Simultaneous Equation Econometrics: Some Weak-Instrument and Time-Series Issues*

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1. Introduction

In a recent paper entitled "Simultaneous Equation Econometrics: The Missing Example," Epple and McCallum (2006; E&M hereafter) begin by documenting that—surprisingly—existing textbooks include no example using actual data of a basic supply-demand system in which simultaneous-equation estimation yields results that are satisfactory and superior to those provided by (inconsistent) ordinary least squares.[1] They then provide an example, based on the U.S. market for chicken broilers, that is claimed to satisfy these criteria. Only a few specification tests were applied, despite the development in recent years of several valuable new types of tests. Our contributions in the present paper are two-fold. (a) We further the pedagogical enterprise begun by E&M, using their model as a platform for illustrative application of some specification tests relating to exogeneity and strength of instruments. (b) We employ their model to address some potential issues concerning the famous "spurious regression" concept.

Exogeneity and weak-instrument tests are important in principle for studies of the type that the E&M paper presents. It transpires, however, that in fact their paper's chicken broiler example fares quite well when subjected to these (and other) tests, better than one might reasonably hope for. With regard to the time-series tests of point (a), it is clear that in principle a number of researchers have concerns about issues related to those tests. It will be argued below, however, that there is room for substantial reservation about any suggestion that the estimates reported in E&M (2006) are in this regard unsatisfactory. It is the purpose of the present paper to develop substantive positions on both points (a) and (b). In the process of doing so, this paper significantly extends and

[1] What is here meant by "satisfactory" is that the results should "... feature theoretically appropriate signs on each of the estimated structural parameters with all of the important estimates being significantly different from zero at conventional significance levels" (Epple and McCallum, 2006, p. 374).

enriches the discussion provided in E&M (2006). In what follows, we begin in Section 2 by examining the posited exogeneity of instruments used in the two-stage least squares estimation and also their strength. Next, we argue that limited-information maximum likelihood (LIML) estimation provides significant advantages over two-stage least squares, one being the possibility of admitting robust standard errors. Accordingly, such estimation is conducted in Section 3, with encouraging results. In Section 4, we turn to a different concern, namely, the possibility that the broiler industry time-series estimates for the supply equation suffer from the "spurious regression" malady. We argue, however, that such is not the case—doing so by presenting a small Monte-Carlo study illustrating that such maladies typically arise under residual autocorrelation much different from that of the study in question. Finally, Section 5 provides a short conclusion.

2. Instrument Tests (TSLS)

It is inarguable that a good simultaneous equations example should pass muster when evaluated by modern econometrics tests. In addition, an example of the relevant type would be more valuable if it also illustrated the use of such tests. Consequently, this portion of the present paper seeks to do both. We will demonstrate that the estimates in the simultaneous-equation chicken broiler example are quite robust and, hence, that the example as presented in E&M (2006) can be used by instructors who choose not to cover some or all the various tests applied below. For those who do wish to use some or all of these tests, it will be shown that the model can productively be employed to illustrate the use and value of these tests. We proceed first with the system's supply equation and then follow with the demand equation. Equations reproduced from E&M (2006) are reported with the same equation numbers used there. Other equations below are labeled with a continuation of the numbers used in E&M (2006). The last equation in E&M (2006) is (16), and accordingly the first of the new equations below is numbered (17).

Supply Equation

For a self-contained presentation, we reproduce here the broiler supply equation, which was in E&M (2006) numbered (13):

$$(13) \quad qprod^A = 2.030 + 0.221\, p - 0.146\, pf + 0.0184\, time + 0.631\, qprod^A(-1)$$

$$(0.695) \quad (0.106) \quad (0.052) \quad (0.0063) \quad (0.125)$$

$$R^2 = 0.996 \quad SE = 0.0351 \quad DW = 2.011 \quad T = 40$$

Here $qprod^A$, p, and pf, and are respectively the natural logarithms of annual chicken production, the price index for chicken, and the price index for chicken feed. The remaining variable, *time*, is a time trend. This equation is estimated using annual data for 1960 through 1999. We approximated exports as follows: $expts = qprod^A - q^A$, where q^A is domestic consumption. Instruments included from the demand equation were then Δy, Δpb, Δpop, $p(-1)$, $q^A(-1)$, *expts*. Here *y*, *pb*, and *pop* are respectively the natural logarithms of income, the price of beef, and population.

