# The Market Price of Fiscal Uncertainty<sup>\*</sup>

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#### Abstract

Recent fiscal interventions have raised concerns about US public debt, future distortionary tax pressure and long-run growth potential. We explore the long-run implications of public financing policies aimed at short-run stabilization when: (i) agents are sensitive to model uncertainty as in Hansen and Sargent (2007), and (ii) growth is endogenous as in Romer (1990). We find that tax-smoothing policies promoting shortrun stabilization generate a trade-off by simultaneously reducing the price of model uncertainty and significantly increasing the amount of long-run risk. Ultimately these tax policies depress innovation and long-run growth and may produce welfare losses.

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# 1 Introduction

The current situation of fiscal stress has increased doubts about the future dynamics of US public debt. As shown in figure 1, projections from the congressional budget office (CBO) about the US debt-output ratio span an increasingly wide range over the next decades, leaving room for substantial uncertainty. Given the distortionary nature of the main tax instruments used to finance the government budget, it is natural to wonder to which extent such uncertainty can impact consumption and investment decisions and, more broadly, the long-term prospects of the economy. In a nutshell, figure 1 raises the question of how the formation of beliefs and revisions about the likelihood of different fiscal scenarios could alter economic outcomes.

In this paper we study the impact of fiscal policy for long-term growth in a model in which agents are uncertain about the effective probability distribution of fiscal prospects characterized by a different mix of debt and tax pressure. More precisely, we assume that agents are exposed to model uncertainty as in Hansen and Sargent (2007), meaning that they do not completely trust their approximating model and are willing to optimally slant probabilities toward the 'worst case' scenario.

We examine implications of such 'worst case' beliefs about future fiscal policies for growth and macroeconomic dynamics in a stochastic version of the Romer (1990) endogenous growth model assuming that the government finances exogenous expenditure using debt and distortionary taxes on labor income. By doing so, we are able to analyze the link between fear of misspecification of future fiscal distortions, short-run fluctuations and—in contrast to several other studies—long-term growth prospects.

Looking at US data, we focus on both government expenditure and tax processes that are persistent and volatile and give rise to substantial fiscal uncertainty at both short and long horizons. We use our empirical estimates to calibrate the true data generating process which coincides with the approximating model of the economy. We then model agents' distrust of the approximating model by introducing endogenous martingale distortions as in Hansen



Fig. 1: CBO Projections of Future US Debt

Notes - This figure shows Federal Debt Held by the Public Under CBO's Long-Term Budget Scenarios. The top panel refers to the CBO Long-Term Budget Outlook issued in 2005. The bottom panel is based on the 2010 outlook. See www.cbo.gov.

and Sargent (2007). Intuitively, our agents want to make choices that perform reasonably well over a set of nearby pessimistically distorted processes for government expenditures, productivity and hence taxes. In our production economy, these beliefs have important implications for consumption, investment and ultimately long-term growth and macroeconomic performance.

Using this robust control approach, we have the following results. First, using Monte Carlo simulations, we back out the implied 'worst case' process for future taxes and show that perceived taxes are increasingly higher than realized taxes as aversion to model uncertainty becomes more severe. This implies that, relative to the benchmark beliefs, agents face stronger perceived distortions and incur significant welfare losses expressed in life-time consumption units. This is because in our setting with endogenous growth, tax rates directly affect the economy's long-term growth rate. Worst case beliefs about future tax rates ultimately let the agents expect extended and persistent slumps in macroeconomic performance. Accordingly, they fear dimmer long-term growth prospects than those suggested by the benchmark model and experience substantial welfare losses.

Following Barillas, Hansen, and Sargent (2009), we link welfare losses associated with worst case beliefs to the market price of model uncertainty, which in our model is bundled up with the *market price of fiscal uncertainty*. We think of our agents as averse to uncertainty about the 'true' fiscal policy model, as opposed to Tallarini (2000) and Alvarez and Jermann (2004) who study welfare losses arising from aversion to consumption risk.

In order to show that there exists a deep difference between fiscal risk and fiscal uncertainty, we examine the implications of commonly observed countercyclical fiscal policies seeking to stabilize short-run fluctuations by means of public debt or, equivalently, tax smoothing in the sense of Barro (1979). Using exogenously specified fiscal policy rules (similarly to Dotsey (1990), Ludvigson (1996), Schmitt-Grohe and Uribe (2005), Schmitt-Grohe and Uribe (2007), Davig, Leeper, and Walker (2009), Leeper, Plante, and Traum (2009) and Li and Leeper (2010)), we show that when growth is endogenous, financing policies that are welfare enhancing under time-additive CRRA preferences can turn into a source of relevant welfare losses under aversion to model uncertainty.

Intuitively, tax cuts stabilize the economy in the short-run upon the realization of adverse exogenous shocks. This reduction in short-run consumption risk is a desirable benefit for both risk- and model uncertainty-averse agents. However, the subsequent financing needs associated with long-run budget balance produce uncertainty about future distortionary taxation that affects mainly agents seeking robustness. When tax distortions endogenously affect growth rates, this leads to more uncertainty about long-term growth prospects. In contrast to agents with CRRA preferences, agents that care about the discounted value of future entropy are averse to such long-run uncertainty.

In a Romer (1990) economy, this aversion to long-run uncertainty has important implications for the unconditional average of consumption growth as investments depend on the market value of cash-flows of new products created through innovation. By increasing longrun uncertainty, countercyclical fiscal policies depress the present value of future cash-flows and hence the incentive to grow. After disciplining the aversion to model uncertainty to reproduce key feature of both U.S. consumption and wealth-consumption ratio as measured by Lustig, Van Nieuwerburgh, and Verdelhan (2010) and Alvarez and Jermann (2004), we find that such growth loss outweighs the benefits of short-run stabilization, implying that common tax smoothing prescriptions obtained with time-additive preferences (see, among others, Aiyagari, Marcet, Sargent, and Seppala (2002)) are no longer optimal in settings with concerns for robustness. Basically, counter-cyclical deficit policies reduce entropy by reducing short-run volatility, but at the cost of increasing the amount of long-run risk embedded in patents' cash-flows. Stabilization comes at the cost of undermining long-run growth.

### 1.1 Related Literature

Karantounias (with Lars Peter Hansen and Thomas J. Sargent) (2011) and Karantounias (2011) consider fiscal policy in a robust setting. In contrast to us, they focus on optimal Ramsey taxation and optimal expectations management and abstract from endogenous growth, i.e., the key channel of our welfare analysis. These papers provide theoretical foundations for robust optimal fiscal policy, but they do not feature any trade-off between stabilization and long-run growth arising from the incentives to innovate. In terms of fiscal policy modeling, our specifications are more similar to those in Schmitt-Grohe and Uribe (2007).

The methodology of our welfare analysis has been proposed by Barillas, Hansen, and Sargent (2009). We differ from them because of our focus on changes in the price of fiscal policy uncertainty in a general equilibrium model with endogenous growth. Barillas, Hansen, and Sargent (2009), instead, follow Tallarini (2000) and focus on aggregate consumption fluctuations not explicitly related to policy interventions.

