Nayar (2022), who introduce information frictions into a multi-country, multi-sector model with global value chains. As in our paper, firms do not observe the shocks and only have access to imperfect signals about productivities of various industries and countries. The two models, however, have an important difference. Whereas Bui et al. (2022) assume that firms choose their primary factor inputs (e.g., labor) under incomplete information, we assume that firms need to choose their intermediate inputs before learning the realizations of the shocks. This distinction is consequential: as Bui et al. (2022) illustrate, when firms face no frictions in choosing their intermediate inputs, the equilibrium only depends on iteration of firms' cross-sectional averages expectations of their suppliers' decisions. In contrast, in our model, the equilibrium depends not only on firms' forecasts of their suppliers' forecasts, but also on their forecasts of their customers' forecasts.

Finally, our paper is related to the growing body of works that studies network interactions in the presence of incomplete information. Examples include Calvó-Armengol, de Martí, and Prat (2015), de Martí and Zenou (2015), Bergemann, Heumann, and Morris (2017), and Golub and Morris (2018). We complement this literature, which is mostly focused on reduced-form games over networks, by focusing on a micro-founded, general equilibrium macro model where firms' decisions and outcomes are interlinked with one another as a result of the economy's disaggregated production network structure. We find that since firms need to make forecasts about both their upstream and downstream supply chain partners, the network interactions take a more complex form than what is usually assumed in the more reduced-form models.

Outline. The rest of the paper is organized as follows. Section 2 sets up the environment and defines the equilibrium concept. Section 3 contains the main characterization result of the paper, where we show how information frictions and real rigidities determine the propagation of supply and demand shocks. We present a quantitative analysis of the model in Section 4. All proofs and some additional technical details are presented in the Appendix.

2 Model

In this section, we present a multisector model that forms the basis of our analysis. The model, which is in the spirit of general equilibrium models of Long and Plosser (1983) and Acemoglu et al. (2012), closely follows the framework in La'O and Tahbaz-Salehi (2022). As our main point of departure from the prior literature, we assume that firms may have to make some of their intermediate input quantity decisions under incomplete information about the realizations of the shocks.

2.1 Firms and Production

Consider an economy consisting of n industries indexed by $i \in \mathcal{N} = \{1, 2, \ldots, n\}$. Each industry consists of two types of firms: (i) a unit mass of monopolistically-competitive firms, indexed by $k \in [0, 1]$, producing differentiated goods and (ii) a competitive producer whose sole purpose is to aggregate the industry's differentiated goods into a single sectoral output. The output of each industry can be either consumed by the households or used as an intermediate input for production by firms in other industries.

The monopolistically-competitive firms within each industry use a common constant-returns-to-scale technology to transform labor and intermediate inputs into their differentiated products. More specifically, the production function of firm $k \in [0, 1]$ in industry i is given by

$$y_{ik} = z_i \zeta_i l_{ik}^{\alpha_i} \prod_{j=1}^{n} x_{ij,k}^{a_{ij}}, \tag{1}$$

where y_{ik} is the firm's output, l_{ik} is the firm's labor input, $x_{ij,k}$ is the quantity of sectoral commodity j purchased by the firm, and z_i is an industry-specific productivity shock. The constant $\alpha_i > 0$ denotes the share of labor in industry i's production technology, $a_{ij} \geq 0$ parameterizes the importance of good j in the production technology of firms in industry i, and $\zeta_i = \alpha_i^{-\alpha_i} \prod_{j=1}^{n} a_{ij}^{-a_{ij}}$ is a normalization constant. As is standard in this literature, we summarize input-output linkages in this economy by matrix $\mathbf{A} = [a_{ij}]$, which with some abuse of terminology, we refer to as the economy's *input-output matrix*. We also define the economy's *Leontief inverse* as $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$, whose (i, j) element captures the role of industry j as a direct or indirect intermediate input supplier to industry i. Throughout the paper, we normalize the steady-state value of all (log) productivity shocks to 0, i.e., $\log z_i^{\text{ss}} = 0$ for all i.

Given the production technology (1), the nominal profits of firm k in industry i are given by

$$\pi_{ik} = (1 - \tau_i) p_{ik} y_{ik} - w l_{ik} - \sum_{j=1}^{n} p_j x_{ij,k}, \tag{2}$$

where p_{ik} is the nominal price charged by the firm, p_j is the nominal price of industry j's sectoral output, w denotes the nominal wage, and τ_i is an industry-specific revenue tax or subsidy levied by the government.

As already mentioned, each industry also contains a competitive producer, which transforms the differentiated products produced by the unit mass of firms in that industry into a sectoral good using a constant-elasticity-of-substitution (CES) production technology, with elasticity of substitution $\theta_i > 1$:

$$y_i = \left(\int_0^1 y_{ik}^{(\theta_i - 1)/\theta_i} \mathrm{d}k \right)^{\theta_i/(\theta_i - 1)}.$$

The sole purpose of this producer is to ensure that each industry produces a single sectoral good, while at the same time allowing for monopolistic competition among firms within the same industry. Throughout the paper, we assume that the industry-specific tax in (2) is set to $\tau_i = 1/(1 - \theta_i)$. As is well-known, this choice undoes the effect of monopolistic markups and ensures that the distortions in the economy are not due to firms' market power.

2.2 Households

In addition to the firms, the economy consists of a representative household, with preferences

$$U(C, L) = \log C - \chi L$$

where C and L denote the household's final consumption basket and total labor supply, respectively, and $\chi > 0$ is a constant that parameterizes the representative household's disutility of labor supply. The representative household's final consumption basket is a Cobb-Douglas aggregator of the sectoral goods produced in the economy,

$$C = \prod_{i=1}^{n} (c_i/\beta_i)^{\beta_i},$$

where c_i is the amount of good i consumed and $(\beta_1, \ldots, \beta_n)$ are nonnegative constants that measure various goods' shares in the household's consumption basket, normalized such that $\sum_{i=1}^{n} \beta_i = 1$. The representative household's budget constraint is therefore given by

$$PC = \sum_{i=1}^{n} p_i c_i = w \sum_{i=1}^{n} \int_0^1 l_{ik} \mathrm{d}k + \sum_{i=1}^{n} \int_0^1 \pi_{ik} \mathrm{d}k + T,$$

where $P = \prod_{i=1}^{n} p_i^{\beta_i}$ is the nominal price of the household's consumption bundle, w denotes the nominal wage, π_{ik} is given by (2), and T denotes lump-sum transfers from the government. To ensure that the government's budget constraint is satisfied, we assume that $T = \sum_{i=1}^{n} \tau_i \int_0^1 p_{ik} y_{ik} \mathrm{d}k$.

Finally, we assume that the representative household is subject to the following cash-in-advance constraint: $PC = m$, where m denotes the nominal aggregate demand in the economy. In what follows, we interpret a decrease in m as a negative aggregate demand shock. While the most natural source of such a shock is a monetary policy shock (say, due to a reduction in money supply), as shown by Baqaee and Farhi (2022) in a simple dynamic extension of the model, a decrease in expected future output, an increase in nominal interest rate, an increase in the household's discount factor, or a decrease in future prices—which can be thought of as a proxy for forward guidance—can all generate effects that are isomorphic to a decrease in m.

2.3 Real Rigidities and Information Frictions

While the benchmark models of production networks assume that firms can adjust their input and output quantities in response to supply and demand shocks, in reality, many firms may have limited ability to do so, at least in the short run. For example, many production processes, especially in manufacturing, require advance planning and setting up production lines that cannot be ramped up or down instantaneously. Similarly, while firms may be able to produce using different mixes of inputs, switching from one mix to another may require reorganizing the production process, a potentially costly and time-consuming endeavor. Additionally, firms may not even have access to all the relevant information that would be necessary for adopting their production plans in response to economic disturbances.

We model the presence of such "real rigidities" by following Angeletos, Iovino, and La'O (2016) and Angeletos and La'O (2020) and assuming that firms make (some or all of) their intermediate input decisions under incomplete information about supply and demand shocks. Specifically, we assume that the economy lasts for two periods, $t \in \{0,1\}$. At $t=0$, firms in industry i receive a common signal $\omega_i \in \Omega_i$ about the realizations of the supply and demand shocks (z,m), where $z=(z_1,\dots,z_n)$. Given ω_i, each firm k in industry i chooses the intermediate input quantities $x_{ij,k}$ for any input $j \in \mathcal{R}_i$ at $t=0$, where $\mathcal{R}_i \subseteq \mathcal{N}$ denotes the set of *rigid* inputs of industry i. The productivity and demand shocks are then observed by all firms at $t=1$, which is when firms set prices, choose their labor input l_{ik}, and choose the remainder of their intermediate inputs, $\{x_{ij,k}\}_{j\in\mathcal{F}_i}$, where $\mathcal{F}_i=\mathcal{N}\setminus\mathcal{R}_i$ denotes the set of *flexible* inputs of industry i. Production and consumption also take place at $t=1$.