Weak instruments test: The weak instrument test employs the F-statistic for the null hypothesis that the coefficients of the instruments are zero in a regression of the RHS endogenous variable against the exogenous variables. That regression is:

(17) $p =$

$$-1.05 - .012 time + 1.45\Delta y + .21 pf + .45\Delta pb + 4.31\Delta pop + .61 p(-1) + .21 qprod^A(-1) + .13\, expts$$

$$(1.65) \quad (.01) \quad (.73) \quad (.08) \quad (.18) \quad (6.67) \quad (.14) \quad (.23) \quad (.50)$$

The instruments in the supply equation are Δy, Δpb, Δpop, p(−1), and expts. The null hypothesis that the coefficients of these five variables are zeros yields an F-value of 6.77.

Stock and Watson (2007) suggest that an F-value less than 10 may signal weak instruments. In such cases, Stock and Watson suggest removing the weakest of the instruments. In (17), these are *Δpop* and *expts*, which have p-values of 0.52 and 0.80 respectively. We then re-estimate the equation in (17) without these two variables, obtaining

$$p = -1.75 - .016time + 1.35\Delta y + .19pf + .43\Delta pb + .64p(-1) + .32qprod^A(-1) \tag{18}$$

$$\quad (1.07)\ (.009) \qquad (.61) \quad (.07) \quad (.18) \qquad (.12) \qquad (.19)$$

Testing the significance of the remaining instruments, we obtain an F-value equal to 11.16, somewhat above the value of 10 suggested by Stock and Watson. Then re-estimating our supply equation without *Δpop* and *expts* as instruments, we obtain:

$$qprod^A = 1.998 + 0.203\ p - 0.141\ pf + 0.0177\ time + 0.640\ qprod^A(-1) \tag{19}$$

$$\quad (0.684) \quad (0.106) \quad (0.0512) \qquad (0.00626) \qquad (0.124)$$

$$R^2 = 0.996 \qquad SE = 0.0345 \quad DW = 2.043$$

A comparison of (19) with the original E&M (2006) supply equation (13) reveals that the results have changed relatively little. Thus, our supply equation estimates are evidently relatively robust to removal of the two weakest instruments.

Testing Endogeneity of Price: To test the endogeneity of price in the supply equation, we retrieve the residuals from (18), denoting them $\hat{\varepsilon}_{18}$, and include them in an ordinary least squares regression along with the variables in (19). In this regression, variable $\hat{\varepsilon}_{18}$ is highly significant, with a p-value of 0.0027. Hence, the test indicates that price is indeed endogenous in the supply equation.

Testing Exogeneity of the Instruments: We next test exogeneity of instruments by testing the over-identifying restrictions in (19), using Hansen's (1982) J-statistic. We first regress the residuals from (19) against the exogenous variables (i.e., the set of variables on the right-hand-side of 18). We then compute the F-statistic obtained by testing the hypothesis that the coefficients of the three instruments (the last three variables in 18) are zeros, obtaining an F-value of 0.074. Multiplying by the number of over-identifying restrictions, two, we obtain the J-statistic value 0.148. Asymptotically, this statistic is distributed as a chi-square with two degrees of freedom. The associated p-value is 0.93. Thus, we find no evidence suggesting rejection of the hypothesis that the instruments are exogenous.