More broadly, our paper is related to a long list of studies in macro and growth that examine the effects of fiscal policy on the macroeconomy. While several authors have examined stochastic fiscal policies in real business cycle models (Dotsey (1990), Ludvigson (1996), Davig, Leeper, and Walker (2009), Leeper, Plante, and Traum (2009)), we focus on long-run growth in the spirit of King and Rebelo (1990), and Rebelo (1991). Our approach is novel in two respects: (i) we explicitly show that there is a trade-off between fiscal policies aimed at short-run stabilization and long-run growth, and (ii) we study and interpret this trade-off in the context of robustness.

We acknowledge that fiscal policy has multiple dimensions we abstract from. For example, we choose to exclude from our analysis learning about the government fiscal policy (Pastor and Veronesi (2010), Pastor and Veronesi (2011)), and productive expenditure (Ferrière and Karantounias (2011)).

The remainder of this paper is organized as follows. In section 2 we introduces our model and discussing robust preferences, endogenous growth and the role of government. In section 3, we briefly detail our calibration approach. Our main results are presented in section 4. Section 5 concludes.

## 2 Model

In this section we describe in detail the stochastic model of endogenous growth that we use to examine the link between long-run growth, fiscal uncertainty and concerns for robustness. As in Romer (1990), the only source of sustained productivity growth is related to the accumulation of new patents on innovations that facilitate the production of the final good. In this class of models, the speed of patent accumulation, i.e., the growth rate of the economy, depends on the market value of the additional cash-flows generated by such innovations. Given that our representative agent has concerns for robustness, the market value of a patent is sensitive to fear about misspecification. Since households price uncertain payoffs using the worst case distribution, doubts about both future taxation and patents' cash-flows generate a premium for exposure to model uncertainty that affects incentives to innovate and growth in the long-run. For simplicity, we abstract from physical capital accumulation. The production of the final good is assumed to depend only on three elements: (i) an exogenous stochastic and stationary productivity process, (ii) the stock of patents, and (iii) the endogenous amount of labor supplied. In our model, labor income is taxed proportionally by the government to finance an exogenous stochastic expenditure stream.

#### 2.1 Household

We assume that the representative household has the following preferences:

$$U_t = (1 - \beta) \log u_t - \beta \theta \log E_t \left[ e^{-\frac{U_{t+1}}{\theta}} \right],$$

defined over a CES aggregator,  $u_t$ , of consumption,  $C_t$ , and leisure,  $1 - L_t$ :

$$u_t = \left[\kappa C_t^{1-\frac{1}{\nu}} + (1-\kappa)[A_t(1-L_t)]^{1-\frac{1}{\nu}}\right]^{\frac{1}{1-\frac{1}{\nu}}}.$$

We let  $L_t$  denote labor and  $\nu$  the degree of complementarity between leisure and consumption, respectively. Leisure is multiplied by  $A_t$ , our measure of standards of living, to guarantee balanced growth when  $\nu \neq 1$ .

As in Hansen and Sargent (2007), these preferences can be derived in a setting in which agents have a concern for robustness, i.e., they fear that their approximating model is misspecified and seek policies that perform reasonably well across a set of plausible 'nearby' models twisted in a pessimistic way. Specifically, our agent makes investment decisions based on a probability measure,  $\tilde{\pi}_{t+1}$ , optimally slanted towards the 'worst' states. To be precise and fix notation, let  $\pi_{t+1}$  denote the conditional probability of state  $s_{t+1}$  at time t induced by the approximating model. As in Hansen and Sargent (2007), such distorted probability can be linked to  $\pi_{t+1}$  as follows:

$$\widetilde{\pi}_{t+1} = \pi_{t+1} \cdot m_{t+1},$$

where  $m_{t+1}$  is the increment of the martingale  $M_{t+1} \equiv M_t \cdot m_{t+1}$ , and its expression in equilibrium is:

$$m_{t+1} = \frac{e^{-\frac{U_{t+1}}{\theta}}}{E\left[e^{-\frac{U_{t+1}}{\theta}}\right]}.$$

In what follows, we denote expectations under the true and the distorted probability measure as  $E[\cdot]$  and  $\tilde{E}[\cdot]$ , respectively. We also express the parameter controlling the degree of concern for robustness,  $\theta$ , in terms of a separate parameter,  $\gamma$ , such that  $\theta = -\frac{1}{1-\gamma}$ . This expression for  $\theta$  reflects the close relationship between preference for robustness and risk aversion, as in Tallarini (2000).

In each period, the household chooses labor,  $L_t$ , consumption,  $C_t$ , equity shares,  $Z_{t+1}$ , and public debt holdings,  $B_{t+1}$ , to maximize utility subject to the following budget constraint:

$$C_t + Q_t Z_{t+1} + B_t = (1 - \tau_t) W_t L_t + (Q_t + \mathcal{D}_t) Z_t + (1 + r_{f,t-1}) B_{t-1},$$
(1)

where  $\mathcal{D}_t$  denotes aggregate dividends (specified in equation (13)) and  $Q_t$  is the market value of an equity share. Wages,  $W_t$ , are taxed at a time-varying rate  $\tau_t$ .

In our setup the stochastic discount factor in the economy is given by

$$\Lambda_{t+1} = \beta \left(\frac{u_{t+1}}{u_t}\right)^{\frac{1}{\nu} - 1} \left(\frac{C_{t+1}}{C_t}\right)^{-1/\nu} \frac{\exp(-U_{t+1}/\theta)}{E_t [\exp(-U_{t+1}/\theta)]},\tag{2}$$

Optimality implies the following asset pricing conditions:

$$Q_t = \mathbb{E}_t [\Lambda_{t+1} (Q_{t+1} + \mathcal{D}_{t+1})],$$
  

$$1 = \mathbb{E}_t [\Lambda_{t+1} (1 + r_{f,t+1})].$$

In equilibrium, the representative agent holds the entire supply of equities (normalized to be one for simplicity, i.e.,  $Z_t = 1 \quad \forall t$ ) and bonds. The stochastic discount factor can be decomposed as follows:

$$\Lambda_{t+1} \equiv \Lambda_{t+1}^R \Lambda_{t+1}^U,$$

with

$$\Lambda_{t+1}^R \equiv \beta \left(\frac{u_{t+1}}{u_t}\right)^{\frac{1}{\nu}-1} \left(\frac{C_{t+1}}{C_t}\right)^{-1/\nu}$$

and

$$\Lambda_{t+1}^U \equiv \frac{\exp(-U_{t+1}/\theta)}{E_t[\exp(-U_{t+1}/\theta)]}$$

The first component,  $\Lambda_{t+1}^R$ , is the familiar stochastic discount factor obtained under expected utility with RRA=IES=1. On the other hand,  $\Lambda_{t+1}^U$ , is the minimizing martingale increment associated with the robust agent's problem. When  $\theta$  approaches infinity ( $\gamma \rightarrow 1$ ), that component goes to unity, and we recover the stochastic discount factor obtained under expected utility.

