Phasing out the GSEs *

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September 30, 2015

Abstract

We develop a new model of the mortgage market where both borrowers and lenders can default. Risk tolerant savers act as intermediaries between risk averse depositors and impatient borrowers. The government plays a crucial role by providing both mortgage guarantees and deposit insurance. Underpriced government mortgage guarantees lead to more and riskier mortgage originations as well as to high financial sector leverage. Mortgage crises occasionally turn into financial crises and government bailouts due to the fragility of the intermediaries' balance sheets. Increasing the price of the mortgage guarantee "crowds in" the private sector, reduces financial fragility, leads to fewer but safer mortgages, lowers house prices, and raises mortgage and risk-free interest rates. Due to a more robust financial sector, consumption smoothing improves and aggregate welfare increases. While borrowers are nearly indifferent to a world with or without mortgage guarantees, savers are substantially better off. While aggregate welfare increases, so does wealth inequality.

JEL: G12, G15, F31.

Keywords: financial intermediation, housing policy, government bailouts

*First draft: February 2015. We thank seminar participants at UT Austin, NYU Stern, George Washington-Federal Reserve Board joint seminar, U.C. Berkeley Haas, the Einaudi Institute in Rome, the ECB-Bundesbank-Goethe Joint Lunch seminar, the Frankfurt School of Management and Finance, the University of Michigan, the University of Missouri, and conference participants at the Yale Cowles Foundation conference, the UBC ULE Summer Symposium, the Society for Economic Dynamics in Warsaw, the National Bureau of Economic Research Summer Institute Public Economics and Real Estate Group, and the World Congress of the Econometric Society in Montreal. We also thank Ralph Koijen and Edward Kung for detailed comments.

1 Introduction

Government and quasi-government entities dominate mortgage finance in the U.S. Over the past five years, the government-sponsored enterprises, Fannie Mae and Freddie Mac, and the Federal Housing Administration have stood behind 80% of the newly originated mortgages.[1] Ever since the collapse of the GSEs in September of 2008 and the conservatorship which socialized housing finance, there have been many proposals to bring back private capital.[2] The main idea of these policy proposals is to dramatically reduce the size and scope of the government guarantee on standard (conforming) mortgages. Because the proposed reform would turn a largely public into a largely private housing finance market, there is both uncertainty and concern about its impact on house prices, the availability of mortgage finance, financial sector stability, and ultimately welfare.[3]

Understanding the economic impact of wholesale mortgage finance reform requires a general equilibrium model. Such a model must recognize the important role that residential real estate and mortgage markets have come to play in the financial system and the macro-economy of rich countries (Jorda, Schularick, and Taylor (2014)). It must also recognize the large footprint of the government. This paper proposes a new general equilibrium model of the housing and mortgage markets where the interaction of the financial sector and the government plays a central role.

In our benchmark model, the government provides mortgage default insurance at low cost. The financial sector issues mortgages to borrowers and decides for how many of those mortgages to buy the government guarantee. The model ascribes the dominance of the guaranteed

[1]Currently, of the $9.85 trillion stock of residential mortgages, 57% are Agency Mortgage-backed Securities guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. Private-label mortgage backed securities make up less than 8% of the stock. The rest is unsecuritized first liens held by the GSEs and the banking sector (28%) and second liens (7%). Acharya, Richardson, Van Nieuwerburgh, and White (2011) provide an in-depth discussion of the history of the GSEs, their growth, and collapse.

[2]The Obama Administration released a first report along these lines in February 2011. The bills proposed -but not passed- by Corker-Warner in 2013 and Johnson-Crapo in 2014 provide the most recent attempts at legislative reform.

[3]The financial and real estate industries, the Mortgage Bankers Association, and consumer advocate groups have all vehemently argued to keep a form of government guarantee in place. They favor a solution with a public mortgage guarantor that would succeed Fannie and Freddie, and fear that there may "not be enough private capital" in the mortgage market in a fully private system. They argue a private market solution would jeopardize the stable provision of mortgage credit for a broad cross-section of households. In contrast, the Congressional Budget Office recently argued that a privatization of housing finance would have minimal impact (CBO 2014).

mortgage market to the low insurance premium (guarantee fee or g-fee) that the government charges banks for that insurance, as well as to the small amount of regulatory capital that banks must hold against guaranteed mortgage bonds. As in the real world, mortgages are long-term, prepayable, and defaultable. A novel model ingredient is that the financial sector enjoys a government bailout guarantee, which is equivalent to deposit insurance in our setting. Deposit insurance is an important feature of any financial system that the literature on mortgage finance has not considered hitherto.[4]

An interesting feature of the model is the interaction between mortgage guarantees, which reduce the risk of banks' assets, and deposit insurance, which reduces the risk of their liabilities. As relatively risk tolerant agents, bankers in the model desire a high return-high risk balance sheet. But taking advantage of the underpriced mortgage guarantee lowers the risk of banks' assets. This prompts banks to increase leverage. The favorable regulatory capital treatment of guaranteed mortgage bonds enables such high leverage. Deposit insurance further propels leverage, since it makes banks' lenders, the depositors, less sensitive to the risk of a banking collapse. A second way in which banks increase risk is through their mortgage origination decisions. They grow the size of the mortgage portfolio and increase its riskiness, by raising mortgage debt-to-income and loan-to-value ratios. Because of the dual guarantees, equilibrium mortgage rates are low, making borrowers willing to take on higher mortgage debt. In sum, the government's underpriced mortgage guarantee distorts financial sector leverage and leads to a larger financial sector and lower underwriting standards as banks pursue their desired risk-reward ratio. Deposit insurance amplifies these effects because it immunizes depositors from banks' elevated risk-taking and removes their incentives to discipline the bankers.

Ex-post, a riskier mortgage portfolio produces higher mortgage default rates and losses. These losses produce deadweight costs of foreclosures, a first source of welfare losses. Given banks' low net worth and high leverage, housing crises occasionally turn into financial crises, defined as bank insolvencies. Thus, the economy with underpriced mortgage guarantees generates financial sector fragility. It also displays high house price volatility. The government absorbs most of the default-induced losses as part of the mortgage default insurance contract and bails

[4]Deposit insurance can be thought of more broadly as encompassing implicit government guarantees for short-term financial sector liabilities such as money market funds, asset-backed commercial paper, repurchase agreements, etc. Indeed, the government stepped in to bail out these markets in the Fall of 2008 and in the Spring of 2009.

out the banks if they are insolvent. The government issues debt to absorb the costs of the guarantee payouts. Government debt is both high and volatile as a result. The government's dual intervention brings two important advantages. First, the government's ability to issue debt in bad times allows society to spread out the fiscal costs of mortgage defaults over time. Second, the mortgage guarantees (partially) protect banks' balance sheets from mortgage losses and enable banks to continue their intermediation function. However, depositors must hold the additional government debt in equilibrium. Since they are risk averse, they dislike volatility in government debt since it gives rise to volatility in their consumption. The net effect of these opposing forces is that equilibrium consumption smoothing is poor in the economy with GSEs. This is the second and main source of welfare loss.

Capturing the spirit of the proposed mortgage finance reforms, our main policy experiment is to start from the low g-fees observed until recently and increase the cost of government mortgage insurance. Naturally, we find that higher guarantee fees "crowd in" the private sector in that they induce a shift of banks' assets from guaranteed to private mortgage bonds. This portfolio shift increases the riskiness of bank assets, making it unnecessary for banks to max out their allowed leverage capacity or to increase the size and riskiness of their mortgage portfolio. Intermediary net worth is higher on average so that banks have more "skin in the game." Ex-post, mortgage default and loss rates are lower in the private sector economy. Fewer mortgage crises turn into financial crises because of the sturdier bank balance sheets. Because of sufficient intermediary capital, banks are able to continue lending even during housing crises. If anything, the provision of mortgage credit becomes more stable in the model with than the one without government guarantees. This result dispels the notion that the GSEs are needed to guarantee stable access to mortgage finance. The key insight is that abolition of mortgage guarantees leads banks to take less risk, moving them farther from their leverage constraints. That improves the economy's ability to allocate resources to the highest marginal utility user, improving consumption smoothing and welfare.

At g-fees that are high enough to crowd out the mortgage guarantee completely, we find that house prices are lower by 6.3%, the mortgage market is smaller by 8.7%, and intermediary leverage is lower by 7.3 percentage points. Mortgages are safer: debt-to-income ratios are 6.2% lower. The upshot in the "private market solution" is that the financial system is less fragile:

the incidence of mortgage defaults and realized mortgage losses are only half as large and bank defaults (financial sector bailouts) are almost completely eliminated. The overall effect of phasing out the GSEs is an increase of social welfare of 0.63% in consumption equivalence units. Borrowers welfare is unchanged while both types of saver households gain substantially. Borrowers benefit from the improved risk sharing in the economy without guarantees, but they lose their mortgage subsidy, face higher mortgage rates, and tighter lending standards. Depositors benefit from higher interest rates and gain 1.3%. Risk-takers (bankers) also gain substantially (1.7%). Thus, while abolishing the GSEs is essentially a Pareto improvement, it increases wealth inequality.

At intermediate levels for the g-fee, we observe that the government guarantee is only taken up in bad times. This dovetails with the "mortgage insurer of last resort" option in the Obama Administration proposal which envisions calling on the government only in crises.

We study the effect of raising the regulatory capital requirement on guaranteed bonds to the same level as that on non-guaranteed bonds. This exercise help us understand how much of the financial sector fragility can be undone with higher capital requirements (macroprudential policy) alone. We find that higher capital requirements eliminate the excessive risk taking by banks. However, the continued presence of underpriced mortgage guarantees continues to lead to large and volatile government debt, and poor risk sharing. The welfare benefits are small suggesting that macro-prudential policy cannot replace GSE reform.

The last part of the paper uses the model to quantitatively evaluate the idea of replacing a full government guarantee by a catastrophic government guarantee (Scharfstein and Sunderam (2011)). The 2014 Johnson-Crapo bill proposes to put 10% private capital in front of a catastrophic government guarantee. The first 10% of losses in the event of a mortgage default would be born by the private sector; the government would step in only when losses exceed that threshold. In our model, this policy amounts to redefining the payoff of the guaranteed mortgage bond. We find that the catastrophic government insurance generates a slightly larger welfare gain than a full phase-out of the guarantee. Since the underpricing of the guarantee is corrected, borrower leverage is lower and the mortgage market is comparable to the economy without guarantees. However, intermediaries still enjoy the insurance against severe mortgage crisis. This improves risk sharing compared to the no-guarantee economy. It demonstrates that a correctly designed government-provided insurance policy can improve welfare.

Related Literature Our paper contributes to several strands of the literature on housing, finance, and macro-economics. Unlike recent quantitative work that explores the causes and consequences of the housing boom,[5] this paper focuses on the current and future state of the housing finance system and the role the government plays in this system. It shares with these models a focus on quantitative implications and on general equilibrium considerations. In particular, house prices and interest rates are determined in equilibrium rather than exogenously specified. We simplify by working in an endowment economy with a constant housing stock.[6]

Like another strand of the literature, our model features borrowers defaulting optimally on their mortgages.[7] Unlike most of that literature, our lenders are not risk-neutral but risk averse. A default risk premium is priced into the mortgage contract. It is time-varying and depends on the covariance of the risk taker's intertemporal marginal rate of substitution with the payoff on the mortgage loan. We assume that lenders impose maximum LTV ratios on borrowers, chosen to match observed mortgage debt/income ratios and default rates in normal and mortgage crisis times. Unlike most of the literature, our mortgage contract is a long-term contract. This is important because of the centrality of the 30-year fixed-rate mortgage in the debate on U.S. housing finance reform. We calibrate our mortgage contract to exhibit the same amount of interest rate risk as the outstanding pool of agency mortgage-backed securities. Our setting is ideally suited to study the interaction of default and prepayment risk. Government policy affects both of these risks.

The main difference between our housing finance model and the literature is our focus on the interplay between the financial sector and the government. A recent literature in asset pricing has emphasized the central role of financial intermediaries in the crisis.[8] Usually, intermediaries

[5]Examples include Ortalo-Magné and Rady (2006), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Landvoigt, Piazzesi, and Schneider (2015), Landvoigt (2012), Chu (2014). See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

[6]The role of housing supply and construction are studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Chatterjee and Eyigungor (2009), Hedlund (2014), and Boldrin, Garriga, Peralta-Alva, and Sanchez (2013).

[7]Recent examples of equilibrium models with default are Corbae and Quintin (2014), Garriga and Schlagenhauf (2009), Chatterjee and Eyigungor (2009), Jeske, Krueger, and Mitman (2013), Landvoigt (2012), Arslan, Guler, and Taskin (2013), and Hedlund (2014).

[8]Recent examples include Brunnermeier and Sannikov (2012), He and Krishnamurty (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), and Maggiori (2013). Brunnermeier, Eisenbach, and Sannikov

have access to a different technology from other agents. In our model, as in Drechsler, Savov, and Schnabl (2014), intermediaries arise from differences in risk aversion instead. The least risk averse savers choose to issue short-term debt ("deposits") to the more risk averse savers. Like in the part of the literature that emphasizes debt constraints, our intermediaries face borrowing (margin) constraints which link the amount of short-term liabilities to the collateral value of their assets. In this class of models, the net worth of the financial sector is the key state variable which governs risk sharing and asset prices. In our model, intermediary wealth is also an important state variable, but it is not the only one. The wealth of the depositors, the wealth of the borrowers, and the outstanding amount of government debt all have important effects on equilibrium allocations and prices.

Unlike most of the intermediary asset pricing literature, we explicitly model the intermediary's decision to default. When intermediary net worth threatens to go negative, intermediaries can choose to offload all assets and liabilities onto the government. The government bailout option is equivalent to deposit insurance in the model. As emphasized above, the interaction of the mortgage guarantee and this bailout guarantee is one of the most interesting and novel aspects of our analysis. By studying the role of the financial sector in the provision of mortgages we capture the stylized fact, pointed out by Jorda, Schularick, and Taylor (2014), that "over a 5-year window run ups in mortgage lending and run ups in house prices raise the likelihood of a subsequent financial crises. Mortgage and house price booms are predictive of future financial crises, and this effect has also become much more dramatic since WW2." We ask how government intervention in the form of asset (mortgage) guarantees or liability (deposit) guarantees affects this nexus.

Our result that the economy without the government guarantee for mortgages features a better capitalized financial sector, a more stable financial system, and higher welfare echoes the arguments made in the literature on capital regulation of financial institutions.[9] Our work contributes a quantitative general equilibrium model to that discussion, with an emphasis on the role of mortgages and mortgage guarantees. The framework should be useful to investigate the quantitative implications of deposit insurance and higher capital requirements as well. See

(2013) provides a review of this literature.

[9]See for example, Kashyap, R., and Stein (2008), Hart and Zingales (2011), Maddaloni and Peydro (2011), Admati and Hellwig (2013), and Admati, DeMarzo, Hellwig, and Pfleiderer (2013).

Begenau (2015) for a similar approach in this literature.

Finally, our paper contributes to the literature that quantifies the effect of government policies in the housing market. Most work focuses on studying the effect of abolishing the mortgage interest rate tax deductibility and the tax exemption of imputed rental income of owner-occupied housing.[10] When house and rental prices are determined endogenously, such policy changes tend to lower house prices and price-rent ratios and cause an increase in home ownership rates. They usually redistribute consumption from the rich to the poor and increase welfare. Studying the GSE subsidies, Jeske, Krueger, and Mitman (2013) reach a similar conclusion regarding welfare. Our work differs from Jeske et al. (2013) in its emphasis on the role of the financial sector and the interaction between the mortgage guarantee and the bailout guarantee. We emphasize heterogeneity between borrowers, banks, and savers. In our model, inequality increases after the abolition of the GSE subsidy. They emphasize heterogeneity among borrowers and obtain a reduction in inequality from the abolition.[11] In terms of model setup, there are several more differences. Jeske et al. model mortgage guarantees as a tax-financed mortgage interest rate subsidy. In our model, the government sells mortgage insurance to the private sector. Second, our mortgages are long-term in nature while theirs are one-period contracts. Third, our lenders are risk-averse, while theirs are risk-neutral. Fourth, our model features aggregate but no idiosyncratic income risk, while their model has no aggregate risk and emphasizes idiosyncratic risk. Fifth, their model has a simple construction sector and constant house prices, while our model has a fixed housing supply and endogenous house prices. Sixth, their model has home ownership choice while ours does not. Seventh, our government can issue debt while theirs has to balance the budget every period. The two papers complement each other nicely.

[10]For example, Gervais (2002), Chambers, Garriga, and Schlagenhauf (2009), Floetotto, Kirker, and Stroebel (2012), and Sommer and Sullivan (2013).

[11]In a recent model similar to Jeske, Krueger, and Mitman (2013), Gete and Zecchetto (2015) argue that the poor, high-credit risk households suffer a disproportionate increase in the cost of mortgage credit from an abolition of GSEs, offsetting the reduction in inequality emphasized by Jeske et al.

2 The Model

2.1 Endowments, Preferences, Technology, Timing

Endowments The model is a two-good endowment economy with a non-housing and a housing Lucas tree. The fruit of the non-housing tree, *output* Y_t, grows and its growth rate is subject to aggregate shocks. The different households are endowed with a fixed and non-tradeable share of this tree. This endowment can be interpreted as labor income. The size of the housing tree (housing stock) grows at the same stochastic trend as output. The total quantity of housing shares is fixed and normalized to $\bar{K}$. The housing stock yields fruit (housing services) proportional to the stock.

Preferences The model features a government and three groups of households. Impatient households will play the role of borrowers in equilibrium (denoted by superscript B), while patient households will turn out to be savers. There are two type of savers, differentiated by their risk aversion coefficient; we refer to the less risk averse savers as "risk takers" (denoted by superscript R) and the more risk averse as "depositors" (denoted by D). Thus, for the rate of impatience we assume that $\beta_R = \beta_D > \beta_B$, and for the coefficient of relative risk aversion we assume that $\sigma_R < \sigma_B \leq \sigma_D$. All agents have Epstein-Zin preferences over the joint consumption bundle which is a Cobb-Douglas aggregate of housing and non-housing consumption with aggregation parameter θ.

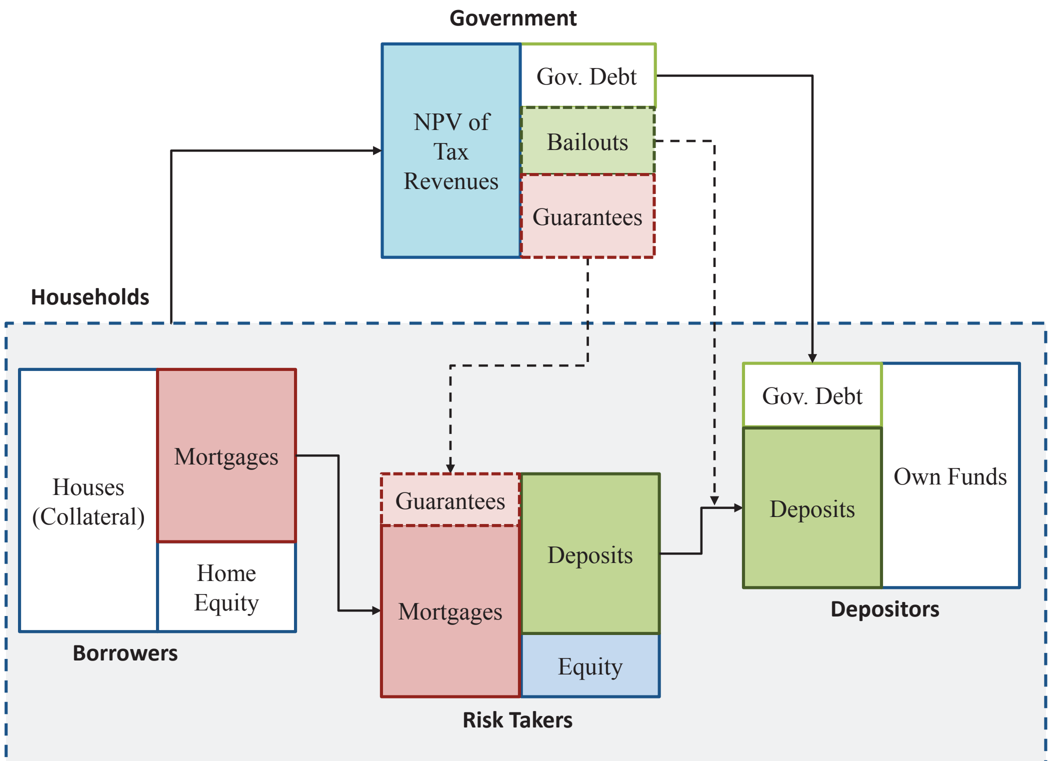
$$U_t^j = \left\{(1-\beta_j)\left(u_t^j\right)^{1-1/\nu} + \beta_j\left(\mathrm{E}_t\left[(U_{t+1}^j)^{1-\sigma_j}\right]\right)^{\frac{1-1/\nu}{1-\sigma_j}}\right\}^{\frac{1}{1-1/\nu}} \tag{1}$$

$$u_t^j = \left(C_t^j\right)^{1-\theta}\left(A_K K_{t-1}^j\right)^{\theta} \tag{2}$$

C_j^t is numeraire non-housing consumption and the constant A_K specifies the housing services from owning the housing stock, expressed in units of the numeraire. All agents share the same elasticity of intertemporal substitution ν.

Figure 1 depicts the balance sheets of the different agents in the economy and the flows of funds between them.

Figure 1: Balance sheets of agents in model economy



Technology In addition to housing, there are three assets in the economy. The first is a one-period short-term bond. The second is a mortgage bond, which aggregates the mortgage loans made to all borrower households. The third is mortgage insurance which the government sells to the private market. The guarantee turns a defaultable long-term mortgage bond into a default-free government-guaranteed mortgage bond. The government exogenously sets the price of the guarantee.

Borrowers experience housing depreciation shocks and may choose to default on their mortgage. There is no recourse; savers and possibly the government (ultimately the tax payers) bear the loss depending on whether mortgage loans are held in the form of private or government-guaranteed mortgage bonds, respectively. A novel model ingredient is that risk takers may also choose to default and declare bankruptcy. Default wipes clean their negative wealth position with no further consequences; the losses are absorbed by the government in a "financial sector bailout."

