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On the Social Usefulness of Fractional Reserve Banking *

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ABSTRACT

This paper presents a simple means-of-payment-in-advance model where households can purchase the single consumption good with either deposits from price taking banks or currency. We show that even if facilitating trade through deposits costs more than using currency, such trade can occur in equilibrium if banks face a low enough reserve ratio. Thus it is possible that fractional reserve banking occurs in the unique equilibrium, but is strictly dominated by the equilibrium associated with a 100% reserve ratio. We further show that if households can, with some probability, access all of their wealth just prior to making purchases, the social benefits of fractional reserve banking converge to zero as this probability approaches one.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction:

Fractional reserve banking is, to put it mildly, problematic. Banks with fractional reserves are subject to runs and panics with arguably enormous external effects. Further, banks are costly. Even abstracting from runs and panics, the banking sector uses up real resources — labor and capital — which could be put to alternative uses. But fractional reserve banking is also ubiquitous. It appears to occur throughout history, with our without bailouts, making it difficult to argue that this ubiquity is due simply to bailouts or government subsidies. Usually, the ubiquity of an economic arrangement itself argues that this arrangement serves a valuable social purpose. Historically, one such purpose is that banks have allowed individuals and firms to pay for goods and services through their provision of bank checks and other widely accepted claims. Therefore, those individuals and firms haven't had to resort to costly barter or non-interest bearing specie trade.

The observation that individuals and firms voluntarily choose to use bank deposits as means of payment clearly implies that banks and bank deposits serve a *privately* useful function. Here we argue that the use of bank deposits may have *social* costs. In one simple example we provide, banks are worse than useless in that they use up real resources but provide no societal benefit. The more general point we wish to make is that the private benefits from creating private payments systems may exceed the social benefits. We go on to argue that as technology changes to allow households to more easily (or less expensively) access all of their wealth when needed, the benefits associated with fractional reserve banking decrease, and thus now, or in the near future, the benefits of fractional reserve banking may no longer justify its associated risk to society.

The core of our argument rests on two simple ideas. If monetary policy deviates from the Friedman Rule of setting nominal interest rates to zero, private agents have incentives to set up alternative payment systems like bank deposits, which pay interest on the means of payment. In a competitive environment with free entry, these alternative systems are inherently fragile in the sense that they are subject to bank runs which impose social costs. These social costs are not internalized by private individuals and firms. Whether a bank deposit like system is socially desirable then comes down to a comparison of the benefits of bank deposit like systems stemming from the higher interest rates on bank deposits to the social costs from bank runs and other costs associated with banking.

At a more subtle theoretical level, competitive equilibria are inefficient in our set up because our model has so-called pecuniary externalities. We assume that transactions in goods markets are anonymous so that producers and consumers must use a trusted means of payment to make transactions. These trusted means of payment can consist either of fiat money (or, alternatively specie) or bank deposits. We model the anonymity of transactions by assuming that households face a means-of-payment constraint in making at least some of their purchases. (See Lagos-Wright (2005) for a similar model of the constraints imposed by anonymity.) Formally, this constraint implies that the price of consumption goods enters the consumption sets of households. Since prices enter consumption sets, the usual welfare theorems do not hold and competitive equilibria are are not necessarily efficient. If the monetary authority follows the Friedman Rule, the means of payment constraint is slack and the welfare theorems hold. If the monetary authority deviates from the Friedman Rule, equilibria are inefficient.

This inefficiency implies that private agents have incentives to set up interest bearing private means of payments such as deposits. Deposit creation drives up the effective money supply and raises prices. Private agents do not internalize this rise in the aggregate price level. Further, this deposit creation creates liabilities on bank balance sheets. These liabilities are matched by assets which consist of currency and productive capital held by banks. Banks use the income generated by productive capital to pay interest on deposits. We assume that bank runs disrupt the ability of bank managers to generate income from the productive capital they hold. This disruption implies that banks are necessarily fragile in the sense that if all depositors demand their funds, they will not be able to meet the interest obligations on their deposits. Thus, if all other agents demand their deposits it is rational for any given individual to do so as well so that bank runs are an ever present possibility. The loss of income from capital deployed in banks is a cost imposed on society.

Our paper is related to a large literature in money and banking. Our paper has in common with Aiyagari, Braun, and Eckstein (1998) that private money creation uses real resources so that inflation stimulates the demand for private money and has real costs. They focus on the welfare costs of inflation while our focus is on examining the benefits and costs of changes in reserve requirements. Our paper is also related to Monnet and Sanches (2011) who show, among other results, that 100% reserve requirements may be undesirable. Their result derives from lack of commitment of bankers to repay depositors. We also show that 100% reserve requirements may be undesirable, but our results derive from comparing the costs of private money creating versus the insurance benefits associated with private money. He and Wright (2005) develop a search model of fractional reserve banking, where, like ours, fractional reserve banking occurs in equilibrium, but do not focus on welfare implications.

Other papers, see for example Gu et.al. (2012) have developed theories of banking using mechanism design approaches. Almost by construction, such theories imply that allocations are efficient. While we believe such an approach is very useful, in our paper we focus on inefficiencies that could arise when private and social interests do not coincide.

Finally, our paper is related to an extensive literature on so called "pecuniary externalities" that arise when prices enter consumption or production sets. See, for instance, Kehoe and Levine (1993), Kiyotaki and Moore (1997), Lorenzoni (2008), and Hart and Zingales (2011). Perhaps the most closely related of these is Hart and Zingales (2011) who develop a four-period model in which they argue that private liquidity provision can be inefficiently high due to a pecuniary externality.

They do not analyze the welfare effects of fractional reserve banking.

2. The Model:

The model is an infinitely repeated, means-of-payment-in-advance economy. There are a unit continuum of identical households, a continuum of price-taking banks, and a monetary authority which makes lump sum monetary transfers to households. Time is discrete and is denoted $t \in \{0, 1, 2, \ldots\}$.

The commodities in the economy are a single non-storable consumption good, k "trees" as well as fiat currency. If a tree is held by a household, it yields y units of the single non-storable consumption good. If a tree is held by a bank, it yields y units of the consumption good if the bank is not hit with a "run shock" and zero units of the consumption good is hit with a run shock. Let $\xi(1)$ be the constant fraction of banks hit with run shocks and thus declared insolvent.

