# Online Appendix to "Opacity, Credit Rating Shopping and Bias"

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# 1. Outline

In this online appendix we solve an alternative model in which the fundamental and the ratings are normally distributed. For ease of comparison, this document has the same structure of the paper "Opacity, Credit Rating Shopping and Bias." Propositions 1 to 4 in this online appendix provide the counterpart to Propositions 1 to 4 in the paper. Propositions 5 and 6 in this online appendix provide the counterpart to, respectively, Propositions 6 and 7 in the paper. This online appendix is organized as follows. Section 2 describes the setup in this alternative model. Section 3 provides the analysis of the transparent market equilibrium. Section 4 provides the analysis of the opaque market equilibrium. The Appendices contain all proofs.

# 2. Setup

**The economy** An *issuer* is endowed with one unit of an asset with random payoff

$$X \sim N(\mu_X, \sigma_X^2). \tag{1}$$

We assume the asset is risky, $\sigma_X^2 > 0$. The issuer is risk neutral, and has an exogenous holding cost for the asset equal to $\kappa$. Ex-ante, $\kappa$ is equally likely to take one of two values $\{\kappa_L, \kappa_H\}$, where $\kappa_L > 0$, $\kappa_H = \kappa_L + \Delta$ and $\Delta > 0$. The realization of $\kappa$ is independent of $X$. The issuer can either hold the asset or sell it to risk-neutral investors.

The issuer can influence investors' valuation of the asset by conveying information to the market via *credit rating agencies* (CRAs). We assume there are two such agencies, endowed with the same rating technology: $\text{CRA}_i$ can produce at a cost $c_i \geq 0$ an unbiased noisy signal, or *rating*

$$r_i = X + \varepsilon_i, \tag{2}$$

for $i = 1, 2$, with $\varepsilon_i$ independent of $X$, i.i.d. and

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2). \tag{3}$$

We assume that ratings are noisy signals of the fundamental, $\sigma_\varepsilon^2 > 0$. Hence, CRAs have access to equivalent but independent rating technologies. CRAs are allowed to be heterogenous with respect to their cost parameters, and we let $c_1 < c_2 < \kappa_L$. Each CRA maximizes profits by setting the fee $c_i$ at which the issuer can purchase its rating, with the constraint that $f_i \geq c_i$.

**Rating process, timing of events and equilibrium definition** The specification of the rating process, timing and equilibrium are as in the baseline model in the paper "Opacity, Credit Rating Shopping and Bias"–with the natural modifications due to the differences in the setup.

The issuer can approach CRA$_i$ and purchase its rating at fee $c_i$; in which case the CRA produces the rating $r_i$, which is communicated to the issuer. At this point the issuer owns the rating and can either withhold it or make it public through the rating agency. Only in the latter case would investors observe the rating. The rating process is defined as *transparent* or *opaque,* depending on whether or not the act of purchasing a rating is observable by investors. If the market is opaque, investors only observe the purchased ratings that the issuer decides to publish voluntarily.

The timing of the model is as follows.

1. CRAs simultaneously post fees $f_1, f_2$. Fees are observed by all players.

2. The holding cost $\kappa$ realizes and is observed by all players.

3. The issuer shops for ratings. The issuer can shop sequentially, that is, can purchase a first rating, and decide whether to purchase the second rating after observing the value

of the first rating.

4. The issuer decides which ratings to disclose (if any).

5. The asset is sold to investors for the price $p$.

6. The payoff $X$ is realized and consumption takes place.

At any point in time (before stage 5) the issuer can decide to quit and retain the asset, in which case trade does not occur. The value of the issuer's outside option at a given stage is the risk-neutral valuation of the asset (conditional on the information that she has acquired) net of the holding cost.

A pair of rating fees and a realization of the holding cost induce a subgame. On a subgame, a strategy profile for the issuer is a set of contingent plans on (i) which ratings to purchase at stage 3, (ii) which of the purchased ratings to disclose at stage 4 and (iii) whether to sell the asset to investors in stage 5. A system of investor beliefs is a specification of beliefs on the value of ratings that are not disclosed both on- and off- the equilibrium path. The solution concept implemented is Perfect Bayesian Equilibrium. A strategy profile for the issuer and a system of investor beliefs are an *equilibrium of the subgame* induced by the triple $(f_1, f_2, \kappa) \in \mathbb{R}_+^2 \times \{\kappa_{L,}\kappa_H\}$ if investor equilibrium beliefs are consistent with the issuer's strategy and the issuer's strategy is sequentially optimal given rating fees and investor beliefs. An *equilibrium of the overall game* is a list of equilibria in every subgame induced by each triple $(f_1, f_2, \kappa) \in \mathbb{R}_+^2 \times \{\kappa_{L,}\kappa_H\}$ and a pair of fees $(f_1^*, f_2^*)$ that constitute a Nash equilibrium of the game played by CRAs in stage 1.

**Value of information and efficiency** Unlike the baseline model in our paper, ratings have no intrinsic social value in this setup with risk neutrality and no investment choice. To ease comparison across the two setups, we make the following assumption:

**Assumption 1.** *Investors can purchase the asset only if one rating has been disclosed.*

Assumption 1 is a reduced form for exogenous certification benefits of credit ratings. It implies that trade between the issuer and investors takes place–and gains from trade realize–only if one rating is produced (regardless of its realization). Since the gains from trade exceed the cost of production of a rating ($c_i < \kappa_L$), producing one rating increases welfare. Producing a second rating, by contrast, has no social value. In the socially optimal outcome, gains from trade realize (for $\kappa \in \{\kappa_H, \kappa_L\}$) at the minimum information production cost. Since $c_1 < c_2$, then–as in the baseline model–an equilibrium is *efficient* if only the first rating is produced and disclosed to investors, and is inefficient otherwise.