A few remarks are in order. First, note that since firms choose their rigid intermediate inputs before the realization of shocks, these input choices are subject to a measurability constraint: $\{x_{ij,k}\}_{j\in\mathcal{R}_i}$ can be contingent on ω_i, but not on (z,m). While related, this measurability constraint on quantities is distinct from measurability constraints on nominal prices, which are a source of *nominal* rigidities as opposed to real ones.[5] Second, it is immediate to see that if either (i) all firms observe perfectly informative signals about the realizations of the shocks, i.e., $\omega_i=(z,m)$ for all i; or (ii) all inputs of all firms are flexible, i.e., $\mathcal{F}_i=\mathcal{N}$ for all i, then the above framework reduces to a standard production network model, such as Acemoglu et al. (2012). Third, note that firms face no frictions in adjusting their labor input, as we assume they choose l_{ik} at $t=1$. This is to ensure that at least one input is free to adjust in response to realized demand, as otherwise markets may fail to clear.[6]

We conclude this discussion by introducing a measure for firms' uncertainty about the shocks' realizations. For any given pair of industries i and j, define

$$\kappa_{ij}=\frac{\mathbb{E}[\mathrm{var}_i(\log z_j)]}{\mathrm{var}(\log z_j)}, \tag{3}$$

where $\log z_j$ is the log productivity shock to industry j, $\mathrm{var}_i(\cdot)$ denotes the variance conditional on the information set of firms in industry i, and $\mathbb{E}[\cdot]$ and $\mathrm{var}(\cdot)$ denote the unconditional expectation and variance operators, respectively. The interpretation of κ_{ij} as a measure of uncertainty is fairly natural: it captures the (ex ante) volatility of $\log z_j$ conditional on i's information set as a fraction of its unconditional volatility. By the law of total variance, κ_{ij} is always in the unit interval $[0,1]$ and obtains it maximum value of 1 if firms in industry i receive no informative signals about the realization of $\log z_j$ (in which case, $\mathrm{var}(\mathbb{E}_i[\log z_j])=0$). At the other end of the spectrum, $\kappa_{ij}=0$ if

[5] See Mankiw and Reis (2002), Mackowiak and Wiederholt (2009), and La'O and Tahbaz-Salehi (2022) for examples of models with information frictions as a source of nominal rigidities.

[6] This is also the key modeling distinction between our framework and that of Bui, Huo, Levchenko, and Pandalai-Nayar (2022), who assume that labor input decisions are made under incomplete information, while all intermediate input quantities can adjust freely in response to shocks.

firms in industry i face no uncertainty about the shock to industry j (i.e., $\text{var}_i(\log z_j) = 0$).[7] We can define a similar object to measure firms' uncertainty about the realization of the demand shock:

$$\kappa_{im} = \frac{\mathbb{E}[\text{var}_i(\log m)]}{\text{var}(\log m)}, \tag{4}$$

where m is the nominal aggregate demand.

2.4 Downward Nominal Wage Rigidities

While firms can set their prices p_{ik} flexibly at $t = 1$ after observing productivity and demand shocks, we allow for downward nominal wage rigidities by assuming that the nominal wage w cannot fall below an exogenously-specified value $\bar{w}$. This restriction on nominal prices means that there are two possibilities. One possibility is that $w > \bar{w}$ and the labor market clears. The other possibility is that the constraint on the nominal wage binds (so that $w = \bar{w}$), in which case the labor market is slack and does not clear, in the sense that the total demand for labor falls short of what the representative household is willing to supply at that wage. Taken together, the two possibilities imply that the labor market is in equilibrium if

$$(w - \bar{w})\left(L - \sum_{i=1}^{n}\int_0^1 l_{ik}\mathrm{d}k\right) = 0, \qquad w \geq \bar{w}, \qquad L \geq \sum_{i=1}^{n}\int_0^1 l_{ik}\mathrm{d}k, \tag{5}$$

where L denotes the household's labor supply and l_{ik} is the labor demand of firm k in industry i. Clearly, the special case that $\bar{w} = 0$ corresponds to an economy with no nominal rigidities.

2.5 Equilibrium

With the various model ingredients in hand, we are now ready to define our solution concept.

Definition 1. An *equilibrium* is a collection of nominal prices, nominal wage, and quantities such that

(i) at $t = 0$, monopolistically-competitive firms in each industry choose their rigid intermediate input quantities to maximize expected real value of their profits given their information;

(ii) at $t = 1$, firms set their nominal prices and choose their labor and flexible intermediate inputs to maximize profits, taking the realized demand and their rigid input quantities as given;

(iii) the competitive producer in each industry chooses inputs to maximize its profits given prices;

(iv) the representative household chooses consumption and labor supply to maximize utility subject to its budget constraint;

(v) the labor market is in equilibrium, i.e., condition (5) is satisfied;

[7] In the special case that all shocks and signals are jointly normally distributed, κ_{ij} takes a familiar form in terms of the signal-to-noise ratio. In particular, suppose firm i observes a single signal given by $\omega_i = \log z_j + \epsilon_i$, where $\log z_j$ and ϵ_i are independent and normally distributed with variances σ_z^2 and σ_ϵ^2, respectively. In that case, $\kappa_{ij} = \sigma_\epsilon^2/(\sigma_z^2 + \sigma_\epsilon^2)$. The expression in (3) generalizes this concept to any arbitrary joint distribution of shocks and signals.

(vi) all sectoral good markets clear, i.e.,

$$y_i = c_i + \sum_{j=1}^{n} \int_0^1 x_{ji,k} \mathrm{d}k. \tag{6}$$

Equilibrium conditions (ii)–(vi) are all standard—capturing firm and household optimizing behavior and consistency restrictions on quantities—with the measurability constraints on the rigid quantities captured by condition (i). Note that while firms in our model are subject to quantity adjustment frictions, they nonetheless optimally plan their production process in anticipation of future shocks subject to their information sets.

3 Equilibrium Characterization

In this section, we provide a characterization of the equilibrium in terms of model primitives, including the set of rigid and flexible intermediate inputs and the information sets of firms in each industry. To present this result, we let $\lambda_i = p_i y_i / PC$ denote industry i's *Domar weight*, defined as its sales as a fraction of GDP. We also use $\mathbb{E}_i[\cdot]$ to denote the expectation operator with respect to the information set of firms in industry i. We have the following result:

Proposition 1. *Equilibrium nominal prices and Domar weights solve the system of equations:*

$$p_i = \frac{1}{z_i} w^{\alpha_i} \prod_{j \in \mathcal{F}_i} p_j^{a_{ij}} \prod_{j \in \mathcal{R}_i} \left(m \frac{\mathbb{E}_i[p_j/m]}{\mathbb{E}_i[\lambda_i]/\lambda_i} \right)^{a_{ij}} \tag{7}$$

and

$$\lambda_i = \beta_i + \sum_{j: i \in \mathcal{F}_j} a_{ji} \lambda_j + \sum_{j: i \in \mathcal{R}_j} a_{ji} \mathbb{E}_j[\lambda_j] \frac{p_i/m}{\mathbb{E}_j[p_i/m]} \tag{8}$$

for all $i \in \mathcal{N}$, *where* m *is the nominal aggregate demand and* w *is the nominal wage.*

Proposition 1 provides a system of $2n$ equations and $2n$ unknowns that expresses sectoral Domar weights $(\lambda_1, \dots, \lambda_n)$ and nominal prices $(p_1, \dots, p_n)$ in terms of the firms' information sets, the realized productivity shocks, the nominal wage, and nominal aggregate demand, m.

Focusing on equation (7), it is easy to verify that if firms in industry i are not subject to real rigidities—either because all their inputs are flexible or because they receive completely informative signals—then (7) reduces to $p_i = \frac{1}{z_i} w^{\alpha_i} \prod_{j=1}^{n} p_j^{a_{ij}}$. In other words, the nominal price of industry i is simply equal to its nominal marginal cost, as anticipated. More generally, however, as equation (7) indicates, the nominal price of industry i depends on industry i's expectation of the prices of its rigid intermediate inputs relative to nominal aggregate demand $\mathbb{E}_i[p_j/m]$, as well as its expectation of its own equilibrium Domar weight, $\mathbb{E}_i[\lambda_i]$. To see the intuition for these dependencies, note that either an increase in $\mathbb{E}_i[p_j/m]$ (which is the expected relative price of input j) or a decrease in $\mathbb{E}_i[\lambda_i]$ (as a proxy for demand for i's output) result in a reduction in the quantity x_{ij} of good j that firms in industry i demand at $t = 0$. Given that this quantity is sunk by the time firms set their prices at $t = 1$, a lower x_{ij} is akin to a lower productivity from the point of view of firms at $t = 1$, thus inducing firms in industry i to set a higher nominal price.

The intuition underlying equation (8) is similar. Recall that the Domar weight of industry i—which represents that industry's size in equilibrium—would be larger when it faces higher demand from its downstream customers (larger x_{ji}'s). As already discussed, the demand from a customer j that is subject to real rigidities is increasing in $\mathbb{E}_j[\lambda_j]$ and is decreasing in $\mathbb{E}_j[p_i/m]$. Therefore, as is evident from (8), λ_i increases in its customers' expectations of their size and decreases in their expectations of i's price. Finally, note that, if none of i's customers are subject to real rigidities, then equation (8) implies that $\lambda_i = \beta_i + \sum_{j=1}^{n} a_{ji}\lambda_j$, as would be the case in the benchmark models of production networks in the absence of information frictions (Carvalho and Tahbaz-Salehi, 2019; Baqaee and Rubbo, 2022).

As already mentioned, Proposition 1 characterizes equilibrium nominal prices and Domar weights in terms of the nominal wage and nominal aggregate demand. Given prices and Domar weights, one can then characterize the entire allocation in terms of m and w. In particular, from the definition of λ_i, it follows immediately that the output of industry i is given by $y_i = \lambda_i m/p_i$. Also, as we show in the proof of Proposition 1, industry i's demand for its flexible and rigid intermediate inputs are given by

$$x_{ij,k} = \begin{cases} a_{ij}m\lambda_i/p_j & \text{if } j \in \mathcal{F}_i \\ a_{ij}\mathbb{E}_i[\lambda_i]/\mathbb{E}_i[p_j/m] & \text{if } j \in \mathcal{R}_i, \end{cases}$$

respectively. Finally, the household's first-order conditions imply that $c_i = \beta_i m/p_i$.

It is important, however, to note that while nominal aggregate demand m is a model primitive (and a proxy for demand shocks), the nominal wage w is an endogenous object that is determined in equilibrium. The following simple lemma expresses the nominal wage in terms of model primitives, namely, nominal aggregate demand and the minimum nominal wage:

Lemma 1. *The nominal wage is given by* $w = \max\{\chi m, \bar{w}\}$.