The fiat currency can be held directly by households (m), in which case it can be used to facilitate trade, or be held by banks (denoted g^b) and used as reserves. Thus for each date $t \geq 0$,

$$m_t + g_t^b = G_t, \tag{1}$$

where G_t is the aggregate supply of currency at date t. We assume $G_0 = 1$ and $G_{t+1} = (1 + \eta)G_t$. At each date t, a bank can create deposits, d_t. Deposits are subject to a legal reserve constraint that $\alpha d_t \leq g_t^b$, where g_t^b is the bank's holdings of currency, and $0 < \alpha \leq 1$ is a reserve ratio. The liabilities of banks consist of deposits, and their assets consist of currency and trees. Banks may trade deposits either for currency or for trees. Since our model has free entry into banking in any given period, the net worth of banks is zero and thus $d = g^b + p^k k^b$.

Households cannot eat the fruit of their own trees but must instead trade with other households or banks. Households face two idiosyncratic shocks labelled $\theta \in \Theta$ and $z \in \{0, 1\}$. The shock

θ is a preference shock which occurs with probability $\pi(\theta)$. A high value of θ is associate with a high marginal utility of consumption for any level of consumption. The shock $z = \{0, 1\}$ determines the extent of access to household wealth. If $z = 0$ (which occurs with probability $\sigma(0)$) households must purchase consumption with either currency or deposits. If $z = 1$, households can use currency, deposits or the rest of their wealth to make purchases.

If a household purchases a unit of consumption with currency, (the selling household receives currency in exchange for the consumption good), we assume it occurs with a physical cost ϕ^m in terms of the consumption good. Specifically, if a household purchases $(1 + \phi^m)c$ units of the consumption good with currency, it only can actually consume c units of this purchase, with $\phi^m c$ units of the consumption good used up facilitating the trade. Likewise, if such trade occurs using bank deposits, (the selling household receives bank deposits in exchange for the consumption good), it occurs with a physical cost $\phi^d > \phi^m$ and if trade occurs using neither currency nor deposits (which can happen only if $z = 1$), it occurs with physical cost $\phi^b > \phi^d$. These costs (ϕ^m, ϕ^d, ϕ^b) stand in for the real resources (labor and capital) of running various systems for facilitating trade and the assumption that $\phi^b > \phi^d > \phi^m$ captures the idea that more sophisticated means-of-payment systems uses more real resources. Formally, they are simply iceberg costs.

Households care about streams of consumption according to the per-period utility function $u(c_t, \theta_t)$ where c_t is consumption and $\theta_t \in \Theta$ is a publicly observed idiosyncratic i.i.d. preference shock. (Assume $\lim_{c \to 0} u_c(c, \theta) = \infty$ for all θ.)

A. Markets:

At the beginning of each period, t, we assume that households and banks can trade trees, currency, and deposits. They can also trade contingent promises to deliver currency at the beginning of the next period as a function of their (yet to be realized) taste shock $\theta_t \in \Theta$ and liquidity shock $z_t \in \{0, 1\}$. Let $q_t(\theta_t, z_t)$ be the price of claim to a unit of (θ_t, z_t)-contingent currency next period and let $x_t(\theta_t, z_t)$ denote the quantity of (θ_t, z_t)-contingent currency a household purchases. (If $x_t(\theta_t, z_t) < 0$, then the household has sold a promise to deliver currency if its shocks turn out to be (θ_t, z_t).) Market clearing in these promises implies

$$\sum_{\theta,z} \pi(\theta)\sigma(z)x(\theta, z) = 0. \tag{2}$$

We also assume households are free at this point to obtain currency directly from the bank, one for one, by presenting deposits to the bank. That is, a deposit is considered to be a legal right to trade deposits for currency at this point in time with the bank, one-for-one. Households and banks may also trade currency and new deposits, again on a one-for-one basis. We further assume, as part of this market structure, that a deposit entails a legal requirement on the bank to pay a market determined rate of interest, r_t, to the depositor, one period later, if it has the resources to do so, which will occur only if the bank is not hit with a run shock. These assumptions deliver the household budget constraint in this market as

$$m_t + d_t + p_t^k k_t^c + \sum_{\theta_t, z_t} q_t(\theta_t, \psi_t)x_t(\theta_t, z_t) \le w_t, \tag{3}$$

where w_t is the household's beginning of period wealth in terms of trees, currency and deposits, p_t^k is the price of a tree, and k_t^c is the quantity of trees the household chooses to purchase.

After the financial markets close, households then split into shopper-seller pairs. Sellers receive yk_t^c units of the consumption good from the household's trees and shoppers realize the household's i.i.d. preference shock $\theta_t \in \Theta$ and access-to-wealth shock $z_t \in \{0, 1\}$ and banks realize their run shocks. We assume at this point that shoppers can demand immediate payment in currency for their deposits in any bank, but only do so for the $\xi(1)$ fraction of banks hit with run shocks. To justify this as maximizing behavior for shoppers, we assume if a measure one of shoppers demands such payment, the bank is insolvent and its trees bear no fruit, any household whose shopper does not demand payment receives nothing, and households whose shopper demanded payment receive the amount of their deposit back at the beginning of the next period (by the bank selling its assets) but without an interest payment.

Sellers inelastically supply yk_t^c units to market. Shoppers pay p_t^c for the consumption good regardless of the form of payment. (Sellers are indifferent between currency and deposits from either solvent or insolvent banks as payment since all three trade for one unit of currency at the beginning of the next period). Let $c_t^m(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) using currency, $c_t^d(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) household using deposits, and $c^b(\theta_t, z_t)$ denote the consumption purchases of a type (θ_t, z_t) household using the household's general wealth. Shoppers face means-of-payment-in-advance constraints such that for all (θ_t, z_t)

$$p_t^c(1 + \phi^m)c_t^m(\theta_t, z_t) \leq m_t, \tag{4}$$

$$p_t^c(1 + \phi^d)c_t^d(\theta_t) \leq d_t, \tag{5}$$

and for all θ_t,

$$c_t^b(\theta_t, 0) = 0. \tag{6}$$

Note that the timing does not allow the household's allocation of its wealth between trees, currency, and deposits to depend on its preference shock θ, access shock z, or which banks are hit by solvency shocks. After trade, households receive utility from consumption given by $u(c_t^m(\theta_t, z_t) + c_t^d(\theta_t, z_t) + c_t^b(\theta_t, z_t), \theta_t)$.

