Management Mathematics: The Audacity of BOPE∗

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Abstract

Can mathematical research from a business school professor that is published in academic journals also substantially improve actual industrial practice in global supply chains? Yes. Can mathematics and operations management help improve fairness in access to organ transplants? Yes, again. Can mathematics developed for business applications facilitate the development of new types of quantum hardware? Also, yes. I have three main messages in this invited essay: (1) Free yourself from self-imposed constraints and explore a wider range of new problems and arcane mathematics; (2) Be bold and imaginative in developing novel solutions; and (3) Implement your ideas in practice, scaling them with a channel partner.

Keywords: Supply Chains, Healthcare, Quantum, Software Entrepreneurship, Social Enterprise.

∗Business Optimizing Professor-Entrepreneur.
Suburb outside Boston.

00:00. Patient receives call from Mayo Clinic in Rochester (MN).
00:01. Calls OrganJet.
01:36. Wheels up, on Learjet 45 [Range 2250 miles at Mach 0.8].
04:46. Reaches Mayo.

Table 1: OrganJet: A liver transplant patient near Boston is transported to Mayo Clinic (in Rochester, Minnesota), on-demand, in under five hours.

1 The Importance of Being Audacious

adjective: audacious

1. showing a willingness to take surprisingly bold risks.

What propels my professional career is a preoccupation with conceiving and implementing novelty as in through my social enterprise, OrganJet (Ata, Skaro and Tayur, 2016); see Table 1. I am as attracted to identifying new problems to tackle as to employing new mathematical methodologies, that can be (and frequently actually are) implemented in practice, at scale, with measurable results. This invited essay showcases the major themes of my professional career over the last three decades: novelty in methodology, deliberate problem selection, implementing solutions in practice, and identifying mechanisms (and associated channel partners) to scale such implementations. I have been inspired by Emerson (1841):

There is no outside, no enclosing wall, no circumference to us. The only sin is limitation.

2 Novelty Matrix

Figure 1 lays out some examples in the familiar 2x2 box framework, with problem choice and solution methodology as the two axes. The problem choice can be canonical or contemporary (which I will take as a synonym for novel), the solution method can be classical or novel. In this section, I will highlight some of my activities in the three boxes where there is at least one source of novelty.
2.1 Canonical Problem Choice, Novel Solution Method

2.1.1 Serial Production Lines, Sample Path Comparisons

I first studied serial production lines (Tayur, 1993b), which was by then a fairly well studied problem, but was done so almost entirely through Markovian models or via simulation. Differently, I tackled this canonical problem using sample path methods and stochastic comparisons. I found structural results that hold in good generality, and which help in significantly reducing the computational burden when designing actual production lines. Additionally, it was particularly amusing that the many numerical studies and simulations conducted through Markovian models in published papers were actually done on provably non-optimal solutions! Some years later, I helped implement a Kanban system at GE (Tayur, 2000).

2.1.2 Integer Programming, Algebraic Geometry

Around that time, I was also quite enamored by Algebraic Geometry and Grobner basis, concepts (well) outside operations research, and found that it was possible to study integer programs, using ideas from polynomial ideals (Tayur, Thomas and Natraj, 1995). In two different ways (Bertsimas, Perakis and Tayur, 2000), in fact! So, here was another example of coming up with methodological innovations for the most fundamental of hard mathematical programming problems, integer programming, and conceiving two different approaches, which were not incremental to, nor intimated by, the mainstream methods (such as Branch and Bound).
2.1.3 Multi-stage Production-Inventory Models, IPA

Continuing to focus on canonical problems, in my summer internship at IBM Somers, I had the good fortune to be exposed to real-world issues in what is now called supply chain management: Planning global inventories inside a multi-national enterprise, using multi-stage, discrete-time, stochastic production inventory models. I immediately realized that, for any model that provides useful outputs, the production capacity (Tayur, 1993a) needs to be constrained, clearly violating the celebrated Clark-Scarf model. Through sample path derivatives obtained via IPA (infinitesimal perturbation analysis) (Glasserman and Tayur, 1995), with few assumptions (especially of stationarity or having specific, distributional forms of demand, or even limited to the topology of pure serial or pure assembly or pure distribution networks), this novel methodological advance allowed, for the first time since the origins of modern inventory theory (in the 1950s) to actually provide industries with sophisticated solutions that could be implemented at scale (Tayur, 2013) with vastly superior performance measured on business metrics (service levels, working capital investment), consequential to both the income statement and the balance sheet (thereby increasing shareholder value and stock price, in publicly traded companies (Troyer et al., 2005)), in addition to satisfying academic requirements (Glasserman and Tayur, 1994) (such as being unbiased and guaranteeing convergence).