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Income shocks for all types of agents and housing depreciation shocks for borrower households are realized.

2. Risk takers (financial intermediaries) decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown.[12] Risk takers know its probability distribution and maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[13]

3. Borrowers decide on mortgage default.

4. Risk takers' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. Borrowers choose how much of the remaining mortgage balance to prepay (refinance). All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves and predict the bankruptcy decisions of borrowers and risk takers. We now describe each of the three types of household problems and the government problem in detail.

2.2 Borrower's Problem

Mortgages As in reality, mortgage contracts are long-term, defaultable, and prepayable. The mortgage is a long-term contract, modeled as a perpetuity. Bond coupon (mortgage) payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the mortgage. A mortgage can default, in which case the lenders have recourse to the housing collateral. We introduce a

[12]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)). Additionally, uncertainty about the consequences of a systematic banking crisis and insolvency may seem quite reasonable.

[13]The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

"face value" $F = \frac{\alpha}{1-\delta}$, a fixed fraction α of the mortgage payments (per unit of mortgage bond), at which the mortgage can be prepaid. Prepayment incurs a cost detailed below. Mortgage payments can be deducted from income for tax purposes at a rate $\tau_t^m = (1-\alpha)\tau_t$, where τ_t is the income tax rate and the fraction $(1-\alpha)$ reflects the interest component of mortgage payments .

Borrower Default There is a representative family of borrowers, consisting of a measure one of members. Each member receives the same stochastic labor income $Y_t^B \propto Y_t$, chooses the same quantity of housing k_{t-1}^B s.t. $\int_0^1 k_{t-1}^B di = K_{t-1}^B$, and the same quantity of outstanding mortgage bonds a_t^B s.t. $\int_0^1 a_t^B di = A_t^B$.

After having received income and having chosen house and mortgage size, each family member draws an idiosyncratic housing depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the value of the house by $(1-\omega_{i,t})p_t k_{t-1}^B$. The value of the house after stochastic depreciation is $\omega_{i,t} p_t k_{t-1}^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = \left(\mathrm{Var}_i[\omega_{i,t}]\right)^{0.5}$, where the latter varies over time. The variable $\sigma_{t,\omega}$ governs the mortgage credit risk in the economy; it is the second exogenous aggregate state variable (in addition to aggregate income growth).

Each borrower household member then optimally decides whether or not to default on the mortgages. The houses that the borrower family defaults on are turned over to (foreclosed by) the lender. Let the function $\iota(\omega) : [0,\infty) \to \{0,1\}$ indicate the borrower's decision to default on a house of quality ω. We conjecture and later verify that the optimal default decision is characterized by a threshold level ω_t^*, such that borrowers default on all houses with $\omega_{i,t} \le \omega_t^*$ and repay the debt for all other houses. Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)p_t K_{t-1}^B$ to be the value to the borrowers of the residual (non-defaulted) housing stock after default decisions have been made. We have:

$$Z_A(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\omega\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*]\ \mathrm{E}[\omega_{i,t}\,|\,\omega_{i,t} \ge \omega_t^*] \tag{4}$$

After making a coupon payment of 1 per unit of remaining outstanding mortgage, the amount of outstanding mortgages declines to $\delta Z_A(\omega_t^*) A_t^B$.

Prepayment Next, the households can choose to prepay a quantity of the outstanding mortgages R_t^B by paying the face value F per unit to the lender. We denote by $Z_t^R \equiv R_t^B / A_t^B$ the ratio of prepaid mortgages to beginning-of-period mortgages. Prepayment incurs a monetary cost Ψ. We use an adjustment cost function $\Psi(R_t^B, A_t^B)$ that is convex in the fraction prepaid Z_t^R, capturing bottlenecks in the mortgage refinance infrastructure when too large a share of mortgages are prepaid at once.

Borrower Problem Statement The borrower family's problem is to choose consumption C_t^B, housing K_t^B, default threshold ω_t^*, prepayment quantity R_t^B, and new mortgage debt B_t^B to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + (1-\tau_t^m) Z_A(\omega_t^*) A_t^B + p_t K_t^B + F R_t^B + \Psi(R_t^B, A_t^B) \leq (1-\tau_t) Y_t^B + G_t^{T,B} + Z_K(\omega_t^*) p_t K_{t-1}^B + q_t^m B_t^B, \tag{5}$$

an evolution equation for outstanding mortgage debt:

$$A_{t+1}^B = \delta Z_A(\omega_t^*) A_t^B + B_t^B - R_t^B, \tag{6}$$

a maximum loan-to-value constraint:

$$F A_{t+1}^B \leq \phi p_t K_t^B. \tag{7}$$

and a double constraint on the amount of mortgages that can be refinanced:

$$0 \leq R_t^B \leq \delta Z_A(\omega_t^*) A_t^B. \tag{8}$$

Outstanding mortgage debt at the end of the period (equation 6) is the sum of the remaining mortgage debt after default and new borrowing B_t^B minus prepayments. The borrower household uses after-tax labor income, net transfer income from the government ($G_t^{T,B}$), residual housing wealth, and new mortgage debt raised to pay for consumption, mortgage debt service net of mortgage interest deductibility, new home purchases, prepayments $F R_t^B$ and associated prepayment costs $\Psi(R_t^B, A_t^B)$. New mortgage debt raised is $q_t^m B_t^B$, where q_t^m is the price of one unit of mortgage bonds in terms of the numeraire good.

The borrowing constraint in (7) caps the face value of mortgage debt at the end of the period, FA_{t+1}^B, to a fraction of the market value of the underlying housing, $p_t K_t^B$, where ϕ is the maximum loan-to-value ratio. With such a constraint, declines in house prices (in bad times) tighten borrowing constraints. It is the first of two occasionally binding borrowing constraints in the model.

The refinancing constraints in equation (8) ensure that the amount prepaid is between 0 and the outstanding balance after the default decision was made. Equivalently, the share prepaid, Z_t^R, must be between 0 and $\delta Z_A(\omega_t^*)$.

2.3 Risk Takers

Next we study the problem of the risk taker households, who lend to borrower households and borrow from depositor households. Hence, we refer to this household type as intermediaries.[14]

Risk-taker Default After shocks to income and housing depreciation have been realized, the risk taker (financial intermediary) chooses whether or not to declare bankruptcy. Risk takers who declare bankruptcy have all their assets and liabilities liquidated.[15] They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, risk takers do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \to \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Risk takers choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that risk takers default for all realizations for which the utility cost exceeds the threshold. As we explain below, risk taker default leads to a government bailout.

[14]Note that we could separately model risk taker households as the shareholders of the banks and the banks they own. For simplicity we combine the two balance sheets.

[15]The mortgages are bonds that trade in a competitive market. They are sold during the liquidation and bought by the banks that start off the following period with zero financial wealth and the exogenous income stream.

After the realization of the penalty, risk takers execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem, where they allocate their wealth between a short-term risk-free bond, a private mortgage bond, and a government-guaranteed mortgage bond.

Private Mortgage Bond A private mortgage bond is a simple pass-through vehicle, aggregating the mortgages of the borrowers. The coupon payment on performing mortgages in the current period is $A_t^B Z_A(\omega_t^*)$, which is the number of mortgage bonds times the fraction that is performing times the coupon payment of 1 per unit of performing bond. For mortgages that go in foreclosure, the risk taker repossesses the homes. These homes are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_{t-1}^B$, where ζ is the fraction of home value destroyed in a foreclosure. It represents a deadweight loss to the economy. Thus, the total payoff per unit of private mortgage bond is:

$$M_{t,P} = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_{t-1}^B}{A_t^B}.$$

The price of the bond is q_t^m per unit.

Government-guaranteed Mortgage Bond A government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as a private mortgage bond. The only difference is that it carries no mortgage default risk because of the government guarantee. To prevent having to keep track of an additional state variable, we model guarantees as one-period default insurance.[16] Combining one unit of a private mortgage bond with one unit of default insurance creates a mortgage bond that is government-guaranteed for one period. One unit of a government-guaranteed mortgage bond has the following payoff:

$$M_{t,G} = 1 + (1 - Z_A(\omega_t^*))\delta F$$

The first term is the coupon of 1 on all loans in the pool, performing and non-performing. The second term is compensation for the loss in principal of defaulted loans. Owners of guaranteed loans receive a principal repayment F. The price of the bond is $q_t^m + \gamma_t$ per unit. The government

[16] Rolling over default insurance every period for the life of the loan is the equivalent to the real-world long-term guarantees provided by Fannie Mae and Freddie Mac. Having the choice of renewal each period makes our guarantees more flexible, and hence more valuable, than those in the real world.

sets the price of insurance γ_t per unit of bond to the government. The choice between guaranteed and private mortgage bonds is the main choice of interest in the paper.

Risk-taker wealth Denote risk-taker financial wealth at the start of period t by W_t^R. This financial intermediary net worth is a key state variable.

$$\begin{aligned} W_t^R = &(M_{t,P} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,P}^R \\ &+ (M_{t,G} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,G}^R + B_{t-1}^R. \end{aligned}$$

It consists of the market value of the portfolio of private and guaranteed bonds bought last period, as well as the short-term bonds from last period which mature this period. When the holdings of short-term bonds are negative, the last term is short-term debt which must be repaid this period. Since the mortgage guarantee is valid for only one period, both private and government-guaranteed bonds bought last period trade for the same price q_t^m. Voluntary (i.e., rate-induced rather than default-induced) mortgage prepayments "come in at par" F. Since such prepayments only happen when $q_t^m > F$, they represent a loss to the intermediary. If the portfolio consists entirely of guaranteed bonds, prepayments constitute an important source of risk driving fluctuations in risk taker net worth.

Consumption-Portfolio Choice Problem While intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^R = \frac{\left(C_t^R\right)^{1-\theta}\left(A_K K_{t-1}^R\right)^{\theta}}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Entering with wealth W_t^R, the risk taker's problem is to choose consumption C_t^R, holdings of private mortgage bonds $A_{t+1,P}^R$, holdings of government-guaranteed mortgage bonds $A_{t+1,G}^R$, and short-term bonds B_t^R to maximize life-time utility U_t^R in (1), subject to the budget constraint:

$$C_t^R + q_t^m A_{t+1,P}^R + (q_t^m + \gamma_t)A_{t+1,G}^R + q_t^f B_t^R + (1-\mu_\omega)p_t K_{t-1}^R \leq (1-\tau_t)Y_t^R + G_t^{T,R} + W_t^R, \quad (9)$$

and the following constraints:

$$A^R_{t+1,P} \geq 0, \tag{10}$$

$$A^R_{t+1,G} \geq 0, \tag{11}$$

$$-q^f_t B^R_t \leq q^m_t \varkappa(\xi_P A^R_{t+1,P} + \xi_G A^R_{t+1,G}). \tag{12}$$

The budget constraint (9) shows that the risk-taker uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, purchases of private and government-guaranteed mortgage bonds and short-term bonds, and for housing repairs which undo the effects of depreciation. We do not allow for negative positions in either long-term mortgage bond (equations 10 and 11). A key constraint in the model is (12). A negative position in the short-term bond is akin to the risk taker issuing short-term bonds, or equivalently deposits. The negative position in the short-term bond must be collateralized by the market value of the risk taker's holdings of long-term mortgage bonds. The parameters ξ_P and ξ_G together with $\varkappa$ determine how useful private and government-guaranteed mortgage bonds are as collateral. In the calibration, we will assume that guaranteed mortgages are better collateral: $\xi_G > \xi_P$. The constraint captures the reality of Basel II/III-type risk weights that restrict intermediary leverage.[17]

2.4 Depositors

The second type of savers, depositors, receive labor income, $Y^D_t \propto Y_t$, own a fixed share of the housing stock K^D_t, and solve a standard consumption-savings problem. Entering with wealth $W^D_t = B^D_{t-1}$, the depositor's problem is to choose consumption C^D_t and holdings of short-term bonds B^D_t to maximize life-time utility U^D_t in (1), subject to the budget constraint:

$$C^D_t + q^f_t B^D_t + (1-\mu_\omega)p_t K^D_{t-1} \leq (1-\tau_t)Y^D_t + G^{T,D}_t + W^D_t, \tag{13}$$

[17]The short-term borrowing is akin to a repo contract. It allows the intermediary to buy a mortgage bond by borrowing a fraction ξ of the purchase price while only using a fraction $1-\xi$ of the purchase price, the margin requirement, of her own capital. One can think of the guaranteed bond as a private mortgage bond plus a government guarantee (a credit default swap or mortgage insurance). Implicit in constraint (12) is the assumption that the government guarantee itself is an off-balance sheet item that cannot be collateralized.

and short-sales constraints on bond holdings:

$$B_t^D \quad \geq 0, . \tag{14}$$

The budget constraint (13) is similar to that of the risk taker. We also do not allow depositor's to take a negative position in the short-term bond (14), consistent with our assumption that the depositor must not declare bankruptcy.

2.5 Government

We model the government as set of exogenously specified tax, spending, bailout, and debt issuance policies.[18] Government tax revenues, T_t, are labor income tax receipts minus mortgage interest deduction tax expenditures plus mortgage guarantee fee income:

$$T_t = \tau_t Y_t - \tau_t^m Z_A(\omega_t^*) A_t^B + \gamma_t (A_{t,G}^R + A_{t,G}^D)$$

Government expenditures, G_t are the sum of payoffs on mortgage guarantees, financial sector bailouts, other exogenous government spending, G_t^o, and government transfer spending G_t^T:

$$G_t = (M_{t,G} - M_{t,P}) A_{t,G}^R - D(\rho_t) W_t^R + G_t^o + G_t^T$$

The mortgage guarantee pays to the risk takers the difference in cash-flow between a guaranteed and a private mortgage bond, for each unit of guaranteed bond they purchase. The bailout to the financial sector equals the negative of the financial wealth of the risk taker, W_t^R, in the event of a bankruptcy ($D(\rho_t) = 1$). By bailing out the intermediaries, the government renders intermediaries' liabilities, deposits, risk-free. In the model, risk taker bankruptcies, limited liability for risk takers, and deposit insurance are equivalent.

The government issues one-period risk-free debt. Debt repayments and government expen-

[18]We consolidate the role of the GSEs and that of the Treasury department into one government, reflecting the reality as of September 2008.

ditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{15}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^D B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^D$ is the SDF of the depositor.[19] Because of its unique ability to tax and repay its debt, the government can spread out the cost of mortgage default waves and financial sector rescue operations over time. We are interested in understanding whether the government's ability to tax and issue debt leads to an increased stability of mortgage credit provision in the world with government guarantees.

Government policy parameters are $\Theta_t = (\tau_t, \gamma_t, G_t^o, \phi, \xi_G, \xi_P, \mu_\rho)$. The parameters ϕ in equation (7) and (ξ_G, ξ_P) in equation (12) can be thought of as macro-prudential policy tools which govern household and intermediary leverage. We added the parameter μ_ρ that governs the mean utility cost of bankruptcy to risk takers to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy. This cost directly affects the strength of deposit insurance but could also include reputational costs of bank defaults.

2.6 Equilibrium

Given a sequence of aggregate income and house valuation shocks $\{Y_t, \sigma_{\omega,t}\}$ and utility costs of default shocks $\{\rho_t\}$, and given a government policy $\{\Theta_t\}$, a competitive equilibrium is an allocation $\{C_t^B, K_t^B, B_t^B, R_t^B\}$ for borrowers, $\{C_t^R, A_{t,P}^R, A_{t,G}^R, B_t^R\}$ for risk takers, $\{C_t^D, B_t^D\}$ for depositors, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrowers, depositors, and risk-takers maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

[19] We show below that the risk averse saver is the marginal agent for short-term risk-free debt.

The market clearing conditions are:

$$\text{Risk-free bonds: } B_t^G = B_t^D + B_t^R \tag{16}$$

$$\text{Mortgages: } A_t^B = A_{t,G}^R + A_{t,P}^R \tag{17}$$

$$\text{Housing tree shares: } K_t^B + K_t^R + K_t^D = \bar{K} \tag{18}$$

$$\text{Consumption: } Y_t = C_t^B + C_t^R + C_t^D + (1-\mu_{t,\omega})p_t\bar{K} + G_t^o + \zeta(\mu_{t,\omega} - Z_K(\omega_t^*))\frac{p_t K_{t-1}^B}{A_t^B} + \Psi(R_t^B, A_t^B)$$

The last equation states that total non-housing resources equal the sum of non-housing consumption expenditures and home renovations by the households, discretionary spending by the government, and lost resources due to the deadweight costs of foreclosure and mortgage refinancing.

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights ℓ:

$$\mathcal{W}_t(\cdot;\Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^R V_t^R,$$

where the $V^i(\cdot)$ functions are the value functions defined in the appendix. A nice feature of value functions under Epstein-Zin preferences is that they are homogeneous of degree one in consumption. Thus, a $\lambda\%$ increase in the value function from a policy change is also a $\lambda\%$ change in consumption units.

3 Model Solution and Calibration

Appendix A presents the Bellman equations for each of the three household types and derives first-order conditions for optimality. We highlight some key features of the solution here by inspecting these FOC. We then turn to the calibration.

3.1 First Order Conditions

Borrower FOCs First, since borrowers are the only households freely choosing their housing position, their choice pins down the price of housing in the economy. Let $\tilde{\mathcal{M}}^i_{t,t+1}$ be the intertemporal marginal rate of substitution (or stochastic discount factor) for agent $i \in \{B, D, R\}$, with expressions provided in the Appendix. At the optimum, house prices satisfy the recursion:

$$p_t \left[1 - \tilde{\lambda}^B_t \phi\right] = \mathrm{E}_t \left[\tilde{\mathcal{M}}^B_{t,t+1} e^{g_{t+1}} \left\{p_{t+1} Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta) K^B_t}\right\}\right] \tag{19}$$

The marginal cost of housing on the left-hand side consists of the house price p_t minus a term which reflects the collateral benefit of housing; an extra unit of housing relaxes the maximum LTV constraint (7). The right hand side captures the expected discounted future marginal benefits which depends on the resale value of the non-defaulted housing stock as well as on the dividend from housing, which is the intratemporal marginal rate of substitution between housing and non-housing goods.

Second, we analyze the borrower's optimal foreclosure decision. In the appendix, we show that the optimal default threshold is given by:

$$\omega^*_t = \frac{(1 - \tau^m_t + \delta q^m_t - \delta \tilde{\lambda}^{RB}_t) A^B_t}{p_t K^B_{t-1}}.$$

At the threshold level ω^*_t, the cost from foreclosure (and mortgage debt relief), which is the loss of a house valued at $\omega^*_t p_t K^B_{t-1}$, exactly equals the expected cost from continuing the service the mortgage (including the option to default in the future which is encoded in q^m_t) and keeping the house. The cutoff has an intuitive interpretation. It is the aggregate loan-to-value ratio of the borrowers, with both mortgage debt and housing valued at market prices. When the *market* leverage of the borrower increases, the house value threshold ω^*_t rises and default becomes more likely. Note that when the borrower exercises her prepayment option to its maximum extent, $\tilde{\lambda}^{RB}_t > 0$ and default becomes less likely. Hence the default option and the prepayment option interact. A valuable refinancing option gives the borrower incentives to postpone a default decision as in Deng, Quigley, and Van Order (2000).

Third, the optimal share of outstanding mortgages that the borrower chooses to prepay,

$Z_t^R = R_t^B / A_t^B$ is given by:

$$\psi Z_t^R = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB} \tag{20}$$

This balances the marginal cost of refinancing on the left-hand side with the marginal benefit on the right-hand side. For an internal prepayment choice, the marginal benefit is to increase the value of mortgage debt raised by $q_t^m - F$. Intuitively, when current mortgage rates are lower than when the mortgage was originated, the mortgage is a premium bond and trades at a price q^m above par value F. By refinancing a marginal unit of debt, the borrower gains $q^m - F$. If $q^m - F$ is large enough, the borrower will want to refinance all outstanding debt ($Z_t^R = \delta Z_A(\omega^*)$). The multiplier on the refinancing upper bound activates ($\tilde{\lambda}_t^{RB} > 0$). Conversely, when $q_t^m < F$, refinancing is not useful and the multiplier on the lower refinancing bound, $\tilde{\mu}_t^{RB} > 0$, turns positive to keep $Z_t^R = 0$.

Fourth, from the borrower's first order condition for A^B, we can read off the demand for mortgage debt.

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*) \left(1 - \tau^m - \frac{\psi \left(Z_{t+1}^R \right)^2}{2 Z_A(\omega_{t+1}^*)} - \delta \tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m \right) \right]. \tag{21}$$

A unit of mortgage debt obtained generates an amount q_t^m today but uses up some borrowing capacity, which is costly when the borrower's loan-to-value constraint binds ($\tilde{\lambda}_t^B > 0$). The non-defaulted part of the debt must be serviced in future periods, modulo a mortgage interest tax deduction, as long as it is not prepaid.

Depositor FOC The risk averse saver buys short-term debt issued by the risk taker. This debt is equivalent to government debt by virtue of the deposit insurance. The depositor's first-order condition for the short-term bond, assuming the short-sales constraint is not binding, is:

$$q_t^f = \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^D \right]$$

The depositor's precautionary savings incentives are a crucial force determining equilibrium risk-free interest rates.

Risk Taker FOCs Next, we turn to the risk taker's default decision. The risk taker will optimally default whenever the utility costs of doing so is sufficiently small: $\rho_t < \rho_t^*$. The threshold depends on her wealth W_t^R and the state variables $\mathcal{S}_t^R$ that are exogenous to the risk taker, including the wealth of the borrower and of the depositor, and the outstanding amount of government debt. At the threshold, she is indifferent between defaulting and offloading her (negative) wealth onto the government or carrying on:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R),$$

where the value function is defined in Appendix A.

