Opacity, Credit Rating Shopping and Bias[*]

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June 29, 2015

Abstract

We develop a rational expectations model in which an issuer purchases credit ratings sequentially, deciding which to disclose to investors. Opacity about contacts between the issuer and rating agencies induces potential asymmetric information about which ratings the issuer obtained. While the equilibrium forces disclosure of ratings when the market knows these have been generated, endogenous uncertainty about whether there are undisclosed ratings can arise and lead to selective disclosure and rating bias. Although investors account for this bias in pricing, selective disclosure makes ratings noisier signals of project value, leading to inefficient investment decisions. Our paper has implications for regulatory disclosure requirements.

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1. Introduction

In the aftermath of the financial crisis there has been considerable spotlight on credit rating accuracy and potential upward bias in ratings, especially given that the issuer pays for ratings and can publish and disclose selectively those for the marketplace to consider in evaluating complex financial instruments.[1] This context raises fundamental questions about the form of equilibrium. To what extent are communications prior to disclosure of ratings publicly available? Under what conditions are ratings disclosed selectively in equilibrium, reflecting bias to which investors would react? This is central to understanding the nature of those credit ratings that are disclosed and the implications for asset pricing.

While rating bias has emerged in the literature under a variety of assumptions in which investors react myopically to ratings, we address whether the incentive to shop for ratings and disclose selectively disappears under rationality. For example, does rationality by investors guarantee unbiased ratings? Would enforcement of mandatory disclosure of the issuer's interaction with the rating agency ensure unbiased ratings? Traditional frictions–such as (a) asymmetric information in which the issuer has better information exogenously than investors or (b) moral hazard in which the rating agency has the incentive to distort the ratings to attract additional ratings fees–would lead potentially to bias. Instead, we abstract from moral hazard and assume that rating agencies report their ratings truthfully and focus on a more subtle form of asymmetric information.

A possible source of rating bias is the ability of issuers to obtain indicative ratings from rating agencies without being required to disclose all contacts (or the ratings). Of course, disclosure of such indicative ratings could take a variety of forms such as mandatory disclosure of the information provided by the rating agency, disclosure of the contact of a particular rating agency by the issuer in the specific context (e.g., the underlying information might be complex and it could be too difficult to mandate disclosure of the fundamental information), or, as had been the case historically, the contact could be viewed as private. This discussion

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[1]Griffin and Tang (2012) document empirically the potential subjectivity in ratings. The potential for bias and more specifically, the apparent inverse relationship between rating standards and the success of a rating firm is illustrated dramatically by a statistic in Lucchetti (2007), who reported that Moody's market share "dropped to 25% from 75% in rating commercial mortgage deals after it increased standards." The ongoing relevance and pervasiveness of rating shopping after the financial crisis is illustrated by Neumann (2012).

suggests a number of policy issues, including what types of disclosure should be required and the incentives between the stages of purchasing an indicative rating and disclosing that rating. It also emphasizes the importance of the form of equilibrium. As our analysis illustrates, under rational expectations disclosure of contacts by the issuer with the rating agency is very powerful and can eliminate rating bias arising from selective disclosure.[2]

We develop a rational expectations framework in which the issuer conveys information to the market by using ratings agencies to improve the precision of the market's information, which in turn can enhance the efficiency of project choice. We examine the impact of transparency at the ratings stage. In addition to mandatory disclosure of all ratings (or equivalently, all contacts for ratings are transparent) we analyze an opaque alternative in which the contacts are not known. If the disclosure of indicative ratings already purchased is costless, then in the transparent case all purchased ratings are disclosed, implying that rating shopping and rating bias do not arise. In the spirit of the literature on voluntary disclosure of information (e.g., Grossman and Hart (1980), Grossman (1981) and Milgrom (1981)), when it is common knowledge that a set of ratings have been purchased, then all of those must be disclosed in equilibrium to avoid a harsh inference about any undisclosed ratings.

In contrast, in the absence of any disclosure requirement about ratings contacts (the "opaque" case) we show how endogenous uncertainty can emanate from the rating process. Then investors would not know whether ratings are undisclosed because they were unavailable, or because the ratings were sufficiently adverse. As a consequence, the issuer can avoid a completely adverse inference (which Milgrom (2008) terms maximally skeptical) as suggested by the "unraveling principle" and full disclosure; instead discretionary or selective disclosure would arise in equilibrium. Rating shopping and rating bias would then occur in the opaque case whenever the equilibrium entails publication of fewer ratings than the number of indicative ratings purchased—as the issuer would then choose selectively which ratings to publish, choosing the highest ratings obtained.[3]

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[2]In our setting "selective disclosure" refers to the issuer selectively disclosing to the market a subset of the credit ratings that they have received. In some other contexts (as in the case of Regulation FD), selective disclosure refers to disclosure to a subset of investment advisors or other market participants rather than disclosure to the entire marketplace.

[3]Neumann (2012) points out that it is unusual for a rating firm to publish a detailed report for a deal that it didn't rate ultimately. Publications of such reports would be typical in a transparent world, but not in an

We focus upon a game in which there is opacity about whether rating agencies have been approached by the issuer and have provided the issuer an indicative rating. We assume that there are two rating agencies, which leads to the potential for selective disclosure in a situation in which a single rating is disclosed. The disclosure of a single rating could reflect either an optimal decision to obtain only a single rating or selective disclosure of the more favorable rating when two ratings have obtained. This reflects the issuer being at the margin of obtaining a second rating, leading to endogenous uncertainty. Of course, if the equilibrium were transparent, uncertainty would not arise.

Under appropriate conditions, the efficient benchmark consists of only one rating being produced (from the low-cost rating provider) and the issuer then proceeding with the investment project provided that it receives a high realization of the rating. However, when information about which ratings have been purchased is private, issuers cannot refrain from shopping for more ratings and disclosing selectively, unless rating fees are sufficiently high. These incentives result in inefficient overproduction of information, which is one source of inefficiency in our framework, enabling ratings agencies to extract rents.[4] Somewhat paradoxically, issuers are worse-off because of their private information.

Another important source of inefficiency in the environment with selective decision is how the underlying disclosure decision distorts the investment decision. The disclosure choice and the ratings of the projects are intertwined with the determination of the real investment decision; these are made by the issuer on behalf of investors. The issuer's investment choice reflects its evaluation of the anticipated price of the project rather than its own assessment of the project's net present value. In turn, the price of the project depends on the information that the issuer discloses to investors. While rational expectations preclude the possibility that investors may be systematically wrong and ensure that investors will be correct on average (and pricing is unbiased), selective disclosure and the lack of transparency allow the possibility that investors have incorrect beliefs in particular states. To the extent that such states are associated with inefficient investment, selective disclosure is detrimental for welfare. More specifically, in our parameterization the nature of the inefficiency is one in which the issuer hides adverse ratings information from investors and sells them the negative NPV project

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opaque setting.

[4]A related result on how voluntary disclosure results in socially excessive incentives to acquire information is in Shavell (1994). The contrast with Shavell (1994) is discussed later in the introduction.

at a profit. In this context rating shopping facilitates investments that would not pass the traditional net present value standard.

Our analysis shows that, provided rating costs are not too large, any opaque equilibrium will be inefficient (Proposition 2) due to the potential investment inefficiency that emerges as well as the overproduction of information, while the transparent equilibrium does not entail rating bias and is efficient. This conclusion leads to an important normative implication of our paper, namely, that issuers should be required to disclose their receipt of indicative ratings.[5] The current framework based upon the Dodd-Frank rules adopted by the SEC in summer 2014 (Rule 17g–7(a), effective June 15, 2015) does not require disclosure of indicative ratings when relevant to understanding rating shopping, but only disclosure after the decision to purchase and disclose a rating.[6] In this sense while the disclosure requirements for ratings were enhanced as a result of Dodd-Frank, the recent regulatory rule changes do not appear to alter fundamentally the context for disclosure of indicative ratings. Indeed, the SEC formally proposed such a rule to require disclosure of indicative ratings in fall 2009 and such a requirement was discussed by the executive branch and legislators in the debate on financial reform, but the SEC's proposed regulation was not adopted and became a lower regulatory priority once it was not included as part of the Dodd-Frank Act's explicit requirements for credit rating agency regulation (while other aspects of Dodd-Frank's requirements for credit rating agencies appeared to have squeezed the proposal from the regulatory agenda).

Rating shopping has had a number of other impacts on the policy debate. For example, the New York State Attorney General's 2008 settlement with the three major rating agencies mandating fees at the indicative ratings stage (though not barring them at the disclosure stage) attempted to reduce rating shopping (Office of New York State Attorney General, 2008).[7] Critics of rating agencies have suggested that rating shopping and the ability of the issuer to choose its rating agencies represent an important conflict of interest that distorts the ratings process. One of our paper's messages is that ex ante fees reduce (but do not eliminate)

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[5]We use the terminology indicative rating to refer to a "preliminary assessment" of a rating prior to a decision by the issuer to go ahead with that rating agency.

[6]The regulatory framework does require disclosure of "preliminary" ratings, which are credit ratings that are anticipated to be publicly provided.

[7]The Cuomo plan prevented the imposition of rating fees that are contingent upon the rating provided by the rating agency.

the incentive to shop for ratings.

The prior literature has highlighted that investor naïveté is one condition under which rating shopping and selective disclosure obtain without rational expectations, while investors are systematically deceived (Skreta and Veldkamp (2009), Bolton, Freixas and Shapiro (2012)). Recent empirical evidence in Kronlund (2014) suggests that rating shopping has distorted the actual ratings on corporate bonds (ratings are relatively higher for issues that are more likely to experience rating shopping). Yet interestingly, Kronlund's evidence suggests that investors adjust for this in market pricing.[8] In effect, the market pricing reflects the potential winner's curse associated with the choice of rating agencies (also see discussion in Sangiorgi, Sokobin and Spatt (2009)). This evidence suggests the advantage of utilizing a rational framework, rather than one based upon naïve or myopic pricing.

Our paper contributes to the literature on discretionary disclosure as the source of asymmetric information arises endogenously through the equilibrium choices in our setting (e.g., compare to Dye (1985), Shin (2003) and Acharya, DeMarzo and Kremer (2011)), rather than being exogenously specified, as would be traditional in many frameworks with asymmetric information. In these papers the source of uncertainty (whether the manager has a signal or is uninformed) is exogenous. Two papers in this literature, which point to some contrasts with our specification, are those of Matthews and Postlewaite (1985) and Shavell (1994). In their settings whether sellers obtain information on their products is private information, as in our opaque regime. As testing is costless in Matthews and Postlewaite (1985), full disclosure is universal absent disclosure requirements (we would have full disclosure in our setting, if the costs were zero).[9] In Shavell (1994), the cost of collecting information is random and privately known by the seller. As a result, not all sellers decide to obtain information and those who do disclose selectively. While uncertainty about the cost of information is natural in several

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[8]Further evidence on rational pricing of credit ratings in the context of competition is offered by Becker and Milbourn (2011). They find that higher competition between credit rating agencies, as measured by Fitch's market share, is associated with higher ratings. However, they find that the correlation between bonds' yield spreads and credit ratings decreases when competition is high, consistent with competition reducing the information content of ratings.

[9]However, under some conditions in Matthews and Postlewaite (1985) the seller prefers buyers to be uninformed and will not test when disclosure rules are in effect, because the full disclosure rule acts as a commitment not to test. When it would be socially desirable for consumers to be informed, this leads to the conclusion that full disclosure rules would not be desirable, a conclusion not obtained in our setting.

settings, it is less applicable to the context of credit ratings, where information about the rating agencies' fee schedules is largely public.

In contrast to these papers, in our setup the issuer can obtain information from multiple sources, and the cost of information (the rating fee) is not exogenously specified, but determined as a strategic variable by the rating agencies.[10] Additionally, in our setting we show that opacity arises endogenously after modeling explicitly the market for information (Proposition 7). Intuitively, rating agencies internalize the issuer's incentives to disclose ratings selectively; opacity prevents the issuer from committing not to shop for ratings and provides a source of rents to the rating agencies. It is important to emphasize that our framework highlights the empirical implications for rating shopping (especially, see Section 6), unlike the earlier theoretical literature on disclosure.

A related literature initiated by Lizzeri (1999) studies the revelation of information by certification intermediaries. In contrast to our paper, in which the focus is on the issuer's incentives to disclose, the focus of this literature is on the strategic disclosure decisions of the certification intermediaries. In Lizzeri, a monopolistic intermediary discloses information only to the point of inducing efficient trade and captures all the surplus. The result follows from the ability of the information intermediary to manipulate information, rather than from the opaque assumption; the rating process is transparent in Lizzeri. Furthermore, competition among certification intermediaries can result in full disclosure and zero profits for the intermediaries (full disclosure is the unique equilibrium in our model when the market is transparent). In a related paper, Faure-Grimaud, Peyrache and Quesada (2009) take a mechanism design approach and find conditions under which the optimal ex post efficient contract is implemented by the firm owning its rating. While they show that full disclosure is not robust in their setting, ratings reveal the asset value perfectly, so there is no incentive for a firm to purchase multiple ratings and rating bias does not arise. Another major difference with our setup is that, in these papers, the object for sale is an existing asset that the seller has some private information about, so that (i) the information that is generated by certification intermediaries has no value ex ante and (ii) selective disclosure has no real implications. By contrast, in our paper, the seller uses the ratings that are produced by the rating agencies to discover the prof-

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[10]The role of a market for information for the emergence of selective disclosure in equilibrium is further discussed in Section 5.1.

itability of the investment project; because the issuer's (investment and disclosure) decisions are linked, selective disclosure distorts the investment choice and has real effects.

Other papers adopt rational expectations to focus on specific aspects of credit ratings precision and regulatory policy rather than shopping.[11] In these papers, the informativeness of credit ratings emerges as a result of strategic information acquisition and disclosure decisions of the rating agencies. In contrast to these papers, in our model the rating technology is exogenous and rating agencies reveal their information truthfully. However, rating shopping and selective disclosure "garble" the meaning of the ratings that are disclosed; in equilibrium, the information on projects' quality that is produced by the rating agencies is transmitted to investors with noise. This noise, which is endogenous to the rating process, lowers investor belief accuracy and results in inefficient investment decisions.

Our paper is organized as follows. Section 2 describes the underlying specification of the model and the disclosure policy in the presence of common knowledge. Section 3 addresses the equilibrium in a transparent market and Section 4 examines the equilibrium in an opaque market. Section 5 provides some discussion of the modeling assumptions and extensions. Section 6 discusses some empirical implications of the model and ties these to the empirical literature on credit ratings. Section 7 concludes. The Appendices contain details omitted from the main text, including proofs.

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[11]Mathis, McAndrews and Rochet (2009) study the role of reputational concerns of rating agencies in determining ratings precision. Reputational concerns are also modeled in Fulghieri, Strobl and Xia (2014)–who study the impact of unsolicited ratings on ratings standards, and Bar-Isaac and Shapiro (2013)–who study how ratings quality varies over the business cycle. Opp, Opp and Harris (2013) examine information acquisition and the impact of rating contingent regulation. Kashyap and Kovrijnykh (2014) study the effect of the payment model on the precision of ratings in an optimal contracting framework. Goel and Thakor (2015) study ratings coarseness in a cheap talk framework. Kartasheva and Yilmaz (2013) analyze the determinants of the precision of ratings in equilibrium and examine the implications for a number of policy issues.

