

Collateral Scarcity, Inflation, and the Policy Trap: A New Monetarist Perspective

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Abstract

We construct a model that captures New Monetarist ideas in order to explain post-Great Recession observations on inflation, nominal interest rates, and real interest rates. When finance constraints bind, the model can deliver low real interest rates and positive rates of inflation at the zero lower bound. Optimal monetary policy in the face of a financial crisis shock implies a positive nominal interest rate. The model reveals some novel perils of Taylor rules.

1 Introduction

In the United States, short-term nominal interest rates have been close to zero since late 2008. Thus, the zero lower bound has been a reality for the Fed for close to six years. In standard monetary models, a central bank policy rule that keeps the nominal interest rate at zero forever is a Friedman rule (Friedman 1969). Typically, however, the Friedman rule is associated with deflation. For example, in versions of the neoclassical growth model with no aggregate uncertainty and a role for money, the Friedman rule will imply deflation at the rate of time preference. But, at least since early 2010, the inflation rate in the U.S. has varied roughly between 1% and 3% on a year-over-year basis, as shown Figure 1. The flip side of those two observations – near-zero short-term nominal interest rates and positive inflation – is that real interest rates have been persistently low since the Great Recession. Figure 2 shows the difference between the fed funds rate and the PCE inflation rate, which has been at post-1980 lows, and usually negative, since the Great Recession. The five-year TIPS yield in Figure 3 tells a similar story.

What are we to make of these observations, and what are the implications for monetary policy? A typical approach to explaining the persistence of low

real interest rates in New Keynesian (NK) models (e.g. Werning 2011) is to introduce a preference shock – an increase in the representative agent’s discount factor – which lowers the “natural real rate of interest.” Then, it can be optimal for a central banker correcting sticky price frictions to set the nominal interest rate at the zero lower bound. The zero lower bound then represents a constraint on policy, and NK models are thus used to argue that the real interest rate is too high relative to what is optimal. The NK approach is then to find policy remedies in central bank forward guidance (Werning 2011, Woodford 2012) or increases in government spending (Eggertsson and Krugman 2012). But baseline NK models have difficulty in explaining recent inflation experience in the United States. A cornerstone of NK models is a Phillips curve, which posits a negative relationship between the inflation rate and the output gap – the difference between output if prices were flexible, and actual output. Given the size of perceived output gaps after the Great Recession, inflation appears to have been inexplicably high. Further, a Phillips curve is hard to detect post-Great Recession, for example in the data depicted in Figure 4, where we plot the PCE inflation rate against the difference between the unemployment rate and the Congressional Budget Office’s measure of the “natural unemployment rate.” A Phillips curve would show up in the figure as a negative correlation.

Old Monetarism does not fare any better than NK theory in helping us understand post-Great Recession experience in the U.S. The behavior of base money is clearly absurdly out of line with price level behavior, as shown in Figure 5. Further, monetary observations seem hard to reconcile with stable money demand functions. For example, in the post-2009Q1 period, with the short-term nominal interest rate essentially constant and near zero, Figure 6 shows that the velocity of M1 has fallen by almost one third.

To address these issues, we build on ideas from the *New Monetarist* literature, principally Williamson (2012, 2014a, 2014b). New Monetarism is surveyed extensively in Williamson and Wright (2010a, 2010b). The basic idea is that explicit modeling of financial structure and monetary arrangements is a fruitful approach to understanding the role of monetary policy and the propagation of financial disturbances, among other issues. In this paper we take a somewhat different approach to New Monetarist ideas by building on cash-in-advance models.¹ This might seem to be at odds with the general New Monetarist approach, but we think not, for reasons discussed in the paper. We also think that what we do here will make the ideas accessible to a wider audience.

The model we construct is highly tractable, and has the property that exchange is intermediated by an array of assets. In the model, economic agents are arranged in large households – a device in the spirit of Lucas (1990) or Shi (1997), for example. Households trade in asset markets and goods markets, and can make transactions using money, government bonds, and credit, though money can be used in a wider array of transactions than can other assets. The model is intended to capture how assets are intermediated and used

¹Bonsal and Coleman (1996) is a related model that permits government bonds to be used in transactions, though Bonsal and Coleman focus on asset pricing implications in an endowment economy, and are not concerned with monetary policy.

by the banking system for transactions purposes, though the large household construct allows us to abstract from the details of banking arrangements, which are considered explicitly in Williamson (2012, 2014a, 2014b).

If the asset constraints of households in the model do not bind, the model behaves in a conventional way, i.e. much like Lucas and Stokey (1987), in terms of how assets are priced, and the relationship between real and nominal interest rates. As well, if asset constraints do not bind, even at the zero lower bound on the nominal interest rate, then a Friedman rule for monetary policy is optimal. However, if finance constraints bind, the behavior of the model is entirely different. The binding asset market constraint imparts a liquidity premium to government bonds, and bonds bear a low real return to reflect that. In general equilibrium, the asset market constraint binds because government bonds are in short supply. This occurs given the fiscal policy rule in place. We assume that the fiscal authority acts to set the real value of the consolidated government debt exogenously, and then the job of the central bank is to determine the composition of that consolidated government debt, through open market operations.

When the asset market constraint binds, lower nominal interest rates will reduce output, and consumption, and will reduce welfare when the nominal interest rate is close to zero. Thus, a binding asset market constraint implies that the zero lower bound is not optimal. If we consider a financial shock (which we can interpret as a financial crisis shock), this can make the asset market constraint bind, or will tighten the asset market constraint if it binds in the absence of the shock. A financial crisis shock will then lower the real interest rate, but the optimal monetary policy response (given fiscal policy) is *not* to go to the zero lower bound, in contrast to what occurs in NK models.

If the asset market constraint binds, at the zero lower bound the inflation rate is higher the tighter is the asset market constraint. Thus, for a sufficiently tight asset market constraint, the inflation rate need not be negative at the zero lower bound, and the inflation rate will fluctuate the household's borrowing constraint.

We examine the properties of Taylor rules in this environment. In the model, the Taylor rule is certainly not optimal, so the idea is to understand how a misguided policy maker armed with a Taylor rule might go wrong – or possibly not so wrong – in pursuing a Taylor rule policy. We assume that the central banker cares only about inflation, and consider rules that account for long-run real interest rates in different ways. Also, the rules we consider allow us to consider how the Taylor principle – a more-than-one-for-one response of the nominal interest rate to an increase in the inflation rate – might matter.

Though our model is somewhat different from the examples considered by Benhabib et al. (2001), we obtain results with a similar flavor in the case in which the asset market constraint does not bind. In particular, there exist two steady state equilibria, a liquidity trap (zero-lower-bound) equilibrium in which the central bank forever undershoots its inflation target, and another with a strictly positive nominal interest rate in which the inflation target is achieved. Dynamics depend on how the central banker accounts for the long-run real

interest rate in the Taylor rule. If the central banker accounts for endogeneity in the real interest rate, then under the Taylor principle there exists a continuum of equilibria (much as in Benhabib et al. 2001) that converge to the liquidity trap equilibrium, and the other steady state is unstable in the usual sense. When the Taylor principle does not hold, there exists a continuum of equilibria that converge to the steady state in which the central bank achieves its inflation target. So, things can go wrong with the Taylor rule when the finance constraint does not bind, in ways that are by now well understood, thanks to Benhabib et al. (2001).

When the asset market constraint binds, the Taylor rule can go wrong in other ways, but it may not go wrong in the same ways as with a nonbinding constraint. A lot depends on what the central banker understands about the determinants of the real interest rate. It is possible that the liquidity trap steady state does not exist. This occurs if the asset market constraint is sufficiently tight at the zero lower bound which produces a high enough inflation rate to push the central banker off the zero lower bound. However, if the central banker thinks – incorrectly – that the long-run real interest rate is equal to the rate of time preference, then it is impossible for the central banker to hit his or her inflation target if he or she adheres to the Taylor rule. A Taylor-rule central banker who understands the determinants of the real interest rate correctly will face the same problems as in Benhabib et al. In particular, under the Taylor principle there exists a continuum of equilibria that converge in finite time to the liquidity trap equilibrium in which the central banker undershoots his or her inflation target. If the Taylor principle does not hold, then the intended steady state is stable in the usual sense.

This basic model does not have short run liquidity effects of monetary policy. Such effects are present in NK models, and also in a class of segmented markets models studied by Lucas (1990), Fuerst (1992), Alvarez and Atkeson (1997), Alvarez, Lucas and Weber (2001), Alvarez, Atkeson, and Kehoe (2002), and Williamson (2006, 2008). A short run liquidity effect occurs when a reduction in the short-term nominal interest rate by the central bank leads to an increase in the inflation rate, and possibly to a reduction in the real interest rate and an increase in real economic activity.

Particularly since the Taylor rule presumes the existence of short run liquidity effects, it seems useful to adapt our model to include them, and that is what we do in the second part of the paper. The resulting model resembles most closely Alvarez, Lucas, and Weber (2001), but with a more elaborate asset market structure. There are two groups of households in the model, trader households, and non-trader households. Trader households make purchases of goods using credit and government bonds, and they trade on financial markets, while non-traders live in a cash-only world. For convenience we drop production, and consider an economy in which households have fixed endowments each period.

As in the basic model, asset market constraints may or may not bind in equilibrium, but this applies only to trader households; non-traders are constrained in a simple cash-in-advance fashion. We first look at what happens when the

asset market constraint does not bind. Supposing that the central banker experiments in a random fashion, he or she could see a negative correlation between the nominal interest rate and inflation, and a positive correlation between the real interest rate and the nominal interest rate, because of the short-run liquidity effect. However, as the Fisher relation holds in this model in the long run, the central banker can increase the inflation rate permanently only if the nominal interest rate increases. Indeed, if the nominal interest rate increases once-and-for-all-time in an anticipated fashion, then there is an equilibrium in which the inflation rate increases monotonically to its higher level. In response to the same experiment, the real interest rate jumps up when the nominal interest rate increases, then falls monotonically, with no effect on the real rate in the steady state.

Things work very differently if the asset market constraint binds. In that case, an increase in the nominal interest rate always leads to an increase in the inflation rate – in the short run or long run – in spite of market segmentation which is working in favor of a short-run liquidity effect. In this case, Taylor rules can go dramatically wrong.

The remainder of the paper proceeds as follows. In the second section the baseline model is set up, and equilibria are constructed and their properties studied in Section 3. Section 4 involves a study of the model’s behavior under Taylor rules for monetary policy. Then, in Section 5, a related segmented markets model is studied.

2 Model

There is a continuum of households with unit mass, each of which consists of a continuum of consumers with unit mass, and a worker/seller. Each household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 u(c_t(i)) di - \gamma n_t \right], \quad (1)$$

where $c_t(i)$ denotes the consumption of the i^{th} consumer in the household, and $i \in [0, 1]$, with consumer names uniformly distributed over the unit interval. In (1), $0 < \beta < 1$, $\gamma > 0$, and n_t is the labor supply of the worker in the household. The household possesses a technology that permits one unit of the perishable consumption good to be produced with each unit of labor supplied by the worker in the household. The household cannot consume its own output, but can consume the output of any other household.

The household enters each period with a portfolio of assets, and then trades on a competitive asset market. The worker/seller then supplies labor and produces output. Then, the worker/seller takes the produced output of the household, n_t , and chooses one of two distinct competitive markets on which to sell it. In market 1, only money is accepted in exchange for goods, as there is no technology available for verifying the existence of other assets that the buyer of goods may hold in his or her portfolio, and no technology for collecting on

debts. In market 2, government bonds are accepted in exchange, and buyers of goods can also use a limited amount of within-period credit.

Before goods markets open, consumers in the household are randomly allocated (by nature), with θ consumers allocated to market 1 and $1 - \theta$ consumers to market 2. The household knows in advance which market each consumer will be trading in, and therefore knows what assets to allocate to each consumer. We can then write the preferences of the household as

$$E_0 \sum_{t=0}^{\infty} \beta^t [\theta u(c_t^1) + (1 - \theta)u(c_t^2) - \gamma n_t], \quad (2)$$

where c_t^j denotes the consumption of consumers in the household who trade in market j . Note that consumption takes place when the consumer trades on the goods market, i.e. consumers in the household cannot share consumption goods.

The household begins each period with m_t units of money carried over from the previous period, along with b_t^a maturing government bonds acquired in the asset market of the previous period, and b_t^g maturing government bonds acquired in the goods market of the previous period. Here, m_t , b_t^a , and b_t^g are measured in units of $t - 1$ consumption good 1. The household also receives a money transfer τ_t from the government in the asset market, defined in units of current consumption good 1. The household then takes beginning-of-period wealth, and trades on the asset market to obtain the money and bonds that it will distribute to consumers in the household to make purchases. The asset market constraint for the household is

$$\theta c_t^1 + q_t b_t^2 + q_t b_{t+1}^a + m_t^2 \leq \frac{p_{t-1}}{p_t} (m_t + b_t^a + b_t^g) + \tau_t, \quad (3)$$

where q_t denotes the price of government bonds in terms of money, b_t^2 and m_t^2 are government bonds and money, respectively, that are given to consumers in the household who purchase goods in competitive market 2, and b_{t+1}^a denotes bonds that will be held over by the household until period $t + 1$. The price p_t denotes the price of good 1 in terms of money. In the analysis that follows, some nonnegativity constraints will be implicit, but it will prove critical to explicitly account for the nonnegativity constraint on bonds held over from the current asset market until period $t + 1$, i.e.

$$b_{t+1}^a \geq 0. \quad (4)$$

Constraint (4) implies limited commitment, in that the household cannot commit to pay off debt in future periods.

The household can borrow on behalf of consumers who purchase goods in market 2, and these consumers can also make purchases with money and bonds. The household's within-period debt is constrained, in that it can pay back at most κ_t at the end of the period, where κ_t is exogenous. As well, credit transactions are not feasible in market 1. Total purchases by consumers who purchase in market 2 are then constrained by

$$(1 - \theta)c_t^2 = b_t^2 + m_t^2 + \kappa_t. \quad (5)$$

Note that bonds are not discounted when accepted in exchange, since either one bond or one unit of money is a claim to one unit of money at the beginning of period $t + 1$, from the point of view of the seller in the goods market. However, bonds can trade at a discount on the asset market, i.e. we can have $q_t < 1$. We also assume in (5) that the household always borrows up to its credit limit, and we will later derive a condition that assures this in equilibrium. Note that a claim to one unit of consumption goods at the end of the period trades for one unit of consumption goods in market 2.

The agent's budget constraint is

$$\theta c_t^1 + q_t(1-\theta)c_t^2 + m_{t+1} + b_{t+1}^g + q_t b_{t+1}^a = \frac{p_{t-1}}{p_t} (m_t + b_t^a + b_t^g) + \tau_t + n_t + q_t \kappa_t - \kappa_t \quad (6)$$

In equation (6), m_{t+1} denotes money held over until period $t + 1$, and the quantity b_{t+1}^g denotes bonds received in payment for goods sold by the household, or in settlement of within-period credit. Note that the price of good 2 in terms of good 1 is q_t , which is implicit from (3) and (5). As for equation (5), note in (6) that we have assumed that the household borrows up to its credit limit for consumers who purchase in market 2.

The government's budget constraints are

$$\bar{m}_0 + q_0 \bar{b}_0 = \tau_0 \quad (7)$$

$$\bar{m}_t - \frac{p_{t-1}}{p_t} \bar{m}_{t-1} + q_t \bar{b}_t - \frac{p_{t-1}}{p_t} \bar{b}_{t-1} = \tau_t, \quad t = 1, 2, 3, \dots, \quad (8)$$

where \bar{m}_t and \bar{b}_t are, respectively, the quantities of money and bonds outstanding (net of government bonds held by the central bank). Note that we have assumed that there are no government liabilities (money or bonds) outstanding at the beginning of period 0.