Second, the risk taker can invest in both government guaranteed and private MBS. The respective first-order conditions are:

$$\begin{aligned} q_t^m + \gamma_t &= \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*) q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] + q_t^m \varkappa \xi_G \tilde{\lambda}_t^R \\ q_t^m &= \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R \left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*) q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] + q_t^m \varkappa \xi_P \tilde{\lambda}_t^R. \end{aligned}$$

Absent binding risk taker borrowing constraints ($\tilde{\lambda}_t^R = 0$), the marginal cost of a guaranteed mortgage bond is the price q_t^m plus the guarantee fee γ_t (expressed as a price) while the benefit is the expected discounted value of the bond tomorrow, which consists of the coupon payment and the repayment of principal in case of default (both are in M_G) plus the resale value of the non-defaulted portion of the mortgage bond. When there are prepayments, the market value of the bond is adjusted for the difference between the market value and the face value, on the share of mortgages that gets prepaid. If the collateral constraint is binding, the benefit is increased by the relaxation of the borrowing constraint, and depends on the haircut ξ_G for guaranteed mortgages. The first-order condition for private mortgages is similar, without the guarantee fee term, with a different collateral requirement term (ξ_P), and a different mortgage payoff M_P.

An equivalent way of restating the risk taker's choice is in terms of how many units of mortgages to originate to borrowers, and for how much of these holdings to buy default insurance from the government. The optimal amount of default insurance to buy solves:

$$\gamma_t = \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{G,t+1} - M_{P,t+1} \right) \right] + \tilde{\lambda}^R_t \varkappa q^m_t \left(\xi_G - \xi_P \right)$$

Risk takers will buy insurance until the marginal cost of insurance on the left equals the marginal benefit. An extra unit of default insurance increases the payoff of the mortgage and it increases the collateralizability of a mortgage, a benefit which only matters when the borrowing constraint binds. A binding risk taker leverage constraint increases demand for mortgage bonds, and especially for guaranteed bonds given their low risk weight (high ξ_G).

3.2 Calibration

The parameters of the model and their targets are summarized in Table 1.

Aggregate Income The model is calibrated at annual frequency. Aggregate endowment or labor income Y_t follows:

$$\begin{aligned} Y_t &= Y_{t-1} \exp(g_t) \\ g_t &= \rho_g g_{t-1} + (1 - \rho_g)\bar{g} + \epsilon_t, \qquad \epsilon_t \sim iid\, \mathcal{N}(0, \sigma_g) \end{aligned}$$

We scale all variables by permanent income in order render the problem stationary. Given the persistence of income growth, g_t becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst (1995). The procedure matches the mean, volatility, and persistence of GDP growth by choosing both the grid points and the transition probabilities between them. We use annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2014 excluding the war years 1940-1945. The resulting mean is 1.9%, the standard deviation is 3.9%, and the persistence is 0.42. The states, the transition probability matrix, and the stationary distribution are listed in Appendix B.1.

Foreclosure crises The stochastic depreciation shocks or idiosyncratic house value shocks, $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters

Table 1: Calibration

This table reports the parameter values of our model.

	Parameter	Description	Value	Target
		Exogenous Shocks		
1	$\bar{g}$	mean income growth	1.9%	Mean rpc GDP gr 1929-2013
2	σ_g	volatility income growth	3.9%	Vol rpc GDP gr 1929-2013
3	ρ_g	persistence income growth	0.41	AC(1) rpc GDP gr 1929-2013
4	μ_ω	mean idiosync. house value shock	2.5%	Housing depreciation Census
5	σ_ω	volatility idiosync. house value shock	{0.10,0.14}	Mortgage default rates (Appendix B.2)
6	$p^\omega_{LL}, p^\omega_{HH}$	transition prob	0.2,0.99	Frequency and duration of mortgage crises
		Population, Income, and Housing Shares		
7	ℓ^i, $i \in \{B, D, R\}$	population shares	{47,51,2}%	Population shares SCF 1995-2013
8	Y^i, $i \in \{B, D, R\}$	income shares	{38,52,10}%	Income shares SCF 1995-2013
9	K^i, $i \in \{B, D, R\}$	housing shares	{39,49,12}%	Housing wealth shares SCF 1995-2013
		Mortgages		
10	ζ	deadweight loss of foreclosure	{0.25,0.425}	Mortgage severities (Appendix B.2)
11	δ	average life of mortgage pool	0.95	Duration Fcn. (Appendix B.3)
12	α	guarantee payout fraction	0.52	Duration Fcn. (Appendix B.3)
13	ϕ	maximum LTV ratio	0.65	Borrowers' mortgage debt-to-income SCF 1995-2013
14	ψ	refinancing cost parameter	8	Mean Conditional Prepayment Rate
		Preferences		
15	σ^B	risk aversion borrower	8	Vol household mortgage debt to GDP 1985-2014
16	β^B	time discount factor borrower	0.88	Mean housing wealth to GDP 1985-2014
17	θ^B	housing expenditure share	0.20	Housing expenditure share NIPA
18	σ^D	risk aversion depositor	20	volatility risk-free interest rate 1985-2014
19	$\beta^D = \beta^R$	time discount factor savers	0.98	Mean risk-free interest rate 1985-2014
20	σ^R	risk aversion risk taker	1	Standard Value
21	ν	intertemp. elasticity of subst.	1	Standard Value
		Government Policy		
22	τ	income tax rate	19.83%	BEA govmt revenues to trend GDP 1946-2013
23	G^o	exogenous govmt spending	15.8%	BEA govmt spending to trend GDP 1946-2013
24	G^T	govmt transfers to agents	3.41%	BEA govmt net transfers to trend GDP 1946-2013
25	$\varkappa$	margin	98%	Financial sector leverage Flow of Funds 1985-2014
26	ξ_G	margin guaranteed MBS	1.6%	Basel 2/3 regulatory capital charge agency MBS
27	ξ_P	margin private MBS	8%	Basel 2/3 regulatory capital charge non-agency mortgages

$(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot; \chi_{t,0}, \chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean μ_ω constant at 0.975, implying annual depreciation of housing of 2.5%, a standard value, and to let the cross-sectional standard deviation $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ govern the aggregate mortgage credit risk and represent the second source of exogenous aggregate risk. We refer to states with the high value for $\sigma_{t,\omega}$ as mortgage crises or foreclosure crises.

We set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.10, 0.14)$ and the deadweight losses of foreclosure $(\zeta_H, \zeta_L) = (0.25, 0.425)$ in order to match the mortgage default rates and severities (losses given default) in normal times and in mortgage crises. In the benchmark model with low guarantee fees, and given all other parameter choices, these parameters imply equilibrium mortgage default rates of 1.6% in normal times and 12.7% in mortgage crises. The unconditional default rate is 2.7%. They imply equilibrium severities of 28.2% in normal times and 47.0% in crises. Mortgage default and severity rates combine to produce unconditional mortgage loss rates of 1.0% per year; 0.46% in normal times and 6.1% in crises. Appendix B.2 discusses the empirical evidence and argues that these numbers are a good match for the data. We note that the values for σ_ω are in line with standard values for individual house price shocks (Landvoigt, Piazzesi, and Schneider (2015)). Our unconditional severities are 30%, in line with typical values in the literature (Campbell, Giglio, and Pathak (2011)).

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^ω_{LL} and p^ω_{HH}, to match the frequency and length of mortgage crises. Based on the argument by Jorda et al. (2014) that most financial crises after WW-II are related to the mortgage market and the historical frequency of financial crises in Reinhart and Rogoff (2009), we target a 10% probability of mortgage crises. Conditional on a crisis, we set the expected length to 2 years, based on evidence in Jorda et al. and Reinhart and Rogoff. Thus, the model implies that not all recessions are mortgage crises, but all mortgage crises are recessions.[20]

[20]In a long simulation, 33% of recessions are mortgage crises. This compares to a fraction of 6/22 (≈27%) in Jorda et al. (2014). The correlation between $\sigma_{t,\omega}$ and g_t is -0.42. The model generates persistence in the

Population and wealth shares To pin down the labor income and housing shares for borrowers, depositors, and risk takers, we calculate a net fixed-income position for each household in the Survey of Consumer Finance (SCF).[21] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, we consider a household to be a saver, otherwise it is a borrower. For savers, we calculate the amount of risky assets, defined as their holdings of stocks, business wealth, and real estate wealth, as well as the share of these risky assets in total wealth. We define risk takes as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 47\%$, $\ell^D = 51\%$, and $\ell^R = 2\%$. Based on this classification and the same SCF data, borrowers receive 38% of aggregate income and own 39% of residential real estate. Depositors receive 52% of income and 49% of housing wealth. Finally, risk takers receive 10% of income and 12% of housing wealth. By virtue of the calibration, the model thus matches basic aspects of the observed income and wealth inequality.

Mortgages In our model, a government-guaranteed MBS is a geometric bond. The issuer of one bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. The same is true if the borrower refinances the mortgage. We estimate values for δ and F such that the duration of the geometric mortgage in the model matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a range of historically observed mortgage rates. This novel procedure, detailed in Appendix B.3, recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. We find that values of $\delta = 0.95$ and $\alpha = 0.52$ imply a relationship between price and mortgage rate for the geometric mortgage that closely matches the price-rate relationship for a real-life MBS pool consisting of fixed-rate mortgages issues across a range of vintages. The average duration in model and data of the mortgage (pool) is about 4 years. Like the real-life MBS pool, the geometric mortgage price is convex in rates when rates are high (the

mortgage default rate of 0.02 in the low g-fee economy and 0.08 in the high g-fee economy. The persistence depends on, among other things, the persistence of $\sigma_{t,\omega}$ which is 0.478.

[21] We use all survey waves from 1995 until 2013 and average across them.

prepayment option is out-of-the-money) and concave when rates are low ("negative convexity" when the prepayment option is in-the-money). Thus, the geometric mortgage has the same interest rate risk (duration) of real-life mortgages for different interest rate scenarios. Despite its simplicity, the perpetual mortgage with prepayment captures the key features of real-life guaranteed MBS pools.

Borrowers can obtain a mortgage with face value up to a fraction ϕ of the market value of their house. We set the LTV ratio parameter $\phi = 0.65$ to match the average mortgage debt-to-income ratio for borrowers in the SCF of 130%. The calibration produces an unconditional mortgage debt-to-income ratio among borrowers of 148% in the benchmark model, somewhat overshooting the target. In the benchmark model, borrowers' mean loan-to-value ratios are 63.8% in book value and 76.2% in market value terms.

We set the marginal prepayment cost parameter ψ so that the benchmark model generates reasonable conditional prepayment speeds. We target an average speed in the range of 15-20% annually.[22]

Government parameters Government fiscal policy consists of mortgage guarantee policy, a financial sector bailout policy, and general taxation and spending policies. In our desire to have a quantitatively meaningful model, we believe it is important to also capture non-housing realted fiscal policy. After the conservatorship of Fannie Mae and Freddie Mac in September 2008, the merger of the GSEs and the Treasury Department became a reality.

Starting with the guarantee policy, our parameter γ specifies the cost of a guarantee, expressed in the same units as the price of the mortgage. Real-world guarantee fees are expressed as a surcharge to the interest rate. We consider several values for γ with implied g-fees ranging from 20 to 300 basis points. Freddie Mac's management and g-fee rate was stable at around 20bps from 2000 to 2012. Similarly, Fannie Mae's single-family effective g-fee was also right around 20bps between 2000 and 2009. Thus, our benchmark model is the 20 basis point g-fee economy. The main policy experiment in the paper is to raise γ and investigate the effects from higher guarantee fees. Interestingly, Freddie Mac has increased its g-fee gradually from 20bps

[22]CPR rates depend strongly on the interest rate and mortgage vintage, even among conventional 30-year mortgages. For example, in December 2009, CPRs ranged from 6% to 34%. In October 2013, they ranged from 14% to 24% (SIFMA prepayment tables, benchmark scenario). In 2003, CPRs were as high as 80%.

at the start of 2012 to 32 bps at the end of 2014, while Fannie Mae has increased its g-fee from 20bps at the start of 2009 to 41 bps at the end of 2014 (Urban Institute Housing Finance Policy Center, December 2014 update). Fannie's g-fees on new single-family originations currently average 63bps.

We set the proportional income tax rate equal to $\tau = 20.4\%$ in order to match average discretionary tax revenue to trend GDP in U.S. data. The discretionary tax revenue in the 1946-2013 data of 19.97% is after mortgage interest deductions, which is about 0.43% of trend GDP. Hence, we set a tax rate before MID of 20.4%.[23] As explained before, the model features mortgage interest rate deductibility at the income tax rate. Because our geometric mortgages do not distinguish between interest and principal payments, we assume that the entire mortgage payment is deductible but at a lower rate, $\tau^m = (1 - \alpha) \times \tau$.[24] Tax revenues are pro-cyclical, as in the data. Every dollar of income is taxed at the same tax rate. Risk takers are only 2% of the population but pay 10% of the income taxes since they earn 10% of the income.

We set exogenous government spending equal to $G^o = 0.163$ (times trend GDP of 1) in order to match average exogenous government spending to trend GDP in the 1946-2013 U.S. data of 16.3%.[25] This exogenous spending is wasted. We also allow for transfer spending of 3.18% of GDP, which equals the net transfer spending in the 1946-2013 data. This spending is distributed lump-sum to the agents in proportion to their population share. As a fraction of *realized* GDP, expenditures fluctuate, mimicking their counter-cyclicality in the data.

We can interpret the risk-taker borrowing constraint parameters, $\varkappa$, ξ_G and ξ_P as regulatory capital constraints set by the government. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and performing" enjoy a 50% risk weight and all others

[23] In our numerical work, we keep the ratio of government debt to GDP stationary by decreasing tax rates $\underline{\tau_t}$ when debt-to-GDP threatens falls below $\underline{b^G} = 0$ and by increasing tax rates when debt-to-GDP exceedd $\overline{b^G} = 1.2$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -30%. Tax rates are gradually and convexly increased until they hit 50% ay a debt-to-GDP ratio of 160%. Our simulations never reach the -30% and +160% debt/GDP states. These profligacy and austerity tax policies do not affect the amount of resources that are available for private consumption in the economy.

[24] As discussed in Appendix B.3, the sum of all mortgage payments is $1/(1 - \delta)$ and $F = \alpha/(1 - \delta)$ is the payment of "principal." Hence, the fraction of "interest payments" is the fraction $(1 - \alpha)$. In the equilibrium with low g-fees, the mortgage interest deductibility expense is 0.46% of trend GDP, very close to the target.

[25] The data are from Table 3.1 from the BEA. Exogenous government spending is defined as consumption expenditures (line 18) plus subsidies (line 27) minus the surplus of government enterprises (line 16). It excludes interest service on the debt and net spending on social security and other entitlement programs. Government revenues are defined as current receipts (line 1), which excludes social security tax receipts. Trend GDP is calculated with the Hodrick Prescott Filter.

a 100% risk weight. Agency MBS receive a 20% risk weight. Given that we think of the non-guaranteed mortgage market as the subprime and Alt-A market, a capital charge of 8% (100% risk weight) seems most appropriate for ξ_P. Given that the government guaranteed mortgages are the counterpart to agency MBS, we set a capital charge of 1.6% (20% risk weight) for ξ_G. We set the additional margin $\varkappa$ to match average leverage ratios of the financial sector, given all other parameters. Since mortgage assets are predominantly held by leveraged financial institutions, we calculate leverage for those kinds of institutions. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[26] Since the non-mortgage portfolio of these institutions have higher risk weights than their mortgage portfolio, we find that $\varkappa < 1$.

Utility cost of risk-taker bankruptcy The model features a random utility penalty that risk takers suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.05$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative financial intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of the risk-taker) depends on the frequency of foreclosure crises, and the endogenous choices (asset composition and liability choice) of the risk taker.

Preference parameters Preference parameters are harder to pin down directly by data since they affect many equilibrium quantities and prices simultaneously. However, the discussion of the first-order conditions above helps us connect the various parameters to specific equilibrium

[26]Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations (Fed Bailout entities e.g. Maiden Lanes), GSEs, Agency- and GSE-backed Mortgage pools (before consolidation), Issuers of ABS, REITs, and Life and Property-Casualty Insurance Companies. Krisnamurthy and Vissing-Jorgensen (2011) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours except that we add insurance companies and take out money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

objects they have a disproportionate effect on.

The coefficients of risk aversion are $\sigma_R = 1$, $\sigma_B = 8$, and $\sigma_D = 20$. The annual subjective time discount factors are $\beta_R = \beta_D = 0.98$ and $\beta_B = 0.88$. Risk aversion and the time discount factor of the depositor disproportionately affect the short-term interest rate and its volatility. The benchmark model generates a mean one-year real risk-free interest rate of 1.1% with a standard deviation of 3.0%. In the data, the mean real interest rate is 1.20% with a volatility of 1.97% over the period 1985-2014.[27] The borrower's discount factor governs mortgage debt and ultimately house prices. In the model, housing wealth to trend GDP is 2.24, while in the Flow of Funds data (1985-2014) it is 2.41.[28] Borrower risk aversion is set to target the volatility of the annual change in household mortgage debt to GDP (Flow of Funds and NIPA), which is 4.2% in the 1985-2014 data. Our low g-fee economy produces a volatility of 1.8%. The risk takers have log period utility and their subjective discount factor is set equal to that of the depositors. We set the elasticity of inter-temporal substitution equal to 1 for all agents, a common value in the asset pricing literature.

3.3 Computation

This is a complicated model to solve given the presence of occasionally binding constraints for both borrowers and risk takers. We provide a non-linear global solution method, policy time iteration, which is a variant of the parameterized expectations approach. As explained in more detail in computational Appendix C, policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of non-linear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities. Kubler and Schmedders (2003) show that there exist stationary equilibria in this class of models when all exogenous state variables follow Markov chains, as is the case here. Our solution method is a variant of theirs.

[27]To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of professional Forecasters. The mean interest rate is sensitive to the sample period. Over the period 1990-2014, the mean is 0.72% and over the period 1998-2014, it is only 0.21%.

[28]The number in the data includes the real estate owned by the corporate sector since our model is a model of the entire economy but does not include real estate-owning firms. Real estate owned by the household and non-corporate sector is 1.51 times GDP on average over this period.

4 Main Results: Phasing out the GSEs

The main experiment in the paper is to compare an economy with and without government-guaranteed mortgages. More precisely, we compute a sequence of economies that differ by the guarantee fee γ_t that the government charges for providing the default insurance. All economies feature a government bailout guarantee to the financial sector (risk takers), or equivalently, deposit insurance. We evaluate how equilibrium prices and quantities, and ultimately welfare are affected by an increase in g-fees. We do so based on a 10,000-period simulation of each model. Tables 2 and 3 summarize the results, reporting unconditional means and standard deviations across the 10,000 simulations.

Our benchmark model is one where the government provides the mortgage guarantee relatively cheaply. We set γ_t to a value that implies an annual rate spread of 20bps.[29] We think of this "low g-fee" case as representing actual g-fees observed between the late 1990s and the late 2000s. We refer to this calibration as the "low g-fee" or "20bps g-fee" economy. We also compute several intermediate g-fee economies with g-fees around 55, 75, 100, and 150 basis points. In the interest of space, we only report detailed results for the 55bps and 100bps g-fee economies. The former is of particular interest since it reflects the level of g-fees observed today. In the latter, guaranteed mortgages are dominant in mortgage crises while private bonds are dominant in normal times. This outcome is reminiscent of Option B in the Obama Administration's policy document of February 2010 which envisions setting the g-fee high enough so that it is only taken up in crises. Finally, we report on a high g-fee economy, the 275bps g-fee economy, where guarantees are expensive enough that they are never bought. In this last economy, the GSEs are "phased out."

4.1 Prices

The first panel of Table 2 shows that interest rates are low and house prices high in the benchmark low g-fee economy. The cheap mortgage guarantees lead to mortgage interest rates

[29] The interest rate on private bonds can be calculated as $r_{P,t} = \log\left(\frac{1}{q_t^m} + \delta\right)$, and the rate on guaranteed bonds is $r_{G,t} = \log\left(\frac{1}{q^m + \gamma_t} + \delta\right)$. The effective g-fee, quoted as a difference in rates, is therefore given by $r_{P,t} - r_{G,t}$.

of 3.5%, 22 basis points lower than in the high g-fee economy. This magnitude of subsidy to mortgage rates is similar to what the empirical research has inferred form the spread between conforming and jumbo mortgage loans.

Short-term interest rates vary more substantially across economies, with the low g-fee economy displaying real short-term interest rates that are 1.1% per year, 70 basis points lower than the 1.8% in the high g-fee economy. The model is successful in generating low real short-term interest rates. We argue below that a key reason for low real interest rates is that the low g-fee economy is risky. Depositors, who often are the marginal agents in the risk-free bond market and who are the most risk averse of all agents, have strong precautionary savings motives which push down interest rates.

Faced with low mortgage rates, borrowers who are the marginal agents in the housing sector demand more housing. Given a fixed housing supply (relative to trend growth), increased housing demand results in higher house prices. The low g-fee economy's house prices are 6.3% higher than in the high g-fee economy. Phasing out the GSEs would lead to a non-trivial decline in house prices. House prices are also substantially more volatile in the low g-fee economy: 14% annual standard deviation compared to 12% in the high g-fee economy. This is a consequence of the higher volatility in the demand for mortgage debt in the low g-fee economy, as discussed further below.