2. The Model

2.1. The economy

Players. We consider a two-period economy populated by a representative issuer, a unit mass of investors and two credit rating agencies (CRAs). All players are risk neutral and maximize expected profits.

The issuer has access to a risky investment project that requires investment of one unit of the consumption good in the first period and pays off y units in the second period. The payoff has a binary structure: the project returns $R > 1$ units of the consumption good in the event of success and returns zero in the event of failure. There are two types of projects, good and bad, that differ in their probability of success. Good (bad) projects have success probability equal to π_g (π_b), with $1 \geq \pi_g > \pi_b \geq 0$. We assume $\pi_g R - 1 > 0 > \pi_b R - 1$, so that only good projects have positive NPV. There is no ex ante asymmetric information: all agents (including the issuer) share the same prior that with probability $\gamma \in (0,1)$ the project is of the good type and with probability $1 - \gamma$ the project is of the bad type.

The issuer cares about first-period consumption only. Conditional on investing, the issuer can create a claim to the payoff of the investment project and sell this asset to investors. Investors care about consumption in both periods, have zero discount rate, and each investor is endowed with $m \geq \pi_g R$ units of the consumption good in the initial period. This setup implies that if the issuer makes the investment and sells the asset, its market price equals investor expectations of the payoff.

Rating technology. Credit rating agencies (CRAs) have access to a technology that allows production of information about the project's type. At some cost $c_i \geq 0$, each CRA$_i$ can produce a signal, or rating, $r_i \in \{H, L\}$, such that

$$\Pr(r_i = H \,|\, g) = \Pr(r_i = L \,|\, b) = \frac{1}{2} + e. \tag{1}$$

The constant $e \in [0, 1/2]$ measures ratings precision: a rating is pure noise if $e = 0$ while it reveals the project's type with certainty if $e = 1/2$. Conditional on the firm's type, each rating is independent. We assume ratings are informative ($e > 0$), but noisy ($e < 1/2$).

These assumptions imply that CRAs have access to equivalent but independent rating technologies, and that ratings precision is the same across project types.[12] CRAs are allowed to be heterogenous with respect to their cost parameters.

Notation. We denote by $E_\iota(y)$, π_ι and NPV_ι, respectively, the expectation of the payoff y, the probability of success and the NPV of the investment, all conditional on the information set ι. That is,

$$NPV_\iota = \pi_\iota R - 1 = E_\iota(y) - 1.$$

Prior (unconditional) information is denoted by $\iota = 0$, so that $E_0(y) = \pi_0 R$, $\pi_0 = \gamma \pi_g + (1 - \gamma) \pi_b$ and $NPV_0 = \pi_0 R - 1$. Further, we denote by p_H the unconditional probability that a rating takes a high value and we denote by $p_{H|H}$ ($p_{H|L}$) the probability that a rating takes a high value conditional on the realization of the other rating being high (low).

By "split ratings" we refer to any event in which the two ratings generated by the rating agencies have different values. We remark that in the current setup, the probability of success of the project conditional on split ratings is (i) independent of which of the two ratings takes the high value, and (ii) coincides with the prior probability of success.

Distribution of types. Let $\bar{\gamma}$ denote the threshold value for the unconditional probability of the good type such that the average project has positive NPV if and only if $\gamma \geq \bar{\gamma}$. That is, $\bar{\gamma}$ solves $E_0(y) = 1$, or

$$(\bar{\gamma} \pi_g + (1 - \bar{\gamma}) \pi_b) R = 1. \tag{2}$$

We assume $\gamma < \bar{\gamma}$, so that the average project has negative NPV. As a result, the input from CRAs is necessary to generate investment and trade in the economy.[13]

Social value of ratings. Consider a total surplus maximizing social planner who is uninformed and thus relies on credit ratings to screen projects before making investment decisions.

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[12]The assumptions in (1) on ratings precision being symmetric greatly simplify the exposition. However, our results do not hinge either on ratings precision being symmetric across types, or–with the exception of results in Section 4.4.1–on ratings precision being symmetric across CRAs.

[13]This assumption simplifies the exposition by making the project negative NPV conditional on a low rating, regardless of the informativeness of the rating technology and regardless of the realization of the other rating. The assumption does not, however, affect the main qualitative features of our analysis.

Since the planner would only invest in positive NPV projects, the value of the planner's problem conditional on information set ι is simply NPV_ι^+. (Here x^+ denotes the positive part of x). We define the *marginal social benefit of a rating* to be its marginal decision value provided to the planner, that is, the expected increase in the value of the planner's problem that is brought about by screening through the additional rating.

Since the average NPV is negative, the initial value of the planner's problem (i.e., conditional on no ratings) is nil. Then, the marginal social benefit of a first rating, v_I say, is given by

$$v_I = p_H NPV_H^+. \tag{3}$$

Intuitively, a first rating improves the planner's investment decision only if a high rating results in the project having positive NPV.

Next, consider whether producing a second rating, r_2 say, adds value to the planner's decision problem after r_1 has been produced. Conditional on $r_1 = L$, the project has negative NPV regardless of the realization of r_2, and the second rating has no social value in this case. In contrast, if $r_1 = H$, a second rating will improve the planner's investment decision if $NPV_{H,H} > 0$, in which case it is optimal to invest in the project if and only if $r_2 = H$. The value of the planner's problem conditional on $r_1 = H$ is NPV_H^+. Hence, the marginal social benefit of a second rating, v_{II} say, conditional on a first rating being high, is

$$v_{II} = p_{H|H} NPV_{H,H}^+ - NPV_H^+. \tag{4}$$

Ratings are *substitutes* if $v_{II} < v_I$, that is, if the marginal social value of a rating decreases in the number of ratings that have already been produced.[14,15]

The socially optimal choice of ratings ultimately depends on the configuration of rating precision and production cost parameters. Although each rating may be valuable in isolation, producing all ratings for all assets is unlikely to be socially optimal in practice. In our framework with two CRAs, we capture these ideas with the assumption that, while each rating has social value individually, it is socially optimal to produce only one rating. These features

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[14]A related (but static) notion of substitutability of signals is given in Börgers, Hernando-Veciana and Krähmer (2013).

[15]Lemma A.2 in Appendix A shows that there exists a value $\bar{e} < 1/2$ such that $v_{II} < v_I$ if and only if $e > \bar{e}$. Intuitively, if ratings are sufficiently informative about the project's type, the added value of screening through a second rating is relatively low.

obtain in our model if ratings are substitutes and costs are such that $c_i < v_I$ for $i = 1, 2$ but $\max\{c_1, c_2\} > v_{II}$. For concreteness, we let CRA$_1$ be the most efficient provider of credit ratings by assuming $c_1 < c_2$.

Summary of parameter restrictions. We now restate more formally the parameter restrictions under which we develop our results. For given π_g, π_b and R such that $NPV_g > 0 > NPV_b$, for the remaining exogenous parameters we assume:

Assumption 1. The parameter values γ, e, c_1, c_2 satisfy $\gamma < \overline{\gamma}$, $c_1 < c_2 < v_I$ and $c_2 > v_{II}$.

Efficient benchmark and economic surplus. Under Assumption 1, the efficient benchmark consists of having only the first rating being produced and making the investment only conditional on the realization of this rating being high. Define further the *economic surplus* to be the sum of all agents' ex ante utilities, net of initial endowments. Economic surplus under the first best, efficient allocation, equals $v_I - c_1 > 0$. We shall refer to $v_I - c_1$ as the *potential surplus of the economy.* In the following analysis, we say that an equilibrium is *efficient* if its associated economic surplus equals the potential surplus of the economy, and is *inefficient* otherwise.

2.2. Rating process

In the market economy, the rating process is as follows. CRAs set rating fees, f_1, f_2, under the constraint $f_i \geq c_i$. The issuer can approach CRA$_i$ and purchase its rating by paying f_i; in which case CRA$_i$ produces the rating r_i, which is communicated to the issuer. At this point the issuer owns the rating and can either withhold it or make it public through the rating agency. Only in the latter case would investors observe the rating. A crucial feature of the rating process that plays a major role in our analysis is its degree of transparency. The rating process is defined as *transparent* or *opaque,* depending on whether or not the act of purchasing a rating is observable by investors. If the market is opaque, investors only observe the purchased ratings that the issuer decides to publish voluntarily.[16]

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[16]It is implicit in our framework that, if the market is opaque, the number of purchased ratings is not verifiable. This follows from the discretion that parties have in practice as to what constitutes a rating or just

The timing of the model is as follows. All decisions are made in the first period, that is divided in the following four stages:

Stage 1 CRAs simultaneously post fees; fees are observed by all players.

Stage 2 The issuer shops for ratings. The issuer can shop sequentially, that is, can purchase a first rating, and decide whether to purchase the second rating after observing the value of the first rating.

Stage 3 Once all purchased ratings are observed by the issuer, the issuer decides which ratings to disclose (if any) to investors.

Stage 4 The issuer decides whether to make the investment and whether to sell the asset.

The issuer consumes and leaves the market at the end of stage 4. In the second period, the payoff from the investment is realized and investors' consumption takes place.

The model assumes ex ante symmetric information between the issuer and investors. However, as the game unfolds, information becomes asymmetric. The extent of this asymmetry depends on the degree of transparency of the rating process. If the market is transparent, there can only be asymmetric information about the undisclosed ratings that are known to have been created. In contrast, if the market is opaque, asymmetric information is more severe as investors don't know whether an undisclosed rating has been created.

Without loss of generality, we assume that, when indifferent between making the investment and selling in stage 4, the issuer makes the investment and sells.

2.3. Equilibrium definition

A pair of rating fees induces a subgame. On a subgame, a strategy profile for the issuer is a set of contingent plans on (i) which ratings to purchase at stage 2, (ii) which of the purchased ratings to disclose at stage 3 and (iii) whether to make the investment and sell the asset to investors in stage 4. A system of investor beliefs is a specification of beliefs on the value of ratings that are not disclosed both on- and off- the equilibrium path. The solution concept

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a preliminary assessment.

implemented is Perfect Bayesian Equilibrium, or *equilibrium* hereafter. A strategy profile for the issuer and a system of investor beliefs are an *equilibrium of the subgame* induced by (f_1, f_2) if investor equilibrium beliefs are consistent with the issuer's strategy and the issuer's strategy is sequentially optimal given rating fees and investor beliefs. An *equilibrium of the overall game* is a list of equilibria in every subgame induced by each pair (f_1, f_2) and a pair of fees (f_1^*, f_2^*) that constitute a Nash equilibrium of the game played by CRAs in stage 1. An equilibrium is said to be *essentially unique* if all equilibria lead to the same payoffs for all players.

2.4. Disclosure, pricing and investment

A key aspect of the model is that the issuer's (investment and disclosure) decisions and the pricing of the asset are intertwined. The reason lies in the issuer's incentives: its investment decisions are based not on its own valuation of the NPV of the project, but on its anticipation of the price of the asset. In turn, the price of the asset depends on investor expectations of the value of the project, and these expectations are based on the information disclosed by the issuer. Through this investment channel, the issuer's disclosure decisions are relevant for welfare.

With rational expectations, full disclosure is a natural benchmark for the issuer's disclosure rule. In a *full disclosure strategy* the issuer sells the asset in stage 4 only if all purchased rating information is disclosed in stage 3. An equilibrium of the subgame in which the issuer follows a full disclosure strategy is a *full disclosure equilibrium*.

With full disclosure, trade always occurs under symmetric information between the issuer and the investors. Hence, the issuer will not make the investment unless the project is positive NPV conditional on the purchased credit rating information, for otherwise the asset price would not cover the investment cost. In other words, full disclosure implies that pricing is "strong form efficient"; the anticipation of this informationally efficient pricing leads the issuer to socially optimal investment decisions.[17]

While full disclosure implies that the equilibrium *use* of the information that is produced is efficient, the assumed ownership structure of ratings implies that the issuer always has the

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[17]For example, under Assumption 1, in a full disclosure equilibrium the issuer makes the investment if and only if all the purchased ratings are high (see Lemma A.3-(i) in Appendix A)–just as the planner would do conditional on the same information.

option of disclosing information selectively (e.g., disclose only if a rating is high). Selective disclosure induces a selection bias in the ratings that are published. Rating bias has real effects if it leads to investment decisions that are individually rational but socially inefficient.

We will refer to *effective selective disclosure* as an event in which the issuer hides negative credit rating information from investors, makes the investment in a negative NPV project and sells it at a profit to investors. An *equilibrium with selective disclosure* is one in which effective selective disclosure is on the equilibrium path.

3. Transparent Market Equilibrium

This section describes the equilibrium of the model under the assumption that the rating process is transparent. This case will provide a benchmark against which we can compare the model's predictions in the opaque case.

We begin by illustrating the following key property of equilibrium disclosure in the transparent market:

Lemma 1. (Unraveling.) *Any equilibrium of the transparent market features full disclosure of purchased credit rating information.*

Lemma 1 is a manifestation of the unraveling principle, well known from the literature on voluntary disclosure of information (e.g., Grossman and Hart (1980), Grossman (1981) and Milgrom (1981)). The idea behind this result is the following. When investors observe which ratings are purchased by the issuer, investor beliefs on ratings that were purchased but not disclosed in equilibrium *must* be that undisclosed ratings are of the worst type. If they were not, consistency of beliefs would require the issuer to withhold positive information in some other states, but this cannot be part of an optimal strategy for the issuer, as an issuer who withholds a high rating would get a better price by disclosing it.

An immediate consequence of Lemma 1 is that effective selective disclosure is incompatible with equilibrium in the transparent market. Unraveling further implies that, in a transparent market equilibrium, the issuer's value for an additional rating coincides with the planner's.[18]

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[18]More specifically, in a transparent market equilibrium: (i) the issuer does not purchase a second rating if

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This results despite the fact that the issuer values a rating only to the extent that its disclosure leads to more favorable pricing of the asset. Intuitively, transparency of contacts makes the option to disclose selectively worthless in equilibrium, which aligns private and social valuation of information. (Of course, the issuer trades off the value of credit rating information against the fees, not the rating production costs.)

Lemma B.1 in Appendix B solves for the equilibrium of the subgame for exogenous values of the rating fees. In an equilibrium, depending on the fees, either no rating or a single one is purchased. Which rating will be purchased ultimately depends on the equilibrium rating fees set by CRAs in the initial stage, which is described by the next proposition:

Proposition 1. (Transparent market equilibrium.) *In the (essentially) unique equilibrium of the overall game, CRAs set $f_1^* = f_2^* = c_2$; the issuer purchases r_1 and makes the investment if and only if $r_1 = H$, in which case the issuer discloses r_1 and sells the asset to investors. The equilibrium is supported by off-equilibrium-beliefs that if a rating is purchased but not disclosed, the rating is low.*

Transparency of contacts renders competition among CRAs effective; competition drives down rating fees and ensures that the issuer's equilibrium choice of ratings coincides with the planner's. As the equilibrium use of information is efficient under full disclosure as explained, Proposition 1 has the following welfare implication:

Corollary 1. *The transparent market equilibrium is efficient.*

We remark that the efficiency result in the transparent setup relies on the unraveling principle, not on our specific parametric assumptions. The unraveling result is undone if there are disclosure costs (e.g., Verrecchia, 1983) or some exogenous source of uncertainty about whether a player has information to disclose (e.g., Dye, 1985). As we do not make such assumptions, rating bias and selective disclosure would not arise in this framework unless some

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either the first purchased rating is low or, if the first purchased rating is high, if the fee for the second rating exceeds v_{II} in Eq. (4) (see Lemma A.3-(ii),(iii)), and (ii) the issuer purchases one rating only if this rating's fee does not exceed v_I in Eq. (3) (see Lemma B.1-(iii)).

degree of uncertainty arises endogenously in equilibrium.