At the end of period t, households receive transfers $x_t(\theta_t, z_t)$, interest on solvent bank deposits $r_t\xi(0)d_t$ and a lump-sum currency transfer from the monetary authority η. (Note that interest is paid on deposits held *before* the market for the consumption good. This assumption ensures that sellers equally value currency and deposits.) For deposits in banks hit with an insolvency shock, we assume insolvent banks return their assets to depositors at the beginning of the next period. For each such bank, $d_t = g_t^b + p_t^k k_t^b$. Thus in any stationary equilibrium (where $p_t^k = p_{k,t+1}$), the insolvent bank has exactly enough assets to repay depositors (without interest). Thus a household starts period $t+1$ with wealth (per unit of date $t+1$ currency)

$$\begin{aligned} w_{t+1}(\theta_t, z_t) \quad = \quad & \frac{1}{1+\eta}\Big(\eta + x_t(\theta_t, z_t) + (1+r_t)\xi(0)d_t + \xi(1)d_t + p_t^c y k_t^c + p_t^k k_t^c \\ & -p_t^c((1+\phi^m)c_t^m(\theta_t, z_t)) + (1+\phi^d)c_t^d(\theta_t, z_t) + (1+\phi^b)c_t^b(\theta_t, z_t))\Big). \end{aligned} \tag{7}$$

3. Allocations:

Let $h^t = (\theta_0, z_0, \ldots, \theta_t, z_t)$. An allocation (with prices) is a sequence

$$\{c_t^m(h^t), c_t^d(h^t), c_t^b(h^t), m_t(h^{t-1}), d_t(h^{t-1}), x_t(\theta_t, z_t)(h^{t-1}), g_t^b, q_t(\theta, z), p_t^c, p_t^k, r_t\}_{t=0}^{\infty}. \tag{8}$$

An allocation is resource feasible if for all $t \geq 0$

$$\sum_{h^t} \pi(\theta_0)\ldots\pi(\theta_t)\sigma(z_0)\ldots\sigma(z_t)((1+\phi^m)c_t^m(h^t) + (1+\phi^d)c_t^d(h^t) + (1+\phi^b)c_t^b(h^t)) = (k_t^c + \xi(0)k_t^b)y, \tag{9}$$

$$\sum_{h^{t-1}} \pi(\theta_0)\ldots\pi(\theta_{t-1})\sigma(z_0)\ldots\sigma(z_t)\Big(m_t(h^{t-1})\Big) + g_t^b = G_t, \tag{10}$$

$$\sum_{h^{t-1}} \pi(\theta_0)\ldots\pi(\theta_{t-1})\sigma(z_0)\ldots\sigma(z_t)\Big(k_t^c(h^{t-1})\Big) + k_t^b = k, \tag{11}$$

and

$$\sum_{h^{t-1}} \pi(\theta_0)\dots\pi(\theta_{t-1})\sigma(z_0)\dots\sigma(z_t)x_t(\theta_t, z_t)(h^{t-1}) = 0. \tag{12}$$

An allocation is a competitive equilibrium if it is resource feasible and solves the household and bank problems outlined below. To simplify somewhat, we only consider *stationary* allocations where prices p_t^c, p_t^k, r_t, and $q_t(\theta, z)$ are all constants.

A. Household Problem:

Given constant prices, the household problem is recursive (in beginning of period wealth relative to the aggregate currency supply) and can be expressed as

$$\begin{aligned} V(w) \quad \equiv \quad & \max_{c^m(\theta,z),c^d(\theta,z),c^b(\theta,z),d,m,k^c,x(\theta,z)} \qquad (13) \\ & \sum_{\theta} \pi(\theta)\sigma(z)[u(c^m(\theta, z) + c^d(\theta, z) + c^b(\theta, z), \theta) + \beta V(w'(\theta, z))] \end{aligned}$$

subject to

$$m + d + p^k k^c + \sum_{\theta} q(\theta, z)x(\theta, z) \leq w, \tag{14}$$

$$p^c(1 + \phi^m)c^m(\theta, z) \leq m, \text{ for all } (\theta, z) \tag{15}$$

$$p^c(1 + \phi^d)c^d(\theta, z) \leq d, \text{ for all } (\theta, z) \tag{16}$$

$$\begin{aligned} w'(\theta, z) \quad = \quad & \frac{1}{1+\eta}\Big(\eta + x(\theta, z) + (1 + r)\xi(0)d + \xi(1)d + p^c y k^c + p^k k^c \qquad (17) \\ & -p^c((1 + \phi^m)c^m(\theta, z)) + (1 + \phi^d)c^d(\theta, z) + (1 + \phi^b)c^b(\theta, z))\Big) \end{aligned}$$

as well as $c^b(\theta, 0) = 0$ for all θ and that all choice variables other than $x(\theta, z)$ be non-negative. Let μ be the Lagrange multiplier on the budget constraint (14) and $\pi(\theta)\sigma(z)\gamma^m(\theta, z)$ and $\pi(\theta)\sigma(z)\gamma^d(\theta, z)$ be the Lagrange multipliers on the means of payment in advance constraints (15) and (16).

B. Bank Problem:

At the beginning of each period, a bank is free to create and sell deposits, d, subject to constraints that $\alpha d \leq g^b$, where g^b is the bank's reserve holdings of currency and subject to $d = g^b + p^k k^b$ or that proceeds from selling deposits must go either to purchasing reserves or trees. Further, the creation of a deposit d obligates the bank to pay rd to the depositor of record when the deposit was sold if it is solvent, and pay d to current holder of the deposit in the following period regardless of whether it is solvent. The banks maximization problem is to choose (d, g^b, k^b) to solve

$$\max_{d, g^b, k^b} \; d - g^b - p^k k^b + p^c \xi(0) y k^b - q(\xi(0) r d + d - g^b - p^k k^b) \tag{18}$$

subject to $\alpha d \leq g^b$ and $d = g^b + p^k k^b$, where $q = \sum_{\theta, z} q(\theta, z)$ is the price of next period currency in terms of current period currency.

4. Stationary Equilibria:

To characterize stationary equilibria, we first show that our assumption that households can trade one-period-ahead contingent claims implies household wealth is the same at the beginning of all dates (and thus we do not need to track distributions of household wealth.)