3 Equilibrium

Let λ_t^1 , λ_t^2 , and μ_t denote, respectively, the multipliers associated with constraints (3), (5), and (6). The consumer chooses c_t^1 , c_t^2 , n_t , b_t^2 , m_t^2 , b_{t+1}^a , m_{t+1} , and b_{t+1}^g , in the current period. Then, from the household's optimization problem, we get

$$u'(c_t^1) - \lambda_t^1 - \mu_t = 0, \quad (9)$$

$$u'(c_t^2) - \lambda_t^2 - q_t \mu_t = 0, \quad (10)$$

$$-\gamma + \mu_t = 0, \quad (11)$$

$$-q_t \lambda_t^1 + \lambda_t^2 = 0, \quad (12)$$

$$-\lambda_t^1 + \lambda_t^2 \leq 0, \quad (13)$$

$$-q_t(\lambda_t^1 + \mu_t) + \beta E_t \left[\frac{p_t}{p_{t+1}} (\lambda_{t+1}^1 + \mu_{t+1}) \right] \leq 0, \quad (14)$$

$$-\mu_t + \beta E_t \left[\frac{p_t}{p_{t+1}} (\lambda_{t+1}^1 + \mu_{t+1}) \right] = 0. \quad (15)$$

First, from (12) and (13), note that if $q_t < 1$, then consumers will not purchase good 2 with money, as it is cheaper to pay with bonds if bonds trade at a discount on the asset market. Reducing (9)-(15) to something we can work with, we get

$$-\gamma + \beta E_t \left[\frac{p_t}{p_{t+1}} u'(c_{t+1}^1) \right] = 0, \quad (16)$$

$$u'(c_t^2) - q_t u'(c_t^1) = 0, \quad (17)$$

$$u'(c_t^2) - \gamma \geq 0, \quad (18)$$

and from (16) and (17) we can derive

$$q_t = \frac{u'(c_t^2)}{\gamma} \beta E_t \left[\frac{p_t}{p_{t+1}} \frac{u'(c_{t+1}^1)}{u'(c_t^1)} \right] \quad (19)$$

$$1 = \frac{u'(c_t^1)}{\gamma} \beta E_t \left[\frac{p_t}{p_{t+1}} \frac{u'(c_{t+1}^1)}{u'(c_t^1)} \right] \quad (20)$$

Equations (19) and (20) price bonds and money, respectively. In each equation, the left-hand side is the price of the asset, and the right-hand side is a liquidity premium multiplied by the “fundamental,” which would be the value of the asset if it were not useful in exchange. Note that the liquidity premium for bonds is the inefficiency wedge for good 2, while the liquidity premium for money is the inefficiency wedge for good 1. Under any circumstances, at the zero lower bound on the nominal interest rate ($q_t = 1$), the liquidity premia on bonds and money must be equal.

We can also determine the real interest rate, as follows. Suppose a real bond that sells at price s_t^a , in units of consumption good 1, in the asset market, and pays off one unit of consumption good 1 in the asset market of period $t+1$. Also suppose that this asset is accepted in exchange, just as nominal bonds are. Its price at the end of the period – the price a firm is willing to take for the real bond in exchange for consumption good 2 – is given by s_t^g . Then,

$$-s_t^a \lambda_t^1 + \lambda_t^2 s_t^g = 0 \quad (21)$$

$$-s_t^g \mu_t + \beta E_t (\lambda_{t+1}^1 + \mu_{t+1}) = 0. \quad (22)$$

Therefore, from (9), (11), (12), (17), (21), and (22), we get

$$s_t^a = \frac{u'(c_t^2)}{\gamma} \beta E_t \left[\frac{u'(c_{t+1}^1)}{u'(c_t^1)} \right]. \quad (23)$$

Note, in equation (23), as in (19), that we can write the price of the real bond as a liquidity premium, multiplied by the fundamental.

We will specify fiscal policy as setting the real value of the consolidated government debt each period, V_t , i.e.

$$V_t = \bar{m}_t + q_t \bar{b}_t, \quad (24)$$

where V_t is exogenous. Then, from (8),

$$\begin{aligned} \tau_0 &= V_0, \\ \tau_t &= V_t - \frac{p_{t-1}}{p_t} V_{t-1} - \bar{b}_{t-1} \left(\frac{p_{t-1}}{p_t} - q_{t-1} \right), \end{aligned} \quad (25)$$

so the period 0 transfer to the private sector is exogenous, but the transfer in each succeeding period is endogenous, and in general will depend on monetary policy, which affects prices. Thus, fiscal policy responds passively to monetary policy so as to achieve a particular time path for the total value of the consolidated government debt. Monetary policy consists of setting a target q_t for the price of government bonds, and this target is then supported by open market operations. The relationship between fiscal and monetary policy here is the same as in Williamson (2014a, 2014b). As it turns out, the real value of the consolidated government debt will play a critical role in our model, and for the key results, so it proves convenient (and realistic, we think) to specify the fiscal policy rule as setting the real value of the consolidated government debt exogenously. Then, monetary policy is about determining the composition of the consolidated government debt so as to achieve a particular price for government debt in financial markets. But, a key element in how monetary policy affects inflation, for example, will be determined by the nature of the fiscal policy rule.

From (3), (5), (6), (16)-(18), (24) and market clearing, an equilibrium is then a stochastic process $\{c_t^1, c_t^2, \pi_{t+1}\}_{t=0}^\infty$ solving

$$-\gamma + \beta E_t \left[\frac{u'(c_{t+1}^1)}{\pi_{t+1}} \right] = 0, \quad (26)$$

$$u'(c_t^2) - q_t u'(c_t^1) = 0, \quad (27)$$

$$u'(c_t^2) - \gamma = 0 \text{ and } V_t + q_t \kappa_t \geq \theta c_t^1 + (1 - \theta) q_t c_t^2, \quad (28)$$

or

$$u'(c_t^2) - \gamma \geq 0 \text{ and } V_t + q_t \kappa_t = \theta c_t^1 + (1 - \theta) q_t c_t^2, \quad (29)$$

given a stochastic process $\{V_t, q_t, \phi_t\}_{t=0}^\infty$, with $q_t \leq 1$. Here $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation rate. Note that the price level in period 0 is irrelevant, but it is determined by the fiscal authority in period 0, i.e. the fiscal authority follows a policy rule that sets exogenously the path for the consolidated government debt, in real terms, and the period 0 price level is then determined by the quantity of nominal debt issued by the fiscal authority in period 0.

In (28) and (29) note that, if $u'(c_t^2) - \gamma = 0$, then the value government debt plus the credit limit is more than sufficient to finance purchases in market 2, so this is the case in which constraint (4) does not bind. But, if $u'(c_t^2) - \gamma > 0$,

i.e. if exchange in market 2 is inefficient, then the value of consumption of both goods consumed is constrained by the real quantity of consolidated government debt plus the credit limit, and the nonnegativity constraint (4) binds.

Given the quasilinear utility function, arriving at an equilibrium solution is easy, in that we can solve period-by-period. First, (27)-(29) solve for c_t^1 and c_t^2 given q_t , V_t , and ϕ_t . Then, we can solve for the inflation rate from (26), i.e.,

$$\pi_t = \frac{\beta u'(c_t^1)}{\gamma}. \quad (30)$$

Let *unconstrained equilibrium* and *constrained equilibrium*, denote the cases where (28) and (29) apply, respectively, i.e. in which the nonnegativity constraint (4) binds, and does not bind, respectively.

3.1 Unconstrained Equilibrium

An unconstrained equilibrium has standard properties that we would find in typical cash-in-advance cash good/credit good models, e.g. Lucas and Stokey (1987). In an unconstrained equilibrium, exchange in market 2 is efficient, as $u'(c_t^2) - \gamma = 0$. For convenience, let c^* denote the efficient consumption quantity, which solves

$$u'(c^*) = \gamma. \quad (31)$$

Thus, from (19) and (20), there is no liquidity premium associated with bonds, but there is a standard liquidity premium associated with money. From (30) and (26), c_t^1 solves

$$u'(c_t^1) = \frac{\gamma}{q_t}, \quad (32)$$

i.e. the inefficiency in market 1, and the liquidity premium on money (from equation (20)) are associated with a positive nominal interest rate ($q_t < 1$). From (30) and (32), we can solve for the gross inflation rate:

$$\pi_t = \frac{\beta}{q_t}. \quad (33)$$

Equation (33) is the Fisher relation – the nominal interest rate increases approximately one-for-one with an increase in the inflation rate. Further, from (23), we get

$$s_t^a = q_t \beta E_t \left[\frac{1}{q_{t+1}} \right]. \quad (34)$$

This equilibrium is unconstrained, as the consolidated government debt V_t and the credit limit κ_t are irrelevant for the solution. Note that, from (28),

$$V_t + q_t \kappa_t \geq \theta c_t^1 + (1 - \theta) q_t c_t^2 \quad (35)$$

must be satisfied for the unconstrained equilibrium to exist.

We can illustrate the unconstrained equilibrium in Figure 7. With a positive nominal interest rate, i.e. $q_1 < 1$, the equilibrium is at point *A*, where the

locus defined by (27), dropping time subscripts, intersects $c_2 = c^*$. An efficient allocation is B , which will be the equilibrium allocation when $q = 1$. At point B , $c_1 = c_2 = c^*$, which is the conventional Friedman rule allocation.

3.2 Constrained Equilibrium

The constrained equilibrium is the interesting case, in which, from (3), (4) with equality, (5), (24), and market clearing, c_t^1 and c_t^2 solve

$$V_t + q_t \kappa_t = \theta c_t^1 + (1 - \theta) q_t c_t^2 \quad (36)$$

and (27). In the constrained equilibrium, comparative statics, dropping t subscripts for convenience, gives

$$\frac{dc_1}{dq} = \frac{\phi y u''(c_2) - u'(c_2)(1 - \theta) \left[\frac{c_2 u''(c_2)}{u'(c_2)} + 1 \right]}{\theta u''(c_2) + q^2(1 - \theta) u''(c_1)} \quad (37)$$

$$\frac{dc_2}{dq} = \frac{\theta u'(c_1) + q u''(c_1) [\kappa - (1 - \theta) c_2]}{\theta u''(c_2) + q^2(1 - \theta) u''(c_1)} \quad (38)$$

We will assume that

$$\kappa < (1 - \theta) c_2, \quad (39)$$

i.e. that the quantity of credit supported by capital income is not sufficient to purchase all the goods supplied in market 2. Then, (23) and (38) imply that $\frac{dc_2}{dq} < 0$. The sign of $\frac{dc_1}{dq}$ is in general ambiguous, but, if $-\frac{c_2 u''(c_2)}{u'(c_2)} < 1$, then $\frac{dc_1}{dq} > 0$. Thus, provided there is not too much curvature in the utility function, a lower nominal interest rate (higher q) reduces consumption in market 2 and increases consumption in market 1. Essentially, this is due to the open market purchase that is required to support a lower nominal interest rate. The open market purchase of government bonds reduces bonds available for transactions in market 2, but increases money available for transactions in market 1.

We can illustrate the constrained equilibrium in Figure 7. Here, V_0 is the locus defined by (36) in the case in which $q = 1$, while V_1 is the same locus when $q = q_1 < 1$. Then, a reduction in q from 1 to q_1 shifts the equilibrium from D to E . Consumption in market 2, c_2 , must increase, while c_1 may rise or fall.

As well, it is useful to look at the effect of an increase in the nominal interest rate on real GDP, which we can express as

$$y_t = \theta c_t^1 + (1 - \theta) c_t^2. \quad (40)$$

Then, from (37) and (38),

$$\frac{dy}{dq} = \frac{[\kappa - (1 - \theta) c_2] [\theta u''(c_2) + (1 - \theta) q u''(c_1)] + u'(c_1) \theta (1 - \theta) (1 - q)}{\theta u''(c_2) + q^2 (1 - \theta) u''(c_1)}.$$

Therefore, under assumption (39), $\frac{dy}{dq} < 0$, so output goes down when the nominal interest rate goes down. We could also do a welfare calculation to find

the optimal monetary policy in this economy under a constrained equilibrium, taking fiscal policy as given. We can do a period-by-period maximization of period utility for the representative household, writing our welfare measure as

$$W = \theta u(c_1) + (1 - \theta)u(c_2) - \gamma [\theta c_1 + (1 - \theta)c_2 - y]$$

Then, differentiating, we get

$$\frac{\partial W}{\partial q} = \theta [u'(c_1) - \gamma] \frac{dc_1}{dq} + (1 - \theta) [u'(c_2) - \gamma] \frac{dc_2}{dq} \quad (41)$$

So, at the zero lower bound ($q = 1$), where $c_1 = c_2 = V + \kappa$,

$$\frac{\partial W}{\partial q} = [u'(V + \kappa) - \gamma] \frac{dy}{dq} < 0.$$

Therefore, given (39), $q < 1$ is optimal, so optimal monetary policy is away from the zero lower bound if the equilibrium is constrained. In general, from (41), (37), and (38), we can write

$$\frac{\partial W}{\partial q} = \frac{[\theta u''(c_2) + q(1 - \theta)u''(c_1)] [u'(c_1) - \gamma] [\kappa - (1 - \theta)c_2] - \gamma \theta (1 - \theta) u'(c_1) (1 - q)}{\theta u''(c_2) + q^2 (1 - \theta) u''(c_1)}.$$

Therefore, we can say, in general, that welfare increases as the nominal interest rate increases, so long as the nominal interest rate is close to zero, i.e. q is close to 1.

Thus, since c_2 is strictly decreasing in q , from (38), therefore a constrained equilibrium exists if and only if the equilibrium is constrained at the zero lower bound. In a constrained equilibrium at the zero lower bound, $c_1 = c_2 = V + \kappa$. Therefore, from (29), a constrained equilibrium exists for some q if and only if

$$\frac{u'(V + \kappa)}{\gamma} > 1, \quad (42)$$

i.e. if and only if $V + \kappa$ is sufficiently small. Furthermore, if (42) holds, then if q is sufficiently small, the equilibrium will be unconstrained. To be more precise, if (42) holds, then the equilibrium is constrained for $q \in (\hat{q}, 1]$, and unconstrained for $q \leq \hat{q}$, where (\hat{q}, \hat{c}_1) solve

$$u'(\hat{c}_1) = \frac{\gamma}{\hat{q}}, \quad (43)$$

$$V + \hat{q}\kappa = \theta \hat{c}_1 + (1 - \theta) \hat{q} c^*. \quad (44)$$

How does monetary policy affect the real interest rate in a constrained equilibrium? One way to think about this is to consider an equilibrium that is constrained, with $V_t = V$ and $\kappa_t = \kappa$ for all t , so that $c_t^1 = c_1$ and $c_t^2 = c_2$ for all t . Then, from (23),

$$s_t^a = \frac{u'(c_2)\beta}{\gamma}. \quad (45)$$

Therefore, from (45), the real interest rate depends on the inefficiency wedge (the ratio $u'(c_2)/\gamma$) in market 2, i.e. on the liquidity premium on real bonds. Thus, from (38), a decrease in the nominal interest rate by the central bank also reduces the real interest rate, as this reduces the supply of bonds, tightens the finance constraint, and increases the liquidity premium on government debt.

3.3 Government Debt and Credit Constraints

What happens if κ_t changes? For example, we might think of a decrease in κ_t as capturing some of what occurred during the financial crisis. In an unconstrained equilibrium, a change in κ_t has no effects at the margin, as the credit limit and government debt are large enough to support efficient exchange in market 2. Therefore, given monetary policy, there is no change in consumption or output.

However, from (43) and (44), a decrease in κ acts to reduce \hat{q} , so there is an increase in the critical value for the nominal interest rate, below which the equilibrium will be constrained. Therefore, a discrete decrease in κ could result in a constrained equilibrium in a case in which the equilibrium was unconstrained before the change in κ . Further, given q , the constrained equilibrium will have lower consumption in both markets and lower real output if κ decreases. In Figure 8, the equilibrium is initially at A , at the intersection between the upward-sloping locus defined by (27), and the downward-sloping locus defined by (36). Then, holding monetary policy constant, so q is constant, the locus defined by (36) shifts from V_0 to V_1 , so c_1 , c_2 , and total real GDP fall.

