

A Probability-Based Stress Test of Federal Reserve Assets and Income

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Abstract

To support the economic recovery, the Federal Reserve amassed a large portfolio of long-term bonds. We assess the Fed's associated interest rate risk—including potential losses to its Treasury and mortgage-backed securities holdings and declines in the Fed's remittances to the Treasury. In assessing this interest rate risk, we use probabilities of alternative interest rate scenarios that are obtained from a dynamic term structure model that respects the zero lower bound on yields. The resulting probability-based stress tests indicate that large portfolio losses or a cessation of remittances to the Treasury are unlikely to occur over the next few years.

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1 Introduction

In late 2008, in response to a severe financial crisis and recession, the Federal Reserve reduced its target for a key policy rate—the overnight federal funds rate—to a range between 0 and 25 basis points. To provide additional monetary stimulus to spur economic growth and avoid deflation, the Fed then conducted three rounds of large-scale asset purchases—commonly referred to as quantitative easing (QE). These actions left its portfolio of longer-term securities several times larger than its pre-crisis level. Although the Fed’s securities portfolio carries essentially no credit risk, its market value can vary over time, and the greater size of the Fed’s portfolio exposes it to greater interest rate risk including, that is, unusually large financial gains and losses from interest rate fluctuations. Furthermore, the Fed’s purchases have shifted the composition of the portfolio toward longer-maturity securities, which increases the sensitivity of its market value to interest rate changes and the risk that increases in *longer-term* interest rates will erode the market value of the Fed’s portfolio—that is, balance sheet risk. In fact, the Fed’s balance sheet is not market to market, so such declines in market value would constitute unrealized capital losses, which would only become realized if the securities were sold. Still, this larger balance sheet risk has raised policy concerns.¹ For example, former Fed Governor Frederic Mishkin (2010) argued that “major holdings of long-term securities expose the Fed’s balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence.” Similarly, former Fed Vice Chairman Donald Kohn (2014) worried: “As long-term rates rise, the Federal Reserve will have mark-to-market losses on its balance sheet. These losses are not a threat to the Federal Reserve’s ability to tighten nor do they have any economic significance, but losses could be used as a political weapon by those who seek to curtail the Federal Reserve’s independence or limit its powers.”

The Fed also faces another form of interest rate risk: the risk that increases in *short-term* interest rates, notably the interest rate that the Fed pays on bank reserves, will significantly increase the funding cost of the Fed’s securities portfolio—that is, income risk. Because the Fed’s interest income is generated from fixed coupon payments on longer-maturity securities, rising short-term interest rates and increased payments on reserves would reduce the Fed’s net interest income, which in turn would lower the Fed’s remittances to the U.S. Treasury. Under extreme circumstances, the remittances could fall to zero. While the Fed’s ability to conduct monetary policy operations under such adverse conditions would not be directly impeded, concerns have been raised about the attendant

¹Such central bank balance sheet concerns have been felt by other central banks. The Bank of Japan has at times in the past limited its bond purchases from a fear that capital losses could tarnish its credibility, while the Bank of England obtained an explicit indemnity from the British Treasury in advance for any capital losses stemming from QE, as detailed in McLaren and Smith (2013). In response, the literature on central bank financial accounting has recently grown in tandem with central bank balance sheets and notably includes Bindseil et al. (2009), Archer and Moser-Boehm (2013), Hall and Reis (2013), and Del Negro and Sims (2014).

political fallout from lower remittances just as with large capital losses (realized or unrealized).² For example, such worries were noted in the minutes of the March 20, 2013 Federal Open Market Committee meeting, which stated that “[s]ome participants were concerned that a substantial decline in remittances might lead to an adverse public reaction or potentially undermine Federal Reserve credibility or effectiveness.”³

To understand and assess the Fed’s balance sheet and income risks, it is crucial to quantify them. Two recent papers—Carpenter et al. (2013) and Greenlaw et al. (2013), henceforth GHHM—have made great progress in doing so. Both studies generated detailed projections of the market value and cash flow of the Fed’s assets and liabilities under a few specific interest rate scenarios. In essence, their projections are akin to the “stress tests” that large financial institutions undergo to gauge whether they have enough capital to survive adverse economic scenarios.⁴ As is common, these stress tests do not place probabilities on the alternative interest rate scenarios but simply consider, say, shifting the level of the entire yield curve up or down from its baseline projection by 100 basis points. Clearly, it is also of great interest to know what probabilities should be attached to the range of considered outcomes.⁵ Attaching likelihoods to the alternative scenarios—or more generally, looking at the entire distributional forecast—results in what we term probabilistic or “probability-based” stress tests. The additional information about the probability distribution of interest rate scenarios allows us to provide new assessments of the likelihood of certain interest rate risk events. In this paper, we illustrate such a probabilistic methodology by examining potential mark-to-market losses on the Fed’s Treasury and mortgage-backed securities (MBS) holdings as well as the potential cessation of its remittances to the Treasury. Importantly, having information from probability distributions enables us to examine the likelihood of certain events, such as the possibility that losses on the Fed’s securities holdings will exceed a certain threshold or that net interest income will be negative for more than one year.

A key component of our probability-based stress test methodology is a dynamic term structure model that generates yield curve projections consistent with historical interest rate variation. Since nominal yields on Treasury debt are near their zero lower bound (ZLB), we use the shadow-rate, arbitrage-free Nelson-Siegel (AFNS) model class developed by Christensen and Rudebusch (2014)

²Importantly, regardless of its portfolio losses or income expenses, the Fed still has operational control of short-term interest rates because its ability to pay interest on bank reserves allows it to conduct monetary policy independently of the size of its balance sheet. Unrealized portfolio losses do not affect the Fed’s reported balance sheet, which is not marked to market. While realized portfolio losses are recorded, any resulting negative net income would not diminish the Fed’s capital; instead, the Fed would maintain its capital by reducing projected future remittances to the Treasury via creation of a deferred asset.

³See also Rudebusch (2011), Dudley (2013) and Goodfriend (2014).

⁴Stress testing financial institutions, and the financial system more broadly, has taken on great importance in the wake of the financial crisis; see Schuermann (2013) and Borio et al. (2014). Our analysis is directly related to interest rate risk stress testing, which is discussed by Drehmann et al. (2010) and Abdymomunov and Gerlach (2013).

⁵Berkowitz (2000) and Pritsker (2011) make a similar point regarding bank stress tests. In contrast, Borio et al. (2014) express the common view that stress tests should focus only on a few scenarios.

to generate the requisite, potentially asymmetric, distributional interest rate forecasts. Shadow-rate models are latent-factor models in which the state variables have standard Gaussian dynamics, but the standard short rate is replaced by a shadow short rate that may be negative, as in the spirit of Black (1995). The model-generated observed short rate and yield forecasts thus respect the ZLB. Despite its inherent nonlinearity, shadow-rate AFNS models remain as flexible and empirically tractable as standard AFNS models. Critically for our purposes, we demonstrate that these models are able to accurately price the Fed’s portfolio of Treasury securities.

For our empirical assessment of the Fed’s balance sheet risk, we generate Treasury yield curve projections using the shadow-rate AFNS model favored by Christensen and Rudebusch (2013, henceforth CR) in their analysis of U.S. Treasury yields near the ZLB. We examine distributional forecasts of the value of the Fed’s Treasury and MBS securities that are based on 10,000 yield curve simulations. Our simulation results indicate that potential losses on the Fed’s securities holdings are unlikely to be large. In particular, based on the Fed’s Treasury holdings as of the second quarter of 2014, the projected median value of the portfolio does not fall below face value over the three-year horizon of our exercise; indeed, such a projected securities valuation shortfall only occurs at about the tenth percentile of the simulated distribution. With respect to the joint holdings of Treasury and MBS securities, the added exposure from the MBS holdings does raise the portfolio’s interest rate sensitivity and thus risk. However, even then, only the projected portfolio value at the 25th percentile falls below its face value.

To assess the Fed’s income risk, we use model-based yield curve projections to generate distributional projections of the Fed’s remittances to the Treasury up to seven years ahead. In more than 90 percent of the simulations, remittances are projected always be positive over the seven-year horizon. Even at the lower fifth percentile of the distribution of outcomes, the cumulative remittance shortfall (i.e., the Fed’s deferred asset) peaks at less than \$5.0 billion in 2018. Accordingly, our probability-based stress test suggests that the risk of a significant halt of remittances to the Treasury is remote. A probability-based approach also allows us to assess the distribution of cumulative remittances, and it appears that the Treasury likely will receive more remittances in total with the Fed’s QE purchases than it would have otherwise.

Finally, an important caveat to our analysis is that we are not conducting a comprehensive assessment of the costs and benefits of the Fed’s program of QE, as discussed by Rudebusch (2011). Indeed, our probability-based stress test captures only part of the financial consequences of the Fed’s securities purchases and, notably, excludes two key fiscal benefits accruing to the Treasury as longer-term interest rates were pushed lower by the Fed’s securities purchases.⁶ First, the lower interest rates likely resulted in higher output and household income, which boosted federal tax revenue

⁶Regarding the effect on yields, see Gagnon et al. (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2014) among many others.

and reduced federal outlays. Second, the lower interest rates associated with QE helped lower the Treasury’s borrowing costs for issuing new debt. Furthermore, it is important to stress that any kind of financial or fiscal accounting of the type we are conducting is ancillary to the Fed’s mission. The Fed’s statutory goal for setting monetary policy is to promote maximum employment and price stability, and these macroeconomic goals are the fundamental metrics for judging monetary policy. Financial considerations—even potentially large capital losses—are secondary.

The rest of the paper is structured as follows. Section 2 describes the evolution of the Fed’s securities portfolio since the onset of the financial crisis and our data sample. Section 3 describes the shadow-rate AFNS model. Section 4 presents our probability-based stress tests of the Fed’s Treasury and MBS holdings. Section 5 details our probability-based stress test of the Fed’s remittances to the U.S. Treasury, and Section 6 concludes.

