Propagation of Shocks in An Input-Output Economy: Evidence From Disaggregated Prices*

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Abstract

This paper empirically evaluates the predictions for the cross-sectional price change distribution made by input-output models with sticky prices. Using disaggregated industry-level data, we find that the response of price to shocks is consistent with the price sensitivities predicted by the input-output model. Moreover, moments of sectoral price change distribution vary over time in response to the evolution of the network structure. Finally, through a quantitative analysis, we disentangle sectoral demand and supply shocks during the pandemic period. Running counterfactual analyses, we find that sectoral supply shocks and the propagation of shocks through the production network contributed significantly to the inflation surge in 2021-2022.

Keywords: nominal rigidity, production network, propagation of shocks, inflation

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1 Introduction

How prices respond to shocks has long been understood to be crucial to determine how monetary policy works. Sticky prices naturally play an important role in this, and there is a long literature studying how price stickiness affects the transmission of monetary shocks (e.g. Caplin and Spulber, 1987; Golosov and Lucas, 2007, among many others). It has also been known for some time that when the economy operates under a production network with input-output linkages across firms and sectors, the real effects of monetary shocks can be amplified (see, for example, Basu (1995) or Nakamura and Steinsson (2010)). In recent years there has also been a great deal of theoretical work done to develop rich production network models with sticky prices and derive implications for monetary non-neutrality and for the conduct of monetary policy (La’O and Tahbaz-Salehi, 2022; Pasten et al., 2020; Rubbo, 2020). However, while there is broad agreement that a production network plays an important role in the transmission of shocks, there is little direct or indirect evidence for the strength of the mechanisms responsible for this.\footnote{One notable exception is Ghassibe (2021b), who provides an empirical test for how important input-output linkages are for the transmission of monetary shocks using high-frequency disaggregated consumption data. Our paper instead makes a series of tests for these mechanisms using industry-level price data.}

In this paper, we empirically evaluate the predictions of the input-output models with sticky prices. While these models have implications for monetary non-neutrality and monetary policy generally, they also make clear predictions about how individual sectoral prices should respond to aggregate and sectoral shocks through the input-output linkages. Through several empirical tests, we evaluate to what extent the mechanisms that are key to models of production networks are present in actual price-setting behavior.

First, the I-O models with price rigidity imply that the degree to which sectoral prices respond to shocks is determined by a set of factors related to the input-output structure of the economy and to differential price rigidity across sectors (see for example Afrouzi and Bhattacharai, 2022; La’O and Tahbaz-Salehi, 2022; Pasten et al., 2020; Rubbo, 2020). Shocks propagate along the supply chain and affect the prices of different sectors through direct and indirect connection. Specifically, price setting is determined by the following key equation in matrix form:

\[
d\log p = \Phi (I - \Omega \Phi)^{-1} [D(\alpha) d\log w - d\log z],
\]

\[\text{Equation 1}\]
which relates price changes to factors determined by the production network ($\Omega$, the matrix of input shares) and price stickiness ($\Phi$, a diagonal matrix of price adjustment flexibility). Thus, sectoral wage ($w$) and technology ($z$) changes pass-through to prices of connected sectors through the I-O structure (along with the diagonal matrix of labor share $D(\alpha)$). We use this equation as the basis for which to test the model’s predictions for sectoral price changes. In particular, the relationship between actual price changes and the model-implied price sensitivity vector, $\Phi(I - \Omega\Phi)^{-1}\alpha$.

To evaluate the theoretical prediction on sectoral price responses, we conduct three types of empirical tests. First, we test whether the predictions of the equation above can explain changes in the shape of the sectoral distribution of prices. We find that these predictions are borne out by sectoral prices from the past 25 years. In particular, moments of the sectoral price change distribution varies over time in response to the evolution of the network structure. In addition, the correlation between sectoral price changes and the model-implied sectoral price sensitivity to aggregate shocks is highly procyclical. These patterns support the model’s prediction. They also imply that the I-O structure could matter for how we interpret information from the shape of the distribution of price changes in sticky price studies. For instance, although skewness and kurtosis of the price change distribution provide implications on the nature of price rigidities in models without an I-O structure (see Alvarez et al., 2016; Baley and Blanco, 2021), the observed moments in the data actually co-move with the I-O structure. In the second type of tests we directly estimate the price responsiveness of different sectoral prices using identified monetary policy shocks from the high-frequency identification literature (Jarociński and Karadi, 2020; Nakamura and Steinsson, 2018), and determine whether the most price-sensitive sectors are those predicted by the model. Here, we find some evidence in support of the model, but less compelling than in the first set of tests.

Finally, we study whether sectoral shocks propagate to other sectors and affect sectoral prices in the way that the model predicts. Using measures of sectoral productivity, we find strong downstream propagation of supply shocks on sectoral prices. Essentially, supply shocks transmit downstream from suppliers to customers along the supply chain through the adjustment of input costs. To test the propagation of demand shocks, we use measures of trade exposure by industry (following the literature on the effects of Chinese imports, as in Acemoglu et al., 2016a,b; Autor et al., 2013, for example). Here, we find strong evidence of both downstream and upstream propagation.
of demand shocks. The upstream propagation reflects how changes in sales propagate through demand for inputs. These findings strongly support the prediction of the model. It is also interesting to notice that in these two exercises, we find stronger evidence for the effect of shocks to sectors along the production network than for the effect of shocks to the sector in question.

Taken together, we believe that our analyses provide support for the importance of input-output linkages and price rigidity in determining sectoral price changes. We then perform a quantitative analysis using the I-O model to determine what possible roles the structure of the production network played in the large rise in inflation seen in the U.S. since early 2021. The pandemic period has brought a great deal of attention to various kinds of supply disruptions, and in particular to problems in global supply chains. We calibrate our model to the empirical input-output structure and to the changes in sectoral employment, consumption, and prices since the start of the COVID-19 pandemic. By solving the model under various counterfactuals regarding the presence of different kinds of shocks and the presence or absence of the input-output structure, we find that the reduction of production possibilities at the sectoral-level and the input-output structure have been major contributors to the surge in inflation. Thus, our paper presents evidence that price stickiness and input-output linkages work together to play a crucial role in the transmission of shocks to prices, and documents how this sort of transmission might have amplified the response of inflation during the pandemic period.

Related Literature. This paper relates to a few strands of literature. First, it fits most directly into the recent line of work on multi-sector models with sticky prices and production networks (such as Afrouzi and Bhattarai, 2022; Ghassibe, 2021a,b; La’O and Tahbaz-Salehi, 2022; Pasten et al., 2020; Rubbo, 2020). The model that we consider is similar to the ones featured in these studies. Most of the existing work in this field has focused quite intensively on understanding and measuring the extent to which the transmission of monetary policy is affected by the economy’s network structure, or on how this network structure affects what is the optimal monetary policy (particularly La’O and Tahbaz-Salehi, 2022; Rubbo, 2020). Our paper investigates the I-O structure and price stickiness from a different angle: their joint implication for the cross-sectional distribution of price responses to shocks. Because these models make strong predictions about how sectors respond to aggregate and sectoral shocks, our empirical tests provide an important validation for this class of models.
Our paper is also connected to the literature on the propagation of shocks in a network economy. There have been papers theoretically or empirically studying how shocks propagate along the supply chain or financial linkages (for example Acemoglu et al., 2016a, 2015; Baqae, 2018; Luo, 2020). However, most of these papers focus on the response of production and employment. Our focus is naturally on the price effect of shocks.

Our analysis on the propagation of demand shocks relates to the literature on the effects of increasing import competition from China and use changes in Chinese import penetration as demand shocks. Papers such as (Acemoglu et al., 2016b;Autor et al., 2013, 2019; Pierce and Schott, 2016) find that the rise in imports from China in the past few decades had a very large negative effect on employment in the U.S. manufacturing sector. Jaravel and Sager (2022) have also studied the effect of Chinese imports on prices in the U.S. Their analysis has focused on consumer prices and on evaluating the predictions of models of international trade. We instead use PPI prices and consider the propagation of these shocks: that is, how shocks to a group of sectors can affect prices in another sector through input-output linkages.

Finally, our quantitative analysis contributes to the growing literature on the macroeconomic consequences of the disruptions caused by the Covid-19 pandemic. In particular, we follow other papers in highlighting the role of sectoral shocks in raising inflation. For example, Guerrieri et al. (2021) develop a multi-sector model in which a demand reallocation shock raises inflation, and they study optimal monetary policy in such a setting. Ferrante et al. (2022) present a model similar to ours, with multiple sectors, heterogeneous price stickiness, and input-output linkages. They consider a shock that shifts demand away from services and towards goods and find that in their model such a shock can explain a large share of the inflation seen in the pandemic recovery. Nonetheless, our analysis focuses on testing the importance of input-output linkages in generating heterogeneous price changes at the cross-sectional level. We disentangle sectoral supply and demand shocks using the observed price changes and then study the factors that contribute to the inflation surge in 2021-2022.

The remainder of the paper is organized as follows. Section 2 presents a production network model with sticky-price and derives predictions for how sectoral prices respond to aggregate and sectoral shocks. Section 3 briefly describes the data used in our empirical tests and in the calibration of our quantitative model. Section 4 describes the results of our various empirical tests of the
model’s predictions for sectoral prices. Section 5 presents our quantitative exercise in which we calibrate a modified version of our model to the pandemic period, and run counterfactual analyses to estimate the contribution of the production network and of different shocks to inflation. Finally, Section 6 concludes.

2 Price Distribution in An Input-Output Economy

In this section, we discuss the I-O model prediction of price response to shocks that will be evaluated in Sections 4 and 5. Our model is based on the the I-O model with price rigidity presented by La’O and Tahbaz-Salehi (2022), with the CES production function and separated labor markets across sectors.

2.1 Model Set-up

Consider a static economy with \( N \) sectors. In each sector \( i \), a continuum of firms \( k \) (distributed from 0 to 1) produce differentiated products. They face a CES production function, given by

\[
y_{ik} = z_i \left[ \alpha_i^{1/\theta} l_{ik}^{\theta-1} + (1 - \alpha_i)^{1/\theta} \sum_j \omega_{ij}^{1/\theta} x_{ij,k}^{\theta-1} \right]^{\theta/(\theta-1)},
\]

where \( l_{ik} \) is the labor input, \( x_{ij,k} \) denotes the amount of intermediate inputs produced by sector \( j \) used by the firm. \( z_i \) is the sectoral TFP shock. \( \alpha_i \) is the labor share. \( \omega_{ij} \) refers to the share of input \( j \) over the total intermediate inputs. \( \theta \) is the elasticity of substitution across production inputs. Also, there is a final producer in each industry that aggregates these differentiated products to one sectoral product. Appendix B.1 provides additional details on the production sector of the model.

We introduce nominal rigidities using the noisy information structure following La’O and Tahbaz-Salehi (2022). A fraction \( \phi_i \) of firms in industry \( i \) receive perfect information about the realized shocks, while a fraction \( 1 - \phi_i \) of firms in industry \( i \) receive no signals during a given time period.\(^2\) Thus, the sectoral price \( p_i \) deviation from the steady state level follows:

\[
\tilde{p}_i = \phi_i m c_i.
\]

\(^2\)The nominal rigidity set-up is close to the Calvo setting (similar as Rubbo, 2020) and the solution of the sectoral price vector (Equation 1) is the same under this alternative price setting.
Marginal cost changes \((\tilde{mc}_i)\) will only partially pass-through into prices, and \(\phi_i\) proxies the sectoral level of price stickiness. The higher is \(\phi_i\), the more flexible price adjustment is. Variables with tilde refer to log deviation from steady state values.

Households solve a standard utility maximization problem with risk aversion \(\gamma\) and the elasticity of labor supply \(\eta\). The consumption bundle aggregates differentiated sectoral products by a CES aggregator with elasticity of substitution \(\varepsilon\). The consumption share of each product \(i\) is denoted as \(\beta_i\). Additional details are provided in Appendix B.1.

### 2.2 Propagation of Shocks

In this subsection, we discuss the model predictions on the propagation of shocks that we will evaluate in the empirical analysis.

First, we denote the nominal expenditure of the economy as \(E \equiv CP\). We denote the Domar weight of sector \(i\) as \(\lambda_i \equiv \frac{p_i y_i}{E}\), which is the sales share as a fraction of GDP. \(\Phi\) denotes a diagonal matrix of price stickiness with diagonal element \(\phi_i\). \(D(\alpha)\) denotes a diagonal matrix with diagonal element \(\alpha_i\). Moreover, we define two I-O matrices. The input matrix \(\Omega\) has entries \((1 - \alpha_i)\omega_{ij}\), which capture the amount of \(j\) used as an input in producing product \(i\) (i.e., \(\frac{\text{Sales}_{j \rightarrow i}}{\text{Sales}_i}\)). The output matrix \(\hat{\Omega}\) has entries \((1 - \alpha_i)\omega_{ij}\frac{\tilde{p}_i}{\tilde{p}_j}\frac{\tilde{y}_i}{\tilde{y}_j}\), which corresponds to the proportion of sales from \(j\) to \(i\) relative to the total sales of \(j\) (i.e., \(\frac{\text{Sales}_{j \rightarrow i}}{\text{Sales}_j}\)). Variables with bars refer to steady state values. Variables without subscript \(i\) refer to vectors of the corresponding sectoral values. For instance, \(\tilde{p}\) refers to a vector of \(\tilde{p}_i\).

We focus on the changes of sectoral product prices subject to three types of shocks (1) sectoral TFP shocks, \(\tilde{z}_i\), (2) sectoral demand shocks \(\tilde{\beta}_i\), and (3) aggregate expenditure shocks, \(\tilde{E}\). And, we consider first-order approximations of the endogenous equilibrium.