To see this, first consider the first order condition with respect to $x(\theta, z)$ in the household problem, or

$$\beta \pi(\theta) \sigma(z) V'(w'(\theta, z)) \frac{1}{1 + \eta} = q(\theta, z) \mu. \tag{19}$$

If $q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta)$, this becomes

$$V'(w'(\theta)) = \mu, \tag{20}$$

for all θ. Thus given $q(\theta) = \beta\pi(\theta)\sigma(z)/(1+\eta)$, $w'(\theta)$ is independent of θ. Further, standard envelope arguments imply $V'(w) = \mu$, thus

$$V'(w'(\theta, z)) = V'(w), \tag{21}$$

for all (θ, z), which implies $w'(\theta, z) = w$ for all (θ, z). In words, when $q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta)$, a condition of household optimization is that household wealth is constant over all dates and all histories of preference and access-to-the-bank shocks. That

$$q(\theta, z) = \beta\pi(\theta)\sigma(z)/(1 + \eta) \tag{22}$$

is from now on imposed. Note that this implies that the non-contingent price of one-period-ahead currency in terms of current currency $q = \frac{1+\eta}{\beta}$.

Next, we show that profit maximization by banks implies a simple cutoff rule that the interest rate r must satisfy. Substituting the bank's reserve requirement, $\alpha d_t \leq g_t^b$, at equality and its flow of funds condition $d = g^b + p^k k^b$, into the bank's objective function (18), the bank's problem is to maximize

$$d\Big[(1 - \alpha)(1 - q) - qr\Big], \tag{23}$$

which is linear in d. Thus a necessary condition for maximization (which also implies zero profits) is that the bracketed expression in (23) equal zero, or

$$r = (1 - \alpha)(\frac{1}{q} - 1) = (1 - \alpha)(\frac{1 + \eta}{\beta} - 1), \tag{24}$$

or that the interest rate on deposits equals $(1-\alpha)$ times the nominal interest rate.

A. Further Characterization

Lemma 1. *There exists a cutoff interest rate on deposits* $r^* \equiv \frac{1}{\xi(0)}\frac{1+\eta}{\beta}\frac{\phi^d-\phi^m}{1+\phi^m}$ *such that if* $r > r^*$, *household optimization implies* $m = 0$ *and* $c^m(\theta,z) = 0$ *for all* (θ,z) *and if* $r < r^*$, $d = 0$ *and* $c^d(\theta,z) = 0$ *for all* (θ,z).

Proof. Consider the household problem. The first order condition with respect to d (ignoring non-negativity and imposing (20)) delivers

$$(1-\beta\frac{1+\xi(0)r}{1+\eta})\mu = \sum_{\theta,z}\pi(\theta)\sigma(z)\gamma^d(\theta,z). \tag{25}$$

Let $c(\theta,z) = c^m(\theta,z) + c^d(\theta,z) + c^b(\theta,z)$. The first order condition with respect to $c^d(\theta,z)$ and imposing (20) delivers

$$u_c(c(\theta,z),\theta) = p^c(1+\phi^d)(\frac{\beta\mu}{1+\eta} + \gamma^d(\theta,z)), \tag{26}$$

which implies

$$\sum_{\theta,z}\pi(\theta)\sigma(z)u_c(c(\theta,z),\theta) = p^c(1+\phi^d)(\frac{\beta\mu}{1+\eta} + \sum_{\theta,z}\pi(\theta)\sigma(z)\gamma^d(\theta,z)), \tag{27}$$

Together, (25) and (27) imply

$$\sum_{\theta,z}\pi(\theta)\sigma(z)u_c(c(\theta,z),\theta) = p^c(1+\phi^d)\mu(1-\frac{\beta\xi(0)r}{1+\eta})). \tag{28}$$

Next, the first order condition with respect to m (again ignoring non-negativity and imposing (20)) delivers

$$(1-\frac{\beta}{1+\eta})\mu = \sum_{\theta,z}\pi(\theta)\sigma(z)\gamma^m(\theta,z), \tag{29}$$

and the first order condition with respect to $c^m(\theta, z)$ and imposing (20) delivers

$$u_c(c(\theta, z), \theta) = p^c(1 + \phi^m)(\frac{\beta\mu}{1+\eta} + \gamma^m(\theta, z)), \tag{30}$$

which implies

$$\sum_{\theta,z} \pi(\theta)\sigma(z)u_c(c(\theta, z), \theta) = p^c(1 + \phi^m)(\frac{\beta\mu}{1+\eta} + \sum_{\theta,z} \pi(\theta)\sigma(z)\gamma^m(\theta, z)), \tag{31}$$

Together, (29) and (31) imply

$$\sum_{\theta,z} \pi(\theta)\sigma(z)u_c(c(\theta, z), \theta) = p^c(1 + \phi^m)\mu. \tag{32}$$

Note that the left hand sides of equation (28) and (32) are identical. Thus m and d can both be positive only if the right hand sides are equal or

$$(1 + \phi^d)(1 - \frac{\beta\xi(0)r}{1+\eta})) = 1 + \phi^m. \tag{33}$$

Let r^* solve (33) or

$$r^* = \frac{1}{\xi(0)} \frac{(1+\eta)}{\beta} \frac{\phi^d - \phi^m}{1 + \phi^d}. \tag{34}$$

The result follows immediately. □

An equilibrium with $d = 0$ and $m > 0$ is referred to as a *monetary equilibrium.* An equilibrium with $d > 0$ and $m = 0$ is referred to as a *banking equilibrium.* Note that this cutoff interest rate is independent of $\sigma(0)$ - the probability that a household faces a means-of-payment in advance constraint.