4.2 Borrowers

Low G-fees Faced with high house prices and low mortgage rates, borrowers demand a lot of mortgage debt in the low g-fee equilibrium. The steady state stock of mortgages outstanding is high (63.4% of GDP in market value terms or 0.053 units A^B). Mortgage debt is also more volatile in the low g-fee, with the economy experiencing large drops during mortgage crises. The average borrower LTV ratio is 63.8% and borrowers' mortgage debt-to-income is 1.49 on average, both are close to the averages in the recent SCF data. We recall that the optimal mortgage default policy for the borrower family depends on the mark-to-market LTV ratio. The higher that ratio, the higher the mortgage default rate. That ratio is the highest and thus the mortgages are the riskiest in the low g-fee economy. The average mortgage default rate is

Table 2: Phasing Out the GSEs: Main Results

	20 bp g-fee		**55 bp g-fee**		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
Prices								
Risk free rate	1.13%	3.00%	1.20%	2.91%	1.50%	3.00%	1.83%	3.14%
Mortgage rate	3.52%	0.24%	3.59%	0.25%	3.67%	0.26%	3.74%	0.26%
House price	2.240	0.142	2.199	0.131	2.152	0.122	2.100	0.120
Borrower								
Mortgage debt	0.053	0.001	0.052	0.001	0.051	0.001	0.050	0.001
Borrower LTV	63.79%	3.93%	63.80%	3.67%	63.81%	3.46%	63.79%	3.50%
Market value of debt LTV	75.72%	6.47%	75.12%	6.21%	74.49%	6.04%	73.91%	6.02%
Borrower debt to income	1.489	0.040	1.462	0.030	1.432	0.024	1.397	0.025
Debt/income growth	0.04%	2.86%	0.04%	2.73%	0.04%	2.79%	0.04%	2.87%
Default rate	2.74%	6.20%	2.32%	5.15%	1.97%	4.21%	1.76%	3.92%
Rate-induced prepayment rate	15.83%	4.24%	14.49%	4.27%	13.11%	4.42%	11.92%	4.38%
Loss Given Default	30.15%	5.76%	30.10%	5.74%	30.06%	5.73%	30.02%	5.72%
MTM Loss Given Prepayment	10.78%	2.59%	9.95%	2.65%	9.08%	2.80%	8.31%	2.81%
Loss rate private	1.04%	2.75%	0.88%	2.25%	0.75%	1.82%	0.68%	1.72%
Loss rate guaranteed	0.38%	0.88%	0.31%	0.72%	0.25%	0.57%	0.21%	0.50%
Loss rate portfolio	0.38%	0.88%	0.31%	0.73%	0.61%	1.56%	0.67%	1.72%
Risk-Taker								
Market value of bank assets	0.634	0.018	0.617	0.014	0.599	0.013	0.579	0.013
Fraction guaranteed	99.96%	0.55%	98.41%	5.48%	10.88%	28.77%	0.58%	5.12%
Risk taker leverage	95.59%	0.92%	95.22%	1.21%	88.97%	1.62%	88.27%	1.77%
Risk taker wealth	0.029	0.012	0.031	0.013	0.068	0.016	0.069	0.016
Fraction $\lambda^R > 0$	32.66%	46.90%	35.07%	47.72%	28.73%	45.25%	19.95%	39.97%
Bankruptcy frequency	0.27%	5.19%	0.06%	2.45%	0.16%	4.00%	0.22%	4.69%
Return on RT wealth[a]	3.56%	35.74%	3.53%	36.11%	3.87%	17.75%	3.85%	18.07%
Government								
Government debt / GDP	14.96%	21.81%	−0.57%	9.95%	−4.70%	4.83%	−6.15%	3.26%
Actuarially Fair g-fee	0.77%	0.43%	0.67%	0.40%	0.59%	0.38%	0.54%	0.31%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The model in the first 2 columns has a mortgage guarantee fee of 20 basis points (20 bp g-fee), the model in columns 3 and 4 has an average g-fee of 55 basis points, the model in columns 5 and 6 has an average g-fee of 100 basis points, and the model in the last two columns has an average g-fee of 275 basis points.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

2.7%, and the loss rate from mortgage default is 1.0%, both match the data. Loss rates from mortgage defaults are 6.1% in housing crises but less than 0.5% in normal times. Borrowers prepay 15.8% of non-defaulted mortgages on average. Prepayments are higher in crises (18.7%), when interest rates are lower.

In the low g-fee economy, losses due to mortgage default are largely born by the government, not by the intermediaries. Defaults act as prepayments to the lenders who hold guaranteed bonds. Both default-induced and rate-induced prepayments come in at par and cause a loss to the intermediaries. The average mark-to-market loss rate given a prepayment is 10.8% (12.6% in crises). The intermediaries' portfolio loss rate is 0.4% on average (0.23% in normal times and 1.68% in crises) in the low g-fee economy. These portfolio losses are nearly entirely due to prepayment-related (both default- and rate-induced) losses rather than credit losses/mortgage arrears.

High G-fees How does the borrower problem change for higher g-fees? The next six columns of Table 2 consider economies with increasing g-fees.

When g-fees and mortgage rates rise, the size of the mortgage market shrinks. The mortgage market also becomes safer. The smaller mortgage market leads to a lower borrower debt-to-income ratio of 1.40 in the high g-fee economy. The lower mark-to-market LTV ratio, due to lower mortgage prices q^m, implies a lower mortgage default rate. In the high g-fee economy, the mortgage default rate is 1.8%, a reduction by almost 40% compared to the low g-fee economy. This translates in a loss rate of 0.7% unconditionally. The loss rate during crises is higher in the high g-fee economy (4.2% in crises) than in the low g-fee economy (1.7%) because intermediaries now bear all the credit risk themselves. Prepayment rates also fall as the g-fee increases, from 15.8% CPR in the low g-fee to 11.9% CPR in the high g-fee economy. This is because equilibrium mortgage rates are higher in the high g-fee economy, reducing the benefit from refinancing. Conditional on prepayment, the mark-to-market losses to intermediaries from prepayment also shrink because of the smaller gap between q_t^m and F. In sum, because banks no longer buy mortgage guarantees when the latter are expensive, default-related losses must be fully absorbed by the banks. By choosing lower leverage and higher equity capital, banks are able to withstand such credit losses as well as the losses due to prepayment.

An oft-invoked rationale for government guarantees is to provide stability of mortgage finance at all times. Indeed, Fannie Mae was founded in the Great Depression when a massive default wave of banks threatened the supply of mortgage credit. Our measure of the stability of the provision of mortgage credit is the standard deviation of mortgage debt to income growth. This volatility is 2.9% in the low g-fee economy when the mortgage guarantees are in full force. Surprisingly, the volatility of changes in mortgage debt/income initially *decreases* to 2.7% as the g-fee increases to 55bp. The volatility inches back up to end up unchanged in the high g-fee economy compared to the low g-fee economy. Even in crisis periods, the decline in mortgage credit relative to income is smaller in absolute value in the high g-fee economy than in the low g-fee economy. The financial sector in the private market solution is well-enough capitalized and has low enough leverage that it can better guarantee the stable provision of mortgage credit over time than a system with a government backstop, where banks are poorly capitalized and prone to occasional collapses. The popular fear that a private mortgage system would lead to large swings in the availability of mortgage credit, especially in bad times, is unwarranted in our model.

4.3 Risk Takers

The third panel of Table 2 reports on the risk takers. As financial intermediaries, they make long-term mortgage loans to impatient borrower households and borrow short-term from patient depositor households. They play the traditional role of maturity transformation. Given their low risk aversion, they are relatively willing to bear fluctuations in their net worth in the process of intermediation.

Low G-fees In the low g-fee economy, risk takers hold nearly all of their assets in the form of government-guaranteed bonds. They buy mortgage guarantees both in normal times and in mortgage crises (high σ_ω) states, taking advantage of the cheap mortgage guarantees provided by the government. As a result, the asset side of their balance sheet is largely shielded from mortgage default risk.

Bearing little default risk on their assets and facing a low interest rate on safe deposits, banks use substantial leverage in order to achieve their desired risk-return combination. The intermediary leverage constraint allows banks who exclusively hold guaranteed mortgage bonds to have a maximum leverage ratio of 96.4%. Banks hit that constraint in 32.7% of the periods; the average bank leverage (market value of debt to market value of assets) ratio is 95.6% (matching the data). Average risk taker wealth is modest, at 2.9% of trend GDP. Banks have little "skin in the game." The constraint binds more frequently in normal times (34.8%) than in crises (14.7%) because of precautionary deleveraging in crises.

In addition to taking risk through leverage, banks in the low g-fee economy have a larger balance sheet of mortgages. Due to the combination of low risk taker wealth, high leverage, and a large and risky mortgage portfolio, the banking system is fragile. When adverse income shocks or mortgage credit shocks hit, risk taker net worth may fall below zero. The reason this can happen despite the prevalence of guaranteed mortgages on intermediaries' balance sheets is because of mark-to-market losses on guaranteed bonds. Mortgage defaults act as prepayments for holders of guaranteed bonds. Since agency bonds usually trade above par prior to the prepayment, and prepayments come in at par, the prepayment constitutes a loss for the holder or agency MBS. Put differently, prepayments happen when interest rates are low and reinvestment opportunities are poor. We find that the low g-fee economy has the highest amount of prepayment risk. It has the highest prepayment rate and the largest mark-to-market loss conditional on a prepayment. By curbing banks' default risk exposure, the GSE subsidies are inadvertently increasing the exposure of banks to prepayment risk.

When intermediary net worth turns negative, the government steps in to bail out the financial sector. Such financial crises happen in 0.27% of the simulation periods. The table reports the return on risk-taker wealth. It is 3.6% excluding the bankruptcy events, but 3.3% including such events (unreported). This difference illustrates the option value introduced by the possibility to go bankrupt.

Higher G-fees As g-fees rise, the composition of the risk taker portfolio shifts towards private bonds. In the 55bp economy, guaranteed bonds still make up almost the complete portfolio (98.4%). This is consistent with the situation today, where g-fees have risen to about 55bp and yet guaranteed bonds continue to dominate. But when g-fees go up to 100 basis points, banks guarantee only 11% of their portfolio. This dramatic reversal occurs because risk takers buy the guarantee only in crises, when guaranteed bonds constitute 93% of their portfolio. In good times they prefer an all-private portfolio. The state uncontingent g-fee is too cheap to forego in bad times but too expensive in normal times. This 100 basis point g-fee economy is reminiscent of Option B of the Obama Administration housing reform plan, which envisions a g-fee level that is high enough so that it would only be attractive in bad times. In the high g-fee economy in the last two columns, risk takers shift exclusively towards holding private MBS. They do not buy default insurance from the government, neither in good nor in bad times. The 275 basis point g-fee is high enough to "crowd-in" the private sector at all times. The 275bp g-fee economy implements Option A in the Obama plan which envisions an entirely private mortgage market.

Our main result is that increasing the g-fee lowers the riskiness of the financial sector. Risk taker leverage falls from 95.6% in the 20bp and 95.2% in the 55bp economy to 89.0% in the 100bp economy, and 88.3% in the high g-fee economy. As the portfolio shifts towards private mortgages, the risk taker's collateral constraint becomes tighter because private mortgages carry higher regulatory capital requirements ($\xi_P > \xi_G$). But this is not the main driver of the lower leverage. Rather, banks choose to stay away from their leverage constraint in most periods; their leverage constraint binds in less than 20% of periods compared to 33% in the low g-fee economy.

Table 2 also shows that intermediaries have much higher average net worth of 7% of GDP in the high g-fee economy, more than double the 2.9% in the low g-fee economy. The higher equity capital buffer is large enough to prevent most bank insolvencies and concomitant bank bailouts, with the intermediary g-fee cases showing the lowest bankruptcy frequencies.

Why do banks choose to reduce leverage and increase equity capital buffers? The shift towards private mortgages increases the overall riskiness of banks' portfolios. Thus, there is less need to lever up in order to achieve the desired risk-return relationship for intermediary wealth. Furthermore, banks also reduce the overall size of their mortgage portfolio. The market value of their assets falls by 8.7% between the low and high g-fee economies. And the mortgages they make are safer with lower market-based LTV ratios and lower borrower debt-to-income ratios. The higher cost of leverage (higher short-term interest rate) and the smaller spread between mortgage rates and deposit rates (192 basis points in the high g-fee versus 239 in the low g-fee economy) also help explain the banks' choices.

4.4 Depositors and Government

Low G-fees Depositors have high risk aversion and thus loathe large fluctuations in consumption across states of the world. Their strong precautionary savings demand makes them willing to lend to intermediaries and the government at low interest rates. Of course, deposit insurance is important because it makes depositors' claims on the banks risk free, independent on the riskiness of the bank's balance sheet.

In the low g-fee economy, the high mortgage default rate leads to large mortgage losses which are mostly absorbed by the government, as explained above. Furthermore, there are occasional bank insolvencies and bailouts. Both lead to a surge in government expenditures, financed with government debt. Government debt to trend GDP is 15.0% on average with a high standard deviation of 21.8%. Tax revenues only slightly exceed government discretionary and transfer spending, and in the aftermath of a severe mortgage crisis it takes many years of small surpluses as well as the absence of a new mortgage crisis to reduce government debt back to steady state levels.

Depositors must hold not only the debt issued by the banks (deposits) but also the debt issued by the government. High risk taker leverage and high government debt in the low g-fee economy lead to high equilibrium holdings of short-term debt by depositors. All else equal, the large supply should result in a low price of short-term debt, or equivalently, a high interest rate to induce the depositor to hold all that debt. Indeed, the interest rate during income contractions that coincide with mortgage crises (-1.7%) is more than a percentage point higher than the interest rate during contractions that do not coincide with crises (-2.8%). Yet on average the low g-fee economy exhibits a *low* average equilibrium interest rate. The reason is that the precautionary savings demand more than offsets the supply effect.

During mortgage crises government debt shoots up as the government pays out on mortgage guarantees and occasionally on bank bailouts. This increase in the supply of short-term debt by the government is only somewhat offset by lower risk taker leverage, so that on net the supply of bonds grows during crises. The ultra low real interest rates in crises make debt issuance attractive for the risk taker and government alike. The depositor absorbs this debt increase in equilibrium by increasing savings and reducing consumption.

In sum, by protecting the financial sector from mortgage defaults, the government shifts more of the consumption fluctuations across states of the world onto the depositor. By virtue of the depositor's high risk aversion, she is more unwilling to bear such consumption fluctuations than the risk taker. The depositor responds by saving a lot more at all times to absorb at least some of the fluctuations in government debt with existing savings. The result is a low equilibrium interest rate and low average financial income for the depositor.

High G-fees In the high g-fee economy, we have the same fraction of mortgage crises. But these crises result in much lower mortgage loss rates. Bank insolvencies become rarer because of smaller and safer mortgage portfolios, lower bank leverage and higher bank equity. Furthermore, the mortgage losses are no longer borne by the government but rather absorbed by the intermediaries' balance sheet. Average government debt falls as does its volatility. Average government debt even becomes slightly negative.

The lower equilibrium supply of both government debt and risk taker debt (deposits) would result in a lower interest rate if savers were risk neutral.[30] But overall interest rates are *higher* in the high g-fee economy because the precautionary savings effect again dominates (given the high risk aversion of the depositors). A safe economy without financial fragility and therefore with low and predictable government debt induces depositors to scale back their precautionary saving demand. The fall in demand for safe assets exceeds the decline in the supply, explaining higher equilibrium real interest rates.

In summary, when the g-fee is high enough, risk takers are well enough capitalized and their intermediation capacity is rarely impaired. They bear and hence internalize all mortgage default risk. In contrast, in the low g-fee economy, mortgage crisis episodes frequently disrupt risk takers' intermediation function. During these crises, the risk free rate drops sharply and government debt increases sharply, effectively making depositors bear a greater part of the mortgage default risk.

[30] Indeed, the large difference between crisis and non-crisis interest rates attributable to additional supply of risk-free debt, holding aggregate income constant, entirely disappears.

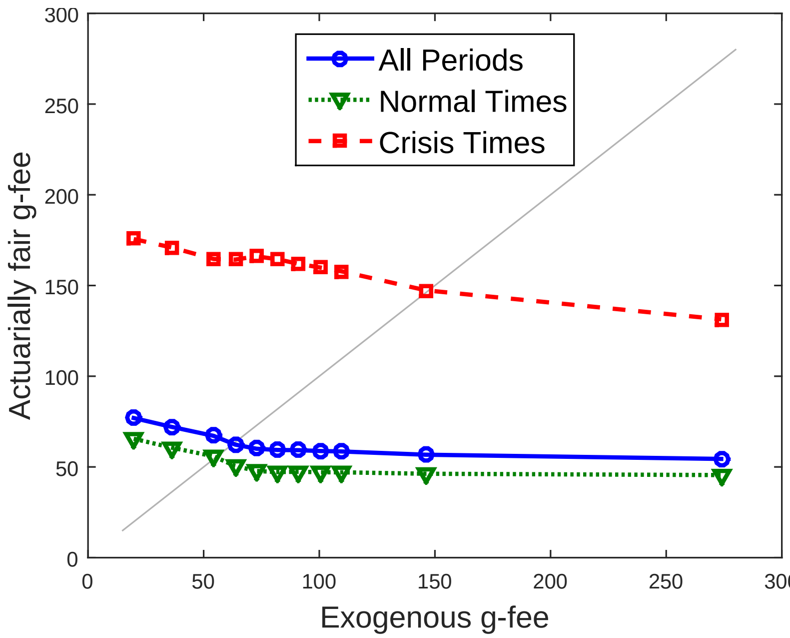
4.5 Actuarially Fair G-fees

It is possible to compute actuarially fair guarantee fees in the equilibrium of our model. It is the fee that a hypothetical risk-neutral agent with the same degree of patience as the savers would charge for the mortgage guarantee payment upon a default ($M_G - M_P$). The actuarially fair g-fee depends on the model in which it is computed. In the 20bp g-fee economy, whose equilibrium displays financial fragility, the actuarially fair g-fee is 77 basis points (last row of Table 2), proving that the 20bp g-fee is massively underpriced. The underpricing makes guarantees attractive to risk takers as we showed above.

Figure 2 shows the actuarially fair g-fee for the low and high g-fee economies as well as several intermediate economies. The solid line shows the unconditional average, the dashed line the actuarially fair g-fee in crises, and the dotted line the fair g-fee in normal times. The figure also draws in the 45-degree line along which actual and actuarially fair g-fees are equal. First, we note that the actuarially fair g-fee is decreasing in the exogenous g-fee (solid line with circles). The higher the g-fee, the safer the mortgages are, and the more stable the financial sector. Hence, to break even, a risk neutral insurer could charge a lower average rate. The actuarially fair g-fee declines from 77 basis points in the 20bp economy to 54 basis points in the 275bp economy. The fixed point where the actual and fair g-fees equate is between the 55bp and 100bp economies, somewhere around 60 basis points. Relative to the risk-neutral benchmark, mortgage guarantees are overpriced *on average* in the economies with a g-fee above 60 basis points and unconditionally underpriced in economies with g-fees below 60bp.

The figure also makes clear that the actuarially fair g-fee is state contingent. During mortgage crises (high σ_ω states), the mortgage loss rate is a lot higher and a much higher g-fee must be charged to break even (dashed line with squares). For g-fees below 150bp, risk takers would always want to buy guarantees in crisis times. But risk takers are not risk neutral but risk averse. It turns out, we must go out to 275 basis points to make guarantees expensive enough so that they are almost never bought in any of the states of the world. The 100bp economy is an interesting one. Risk takers overwhelmingly buy the mortgage guarantee in crisis periods but overwhelmingly hold private mortgage bonds in normal times. The actuarially fair g-fee in that economy is 158bp in crisis times while it is 47bp in normal times. Thus, the 100bp non-state contingent guarantee fee is attractively priced only in crises.

Figure 2: Actuarially Fair G-Fees



The graphs show the actuarially fair g-fee (y-axis) for seven economies that differ by their exogenously given g-fee (x-axis). The solid line with circles denotes the average g-fee across all periods in a long simulation. The dotted line with triangles denotes the average g-fee during normal times whereas the dotted line with squares denotes the average g-fee during mortgage crises (high σ_ω) times.

Private Mortgage Insurance An interesting question is whether there is there scope for welfare-enhancing private mortgage insurance. In the low g-fee equilibrium, private mortgage insurance could not compete with the government's underpriced guarantees. In the high g-fee equilibrium, risk takers would buy a mortgage guarantee in crisis periods if it were available at all times for a state-uncontingent 54bps. This logic is incomplete, however. The presence of the guarantee would change the equilibrium of the economy, reintroducing moral hazard, and leading to larger and riskier intermediary balance sheets. The increase in risk would increase the actuarially fair g-fee. Second, the provider of private mortgage insurance would go bankrupt since she would need to charge the much higher crisis-only guarantee fee of 131 basis points if banks only bought the guarantee in crisis times. We would have to take a stance on the particulars of the private mortgage insurance sector. Should it not have the same preferences as the risk takers (being also part of the leveraged financial sector)? Should it not enjoy the same bailout guarantees as the banks (cfr. AIG)? Clearly a thorough analysis of this question would require adding a fifth balance sheet for the PMI sector. Such an extension adds substantial numerical complexity and is beyond the scope of this paper. Suffice to say that if

Table 3: Phasing Out the GSEs: Welfare and Risk Sharing

	50 bp g-fee		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev
Aggregate Welfare[a]	+0.14%	+0.31%	+0.20%	+0.16%	+0.63%	+0.23%
Value function borrower[a]	+0.05%	−0.24%	+0.04%	−0.95%	+0.04%	−1.37%
Value Function depositor[a]	+0.23%	−0.27%	+0.37%	+0.12%	+1.32%	+0.94%
Value function risk taker[a]	+0.57%	+4.32%	+1.10%	+4.36%	+1.69%	+6.86%
Consumption borrower	−0.20%	−3.88%	−0.47%	−8.06%	−0.56%	−9.28%
Consumption depositor	+0.81%	−4.09%	+1.52%	−13.80%	+2.15%	−20.68%
Consumption risk taker	+0.42%	−11.35%	+1.47%	−16.20%	+1.79%	−8.94%
MU ratio borrower/risk taker[b]	−2.02%	−5.19%	−19.61%	−18.08%	−13.96%	−22.61%
MU ratio risk taker/depositor[b]	+1.16%	−11.11%	+1.22%	−20.90%	+1.21%	−7.69%
DWL from Foreclosure	−16.55%	−17.99%	−30.28%	−34.34%	−39.39%	−39.95%
DWL from Prepayment	−16.47%	−9.18%	−31.19%	−16.71%	−43.36%	−25.74%
Maintenance Costs	−1.83%	−22.53%	−3.92%	−34.98%	−6.23%	−34.00%

The table reports percent changes in unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models relative to the 20 bp g-fee benchmark. The model in the first 2 columns has a mortgage guarantee fee of 55 basis points (50 bp g-fee). The model in columns 3 and 4 has an average g-fee of 100 basis points, and the model in the last two columns has an average g-fee of 275 basis points.
[a]: With unit EIS the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore differences in values have a direct interpretation as consumption-equivalent welfare differences.
[b]: Marginal utility ratios are calculated as the difference of the logarithm of marginal utilities.

the PMI sector has the same preferences as the intermediaries and enjoys the same treatment by the government, we can simply think of the current intermediary sector as the consolidated balance sheet of both banks and mortgage insurers. Since the quantity of mortgage insurance would both be an asset and a liability of the consolidated sector, its equilibrium size would be indeterminate. The equilibrium allocations and prices (including the actuarially fair g-fee) would be the same as the economy we currently compute.