**Proposition 1.** Changes in the sectoral product prices and sectoral wages can be approximated to the first order by the following equations:

\[
\tilde{p} = \Phi(I - \Omega \Phi)^{-1}(D(\alpha)\tilde{\omega} - \tilde{z})
\]

\[
\tilde{\omega} = \left(\frac{\theta - 1}{\theta}\right)\tilde{z} + \frac{1}{\theta}(\tilde{\lambda} + \tilde{E}) + \left(\Phi^{-1} - \frac{1}{\theta}I\right)\tilde{p} - \frac{1}{\theta}\tilde{l}
\]
and the Domar weight and sectoral labor responses follow

\[
\hat{\lambda} = [I - \hat{\Omega}']^{-1} \left\{ (\theta - 1)\hat{\Omega}' \hat{z} + D \left( \beta \frac{\hat{z}}{\lambda} \right) \hat{p} + Q\hat{p} - D \left( \beta \frac{\hat{p}}{\lambda} \right) (1 - \varepsilon)\hat{P} \right\},
\]

(3)

\[
\hat{\ell} = -\gamma \eta \hat{E} \cdot 1_N + \eta (\gamma - 1) \hat{P} \cdot 1_N + \eta \hat{w},
\]

(4)

with aggregate price \( \hat{P} = \beta' \hat{p} \) and \( Q \equiv \left[ \hat{\Omega}' \left( \theta \Phi^{-1} - I \right) + D \left( 1 - \theta + (\theta - \varepsilon) \bar{\beta} \lambda \right) \right] \).

Three effects are responsible for the propagation of shocks in this economy: (1) the I-O structure, \( \Omega \) and \( \hat{\Omega}' \), (2) price stickiness, \( \Phi \), and (3) elasticity of substitution in production, \( \theta \). All types of shocks propagate in the I-O economy and impact sectoral prices through these three effects. In particular, we can express the sectoral price change vector as:

\[
\hat{p} = \Theta^{-1} \left\{ -A_z \hat{z} + A_\beta \hat{\beta} + A_E \hat{E} \cdot 1_N \right\},
\]

(5)

with \( \Theta = [I - \frac{\theta^{-1}}{1 + \theta^{-1}} \Phi(I - \Omega \Phi)^{-1} D(\alpha)F]^3 \).

\[
A_z \equiv \Phi(I - \Omega \Phi)^{-1} D(\alpha) \left[ D(\frac{1}{\alpha}) - \frac{1 - \theta^{-1}}{1 + \theta^{-1}} - \frac{1}{1 + \theta^{-1}} (I - \hat{\Omega}')^{-1} \hat{\Omega}' \right],
\]

\[
A_\beta \equiv \frac{\theta^{-1}}{1 + \theta^{-1}} \Phi(I - \Omega \Phi)^{-1} D(\alpha) (I - \hat{\Omega}')^{-1} D \left( \frac{\beta}{\lambda} \right),
\]

\[
A_E \equiv \frac{\theta^{-1}(1 + \gamma \eta)}{1 + \theta^{-1}} \Phi(I - \Omega \Phi)^{-1} D(\alpha),
\]

Ignoring higher order terms from \( \Theta^{-1} \), we have \( A_z, A_\beta \) and \( A_E \) capturing the sensitivity of sectoral prices to sectoral TFP, sectoral demand and aggregate expenditures shocks correspondingly.

(1) The I-O Structure. All shocks propagate upstream and downstream through the supply chain. On the one hand, shocks transmit downstream through \( (I - \Omega)^{-1} \), the Leontief inverse matrix. Mathematically, this matrix equals the geometric summation of the input matrix \( \Omega \) and captures the direct as well as indirect uses of inputs in the production network. Thus, the Leontief inverse matrix captures how sectoral price changes propagate downstream through costs. For example, a TFP shock of a supplier \( j \) affects the price of customer \( i \). The direct impact of the shock depends on the input share of production \( j \) in the production of \( i \) (captured by \( \omega_{ij} \)). The indirect impact of the shock may depend on a third sector \( k \) that uses product \( j \) and supplies sector \( i \), which

\[
\Phi \equiv \left[ (I - \hat{\Omega}')^{-1} Q + (\theta \Phi^{-1} - I) - \eta (\gamma - 1) \left[ I + \frac{1}{\eta(\gamma - 1)} (I - \hat{\Omega}')^{-1} D \left( \frac{\hat{z}}{\lambda} \right) (1 - \varepsilon) \right] B \right].
\]

And, \( B \) being an \( N \times N \) matrix with each row equals \( \hat{\beta}' \).
is captured by \( \omega_{ik}\omega_{kj} \).

On the other hand, shocks transmit upstream through \( (I-\hat{\Omega}')^{-1} \), the sale-based Leontief inverse matrix. It reflects the upstream propagation of shocks and reflects how changes in sales propagate upstream through demand. In Section 4 we separately test the upstream and downstream propagation of shocks empirically.

(2) Price Stickiness. Price rigidity directly affect price response to marginal cost changes. In addition, the I-O structure amplifies price stickiness due to strategic complementarity, and price rigidity restricts the downstream pass-through of prices. Equation 1 shows that \( \Phi \) plays an important role in the sectoral price responses to cost changes. Indeed, \( (I-\Omega\Phi)^{-1} \) is the price-flexibility-adjusted Leontief inverse matrix. Similar to the Leontief inverse matrix, it reflects the downstream propagation of shocks through the input matrix \( \Omega \). However, price rigidity has no direct effect on the upstream pass-through of prices, but it indirectly affect the upstream propagation through its impact on the Domar weight captured by the high order terms in \( \Theta \).

(3) Elasticity of Substitution. The elasticity of substitution affects sectoral price responses to sectoral shocks. As discussed in the literature on the micro origin of macro fluctuations (e.g. Atalay, 2017; Horvath, 2000), a lower elasticity of substitution across sectoral products (as production inputs) leads greater sectoral comovements. Empirical estimates of this elasticity are very low, suggesting that sectoral products are complements (see Atalay, 2017). Equation 5 indicates that a lower elasticity of substitution generates greater sectoral price response to all types of shocks. Indeed, the sensitivity values given by \( A_z, A_\beta \) and \( A_E \) decrease with \( \theta \).

For example, a clear finding is that TFP shocks only propagate downstream in the Cobb-Douglas setting (i.e. \( \theta = 1, \varepsilon = 1 \)). Nonetheless, in a CES setting, sectoral TFP shocks also propagate upstream. In addition, a reduction in \( \theta \) amplifies both the upstream and downstream propagation of TFP shocks. This has an intuitive explanation: Suppose both sector \( i \) and sector \( k \) supply intermediate inputs to sector \( j \). If the input elasticity of sector \( j \), \( \theta_j \), is large, sector \( j \) can easily substitute towards product \( i \) if \( p_k \) increases, which weakens the impact on sector \( j \)’s price. If, instead, \( \theta_j \) is small, sector \( j \) has limited ability to substitute its inputs away from \( k \), and the effect on \( p_i \) is larger. Simply put, the transmission of shocks across sectors is stronger when the flexibility of adjusting the input structure is weaker, which is the case when the elasticity of substitution is

\[ \text{In a Cobb-Douglas setting, the } \hat{\Omega}' \text{ term in } A_z \text{ disappears.} \]
low. Through the same intuition, we predict that a lower elasticity of substitution across sectoral products also generates larger sectoral price dispersion in general.

*Sectoral Price Response to Aggregate Shocks.* Equation 5 also shows that the sensitivity of the sectoral price change distribution to aggregate shocks depends on the following important vector

\[ \Psi \equiv \Phi(I - \Omega \Phi)^{-1} \alpha. \]

We further denote the \( i \)-th entry of \( \Psi \) as \( \psi_i \). In the next section, we will empirically evaluate the relationship between the distribution of sectoral price changes, and the distribution of entry values in this vector.\(^5\)

Last but not least, it is worth noting that sectoral wage heterogeneity is crucial in generating upstream propagation of shocks. As shown in the next subsection, in an economy with homogeneous wages, the wage rate does not depend on the Domar weight. Thus, aggregate expenditure shocks and sectoral TFP shocks affect prices through downstream propagation only, while sectoral demand shocks have no price impact. However, in Section 4, we find that demand shocks affect sectoral prices through both upstream and downstream propagation in the empirical analysis. Thus, wage heterogeneity is important in generating price change patterns that are consistent with the data.

### 2.3 Aggregate Inflation Impact of Shocks

In this subsection, we study the aggregate price response to shocks. In particular, we focus on the role of the I-O structure. The predictions of this subsection have implications for the quantitative analysis of the high inflation episode from 2021 to 2022 presented in Section 5. In order to illustrate the aggregate inflation response in clean analytical expressions, we consider economies with homogeneous wages across sectors in this subsection.

First, we find that the I-O structure attenuates the aggregate price impact of aggregate expenditure shocks, thus it amplifies monetary non-neutrality. The aggregate price response to aggregate

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\(^5\)Price responses to aggregate supply shocks is captured by \( A_z \), which contains an additional term in the square bracket other than \( \Psi \). To the extent that aggregate demand shocks have more important factors in price setting than aggregate supply shocks, we focus on \( \Psi \) to investigate the connection between the I-O structure and the price change distribution.
expenditure shock in this economy follows,

\[
\frac{\partial P}{\partial E} = \frac{(\eta^{-1} + \gamma)\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)\mathbf{1}}{1 + [\eta^{-1} - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)\mathbf{1}},
\]

which is consistent with the prediction of La’O and Tahbaz-Salehi (2022).

To investigate the role of the I-O structure, we compare the benchmark economy with a horizontal economy. In the absence of an I-O structure, the horizontal economy has \( \Omega^H = 0 \), \( \alpha_i^H = 1 \) and the same level of price adjustment flexibility \( \phi_i \). The equilibrium of this economy depends on the choice of the consumption share. We set \( \beta^H = \beta \). It is straightforward to show that the response of aggregate prices to aggregate expenditure shocks is larger in the horizontal economy than in the I-O economy. Thus, the I-O structure increases monetary non-neutrality. This result has been discussed by various papers on price rigidity in an I-O economy (see Afrouzi and Bhattarai, 2022; La’O and Tahbaz-Salehi, 2022; Rubbo, 2020). Fundamentally, it is the strategic complementarity in firms’ price-setting behavior that amplifies monetary non-neutrality. Intermediate inputs in the I-O structure generate a substantial amount of strategic complementarity (see Nakamura and Steinsson, 2010).

In addition, the I-O structure generates ambiguous effect on the aggregate price response to TFP shocks. The aggregate price response to sectoral TFP shocks follows,

\[
\frac{\partial P}{\partial z} = \frac{\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)B_w}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)\mathbf{1}} - \beta'\Phi(I - \Omega\Phi)^{-1},
\]

where \( B_w = [\beta'_w; \cdots; \beta'_w] \) with \( \beta_w \equiv \eta(\gamma - 1)\beta'\Phi(I - \Omega\Phi)^{-1} - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)(I - \Omega)^{-1} \).

Unlike the response to aggregate expenditure shocks, the I-O structure does not always attenuate the aggregate price response to TFP shocks. As discussed in Appendix B.4, the I-O structure amplifies the aggregate price response to the TFP shocks of the upstream sectors and attenuates the aggregate price response to the TFP shocks of the downstream sectors when \( \beta^H = \beta \). In the I-O economy, the centrality of upstream sectors, as a measure of their relative importance to the aggregate economy, is larger than it is in the horizontal economy. Also, the downstream propagation of supply shocks is stronger than upstream. Thus, the I-O structure amplifies the supply shocks of upstream sectors. However, this finding depends on the construction of the horizontal economy,
especially $\beta_H$.\textsuperscript{6,7}

Another important factor that affects sectoral price and aggregate price responses to shocks is the role of distortions. There are two types of distortions that result in an inefficient allocation of resources and decreases productivity. First, price dispersion created by price rigidity generates within-sector distortions. Second, the wedge between sectoral prices and marginal costs creates across-sector distortions. We present only results based on first-order approximations in the current section. Thus, within-sector distortions disappear. In Section 5, we solve an exact version of the model in levels, and we postpone the discussion about distortion to that section.

3 Data

In order to test the model’s implications on sectoral price setting and the response to different shocks, we collect data by industry from a number of sources. Most of the empirical analysis is based on the industries in the detailed I-O tables produced by the Bureau of Economic Analysis (henceforth BEA), and we will refer to these as detailed industries or I-O detailed industries. The quantitative exercises in Section 5 are based on a model calibrated to a broader set of industries, which we will refer to as industry groups. Roughly speaking, detailed industries correspond to 6-digit NAICS industries, while industry groups correspond to 3-digit NAICS industries. This section describes in detail the data that we use and how we match industries across different data sources.

3.1 I-O Structure and Price Stickiness

We use the make-use tables of the I-O Accounts created by the Bureau of Economic Analysis (BEA) to estimate the economy’s I-O structure.\textsuperscript{8} The detailed make-use tables are published every

\textsuperscript{6}If instead, we set $\beta_H = [\beta'(I - \Omega)^{-1}D(\alpha)]' = [\lambda D(\alpha)]'$. The horizontal economy with this $\beta_H$ is allocationally equivalent to the benchmark I-O economy at the steady state, with identical sectoral labor allocation and an identical level of aggregate output. Under this setting, the I-O structure amplifies the aggregate price response to the TFP shocks of the downstream sectors and attenuates the aggregate price response to the TFP shocks of the upstream sectors. Appendix B.4 provides additional details.

\textsuperscript{7}Afrouzi and Bhattarai (2022) also find that negative supply shocks in the “computers and electronics industry” propagates downstream like a markup shock to these downstream sectors. The I-O structure amplifies this propagation effect.

\textsuperscript{8}The BEA report a “make” table and a “use” table that consists industry-commodity production and usages. We convert the “make” and “use” tables to the industry-by-industry I-O matrix following the approach of Pasten et al.
five years, and with a long lag. At the time of writing, the latest available set of tables is from 2012. The exact categorization of industries can vary slightly across releases of the I-O tables. These tables report the use and production of commodities for each of 417 industries, in dollar terms. We can therefore use these tables to estimate the intermediate input shares that determine the I-O structure in the model. The tables also report the total sales, value added, and labor compensation of each sector, along with other variables relevant to the national income accounts.

In addition to the I-O structure, we seek measures of prices, wages, employment, and price stickiness for each industry. We use the frequency of price change estimates by industry of Pasten et al. (2020), and which those authors generously made available to us. These estimates are based on the micro data underlying the U.S. Producer Price Index (PPI), and are reported by detailed industry in the 2002 I-O tables. For this reason, our empirical analysis is based on the detailed industries in the 2002 tables. There are frequency of price change estimates for 341 detailed industries, and we have been able to match 337 of those to an adequate price index. These 337 detailed industries form the sample for our empirical analysis. The production network is estimated using the flows reported in the 2002 I-O tables using only those 337 detailed industries. Based on the 2002 I-O tables, these 337 industries account for 60% of total value added in the economy, and 73% of value added excluding the federal government.