Taken together, Proposition 1 and Lemma 1 provide a complete (albeit implicit) characterization of equilibrium in the presence of information frictions and real rigidities. Unfortunately, in general—and unless one imposes some discipline on the economy's information structure—equations (7) and (8) do not lend themselves to closed-form solutions. This is due to two sources of complexity in the model. First, as in Golub and Morris (2018), La'O and Tahbaz-Salehi (2022), and Bui et al. (2022), the presence of network interactions in the economy means that the equilibrium depends not only on the firms' first-order expectations, but also on their expectations of higher order. This is because firms need to forecast their Domar weights and input prices, which depend not only on the firms' own forecasts of the realized shocks, but also of their forecasts of other firms' forecasts, and so on. Second, and in contrast to the prior literature, the relevant higher-order expectations do not have a simple iterative representation in terms of cross-sectional (weighted) averages of firms' first-order expectations. To see this, note that, according to (7), the nominal price set by industry i depends on i's expectation of its input prices—an object that depends on the actions of its *upstream* suppliers—as well as on i's expectation of its own Domar weight—an object that is determined by the demand from its *downstream* customers. This means that the equilibrium depends on iterations of expectations both upstream and downstream over the network, significantly complicating how information frictions interact with the production network structure.[8]

In order to explore the implications of Proposition 1 in a transparent manner, in the remainder of this section, we study various special cases of the general setting in Section 2 by focusing on particular information structures.

3.1 Single Rigid Industry

As a first special case of the general setting in Section 2, we assume that the economy consists of a single rigid industry. Specifically, we assume that firms in industry r are the only firms in the economy with incomplete information about the realizations of productivity shocks $(z_1, \ldots, z_n)$ and the aggregate demand shock, m. Firms in all other industries are not subject to real rigidities and make all their decisions at $t = 1$, that is, $\mathcal{R}_i = \varnothing$ for all $i \neq r$. Focusing on such a case allows us to identify the role played by real rigidities and information frictions at each industry separately.

We also impose the following assumption on the economy's production network structure:

Assumption 1. $\ell_{ir}\ell_{ri} = 0$ for all industries $i \neq r$.

To interpret the above assumption, recall that the (i, j) element of the economy's Leontief inverse, ℓ_{ij}, captures the extent to which industry i relies on industry j as a (direct or indirect) input supplier. Therefore, under Assumption 1, there is no industry i in the economy that is simultaneously upstream and downstream to r. We impose this assumption to tease out the role of upstream and downstream relationships vis-à-vis industry r in the most transparent manner.

Finally, to investigate the impact of supply shocks separately from that of demand shocks, we first assume that $m \geq \bar{w}/\chi$, which in view of Lemma 1, guarantees that the downward nominal wage rigidity constraint does not bind, making shocks to nominal aggregate demand neutral for real outcomes. We have the following result:

Proposition 2. *Suppose r is the only rigid industry, Assumption 1 is satisfied, and $m \geq \bar{w}/\chi$. Then,*

$$\log C = \log C^* - \lambda_r^{\text{ss}} \sum_{j\in\mathcal{R}_r} \sum_{i=1}^{n} a_{rj}\ell_{ji}(\log z_i - \mathbb{E}_r[\log z_i]) - \frac{1}{2}\lambda_r^{\text{ss}} \sum_{j\in\mathcal{R}_r} a_{rj}\text{var}_r\left(\sum_{i=1}^{n} \ell_{ji}\log z_i\right) \tag{9}$$

to a second-order approximation, where $\log C^* = \sum_{i=1}^{n} \lambda_i^{\text{ss}} \log z_i - \log\chi$ is log output in the absence of information frictions and λ_i^{ss} is the steady-state Domar weight of industry i.

Proposition 2 characterizes the impact of productivity shocks on aggregate output when firms in industry r face information frictions in setting up their production processes. It is immediate to see that the last two terms on the right-hand side of (9) vanish when firms in industry r have complete information about the realizations of productivity shocks—and hence make all their price

[8] See the economy in Subsection 3.3 for a more detailed discussion.

and quantity decisions with full anticipation of all shocks. In such a case, the expression in (9) reduces to the standard result in the literature, according to which the impact of a productivity shock to industry i on aggregate output is equal to its pre-shock Domar weight, λ_i^{ss}. However, equation (9) also shows that such a simple relationship no longer holds if firms in industry r face uncertainty about the productivity of their upstream supply chains. Specifically, the result in Proposition 2 leads to the following observations.

First, it is easy to see that an increase in the rigidity of industry r—as proxied by higher uncertainty faced by firms in that industry—translates into lower aggregate output (both in expectation and ex post). This is, of course, intuitive: the fact that firms in industry r make (some or all of) their intermediate input decisions under incomplete information about productivity shocks induces them to rely more heavily on less volatile suppliers, even if this comes at the cost of forgoing more efficient ones.

Second, the last term on the right-hand side of (9) indicates that the negative impact of information frictions on aggregate output is increasing in industry r's Domar weight, as well as in r's uncertainty about the supply chains of each of its rigid intermediate input, $\text{var}_r\left(\sum_{i=1}^{n} \ell_{ji} \log z_i\right)$. Not surprisingly, this uncertainty is weighted by the importance of the rigid supplier industry j in r's production technology (as captured via the expenditure share a_{rj}). This means that, all else equal, a rigid industry r functions as a tighter "production bottleneck" for the entire economy if it is simultaneously (i) an overall large supplier in the economy (as captured by the larger Domar weight, λ_r^{ss}) and (ii) an important direct or indirect customer of other firms in the economy (proxied by greater a_{rj} and ℓ_{ji}).

Third, one can use Proposition 2 to obtain an expression for how microeconomic shocks translate into macroeconomic outcomes. Under the assumption that industry-level productivity shocks are independent, equation (9) implies that the slope coefficient of regression

$$\log C = \gamma_0^z + \gamma_i^z \log z_i + \varepsilon_i \tag{10}$$

—which captures the (average) first-order impact of shocks to industry i on aggregate output—is given by[9]

$$\gamma_i^z = \lambda_i^{\text{ss}} - \left(\lambda_r^{\text{ss}} \sum_{j \in \mathcal{R}_r} a_{rj} \ell_{ji}\right) \kappa_{ri}, \tag{11}$$

where $\kappa_{ri} = \mathbb{E}[\text{var}_r(\log z_i)] / \text{var}(\log z_i)$ parameterizes the uncertainty of firms industry r about shocks to industry i (as defined in (3)). Equation (11) establishes that, in the presence of real rigidities induced by information frictions, (i) the first-order impact of productivity shocks is dampened compared to the fully flexible benchmark (under which the first-order impact of a shock to industry i is equal to λ_i^{ss}), (ii) this dampening effect is stronger the higher the degree of real rigidities, and (iii) the extent to which shocks to industry i shape aggregate outcomes depends on the size of the rigid industry r and the extent to which r is exposed (directly or directly) to those shocks (as captured by $\sum_{j \in \mathcal{R}_r} a_{rj} \ell_{ji}$).

[9] γ_i^z serves as the counterpart to $\mathrm{d}\log C / \mathrm{d}\log z_i$ in our incomplete-information economy.

We next turn to the implications of demand shocks for output and inflation. To ensure that such shocks have a non-trivial impact on real variables, we focus on the case in which the downward nominal wage rigidity constraint bind. We have the following counterpart to Proposition 2.

Proposition 3. *Suppose r is the only rigid industry and Assumption 1 is satisfied. If $m < \bar{w}/\chi$, and in the absence of productivity shocks,*

$$\log C = \log m - \log \bar{w} - \left(\log m - \mathbb{E}_r[\log m] + \frac{1}{2}\mathrm{var}_r(\log m)\right)\lambda_r^{\mathrm{ss}} \sum_{j\in\mathcal{R}_r} a_{rj} \tag{12}$$

$$\log P = \log \bar{w} + \left(\log m - \mathbb{E}_r[\log m] + \frac{1}{2}\mathrm{var}_r(\log m)\right)\lambda_r^{\mathrm{ss}} \sum_{j\in\mathcal{R}_r} a_{rj} \tag{13}$$

to a second-order approximation.

This result shows that the real rigidities induced by firms' uncertainty about the realization of demand shocks—as captured via $\mathrm{var}_r(\log m)$—reduce aggregate output, while increasing the price level (which serves as a proxy for inflation in our model).

As with supply shocks, one can also use Proposition 3 to characterize the (average) first-order impact of demand shocks on output and inflation by calculating the slope coefficients of the following regressions:

$$\log C = \gamma_0 + \gamma^m \log m + \varepsilon_m$$
$$\log P = \delta_0 + \delta^m \log m + \varepsilon_m.$$

Using equations (12) and (13), it is easy to verify that

$$\gamma^m = 1 - \delta^m \qquad \text{and} \qquad \delta^m = \left(\lambda_r^{\mathrm{ss}} \sum_{j\in\mathcal{R}_r} a_{rj}\right)\kappa_{rm}, \tag{14}$$

where $\kappa_{rm} = \mathbb{E}[\mathrm{var}_r(\log m)]/\,\mathrm{var}(\log m)$ parameterizes the uncertainty of firms in industry r about the realization of $\log m$, as defined in (4). Given that $\delta_m > 0$ whenever $\kappa_r(\log m) > 0$, it follows immediately that the real rigidities induced by information frictions dampen the real effect of positive aggregate demand shocks, while increasing their inflationary effects. Not surprisingly then, both of these effects are determined by the size of the rigid industry, λ_r^{ss}, and the extent to which it relies on rigid intermediate inputs, $\sum_{j\in\mathcal{R}_r} a_{rj}$.

It is also instructive to compare the coefficients γ^m and δ^m in (14) with the results of Baqaee and Farhi (2022), who find that, in a Cobb-Douglas economy with complete markets and a single factor of production subject to nominal rigidities, aggregate demand shocks translate one-for-one to aggregate output, with no impact on inflation. They also show that, holding sectoral Domar weights constant, the details of the economy's production network structure are irrelevant for how aggregate demand shocks impact aggregate output. As is evident from (14), neither statement is no longer true in the presence of information frictions: the real effect of aggregate demand shocks is dampened, a positive demand shock leads to an increase in the price level, and both effects depend on the production network structure.