4.6 Welfare

Aggregate welfare, measured as the population-weighted average of the value functions of the three types of agents, increases in the g-fee. The first row of Table 3 shows that it is 0.63% higher in the high g-fee economy than in the benchmark low-g-fee economy. In unreported results for a series of intermediate g-fee economies, we confirm that aggregate welfare increases monotonically in the g-fee. With unit EIS, the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore, increases in aggregate welfare can be directly interpreted as consumption-equivalence gains. We consider the 0.63 percent improvement in consumption equivalence terms from GSE reform to be a substantial effect.

There are two effects that help understand the aggregate welfare gain: an improvement in risk sharing and a reduction in deadweight losses. First, risk sharing between the different types of agents generally improves as g-fees increase. To measure the extent of the improvement, we compute the ratios of (log) marginal utilities between the different types. If markets were complete, agents would be able to achieve perfect risk sharing by forming portfolios that keep these ratios constant. Hence, larger volatilities of these marginal utility (MU) ratios indicate worse risk sharing between the different types of agents. Table 3 lists the average MU ratios and their volatilities for borrowers/risk takers, and risk takers/depositors, as these are the pairs of agents that directly trade with each other. The volatilities of both ratios is lower in the high g-fee economy than in the low g-fee economy. The MU ratio volatility between borrower and risk taker falls by 22.6%. The decline in the MU ratio volatility between risk taker and depositor is 7.7%. We see similarly large improvements in risk sharing if we look at consumption volatility for the three types of agents: -9.3% for the borrower, -20.7% for the depositor, and -8.9% for the risk taker. While the risk sharing between borrower and risk taker improves monotonically as g-fees rise, the risk sharing between the risk taker and the depositor is highest at an intermediate g-fee closest to the actuarially fair g-fee (around 65bp). Similarly, the volatility of consumption for the risk taker is the lowest at that g-fee. At low g-fee levels, the high leverage and risk-taking of intermediaries makes their consumption volatile. As the g-fee rises from 20bps to about 65bps, leverage falls sharply but the risk taker's portfolio is still largely protected by government guarantees. However, as g-fees further above 65bps, the risk taker portfolio tilts towards private bonds, and this makes consumption volatility rise again (despite further reductions in leverage). Intermediary wealth is a crucial driver of the overall degree of risk sharing between the agents in the economy. In the private economy, banks are better able to provide consumption smoothing services to both borrowers and depositors because they are better capitalized and less fragile. Improved risk sharing also increases the risk-free rate and therefore mean consumption for savers.

The second source of the welfare gain is a reduction in deadweight losses. The first deadweight loss is the one associated with mortgage foreclosures. Lower deadweight costs leave more resources for private consumption each period. This deadweight loss is 0.59% of GDP in the 20bps economy and 0.36% of GDP in the 275bp economy. While the deadweight loss falls by 39%, it remains modest. The economy also benefits from lower deadweight losses from prepayment costs in high g-fee economies since prepayment rates are lower. These losses are of the same magnitude as those from foreclosures and fall by about the same percentage. The final deadweight loss is associated with house maintenance. Because maintenance expenses are proportional to the value of the house, they fall with g-fees because house prices fall with g-fees. The decline is 6% in percentage terms and accounts for about half of the overall decline in all three DWL sources combined. A reduction in housing consumption leaves more resources for non-housing consumption. As we see in the table, the increases in resources goes to the depositor and risk taker, both of which increase mean consumption. The borrower's consumption declines because she faces higher mortgage interest rates in the high g-fee economy.

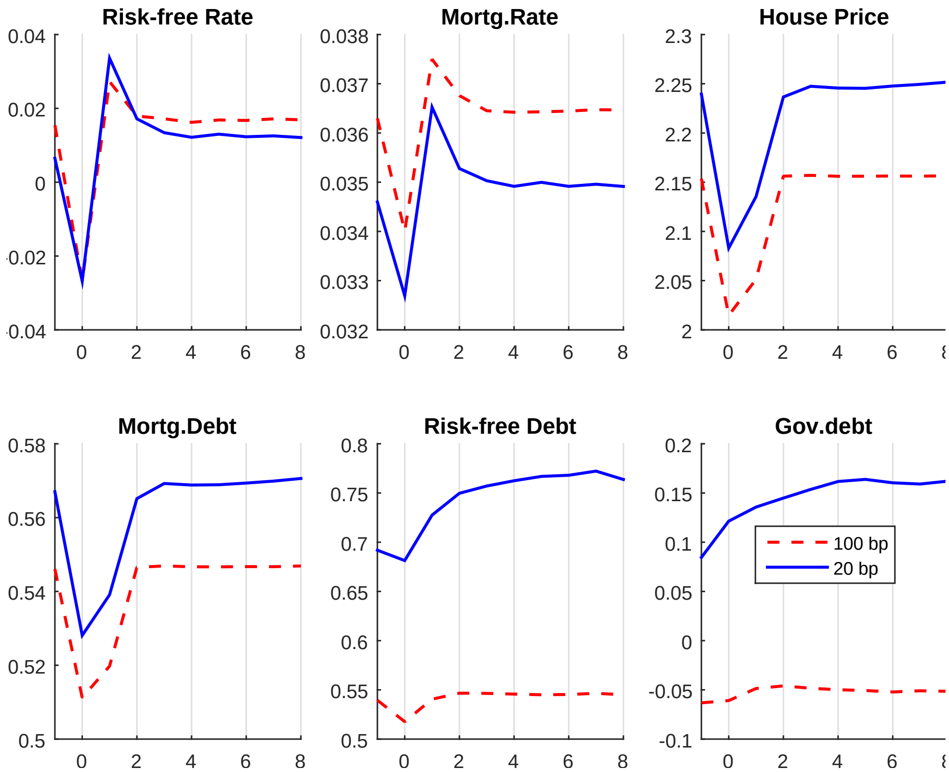
What are the distributional consequences of a mortgage market privatization? Close inspection of the value function of each of the three household types shows that the borrower's welfare stays almost constant between the low and the high g-fee economies (+0.04%), while depositor welfare (+1.32%) and risk taker welfare (+1.69%) both increase substantially.[31] The near-absence of a welfare loss for borrowers is surprising since taking away underpriced mortgage guarantees increases mortgage rates and lowers property values. The important offset to a decline in her consumption comes from the improvement in risk sharing. In conclusion, while GSE reform is a Pareto improvement, it redistributes wealth from borrowers to savers thereby raising inequality.

4.7 Inspecting the Mechanism: Crises Periods

To gain further intuition for the workings of the model, we study transitions from normal times to mortgage crises. Specifically, we select all simulation periods in which the economy is in normal times at time $t = -1$ ($\sigma_{t,\omega} = \sigma_{L,\omega}$) and in a mortgage crisis in period $t = 0$ ($\sigma_{t,\omega} = \sigma_{H,\omega}$). We plot the economy from period -1 to period +8. We recall that the average mortgage crisis lasts 2 periods (period 1 and 2); some are longer and some shorter. Also recall that all mortgage

[31] Given the homogeneity properties of the value function, log changes in value functions are directly interpretable as consumption equivalence changes and hence directly comparable across agents.

Figure 3: Mortgage crisis episode: interest rates, house price, and debt quantities

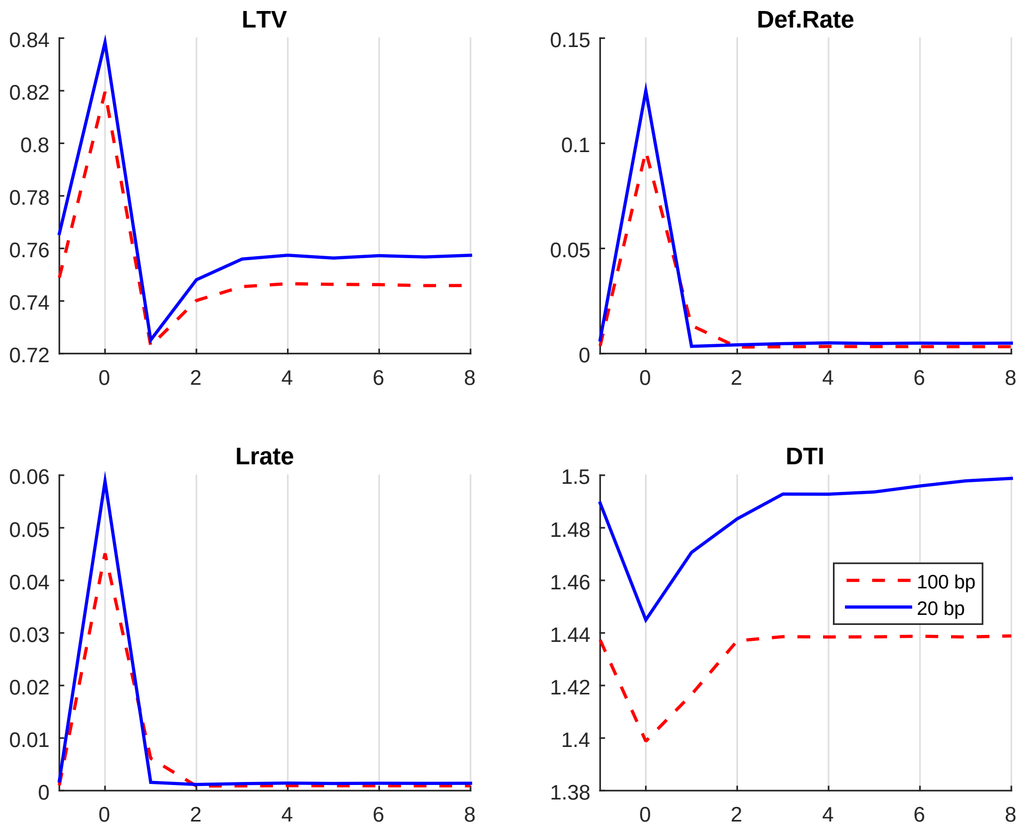


The graphs show the median path of the economy through a mortgage crisis episode starting at time 0. The crisis episodes are taken from the same 10,000 period simulation used to compute the moments in tables 2 and 3. **Blue solid line:** 20bp economy **red dashed line:** 100bp economy.

crises coincide with low aggregate output growth. The graphs show the median path over all such episodes and compare the benchmark low g-fee economy to the intermediate 100bp g-fee economy for the exact same sequence of shocks.

There is a gradual but substantial increase in government debt during the crisis and for several periods following it. The increase is close to 8% points of trend GDP. The payouts on mortgage insurance and possibly bank bailouts increase the deficit. So do the pro-cyclical tax revenues and the counter-cyclical exogenous government expenditures. The increase in interest rates during the recovery raises the interest expense to the government and further increases the debt. The middle panel in the bottom row shows the risk-free debt held by the depositor. By market clearing, it is the sum of the deposits issued by the banks and the debt issued by the government. During and in the aftermath of a crisis, the depositor must increase total bond holdings and absorb the increased supply of government debt. Given that these are high marginal utility states for the (highly risk averse) depositor, her welfare suffers.

Figure 4: Mortgage crisis episode: borrowers

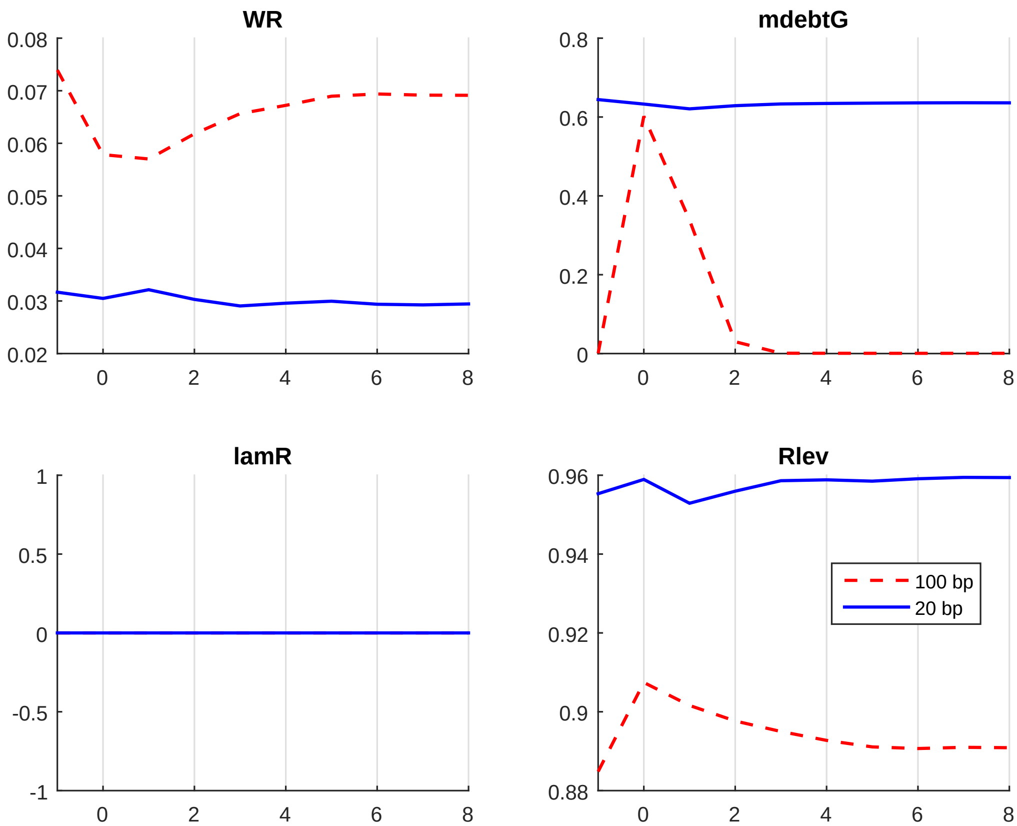


The graphs show the median path of the economy through a mortgage crisis episode starting at time 0. The crisis episodes are taken from the same 10,000 period simulation used to compute the moments in tables 2 and 3. **Blue solid line:** 20bp economy **red dashed line:** 100bp economy.

Figure 4 shows that mortgage default and loss rates spike during the crisis, but are mostly back to pre-crisis levels by period 1 (the second year of crisis). The increase in default rates arises because the mark-to-market LTV ratio of the mortgage portfolio spikes. This occurs both because the market value of debt increases (mortgage interest rates fall) and the market value of houses falls, as we saw in Figure 3. Later on, in period +1, the price of mortgage debt falls while house prices partially recover. The resulting drop in the LTV ratio lowers default rates. The last panel shows that the mortgage debt-to-income ratio falls precipitously in the crisis as lenders reduce mortgage originations. The reduction is smaller in the high g-fee economy (-2.7%) than in the low g-fee economy (-3%), contrary to the argument that GSEs are necessary to prevent a collapse in provision of mortgage credit during a crisis.

Figure 5 shows that risk taker wealth is much lower in the low g-fee economy and falls even further from period -1 to period +1. As a result, their ability to intermediate funds between

Figure 5: Mortgage crisis episode: risk takers



The graphs show the median path of the economy through a mortgage crisis episode starting at time 0. The crisis episodes are taken from the same 10,000 period simulation used to compute the moments in tables 2 and 3. **Blue solid line:** 20bp economy **red dashed line:** 100bp economy.

depositors and borrowers is impaired. As the crisis eases in period +1, risk takers in the low g-fee economy have very low wealth but want to start lending again to meet borrower demand. Risk takers are close to their borrowing constraint (Rlev) throughout in the low g-fee economy. Financial leverage dynamics are more pronounced in the 100bp g-fee economy, where risk takers choose to stay away from binding constraints after the crisis dissipates. Leverage increases in the first period of a crisis (time 0) as risk takers have to absorb losses on their mortgage portfolios. Their net worth is sufficient to absorb the negative shock due to mortgage defaults. As pointed out before, banks' portfolios shift towards guaranteed bonds (mdebtG) during the crisis in the 100bp economy. Once the crisis wanes, they shift back to holding private bonds as they slowly restore their net worth.

5 Alternative Policy Experiments

Our main policy experiment consisted of raising the g-fees. We now study two alternative policies and compare their welfare consequences to those in the main policy experiment. The first experiment explores the effect of macro-prudential policy. The second experiment studies a recent legislative proposal to "put private capital in front of a government guarantee." These exercises help to further illuminate the interaction of government guarantees, deposit insurance, and risk taker leverage.

5.1 Higher Regulatory Capital Charges

The first policy raises the regulatory capital weight on guaranteed mortgage bonds. Specifically, we lower ξ_G from its benchmark value of 0.984 (1.6% capital charge) to $\xi_G = \xi_P = 0.92$ (8% capital charge). We only change this one parameter and keep the low 20bp g-fees in place. The question is whether eliminating the favorable regulatory treatment of government-guaranteed bonds can be a (partial) substitute for higher g-fees. Since this reform tightens the borrowing constraint on financial intermediaries, it is a macro-prudential policy that reduces bank leverage. The first two columns of Table 4 report the results.

Because the macro-prudential policy does not change the low cost of the mortgage guarantees, which continue to be severely underpriced at 20bps, guaranteed bonds retain their dominant position in banks' portfolio. Private bonds constitute 3% of total assets. The overall size of bank assets is only modestly smaller than in the benchmark economy (by 1%). With a tighter regulatory capital constraint, banks have more "skin in the game." The bankruptcy option now becomes less valuable (compared to the benchmark case with $\xi_G = .984$) because it involves a greater loss of own capital upon bankruptcy. Risk takers become more cautious and choose to stay away from the constraint more often. Risk taker leverage is substantially lower at 88.8% (compared to 95.6% in the benchmark economy and close to the high g-fee economy leverage of 88.3%) and the constraint binds in 23.8% of periods. Risk taker wealth is 7.2% of GDP, more than twice the level in the low g-fee economy and even somewhat higher than in the high g-fee economy. Bankruptcies are nearly eliminated because higher risk taker net worth can absorb the shocks.

When risk takers are forced to have a bigger equity cushion, they self-insure better against very bad states. This reduces the likelihood of low consumption. While risk taker consumption volatility goes up, the increase in volatility comes to a large degree from upside volatility. They can grant the same amount of mortgages with less debt by having greater own wealth. Their consumption is higher on average since they earn a higher spread between mortgages and deposit rates (262 vs 239 bps). In sum, risk takers enjoy substantial welfare gains from this reform increasing consumption-equivalent welfare by 1.74%. This welfare increase for risk takers is essentially the same as the one we saw in our main policy experiment where we raised the g-fee (+1.69%). In sum, tighter macroprudential policy is as successful in reducing financial fragility as high g-fees. Interestingly, reducing the ability of financial firms to lever up benefits them.

On the borrower side, the economy with tighter capital requirements looks similar to the low g-fee economy. Mortgage rates, house prices, default rates, and mortgage losses are almost unchanged. Borrowers see almost no change in welfare (+0.03%), similar to the transition from a low g-fee to a high g-fee economy (+0.04%).

The reason overall welfare only goes up by 0.07% in aggregate is because the depositors only benefit modestly (0.10%) and constitute a large share of the population. In the main policy experiment, they gained +1.32%. The low welfare gains for depositors can be understood from the low interest rates they earn in this experiment (20 basis points lower than in the low g-fee economy and 90 basis points lower than in the high g-fee economy). These low rates make depositor mean consumption much lower than in the high g-fee economy. The reason for the low interest rates is the same as in the benchmark economy: depositors must absorb high and variable government debt. Because they loathe fluctuations in consumption across states of the world, they display strong precautionary savings resulting in low rates. Macro-prudential policy does not affect the overall size and riskiness of the mortgage market since banks who continue to enjoy the guarantee hold essentially the same mortgage portfolio as in the low g-fee economy. The fiscal implications are the same and lead to poor consumption insurance for depositors.

Table 4: The Role of Capital Requirements and Catastrophic Insurance

	20 bp g-fee		**275 bp g-fee**		$\xi_G = 92\%$		**JC 10%**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.13%	3.00%	1.83%	3.14%	0.92%	2.87%	1.89%	3.24%
Mortgage rate	3.52%	0.24%	3.74%	0.26%	3.54%	0.26%	3.75%	0.27%
House price	2.240	0.142	2.100	0.120	2.222	0.139	2.101	0.119
				Borrower				
Mortgage debt	0.053	0.001	0.050	0.001	0.053	0.002	0.050	0.001
Borrower LTV	63.79%	3.93%	63.79%	3.50%	63.81%	4.10%	63.81%	3.46%
Market value of debt LTV	75.72%	6.47%	73.91%	6.02%	75.58%	6.65%	73.91%	6.03%
Borrower debt to income	1.489	0.040	1.397	0.025	1.477	0.044	1.398	0.024
Debt/income growth	0.04%	2.86%	0.04%	2.87%	0.06%	3.48%	0.04%	2.96%
Default rate	2.74%	6.20%	1.76%	3.92%	2.76%	6.89%	1.76%	3.87%
Rate-induced prepayment rate	15.83%	4.24%	11.92%	4.38%	15.47%	4.52%	11.88%	4.48%
Loss Given Default	30.15%	5.76%	30.02%	5.72%	30.15%	5.78%	30.02%	5.72%
MTM Loss Given Prepayment	10.78%	2.59%	8.31%	2.81%	10.55%	2.79%	8.28%	2.88%
Loss rate private	1.04%	2.75%	0.68%	1.72%	1.07%	3.18%	0.68%	1.69%
Loss rate guaranteed	0.38%	0.88%	0.21%	0.50%	0.37%	0.90%	0.21%	0.50%
Loss rate portfolio	0.38%	0.88%	0.67%	1.72%	0.38%	0.90%	0.56%	1.45%
				Risk Taker				
Market value of bank assets	0.634	0.018	0.579	0.013	0.628	0.021	0.579	0.014
Fraction guaranteed	99.96%	0.55%	0.58%	5.12%	97.00%	14.12%	28.81%	30.00%
Risk taker leverage	95.59%	0.92%	88.27%	1.77%	88.83%	1.28%	91.60%	2.40%
Risk taker wealth	0.029	0.012	0.069	0.016	0.072	0.015	0.050	0.019
Fraction $\lambda^R > 0$	32.66%	46.90%	19.95%	39.97%	23.80%	42.59%	76.90%	42.15%
Bankruptcy frequency	0.27%	5.19%	0.22%	4.69%	0.04%	2.00%	0.36%	5.99%
Return on RT wealth[a]	3.56%	35.74%	3.85%	18.07%	4.94%	18.40%	2.88%	26.42%
				Government				
Government debt / GDP	14.96%	21.81%	−6.15%	3.26%	13.98%	21.03%	−6.38%	3.27%
Actuarially Fair g-fee	0.77%	0.43%	0.54%	0.31%	0.80%	0.43%	0.02%	0.01%
				Welfare				
Aggregate Welfare	0.279	0.008	+0.63%	+0.23%	+0.07%	−0.34%	+0.67%	+0.48%
Value Function borrower	0.319	0.010	+0.04%	−1.37%	+0.03%	−0.42%	+0.05%	−1.17%
Value Function depositor	0.249	0.006	+1.32%	+0.94%	+0.10%	−0.16%	+1.38%	+1.27%
Value function risk taker	0.083	0.000	+1.69%	+6.86%	+1.74%	−8.83%	+1.23%	+42.92%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two models (first 4 columns) are the benchmark and high g-fee models from Table 2. The model in columns 6 and 7 two columns has a capital charge for guaranteed bonds set to 8% (same as for private bonds). The last 2 columns report results for an economy where the government guarantees only losses in excess of 10%. Like in the benchmark economy, guarantees in the last two models are both prices at 20 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

5.2 Catastrophic Insurance

The last policy experiment we study is inspired by recent legislative proposals in the U.S. Senate Banking Committee. We refer to these experiments as the Johnson-Crapo or JC experiment.[32] The spirit of the proposed reform is to force mortgage lenders to hold a substantial buffer of private capital in order to better protect the tax payer. The idea is that by having more private capital at risk, mortgage underwriting would be more prudent; moral hazard would be diminished. The government guarantee would be a catastrophic guarantee, kicking in only after the private capital is wiped out. Johnson-Crapo proposed that the first 10% of losses due to mortgage default would be absorbed by the private sector. The industry has argued that 10% is too large, and has proposed to replace it by a 5% private loss. We evaluate both proposals in our framework. We are the first to provide a detailed quantitative analysis of Johnson-Crapo, including all the general equilibrium effects on risk taking, interest rates, house prices, and the distributional effects for the various types of tax payers.