3.2 Sectoral Prices

Sectoral prices are taken from the Bureau of Labor Statistics (BLS)’s Producer Price Index (PPI) program. The PPI seems most appropriate for our analysis for two reasons. First, the PPI is reported by industry, which makes it considerably easier to match with I-O detailed categories. Second, and perhaps more importantly, the PPI aims to measure the prices of goods and services produced by firms based on their sales to other firms, which is much more closely related to the flows between firms that characterize the production network structure of the economy. As mentioned above, we are able to match 337 detailed industries with relevant PPI industry prices.

At the industry group level, the BEA reports the make-use tables from 1997-2020 at an annual

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9In contrast, the Consumer Price Index (CPI) is reported by consume expenditure category, which is only loosely related to industries. In addition, the CPI is based entirely on consumer prices, which can have very different dynamics from the prices that producers of goods and services charge in inter-firm flows.
frequency. These tables feature 71 industry groups, and we identify 66 that can be merged with the price and stickiness data used in the rest of the empirical analysis. We follow Baqae and Farhi (2022) in matching PPI series to industries somewhat more disaggregated than industry groups, and then compute price changes by industry group as the weighted average of the corresponding PPI price changes using industry sales as weights. To obtain estimates of the frequency of price change by industry group, we use the detailed industry frequency estimates and collapse them to industry group using industry sales as weights. Because some of the industry groups do not have a relevant frequency estimate available, we augment our list of estimated frequencies with estimates from the CPI used in Nakamura et al. (2018) and Luo and Villar (2021b). We do this for industry groups in education, entertainment, restaurants, and a few other services industries.

### 3.3 Employment and Wages

Measures of wages and employment are taken mostly from the BLS’ Current Employment Statistics (CES, also known as the establishment survey), which publishes estimates of payroll employment, hours, and average hourly earnings. Employment is measured as total payroll employment by industry, and wages are measured using average hourly earnings for production and nonsupervisory workers.

For our quantitative analysis based on broader industry groups, we need to go beyond CES to measure wages and employment, as some industry groups that we use in the quantitative analysis are not measured in CES. For those industry groups, we turn to the Quarterly Census of Employment and Wages (QCEW), which collects counts of employment and total earnings for all workers covered by the unemployment insurance system. This covers the near-universe of employment in the U.S, and gives estimates of employment and average weekly wages for all NAICS industries as narrow as 6-digit.

### 3.4 Measures of Shocks

In order to test the price responsiveness of different sectors to aggregate demand shocks, we use monetary policy shocks estimated with the high-frequency approach of Swanson (2021). These shocks are constructed based on changes in interest rates and asset prices in a short period sur-
rounding scheduled Federal Open Market Committee (FOMC) announcements. The shocks are decomposed into separate components corresponding to policy rates, forward guidance, and large-scale asset purchases (LSAPs), and we will consider these shocks separately. The shocks are available for the period 1991-2019, with about eight observations per year, each corresponding to an FOMC meeting. We sum observations in each quarter to create a quarterly series that we use in our analysis. An important advantage that these shocks have over the monetary shocks constructed by Romer and Romer (2004) is that they can be estimated during the zero lower bound period between 2009 and 2015. Because many of the industries in our sample only have PPI data starting in 2003, extending the sample past 2009 is very helpful.

We also test the upstream and downstream propagation of shocks, or the extent to which firms in industries connected to a given industry through the production network affect the sectoral price in that industry. As shown by Proposition 1, our model predicts that the industry-specific shocks facing an industry can affect the sectoral price of any other industry through I-O linkages. This type of analysis requires demand and supply shocks measured by industry, and we use the trade and TFP shocks also used by Acemoglu et al. (2016a) who similarly study how the propagation of these and other shocks affect employment and output in different industries. Trade shocks consist of the change in Chinese import penetration by industry. Chinese import penetration is defined as the value of imports from China as a share of the size of the U.S. market (industry shipments plus net industry imports, measured in an initial period which in our case is 1991) for a particular industry:

\[ \text{Trade}_{j,t} = -\Delta \frac{\text{U.S. Imports from China}_{j,t}}{\text{U.S. market size}_j} \] (6)

Because changes in Chinese imports are potentially correlated with various U.S. industry factors, we also consider specifications in which we instrument for this trade shock with Chinese exports to other developed countries as in much of the large literature on the “China trade shock” which has found numerous effects of increasing imports from China on various aspects of the U.S. economy and society.\(^\text{10}\) We construct trade shocks using the trade and market size data featured in Acemoglu et al. (2016b) and Autor et al. (2019), and TFP shocks as changes in TFP as measured in

\(^{10}\)To cite just a few examples in this literature: Acemoglu et al. (2016b); Autor et al. (2013, 2019); Jaravel and Sager (2022). Jaravel and Sager (2022) study the effect of increased trade with China on consumer prices in the U.S.. We instead focus on how these changes to trade patterns, by affecting demand in different industries, propagate to affect prices according to the production network of the economy.
the NBER-CES manufacturing database (Becker et al., 2021), following Acemoglu et al. (2016a). Finally, note that these shocks can only be constructed for manufacturing industries. As a result, this analysis is restricted to manufacturing industries.

With the data available, we obtain annual measures for these different shocks for 266 detailed I-O industries for the period 1991-2014. Although our main analysis is based on annual data, we also consider shocks based on changes in prices and trade patterns over longer periods of time. We specifically also consider “long-difference” versions of these shocks, which are changes in Chinese import penetration over five years, for a time sample covering 1991-2011. The longer differences can potentially smooth through some of the factors that drive year-to-year changes in trade patterns and prices that might represent noise from the perspective of our analysis. Here we also follow some of the literature on Chinese import competition (such as (Acemoglu et al., 2016a,b; Jaravel and Sager, 2022)) in considering changes in Chinese import penetration over longer time horizons.

4 Empirical Analysis

In this section, we empirically evaluate the model’s predictions on how the production network structure affects the price change distribution in an environment with price frictions. Balke and Wynne (2000) were the first to evaluate how the network structure affects the cross-sectional distribution of prices, although in a flexible price setting. Following this line of work, we further investigate the interaction between the network structure, price rigidity, and price setting. We conduct a series of tests to determine whether sectoral prices respond to aggregate and sectoral shocks in a way that is consistent with the model predictions, and to assess the presence of upstream and downstream propagation of shocks on sectoral prices.

4.1 The Distribution of Price Changes and the Network Structure

We first look for evidence that the network structure affects the distribution of price changes over time. Section 2.2 shows that the vector $\Psi$ connects the network structure with the vector of sectoral price changes. If there are only aggregate shocks, the direct effect of shocks works through $\Psi$. Thus, we would expect the distribution of sectoral price changes to correlate with the distribution of $\psi_i$. We empirically investigate this prediction through two exercises.
First, we study whether moments of the distribution of sectoral price change correlated with those of $\Psi$ overtime. Figure 1 presents the scatter plot of the moments of the distribution of price changes and those of $\psi_i$ for the more aggregated industry groups.\textsuperscript{11} Each year from 1997-2020, we have 66 observations of sectoral price changes and 66 $\psi_i$. For price changes, we first calculate moments of the price change distribution each month. Then, we construct moments at annual frequency by averaging moments at the monthly level within each year. We compare the moments price change and the moments of the $\psi_i$ distribution. The three moments we calculate are, the standard deviation, the absolute value of skewness, and the kurtosis.\textsuperscript{12} Notice that we have a constant price flexibility measure overtime, but time-varying I-O structure. Thus, this exercise, in particular, reflects the role of the I-O structure in the price change distribution.

Figure 1 shows how the evolution of the network structure affects price change patterns over time, using $\Psi$ as measures of the model-implied price sensitivity. We see a positive relation for the standard deviation, skewness, and kurtosis of log price changes. These relations suggest that the network structure does have an influence over the distribution of price changes. It is also worth noting that in practice various sectoral shocks are likely to be important in shaping the distribution of price changes, which would obscure the relation with the network distribution. Furthermore, it is worth noting that the kurtosis of price changes, or of any distribution, is difficult to estimate precisely in samples as small as the one we are using here (some of the small sample issues likely also apply to estimates of the skewness). As a result the empirical relation for kurtosis shown in Figure 1 is likely more affected by measurement error than for the other moments. Nevertheless, there does appear to be a positive relation even for the kurtosis.

Thus, the I-O structure has a significant impact on the shape of the price change distribution. Our finding is in line with research on the interaction between the sectoral structure and the price change distribution. First, Balke and Wynne (2000) find that an input-output economy with flexible prices can generate the observed positive correlation between inflation and skewness of sectoral price changes. Therefore, the observed moments of the price change distribution do not purely

\textsuperscript{11}We use the more aggregated industry groups instead of the detailed industries for this analysis because estimates of the I-O tables are available at the annual frequency for industry groups, but only every five years for detailed industries.

\textsuperscript{12}We choose the absolute value of skewness as the third moment investigation, because skewness of the price change distribution changes signs over time depending on the direction of shocks. However, skewness of $\psi_i$ is always positive.
reflect sluggishness in the adjustment of individual prices in response to shocks. Moreover, recent work by Cotton and Garga (2022) shows that the shift in industrial composition contributes to the reduction in monthly frequency of price change, since the economy shifts from primary and secondary industries toward service industries.

Second, we study the cyclicality of the correlation between sectoral price changes and $\Psi$. The model predicts that the correlation should be positive when demand grows, and negative when demand contracts, so the correlation should be pro-cyclical. For this exercise, we use the estimates of $\Psi$ and price changes by detailed industry. $\Psi$ is based on the 2002 BEA I-O tables and is fixed at the 2002 value. Sectoral price changes are monthly from Jan 1999 to May 2022, and we use 12-month log price changes. As a snapshot, Figure 2 shows a scatterplot of $\Delta p$ against $\Psi$ at two specific time periods. Panel (a) shows a strong negative correlation between sectoral price changes and $\Psi$ at July 2009, during the depths of the Great Recession. Panel (b) shows a strong positive correlation between $\Delta p$ and $\Psi$ at April 2021, as the pandemic recovery was starting. Moreover, Figure 3 plots the correlation between sectoral price changes and $\Psi$ over time. We find that the correlation is highly procyclical. The cyclicity essentially follows the cyclical movements of aggregate price levels in general. Both figures support the prediction that the network structure plays an important role in determining the sectoral price response to shocks. In addition, we also plot the correlations of $\Delta p$ and version of $\Psi$ without price rigidity (i.e. with $\Phi = I$). Noticing the gap between the solid and dashed curves in Figure 3, this suggests that heterogeneity of the sectoral price rigidity also plays an important role in the sectoral price change distribution.

We attempt to more formally assess the cyclicality of the correlation between price changes and
Ψ by relating this correlation to changes in employment. We compute the correlation for 3-month log changes of detailed industry prices, giving us a quarterly series of the correlation between price changes and the network structure. To determine whether this correlation is cyclical, we construct a measure of employment in the 337 detailed industries in our sample.\footnote{The employment measures is based on changes in employment among industries that remain in the sample. This helps deal with the fact that some industries enter the sample after others.}

Figure 4 shows a scatter plot with the correlation between 3-month price changes and the network structure vector on the x-axis, against 3-month log changes in our measure of industry employment. Although employment changes are relatively small in most periods, there is a positive relation.\footnote{Because our sample of industries over-represents manufacturing industries relative to the overall economy the cycle implied by this employment measure is somewhat different from the overall U.S. economic cycle. Notably, much of the early part of our time sample sees a steady decline in employment as manufacturing employment was declining during this period.} In particular, the recessionary periods in which employment declined the most clearly featured a negative correlation between price changes and the network structure. Note that the presence of sectoral shocks would also tend to obscure this relation, as they would lead to sectoral prices changing in a way not necessarily related to the network structure, even conditional on the state of the business cycle.

Besides providing some evidence to support the model’s predictions, these patterns are also relevant to the literature on higher order moments of the price change distribution. For example, studies such as Baley and Blanco (2021) and Alvarez et al. (2016) find that moments such as the dispersion
sion and kurtosis of the adjustment distributions of individual economic agents (price setting in the case of Alvarez et al. (2016), capital adjustment for Baley and Blanco (2021)) provide important information about how the overall economy adjusts to aggregate shocks. Most of the studies that use moments of price changes to evaluate monetary non-neutrality or sticky price models are abstracted from the production network structure (see for example Costain and Nakov, 2011; Luo and Villar, 2021a; Midrigan, 2011; Nakamura and Steinsson, 2008, and many others). Our model predicts that the I-O linkages across sectors of the economy can affect the shape of the distribution of sectoral price changes, and the empirical correlations that we present suggest that there is some support for this in the data. This implies that the structure of the economy’s production network could matter for how we interpret information from the shape of the distribution of price changes. In this way, the observed price change distribution reflects more than the sluggish price-setting behavior of firms.  

**4.2 Responses of sectoral prices to monetary policy shocks**

In this section we estimate the sensitivity of sectoral prices to monetary policy shocks and evaluate whether these sensitivities are consistent with the model-implied sensitivities $\Psi$. We take this

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15Papers that use empirical estimates of higher moments to draw inference about aggregate responses to shocks, such as Alvarez et al. (2016) and Baley and Blanco (2021), focus on the distribution of individual and not sectoral responses. Furthermore, Alvarez et al. (2016) estimate the kurtosis of price changes controlling for sectoral heterogeneity. Even though sectoral factors do not necessarily impact estimates of the distribution of price changes or adjustments across individuals, we believe that our results still have some relevance. Indeed, our results have shown how input-output linkages across sectors can affect the distribution of price changes across sectors. Although our analysis is based on sectors, it is also likely that input-output linkages exist across individual firms even within sectors. Our analysis suggests that such linkages could also affect the distribution of adjustments across individual firms.
prediction to the data by estimating the price response of the 337 I-O detailed categories to the monetary shocks constructed by Swanson (2021). The price response is estimated using local projections following Jordà (2005), based on the following sets of regressions:

$$\pi_{k,t-1,t+h} = \alpha + \beta_{k,h} \eta_t + \sum_{j=1}^{3} \delta^j \pi_{k,t-j} + \sum_{j=1}^{3} \gamma^j \eta_{t-j} + \beta^{Controls} X_t + \varepsilon_t,$$

for h=0,...,12. $\pi_{t-1,t+h}$ is the change in the log price level for detailed industry k between period t-1 and t+h, $\pi_k$ is the log change in price k between t and t-1, and $\eta_t$ is the exogenous monetary policy shock. $X_t$ is a vector of controls, which include credit spreads and changes in commodities prices.\(^\text{16}\) The regression is run separately for each detailed industry on quarterly data for all periods available in each industry. We are interested in the sets of parameters $\beta_{h,k}$, which give the cumulative price response of industry k after h quarters. To allow for the possibility that the price response reverts at least partially to zero before 12 quarters, for each industry we keep the $\beta_{h,k}$ with largest magnitude beyond $h = 4$.