**Notation.** We define

$$\sigma^2_{X|r} := \mathrm{Var}\left(X|\, r_i\right); \quad \sigma^2_{X|2r} := \mathrm{Var}\left(X|\, r_1, r_2\right); \quad \sigma^2_{r|r} := \mathrm{Var}\left(r_i|\, r_{-i}\right).$$

Then, given the assumptions in Eqs. (1)-(3), we have the following standard results for conditional moments:

$$\begin{array}{rlrl} \mathrm{E}\left(X|\, r_i\right) = \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2}+\sigma_\varepsilon^{-2}}(r_i - \mu_X); & & \sigma^2_{X|r} = \left(\sigma_X^{-2} + \sigma_\varepsilon^{-2}\right)^{-1} & \\ \mathrm{E}\left(X|\, r_1, r_2\right) = \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2}+2\sigma_\varepsilon^{-2}}\left[(r_2 - \mu_X) + (r_2 - \mu_X)\right] & & \sigma^2_{X|2r} = \left(\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}\right)^{-1} & . \\ \mathrm{E}\left(r_i|\, r_{-i}\right) = \mathrm{E}\left(X|\, r_i\right); & & \sigma^2_{r|r} = \sigma^2_{X|r} + \sigma^2_\varepsilon & \end{array} \tag{4}$$

We denote CDF and PDF of a standard normal with, respectively, $\Phi(\cdot)$ and $\phi(\cdot)$. Finally, we denote

$$h(x) = \frac{\phi(x)}{\Phi(x)}.$$

**Full disclosure benchmark** As in the main paper, a strategy for the issuer is said to be a *full disclosure strategy* if the issuer sells the asset in stage 5 only if all purchased rating information is disclosed in stage 4. We define a *full disclosure equilibrium* to be an equilibrium of the subgame in which the issuer follows a full disclosure strategy.

**Selective disclosure** Selective disclosure is defined as an event in which the issuer hides a rating from investors and sells for a better price than if that rating was disclosed. An *equilibrium with selective disclosure* is an equilibrium of the subgame in which selective disclosure is on the equilibrium path.

# 3. Transparent Market Equilibrium

The following lemma follows from the usual unraveling argument in the transparent market and is provided without proof.

**Lemma 1**. (Unraveling.) *Any equilibrium of the transparent market features full disclosure of purchased credit rating information.*

The next proposition describes the equilibrium outcome in the transparent market when rating fees are endogenous.

**Proposition 1**. (Transparent market equilibrium.) *In the unique equilibrium of the overall game, CRAs set $f_1^* = f_2^* = c_2$, the issuer purchases and discloses $r_1$ and sells the asset to investors. The equilibrium is supported by worst-case off-equilibrium-beliefs on ratings that are purchased but not disclosed.*

Proposition 1 has the following welfare implication:

**Corollary 1**. *The transparent market equilibrium is efficient.*

# 4. Opaque Market Equilibrium

## *4.1. Private value of information*

Consider whether the issuer's strategy from Proposition 1 can be part of an equilibrium in the opaque case. Conditional on $r_1$, the issuer could deviate, purchase $r_2$ and disclose this additional rating selectively. Then, the issuer could sell for a price of $E(X|\, r_1, r_2) > E(X|\, r_1)$ in case $r_2$ turns out to be higher than $E(X|\, r_1)$,[1] and sell for $E(X|\, r_1)$ (by disclosing $r_1$ but not $r_2$) in case $r_2$ turns out to be lower than $E(X|\, r_1)$. (In the latter case investors would not detect the deviation and would, in fact, overpay for the asset.) Anticipating this, the issuer's expected profits from purchasing the second rating and disclosing it selectively equal

$$-f_2 + E\left[\max\{E(X|\, r_1), E(X|\, r_1, r_2)\}|\, r_1\right] = -f_2 + E(X|\, r_1) + \sqrt{\frac{\sigma^2_{X|r} - \sigma^2_{X|2r}}{2\pi}}. \quad (5)$$

(The equality in Eq. (5) is proved in Appendix B.) Since equilibrium payoffs, conditional on disclosing $r_1$, amount to $E(X|\, r_1)$, then the issuer has an incentive to shop for the second rating if $f_2 < \widehat{v}$, where

$$\widehat{v} := \sqrt{\frac{\sigma^2_{X|r} - \sigma^2_{X|2r}}{2\pi}}. \quad (6)$$

[1] Inspection of the expressions for the conditional expectations in Eq. (4) immediately reveals that $E(X|\, r_1, r_2) > E(X|\, r_1) \Leftrightarrow r_2 > E(X|\, r_1)$.

## *4.2. Inefficiency of opaque market equilibrium*

Let the inefficiency threshold $\bar{c}_2$ be defined as

$$\bar{c}_2 := \min\{\widehat{v}, \Delta\}.$$

We have:

**Proposition 2**. (Inefficiency of opaque market equilibrium.) *For $c_2 < \bar{c}_2$, any equilibrium of the overall game in the opaque market is inefficient.*

The intuition for Proposition 2 is the same as in the benchmark model: for the efficient benchmark to be an equilibrium of the overall game in the opaque market, it is necessary that $f_2^*$ satisfies the "no-shopping condition" $f_2^* \geq \widehat{v}$, as explained. However, under the condition in the proposition, this no-shopping condition cannot be part of an equilibrium of the overall game in which $\mathrm{CRA}_2$ makes zero profits.

## *4.3. Framework for equilibrium selective disclosure*

The following definition illustrates the equilibrium upon which we will focus:

**Definition 1**. *An equilibrium features* rating shopping and selective disclosure of the second rating *if the following strategy is optimal for the issuer:*

Stage 3 *Purchase $r_1$ first and then purchase $r_2$ with "shopping probability" $q \in (0, 1)$.*

Stage 4 *Always disclose $r_1$; disclose $r_2$ if and only if $r_2 > s(r_1)$, for a disclosure function $s(\cdot)$.*

Stage 5 *Sell the asset to investors.*

The equilibrium strategy from Definition 1 features selective disclosure: conditional on having purchased both ratings, the issuer hides from investors the second rating if this is not favorable enough. Specifically, in Appendix B we show that the disclosure function takes the following form:

$$s(r_1) = E\left(X|\, r_1\right) - \bar{r}, \tag{7}$$

for some positive constant $\bar{r}$. When only $r_1$ is disclosed, investors will use Bayes' Law–in a way that is consistent with the issuer's strategy–to determine the fair value of the asset, that is, the valuation of the asset that is consistent with the probability that the second rating was purchased but not disclosed. In Appendix B we show that this adjustment is reflected in the equilibrium price, $p_q\left(r_1\right)$ say, as follows:

$$p_q\left(r_1\right) = E\left(X|\, r_1\right) - q_P\sqrt{\sigma^2_{X|r} - \sigma^2_{X|2r}}\,h\left(-\frac{\bar{r}}{\sigma_{r|r}}\right), \tag{8}$$

where $q_P$ denotes the posterior probability of selective disclosure and is derived in Eq. (B.2) in Appendix B. The first term in the R.H.S. of Eq. (8) corresponds to the price investors would be willing to pay conditional on a single rating being disclosed with value $r_1$ if they were to take this rating at face value. The second term in the R.H.S. of Eq. (8) measures a "selective disclosure discount," as investors rationally adjust pricing downward to reflect the possibility that a low rating is not being disclosed.