3.2 Public Information

By focusing on an economy with a single rigid industry, the results in Subsection 3.1 abstract from the possibility that firms may be subject to multiple rigid suppliers and customers. In this subsection, we apply the general result in Proposition 1 to an economy in which firms in all industries have access to same public information about the shocks (that is, $\omega_i = \omega$ for all industries i) and are subject to real rigidities in all their intermediate inputs (that is, $\mathcal{F}_i = \varnothing$ for all i). Focusing on such an economy allows us to explore how the impact of real rigidities can build up over production chains. We additionally impose the following condition on the production network structure:

Assumption 2. Each industry is either an intermediate good producer or a final good producer.

While not necessary, Assumption 2 simplifies the final expressions significantly. As before, we also investigate the impacts of supply and demand shocks separately.

Starting with supply shocks, and assuming that aggregate nominal demand is large enough so that downward nominal wage rigidity constraint does not bind, we have the following result:

Proposition 4. *Suppose all firms share a common information set. If Assumption 2 is satisfied, and* $m \geq \bar{w}/\chi$, *then*

$$\log C = \log C^* + \sum_{k\in\mathcal{I}} \lambda_k^{\mathrm{ss}}(\mathbb{E}_p[\log z_k] - \log z_k) - \frac{1}{2}\sum_{k\in\mathcal{I}}(\lambda_k^{\mathrm{ss}}/\alpha_k)\mathrm{var}_p(\log z_k) \tag{15}$$

to a second-order approximation, where $\mathcal{I}$ *denotes the set of intermediate-input producing industries and* $\mathbb{E}_p[\cdot]$ *is the expectation operator conditional on the public signal,* ω.

As in Proposition 2, the above result illustrates that an increase in the extent of real rigidities reduces aggregate output. The intuition is also similar: the fact that intermediate input decisions are sunk by the time firms observe the realized productivities means that they shift their demand from more uncertain suppliers to more reliable ones, even if it comes at the cost of lower (expected) productivity. Not surprisingly, the characterization in (15) also shows that uncertainty about shocks to industries that (i) are larger in steady-state (larger λ_k^{ss}) and (ii) rely more on intermediate inputs (smaller α_k) is more important. Furthermore, as the last term on the right-hand side of (15) indicates, depending on the expenditure shares, this effect can be arbitrarily large: if some industry k relies heavily on rigid intermediate inputs (so that α_k is small), then the resulting drop in aggregate output can be substantial.

We can also use (15) to obtain the slope coefficient of the regression in (10):

$$\gamma_i^z = \lambda_i^{\mathrm{ss}}(1 - \kappa_i),$$

where $\kappa_i = \mathbb{E}[\mathrm{var}_p(\log z_i)]/\mathrm{var}(\log z_i)$ parameterizes the uncertainty about the realization of $\log z_i$ given the public signal. Contrasting the above expression with (11) illustrates that, not surprisingly, the aggregate impact of productivity shocks becomes smaller as more industries are subject to information frictions.

Next, turning to demand shocks in this setting, we can establish the following result:

Proposition 5. *Suppose all firms share a common information set. If Assumption 2 is satisfied and $m < \bar{w}/\chi$, then, in the absence of productivity shocks,*

$$\log C = \log m - \log \bar{w} - \left(\log m - \mathbb{E}[\log m] + \frac{1}{2}\operatorname{var}(\log m)\right) \sum_{k \in \mathcal{I}} \lambda_k^{\text{ss}} \alpha_k \tag{16}$$

$$\log P = \log \bar{w} + \left(\log m - \mathbb{E}[\log m] + \frac{1}{2}\operatorname{var}(\log m)\right) \sum_{k \in \mathcal{I}} \lambda_k^{\text{ss}} \alpha_k \tag{17}$$

to a second-order approximation, where $\mathcal{I}$ denotes the set of intermediate-input producing industries.

3.3 Dispersed Information and Higher-Order Expectations

While allowing for incomplete information throughout the economy, the information structure studied in Subsection 3.2 ignores the possibility of dispersed information, as all signals are public and shared among all firms. As such, firms can use these public signals to coordinate not only with their immediate suppliers and customers, but also with potentially distant firms on their supply chains. We therefore conclude this section by studying the implications of heterogeneity in the firms' information sets. In order to keep the analysis tractable, we focus on a simple production network structure.

Consider an economy consisting of three industries organized on a vertical production chain, where industry 3 is the sole input supplier to industry 2 (with expenditure share a_2) and industry 2 is the sole input supplier to industry 1 (with expenditure share a_2). Furthermore, assume that industry 3 only uses labor for production ($\alpha_3 = 1$) and that industry 1 is the only industry that sells to the households ($\beta_1 = 1$). We have the following result:

Proposition 6. *Suppose $m < \bar{w}/\chi$. In the absence of productivity shocks*

$$\log C = \log m - \log \bar{w} - a_1(1-a_2) \sum_{s=0}^{\infty} a_2^s \Big(\log m - (\mathbb{E}_1\mathbb{E}_2)^s \mathbb{E}_1[\log m] \Big) \tag{18}$$

$$\log P = \log \bar{w} + a_1(1-a_2) \sum_{s=0}^{\infty} a_2^s \Big(\log m - (\mathbb{E}_1\mathbb{E}_2)^s \mathbb{E}_1[\log m] \Big) \tag{19}$$

to a first-order approximation, where $(\mathbb{E}_1\mathbb{E}_2)^{s+1}[\cdot] = \mathbb{E}_1\mathbb{E}_2[(\mathbb{E}_1\mathbb{E}_2)^s[\cdot]]$.

Proposition 6 illustrates that, when information is dispersed, the impact of aggregate demand shocks on aggregate output and the price level depends on not just firms' first-order expectations but also on all expectations of higher order. As is well-known, higher-order beliefs adjust more sluggishly than first-order beliefs (Angeletos and Huo, 2021). Therefore, equations (18) and (19) underscore how rigidities build up over the production chain, dampening the real effects of positive monetary shocks, while amplifying their inflationary effects.

That firms' higher-order expectations can matter for aggregate economic outcomes is not, in and of itself, novel, and it is line with prior work such as Golub and Morris (2018), Angeletos and Lian (2016, 2018), and La'O and Tahbaz-Salehi (2022), among others. What distinguishes the expressions in Proposition 6 from the results in prior work is how these higher-order expectations matter for aggregate output and price level. In particular, even though goods flow only in one direction in this economy—from upstream suppliers to their downstream customers—the macroeconomic variables in (18) and (19) depend on the iterated expectations in both directions: the supplier's expectations of the customer's expectation and vice versa. This is a consequence of the fact that, when choosing their input quantities, firms need to form forecasts not only about their suppliers' input prices, but also about demand from their customers.[10] But of course the same logic applies to those customers and suppliers as well, resulting in an infinite regress of expectations between firms in industries 1 and 2.

4 Quantitative Analysis

In this section, we use our theoretical results from the previous section to quantify the effect of information frictions and real rigidities on aggregate output in a calibrated version of the model.

We calibrate the model by relying on two sources of data. The first dataset we rely on is the 2021 Input-Output tables, constructed by the Bureau of Economic Analysis (BEA). These tables provide intermediate input expenditure by various industries, as well as each industry's contribution to final uses. The second dataset is the March 2022 release of the BEA/BLS Integrated Industry-level Production Account (ILPA). This dataset contains total factor productivity estimates for each industry over the 1987–2019 period.

We merge the two datasets at the summary level, which includes 71 industries corresponding to 3-digit North American Industry Classification System (NAICS). We exclude Federal, state, and local government industries, thus ending up with a total of 66 industries.

Calibration. We calibrate our two-period model at a quarterly frequency. This amounts to assuming that firms are unable to adjust their rigid intermediate inputs within a quarter after the realizations of the shocks.

To calibrate the model, we first combine the The Make of Commodities by Industries Table (the Make Table) with the The Use of Commodities by Industries Table (the Use Table) to calculate the expenditure share of each industry on any other industry. We treat the resulting matrix as the counterpart to the input-output matrix $\mathbf{A} = [a_{ij}]$ in Section 2. We calibrate the labor expenditure shares α_i to match industry i's expenditure as a compensation of employees. The vector of consumption shares $\beta = (\beta_1, \dots, \beta_n)$ is constructed to match the share of final uses of each industry's output. Given that our simple model does not feature capital, we construct an industry's Domar weights as the ratio of that industry's total sales as a fraction of total compensation of employees (which corresponds to aggregate value added in the model).

In order to obtain an estimate for the macroeconomic impact of real rigidities and information frictions, we need to take a stance on the firms' uncertainty about realized productivity shocks. We construct the variance-covariance matrix of $(\log z_1, \dots, \log z_n)$ by first detrending the TFP process

[10] Importantly, as one can see from equations (7) and (8), the suppliers' and customers' expectations do not appear symmetrically.

for each industry (obtained from the ILPA dataset) and then setting the variance-covariance matrix of the log-productivity shocks equal to the empirical variance-covariance matrix of the detrended processes. This amounts to assuming that while firms observe all past productivity shocks and are aware of their corresponding trends, they do not observe productivity innovations at the beginning of each quarter.

For this calibration exercise, we ignore the role of downward nominal wage rigidities by setting the minimum nominal wage to $\bar{w} = 0$.