The above decomposition then suggests a natural interpretation for the above pricing relationships: assets are priced by  $\Lambda_{t+1}^R$  but under the worst case distribution. In this sense, the standard asset pricing equation for any return  $R_{t+1}$ , can be rewritten as

$$1 = \mathbb{E}_t[\Lambda_{t+1}R_{t+1}]$$
$$= \widetilde{\mathbb{E}}_t[\Lambda_{t+1}^R R_{t+1}].$$

In this economy, the maximum conditional Sharpe-ratio is  $\frac{\sigma_t(\Lambda_{t+1})}{E_t(\Lambda_{t+1})}$ , which we decompose and interpret in robustness terms. Specifically, in what follows we refer to  $\frac{\sigma_t(\Lambda_{t+1}^R)}{E_t(\Lambda_{t+1}^R)}$  as the market price of risk, while  $\frac{\sigma_t(\Lambda_{t+1}^U)}{E_t(\Lambda_{t+1}^U)}$  denotes the market price of model uncertainty. We find this terminology more appropriate, as  $\sigma_t(\Lambda_{t+1}^U)$  goes to zero when the concerns for robustness disappear even though well-defined risks remain. As in our economy tax rate risk is bound up with both productivity and expenditure risk, in what follows we often refer to the market price of model uncertainty as market price of fiscal uncertainty. We conclude this section pointing out that the intratemporal optimality condition on labor takes the following form:

$$\frac{1-\kappa}{\kappa} A_t^{(1-1/\nu)} \left(\frac{C_t}{1-L_t}\right)^{1/\nu} = (1-\tau_t) W_t, \tag{3}$$

and implies that the household's labor supply is directly affected by fiscal policy.

## 2.2 Technology

Final Good Firm. There is a representative and competitive firm that produces the single final output good in the economy,  $Y_t$ , using labor,  $L_t$ , and a bundle of intermediate goods,  $X_{it}$ . We assume that the production function for the final good is specified as follows:

$$Y_t = \Omega_t L_t^{1-\alpha} \left[ \int_0^{A_t} X_{it}^{\alpha} di \right]$$
(4)

where,  $\Omega_t$  denotes an exogenous stationary stochastic productivity process

$$\log(\Omega_t) = \rho \cdot \log(\Omega_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),$$

and  $A_t$  is the total measure of intermediate goods in use at date t.

This competitive firm takes prices as given and chooses intermediate goods and labor to maximize profits as follows:

$$D_{t} = \max_{L_{t}, X_{it}} Y_{t} - W_{t}L_{t} - \int_{0}^{A_{t}} P_{it}X_{it}di,$$

where  $P_{it}$  is the price of intermediate good *i* at time *t*. At the optimum:

$$X_{it} = L_t \left(\frac{\Omega_t \alpha}{P_{it}}\right)^{\frac{1}{1-\alpha}}, \quad \text{and} \quad W_t = (1-\alpha)\frac{Y_t}{L_t}.$$
(5)

Intermediate Goods Firms. Each intermediate good  $i \in [0, A_t]$  is produced by an infinitesimally small monopolistic firm. Each firm needs  $X_{it}$  units of the final good to produce  $X_{it}$  units of its respective intermediate good i. Given this assumption, the marginal cost of an intermediate good is fixed and equal to one. Taking the demand schedule of the final good producer as given, each firm chooses its price,  $P_{it}$ , to maximize profits,  $\Pi_{it}$ :

$$\Pi_{it} \equiv \max_{P_{it}} P_{it} X_{it} - X_{it}.$$

At the optimum, monopolists charge a constant markup over marginal cost:

$$P_{it} \equiv P = \frac{1}{\alpha} > 1.$$

Given the symmetry of the problem for all the monopolistic firms, we get:

$$X_{it} = X_t = L_t (\Omega_t \alpha^2)^{\frac{1}{1-\alpha}},$$

$$\Pi_{it} = \Pi_t = (\frac{1}{\alpha} - 1)X_t.$$
(6)

Equation (4) and (6) allow us to express final output in the following compact form:

$$Y_{t} = \frac{1}{\alpha^{2}} A_{t} X_{t} = \frac{1}{\alpha^{2}} A_{t} L_{t} (\Omega_{t} \alpha^{2})^{\frac{1}{1-\alpha}}.$$
(7)

Since both labor and productivity are stationary, the long run growth rate of output is determined by the expansion of intermediate goods variety,  $A_t$ . This expansion is originated in the research and development sector that we describe below.

**Research and Development.** Innovators develop new intermediate goods for the production of final output and obtain patents on them. At the end of the period, these patents are sold to new intermediate goods firms in a competitive market. Starting from next period on, the new monopolists produce the new varieties and make profits. We assume that each existing variety dies, i.e., becomes obsolete, with probability  $\delta \in (0, 1)$ . In this case, its production is terminated. Given these assumptions, the cum-dividend value of an existing variety,  $V_{it}$ , is equal to the present value of all future expected profits and can be recursively expressed as follows:

$$V_{it} = \Pi_{it} + (1 - \delta) E_t \left[ \Lambda_{t+1} V_{it+1} \right]$$
(8)

Let  $1/\vartheta_t$  be the marginal rate of transformation of final goods into new varieties. The free-entry condition in the R&D sector implies that in equilibrium:

$$\frac{1}{\vartheta_t} = E_t \left[ \Lambda_{t+1} V_{t+1} \right]. \tag{9}$$

The left-hand side of the free-entry condition measures the marginal cost of producing an extra variety. The right-hand side, instead, is equal to the end-of-the-period market value of the new patents. Equation (9) is extremely relevant in this class of models because it implicitly pins down the optimal level of investment in R&D and ultimately the growth rate of the economy. To better explain this point, let  $S_t$  denote the units of final good devoted to R&D investment, and notice that in our economy the total mass of varieties evolves according to

$$A_{t+1} = \vartheta_t S_t + (1-\delta)A_t,^1 \tag{10}$$

from which we obtain

$$\frac{A_{t+1}}{A_t} - 1 = \vartheta_t \frac{S_t}{A_t} - \delta.$$

<sup>&</sup>lt;sup>1</sup>This dynamic equation is consistent with our assumption that new patents survive for sure in their first period of life. If new patents are allowed to immediately become obsolete, equation (9) and (10) need to be replaced by  $A_{t+1} = (1 - \delta)(\vartheta_t S_t + A_t)$  and  $\frac{1}{\vartheta_t} = E_t [\Lambda_{t+1}(1 - \delta)V_{t+1}]$ , respectively. Our results are not sensitive to this modeling choice.

As often done in the literature, we impose

$$\vartheta_t = \chi \left(\frac{S_t}{A_t}\right)^{\eta - 1} \quad \eta \in (0, 1), \tag{11}$$

in order to capture the idea that concepts already discovered make it easier to come up with new ideas,  $\partial \vartheta / \partial A > 0$ , and that R&D investment has decreasing marginal returns,  $\partial \vartheta / \partial S < 0$ .

Combining equations (9)—(11), we obtain the following optimality condition for investment:

$$\frac{1}{\chi} \left(\frac{S_t}{A_t}\right)^{1-\eta} = E_t \left[\sum_{j=0}^{\infty} \Lambda_{t+j|t} (1-\delta)^j \left(\frac{1}{\alpha} - 1\right) (\Omega_{t+j} \alpha^2)^{\frac{1}{1-\alpha}} L_{t+j}\right]$$
(12)

where  $\Lambda_{t+j|t} \equiv \prod_{s}^{j} \Lambda_{t+s|t}$  is the *j*-steps ahead pricing kernel and  $\Lambda_{t|t} \equiv 1$ . Equation (12) suggests that the extent of innovation intensity in the economy,  $S_t/A_t$ , is directly related to the discounted value of future profits and, ultimately, future labor conditions. When agents expect labor above steady state, they will have an incentive to invest more in R&D, ultimately boosting long-run growth. Vice versa, when agents expect labor to remain below steady state, they will revise downward their evaluation of patents and will reduce their investment in innovation and, therefore, future growth. We discuss this intuition further in section 2.3.