To analyze Johnson-Crapo, we change our definition of the government guarantee. When risk takers buy a mortgage guarantee, the guarantee pays out only if the loss rate on the mortgage pool exceeds 10%. If the loss is less than 10%, the guarantee is worthless and the guaranteed bond has the same payoff as a private mortgage bond. If the loss is higher than the threshold, the guaranteed bond pays out an amount equal to the losses above the threshold. For ease of comparison, we keep the regulatory capital advantages of guaranteed bonds from the benchmark economy. Also for ease of comparison, we assume that the government offers the catastrophic insurance at 20 basis points.[33] We compute the actuarially fair cost, *at the new equilibrium*. The last two columns of Table 4 present the results.

The JC economy is similar to the high g-fee economy in several aspects. It has lower house prices, higher mortgage rates, and a smaller mortgage market. In market value terms, risk taker mortgage assets are 8.7% lower in the JC 10% economy than in the benchmark low g-fee

[32] The "Housing Finance Reform and Taxpayer Protection Act of 2014" introduced by senators Corker and Warner preceded the draft bill introduced by Senators Johnson and Crapo and voted in the Senate Committee on Banking, Housing, and Urban Affairs on May 15, 2014. The 13-9 vote was not strong enough to force a full senate floor vote.

[33] This assumption does not significantly affect results. In the appendix, we report results from two additional experiments. In the first one, the catastrophic insurance is priced at 5bp. The second one keeps the g-fee at 20 bp but provides insurance for losses in excess of 5%, rather than 10% percent. Results are qualitatively similar.

economy. House prices fall by 6.2%. There is a substantial reduction in mortgage default rates, just like in the high g-fee economy.

In the JC economy, the guarantee's actuarially fair cost is 2 basis points. Insurance is quite *over*priced at 20bp. As a result, risk takers hold fewer guaranteed bonds (29% of their portfolio). While the guarantee is less generous, the insured mortgages are also endogenously less risky, so that the loss rate on *guaranteed* bonds is lower than in the low g-fee case. Risk takers' portfolio loss rate is 0.56%, which is 47% higher than in the benchmark economy but 17% lower than in the high g-fee economy when only private mortgages are held, as the guaranteed bonds are still substantially safer than uninsured bonds. Absent the severely underpriced guarantee, borrower leverage is lower, and lower default and loss rates on mortgages require smaller and less frequent guarantee payouts. As a result, government debt is much lower in the JC economies.

Risk taker leverage is lower at 91.6%, suggesting that the increased losses they must bear reduce their appetite for high leverage. The protection offered by the catastrophic guarantee increases their appetite to resume lending after a mortgage crisis and they run more often into binding constraints (77% on average, 88% in crises). The return on risk takers' wealth is now lower (2.9% excluding bankruptcy periods) but also less volatile, with volatility dropping by 26%.

Interestingly, aggregate welfare in the JC 10% case is slightly higher than in the high g-fee economy (+0.67% versus +0.63%). Borrowers' welfare changes in the almost identical way as from a phase-out of the guarantee (+0.05%). Depositors gain more than in the main experiment (+1.38% vs. +1.32%), while risk takers gain slightly less (+1.23% versus +1.69%). The gain for depositors comes from the much lower and less volatile government debt, just as in the main experiment. In addition, risk takers are better able to help depositors smooth consumption. This ability is higher in the JC economy than in the high g-fee economy because the catastrophic guarantee protects the banks in very adverse states of the world. Equilibrium interest rates reflect the safe environment for depositors and are 6bp higher than in the high g-fee economy.

By the same token, risk takers benefit form the catastrophic insurance. While in the high g-fee economy the 0.1-percentile of risk taker consumption is 0.045 (relative to trend GDP), the same percentile is much higher at 0.051 in the JC 10% economy (mean risk taker consumption is approximately 0.075 in both economies). The volatility of risk taker consumption falls substantially, besting the change in the main experiment. The mean consumption gain for the risk taker is not as high as in the main experiment, in contrast, because the return on bank equity and the mortgage spread are not as high in the JC 10% economy as in the high g-fee economy. This in turn is due to the higher risk-free rate in the JC economy.

6 Conclusion

Our main findings are that underpriced, government-provided mortgage default insurance distorts the incentives of the financial sector in a way that leads it to take more risk in the mortgages it originates and in the leverage it takes on. While the policy leads to higher house prices, lower mortgage rates, and lower interest rates, it also leads to more frequent mortgage defaults, and financial crises when banks become insolvent. Even though the government can mitigate the fallout from such crises by spreading out the costs over time, the allocation of risk remains suboptimal. We document a substantial welfare gain from moving to a private mortgage system, a transition which can be effectuated by raising the cost of the government mortgage guarantees. The private market provides a safer financial sector with fewer mortgage foreclosures and better intermediation between borrowers and savers. While the policy change is a Pareto improvement, it benefits depositors and bankers more and raises wealth inequality. We find that recent policy proposals in which the government only provides catastrophic loss insurance behind private loss-bearing capacity realize as high a welfare gain as a complete phase out phase-out.

The paper brings together the literatures of financial intermediary-based asset pricing and housing finance. It introduces in the role of the government into the former and the importance of the financial sector into the latter. New is the possibility of default for both borrowers and banks and the government's provision of bailout guarantees to the creditors' of the banking sector. An important ingredient is the interaction of such guarantees on banks' liabilities with the mortgage guarantee on banks' assets.

The model is a natural laboratory to explore the effects of government purchases of mortgages. The GSEs were a large buyer of guaranteed and non-guaranteed mortgages, accumulating a combined portfolio of $1.7 trillion dollars by 2007. Since then, they have reduced their holdings by 50%. The Federal Reserve was a large buyer of guaranteed mortgage bonds, accumulating $1.8 trillion as part of its QE1 and QE3 programs during and in the aftermath of the financial crisis. It is merely a matter of time before the Fed will start to shrink the size of these holdings. Thus, over the next several years, the U.S. is likely to see a major change from governmental to private ownership of at least 25% of the secondary mortgage market, one of the largest fixed income markets in the world. A complete understanding of such dramatic shift on the mortgage market, house prices, bond yields, the macro-economy, and the financial sector remains an important challenge for future research.

There are several other promising avenues for further exploration. The model currently abstracts from the choice between owning and renting. Abolishing the mortgage guarantees may well affect the home ownership rate. If house price-to-rent ratios fall in the aftermath of the policy reform, as they do in recent models that study the abolition of mortgage interest rate deductibility, phasing out the GSEs may well boost the home ownership rate. A second ingredient our work abstracts from is the feedback effect from the mortgage lending complex to the rest of the financial sector and to the real economy. In a world with subsidized mortgage lending, lending to capital-constrained entrepreneurs with productive investment opportunities may get crowded out, adding to the welfare cost of government mortgage and financial sector bailout guarantees.

References

Acharya, V. V., M. Richardson, S. Van Nieuwerburgh, and L. J. White (2011): *Guaranteed To Fail: Freddie, Fannie, and the Debacle of U.S. Mortgage Finance*. Princeton University Press.

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2013): "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Socially Expensive," Working Paper Stanford GSB.

Admati, A. R., and M. Hellwig (2013): *The Bankers' New Clothes: What's Wrong With Banking and What To Do About It*. Princeton University Press.

Adrian, T., and N. Boyarchenko (2012): "Intermediary leverage cycles and financial stability," *Working paper*.

Arslan, Y., B. Guler, and T. Taskin (2013): "Joint Dynamics of House Prices and Foreclosures," Working Paper, Indiana University.

Begenau, J. (2015): "Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model," Harvard Business School Working Paper, No. 15-072.

Boldrin, M., C. Garriga, A. Peralta-Alva, and J. M. Sanchez (2013): "Reconstructing the Great Recession," Federal Reserve Bank of St. Louis Working Paper 2013-006B.

Brunnermeier, M. K., T. Eisenbach, and Y. Sannikov (2013): *Macroeconomics with Financial Frictions: A Survey*Cambridge University Press, New York.

Brunnermeier, M. K., and Y. Sannikov (2012): "A Macroeconomic Model with a Financial Sector," Working Paper, Princeton University.

Campbell, J. Y., S. Giglio, and P. Pathak (2011): "Forced Sales and House Prices," *American Economic Review*, 101, 2108–31.

Chambers, M., C. Garriga, and D. E. Schlagenhauf (2009): "Housing Policy and the Progressivity of Income Taxation," *Journal of Monetary Economics*, 56(8), 1116–1134.

Chatterjee, S., and B. Eyigungor (2009): "Foreclosures and House Price Dynamics: A Quantitative Analysis of the Mortgage Crisis and the Foreclosure Prevention Policy," Federal Reserve Bank of Philadelphia Working Paper 09-22.

Chu, Y. (2014): "Credit Constraints, Inelastic Supply, and the Housing Boom," *Review of Economic Dynamics*, 17(1), 52–69.

Corbae, D., and E. Quintin (2014): "Leverage and the Foreclosure Crisis," *Journal of Political Economy*, forthcoming.

Davis, M. A., and S. Van Nieuwerburgh (2015): *Handbook of Regional and Urban Economics*chap. Housing, Finance, and the Macroeconomy, p. Chapter 12. North Holland.

Deng, Y., J. M. Quigley, and R. Van Order (2000): "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options," *Econometrica*, 68(2), 275–307.

Drechsler, I., A. Savov, and P. Schnabl (2014): "A Model of Monetary Policy and Risk Premia," *Working paper*.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2015): "The Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, Unpublished paper, New York University.

Floetotto, M., M. Kirker, and J. Stroebel (2012): "Government Intervention in the Housing Market: Who Wins, Who Loses?," Working Paper, Stanford University.

Gârleanu, N., and L. H. Pedersen (2011): "Margin-based Asset Pricing and Deviations from the Law of One Price," *Review of Financial Studies*.

Garriga, C., and D. E. Schlagenhauf (2009): "Home Equity, Foreclosures, and Bailouts," Working Paper, Federal Reserve Bank of St. Louis.

Gervais, M. (2002): "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49(7), 1461–1489.

Gete, P., and F. Zecchetto (2015): "Distributional Implications of Government Guarantees in Mortgage Markets," Working Paper Georgetown.

Hansen, G. D. (1985): "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309–327.

Hart, O., and L. Zingales (2011): "A new capital regulation for large financial institutions," *American Law and Economics Review*, 13, 453–490.

He, Z., and A. Krishnamurty (2013): "Intermediary asset pricing," *American Economic Review*, 103 (2), 732–770.

Hedlund, A. (2014): "The Cyclical Dynamics of Illiquid Housing, Debt, and Foreclosures," Working Paper, University of Missouri.

Jeske, K., D. Krueger, and K. Mitman (2013): "Housing, Mortgage Bailout Guarantees and the Macro Economy," *Journal of Monetary Economics*, 60(8).

Jorda, O., M. Schularick, and A. Taylor (2014): "Betting the House," NBER Working Paper No. 20771.

Judd, K. L. (1998): *Numerical Methods in Economics*, vol. 1 of *MIT Press Books*. The MIT Press.

Judd, K. L., F. Kubler, and K. Schmedders (2002): "A solution method for incomplete asset markets with heterogeneous agents," Working Paper, SSRN.

Kashyap, A., R. R. R., and J. Stein (2008): "Rethinking Capital Regulation," in *Federal Reserve Bank of Kansas City Symposium on Maintaining Stability in a Changing Financial System*, pp. 431–471.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2011): “Winners and Losers in Housing Markets,” *Journal of Money, Credit and Banking*, 43(2-3), 255–296.

Kubler, F., and K. Schmedders (2003): “Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral,” *Econometrica*, 71(6), 1767–1793.

Landvoigt, T. (2012): “Aggregate Implications of the Increase in Securitized Mortgage Debt,” Working Paper, Stanford University.

Landvoigt, T., M. Piazzesi, and M. Schneider (2015): “The Housing Market(s) of San Diego,” *American Economic Review*, 105(4), 1371–1407.

Maddaloni, A., and J.-L. Peydro (2011): “Bank Risk-taking, Securitization, Supervision, and Low Interest Rates: Evidence From The Euro-area And The US Lending Standards,” *Review of Financial Studies*, 24 (6), 2121–2165.

Maggiori, M. (2013): “Financial intermediation, international risk sharing, and reserve currencies,” *Working paper*.

Ortalo-Magné, F., and S. Rady (2006): “Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints,” *Review of Economic Studies*, 73(2), 459–485.

Reinhart, C. M., and K. Rogoff (2009): *This Time is Different*. Princeton University press.

Rouwenhorst, K. G. (1995): “Asset Pricing Implications of Equilibrium Business Cycle Models,” in *rontiers of Business Cycle Research*, ed. by T. Cooley. Princeton University Press.

Scharfstein, D., and A. Sunderam (2011): *The Future of Housing Finance*chap. The Economics of Housing Finance Reform. Brookings Institution Press.

Sommer, K., and P. Sullivan (2013): “Implications of U.S. Tax Policy for House Prices, Rents and Homeownership,” Working Paper, Federal Reserve Board of Governors.

A Model Appendix

We reformulate the problem of risk taker, depositor, and borrower to ensure stationarity of the problem. We do so by scaling all variables by permanent income.

A.1 Borrower problem

A.1.1 Preliminaries

We start by defining some preliminaries.

$$\begin{aligned} Z_A(\omega_t^*) &= [1 - F_\omega(\omega_t^*;\chi)] \\ Z_K(\omega_t^*) &= [1 - F_\omega(\omega_t^*;\chi)]\mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*;\chi\right] \end{aligned}$$

and $F_\omega(\cdot;\chi)$ is the CDF of $\omega_{i,t}$ with parameters χ. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$\begin{aligned} \mu_\omega &= \mathrm{E}_i[\omega_{i,t};\chi_0,\chi_1] = \chi_0\chi_1 \\ \sigma_{t,\omega}^2 &= \mathrm{Var}_i[\omega_{i,t};\chi_0,\chi_1] = \chi_0\chi_1^2 \end{aligned}$$

From Landsman and Valdez (2004, equation 22), we know that

$$\mathrm{E}[\omega|\omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega};\chi_0+1,\chi_1)}{1 - F_\omega(\bar{\omega};\chi_0,\chi_1)}$$

so the closed form expression for Z_K is

$$Z_K(\omega_t^*) = \mu_\omega\left[1 - F_\omega(\omega_t^*;\chi_0+1,\chi_1)\right]$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$\begin{aligned} \frac{\partial Z_K(\omega_t^*)}{\partial\omega_t^*} &= \frac{\partial}{\partial\omega_t^*}\int_{\omega_t^*}^{\infty} \omega f_\omega(\omega)d\omega = -\omega_t^* f_\omega(\omega_t^*), \\ \frac{\partial Z_A(\omega_t^*)}{\partial\omega_t^*} &= \frac{\partial}{\partial\omega_t^*}\int_{\omega_t^*}^{\infty} f_\omega(\omega)d\omega = -f_\omega(\omega_t^*), \end{aligned}$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters (χ_0, χ_1).

Prepayment Cost Let

$$\Psi(R_t^B, A_t^B) = \frac{\psi}{2}\left(\frac{R_t^B}{A_t^B}\right)^2 A_t^B$$

Then partial derivatives are

$$\Psi_R(R_t^B, A_t^B) = \psi\frac{R_t^B}{A_t^B} \tag{22}$$

$$\Psi_A(R_t^B, A_t^B) = -\frac{\psi}{2}\left(\frac{R_t^B}{A_t^B}\right)^2 \tag{23}$$

A.1.2 Statement of stationary problem

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, W_t^R, W_t^D, B_{t-1}^G\right)$ represent state variables exogenous to the borrower's decision. We consider the borrower's problem in the current period after income and house depreciation shocks have been realized, after the risk taker has chosen a default policy, and after the risk taker's random utility penalty is realized. Then the borrower's value function, transformed to ensure stationarity, is:

$$V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_t^B, \omega_t^*, R_t^B, B_t^B\}} \left\{ (1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \right.$$
$$\left. + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^B = (1-\tau_t)Y_t^B + G_t^{T,B} + Z_K(\omega_t^*)p_t K_{t-1}^B + q_t^m B_t^B - (1-\tau_t^m)Z_A(\omega_t^*)A_t^B - p_t K_t^B - F R_t^B - \Psi(R_t^B, A_t^B) \tag{24}$$

$$A_{t+1}^B = e^{-g_{t+1}}\left[\delta Z_A\left(\omega_t^*\right)A_t^B - R_t^B + B_t^B\right] \tag{25}$$

$$\phi p_t K_t^B \geq F\left[\delta Z_A\left(\omega_t^*\right)A_t^B - R_t^B + B_t^B\right] \tag{26}$$

$$0 \leq R_t^B \leq \delta Z_A\left(\omega_t^*\right)A_t^B \tag{27}$$

$$\mathcal{S}_{t+1}^B = h(\mathcal{S}_t^B) \tag{28}$$

where the functions Z_K and Z_A are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$\begin{aligned}\tilde{V}^B\left(K_{t-1}^B, A_t^B, \mathcal{S}_t^B\right) &= F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) \mid \rho < \rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) \mid \rho > \rho_t^*\right] \\ &= F_\rho(\rho_t^*)V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1-F_\rho(\rho_t^*))\,V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)), \end{aligned} \tag{29}$$

where (29) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$V_t^B \equiv V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B),$$
$$V_{A,t}^B \equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B},$$
$$V_{K,t}^B \equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial K_{t-1}^B}.$$

Therefore the marginal values of borrowing and of housing of $\tilde{V}^B(\cdot)$ are:

$$\tilde{V}_{A,t}^B = F_\rho(\rho_t^*)\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial A_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*))}{\partial A_t^B}$$
$$\tilde{V}_{K,t}^B = F_\rho(\rho_t^*)\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial K_{t-1}^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*))}{\partial K_{t-1}^B}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}$$

Recall that

$$u_t^B = \left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}$$

A.1.3 First-order conditions

New mortgages The FOC for new mortgage loans B_t^B is:

$$\begin{aligned}
0 = &\frac{1}{1-1/\nu}\left\{(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \right.\\
&\left.+\beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}-1} \times\\
&\times\left\{(1-1/\nu)(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^{\theta}(C_t^B)^{-\theta}q_t^m + \right.\\
&+\beta_B\frac{1-1/\nu}{1-\sigma_B}\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}-1}\times\\
&\left.\times E_t\left[(1-\sigma_B)\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{-\sigma_B}e^{g_{t+1}}\tilde{V}_{A,t+1}^B e^{-g_{t+1}}\right]\right\} - \lambda_t^B F
\end{aligned}$$

where λ_t^B is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$\begin{aligned}
&q_t^m\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\
&\lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}
\end{aligned} \tag{30}$$

Observe that we can rewrite equation (30) as:

$$q_t^m = \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}\left\{\lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\right\}.$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of current consumption:

$$\tilde{\lambda}_t^B = \lambda_t^B\frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Then we can solve for the mortgage price as:

$$q_t^m = \tilde{\lambda}_t^B F - \beta_B\frac{C_t^B\left\{\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\right\}}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}. \tag{31}$$

Houses The FOC for new purchases of houses K_t^B is:

$$0 = \frac{1}{1-1/\nu}(V_t^B)^{1/\nu} \times \Big\{ -(1-1/\nu)(1-\beta_B)(u_t^B)^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^\theta (C_t^B)^{-\theta} p_t + \\ + \frac{1-1/\nu}{1-\sigma_B}\beta_B (CE_t^B)^{\sigma_B - 1/\nu} \mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B} e^{g_{t+1}} \tilde{V}_{K,t+1}^B] \Big\} + \lambda_t^B \phi p_t.$$

Simplifying, we get:

$$p_t \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \lambda_t^B \phi p_t + \beta_B \mathrm{E}_t[e^{(1-\sigma_B)g_{t+1}}(\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \tag{32}$$

Default Threshold Taking the first-order condition with respect to $\omega_t^\star$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$f_\omega(\omega_t^*)\left[\omega_t^* p_t K_{t-1}^B - (1-\tau_t^m)A_t^B\right]\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \delta A_t^B f_\omega(\omega_t^*)\left\{\lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}_{t+1}^B\right)^{-\sigma_B}\tilde{V}_{A,t+1}^B\right] \times (CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}\right\}.$$