Heteroskedasticity and Autocorrelation: Testing for the presence of heteroskedasticity utilizing the White (1980) test, we reject the null hypothesis of homoskedasticity of the residuals in (19) with p-values of 0.03 and 0.010 with and without cross terms. Estimating the model allowing for first-order serially correlated errors, we obtain a coefficient of –0.14 with p-value 0.54. However, testing for autocorrelation using the Breusch-Godfrey test (Breusch, 1978; Godfrey, 1978) with two lagged terms, we obtain a p-value of 0.046. Given the test results for heteroskedasticity and autocorrelation, it is prudent to compute the standard errors of the coefficients in (19) using Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Comparing the resulting standard errors in (20) to those in (19), we see that the change in standard errors is modest.

$$\text{(20)} \quad \underset{}{qprod^A} = \underset{(0.751)}{1.999} + \underset{(0.107)}{0.203\,p} - \underset{(0.077)}{0.141\,pf} + \underset{(0.0062)}{0.0177\,time} + \underset{(0.130)}{0.640\,qprod^A(-1)}$$

Stability: Finally we apply the Ramsey (1969) RESET test to (19). With one fitted term, we obtain a p-value of 0.89 and with two fitted terms a p-value of 0.91. Thus, we do not find evidence against the specified functional form.

Implications of Tests of the Supply Equation

We conclude that the amended supply equation in (20) satisfies all of the criteria that we have mentioned. Moreover, the instrumental-variable estimates in (20) do not differ in any important respect from the original supply equation (13) estimated by two-stage least squares. Thus, the E&M (2006) supply equation can be used with reasonable confidence in teaching simultaneous econometrics and two-stage least squares estimation. The steps leading to the amended supply equation (19) provide an illuminating application of the weak-instruments test, and that equation in turn provides a useful framework for exhibiting modern econometric tests.[2]

Demand Equation

The demand equation numbered (12) in E&M (2006) is repeated for convenience:

$$\Delta q = \underset{(0.142)}{0.841}\, \Delta y - \underset{(0.086)}{0.397}\, \Delta p + \underset{(0.093)}{0.274}\, \Delta pb \tag{12}$$

$R^2 = 0.299$ SE = 0.0251 DW = 1.920 T = 40

The instruments for this demand equation are *pf, time,* Δpop*, p(–1),* $qprod^A(-1)$*,* and *expts*. These variables have all been defined above.

Weak instruments test: We regress the RHS endogenous variable in (12), Δp, against the exogenous variables (Δy, Δpb, *pf*, *time*, Δpop, *p(–1)*, $qprod^A(-1)$, *expts*) and a constant. The instruments in the demand equation are the last six variables in the list in the preceding sentence and the constant. The null hypothesis that the coefficients of

[2] For a useful non-technical exposition concerning weak instruments, robust standard errors, and other related matters, the reader is referred to Hill, Griffiths, and Lim (2008), Murray (2006), or Stock and Watson (2007).

these six variables are zeros yields an F-value of 3.83, which is well short of the F-value of 10 suggested by Stock and Watson (2007). From the p-values for these variables, we conclude that the five weakest are the constant, *Δpop*, *time*, $qprod^A$, and *expts*. Repeating the weak instruments test after deleting these five variables, we obtain an F-value equal to 9.39, still somewhat short of the value of 10, though not markedly so. In E&M (2006, p. 379) the logarithm of meat exports, *mx*, is utilized as an instrument for the change in chicken exports. Using the change in this variable and the change in *pf*, we estimate the equation for the weak instrument test, regressing Δp against (*Δy*,*Δpb*, *Δpf*, *p(–1)*,Δ*mx*). Testing the three instruments, the last three variables in the preceding list, we obtain an F-value = 11.40, which exceeds the value of 10 suggested by Stock and Watson.

Re-estimating the demand equation with these instruments, we obtain:

$$\Delta q = \underset{(0.138)}{0.830}\,\Delta y - \underset{(0.076)}{0.336}\,\Delta p + \underset{(0.088)}{0.225}\,\Delta pb \tag{21}$$

$R^2 = 0.289$ SE = 0.0244   DW = 1.820   T = 39

Comparing this result to equation (12) above, we see that estimating the demand equation with stronger instruments produces modestly lower price and cross-price elasticities.