More formally, from (27) and (36),

$$\frac{dc_1}{d\kappa} = \frac{qu''(c_2)}{\theta u''(c_2) + q^2(1-\theta)u''(c_1)} > 0, \quad (46)$$

$$\frac{dc_2}{d\kappa} = \frac{q^2u''(c_2)}{\theta u''(c_2) + q^2(1-\theta)u''(c_1)} > 0. \quad (47)$$

Therefore, a reduction in ϕ reduces consumption in both markets, and lowers real output. As well, from (27) and (36), we get

$$\frac{dc_1}{dV} = \frac{1}{q} \frac{dc_1}{d\kappa}, \quad (48)$$

$$\frac{dc_2}{dV} = \frac{1}{q} \frac{dc_2}{d\kappa}. \quad (49)$$

Therefore, a decrease in the real quantity of consolidated government debt has the same qualitative effect as a reduction credit limit. Put another way, a reduction in the credit limit can be mitigated or eliminated if the fiscal authority acts to increase the quantity of government debt. However, one of our goals in this paper is to examine the effects of monetary policy in the face of suboptimal behavior of the fiscal authority which, in this case, will not act to relax asset market constraints when that is appropriate.

We can also examine how changes in κ and V affect the real interest rate. If, as above, we look at cases where $\kappa_t = \kappa$ and $V_t = V$ for all t , then from (45), (47), and (49), a decrease in κ or V will lower the real interest rate, because this increases the inefficiency wedge in market 2, and therefore increases the liquidity premium on government bonds.

As well, note from (30) that a decrease in κ or V will increase the inflation rate, given q . Because the asset market constraint tightens, increasing the liquidity premium on government bonds and lowering the real interest rate, the inflation rate must rise since the nominal interest rate is being held constant in these experiments.

These results – that a reduction in credit limits will reduce consumption and output, reduce the real interest rate, and lead to an increase in the inflation rate (given the nominal interest rate) – are consistent with observations on the U.S. economy following the financial crisis. Post-2008, the real interest rate on government debt was low, and it may seem surprising, given the zero-lower-bound policy of the Fed, that the inflation rate was still positive. However, this is consistent with what our model predicts.

To see more clearly where these results are coming from, we consider in the next subsection what happens at the zero lower bound in a constrained equilibrium.

3.4 Liquidity Trap

It is useful to examine specifically the properties of the model when the nominal interest rate is set to zero by the central bank, or $q_t = 1$, so that we have a liquidity trap equilibrium. From (27), this implies that $c_t^1 = c_t^2$, so consumption is equalized across markets. If (42) does not hold, so that the real value of the consolidated government debt plus the credit limit is sufficiently large, then the liquidity trap equilibrium is unconstrained, so from (28), we have $u'(c_t^1) = u'(c_t^2) = \gamma$ and, from (30), $\pi_t = \beta$. Therefore, if assets used in exchange are sufficiently plentiful, then a liquidity trap equilibrium has conventional properties. Exchange in markets 1 and 2 is efficient, and there is deflation at the rate of time preference.

However, a constrained liquidity trap equilibrium has very different properties. If (42) holds, then from (36),

$$c_t^1 = c_t^2 = y_t = V_t + \kappa_t \tag{50}$$

so consumption and output are determined by the value of the consolidated government debt plus the credit limit. This shows, in the most obvious way, the non-Ricardian nature of the constrained equilibrium. In a liquidity trap, increases in the quantity of government debt (in real terms) are not neutral, and will increase output and consumption one-for-one. As well, from (30), the inflation rate in a liquidity trap, if the equilibrium is constrained, is given by

$$\pi_t = \frac{\beta u'(V_t + \kappa_t)}{\gamma}, \tag{51}$$

so the inflation rate increases with the inefficiency wedge in goods markets, which determines the liquidity premium on all assets – money and bonds. Basically, a lower quantity of government debt plus the credit limit implies a larger inefficiency wedge in goods markets, a larger liquidity premium on assets used in exchange, and a larger inflation rate. There need not be deflation in a liquidity trap.

To understand exactly what is going on, it helps to consider what the fiscal authority is doing in the constrained equilibrium at the zero lower bound. From (25),

$$\tau_t = V_t \left(1 - \frac{1}{\pi_t} \right), \quad (52)$$

so given the fiscal authority's rule, which sets V_t exogenously, at the zero lower bound, the real transfer rebated to the private sector is determined by the real value of the government debt, and the inflation rate. Basically, in (52), V_t is the tax base, and $1 - \frac{1}{\pi_t}$ is the tax rate, and effectively the real transfer is seigniorage on the consolidated government debt. So, if we substitute using (51) in (52), we get

$$\tau_t = V_t \left(1 - \frac{\gamma}{\beta u'(V_t + \kappa_t)} \right), \quad (53)$$

and this tells us the real transfer required to support the fiscal authority's policy given monetary policy at the zero lower bound, in a constrained equilibrium.

So, assuming that $-c \frac{u''(c)}{u'(c)} = \alpha$, a constant, and $\frac{\gamma}{\beta}(V + \kappa)^\alpha < 1$, then the transfer τ_t , as a function of V_t , can be depicted as in Figure 9. Thus, in this case, if the inflation rate is close to zero (so that τ_t is close to zero) and positive, then a reduction in V_t will lead to an increase in τ_t , i.e. an increase in the government deficit. In a stationary equilibrium with $V_t = V$ for all t , and $q_t = 1$ for all t , then, a permanent decrease in V implies that the inflation rate must increase permanently. To accomplish this, the nominal quantity of consolidated government liabilities must be increasing at a higher rate. Thus, suppose the fiscal authority wants to accomplish a permanent reduction in the real quantity of consolidated government debt outstanding. Then, given monetary policy at the zero lower bound, the nominal value of the consolidated government debt must be increasing at a higher rate. With assumptions that give the configuration in Figure 9, if the inflation rate is currently sufficiently close to zero, then the deficit must rise in real terms to support this.

4 Taylor Rule

Thus far, we have established the operating characteristics of this model economy, and have characterized optimal monetary policy, given a fiscal policy rule which is in general suboptimal. In this section, we want to understand what will happen in this economy if a central banker adopts a standard type of policy rule – a Taylor rule. We know at the outset that the Taylor rule will be suboptimal

here, in general, but we wish to understand what types of pitfalls would meet a Taylor rule central banker in this context.

For simplicity we assume that the central banker cares only about inflation, and the Taylor rule takes the form

$$\frac{1}{q_t} = \max[\pi_t^\alpha (\pi^*)^{1-\alpha} x_t, 1] \quad (54)$$

Here, $\frac{1}{q_t}$ is the gross nominal interest rate, π^* is the central bank's target gross inflation rate, and x_t is the adjustment the central bank makes for the real rate of return on government debt. In general, $\alpha > 0$, and if $\alpha > 1$ the rule falls the "Taylor principle," whereby deviations of the inflation rate from its target are met with an aggressive response by the central bank.

4.1 Unconstrained Equilibrium

First, suppose that (42) does not hold, so that the equilibrium is unconstrained, for all $q_t \leq 1$. Also suppose, in standard fashion, that $x_t = \frac{1}{\beta}$, i.e. the long-run "natural" real interest rate is the rate of time preference, so from (30)-(??) and (54), we can solve for equilibrium q_t from

$$1 = \max \left[\left(\frac{q_t \pi^*}{\beta} \right)^{1-\alpha}, q_t \right]. \quad (55)$$

Note in particular that there are no dynamics associated with this Taylor rule, which is in part due to our assumption of quasilinear preferences. Assume $\pi^* \geq \beta$, so that the target inflation rate is larger than minus the rate of time preference. Then, if the Taylor rule follows the Taylor principle, so $\alpha > 1$, there are two equilibrium solutions to (55), as depicted in Figure 10. The two solutions are $q_t = \frac{\beta}{\pi^*}$, which implies that $\pi = \pi^*$ and the central bank achieves its target rate of inflation, and $q_t = 1$, which is the liquidity trap solution for which $\pi = \beta < \pi^*$. In the liquidity trap equilibrium the central banker sees a low inflation rate, and responds aggressively by setting the nominal interest rate as low as possible, which ultimately has the effect of producing a low inflation rate. This is a well-known property of monetary models (see Benhabib et al. 2001) – under the Taylor principle there are multiple steady states, including the liquidity-trap steady state. In this particular model, in an unconstrained equilibrium, the Taylor rule does not impart any dynamics to the economy (in contrast to Benhabib et al. 2001), but we will show in what follows how dynamic equilibria arise with other forms of the monetary policy rule.

If, however, $\alpha < 1$, then, as in Figure 11, there is a unique equilibrium with $q = \frac{\beta}{\pi^*}$, and the central banker always achieves his or her inflation target. Thus, in this particular model, in an unconstrained equilibrium the Taylor principle is not a good idea, as the central banker will not achieve his or her inflation target. Note that, if $\pi^* = \beta$, then this maximizes welfare in the unconstrained equilibrium, and the equilibrium is unique – this is just the Friedman rule solution.

In the Taylor rule the term x_t makes an adjustment for the real interest rate, to account for the fact that the Fisher relation must hold in the long run. With an appropriately chosen x_t term, there is at least the possibility that the Taylor rule will lead to convergence to the central bank's inflation target in the long run. But, what if the central bank accounted explicitly for endogeneity in the real interest rate? In particular, suppose that the central bank chooses

$$x_t = \frac{1}{s_t^a}, \quad (56)$$

where s_t^a is the price of a real bond, as determined in (23). In this case, the central bank recognizes that the real interest rate is endogenous, and sets the nominal interest rate in line with fluctuations in the real interest rate. Suppose, for convenience, that we consider only deterministic dynamic equilibria. Substituting in (54) using (23), (28), (30), (33) and (56), we get

$$\pi_{t+1} = \max \left\{ \pi_t^\alpha (\pi^*)^{1-\alpha}, \beta \right\}, \quad (57)$$

which is a nonlinear first-order difference equation in the gross inflation rate π_t , which we can use to solve for an equilibrium. An unconstrained dynamic equilibrium satisfying this version of the Taylor rule is a sequence $\{c_t^1, q_t, c_t^2, \pi_t\}_{t=0}^\infty$ satisfying (30), (33), (57), and $c_t^2 = c^*$ for all t .

First, suppose that $\alpha > 1$. Then there are two steady states, just as for the Taylor rule with $x_t = \beta^{-1}$, and these are the same steady states as for the simpler Taylor rule – one where the central bank meets its inflation target, and the liquidity trap equilibrium. In the high-inflation steady state, the gross inflation rate is $\pi = \pi^*$, so the central bank achieves its inflation target, c_t^1 solves

$$u'(c_t^1) = \frac{\pi^* \gamma}{\beta}, \quad (58)$$

and $q_t = \frac{\beta}{\pi^*}$. In the liquidity trap steady state, $\pi = \beta$, so the central bank falls short of its inflation target, $c_t^1 = c^*$, and $q_t = 1$.

In contrast to the case with $x_t = \beta^{-1}$ though, there are nonstationary equilibria. With $\alpha > 1$, there exists a continuum of nonstationary equilibria that converge to the liquidity trap steady state in finite time. In each of these equilibria, $\beta < \pi_0 < \pi^*$, and

$$\pi_{t+1} = \max \left\{ \pi_t^\alpha (\pi^*)^{1-\alpha}, \beta \right\},$$

for $t = 1, 2, \dots$, with

$$q_t = \frac{\beta}{\pi_t}. \quad (59)$$

In Figure 12, B is the high-inflation steady state, A is the liquidity trap steady state, and we have depicted one of the nonstationary equilibria, for which the initial gross inflation rate is π_0 , and there is convergence to the liquidity trap steady state in period 4.

Second, if $\alpha < 1$, then there is a unique steady state with $\pi = \pi^*$, c_t^1 solving (58), and $q_t = \frac{\beta}{\pi^*}$. As well, there exists a continuum of nonstationary equilibria that converge in the limit to the steady state equilibrium. For each of these equilibria, $\beta \leq \pi_0 < \infty$,

$$\pi_{t+1} = \pi_t^\alpha (\pi^*)^{1-\alpha}$$

for $t = 1, 2, \dots$, c_t^1 solves (58), and q_t is given by (59). In Figure 13, we show the case $\alpha < 1$, where A is the steady state, and we show one of the nonstationary equilibria, for which the initial gross inflation rate is π_0 , and there is convergence in the limit to the steady state.

So, the ‘‘Taylor principle’’ (the case $\alpha > 1$) does not have anything in particular to recommend it in this standard unconstrained case. The Taylor principle yields a liquidity trap steady state in which the central banker falls short of his or her inflation target, and this steady state is stable, in the sense that there exists a continuum of nonstationary equilibria that converge to the liquidity trap steady state in finite time. However, if the Taylor principle does not hold, with $\alpha < 1$, there is a unique steady state equilibrium in which the central banker achieves his or her inflation target, and this steady state is stable.

4.2 Constrained Equilibrium

Next, suppose that (42) holds, so that the equilibrium will be constrained for sufficiently large q . Therefore, if (42) holds and $R_t = \beta^{-1}$, then there is a static equilibrium solution and (54) becomes

$$\frac{1}{q} = \max \left\{ \left[\frac{u'(c_1)}{\gamma} \right]^\alpha \left(\frac{\pi^*}{\beta} \right)^{1-\alpha}, 1 \right\}, \quad (60)$$

and then an equilibrium consists of (c_1, c_2, q) solving (60),

$$u'(c_2) - qu'(c_1) = 0, \quad (61)$$

and

$$V + q\kappa = \theta c_1 + (1 - \theta)qc_2. \quad (62)$$

Then, from (50), a liquidity trap equilibrium, with $q = 1$ has $c_1 = c_2 = V + \kappa$. Therefore, from (60), this is an equilibrium if and only if

$$\frac{u'(V + \kappa)}{\gamma} \leq \left(\frac{\pi^*}{\beta} \right)^{1-\frac{1}{\alpha}}. \quad (63)$$

But, if $\alpha > 1$, then (63) will not hold for $V + \kappa$ sufficiently small, i.e. if government debt plus the credit limit is sufficiently small in a liquidity trap, which in turns makes the inflation rate high in a liquidity trap. If $\alpha < 1$, then since $\pi^* \geq \beta$, (63) and (42) cannot both hold, so the liquidity trap is not an equilibrium. Therefore, the inflation rate must be sufficiently low at the zero lower bound in order for the liquidity trap to be an equilibrium, and the Taylor

principle must hold, i.e. $\alpha > 1$. Indeed, if the liquidity trap equilibrium exists, the inflation rate at the zero lower bound equilibrium is smaller than the target inflation rate, from (63).

If an equilibrium away from the zero lower bound exists, it is straightforward to show that, in general, $\pi \neq \pi^*$ in this equilibrium, i.e. the central bank does not achieve its inflation target, which is also true in the liquidity trap equilibrium, if it exists. There also are potentially multiple solutions to (60)-(62) with $q < 1$, though constructing simple examples seems difficult. The key problem here is that the policy maker does not correctly understand what is determining the real interest rate, and constructs the Taylor rule under the incorrect assumption that the long run real rate is equal to the rate of time preference.

Next, suppose that (42) holds, and the central bank follows a Taylor rule that accounts for the endogeneity in the real interest rate. Then, from (23), (27), and (30),

$$x_t = \frac{1}{q_t \pi_{t+1}}. \quad (64)$$

Then, from (55), (64), (27), and (36), we can express the Taylor rule as

$$\pi_{t+1} = \max \left\{ \pi_t^\alpha (\pi^*)^{1-\alpha}, \frac{\beta u'(V + \kappa)}{\gamma} \right\}, \quad (65)$$

supposing for convenience that $V_t = V$ and $\kappa_t = \kappa$ for all t . Then, an equilibrium consists of a sequence $\{c_t^1, c_t^2, q_t, \pi_t\}_{t=0}^\infty$ solving (65), (27), (30) and (36). Solving for an equilibrium involves first finding a solution to the difference equation (65), then solving for $\{c_t^1, c_t^2, q_t\}_{t=0}^\infty$ from (27), (30) and (36).