4. Opaque Market Equilibrium

This section analyzes the case in which investors cannot observe the number of purchased ratings. In the opaque market, asymmetric information is more severe: the issuer has private information about which ratings are purchased. As a consequence of this informational advantage, the issuer has an incentive to shop for ratings and disclose selectively. This section shows that the impact of these incentives on the equilibrium are substantial.

4.1. Private value of information

A key consequence of opacity that contrasts with the transparent case is the disconnect between the private and social value of information. Consider whether the issuer's strategy from Proposition 1 can be part of an equilibrium in the opaque case. Conditional on $r_1 = H$, the issuer could deviate, purchase r_2 and disclose this additional rating selectively. Then, the issuer could sell for a price of $E_{H,H}(y) > E_H(y)$ in case r_2 turns out to be high, and sell for $E_H(y)$ (by disclosing r_1 but not r_2) in case r_2 turns out to be low. (In the latter case investors would not detect the deviation and would, in fact, overpay for the asset.) Anticipating this, the issuer's expected profits from purchasing the second rating and disclosing it selectively equal

$$-f_2 + p_{H|H}\left(E_{HH}(y) - 1\right) + \left(1 - p_{H|H}\right)\left(E_H(y) - 1\right). \tag{5}$$

Since equilibrium profits, conditional on disclosing $r_1 = H$, amount to $E_H(y) - 1$, then the issuer has an incentive to shop for the second rating if $f_2 < \widehat{v}$, where

$$\widehat{v} := p_{H|H}\left(E_{H,H}(y) - E_H(y)\right). \tag{6}$$

The R.H.S. of (6) measures the issuer's (private) value for a second rating. By comparing (6) and (4), it is immediate to see that $\widehat{v}$ exceeds the planner's value for a second rating:

$$\widehat{v} - v_{II} = \left(1 - p_{H|H}\right) NPV_H > 0.$$

Intuitively, the difference $\widehat{v} - v_{II}$ measures the ex ante value to the issuer of the option to disclose selectively the second rating, and this value is strictly positive.

This example illustrates how the information friction induced by opaqueness bears upon the issuer's incentives: the issuer cannot refrain from shopping for more ratings and disclosing selectively unless shopping costs (the fees) are high enough. This feature of our model is consistent with the principle that selective reporting encourages excessive acquisition of information (e.g., Shavell, 1994). In our model, however, whether this distortion results in inefficiencies or selective disclosure depends on the equilibrium that emerges when rating fees are set strategically by CRAs.

4.2. Inefficiency of opaque market equilibrium

Under opacity, the model features multiple equilibria of the overall game. Regardless of this multiplicity, a relevant question to ask is whether the efficient outcome can be sustained, if not as the unique equilibrium (as in the transparent case), at least as one of the possible equilibria. The following analysis shows when the answer to this question is negative, and therefore any equilibrium in the opaque market is inefficient. Specifically, let the inefficiency threshold $\bar{c}_2$ be defined as

$$\bar{c}_2 := \min\{p_H v_{II} + (1 - p_H)v_I, \widehat{v}\}. \tag{7}$$

We have:

Proposition 2. (Inefficiency of opaque market equilibrium.) *Assume $c_2 < \bar{c}_2$. Then, any equilibrium of the overall game in the opaque market is inefficient.*

The intuition behind Proposition 2 is as follows. For the efficient benchmark to be an equilibrium of the overall game in the opaque market, it is necessary that f_2^* satisfies the "no-shopping condition" $f_2^* \geq \widehat{v}$, as explained. However, under the condition in the proposition, this no-shopping condition cannot be part of an equilibrium of the overall game in which CRA_2 makes zero profits. In fact, as the proof of the proposition shows, CRA_2 makes positive expected profits in *any* equilibrium of the subgame for $f_1 \leq v_I$ and $f_2 \in (c_2, \bar{c}_2)$. Hence, CRA_2 has an incentive to undercut, which contradicts $f_2^* \geq \widehat{v}$ being a best response.[19]

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[19]For some parameter values, the restriction in Proposition 2 that $c_2 < \bar{c}_2$ reflects a more stringent condition for the inefficiency than the requirement that c_2 falls below the no-shopping threshold $\widehat{v}$. This result is in

4.3. Framework for equilibrium selective disclosure

After establishing the conditions under which the efficient benchmark is not an equilibrium in the opaque market (Proposition 2), we turn to the question of whether there exists an equilibrium with selective disclosure.

The following definition illustrates the equilibrium upon which we will focus:

Definition 1. *An equilibrium features* rating shopping and selective disclosure of the second purchased rating *if the following strategy is optimal for the issuer:*

Stage 2 *Purchase r_1 first and then, if and only if $r_1 = H$, purchase r_2 with "shopping probability" $q \in (0,1)$.*

Stage 3 *Disclose only the ratings with a high realization.*

Stage 4 *Make the investment and sell if and only if (at least) one high rating was disclosed.*

The equilibrium strategy from Definition 1 features selective disclosure: conditional on split ratings in stage 2, the issuer publishes the first (and favorable) rating but hides the second (and unfavorable) rating. Lemma 1 implies that there is no selective disclosure in the transparent market, so this strategy cannot be part of an equilibrium in the transparent case. In the opaque case, however, conditional on only $r_1 = H$ being disclosed, investors in equilibrium are truly uncertain as to whether the unobserved rating reflects selective disclosure. Because of this uncertainty, the unraveling principle fails to hold, and the option to disclose selectively is viable. This mechanism–uncertainty about whether a player has information to disclose–is the same as in Dye (1985). Differently from Dye (1985), this uncertainty is endogenous in our setup as it arises from the issuer's optimal information acquisition decisions.

The following features of the equilibrium described in Definition 1 contrast with the transparent market equilibrium.

First, when only $r_1 = H$ is disclosed, trade occurs under asymmetric information. In this case investors will use Bayes' Law–in a way that is consistent with the issuer's strategy–to figure

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contrast to the case of exogenous rating fees (i.e., assuming $f_i = c_i$), where the distortion induced by selective reporting always results in an inefficient outcome when $c_2 < \widehat{v}$.

out the probability that the second rating was purchased but not disclosed. This probability is then reflected by the equilibrium price, $E_{H,q}(y)$ say, as follows:

$$E_{H,q}(y) = E_H(y) - q_P(E_H(y) - E_{H,L}(y)),\qquad(8)$$

where q_P denotes the posterior probability of selective disclosure and is derived in Eq. (C.2) in Appendix C. The price function in Eq. (8) is intuitive. The first term in the R.H.S. of Eq. (8) corresponds to the price investors would be willing to pay conditional on a high rating if they were to take the rating at face value. The second term in the R.H.S. of Eq. (8) measures a "selective disclosure discount," as investors rationally adjust pricing downward to reflect the possibility that a low rating is not being disclosed. Intuitively, q_P is increasing in the shopping probability q: the larger is q, the larger the discount.

The second feature relates to the issuer's investment decision in the state when selective disclosure occurs; this investment decision is individually optimal but socially inefficient. In fact, conditional on the acquired credit rating information, the project has negative NPV. However, in this situation the issuer withholds the low rating from investors and makes the investment anticipating that the asset will be overpriced.[20]

Figure 1 illustrates further properties of the equilibrium from Definition 1 in the event that only $r_1 = H$ is disclosed. The right panel shows the informational content of the rating–from the point of view of investors–as a function of the shopping probability q. Selective disclosure "garbles" the meaning of the rating and results in lower investor belief accuracy. Larger values of q correspond to a larger likelihood that a low rating has not been disclosed, and therefore to a larger probability that the issuer has invested in a negative NPV project. These ex ante inefficient investment decisions result in a higher ex post frequency of defaults, as shown in the left panel of the figure.

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[20] Of course, since investors break even on average, then the asset must be underpriced in some other state, that is, when only $r_1 = H$ is disclosed but the issuer is *not* disclosing selectively. This underpricing, however, is a pure transfer from the issuer to investors.
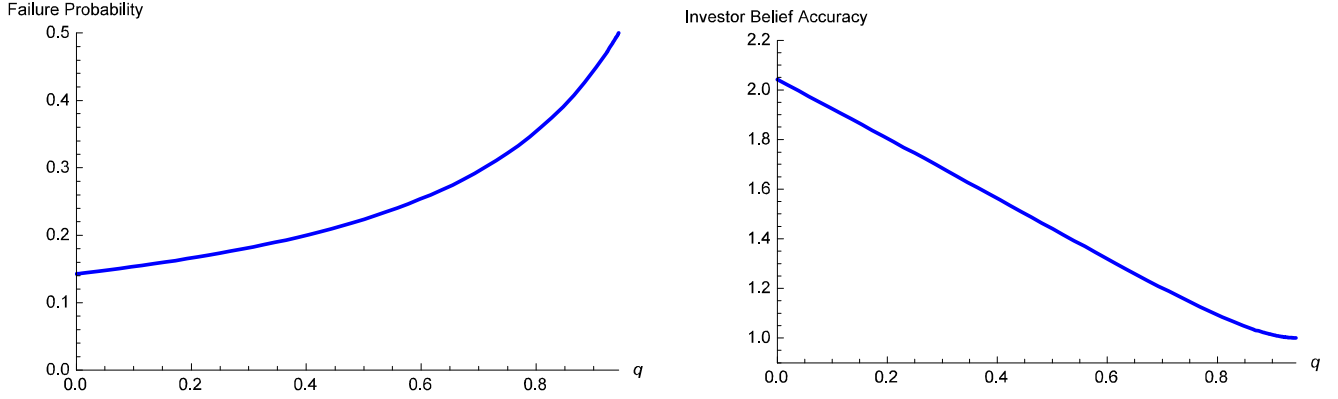
FIGURE 1. Equilibrium failure probability (left panel) and investor belief accuracy (right panel) conditional on $r_1 = H$. Investor belief accuracy is defined as $Var(y|\,r_1 = H)^{-1}$. Exogenous parameter values: $y = 2$; $\pi_g = 1$; $\pi_b = 0$; $\gamma = e = 0.4$. For these parameter values, $\bar{\gamma} = 0.5$, $\bar{e} = 0.19$ and $\bar{q} = 0.94$; unconditional failure probability and investor belief accuracy are, respectively, $1 - \pi_0 = 0.6$ and $Var(y)^{-1} = 1.04$.

Conditions must be satisfied for the strategy in Definition 1 to be optimal for the issuer, as we discuss next and derive in detail in Lemma C.1 in Appendix C.

First, the "pooling" price in Eq. (8) must be large enough to induce the issuer to make the investment, which imposes an upper bound on the shopping probability, $q \leq \bar{q}$.[21]

Second, the issuer must be indifferent between purchasing r_2 or not in stage 3 conditional on $r_1 = H$. This indifference condition requires the fee for the second rating, f_2, to satisfy

$$-f_2 + p_{H|H}\left(E_{H,H}\left(y\right) - 1\right) + \left(1 - p_{H|H}\right)\left(E_{H,q}\left(y\right) - 1\right) = E_{H,q}\left(y\right) - 1, \qquad (9)$$

which can be rearranged as

$$f_2 = p_{H|H}\left(E_{H,H}\left(y\right) - E_{H,q}\left(y\right)\right). \qquad (10)$$

The L.H.S. of the first equality in Eq. (9) is the expected value of purchasing the second rating net of its fee: it encompasses the anticipation for a higher price in case r_2 turns out to be high

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[21]Equivalently, the pool of issuers that only disclose $r_1 = H$ in equilibrium must be such that the project has non-negative NPV.

as well as the option to disclose selectively and sell an overvalued asset in case r_2 turns out to be low. The R.H.S. of the same equation corresponds to the issuer's profits if r_2 is not purchased, in which case the issuer would be selling an undervalued asset.

Finally, the issuer must not have incentives to purchase or disclose ratings in a different way, which is guaranteed if f_1 is not too high and not greater than f_2 and if investors react to off-equilibrium moves with worst-case beliefs on undisclosed ratings.

The next proposition provides sufficient conditions for shopping and selective disclosure to be an equilibrium outcome when fees are determined endogenously. We have:

Proposition 3. (Equilibrium selective disclosure.) *Assume $c_2 < \widehat{v}$. Then, there is a strictly positive constant $\bar{c}_1$ such that, for all $c_1 \leq \bar{c}_1$, there is an equilibrium of the overall game with rating shopping and selective disclosure of the second purchased rating.*

When contacts between issuers and CRAs are opaque and Propositions 2 and 3 hold, selective disclosure emerges as an equilibrium while the efficient outcome does not. The next proposition facilitates the comparison by showing how the various parametric restrictions are jointly fulfilled as long as simple conditions on the fraction of good projects and rating production costs are satisfied. We have:

Proposition 4. (Joint parameter restrictions.) *For any $e \in (0, 1/2)$ there exist values $\widetilde{c} > 0$ and $\widetilde{\gamma} < \overline{\gamma}$ such that, for all $c_1 < c_2 < \widetilde{c}$ and $\gamma \in (\widetilde{\gamma}, \overline{\gamma})$, all parameter restrictions in Assumption 1 and Propositions 2 and 3 simultaneously hold.*

4.4. Alternative equilibria and welfare comparison

4.4.1. A symmetric equilibrium with selective disclosure

If the rating process is opaque, the issuers' incentives to shop for ratings and disclose selectively may result in inefficient investment, overproduction of information and biased ratings, even in the absence of any other friction. Our model portrays these effects with a specific equilibrium, but the insights of our model carry over to equilibria other than the one we described in Definition 1. Consider the following alternative equilibrium:

21

Definition 2. *An equilibrium features* rating shopping and selective disclosure of the first purchased rating *if the following strategy is optimal for the issuer:*

Stage 2 *Purchase a first rating, r_i say, randomly selected with equal probability among r_1 and r_2, and then, if and only if $r_i = L$, purchase r_{-i} with "shopping probability" $q \in (0, 1]$.*

Stage 3 *Disclose only the ratings with a high realization.*

Stage 4 *Make the investment and sell if and only if one high rating was disclosed.*

In contrast to the equilibrium in Definition 1, the issuer now shops for a second rating only if the first purchased rating has a low realization, and at most one high rating is disclosed in equilibrium. The issuer's strategy in Definition 2 captures the intuitive idea that an issuer might want to shop for credit ratings until satisfied with a sufficiently good outcome, in which case it will stop and disclose the highest rating obtained.