Corollary 1. *There exists a cutoff reserve ratio* $\alpha^* \equiv 1 - \frac{1}{\xi(0)} \frac{1+\eta}{1+\eta-\beta} \frac{\phi^d-\phi^m}{1+\phi^d}$ *such that if* $\alpha > \alpha^*$*, a* *banking equilibrium cannot exist, and if $\alpha < \alpha^*$, a monetary equilibrium cannot exist.*

Proof. Since r is a function of the reserve ratio α, one can solve for α^* - the cutoff reserve ratio by equating the right hand sides of (24) and (34) to derive

$$\alpha^* \equiv 1 - \frac{1}{\xi(0)} \frac{1+\eta}{1+\eta-\beta} \frac{\phi^d - \phi^m}{1+\phi^d}. \tag{35}$$

□

Corollary 2. *If $\frac{1}{\xi(0)} \frac{\phi^d-\phi^m}{1+\phi^d} > 1 - \frac{\beta}{1+\eta}$, then a banking equilibrium cannot exist. In particular, if $\frac{1+\eta}{\beta} = 1$, or the net nominal interest rate equals zero, then a banking equilibrium cannot exist. If $\frac{1}{\xi(0)} \frac{\phi^d-\phi^m}{1+\phi^d} < 1 - \frac{\beta}{1+\eta}$, then there exists a sufficiently low reserve ratio $\alpha > 0$ such that a monetary equilibrium does not exist.*

Proof. For households to be willing to transact using deposits, one needs

$$r \geq r^* = \frac{1}{\xi(0)} \frac{1+\eta}{\beta} \frac{\phi^d - \phi^m}{1+\phi^d}. \tag{36}$$

For banks to be willing to create deposits, they need $r \leq (1-\alpha)(\frac{1+\eta}{\beta} - 1)$. Thus for a banking equilibrium to exist one needs

$$\frac{1}{\xi(0)} \frac{1+\eta}{\beta} \frac{\phi^d - \phi^m}{1+\phi^d} \leq (1-\alpha)(\frac{1+\eta}{\beta} - 1). \tag{37}$$

Since the right hand side of (37) is decreasing in α, setting $\alpha = 0$ increases the right hand side as much as possible and relaxes (37) as much as possible. Thus if $\frac{1}{\xi(0)} \frac{1+\eta}{\beta} \frac{\phi^d-\phi^m}{1+\phi^d} > \frac{1+\eta}{\beta} - 1$, which simplifies (somewhat) to $\frac{1}{\xi(0)} \frac{\phi^d-\phi^m}{1+\phi^d} > 1 - \frac{\beta}{1+\eta}$, banking is incompatible with either household optimization or non-negative profits for banks for all $\alpha \geq 0$. Likewise, if $\frac{1}{\xi(0)} \frac{\phi^d-\phi^m}{1+\phi^d} < 1 - \frac{\beta}{1+\eta}$, then α can be set low enough (but still positive) such that (37) can be satisfied as a strict inequality. □

Corollary 3. *If $\sigma(0) > 0$ and $\alpha \neq \alpha^*$, then either $m > 0$ and $d = 0$ or $d > 0$ and $m = 0$.*

Proof. For any given θ, that $\sigma(0) > 0$ implies $u(c(\theta, 0), \theta)$ has positive weight in the household objective function. That $\lim_{c \to 0} u_c(c, \theta) = \infty$ implies $c(\theta, 0) > 0$. That $c(\theta, 0) = c^m(\theta, 0) + c^d(\theta, 0)$ (from $c^b(\theta, 0) = 0$) then implies either $c^m(\theta, 0) > 0$ or $c^d(\theta, 0) > 0$. Proposition 1 and the means-of-payment in advance constraints (15) and (16) then imply the result. □

It is a straightforward to prove a stationary equilibrium always exists. The results above imply that if $\alpha > \alpha^*$ and $\sigma(0) > 0$ then this equilibrium is unique and a monetary equilibrium. If $\alpha < \alpha^*$ and $\sigma(0) > 0$, then the unique equilibrium is a banking equilibrium. Thus we have shown that a private means-of-a-payment system that uses deposits is viable if and only if reserve requirements are sufficiently low. We have also shown that if reserve requirements are sufficiently low, private incentives to set up interest bearing means-of-payment systems are sufficiently strong that bank deposits are used in transactions.

5. Welfare in the $\sigma(0) = 1$ Economy:

To see the welfare effects of deposits substituting for currency, first consider the case where $\sigma(0) = 1$, or households must pay for all consumption with either currency or deposits. If Ω is a singleton (no preference shocks), then characterization and welfare analysis is greatly simplified, thus we consider this case first.

As noted in the previous section, if $\alpha > \alpha^*$, $d = 0$. Bank optimization then implies $g^b = 0$ and $k^b = 0$ (banks hold no reserves or capital). Market clearing for currency (equation (1)) implies all currency is held by households, or $m = 1$, where the currency supply is normalized to unity each period. Goods market clearing implies $c = yk/(1 + \phi^m)$ and the cash-in-advance constraint implies $p^c = \frac{1}{yk}$.

Thus the stationary monetary equilibrium with no ability to purchase consumption without currency and no preference shocks is quite simple. Each period, a household enters with one unit of currency and one tree and thus wealth $w = 1 + p^k k = 1 + \frac{1}{1-\beta}$ (from $p^k = p^c \frac{y}{1-\beta}$ and $p^c = \frac{1}{yk}$). It makes no trades in the initial financial market. The household's shopper goes off and purchases yk units of the consumption good while the household's seller sells the household's endowment, yk, for one unit of currency. The household is then handed η units of currency by the monetary authority. At the beginning of the next period, the household holds $(1+\eta)$ units of currency in terms of the previous period's numeraire, or one unit of currency in terms of the current numeraire, and the same tree.

Next, suppose $\alpha < \alpha^*$ and thus $m = 0$. Market clearing for currency (equation (1)) implies all currency is held as bank reserves, or $g^b = 1$ and $d = 1/\alpha$. Goods market clearing implies $(1+\phi^d)c = y(\xi(0)k^b + k^c)$, and the cash-in-advance constraint, $p^c(1+\phi^d)c = d$ implies $p^c = \frac{1}{\alpha y(\xi(0)k^b + k^c)}$. That $d + p^k k^c = 1 + p^k k$ and $p^k = p^c y/(1-\beta)$ then implies $k^c = k\frac{1-(1-\alpha)(1-\beta)\xi(0)}{1+(1-\alpha)(1-\beta)\xi(1)} < 1$.

Thus the stationary banking equilibrium can be described as follows: Each period, a household again enters the period with wealth $w = 1 + p^k k$ in terms of the current currency supply. At the initial financial market, the household sells $k^b = k - k^c$ units of capital , leaving the financial market with $d = 1/\alpha$ deposits and k^c trees. The household's shopper then goes off and purchases yk^c units of consumption from other households and $\xi(0)yk^b$ units of consumption from banks, the proceeds of which are paid back to the household as interest. Since one can consider all banks (both solvent and insolvent) as liquidating at the beginning of the next period, handing households all trees and currency in return for their deposits, households start the next period with the same wealth as before: k trees and one (renormalized) unit of currency.