Real rigidities and Supply Chain Bottlenecks. As a first exercise, we use the calibrated model and the result in Proposition 2 to determine the industries that can act as supply chain bottlenecks due to information frictions and real rigidities. Recall from Proposition 2 that if industry r is the only industry that is subject to information frictions, then

$$\mathbb{E}[\log C] - \log C^{\text{ss}} = -\frac{1}{2}\lambda_r^{\text{ss}} \sum_{j \in \mathcal{R}_r} a_{rj} \text{var}_r \left(\sum_{i=1}^{n} \ell_{ji} \log z_i \right),$$

where C^{ss} denotes the steady-state level of output and we are using the assumption that $\mathbb{E}[\log C^*] = \log C^{\text{ss}}$. We therefore use the calibrated model to calculate the right-hand side of the above equation for all industries and obtain an estimate for the reduction in expected consumption if that industry were the only sector that were subject to information frictions.

Figure 1 reports the results for the 15 industries that result in the largest drop in expected output. At the top of the list is 'Petroleum and coal products', with a drop in GDP equal to 0.05% of steady-state output, followed by 'Funds, trusts, and other financial vehicles', and 'Construction'. The reason these industries appear to be the most important is because they simultaneously exhibit large Domar weights, while at the same time relying on a wide range of intermediate inputs.

Another notable observation that emerges from Figure 1 is the relatively small magnitude of the output loss, even for an industry like Petroleum and coal products. This is of course to be expected, as these numbers are calculated under the assumption that only one industry is subject to rigidity at the time.

Real rigidities and output loss. As a second exercise, we quantify the role of real rigidities for aggregate output under the assumption that all sectors are subject to information frictions. To this end, we assume a symmetric information structure and apply our result in Proposition 2 to obtain an estimate for the expected loss in output. Specifically, note that equation (9) implies that

$$\mathbb{E}[\log C] - \log C^{\text{ss}} = -\frac{1}{2} \sum_{k \in \mathcal{I}} (\lambda_k^{\text{ss}} / \alpha_k) \mathbb{E}[\text{var}_p(\log z_k)],$$

where C^{ss} denotes the steady-state level of output. The right-hand side of the above equation therefore provides a measure for the drop in expected output as a fraction of steady-state consumption. Using the calibrated model to calculate this expression, we find that the interaction of information frictions and real rigidities with the economy's production network structure results

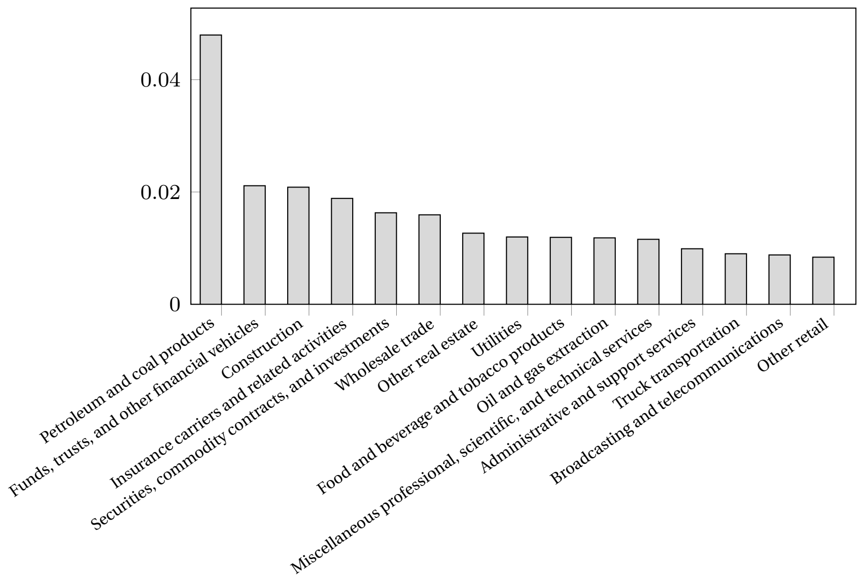

Figure 1. Output loss as a percentage of steady-state consumption due to real rigidities

in 1.3% drop in steady-state consumption. That this number is significantly larger than the ones reported in Figure 1 is due not only to the obvious fact that now all sectors are assumed to be subject to information frictions, but also the fact that the effects of rigidities get amplified over the production network

5 Conclusions

In this paper, we develop a production network model, in which firms are subject to information frictions and real rigidities. The presence of such frictions means that (i) firms may be restricted in how effectively they can adjust their intermediate input quantities in response to changes in the economic environment and (ii) firms may have to choose their quantities only with incomplete information about the realizations of shocks. Our main theoretical result provides an implicit characterization of equilibrium nominal prices and Domar weights in terms of model primitives, namely, the economy's production network structure, the firms' information sets, and the set of rigid intermediate inputs. While only implicit, this result illustrates that equilibrium prices and quantities are determined by the firms' expectations of their upstream input prices as well as their expectations' of their downstream demand. We then considered various special cases of this economy to obtain closed-form solutions for how supply and demand shocks impact aggregate output and inflation.

A few insights emerge from the model. First, the presence of the real rigidities induced by information frictions results in an unambiguous drop in aggregate output, as firms decide to shift demand from more efficient suppliers towards those that are less volatile. Second, these frictions in turn reduce the passthrough of productivity shocks to aggregate output and dampen the real aggregate effect of positive demand shock, while at the same time, increasing their inflationary effect compared to a benchmark model without real rigidities.

While aimed at incorporating two realistic frictions into an otherwise standard model of production networks, the model developed in this paper is nonetheless still very stylized. First, one implicit assumption is that firms' face the same degree of rigidity irrespective of whether they decide to increase various quantities or decrease them. In reality, such frictions are most likely asymmetric. Second, we abstracted from adjustment costs by assuming that once the intermediate input quantity decisions are made, they are completely sunk and cannot be changed. But it is easy to imagine various scenarios in which firms can adjust their production processes by paying some adjustment costs. We leave exploring the implications of these realistic features for future research.

A Appendix: Proofs and Derivations

Proof of Proposition 1

We characterize the equilibrium via backward induction. Starting with the firms' decisions at $t = 1$, recall that firms optimally choose their labor input and flexible intermediate input quantities to meet the realized demand. Taking the prices, its realized demand, and their rigid input demands as given, firm k in industry i faces the following cost-minimization problem:

$$\min_{l_{ik},\{x_{ij,k}\}_{j\in\mathcal{F}_i}} \quad wl_{ik} + \sum_{j\in\mathcal{F}_i} p_j x_{ij,k}$$
$$\text{subject to} \quad y_{ik} = z_i \zeta_i l_{ik}^{\alpha_i} \prod_{j=1}^{n} x_{ij,k}^{a_{ij}}.$$

Solving this problem implies that the firm's expenditure on labor and flexible input demands are given by

$$wl_{ik} = \alpha_i (y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i} a_{ij})} \tag{A.1}$$
$$p_j x_{ij,k} = a_{ij} (y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i} a_{ij})} \qquad \text{for all } j \in \mathcal{F}_i \tag{A.2}$$

respectively, where Q_{ik} only depends on the firm's productivity, its input prices, the nominal wage, and the intermediate input decisions that are sunk by $t = 1$:

$$Q_{ik} = z_i w^{-\alpha_i} \prod_{j\in\mathcal{F}_i} p_j^{-a_{ij}} \prod_{j\in\mathcal{R}_i} (x_{ij,k}/a_{ij})^{a_{ij}}. \tag{A.3}$$

Therefore, the firm faces the following problem when deciding on its nominal price at $t = 1$:

$$\max_{p_{ik}} \quad (1-\tau_i) p_{ik} y_{ik} - wl_{ik} - \sum_{j\in\mathcal{F}_i} p_j x_{ij,k} \tag{A.4}$$
$$\text{subject to} \quad y_{ik} = (p_{ik}/p_i)^{-\theta_i} y_i \tag{A.5}$$

as well as the labor and intermediate input demand constraints (A.1) and (A.2). The first-order conditions of this optimization implies that

$$(1-\tau_i)(1-\theta_i)(p_{ik}/p_i)^{-\theta_i} y_i - (y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i} a_{ij})} \frac{1}{y_{ik}} \frac{\mathrm{d}y_{ik}}{\mathrm{d}p_{ik}} = 0. \tag{A.6}$$

Solving this optimization problem implies that the nominal price set by firm k in industry i is given by

$$p_{ik} = \left((p_i^{\theta_i} y_i)^{\sum_{j\in\mathcal{R}_i} a_{ij}} Q_{ik}^{-1} \right)^{1/(1+(\theta_i-1)\sum_{j\in\mathcal{R}_i} a_{ij})}, \tag{A.7}$$

where Q_{ik} is given by (A.3) and we are using the assumption that $\tau_i = 1/(1-\theta_i)$. With the firm's price and quantity decisions at $t = 1$ in hand, we can now turn to the rigid intermediate input decisions of the firm at $t = 0$. Recall that firms choose their rigid intermediate inputs in order to maximize the expected real value of their profits given their information set. Therefore, firm k in industry i faces the following optimization problem at $t=0$:

$$\max_{\{x_{ij,k}\}_{j\in\mathcal{R}_i}} \quad \mathbb{E}_i\left[\frac{U'(C)}{P}\left((1-\tau_i)p_{ik}y_{ik} - wl_{ik} - \sum_{j=1}^{n} p_j x_{ij,k}\right)\right]$$

subject to constraints (A.1)–(A.2), (A.5), and (A.7), where $\mathbb{E}_i[\cdot]$ denotes the expectation operator with respect to the information set of firms in industry i, $U'(C)=1/C$ is the household's marginal utility, and P is the price of the consumption good bundle. Note that, $PC=m$. Therefore, the first-order condition of the firm's problem at $t=0$ is given by

$$\mathbb{E}_i\left[\frac{1}{m}\left((1-\tau_i)(1-\theta_i)(p_{ik}/p_i)^{-\theta_i}y_i - (y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i}a_{ij})}\,\frac{1}{y_{ik}}\frac{\mathrm{d}y_{ik}}{\mathrm{d}p_{ik}}\right)\frac{\mathrm{d}p_{ik}}{\mathrm{d}x_{ij,k}}\right]$$
$$+\mathbb{E}_i\left[\frac{1}{m}\left((y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i}a_{ij})}\,\frac{1}{Q_{ik}}\frac{\mathrm{d}Q_{ik}}{\mathrm{d}x_{ij,k}} - p_j\right)\right]=0.$$