IPA has wide-ranging applications (Table 2) including: periodic demand (Kapuściński and Tayur, 1998), quoting lead times (Kapuscinski and Tayur, 1998), stochastic cyclic scheduling (Anupindi and Tayur, 1998), understanding the value of information sharing (Gavirneni, Kapuscinski and Tayur, 1999), re-entrant flow shops (semi-conductor manufacturing) (Bispo and Tayur, 2001), and risk sharing in global supply chains (Scheller-Wolf and Tayur, 2009).

<table>
<thead>
<tr>
<th>Topic</th>
<th>Infinitesimal Perturbation Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-echelon Inventory</td>
<td>Glasserman and Tayur (1995)</td>
</tr>
<tr>
<td>Non-stationary Demand</td>
<td>Kapuscinski and Tayur (1998)</td>
</tr>
<tr>
<td>Lead Time Quotation</td>
<td>Kapuscinski and Tayur (1998)</td>
</tr>
<tr>
<td>Postponement</td>
<td>Swaminathan and Tayur (1998)</td>
</tr>
<tr>
<td>Cyclic Schedule</td>
<td>Anupindi and Tayur (1998)</td>
</tr>
<tr>
<td>Value of Information</td>
<td>Gavirneni, Kapuscinski and Tayur (1999)</td>
</tr>
<tr>
<td>Re-entrant Flow</td>
<td>Bispo and Tayur (2001)</td>
</tr>
<tr>
<td>Risk Sharing in SC</td>
<td>Scheller-Wolf and Tayur (2009)</td>
</tr>
<tr>
<td>Omni-channel</td>
<td>Jia, Karp, Ravi and Tayur (2021)</td>
</tr>
</tbody>
</table>

**Table 2:** Various Applications of IPA.
2.2 Contemporary Problem, Classical Solution Method: Private Jets, MILP

I also had (in 1994) the opportunity of working on a new business model (Keskinocak and Tayur, 1998), fractional jets, which had to be scheduled. The methodology and framework we used was somewhat routine: Mixed Integer-Linear Programming (MILP). The important aspect, beyond having identified a new problem, was that our solution was implemented, and was crucial for the effective daily operation of the business. Since then, other such companies have developed similar algorithms and added crew scheduling (Yang et al., 2008) and use it for on-going operations. Other examples of MILP models that have been implemented in practice include Bundling and Pricing (Shunko et al., 2018) at Caterpillar, Product Portfolio Optimization (Yunes et al., 2007) at Deere, Production Planning (Mehrotra et al., 2011) at ConAgra Foods, and Scheduling Advertisements in Video Games (Turner et al., 2011) at Microsoft; see Table 3.

<table>
<thead>
<tr>
<th>Application Area</th>
<th>MILP model Publication</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yang et al (2008)</td>
</tr>
<tr>
<td>Production Planning</td>
<td>Mehrotra et al (2013)</td>
</tr>
<tr>
<td>Ads in Video Games</td>
<td>Turner et al (2011)</td>
</tr>
</tbody>
</table>

Table 3: Various Implementations of MILP (and extensions).

2.3 Contemporary Problems, Novel Method

2.3.1 Vanilla Boxes and Rapid-Response Supply Chain, IPA

The other new problem that attracted my attention was in managing broader product variety through delayed differentiation, also called postponement, using vanilla boxes (Swaminathan and Tayur, 1998). This was formulated as a two-stage, Stochastic Program with Recourse. Using (the then recently developed) IPA as the computational methodology, IBM applied this to its PC line of business, in Raleigh, North Carolina (to compete against Dell, who had just invented a new method of delivering products to consumers, now called the direct to consumer model), saving them over $600 million annually.

What made IPA (and my academic research) widely known to business executives was a Fortune article (Seikman, 2000) that described our implementation at Caterpillar (Rao, Scheller-Wolf and Tayur, 2000), which was another new, that is, contemporary, problem, that of designing
a rapid-response supply chain for a new line of (small construction) products, such as Skid-Steel Loader (SSL) and Compact Wheel Excavator (CWL) (to compete against an entrenched market leader, Ingersoll-Rand, that makes Bobcats). To this day, this excerpt brings me great joy:

Among the techniques the Carnegie-Mellon group used to attack this complex problem was so-called infinitesimal perturbation analysis, for which no complete explanation is possible for the faint-hearted or mathematically disadvantaged.