2 The Fed’s Securities Portfolio

We start with a brief description of the Fed’s securities holdings and the associated yield data. Figure 1 shows the evolution of the assets of the Federal Reserve System at a weekly frequency since the start of 2008. In the early stages of the financial crisis, the Fed’s balance sheet expanded through various emergency lending facilities, most notably the Term Auction Facility.⁷ In the figure, this lending appears in the “Other Assets” category, which as of June 2014 represented less than 5 percent of the Fed’s assets. The “Non-Treasury Securities” category is composed almost exclusively of agency MBS, much of which was purchased during the Fed’s first and third large-scale asset purchase programs (QE1 and QE3). As of June 25, 2014, the MBS portfolio totaled \$1.66 trillion and represented 40.5 percent of the securities held outright. As shown in Table 1, this MBS portfolio is made up of many small, heterogeneous, difficult-to-value securities. For example, about 8.3 percent of the portfolio was spread across 53,193 securities, each with a holding of less than \$10 million.

The “Treasury Securities” category experienced a large expansion during the second and third purchase programs (QE2 and QE3).⁸ As of June 25, 2014, the Fed’s nominal Treasury portfolio totaled \$2.28 trillion, represented 55.7 percent of the securities held outright, and was spread across 237 different securities. The long duration of these securities is also relevant for assessing balance sheet risk. From September 2011 through the end of 2012, the Fed conducted a Maturity Extension Program that sold Treasury securities with remaining maturities of three years or less and purchased a similar amount of Treasury securities with remaining maturities of six to thirty years. As a result of this policy, the Fed sold almost all of its short-term Treasury securities, so Treasuries with less than

⁷See Christensen et al. (2014) for details on this facility.

⁸We ignore the small amount of inflation-indexed, Treasury inflation protected securities (TIPS) discussed in Christensen and Gillan (2014). As of June 25, 2014, TIPS totaled \$97 billion in principal and another \$16 billion in accrued inflation compensation.

MBS distribution	#	Official account	
		Face value	Percent
All	68,557	1,663.90	100.00
\$1 bill. or more	197	630.17	37.87
\$100 mill.-\$1 bill.	2,170	510.27	30.67
\$50-\$100 mill.	2,168	150.76	9.06
\$10-\$50 mill.	10,829	234.64	14.10
\$5-\$10 mill.	9,300	65.08	3.91
\$1-\$5 mill.	27,005	66.25	3.98
\$1 mill. or less	16,888	6.73	0.40

Table 1: **Fed’s MBS Holdings.**

The table reports the composition of the Fed’s MBS holdings as of June 25, 2014. Note that the face values are reported in billions of dollars.

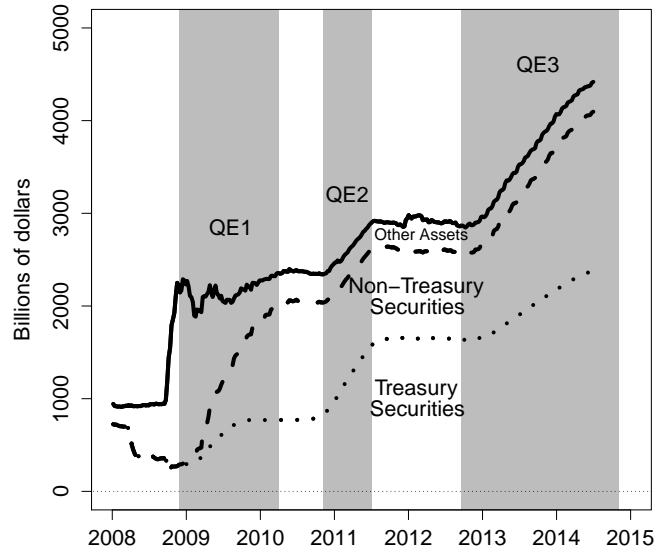


Figure 1: **Assets of the Federal Reserve System.**

Illustration of the total assets of the Federal Reserve System broken down into Treasury securities, non-Treasury securities, and other assets. The data are weekly covering the period from January 2, 2008, to June 25, 2014.

three years to maturity represented only 0.25 percent of its Treasury securities holdings at year-end 2012. Due to maturity reduction of the remaining part of the Treasury portfolio, this category’s share has increased to 13.28 percent by June 25, 2014, but still remains far below its historical level.

As is usual, this description of the Fed’s portfolio is based on the face value of the securities

	Face value	Amortized value	Fair value
All securities	4,105	4,299	4,390
All Treasuries	2,397	2,541	2,614
Nominal Treasuries	2,284	n.a.	n.a.
MBS	1,664	1,713	1,727

Table 2: **Fed’s Portfolio Value Based on Different Accounting Measures.**

The table reports the value of the Fed’s securities holdings as of the second quarter of 2014 according to three different accounting measures as explained in the text and for four categories: all securities, all Treasury securities, nominal Treasury securities, and MBS. Note that the face value measure is from the H.4.1 statistical release and dated June 25, 2014, while the amortized value and fair value measures are from the unaudited Federal Reserve Banks Combined Quarterly Financial Report for the second quarter of 2014 and dated June 30, 2014. All numbers are measured in billions of dollars.

held, as shown in the first column of Table 2 as of June, 2014.⁹ However, there are two other accounting methods also shown in Table 2 that can be used to measure the size of the Fed’s securities holdings: amortized historical cost and fair (or market) value. Historical cost values securities by their purchase prices; thus, it reflects any premiums or discounts paid relative to the securities’ face values. However, the Fed does not report the actual historical cost of securities purchases; instead, it reports an amortized historical cost, which adjusts the acquisition cost basis of the securities for amortization of premiums or accretion of discounts over the maturity of the bonds on a straight-line basis.¹⁰ The Fed has long argued that such (amortized) historical-cost accounting more accurately reflects the quantity of reserves in the banking system and is especially appropriate given the Fed’s macroeconomic policy objectives (not profit oriented) and the buy-and-hold securities strategy the Fed has traditionally followed. Thus, the Fed only registers capital gains and losses when securities are sold. In contrast, the fair value approach records the market value of the securities at a given point in time. The Fed reports the fair value of its holdings on a quarterly basis (though not for the nominal Treasuries category that we project), which allows calculation of unrealized capital gains and losses on its securities portfolio.¹¹ In our analysis, we project forward the fair value of the Fed’s portfolio and use as a benchmark for comparison the face value of the securities. Amortized historical cost would be an alternative benchmark, but it is infeasible to project that cost into the future. The Fed only reports the aggregate remaining unamortized premiums and discounts, and our security-by-security accounting of premiums and discounts would require information about both the purchase date and the purchase price of each unit of the securities held by the Fed in order to make

⁹Face values for aggregate holdings are reported in the H.4.1 release and for individual securities at http://www.newyorkfed.org/markets/soma/sysopen_accholdings.html.

¹⁰To be specific, U.S. Treasury and federal agency debt securities are amortized on a straight-line basis, while mortgage-backed securities are amortized on an effective-interest basis.

¹¹See http://www.federalreserve.gov/monetarypolicy/bst_fedfinancials.htm#quarterly.

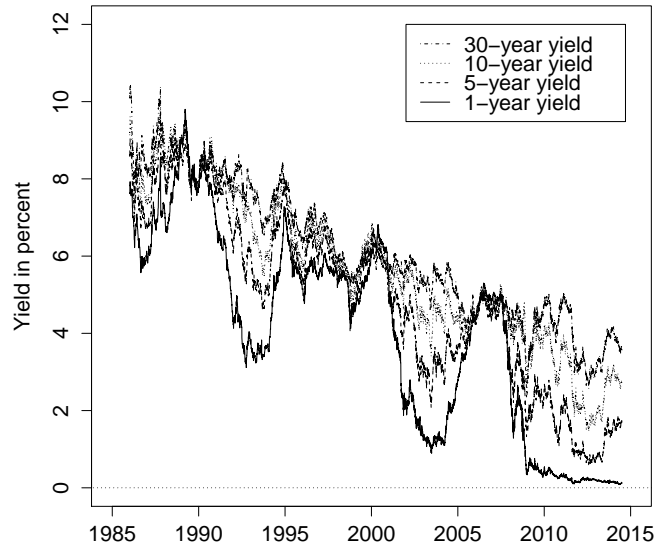


Figure 2: **Time Series of Treasury Bond Yields.**

Illustration of the daily Treasury zero-coupon bond yields covering the period from January 2, 1986, to June 25, 2014. The yields shown have maturities in 1 year, 5 years, 10 years, and 30 years, respectively.

the appropriate amortization. This complexity (especially as applied to the MBS) is a key reason why we use the face value of the Fed’s holdings as a straightforward benchmark for comparison to their estimated market values. In addition, if most salient balance sheet risk that the Fed faces from an enlarged portfolio is political in nature, then the valuation shortfall relative to face value is an obvious calculation that is easy for the public to understand. Finally, the relatively small difference between the face value and amortized cost approaches—on the order of 5% for all securities shown in Table 2—suggests that a comparison based on the face value of securities should not be much affected by the accounting treatment.

As noted in the introduction, this enlarged portfolio of longer-term securities greatly increases the Fed’s interest rate risk. To model the market value of the Fed’s Treasury and MBS holdings, we use the data set of zero-coupon Treasury yields described in Gürkaynak et al. (2007).¹² We use daily yields from January 2, 1986, to June 25, 2014, for the following 11 maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year.¹³ As shown in Figure

¹²For each business day, a zero-coupon yield curve is fitted to price a large pool of underlying off-the-run Treasury bonds. For up-to-date data, see the related website <http://www.federalreserve.gov/pubs/feds/2006/index.html>.

¹³The longest maturity Treasury yields are not available prior to November 25, 1985. Also, between October 2001 and February 2006, the U.S. Treasury temporarily halted its issuance of 30-year bonds, but this has only a minuscule effect on our estimation results, which are primarily determined by the yields with 10 years or less to maturity.