**Stock Market.** Given the multi-sector structure of the model, various assumptions on the constituents of the stock market can be adopted. We assume that the stock market value includes all the production sectors described above, namely, the final good, the intermediate goods and the R&D sector. Taking into account the fact that both the final good and the R&D sector are competitive, aggregate dividends are simply equal to monopolistic profits net of investment:

$$\mathcal{D}_t = \Pi_t A_t - S_t. \tag{13}$$

In equilibrium, the ex-dividend stock market value  $Q_t$  can be rewritten as follows:

$$Q_t = (V_t - \Pi_t)A_t = \frac{1 - \delta}{\vartheta_t}A_t.$$

**Government.** The government faces an exogenous and stochastic expenditure stream,  $G_t$ , that evolves as follows:

$$\frac{G_t}{Y_t} = \frac{1}{1 + e^{-gy_t}},\tag{14}$$

where

$$gy_t = (1 - \rho)\overline{gy} + \rho_g gy_{t-1} + \epsilon_{G,t}, \quad \epsilon_{G,t} \sim N(0, \sigma_g y).$$

This specification ensures that  $G_t \in (0, Y_t) \quad \forall t$ . In order to finance this expenditure, the government can use tax income,  $T_t = \tau_t W_t L_t$ , or public debt according to the following budget constraint:

$$B_t = (1 + r_{f,t-1})B_{t-1} + G_t - T_t.$$
(15)

**Aggregate Resource Constraint.** In this economy, the final good market clearing condition implies:

$$Y_t = C_t + S_t + A_t X_t + G_t.$$

Final output, therefore, is used for consumption, R&D investment, production of intermediate goods, and public expenditure.

#### 2.3 Some Properties of the Equilibrium

Combining equations (9)—(12), we obtain the following expression for growth rate in the economy:

$$\frac{A_{t+1}}{A_t} = 1 - \delta + E_t \left[ \chi^2 \Lambda_{t+1} V_{t+1} \right]^{\frac{1-\eta}{\eta}}$$

$$= 1 - \delta + E_t \left[ \chi^2 \sum_{j=1}^{\infty} \Lambda_{t+j|t} (1-\delta)^{j-1} \left(\frac{1}{\alpha} - 1\right) (\Omega_{t+j} \alpha^2)^{\frac{1}{1-\alpha}} L_{t+j} \right]^{\frac{1-\eta}{\eta}}.$$
(16)

The relevance of equation (16) is twofold, since it enables us to discuss both the interaction between preferences for robustness and endogenous growth, and the role played by the tax system.

First, we point out that in this framework, growth is a monotone transformation of the discounted value of future profits. This implies that the average growth in the economy is endogenously negatively related to both the discount rate used by the household and the amount of perceived uncertainty. When the household has standard time additive preferences, only actual uncertainty matters for the determination of the value of a patent. When the agent has multiplier preferences, instead, optimal growth depends also on the endogenous amount of perceived volatility in expected long-run profits under the worst case distribution.

Second, since profits are proportional to labor, and labor supply is sensitive to the tax rate, a fiscal system based on tax smoothing ultimately introduces long-lasting fluctuations in future profits and tends to depress patent's value slowing down the entire economy. The welfare implications of these changes in long-run growth are the main object of our analysis.

## 3 Calibration

We report our benchmark calibration in table 1, and the implied main statistics of the model in table 2. We calibrate our productivity process to replicate several key properties of US consumption growth over the long sample 1929–2008. We choose a long sample to

Description	Symbol	Value
Preference Parameters		
Consumption-Labor Elasticity	ν	0.72
Utility Share of Consumption	$\kappa$	0.11
Discount Factor	eta	0.997
Robustness Concern	$\theta$	0.111
Technology Parameters		
Elasticity of Substitution Between Intermediate Goods	$\alpha$	0.7
Autocorrelation of Productivity	ρ	0.97
Scale Parameter	$\chi$	0.52
Survival rate of intermediate goods	$1-\delta$	0.97
Elasticity of New Intermediate Goods wrt R&D	$\eta$	0.83
Standard of Deviation of Technology Shock	$\sigma$	0.006
Government Expenditure Parameters		
Level of Expenditure-Output Ratio $(G/Y)$	$\overline{gy}$	-2.2
Autocorrelation of $G/Y$	$ ho_G$	0.98
Standard deviation of $G/Y$ shocks	$\sigma_G$	0.008

 Table 1: Calibration

Notes - This table reports the benchmark quarterly calibration of our model. All parameters are chosen according to the discipline proposed by Kung and Schmid (2010).

better capture long-run growth dynamics. Under our benchmark calibration, average annual consumption growth is 2.8%, while the volatility is about 2.6%.

The parameters for the government expenditure-output ratio are set to have an average share of 10% at the deterministic steady-state and an annual volatility of 4%, consistent with U.S. annual data over the sample 1929–2008.

The robustness parameter  $\theta$ , and subjective discount factor  $\delta$  are set to replicate the low historical average of the risk-free rate and the consumption claim risk premium estimated by Lustig, Van Nieuwerburgh, and Verdelhan (2010). Replicating these asset-pricing moments is important because it imposes a strict discipline on the way in which innovations are priced and average growth is determined. All other parameters are chosen consistently with the endogenous growth literature (see Kung and Schmid (2010) for a broader discussion).

## 4 The Market Price of Fiscal Uncertainty

In this section we study the link between concerns for robustness, fiscal uncertainty and growth. In an environment with endogenous growth, concerns about future taxation will not only affect perceived future growth, but also realized dynamics of future growth. Because agents with a concern for robustness consider discounted relative entropy of perturbations to their benchmark model, such long-horizon dynamics will be reflected in welfare through the model uncertainty component.

We proceed in two steps. In the next section, in order to illustrate the main mechanisms at work in our setup, we assume that the government is committed to a zero-deficit policy, i.e, it does not issue any debt. In this setup, taxes are a purely exogenous stochastic process mimicking the dynamics of government expenditures. This case serves as a useful benchmark highlighting the basic features of our model. In a second step, we examine the effectiveness of common countercyclical tax smoothing policies when agents have robustness concerns and growth is endogenously determined through innovation.

## 4.1 Zero-Deficit Policies

According to equation (15), a zero-deficit policy implies  $G_t = T_t \quad \forall t$ . Exogenous shocks to the expenditure-output ratio, therefore, are fully absorbed in the tax rate in each period and each state of the world. Under this policy the properties of the tax rate process are determined solely by the properties of both the exogenous productivity and public expenditure shocks.

In table 2 we report various moments from simulations of our model computed both under the true and the distorted measure. We focus on varying degrees of robustness concerns as captured by detection error probabilities. Column 2 refers to our benchmark calibration. The other columns are obtained by progressively reducing  $\gamma$  while keeping the other parameters fixed.