...the Carnegie-Mellon solutions are not what Cat would have come up with on its own. A couple of special tool-distribution centers, which the company had planned to build, were found unnecessary. Just as important, the response time in the system was sufficiently fast that the inventories that the dealers would have to carry were not high enough to require a subsidy from Caterpillar. [Carnegie Mellon] gave us the highest response, lowest cost, lowest inventory [solution]...

2.3.2 Alpha Optimal Combination (AOC), Quantum-inspired algorithms

Were the algebraic geometry algorithms developed for integer programs implemented anywhere? Not at that time, but, I have now-twenty-five years later!-repurposed them into a quantum-inspired algorithm (Sodhi and Tayur, 2023), and implemented it at QuantBot Technologies, a multi-billion dollar Hedge Fund Manager, in a contemporary problem, Alpha Optimal Combination (AOC). Here alphas, representing positive market signals, that are noisy and correlated, derived daily from various machine-learning (ML) algorithms, from various research sub-groups, need to be combined optimally (by the Chief Investment Officer, CIO) to yield the best total daily return subject to several constraints on risk.

2.4 Frivolous-Gravitas (FG) Spectrum

Recently, I had the opportunity to work on omni-channel fulfillment (Jia et al., 2021), a contemporary issue in retail operations, using, what I now call a classical method: IPA. This work was done in collaboration with a startup called Onera Commerce (co-founded by a CMU alum and acquired by Tools Group), whose clients include Dick’s Sporting Goods and Saks Fifth Avenue. Likewise, motivated by Rent-the-Runway (IPO in 2021), a new business model that rents out high-end fashion, we analyzed their operations (Slaugh, Biller and Tayur, 2016) using sample path comparisons!

Contemporary problems analyzed with novel solution methods, in particular machine
learning (ML) approaches, include Liquid Biopsy (through adaptive sequential hypothesis testing) (Gan et al., 2021) and Split Liver Transplantation (through multi-armed bandits) (Tang et al., 2021).

Why always be so serious? In Frivolous-Gravitas (FG) spectrum, video games (Turner, Scheller-Wolf and Tayur, 2011) may be considered discretionary and frivolous (although gaming is a big business), in contrast to organ transplantation and cancer genomics, which are usually thought of as having gravitas and essential. Our dynamic scheduling of advertisements in video games was done in collaboration with Massive Incorporated, a startup, that was acquired by Microsoft (reportedly for $400 million): Would you consider that frivolous? Private Jets can be likewise considered discretionary; but, making them affordable and widely available for life-saving transplants? Audacious, isn’t it?

3 The Engineer as an Entrepreneur

Identifying new problems: yes, it’s fun. Formulating them creatively and solving them innovatively is even more fun. Publishing them in prestigious journals and winning academic awards (Table 4)-sure, why not? Editing research oriented books in supply chain management (Tayur, Ganeshan and Magazine, 1999) and healthcare (Dai and Tayur, 2018) for the academic community? Yes, it is important.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Mathematics</th>
<th>First Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overtesting</td>
<td>Queuing Games</td>
<td>CHOM (2012)</td>
</tr>
<tr>
<td>OrganJet</td>
<td>Queuing Games</td>
<td>Pierskalla (2015)</td>
</tr>
<tr>
<td>Liquid Biopsy</td>
<td>ML</td>
<td>Pierskalla (2021)</td>
</tr>
<tr>
<td>Combating Child Labor</td>
<td>Game Theory</td>
<td>MSOM (2022)</td>
</tr>
<tr>
<td>Split Liver Transplant</td>
<td>Fluid Model</td>
<td>PSOR (2022)</td>
</tr>
</tbody>
</table>

Table 4: Academic Prizes. CHOM: College of Healthcare Operations Management of POM Society. Pierskalla Award for Best Paper, from INFORMS Healthcare Application Society. MSOM Best Paper Award, from MSOM Society. PSOR: Public Sector Operations Research, from INFORMS.