Lemma B.1 in Appendix B derives the conditions for the strategy in Definition 1 with the disclosure function in Eq. (7) to be an equilibrium of the subgame. The next proposition provides sufficient conditions for shopping and selective disclosure to be an equilibrium outcome when fees are determined endogenously. We have:

**Proposition 3.** (Equilibrium selective disclosure.) *Assume $c_2 < \widehat{v}$. There exists a values $\bar{\kappa}$ such that, for all $\kappa_L \geq \bar{\kappa}$, there is an equilibrium of the overall game with rating shopping and selective disclosure of the second rating. In equilibrium, the disclosure function $s(\cdot)$ is as in Eq. (7).*

When contacts between issuers and CRAs are opaque and Propositions 2 and 3 hold, selective disclosure emerges as an equilibrium while the efficient outcome does not. The next proposition facilitates the comparison by showing how the various parametric restrictions are jointly fulfilled as long as simple conditions are satisfied. We have:

**Proposition 4.** (Joint parameter restrictions.) *Assume $\kappa_L \geq \bar{\kappa}$. Then Propositions 2 and 3 simultaneously hold for all $c_1 < c_2 < \bar{c}_2$.*

## *4.4. Full disclosure equilibria and welfare comparison*

The following proposition provides welfare comparison across equilibria. The economic surplus generated in an equilibrium is defined as the sum of the equilibrium expected payoffs of all players net of the outside option of the issuer.

**Proposition 5.** (Welfare comparison.) *There exists $\bar{c}_1 > 0$ such that if parameter values are as in Proposition 4 and $c_1 < \bar{c}_1$, the equilibrium of Proposition 3 generates larger economic surplus than any full disclosure equilibrium of the overall game.*

### *4.5. Endogenous opacity*

Consider an extended game in which each CRA can independently choose the transparency regime as a strategic variable. The contract would therefore specify both the fee at which the rating is sold, and whether the CRA would communicate to investors that the rating has been purchased (transparent regime) or not (opaque regime). Contracts are then announced simultaneously by CRAs in stage 1 and such announcement is observed by all players. The rest of the game is unchanged. The next proposition establishes whether the equilibrium outcomes described in Propositions 1 and 3 are robust to this extension.

**Proposition 6**. (Endogenous opacity.) *When the degree of transparency is endogenously determined, then:*

**i)** *For $c_2 < \widehat{v}$, the transparent market equilibrium of Proposition 1 is not an equilibrium of the extended game because $CRA_2$ has an incentive to make its rating opaque and set a higher fee;*

**ii)** *The opaque market equilibrium of Proposition 3 is an equilibrium of the extended game.*

# Appendices

## Appendix A: Proofs for Section 3.

**Lemma A.1**. (Equilibrium of the subgame in the transparent market) *Let the issuer's holding cost be $\kappa_\tau$ for $\tau \in \{L, H\}$ and let fees be $f_1, f_2$.*

**(i)** *If $f_1, f_2$ are strictly positive and the market is transparent, there is no equilibrium of the subgame in which the issuer purchases a second rating.*

**(ii)** *If $f_i < f_{-i}$, $CRA_{-i}$ makes zero profits in any equilibrium of the subgame.*

**(iii)** *If $min\{f_1, f_2\} > \kappa_\tau$, expected profits are zero for both CRAs.*

**(iv)** *If $f_i < f_{-i}$ and $f_i < \kappa_\tau$ there is a unique equilibrium of the subgame; in this equilibrium $CRA_i$'s profits amount to $f_i - c_i$ and $CRA_{-i}$ makes zero profits.*

**Proof of part (i).** By contradiction, assume there is an equilibrium of the subgame in which the issuer purchases a first rating, $r_i$ say, and then (possibly only for some realizations of $r_i$) purchases $r_{-i}$. Conditional on the realization of $r_i$, by Lemma 1, the issuer anticipates that if it purchases a second rating, then (for any realization of $r_{-i}$) it will disclose both ratings and sell for a price of $\mathrm{E}(X|\, r_i, r_{-i})$. Hence, the issuer's expected continuation payoffs conditional on $r_i$ equal

$$-f_{-i} + E\left(E\left(X|\, r_i, r_{-i}\right)|\, r_i\right) = -f_{-i} + E\left(X|\, r_i\right). \tag{A.1}$$

But then, since in a transparent market the issuer can deviate and sell the asset for $E\left(X|\, r_i\right)$ without purchasing $r_{-i}$, it follows that purchasing the second rating is not sequentially rational for all $f_{-i} > 0$. ■

**Proof of part (ii).** Follows from part (i) and the fact that, if the issuer purchases one rating in equilibrium, sequential optimality requires this rating to be the cheaper rating. ■

**Proof of part (iii).** Follows from the fact that, anticipating full disclosure of purchased ratings (Lemma 1), the issuer's expected continuation payoff (when one rating is purchased) are not greater than $\mu_X - \min\{f_1, f_2\}$, while the issuer's outside option is equal to $\mu_X - \kappa_\tau$. Hence, for $min\{f_1, f_2\} > \kappa_\tau$, the issuer is better off by retaining the asset. ■

**Proof of part (iv).** By purchasing and disclosing the cheapest rating, the issuer's expected profits equal $\mu_X - f_i$, and exceed both the value of the issuer's outside option of retaining the asset, $\mu_X - \kappa_\tau$ (because $f_i < \kappa_\tau$) and the expected profits of purchasing $r_{-i}$, $\mu_X - f_{-i}$ (because $f_i < f_{-i}$). By part (i), a second rating is never purchased in equilibrium. Hence, in the equilibrium of the subgame is unique: the issuer purchases and discloses the cheapest rating. This completes the proof. ■

**Proof of Proposition 1.** First, we rule out any candidate equilibrium fees $f_1^*, f_2^*$ that satisfy $f_1^* \neq f_2^*$. By Lemma A.1-(ii), for $f_1^* \neq f_2^*$, the CRA that sets the larger fee, CRA$_i$ say, must make zero profits. But this cannot be an equilibrium of the overall game because:[2] for $\min\{f_1^*, f_2^*\}> c_2$, CRA$_i$ can undercut to $f \in \left(c_2, \min\{\kappa_L, f_{-i}^*\}\right)$ and make positive profits by Lemma A.1-(iv); for $\min\{f_1^*, f_2^*\}= c_2$, CRA$_{-i}$ can increase its fee to $f \in (c_2, \min\{\kappa_L, f_i^*\})$ and therefore increase its profits by Lemma A.1-(iv); for $\min\{f_1^*, f_2^*\}< c_2$ (which can only be an equilibrium if $\min\{f_1^*, f_2^*\} = f_1^*$), CRA$_1$ could charge a larger fee, $f \in (f_1^*, c_2)$ and therefore increase its profits by Lemma A.1-(iv).