	Data	Benchmark	$p(\theta^{-1}) = 5\%$	$p(\theta^{-1}) = 10\%$
$\sigma(\Delta c)$	2.60	2.67	2.68	2.69
$ACF_1(\Delta c)$	0.44	0.45	0.44	0.43
$\sigma(E_t[\Delta c_{t+1}])$		0.49	0.47	0.46
$ACF_1(E_t[\Delta c_{t+1}])$		0.93	0.93	0.93
$E(\Delta c) - \tilde{E}(\Delta c)$		1.35	0.87	0.57
$ ilde{E}(\epsilon)$		$-1.56e^{-3}$	$-9.98e^{-3}$	$-6.55e^{-4}$
$ ilde{E}(\epsilon_G)$		$2.01e^{-4}$	$1.24e^{-4}$	$0.77e^{-4}$
$\tilde{E}(\tau) - E(\tau)$		0.05	0.02	0.01
$\tilde{E}(\log(V)) - E(\log(V))$		-0.57	-0.39	-0.26
$E(\log \frac{U}{A})$		99.56	99.88	100.08
$E(r^{C,ex})$		1.76	1.15	0.77
$\overline{E(\tau)}$	33.5	33.51	33.51	33.51
$\sigma(\tau)$		2.64	2.63	2.63

 Table 2: Main Statistics under Zero-Deficit

Notes - This table reports the annualized summary statistics obtained simulating our model. The benchmark case corresponds to the calibration in table 1. For the other cases, we adjust  $\theta$  to obtain the indicated detection error probabilities. Detection error probabilities are computed over 100,000 different small samples with 235 quarterly observations. All figures are multiplied by 100, except the first-order autocorrelation, ACF<sub>1</sub>, and the distorted expectations  $\tilde{E}(\epsilon)$  and  $\tilde{E}(\epsilon_G)$ .  $\tilde{E}(\cdot)$  refers to distorted mean, while  $p(\theta^{-1})$  denotes detection error probabilities. The excess returns to the consumption claim are denoted by  $r^{C,ex}$ . Tax rate, value of patents and standardized utility in log units are denoted by  $\tau$ , V and log  $\frac{U}{A}$ , respectively.

Consider first the implied moments for consumption growth, i.e., the main determinant of welfare. The unconditional volatility of consumption is close to its empirical counterpart across all different levels of error detection probabilities. After taking into account timeaggregation, the autocorrelation of annualized consumption growth is modest. On the other hand, the conditional expectation of consumption growth is sizable and extremely persistent, implying that the model generates a fair amount of endogenous long-run consumption risk.

Given the strong impact that long-run risk has on discounted entropy (see Hansen and Sargent (2010)), the gap between the true and the distorted expected growth rate of consumption is sizeable. Furthermore, since our model is very close to be log-linear, we observe distortions only in the first moment of our variables of interest, consistent with Anderson, Hansen, and Sargen (2003) and Bidder and Smith (2011) who document no distortion in second or higher moments. The negative distortion in expected consumption growth is the natural result of the pessimistic expectations about both productivity and government expenditure shocks. Our agent, indeed, slants probabilities toward states in which productivity shocks are negative and government liabilities shocks are positive. In these states, the tax base is low while the liabilities of the government are high. Agents, therefore, expect higher levels of taxation under the undesired worst case scenario. Equation (16) clarifies the implications of these distortions on growth: a higher expected tax rate triggers a permanent decrease in after-tax expected wage, labor supply, future profits and perceived value of patents,  $\tilde{E}(\log(V))$ , ultimately discouraging investment in innovative products.

As robustness concerns increase, the implied decline in the value of patents and growth depresses welfare to a greater extent. Simultaneously, the quest for further robustness increases model uncertainty and hence the premium associated to consumption cash-flow. Our benchmark specification generates a substantial consumption risk premium of about 1.75 in line with the empirical estimates of Lustig, Van Nieuwerburgh, and Verdelhan (2010). This premium is mainly driven by model uncertainty, as shown by the fact that it rapidly decreases when the concern for robustness declines.

We conclude the analysis of our table 2, by pointing out that under our benchmark calibration the average tax rate is roughly 33.5%, consistent with the data. On the other hand, the implied volatility for taxes is moderate, in the order of 2.6%. Our results, therefore, are not driven by an excessively volatile tax rate.

These results can be better understood by inspecting the impulse responses of key quantities after a positive one-standard deviation shock to G/Y. In figure 2 we depict the dynamic response of both short- and long-horizon variables for various degrees of robustness concerns. We distinguish between aversion to model uncertainty, and aversion to risk (the dash-dotted green line). We start by discussing the case of aversion to model uncertainty.

The top-left panel of figure 2 shows that when an adverse government shocks materializes, labor tends to fall. This is due to a substitution effect: under a zero-deficit policy a higher



Fig. 2: Short- and Long-Run Dynamics following adverse G/Y Shock

Notes - This figure shows quarterly log-deviations from the steady state. All deviations are multiplied by 100. The benchmark case corresponds to the calibration in table 1. For the other cases, we adjust  $\theta$  to obtain the indicated detection error probabilities. CRRA corresponds to  $\gamma = 10$ , where  $\gamma$  parameterizes risk aversion rather than aversion to model uncertainty.

level of government expenditures directly translates into a higher tax rate that depresses the supply of labor. This effect gets weaker when the concern for robustness becomes stronger. This has a natural intuition: a higher concern for robustness makes the agent feel more pessimistic and work harder (income effect). However, the top-right panel shows that such more stable short-run dynamics come at the cost of lower expected recovery speed. This is because agents perceive higher expected taxes when the robustness concerns are more severe.

Output and consumption exhibit similar patterns when we focus on their short-run dynamics (left hand side panels): stronger concerns for robustness are associated to more stable short-run responses. Turning our attention to expected output and consumption growth (right hand panels), we can see a more severe fall when aversion to model uncertainty increases. According to equation (16), such result can be explained by studying the two key determinants of aggregate growth, namely expected future profits and stochastic discount factor. Since government expenditures are persistent, the agent anticipates higher expenditures and hence higher tax rates for the long-run. The lower incentives to supply labor generate lower long-run expected profits and hence a severe drop in patents value. Since investments fall, expected growth is automatically revised downward. On the discount rate side, an increase in aversion to model uncertainty amplifies the expectations adjustment just described. The cash-flow channel and the discount rate channel, therefore, work in the same direction and reinforce each other.

In our benchmark calibration the implicit value for  $\gamma$  is 10. The dash-dotted lines in figure 2 refers the case in which we impose  $\gamma = 10$ , but now interpret it as pure aversion to risk in an environment with CRRA preferences. The dynamics of consumption changes drastically when we focus on an economy featuring pure aversion to risk. First of all, upon the realization of an adverse government expenditure shock, labor falls much less. The reason being that in this setting the agent cares only about short-run uncertainty and investment decisions are no longer significantly sensitive to a long-run increase in taxes. Expected long-run growth of output, therefore, falls by less. Long-run consumption growth becomes actually positive, as the agent anticipates that government expenditures will decline as a fraction of output and will leave more resources available for private consumption.