Proposition 1. *Suppose Ω is a singleton (no preference shocks), and $\sigma(0) = 1$. Then the monetary equilibrium dominates the banking equilibrium (or the optimal policy is $\alpha = 1$).*

Proof. In the monetary equilibrium, steady state consumption is equal to $yk/(1+\phi^m)$. In the banking equilibrium, steady state consumption is equal to $y(\xi(0)k^b + k^c)/(1 + \phi^d)$. Given $\phi^d > \phi^m$, the result follows. (The result follows as well if $\phi^d = \phi^m$ and $\xi(0) < 1$.) $\square$

Note that the monetary equilibrium can be enforced here either by setting $\alpha = 1$ (actually $\alpha > \alpha^*$) or by setting $\eta = \beta - 1$ (deflating according to the Friedman rule).

A useful question is why, if consumption is lower in the deposit equilibrium, a household doesn't deviate and use currency in the deposit equilibrium, since its utility in the monetary equilibrium is higher than in the deposit equilibrium. The simple answer is that it can't afford to. The price level is at least $1/\alpha$ times higher in the deposit equilibrium (and exactly $1/\alpha$ times higher as $\xi(0) \to 1$), since the supply of "money" (assets which can be used for trade) is $1/\alpha$ times higher in the deposit equilibrium. Thus a household which used only currency and didn't sell any of its trees to a bank in the banking equilibrium could only afford $1/p^c = \frac{\alpha y(\xi(0)k^b+k^c)}{(1+\phi^m)}$ units of consumption in the first period, as opposed to $y(\xi(0)k^b + k^c)/(1 + \phi^d)$, or roughly $1/\alpha$ times that amount if it doesn't deviate. In subsequent periods, the household could afford to purchase more consumption than its non-deviating neighbors, but consuming the equilibrium level of the consumption good each period is preferable to consuming a small amount in the first period and higher amounts thereafter. Now this is not the only deviation where only currency is used. The deviating household could borrow from other households, for instance, to smooth consumption and nevertheless only use currency. However, the point remains that that using currency in every period to purchase $yk/(1+\phi^m)$ units of consumption as in the monetary equilibrium is not in a household's constraint set when $p^c = \frac{1}{\alpha y(\xi(0)k^b+k^c)}$ as opposed to $p_c = \frac{1}{yk}$. Further, the effect on the price level of the increase in the money supply induced by a household's decision to use deposits is external to that household. This externality, through the means-of-payment-in-advance constraints, is the source of the competitive equilibrium inefficiency.

Does this imply there are no social benefits at all to privately created means-of-payments in this economy? No. When households face preference shocks, interest bearing deposits can be socially beneficial.

To see this, consider when Ω is not a singleton but still assuming $\sigma(0) = 1$. If $\alpha > \alpha^*$, then the actual value of α is irrelevant. Intuitively, if the reserve ratio is high enough to shut down banking, increasing it further has no effects. However, if $\alpha < \alpha^*$ (or that the stationary equilibrium involves only deposits), then increasing α has ambiguous effects on welfare. If the reserve ratio is low enough to allow banking, decreasing it further increases the equilibrium interest rate on deposits. This allows better insurance across θ shocks. In particular, if $\alpha = 0$ (or, more carefully, as $\alpha \to 0$ from above to avoid an infinite price level) the equilibrium allocation has the interest rate on deposits $r = \frac{1+\eta}{\beta} - 1$ — the nominal interest rate in the credit market — and thus mimics the non-cash-in-advance constrained allocation for the economy where each household's per period endowment is $y(\xi(0)k^b + k^c)/(1 + \phi^d)$. For higher values of α (but still low enough to allow banking), each household conserves on deposits (since there is an opportunity cost to holding wealth in the form of deposits), distorting the household's consumption choice $c^d(\theta)$. On the other hand, in a banking equilibrium, $k^c = k\frac{1-(1-\alpha)(1-\beta)}{\xi(0)+(1-\alpha)(1-\beta)\xi(1)}$ which is increasing in α. Thus if the reserve ratio is again low enough to allow banking, decreasing it further increases the equilibrium amount of capital held by banks. Since fraction $\xi(1)$ of trees held by banks yield no fruit, this hurts welfare.

The non-comparability of equilibria across deposit equilibria as α is varied extends to comparing currency versus deposit equilibria. If $\eta = \beta - 1$, (or money growth is negative at the deflationary Friedman rule rate) then full insurance, $u_c(c(\theta), \theta) = u_c(c(\hat{\theta}), \hat{\theta})$ for all $(\theta, \hat{\theta})$ with $\sum_\theta \pi(\theta)c(\theta) = y/(1 + \phi^m)$, is achieved in the monetary equilibrium, which is guaranteed to occur since under this money growth rate $\alpha^* < 0$. However, if $\eta > \beta - 1$, no welfare comparison can, in general, be made between the monetary equilibrium ($\alpha > \alpha^*$) and the various banking equilibria (one for each $\alpha < \alpha^*$). If one considers, as a thought experiment, moving α from just above α^* to just below, several countervailing forces on welfare occur. First, consumption falls from $yk/(1+\phi^m)$ on average per household to $y(\xi(0)k^b + k^c)/(1+\phi^d)$ on average per household. This hurts welfare (assuming either $\xi(0) < 1$ or $\phi^d > \phi^m$). But second, the interest rate on the equilibrium means-of-payment rises from zero (the interest rate on currency) to $r = (1-\alpha^*)(\frac{1+\eta}{\beta}-1) > 0$ – a discontinuous jump as a function of α. This jump in the interest rate causes a discontinuous and beneficial increase in insurance for the reasons outlined in the previous paragraph.

A. Access to the bank possible

We now consider when $\sigma(1) > 0$ or it is possible that a shopper has access to all her wealth after discovering her preference shock. We first show that the assumption that $\phi^b > \phi^d > \phi^m$ delivers that the household's z shock realization does not affect how much the shopper spends using currency (in a monetary equilibrium) or using deposits (in a deposit equilibrium). Instead, if for a particular θ realization the household's means-of-payment in advance constraint is slack, it sets $c^b(\theta, 1) = 0$ even though it is not constrained to do so. On the other hand, if for a particular θ realization the household's means-of-payment in advance constraint binds, the household "tops off" consumption by accessing its wealth.