Second, we rule out any candidate equilibrium fees $f_1^*, f_2^*$ that satisfy $f_1^* = f_2^* = f^*$ and $f^* > c_2$. $f^* > \kappa_H$ is not an equilibrium since both CRAs would make zero profits by Lemma A.1-(ii); CRA$_i$ could profitably deviate to $f \in (c_2, \kappa_L)$ and make positive profits by Lemma A.1-(iv). Any $f^* \in (\kappa_L, \kappa_H]$ is not an equilibrium because either (i) there is one CRA, CRA$_i$ say, that makes zero expected profits (if issuers of type $\kappa_H$ only purchase $r_{-i}$) or (ii) for both CRAs expected profits are strictly less than $\frac{1}{2}(f^* - c_i)$ (if issuers of type $\kappa_H$ randomize over the two ratings); in both cases the CRA that makes lower expected profits has an incentive to undercut to $f^* - \varepsilon$ where $\varepsilon > 0$, as it would then make $\frac{1}{2}(f^* - \varepsilon - c_i)$ by Lemma A.1-(iv) (the deviation is profitable for $\varepsilon$ sufficiently small). A similar argument shows that any $f^* \in (c_2, \kappa_L]$ is not an equilibrium of the overall game. Next, consider equilibrium fees that satisfy $f_1^* = f_2^* = c_2$. We can rule out any equilibrium in which CRA$_1$'s expected profits are strictly less than $c_2 - c_1$, because otherwise CRA$_1$ could deviate to $f = c_2 - \varepsilon$ and make expected profits equal to $c_2 - \varepsilon - c_1$by Lemma A.1-(iv) (the deviation is profitable for $\varepsilon$ sufficiently small). Then, we are left with $f_1^* = f_2^* = c_2$ and an equilibrium of the subgame in which the issuer always purchases $r_1$, discloses $r_1$ and sells. This is an equilibrium of the overall game as neither CRA has an incentive to undercut or to charge a higher fee. ■

## Appendix B: Proofs for Section 4.

**Proof of Eq. (5)** By Eqs. (4), it is immediate that

$$E\left(X|\, r_1\right) < E\left(X|\, r_1, r_2\right) \Leftrightarrow r_2 > \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}}(r_1 - \mu_X) = E\left(X|\, r_1\right),$$

and therefore

$$E\left[\max\{E\left(X|\, r_1\right), E\left(X|\, r_1, r_2\right)\}|\, r_1\right]$$

[2]We remind the reader of the parametric assumption $c_1 < c_2 < \kappa_L$.

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} \max\{E(X|r_1), E(X|r_1,r_2)\} d\Phi\left(\frac{r_2 - E(X|r_1)}{\sigma_{r|r}}\right) \\
&= \int_{-\infty}^{E(X|r_1)} E(X|r_1)\, d\Phi\left(\frac{r_2 - E(X|r_1)}{\sigma_{r|r}}\right) + \int_{E(X|r_1)}^{\infty} E(X|r_1,r_2)\, d\Phi\left(\frac{r_2 - E(X|r_1)}{\sigma_{r|r}}\right) \\
&= \frac{1}{2}\left(E(X|r_1) + \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}\left[(E(X|r_1) - \mu_X) + (r_1 - \mu_X)\right]\right) + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}\sqrt{\frac{\sigma_{r|r}^2}{2\pi}} \\
&= E(X|r_1) + \sqrt{\frac{\sigma_{X|r}^2 - \sigma_{X|2r}^2}{2\pi}},
\end{aligned}
$$

where the third equality follows from standard properties of the truncated normal distribution and the last equality follows from the expressions in Eqs. (4) and simple manipulations (see also Footnote 3). ■

**Proof of Proposition 2.** By contradiction, assume that regardless of the realization of $\kappa$, the issuer's equilibrium strategy is to purchase and disclose $r_1$, so CRA$_2$'s equilibrium profits are zero. This requires $f_2^* \geq \widehat{v}$ (no shopping condition) and $f_1^* \leq \kappa_L$ (issuer's ex ante participation constraint). We will prove that $f_2^* \geq \widehat{v}$ cannot be a best response for CRA$_2$ by showing that, if CRA$_2$ deviates to $\tilde{f}_2 \in (c_2, \bar{c}_2)$, where $\bar{c}_2 := \min\{\widehat{v}, \Delta\}$, the deviation necessarily results in positive expected profits for CRA$_2$. Let

$$F = \{(f_1, f_2) \text{ such that } f_1 \leq \kappa_L \text{ and } f_2 \in (c_2, \bar{c}_2)\}.$$

As a first step, we show that there is no equilibrium of the subgame induced by any $(f_1, f_2) \in F$ and $\kappa = \kappa_H$ in which no rating is purchased. By contradiction, assume there was such an equilibrium. Then, the issuer must retain the asset in equilibrium and therefore make $\mu_X - \kappa_H$. We will show that the issuer has an incentive to deviate and purchase and disclose both ratings: the issuer's ex ante profits from such a deviation are $\mu_X - (f_1 + f_2)$, which exceed the payoff from retaining the asset because

$$\mu_X - (f_1 + f_2) > \mu_X - \kappa_H \Leftrightarrow \kappa_H > (f_1 + f_2) \Leftrightarrow \kappa_L + \Delta > (f_1 + f_2), \tag{B.1}$$

where the last inequality follows from $f_1 \leq \kappa_L$ and $f_2 < \Delta$.