Figure 5 shows scatter plots between the industry’s $\Psi$ and the peak $\beta_{h,k}$, for the three different monetary shocks estimated by Swanson (2021). When estimated using the policy rate and LSAP

\(^{16}\)Studies such as Gertler and Karadi (2015) have shown that a key source of transmission of monetary shocks is through credit costs. In addition, commodity prices are important costs for the manufacturing industries that compose a large share of our sample, and commodity prices are likely also affected by financial conditions. This leads us to control for credit spreads and commodity prices in the local projections, but results are broadly similar if no controls are used.
shocks, there is no relation between and industry’s $\Psi$ and the responsiveness of its sectoral price. However, using the forward guidance shock there is a clear negative relation between $\Psi$ and the peak impulse response. This is consistent with the model’s predictions: the industries with the most price sensitivity to aggregate shocks see their prices decline the most in response to this identified monetary shock.

Of course, our results also show that the expected relation between theoretical and empirical price sensitivity does not hold for all types of monetary shocks. Clearly, the evidence in favor of the model’s predictions would be more compelling if the same relation held for all types of monetary shocks. However, we believe that it is still important that the forward guidance shock produces the predicted relation, as we have some reasons to focus on this shock. First, as mentioned in Section 3, many of the industries in our sample have price data available only after 2003, making the zero lower bound period between 2009-2015 especially important to estimate the impulse responses. During this period, the decomposition of shocks carried out by Swanson (2021) into policy rate, forward guidance, and LSAP components seems particularly important. Policy rates obviously did not change while at the ZLB, and unsurprisingly the policy rate shocks are smaller during this period. It is therefore not surprising that we see no results with these shocks.\footnote{We also carry out this analysis with the monetary shocks estimated by Nakamura and Steinsson (2018) and Jarociński and Karadi (2020), who also use high-frequency identification. Jarociński and Karadi (2020) estimate separate policy rate and “information” components, where the information component represents news revealed about the FOMC’s expectations about the economy. We include scatter plots similar to those in Figure 11 in the Appendix, but do not find the expected relation between impulse response and $\Psi$ across industries. These shocks mostly also produce counter-intuitive effects on aggregate prices, which we show in the Appendix Figure 12.}

We also find it informative to consider the impulse responses of aggregate price levels to the
different shocks to determine which shocks are providing information on aggregate demand relevant to prices. These impulse responses are shown in Figure 6 for two measures of aggregate prices: core PCE prices and the core goods PPI. Both aggregate prices rise in response to the policy rate shock. They also rise modestly in response to the LSAP shock, although core PCE prices seem to decline very slightly about three years after the shock. In contrast, both price indexes decline modestly in response to the forward guidance shock, as expected if these shocks represent an expected increase in interest rates that lowers demand. We therefore find it reassuring that this shock produces more negative industry-level impulse responses for industries with a large $\Psi$ as seen in Figure 5, consistent with the model predictions.

It is also worth highlighting from Figure 5 that many sector’s prices respond positively even to the forward guidance shock, contrary to what one would expect. There is a large literature that has studied the positive response of certain prices to contractionary monetary shocks (often referred to as the “price puzzle”). Our analysis is based on the response up to four years after a shock, which is longer than the horizon over which the price puzzle is usually observed, and some of the positive responses that we estimate are possibly noise as the estimates are not always very precise. It is also possible, however, that certain industries face demand or financing conditions that make them more likely to respond positively to monetary shocks. Balke and Wynne (2007) had already documented that there is considerable dispersion in the response of sectoral prices to monetary
shocks. Mandel et al. (2019) showed that in a production network model with capital requirement constraints some firms will raise prices in response to a monetary tightening, or lower prices after a monetary loosening.

Overall, we find some evidence in support of the model’s prediction about which sectors should be most responsive to aggregate demand shocks. However, the ambiguity of how prices in general respond to these monetary shocks makes the evidence less than fully compelling.

### 4.3 Propagation of shocks through the network structure

We now turn to testing the model’s predictions on the propagation of shocks through the production network. So far in this section we have looked at whether the key network structure vector explains how sectoral prices respond to aggregate shocks, and found some evidence consistent with that. Here, our analysis is based on how one sector’s prices might respond to shocks specific to other sectors to which it is connected through the production network. We will therefore construct “upstream” and “downstream” shocks, which consist of linear combinations of the shocks faced by all sectors, with coefficients determined by the production network.

In particular, we study the propagation of the trade and productivity (TFP) shocks identified by Acemoglu et al. (2016a) and Acemoglu et al. (2016b), as we believe that these variables represent important industry-specific demand and supply shocks that should affect price-setting. As mentioned in Section 3, this analysis is limited to 266 manufacturing detailed industries for which the necessary data is available.

Using the measures of shocks at the detailed industry-level, we construct the measures of upstream and downstream shocks based on the model solutions with the following approach. According to Equation 5, sectoral demand shocks affect prices through $\Theta^{-1}A_\beta$, while sectoral supply shocks affect prices through $\Theta^{-1}A_\sigma$. We can write sectoral price sensitivity to sectoral demand and
supply shocks ($\partial \tilde{p}/\partial \tilde{\beta}$ and $\partial \tilde{p}/\partial \tilde{z}$) as

$$
\frac{\partial \tilde{p}}{\partial \tilde{\beta}} = \frac{\theta^{-1}}{1 + \theta^{-1} \eta} \left[ \Phi D\left(\frac{\alpha \tilde{\beta}}{\lambda}\right)_{own} + \Phi \Omega D\left(\frac{\alpha \tilde{\beta}}{\lambda}\right)_{Downstream\_Prop} + \Phi D(\alpha) \hat{\Omega}' D\left(\frac{\beta}{\lambda}\right)_{Upstream\_Prop} \right] + \text{higher order terms}
$$

$$
\frac{\partial \tilde{p}}{\partial \tilde{z}} = \Phi_{own} (I - \frac{\theta - 1}{\theta + \eta} D(\alpha)) - \frac{\theta - 1}{\theta + \eta} \Phi D(\alpha) \hat{\Omega}' + \Phi \Omega \Phi (I - \frac{\theta - 1}{\theta + \eta} D(\alpha)) + \text{higher order terms}
$$

where the “own” term captures the direct effect from shocks of its own sector, the “Downstream\_Prop” term captures the impacts from shocks to its suppliers; the “Upstream\_Prop” terms captures the impacts from shocks to its customers. Correspondingly, we construct the following three import shock variables

$$
Own^{Trade} = \Phi D\left(\frac{\alpha \tilde{\beta}}{\lambda}\right) \ast (\text{import shock}),
$$

$$
Down^{Trade} = \Phi \Omega \Phi D\left(\frac{\alpha \tilde{\beta}}{\lambda}\right) \ast (\text{import shock}),
$$

$$
Up^{Trade} = \Phi D(\alpha) \hat{\Omega}' \Phi D\left(\frac{\beta}{\lambda}\right) \ast (\text{import shock}),
$$

and the following three TFP shock variables

$$
Own^{TFP} = \Phi \ast (\text{TFP shock}),
$$

$$
Down^{TFP} = \Phi \Omega \Phi \ast (\text{TFP shock}).
$$

$$
Up^{TFP} = \Phi D(\alpha) \hat{\Omega}' \ast (\text{TFP shock}).
$$

It is important to note that the upstream propagation effect of TFP shocks is weaker than the “own” and downstream effects, and the upstream propagation of TFP shocks disappears when $\theta = 1$ (i.e. Cobb-Douglas production technology).

Note that for these calculations we use a matrix $\Phi$ that has been adjusted to reflect an annual frequency of price change to be consistent with the fact that these regressions will be based on annual data (and as the original frequency of price change estimates are at a monthly frequency). These calculations only take into the account the direct I-O linkages between sectors, and not the
indirect effects arising from input industries’ input structure, for example. This can be thought of as a first-order approximation to the model-implied propagation of shocks. Quantitatively, these direct effects are much more important than the indirect effects, given that all entries of \( \Omega \) are smaller than one.

The shocks are similar to the ones constructed by Acemoglu et al. (2016a), but the shocks based on our model take into account all the mechanisms in our model, including price rigidity (which the model in Acemoglu et al. (2016a) does not include). We also consider measures of shocks according to the construction used by Acemoglu et al. (2016a) (we label these ”AAK”). That study found significant upstream effects of demand shocks on employment and production, and significant downstream effects of supply shocks. Both of these results are consistent with the production network model that they presented. Similarly, we will test whether propagation measured in the same way matters for price setting using the following shocks: \( D_{\text{Trade,AAK}} = \Omega \ast (\text{import shock}) \) and \( U_{\text{Trade,AAK}} = \hat{\Omega}' \ast (\text{import shock}) \). Upstream and downstream TFP-AAK shocks are defined in the same way, applying the log change in TFP to the equations above.

We also follow Acemoglu et al. (2016a) in standardizing the trade and TFP shocks by dividing them by their respective standard deviation. Importantly, we implement this standardization before computing the upstream and downstream shocks, and before multiplying by the industry parameters to calculate the “own” shock. As Acemoglu et al. (2016a) note, this ensures the estimated regression coefficients are comparable in the case of the AAK shocks, as they represent the effect of a one-standard deviation increase in an industry’s own shock (for the own shock) or of a one-standard deviation increase in the shock of all customers or suppliers of an industry. Note that this is not the case for the shocks derived from our model, as heterogeneous price rigidity lowers the importance of different industries’ shocks to different degrees. We still test for the significance of the effect of the model-implied upstream and downstream shocks. Finally, we also winsorize the original changes in trade and TFP by setting the values in the lowest 1 percent to equal the first percentile, and setting the values in the highest 1 percent to equal the 99th percentile. Again, we apply this before computing the upstream and downstream shocks.

To illustrate how these different shocks relate to and differ from each other (despite being based on the same underlying variables: changes in trade and in TFP), Table 8 in Appendix shows the correlations between the different trade-based shocks that we use, and Table 9 in Appendix does
the same for the TFP shocks. We emphasize two facts from these tables. First, looking within trade shocks or within TFP shocks, the shocks are generally positively correlated but to varying degrees. The correlations between each model-derived shock and the “AAK” counterpart are particularly strong, which makes sense because they are constructed in a similar way. However, the correlations between “own”, “upstream”, and “downstream” shocks are generally much lower. This illustrates how the production network structure yields significant variation to identify the effect of upstream and downstream shocks separately. The second fact is that the model-implied shocks have a much smaller variance than the corresponding “AAK” shocks. The reason is that the model-implied shocks incorporate price stickiness, which weakens the impact of any given change in demand or supply (as measured by trade or TFP in these cases). This is important to keep in mind when interpreting the magnitude of the estimated coefficients and standard errors for the different shocks.

The Propagation of Import Shocks. Instruments for the own, upstream, and downstream trade shocks are constructed using exports from China to developed countries following the literature on the China trade shock. The regressions for our analysis take the following form:

\[
\Delta p_{i,t} = \delta_t + \sum_{k=1,2} \psi_k \Delta p_{i,t-k} + \beta_k^{own} \text{Own}_{t,t-k}^{Trade} + \beta_k^{up} \text{Up}_{t,t-k}^{Trade} + \beta_k^{down} \text{Down}_{t,t-k}^{Trade} + \epsilon_{i,t} \tag{8}
\]

In the annual regressions, time periods are years, and price changes are calculated as the log change in prices from December to December of the previous year. We include year fixed effects (\(\delta_t\)) to control for aggregate factors, and two lags of each explanatory variable to allow for autocorrelation in inflation and for shocks to have delayed price effects. In the 5-year regressions (which we also denote as ”long differences”), periods are the changes between the following years: 1991, 1996, 2001, 2006, 2011. We use the log price change between the December value of each of those years, and shocks based on changes in import penetration between those years. The long difference regressions do not include lags of price changes or trade shocks, and the model implied shocks are constructed under flexible price setting with \(\Phi = I\).\(^{18}\) We also run similar sets of regressions using the AAK shocks, both annual and long differences. For all of these sets of regressions featuring the trade shocks, we run simple Ordinary Least Squares ones, and two-stage least squares regressions using Chinese trade with other developed countries as an instrument for the trade shocks. The

\(^{18}\)To construct the shocks based on 5-year changes in TFP and trade, we ignore price rigidity as it is unlikely to matter significantly over such a long horizon. However, we also construct trade shocks taking into account sectoral labor shares, consumption shares, and Domar weights, as determined by the model.
regressions featuring the TFP shocks are OLS regressions, as we do not have an instrument for TFP shocks.

Table 1 shows the results of the regressions involving the annual model-derived trade shocks. The table also reports the p-value for the test on the sum of the two lag coefficients for each type of shock being zero. This is trying to capture the significance of the total effect over two years. Based on this, upstream and downstream trade shocks both have significant effects on sectoral price inflation, as predicted by the model. Note that because the trade shock is constructed as the negative of the change in trade exposure, the positive coefficient means that an increase in trade exposure lowers prices, which is as expected. The own shock generally does not have a significant effect. Our baseline results weigh observations based on industry-level value added in 2000. However, the upstream and downstream shocks still have significant effects in unweighted regressions. This sum is significant under all specifications for the downstream shocks. However, the upstream shocks become marginally insignificant when we cluster standard errors, although in the IV regression the coefficient on the second lag of the upstream shock is large and significant, while the coefficient on the first lag is close to zero. Finally, the IV estimates of the downstream and upstream shock coefficients are generally slightly smaller than the OLS estimates. 19

As Acemoglu et al. (2016a) show, their model under the Cobb-Douglas setting predicts that upstream demand shocks should have positive effects on value added and employment, while downstream demand shocks should have no effect, and this is what they find. Due to the different demand structure and price setting constraints, our model features a richer price-setting problem than in the model of Acemoglu et al. (2016a), and thus makes different predictions for the price effect of shocks. In particular, our model predicts that upstream and downstream demand shocks should affect prices. Table 2 presents our regression results with annual AAK shocks. Again relying on the tests for the sum of coefficients on the lagged shocks, we find that downstream shocks have significant positive effects on inflation in almost every specification. The upstream shocks have a smaller effect, which is significant in the OLS regressions but not in the weighted IV regressions. Own shocks have an estimated effect that is close to zero and statistically insignificant, as in the regressions with the model-based shocks.