Lemma 2. *In a monetary equilibrium ($m > 0$, $d = 0$), for all θ, $c^m(\theta, 0) = c^m(\theta, 1)$. Further, if $p^c(1 + \phi^m)c^m(\theta, 1) < m$, then $c^b(\theta, 1) = 0$. Likewise, in a banking equilibrium, ($m = 0$, $d > 0$), for all θ, $c^d(\theta, 0) = c^d(\theta, 1)$ and if $p^c(1 + \phi^d)c^m(\theta, 1) < d$, then $c^b(\theta, 1) = 0$.*

Proof. First suppose a monetary equilibrium and $p^c(1+\phi^m)c^m(\theta, 1) < m$ which implies $\gamma^m(\theta, 1) = 0$. If both $c^m(\theta, 1)$ and $c^b(\theta, 1)$ are positive, this contradicts the first order condition of the household problem with respect to $c^m(\theta, 1)$ (30) and the first order condition for $c^b(\theta, 1)$,

$$u_c(c(\theta, 1), \theta) = p^c(1 + \phi^b)\frac{\beta\mu}{1 + \eta}, \tag{38}$$

since $\phi^b > \phi^m$. Thus if $p^c(1+\phi^m)c^m(\theta,1) < m$ in a monetary equilibrium, $c^b(\theta,1) = 0$.

Still assuming a monetary equilibrium, if $c^b(\theta,1) > 0$, (30) and (38) imply $\gamma^m(\theta,1) > 0$ and thus $c^m(\theta,1) = \frac{m}{p^c(1+\phi^m)}$ (from (15)). That $c^m(\theta,0) \leq c^m(\theta,1)$ (again from (15)), the first-order-conditions with respect to $c^m(\theta,0)$, $c^m(\theta,1)$ and the concavity of u imply $\gamma^m(\theta,0) > \gamma^m(\theta,1) > 0$. Thus $c^m(\theta,0) = \frac{m}{p^c(1+\phi^m)} = c^m(\theta,1)$.

If $c^b(\theta,1) = 0$, then condition (30) for both $z = 1$ and $z = 0$ implies if $\gamma^m(\theta,0) = 0$ and $\gamma^m(\theta,1) = 0$, then $c^m(\theta,0) = c^m(\theta,1)$. If $\gamma^m(\theta,0) > 0$ and $\gamma^m(\theta,1) > 0$, then (15) implies $c^m(\theta,0) = c^m(\theta,1) = \frac{m}{p^c(1+\phi^m)}$. If $\gamma^m(\theta,0) > 0$ and $\gamma^m(\theta,1) = 0$ (and likewise if $\gamma^m(\theta,0) = 0$ and $\gamma^m(\theta,1) > 0$) then condition (30) for both $z = 1$ and $z = 0$ and the concavity of u imply $c^m(\theta,0) < c^m(\theta,1)$ (or likewise $c^m(\theta,0) > c^m(\theta,1)$) which, given the lesser one equals $\frac{m}{p^c(1+\phi^m)}$ violates (15) and is thus a contradiction. Thus $c^m(\theta,0) = c^m(\theta,1)$ for all θ in all monetary equilibria.

This exact argument, using the first order conditions of the household problem for $c^d(\theta,z)$ instead of $c^m(\theta,z)$ and using the fact that $\phi^d < \phi^b$ establishes that in any banking equilibrium, if $p^c(1+\phi^d)c^m(\theta,1) < d$, then $c^b(\theta,1) = 0$, $c^d(\theta,0) = c^d(\theta,1)$ for all θ. □

Next, we show that if ϕ^b is not too large and $\sigma(1) \to 1$, then the monetary equilibrium approximates the welfare associated with full insurance with an endowment of $yk/(1+\phi^b)$. First we show that marginal utilities across θ shocks are approximately equated (that is, when $\phi^b - \phi^m$ is small) when $z = 1$, or the shopper does not face a means-of-payment-in-advance constraint.

Lemma 3. *If $\sigma(1) > 0$, then for all $(\theta,\hat{\theta}) \in \Theta^2$,*

$$\frac{1+\theta^m}{1+\theta^b} \leq \frac{u_c(c(\theta,1),\theta)}{u_c(c(\hat{\theta},1),\hat{\theta})} \leq \frac{1+\theta^b}{1+\theta^m}. \tag{39}$$

Proof. If $\gamma^m(\theta,1) = 0$, the first-order-condition of the household problem with respect to $c^m(\theta,1)$

implies

$$u_c(c(\theta,1),\theta) = p^c(1+\phi^m)\frac{\beta\mu}{1+\eta}. \tag{40}$$

If $\gamma^m(\theta,1) > 0$ and $c^b(\theta,1) > 0$, the first-order-condition of the household problem with respect to $c^b(\theta,1)$ implies

$$u_c(c(\theta,1),\theta) = p^c(1+\phi^b)\frac{\beta\mu}{1+\eta}. \tag{41}$$

If $\gamma^m(\theta,1) > 0$ and $c^b(\theta,1) = 0$, the first order conditions with respect to $c^m(\theta,1)$ and $c^b(\theta,1)$ respectively imply

$$p^c(1+\phi^m)\frac{\beta\mu}{1+\eta} \le u_c(c(\theta,1),\theta) \le p^c(1+\phi^b)\frac{\beta\mu}{1+\eta}. \tag{42}$$

Thus (42) holds for all $(\theta,\hat{\theta})$, implying the result. □

This lemma implies if ϕ^b is sufficiently close to ϕ^m, then for all $(\theta,\hat{\theta})$ marginal utilities $u_c(c(\theta,1),\theta)$ and $u_c(c(\hat{\theta},1),\hat{\theta})$ are approximately equated.

Define the full insurance optimum $c^*(\theta)$ as solving $u_c(c^*(\theta),\theta) = u_c(c^*(\hat{\theta}),\hat{\theta})$ for all $(\theta,\hat{\theta})$ and $\sum_\theta \pi(\theta)c^*(\theta) = yk/(1+\phi^m)$.

Proposition 2. *Suppose $u(c,\theta) = \theta c^\nu/\nu$ and consider a sequence of economies $i = \{1,\ldots,\infty\}$ with $\alpha = 1$ and where $\sigma_i(1) \to 1$. Then for all $\epsilon > 0$, there exists a $\delta > 0$ such that if $\phi^m < \phi^b \le \phi^m + \delta$*

$$\epsilon + \lim_{i\to\infty}\sum_{\theta,z}\pi(\theta)\sigma_i(z)u(c_i(\theta,z),\theta) \ge \sum_\theta \pi(\theta)u(c^*(\theta),\theta), \tag{43}$$

where $c^*(\theta)$ is the full insurance optimum.