Testing Endogeneity of Price: To test the endogeneity of price in the demand equation, we retrieve the residuals from the first-stage equation, denoting them $\hat{\varepsilon}_{21}$, and include them in an ordinary least squares regression with variables in (21) together with the estimated residuals $\hat{\varepsilon}_{21}$. We find that variable $\hat{\varepsilon}_{21}$ is not significant (p-value = 0.82). Hence, we do not find evidence that price is endogenous in the demand equation. This result suggests that it would be acceptable to rely on the ordinary least squares estimates of the coefficients of the demand function, presented in E&M equation (6):

(6) $\Delta q = 0.711\ \Delta y - 0.375\ \Delta p + 0.251\ \Delta pb$

$$(0.150) \qquad (0.058) \qquad (0.068)$$

This suggestion is reinforced by the strong similarity of these OLS estimates, the TSLS estimates in (8), and the IV estimates in (21).

Testing Exogeneity of the Instruments: Before concluding that OLS estimation is appropriate for the demand function, we investigate exogeneity of the instruments by testing the over-identifying restrictions in (21). We regress the residuals from (21) against the exogenous variables ($\Delta y, \Delta pb, pf, p(-1), \Delta mx$). Testing the hypothesis that the coefficients of the three instruments (the last three variables) are zero, we obtain an F-value of 0.747. Multiplying by the number of over-identifying restrictions, two, we obtain a value of 1.49 for Hansen's (1982) J-statistic value. Asymptotically, this statistic is distributed as a chi-square with two degrees of freedom, and the associated p-value equals 0.48. Thus, we find no evidence to reject the hypothesis that the instruments are exogenous.

Heteroskedasticity and Autocorrelation: Testing for the presence of heteroskedasticity utilizing the White (1980) test, we obtain p-values of 0.59 and 0.60 with and without cross terms. Thus, we do not reject the null hypothesis of homoskedasticity of the residuals. Estimating the model allowing for first-order serially correlated errors, we obtain a coefficient of –0.21 with p-value 0.17. Testing for autocorrelation using the Bruesch-Godfrey test (Bruesch, 1978; Godfrey, 1978) with two lagged terms, we obtain a p-value of 0.54. Given the test results for heteroskedasticity and autocorrelation, there is no evidence of departures from the classical assumptions regarding the error terms. However, for completeness, we report below the results obtained by using the Newey-

West (1987) heteroskedasticity and autocorrelation consistent standard errors for equation (6). Comparing the resulting standard errors in (6), we see, as expected, that the change in standard errors is modest.

$$(22)\quad \Delta q = \underset{(0.127)}{0.711}\,\Delta y - \underset{(0.067)}{0.374}\,\Delta p + \underset{(0.053)}{0.251}\,\Delta pb$$

Stability: Applying the Ramsey (1969) RESET test, we obtain p-values of 0.47 and 0.74 respectively with one and two fitted terms. Thus, we find no evidence against the functional form we have chosen.

Implications of Tests of the Demand Equation

Based on our tests of the demand equation, we conclude that ordinary least squares is an appropriate estimation procedure for the demand equation. Moreover, there is no evidence of heteroskedasticity, autocorrelation, or model instability. It is of interest to note that equation (13) can be used as a demand estimation example in elementary treatments of OLS estimation. In using such an example, the instructor need not be concerned that the estimates are subject to simultaneity bias that is being "swept under the rug." Moreover, the estimates provide highly significant and plausible income, price and cross-price elasticities, illustrating three key concepts of elementary demand theory. While the demand equation can be used as a stand-alone example, in more extended treatments the supply-demand example in E&M can then later be employed in developing TSLS and IV estimation.

3. Limited-Information Maximum Likelihood and Robust Standard Errors

As we have shown above, two-stage least squares coefficient and standard error estimates are robust to the presence of absence of the weaker instruments. This is quite reassuring. A further method for enhancing robustness is to adopt an approach to estimation that is less sensitive to the presence of weak instruments. The Limited Information Maximum Likelihood (LIML) estimator of the coefficients has been found to be much less sensitive to the presence of weak instruments than estimators obtained by two-stage least squares (Imbens and Wooldridge, 2007). In addition, standard errors for endogenous variables can be obtained that are robust to the presence of weak instruments (Moreira, 2003). Hence, we now provide results obtained using LIML.