As a second step, we show that there is no equilibrium of the subgame induced by any $(f_1, f_2) \in F$ and $\kappa = \kappa_H$ in which $r_2$ is purchased with probability zero while $r_1$ is purchased with positive probability. By contradiction, assume there was such an equilibrium; then trade must occur with positive probability on the equilibrium path, for otherwise the issuer would be strictly better off by purchasing and disclose both ratings (see (B.1)). For there to be trade, the issuer must disclose $r_1$ in some states. However, because $f_2 < \widehat{v}$, the derivation of the no-shopping condition (Eq. (6) in the text) implies that, conditional on any value of $r_1$, the anticipated profits from purchasing $r_2$ and disclosing selectively exceed equilibrium profits from selling, a contradiction.

The last step is to prove that, on every subgame induced by any $(f_1, f_2) \in F$ and $\kappa = \kappa_H$, an equilibrium exists in which $\text{CRA}_2$ makes positive expected profits. In fact, an equilibrium of the subgame exists in which the issuer purchases and discloses both ratings, supported by worst-case beliefs on undisclosed ratings. In this equilibrium, the issuer's ex ante profits exceed the outside option of retaining the asset by (B.1) and the acquisition and disclosure strategy is optimal under the assume out-of-equilibrium beliefs. Since prob$(\kappa = \kappa_H)$=1/2 and $f_2 > c_2$, $\text{CRA}_2$ makes positive expected profits in this equilibrium. This completes the proof. ■

**Derivation of the equilibrium price in Eq. (8)**. Denote with $q_P$ the posterior probability that $r_2$ was purchased but not disclosed conditional on only $r_1$ being disclosed. When the issuer's strategy is as in Definition 1 for a given shopping probability $q$ and the disclosure function in Eq (7), Bayes' rule gives

$$q_P = \frac{q\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right) + 1 - q}. \tag{B.2}$$

Hence, the equilibrium price that is consistent with the strategy of the issuer equals

$$p_q(r_1) = (1 - q_P)\, E(X|\, r_1) + q_P E(X|\, r_1, r_2 \leq E(X|\, r_1) - \bar{r}).$$

By direct computation of the second conditional expectation in the previous equation we find

$$\begin{aligned} E(X|\, r_1, r_2 \leq E(X|\, r_1) - \bar{r}) &= \int_{-\infty}^{\infty} X \frac{1}{\sigma_{X|r}} \phi\left(\frac{X - E(X|\, r_1)}{\sigma_{X|r}}\right) \frac{\Phi\left(\frac{E(X|r_1) - \bar{r} - X}{\sigma_\varepsilon}\right)}{\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)} dX \\ &= E(X|\, r_1) - \sqrt{\sigma^2_{X|r} - \sigma^2_{X|2r}} \frac{\phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}, \end{aligned}$$

and therefore

$$p_q(r_1) = E(X|\, r_1) - q_P \sqrt{\sigma^2_{X|r} - \sigma^2_{X|2r}} \frac{\phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}. \quad ■$$

**Additional definitions.** Let

$$g(x) := \int_{-\infty}^{x} \Phi(t)\, dt = x\Phi(x) + \phi(x) \tag{B.3}$$

and let $\bar{r}(q)$ be implicitly defined by

$$qg\left(-\frac{\bar{r}}{\sigma_{r|r}}\right) = (1-q)\frac{\bar{r}}{\sigma_{r|r}}. \tag{B.4}$$

Since $g(x) > 0$ for all $x \in \mathbb{R}$ and $\frac{d}{dx}g(x) > 0$, then, for all $q \in (0,1)$, a value $\bar{r}$ satisfying Eq. (B.4) exists, is unique, and is such that $\bar{r} > 0$ and

$$\lim_{q \downarrow 0} \bar{r}(q) = 0; \qquad \lim_{q \uparrow 1} \bar{r}(q) = \infty. \tag{B.5}$$

The implicit function theorem and Eq. (B.4) give

$$\frac{d}{dq}\bar{r}(q) > 0. \tag{B.6}$$

Define further

$$f_{II}(q) := \sqrt{\sigma^2_{X|r} - \sigma^2_{X|2r}}\, g\left(\frac{\bar{r}(q)}{\sigma_{r|r}}\right). \tag{B.7}$$

Since $g(0) = \sqrt{\frac{1}{2\pi}}$, then (B.5)-(B.7) imply

$$\frac{d}{dq}f_{II}(q) > 0; \qquad \lim_{q \downarrow 0} f_{II}(q) = \widehat{v}; \qquad \lim_{q \uparrow 1} f_{II}(q) = \infty. \tag{B.8}$$

**Lemma B.1.** (Optimality of issuer's strategy in the equilibrium of Definition 1.) *Let $q \in (0,1)$ and $(f_1, f_2, \kappa_\tau)$ satisfy:*

$$f_1 + qf_2 \leq \kappa_\tau; \qquad f_2 = f_{II}(q). \tag{B.9}$$

*Then, the issuer's strategy in Definition 1 and investor equilibrium beliefs on undisclosed ratings in Eq. (B.2) are an equilibrium of the subgame induced by $(f_1, f_2, \kappa_\tau)$. The equilibrium is supported by worst-case off-equilibrium beliefs on $r_1$ if $r_1$ is not disclosed.*

**Proof.** First of all we note that, by construction, the posterior probability of selective disclosure in Eq. (B.2) and the equilibrium price in Eq. (8) are consistent with the issuer's strategy from Definition 1.

Next, we take the issuer's rating acquisition strategy and the price in Eq. (8) as given and we verify the conjecture on the disclosure function in Eq. (7). Assume the issuer has purchased both ratings. Since the issuer has the option of disclosing both ratings and $\kappa_\tau > 0$, holding the asset is never optimal. Under the assumed off-equilibrium beliefs, not disclosing $r_1$ is never optimal. Disclosing both ratings dominates disclosing only $r_1$ if

$$E(X|\, r_1, r_2) \quad > \quad p_q(r_1)$$

$$
\begin{aligned}
&\Leftrightarrow \quad E\left(X|\,r_1,r_2\right) > E\left(X|\,r_1\right) - q_P\sqrt{\sigma^2_{X|r}-\sigma^2_{X|2r}}\frac{\phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)} \\
&\Leftrightarrow \quad r_2 > E\left(X|\,r_1\right) - \frac{\left(\sigma_X^{-2}+2\sigma_\varepsilon^{-2}\right)}{\sigma_\varepsilon^{-2}} q_P\sqrt{\sigma^2_{X|r}-\sigma^2_{X|2r}}\frac{\phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{\Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)} \\
&\Leftrightarrow \quad r_2 > E\left(X|\,r_1\right) - \sigma_{r|r}\frac{q\ \phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\ \Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)+1-q},
\end{aligned}
$$

where the last line makes use of (B.2) and Eqs. (4).[3] Hence, the conjecture in Eq. (7) is verified if $\bar{r}$ solves

$$
\bar{r} = \sigma_{r|r}\frac{q\ \phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\ \Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)+1-q}. \tag{B.10}
$$

Simple manipulations show that Eq. (B.10) can be rearranged as Eq. (B.4). Hence, for a given $q$, a value $\bar{r}$ such that the conjecture in Eq. (7) is verified exists, is unique, is strictly positive and satisfies (B.5)-(B.6).