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19 When standard errors are clustered, they are clustered by an input-output table category that roughly corresponds to 4-digit NAICS industries. The 266 manufacturing detailed industries are split into 53 such clusters.
Table 1: Propagation of Sectoral Import Shocks, Annual

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<td>(0.013)</td>
<td>(0.010)</td>
</tr>
<tr>
<td><strong>Upstream, Lag1</strong></td>
<td>0.159***</td>
<td>0.159</td>
<td>0.039***</td>
<td>-0.011</td>
<td>-0.011</td>
<td>0.059**</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.119)</td>
<td>(0.014)</td>
<td>(0.286)</td>
<td>(0.091)</td>
<td>(0.029)</td>
</tr>
<tr>
<td><strong>Upstream, Lag2</strong></td>
<td>0.074</td>
<td>0.074</td>
<td>0.038**</td>
<td>0.151</td>
<td>0.151***</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.050)</td>
<td>(0.016)</td>
<td>(0.256)</td>
<td>(0.056)</td>
<td>(0.024)</td>
</tr>
<tr>
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<td>0.786***</td>
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<td>0.582***</td>
<td>0.547*</td>
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<tr>
<td></td>
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<td>(0.172)</td>
<td>(0.239)</td>
<td>(0.140)</td>
<td>(0.325)</td>
</tr>
<tr>
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<td>0.554***</td>
<td>0.502***</td>
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<td>0.265*</td>
<td>0.531*</td>
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<td></td>
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<td>(0.168)</td>
<td>(0.163)</td>
<td>(0.218)</td>
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<tr>
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Standard errors in parentheses

* p < 0.1, ** p < 0.05, *** p < 0.01
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<td>OLS</td>
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<td>IV</td>
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<td></td>
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<td>-0.007</td>
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<td>(0.001)</td>
<td>(0.011)</td>
<td>(0.006)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Upstream, Lag1</td>
<td>0.028**</td>
<td>0.028***</td>
<td>0.007**</td>
<td>0.005</td>
<td>0.005</td>
<td>0.009**</td>
</tr>
<tr>
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<td>(0.013)</td>
<td>(0.010)</td>
<td>(0.003)</td>
<td>(0.018)</td>
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<td>(0.005)</td>
</tr>
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<td>Upstream, Lag2</td>
<td>0.014*</td>
<td>0.014</td>
<td>0.006**</td>
<td>0.016</td>
<td>0.016***</td>
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<td>(0.014)</td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Downstream, Lag1</td>
<td>0.048***</td>
<td>0.048***</td>
<td>0.032***</td>
<td>0.034**</td>
<td>0.034**</td>
<td>0.030***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.008)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Downstream, Lag2</td>
<td>0.025**</td>
<td>0.025***</td>
<td>0.024***</td>
<td>0.019</td>
<td>0.019***</td>
<td>0.036***</td>
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<td>(0.012)</td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.015)</td>
<td>(0.007)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>p(Sum Own Lags=0)</td>
<td>0.876</td>
<td>0.879</td>
<td>0.931</td>
<td>0.581</td>
<td>0.610</td>
<td>0.619</td>
</tr>
<tr>
<td>p(Sum Upstream Lags=0)</td>
<td>0.006</td>
<td>0.028</td>
<td>0.000</td>
<td>0.233</td>
<td>0.120</td>
<td>0.004</td>
</tr>
<tr>
<td>p(Sum Downstream Lags=0)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
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<tr>
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<td>4053</td>
<td>3912</td>
<td>3912</td>
<td>3912</td>
</tr>
<tr>
<td>R2</td>
<td>0.318</td>
<td>0.318</td>
<td>0.088</td>
<td>0.368</td>
<td>0.368</td>
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<td>No</td>
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<tr>
<td>Clustered SEs</td>
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<td>NAICS4</td>
<td>No</td>
<td>No</td>
<td>NAICS4</td>
<td>No</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* p < 0.1, ** p < 0.05, *** p < 0.01
Table 3 shows the results of the long difference regressions. Because these regressions use 5-year changes, they include fewer time observations and thus a smaller sample. Both upstream and downstream shocks have a consistently significant effect on inflation in the IV regressions and the weighted OLS regressions, while the own trade shock has a very small and insignificant effect. The results of the long difference regressions with the AAK shocks are shown in table 4 and are mostly similar. Again we see significant effects of the upstream and downstream shocks, although the downstream shocks are insignificant in the IV regression with clustered standard errors. Another difference is that here the own shocks have a positive effect on inflation that is significant in the weighted IV regressions. These are the only regressions where we see the own trade shock being significant.

It is noteworthy that we consistently find strong and significant downstream propagation of demand shocks on sectoral inflation with both the model-implied and the more simple to construct AAK shocks. We also find weaker but still present propagation of upstream shocks, although the statistical significance is more sensitive to the regression specification. Note also that because all of the regressions use year fixed effects, the results do not reflect the effect of movements in aggregate demand. Taken together with our results on the shocks derived from our model, this analysis suggests that the network structure plays a key role in how shocks to certain industries propagate to other industries in affecting their prices. To the extent the identification strategy
behind the IV regressions is valid, these results are telling us how exogenous demand throughout the production network causally affects price changes in specific sectors. Finally, we also see that in general the effects appear smaller in unweighted regressions, although they are still often statistically significant. Although it is unclear exactly why this might be, we believe this could indicate some amount of heterogeneity in the importance of these effects across industries.

The Propagation of TFP Shocks. We now turn to our analysis involving TFP shocks. We do not know of an instrumental variables strategy to identify exogenous movement in productivity, so these effects should not be thought of as causal. The regressions take the same form as the trade shock regressions above, except that the shocks are constructed based on changes in TFP. Table 5 presents the regression results, using both the model-based and AAK shocks. In the weighted regressions, we find that own TFP shocks have a significant negative effect on price inflation after two periods, but the sum of the "own" coefficients is always insignificant. Downstream effects are always significant in the weighted regressions, but not in the unweighted ones. Finally, we also find insignificant effects of upstream TFP shocks. This is consistent with the model, which predicts a weaker upstream propagation than downstream propagation.

---

The economic magnitude of these effects is not straightforward to evaluate. Even though the size of the shocks is standardized before computing the upstream and downstream versions, the upstream and downstream shocks are effectively weighted averages of a full set of sectoral shocks, which reduces their variance. This is true especially in the model-implied annual shocks (where price stickiness effectively discounts other sector’s shocks in the calculations), but also to some extent in the AAK shocks.

---

Table 4: Propagation of Sectoral Import Shocks, 5-year changes- AAK Construction

<table>
<thead>
<tr>
<th></th>
<th>(1) OLS</th>
<th>(2) OLS</th>
<th>(3) OLS</th>
<th>(4) IV</th>
<th>(5) IV</th>
<th>(6) IV</th>
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</thead>
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<tr>
<td>Own</td>
<td>0.048</td>
<td>0.048</td>
<td>0.002</td>
<td>0.143**</td>
<td>0.143*</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.021)</td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.082)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Upstream</td>
<td>0.292***</td>
<td>0.292***</td>
<td>0.036</td>
<td>0.368***</td>
<td>0.368***</td>
<td>0.068***</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.059)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.071)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Downstream</td>
<td>0.483***</td>
<td>0.483***</td>
<td>0.300***</td>
<td>0.289***</td>
<td>0.289</td>
<td>0.369***</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.059)</td>
<td>(0.085)</td>
<td>(0.052)</td>
<td>(0.197)</td>
<td>(0.065)</td>
</tr>
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<td>662</td>
<td>662</td>
<td>662</td>
<td>662</td>
<td>662</td>
</tr>
<tr>
<td>R2</td>
<td>0.491</td>
<td>0.491</td>
<td>0.146</td>
<td>0.392</td>
<td>0.392</td>
<td>0.124</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Clustered SEs</td>
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<td>NAICS4</td>
<td>No</td>
<td>No</td>
<td>NAICS4</td>
<td>No</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01
The results of the long difference regressions with TFP shocks are in Table 6. Here, we only use the AAK construction of upstream and downstream shocks because the model implied and the AAK constructions are very close when $\Phi = I$ (that is, when we disregard price stickiness). Again, the downstream shocks have a large and significant negative effect. The own shocks have a small negative effect, although it is insignificant in the weighted regression with unclustered standard errors. These regressions also find a negative effect for the upstream TFP shock, and it is significant in the weighted regressions. However, the effect of the upstream shock is much smaller than that of the downstream shock, which is consistent with the model prediction.

Overall, we find evidence to support the model’s prediction that productivity shocks propagate along the supply chain. Productivity increases propagate downstream (i.e. increases in productivity among suppliers lowers inflation in the using industry). Again, this evidence is clearer than the evidence that productivity increases lower inflation in the own sector. Also, upstream shocks have weaker effects than downstream shocks.

5 A Quantitative Analysis of Post-Pandemic High Inflation Episode

The COVID-19 pandemic has led to a surge in inflation in 2021 and 2022, with inflation reaching levels not seen in about 40 years in the U.S. Multiple factors contribute to this surge, including various types of shocks and the propagation of shocks through the supply chain. Figure 7 presents scatterplots of the sectoral price changes at 2022 Q1 (measured by the log deviation from a log-linear trend estimated over 2016-2019) against measures of sectoral price stickiness and sectoral position in the production network. Sectors with larger price increase tend to have higher levels of price flexibility, higher levels of $\psi_i$ and are relatively upstream.\textsuperscript{21}

In this section, we simulate a quantitative model to investigate the high inflation episode of 2022. We disentangle sectoral supply and demand shocks using observed sectoral-level price changes, employment changes and consumption changes. Using these shocks, we then conduct counterfactual analyses to investigate the role of the I-O structure.

In this section, we simulate a quantitative model to investigate the high inflation episode of

\textsuperscript{21}Downstreamness is measured using the vector of $[(I - \Omega)^{-2} \beta o \lambda^{-1}]^{-1}$. This measure follows Antràs and Chor (2013) and captures how intensive is the product use as a direct input for final-use production. The measure increases with downstreamness.
### Table 5: Propagation of Sectoral TFP Shocks

<table>
<thead>
<tr>
<th></th>
<th>(1) Model</th>
<th>(2) Model</th>
<th>(3) Model</th>
<th>(4) AAK</th>
<th>(5) AAK</th>
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<td>Own, Lag1</td>
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<td>-0.000</td>
<td>0.000</td>
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<tr>
<td></td>
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<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
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<td>-0.005***</td>
<td>-0.005**</td>
<td>-0.004*</td>
<td>-0.004***</td>
<td>-0.003**</td>
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<td>(0.002)</td>
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<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.003)</td>
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<td>-0.012</td>
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<td>-0.002</td>
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<td>(0.007)</td>
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<td>(0.003)</td>
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<td>-0.044***</td>
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<td>-0.031***</td>
<td>-0.031***</td>
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<td>(0.009)</td>
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<td>-0.046***</td>
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<td>-0.038***</td>
<td>-0.007</td>
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<td>(0.010)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>p(Sum Own Lags=0)</td>
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</tr>
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<td>0.000</td>
<td>0.695</td>
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<tr>
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<td>0.371</td>
<td>0.074</td>
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<td>0.367</td>
<td>0.074</td>
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<td>No</td>
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</table>

Standard errors in parentheses

* *p < 0.1, ** p < 0.05, *** p < 0.01

### Table 6: Propagation of Sectoral TFP Shocks, 5-year changes- AAK

<table>
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<tr>
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<th>(3) Unweighted</th>
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<tr>
<td>Own</td>
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<td>-0.042**</td>
<td>-0.020*</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.017)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Upstream</td>
<td>-0.100*</td>
<td>-0.100***</td>
<td>-0.026</td>
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<td>(0.054)</td>
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<td>(0.026)</td>
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<td>-0.307***</td>
<td>-0.307***</td>
<td>-0.143***</td>
</tr>
<tr>
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<td>(0.086)</td>
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<tr>
<td>R2</td>
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<td>No</td>
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Standard errors in parentheses

* *p < 0.1, ** p < 0.05, *** p < 0.01
2022. We disentangle sectoral supply and demand shocks using observed sectoral-level price changes, employment changes and consumption changes. Afterwards, we conduct counterfactual analyses to investigate the roles of the I-O structure, price rigidity and elasticity of substitution.

5.1 The Quantitative Model

First, we update the model presented in Section 2 with a two-period setting and exogenous labor and capital supply. The dynamic feature appears in the households sector. Producers solve an intratemporal problem as in Section 2.1 with the same CES production technology. All households maximize the same intertemporal utility function:

$$\max \ (1 - \rho) \left( \frac{C_1^{1-1/\gamma} - 1}{1-1/\gamma} \right) + \rho \left( \frac{C_*^{1-1/\gamma} - 1}{1-1/\gamma} \right),$$

s.t. $$CP + \frac{C_*P_*}{1+i} = \sum w_il_i + T + \sum \frac{w_il_is}{1+i} + T_*.$$
\( \rho \) is the preference parameter. \( i \) refers to the nominal interest rate, which is controlled by monetary policy. Variables with star represent future levels of the corresponding variables. We further assume that after unexpected shocks in the first period, the economy returns to a long-run equilibrium in the second period (as in Baqaee and Farhi, 2022; Eggertsson and Krugman, 2012).

Thus, changes in output can be expressed as

\[
\tilde{C} = -\gamma \tilde{P} + \tilde{\zeta},
\]

with \( \tilde{\zeta} \equiv \tilde{C}_* - \gamma \left[ \tilde{i} + \frac{\tilde{\rho}}{(1-\rho)^2} - \tilde{P}_* \right] \), which captures the aggregate demand shock. The economy’s equilibrium conditions that we use for the quantitative analysis is presented in Appendix C.