This can be simplified by replacing the term in braces on the right-hand side using the FOC for new loans (31) and solving for ω_t^* to give:

$$\omega_t^* = \frac{A_t^B(1-\tau_t^m + \delta q_t^m - \delta\tilde{\lambda}_t^{RB})}{p_t K_{t-1}^B}, \tag{33}$$

where the rescaled Lagrange multiplier on the upper refinancing bound is:

$$\tilde{\lambda}_t^{RB} = \lambda_t^{RB}\frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Prepayment The FOC for repayments R_t^B is:

$$[F + \Psi_R(R_t^B, A_t^B)]\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ \mu_t^{RB} - \lambda_t^{RB} + \lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}, \tag{34}$$

where λ_t^{RB} is the Lagrange multiplier on the upper bound on R_t^B and μ_t^{RB} is the Lagrange multiplier on the lower bound. Combining with (31), we obtain:

$$\Psi_R(R_t^B, A_t^B) = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB},$$

where we defined the lower bound Lagrange multiplier on refinancing as the original multiplier divided by marginal utility of consumption:

$$\tilde{\mu}_t^{RB} = \mu_t^B \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Recall the definition $Z_t^R = R_t^B / A_t^B$. Using the functional form of Ψ_R from (22), the optimal prepayment fraction is:

$$Z_t^R = \frac{1}{\psi}\left(q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}\right) \tag{35}$$

A.1.4 Marginal Values of State Variables and SDF

Mortgages Taking the derivative of the value function with respect to A_t^B gives:

$$
\begin{aligned}
V_{A,t}^B = &-\left(1-\tau_t^m+\frac{\Psi_A(R_t^B,A_t^B)}{Z_A(\omega_t^*)}\right)Z_A(\omega_t^*)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}\\
&-\delta Z_A(\omega_t^*)\{\lambda_t^B F-\lambda_t^{RB}-\beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B]\times(CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}\}.
\end{aligned}
$$

Note that we can substitute for the term in braces using equation (30) and for Ψ_A using (23):

$$
V_{A,t}^B=-Z_A(\omega_t^*)\left(1-\tau_t^m-\frac{\psi\left(Z_t^R\right)^2}{2Z_A(\omega_t^*)}+\delta q_t^m-\delta\tilde{\lambda}_t^{RB}\right)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}. \tag{36}
$$

Houses Taking the derivative of the value function with respect to K_{t-1}^B gives:

$$
V_{K,t}^B=\left[p_t Z_K(\omega_t^*)+\frac{\theta C_t^B}{(1-\theta)K_{t-1}^B}\right]\frac{1-\theta}{C_t^B}(V_t^B)^{1/\nu}(1-\beta_B)(u_t^B)^{1-1/\nu}. \tag{37}
$$

SDF Define the borrower's intertemporal marginal rate of substitution between t and $t+1$, conditional on a particular realization of ρ_{t+1} as:

$$
\begin{aligned}
\mathcal{M}_{t,t+1}^B(\rho_{t+1}) &= \frac{\partial V_t^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B}=\frac{\partial V_t^B}{\partial V_{t+1}^B}e^{-g_{t+1}}\frac{\partial V_{t+1}^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B}\\
&=(V_t^B)^{1/\nu}\beta_B(CE_t^B)^{\sigma_B-1/\nu}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\frac{\frac{1-\theta}{C_{t+1}^B}(1-\beta_B)(V_{t+1}^B)^{1/\nu}(u_{t+1}^B)^{1-1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}\\
&=\beta_B e^{-\sigma_B g_{t+1}}\left(\frac{C_{t+1}^B}{C_t^B}\right)^{-1}\left(\frac{u_{t+1}^B}{u_t^B}\right)^{1-1/\nu}\left(\frac{V_{t+1}^B}{CE_t^B}\right)^{-(\sigma_B-1/\nu)}
\end{aligned}
$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$
\tilde{\mathcal{M}}_{t,t+1}^B=F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^B(\rho_{t+1}<\rho_{t+1}^*)+(1-F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^B(\rho_{t+1}>\rho_{t+1}^*),
$$

where $\mathcal{M}_{t,t+1}^B(\rho_{t+1}<\rho_{t+1}^*)$ and $\mathcal{M}_{t,t+1}^B(\rho_{t+1}>\rho_{t+1}^*)$ are the IMRSs, conditional on the two possible realizations of state variables.

A.1.5 Euler Equations

Mortgages Recall that $\tilde{V}_{A,t+1}^B$ is a linear combination of $V_{A,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $V_{A,t+1}^B$ given by equation (36). Substituting in for $\tilde{V}_{A,t+1}^B$ in (31) and using the SDF expression, we get the recursion:

$$
q_t^m=\tilde{\lambda}_t^B F+\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*)\left(1-\tau^m-\frac{\psi\left(Z_{t+1}^R\right)^2}{2Z_A(\omega_{t+1}^*)}-\delta\tilde{\lambda}_{t+1}^{RB}+\delta q_{t+1}^m\right)\right]. \tag{38}
$$

Houses Likewise, observe that we can write (32) as:

$$
p_t\left[1-\tilde{\lambda}_t^B\phi\right]=\frac{\beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}
$$

Recall that $\tilde{V}^B_{K,t+1}$ is a linear combination of $V^B_{K,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{K,t+1}$ given by equation (37). Substituting in for $\tilde{V}^B_{K,t+1}$ and using the SDF expression, we get the recursion:

$$p_t \left[1 - \tilde{\lambda}^B_t \phi\right] = \mathrm{E}_t \left[\tilde{\mathcal{M}}^B_{t,t+1} e^{g_{t+1}} \left\{ p_{t+1} Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta) K^B_t} \right\}\right] \tag{39}$$

A.2 Risk Takers

A.2.1 Statement of stationary problem

Denote by W^R_t risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{W}^R_t = (1 - D(\rho_t)) W^R_t,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}^R_t = \left(g_t, \sigma_{\omega,t}, W^D_t, A^B_t, B^G_{t-1}\right)$ denote all other aggregate state variables exogenous to risk takers.

After the default decision, risk takers face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\begin{aligned} V^R(\tilde{W}^R_t, \tilde{\rho}_t, \mathcal{S}^R_t) = \max_{C^R_t, A^R_{t+1,P}, A^R_{t+1,P}, B^R_t} & \left\{ (1-\beta_R) \left[\frac{(C^R_t)^{1-\theta} (K^R_{t-1})^\theta}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu} \right. \\ & \left. + \beta_R \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{V}^R\left(W^R_{t+1}, \mathcal{S}^R_{t+1}\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}} \right\}^{\frac{1}{1-1/\nu}} \end{aligned} \tag{40}$$

subject to:

$$(1-\tau^S) Y^R_t + \tilde{W}^R_t + G^{T,R}_t = C^R_t + (1-\mu_\omega) p_t K^R_{t-1} + q^m_t A^R_{t+1,P} + (q^m_t + \gamma_t) A^R_{t+1,G} + q^f_t B^R_t, \tag{41}$$

$$\begin{aligned} W^R_{t+1} = e^{-g_{t+1}} & \left[(M_{t+1,P} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]) A^R_{t+1,P} \right. \\ & \left. + (M_{t+1,G} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]) A^R_{t+1,G} + B^R_t \right], \end{aligned} \tag{42}$$

$$q^f_t B^R_t \geq -q^m_t (\xi_P A^R_{t+1,P} + \xi_G A^R_{t+1,G}), \tag{43}$$

$$A^R_{t+1,G} \geq 0, \tag{44}$$

$$A^R_{t+1,P} \geq 0, \tag{45}$$

$$\mathcal{S}^R_{t+1} = h(\mathcal{S}^R_t). \tag{46}$$

The continuation value $\tilde{V}^R\left(W^R_{t+1}, \mathcal{S}^R_{t+1}\right)$ is the outcome of the optimization problem risk takers face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{V}^R(W^R_t, \mathcal{S}^R_t) = \max_{D(\rho)} \mathrm{E}_\rho \left[D(\rho) V^R(0, \rho, \mathcal{S}^R_t) + (1 - D(\rho)) V^R(W^R_t, 0, \mathcal{S}^R_t)\right] \tag{47}$$

Define the certainty equivalent of future utility as:

$$CE^R_t = \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{V}^R\left(W^R_{t+1}, \mathcal{S}^R_{t+1}\right)\right)^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}. \tag{48}$$

and the composite within-period utility (evaluated at $\rho = 0$) as:

$$u^R_t = (C^R_t)^{1-\theta} (A_K K^R_{t-1})^\theta.$$

A.2.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0,1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $V^R(W, \rho, \mathcal{S}_t^R)$ defined in (40) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given W_t^R, risk-takers optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (47) as

$$\begin{aligned}\tilde{V}^R(W_t^R, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho \left[\mathbb{1}[\rho < \rho^*]\, V^R(0, \rho, \mathcal{S}_t^R) + (1 - \mathbb{1}[\rho < \rho^*])\, V^R(W_t^R, 0, \mathcal{S}_t^R)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\; \mathrm{E}_\rho \left[V^R(0, \rho, \mathcal{S}_t^R) \,|\, \rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $V^S(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = V^R(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(V^R(W_t^R, 0, \mathcal{S}_t^R)), \tag{49}$$

and by substituting the solution into (47), we obtain

$$\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = F_\rho(\rho_t^*)\mathrm{E}_\rho \left[V^R(0, \rho, \mathcal{S}_t^R) \,|\, \rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R). \tag{50}$$

Equations (40), (49), and (50) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (49) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1 - \beta_R)u_t^R) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_R(CE_t^R)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1 - \beta_R)\log(u_t^R) + \beta_R\log(CE_t^R) - \log(x) - (1 - \beta_R) & \text{if } \nu = 1. \end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned}&q_t^f \frac{1 - \theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ &\lambda_t^R q_t^f + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}\end{aligned} \tag{51}$$

where λ_t^R is the Lagrange multiplier on the borrowing constraint (43).

The first order condition for the government-guaranteed mortgage bond position is:

$$\begin{aligned}&(q_t^m + \gamma_t)\frac{1 - \theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_G q_t^m + \mu_{G,t}^R \\ &+ \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu},\end{aligned} \tag{52}$$

where $\mu_{t,G}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (44).

The first order condition for the private mortgage bond position is:

$$\begin{aligned}&q_t^m \frac{1 - \theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_P q_t^m + \mu_{P,t}^R \\ &+ \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu},\end{aligned} \tag{53}$$

where $\mu^R_{t,P}$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (45).

A.2.3 Marginal value of wealth and SDF

Differentiating (50) gives the marginal value of wealth

$$\tilde{V}^R_{W,t} = (1 - F_\rho(\rho^*_t))\,\frac{\partial V^R(W^R_t, 0, \mathcal{S}^R_t)}{\partial W^R_t},$$

where

$$\frac{\partial V^R(W^R_t, 0, \mathcal{S}^R_t)}{\partial W^R_t} = \frac{1-\theta}{C^R_t}(1-\beta_R)(V^R(W^R_t, 0, \mathcal{S}^R_t))^{1/\nu}(u^R_t)^{1-1/\nu},$$

The stochastic discount factor of risk-takers is therefore

$$\mathcal{M}^R_{t,t+1} = \beta_R e^{-\sigma_R g_{t+1}} \left(\frac{V^R(W^R_{t+1}, 0, \mathcal{S}^R_{t+1})}{CE^R_t}\right)^{-(\sigma_R - 1/\nu)} \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-1} \left(\frac{u^R_{t+1}}{u^R_t}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}^R_{t,t+1} = (1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^R_{t,t+1}$$

A.2.4 Euler Equations

It is then possible to show that the FOC with respect to B^R_t, $A^R_{t+1,G}$, and $A^R_{t+1,P}$ respectively, are:

$$q^f_t = q^f_t \tilde{\lambda}^R_t + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\right], \tag{54}$$

$$q^m_t + \gamma_t = q^m_t \xi_G \tilde{\lambda}^R_t + \tilde{\mu}_{t,G} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{G,t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)\right], \tag{55}$$

$$q^m_t = q^m_t \xi_P \tilde{\lambda}^R_t + \tilde{\mu}_{t,P} + \mathrm{E}_t\left[\tilde{\mathcal{M}}^R_{t,t+1}\left(M_{P,t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F]\right)\right]. \tag{56}$$

A.3 Depositor

We state here a slightly more general problem than in the main text whereby we allow the depositor to also invest in government-guaranteed mortgage bonds in addition to short-term government bonds. The problem in the main text then arises as a special case where we impose the additional constraint that the guaranteed mortgage bond holdings must be non-positive. The Lagrange multiplier on this constraint tells us whether the depositor in the restricted problem would want to hold guaranteed bonds, evaluated at the equilibrium allocation of the restricted model.

A.3.1 Statement of stationary problem

Let $\mathcal{S}^D_t = \left(g_t, \sigma_{\omega,t}, W^R_t, A^B_t, B^G_{t-1}\right)$ be the depositor's state vector capturing all exogenous state variables. Scaling by permanent income, the stationary problem of the depositor -after the risk taker has made default her decision and the utility cost of default is realized- is:

$$V^D(W^D_t, \mathcal{S}^D_t) = \max_{\{C^D_t, B^D_t, A^D_{t+1,G}\}} \left\{(1-\beta_D)\left[\left(C^D_t\right)^{1-\theta}\left(A_K K^D_{t-1}\right)^{\theta}\right]^{1-1/\nu} + \right.$$

$$\left. + \beta_D \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W^D_{t+1}, \mathcal{S}^D_{t+1})\right)^{1-\sigma_D}\right]^{\frac{1-1/\nu}{1-\sigma_D}}\right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$
\begin{aligned}
C_t^D &= (1-\tau_t^S)Y_t^D + G_t^{T,D} + W_t^D - (q_t^m + \gamma_t)A_{t+1,G}^D - q_t^f B_t^D - (1-\mu_{t,\omega})p_t K_{t-1}^D && (57)\\
W_{t+1}^D &= e^{-g_{t+1}}\left[(M_{t+1,G} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F])A_{t+1,G}^D + B_t^D\right] && (58)\\
B_t^D &\geq 0 && (59)\\
A_{t+1,G}^D &\geq 0 && (60)\\
\mathcal{S}_{t+1}^D &= h(\mathcal{S}_t^D) && (61)
\end{aligned}
$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$
\begin{aligned}
V_t^D &\equiv V_t^D(W_t^D, \mathcal{S}_t^D),\\
V_{W,t}^D &\equiv \frac{\partial V_t^D(W_t^D, \mathcal{S}_t^D)}{\partial W_t^D},
\end{aligned}
$$

Denote the certainty equivalent of future utility as:

$$
CE_t^D = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_t^D, \mathcal{S}_t^D)\right)^{1-\sigma_D}\right],
$$

and the composite within-period utility as:

$$
u_t^D = (C_t^D)^{1-\theta}(A_K K_{t-1}^D)^\theta.
$$

Like the borrower, the depositor must take into account the risk-taker's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$
\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.
$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$
\begin{aligned}
&q_t^f\frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} =\\
&\lambda_t^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu} \qquad (62)
\end{aligned}
$$

where λ_t^D is the Lagrange multiplier on the no-borrowing constraint (59).

The first order condition for the government-guaranteed mortgage bond position is:

$$
\begin{aligned}
&(q_t^m + \gamma_t)\frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} =\\
&\mu_{G,t}^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu}, \quad (63)
\end{aligned}
$$

where $\mu_{t,G}^D$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (60).

A.3.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$
V_{W,t}^D = \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu}, \qquad (64)
$$

and for the continuation value function:

$$\tilde{V}_{W,t}^{D} = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}_{t,t+1}^{D}(\rho_t) = \beta_D e^{-\sigma_D g_{t+1}}\left(\frac{V_{t+1}^D}{CE_t^D}\right)^{-(\sigma_D - 1/\nu)}\left(\frac{C_{t+1}^D}{C_t^D}\right)^{-1}\left(\frac{u_{t+1}^D}{u_t^D}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^{D} = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^D(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^D(\rho_{t+1} > \rho_{t+1}^*).$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (62) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\right] \tag{65}$$

where $\tilde{\lambda}_t^D$ is the original multiplier λ_t^D divided by the marginal value of wealth.

Similarly, from (63) we get the Euler Equation for guaranteed mortgages:

$$q_t^m + \gamma_t = \tilde{\mu}_{G,t}^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] \tag{66}$$

A.4 Equilibrium

The optimality conditions describing the problem are (24), (33), (35), (38) and (39) for borrowers, (41), (54), (55), and (56) for risk takers, and (57), (65), and (66) for depositors. We add complementary slackness conditions for the constraints (26) and (27) for borrowers, (43), (44), and (45) for risk-takers, and (59) and (60) for depositors. Together with the market clearing conditions (16), (17), and (18), these equations fully characterize the economy.

B Calibration Appendix

B.1 States and Transition Probabilities

After discretizing the aggregate real per capita income growth process as a Markov chain using the Rouwenhorst method, we obtain the following five states for g:

$$[0.943, 0.980, 1.018, 1.058, 1.101]$$

with 5×5 transition probability matrix:

$$\begin{bmatrix} 0.254 & 0.415 & 0.254 & 0.069 & 0.007 \\ 0.103 & 0.381 & 0.363 & 0.134 & 0.017 \\ 0.042 & 0.242 & 0.430 & 0.242 & 0.042 \\ 0.017 & 0.134 & 0.363 & 0.381 & 0.103 \\ 0.007 & 0.069 & 0.254 & 0.415 & 0.254. \end{bmatrix}$$

We discretize the process for σ^2_ω into a two-state Markov chain that is correlated with income growth g. The two states are:

$$[.078, .203]$$

The transition probability matrix, conditional on being in one of the bottom two g states is:

$$\begin{bmatrix} 0.80 & 0.20 \\ 0.01 & 0.99 \end{bmatrix}$$

The transition probability matrix, conditional on being in one of the top three g states is:

$$\begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}$$

The stationary distribution for the joint Markov chain of g and σ^2_ω is

State	1	2	3	4	5	6	7
g	0.943	0.943	0.980	0.980	1.018	1.058	1.101
σ^2_ω	0.078	0.203	0.078	0.203	0.078	0.078	0.078
Prob.	0.039	0.023	0.167	0.081	0.372	0.255	0.063

From a long simulation, we obtain the following mean, standard deviation, and persistence for g: 1.019, .039, and .42, respectively. We obtain the following mean, standard deviation, and persistence for σ^2_ω: .092, .039, and .46, respectively. We obtain a correlation between g and σ_ω of -0.42.

B.2 Evidence on default rates and mortgage severities

Since not all mortgage delinquencies result in foreclosures (loans can cure or get modified), we use the fraction of loans that 90-day or more delinquent or in foreclosure as the real world counterpart to our model's default rate. Some loans that were 90-day delinquent or more received a loan modification, but many of these modifications resulted in a redefault 12 to24 months later. Given that our model abstracts from modifications, using a somewhat broader criterion of delinquency than foreclosures-only seems warranted.

The observed 90-day plus (including foreclosures) default rate rose from 2% at the start of 2007 to just under 10% in 2010.Q1. Since then, the default rate has been gradually falling back, to 4.7% by 2014.Q3 (Mortgage Bankers Association and Urban Institute). The slow decline in foreclosure rates in the data is partly due to legal delays in the foreclosure process, especially in judicial states like New York and Florida where the average foreclosure process takes up to 1000 days. In other part it is due to re-defaults on modified loans. Since, neither is a feature of the model, it seems reasonable to interpret the abnormally high default rates of the post-2013 period as due to such delays, and to reassign them to the 2010-2012 period. If we assume that the foreclosure rate will return to its normal 2% level by the end of 2016, then such reassignment delivers an average foreclosure rate of 8.5% during the 2007-2012 foreclosure crisis. Absent reassignment, the average default rate would be 5.9% over the 2007-2016 period. Jeske et al. (2014) target only a 0.5% foreclosure rate, but their calibration is to the pre-2006 sample. The evidence from the post-2006 period dramatically raises the long-term mean default rate.

Fannie Mae's 10K filings for 2007 to 2013 show that severities, or losses-given-default, on conventional single-family loans were 4% in 2006, 11% in 2007, 26% in 2008, 37% in 2009, 34% in 2010, 35% in 2011, 31% in 2012, and 24% in 2013. Severities on Fannie's non-conforming (mostly Alt-A and subprime) portfolio holdings exceed 60% in all these years. If anything, the severity rate on Fannie's *non-conforming* holdings is lower than that of the overall non-conforming market due to advantageous selection (Adelino et al 2014). Given that the non-conforming market accounted for half of all mortgage originations in 2004-2007, the severities on conventional loans are too low to accurately reflect the market-wide severities. To take account of this composition effect, we target a market-wide severity rate of 40% in the crisis (2007-2012). We target a severity rate of 15% in non-crisis years (pre-2007 and post-2012), based on Fannie's experience in that period and the much smaller size of the non-conforming mortgage market in those years.

Combining a default rate of 2% in normal times with a severity of 15%, we obtain a loss rate of 0.3% in normal times. Combining the default rate of 8.5% during a foreclosure crisis with the severity of 40% in crises, we obtain a 3.4% loss rate.

To obtain mortgage debt to GDP in normal times and in crisis times, we calculate a time series of household mortgage debt (including debt on multi-family real estate owned by the household sector) and divide by GDP. Since mortgage debt-GDP saw a gradual decrease for reasons related to new technology, such as automated underwriting and securitization, we focus attention on the post-1985 period. Mortgage debt-GDP averages to 54% in the 1985-1999 period. We target this for our normal times value. Mortgage debt-GDP averages to 78% in the 2000-2014 period. We target that number for our crisis number.

B.3 Long-term mortgages

Our model's mortgages are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. Real life mortgages have a finite maturity (usually 30 years) and a principal payment. They also have a vintage (year of origination), whereas our mortgages combine all vintages in one variable. This appendix explains how to map the geometric mortgages in our model into real-world mortgages.