2, Treasury yields were at the lower end of their historical range towards the end of our sample, and short-term yields remained at the effective ZLB on nominal yields.

3 A Shadow-Rate Model of U.S. Treasury Yields

A key ingredient for our probability-based stress test is a data-generating process for the Treasury yield curve, and this section describes the term structure model we use for this purpose. Because short-term interest rates have been near zero since 2009, the proximity of the ZLB affects the pricing of Treasuries and induces a notable asymmetry into distributional forecasts of future yields. To respect the ZLB, we employ a shadow-rate term structure model.

3.1 The Option-Based Approach to the Shadow-Rate Model

The concept of a shadow interest rate as a modeling tool to account for the ZLB can be attributed to Black (1995). He noted that the observed nominal short rate will be nonnegative because currency is a readily available asset to investors that carries a nominal interest rate of zero. Therefore, the existence of currency sets a zero lower bound on yields. To account for this, Black postulated using a shadow short rate, s_t , which is unconstrained by the ZLB, as a modeling tool. The usual observed instantaneous risk-free rate, r_t , which is used for discounting cash flows when valuing securities, is then given by the greater of the shadow rate or zero:

$$r_t = \max\{0, s_t\}. \tag{1}$$

Accordingly, as s_t falls below zero, the observed r_t simply remains at the zero bound.

While Black (1995) described circumstances under which the zero bound on nominal yields might be relevant, he did not provide specifics for implementation. The small set of empirical research on shadow-rate models has relied on numerical methods for pricing.¹⁴ To overcome the computational burden of numerical-based estimation that limits the use of shadow-rate models, Krippner (2013) suggested an alternative option-based approach that makes shadow-rate models almost as easy to estimate as the standard model.¹⁵ To illustrate this approach, consider two bond-pricing situations: one without currency as an alternative asset, and the other that has a currency in circulation with a constant nominal value and no transaction costs. In the world without currency, the price of a shadow-rate zero-coupon bond, $P_t(\tau)$, may trade above par; that is, its risk-neutral expected

¹⁴For example, Kim and Singleton (2012) and Bomfim (2003) use finite-difference methods to calculate bond prices, while Ichiue and Ueno (2007) employ interest rate lattices.

¹⁵Wu and Xia (2014) derive a discrete-time version of the Krippner framework and implement a three-factor specification using U.S. Treasury data. In related research, Priebsch (2013) derives a second-order approximation to the Black (1995) shadow-rate model and estimates a three-factor version thereof, but it requires the calculation of a double integral in contrast to the single integral needed to fit the yield curve in the Krippner framework.

instantaneous return equals the risk-free shadow short rate, which may be negative. In contrast, in the world with currency, the price at time t for a zero-coupon bond that pays \$1 when it matures in τ years is given by $\underline{P}_t(\tau)$. This price will never rise above par, so nonnegative yields will never be observed.

Now consider the relationship between the two bond prices at time t for the shortest (say, overnight) maturity available, δ . In the presence of currency, investors can either buy the zero-coupon bond at price $P_t(\delta)$ and receive one unit of currency the following day or just hold the currency. As a consequence, this bond price, which would equal the shadow bond price, must be capped at 1:

$$\begin{aligned}\underline{P}_t(\delta) &= \min\{1, P_t(\delta)\} \\ &= P_t(\delta) - \max\{P_t(\delta) - 1, 0\}.\end{aligned}$$

That is, the availability of currency implies that the overnight claim has a value equal to the zero-coupon shadow bond price minus the value of a call option on the zero-coupon shadow bond with a strike price of 1. More generally, we can express the price of a bond in the presence of currency as the price of a shadow bond minus the call option on values of the bond above par:

$$\underline{P}_t(\tau) = P_t(\tau) - C_t^A(\tau, \tau; 1), \quad (2)$$

where $C_t^A(\tau, \tau; 1)$ is the value of an American call option at time t with maturity in τ years and strike price 1 written on the shadow bond maturing in τ years. In essence, in a world with currency, the bond investor has had to forgo any possible gain from the bond rising above par at any time prior to maturity.

Unfortunately, analytically valuing this American option is complicated by the difficulty in determining the early exercise premium. However, Krippner (2013) argues that there is an analytically close approximation based on tractable European options. Specifically, Krippner (2013) shows that the ZLB instantaneous forward rate, $\underline{f}_t(\tau)$, is

$$\underline{f}_t(\tau) = f_t(\tau) + z_t(\tau),$$

where $f_t(\tau)$ is the instantaneous forward rate on the shadow bond, which may go negative, while $z_t(\tau)$ is an add-on term given by

$$z_t(\tau) = \lim_{\delta \rightarrow 0} \left[\frac{\partial}{\partial \delta} \frac{C_t^E(\tau, \tau + \delta; 1)}{P_t(\tau + \delta)} \right],$$

where $C_t^E(\tau, \tau + \delta; 1)$ is the value of a European call option at time t with maturity $t + \tau$ and strike price

1 written on the shadow discount bond maturing at $t + \tau + \delta$. Thus, the observed yield-to-maturity is

$$\begin{aligned} \underline{y}_t(\tau) &= \frac{1}{\tau} \int_t^{t+\tau} \underline{f}_t(s) ds \\ &= \frac{1}{\tau} \int_t^{t+\tau} f_t(s) ds + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \rightarrow 0} \left[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s + \delta; 1)}{P_t(s + \delta)} \right] ds \\ &= y_t(\tau) + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \rightarrow 0} \left[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s + \delta; 1)}{P_t(s + \delta)} \right] ds. \end{aligned}$$

Hence, bond yields constrained at the ZLB can be viewed as the sum of the yield on the unconstrained shadow bond, denoted $y_t(\tau)$, which is modeled using standard tools, and an add-on correction term derived from the price formula for the option written on the shadow bond that provides an upward push to deliver the higher nonnegative yields actually observed.

As highlighted by Christensen and Rudebusch (2014), the Krippner (2013) framework should be viewed as an approximation to an arbitrage-free model. The size of the approximation error near the ZLB has been determined in Christensen and Rudebusch (2013, 2014) to be quite modest.¹⁶ Of course, away from the ZLB, with a negligible call option, the model will match the standard arbitrage-free term structure representation.

3.2 The Shadow-Rate AFNS Model

In theory, the option-based shadow-rate result is quite general and applies to any assumptions about the dynamics of the shadow-rate process. However, as implementation requires the calculation of the limit term under the integral, option-based shadow-rate models are limited practically to the Gaussian model class where option prices are available in analytical form. The arbitrage-free Nelson-Siegel (AFNS) representation developed by Christensen et al. (2011, henceforth CDR) is well suited for this extension.¹⁷ Its three factors correspond to the level, slope, and curvature factors commonly observed for Treasury yields and are denoted L_t , S_t , and C_t , respectively. The state vector is thus defined as $X_t = (L_t, S_t, C_t)$.¹⁸

In the shadow-rate AFNS model, the instantaneous risk-free rate is the nonnegative constrained

¹⁶Christensen and Rudebusch (2013, 2014) analyze how closely the option-based bond pricing from their estimated shadow-rate AFNS models matches an arbitrage-free bond pricing that is obtained from the same models using Black's (1995) approach based on Monte Carlo simulations. They consider bonds of maturities out to 10 years. We extended these simulation results to consider bond maturities of 30 years (needed for pricing the longest bonds in the Fed's portfolio). At the 30-year maturity, the approximation errors are understandably larger but still do not exceed 6 basis points, which are notably smaller than the model's fitted errors.

¹⁷For details of this derivation, see Christensen and Rudebusch (2014). For general discussion of the AFNS model, see Diebold and Rudebusch (2013)

¹⁸Note that this factor structure fits U.S. supervisory guidance on stress testing depository institution interest rate risk quite well. As summarized in Supervision and Regulation Letter SR 10-1 (2010), firms are instructed to examine large changes in the level, slope, and shape of the yield curve.

process of the shadow risk-free rate, which is defined as the sum of level and slope as in the original AFNS model class:

$$s_t = L_t + S_t, \quad r_t = \max\{0, s_t\}. \quad (3)$$

Also, the dynamics of the state variables used for pricing under the Q -measure remain as in the regular AFNS model:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \end{pmatrix}, \quad (4)$$

where Σ is the constant covariance (or volatility) matrix.¹⁹

Based on this specification of the Q -dynamics, the yield on the shadow discount bond maintains the popular Nelson and Siegel (1987) factor loading structure

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \quad (5)$$

where $A(\tau)/\tau$ is a maturity-dependent yield-adjustment term. The corresponding instantaneous shadow forward rate is given by

$$f_t(\tau) = -\frac{\partial}{\partial\tau} \ln P_t(\tau) = L_t + e^{-\lambda\tau} S_t + \lambda\tau e^{-\lambda\tau} C_t + A^f(\tau), \quad (6)$$

where the final term is another maturity-dependent yield-adjustment term.

Christensen and Rudebusch (2014) show that, within the shadow-rate AFNS model, the zero-coupon bond yields that observe the zero lower bound, denoted $\underline{y}_t(\tau)$, are readily calculated as

$$\underline{y}_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} \left[f_t(s) \Phi\left(\frac{f_t(s)}{\omega(s)}\right) + \omega(s) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left[\frac{f_t(s)}{\omega(s)}\right]^2\right) \right] ds, \quad (7)$$

where $\Phi(\cdot)$ is the cumulative probability function for the standard normal distribution, $f_t(\tau)$ is the shadow forward rate, and $\omega(\tau)$ takes the following simple form

$$\omega(\tau)^2 = \sigma_{11}^2 \tau + \sigma_{22}^2 \left(\frac{1 - e^{-2\lambda\tau}}{2\lambda}\right) + \sigma_{33}^2 \left(\frac{1 - e^{-2\lambda\tau}}{4\lambda} - \frac{1}{2} \tau e^{-2\lambda\tau} - \frac{1}{2} \lambda \tau^2 e^{-2\lambda\tau}\right),$$

when the volatility matrix Σ is assumed diagonal.