Proof. First note that

$$\sum_{\theta,z}\pi(\theta)\sigma_i(z)u(c_i(\theta,z),\theta) = \sigma_i(0)\sum_\theta \pi(\theta)u(c_i(\theta,0),\theta) + \sigma_i(1)\sum_\theta \pi(\theta)u(c_i(\theta,1),\theta), \tag{44}$$

or expected utility conditional on $z = 0$ plus expected utility conditional on $z = 1$, weighted by the $\sigma(z)$. Uniformly over all i, marginal utilities conditional on $z = 1$, $u_c(c_i(\theta, 1), \theta)$, can be arbitrarily closely equated by choosing ϕ^b sufficiently close to ϕ^m. Further, as $\sigma_i(1) \to 1$, aggregate consumption of those households with $z = 1$ must approach unconditional aggregate consumption from market clearing, and this aggregate consumption is arbitrarily close to aggregate consumption under full insurance if ϕ^b is close to ϕ^m. Thus for all $\epsilon > 0$, there exists $\delta > 0$ such that if $\phi^m < \phi^b \leq \phi^m + \delta$, $\epsilon + \lim_{i\to\infty} \sum_\theta \pi(\theta)u(c_i(\theta, 1), \theta) \geq \sum_\theta \pi(\theta)u(c^*(\theta), \theta)$. Thus unconditional expected utility converges to the utility of full insurance if and only if $\lim_{i\to\infty} \sigma_i(0) \sum_\theta \pi(\theta)u(c_i(\theta, 0), \theta) = 0$.

Consider this last expression θ by θ. If for a particular θ, $\lim_{i=\infty} c_i(\theta, 0) > 0$, then $\lim_{i=\infty} u(c_i(\theta, 0), \theta) > -\infty$ and thus $\lim_{i\to\infty} \sigma_i(0)\pi(\theta)u(c_i(\theta, 0), \theta) = 0$ since $\sigma_i(0) \to 0$. Next suppose $\lim_{i=\infty} c_i(\theta, 0) = 0$ and note if $u(c, \theta) = \theta c^\nu/\nu$ then $u(c, \theta) = c\ u_c(c, \theta)/\nu$. This implies that to show $\lim_{i\to\infty} \sigma_i(0)\pi(\theta)u(c_i(\theta, 0), \theta) = 0$ it is sufficient to show $\lim_{i\to\infty} \sigma_i(0)$ $u_c(c_i(\theta, 0), \theta) < \infty$. This is guaranteed if $\lim_{i\to\infty} \sum_{\theta,z} \pi(\theta)\sigma_i(z)u_c(c_i(\theta, z), \theta) < \infty$, which, if shown, proves the result since then for all θ, $\lim_{i\to\infty} \pi(\theta)\sigma_i(0)u(c_i(\theta, 0), \theta) = 0$.

So suppose $\lim_{i\to\infty} \sum_{\theta,z} \pi(\theta)\sigma_i(z)u_c(c_i(\theta, z), \theta) = \infty$. This implies there exists $\overline{\theta}$ such that $\lim_{i\to\infty} \sum_z \pi(\overline{\theta})\sigma_i(z)u_c(c_i(\overline{\theta}, z), \theta) = \infty$, which can only occur if $c_i^m(\overline{\theta}, 0) = c_i^m(\overline{\theta}, 1) \to 0$ since $c_i(\overline{\theta}, 1) \geq c_i^m(\overline{\theta}, 1)$. The first order condition with respect to $c^b(\overline{\theta}, 1)$ implies for all i if $c_i^b(\overline{\theta}, 1) > 0$ then

$$u_c(c_i(\overline{\theta}, 1), \theta) = p_i^c(1 + \phi^b)\frac{\beta}{1 + \eta}\mu_i \tag{45}$$

which imposing the envelope condition $p_i^c\mu_i = \sum_{\theta,z} \pi(\theta)\sigma_i(z)u_c(c_i(\theta, z), \theta)$ implies

$$u_c(c_i(\overline{\theta}, 1), \theta) = (1 + \phi^b)\frac{\beta}{1 + \eta}\sum_{\theta,z} \pi(\theta)\sigma_i(z)u_c(c_i(\theta, z), \theta). \tag{46}$$

Since the right hand side of (46) goes to infinity, then either $c_i(\overline{\theta}, 1) \to 0$ or $c_i^b(\overline{\theta}, 1) = 0$ arbitrarily far out in the sequence. If for any θ, $c_i(\theta, 1) \to 0$ then either aggregate consumption goes to zero or marginal utilities are not approximately equated, contradicting either the previous lemma or market clearing. Thus $c_i^b(\overline{\theta}, 1) = 0$ arbitrarily far out in the sequence. But recall $c_i^m(\overline{\theta}, 0) = c_i^m(\overline{\theta}, 1) \to 0$. Thus $c_i(\overline{\theta}, 1) \to 0$, also a contradiction. □

An implication of Proposition 2 is that if ϕ^b is sufficiently close to ϕ^m, and $\sigma(1)$ is sufficiently close to one, then welfare under a monetary equilibrium approaches the maximum welfare (over all $\alpha < \alpha^*$) under a deposit equilibrium. This occurs because the only welfare gain in our model to deposits is better insurance over θ shocks. But since full insurance in the monetary economy is achieved as $\sigma(1) \to 1$, then the only social advantage a deposit economy can have is if ϕ^d is sufficiently smaller than ϕ^b to compensate output being lower in the deposit economy than the monetary economy when $\xi(0) < 1$. It is this result which we interpret as implying that while fractional reserve banking may have once been desirable, the benefits of fractional reserve banking may no longer justify its associated costs to society, even though fractional reserve banking occurs in equilibrium with a low enough reserve ratio.

6. Conclusion:

We have developed a model in which privately produced money provides a socially useful insurance role and a privately useful but socially costly medium of exchange. We have shown that if reserve requirements are sufficiently low then private money drives out currency as a medium of exchange. We have further shown that equilibria with fractional reserve banking may be inefficient, thus it may be desirable to set reserve requirements either to 100%. Finally, if the technology for households allows more and more of a household's wealth to be accessible for transactions, the benefits to fractional reserve banking decrease to zero.

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