Equation (A.6) implies that the first term on the right-hand side of the above equation is equal to zero. Furthermore, note that (A.3) implies that $\mathrm{d}Q_{ik}/\mathrm{d}x_{ij,k}=a_{ij}Q_{ik}/x_{ij,k}$. Therefore,

$$x_{ij,k}=\frac{a_{ij}}{\mathbb{E}_i[p_j/m]}\mathbb{E}_i\left[\frac{1}{m}\,(y_{ik}/Q_{ik})^{1/(1-\sum_{j\in\mathcal{R}_i}a_{ij})}\right] \qquad \text{for all } j\in\mathcal{R}_i. \tag{A.8}$$

To simplify the above, note that given that all firms within the same industry are symmetric, they all set the same prices and produce the same quantities, that is, $p_{ik}=p_i$ and $y_{ik}=y_i$. Therefore, we can drop the firm index k from (A.7) and solve for Q_{ik} in terms of the price of firms in industry i:

$$Q_{ik}=(p_i y_i)^{\sum_{j\in\mathcal{R}_i}a_{ij}}/p_i. \tag{A.9}$$

Plugging this expression back into (A.2) and (A.8), we obtain

$$x_{ij,k}=\begin{cases} a_{ij}\lambda_i m/p_j & \text{if } j\in\mathcal{F}_i\\ a_{ij}\mathbb{E}_i[\lambda_i]/\mathbb{E}_i[p_j/m] & \text{if } j\in\mathcal{R}_i,\end{cases} \tag{A.10}$$

where we are using the fact that the Domar weight of industry i is given by $\lambda_i = p_i y_i/m$. This expression together with the market-clearing condition (6) for sectoral good i implies that

$$y_i = c_i + \sum_{j\in\mathcal{F}_i} a_{ji}\frac{\lambda_j}{p_i/m} + \sum_{j\in\mathcal{R}_i} a_{ji}\frac{\mathbb{E}_j[\lambda_j]}{\mathbb{E}_j[p_i/m]}.$$

Multiplying both sides of the above equation by p_i/m and using the fact that $c_i=\beta_i m/p_i$—which is a consequence of the household's optimization problem—then establishes (8).

We next establish (7). To this end, note that equations (A.3) and (A.10) imply that

$$Q_{ik}=z_i w^{-\alpha_i}\prod_{j\in\mathcal{F}_i}p_j^{-a_{ij}}\prod_{j\in\mathcal{R}_i}\left(\mathbb{E}_i[\lambda_i]/\mathbb{E}_i[p_j/m]\right)^{a_{ij}}.$$

Combining the above equation with the expression for Q_{ik} in (A.9) then establishes (7). □

Proof of Lemma 1

To characterize the equilibrium nominal wage, it is most transparent to characterize the nominal wage under the assumption that the Frisch elasticity of labor supply is η and then consider the limit as $\eta \to \infty$.

On the one hand, note that combining (A.1) with the expression for Q_{ik} in (A.9) implies that the labor demand of firm k in industry i is given by $l_{ik} = \alpha_i \lambda_i m/w$. Therefore, aggregate demand for labor in the economy is equal to

$$\sum_{i=1}^{n} \int_{0}^{1} l_{ik} \mathrm{d}k = (m/w) \sum_{i=1}^{n} \alpha_i \lambda_i.$$

On the other hand, note that the first-order conditions of the household's problem imply that total labor supply is given by $L = (m\chi/w)^{-\eta}$. Combining the above two equations therefore implies that the labor market equilibrium condition (5) is given by

$$(w - \bar{w}) \left((m\chi/w)^{-\eta} - (m/w) \sum_{i=1}^{n} \alpha_i \lambda_i \right) = 0, \qquad w \geq \bar{w}, \qquad \chi m/w \leq \left(\frac{1}{\chi} \sum_{i=1}^{n} \alpha_i \lambda_i \right)^{-1/(1+\eta)}.$$

We consider two separate cases. If $w > \bar{w}$, then the first condition above implies that $\chi m/w = (\chi \sum_{i=1}^{n} \alpha_i \lambda_i)^{-1/(1+\eta)}$. Thus, as $\eta \to \infty$, we get $w = \chi m$. This is consistent with the original conjecture as long as $\chi m > \bar{w}$. As the second case, suppose $w = \bar{w}$. In that case, the last inequality above implies that $m\chi/\bar{w} \leq 1$ as $\eta \to \infty$. Putting the two cases together therefore implies that $w = \max\{\chi m, \bar{w}\}$. $\square$

Proof of Propositions 2 and 3

As a first observation, note that given industry r is the only industry subject to information frictions, equation (8) implies that

$$\lambda_i = \beta_i + \sum_{j=1}^{n} a_{ji} \lambda_j + a_{ri} \left(\mathbb{E}_r[\lambda_r] \frac{p_i/m}{\mathbb{E}_r[p_i/m]} - \lambda_r \right) \mathbb{I}_{\{i \in \mathcal{R}_r\}}. \tag{A.11}$$

Taking expectations from both sides of the above equation with respect to the information set of industry r implies that $\mathbb{E}_r[\lambda_i] = \beta_i + \sum_{j=1}^{n} a_{ji} \mathbb{E}_r[\lambda_j]$ for all i. Solving this system of equations for $\mathbb{E}_r[\lambda_i]$ implies that $\mathbb{E}_r[\lambda_i] = \lambda_i^{\text{ss}}$, where is the steady-state Domar weight of industry i. Consequently, we can rewrite equation (A.11) as follows:

$$\lambda_i = \beta_i + \sum_{j=1}^{n} a_{ji} \lambda_j + a_{ri} \left(\lambda_r^{\text{ss}} \frac{p_i/m}{\mathbb{E}_r[p_i/m]} - \lambda_s \right) \mathbb{I}_{\{i \in \mathcal{R}_r\}},$$

Furthermore, note that the steady-state Domar weights satisfy the following system of equations: $\lambda_i^{\text{ss}} = \beta_i + \sum_{j=1}^{n} a_{ji} \lambda_j^{\text{ss}}$ for all i. Subtracting this equation from the previous one therefore implies that

$$\Delta_i = \sum_{j=1}^{n} a_{ji} \Delta_j + a_{ri} \left(\lambda_r^{\text{ss}} \frac{p_i/m}{\mathbb{E}_r[p_i/m]} - \lambda_s \right) \mathbb{I}_{\{i \in \mathcal{R}_r\}}.$$

where $\Delta_i = \lambda_i - \lambda_i^{\text{ss}}$. Solving the above system of equations for Δ_i and setting $i = r$, we obtain

$$\lambda_r - \lambda_r^{\text{ss}} = \sum_{i=1}^{n} \ell_{ir} a_{ri} \left(\lambda_r^{\text{ss}} \frac{p_i/m}{\mathbb{E}_r[p_i/m]} - \lambda_s \right) \mathbb{I}_{\{i \in \mathcal{R}_r\}} = 0, \tag{A.12}$$

where the second equality is a consequence of Assumption 1. This establishes that the equilibrium Domar weight of industry r is equal to its steady-state value.

Next, we turn to determining equilibrium prices. By equation (7), the (log) nominal price of industry $i \neq r$ is given by

$$\log p_i = -\log z_i + \alpha_i \log w + \sum_{j=1}^{n} a_{ij} \log p_j.$$

Let $\tilde{p} \in \mathbb{R}^{n-1}$ denote the vector of nominal prices for all industries $i \neq r$ and let $\tilde{\mathbf{A}} \in \mathbb{R}^{(n-1)\times(n-1)}$ denote the sub-block of the input-output matrix $\mathbf{A}$ corresponding to all industries except for r. Writing the above equation in vector form therefore implies that $\log \tilde{p} = -\log \tilde{z} + \tilde{\alpha} \log w + \tilde{\mathbf{A}} \log \tilde{p} + \tilde{a}_r \log p_r$, where $\tilde{\alpha}$ and $\tilde{z}$ denote the vectors of labor shares and productivity shocks for all $i \neq r$ and $\tilde{a}_r \in \mathbb{R}^{n-1}$ is a vector with elements a_{is} for all $i \neq r$. Consequently,

$$\log \tilde{p} = \tilde{\mathbf{L}} \tilde{\alpha} \log w - \tilde{\mathbf{L}} \log \tilde{z} + \tilde{\mathbf{L}} \tilde{a}_r \log p_r,$$

where $\tilde{\mathbf{L}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$. Under Assumption 1, the elements of $\tilde{\mathbf{L}}$ can be expressed in terms of the elements of the economy's Leontief inverse $\mathbf{L}$. In particular, $\tilde{\ell}_{ij} = \ell_{ij} - \ell_{ir}\ell_{rj}$ for all $i, j \neq r$. Hence,

$$\log p_i = \log w \sum_{j \neq r} (\ell_{ij} - \ell_{ir}\ell_{rj})\alpha_j - \sum_{j \neq r} (\ell_{ij} - \ell_{ir}\ell_{rj}) \log z_j + \log p_r \sum_{j \neq s} (\ell_{ij} - \ell_{ir}\ell_{rj}) a_{jr}$$

for all $i \neq r$. Consequently,

$$\log p_i = (1 - \ell_{ir}) \log w + \ell_{ir} \log p_r - \sum_{j \neq r} (\ell_{ij} - \ell_{ir}\ell_{rj}) \log z_j \tag{A.13}$$

for all $i \neq r$, where we are using the fact that $\sum_{j=1}^{n} \ell_{ij}\alpha_j = 1$ for all i and $\ell_{rr} = 1$, the latter of which is a consequence of Assumption 1. With the above in hand, we can therefore obtain an expression for aggregate output in terms of the nominal price of industry r. In particular, the fact that $\log C = \log m - \sum_{i=1}^{n} \beta_i \log p_i$ implies that

$$\log C = \log(m/w) - \lambda_r^{\text{ss}} \log(p_r/w) + \sum_{j \neq r} (\lambda_j^{\text{ss}} - \lambda_r^{\text{ss}} \ell_{rj}) \log z_j, \tag{A.14}$$

where λ_j^{ss} denotes the steady-state Domar weight of industry j. Next, note that setting $i = r$ in equation (7) implies that

$$\log p_r = -\log z_r + \alpha_r \log w + \sum_{j \in \mathcal{F}_r} a_{rj} \log p_j + \log m \sum_{j \in \mathcal{R}_r} a_{rj} + \sum_{j \in \mathcal{R}_r} a_{rj} \log \mathbb{E}_r[p_j/m],$$

where we are also using the fact that $\lambda_r = \lambda_r^{\rm ss}$, as established in (A.12). Hence, to a second-order approximation around steady-state,

$$\log p_r = -\log z_r + \alpha_r \log w + \sum_{j\in\mathcal{F}_r} a_{rj}\log p_j + \log m \sum_{j\in\mathcal{R}_r} a_{rj} + \sum_{j\in\mathcal{R}_r} a_{rj}\mathbb{E}_r[\log(p_j/m)] + \frac{1}{2}\sum_{j\in\mathcal{R}_r} a_{rj}\mathrm{var}_r(\log(p_j/m)).$$