Our supply equation estimated by LIML is:

$$(23) \quad \underset{}{qprod^A} = \underset{(0.703)}{2.140} + \underset{(0.114)}{0.286}\, p - \underset{(0.053)}{0.163}\, pf + \underset{(0.0065)}{0.0205}\, time + \underset{(0.128)}{0.600}\, qprod^A(-1)$$

Comparing these coefficient estimates to those in (13) above, we see that the own-price and cross-price elasticities have increased somewhat in magnitude. Overall, the coefficient estimates are reassuringly similar to those in equation (13). Beneath the coefficients, we have reported the standard errors obtained using the conventional normal approximation. Using the conventionally calculated standard error, the p-value for the endogenous variable, *p*, is 0.017. The p-value for this endogenous variable using Moreira's (2003) robust Conditional Likelihood Ratio test is 0.0019. Thus, the endogenous variable is even more significant using this robust test.

The broiler demand equation estimated by LIML is:

$$(24) \quad \Delta q = \underset{(0.163)}{0.881}\, \Delta y - \underset{(0.134)}{0.546}\, \Delta p + \underset{(0.121)}{0.373}\, \Delta pb$$

The LIML estimates of the coefficients are somewhat larger in magnitude than the two-stage least squares estimates. Overall, as with the supply equation, the coefficients are reassuringly similar to their two-stage least squares counterparts. The conventionally calculated standard errors are in parentheses. The endogenous variable, Δp, has a p-value of .000 using either the conventionally calculated standard error or Moreira's robust test. The results from LIML estimation provide further support for our simultaneous equations model.

4. Time Series Issues

We now turn to a different topic, namely, a concern relating to the time series properties of the data and implications for estimated relationships. The issue is whether the supply function represents a case of the much-discussed "spurious regression" phenomenon (Granger and Newbold, 1974). This possibility must be carefully considered, however, because the Granger-Newbold examples—and others prominent in the literature—typically feature strong serial correlation in the residuals of the estimated equations. The E&M supply function, by contrast, has a DW statistic of 1.87 and shows no significant residual autocorrelation in a LM test (as noted in E&M footnote 7).

In a paper that considers related issues, McCallum (1993) has implicitly argued that concern for autocorrelated residuals is crucial in alleged cases of spurious correlation. His position is that in any time series study an investigator with even elementary training in econometrics should not be satisfied with a time series regression in which strong serial correlation of the residuals is apparent—especially in cases in which the estimated relation includes a lagged endogenous variable as a regressor. At a minimum, a conscientious and competent investigator would re-estimate the relation while including some "correction" for autocorrelated disturbances such as the iterated Cochrane-Orcutt (1949) procedure or related procedure such as those employing nonlinear constraints, as outlined in Davidson and MacKinnon (1993, pp. 331-341). In cases similar to the basic Granger-Newbold examples, McCallum (1993) implies, the spurious findings of nonexistent relationships will tend to be eliminated by this procedure.

To provide support for McCallum's argument, which was more suggestive than conclusive, consider the simulation results reported in Table 19.1 of Davidson and MacKinnon (1993, p. 672). There the standard case of a spurious regression between two independently-generated random-walk variables is examined in that table's column 2, which shows much greater rejection frequencies than the actual 0.05 for the true hypothesis of a zero slope coefficient—this is the "spurious" finding.[3] For convenience, we report some of the Davidson and MacKinnon frequencies in the first row of Table 1, where T designates sample size for the regressions studied via numerous replications. Of course we cannot use the same data as that generated by Davidson and MacKinnon, but

Table 1
Simulation Results Regarding Spurious Regression:
Relative frequency of rejections of tested hypotheses

	T = 50	T = 100	T = 250	T = 500	T = 1000
D&M, Table 19.1, col.2	0.662	0.760	0.847	0.890	0.928
BEM, repl. [frac DW<1]	0.668 [0.997]	0.769 [1.000]	0.859 [1.000]	0.896 [1.000]	0.920 [1.000]
BEM, with AR(1)	0.0875	0.0659	0.0571	0.0537	0.0527
BEM, non-RW case (.8)	0.339	0.350	0.360	0.360	0.358
BEM, with AR(1)	0.0645	0.0564	0.0484	0.0503	0.0516

we have generated results using the same simulation setup. Our rejection frequencies, analogous to those of Davidson and MacKinnon, are shown in row 2; they indicate