We model responses of the economy from 2020-2022 with a combination of supply and demand shocks, in a similar fashion as Baqaee and Farhi (2022). First, we consider supply shocks to be changes in the economy’s production possibilities. On the one hand, the pandemic imposed production restrictions due to voluntary or mandated policies, such as lock-downs, business closures, and social distancing. Relatedly, certain sectors are naturally less capable of accommodating remote work productively. At the same time, the same type of restrictions reduced the supply of labor. Thus, supply shocks include sectoral TFP shocks (\( \tilde{z}_i \)) and sectoral labor supply shocks (\( \tilde{l}_i \)).

Second, we consider the aggregate demand shock as a combination of monetary policy shocks and inter-temporal preference shocks, captured by \( \tilde{\zeta} \). Finally, households’ preferences over different consumption goods changed during the pandemic and the pandemic recovery. This can be captured by the change of expenditure share (\( \tilde{\beta}_i \)). \( \tilde{\beta}_i \) can be referred to as sectoral demand shocks with \( \sum_i \tilde{\beta}_i = 0 \).

### 5.2 Calibration

Parameters. Our quantitative model is calibrated at the industry group level containing 66 industries. We use 2019 BEA I-O tables to construct the I-O matrix. The BEA tables are also used to calculate factor shares (\( \alpha_i \)) as well as the consumption share at steady state (\( \tilde{\beta}_i \)). The degree of industry’s price adjustment flexibility still captured by \( \phi_i \) (from Pasten et al., 2020). Following the line of work that estimates elasticity of substitutions (and Baqaee and Farhi, 2022), we set the elasticity of substitution across intermediates to be 0.2. The elasticity of substitution between labor
and capital as well as between value-added and intermediate inputs is 0.6. Household preference are Cobb-Douglas.

**Demand Shocks.** The aggregate demand shock is calibrated to match the change of nominal GDP from its linear trend from 2016-2019. Sectoral demand shocks are calibrated to match the change in consumption shares from their 2019 annual average to shares of 2022 Q1 using the PCE data. The calibrated sectoral demand shocks are presented in Appendix Figure 13.

**Supply Shocks.** First of all, sectoral labor supply is exogenous and is calibrated using employment data from CES and QCEW (as discussed in Section 3.3). Specifically, we calculate the log difference between the level of sectoral employment of 2022 Q1 and the level of sectoral employment in 2019 Q4. The calibrated employment change is illustrated in Appendix Figure 13. Nonetheless, sectoral TFP shocks will be estimated using the model by targeting sectoral price changes. Sectoral price change is calculated as the observed sectoral price deviation at 2022 Q1 from their pre-pandemic linear trends of 2016-2019. The calculated sectoral price change is presented in Figure 14 in Appendix E.

Finally, as a test of out-of-sample fitness, we will compare the model implied sectoral wage changes with the actual wage changes. The actual wage change is constructed using wage data from CES and QCEW, as described in Section 3.3. We calculate sectoral wage changes from their pre-pandemic linear trends of 2016-2019.

### 5.3 Results and Counterfactuals

Having solved the model in levels, we now discuss the model solution. First, we check the model’s out-of-sample fit in order to evaluate the model’s performance. Figure 15 in Appendix E shows a scatterplot of wage changes implied by the model against the actual wage changes. The size of the dots captures the relative employment size of each sector. Overall, the model performs reasonably well in predicting the wage changes quantitatively.22

The estimated sectoral TFP shocks are listed in Appendix Figure 13. Figure 8 presents the

---

22The three outlier sectors are relatively small size sectors in terms of employment. The two at the bottom right corner are both energy related sectors. Employment declined significantly in these two industries (see Figure 13) and remained depressed in early 2022, which drives the high wage increase in simulated data. However, the observed wage has declined. Many factors may contribute to the bad fit of these industries, such as idiosyncratic labor demand shocks, trade impacts or commodity price shocks.
scatter plot of the sectoral TFP and labor supply shocks against downstreamness of each sector and their price adjustment frequency. Panels (a) and (b) of Figure 8 show that shocks are dispersed at various levels of frequency. Panel (c) of Figure 8 shows that sectoral TFP shocks of upstream sectors are dispersed across zero with relatively large size, while the shocks of the downstream sectors are concentrated at the negative range with small size. Panel (d) of Figure 8 shows that upstream sectors experienced stronger negative labor supply shocks than downstream sectors.

Figure 8: Estimated Supply Shocks

Note: Size of scatter reflects the relative size of aggregate price sensitivity to sectoral shocks $\beta' \Psi$. Sectoral price frequency is calculated at the 2-year flexibility level.

To decompose the origin of the high inflation, we conduct several counterfactual analyses. First, we solve the model under various shocks separately, and we compare results with those under a horizontal structure— that is, without an I-O structure. Figure 9 presents the simulated inflation rate under various types of shocks with and without the I-O structure. The horizontal economy has $\Omega^H = 0$, $\alpha_i^H = 1$, and the same $\Phi$ and $\beta$ as the I-O economy, as discussed in Section 2.3. Blue bars present results of the benchmark economy with the calibrated I-O matrix. The aggregate price level is above its trend by 6.82% in the first quarter of 2022. About 60% of the
high inflation rate comes from the reduction of production possibilities (captured by the TFP shock in the model). About 20% of the high inflation rate comes from labor supply reductions. Finally, 20% of the inflation rate is accounted for by the aggregate demand shocks. Red bars present results of the counterfactual horizontal economy. It appears that the I-O structure plays an important role in amplifying the price impact of shocks, in particular the TFP shocks. Without the I-O structure, the inflation rate would decrease by 23% from the observed level.

Figure 9: Inflation of Various Scenarios

<table>
<thead>
<tr>
<th>Sector</th>
<th>Input-Output</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>6.82%</td>
<td>5.24%</td>
</tr>
<tr>
<td>Sectoral TFP</td>
<td>4.18%</td>
<td>1.81%</td>
</tr>
<tr>
<td>Sectoral Labor</td>
<td>1.25%</td>
<td>1.16%</td>
</tr>
<tr>
<td>Aggregate Demand</td>
<td>2.92%</td>
<td>1.81%</td>
</tr>
<tr>
<td>Sector Demand</td>
<td>0.30%</td>
<td>0.61%</td>
</tr>
</tbody>
</table>

Table 7 presents the estimated inflation and sectoral price dispersion in two sets of counterfactual analyses. Panel (a) present results of the benchmark economy under different types of shocks separately, the same as the results presented in Figure 9. There are two important findings. First, as discussed previously, sectoral TFP shocks play a dominant role in the inflation surge. Moreover, the three types of sectoral shocks (TFP, Labor supply and sectoral demand shocks) account for the sectoral price dispersion.

As discussed in Sections 2.2 and 2.3 there are three factors that affect the response of prices to shocks: the I-O structure, price rigidity and elasticity of substitution across sectoral products. In order to investigate the role of these factors in the 2021-2022 inflation surge, Table 7 Panel (b) presents price response under alternative settings: flexible prices, Cobb-Douglas economy and horizontal economy. It is important to note that the aggregate price response depends on the relationship between sectoral shocks and these factors.

Table 7 Panel (b) shows that all three factors amplify the aggregate price response to shocks. First, the difference between the benchmark economy and the Cobb-Douglas economy reflects the role of elasticity of substitution across products. Consistent with the prediction of Section 2.2, we
find that complementarity amplifies price responses and increases sectoral price dispersion.

Second, the comparison between the benchmark economy and the horizontal economy informs the role of the I-O structure. The I-O structure facilitates the propagation of shocks, and it also amplifies the distortion effect of price rigidity. From the perspective of the propagation of shocks, the I-O structure amplifies the aggregate price effect of TFP shocks and attenuates the effect of labor supply shocks, as shown in Figure 9. Moreover, as will be discussed shortly, the I-O structure amplifies the distortion effect, which lowers production and increases inflation.

Third, the difference between the benchmark economy and the economy with flexible prices reflects the role of price rigidity. Price rigidity restricts the downstream passthrough of shocks, but it also generates distortions within and across sectors. On the one hand, panels (a) and (b) of Figure 8 show that shocks are dispersed at various levels of frequency. Price rigidity weakens the passthrough of both positive and negative shocks. The overall effect of passthrough is not clear. On the other hand, price rigidity generates distortions in the production and creates inflationary pressure at the aggregate level, as will be discussed next.

**Sectoral Distortion.** Price adjustment frictions generate distortions across firms and sectors, which results in an inefficient reallocation of resources and lowers production (see Baqaee and Farhi, 2019; Bigio and La’O, 2020; Jones, 2013; Liu, 2019, etc.). To a certain extent, distortion works like a negative TFP shock, which creates additional inflationary pressure. There are two types of distortions in this economy: within-sector distortion and across-sector distortion. First, price stickiness generates price dispersion within each sector, which reduces sectoral productivity (see Nakamura et al., 2018). Second, sectoral wedges between prices and marginal costs further result in an inefficient reallocation of resources across sectors, and these wedges compound through the I-O structure (see Bigio and La’O, 2020).

In fact, price rigidity generates two types of distortion effects in an I-O economy, effect on aggregate TFP and effect on the labor wedge. Given that factor supplies are exogenous in the model, distortion only work through the first effect. Although sectoral distortion has zero first-order effects on TFP near efficiency (steady state), this effect is captured in our results by solving

---

23 Consider the model of Section 2.1 with exogenous labor supply (i.e. $\eta = 0$). The first order response of aggregate price to TFP shocks is $\beta' \Phi (I - \Omega \Phi)^{-1}$. The I-O structure amplifies the price effect of TFP shocks because $\beta' \Phi (I - \Omega \Phi)^{-1} > \beta' \Phi$. The first order response of aggregate price to aggregate labor supply shocks is weaker in the I-O economy. However, the response to sectoral labor supply shocks is case dependent, see proof in Appendix app: agg response.
the model at levels and at large deviations from steady state. Thus, the distortion effect together with the propagation of shocks generates a larger inflation in the benchmark economy than the economy with flexible price adjustment and the economy without the I-O structure. Furthermore, it is also important to notice that the Calvo price stickiness feature in the model generates larger distortion than a menu cost setting. Thus, the distortion effect shown here might be overestimated.

Table 7: Inflation and Sectoral Price Dispersion

<table>
<thead>
<tr>
<th>Panel (a): Benchmark economy with various types of shock</th>
<th>Benchmark all shocks</th>
<th>TFP</th>
<th>Labor supply</th>
<th>Aggregate demand</th>
<th>Sectoral demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.068</td>
<td>0.042</td>
<td>0.013</td>
<td>0.012</td>
<td>0.003</td>
</tr>
<tr>
<td>Price dispersion</td>
<td>0.139</td>
<td>0.130</td>
<td>0.075</td>
<td>0.001</td>
<td>0.067</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel (b): Various setting with all types of shocks</th>
<th>Benchmark flexible price</th>
<th>Cobb-Douglas economy</th>
<th>Horizontal economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.068</td>
<td>0.062</td>
<td>0.063</td>
</tr>
<tr>
<td>Price dispersion</td>
<td>0.139</td>
<td>0.136</td>
<td>0.088</td>
</tr>
</tbody>
</table>

6 Conclusion

In this paper, we investigate the propagation of shocks in an input-output economy with price rigidity. The I-O structure, price adjustment rigidity and the elasticity of substitution across products facilitate the propagation of shocks along the supply chain. Using disaggregated sectoral-level price data, we find empirical support for the pass-through of shocks to prices as predicted by the production network theory. We also find a close connection between the distribution of sectoral price changes and the distribution of the I-O structure-implied price sensitivity to shocks, which implies that the I-O structure matters in the interpretations of the distribution of price changes in the sticky price literature. Finally, we investigate the high inflation episode of the Covid-19 pandemic recovery through a quantitative analysis. We disentangle various types of shocks during the pandemic. We find that the I-O structure amplifies the pandemic related shocks, especially the sectoral TFP shocks. In addition to the distortion effect amplified by the I-O structure, production
network contributes 23% of the observed inflation.
References


A Additional Data Information

In this section we provide additional detail about the collection of empirical measures for different economic variables, and their matching to the industry classifications that we use in our empirical and quantitative analyses.

A.1 Sectoral Prices

As mentioned in Section 3, we use price measures from the PPI for our empirical analysis. The PPI is reported by industry and by commodity, and we use industry PPIs for almost all detailed industries. Our approach is to use industry PPIs where possible. For a handful of industries, mostly in agriculture, there is no industry PPI but a commodity PPI closely corresponding to the industry’s output is available. In those cases we use the commodity PPI as the price measure. Because industry PPIs correspond to NAICS industries and the detailed industries from the I-O tables correspond to one or a few NAICS industries it is quite easy to match PPIs to detailed industries in most cases. Detailed industries mostly correspond to 6-digit NAICS categories, but some industries correspond to 5-, 4-, or 3- digit NAICS industries. When a PPI is available for the corresponding detailed industry, we simply match them. For detailed industries corresponding to more than one NAICS category with an available PPI, we construct the detailed industry’s log price change as the average of the log price changes of the available PPIs corresponding to that detailed industry.

In several cases, however, this is not simple because a PPI might not be available for the specific NAICS industry corresponding to a particular detailed industry. In those cases, we search through the available PPIs and match the detailed industry with the most relevant available PPI. Out of the 341 detailed industries for which we have price stickiness measures, there were 4 for which we did not think an adequate PPI existed, which is why our analysis is based on 337 detailed industries. Fundamentally, the difficulty in obtaining price measures for detailed industries is that
the PPI does not cover the universe of industries. Indeed, while PPIs are available for almost every manufacturing detailed industry, the coverage of service industries is considerably more sparse.\textsuperscript{24}

Although we have described the reasons why the PPI is generally more appropriate for our analysis than the CPI, it is possible that some of the industries for which a PPI is unavailable could be well measured by the CPI. In cases where industry sales are made almost exclusively to consumers as final consumption, the CPI price would capture the price of industry output quite accurately. Some examples could include restaurants, entertainment, and some educational services. However, we do not attempt to do this in our main empirical analysis and leave it as a possible future extension.