Figure 2 shows that the dynamics of macroeconomic quantities depend crucially on the entertained interpretation of  $\gamma$  as capturing aversion to either model uncertainty or risk. To be more precise about this point, in table 3 we show volatility and composition of the pricing kernel  $\Lambda$  for all the four calibrations used in figure 2.

Our benchmark model generates a maximum Sharpe-ratio of 0.28, within with the Hansen and Jagannathan (1991) bound. Across all our different calibrations of  $\theta$ , almost all the

	Benchmark	$p(\theta^{-1}) = 5\%$	$p(\theta^{-1}) = 10\%$	CRRA
$\sigma(\Lambda)/E(\Lambda)$	0.28	0.18	0.13	0.09
$\sigma(\Lambda^U)/E(\Lambda^U)$	0.26	0.17	0.11	0.00

 Table 3: Market Price of Risk

Notes - This table reports market price of risk and fiscal uncertainty under different degrees of robustness concerns. The benchmark case corresponds to the calibration in table 1. For the other cases, we adjust  $\theta$  to obtain the indicated detection error probabilities. The last column refers to the case in which the agent has time-additive CRRA preferences with relative risk aversion equal to 10. Detection error probabilities are computed over 100,000 different small samples with 235 quarterly observations.

volatility of the pricing kernel can be attributed to model uncertainty. Intuitively, our endogenous growth model generates persistent variations in expected consumption growth that are a source of serious concern for an agent seeking robustness since such low-frequency dynamics are hard to detect in short sample. These persistent variations in expected consumption growth are a source of long-run risk (Bansal and Yaron (2004)) endogenously related to investment and public expenditure shocks.

With standard time-additive CRRA preferences, the agent is not concerned by long-run model uncertainty and for this reason all the pricing kernel volatility is related to short-run consumption volatility. Even when the relative risk aversion is calibrated to a value as high as 10, the market price of risk remains small, as the agent manages to hedge a substantial amount of short-run consumption risk through investments.

Summarizing, we find that fiscal uncertainty in an endogenous growth setting with robustness concerns leads to higher perceived taxation, lower perceived growth and welfare losses. These welfare losses are intimately connected to the volatility of the stochastic discount factor, which is almost exclusively driven by model uncertainty. These findings suggest that even a small alteration of tax dynamics can produce substantial changes in growth and welfare. In the next section we connect model uncertainty to more general public financing policies aimed at stabilizing the economy over the short-run and show that they may actually be sub-optimal with respect to a simple zero-deficit policy.

#### 4.2 Public Debt and Endogenous Tax Uncertainty

In the previous section, we focused on a tax process,  $T_t$ , that perfectly mimics the properties of the exogenously specified expenditure process described by  $gy_t$ . In this section, instead, we allow the government to run deficit and surpluses according to the following rule on debt-output ratio:

$$\frac{B_t^G}{Y_t} = \rho_B \frac{B_{t-1}^G}{Y_{t-1}} + \epsilon_{B,t}$$

$$\epsilon_{B,t} = \phi_B \cdot (\log L_{SS} - \log L_t),$$
(17)

where  $L_{SS}$  is the steady state level of labor, and  $\rho_B \in (0, 1)$  and  $\phi_B \ge 0$  measure the inverse of the speed of repayment of debt and the intensity of the policy, respectively.

When  $\phi_B > 0$ , our simple debt policy rule captures the behavior of a government that is concerned about employment and wants to minimize labor fluctuations. In particular, the government cuts labor taxes (increases debt) when labor is below steady state and increases them (reduces debt) in periods of boom for the labor market. The convenience of working with this policy is twofold. First, with time-additive preferences, this simple policy improves welfare against a zero-deficit policy, ie, against the no tax smoothing case,  $\phi_B = 0$ . This is relevant because it implies that we are working with a policy that can bring the economy closer to the Ramsey second best, at least with time additive preferences. We prove and explain this point in detail in section 4.2.2. Second, this policy rule allows us to focus only on the two most important dimensions of a tax system, namely the intensity of tax-smoothing,  $\phi_B$ , and its persistence,  $\rho_B$ .

The condition  $\rho_B < 1$  ensures that the public administration wants to keep the debtoutput ratio stationary. In the language of Leeper, Plante, and Traum (2009), we anchor expectations about debt and rule out unsustainable paths. Since in our economy with

#### a) positive expenditure shock

b) negative productivity shock



Fig. 3: Impulse response of Tax Rate and Debt

Notes - This figure shows quarterly log-deviations from the steady state for government expenditure-output ratio (G/Y), debt-output ratio (B/Y) and labor tax ( $\tau$ ). Panel a) refers to an adverse shock to government expenditure. Panel b) refers to a negative productivity shock. All deviations are multiplied by 100. All the parameters are calibrated to the values used in Tables 1. The zero-deficit policy is obtained by imposing  $\phi_B = 0$ . The tax-smoothing policy is obtained setting  $\rho_B^4 = .975$  and  $\phi_B = 0.25\%$ .

recursive preferences the following holds:

$$E\left[\frac{1+r_{f,t}}{exp\{\Delta y_{t+1}\}}\right] < 1,$$

the unconditional average of both debt and deficit is zero. Under this policy, therefore:

$$E[\tau_t] = E\left[\frac{G_t}{W_t L_t}\right] = \frac{1}{\alpha} E\left[\frac{G_t}{Y_t}\right].$$

In absence of uncertainty,  $E[\tau_t]$  depends only on  $\alpha$  and  $\overline{gy}$ . In the model with uncertainty, in contrast,  $E[\tau_t]$  becomes an inverse function of the average amount of labor that the household optimally supplies.

The dynamics of  $\tau_t$  around its unconditional mean,  $E[\tau_t]$ , are implicitly determined by (15) and (17). Given  $\phi_B > 0$ , panel a) and b) of figure 3 show the response of the tax

rate after a positive shock to government expenditures and a negative shock to productivity, respectively. According to (17), in both cases the government responds to these shocks by initially lowering the tax rate below the level required to have a zero deficit. Over the long-horizon, instead, the government increases taxation above average in order to run surpluses and repay debt. Since this is true also with time-additive preferences, for the sake of brevity we plot only the responses under our benchmark calibration.

The main goal of the remainder of this section is to illustrate that with robustness preferences the welfare implications of commonly used tax smoothing rules are quite different from those normally obtained with time-additive preferences. In what follows, first we describe the impact of this fiscal policy on macroeconomic aggregates by looking at impulse response functions. Second, we show that our simple tax smoothing policy generates welfare benefits with respect to a simple zero-deficit rule when the agent has standard preferences with risk aversion. Third, we show that when the agent is averse to model uncertainty the same tax-smoothing policy may generate, in contrast, significant welfare costs.

#### 4.2.1 Short-run dynamics and long-run expectations

Keeping the behavior of the tax rate in mind, we now turn our attention to the behavior of labor, output and consumption growth upon the realization of an adverse government expenditure shock. The left-hand panels of figure 4 show the short-run dynamics of these macroeconomic quantities, while the right-hand panels depict the response of the conditional expectations. We point out two relevant differences. First, the responses of  $l_t$ ,  $\Delta y_t$  and  $\Delta c_t$ upon the realization of an adverse expenditure shock are less pronounced than those observed in figure 2 under zero-deficit. This implies that our exogenous policy accomplishes the task for which it is designed, i.e., it reduces short-run fluctuations.