As in the affine AFNS model, the shadow-rate AFNS model is completed by specifying the price of risk using the essentially affine risk premium specification introduced by Duffee (2002), so the risk

¹⁹As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

premium Γ_t is defined by the measure change

$$dW_t^Q = dW_t^P + \Gamma_t dt,$$

with $\Gamma_t = \gamma^0 + \gamma^1 X_t$, $\gamma^0 \in \mathbf{R}^3$, and $\gamma^1 \in \mathbf{R}^{3 \times 3}$. Therefore, the real-world dynamics of the state variables can be expressed as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P. \quad (8)$$

In the unrestricted case, both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q -measure just as in the original AFNS model.

In state-space form, the model is characterized by a standard Gaussian affine transition equation (8) and a measurement equation (7), where measurement errors assumed *i.i.d.* with standard deviations unique to each yield maturity are added in the model estimation. Finally, we note that, due to the nonlinear measurement equation for the yields in the shadow-rate AFNS models, their estimation is based on the extended Kalman filter as described in Christensen and Rudebusch (2014).

3.3 The B-CR Model

In this subsection, we briefly describe the specific model underlying our analysis, which is the shadow-rate equivalent of the AFNS model preferred by Christensen and Rudebusch (2012). Using both in-sample and out-of-sample performance measures, the authors determined that the zero-value restrictions on the K^P matrix in the following dynamic system for the P -dynamics were empirically warranted; i.e.,

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left(\begin{pmatrix} 0 \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \end{pmatrix}, \quad (9)$$

where the covariance matrix Σ is assumed diagonal and constant. Throughout, we refer to the shadow-rate AFNS model given by equations (3), (4), and (9) as the B-CR model.

Note that the level factor is restricted to be an independent unit-root process under both probability measures. As discussed in Christensen and Rudebusch (2012), this restriction helps improve forecast performance independent of the specification of the remaining elements of K^P .²⁰ Second, we test the significance of the four parameter restrictions imposed on K^P in the model relative to

²⁰As described in Bauer et al. (2012), bias-corrected K^P estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias. Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta_1^P = 0$. In model estimation, factor nonstationarity is handled as described in Christensen and Rudebusch (2012).

K^P	$K_{\cdot,1}^P$	$K_{\cdot,2}^P$	$K_{\cdot,3}^P$	θ^P		Σ
$K_{1,\cdot}^P$	10^{-7}	0	0	0	σ_{11}	0.0043 (0.0000)
$K_{2,\cdot}^P$	0.4240 (0.1695)	0.3914 (0.1182)	-0.4799 (0.1059)	0.0386 (0.0355)	σ_{22}	0.0086 (0.0001)
$K_{3,\cdot}^P$	0	0	0.4249 (0.1661)	-0.0296 (0.0119)	σ_{33}	0.0264 (0.0002)

Table 3: **Parameter Estimates for the B-CR Model.**

The estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix are shown for the B-CR model. The estimated value of λ is 0.4868 (0.0010). The numbers in parentheses are the estimated parameter standard deviations. The data used in model estimation cover the period from January 2, 1986, to January 2, 2013.

the corresponding model with an unrestricted K^P matrix.²¹ The test results from rolling weekly re-estimations show that the four parameter restrictions have been either statistically insignificant or at most borderline significant since the early 2000s.²² Thus, the B-CR model is flexible enough to capture the relevant information in the data. Third, and more importantly, CR extend the analysis of Christensen and Rudebusch (2012) to encompass the most recent period with yields near the ZLB. They document that the B-CR model outperforms its standard AFNS model equivalent in that period in terms of forecasting future policy rates in real time and matching the compression in yield volatility. To summarize, the B-CR model has desirable dynamic properties in the current yield environment in addition to enforcing the ZLB.

4 Stress Testing the Fed’s Securities Holdings

In this section, we conduct probability-based stress tests of the Fed’s securities portfolio using model-based distributional forecasts of the yield curve over three-year forecast horizons at two different dates: the start of 2013 and mid-2014. The earlier date is one we used in an initial preliminary working paper version of this article, so the underlying projection can be considered a real-time forecast. It provides both interesting historical perspective (that is, what were the risks at the beginning of 2013) and an opportunity for model validation (by comparing subsequent realizations of the portfolio values to the real-time projections). The later date gives the most up-to-date reading on the risks posed by the Fed’s balance sheet. We conduct stress tests at these two dates on Fed’s holdings of Treasuries alone and on the Fed’s combined portfolio of Treasury and MBS securities.

Maturity in months	B-CR model		
	Mean	RMSE	$\hat{\sigma}_\varepsilon(\tau_i)$
3	-2.62	10.28	10.31
6	-0.04	0.17	0.68
12	2.80	6.63	6.64
24	2.94	5.21	5.29
36	0.01	0.74	1.46
60	-5.35	8.18	8.28
84	-6.61	11.05	11.08
120	-3.66	9.32	9.30
180	1.74	4.70	4.69
240	1.49	11.19	11.22
360	-10.23	33.69	33.73
Max log L	417,381.9		

Table 4: **Summary Statistics of the Fitted Errors.**

The mean and root mean squared fitted errors (RMSE) as well as the estimated yield error standard deviations for the B-CR model are shown. All numbers are measured in basis points. The data covers the period from January 2, 1986, to January 2, 2013.

4.1 Model Estimation Results and Yield Curve Fit

In our analysis below, we employ estimates of the B-CR model over three different samples to provide real-time yield curve forecasts. All three samples start in January 1986 and end on January 2, 2013, December 31, 2013, or June 25, 2014. The estimates from the middle sample endpoint will be used for the income projections in the next section. Here, we use the estimates corresponding to the first and third sample endpoints for analysis of the Fed’s balance sheet. The estimated model parameters using data through the start of 2013 are reported in Table 3. The summary statistics of the fit of the model to the yield curve are given in Table 4 and indicate a very good fit to the entire yield curve up to a maturity of about 20 years with some deterioration in the fit for the 30-year yield. Not surprisingly, parameter estimates and goodness of fit are very similar for the other two samples and they are not reported here.

The estimated B-CR model captures the important dynamics of the term structure in a parsimonious, theoretically consistent framework. However, to properly stress test the Fed’s portfolio, the model must also accurately price the wide variety of securities in the Fed’s portfolio. To examine the pricing performance of the B-CR model, we first calculate the model-implied value of the 241 Treasury securities that were in the Fed’s portfolio as of January 2, 2013. Specifically, on that date, we determine the remaining cash flow of each Treasury security in the Fed’s portfolio and discount

²¹That is, we test the hypotheses $\kappa_{12}^P = \kappa_{13}^P = \kappa_{31}^P = \kappa_{32}^P = 0$ jointly using a standard likelihood ratio test.

²²Figure 1 in Christensen and Rudebusch (2012) provide similar evidence for the corresponding AFNS model.

Maturity	#	Official account		Market value as of January 2, 2013			
		Face value	Percent	Bloomberg	Percent	B-CR model	Percent
All	241	1,580	100.00	1,847	100.00	1,848	100.00
3 years or less	93	4	0.25	4	0.24	4	0.24
4-6 years	72	630	39.87	699	37.82	697	37.70
7-10 years	38	577	36.53	663	35.87	659	35.64
11 or more years	38	369	23.35	482	26.07	488	26.42

Table 5: **Value of the Fed’s Treasury Securities Portfolio as of January 2, 2013.**

The table reports the distribution of the 241 Treasury securities in the Fed’s portfolio as of January 2, 2013 across maturity buckets, using three different valuation methods. The first method is the official account based on the bonds’ principal values. The second method is to calculate the market value based on bond prices from Bloomberg. The third method is to calculate the market value based on the estimated B-CR model as of January 2, 2013. The reported bond values are measured in billions of dollars.

that cash flow using the fitted yield curve from the B-CR model (estimated with the sample ending on January 2, 2013). This computation provides the net present value of each security as of that date, which is then multiplied by the notional amount of the Fed’s holdings of that security. Finally, we sum the net present values of all 241 Treasury securities in the Fed’s portfolio.²³ We emphasize that this exercise is a strong test of the B-CR model as it has to match raw bond price data that were not directly used in the model estimation.

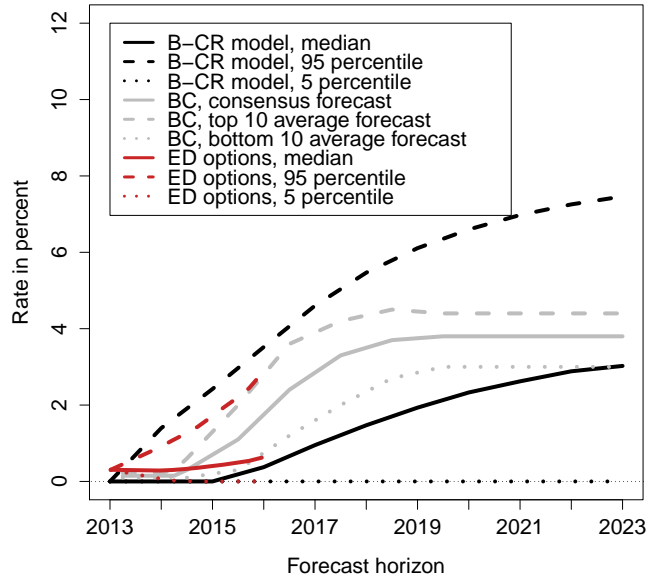
Table 5 reports the total value of the Fed’s Treasury securities portfolio and its distribution across maturity buckets. The first column shows the number of securities in each maturity bucket. The second and third columns report the official account based on the face value of the securities. For comparison, the following two columns reflect the market value of the portfolio based on bond prices from Bloomberg. The last two columns contain the market value of the portfolio implied by the B-CR model estimated with data up to January 2, 2013. The Bloomberg and B-CR estimated portfolio values are nearly identical. Similarly, Table 6 reports the same exercise for the 237 securities that were in the Fed’s Treasury portfolio as of June 25, 2014 (with the B-CR model estimated on data up through that day). Once again, both portfolio values are essentially the same. These small valuation differences support application of this model for our stress test.