A.2 Employment and Wages

The measures of industry employment and wages that we use in the empirical and quantitative analyses come mostly from the BLS’ Current Employment Statistics (CES). Because CES is reported by NAICS industry code, it is again quite simple to match employment and wages to I-O detailed categories. The only complication in this process is that many 6-digit NAICS industries do not have estimates in CES. When this is the case, we match a detailed industry to the narrowest available CES industry that covers the detailed industry. Specifically, we try to match by 5-digit industry code, and if that is not possible we try by 4-digit code, and then 3-digit code. Although there are some broad industries for which wages are not measured in CES, such as education and public sector establishments generally, all of the 337 detailed industries in our analysis have a corresponding 3-digit (or narrower) industry that provides wage estimates.

Wages are measured by the average hourly earnings of production and nonsupervisory workers. This includes all wages and salaries (including overtime, paid leave, incentive pay, and regular bonuses). It excludes commissions, bonuses not paid each pay period, stock options, or fringe benefits such as employer contributions to insurance or retirement plans. Employees in any kind of managerial role are also excluded. Average hourly earnings for all employees are also available. However, those estimates are only available starting in 2006. We also believe that excluding managerial employees gives a more accurate estimate of firms’ labor costs to producing output. Other measures of wages and compensation are produced by the BLS, such as Compensation per Hour and the Employment Cost Index. While these measure compensation more comprehensively, they are not broken down into detailed industry.

For the quantitative analysis based on industry groups, we need to use employment and wages from the Quarterly Census of Employment and Wages (QCEW) for certain industry groups not captured by CES. In these cases, we use average weekly earnings as the measure of wages. Al-

\textsuperscript{24}Most service PPI series also begin later than the non-service series, as most service PPIs begin in 2003.
though the units between the two wage data sources are different (hourly earnings in CES, weekly earnings in QCEW), since we are interested in log changes over periods of time the effect of these differences are likely second order.

Finally, we could in principle use QCEW average weekly earnings for all the detailed industries. We choose to nevertheless use CES hourly earnings where possible mainly because it allows us to focus on the earnings of production workers. Given the concentration of labor income among very high earners (as emphasized by Piketty and Saez, 2003, for example), QCEW earnings will in some cases be influenced by changes in bonuses and stock options paid to high-wage employees such as managers. Also note that to measure employment CES is essentially equivalent to the QCEW. That is because after the annual benchmarking process, which usually happens at the start of each year, CES total and industry employment estimates are aligned with QCEW employment counts for the previous year.

### A.3 Trade and TFP shocks

Our empirical analysis on propagation is based on trade and productivity (TFP) shocks similar to the ones used by Acemoglu et al. (2016a) and Acemoglu et al. (2016b). Both types of shocks need to be measured by detailed industry and year. The trade shocks require Chinese exports to the U.S. (and to other countries for the instruments to these shocks) by industry and a measure of market size by industry. The TFP shocks require a measure of TFP by industry and year.\(^{25}\)

The trade, market size, and TFP data is easily made available at an annual frequency by the authors mentioned above and the NBER-CES database (Becker et al. (2021)). However, industries are categorized according to 4-digit 1987 SIC codes. We therefore use the BEA’s SIC-NAICS correspondence to match SIC codes to 6 digit NAICS codes, and then match the NAICS codes to the I-O detailed industry codes that form the basis of our empirical analysis. Many detailed industries correspond to multiple 4-digit SIC codes. In these cases, we construct the trade shock by summing imports and market size by detailed industry and dividing to obtain import penetration by detailed industry. For TFP we construct log averages weighted by real value added. Using all the available SIC industries, we are left with 266 I-O detailed industries for which we can measure trade shocks and TFP, all of them in manufacturing sectors. As a result, this part of our analysis is restricted to manufacturing.

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\(^{25}\)The instruments are constructed as changes in Chinese exports to a total of seven countries: Australia, Denmark, Finland, Germany, Japan, New Zealand, and Switzerland. These countries were selected due to the availability of detailed trade data extending far enough in time.
B Proofs

B.1 Model Set-Up

Consider a static economy with $N$ sectors. In each sector, a continuum of firms $k$ (distributed from 0 to 1) produce differentiated products. They face a CES production function, given by

$$y_{ik} = z_i \left[ \alpha_i^{1/\theta} \frac{\theta}{\theta - 1} l_{ik}^\theta + \left(1 - \alpha_i\right)^{1/\theta} \sum_j \omega_{ij} \frac{\theta}{\theta - 1} x_{ij,k}^{1/\theta} \right]^\frac{\theta}{\theta - 1},$$

where $l_{ik}$ is the labor input, $x_{ij,k}$ denotes the amount of intermediate inputs produced by firms in sector $j$. $z_i$ is the sectoral TFP shock. $\theta$ is the elasticity of substitution across production inputs.

Further, there is a final producer in each industry $i$ aggregates differentiated products to one sectoral product following $y_i = \left(\int_k y_{ik} \right)^{\epsilon/(\epsilon-1)}$. $\epsilon$ captures the elasticity of substitution across products within each industry. Thus, firms have constant desired markup given by $\frac{\epsilon}{\epsilon - 1} > 1$.

All firms set their price optimally to maximize their profit based on their expected marginal cost:

$$\max_{p_{ik}} \mathbb{E}_{ik}[(1 - \tau)p_{ik} - mc_{ik}]y_{ik}$$

where $p_i$ are the sector-level price index, $mc_{ik}$ denotes marginal cost and $\tau$ is an industry-specific revenue tax or subsidy levied by the government. $\tau$ is set to eliminate the distortions that arise under the CES demand structure in each industry.

It is easy to show that the realized sector-level marginal cost follows

$$mc_i = z_i^{-\frac{1}{\theta}} \left[ \alpha_i w_i^{1-\theta} + \left(1 - \alpha_i\right) \sum_{j=1}^n \omega_{ij} (p_j)^{1-\theta} \right]^{\frac{1}{\theta - 1}}$$

where $w_i$ denotes the nominal wage of sector $i$.

We introduce nominal rigidities using the noisy information structure following La’O and Tahbaz-Salehi (2022). A fraction $\phi_i$ of firms in industry $i$ receive perfect information about the realized shocks, while a fraction $1 - \phi_i$ of firms in industry $i$ receive no signals during a given time period.\(^{26}\) Thus, the sectoral price $p_i$ deviation from the steady state level follows:

$$\tilde{p}_i = \phi_i \tilde{mc}_i.$$

Marginal cost changes $\tilde{mc}_i$ will only partially pass-through into prices, and $\phi_i$ proxies the sectoral

\(^{26}\)The nominal rigidity set-up is close to the Calvo setting (similar as Rubbo, 2020) and the solution of the sectoral price vector (Equation 1) is the same under this alternative price setting.
level of price stickiness. The higher is $\phi_i$ the more flexible price adjustment is. Variables with tilde refer to log deviation from steady state value.

In addition, households solve the following problem

$$\max \frac{C^{1-\gamma}}{1-\gamma} - \sum_i \chi_i \frac{k_i^{1+1/\eta}}{1 + 1/\eta},$$

s.t. $PC = \sum_i w_il_i + T + \Pi$

where $\chi$ ensures the steady state labor share by industry is consistent with the data. $T$ is the transfers from the government. $\Pi$ is the transfer from firms. Further assume that preferences over consumption goods is given by a CES aggregator.

$$C = \left( \sum_i \beta_i^{1/\epsilon} c_i^{\epsilon-1/\epsilon} \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $c_i$ is the consumption of goods produced by sector $i$, $\beta_i$ is the preference parameter, and $\epsilon$ is the elasticity of substitution. Thus, the aggregate price follows

$$P = \left( \sum_i \beta_i p_i^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

### B.2 Proof of Proposition 1

First order condition on labor delivers,

$$mc_i \left( z_i^{\theta-1} y_{ik}^{1/\theta} \alpha_i^{1/\theta} i_{ik}^{1-1/\theta} \right) = w_i$$  \hspace{1cm} (9)

First order condition on intermediate inputs delivers,

$$p_j = mc_i g_{ik}^{1/\theta} z_i^{1-1/\theta} (1 - \alpha_i)^{1/\theta} \omega_{ij}^{1/\theta} x_{ijk}^{1-1/\theta}$$ \hspace{1cm} (10)

Plug equations 9 and 10 into the production function, we can show that the marginal cost follows

$$mc_i = z_i^{-1} \left[ \alpha_i w_i^{1-\theta} + (1 - \alpha_i) \sum_{j=1}^n \omega_{ij} (p_j)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$ \hspace{1cm} (11)
From equation \(10\), we have
\[
p_j = mcy^{1/\theta}z^{1-1/\theta}(1 - \alpha_i)^{1/\theta}\omega^{1/\theta}_{ijk}x_{ijk}^{-1/\theta}.
\]
Thus, we have
\[
p_jx_{ij} = p_j\int_k x_{ijk} = \left(\frac{mc_i}{p_j}\right)^{\theta}p_jz_{ik}^\theta(1 - \alpha_i)\omega_{ijk}d_i,
\]
where \(d_i \equiv \int \frac{y_{ik}}{y_i}dk = \int \left(p_{ik}/p_i\right)^{-\epsilon}dk\).

Thus, the Domar weight follows,
\[
\lambda_i = \beta_i \left(p_i/p\right)^{1-\epsilon} + \sum_j \lambda_j \left((1 - \alpha_j)\omega_{ji}z_j^\theta \left(\frac{mc_j}{p_i}\right)^\theta\frac{p_i}{p_j}d_j\right)
\]

Labor share \(\lambda^L_i = w_i/l_i/E\) follows,
\[
\lambda^L_i = \lambda_i\alpha_i\omega_i^{-\theta i}mc_iw_i^{-\theta i}p_i^{-1}\int_k \frac{y_{ik}}{y_i} = \lambda_i\alpha_i\omega_i^{-\theta i} \left(\frac{mc_i}{w_i}\right)^\theta\frac{w_i}{p_i}d_i
\]

Log-linearize Equation \(9\), we get Equation \(2\):
\[
\tilde{w} = \left(\frac{\theta - 1}{\theta}\right)\tilde{z} + \frac{1}{\theta}(\tilde{\lambda} + \tilde{E}) + \left(\Phi^{-1} - \frac{1}{\theta}I\right)\tilde{p} - \frac{1}{\theta}\tilde{l}.
\]

Further define consumption share as \(\lambda_{0i} = \frac{p_{0i}}{E}\). Then, from goods market clearing condition, we have
\[
p_iy_i = p_ic_i + \sum_j p_jx_{ji}.
\]
Thus,
\[
\lambda_i = \lambda_{0i} + \sum_j \lambda_j \frac{p_jx_{ji}}{p_jy_j}.
\]

Log-linearize the above equation and denote \(\hat{\omega}_{ji} = \omega_{ji}\tilde{\lambda}_i/\tilde{\xi}_i\) delivers,
\[
\hat{\xi}_i = \sum_j (1 - \alpha_j)\hat{\omega}_{ji}\left[\hat{\lambda}_j + (\theta - 1)\hat{z}_j + \theta m\omega_j - (\theta - 1)\hat{p}_i - \hat{p}_j\right] + \frac{\beta_i}{\lambda_i}(1 - \varepsilon)(\hat{p}_i - \tilde{P}) + \frac{\beta_i}{\lambda_i}\hat{\beta}_i.
\]

Thus in matrix form, we get Equation \(3\)
\[
\tilde{\lambda} = [I - \hat{\Omega}']^{-1} \left[(\theta - 1)\hat{\Omega}^\prime \tilde{z} + \left(\hat{\Omega}^\prime D\left(\frac{\theta}{\phi} - 1\right) + D\left(1 - \theta + (\theta - \varepsilon)\frac{\beta}{\lambda}\right)\right)\tilde{p}\right] - [I - \hat{\Omega}']^{-1} D\left(\frac{\beta}{\lambda}\right) [(1 - \varepsilon)\tilde{P} - \tilde{\beta}]
\]
Finally, from households’ problem, we have

\[ \chi_i^{1/\eta} = C^{-\gamma}w_i/P, \]

so we get Equation 4

\[ \tilde{l} = -\gamma\eta\tilde{E} \cdot 1_N + \eta(\gamma - 1)\tilde{P} \cdot 1_N + \eta\tilde{w}. \]

### B.3 Sectoral Price Response to Shocks

This subsection shows the proof for Equation 5. From Proposition 1, we have

\[
\tilde{w} = \left( \frac{\theta - 1}{\theta} \right) \tilde{z} + \frac{1}{\theta} (\tilde{\lambda} + \tilde{E}) + \left( \Phi^{-1} - \frac{1}{\theta} I \right) \tilde{p} + \frac{1}{\theta} \gamma \eta \tilde{E} - \frac{1}{\theta} \eta(\gamma - 1)\tilde{P} \cdot 1_N - \frac{1}{\theta} \eta \tilde{w} 
\]

\[
= \frac{\theta^{-1}}{1 + \theta^{-1} \eta} \left[ (I - \tilde{\Omega}^{-1})^{-1} Q + (\theta \Phi^{-1} - I) \right] \tilde{p} + \frac{\theta^{-1}}{1 + \theta^{-1} \eta} (I - \tilde{\Omega})^{-1} D \left( \frac{\beta}{\lambda} \right) \tilde{\beta} 
\]

\[
+ \frac{\theta^{-1}}{1 + \theta^{-1} \eta} (I - \tilde{\Omega})^{-1} \tilde{\Omega}' (\theta - 1) \tilde{z} + \frac{\theta - 1}{\theta + \eta} \tilde{z} 
\]

\[
+ \frac{\theta^{-1}(1 + \gamma \eta)}{1 + \theta^{-1} \eta} \tilde{E} \cdot 1_N - \frac{\theta^{-1}(\gamma - 1)}{1 + \theta^{-1} \eta} \tilde{P} \cdot 1_N - \frac{\theta^{-1}}{1 + \theta^{-1} \eta} (I - \tilde{\Omega})^{-1} D \left( \frac{\beta}{\lambda} \right) (1 - \varepsilon)\tilde{P} \cdot 1_N. 
\]

Thus,

\[
\Theta \tilde{p} = -\Phi(I - \Omega \Phi)^{-1} \left( -\frac{\theta - 1}{\theta + \eta} D(\alpha) - \frac{\theta - 1}{\theta + \eta} D(\alpha)(I - \tilde{\Omega})^{-1} \tilde{\Omega}' + I \right) \tilde{z} 
\]

\[
+ \frac{\theta^{-1}}{1 + \theta^{-1} \eta} \Phi(I - \Omega \Phi)^{-1} D(\alpha)(I - \tilde{\Omega})^{-1} D \left( \frac{\beta}{\lambda} \right) \tilde{\beta} 
\]

\[
+ \frac{\theta^{-1}(1 + \gamma \eta)}{1 + \theta^{-1} \eta} \Phi(I - \Omega \Phi)^{-1} D(\alpha) \tilde{E} \cdot 1_N, 
\]

where \( \Theta = [I - \frac{\theta^{-1}}{1 + \theta^{-1} \eta} \Phi(I - \Omega \Phi)^{-1} D(\alpha)F] \) and

\[
F \equiv \left[ (I - \tilde{\Omega})^{-1} Q + (\theta \Phi^{-1} - I) - \eta(\gamma - 1) \left( I + \frac{1}{\eta(\gamma - 1)} (I - \tilde{\Omega})^{-1} D \left( \frac{\beta}{\lambda} \right) (1 - \varepsilon) \right) \right] B. 
\]

\( B \) is an \( N \times N \) matrix with each row equals \( \tilde{\beta}' \).