As a second step, we take the issuer's disclosure strategy and the price in Eq. (8) as given and verify that the conjectured rating acquisition strategy is optimal. Conditional on $r_1$ being purchased, if the issuer discloses $r_1$ and sells, the issuer's payoff equals $p_q\left(r_1\right)$. On the other hand, the expected continuation payoff by purchasing $r_2$ equals

$$
-f_2 + E\left(\max\{p_q\left(r_1\right), E\left(X|\,r_1,r_2\right)\}|\,r_1\right).
$$

Hence, the issuer is indifferent between disclosing $r_1$ or purchasing $r_2$ if

$$
\begin{aligned}
p_q\left(r_1\right) \quad &= \quad -f_2 + E\left(\max\{p_q\left(r_1\right), E\left(X|\,r_1,r_2\right)\}|\,r_1\right) \\
&\Leftrightarrow \quad f_2 = \int_{-\infty}^{\infty}\max\{0, E\left(X|\,r_1,r_2\right)-p_q\left(r_1\right)\}d\Phi\left(\frac{r_2-E\left(X|\,r_1\right)}{\sigma_{r|r}}\right).
\end{aligned} \tag{B.11}
$$

[3] Using Eqs. (4), it is immediate to verify the following equality:

$$
\frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2}+\sigma_\varepsilon^{-2}} = \sqrt{\frac{\sigma^2_{X|r}-\sigma^2_{X|2r}}{\sigma^2_{r|r}}};\ \text{or equivalently}\quad \frac{\sigma^2_{X|r}}{\sigma_{r|r}} = \sqrt{\sigma^2_{X|r}-\sigma^2_{X|2r}}.
$$

Using the disclosure rule in Eq. (7), the previous equality becomes

$$
\begin{aligned}
f_2 &= \int_{E(X|r_1)-\bar{r}}^{\infty} \left(E\left(X|\, r_1, r_2\right) - p_q\left(r_1\right)\right) d\Phi\left(\frac{r_2 - E\left(X|\, r_1\right)}{\sigma_{r|r}}\right) \\
&= \left(E\left(X|\, r_1\right) - p_q\left(r_1\right)\right)\Phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right) + \sqrt{\sigma_{X|r}^2 - \sigma_{X|2r}^2}\,\phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right) \\
&= \sqrt{\sigma_{X|r}^2 - \sigma_{X|2r}^2}\left(\frac{q\ \phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\ \Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right) + 1 - q}\Phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right) + \phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right)\right) \\
&= \sqrt{\sigma_{X|r}^2 - \sigma_{X|2r}^2}\left(\frac{\bar{r}}{\sigma_{r|r}}\Phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right) + \phi\left(\frac{\bar{r}}{\sigma_{r|r}}\right)\right) \\
&= \sqrt{\sigma_{X|r}^2 - \sigma_{X|2r}^2}\, g\left(\frac{\bar{r}}{\sigma_{r|r}}\right),
\end{aligned}
$$

where the second equality follows from standard properties of the truncated normal distribution (and the equality in Footnote 3), the third equality uses Eq. (8) and Eq. (B.2), the fourth equality makes use of Eq. (B.10) and the last equality follows from the definition of the $g$ function in Eq. (B.3). This proves that if $f_2 = f_{II}(q)$ in (B.9), it is optimal to purchase the second rating with probability $q$.

Next, we show that the issuer's ex-ante participation constraint is met if the first condition in (B.9) holds. Because the equilibrium asset price reflects consistent beliefs, the expected selling price when the issuer follows the strategy in Definition 1 equals the unconditional mean of the asset. Hence, the ex ante continuation equilibrium profits of the issuer, conditional on the realization of $\kappa$, equal $-f_1 + \mu_X - qf_2$, and the ex ante participation constraint requires

$$-f_1 + \mu_X - qf_2 \geq \mu_X - \kappa_\tau \Leftrightarrow f_1 + qf_2 \leq \kappa_\tau. \tag{B.12}$$

We now show that the issuer has no incentive to deviate from the strategy in Definition 1 by retaining the asset when only $r_1$ is purchased. By the indifference condition (B.11), this interim participation constraint is satisfied if the payoff from disclosing $r_1$ and selling exceeds the outside option of retaining the asset, that is, if

$$p_q\left(r_1\right) \geq E\left(X|\, r_1\right) - \kappa_\tau \Leftrightarrow \kappa_\tau \geq \sqrt{\sigma_{X|r}^2 - \sigma_{X|2r}^2}\frac{q\ \phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\ \Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right) + 1 - q}. \tag{B.13}$$

Since the indifference condition (B.11) holds for all $r_1$, then it also holds in expectation. Hence, the issuer's ex ante equilibrium continuation profits satisfy

$$-f_1 + \mu_X - qf_2 = -f_1 + E\left(p_q\left(r_1\right)\right) = -f_1 - f_2 + E\left(\max\{p_q\left(r_1\right), E\left(X|\, r_1, r_2\right)\}\right). \tag{B.14}$$

Then, using the first equality in (B.14), the ex-ante participation constraint (B.12) is equivalent to

$$-f_1 + E\left(p_q\left(r_1\right)\right) \geq \mu_X - \kappa_\tau \Leftrightarrow \kappa_\tau \geq f_1 + \sqrt{\sigma^2_{X|r} - \sigma^2_{X|2r}} \frac{q\ \phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right)}{q\ \Phi\left(-\frac{\bar{r}}{\sigma_{r|r}}\right) + 1 - q}. \tag{B.15}$$

Of course, if (B.15) is satisfied, then the interim participation constraint in (B.13) holds.