First, if

$$\pi^* < \frac{\beta u'(V + \kappa)}{\gamma},$$

then an equilibrium does not exist. In this case, the inflation target is less than the inflation rate in the liquidity trap equilibrium, so no steady state equilibrium exists, and there are no nonstationary equilibria. However, if

$$\pi^* \geq \frac{\beta u'(V + \kappa)}{\gamma}, \quad (66)$$

then there exist two steady state constrained equilibria. In the high-inflation equilibrium, $\pi_t = \frac{\beta u'(c_t^1)}{\gamma} = \pi^*$, so the central bank achieves its inflation target.

In the liquidity trap steady state, $\pi_t = \frac{\beta u'(V + \kappa)}{\gamma} \leq \pi^*$, from (66), so the central bank in general falls short of its inflation target.

In terms of nonstationary equilibria, we get similar results to the unconstrained case with an endogenous real interest rate incorporated into the Taylor rule. In particular, if $\alpha > 1$ (the Taylor principle holds), then there exist a continuum of equilibria with $\pi_0 \in \left(\frac{\beta u'(V + \kappa)}{\gamma}, \pi^* \right)$ that all converge in finite time to the liquidity trap equilibrium. However, if $\alpha < 1$, then there exists a continuum

of equilibria with $\pi_0 \in \left[\frac{\beta u'(V+\kappa)}{\gamma}, \infty \right)$ that all converge to the high-inflation equilibrium.

We illustrate these results in Figures 14 and 15. Figure 14 shows the case $\alpha > 1$, where A is the high-inflation steady state, and B is the liquidity trap steady state. The figure shows one of the nonstationary equilibria, with the initial condition π_0 , converging to the liquidity trap steady state in period 2. Figure 15, shows the case $\alpha < 1$, where A is the unique steady state in which $\pi_t = \pi^*$ for all t . We also show one of the nonstationary equilibria, which follows the path $\{\pi_0, \pi_1, \pi_2, \dots\}$, and converges in the limit to the steady state.

Thus, our results are again similar to the unconstrained case. The Taylor principle tends to yield poor dynamic properties, in that there are many equilibria that converge to the liquidity trap equilibrium in which the central bank fails to achieve its inflation target.

5 Market Segmentation and Liquidity Effects

The structure of the Taylor rule seems in general to be aimed at correcting some short run distortion, while achieving an inflation target in the long run. Further, implicit in the rule is the idea that the short run distortion is corrected by setting the nominal interest rate lower the larger is the distortion. The idea seems to be that there is a short run liquidity effect. That is, the first-round effects of a decrease in the nominal interest rate are a decrease in the real interest rate, and possibly increases in the inflation rate and in real economic activity. New Keynesian (NK) models (e.g. Woodford 2003) certainly have these properties – in a typical NK model, a reduction in the nominal interest rate lowers the real interest rate and increases output and the inflation rate.

However, there are other types of models that generate short-run liquidity effects, in particular the class of segmented markets models, including Lucas (1990), Fuerst (1992), Alvarez and Atkeson (1997), Alvarez, Lucas and Weber (2001), Alvarez, Atkeson, and Kehoe (2002), and Williamson (2006, 2008). In such models, monetary policy is non-neutral in the short run because of a distribution effect of monetary policy. Only some economic agents are on the receiving end of central bank intervention in financial markets, and this can result in a decrease in the real interest rate, an increase in inflation, and possibly an increase in aggregate output, as the result of monetary policy easing. For our purposes, segmented markets frameworks are useful, as it is fairly straightforward to adapt the model we have worked with thus far to incorporate segmented-markets liquidity effects. The resulting model most closely resembles Alvarez, Lucas and Weber (2001).

For convenience, we take production out of the model. Instead of producing output for sale, each household receives an endowment y each period of perishable consumption goods, and each household has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j),$$

where c_t^j denotes the consumption of type j , where $j = T$ denotes a *trader* and $j = N$ denotes a *non-trader*. In the population, there is a mass σ of traders, and a mass $1 - \sigma$ of non-traders, where $0 < \sigma < 1$. Traders participate in asset markets, and in the goods market they always carry out exchange with households that accept government bonds, money, and some credit, in exchange. Each trader household can pledge $\phi_t y$ of its end-of-period income to support intra-period credit to purchase goods from other households. Traders also each receive a lump-sum transfer each period of $\frac{\tau_t}{\sigma}$ in the asset market. Non-traders cannot trade on asset markets, and in goods markets they trade with other households that take only money in exchange. Non-traders do not pay taxes.

A trader then faces the constraints

$$q_t c_t^T + q_t b_{t+1}^a \leq \frac{p_{t-1}}{p_t} (m_t^T + b_t^g + b_t^a) + \frac{\tau_t}{\sigma} + q_t \phi_t y, \quad (67)$$

and

$$q_t c_t^T + m_{t+1}^T + b_{t+1}^g + q_t b_{t+1}^a = \frac{p_{t-1}}{p_t} (m_t^T + b_t^g + b_t^a) + \frac{\tau_t}{\sigma} + q_t \phi_t y + (1 - \phi_t) y. \quad (68)$$

Inequality (67) is the trader's finance constraint, and (68) is the budget constraint. The trader enters period t with m_t^T money balances, b_t^g bonds acquired in the goods market of the previous period, and b_t^a bonds acquired in the asset market of the previous period, as defined for the previous version of the model. In the asset market of period t , the trader acquires b_{t+1}^a bonds to be held until period $t+1$, and also acquires bonds and money to make purchases in the goods market. Note that money will be acquired only if $q_t = 1$. One unit of either money or bonds is accepted on the same terms in the goods market, as they both pay off one unit of money in period $t+1$. Thus, effectively the price of goods for the trader is q_t . If the trader purchases goods with credit, then a claim to one unit of goods at the end of the period trades for one unit of goods, which is why the quantity borrowed, $\phi_t y$, is multiplied by q_t in the constraints. As well, the trader can borrow at most $\phi_t y$. We have assumed implicitly in (68) that the trader always borrows up to the borrowing limit, and we will show where this assumption matters in the analysis below. The trader household sells its endowment y on the goods market for money, bonds, and credit (it does not matter which means of payment the household accepts) and decides how much money and bonds, m_{t+1}^T and b_{t+1}^g , respectively, to carry forward into the next period.

Similarly, a non-trader's constraints are

$$c_t^N \leq \frac{p_{t-1}}{p_t} (m_t^N + b_t^N), \quad (69)$$

and

$$c_t^N + m_{t+1}^N + b_{t+1}^N = \frac{p_{t-1}}{p_t} (m_t^N + b_t^N) + y. \quad (70)$$

Note that non-trader households purchase consumption goods with money, they do not trade on the asset market, and they do not pay taxes.

We will make a different assumption about fiscal policy here. In particular, assume that $\{\tau_0, \tau_1, \tau_2, \dots\}$ is exogenous. As it turns out, our former assumption about fiscal policy (real quantity of consolidated debt exogenous) is not a feasible policy rule in this economy. Also suppose that the finance constraints (67) and (69) hold with equality. Then, (69) and (70) imply

$$c_t^N = \frac{y}{\pi_t}, \quad (71)$$

and recall that π_t is the gross inflation rate. As well, market clearing in the goods market gives

$$\sigma c_t^T + (1 - \sigma)c_t^N = y, \quad (72)$$

so from (71) and (72), we can solve for the consumption of trader households as a function of the gross inflation rate.

$$c_t^T = \frac{y \left[1 - \frac{(1-\sigma)}{\pi_t} \right]}{\sigma} \quad (73)$$

Thus, no matter what policy regime is in place, from (71) and (73) inflation will lead to a distribution effect, with higher inflation leading to higher consumption of trader households and lower consumption of non-trader households. This effect is typical of segmented markets models. The interesting part of the model is how we connect constraints in asset markets to monetary policy, fiscal policy, and inflation.

As in the version of our model with production, there will be two cases to be concerned with, one in which the traders hold some bonds over from the asset market in period t until period $t + 1$, and another in which all bonds are used in transactions by traders during the period. When some bonds are held over until the succeeding period, optimization by traders implies

$$u'(c_t^T) = \beta E_t \left[\frac{u'(c_{t+1}^T)}{\pi_{t+1} q_{t+1}} \right]. \quad (74)$$

Note the difference in (74) from typical bond pricing relationships in representative agent models and, for example, in Alvarez, Lucas, and Weber (2001). In the latter model, $q_t u'(c_t^T)$ appears on the left-hand side, while the quantity inside the expectation operator on the right-hand side is $\frac{u'(c_{t+1}^T)}{\pi_{t+1}}$. The difference here is that the trader households are paying for consumption goods with government bonds. Giving up the consumption equivalent of one bond in the present implies that the trader foregoes $u'(c_t^T)$, not $q_t u'(c_t^T)$. And then, the payoff in the future will be the marginal utility of the quantity of goods this will purchase, which is $\frac{u'(c_{t+1}^T)}{\pi_{t+1} q_{t+1}}$, not $\frac{u'(c_{t+1}^T)}{\pi_{t+1}}$.

Then, if we substitute for c_t^T in (74) using (73), we get

$$u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_t} \right]}{\sigma} \right) = \beta E_t \left[\frac{u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_{t+1}} \right]}{\sigma} \right)}{\pi_{t+1} q_{t+1}} \right], \quad (75)$$

and (75) then solves for inflation given a policy rule for the price of bonds, q_t . As well, we can price a real bond, just as in the previous version of the model, obtaining

$$\frac{s_t}{q_t} u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_t} \right]}{\sigma} \right) = \beta E_t \left[\frac{u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_{t+1}} \right]}{\sigma} \right)}{q_{t+1}} \right], \quad (76)$$

where s_t is the price of a claim to one unit of consumption in period $t + 1$. To guarantee that there is always a sufficiently high quantity of government bonds to finance consumption by traders in this equilibrium, from the government budget constraint (8) and (73),

$$\tau_t \geq y \left[q_t (1 - \sigma \phi_t) - \frac{q_t (1 - \sigma)}{\pi_t} - \frac{\sigma (1 - \phi_{t-1})}{\pi_t} \right] \quad (77)$$

5.1 Example: Random Policy Shocks

Suppose that q_t is i.i.d., so the central bank engages in random monetary policy, drawing q_t from a time-invariant distribution each period. Suppose also that the distribution for q_t is chosen so that (77) holds in all states of the world, and that $\tau_t = \tau$ for all t . Then, in an equilibrium in which π_t is state-dependent, the right-hand side of (75) is a constant, and so consumption of traders, c_t^T , is a constant, and $\pi_t = \pi$, a constant for all t . Then, from (75) the gross inflation rate solves

$$\pi = \beta E_t \left[\frac{1}{q_{t+1}} \right] \quad (78)$$

Equation (78) is essentially a Fisher relation, but on its head. That is, the inflation rate is determined by the anticipated future nominal interest rate, rather than the nominal interest rate being determined by the anticipated future inflation rate.

Further, in this equilibrium, from (76),

$$s_t = \beta q_t E_t \left[\frac{1}{q_{t+1}} \right].$$

Thus, in this regime, the central bank sees anticipated inflation as being “anchored.” Indeed, it seems the inflation rate is sticky – it is a constant – and so the real interest rate fluctuates one-for-one with the nominal interest rate.

Next, instead of announcing the realization of the random nominal interest rate at the beginning of the period, suppose that the central bank announces the policy rate one period in advance. We will continue to assume that the central bank draws the nominal interest rate it will target from a time-invariant distribution. In this case, from (75),

$$q_{t+1}u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_t} \right]}{\sigma} \right) = \beta E_t \left[\frac{u' \left(\frac{y \left[1 - \frac{(1-\sigma)}{\pi_{t+1}} \right]}{\sigma} \right)}{\pi_{t+1}} \right]. \quad (79)$$

In an equilibrium in which π_t is state-dependent, the right-hand side of (79) is a constant. This implies that, the higher is q_{t+1} the larger is π_t , and the larger is consumption in period t of traders relative to non-traders. This resembles a typical segmented-markets liquidity effect, of the type studied by Alvarez, Lucas, and Weber (2001), for example. The difference here is that the liquidity effect is a response to anticipated monetary policy, not a response to a money surprise. As well, the real interest rate depends only on the current nominal interest rate, i.e. from (75) and (76), we can write

$$s_t = q_t \psi,$$

where ψ is a constant. Therefore, the real interest rate roughly moves one-for-one with the current nominal interest rate.

Suppose then, that the central banker experiments at random, hoping to learn how the economy responds to monetary policy. What we have shown is that the central banker will see (depending on how the random policy experiments are announced) either no response of the inflation rate and low real interest rates in response to low nominal interest rates, or increases in inflation and reductions in the real interest rate (with a lag) in response to low nominal interest rates.

5.2 Example: Linearization

Next, assume that $u(c) = \frac{c^{1-\delta}-1}{1-\delta}$, where $\delta > 0$ is the coefficient of relative risk aversion. Then (75) becomes, after taking logs and using a first-order Taylor series approximation,

$$\frac{\delta(1-\sigma)}{\sigma} i_t = \rho + E_t i_{t+1} - E_t R_{t+1} + \frac{\delta(1-\sigma)}{\sigma} E_t i_{t+1}, \quad (80)$$

where i_t is the inflation rate and R_t is the nominal interest rate. Then, if we consider deterministic cases with an exogenous path for the nominal interest rate $\{R_0, R_1, R_2, \dots\}$, the inflation rate $\{i_0, i_1, i_2, \dots\}$ is a solution to the difference equation

$$i_{t+1} = \frac{(R_{t+1} - \rho) \sigma}{\delta(1-\sigma) + \sigma} + \frac{\delta(1-\sigma) i_t}{\delta(1-\sigma) + \sigma}. \quad (81)$$

For example, if $R_t = R$ for all t , then there are many deterministic equilibria, all of which converge monotonically to the steady state with

$$i_t = i = R - \rho,$$

where the Fisher relation holds.

Suppose a monetary policy represented by the nominal interest rate path $R_t = R_1$ for $t = 1, 2, 3, \dots, T - 1$, and $R_t = R_2$ for $t = T, T + 1, T + 2, \dots$, with $R_2 > R_1$. An equilibrium (not the only one) given this policy, is $i_t = R_1 - \rho$ for $t = 0, 1, 2, \dots, T - 1$, and then i_t increases monotonically to $R_2 - \rho$. Thus, the increase in the nominal interest rate, produces an increase in inflation, but it takes time for the adjustment to occur. In particular, from (81) the speed of adjustment is faster the larger is σ (i.e. the greater is the degree of asset market participation) and the smaller is δ (the higher is the intertemporal elasticity of substitution).

Further, a linear approximation to (76) is

$$\frac{\delta(1-\sigma)}{\sigma} i_t + r_t - R_t = \rho - E_t R_{t+1} + \frac{\delta(1-\sigma)}{\sigma} E_t i_{t+1}, \quad (82)$$

where r_t denotes the real interest rate. In our deterministic examples, once we solve for the path for the inflation rate from (81), equation (82) allows us to solve for the real interest rate, i.e.

$$r_t = \rho + \frac{\delta(1-\sigma)(i_{t+1} - i_t)}{\sigma} + R_t - R_{t+1} \quad (83)$$

So, consider the experiment above, in which the nominal interest rate is constant, and then increases once-and-for-all to a higher level. The effect on the real interest rate from (83), and given the equilibrium solution under consideration, is an increase in period T equal to the increase in the nominal interest rate, with the real rate then falling over time as the inflation rate converges to its higher steady state level. We show the equilibrium path under the central bank's policy, of the inflation rate and the real interest rate, in Figure 16.

The story here is that, if a central banker simply learns from random experiments with the policy rate, this will not tell him or her how to increase the rate of inflation permanently. The experiments tell the policy maker that a decrease in the nominal interest rate makes the inflation rate go up and causes the real interest rate to go down. But what it actually takes to raise the inflation rate is a sustained increase in the nominal interest rate, which will lead to a gradual increase in the inflation rate, with the real interest rate first increasing and then falling over time, with no net effect on the real rate in the long run.