As in the equilibrium in Definition 1, the equilibrium in Definition 2 features selective disclosure and socially inefficient investment. In case the first purchased rating is low and the second is high, the issuer hides the low rating from investors and invests in a negative NPV project. In this event, investors buy a highly rated but overpriced asset. Endogenous uncertainty as to whether a second rating was obtained that was not disclosed makes this outcome consistent with equilibrium.[22]

Lemma C.2 in Appendix C details the necessary conditions on rating fees and shopping probability for the strategy in Definition 2 to be an equilibrium on the subgame. The proof of the next proposition gives sufficient conditions for this equilibrium to emerge when fees are endogenous.

Proposition 5. (More equilibrium selective disclosure.) *For a subset of the parameter values for which Proposition 2 holds, there is an equilibrium of the overall game with rating shopping and selective disclosure of the first purchased rating.*

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[22]Of course, selective disclosure is priced so that investors break even on average. (See the equilibrium price function in Eqs. (C.19)-(C.20) in Appendix C).

The equilibrium in Proposition 5 features an investment-and-trade "exuberance" relative to the efficient benchmark. In equilibrium, investors purchase a highly rated security with probability

$$p_H + q^* p_{H\&L},\tag{11}$$

where q^* denotes the shopping probability associated with the equilibrium from Proposition 5 and $p_{H\&L}$ is the probability of split ratings. The second term in (11) is the probability of overinvestment (and subsequent trade), which occurs when the issuer gets a first negative rating but successfully obtains a positive second rating. The incremental investment that arises reflects a low quality investment that would not be undertaken under the efficient benchmark.

4.4.2. Full disclosure equilibria and welfare comparison

The focus of our analysis has been on showing that equilibria with selective disclosure are viable under the opaque assumption. Indeed, under the conditions in Propositions 4 and 5, equilibria with selective disclosure arise in the opaque market while the efficient benchmark does not. However, our analysis does not rule out alternative equilibria, other than the efficient benchmark, that do not feature selective disclosure. If these equilibria exist and are more "reasonable," one might wonder whether our focus on selective disclosure is warranted. To address this concern, we use a total surplus criterion and compare welfare across full disclosure and selective disclosure equilibria of the overall game.

As an illustration, consider a "two-ratings" full disclosure equilibrium in which the issuer purchases r_1 first and then purchases r_2 if and only if $r_1 = H$. As derived in Appendix C, such an equilibrium is *less* efficient than the equilibrium from Proposition 3 if

$$p_H (c_2 - v_{II}) > q^* p_H (c_2 - v_{II}) + q^* p_{H\&L} (1 - E_0 (y)),\tag{12}$$

where q^* denotes the shopping probability associated with the equilibrium from Proposition 3 and $p_{H\&L}$ is the probability of split ratings.

The inequality in (12) is intuitive. With full disclosure, the equilibrium use of information is efficient, as explained in Section 2.4. However, the *production* of information in this two-ratings equilibrium is inefficient because the cost of the second rating exceeds its marginal social benefit. The L.H.S. of (12) measures the ex ante welfare loss associated with such overproduction

of information. The equilibrium of Proposition 3 features overproduction of information too, which results in the welfare loss quantified by the first term in the R.H.S. of (12). In addition, and in contrast to the full disclosure case, the equilibrium *use* of information is also inefficient. When a second rating is produced that is low, this information is not disclosed and production takes place regardless of the project being negative NPV. The second term in the R.H.S. of (12) measures the ex ante welfare loss associated with this inefficient investment decision. However, because the shopping probability q^* is less than one, these inefficiencies materialize less often in the selective disclosure equilibrium than in the full disclosure equilibrium. As a result, we show in Lemma C.3 in Appendix C, the inequality in (12) holds under conditions similar to those in Proposition 4.

As for other full disclosure equilibria, the proof of the next proposition shows that the two-ratings full disclosure equilibrium described in this section is, under certain conditions, the least inefficient among all full disclosure equilibria of the overall game. Hence, the following welfare implication:

Proposition 6. (Welfare comparison.) *For a subset of the parameter values for which Proposition 4 holds, the equilibrium of Proposition 3 generates larger economic surplus than any full disclosure equilibrium of the overall game.*

4.5. Endogenous opacity

So far we have taken the opacity of the rating process as given. We now ask the natural question of whether this opacity can be understood through the lens of our model as the outcome of a choice made by CRAs about the formal interaction with issuers.

More formally, consider an extended game in which each CRA can independently choose the transparency regime as a strategic variable. The contract would therefore specify both the fee at which the rating is sold, and whether the CRA would communicate to investors that the rating has been purchased (transparent regime) or not (opaque regime). Contracts are then announced simultaneously by CRAs in stage 1 and such announcement is observed by all players. The rest of the game is unchanged. The next proposition establishes whether the equilibrium outcomes described in Propositions 1, 3 and 5 are robust to this extension.

Proposition 7. (Endogenous opacity.) *When the degree of transparency is endogenously determined, then:*

i) *For $c_2 < \widehat{v}$, the transparent market equilibrium of Proposition 1 is not an equilibrium of the extended game because CRA_2 has an incentive to make its rating opaque and set a higher fee;*

ii) *The opaque market equilibria of Propositions 3 and 5 are equilibria of the extended game.*

Given our discussion on the different regimes, it is intuitive that the opaque regime emerges as an equilibrium, while the transparent does not. Enabling the issuer with the option to purchase the rating without investors knowing it increases the issuer's private value for the rating. The resulting disconnect between private and social value of information is beneficial to CRAs as it allows them to extract rents that they would not be able to extract if contacts with the issuer were transparent.

5. Discussion and extensions

5.1. The role of a market for information

What is the role of the endogenous rating fees for the result of equilibrium selective disclosure? When only one rating is purchased and disclosed in the equilibria of Sections 4.3 and 4.4.1, the issuer sells at a selective disclosure discount. Hence, the option to disclose a high value for the second rating is more valuable in these cases than in the situation described in Section 4.1, where investor beliefs are held fixed. As a result, the rating acquisition and disclosure strategies in Definitions 1 and 2 are optimal for the issuer only if rating fees satisfy $f_1 \leq v_I$ and $f_2 > \widehat{v}$ (see Lemmas C.1 and C.2 in Appendix C). This implies that rating shopping and selective disclosure describe an equilibrium of the subgame only if the full-disclosure efficient outcome is also an equilibrium of the subgame. However, when fees are determined endogenously, Propositions 4 and 5 show that selective disclosure emerges as an equilibrium of the overall game, while the full disclosure efficient outcome does not. Key to this result is that, when

setting rating fees as a strategic variable, CRAs internalize the issuer's incentives to acquire more information and disclose selectively.

5.2. Noisy ratings

Selective disclosure and endogenous opacity would not arise in the equilibria of our model if ratings revealed the project type with no error ($e = 1/2$). In this case, an extra rating has neither additional information content nor it can possibly result in a better outcome if the first purchased rating is low. The value of shopping for a second rating vanishes as a result if $e = 1/2$. In this sense, a minimal amount of "asset complexity" is necessary for rating shopping and selective disclosure in our framework. On the other hand, Proposition 4 shows that any non-trivial amount of noise in the rating is consistent with equilibrium selective disclosure (under appropriate conditions on the other parameters).

5.3. Continuous distributions

The choice of a binary framework is motivated by analytical tractability. It allows for explicit analysis in a model that features endogenous investment, information acquisitions and disclosure decisions with multiple and imperfectly correlated ratings as well as endogenous pricing of such ratings. The validity of our results, however, is not confined to this particular modeling choice.

In the Online Appendix we solve an alternative model in which the asset payoff and the ratings' distributions are continuous.[23] It is assumed that one (and only one) rating has positive social value in that it allows realization of (exogenous) gains from trade. The main results of the binary framework are shown to be robust to this alternative setup. First, the transparent market equilibrium is efficient, while the opaque market is not (Propositions 1 and 2 in the Online Appendix). Second, while the efficient outcome is not an equilibrium in the opaque case, an equilibrium of the overall game exists that features rating shopping and

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[23] The details of this alternative model are as follows. The issuer is endowed with an asset with random payoff $X \sim N\left(\mu, \sigma_X^2\right)$. There are no investment decisions. Each CRA_i can produce a rating of the form $r_i = X + \varepsilon_i$, where $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$ for $i = 1, 2$ are independent across CRAs and independent of X. The issuer is initially uninformed about X and has an exogenous selling motive. All players are risk neutral.

selective disclosure (Propositions 3 and 4 in the Online Appendix). Similar to the equilibria described in Definitions 1 and 2, in this equilibrium the issuer randomizes over acquiring the second rating and, if a second rating is acquired, the issuer discloses selectively. In the continuous case, however: (i) the issuer is indifferent between purchasing the second rating for any realization of the first rating, and (ii) selective disclosure takes the form of a disclosure rule by which the issuer discloses the second rating if and only if this exceeds a given value that depends on the realization of the first purchased rating. Although inefficient, under some conditions on the parameters this equilibrium generates larger economic surplus than any other full disclosure equilibrium of the overall game (Proposition 5 in the Online Appendix). Finally, when the degree of transparency is determined endogenously as in Section 4.5, the opaque regime emerges as an equilibrium, while the transparent does not (Proposition 6 in the Online Appendix).

6.   Empirical implications

The framework developed in the paper offers a range of interesting empirical implications about the nature of credit ratings in the presence of potential rating shopping, and is particularly relevant for understanding multiple ratings and the information content of published ratings at various levels.[24] The information content in ratings reflects not only the ratings selected for publication and disclosure, but also indicative ratings (even though unobservable) that are not selected for disclosure (also discussed in Sangiorgi, Sokobin and Spatt (2009)). For example, Figure 2 illustrates the comparison between the probability of default for a given rating as a function of the number of ratings and the extent to which these ratings are being disclosed.

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[24]There is considerable evidence with respect to both multiple ratings and split ratings (e.g., see Bongaerts, Cremers and Goetzmann (2012), Livingston, Naranjo and Zhou (2005), Mattarocci (2005) and Livingston, Wei and Zhou (2010)).
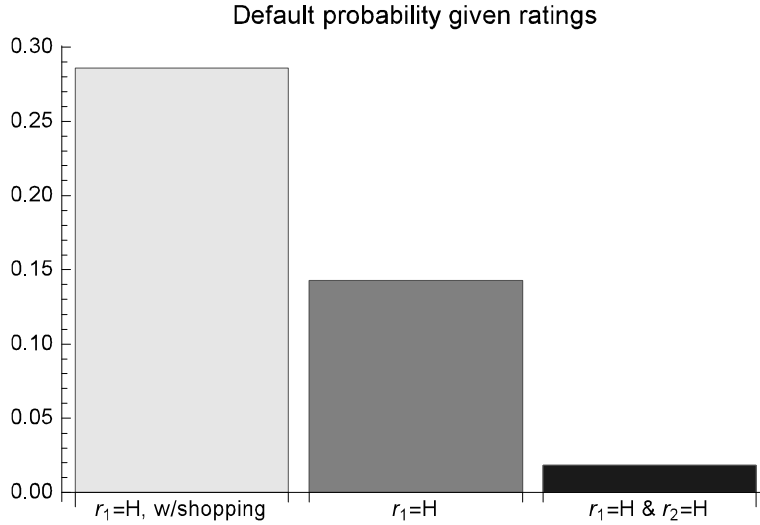
Default probability given ratings

FIGURE 2. The leftmost bar plots the default probability conditional on a single high rating being disclosed in the equilibrium of Proposition 3. The middle and rightmost bars plot the default probability conditional on, respectively, a single high rating with full disclosure, and two high ratings being disclosed. Exogenous parameter values are as in Figure 1 and $c_2 = 0.18$. For these parameter values, the shopping probability in the equilibrium of Proposition 3 is $q^* = 0.68$.

If we compare situations in which a single rating is published with a high value vs. both ratings being published with a high value, a lower number of ratings should predict higher default probabilities and/or future downgrades. The figure decomposes the marginal information content of a second high rating into two parts: (i) the ability to obtain several relatively favorable ratings, and (ii) the presence of fewer unobservable ratings at lower levels. While the first effect would be present even under full disclosure, the second one is specific to the equilibrium with selective disclosure. This emphasizes the fact that selective disclosure increases the sensitivity of the default probability to the number of ratings obtained.

Empirical evidence about credit rating shopping has explored ratings in both the context of traditional bonds (such as corporate bonds) and mortgage-backed securitization tranching. To a degree these are very different contexts–the mortgage-backed securities setting is a relatively newer context with considerable uncertainty (especially due to the import of tranching) as compared to the traditional corporate (or municipal) bonds, for which there is much greater

homogeneity in perspective. However, we do see evidence of the import of selective disclosure in both the corporate bond and tranching contexts. For example, the evidence in Kronlund (2014) highlights that even in the corporate bond context (where the potential for selective disclosure is more limited) that the market's pricing reflects the anticipation of shopping. The tranches underlying structured financing are considerably more complex than standard corporate bonds, so the potential for rating shopping is particularly great within the structured finance market (e.g., Skreta and Veldkamp (2009)). Benmelech and Dlugosz (2010) highlight the collapse of structured finance credit ratings during the financial crisis. It examines the impact of the number of rating agencies that rate an instrument on the subsequent likelihood of a downgrade. Tranches that are rated only by a single agency are most likely to be downgraded and have relatively larger ratings declines.[25] Our analysis highlights (see Sections 4.3 and 4.4.1 as well as Figure 2) that in our opaque equilibrium that when a single rating is published, it is upward biased. The overall empirical findings provide evidence suggestive of shopping and not inconsistent with a rational framework. Examining ex ante pricing, as in Kronlund (2014), is a way to distinguish between the rational and myopic frameworks.

Our formal analysis does highlight why issuers might want to publish multiple ratings, even absent regulatory requirements to obtain multiple ratings. To the extent that investors expect the issuer to solicit multiple ratings, absence of publication suggests adverse information, and implies an information discount. Additionally, multiple ratings could be socially optimal if the additional decision value outweighs the cost of information production. These motives tie closely to the "shopping hypothesis" and "information production hypothesis" in Bongaerts, Cremers and Goetzmann (2012).

At the heart of our equilibrium in the opaque formulation is the incentive to acquire excess ratings for many parameter values because of the potential benefits from selective disclosure for some realizations. For some parameters the ratings are fully disclosed in equilibrium, so in such situations the potential benefits of rating shopping would not manifest itself in actual selective disclosure. Indeed, the prevalence of situations in which two and even three ratings are obtained in practice from the three major rating agencies is a central empirical fact,

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[25]Recent evidence for CDOs that leads to somewhat differing conclusions is provided by Griffin, Nickerson and Tang (2013). They find that CDOs that are rated by a larger set of rating agencies obtain lower yields (consistent with the rational pricing story), but are more likely to default (not consistent with the ex ante pricing nor with the rating shopping perspective).

but it does not suggest that rating shopping is unimportant (unlike the conventional wisdom that rating shopping is associated with the purchase of a single or few ratings). Indeed, our model highlights the inefficiency and overproduction of ratings, coupled with the potential considerable profitability of the rating agencies when issuers derive valuable benefits from the possibility of selective disclosure. Somewhat surprisingly, our formulation demonstrates that the publication of many ratings is compatible with rating shopping.