Replacing for log nominal prices from (A.13) into the above equation leads to

$$\log p_r = -\log z_r + \log w\left(\alpha_r + \sum_{j\in\mathcal{F}_r} a_{rj}\right) - \sum_{j\in\mathcal{F}_r}\sum_{i\neq r} a_{rj}\ell_{ji}\log z_i + \log m\sum_{j\in\mathcal{R}_r} a_{rj} + \sum_{j\in\mathcal{R}_r} a_{rj}\mathbb{E}_r\left[\log(w/m) - \sum_{i=1}^n \ell_{ji}\log z_i\right] + \frac{1}{2}\sum_{j\in\mathcal{R}_r} a_{rj}\mathrm{var}_r\left(\log(w/m) - \sum_{i=1}^n \ell_{ji}\log z_i\right),$$

where once again we are relying on Assumption 1 to simply the above expression. Combining the above with (A.14) therefore implies that

$$\log C = \log(m/w) + \sum_{j=1}^n \lambda_j^{\rm ss}\log z_j + \lambda_r^{\rm ss}\sum_{j\in\mathcal{R}_r} a_{rj}(\log(w/m) - \mathbb{E}_r[\log(w/m)]) - \lambda_r^{\rm ss}\sum_{j\in\mathcal{R}_r}\sum_{i\neq r} a_{rj}\ell_{ji}(\log z_i - \mathbb{E}_r[\log z_i]) - \frac{1}{2}\lambda_r^{\rm ss}\sum_{j\in\mathcal{R}_r} a_{rj}\mathrm{var}_r\left(\log(m/w) + \sum_{i=1}^n \ell_{ji}\log z_i\right).$$

Noting that log aggregate output in the absence of real rigidities due to information frictions is given by $\log C^* = \log(m/w) + \sum_{j=1}^n \lambda_j^{\rm ss}\log z_j$ then establishes (9) and (12). Finally, (13) follows immediately from the observation that $\log P = \log m - \log C$. □

Proof of Propositions 4 and 5

Taking expectations from both sides of (8) with respect to the firms' (common) information set implies that $\mathbb{E}[\lambda_i] = \beta_i + \sum_{j=1}^n a_{ji}\mathbb{E}[\lambda_j]$ for all i. On the other hand, note that the steady-state Domar weights of all industries also satisfy the following system of equations: $\lambda_i^{\rm ss} = \beta_i + \sum_{j=1}^n a_{ji}\lambda_j^{\rm ss}$. Comparing the two equations then implies that

$$\mathbb{E}[\lambda_i] = \lambda_i^{\rm ss} \qquad \text{for all } i.$$

Plugging this back into equation (8), we get

$$\lambda_i = \beta_i + \frac{p_i/m}{\mathbb{E}[p_i/m]}\sum_{j=1}^n a_{ji}\lambda_j^{\rm ss} = \beta_i + \frac{p_i/m}{\mathbb{E}[p_i/m]}(\lambda_i^{\rm ss} - \beta_i).$$

Imposing Assumption 2 now implies that Therefore,

$$\lambda_i/\lambda_i^{\rm ss} = \begin{cases} \dfrac{p_i/m}{\mathbb{E}[p_i/m]} & \text{if } \beta_i = 0 \\ 1 & \text{if } \beta_i > 0. \end{cases} \tag{A.15}$$

Next, taking expectations from both sides (7) implies that

$$\mathbb{E}[p_i/m] = \mathbb{E}[(w/m)^{\alpha_i}\lambda_i^{1-\alpha_i}/z_i]\prod_{j=1}^{n}\left(\mathbb{E}[p_j/m]/\lambda_i^{\text{ss}}\right)^{a_{ij}}.$$

Dividing both sides of this equation by (7) implies that

$$\frac{p_i/m}{\mathbb{E}[p_i/m]} = \frac{(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{\text{ss}})^{1-\alpha_i}/z_i}{\mathbb{E}[(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{\text{ss}})^{1-\alpha_i}/z_i]} \tag{A.16}$$

Therefore, the juxtaposition of equations (A.15) and (A.16) implies that

$$\lambda_i/\lambda_i^{\text{ss}} = \frac{(w/m)z_i^{-1/\alpha_i}}{(\mathbb{E}[(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{*})^{1-\alpha_i}/z_i])^{1/\alpha_i}} \tag{A.17}$$

for all intermediate-input supplier industries. Consequently,

$$(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{\text{ss}})^{1-\alpha_i}/z_i = \frac{(w/m)z_i^{-1/\alpha_i}}{(\mathbb{E}[(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{\text{ss}})^{1-\alpha_i}/z_i])^{(1-\alpha_i)/\alpha_i}}.$$

Taking expectations from both sides, we obtain

$$\mathbb{E}[(w/m)^{\alpha_i}(\lambda_i/\lambda_i^{\text{ss}})^{1-\alpha_i}/z_i] = \mathbb{E}^{\alpha_i}[(w/m)z_i^{-1/\alpha_i}],$$

which once we plug this back into (A.17) and using (A.15) leads to

$$\lambda_i/\lambda_i^{\text{ss}} = \frac{p_i/m}{\mathbb{E}[p_i/m]} = \frac{(w/m)z_i^{-1/\alpha_i}}{\mathbb{E}[(w/m)z_i^{-1/\alpha_i}]} \tag{A.18}$$

for all industries i that are intermediate input producers.

Now that we have the relationship between the price and its expectation, we can solve for the prices. Note that equation (7) implies that

$$\log(p_i/m) = \alpha_i\log(w/m) - \log z_i + (1-\alpha_i)\log(\lambda_i/\lambda_i^{\text{ss}}) + \sum_{j=1}^{n}a_{ij}\log\mathbb{E}[p_j/m]$$

for all i. Therefore,

$$\log(p_i/m) = \alpha_i\log(w/m) - \log z_i + (1-\alpha_i)\log(\lambda_i/\lambda_i^{\text{ss}}) + \sum_{j\in\mathcal{I}}a_{ij}\log\mathbb{E}[p_j/m],$$

where $\mathcal{I}$ denotes the set of intermediate-input producing industries. Now using Assumption 2 one more time implies that

$$\log(p_i/m) = \alpha_i\log(w/m) - \log z_i + (1-\alpha_i)\log(\lambda_i/\lambda_i^{\text{ss}}) + \sum_{j=1}^{n}a_{ij}\log p_j - \sum_{j\in\mathcal{I}}a_{ij}\log\frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]}.$$

Consequently,

$$\log(p_i/m) = \log(w/m) - \sum_{k=1}^{n}\ell_{ik}\log z_k + \sum_{k\in\mathcal{I}}\ell_{ik}(1-\alpha_k)\log(\lambda_k/\lambda_k^{\text{ss}}) - \sum_{k=1}^{n}\sum_{j\in\mathcal{I}}\ell_{ik}a_{kj}\log\frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]}.$$

Replacing for $\lambda_k/\lambda_k^{\rm ss}$ from (A.18) for intermediate-input producing industries, we get

$$\log p_i = \log w - \sum_{k=1}^{n} \ell_{ik} \log z_k + \sum_{k\in\mathcal{I}} \ell_{ik}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]} - \sum_{k=1}^{n}\sum_{j\in\mathcal{I}} \ell_{ik}a_{kj} \log \frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]}.$$

Therefore, aggregate output is

$$\log C = \log(m/w) + \sum_{k=1}^{n} \lambda_i^{\rm ss} \log z_k - \sum_{i=1}^{n}\sum_{k\in\mathcal{I}} \beta_i \ell_{ik}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]}$$
$$+ \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j\in\mathcal{I}} \beta_i \ell_{ik} a_{kj} \log \frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]}.$$

To simplify the above, note that if an industry has $\beta_i > 0$, then clearly i is not an intermediate input supplier, which means $\ell_{ii} = 1$. At the same time, for any $k \neq i$ such that $\ell_{ik} > 0$, we have that k is an intermediate input supplier. Hence,

$$\sum_{i=1}^{n}\sum_{k\in\mathcal{I}} \beta_i \ell_{ik}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]} = \sum_{i=1}^{n}\sum_{k=1}^{n} \beta_i \ell_{ik}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]}$$
$$- \sum_{i=1}^{n} \beta_i(1-\alpha_i) \log \frac{(w/m)z_i^{-1/\alpha_i}}{\mathbb{E}[(w/m)z_i^{-1/\alpha_i}]}.$$