5.2.1 Taylor rule

Now, suppose that the central banker follows a linear Taylor rule

$$R_t = \min[\alpha i_t + (1 - \alpha) i^* + \rho, 0], \quad (84)$$

where i^* is the central banker's inflation target. If we then substitute for R_t in (81) using (84), we obtain

$$\begin{aligned} i_{t+1} &= \frac{(1-\alpha)\sigma i^*}{\delta(1-\sigma)+\sigma} + \left[\frac{\delta(1-\sigma)+\alpha\sigma}{\delta(1-\sigma)+\sigma} \right] i_t, \text{ if } i_t \geq \frac{-\rho+(\alpha-1)i^*}{\alpha}, \\ i_{t+1} &= \frac{-\sigma\rho}{\delta(1-\sigma)+\sigma} + \left[\frac{\delta(1-\sigma)}{\delta(1-\sigma)+\sigma} \right] i_t, \text{ if } i_t \leq \frac{-\rho+(\alpha-1)i^*}{\alpha}. \end{aligned} \quad (85)$$

First, consider the Taylor-principle case, i.e. $\alpha > 1$. Then, as depicted in Figure 17, there are two steady states, the liquidity trap steady state at A , where $i = -\rho$, $R = 0$, and $r = \rho$, and the high-inflation steady state at B , with $i = i^*$, $R = \rho + i^*$, and $r = \rho$. There also exist a continuum of nonstationary equilibria, indexed by $i_0 \in (-\rho, \pi^*)$, that converge to the liquidity trap equilibrium in the limit.

Next, suppose that $\alpha < 1$. Then, there exists a unique steady state with $i = i^*$, $R = \rho + i^*$, and $r = \rho$, and a continuum of nonstationary equilibria that converge to the steady state in the limit, with these equilibria indexed by $i_0 \geq -\rho$. This case is depicted in Figure 18.

5.3 Bonds in Short Supply

We need to consider the case in which traders carry no bonds from one period to the next, i.e. $b_{t+1}^a = 0$, so that all government bonds are used in exchange and (77) is a binding constraint. Then, we can solve for an equilibrium in a static fashion, as (71), (73), and (77) solve for c_t^N , c_t^T , and π_t , given q_t and τ_t .

From (77) we obtain a closed-form solution for the gross inflation rate:

$$\pi_t = \frac{y [q_t(1-\sigma) + \sigma(1-\phi_{t-1})]}{yq_t(1-\sigma\phi_t) - \tau_t}. \quad (86)$$

Therefore, from (86), an increase in q_t (a decrease in the nominal interest rate) results in a decrease in the inflation rate. This contrasts with the short-run liquidity effect that we obtained in the case in which (77) does not bind, whereby a decrease in the nominal interest rate can be associated with an increase in the inflation rate.

Note that a larger deficit (larger τ_t) implies a higher inflation rate. As well, tighter credit in the current period (lower ϕ_t) implies a lower inflation rate, but tighter credit in the previous period (lower ϕ_{t-1}) implies a higher inflation rate.

5.3.1 Taylor Rule

Suppose $x_t = \beta^{-1}$, so that the central banker assumes (incorrectly) that the long-run real interest rate is the rate of time preference. Then, substitute using (86) in (54) to obtain

$$1 = \max \left(G(q_t) \frac{(\pi^*)^{1-\alpha}}{\beta}, q_t \right)$$

where

$$G(q_t) = q_t \left\{ \frac{y [q_t(1 - \sigma) + \sigma(1 - \phi_{t-1})]}{yq_t(1 - \sigma\phi_t) - \tau_t} \right\}^\alpha.$$

Then, we can show that, if α is sufficiently large, then $G'(q_t) < 0$. Further,

$$G(1) = \left\{ \frac{y(1 - \sigma\phi_{t-1})}{y(1 - \sigma\phi_t) - \tau_t} \right\}^\alpha$$

so, if $\phi_{t-1} = \phi_t$, then $G(1) > 1$. This then implies, as shown in Figure 19, that an equilibrium does not exist for this particular Taylor rule, given some mild restrictions. Therefore, a standard Taylor rule is particularly ill-behaved in these circumstances.

6 Conclusion

In the model we have constructed, all consolidated government debt plays a role in exchange, though money is acceptable in exchange under a wider range of circumstances than are government bonds. Then, if asset market constraints do not bind, the economy has standard operating characteristics, which are familiar from the cash-in-advance literature. The economy is Ricardian, a Friedman rule is optimal, and a reduction in the nominal interest rate increases aggregate output. If asset market constraints bind, then the economy is non-Ricardian. Under the assumption that fiscal policy is suboptimal, it is optimal for the central bank to set the nominal interest rate above zero, and lowering the nominal interest rate can reduce consumption and aggregate output.

We examine the properties of the model under Taylor rules, which are generally suboptimal in this environment. The Taylor rule is associated with the most problems in the case in which asset market constraints bind. If the central banker fails to account for the fact that these binding constraints make the real interest rate low, then the Taylor rule will not yield steady states in which the central banker achieves his or her inflation target. In the case in which the central banker corrects for endogeneity in the long-run real interest rate, the Taylor rule encounters familiar perils – there are many equilibria which converge to the zero lower bound on the nominal interest rate if the Taylor principle holds.

We also modify the model to include short-run liquidity effects. In this version of the model, a central banker who experiments with short-run monetary policy may get the idea that reductions in the nominal interest rate are associated with increases in the inflation rate. But what will actually increase the inflation rate permanently is a permanent increase in the nominal interest rate. Given such a policy, the inflation rate increases gradually over time to a higher steady state level. Taylor rules encounter difficulties here as well, including non-existence of equilibrium if asset market constraints bind, or a multiplicity of equilibria converging to the zero lower bound. In instances in which there is convergence to a deflationary steady state, the nominal interest rate can reach

zero while the inflation rate is still positive on a path to a deflationary steady state.

7 References

- Alvarez, F. and Atkeson, A. 1997. "Money and Exchange rates in the Grossman-Weiss-Rotemberg Model," *Journal of Monetary Economics* 40, 619-640.
- Alvarez, F., Atkeson, A. and Kehoe, P. 2002. "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets," *Journal of Political Economy* 110, 73-112.
- Alvarez, F., Lucas, R., and Weber, W. 2001. "Interest Rates and Inflation," *American Economic Review* 91, 219-225.
- Benhabib, J., Schmitt-Grohe, S., and Uribe, M. 2001. "The Perils of Taylor Rules," *Journal of Economic Theory* 96, 40-69.
- Bansal, R. and Coleman, W.J. 1996. "A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles," *Journal of Political Economy* 104, 1135-1171.
- Eggertsson, G. and Krugman, P. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *The Quarterly Journal of Economics* 127, 1469-1513.
- Friedman, M. 1969. *The Optimum Quantity of Money and Other Essays*, Aldine Publishing Company, New York.
- Fuerst, T. 1992. "Liquidity, Loanable Funds, and Real Activity," *Journal of Monetary Economics* 29, 3-24.
- Kehoe, T. and Levine, D. 1993. "Debt-Constrained Asset Markets," *Review of Economics Studies* 60, 865-888.
- Lucas, R. 1990. "Liquidity and Interest Rates," *Journal of Economic Theory* 50, 237-264.
- Lucas, R. and Stokey, N. 1987. "Money and Interest in a Cash-in-Advance Economy," *Econometrica* 55, 491-513.
- Sanches, D., and Williamson, S. 2010. "Money and Credit with Limited Commitment and Theft," *Journal of Economic Theory* 145, 1525-1549.
- Shi, S. 1997. "A Divisible Search Model of Fiat Money," *Econometrica* 65, 75-102.
- Werning, I. 2011. "Managing a Liquidity Trap: Monetary and Fiscal Policy," NBER Working Papers 17344, National Bureau of Economic Research, Inc.

- Williamson, S. 2012. "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," *American Economic Review* 102, 2570-2605.
- Williamson, S. 2006. "Search, Limited Participation, And Monetary Policy," *International Economic Review* 47, 107-128.
- Williamson, S. 2008. "Monetary policy and distribution," *Journal of Monetary Economics* 55, 1038-1053.
- Williamson, S. 2014a. "Scarce Collateral, the Term Premium, and Quantitative Easing," working paper.
- Williamson, S. 2014b. "Central Bank Purchases of Private Assets," working paper.
- Williamson, S., and Wright, R. 2010a. "New monetarist economics: methods," *Review* Federal Reserve Bank of St. Louis, May issue, pages 265-302.
- Williamson, S. and Wright, R. 2010b. "New Monetarist Economics: Models," *Handbook of Monetary Economics*, in: Benjamin M. Friedman & Michael Woodford (ed.), *Handbook of Monetary Economics*, edition 1, volume 3, chapter 2, pages 25-96 Elsevier.
- Woodford, M. 2003. *Interest and Prices*, Princeton University Press, Princeton, NJ.
- Woodford, M. 2012. "Methods of Policy Accommodation at the Interest-Rate Lower Bound," *Proceedings - Economic Policy Symposium - Jackson Hole*, Federal Reserve Bank of Kansas City, 185-288.