7. Conclusion

Our paper uses a model based upon rational expectations to examine conditions under which selective disclosure and rating bias emerge in equilibrium. We highlight the role of the structure of equilibrium and regulatory policy about disclosure of contacts with rating agencies to purchase ratings. Requiring the disclosure of the existence of indicative ratings may be equivalent to requiring disclosure of the indicative ratings themselves, eliminating rating bias and leading to efficient generation and use of credit ratings information in equilibrium. In the absence of requiring disclosure of the contacts about indicative ratings (the opaque analysis), rating bias can emerge from selective disclosure even under rational expectations. The inefficient use of credit rating information reflected in rating bias results in investment decisions that are socially inefficient and leads to misallocation of resources.

The focus in our paper on opacity and the structure of equilibrium also is relevant for a broader range of applications beyond our focus on credit rating agencies. For example, consider the situation in which "test takers" can request that only their highest score on a particular subject or exam be reported to outsiders. This would lead to considerable "excess" testing and the ability of the testing body or institution to extract additional rents from that process.[26] This excess testing can also lead to misallocation of investments in human capital, highlighting the real consequences for the allocation of human resources. More generally, the endogenous structure of economic activity is an important potential source of information asymmetry that influences the analysis of institutional arrangements.

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[26] At a presentation at one institution, we heard that it has such a system and considerable retaking of exams. At another presentation, one graduate of that institution said that she had retaken several exams when she was a student.

Appendices

Notation and preliminaries

Investors' information sets. Denote with $\mathcal{I}_T = \{\{H\}, \{L\}, \{N\}, \{\emptyset\}\}^2$ the possible combinations of ratings being disclosed with either high (H) or low (L) value or being produced but not disclosed (N) or not being produced $(\emptyset)$. Similarly, denote $\mathcal{I}_O = \{\{H\}, \{L\}, \{ND\}\}^2$, where ND denotes the event in which a rating was not disclosed without further information about whether the rating was produced or not, i.e., $ND = \{N, \emptyset\}$. At the end of stage 3, investors will have learned $\iota \in \mathcal{I}_T$ in a transparent market and $\iota \in \mathcal{I}_O$ in an opaque market. We use $\iota = d$ to denote any $\iota \in \mathcal{I}_T$ such that $\iota \in \{(r_1 = d, r_2 = \emptyset), (r_1 = \emptyset, r_2 = d)\}$, for $d = H, L$. Similarly, we use $\iota = d$ to denote any $\iota \in \mathcal{I}_O$ such that $\iota \in \{\{r_1 = d, r_2 = ND\}, \{r_1 = ND, r_2 = d\}\}$, for $d = H, L$, whenever investor beliefs are that the undisclosed rating was not produced.

Conditional moments. Denote project's type with $\tau \in \{g, b\}$ and let $g_\iota = \text{Prob}(\tau = g | \iota)$ be the probability of project's type being good conditional on information set ι. Bayes' rule gives

$$g_H = \frac{\gamma(1/2 + e)}{\gamma(1/2 + e) + (1 - \gamma)(1/2 - e)}; \qquad g_L = \frac{\gamma(1/2 - e)}{\gamma(1/2 - e) + (1 - \gamma)(1/2 + e)} \tag{N.1}$$

$$g_{H,H} = \frac{\gamma(1/2 + e)^2}{\gamma(1/2 + e)^2 + (1 - \gamma)(1/2 - e)^2}; \qquad g_{L,L} = \frac{\gamma(1/2 - e)^2}{\gamma(1/2 - e)^2 + (1 - \gamma)(1/2 + e)^2} \tag{N.2}$$

$$g_{H,L} = g_{L,H} = \gamma \tag{N.3}$$

and therefore

$$E_\iota(y) = \pi_\iota R; \qquad \pi_\iota = g_\iota \pi_g + (1 - g_\iota)\pi_b; \qquad NPV_\iota = E_\iota(y) - 1. \tag{N.4}$$

Furthermore, we have

$$p_H = \gamma(1/2 + e) + (1 - \gamma)(1/2 - e); \qquad p_L = 1 - p_H \tag{N.5}$$

$$p_{H|H} = \left(\gamma(1/2 + e)^2 + (1 - \gamma)(1/2 - e)^2\right)/p_H; \qquad p_{L|H} = 1 - p_{H|H} \tag{N.6}$$

$$p_{H|L} = \left(1/4 - e^2\right)/p_L; \qquad p_{L|L} = 1 - p_{H|L} \tag{N.7}$$

Since $NPV_{L,H} = NPV_0$, we can write

$$NPV_H = p_{H|H} NPV_{H,H} + p_{L|H} NPV_0,$$

and therefore, if $NPV_H > 0$, v_{II} in Eq. (4) simplifies to

$$v_{II} = p_{H|H} NPV_{H,H} - NPV_H = -p_{L|H} NPV_0. \tag{N.8}$$

Appendix A: Proofs for Section 2.

Lemma A.1. *For all $\gamma < \overline{\gamma}$ there exists a value $\hat{e} < 1/2$ such that $NPV_H > 0$ if and only if $e > \hat{e}$.*

Proof. From Eqs. (N.1)-(N.4) it is immediate that $\lim_{e \downarrow 0} NPV_H = NPV_0 < 0$ and $\lim_{e \uparrow 1/2} NPV_H = \pi_g R - 1 > 0$. The claim in the lemma follows by g_H being continuous and strictly increasing in e. ∎

Lemma A.2. *For all $\gamma < \overline{\gamma}$ there exists a value $\bar{e} \in (\hat{e}, 1/2)$ such that $v_I > v_{II}$ if and only if $e > \bar{e}$.*

Proof. Define Let $\delta(e) := v_I - v_{II}$. The definitions for v_I, v_{II} in Eqs. (3) and (N.8), Eqs. (N.1)-(N.7) and $\hat{e}$ from Lemma A.1 imply that $\lim_{e \downarrow \hat{e}} \delta(e) = - \left(p_{H|H} NPV_{H,H} \right) \big|_{e=\hat{e}} < 0$ and $\lim_{e \uparrow 1/2} \delta(e) = \gamma (\pi_g R - 1) > 0$. The claim in the lemma follows by $\delta(e)$ being continuous and strictly increasing in e. ∎

Remark A.1. By Assumption 1 in the text we have $\gamma < \overline{\gamma}$ and $e \in (\bar{e}, 1/2)$ and therefore

$$E_{H,H}(y) > E_H(y) > 1 > E_0(y) > E_L(y) > E_{L,L}(y). \tag{A.1}$$

Full disclosure strategies and full disclosure equilibrium properties

We remind that in Section 2.4.1 we defined a strategy for the issuer to be a *full disclosure strategy* if the issuer sells the asset in stage 4 only if all purchased rating information is disclosed in stage 3, and we defined a *full disclosure equilibrium* to be an equilibrium of the subgame in which the issuer follows a full disclosure strategy.

Lemma A.3. *If Assumption 1 holds, then:*

(i) *In a full disclosure equilibrium of the subgame, the issuer makes the investment and sells in stage 4 if and only if all the purchased ratings are high.*

(ii) *If $\max \{f_1, f_2\} > 0$, there is no full disclosure equilibrium of the subgame in which the issuer purchases a second rating after the realization of the first rating being low.*

(iii) *If $\max \{f_1, f_2\} > v_{II}$ and the market is transparent, there is no full disclosure equilibrium of the subgame in which the issuer purchases a second rating after the realization of the first rating being high.*

Proof of part (i). Follows by the inequalities in (A.1) and the requirement that it must be sequentially rational for the issuer to make the investment and sell in stage 4. ■

Proof of part (ii). Assume there was such an equilibrium. If $f_1 \neq f_2$, it is immediate that the issuer's equilibrium strategy must involve purchasing the cheaper rating first. Conditional on the first purchased rating, r_i say, being low, part (i) in the lemma implies that the issuer's expected continuation profits from purchasing the second rating equal $-f_{-i} < 0$, contradicting sequential optimality of the issuer's strategy. ■

Proof of part (iii). Assume there was such an equilibrium. Conditional on the first purchased rating, r_i say, being high, part (i) in the lemma implies that the issuer's expected continuation profits from purchasing the second rating equal $-f_{-i} + p_{H|H} NPV_{H,H}$. However, in a transparent market the issuer could deviate, disclose $r_i = H$, make the investment and sell without purchasing r_{-i}; the resulting profits from this deviation are $E_H(y) - 1 = NPV_H$. Hence, for it to be sequentially rational for the issuer to get the second rating it must be $f_{-i} \leq p_{H|H} NPV_{H,H} - NPV_H = v_{II}$. As the issuer must be purchasing the cheaper rating first, we have $f_{-i} = \max\{f_1, f_2\} > v_{II}$, a contradiction. ■

Remark A.2. (On full disclosure strategies.) By Lemma A.3-(i), trade does not occur in a full disclosure equilibrium unless all purchased ratings are high. Hence, in a full disclosure equilibrium, the disclosure rule of the issuer conditional on at least one of the purchased ratings being low is irrelevant for players' profits and economic surplus. Without loss of generality, and to simplify notation in the following proofs, we will focus on full disclosure strategies in which ratings are disclosed in stage 3 only if the issuer makes the investment and sells in stage 4.[27]

Notation for full disclosure strategy profiles. We introduce some notation for the full disclosure strategies of the issuer that are relevant for the analysis. Denote with α_0 the strategy of not purchasing any rating and not making the investment; denote with α_i the full disclosure strategy in which the issuer purchases only rating r_i and then discloses, makes the investment and sells if and only if $r_i = H$; denote with $\alpha_{i,-i}$ the full disclosure strategy in which the issuer purchases r_i first, then purchases r_{-i} if and only if $r_i = H$, and then discloses, makes the investment and sells if and only if $r_1 = r_2 = H$. Denote

$$\Gamma = (\alpha_0, \alpha_1, \alpha_2, \alpha_{1,2}, \alpha_{2,1}). \tag{A.2}$$

Let σ denote a mixed strategy over Γ, and let $\Sigma(\Gamma)$ denote the set of mixed strategies over Γ. For each $\sigma \in \Sigma(\Gamma)$, $\sigma(\alpha)$ denotes the probability assigned by σ to playing $\alpha \in \Gamma$.

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[27] We further note that, conditional on at least one rating being low, any other disclosure rule in a full-disclosure strategy would not be optimal if we were to assume an exogenous infinitesimally small disclosure cost.

Remark A.3. **(Equilibrium full disclosure strategies.)** Lemma A.3 implies that α (resp. σ) is the issuer's strategy played ex-post (resp. ex ante) in a full disclosure equilibrium of the subgame only if $\alpha \in \Gamma$ (resp. $\sigma \in \Sigma(\Gamma)$). (See also Remark A.2.)

Issuer's ex ante utility and economic surplus under full disclosure. For all $\alpha \in \Gamma$, denote with $\Pi(\alpha)$ the ex ante expected utility for the issuer, net of the issuer's endowment, when the issuer plays strategy α. The fact that $NPV_\iota = E_\iota(y) - 1$ immediately implies

$$\Pi(\alpha_0) = 0; \qquad \Pi(\alpha_i) = -f_i + p_H NPV_H; \qquad \Pi(\alpha_{i,-i}) = -f_i + p_H \left(-f_{-i} + p_{H|H} NPV_{H,H}\right). \quad \text{(A.3)}$$

For all $\alpha \in \Gamma$, denote with $\mathcal{W}(\alpha)$ the corresponding economic surplus. The definitions of v_I in Eq. (3) and v_{II} in Eq. (N.8) imply

$$\mathcal{W}(\alpha_0) = 0; \qquad \mathcal{W}(\alpha_i) = v_I - c_i; \qquad \mathcal{W}(\alpha_{i,-i}) = v_I - c_i + p_H \left(v_{II} - c_{-i}\right). \qquad \text{(A.4)}$$

For a mixed strategy $\sigma \in \Sigma(\Gamma)$, we have $\Pi(\sigma) = \sum_{\alpha \in \Gamma} \sigma(\alpha) \Pi(\alpha)$ and $\mathcal{W}(\sigma) = \sum_{\alpha \in \Gamma} \sigma(\alpha) \mathcal{W}(\alpha)$.

Remark A.4. **(Issuer's ex ante utility from full disclosure strategies in Γ on- and off-equilibrium.)** For α_0 and $\alpha_{i,-i}$ the specification of investor beliefs are irrelevant to $\Pi(\alpha)$ either because trade does not occur (α_0) or because trade occurs only if both ratings are disclosed ($\alpha_{i,-i}$). For α_i, investor beliefs do not matter if the market is transparent (because investors can observe that r_{-i} has not been purchased); in case the market is opaque, then $\Pi(\alpha_i)$ in Eq. (A.3) above only corresponds to the case in which investor beliefs are consistent with the issuer's strategy.

Appendix B: Proofs for Section 3.

Proof of Lemma 1. We prove Lemma 1 by proving that any event in which the issuer makes the investment and sells without disclosing a purchased rating cannot be consistent with equilibrium in the transparent market. By contradiction, assume the following is on the equilibrium path: the issuer purchases only one rating, say r_1, makes the investment and sells without disclosing. Denote the corresponding equilibrium price as $E_{N,\emptyset}(y)$. Making the investment is sequentially rational for the issuer only if $E_{N,\emptyset}(y) \geq 1$. Since $E_0(y) < 1$, then, for $E_{N,\emptyset}(y) \geq 1$ to reflect consistent beliefs, the issuer's equilibrium strategy must satisfy: (i) conditional on $r_1 = L$, with positive probability the issuer exists the game without selling, and (ii) conditional on $r_1 = H$, with positive probability the issuer makes the investment and sells without disclosing. We will show that any $E_{N,\emptyset}(y) \geq 1$ yields to a contradiction of either (i) or (ii). If $E_{N,\emptyset}(y) > 1$, then (i) cannot be true because, conditional on $r_1 = L$, the issuer is strictly better off by selling without disclosing (and making positive profits, $E_{N,\emptyset}(y) - 1 > 0$) than by exiting the game without selling. If, instead, $E_{N,\emptyset}(y) = 1$, then (ii) cannot be true because, conditional on $r_1 = H$, the issuer would then get a strictly higher price by disclosing and selling (since $E_H(y) > 1$).