4.2 Model-Based Short Rate Projections

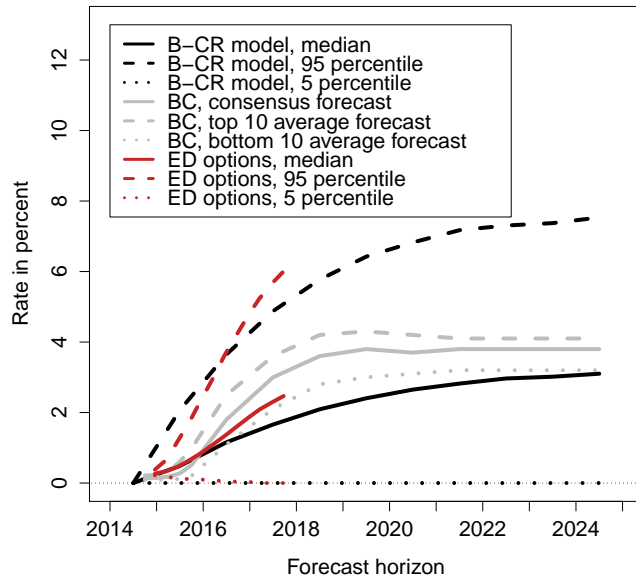
Treasury yield curve projections based on the estimated B-CR model allow us to assign probabilities to specific yield curve outcomes. As the yield function in equation (7) is nonlinear in the state variables, we use Monte Carlo simulations to generate the yield curve distributional projections.²⁴ Specifically, we simulate 10,000 sample paths of the state variables, denoted $\hat{X}_t = (\hat{L}_t, \hat{S}_t, \hat{C}_t)$, up

²³This same method is used throughout this section on other dates to value the Fed’s Treasury holdings.

²⁴We approximate the continuous-time process in equation (9) using the Euler approximation with a uniform Δt increment of 0.0001 measured in years, see Thompson (2008) for an example.



(a) January 2, 2013.



(b) June 25, 2014.

Figure 3: Short Rate Projections.

Panel (a) presents the median and [5%, 95%] range of the fed funds rate from the B-CR model’s simulated interest rate scenarios as of January 2, 2013. The graph also shows the consensus federal funds rate forecast as well as the averages of the top and bottom ten forecasts from the Blue Chip Financial Forecasts (BC) survey released on December 1, 2012. Finally, the graph shows the risk-neutral distribution of three-month LIBOR implied by options on eurodollar futures (ED) with maturities up to three years ahead as of January 2, 2013. Panel (b) shows the corresponding results as of June 25, 2014, with a comparison to the BC survey released on June 1, 2014.

Maturity	#	Official account		Market value as of June 25, 2014			
		Face value	Percent	Bloomberg	Percent	B-CR model	Percent
All	237	2,284	100.00	2,470	100.00	2,473	100.00
3 years or less	83	303	13.28	322	13.03	321	12.99
4-6 years	77	897	39.28	942	38.16	945	38.23
7-10 years	36	524	22.94	557	22.54	558	22.58
11 or more years	41	560	24.50	649	26.28	648	26.19

Table 6: **Value of the Fed’s Treasury Securities Portfolio as of June 25, 2014.**

The table reports the distribution of the 237 Treasury securities in the Fed’s portfolio as of June 25, 2014, across maturity buckets, using three different valuation methods. The first method is the official account based on the bonds’ principal values. The second method is to calculate the market value based on bond prices from Bloomberg. The third method is to calculate the market value based on the estimated B-CR model as of June 25, 2014. The reported bond values are measured in billions of dollars.

to three years ahead, each starting from the filtered values at the end of each of our samples. The simulated state variables are converted into a full yield curve at each quarter during the three-year simulation horizon.

To provide some insight into the variation in the resulting yield curve forecasts, Figure 3(a) compares forecasts of the short rate as of early 2013 from our model, from Blue Chip professional forecasters, and from eurodollar options data. For the model-based forecasts, the figure shows the median, 5th, and 95th percentile values for the simulated short rate for a 10-year forecast horizon.²⁵ For the short rate, the median simulated yield remains at the ZLB for the first two years of the forecast horizon and gradually rises to 3 percent at the 10-year horizon. The upper 95th percentile rises more rapidly and reaches 7 percent 10 years out, while the lower 5th percentile remains at the ZLB throughout. Long-term projections of the federal funds rate from the Blue Chip survey are also shown. The median (or consensus) forecast as well as the averages of the top and bottom 10 forecasts fit well within the range of our simulated projections. Finally, the distribution of future three-month LIBOR implied by options on eurodollar futures with maturities up to three years ahead provides another benchmark for comparison.²⁶ The median, 5th and 95th percentile values from these distributions are shown in Figure 3(a). The model-implied distributions easily encompass the option-based ones, which suggests that the model is able to account adequately for future likely outcomes of short rates over the projection horizon.

Figure 3(b) presents a similar short-rate forecast comparison as of June 25, 2014. The model’s median short-rate projections remain at roughly 3% for the longer horizon. As before, the Blue

²⁵Note that the lines do not represent yield curves from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

²⁶See Bauer (2014) for a description of these data. Note that these are risk-neutral distributions with no correction for risk premiums, while the model-implied distributions that reflect objective probabilities. Also, three-month LIBOR represents unsecured lending at term and has historically been above the short end of the Treasury curve, which accounts for the base difference between these series.

Chip projections are within the model’s range of outcomes, and the consensus projections are now closer to the model projections over the entire period. Similarly, the option-implied distribution for three-month LIBOR is in line with the other two projections, although its 95% percentile bound is a bit higher than the B-CR model after mid-2016.

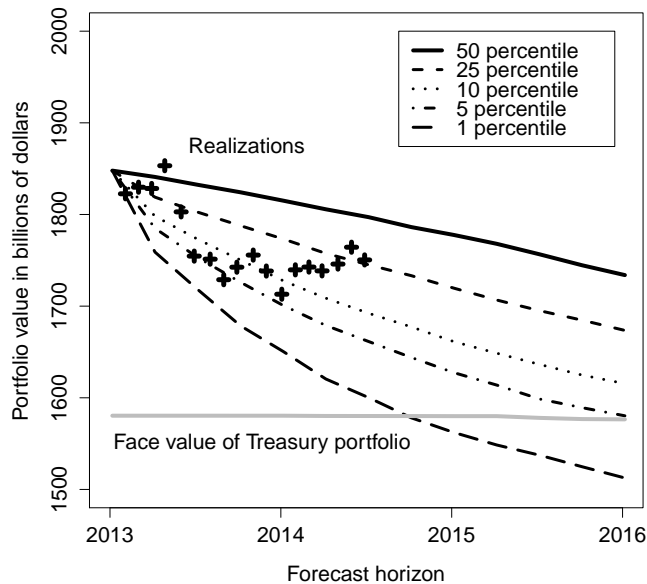
4.3 Stress Testing the Fed’s Treasury Portfolio

We now turn to our probability-based stress tests of the Fed’s Treasury portfolio. Specifically, we use the 10,000 simulated yield curves to price the Fed’s individual securities (which numbered 241 in January 2013 and 237 in June 2014) using the method described above. That is, for each quarter, we use each yield curve associated with one of the 10,000 values of the state variables to price the cash flows associated with each individual security in the Fed’s portfolio and use these prices to calculate the overall portfolio value at the end of that quarter. Taken together, these values provide a distributional forecast of the market value of the Fed’s Treasury portfolio.

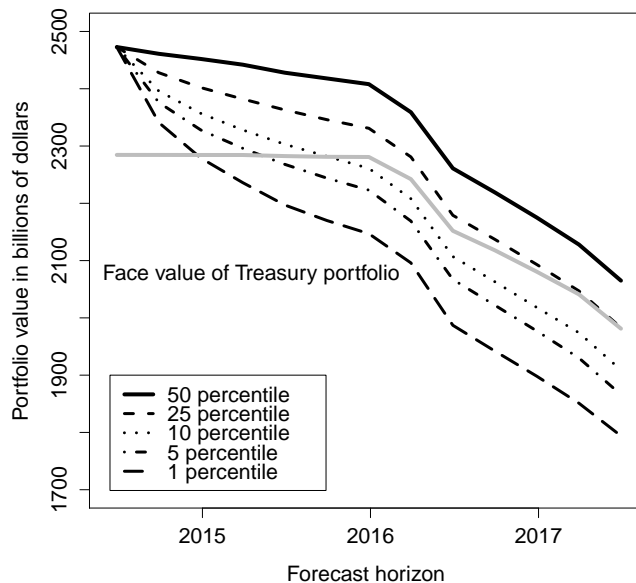
Figure 4(a) presents the lower percentiles and the median of the projected Treasury portfolio value over a three-year horizon as of January 2, 2013—our first real-time projection start date with this model. The median portfolio value declines as the forecast horizon increases as the general upward trend in short rate projections shown in Figure 3(a) leads to a decline in bond values. Still, the results show that the projected value of the Fed’s Treasury holdings was unlikely to fall below face value over the forecast horizon as of January 2, 2013, and at most, losses are expected to occur with only a 5 percent probability by the end of 2015. Also shown in Figure 4(a) as crosses are the actual subsequent monthly realizations of the value of the Treasury portfolio that the Fed was holding as of January 2, 2013.²⁷ According to the model, the sell-off in global financial markets during the summer of 2013 (the “taper tantrum”) represented about a 5% event from the perspective of what could reasonable be expected as of January 2, 2013. However, yields fell thereafter through the first half of 2014. As a consequence, by mid-2014, the net yield changes since early 2013 represented an outcome that the model expected to occur with about 25% chance. Although this is just one out-of-sample path realization, it suggests the model is not unrealistic in its interest rate projections.