Figure 1: Fed Funds Rate and PCE Inflation Rate

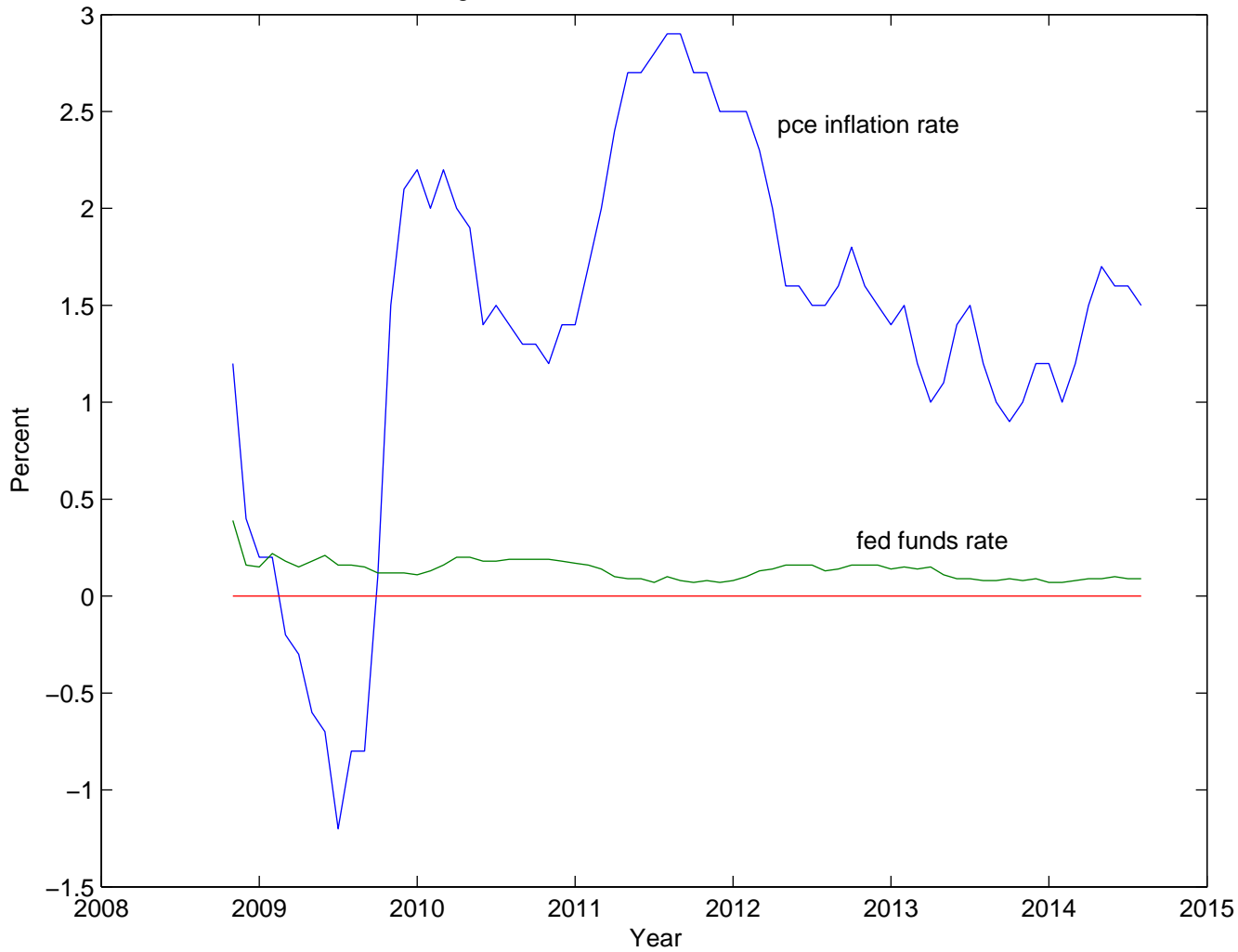


Figure 2: Fed Funds Rate – Inflation Rate

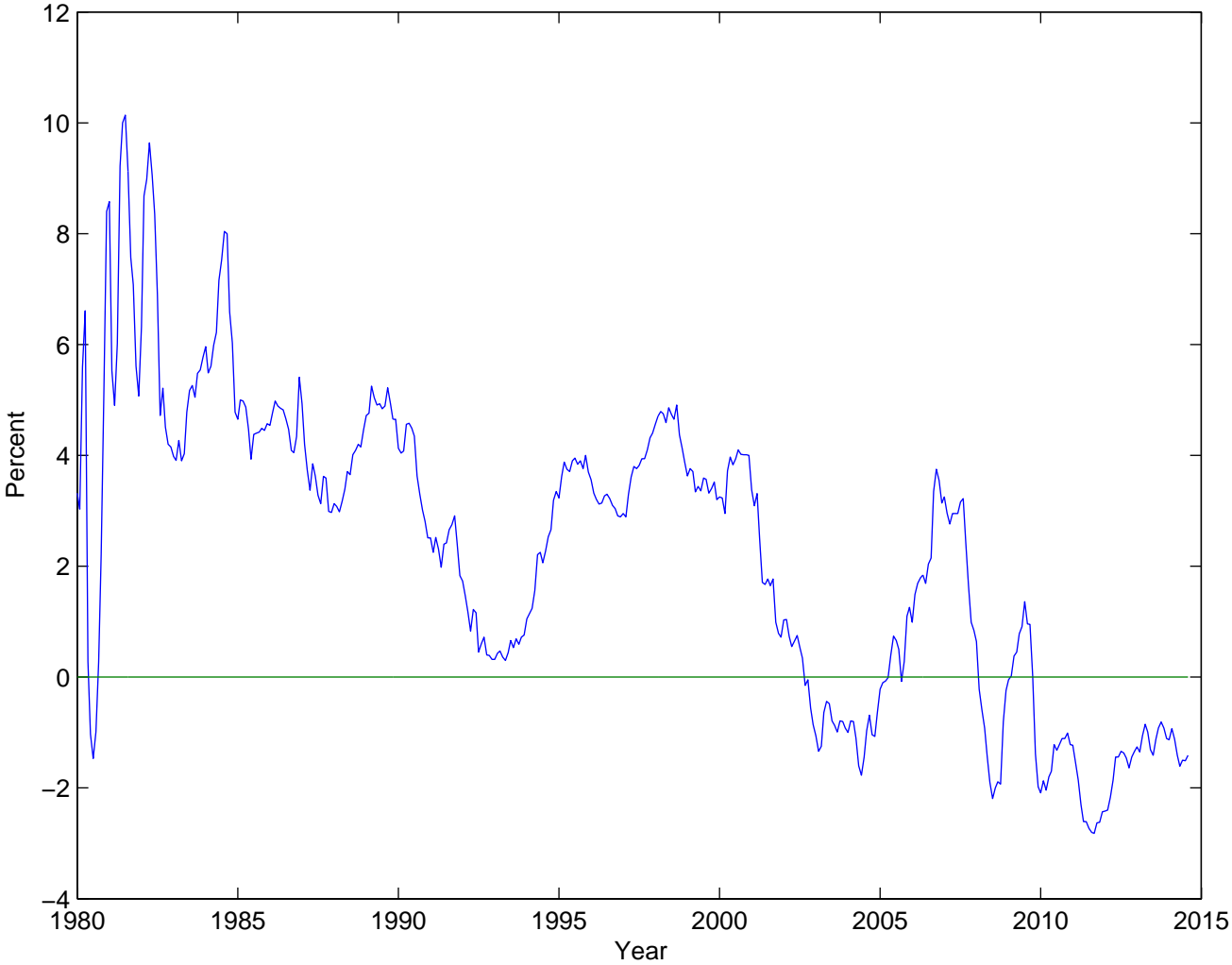


Figure 3: Five-Year TIPS Yield

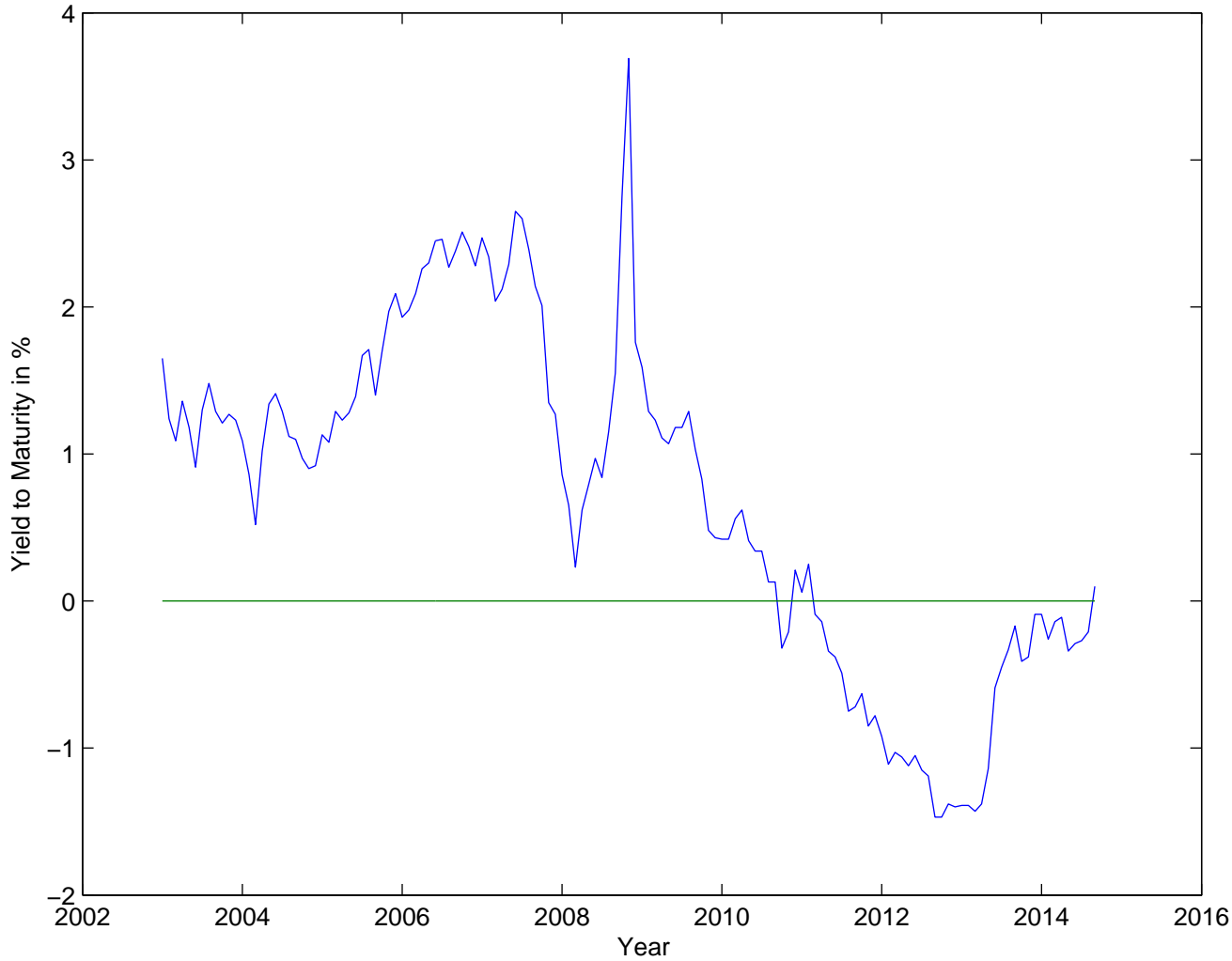


Figure 4: Phillips Curve?