Second, under CRRA the response of the conditional expectations is almost unaltered with respect to the zero-deficit case. Under the robustness case, instead, such adjustment is *amplified* when tax smoothing is active. Specifically, in the economy with robustness



Fig. 4: Impulse Response Functions with Tax-smoothing

Notes - This figure shows impulse response functions under the probability measure induced by the approximating model. All the parameters are calibrated to the values used in Tables 1. The lines depicted in each plot are associated to different levels robustness concerns,  $\theta = -(1 - \gamma)^{-1}$ , and detection error probabilities,  $p(\theta^{-1})$ . Under the the 'Benchmark' calibration,  $\gamma = 10$ . The dashed-dotted line refers to the time-additive CRRA case with relative risk aversion of 10.

concerns, the short-run stabilization comes at the cost of having a more pronounced and pessimistic adjustment of the expectations about future growth. According to equation (16), expectations about growth are just a monotone transformation of the patent's value and ultimately they depend on the properties of profits.

Figure 5 shows what happens to both the intertemporal composition of profits risk and the value of a patent as we change the policy parameters  $(\rho_B, \phi_B)$  under our benchmark calibration. For given  $\rho_B$ , as the intensity of the policy  $\phi_B$  increases, the short-run volatility of profits declines (top-right panel), while simultaneously the long-run component of profits



Fig. 5: Patents' Value and Profits Distribution with Robustness

Notes - This figure shows the average value of patents, E[V], and key moments of profits,  $\Pi$ . All the parameters are calibrated to the values used in Tables 1. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively. Horizontal axis corresponds to different annualized autocorrelation,  $\rho_B^4$ , of debt to output ratio,  $B^G/Y$ ; the higher the autocorrelation, the lower the speed of repayment.

becomes more persistent (bottom-left panel). When the household cares about discounted entropy, more persistent long-run profit fluctuations may generate a substantial increase in the average excess return. In our case, as  $\phi_B$  increases, the government budget constraint impels more severe long-run taxation adjustments which produce long lasting adverse fluctuations in labor and profits. The increased persistence of long-run profits dominates the decline of short-run risk and lets future profits be discounted at a higher rate. This explains why a more intense tax-smoothing policy ultimately depresses patent values (top-left panel) and growth.

Furthermore, figure 5 shows that the negative effects of tax smoothing on patent valuation and growth become more severe when the smoothing attitude,  $\rho_B$ , increases. More persistent tax rate fluctuations amplify long lasting profit risk and depress growth even though more



Fig. 6: Patents' Value and Profits Distribution in the CRRA case

Notes - This figure shows the average value of patents, E[V], and key moments of profits,  $\Pi$ . All the parameters are calibrated to the values used in Tables 1. preferences are time-additive with relative risk aversion set to 10. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively. Horizontal axis corresponds to different annualized autocorrelation,  $\rho_B^4$ , of debt to output ratio,  $B^G/Y$ ; the higher the autocorrelation, the lower the speed of repayment.

short-run stabilization is achieved. With time-additive CRRA preferences, in contrast, the value of the patents improves with tax smoothing, as shown in figure 6. The reason being that there exists no concern about model uncertainty and fiscal stabilization indeed reduces aggregate short-run risk.

Taken together, these results suggest that the intertemporal distribution of tax distortions matters when the agent works under the worst-case scenario. In a model with endogenous growth and robustness concerns, the financing mix of taxes and debt significantly feeds-back on the patent valuation and long-run growth prospects.

This point is perhaps even more evident if we look at figure 7, where we depict the im-





Notes - This figure shows the impulse response function of the cumulative growth of GDP under both a 'Weak' (left panels,  $\phi_B = 0.1\%$ ) and a 'Strong' (right panels,  $\phi_B = 0.25\%$ ) fiscal policy with respect to the zero-deficit case. All the parameters are calibrated to the values used in Tables 1. The lines reported in each plot are associated to different levels of robustness concerns. In the Benchamrk case  $p(\theta^{-1}) = 1.15\%$ . The dashed-dotted line refers to the case of time-additive preferences with RRA=10. The top (bottom) two panels depict the response to an adverse productivity (government expenditure) shock.

pulse response of cumulative output growth upon the realization of both adverse productivity (top panels) and expenditure shocks (bottom panels). The left panels show the difference in cumulative growth between a weak fiscal policy and the zero-deficit policy. The right panels, instead, refer to the policy with strong intensity. While with standard time-additive CRRA preferences there is no interesting dynamics, with robustness concerns, instead, growth dynamics are quite different. In particular, a policy aimed at strong short-run stabilization produces excess growth with respect to a zero-deficit policy only for a limited amount of quarters and then it generates a substantial growth loss for the long-run. Furthermore, this growth loss increases with robustness concerns,  $\theta$ , implying that in the presence of strong aversion to model uncertainty a passive zero-deficit policy becomes increasingly more desir-



Notes - This figure shows impulse response functions under the worst-case scenario. All the parameters are calibrated to the values used in Tables 1. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). Under the 'Active' policy  $\phi_B = 0.25\%$ , while under 'zero-deficit'  $\phi_B = 0$ .

able. A zero-deficit policy, indeed, has the benefit of eliminating an important source of risk, namely future tax pressure necessary for budget consolidation.

So far we have studied the model response under the probability measure induced by the approximating model, but we also find it instructive to have a look at worst-case impulse response functions as in Bidder and Smith (2011). In period 1, we draw a one standard deviation shock from the distorted distribution of government expenditure news. Starting from period 2, shocks go back to their distorted average forever. We depict the response of quantities and expectations over the worst-case average path in figure 8.

Under the worst case scenario, government expenditure shocks are expected to be positive

forever, hence taxes are expected to be systematically higher over time. By the substitution effect, labor tends to fall over time (top left panel). The less severe drop of labor under the active policy is just a reflection of the fact that the government stabilizes the labor market even along the worst case scenario path.

Simultaneously, however, the fall of output growth is amplified (middle-left panel). As mentioned before, this has to do with the riskiness of profits over the long-horizon: under the active policy profits are riskier in the long-run, hence both investment and output growth are discouraged state-by-state and period-by-period.

The difference in consumption growth under the two policies is almost invisible (bottomleft panel), in sharp contrast to that of expected future consumption growth (bottom right panel). Under the active policy, in fact, the agent slants probabilities toward states of the world with slow recovery speed. This is true for consumption, output and labor growth. Focusing on the top-right panel of figure 8, we can see that under zero-deficit the agent expects a stronger rebound of the labor market as labor moves further away from its undistorted unconditional mean. Under the distorted measure, however, the opposite is true: agents expect a further prolonged drop in employment along the worst-case scenario. This has to do with the fact that under the active policy agents expect more severe deficits and even higher tax pressure in the future, implying an endogenous and long-lasting contraction in employment.