Now we turn to an up-to-date stress test of the Fed’s Treasury portfolio. 4(b) provides a distributional forecast of the Fed’s Treasury portfolio as of June 25, 2014 based on an updated set of simulated yield curves. The decline in the portfolio face value over the scenario horizon is due to the maturing of the bonds with the shortest remaining terms. As before, the projected median portfolio value remains above the face value, but relative to the 2013 analysis, the much larger Treasury portfolio that the Fed was holding—due to continued QE3 purchases throughout 2013 and the first

²⁷The realizations plotted in the figure are derived by valuing the Treasury securities that remain outstanding at the end of each month using the fitted yield curve from an updated estimation of the B-CR model at the end of each month. The 18 crosses represent each month from January 2013 through June 2014.



(a) January 2, 2013.



(b) June 25, 2014.

Figure 4: Model-Based Projected Market Value of the Fed's Treasury Securities.

Panel (a) presents the percentiles ranging from 1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10,000$ Monte Carlo simulations of the B-CR model as described in the text. Also shown are the subsequent monthly realizations until June 25, 2014. Panel (b) shows the corresponding model-based simulation results as of June 25, 2014.

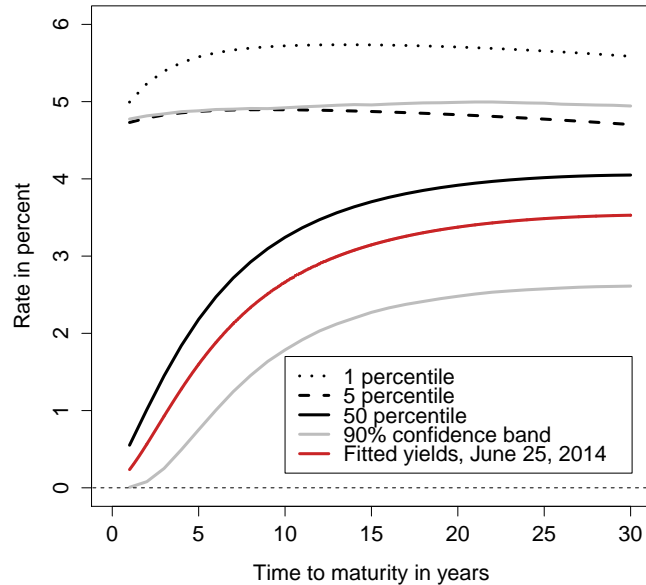


Figure 5: **Projected Yield Curves for Mid-2017.**

Illustration of the projected yield curves that produce the 1 percentile, 5 percentile, and 50 percentile (median) Treasury portfolio values by mid-2017 in Figure 4(b). Also shown is the [5%, 95%] range of yields for each maturity from the B-CR model’s simulated interest rate scenarios by mid-2017 in addition to the fitted yield curve from the B-CR model as of June 25, 2014.

half of 2014—increased the risk of large dollar losses. For this stress test, the probability that the portfolio value falls below face value rises from about 10% at the end of 2015 to 25% at the end of the scenario horizon in mid-2017.

To provide a sense of the projected yields associated with these lower tail outcomes for the portfolio values, Figure 5 shows the projected yield curves that produce the first, fifth, and median percentiles of portfolio values by mid-2017 in Figure 4(b). The figure shows that the projected yield curve that pushes the value of the Fed’s Treasury portfolio below its face value at the first percentile is associated with a federal funds rate above 5% percent and a corresponding dramatic increase in the entire yield curve from its level as of June 25, 2014, also shown in the figure. This simulated yield curve represents a very different state of monetary policy actions and corresponding economic conditions than currently expected. On the other hand, it is also clear that the simulated yield curve generating the median outcome reflects only a slight change from the yield curve as of June 25, 2014; i.e., the simulated curve matches the compression in yield volatilities near the ZLB, which reduces the magnitude of the yield curve changes and the variation in the model’s projected market valuations. Finally, to put the three shown yield curves in the context of the entire distribution of

projected yield curves, the figure also shows the band between the fifth and 95th percentile values for the B-CR model’s yield curve simulations by mid-2017 across all maturities. Clearly, the tail outcomes for the portfolio values by mid-2017 are associated with short- and medium-term yields outside of the 90 percent band of simulated outcomes, even though these curves move with the band at longer maturities.

4.4 Inclusion of MBS Holdings in the Stress Testing Analysis

As noted in Section 2, agency MBS constitute a large fraction of the Fed’s portfolio holdings—roughly 40% as of June 25, 2014. Thus, stress testing the Fed’s Treasury securities alone is a limited exercise. However, using our methodology to value the large number of different MBS held by the Fed (almost 70,000 individual securities as of mid-2014)—and especially taking into account their prepayment option—is a Herculean task. Instead, in this section, we use a simplified approach that converts the expected duration of an MBS portfolio into ten-year Treasury equivalents.²⁸ This dollar-weighted duration measure provides an estimate of the amount of ten-year Treasury bonds that an investor would need to hold in order to be exposed to the same degree of interest rate risk. This approach allows us to simply augment the Treasury portfolio with a large amount of one additional 10-year bond and use our same probability-based stress test methodology. Of course, the MBS prepayment options are not completely well priced in this approach, but they likely represent a relatively small fraction of potential losses in the current rising rate environment.

As shown in Figure 6, the Fed’s MBS holdings as of January 2, 2013, represented \$271.2 billion in ten-year Treasury equivalents. According to the B-CR model, the ten-year par-coupon yield was 190.6 basis points as of January 2, 2013. Thus, to account for the MBS holdings, we add to the Fed’s Treasury portfolio on that date a \$271.2 billion ten-year Treasury with a coupon of 1.906 percent. Doing so, brings the total face value of the augmented portfolio to \$1.85 trillion, while its market value is put at \$2.12 trillion.

Figure 7(a) shows the stress test results as of January 2, 2013 using the B-CR model to simulate the market value of this augmented portfolio of Treasury holdings. As compared to the Treasuries-only stress test in Figure 4(a), the augmented portfolio has a higher probability of falling below its face value by the end of 2015. However, this potential loss remains relatively small with the projected portfolio values dipping below face value with a less than 10% probability by late 2015. Again, subsequent monthly realizations, denoted as crosses, fall very close to the central tendency of the projection.

The interest rate risk of the joint Treasury and MBS portfolio does increase in an updated stress test as of June 25, 2014. By that date, the Fed had increased its notional holdings of Treasuries and MBS by \$704 billion and \$737 billion, respectively. In the absence of an officially reported number,

²⁸See Greenwood et al. (2014) and Li and Wei (2013) for other applications involving ten-year Treasury equivalents.

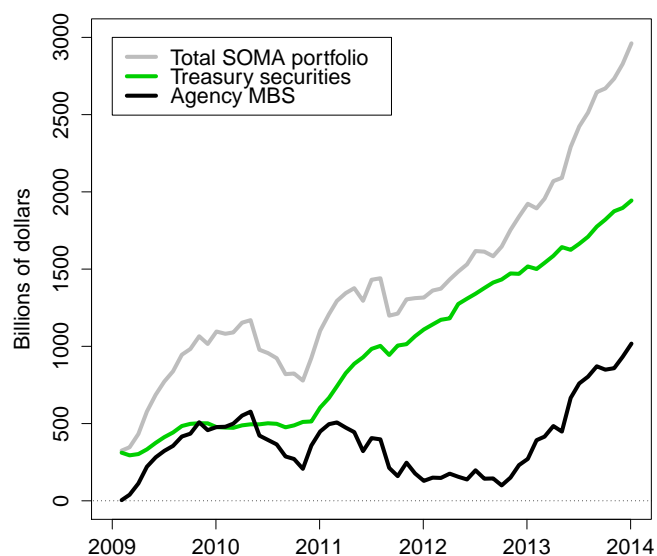
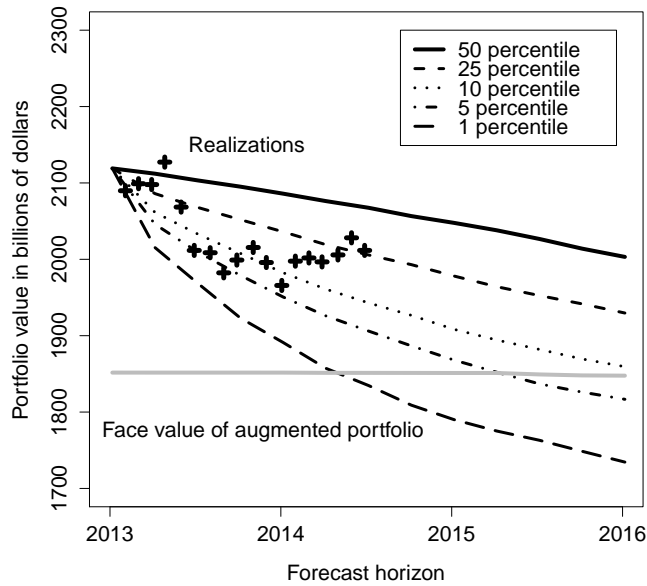