The other relevant cases are if the issuer purchases both ratings and discloses either only one or none; the proof that these events cannot be consistent with equilibrium relies on the same argument and is omitted. ∎

Lemma B.1. (Issuer's equilibrium strategies for exogenous fees.) *Let $f_i \geq c_i$ for $i = 1, 2$. Then:*

(i) *At most one rating is purchased in equilibrium; if a rating is purchased the issuer makes the investment and sells if and only if the purchased rating is high.*

(ii) *α_0 in (A.2) is an equilibrium strategy only if $\min\{f_1, f_2\} \geq v_I$.*

(iii) *α_i in (A.2) is an equilibrium strategy only if $f_i \leq v_I$ and $f_i \leq f_{-i}$ and $f_{-i} \geq v_{II}$.*

(iv) *If the restriction on fees in part (ii) (resp. part (iii)) are satisfied, then α_0 (resp. α_i) is an equilibrium of the subgame supported by worst-case off-equilibrium beliefs on purchased ratings that are not disclosed.*

Proof of part (i). $f_i \geq c_i$ and $c_2 > v_{II}$ imply $\max\{f_1, f_2\} > v_{II}$. Hence, the first part of the statement follows by Lemma 1, Lemma A.3-(ii) and Lemma A.3-(iii). The second part of the statement follows by Lemma 1 and Lemma A.3-(i). ∎

Proof of part (ii). By contradiction, assume α_0 is an equilibrium and $\widehat{f} \equiv \min\{f_1, f_2\} < v_I$. For the sake of the argument, assume $f_1 = \widehat{f}$. The issuer could deviate from α_0 and play α_1; the expected profits from the deviation are $\Pi(\alpha_1) = p_H NPV_H - \widehat{f} = v_I - \widehat{f} > 0$. Since equilibrium profits are $\Pi(\alpha_0) = 0$, the deviation is profitable. ∎

Proof of part (iii). Optimality of α_i requires $\Pi(\alpha_i) \geq 0 \Leftrightarrow f_i \leq p_H NPV_H = v_I$. Moreover, it must be that $f_i \leq f_{-i}$, for otherwise the issuer could play α_{-i}, which gives higher ex ante profits as $\Pi(\alpha_{-i}) > \Pi(\alpha_i) \Leftrightarrow f_i > f_{-i}$. Finally, conditional on $r_i = H$ the issuer could purchase r_{-i}, disclose both ratings and make the investment and sell if and only if $r_1 = r_2 = H$; the issuer has no incentive to so only if $NPV_H \geq -f_{-i} + p_{H|H} NPV_{H,H} \Leftrightarrow f_{-i} \geq v_{II}$. ∎

Proof of part (iv). Under the assumed off-equilibrium beliefs, if the issuer deviates from α_0 (resp. α_i), the selling price will not exceed the investment cost (off-equilibrium) unless all purchased ratings are high. Then, given the conditions on fees in part (ii) (resp. part (iii)), it is immediate to verify that the issuer has no incentive to deviate. ∎

In the proof of Proposition 1 below, we repeatedly use the following corollary of Lemma B.1 together with the parametric assumptions implied by Assumption 1:

Corollary B.1. (C.B.1) If $f_i < f_{-i}$ and $f_i < v_I$ and $f_{-i} > v_{II}$, there is an essentially unique equilibrium of the subgame; in this equilibrium, CRA_i's profits amount to $f_i - c_i$ while CRA_{-i} makes zero profits.

Proof of Proposition 1. First, we rule out any candidate equilibrium fees f_1^*, f_2^* that satisfy $\min\{f_1^*, f_2^*\} \geq v_I$. By Lemma B.1, in an equilibrium of the subgame in which $\min\{f_1^*, f_2^*\} \geq v_I$ either there is at least one CRA, CRA_i say, that makes zero profits, or both make positive profits (if the issuer randomizes over α_1 and α_2), in which case it must be $f_1^* = f_2^* = v_I$. In the former case, CRA_i can undercut to $f \in (c_i, v_I)$ and make positive profits by C.B.1; in the latter case, it must be that each rating is purchased with probability less than one; by deviating to $f = v_I - \varepsilon$, for ε sufficiently small, CRA_i can increase profits by C.B.1. Second, we rule out the case in which $f_i^* \in [c_2, v_I)$ and $f_{-i}^* > f_i^*$, as CRA_i can deviate to $f \in (f_i^*, \min\{v_I, f_{-i}^*\})$ and increase profits by C.B.1. Third, we rule out any f_1^*, f_2^* such that $f_1^* \in [c_1, c_2)$ and $f_2^* \geq c_2$, as CRA_1 can deviate to $f \in (f_1^*, c_2)$ and increase profits by C.B.1. Fourth, we rule out any set of fees such that $f_1^* = f_2^* = f^* \in (c_2, v_I)$: by Lemma B.1, on any such subgame there must be one CRA, CRA_i say, whose rating is purchased with probability less than one; by undercutting to $f = f^* - \varepsilon$, for ε small enough, CRA_i can increase profits by C.B.1. The only subgame left that satisfies $f_i^* \geq c_i$ is $f_1^* = f_2^* = c_2$. On this subgame, we can rule out any equilibrium in which r_1 is purchased with probability less than one, as CRA_1 could increase profits by undercutting to $f = f^* - \varepsilon$, for ε small enough, by C.B.1. This completes the proof. ∎

Appendix C: Proofs for Section 4.

Proof of Proposition 2. By contradiction, assume the issuer's equilibrium strategy is α_1, so CRA_2's equilibrium profits are zero. This requires $f_2^* \geq \widehat{v}$ (no shopping condition) and $f_1^* \leq v_I$ (issuer's ex ante participation constraint). We will prove that $f_2^* \geq \widehat{v}$ cannot be a best response for CRA_2 by showing that, if CRA_2 deviates to $\tilde{f}_2 \in (c_2, \bar{c}_2)$, where $\bar{c}_2$ is defined in Eq. (7), then there is no equilibrium of the subgame in which CRA_2 makes zero profits, while there is (at least) an equilibrium in which CRA_2 makes positive profits. Let

$$F = \{(f_1, f_2) \text{ such that } f_1 \leq v_I \text{ and } f_2 \in (c_2, \bar{c}_2)\}.$$

As a first step, we show that for there is no equilibrium of the subgame induced by any $(f_1, f_2) \in F$ in which no rating is purchased. By contradiction, assume there was such an equilibrium. Since $NPV_0 < 0$, the issuer must not make the investment in equilibrium (i.e., issuer's equilibrium strategy is α_0) and therefore makes zero profits. We will show that the issuer has an incentive to deviate and play $\alpha_{2,1}$; the issuer's ex ante profits from such a deviation are strictly positive because

$$\Pi(\alpha_{2,1}) \geq -f_2 + p_H(-v_I + p_{HH} NPV_{H,H}) > 0 \Leftrightarrow f_2 < p_H v_{II} + (1 - p_H)v_I. \tag{C.1}$$

Note that the weak inequality in (C.1) follows from Eq. (A.3) and the fact that $f_1 \leq v_I$, and the equivalence follows from the definitions of v_I in Eq. (3) and v_{II} Eq. (4); the second strict inequality in (C.1) follows from the definition of $\bar{c}_2$ in Eq. (7) and the fact that $f_2 < \bar{c}_2$.

As a second step, we show that there is no equilibrium of the subgame induced by any $(f_1, f_2) \in F$ in which r_2 is purchased with probability zero while r_1 is purchased with positive probability. By contradiction, assume there was such an equilibrium; then trade must occur with positive probability on the equilibrium path, for otherwise the issuer would be strictly better off by playing $\alpha_{2,1}$ (because $\Pi(\alpha_{2,1}) > 0$, see (C.1)). For there to be trade in this equilibrium, the selling price must not be below one (for it to be sequentially rational for the issuer to sell), which in turn requires (for investor equilibrium beliefs to be consistent) the issuer's equilibrium strategy to be such that the issuer makes the investment and sells with positive probability in stage 4 conditional on $r_1 = H$. However, because the selling price cannot exceed $E_H(y)$ and $f_2 < \hat{v}$, the derivation of the no-shopping condition (Eq. 6) in the text implies that, conditional on $r_1 = H$, the anticipated profits from purchasing r_2 and disclosing selectively exceed equilibrium profits from selling, a contradiction.

The last step is to prove that, on every subgame induced by any $(f_1, f_2) \in F$, at least an equilibrium exists in which CRA_2 makes positive expected profits. Let $(f_1, f_2) \in F$ and $f_2 \leq f_1$ (resp. $f_2 > f_1$). Then, it is immediate to verify that $\alpha_{2,1}$ (resp. $\alpha_{1,2}$) is an equilibrium supported by worst-case beliefs on undisclosed ratings in which issuer's ex ante profits are positive; since r_2 is purchased with positive probability and $f_2 > c_2$, CRA_2 makes positive expected profits. This completes the proof. ∎

Derivation of the equilibrium price in Eq. (8). Denote with q_P the posterior probability that r_2 was purchased but not disclosed conditional on only $r_1 = H$ being disclosed. When the issuer's strategy is as in Definition 1 for a given shopping probability q, Bayes' rule gives

$$q_P = \frac{q \, p_{L|H}}{q \, p_{L|H} + 1 - q}, \tag{C.2}$$

and therefore

$$E_{H,q}(y) = (1 - q_P) E_H(y) + q_P E_{H,L}(y), \tag{C.3}$$

where $E_{H,L}(y) = E_0(y)$. ∎

Define

$$f_I(q) := p_H NPV_{H,q}; \qquad f_{II}(q) := p_{H|H}(E_{H,H}(y) - E_{H,q}(y)), \tag{C.4}$$

where $NPV_{H,q} = E_{H,q}(y) - 1$ and $E_{H,q}(y)$ is given in Eq. (C.3).

Lemma C.1. (Optimality of issuer's strategy in the equilibrium of Definition 1.) *There exists a* $\bar{q} \in (0, 1)$ *such that, for all* $q \leq \bar{q}$, *the issuer's strategy in Definition 1 and investor equilibrium beliefs on undisclosed ratings in Eq. (C.2) are an equilibrium of the subgame induced by* (f_1, f_2) *if*

$f_1 \leq f_I(q)$, $f_2 = f_{II}(q)$ and $f_1 \leq f_2$. *The equilibrium is supported by off-equilibrium beliefs that undisclosed ratings are low.*

Proof. By Eqs. (9) and (10), the indifference condition in stage 3 is satisfied if and only if $f_2 = f_{II}(q)$. For $f_2 = f_{II}(q)$, issuer's ex ante profits from the strategy in Definition 1 equal

$$-f_1 + p_H \left(-f_2 + p_{H|H} NPV_{H,H} + (1 - p_{H|H}) NPV_{H,q}\right) = -f_1 + p_H NPV_{H,q}, \qquad \text{(C.5)}$$

so the issuer's ex ante participation constraint (P.C.) is met if and only if $f_1 \leq f_I(q)$. From Eq. (C.2) it is immediate that

$$\lim_{q \uparrow 1} q_P = 1; \qquad \lim_{q \downarrow 0} q_P = 0; \qquad \frac{\partial q_P}{\partial q} > 0. \qquad \text{(C.6)}$$

Since $E_0(y) < 1 < E_H(y)$, then Eqs. (C.3) and (C.6) imply that there is a unique $\bar{q} \in (0,1)$ such that $E_{H,q}(y) \geq 1 \Leftrightarrow q \leq \bar{q}$. Worst-case off-equilibrium beliefs on undisclosed ratings imply that the asset is sold (off-equilibrium) for a price greater than one only if both ratings are disclosed with a high value. Then, it is immediate that the issuer's strategy from Definition 1 is optimal in stage 4 conditional on the rating acquisition strategy in stage 3, and that the rating acquisition strategy in stage 3 is optimal because: (i) the issuer has no incentive to purchase r_2 if $r_1 = L$ (off-equilibrium price is less than one if $r_1 = L$ is disclosed or if r_1 is not disclosed); (ii) the issuer has no incentive to purchase r_2 prior to r_1 (issuer's expected profits from such deviation is at most $\Pi(\alpha_{2,1})$, where $\Pi(\alpha_{2,1}) \leq \Pi(\alpha_{1,2})$ if $f_1 \leq f_2$ and $\Pi(\alpha_{1,2})$ is lower than equilibrium expected profits in the L.H.S. of Eq. (C.5)), and (iii) the issuer has no incentive to purchase no ratings (off-equilibrium profits are at most zero if no rating is purchased). ∎

Proof of Proposition 3. As a first step in the proof, we provide conditions on q and f_2, that depend on f_1, such that, for all $f_1 \in (0, c_2]$, the conditions in Lemma C.1 hold and the issuer's ex ante P.C. is met with equality. Let $\hat{q} : [0, v_I] \longrightarrow [0, \bar{q}]$ be the inverse function of f_I in Eq. (C.4), such that

$$f = f_I(\hat{q}(f)), \qquad \text{(C.7)}$$

and let the function $\hat{f} : [0, v_I] \longrightarrow [\hat{v}, p_{HH} NPV_{H,H}]$ be the composition of f_{II} and $\hat{q}$, that is,

$$\hat{f}(f) = f_{II}(\hat{q}(f)).$$

Using the definitions of f_I and f_{II} in Eq. (C.4) and the definition of $\hat{q}$ in Eq. (C.7), we have

$$\hat{f}(f) = p_{H|H} \left(NPV_{H,H} - NPV_{H,\hat{q}(f)}\right) = p_{H|H} \left(NPV_{H,H} - \frac{f}{p_H}\right). \qquad \text{(C.8)}$$

The second equality in Eq. (C.8) and the definition of $\hat{v}$ imply

$$\hat{f}(0) = p_{H|H} NPV_{H,H}; \qquad \hat{f}(v_I) = \hat{v}; \qquad \frac{d}{df}\hat{f}(f) < 0. \qquad \text{(C.9)}$$

Furthermore, since $\widehat{f}(f_1) \geq \widehat{v}$ for all $f_1 \leq v_I$ and $c_2 < \min\{\widehat{v}, v_I\}$, then $f_1 < \widehat{f}(f_1)$ for all $f_1 \leq c_2$. Hence, all $f_1 \in (0, c_2]$ and $(f_2, q) = \left(\widehat{f}(f_1), \widehat{q}(f_1)\right)$ are such that: (i) the conditions in Lemma C.1 are satisfied, and (ii) the issuer's ex ante P.C. holds as an equality by definition of $\widehat{q}$ in Eq. (C.7).