What is even more enjoyable and fulfilling is having companies actually implement your recommendations/algorithms, and being compensated handsomely (Camm and Tayur, 2010), proportionally to, even half of, the value that your implementation created (Tardif et al., 2010). This has been the story of my research career over and over again, whether in manufacturing, in supply chains, in private jets, or in video games.
3.1 Software Entrepreneurship

How to go from one-off commercial projects and create a repeatable model? How to change industrial practice worldwide? One is to partner with a (major) consulting firm; for instance, I have a good long-term relationship with McKinsey & Company. This, however, is still not very scalable, nor is it particularly satisfying to me. A second approach is to be a software entrepreneur, which I was, as Founder and CEO of SmartOps Corporation (acquired by SAP), and when I created the market for Enterprise Inventory Optimization (EIO) software. Over 700 global companies (Caterpillar, for example (Keene et al., 2006)), across a variety of industries, in over 50 countries (see some examples in Europe, Figure 2), use EIO on an ongoing manner, connected to their enterprise information technology (IT) backbone.

3.2 Social Entrepreneurship

I opened this essay with OrganJet, my second entrepreneurial venture (Roth, 2012), a social enterprise that focuses on democratizing access to transplants by providing on-demand and affordable access to private jets (see Figure 3). How to analyze the equilibrium consequences of this innovation? Each transplant center can be considered as an over-crowded queue with abandonment, and multiple listing as selfish routing in a network of such queues. The equilibrium is studied through fusing game theory with queuing: rational queues or queuing games (a mathematical technique also used in studying Over testing in healthcare) (Dai, Akan and Tayur, 2016). Analysis using real data shows that OrganJet increases the number of transplants, through an increase in organs used (and by reducing the discards) and transplants the ones in need in a timely manner, thus reducing geographic inequity and the number of deaths on the waiting list. This is perhaps the most dramatic demonstration of Rawlsian concept (Rawls, 1985) of fairness, made possible through matching demand with supply using private jets on-demand. For over 12 years, OrganJet has provided hundreds of people an opportunity to obtain transplants earlier and has made their lives better. I am very happy to be contributing to Market Design (Roth, 2002) by re-imagining the US Transplant System (Table 5).
Figure 2: Some examples of SmartOps EIO in Europe.

Figure 3: OrganJet: Flight paths due to multiple-listing.
3.3 Scaling: Matching Content with Channel

I think a lot about how the content I am developing can be scaled; that is, what is the channel for its widespread dissemination beyond publications (Table 6)?

Beyond OrganJet (which is a Case Study (Battilana and Weber, 2013), as is SmartOps (Wilcox and Yemen, 2011), both distributed by Harvard), I am most proud of the Nudge Videos that are being used at many OPOs (Organ Procurement Organizations) to increase second-person consent from next-of-kin (NOK) (Kush and Tayur, 2022), an initiative that was highlighted by the Obama White House in 2016, and is now in use at various OPOs. Both OrganJet and Nudge Videos are examples of Maximally Inverse thinking, a phrase that I coined: OrganJet takes the patient to the organ, rather than bringing the organ to the patient; Nudge Videos influence the NOK after the potential donor is deceased, the last possible point in time when one could be helpful, in contrast to nudging first-time drivers at the Department of Motor Vehicles (DMV), likely the earliest time when one could choose to become a donor.

<table>
<thead>
<tr>
<th>Patient/ Society Issue</th>
<th>Market Design Solutions Requiring No Reforms</th>
<th>Proposed Reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geographic disparity</td>
<td>OrganJet</td>
<td></td>
</tr>
<tr>
<td>Size/ Gender Inequity</td>
<td>Split Liver</td>
<td>Exception Points by Height</td>
</tr>
<tr>
<td></td>
<td>Transplantation (SLT)</td>
<td></td>
</tr>
<tr>
<td>Supply shortage</td>
<td>Nudge Videos</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Market Design: Re-imagining the US Transplant System.

I now teach this way of thinking to award-winning social entrepreneurs, from more than 20 countries, annually since 2017, at the Harvard-WEF Schwab Social Entrepreneurship Leadership Program at Harvard Kennedy School (HKS).