Figure 6: **SOMA Portfolio in Ten-Year Treasury Equivalents.**

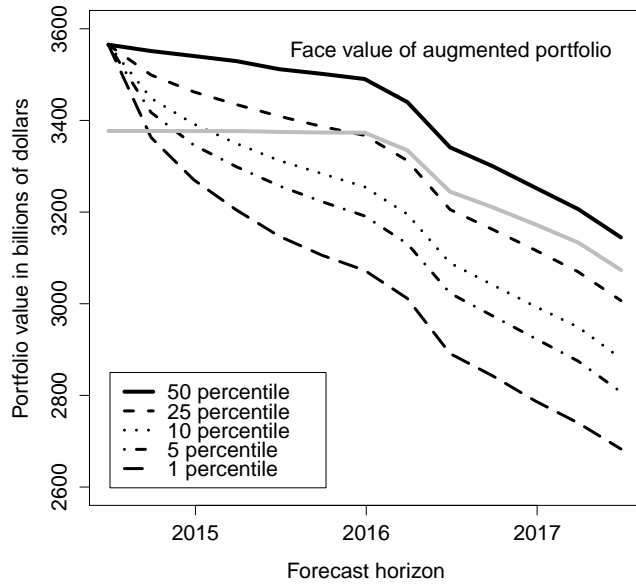
Illustration of the ten-year Treasury equivalents of the entire SOMA portfolio as well as those of the Fed’s Treasury securities and agency MBS holdings. The data is monthly, cover the period from January 2009 to December 2013, and is taken from Chart 9 in the 2013 Annual SOMA Report.

we must determine the ten-year Treasury bond equivalent of the MBS holdings at that date. Since yields have not changed much, on net, since the end of 2013, we make the simplifying assumption that the average duration of the MBS portfolio has also not changed since the end of 2013, when it was reported to be 5.8 years.²⁹ According to the B-CR model, as of June 25, 2014, the duration of a ten-year par-coupon Treasury bond was 8.834 years, while its coupon was 2.585 percent. Hence, as the Fed’s MBS holdings totalled \$1,663.9 billion in notional, the interest rate sensitivity of the MBS portfolio can be approximated by replacing it with $1,663.9 \times 5.8 / 8.8336 = \$1,092.5$ billion of ten-year Treasury bonds with a coupon of 2.585 percent. The results of this stress test are shown in Figure 7(b). The continued purchase of MBS during 2013 and the first half of 2014 clearly increased the portfolio’s interest rate risk. While the median projected value remains above the portfolio face value up through the second quarter of 2017, about 30% of the simulated yield curve paths cause the projected value to fall below face value. The value of the Fed’s securities portfolio has become more sensitive to yield outcomes under the QE purchase program.

²⁹See Chart 7 on page 6 of the Federal Reserve Bank of New York’s report on *Domestic Open Market Operations During 2013*.



(a) January 2, 2013.



(b) June 25, 2014.

Figure 7: Model-Based Projected Market Value of the Fed’s MBS Augmented Treasury Portfolio.

Panel (a) shows the percentiles ranging from 1% to 50% in the distribution of the market value of the Fed’s MBS augmented Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10,000$ Monte Carlo simulations of the B-CR model as of January 2, 2013. Also shown are the subsequent monthly realizations until June 25, 2014. Panel (b) shows the corresponding updated projections as of June 25, 2014, as described in the text.

Variable	GHHM assumptions (p. 64)	Our assumptions
1. Asset purchases	Continue at current pace through December 2013, then slow to maintenance levels and end in 2014.	Purchases through 2014 match Primary Dealer Survey as of June 2014 and end in 2014.
2. Asset sales	No Treasury sales. MBS sales start in late 2015 and are completed in 2019.	No Treasury or MBS sales.
3. MBS prepayment	Calibrated to current market expectations.	Same.
4. Liabilities	Currency grows at 7% annual rate (2 percentage points above Blue Chip forecast for nominal GDP growth per historical experience); required reserves grow at 4% annual rate.	Same.
5. Interest rates	Driven by Blue Chip forecasts.	B-CR model projections.
6. Fed capital	Grows at 10% annual rate per historical average.	Same.
7. Operating expenses	Grow on historical trend.	Same.
8. Principal reinvestment	Reinvestment in Treasuries through 2014 in established maturity ranges.	Reinvestment in Treasuries through 2015 in established maturity ranges.

Table 7: **Assumptions Underlying Balance Sheet and Income Projections.**

5 Stress Testing the Fed’s Net Income

Our probability-based stress testing approach can also be applied to questions regarding the Fed’s income risk; that is, the sensitivity of its net income to alternative interest rate scenarios. The Fed’s interest income is relatively fixed by the set coupon income payments from its securities holdings (although MBS payments are sensitive to prepayment risk). However, the Fed’s interest expenses will vary directly with the level of short-term interest rates through payment of interest on bank reserves. The primary concern is that certain interest rate outcomes could lead the Fed’s net income to decline sufficiently that it would halt remittances (i.e., payments of excess net interest income beyond operational expenses) to the Treasury Department. For example, Carpenter et al. (2013) and GHHM consider whether the Fed’s remittances would remain positive under several specific interest rate scenarios. In this section, we address this policy question using our model-based approach to generating yield curve distributions in conjunction with the accounting framework of GHHM, which encompasses the projection horizon of 2013 through 2020 at an annual frequency.³⁰

As detailed in Table 7, we adopt the GHHM assumptions regarding future MBS prepayment, liability growth, capital accretion, and operating expenses. However, we do update and alter several

³⁰We greatly appreciate the authors’ sharing of their model with us for the purposes of this analysis. In the presentation of our stress testing results, we rely on projected values over the period from 2014 through 2020 as of year-end 2013 generated from a separate set of yield curve simulations.

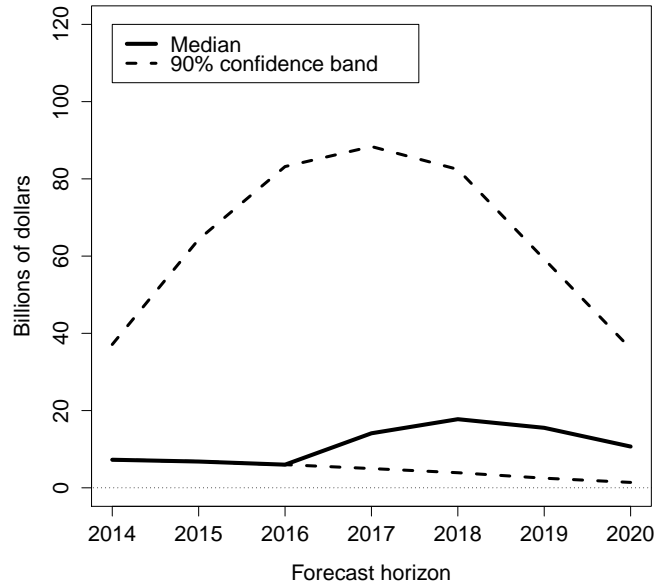


Figure 8: **Projections of Annual Fed Interest Expenses.**

Illustration of the median and the 5th and 95th percentiles of the projected interest expenses based on the CLR baseline scenario combined with $N = 10,000$ Monte Carlo simulations of the B-CR model.

other assumptions. First, we link the expected path of Federal Reserve asset purchases directly to the publicly announced results of the New York Fed Primary Dealer Survey as of June 2014, which has purchases ending in October 2014.³¹ Second, we assume that the Fed does not sell any securities through 2020 (which is consistent with the announced plans in the FOMC’s “Policy Normalization Principles and Plans” released on September 17, 2014). Third, we extend the assumed reinvestment path for bond principal payments into Treasuries of various maturities up through the end of 2015. A final modification is that we set the path for the interest on excess reserves (IOER) rate (i.e., the rate the Fed pays on the reserves that banks hold and is its main interest expense) equal to the overnight rate as implied by our yield curve simulations.³² Given the variation in our simulated short rates, this results in important variation in the Fed’s interest expenses going forward. Indeed, as shown in Figure 8, a 90 percent confidence interval for the Fed’s annual interest expenses ranges at its widest point in 2017 ranges from \$5 billion to almost \$90 billion.

Figures 9 and 10 present the key results of our simulation-based approach using this set of assumptions. Figure 9 presents the projected range of the Fed’s remittances to the Treasury over the

³¹The survey is available at: http://www.newyorkfed.org/markets/primarydealer_survey_questions.html.

³²In this exercise, the overnight rate is approximated by an instantaneous short rate given by $r_t = \max\{0.25\%, s_t\}$; i.e., we impose a minimum of 25 basis points for the IOER rate consistent with current practice.

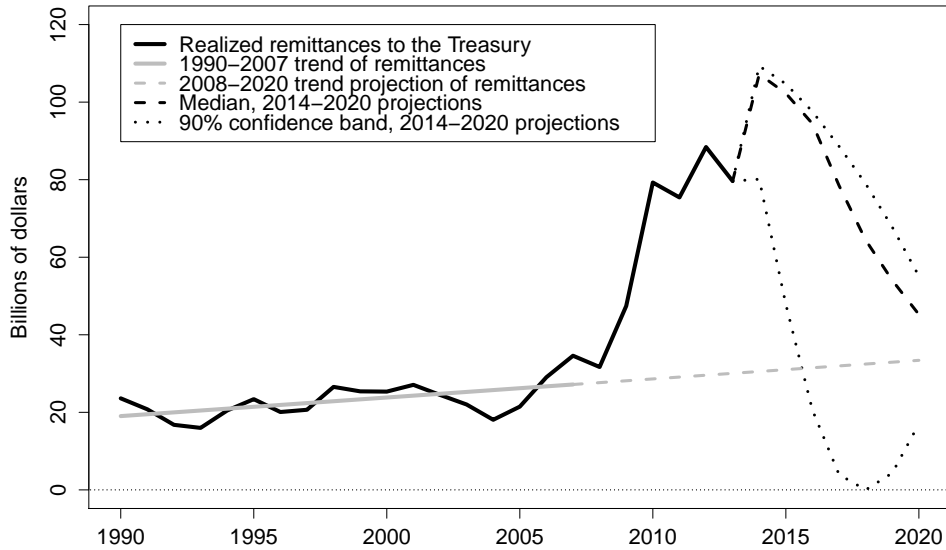


Figure 9: **Actual and Projected Remittances to the U.S. Treasury.**

The solid black line shows the realized remittances to the U.S. Treasury over the period from 1990 to 2013. The solid gray line indicates the simple linear trend of the remittances from 1990 to 2007, while the dashed gray line shows the extrapolation of that trend to the 2008-2020 period. Also shown are the median (dashed black line) and the 5th and 95th percentiles (dotted black lines) of the projected payments to the U.S. Treasury over the 2014-2020 period based on the CLR baseline scenario combined with $N = 10,000$ Monte Carlo simulations of the B-CR model.

period from 2014 to 2020. The median value (represented as the black, dashed line) declines over the period, both as interest income declines based on maturing securities and as interest rate expenses rise due to interest rates rising from their current values near the ZLB. However, by the end of the forecast horizon, these declines are back in line with linearly projected remittances (the gray dashed line) based on data from 1990 to 2007. Remittances are only projected to stop with a 5% probability in 2018 before resuming again.