Our model's mortgage refers to the entire pool of all outstanding mortgages. In reality, this pool not only consists of newly issued 30-year fixed-rate mortgages (FRMs), but also of newly issued 15-year mortgages, other mortgage types such as hybrid adjustable-rate mortgages (ARMs), as well as all prior vintages of all mortgage types. This includes, for example, 30 year FRMs issued 29 years ago. The Barclays U.S. Mortgage Backed Securities (MBS) Index is the best available measure of the overall pool of outstanding government-guaranteed mortgages. It tracks agency mortgage backed pass-through securities (both fixed-rate and hybrid ARM) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC). The index is constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on program, coupon and vintage. For this MBS index we obtain a time series of monthly price, duration (the sensitivity of prices to interest rates), weighted-average life (WAL), and weighted-average coupon (WAC) for January 1989 until December 2014.

Our calibration strategy is to choose values for δ and F so that the relationship between price and interest rate (duration) is the same for the observed Barclays MBS Index and for the model's geometric bond. We proceed in two steps. In the first step, we construct a simple model to price a pool of MBS bonds and calibrate it to match the observed time series of MBS durations. With this auxiliary model in hand, we then choose the two parameters to match the price-rate curve in the auxiliary model and the geometric mortgage model.

B.3.1 Step 1: A simple MBS pricing model

Changes in duration of the Barclays MBS index are often driven by changes in the index composition. As mortgages are prepaid and new ones are issued with different coupons, both the weighted-average-life and weighted-average-coupon of the Index change significantly. Any model that wants to have a chance at matching the observed durations must account of these compositional changes.

For simplicity, we assume that all mortgages are 30-year fixed-rate mortgages. We construct a portfolio of MBS with *remaining* maturities ranging from 1 to 360 months. Each month, a fraction of each MBS prepays. We assume that the prepayment rate is given by a function $CPR(c - r)$ which depends on the "prepayment incentive" of that particular MBS, defined as the difference between the original coupon rate of that mortgage and the current mortgage rate. We assume that every prepayment is a refinancing: a dollar of mortgage balance prepaid result in a dollar of new mortgage balance originated at the new mortgage rate. In addition, each period an exogenously given amount of new mortgages are originated with a coupon equal to that month's mortgage rate to reflect purchase originations (as opposed to refinancing originations).

In a given month t, each mortgage i has starting balance bal_t^i, pays a monthly mortgage pmt_t^i of which int_t^i is interest and $prin_t^i$ is scheduled principal, where i is the remaining maturity of the mortgage, i.e., the mortgage was originated at time $t - (360 - i) - 1$. Denote the unscheduled principal payments, or prepayments, by prp_t^i. Let SMM_t^i be the prepayment rate in month t on that mortgage. The evolution equations for actual mortgage cash flows are:

$$\begin{aligned}
int_t^i &= \frac{c_{t-(360-i)-1}}{12} \times bal_t^i \\
prin_t^i &= pmt_t^i - int_t^i \\
prp_t^i &= SMM_t^i(bal_t^i - prin_t^i) \\
bal_{t+1}^{i-1} &= (1 - SMM_t^i)(bal_t^i - prin_t^i) \\
pmt_{t+1}^{i-1} &= (1 - SMM_t^i)pmt_t^i
\end{aligned}$$

The initial payment is given by the standard annuity formula, normalizing the amount borrowed to 1.

$$\begin{aligned}
pmt_t^{360} &= \frac{\frac{c_{t-1}}{12}}{1 - (1 + c_{t-1}/12)^{-360}} \\
bal_t^{360} &= 1 + \sum_{i=1}^{360} prp_{t-1}^i
\end{aligned}$$

The last equation says that the initial balance of new 30-year FRMs is comprised on 1 unit of purchase originations, an exogenously given flow of originations each period, plus refinancing originations which equal all prepayments from the previous period.

Furthermore, at every month t we compute *projected* cash flows on each mortgage assuming mortgage rates stay constant from t until maturity i. These projected cash flows follow the same evolution equations as presented above. Denote these projected cash flows with a tilde over the variable.

We can then compute the price P_t, (modified) duration Dur_t, and weighted-average-life WAL_t of the MBS *portfolio* comprised of all vintages:

$$P_t = \sum_{i=1}^{360} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s}$$

$$Dur_t = \frac{1}{1 + \frac{r_t}{12}} \sum_{i=1}^{360} \frac{1}{P_t^i} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s} s$$

$$WAL_t^i = \sum_{i=1}^{360} \frac{\sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}) s}{\sum_{i=1}^{360} \sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})}$$

What remains to be specified is our prepayment model delivering the single-month mortality SMM_t^i used above. Following practice, we assume an annual constant prepayment rate (CPR) which is a S-shaped function of the rate incentive: $CPR_t^i = CPR(r_t - c_{t-(360-i)-1})$:

$$CPR(x) = \underline{CPR} + (\overline{CPR} - \underline{CPR}) \left(1 - \frac{\exp(\psi(x - \bar{x}))}{1 + \exp(\psi(x - \bar{x}))}\right)$$

The annual CPR implies a monthly SMM $SMM_t^i = factor_i \times (1 - (1 - (CPR_t^i)^{1/12})$. The multiplicative $factor_i$ allows us to deal with slow prepayments early in the life of the mortgage (the "ramp-up" phase) and late in the life of the mortgage (the "burn-out" phase). For simplicity, we make $factor_i$ linearly increasing from 0 in month 1 (when $i = 360$) to 1 in month 30, flat at 1 between month 30 and month 180 and linearly decreasing back to 0 between months 180 and month 360. We choose the CPR curve parameters $\{\underline{CPR}, \overline{CPR}, \psi, \bar{x}\}$ to minimize the sum of squared errors between the time series of model-implied duration $\{Dur_t\}$ and observed duration on the Barclays index.

To produce the time-series of model-implied duration $\{Dur_t\}$, we feed in the observed 30-year conventional fixed rate mortgage rate (MORTGAGE30US in FRED), $\{r_t\}$. We initialize the portfolio many years before the start of our time series data to ensure that the model is in steady state by the time our time series for the Barclays index starts. Specifically, we start the computation in April 1903 by issuing 1 MBS. By March 1933, we have a complete portfolio of 360 fixed-rate amortizing mortgages, maturing any month from April 1933 to March 1963.

The left panel of Figure 6 shows the observed time series of duration on the Barclays MBS index plotted against the model-implied duration on the MBS pool. The two time series track each other quite closely despite several strong modeling assumptions. The resulting CPR curve looks close to historical average prepayment behavior on agency MBS, as prepayment data from SIFMA indicate. CPR is slightly above 40% when the rate incentive is 200 basis points or more, about 15% when the rate incentive is zero, and slightly above 5% when the rate incentive is below -200 basis points.

B.3.2 Step 2: Matching MBS pool to perpetual mortgage in our model

With a well-calibrated auxiliary model for a MBS pool, we now proceed to match key features of that auxiliary model's MBS pool to the mortgage in our model, which is a geometrically declining perpetuity.

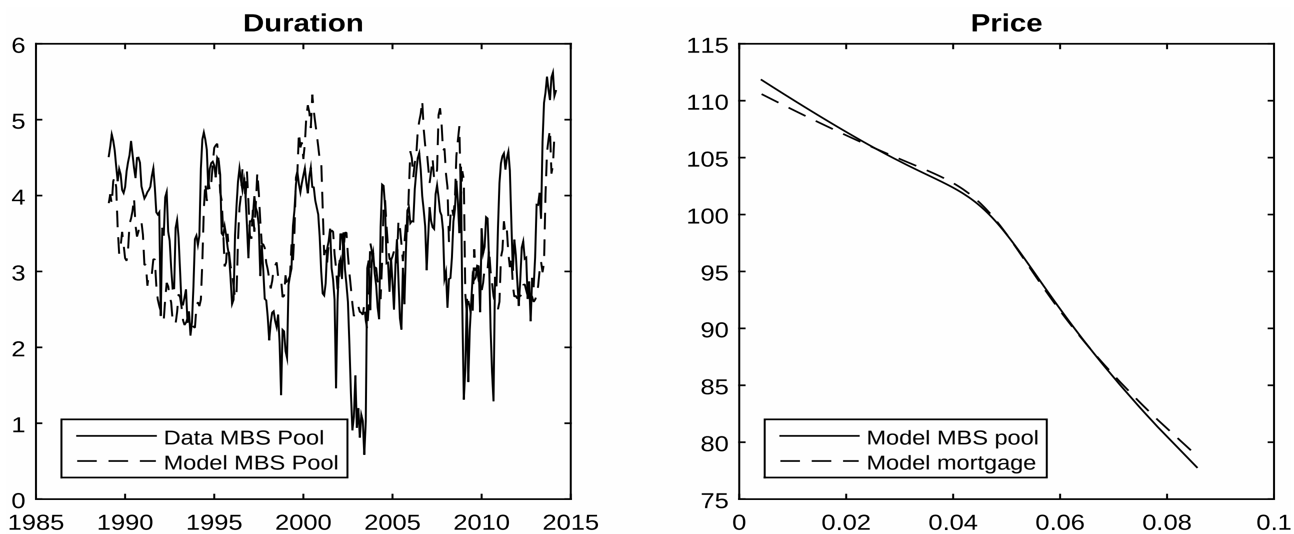
We start by computing the price $P(r)$ of a fixed-rate MBS with maturity T and coupon c as a function of the current real MBS rate r, using the constant prepayment rate function $\hat{CPR}(r) = CPR(r - c)$ obtained from step 1. For T and c we use the time-series average of the weighted-average maturity and weighted-average real coupon, respectively, from the model-implied MBS pool obtained in step 1.[34]

We can write the steady-state price of a guaranteed geometric mortgage with parameters (δ, F) and a per-

[34]To get real mortgage rates from nominal mortgage rates, we subtract realized inflation over the following year. To get real coupons and MBS rates from real mortgage rates, we subtract 50 bps to account for servicing and guarantee fees.

Figure 6: Matching Mortgages in Model to Data

The left panel plots the observed time series of duration on the Barclays MBS index (solid line) plotted against the duration on the model-implied MBS pool (dashed-line). The right panel plots the mortgage price-interest rate relationship for the model-implied MBS pool (solid line) and the model-implied geometrically declining perpetual mortgage (dashed line). Prices on a \$100 face value mortgage are on the vertical axis, while interest rates are on the horizontal axis. The Barclays MBS index data are from Bloomberg for the period 1989 until 2014 (daily frequency). The calculations also use the 30-year fixed-rate mortgage rate from FRED.



period fee γ paid for the life of the loan recursively as:

$$Q(r,\gamma) + \gamma = \frac{1}{1+r}\left(1 + C\hat{P}R(r)\delta F + (1 - C\hat{P}R(r))\delta(Q(r,\gamma)+\gamma)\right)$$

Solving for $Q(r,\gamma)$, we get

$$Q(r,\gamma) = \frac{1 + C\hat{P}R(r)\delta F}{1 + r - \delta(1 - C\hat{P}R(r))} - \gamma. \tag{67}$$

Note that the fee γ in equation 67 is quoted in units of the guaranteed bond's price. However, in the data MBS pool we observe a guarantee and servicing fee of approximately 50 bp on average that is charged as a spread on top of a bond's yield. During the calibration, we thus need to use the net-of-fees rate for the MBS pool and the gross-of-fees rate for the geometric bond.

The stage 2 calibration determines how many units X of the geometric mortgage with parameters (δ, F) one needs to sell to hedge one unit of the MBS against parallel shifts in interest rates, across the range of historical mortgage rates:

$$\min_{\delta,F,X,\gamma} \int [P(r) - XQ(r+0.005,\gamma)]^2 dr,$$

subject to

$$\log\left(\frac{1}{Q} + \delta\right) = \log\left(\frac{1}{Q+\gamma} + \delta\right) + 0.005. \tag{68}$$

The equality constraint 68 determines the price-fee γ that corresponds to the 50 bps rate-fee. The LHS is the gross-of-fees mortgage rate and the RHS is the equivalent net-of-fees mortgage rate plus the 50 bps fee[35]. Generally the equivalent price-fee will depend on the level of the price, which is endogenous to the minimization problem. Thus the constraint determines γ as the equivalent price-fee when the MBS trades at par (with price 1) so that $Q = 1/X$.

[35]The yield of a geometric bond with price Q and duration parameter δ is $r = \log\left(\frac{1}{Q} + \delta\right)$.

We estimate values of $\delta = 0.948$, $F = 9.910$, which implies $\alpha = 0.520$, and $X = 0.1080$. For the model calibration, we only need δ and α. The right panel of Figure 6 shows that the fit is excellent. The average error is only 0.34% of the MBS pool price.

In conclusion, despite its simplicity, the perpetual mortgage in the model captures all important features of real life mortgages (or MBS pools). The relationship between price and interest rate is convex when rates are high and concave ("negative convexity") when rates are low, which is when the prepayment option is in the money. It matches the interest rate risk (duration) of real-life mortgages, for different interest rate scenarios.

C Computational Solution

The computational solution of the model is implemented using what Judd (1998) calls "time iteration" on the system of equations that characterizes the equilibrium of the economy defined in appendix section A.4. The general solution approach for heterogeneous agent models with incomplete markets and portfolio constraints that we employ in this paper is well described by Kubler and Schmedders (2003).

The procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** The unknown functions of the state variables that need to be computed are the set of endogenous objects specified in the equilibrium definition. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. There is an equal number of unknown functions and nonlinear functional equations. To approximate the unknown functions in the space of the two exogenous state variables $[Y_t, \sigma_{\omega_t}]$ and four endogenous state variables $[A_t^B, W_t^R, W_t^S, G_t]$, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any three other state variables. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding, and we test the accuracy of the approximation by computing relative Euler equation errors.

2. **Iteratively solve for the unknown functions.** Given an initial guess $\mathcal{C}^0(\mathcal{S})$ to compute tomorrow's optimal policies as functions of tomorrow's states, solve the system of nonlinear equations for the current optimal policies at each point in the discretized state space. Expectations are computed using quadrature methods. Using the solution vector for current policies, compute the next iterate of the approximation $\mathcal{C}^1(\mathcal{S})$ and repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Judd, Kubler, and Schmedders (2002) show how Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose.

3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed.

In a long simulation, errors in the nonlinear equations are low. Table 5 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 99.5^{th} percentiles.

Table 5: Computational Errors

	Percentile				
	50th	75th	95th	99th	Max
(38)	0.0010	0.0020	0.0085	0.0130	0.0239
(39)	0.0007	0.0014	0.0058	0.0088	0.0164
(56)	0.0008	0.0050	0.0157	0.0266	0.0691
(54)	0.0008	0.0055	0.0167	0.0294	0.0746
(65)	0.0002	0.0004	0.0009	0.0016	0.0083
(26)	0.0002	0.0007	0.0033	0.0058	0.0313
(43)	0.0003	0.0006	0.0025	0.0034	0.0086
(59)	0.0030	0.0042	0.0084	0.0092	0.0230
(16)	0.0002	0.0003	0.0010	0.0022	0.0088
(45)	0.0000	0.0000	0.0000	0.0008	0.0108
(27)	0.0000	0.0001	0.0003	0.0014	0.0103
(55)	0.0008	0.0051	0.0160	0.0280	0.0736
(44)	0.0000	0.0000	0.0001	0.0003	0.0029
(42)	0.0001	0.0001	0.0003	0.0013	0.0279
(15)	0.0003	0.0005	0.0020	0.0023	0.0487

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the 20 bps g-fee model. The first 13 equations define policy functions. They are a subset of the 22 equations that define the equilibrium. The last two equations define evolutions of risk-taker wealth and government debt, respectively.

D Additional Experiments

Our main policy experiment consisted of raising the g-fees. In the main body of the paper, we also reviewed two alternative experiments. The first changed capital requirements on guaranteed mortgages. The second considered a legislative proposal to to "put private capital in front of a government guarantee" by limiting guarantees to losses in excess of 10%. We now study three more policy experiments and compare their welfare consequences to those in the main policy experiment. The first experiment explores the effect of limited liability. The next two experiments study alternate ways to make guarantees operative only when losses are catastrophic. These exercises help to further illuminate the interaction of government guarantees, deposit insurance, and risk taker leverage.

D.1 Limited Liability

The second alternative policy we consider is one that weakens deposit insurance. The knowledge that they (and their depositors) will be bailed out by the government if their net worth turns negative leads banks to take on more risk. We weaken deposit insurance, or equivalently weaken limited liability for banks, by increasing the mean (μ_ρ) of the utility penalty that banks incur for insolvency. The third and fourth columns of Table 6 labeled "high μ_ρ" report the results.

Many of the effects are similar as for the tighter leverage constraint. Guarantees remain very valuable and dominate the portfolio, even more so than in the previous experiment. Portfolio delinquency and loss rates are close to the benchmark low g-fee economy. One big difference to the main experiment is that weaker limited liability does not lead banks to reduce leverage. Risk taker net worth only increases marginally. Still, this small increase in net worth, combined with the higher utility cost of bankruptcy is enough to eliminate all bankruptcies. As a result of the high leverage and government debt, depositors must hold substantial amounts of safe assets and interest rates are a bit higher than the benchmark model as a result. The real short rate increases by 2 basis points to 1.15%.

We find no significant effect on aggregate welfare from this policy. It has an aggregate welfare loss of 0.02%. Borrowers' welfare is unaffected and both risk takers and depositors lose slightly. In sum, while increasing the costs of bank bankruptcy is successful at eliminating bank bankruptcies, it has a small negative aggregate welfare effect. This demonstrates that intermediary bankruptcies are not the driving force behind our welfare results. The key issue rather is the underpricing of the guarantee, which is as paramount in this economy as in the low g-fee benchmark.

D.2 Catastrophic Insurance

Columns 6 and 7 of Table 6 report results for a catastrophic insurance policy which "kicks in" at losses of 5%. Welfare increases are smaller than when the private sector loss is capped at 10% (+0.56% vs +0.67%). This increase in welfare is smaller than that from a complete phase-out. Thus the largest welfare gains are obtained when the private sector bears enough losses to reduce its risk taking, but not so much as to debilitate its intermediation function which is important to achieve the best distribution of aggregate risk in the economy.

The last two columns of Table 6 report results for a catastrophic insurance experiment in which losses are capped at 10% but where the insurance is offered at a much lower price of 5 bp instead of the 20bp discussed in the main text. This policy has higher welfare gains of +0.69%, compared to +0.67% for the 20bp catastrophic guarantee and +0.63% for the full phase-out. The main difference with the more expensive catastrophic guarantee is that because the guarantee is cheaper (and closer to the actuarially fair cost of 2bp), risk takers are much more likely to purchase it. This protects them better to unexpected catastrophic shocks than in the 20bp JC economy. It further improves risk sharing and raises interest rates.

Table 6: The Role of Limited Liability and Catastrophic Insurance

	20 bp g-fee		**High μ_ρ**		**JC 5%**		**JC 10%, 5 bp**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.13%	3.00%	1.15%	3.07%	1.82%	3.22%	1.92%	3.21%
Mortgage rate	3.52%	0.24%	3.52%	0.24%	3.73%	0.26%	3.74%	0.25%
House price	2.240	0.142	2.240	0.142	2.113	0.121	2.106	0.123
				Risk Taker				
Market value of bank assets	0.634	0.018	0.634	0.018	0.584	0.013	0.581	0.012
Fraction guaranteed	99.96%	0.55%	99.92%	0.89%	54.53%	34.10%	75.37%	24.19%
Risk taker leverage	95.59%	0.92%	95.28%	1.05%	93.31%	2.33%	94.90%	1.56%
Risk taker wealth	0.029	0.012	0.031	0.013	0.040	0.018	0.030	0.014
Fraction $\lambda^R > 0$	32.66%	46.90%	27.17%	44.49%	85.20%	35.51%	98.82%	10.80%
Bankruptcy frequency	0.27%	5.19%	0.00%	0.00%	0.02%	1.41%	0.44%	6.62%
Return on RT wealth[a]	3.56%	35.74%	2.84%	34.69%	2.71%	30.95%	3.50%	37.00%
				Borrower				
Mortgage debt	0.053	0.001	0.053	0.001	0.050	0.001	0.050	0.001
Borrower LTV	63.79%	3.93%	63.79%	3.90%	63.81%	3.48%	63.78%	3.50%
Market value of debt LTV	75.72%	6.47%	75.72%	6.47%	74.07%	6.04%	73.96%	6.00%
Borrower debt to income	1.489	0.040	1.488	0.040	1.406	0.025	1.400	0.024
Debt/income growth	0.04%	2.86%	0.04%	2.90%	0.04%	2.89%	0.04%	2.75%
Default rate	2.74%	6.20%	2.75%	6.18%	1.82%	4.01%	1.76%	3.79%
Rate-induced prepayment rate	15.83%	4.24%	15.81%	4.30%	12.22%	4.43%	12.07%	4.28%
Loss Given Default	30.15%	5.76%	30.15%	5.76%	30.03%	5.72%	30.03%	5.72%
MTM Loss Given Prepayment	10.78%	2.59%	10.77%	2.62%	8.51%	2.83%	8.42%	2.74%
Loss rate private	1.04%	2.75%	1.05%	2.73%	0.70%	1.75%	0.67%	1.64%
Loss rate guaranteed	0.38%	0.88%	0.38%	0.88%	0.22%	0.52%	0.21%	0.49%
Loss rate portfolio	0.38%	0.88%	0.38%	0.88%	0.42%	1.12%	0.34%	0.83%
				Government				
Government debt / GDP	14.96%	21.81%	15.37%	22.05%	−6.03%	3.76%	−6.06%	3.95%
Actuarially Fair g-fee	0.77%	0.43%	0.77%	0.43%	0.11%	0.08%	0.02%	0.01%
				Welfare				
Aggregate Welfare	0.279	0.008	−0.02%	+0.05%	+0.56%	+0.49%	+0.69%	+0.41%
Value Function borrower	0.319	0.010	0.00%	+0.02%	+0.05%	−1.02%	+0.04%	−1.14%
Value Function depositor	0.249	0.006	−0.04%	+0.12%	+1.16%	+1.07%	+1.46%	+1.06%
Value function risk taker	0.083	0.000	−0.24%	+6.44%	+0.96%	+40.15%	+1.02%	+8.94%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two columns are the benchmark model from Table 2. The next two columns have a higher mean utility cost of default μ_ρ. The model in columns 6 and 7 report results for an economy where the government offers catastrophic insurance i.e. guarantees only losses in excess of 5%. The last 2 columns report results for a catastrophic insurance economy where the attachment point is 10%, like in Table 4, but at a lower price of 5 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.