Finally, we show that the issuer has no incentive to purchase $r_2$ first (instead of $r_1$). Assume the issuer purchases $r_2$ first. Given the assumed off-equilibrium beliefs, it cannot be optimal to disclose only $r_2$, and the issuer has two options: either retain the asset, which gives

$$E\left(X|\, r_2\right) - \kappa_\tau, \tag{B.16}$$

or purchasing also $r_1$ and disclosing selectively, which gives an expected continuation payoff of

$$-f_1 + E\left(\max\{p_q\left(r_1\right), E\left(X|\, r_1, r_2\right)\}|\, r_2\right) \geq -f_1 + E\left(E\left(X|\, r_1, r_2\right)|\, r_2\right) = -f_1 + E\left(X|\, r_2\right). \tag{B.17}$$

Comparing (B.16)-(B.17), it is immediate that the issuer will also purchase $r_1$ if $f_1 < \kappa_\tau$, which in turn is implied by (B.15). Therefore, ex ante, expected profits from purchasing $r_2$ first, then purchasing $r_1$ and then disclosing selectively, equal

$$-f_1 - f_2 + E\left(\max\{p_q\left(r_1\right), E\left(X|\, r_1, r_2\right)\}\right),$$

which coincide with the expected equilibrium continuation profits by the second equality in (B.14). ■

**Lemma B.2.** *If $f_1 + f_2 \geq \kappa_\tau$, the exists an equilibrium of the subgame in which the issuer purchases no ratings and retains the asset. The equilibrium is supported by worst-case off-equilibrium beliefs on undisclosed ratings.*

**Proof** Assume the issuer deviates and purchases one rating, $r_i$ say. Under Assumption 1 and the assumed worst case beliefs, the issuer must disclose both ratings for trade to occur. Hence, the expected continuation payoffs from purchasing $r_{-i}$ and disclosing both ratings equal

$$-f_{-i} + E\left(E\left(X|\, r_{-i}, r_i\right)|\, r_i\right) = -f_{-i} + E\left(X|\, r_i\right).$$

Since the outside option of retaining the asset is worth $-\kappa_\tau + E\left(X|\, r_i\right)$, the issuer will purchase $r_{-i}$ only if $f_{-i} \leq \kappa_\tau$. If $f_{-i} > \kappa_\tau$, it is not sequentially optimal to purchase $r_{-i}$ after having purchased $r_i$, and the anticipated payoff from the deviation equals $-f_i - \kappa_\tau + \mu_X \leq -\kappa_\tau + \mu_X$, so the issuer has no incentive to deviate. If $f_{-i} \leq \kappa_\tau$ and the issuer purchases $r_{-i}$ after having purchased $r_i$, and discloses both ratings, the anticipated payoff from the deviation equals $-f_1 - f_2 + \mu_X$, which, since $f_1 + f_2 \geq \kappa_\tau$ by assumption, does not exceed the equilibrium payoff $-\kappa_\tau + \mu_X$. We conclude that retaining the asset is an equilibrium of the subgame for $f_1 + f_2 \geq \kappa_\tau$. ■

**Proof of Proposition 3.** Let $(f_1^*, f_2^*)$ and $q^*$ be such that

$$f_1^* + q^* f_2^* = \kappa_H \tag{B.18}$$
$$f_2^* = f_{II}(q^*) \tag{B.19}$$

and

$$f_1^* = \widehat{v}. \tag{B.20}$$

Eqs. (B.19)-(B.20) imply $f_i^* > c_i$ for $i = 1, 2$. (Recall that $f_{II}(q) \geq \widehat{v}$ for all $q \geq 0$ by (B.8), and that $\widehat{v} > c_2 > c_1$ by assumption.) Using Eqs. (B.18)-(B.19) to substitute for $f_1^*$ and $f_2^*$, we can write (B.20) as

$$q^* f_{II}(q^*) = \kappa_H - \widehat{v}. \tag{B.21}$$

Since $f_{II}(q)$ is strictly increasing in $q$ and such that $\lim_{q \uparrow 1} f_{II}(q) = \infty$ (see (B.8)), it is immediate that for any $\kappa_H > \widehat{v}$ there exists a value $q^* \in (0, 1)$ such that (B.21) holds. It also follows that $q^*$such that (B.21) holds satisfies

$$\frac{d}{d\kappa_H} q^* > 0; \qquad \lim_{\kappa_H \to \infty} q^* = 1. \tag{B.22}$$

Assume, then, that $\kappa_L \geq \widehat{v}$ and therefore $\kappa_H > \widehat{v}$ for all $\Delta > 0$. We now derive conditions such that no CRA has an incentive to deviate when fees satisfy Eqs. (B.18)- (B.20) and the equilibrium of the subgame for $\kappa = \kappa_H$ is such that the issuer's strategy is as in Definition 1 for $q = q^*$, while the equilibrium of the subgame for $\kappa = \kappa_L$ is such that the issuer purchases (and discloses) $r_1$ but not $r_2$.

We start by showing that no CRA has an incentive to undercut. Assume CRA$_2$ undercuts to some $f_2' \in [\widehat{v}, f_{II}(q^*))$. Since $f_1^* = \widehat{v}$ by (B.20) and $f_2' \geq \widehat{v}$, there exists an equilibrium of the subgame in which CRA$_2$ makes zero profits as the issuer (for both $\kappa = \kappa_L$ and $\kappa = \kappa_H$) purchases (and discloses) $r_1$ but not $r_2$; the equilibrium is supported by worst-case beliefs in case $r_1$ is not disclosed. If CRA$_2$ undercuts to some $f_2' \in (c_2, \widehat{v})$, the expected profits from this deviation are less than $\widehat{v} - c_2$. Since CRA$_2$'s equilibrium expected profits are equal to prob$(\kappa = \kappa_L) \times q^* (f_{II}(q^*) - c_2)$ and prob$(\kappa = \kappa_L) = 1/2$, then CRA$_2$ has no incentive to undercut if

$$\widehat{v} - c_2 \leq \frac{q^*}{2} (f_{II}(q^*) - c_2),$$

which can be rearranged as

$$(\widehat{v} - c_2)(2 - q^*) \leq q^* (f_{II}(q^*) - \widehat{v}). \tag{B.23}$$