### B.4 Aggregate Price Response to Shocks

From the household budget constraint, we have

\[
PC = \sum_i (w_i L_i) + \sum_{i=1}^n \left( p_i y_i - m_c \int_0^1 y_{ik} dk \right) = \sum_i w_i L_i/ \left( \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right), \quad (12)
\]
where $\varepsilon_i$ is the sectoral wedge

$$\varepsilon_i \equiv \frac{mc_i}{p_i} \int_0^1 (p_{ik}/p_i)^{-\varepsilon} \, dk = \frac{mc_i}{p_i} d_i.$$ 

It is easy to find that

$$\sum_i \lambda_i^* \tilde{\varepsilon}_i = \beta'(I - \Omega)^{-1}(\Phi^{-1} - I)\tilde{p}$$

$$= \beta'(I - \Phi\Omega)^{-1}(I - \Phi)((I - \Omega)^{-1} \text{diag}((I - \Omega)1)\tilde{w} - (I - \Omega)^{-1}\tilde{z})$$

Thus, log-linearize equation 12, one obtain

$$\sum \lambda_i^* (\tilde{w}_i + \tilde{r}_i) = \tilde{E} + \beta'(I - \Phi\Omega)^{-1}(I - \Phi)((I - \Omega)^{-1} \text{diag}((I - \Omega)1)w - (I - \Omega)^{-1}z).$$

where $\lambda_i^* \equiv D(\alpha)(I - \Omega)^{-1}\beta$ denotes the steady state labor share and $\lambda L^*1 = \beta'(I - \Omega)^{-1}(I - \Omega)1 = 1$.

If wage rate is homogeneous across sectors, $\alpha w = \text{diag}((I - \Omega)1)w = (I - \Omega)w$. Thus,

$$[1 - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)1]\tilde{w} = \tilde{E} - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)(I - \Omega)^{-1}z - \sum \lambda_{Li}^*\tilde{r}_i. \quad (13)$$

Plug Equation 4 to 13, we have

$$[1 - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)1]\tilde{w} = \tilde{E} + \gamma\eta\tilde{E} - \eta(\gamma - 1)\beta'\Phi(I - \Omega\Phi)^{-1}(I - \Omega)1_N\tilde{w} - \eta\tilde{w}$$

$$+ \left[\eta(\gamma - 1)\beta'\Phi(I - \Omega\Phi)^{-1} - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)(I - \Omega)^{-1}\right]\tilde{z}$$

Hence,

$$\partial w/\partial E = \frac{1/\eta + \gamma}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1},$$

and

$$\partial w/\partial z = \frac{\eta(\gamma - 1)\beta'\Phi(I - \Omega\Phi)^{-1} - \beta'(I - \Phi\Omega)^{-1}(I - \Phi)(I - \Omega)^{-1}}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1}$$

$$= \frac{\beta_w}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1}.$$ 

Thus,

$$\partial P/\partial E = \frac{(1/\eta + \gamma)\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1},$$

and

$$\partial P/\partial z = \frac{\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)B_w}{1 + [1/\eta - (1 - \gamma)]\beta'(I - \Phi\Omega)^{-1}\Phi(I - \Omega)1} - \beta'\Phi(I - \Omega\Phi)^{-1},$$

where $B_w = [\beta_w; \cdots; \beta_w]$ a matrix with $N$ rows of $\beta_w \equiv \eta(\gamma - 1)\beta'\Phi(I - \Omega\Phi)^{-1} - \beta'(I - \Omega\Phi)^{-1}$. 

8
ΦΩ)^{-1}(I - Φ)(I - Ω)^{-1}.

To show that monetary non-neutrality is stronger in an input-output economy, we construct a horizontal economy with Ω^H = 0 and α_i^H = 1. Further, consumption share in the horizontal economy 1 is β_1^H = β.

\[ β'(I - ΦΩ)^{-1}Φ(I - Ω)1 < β'Φ1 < β'(I - Ω)^{-1}Φ1. \]

Thus, \( \partial P/\partial E \) is larger in the horizontal economy.

Figure 10: A Simple 2-Firm Economy

(a) Horizontal

1

Households

2

(b) Vertical

1

Households

2

However, the comparison of the aggregate price to TFP shocks in the IO economy versus the horizontal economy depends on the location of the TFP shock. Let’s illustrate this using a 2-firm example, with a vertical input-output structure illustrated in panel (b) and a horizontal structure illustrated in panel (a) of figure 10. In the vertical economy, firm 1 supplies to firm 2 and both firms supply to households. In the horizontal economy, there is no input-output linkages between the two firms.

If the horizontal economy has consumption share equal \( β_1^H = β \), it is easy to see that \( P \) is more response to \( z_1 \) in the vertical economy than the horizontal economy, and \( P \) is less response to \( z_2 \) in the vertical economy than in the horizontal economy. If the horizontal economy has consumption share equal \( β_2^H = [D(α)]' = [β'(I - Ω)^{-1}D(α)]' \), the opposite is true. Thus, the result is sensitive to the construction of \( β^H \), because different \( β^H \) assigns different market size to the two firms in the horizontal economy. \( β_2^H \) assigns larger weight to firm 1 than \( β_1^H \) does. Thus, \( z_1 \) shock has a larger effect on aggregate price in the I-O economy than in the this horizontal economy with \( β_1^H \).

Finally, if labor is exogenous, \( η = 0 \). It is very straightforward to show that \( \partial P/\partial E = 1 \) at the first order and \( \partial P/\partial z = -β'Φ(I - ΩΦ)^{-1} \). The aggregate price response to monetary shock is the same in the I-O and the horizontal economy with \( β^H = β \). However, this result is only at first order. The aggregate price response to TFP shocks is stronger in the I-O economy than in the horizontal economy, because \( β'Φ(I - ΩΦ)^{-1} > β'Φ \). Next consider the price response to labor
supply shocks. First, wage responses to labor supply shocks follow:

\[
\frac{\partial \tilde{w}}{\partial \tilde{l}} = \frac{\lambda_L^*}{[1 - \beta'(I - \Phi \Omega)^{-1}(I - \Phi)1]}
\]

where \(\lambda_L^* = D(\alpha)(I - \Omega')^{-1}\beta\). The denominator has a larger value under the I-O structure than the horizontal structure, because \(\beta'(I - \Phi \Omega)^{-1}(I - \Phi)1 < \beta'1\). The comparison of the nominator under the two economies is more complicated. If labor supply shocks are homogeneous across sectors, then we have the nominator has the same size in the I-O and the horizontal economy (given that \(\lambda_L'1 = \beta'1 = 1\)). Overall, wage is less sensitive to aggregate labor supply shocks in the I-O structure. Wage response to sectoral labor supply shocks depends on specific shocks. Then, the aggregate price response follows

\[
\frac{\partial \tilde{P}}{\partial \tilde{l}} = \beta'\Phi(I - \Omega \Phi)^{-1}D(\alpha)\frac{\partial \tilde{w}}{\partial \tilde{l}}.
\]

Because \(\beta'\Phi(I - \Omega \Phi)^{-1}D(\alpha) \cdot 1 < \beta'\Phi1\), aggregate price response to aggregate labor supply shocks is weaker in the I-O structure. Aggregate price response to sectoral labor supply shocks is case dependent.

### C Covid-Economy Equilibrium

We generalize the model with the following production function

\[
y_{ik} = z_i \left( \alpha \frac{1}{\theta} l_{ik}^{\theta - 1} + \alpha \frac{1}{\theta} k_{ik}^{\theta - 1} + (1 - \alpha Li - \alpha Ki)^{1/\theta} M_{ik}^{\theta - 1} \right)^{\theta} + (1 - \frac{1}{\theta}) 1^{\theta - 1}
\]

\(\theta\) is the elasticity of substitution across value added and intermediate input bundle. The intermediate input bundle follow,

\[
M_{ik} = \left( \sum \omega_{ij} x_{ijk}^{\theta - \theta M} M_{ik}^{\theta M - 1} \right)^{\theta M - 1}
\]

\(\theta M\) is the elasticity of substitution among inputs. Equilibrium of the model contains the following equations.

\[
p_{Mi} = \left( \sum \omega_{ij} p_j^{1 - \theta M} \right)^{1 - \theta M}
\]

\[
m_{Ci} = z_i^{-1} \left[ \alpha Li w_{i}^{1 - \theta} + \alpha Ki r_{i}^{1 - \theta} + (1 - \alpha Li - \alpha Ki) p_{Mi}^{1 - \theta} \right]^{1 - \theta}
\]

\[
p_i = \left[ \phi_i m_{Ci}^{1 - \epsilon} + (1 - \phi_i) p_i^{1 - \epsilon} \right]^{1 - \epsilon}
\]
and

$$\lambda_{ij} = \lambda_{M_i} \omega_{ij} \left( \frac{p_{M_i}}{p_j} \right)^{\theta M - 1}$$

$$\lambda_{M_i} = \lambda_i (1 - \alpha_{L_i} - \alpha_{K_i}) z_i^{\theta - 1} \left( \frac{m c_i}{p_{M_i}} \right)^\theta \frac{p_{M_i}}{p_i} d_i$$

$$\lambda_i = \beta_i \left( \frac{p_i}{P} \right)^{1 - \varepsilon} + \sum_j \lambda_j \left( 1 - \alpha_{L_j} - \alpha_{K_j} \right) z_j^{\theta - 1} \left( \frac{m c_j}{p_{M_j}} \right)^\theta \frac{p_{M_j}}{p_j} d_j$$

Labor and capital are exogenous. Suppose the labor supply shock is $z_i^L$, i.e. $l_i = z_i^L * \bar{l}_i$.

$$\lambda_i^L \equiv w_i l_i / E = \bar{\lambda}_i^L * z_i^L w_i / E$$

$$\lambda_i^K \equiv r_i k_i / E = \bar{\lambda}_i^K r_i / E$$

And Equation 9 delivers,

$$\lambda_i^L = \lambda_i \alpha_i z_i^{\theta - 1} \left( \frac{m c_i}{w_i} \right)^\theta \frac{w_i}{p_i} d_i$$

$$\lambda_i^K = \lambda_i \alpha_{K_i} z_i^{\theta - 1} \left( \frac{m c_i}{r_i} \right)^\theta \frac{r_i}{p_i} d_i$$

Finally, households can smooth out two-period income,

$$\lambda_0 = q \zeta \left[ \lambda_0^* + \sum_i (\lambda_i^L + \lambda_i^K) \left/ \left( 1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) \right. \right]$$

where $\zeta$ represent the aggregate demand shock and $q \equiv (1 + (1 + i) * \frac{\rho}{1 - \rho})^{-1}$.

$mc_i, p_i, p_{M_i}, w_i, r_i, \lambda_i, \lambda_{M_i}, \lambda_i^L, \lambda_i^K, \lambda_0$ jointly determine the equilibrium of the economy.

**D Distortion**

There are two types of distortions in this economy, within-sector distortion and across-sector distortion. First, within each sector, price dispersion reduces sectoral productivity. Given labor, capital and intermediate inputs, we denote

$$H_{ik} = \left( \alpha_{L_i}^{1/\theta} l_{ik}^{\theta - 1} + \alpha_{K_i}^{1/\theta} k_{ik}^{\theta - 1} + (1 - \alpha_{L_i} - \alpha_{K_i})^{1/\theta} M_{ik}^{\theta - 1} \right)^{\theta/(\theta - 1)}$$

and

$$H_i = \int H_{ik} dk.$$
Combing firms’ production function with their demand function, we have

\[ z_i H_{ik} = \left( \frac{p_{ik}}{p_i} \right)^{-\epsilon} y_i. \]

Integrating both sides

\[ z_i H_i = \int \left( \frac{p_{ik}}{p_i} \right)^{-\epsilon} dk y_i \]

As mentioned before, we denote price dispersion as \( d_i = \int \left( \frac{p_{ik}}{p_i} \right)^{-\epsilon} dk \). Thus,

\[ y_i = d_i^{-1} z_i H_i \]

Price dispersion \( d_i \) reduces sectoral productivity.

TBA: distortion from the wedge

**E Additional Figures and Tables**
Figure 12: Impulse Responses of Price Levels to Monetary Shocks

(a) Nakamura and Steinsson (2018)
   Surprise Shock

(b) Jarociński and Karadi (2020)
   Information Factor
Table 8: Correlation between different trade based shocks

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<th>Own\textsuperscript{Trade}</th>
<th>UP\textsuperscript{Trade}</th>
<th>Down\textsuperscript{Trade}</th>
<th>Trade Shock</th>
<th>UP\textsuperscript{Trade,AAK}</th>
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Table 9: Correlation between different TFP based shocks

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<th>UP\textsuperscript{TFP}</th>
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Figure 13: Estimated Sectoral Shocks
Figure 14: Sectoral Price Change
Figure 15: Sectoral Wage Change: Model V.S. Data

The figure compares simulated and observed wages across different sectors. The x-axis represents the simulated wage, while the y-axis represents the observed wage. The data points are clustered, indicating a high degree of correlation between the simulated and observed wages.

Key sectors mentioned include:
- Funds, trusts, and other financial vehicles
- Oil and gas
- Support for mining