Consequently,

$$\sum_{i=1}^{n}\sum_{k\in\mathcal{I}} \beta_i \ell_{ik}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]} = \sum_{k=1}^{n} (\lambda_k^{\rm ss} - \beta_k)(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]}$$
$$= \sum_{k\in\mathcal{I}} \lambda_k^{\rm ss}(1-\alpha_k) \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]}.$$

Similarly, note that $a_{kj} > 0$ if and only if j is an intermediate input supplier. Therefore,

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j\in\mathcal{I}} \beta_i \ell_{ik} a_{kj} \log \frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]} = \sum_{j\in\mathcal{I}} \lambda_j^{\rm ss} \log \frac{(w/m)z_j^{-1/\alpha_j}}{\mathbb{E}[(w/m)z_j^{-1/\alpha_j}]}.$$

Therefore, plugging the above two equations into the expression for $\log C$, we get

$$\log C = \log(m/w) + \sum_{k=1}^{n} \lambda_i^{\rm ss} \log z_k + \sum_{k\in\mathcal{I}} \lambda_k^{\rm ss} \alpha_k \log \frac{(w/m)z_k^{-1/\alpha_k}}{\mathbb{E}[(w/m)z_k^{-1/\alpha_k}]}.$$

Approximating the above equation to a second order, we obtain

$$\log C = \log(m/w) + \sum_{k=1}^{n} \lambda_i^{\rm ss} \log z_k + \sum_{k\in\mathcal{I}} \lambda_k^{\rm ss} \alpha_k (\log(w/m) - \mathbb{E}[\log(w/m)]) + \sum_{k\in\mathcal{I}} \lambda_k^{\rm ss} (\mathbb{E}[\log z_k] - \log z_k)$$
$$- \frac{1}{2} \sum_{k\in\mathcal{I}} \lambda_k^{\rm ss} \alpha_k \operatorname{var}\left(\log(w/m) - \frac{1}{\alpha_k} \log z_k\right).$$

Noting that log aggregate output in the absence of real rigidities due to information frictions is given by $\log C^* = \log(m/w) + \sum_{j=1}^{n} \lambda_j^{\rm ss} \log z_j$ then establishes (15) and (16). Finally, (17) follows immediately from the observation that $\log P = \log m - \log C$. □

Proof of Proposition 6

We consider a more general vertical production network with industries 1 through n arranged on a chain, with industry 1 as the final good producer. We then specialize this economy to the case of $n = 3$ in Proposition 6.

Recall from Proposition 1 that nominal prices and Domar weights satisfy the system of equations in (7) and (8). Applying these equations to the vertical production network economy, we obtain

$$p_i = \frac{1}{z_i} w^{1-a_i} \left(m \frac{\mathbb{E}_i[p_{i+1}/m]}{\mathbb{E}_i[\lambda_i]/\lambda_i} \right)^{a_i} \qquad \text{for } 1 \leq i \leq n, \tag{A.19}$$

$$\lambda_{i+1} = a_i \mathbb{E}_i[\lambda_i] \frac{p_{i+1}/m}{\mathbb{E}_i[p_{i+1}/m]} \qquad \text{for } 1 \leq i \leq n-1 \tag{A.20}$$

with the initial conditions that $\lambda_1 = 1$ and the convention that $a_n = 0$. Solving for $\mathbb{E}_i[\lambda_i]/\mathbb{E}_i[p_{i+1}/m]$ from (A.20) and plugging it back into (A.19) implies that $p_i = \frac{1}{z_i} a_i^{a_i} (\lambda_i/\lambda_{i+1})^{a_i} w^{1-a_i} p_{i+1}^{a_i}$. Hence,

$$\log p_i = a_i(\delta_i - \delta_{i+1}) - \log z_i + a_i \log p_{i+1} + (1 - a_i) \log w,$$

for $1 \leq i \leq n$, where $\delta_i = \log \lambda_i - \log \lambda_i^{\text{ss}}$ and we are using the fact that $\lambda_{i+1}^{\text{ss}} = a_i \lambda_i^{\text{ss}}$. Solving the above recursion, we can express nominal prices in terms of Domar weights:

$$\log p_i = \log w - \sum_{j=i}^{n} (a_i a_{i+1} \dots a_{j-1}) \log z_j + \sum_{j=i}^{n-1} (a_i a_{i+1} \dots a_j)(\delta_j - \delta_{j+1}). \tag{A.21}$$

Next, note that, to a first-order approximation, (A.20) can be expressed as

$$\log \lambda_{i+1} = \log a_i + \log(p_{i+1}/m) + \mathbb{E}_i[\log \lambda_i] - \mathbb{E}_i[\log p_{i+1}/m],$$

and as a result,

$$\delta_{i+1} = \mathbb{E}_i[\delta_i] + \log(p_{i+1}/m) - \mathbb{E}_i[\log p_{i+1}/m]. \tag{A.22}$$

We now have a system of linear expectations (A.21) and (A.22) that fully pins down equilibrium nominal prices and Domar weights in terms of the productivity shocks, nominal aggregate demand, and the nominal wage.

Specializing these equations to the case of $n = 3$ and shutting off all productivity shocks, it is immediate that $\log p_3 = \log w$, and hence, we get the following equations:

$$\begin{aligned}
\log p_2 &= \log w + a_2(\delta_2 - \delta_3) \\
\delta_2 &= \log p_2 - \mathbb{E}_1[\log p_2] - \log m + \mathbb{E}_1[\log m] \\
\delta_3 &= \mathbb{E}_2[\delta_2] + \log(w/m) - \mathbb{E}_2[\log(w/m)].
\end{aligned}$$

Replacing for δ_3 into the expression for $\log p_2$, we get

$$\log p_2 = a_2(\delta_2 - \mathbb{E}_2[\delta_2]) + Q + \log m, \tag{A.23}$$

where

$$Q = (1 - a_2)\log(w/m) + a_2\mathbb{E}_2[\log(w/m)]. \tag{A.24}$$

Consequently, we get the following equation for δ_2:

$$\delta_2 = a_2(\delta_2 - \mathbb{E}_2[\delta_2]) - a_2(\mathbb{E}_1[\delta_2] - \mathbb{E}_1\mathbb{E}_2[\delta_2]) + Q - \mathbb{E}_1[Q]$$

Noting that $\mathbb{E}_1[\delta_2] = 0$, we get

$$\delta_2 = -\frac{a_2}{1-a_2}\mathbb{E}_2[\delta_2] + \frac{a_2}{1-a_2}\mathbb{E}_1\mathbb{E}_2[\delta_2] + \frac{1}{1-a_2}(Q - \mathbb{E}_1[Q]). \tag{A.25}$$

Taking expectations with respect to the information set of firms in industry 2 from both sides of (A.25) and solving for $\mathbb{E}_2[\delta_2]$ implies that

$$\mathbb{E}_2[\delta_2] = a_2\mathbb{E}_2\mathbb{E}_1\mathbb{E}_2[\delta_2] + \mathbb{E}_2[Q] - \mathbb{E}_2\mathbb{E}_1[Q].$$

We can thus solve for $\mathbb{E}_2[\delta_2]$ in terms of the infinite regress of expectations as follows:

$$\mathbb{E}_2[\delta_2] = \sum_{s=0}^{\infty} a_2^s(\mathbb{E}_2\mathbb{E}_1)^s(\mathbb{E}_2[Q] - \mathbb{E}_2\mathbb{E}_1[Q]) = \sum_{s=0}^{\infty} a_2^s(\mathbb{E}_2\mathbb{E}_1)^s\mathbb{E}_2[Q] - \sum_{s=0}^{\infty} a_2^s(\mathbb{E}_2\mathbb{E}_1)^{s+1}[Q].$$

Plugging the above expression into (A.25), we get

$$\delta_2 = \frac{1}{1-a_2}\left(\sum_{s=0}^{\infty} a_2^{s+1}(\mathbb{E}_1\mathbb{E}_2)^{s+1}[Q] - \sum_{s=0}^{\infty} a_2^{s+1}(\mathbb{E}_2\mathbb{E}_1)^s\mathbb{E}_2[Q] + \sum_{s=0}^{\infty} a_2^s(\mathbb{E}_2\mathbb{E}_1)^s[Q] - \sum_{s=0}^{\infty} a_2^s\mathbb{E}_1(\mathbb{E}_2\mathbb{E}_1)^s[Q]\right).$$

Hence, combining this equation with (A.23) and using the observations that $\log p_1 = a_1(\delta_1 - \delta_2) + a_1\log p_2 + (1 - a_1)\log w$ and $\delta_1 = 0$, we get the following expression for $\log p_1$:

$$\log(p_1/m) = a_1\left(\sum_{s=0}^{\infty} a_2^s\mathbb{E}_1(\mathbb{E}_2\mathbb{E}_1)^s[Q] - \sum_{s=0}^{\infty} a_2^{s+1}(\mathbb{E}_1\mathbb{E}_2)^{s+1}[Q]\right) + (1 - a_1)\log(w/m).$$

Rearranging terms, we get

$$\log(p_1/m) = a_1\sum_{s=0}^{\infty} a_2^s(\mathbb{E}_1\mathbb{E}_2)^s\mathbb{E}_1\Big[Q - a_2\mathbb{E}_2[Q]\Big] + (1 - a_1)\log(w/m).$$

On the other hand, note that (A.24) implies that $Q - a_2\mathbb{E}_2[Q] = (1 - a_2)\log(w/m)$. Therefore,

$$\log(p_1/m) = a_1(1 - a_2)\sum_{s=0}^{\infty} a_2^s(\mathbb{E}_1\mathbb{E}_2)^s\mathbb{E}_1[\log(w/m)] + (1 - a_1)\log(w/m).$$

By Lemma 1, the assumption that $m < \bar{w}/\chi$ implies that $w = \bar{w}$, in which case the above equation immediately reduces to (19). Furthermore, noting that $\log m = \log(PC)$ then establishes (18). □

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