And finally, how to scale academic entrepreneurship itself? As a Limited Partner (LP) and an advisor in a venture capital (VC) firm, Neotribe Ventures! Half of our investments are in University spin-offs.
<table>
<thead>
<tr>
<th>Content</th>
<th>Channel</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>SmartOps Enterprise Inventory Optimization (EIO)</td>
<td>SAP</td>
<td>2004</td>
</tr>
<tr>
<td>OrganJet &amp; GuadianWings</td>
<td>Harvard</td>
<td>2011</td>
</tr>
<tr>
<td>Nudge Video for NOK consent</td>
<td>White House</td>
<td>2016</td>
</tr>
<tr>
<td><em>Maximally Inverse</em> Thinking</td>
<td>WEF /HKS</td>
<td>2017</td>
</tr>
<tr>
<td>Quantum Integer Programming (QuIP)</td>
<td>NASA/USRA</td>
<td>2018</td>
</tr>
<tr>
<td>Unconventional Computing</td>
<td>TED</td>
<td>2020</td>
</tr>
</tbody>
</table>

**Table 6:** Matching Content with Channel.

### 4 The Second Quantum Revolution

I have had a three-decades long avocational fascination with physics. The possibility of fusing this (until now, a purely intellectual) hobby with my professional career was too tempting to ignore. I created the field of Quantum Integer Programming (QuIP) in 2018 (Bernal, Tayur and Venturelli, 2021). NASA/USRA (Bubenik, 2020) and TED (Tayur, 2021) have been channels to disseminate this work to technical and non-technical audiences, respectively.

Some ways for our OR/OM community to participate in *The Second Quantum Revolution* are listed in Table 7. For the computationally oriented, the one-to-one correspondence between certain combinatorial optimization problems (for example, that underlie cancer genomics (Alghassi et al., 2019)) represented by a Quadratic Unconstrained Binary Optimization (QUBO) model, which maps directly to the Ising model-creates wonderful opportunities to contribute to *quantum computing* (Alghassi, Dridi and Tayur, 2019). For the more mathematically oriented, the connection between Quantum Information Science (QIS) and Semi-definite Programming (SDP) is a natural pathway to learn about *quantum sensing* (Siddhu and Tayur, 2022). Those interested in queuing (Siddhu et al., 2021) and Markov Decision Processes (Kumar et al., 2023) can find exciting applications to tackle in *quantum communications*.

Our community has stayed away from being at the forefront of developing new (computing) hardware, focusing on algorithmic analysis and development. I now have an opportunity (on a DARPA grant with BBN-Raytheon and engineering physicists at Cornell University (Onodera, Ng and McMahon, 2020) to build an entirely new type of non-Turing computing device based on Floquet engineering and superconducting qubits to solve hard combinatorial problems! I invite others in our community to join me in contributing to *The Second Quantum Revolution*. 

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5 What are we doing here?

What is the meaning of one’s professional life (beyond teaching which I have previously discussed (Tayur, 2016))? How can we help bring novelty, such as AI for Medicine (Dai and Tayur, 2022), responsibly into this world? How can we improve fairness in our society (Bernards et al., 2022)? How can we ethically (Kim et al., 2022) improve the lives of many? How can we prevent atrocities (Cho et al., 2019) in global supply chains? How can we advance our civilization through pushing the boundaries of computing hardware (Prabhakar et al., 2023)?

<table>
<thead>
<tr>
<th>Capitalism</th>
<th>Academic Capitalist</th>
<th>Academic</th>
<th>Academic Philanthropist</th>
<th>Philanthropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commonwealth Capital PE</td>
<td>SmartOps</td>
<td>Supply Chain</td>
<td>New Business Models</td>
<td>OrganJet Healthcare</td>
</tr>
<tr>
<td>Neotribe VC</td>
<td>Healthcare Quantum</td>
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Table 8: Academic-Capitalist-Philanthropist.

Table 8 shows how I view my professional activities (Tayur, 2017). They center on Academic research (which over three decades has spanned supply chain, new business models, healthcare, and quantum); they then branch out, both towards Capitalism and towards Philanthropy, through entrepreneurship (SmartOps, OrganJet) and through judicious use of money (Neotribe Ventures and RAGS Foundation), which was itself earned by implementing novelty in a scalable manner with a channel partner (EIO, SmartOps, and SAP).

My contemporary response to Emerson (Emerson, 1837) is summarized in Figure 4: it provides a framework of Transcendental Engagement that I have developed for myself, to shape my own professional life, that rests on four pillars: intellectual enjoyment (bringing novel concepts into this
world), economic vibrancy (through implementing and facilitating novelty), spiritual nourishment (improving fairness and equity, combating atrocities), and philosophical reflection; that is, *How can we, through our professional craft, leave worthwhile footprints on the sands of time?*

![Figure 4: Transcendental Engagement.](image)

**Acknowledgements.** There are way too many people who have been my partners and collaborators - from Industry, Academia, Government - in this journey to list by name. Indeed, to just mention briefly, even a couple of sentences each, on how they contributed will take more pages than the main text. Thanks to all.
References