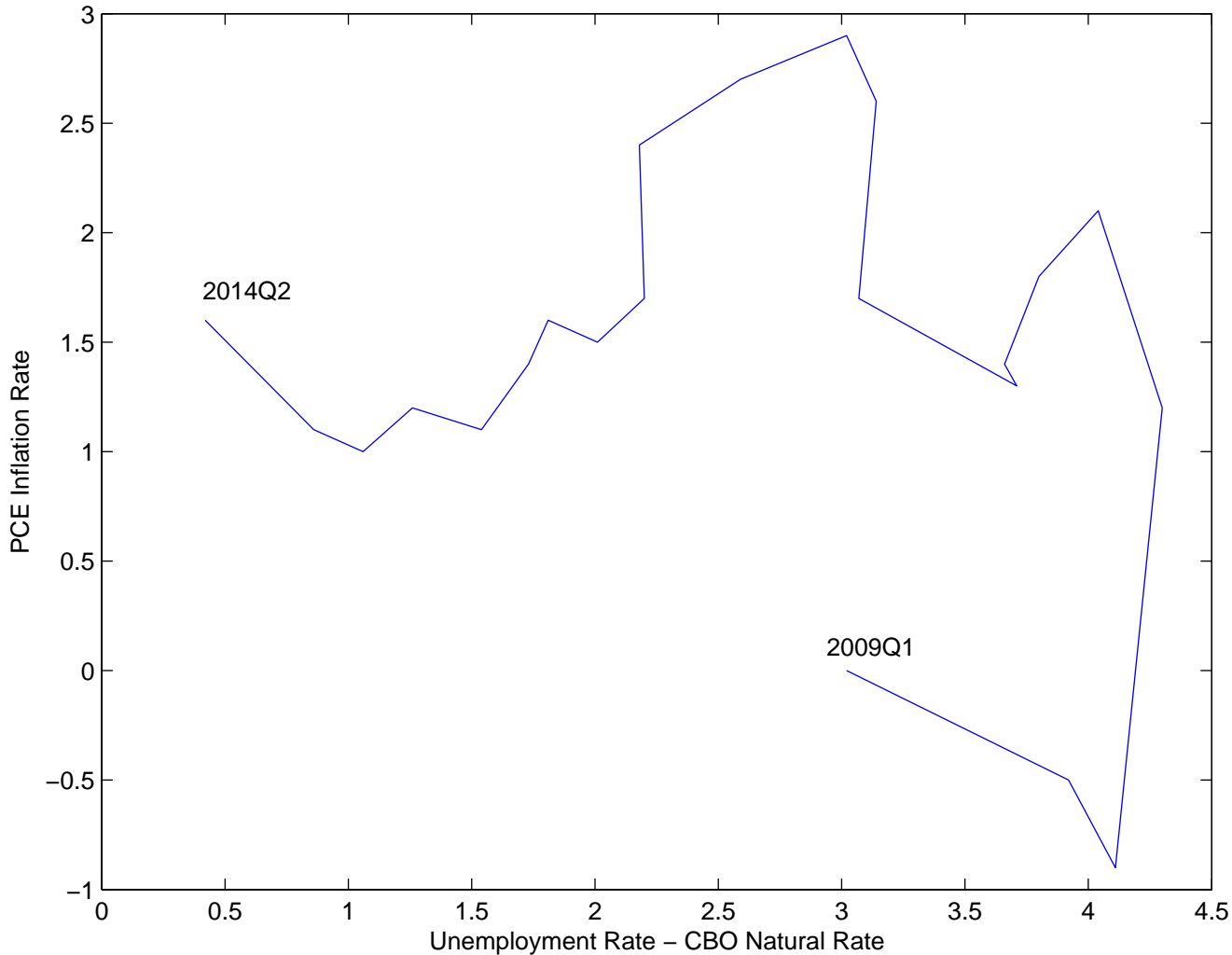


Figure 5: Price Level and Money Base

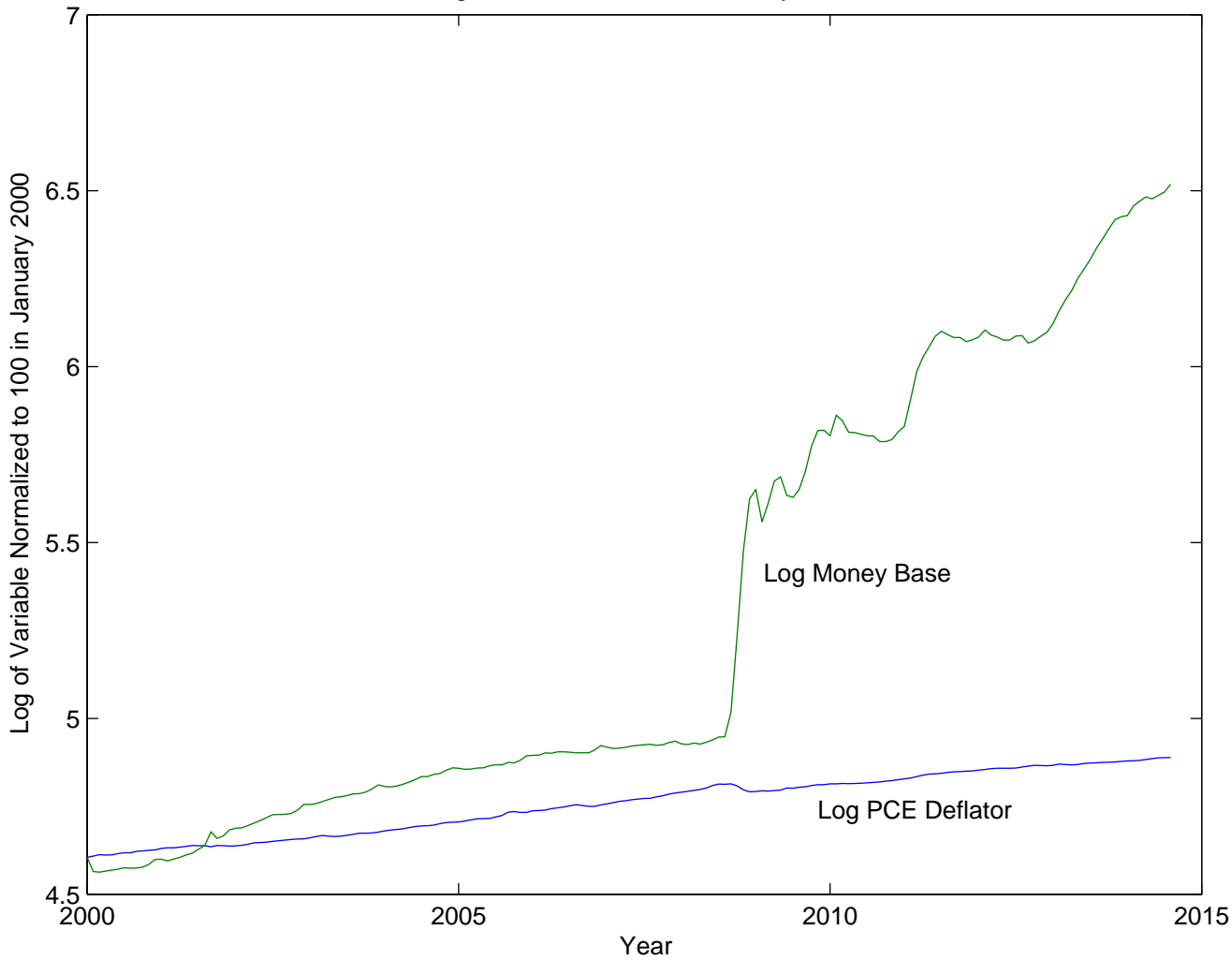


Figure 6: Velocity of M1

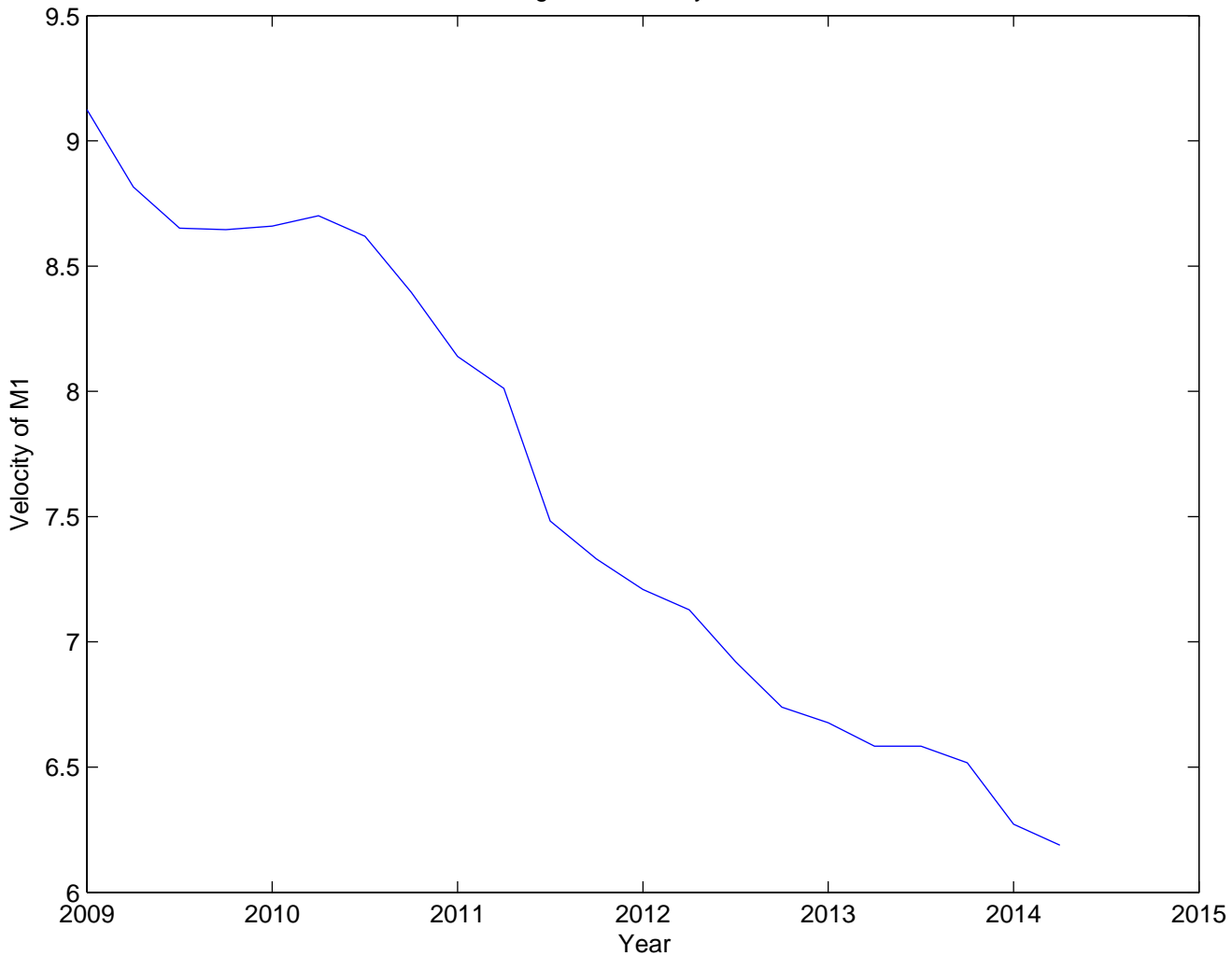


Figure 8: A Decrease in V or κ , Constrained Equilibrium

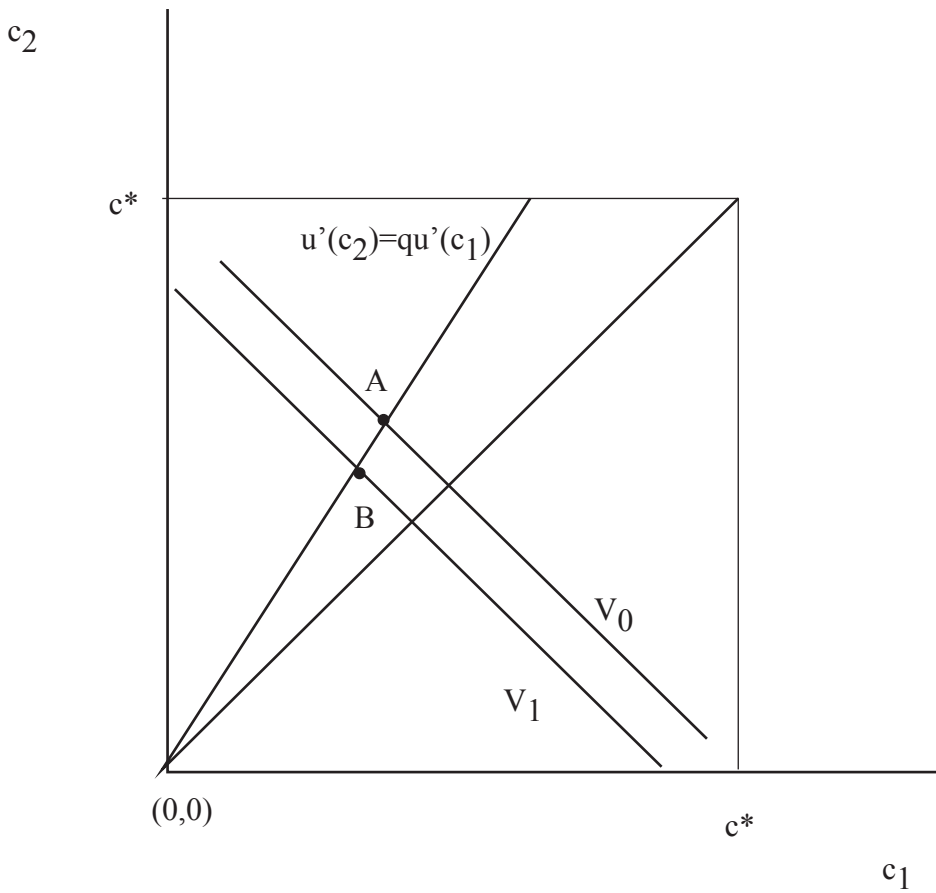


Figure 9: Government Deficit as a Function of V_t

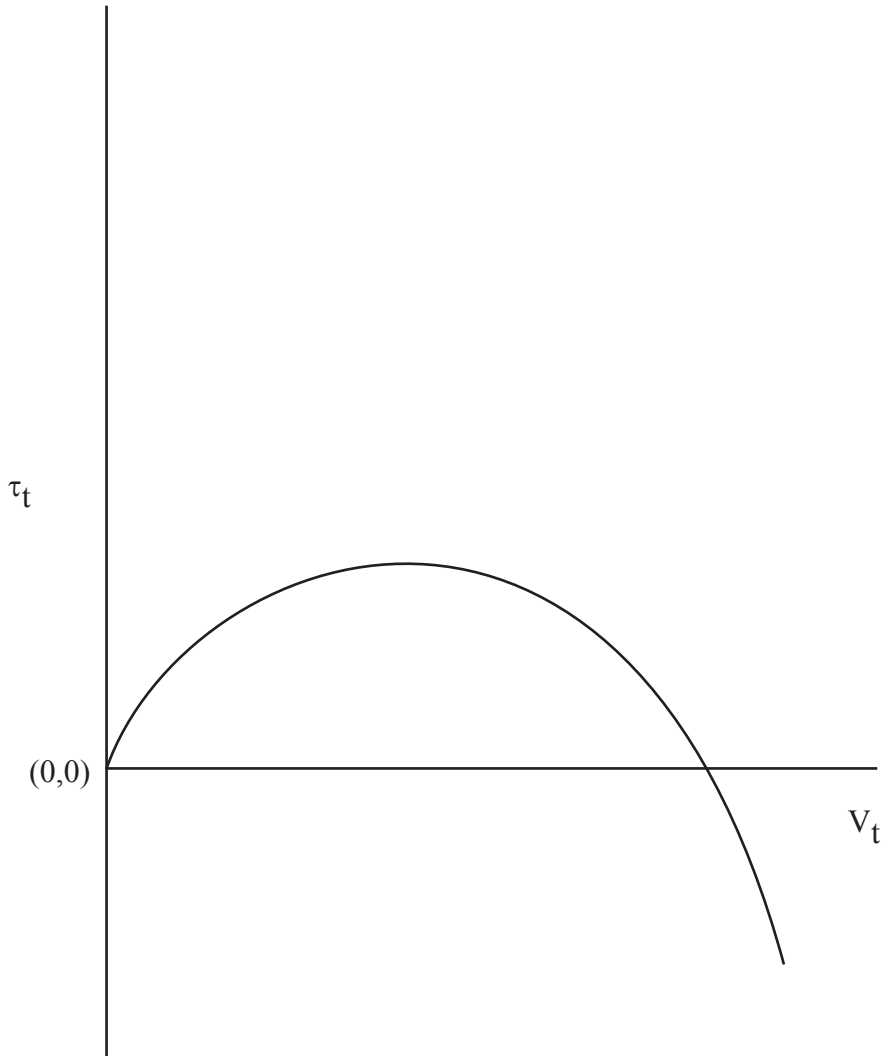


Figure 10: Taylor Rule Equilibrium, Unconstrained, $\alpha > 1$

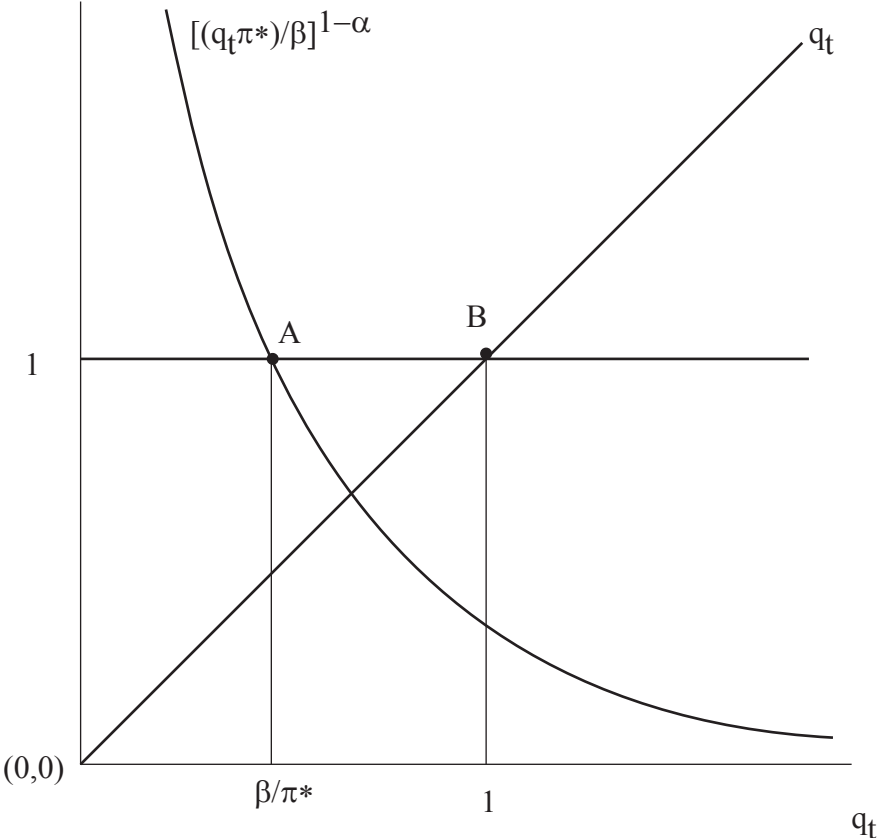


Figure 11: Taylor Rule Equilibrium, Unconstrained, $\alpha < 1$

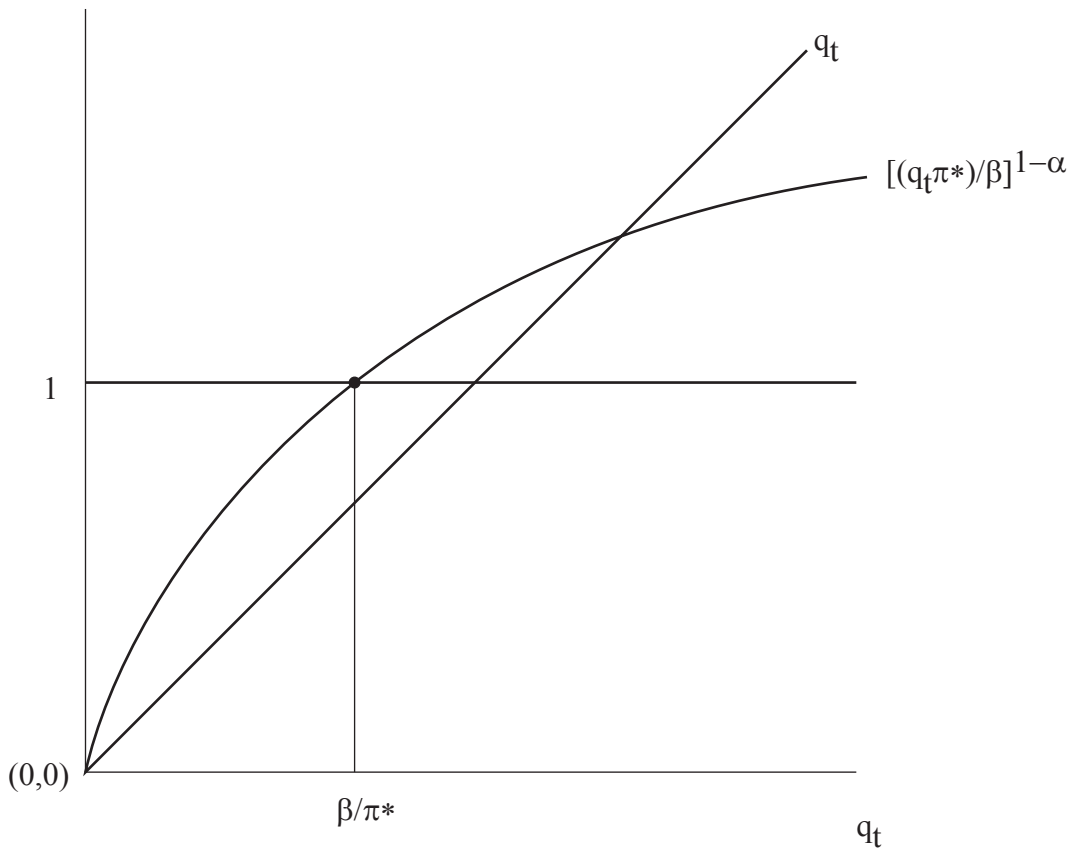


Figure 12: Taylor Rule Equilibrium, Unconstrained, $\alpha > 1$, Endogenous Real Interest Rate

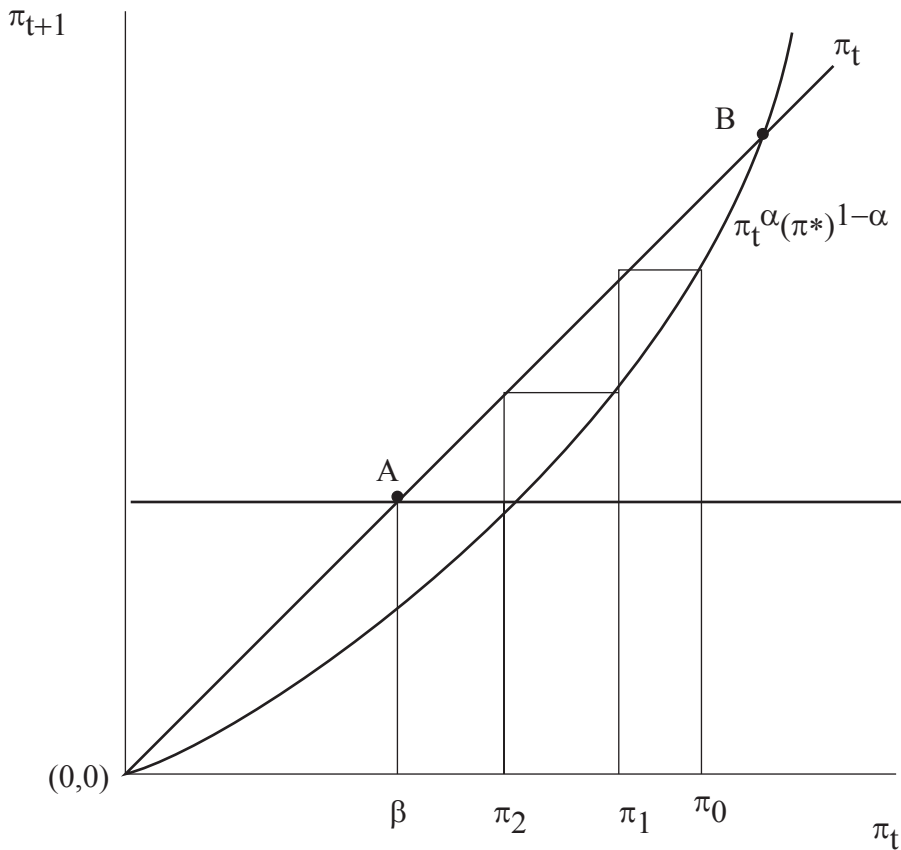


Figure 13: Taylor Rule Equilibrium, Constrained, $\alpha < 1$, Endogenous Real Interest Rate

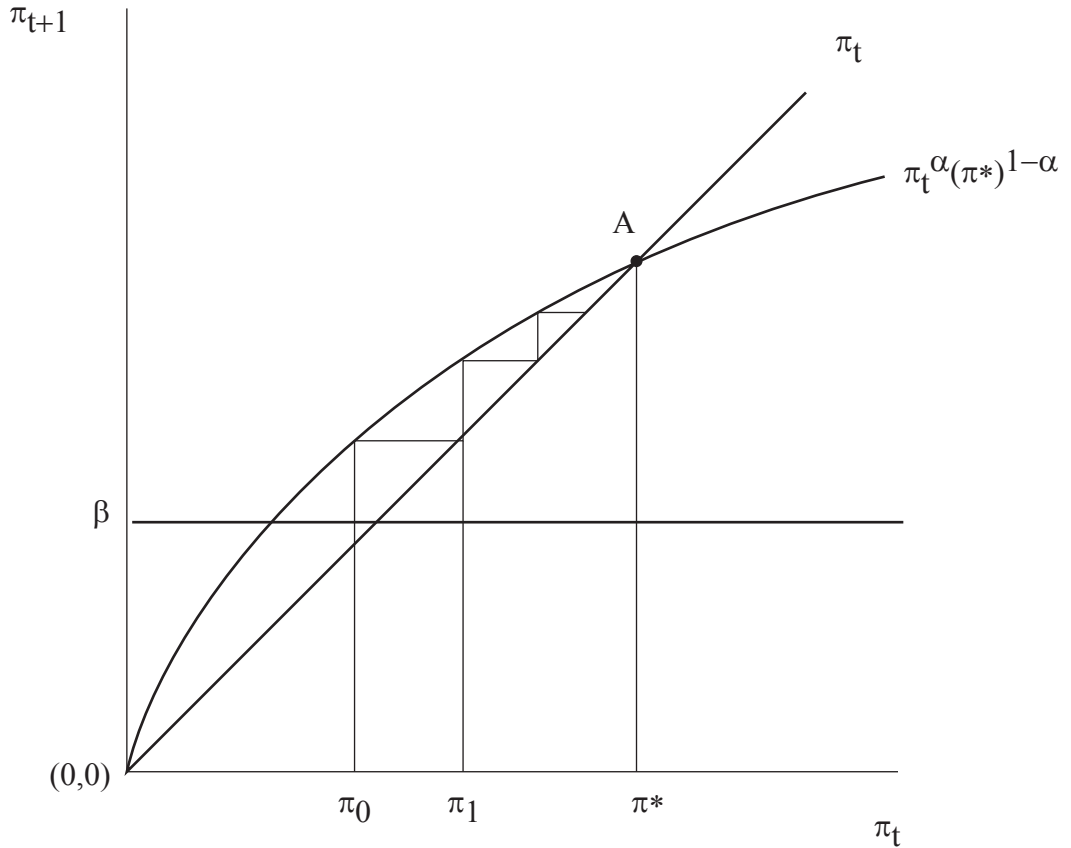


Figure 14: Taylor Rule Equilibrium, Constrained, $\alpha > 1$, Endogenous Real Interest Rate