As a second step, we solve for the function $\widehat{q}$ and the constant $\overline{q}$. We can solve for $\widehat{q}$ in Eq. (C.7) using the definitions of f_I in Eq. (C.4) and q_P in Eq. (C.2) and $E_{H,q}(y)$ in Eq. (C.3): straightforward computations give

$$\widehat{q}(f) = \frac{NPV_H - \frac{f}{p_H}}{p_{H|H}\left(NPV_{H,H} - \frac{f}{p_H}\right)}.\tag{C.10}$$

By definition of $\widehat{q}$ and $\overline{q}$ we have

$$q(0) = \overline{q}; \qquad \widehat{q}(v_I) = 0; \qquad \frac{d}{df}\widehat{q}(f) < 0,\tag{C.11}$$

and therefore Eq. (C.10) implies

$$\overline{q} = \frac{NPV_H}{p_{H|H}NPV_{H,H}}.\tag{C.12}$$

As a third step, we show conditions for (f_1^*, f_2^*) to be an equilibrium of the overall game. First, for any (f_1^*, f_2^*) such that $f_1^* \leq c_2$ and $f_2^* = \widehat{f}(f_1^*)$, we show that no CRA wants to deviate to a higher fee. We do this by showing that if CRA_i deviates to $f_i > f_i^*$, there exists an equilibrium of the subgame in which the issuer plays α_0–that is, in which no rating is purchased and CRA_i makes zero profits–supported by worst case off-equilibrium beliefs on undisclosed ratings. Under worst-case beliefs on undisclosed ratings, the issuer's expected off-equilibrium payoff if it deviates from α_0 is at most $\Pi_d \equiv \max\{\Pi(\alpha_{1,2}), \Pi(\alpha_{1,2})\}$, where

$$\Pi_d < -f_1^* + p_H\left(-\widehat{f}(f_1^*) + p_{H,H}NPV_{H,H}\right) \leq 0,$$

where the first inequality follows because one CRA deviated to a higher fee and, as derived in the first step, $\widehat{f}(f) > f$ for all $f \leq c_2$; the second weak inequality follows by comparison with the L.H.S. of Eq. (C.5) which, by construction of $\widehat{f}$ and $\widehat{q}$, is equal to zero. Second, we show that CRA_2 does not want to undercut by showing that for all subgames induced by (f_1^*, f_2) such that $f_2 \in (c_2, f_2^*)$, there exists an equilibrium of the subgame in which CRA_2 makes lower expected profits. We consider two cases. In case CRA_2 undercuts to $f_2 \in [\widehat{v}, f_2^*)$, there exists an equilibrium of the subgame in which the issuer's equilibrium strategy is α_1 and CRA_2 makes zero profits ($f_1^* \leq c_2$ implies $\Pi(\alpha_1) > 0$ and the no shopping condition for the second rating is satisfied as $f_2 \geq \widehat{v}$; the equilibrium is supported by worst case off-equilibrium beliefs in case r_1 is not disclosed). In case CRA_2 undercuts to $f_2 \in (c_2, \widehat{v})$, then, either $\Pi(\alpha_{1,2}) \leq 0$ and there exists an equilibrium of the subgame in which the issuer's strategy is α_0 (supported by worst-case beliefs on undisclosed ratings), or $\Pi(\alpha_{1,2}) > 0$ and there exists an equilibrium of the subgame in which the issuer's strategy is $\alpha_{1,2}$ (supported by worst-case beliefs on

undisclosed ratings) and CRA$_2$'s expected profits amount to $p_H (f_2 - c_2)$. Therefore, since $f_2 < \widehat{v}$, it suffices to show that

$$p_H (\widehat{v} - c_2) \le p_H \widehat{q} (f_1^*) \left(\widehat{f} (f_1^*) - c_2 \right) \iff \Phi (f_1^*, c_2) \le 0,$$

where

$$\Phi (f, c_2) := \widehat{v} - c_2 - \widehat{q} (f) \left(\widehat{f} (f) - c_2 \right). \tag{C.13}$$

Using Eqs. (C.8)-(C.12), it is immediate to verify that, for all $c_2 < \widehat{v}$,

$$\Phi (0, c_2) = -c_2 (1 - \overline{q}) + v_{II} - \frac{p_{H,H}}{p_H} v_I < 0; \qquad \Phi (v_I, c_2) = \widehat{v} - c_2 > 0,$$

and

$$\frac{\partial}{\partial f} \Phi (f, c_2) = - \left(\widehat{f} (f) - c_2 \right) \frac{d}{df} \widehat{q} (f) - \widehat{q} (f) \frac{d}{df} \widehat{f} (f) > 0,$$

implying that there exists a unique $\overline{f} (c_2) \in (0, v_I)$ such that $\Phi (f, c_2) \le 0$ for all $f \in [0, \overline{f} (c_2)]$. Implicit differentiation of Φ gives that $\overline{f} (c_2)$ is strictly increasing in c_2. Defining further

$$\overline{c}_1 := \min\{c_2, \overline{f} (c_2)\}, \tag{C.14}$$

then $f_1^* = \overline{c}_1$, $f_2^* = \widehat{f} (\overline{c}_1)$ is an equilibrium of the overall game in which the issuer follows the strategy from Definition 1 with shopping probability $q^* \equiv \widehat{q} (\overline{c}_1)$. Since $c_2, \overline{f} (c_2)$ are strictly positive and less than v_I, then $\overline{c}_1 \in (0, v_I)$ and therefore $q^* \in (0, \overline{q})$ by (C.11). Since CRA$_1$ must make non-negative profits, this equilibrium exists if and only if $c_1 \le \overline{c}_1$. $\blacksquare$

Proof of Proposition 4. Fix some $e \in (0, 1/2)$. By definition of $\overline{\gamma}$ in Eq. (2) we have $NPV_0|_{\gamma = \overline{\gamma}} = 0$ and therefore

$$v_{II}|_{\gamma = \overline{\gamma}} = 0; \qquad v_I|_{\gamma = \overline{\gamma}} = (p_H NPV_H)|_{\gamma = \overline{\gamma}} > 0, \tag{C.15}$$

which imply

$$\overline{e}|_{\gamma = \overline{\gamma}} = 0; \qquad NPV_H|_{\gamma = \overline{\gamma}} = \left(p_{H|H} NPV_{H,H} \right)\big|_{\gamma = \overline{\gamma}}; \qquad \widehat{v}|_{\gamma = \overline{\gamma}} = \left((1 - p_{H|H}) NPV_H \right)\big|_{\gamma = \overline{\gamma}} > 0. \tag{C.16}$$

Define further

$$\widehat{c} \equiv \lim_{\gamma \uparrow \overline{\gamma}} \overline{c}_2; \qquad \overline{\Phi} \equiv \lim_{\gamma \uparrow \overline{\gamma}} \Phi (c_2, c_2). \tag{C.17}$$

By the definition of $\overline{c}_2$ in Eq. (7) and (C.15)-(C.16), we have $\widehat{c} > 0$. Using Eq. (C.13), Eqs. (C.8) and (C.10) and the equalities in Eqs. (C.15)-(C.16), it is immediate to verify that, for $c_2 \le v_I|_{\gamma = \overline{\gamma}}$, $\overline{\Phi} (c_2, c_2) < 0 \Leftrightarrow Q (c_2) < 0$, where

$$Q (c_2) := \left. \left(-c_2^2 \left(p_H + (1 - p_H) p_{H|H} \right) + c_2 v_I \left(1 + p_{H|H}^2 \right) - v_I^2 p_{H|H} \right) \right|_{\gamma = \overline{\gamma}}. \tag{C.18}$$

Since $Q(c_2)$ is quadratic in c_2, concave, and such that $Q(0) < 0$ and $\frac{d}{dc_2}Q(0) > 0$, then there is a value $\overrightarrow{c} > 0$ such that $Q(c_2) < 0$ for all $c_2 < \overrightarrow{c}$. (If $\max_{c_2} Q(c_2) < 0$, then $\overrightarrow{c} = \infty$.) Let $\check{c} \equiv \min\{\overrightarrow{c}, v_I|_{\gamma=\overline{\gamma}}\}$. By definition of Φ in Eq. (C.13) and by definition of $\overline{f}$ and $\bar{c}_1$ in Eq. (C.14), $\bar{\Phi}(c_2, c_2) < 0$ implies that $c_2 < \lim_{\gamma\uparrow\overline{\gamma}} \overline{f}(c_2)$ and therefore $c_2 = \lim_{\gamma\uparrow\overline{\gamma}} \bar{c}_1$ for all $c_2 < \check{c}$. Finally, define $\tilde{c} \equiv \min\{\check{c}, \hat{c}\}$. By continuity of $\bar{e}$, $\bar{c}_2$, v_{II}, $\hat{v}$ and $\Phi(c_2, c_2)$ in γ, there exists a value $\tilde{\gamma} < \overline{\gamma}$ such that, for all $\gamma \in (\tilde{\gamma}, \overline{\gamma})$ and $c_1 < c_2 < \tilde{c}$, the following conditions simultaneously hold: (i) $e \in (\bar{e}, 1/2)$; (ii) $c_2 \in (v_{II}, \bar{c}_2)$ (and therefore, since $\bar{c}_2 < v_I$ and $\bar{c}_2 \leq \hat{v}$, such that the conditions $c_1 < c_2 < v_I$ in Assumption 1 and $c_2 < \bar{c}_2$ in Proposition 2 and $c_2 < \hat{v}$ in Proposition 3 hold); and (iii) $c_2 = \bar{c}_1$ (and therefore, since $c_1 < c_2$, such that the condition $c_1 \leq \bar{c}_1$ in Proposition 3 holds). ■

Derivation of the equilibrium price in the equilibrium of Definition 2. Denote with $\hat{q}_P$ the posterior probability that a rating with a low realization was purchased but not disclosed conditional on only one rating being disclosed with a high value. When the issuer's strategy is as in Definition 2 for a given shopping probability q, Bayes' rule gives

$$\hat{q}_P = \frac{p_L\, q\, p_{H|L}}{p_H + p_L\, q\, p_{H|L}}, \tag{C.19}$$

and therefore, denoting the equilibrium price with $E_{\hat{H},q}(y)$, we have

$$E_{\hat{H},q}(y) = (1 - \hat{q}_P)\, E_H(y) + \hat{q}_P E_{H,L}(y), \tag{C.20}$$

where $E_{H,L}(y) = E_0(y)$.

Define
$$f_L(q) := p_{H|L} NPV_{\hat{H},q}; \qquad f_H(q) := p_{H|H}\left(E_{H,H}(y) - E_{\hat{H},q}(y)\right), \tag{C.21}$$

where $NPV_{\hat{H},q} = E_{\hat{H},q}(y) - 1$ and $E_{\hat{H},q}(y)$ is given in (C.20).

Lemma C.2. (Optimality of issuer's strategy in the equilibrium of Definition 2.)

(i) *If $f_H(0) < f_L(0)$, there exists a $\overline{q} \in (0,1]$ such that, for all $q \leq \overline{q}$, the issuer's strategy in Definition 2 and investor equilibrium beliefs on undisclosed ratings in (C.19) are an equilibrium of the subgame induced by (f_1, f_2) if $f_1 = f_2 = f_L(q)$. The equilibrium is supported by off-equilibrium beliefs that undisclosed ratings are low.*

(ii) *Let π_g, π_b and R be such that $NPV_g + NPV_b < 0$. Then, for all $e \in (0, 1/2)$, there exists a value $\gamma_l < \overline{\gamma}$ such that, for all $\gamma \in (\gamma_l, \overline{\gamma})$, we have $f_H(0) < f_L(0)$.*

Proof of part-(i). Take $f_1 = f_2 = f$ and q as given, and assume that

$$NPV_{\hat{H},q} \geq 0. \tag{C.22}$$

41

Consider the case case in which the issuer purchases a first rating, r_i say, and $r_i = H$. The issuer has no incentives to purchase a second rating if

$$-f + p_{H|H}NPV_{H,H} + (1 - p_{H|H})NPV_{\hat{H},q} \leq NPV_{\hat{H},q} \iff f \geq f_H(q). \tag{C.23}$$

Hence, if (C.22) and (C.23) hold, and given worst-case off-equilibrium beliefs, disclosing $r_i = H$, making the investment and selling is optimal for the issuer. Assume instead that the first purchased rating was low, $r_i = L$, and that the issuer also purchased the second rating; worst-case off-equilibrium beliefs and (C.22) immediately imply that the conjectured disclosure and investment strategy in Definition 2 is optimal. Anticipating this, one step back, if $r_i = L$, the issuer is indifferent between purchasing r_{-i} or not if and only if

$$-f + p_{H|L}NPV_{\hat{H},q} = 0 \iff f = f_L(q). \tag{C.24}$$

For $f = f_L(q)$, the issuer's ex ante expected equilibrium profits from the strategy of Definition 2 equal

$$-f + p_H NPV_{\hat{H},q} \geq 0 \iff f \leq p_H NPV_{\hat{H},q}. \tag{C.25}$$

Since $p_H > p_{H|L}$, then the issuer's ex ante P.C. in (C.25) is met if (C.24) holds. Finally, with equal fees, the issuer is ex ante indifferent between purchasing first r_1 or r_2, and randomizing with equal probability is optimal.

The definitions in (C.19)-(C.21) imply that $NPV_{\hat{H},q}$ and $f_L(q)$ are decreasing and continuous in q and $f_H(q)$ is increasing and continuous in q. Since $NPV_{\hat{H},0} = NPV_H > 0$, then, if $f_H(0) < f_L(0)$, it is immediate that there exists a $\bar{q} \in (0,1]$ such that the conditions (C.22)-(C.25) hold for all $q \leq \bar{q}$ if $f_1 = f_2 = f_L(q)$. ∎

Proof of part-(ii). Fix $e \in (0, 1/2)$. By the definitions in Eqs. (C.19)-(C.21) and (C.15)-(C.16), we have

$$\lim_{\gamma \uparrow \bar{\gamma}} (f_L(0) - f_H(0)) = \left(NPV_H(p_{H|H} + p_{H|L} - 1) \right)\big|_{\gamma = \bar{\gamma}},$$

and straightforward algebra shows that $(p_{H|H} + p_{H|L} - 1)\big|_{\gamma = \bar{\gamma}} > 0 \iff NPV_g + NPV_b < 0$. Since $NPV_H > 0$ for $e > \bar{e}$ and $\bar{e}\big|_{\gamma = \bar{\gamma}} = 0$, the statement in the lemma follows by continuity of $f_H(0)$, $f_L(0)$ and $\bar{e}$ in γ. ∎

Proof of Proposition 5 We prove Proposition 5 by proving the following proposition:

Proposition C.1. (Restatement of Proposition 5.) *For all π_g, π_b and R such that $NPV_g + NPV_b < 0$, there exist values $e_l > 0$, $\hat{c} > 0$, $q_L > 0$ and $\gamma_L < 1/2$ such that, if $c_1 < c_2 < \hat{c}$ and $e < e_l$ and $\gamma \in (\gamma_L, \bar{\gamma})$, then: (i) Proposition 2 holds and (ii) there is an equilibrium of the overall game in which CRAs set $f_1^* = f_2^* = f_L(q_L)$ and the issuer's strategy is as in Definition 2.*

Proof. We will assume initially that $c_1 < c_2 < \bar{c}_2$ and $f_H(0) < f_L(0)$ hold (and later derive conditions under which this assumption is satisfied). Assume the equilibrium on the subgame induced by $f_1 = f_2 = f_L(q)$–for some $q \leq \bar{q}$ where $\bar{q}$ is as in Lemma C.2 (i)–conforms to Definition 2. Notice that $f_H(0) = \hat{v}$ and $f_L(0) < v_I$. Hence, since $\frac{d}{dq} f_L(q) < 0$ and $\frac{d}{dq} f_H(q) > 0$ and $f_H(0) < f_L(0)$, we have

$$\hat{v} < f_L(q) < v_I \text{ for all } q \leq \bar{q}. \tag{C.26}$$

Since $c_i < \bar{c}_2$ and $\bar{c}_2 \leq \hat{v}$, then Eq. (C.26) implies that CRAs make positive expected profits in equilibrium. Next, we derive conditions under which CRAs have no incentive to deviate.

First, we show CRAs do not want to charge higher fees: if CRA$_i$ deviates by setting $f_i > f_L(q)$, then Eq. (C.26) implies that there is an equilibrium of the subgame in which the issuer's strategy is α_{-i} (supported by worst-case beliefs on undisclosed ratings) in which CRA$_i$ makes zero profits.