[3] The innovations are standard normal variates.

clearly that our simulation study reproduces the drastically incorrect rejection tendency noted by Davidson and MacKinnon.[4] Also in row 2, we report in brackets the fraction of times in which the regression's DW (Durbin-Watson, 1951) statistic is below 1.0, a value that implies extremely strong autocorrelation of the estimated residuals. As is clear from these values, almost all of the regressions in the Davidson and MacKinnon version of the basic Granger-Newbold example feature very strong autocorrelation. So next we ask, what would happen if the econometrician re-estimated his equation using some standard technique of the iterated Cochrane-Orcutt (1949) type, i.e., one designed to take account of first order serial correlation of residuals. In row 4, we report our results based on calculations provided by the "AR(1)" procedure built into EViews.[5] As can be seen, when the simulated equation is estimated with EViews while specifying an AR(1) disturbance, rather than presuming white noise disturbances, the proportion of rejection frequencies falls to 0.0875 and 0.0659—rather than 0.668 and 0.769—for sample sizes of T = 50 and T = 100, and to values very close to the true 0.050 for larger sample sizes.

In a fairly recent expository paper, Granger (2001) has emphasized that spurious estimated relationships occur not only between (or among) random-walk or "integrated" variables, but also stationary but strongly autocorrelated time series variates. We illustrate this point in row 4 of Table 1 by means of two independent series each of which is generated by an autoregressive process, specifically, a first-order autoregression with AR parameter value of 0.8. The rejection frequencies shown in row 4 do not rise with sample size, as in rows 1 and 2, but the frequency of hypothesis rejections is greater than

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[4] We have followed Davidson and MacKinnon in using n = 10,000 as our number of replications over which to average the results.

[5] The EViews AR(1) estimation procedure differs somewhat from the iterative Cochrane-Orcutt procedure. In particular, it assumes an AR(1) disturbance process and then uses nonlinear estimation of parameters, including the AR parameter, as mentioned above.

1/3 in all of our cases, over six times as large as the true rejection probability. But, as before, almost all of the test regressions have DW statistic values smaller than 1.0. Accordingly, we again consider the outcomes that occur when the investigator employs the EViews AR(1) procedure. Clearly, row 5 shows that in this case the rejection frequencies become very close to the true values built into our simulation study. Thus these results, like those of row 3, support the suggestion that the “spurious regression” phenomenon is actually not a matter of concern when there is no evidence of autocorrelated disturbances—as in the supply function estimation.

Also of concern are unit root tests on individual variables and the popular concept of cointegration. Of course, many interesting econometric issues exist that involve unit roots and cointegration, but relevant tests are problematic and for some of these issues the actual difficulties have to do with stochastic behavior that involves autoregressive roots close to, but not exactly equal to, 1.0. In any case, the issues are of doubtful relevance in terms of the findings reported in the E&M (2006) study. The main issues in the context of that study would seem to be (i) whether the E&M supply function is sensibly specified and (ii) whether that equation’s residuals indicate serial correlation of the disturbances. The latter has already been discussed. Regarding the former, it should be noted that the supply function is not misspecified due to the inclusion of TIME. That variable is included in the supply equation not for detrending purposes but as a proxy for the state of technology in broiler production, an unobservable variable that is theoretically of great importance in the relationship in question. While use of the TIME proxy is certainly not fully satisfactory, it seems greatly preferable to the alternative of omitting the technology variable altogether. Moreover, the inclusion of TIME as an exogenous variable creates no statistical problems such as inconsistent standard errors and non-standard distributions. The discussion in Chapter 16 of Hamilton (1994) indicates that while different modes of convergence are needed in the study of asymptotic distributions, the usual tests and confidence intervals are appropriate when time trends are included in linear models. Hamilton's results pertain to OLS regressions, but the logic would appear to carry over to instrumental variable estimators as well.