Figure 10 shows the corresponding projected range of cumulative negative remittances or, in accounting terms, the deferred asset.³³ As before, our results imply zero remittances and a need for the deferred assets for the lower fifth percentile of outcomes. In these cases, the low point is only in 2018, and the deferred asset does not exceed \$5 billion. The figure also presents the lower first percentile of simulated outcomes, which consists of a much longer period of paused remittances and

³³The Fed's policy is to remit all net income after expenses, dividends, and additions to capital to the U.S. Treasury. If earnings are insufficient to cover these costs, the Fed creates new reserves against a deferred asset, which represents a claim on future earnings and remittances to the Treasury.

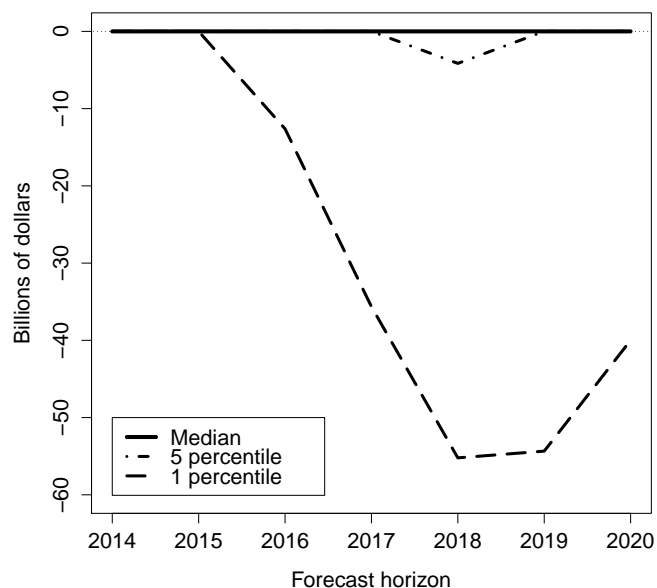


Figure 10: **Projected Cumulative Negative Remittances (i.e., the Deferred Asset).**

Illustration of the median and the 5th percentile of projections of the Fed’s deferred asset based on the CLR baseline scenario combined with $N = 10,000$ Monte Carlo simulations of the B-CR model.

a much greater magnitude of deferred assets. However, as of year-end 2013, our probabilistic results suggest that the Federal Reserve is unlikely to stop earning net interest income and making Treasury remittances over the next seven years under reasonable assumptions.

To provide further insight on the use of deferred assets, Figure 11 shows our simulated probability distribution of the maximum deferred asset amount over the forecast horizon up through 2020. The results are heavily left-skewed with more than 92 percent of the probability mass at zero—that is, no cessation of remittances. For the rest of the distribution, the probability of observing a maximum deferred asset of less than \$10 billion is 2.4 percent, and the probability of a maximum value greater than \$10 billion is just shy of 5 percent.

As a final exercise, we try to assess the cumulative remittances to the Treasury from the Fed’s expansion of its balance sheet starting in 2008. Figure 12 shows our simulated probability distribution for the cumulative remittances from 2008 to 2020 net of the projected linear trend based on remittances from 1990 to 2007. The trend totals nearly \$400 billion in cumulative remittances during the 2008-2020 period—about \$30 billion per year. This amount, which is a benchmark for the absence of any QE programs or balance sheet expansion by the Fed, is then deducted from the sum of projected remittances (including the known 2008-2013 remittances of \$402 billion) in each of the 10,000 simulation runs to produce the distribution.

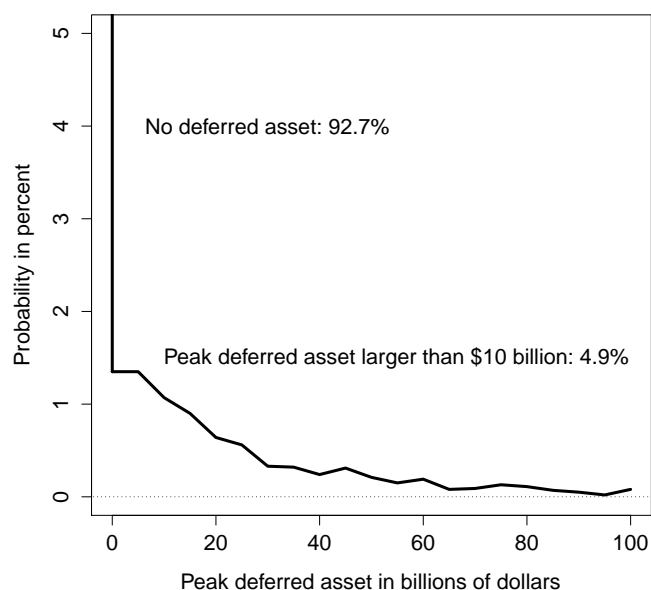


Figure 11: **Simulated Distribution of Maximum Deferred Asset over Forecast Horizon.** The graph depicts the simulated probability distribution function for the maximum deferred asset over the seven-year forecast horizon. More than 92 percent of the values are zero.

There are two things to note in the figure. First, with near certainty, the expansion of the Fed’s balance sheet is likely to generate hundreds of billions of dollars in excess remittances to the U.S. Treasury over the entire 2008-2020 period. Thus, the extraordinary monetary policy initiatives most likely will have provided a direct financial benefit to the Treasury, in addition to any indirect benefits from improved economic outcomes noted in the introduction. Second, the chance that these policies will ultimately produce below-trend net remittances is near zero according to this exercise, as the smallest outcome across the 10,000 simulations is \$51 billion in remittances above trend.

6 Conclusion

Financial stress tests, including those that have examined the Fed’s financial position, usually only consider a small number of hand-picked scenarios. The selection of a few ad hoc scenarios introduces a substantial degree of arbitrariness into the stress test and makes it difficult to judge the plausibility of the results. Our methodological contribution is to introduce a probabilistic structure into a stress test of the Fed’s balance sheet and income risks. We argue that attaching likelihoods to adverse outcomes based on interest rate fluctuations is a crucially important addition to the policy debate.

In terms of substantive results, we use a model-based approach to generate Treasury yield curve

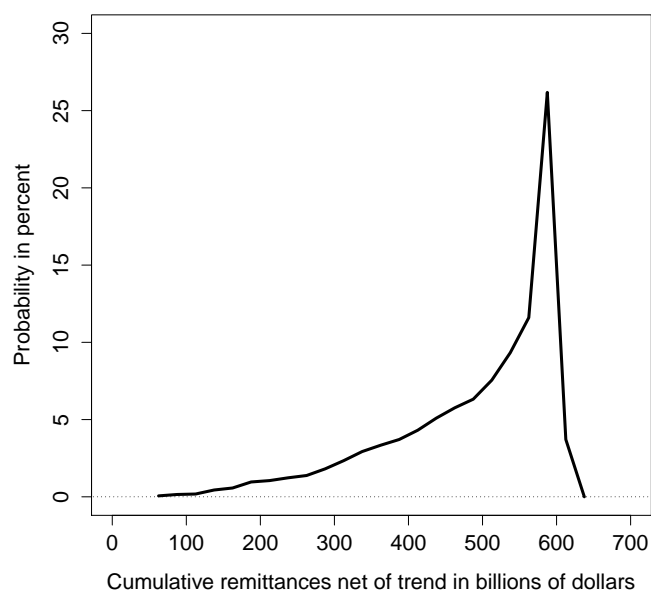


Figure 12: **Simulated Distribution of Cumulative Remittances Net of Trend.**

The graph depicts the simulated probability distribution function for the cumulative remittances by the Fed to the U.S. Treasury from 2008 to 2020 net of the projected trend of remittances from the 1990-2007 period.

projections. Our stress test indicates that in all likelihood the potential losses to the Fed’s Treasury and agency MBS holdings over the next several years will be quite moderate. We also generate more comprehensive projections of the Fed’s future income and find a small chance of a temporary halt in the remittances to the Treasury; furthermore, the magnitude of the deferred asset created during this period likely would be modest. In addition, cumulative remittances to the Treasury over the period from 2008 to 2020 are almost surely to be greater than in a counterfactual scenario in which remittances were a linear projection of what they were from 1990 through 2007. In summary, our probability-based scenario analysis provides generally reassuring results regarding questions related to the financial costs of the Fed’s balance sheet policy.

Of course, our analysis leaves room for further research. For example, the analysis relies on historical data to estimate forecast distributions, and these may not be completely appropriate for assessing all future circumstances. Consideration of a distribution with fatter tails may be warranted. Also, we do not consider distributional projections of all possible relevant conditioning factors—such as the inflation path or possible asset sales by the Fed. Finally, as noted earlier, unlike for a stress test of a commercial enterprise, it is the political consequences of the financial costs that are of key concern to the Fed. It may be that in those states of the world in which the Fed bears large losses owing to higher long rates, economic growth is also likely to be strong, which may mitigate the political risks.

Alternatively, in scenarios with very small Fed remittances to the Treasury because of a high IOER rate, substantial payments of interest on reserves would be paid to large commercial banks, likely boosting political risk. Further research could expand and refine our probabilistic structure in these directions.

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