Second, we determine conditions under which CRAs have no incentive to undercut. Define $f_0(q)$ as the value of f_i that solves $\Pi(\alpha_{i,-i}) = 0$ when $f_{-i} = f_L(q)$, that is,

$$f_0(q) := p_H \left(-f_L(q) + p_{H|H} NPV_{H,H} \right). \tag{C.27}$$

As we now explain, the following condition is sufficient for CRA$_i$ to have no incentives to undercut:

$$f_0(q) - c_i \leq \left(\frac{1 + p_L q}{2} \right) (f_L(q) - c_i). \tag{C.28}$$

The L.H.S. of Eq. (C.28) equals the upper bound to CRA$_i$'s expected profits if it deviates to $f_i < f_0(q)$, and the R.H.S. of Eq. (C.28) equals CRA$_i$'s expected profits in equilibrium. Hence, if Eq. (C.28) holds, CRA$_i$ has no incentive to undercut to any fee lower than $f_0(q)$. Furthermore, Eq. (C.28) holds only if $f_0(q) \leq f_L(q)$ and therefore, by construction of $f_0(q)$, any subgame induced by $f_i \in [f_0(q), f_L(q))$ and $f_{-i} = f_L(q)$ has an equilibrium in which no rating is purchased, supported by worst case beliefs on undisclosed ratings. A sufficient condition for Eq. (C.28) to hold for all $c_i \geq 0$ is that $\vartheta(q) \leq 0$, where

$$\vartheta(q) := f_0(q) - \left(\frac{1 + p_L q}{2} \right) f_L(q). \tag{C.29}$$

By Eqs. (C.15)-(C.16), (C.21) and (C.27), it is immediate to verify that

$$\lim_{\gamma \uparrow \bar{\gamma}} \vartheta(0) = e \frac{H_1(e)}{H_2(e)} \lim_{\gamma \uparrow \bar{\gamma}} NPV_H,$$

where $NPV_H|_{\gamma = \bar{\gamma}} > 0$ for all $e > 0$ and, if $NPV_g + NPV_b < 0$, then $H_2(e) < 0$ and $H_1(e) = h_0 + h_1 e + h_2 e^2$ is such that $H_1(0) > 0$ and $H_1(1/2) < 0$. Hence, if $NPV_g + NPV_b < 0$, there exists a value $e_l \in (0, 1/2)$, that only depends on π_g, π_b and R, such that, for all $e \in (0, e_l)$ there exists a $\gamma_1 < \bar{\gamma}$ such that $\vartheta(0) < 0$ for all $\gamma \in (\gamma_1, \bar{\gamma})$. Continuity of ϑ in q further implies that, for all $\gamma \in (\gamma_1, \bar{\gamma})$, there exists a $\tilde{q} > 0$ (that depends on π_g, π_b, R, e and γ), such that Eq. (C.28) holds for both CRAs for all $q \leq \tilde{q}$.

43

Finally, we determine q_L and γ_L in the statement of the proposition. Assume $c_1 < c_2 < \hat{c}$, where $\hat{c}$ is defined in Eq. (C.17); by continuity of $\bar{c}_2$ in γ and Lemma C.2-(ii), there is a $\gamma_2 < \bar{\gamma}$ such that $c_1 < c_2 < \bar{c}_2$ and $f_H(0) < f_L(0)$ hold (justifying our initial assumption) for all $\gamma \in (\gamma_2, \bar{\gamma})$. Letting $q_L \equiv \min\{\widetilde{q}, \bar{q}\}$–where $\bar{q}$ is as in Lemma C.2-(i)–and $\gamma_L \equiv \max\{\gamma_1, \gamma_2\}$ completes the proof of Proposition C.1 and, therefore, the proof of Proposition 5. ∎

Derivation of the welfare condition in Eq. (12). Let $\mathcal{W}^*$ denote the economic surplus associated with the equilibrium from Proposition 3. From Eq. (A.4) and Definition 1, we have

$$\mathcal{W}(\alpha_{1,2}) < \mathcal{W}^* \iff -c_1 + p_H \left(-c_2 + p_{H|H} NPV_{H,H}\right) < -c_1 + p_H \left(-q^* c_2 + NPV_H\right). \qquad \text{(C.30)}$$

Using the definition of v_{II} in the first equality in Eq. (N.8), the second inequality in Eq. (C.30) can be rearranged as

$$p_H \left(c_2 - v_{II}\right) > p_H q^* c_2, \qquad \text{(C.31)}$$

which, using the second equality in Eq. (N.8), can be rewritten as in the text as

$$p_H \left(c_2 - v_{II}\right) > q^* p_H \left(c_2 - v_{II}\right) + q^* p_{H\&L} \left(1 - E_0\left(y\right)\right),$$

where $p_{H\&L} \equiv p_H p_{L|H}$. ∎

Lemma C.3. *Let parameters be as in Proposition 4, that is, let $e \in (0, 1/2)$, $c_1 < c_2 < \tilde{c}$ and $\gamma \in (\widetilde{\gamma}, \bar{\gamma})$. Then, there exists $\widehat{\gamma} \in [\widetilde{\gamma}, \bar{\gamma})$ such that the inequality in Eq. (12) holds for all $\gamma \in (\widehat{\gamma}, \bar{\gamma})$.*

Proof. In the proof of Proposition 4 we established that, for all $\gamma \in (\widetilde{\gamma}, \gamma)$, we have $c_2 = \bar{c}_1$ and therefore $q^* = \widehat{q}(c_2)$. Define $\widetilde{p}_H \equiv \lim_{\gamma \uparrow \bar{\gamma}} p_H$ and $\tilde{q} \equiv \lim_{\gamma \uparrow \bar{\gamma}} \widehat{q}(c_2)$, where Eq. (N.5) implies $\widetilde{p}_H > 0$ and Eqs. (C.10) and (C.15)-(C.16) imply $\tilde{q} \in (0, 1)$. Then, by Eq. (C.15) we have

$$\lim_{\gamma \uparrow \bar{\gamma}} p_H \left(c_2 \left(1 - q^*\right) - v_{II}\right) = \widetilde{p}_H c_2 \left(1 - \tilde{q}\right) > 0. \qquad \text{(C.32)}$$

By continuity of $p_H, \widehat{q}(c_2)$ and v_{II} in γ, the limit in (C.32) implies that there exists a value $\widehat{\gamma} \in [\widetilde{\gamma}, \bar{\gamma})$ such that the inequality in (C.31) holds for all $\gamma \in (\widehat{\gamma}, \bar{\gamma})$. ∎

Lemma C.4. (Necessary conditions for optimality of full disclosure mixed-strategies) *Let $\sigma \in \Sigma(\Gamma)$ and assume investor beliefs are consistent with σ. Then:*

(i) *$\sigma \in \Sigma(\Gamma)$ is optimal for the issuer only if $\min\{\sigma(\alpha_i), \sigma(\alpha_{i,-i})\} = 0$, for $i = 1, 2$.*

(ii) *If, moreover, $v_I < \widehat{v}$, then $\sigma \in \Sigma(\Gamma)$ is optimal for the issuer only if:*

    (a) *$\min\{\sigma(\alpha_1), \sigma(\alpha_2)\} = 0$, and*

    (b) *$\min\{\sigma(\alpha_i), \sigma(\alpha_{-i,i})\} = 0$, for $i = 1, 2$.*

Proof part-(i). Assume $\sigma \in \Sigma(\Gamma)$ is an equilibrium strategy and is such that both $\sigma(\alpha_i)$ and $\sigma(\alpha_{i,-i})$ are strictly positive. Assume ex-post the issuer plays $\alpha_{i,-i}$ and it turns out that $r_i = H$ and $r_{-i} = L$, so equilibrium profits are zero. The issuer has an incentive to deviate from full disclosure by disclosing only $r_i = H$, making the investment and selling, which yields (off-equilibrium) profits equal to $E_H(y) - 1 > 0$. ∎

Proof of part-(ii.a). Assume $\sigma \in \Sigma(\Gamma)$ is an equilibrium strategy and is such that both $\sigma(\alpha_1)$ and $\sigma(\alpha_2)$ are strictly positive. Then, the issuer's ex ante P.C. requires $\Pi(\alpha_i) \geq 0 \Longleftrightarrow f_i \leq v_I$ for $i = 1, 2$. If $v_I < \hat{v}$, then $f_i < \hat{v}$ for $i = 1, 2$ and the no-shopping condition for the second rating is violated. Hence, if the issuer ex-post plays α_i and it turns out that $r_i = H$, the issuer has an incentive to deviate, purchase r_{-i} and disclose r_{-i} selectively (see the derivation of $\hat{v}$ in Eq. (6)). ∎

Proof of part-(ii.b). Assume $\sigma \in \Sigma(\Gamma)$ is an equilibrium strategy and is such that both $\sigma(\alpha_i)$ and $\sigma(\alpha_{-i,i})$ are strictly positive. Then, optimality of σ requires: (i) the issuer to be ex ante indifferent between α_i and $\alpha_{-i,i}$, and therefore $\Pi(\alpha_i) = \Pi(\alpha_{-i,i}) \Longleftrightarrow f_{-i} = f_i(1 - p_H) + v_{II}p_H$, and (ii) the ex ante P.C. to be satisfied, which requires $\Pi(\alpha_i) \geq 0 \Longleftrightarrow f_i \leq v_I$. Combining the last two conditions we obtain $f_{-i} < v_I$. Then, $v_I < \hat{v}$ implies $f_{-i} < \hat{v}$; the rest of the proof is as in part (ii.a). ∎

Lemma C.5. *Let y, π_g and π_b be such that $NPV_g + NPV_b > 0$. Then, there exists a value $\widetilde{e} \in (0, 1/2)$ such that, for all $e \in (0, \widetilde{e})$ there is a $\widehat{\widehat{\gamma}} < \overline{\gamma}$ for which $v_I < \hat{v}$ for all $\gamma \in (\widehat{\widehat{\gamma}}, \overline{\gamma}]$.*

Proof. Let $\Delta(e) \equiv \lim_{\gamma \uparrow \overline{\gamma}} \hat{v} - v_I$. Using Eqs. (C.15)-(C.16) and simplifying,

$$\Delta(e) = e \frac{G_1(e)}{G_2(e)} \left. NPV_H \right|_{\gamma = \overline{\gamma}},$$

where $\left. NPV_H \right|_{\gamma = \overline{\gamma}} > 0$ for all $e > 0$ and $G_2(e) > 0$ for all $e \in [0, 1/2]$ and $G_1(e) = a_0 + a_1 e$ is such that

$$G_1(1/2) < 0; \quad a_0 = (\pi_g - \pi_b)(NPV_g + NPV_b).$$

Hence, $a_0 > 0 \Longleftrightarrow NPV_g + NPV_b > 0$. Let $\widetilde{e} = -a_0/a_1$. Clearly, under the conditions in the lemma, $\Delta(e) > 0$ for all $e \in (0, \widetilde{e})$. The statement in the lemma follows by continuity of $\hat{v}$ and v_I in γ. ∎

Proof of Proposition 6. Let parameters be such that Proposition 4, Lemma C.3 and Lemma C.5 hold, that is, let $e, c_1, c_2, y, \pi_g, \pi_b$ and γ be such that $e \in (0, \widetilde{e})$, $c_1 < c_2 < \tilde{c}$, $NPV_g + NPV_b > 0$ and $\gamma \in (\check{\gamma}, \overline{\gamma})$ where $\check{\gamma} \equiv \max\{\widehat{\gamma}, \widehat{\widehat{\gamma}}\}$. Denote with $\hat{\Sigma}(\Gamma)$ the subset of full disclosure strategies $\Sigma(\Gamma)$ that are are consistent with an equilibrium of the overall game. The proof of Proposition 2 implies that there is no equilibrium of the overall game in which one CRA makes zero expected profits, and therefore:

$$\sigma \in \hat{\Sigma}(\Gamma) \Rightarrow \sigma(\alpha_0) + \sigma(\alpha_i) < 1, \text{ for } i = 1, 2. \tag{C.33}$$

45

Combined with Lemma C.4, (C.33) implies that $\sigma \in \hat{\Sigma}(\Gamma)$ only if $\sigma(\alpha_i) = 0$ for $i = 1, 2$. Then, by (A.4) as well as Remark A.3, it is immediate that $\mathcal{W}(\alpha_{1,2})$ achieves the maximum economic surplus among all equilibria with full disclosure, that is,

$$\mathcal{W}(\alpha_{1,2}) = \max_{\sigma \in \hat{\Sigma}(\Gamma)} \mathcal{W}(\sigma). \tag{C.34}$$

Since $\mathcal{W}^* > \mathcal{W}(\alpha_{1,2})$ by Lemma C.3, the statement in the proposition follows. ∎

Proof of Proposition 7 Part-(i). Recall that in the equilibrium of Proposition 1 $f_1^* = f_2^* = c_2$. Furthermore, $c_2 < \min\{v_I, \widehat{v}\}$ by assumption. Suppose CRA$_2$ deviates from $f_2^* = c_2$ by setting its fee equal to some $f_2 \in (c_2, \min\{v_I, \widehat{v}\})$ and by making its rating opaque. We first show that on the subgame induced by $f_1 = c_2$ and $f_2 \in (c_2, \min\{v_I, \widehat{v}\})$ (such that r_1 is transparent and r_2 is opaque) there is no equilibrium in which no rating is purchased. Since r_1 is transparent, there is no asymmetric information about r_1 if r_1 is not purchased by the issuer. Hence, no-trade (i.e., the issuer's strategy α_0) is not an equilibrium on the subgame because the issuer can deviate and play α_2, which gives ex ante utility of $\Pi(\alpha_2) = v_I - f_2 > 0$. The same argument used in the proof of Proposition 2 shows that there is no equilibrium on the subgame in which r_2 is purchased with probability zero while r_1 is purchased with positive probability. Finally, because $f_1 = c_2 > v_{II}$, α_2 is an equilibrium strategy on the subgame, supported by worst-case off-equilibrium beliefs on r_2 if r_2 is not disclosed and by worst-case off-equilibrium beliefs on r_1 if r_1 is purchased but not disclosed. Since $f_2 > c_2$, CRA$_2$ makes positive profit in this equilibrium and the deviation is profitable. ∎

Proof of Proposition 7 Part-(ii). We first prove that no CRA has an incentive to deviate from the equilibrium of Proposition 3 by making its rating transparent. Recall that $f_1^* \leq c_2$ and $f_2^* > \widehat{v}$ in the equilibrium of Proposition 3. Because $v_I > c_2 > v_{II}$, if CRA$_2$ makes its rating transparent, then for all $f_2 \geq c_2$ there exists an equilibrium of the subgame in which the issuer plays α_1; the equilibrium is supported by worst-case off-equilibrium beliefs on r_1 if r_1 is not disclosed and worst-case off-equilibrium beliefs on r_2 if r_2 is purchased but not disclosed. If CRA$_1$ deviates by making its rating transparent, it can only make higher profits if it sets a higher fee. However, then, the same argument provided in the proof of Proposition 3 shows that there is an equilibrium on the subgame in which no rating is purchased; the equilibrium is supported by worst-case off-equilibrium beliefs on r_2 if r_2 is not disclosed and worst-case off-equilibrium beliefs on r_1 if r_1 is purchased but not disclosed.

Second, we prove that no CRA has an incentive to deviate from the equilibrium of Proposition 5 by making its rating transparent. Recall that $\widehat{v} < f^* < v_I$ in the equilibrium of Proposition 5. Then, if CRA$_i$ deviates by making its rating transparent and setting a fee equal to $f_i \geq f^*$, there is an equilibrium of the subgame in which the issuer plays α_{-i}; the equilibrium is supported by worst-case off-equilibrium beliefs on r_{-i} if r_{-i} is not disclosed and worst-case off-equilibrium beliefs on r_i if r_i is purchased but not disclosed. The same argument used in the proof of Proposition 5 shows that no CRA has an incentive to undercut (the argument used in the proof of Proposition 5 does not hinge on whether the rating of the CRA that undercuts is transparent or opaque.) This concludes the proof. ∎

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