5. Conclusions

The goal in E&M (2006) was to provide a pedagogically useful example of a simultaneous equation supply-demand system featuring results with actual market data. In the present paper, we have suggested that application of modern econometric tests to that model is appropriate. As it transpires, the results of such tests serve to enhance the pedagogical value of the E&M (2006) chicken broiler example. In particular, we see the results leading to the following conclusions:

(i) The coefficient and standard-error estimates from two-stage least squares estimation of the supply-demand model are quite robust. Thus, the model is well-suited to its original purpose of illustrating two-stage least squares estimation of a simple supply-demand model.

(ii) Both the supply and demand equations prove to be useful in illustrating the application of weak-instrument tests. For the supply equation, the weak-instrument test suggests dropping two of the weaker instruments that are included when two-stage least squares is used. For the demand equation as well, the weak-instrument test suggests dropping the weaker instruments. In addition, that test suggests inclusion of an instrument for exports that E&M (2006) had introduced in their paper. When the model is estimated retaining only the stronger instruments, the coefficients and standard errors exhibit only minor change relative to those obtained with two-stage least squares.

(iii) The value of the test for endogeneity of price is well illustrated by the two equations in the model. The test strongly suggests that price is endogenous in the supply equation while providing no evidence of endogeneity in the demand equation. Thus, the two together provide a particularly illuminating application of the test for endogeneity. The tests of instrument exogeneity are well illustrated in both equations, and support the presumed exogeneity of regressors in both.

(iv) Tests for heteroskedasticity and autocorrelation are also well illustrated by the two equations. The White (1980) test points to the presence of heteroskedasticity of the residuals in the supply equation but not in the demand equation. The Breusch-Godfrey Lagrange multiplier test suggests that autocorrelation may be present in the supply but not in the demand equation. Estimation allowing for either AR(1) or MA(1) error components reveals that neither is close to significant in the supply equation. Thus, correction of the standard errors of the coefficients for heteroskedasticity is clearly called for in the supply equation, and prudence suggests correcting for autocorrelation as well. Those corrections result in little change in the estimated standard errors of the coefficients in the supply equation.

(v) The demand equation is apparently a near-ideal vehicle for elementary presentation of ordinary least squares. Having applied the impressive arsenal of tests offered by modern econometrics, we have found no evidence to suggest departing from the use of ordinary least squares estimation with conventional calculation of standard errors of the coefficients. Moreover, the equation provides estimates of three demand elasticities that are instructive for elementary presentations of demand theory.

(vi) The supply equation, by contrast, provides an attractive example for illustrating the value of tests of endogeneity and exogeneity. These tests both demonstrate the need for instrumental variable estimation and the validity of the instruments used. The weak instrument test is also instructive as are tests for heteroskedasticity and autocorrelation. The former illustrates selection among potential instruments. The latter two tests motivate use of standard errors that are corrected for the possible presence of heteroskedasticity and autocorrelation.

As a further investigation of robustness, we presented limited information maximum likelihood estimates of the coefficients and significance tests for the endogenous variables using Moreira's (2003) robust Conditional Likelihood Ratio test. These results further support the specification developed in E&M (2006).

All in all, the simultaneous equation system proposed in E&M (2006) seems well suited to illustrate estimation of a supply-demand system with two-stage least squares or with limited information maximum likelihood. Additional issues relating to time-series properties of the system's variables are discussed in Section 4. Here we will not attempt a full summary of that rather terse discussion. The main point, however, is that "spurious regression" relationships between random-walk or strongly autoregressive variables are generally accompanied by signs of severe autocorrelation in the residuals of the estimated relationships; re-estimation taking account of potential autocorrelation tends to eliminate the appearance of non-existent relationships. In the broiler market case at hand, however, there is no indication of such autocorrelation in the first place. Accordingly, despite the strongly autoregressive nature of broiler price and quantity series, there is no reason based on Granger-Newbold (1974) analysis to suspect that the estimated supply-demand relationships for chicken broilers are spurious.

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