#### 4.2.2 Welfare and growth incentives

We start by focusing on the case of time-additive preferences where  $\gamma$  is a pure measure of risk aversion. In the top left panel of figure 9, we plot welfare costs (benefits) obtained by departing from the zero-deficit policy and implementing counter-cyclical tax-smoothing with different levels of intensity,  $\phi_B$ , and persistence,  $\rho_B$ . The top- and the bottom-right panel show short- and long-run consumption risk as a function of  $\phi_B$  and  $\rho_B$ , respectively. The bottom-left panel shows changes in the unconditional growth rate of consumption with



Fig. 9: Welfare Costs and Consumption Distribution in the CRRA Case

Notes - This figure shows the welfare costs and key moments of consumption growth. All the parameters are calibrated to the values used in Tables 1, except for the fact that preferences are specified as time-additive CRRA with relative risk aversion of 10. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively. Horizontal axis corresponds to different annualized autocorrelation,  $\rho_B^4$ , of debt to output ratio,  $B^G/Y$ ; the higher the autocorrelation, the lower the speed of repayment. Welfare costs are calculated as in Lucas (1987).

respect to a zero-deficit policy. The main message of this figure is simple: with standard preferences our exogenous financing policy is able to reduce short-run consumption risk, promote growth and generate welfare benefits. These results, however, are completely overturned under our benchmark calibration featuring robustness concerns, as shown in figure 10.

The top-left panel of this figure, indeed, shows that standard counter-cyclical financing policies may produce welfare losses which are very sizable, especially relative to the small



Fig. 10: Welfare Costs and Consumption Distribution with Robustness

Notes - This figure shows the welfare costs and key moments of consumption growth. All the parameters are calibrated to the values used in Tables 1. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively. Horizontal axis corresponds to different annualized autocorrelation,  $\rho_B^4$ , of debt to output ratio,  $B^G/Y$ ; the higher the autocorrelation, the lower the speed of repayment. Welfare costs are calculated as in Lucas (1987).

benefits depicted in figure 9. The top-right panel of figure 10 shows that the government is still able to stabilize consumption dynamics in the short-run when using more aggressive fiscal policies (stronger intensity,  $\phi_B$ , or smoothing,  $\rho_B$ ). The problem, however, is that such short-run stabilization comes at the cost of increasing long-run profits risk which turns into more pronounced long-run consumption fluctuations and lower unconditional growth. Since growth is a first order determinant of welfare, the final result is an impoverishment of the household.

As documented in table 4, standard tax-smoothing policies are able to reduce model uncertainty by reducing the volatility of the martingale increment. This also implies that the government is able to reduce pessimistic distortions, as depicted in figure 11, consistent with the results obtained by Barillas, Hansen, and Sargent (2009) in an endowment economy.

	Benchmark	Weak	Strong	Strong
		$ \rho_{B}^{4} = .988 $	$ \rho_{B}^{4} = .988 $	$ \rho_{B}^{4} = .95 $
$\frac{\sigma(\Lambda)}{E(\Lambda)}$	0.2805	0.2789	0.2770	0.2794
$\left(\frac{\sigma(\Lambda^U)}{E(\Lambda^U)}\right) / \left(\frac{\sigma(\Lambda)}{E(\Lambda)}\right)$	0.938	0.941	0.943	0.940

Table 4: Market Price of Risk with Different Policies

Notes - This table reports market price of risk and fiscal uncertainty under different public financing policies. The benchmark case corresponds to the calibration in table 1 under a zero-deficit policy. 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively.

Unfortunately, however, these accomplishments come at the cost of allowing more long-run profits *risk*. We emphasize the word risk because the increase in the persistence of the profits fluctuations is obtained both under the true and the distorted probability measure. As anticipated before, in fact, we find no significant distortion in second moments. The agent, therefore, is perfectly aware of the fact that stronger tax-smoothing boils down to stronger swings in long-run tax rates, labor, profits and growth.

While in a model with exogenous growth reducing model uncertainty automatically produces substantial welfare benefits (Barillas, Hansen, and Sargent (2009), Tallarini (2000)), in an endogenous growth economy reducing model uncertainty can come at the cost of depressing growth for the long-run. More broadly, our welfare results suggest that this trade-off should be taken seriously into account when working on optimal fiscal policy design and that the current attention toward short-run stabilization may be questionable.

# 5 Conclusion

Recent fiscal interventions have raised concerns about public debt, future distortionary tax pressure and long-run growth potential across the globe. The way in which expectations about future taxation and fiscal policies are formed affect both households' and firms' current economic decisions, and—more broadly—current macroeconomic growth. While most of the literature in macroeconomics and growth assumes that agents know the true probability distributions of the future paths of policy instruments, in this paper we consider the



Fig. 11: Pessimistic Distortions and Tax-smoothing

Notes - This figure shows detection error probabilities and pessimistic distortions as a function of the policy parameters  $\phi_B$  and  $\rho_B$ . All other parameters are calibrated to the values used in Tables 1. The two lines reported in each plot are associated to different levels of intensity of the countercyclical fiscal policy described in equation (17). 'Weak' and 'Strong' policies are generated by calibrating  $\phi_B$  to 0.1% and 0.25%, respectively. Horizontal axis corresponds to different annualized autocorrelation,  $\rho_B^4$ , of debt to output ratio,  $B^G/Y$ ; the higher the autocorrelation, the lower the speed of repayment. Detection error probabilities are computed over 100,000 small samples each with 235 quarterly observations.

possibility that agents doubt the probability measure implied by their approximating model. Following Hansen and Sargent (2005), we assume that agents have a concern for robustness and are averse to model uncertainty. We examine the implications of such aversion in a model of endogenous growth in which fiscal policy can alter both short- and long-run economic dynamics.

We show that standard tax-smoothing policies which are welfare enhancing with timeadditive CRRA preferences can turn into a source of large welfare losses when agents have concerns for robustness. The reason being that long-run tax risk implied by public budget balance can increase long-run profits risk and slow down growth forever. Using exogenous tax-smoothing policies, we show that there is a relevant trade-off between model uncertainty and long-run profits risks. Reducing short-run uncertainty through public deficits or surpluses, can reduce pessimistic distortions, but at the cost of bringing about more risk for long-run profits. Since the agent cares about the discounted value of entropy, this depresses the value of patents and hence the incentive to innovate, ultimately reducing endogenous growth for the long-run. When we discipline the aversion to model uncertainty by calibrating the model to reproduce key feature of both U.S. consumption and wealth-consumption ratio as measured by Lustig, Van Nieuwerburgh, and Verdelhan (2010) and Alvarez and Jermann (2004), we find that the loss of growth outweighs the benefits of short-run stabilization.

These results suggest that common tax smoothing prescriptions obtained with timeadditive preferences (see, among others, Aiyagari, Marcet, Sargent, and Seppala (2002)) may no longer be optimal in settings with concerns about robustness and endogenous growth. In a companion paper (Croce, Nguyen, and Schmid (2011)) we show both qualitatively and quantitatively that these results hold also in a broader setup with general Epstein and Zin (1989) preferences.

Future research should integrate business cycle considerations in our model and study the optimality of multiple tax instruments. Furthermore, it is going to be important to study the optimal interaction between monetary and fiscal policy over both the short- and the long-run. Finally, our model abstracts from financial and labor market frictions. Whether these elements could increase or reduce the performance of standard tax-smoothing policies with robustness is a question that we leave for future research.

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