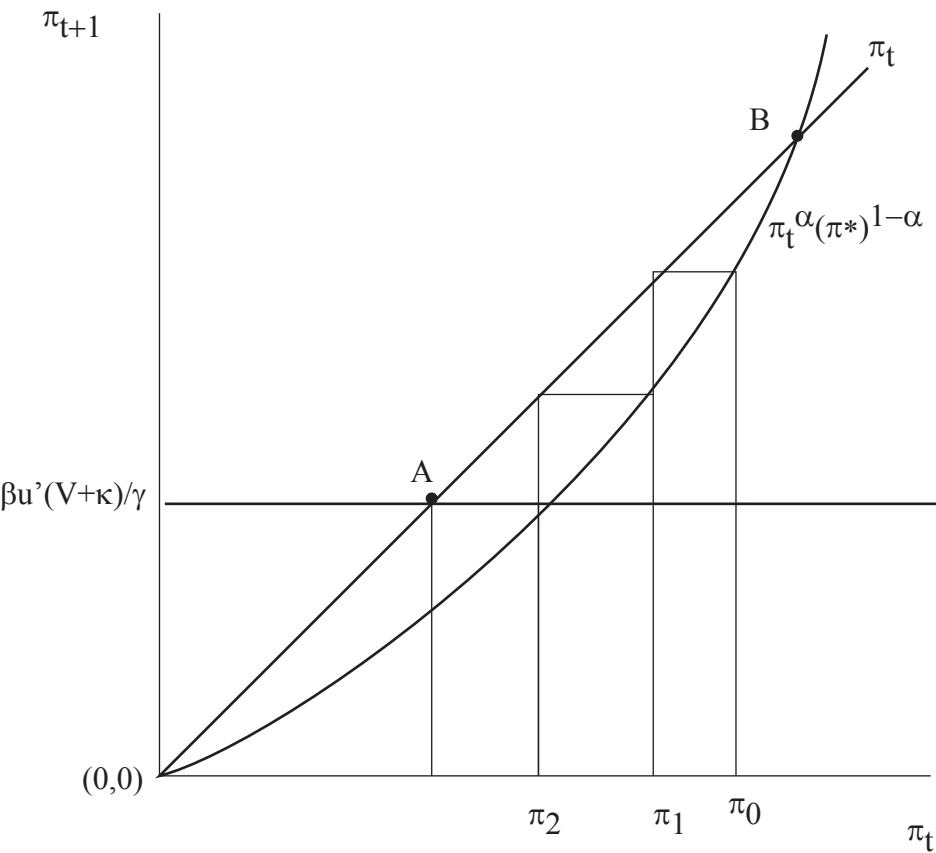


Figure 15: Taylor Rule Equilibrium, Constrained, $\alpha < 1$, Endogenous Real Interest Rate

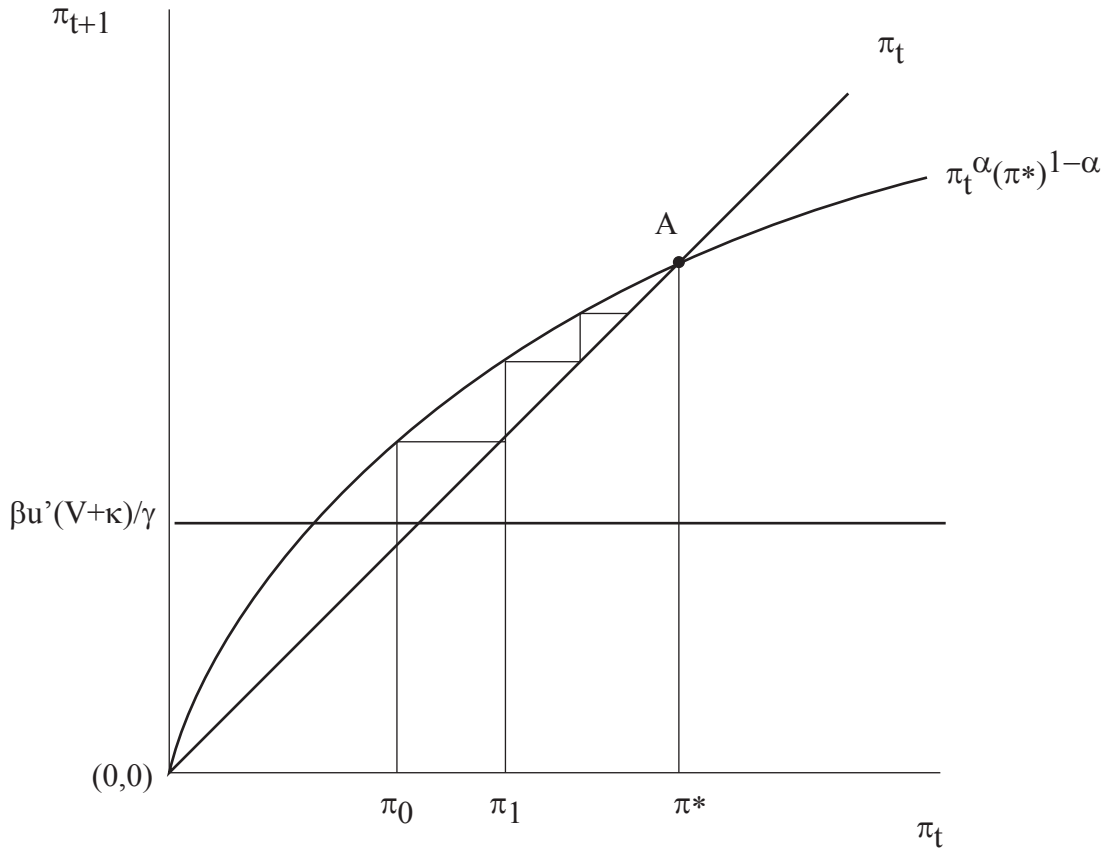


Figure 16: Effects of a Nominal Interest Rate Increase in Period T

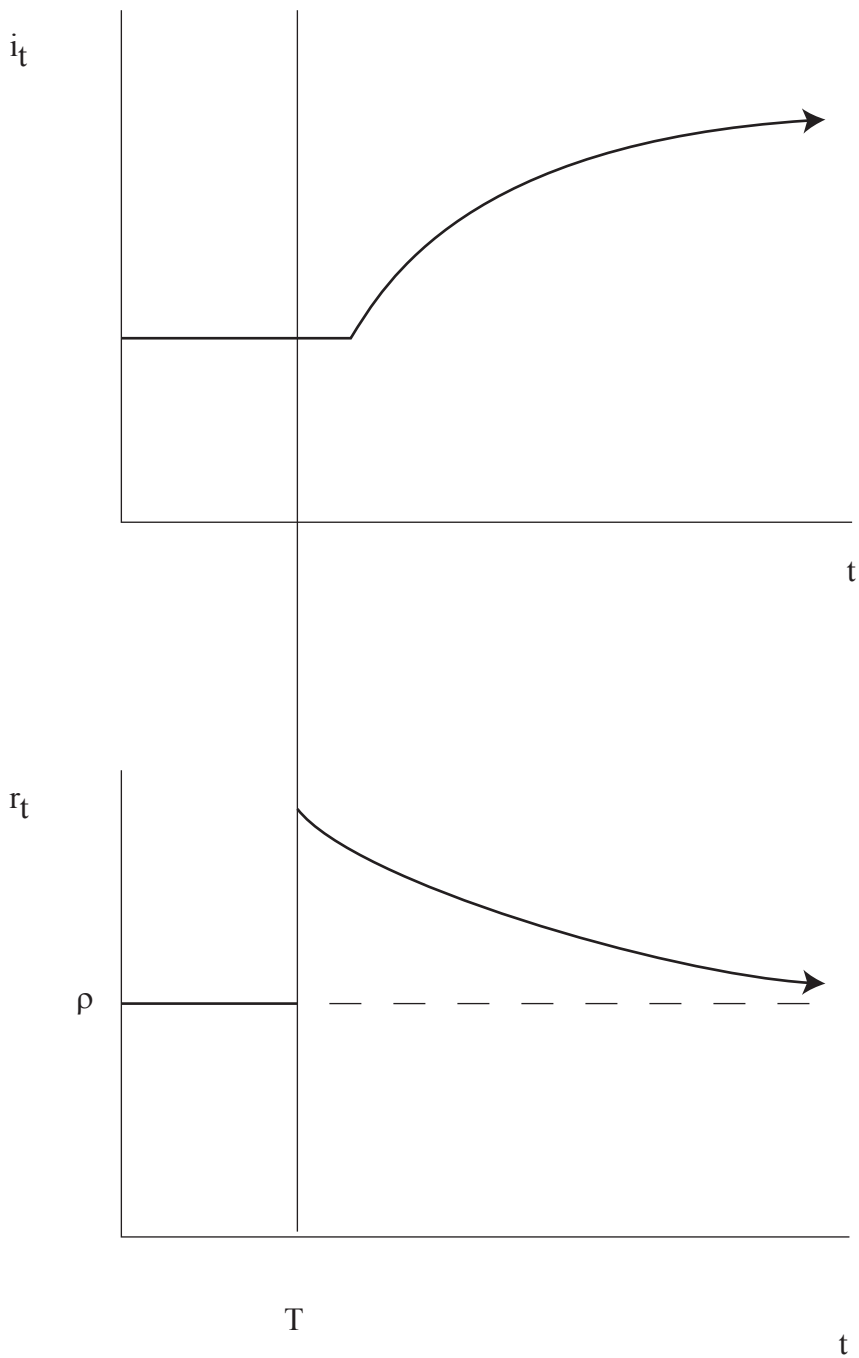


Figure 17: Taylor Rule Dynamics

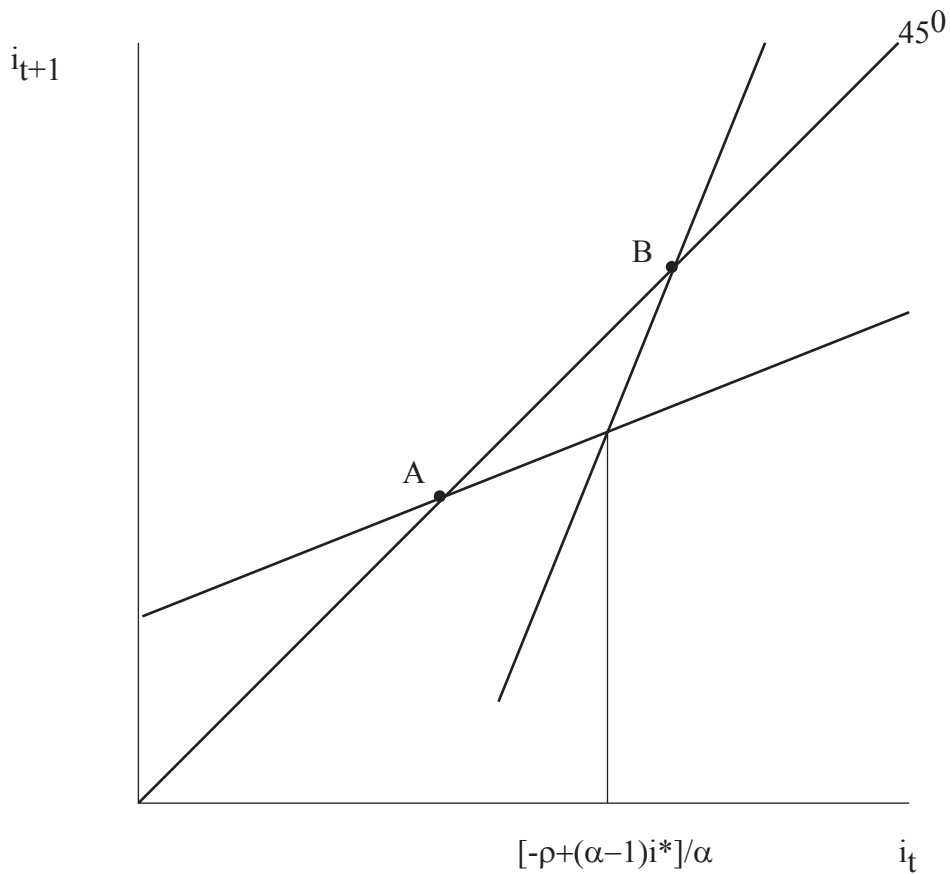


Figure 18: Taylor Rule Dynamics, $\alpha < 1$

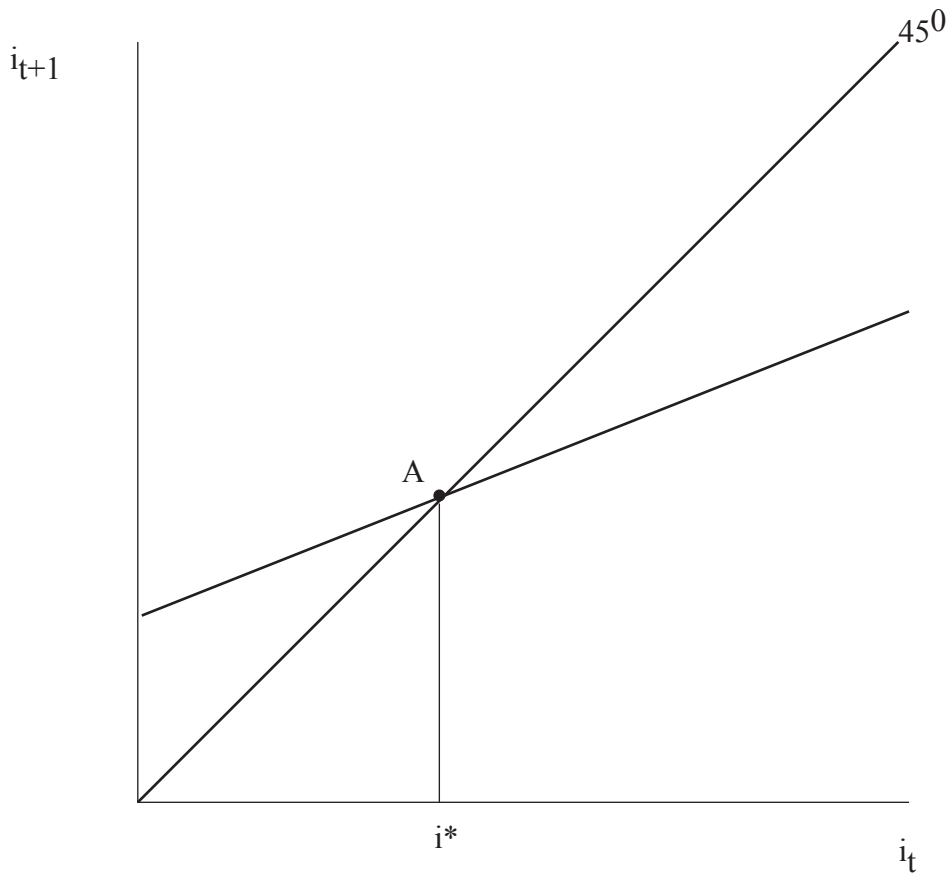


Figure 19: Taylor Rule With Binding Asset Market Constraint