By (B.8), it is immediate that there exists some $\widetilde{q} \in (0, 1)$ such that the inequality

$$(\widehat{v} - c_2)(2 - q) \leq q (f_{II}(q) - \widehat{v}) \tag{B.24}$$

holds for all $q \geq \widetilde{q}$. Let $\breve{q}$ be such that (B.24) holds as an equality when we set $c_2 = 0$, that is,

$$\widehat{v}\,(2 - \breve{q}) = \breve{q}\,(f_{II}\,(\breve{q}) - \widehat{v})\,,$$

or, equivalently

$$\breve{q} f_{II}\,(\breve{q}) = 2\widehat{v}. \tag{B.25}$$

By (B.8), $\breve{q}$ is unique and $\breve{q} < 1$. Furthermore, since $\widetilde{q}$ is such that (B.24) holds as an equality, then $\widetilde{q}$ is strictly decreasing in $c_2$ and therefore $\widetilde{q} < \breve{q}$ for all $c_2 > 0$ and $\widetilde{q} \to \breve{q}$ as $c_2 \to 0$.

Next, let $\overline{\kappa}$ be such that

$$\breve{q} f_{II}\,(\breve{q}) = \overline{\kappa} - \widehat{v} \tag{B.26}$$

and therefore, using (B.25) and (B.25), such that $\overline{\kappa} = 3\widehat{v}$. If we let $\kappa_L \geq \overline{\kappa}$, then, it is immediate that $q^*$ (i.e., the value of $q$ such that (B.21) holds), is such that $q^* \geq \breve{q}$ and therefore $q^* \geq \widetilde{q}$ for all $\Delta \geq 0$ and $c_2 \geq 0$. Hence, (B.23) holds and $\text{CRA}_2$ has no incentive to undercut. Clearly, $\text{CRA}_1$ has no incentive to undercut either, as its rating is purchased with probability one in equilibrium.

Next, we show that no CRA has an incentive to set a larger fee. Since $f_1^* + q^* f_2 = \kappa_H$ by (B.18) and $q^* < 1$, then

$$f_1^* + f_2^* > \kappa_H. \tag{B.27}$$

The inequality in (B.27) and Lemma B.2 imply that, if $\text{CRA}_i$ deviates to $f_i' > f_i^*$, there exists an equilibrium of the subgame that starts with either $\kappa = \kappa_L$ or $\kappa = \kappa_H$ in which the issuer retains the asset and CRAs' profits are zero.

To summarize, for $\kappa_L \geq \overline{\kappa}$ and $(f_1^*, f_2^*)$ and $q^*$ that solve Eqs. (B.18)-(B.20), Lemma B.1 implies that for $\kappa = \kappa_H$ there is an equilibrium of the subgame such that the issuer's strategy is as in Definition 1 for $q = q^*$, while for $\kappa = \kappa_L$ there is an equilibrium of the subgame such that the issuer purchases (and discloses) $r_1$ but not $r_2$ (the equilibrium is supported by worst case off-equilibrium beliefs on $r_1$ if $r_1$ is not disclosed; since $f_1^* = \widehat{v}$ and $\overline{\kappa} > \widehat{v}$, the issuer's participation constraint is trivially satisfied). Furthermore, we have shown that $(f_1^*, f_2^*)$ is a Nash equilibrium of the fee setting game played by the CRAs in the initial stage. This concludes the proof. ■

**Proof of Proposition 4.** If $\kappa_L \geq \overline{\kappa}$, then Proposition 3 holds for all $c_1, c_2$ and $\Delta$ such that $c_1 \leq c_2$ and $c_2 < \widehat{v}$ and $\Delta \geq 0$. Since $\widehat{v} \geq \bar{c}_2$, then Proposition 3 also holds for all $c_1, c_2$ and $\Delta$ such that $c_1 < c_2 < \bar{c}_2$, for which Proposition 2 holds. This concludes the proof. ■

**Further notation for full disclosure strategies.** Denote with $\alpha_0$ the strategy of not purchasing any rating and retaining the asset; denote with $\alpha_i$ the full disclosure strategy in which the issuer purchases only rating $r_i$, discloses and sells the asset; denote with $\alpha_{12}$ the full disclosure strategy in which the issuer purchases both ratings, discloses both ratings and sells. Denote

$$\Gamma = (\alpha_0, \alpha_1, \alpha_2, \alpha_{12})\,. \tag{B.28}$$

Let $\sigma$ denote a mixed strategy over $\Gamma$, and let $\Sigma(\Gamma)$ denote the set of mixed strategies over $\Gamma$. For each $\sigma \in \Sigma(\Gamma)$, $\sigma(\alpha)$ denotes the probability assigned by $\sigma$ to playing $\alpha \in \Gamma$.

**Remark B.1.** In an equilibrium of the subgame in which the issuer's strategy is $\sigma$ and $\sigma(\alpha_i) \in (0,1)$, the equilibrium price upon disclosure of $r_i$ (but not $r_{-i}$) is $E(X|\, r_i)$. This observation is used throughout the proof of Lemma B.3.

The proof of Proposition 5 will make use of the following lemma:

**Lemma B.3**. (Necessary conditions for optimality of full disclosure mixed-strategies) *Let $\sigma \in \Sigma(\Gamma)$ and assume investor beliefs are consistent with $\sigma$. Then, $\sigma \in \Sigma(\Gamma)$ is optimal for the issuer only if*:

**(i)** $\min\{\sigma(\alpha_i), \sigma(\alpha_{12})\} = 0$ *for* $i = 1, 2$.

**(ii)** $\min\{\sigma(\alpha_1), \sigma(\alpha_2)\} = 0$

**Proof part-(i).** Assume $\sigma \in \Sigma(\Gamma)$ is an equilibrium strategy and is such that both $\sigma(\alpha_i)$ and $\sigma(\alpha_{12})$ are strictly positive. Assume ex-post the issuer plays $\alpha_{12}$ and it turns out that $r_{-i} < E(X|\, r_i)$ so that $E(X|\, r_i, r_{-i}) < E(X|\, r_i)$. The issuer has an incentive to deviate from full disclosure by disclosing only $r_i$. ■

**Proof of part-(ii).** Assume $\sigma \in \Sigma(\Gamma)$ is an equilibrium strategy and is such that both $\sigma(\alpha_1)$ and $\sigma(\alpha_2)$ are strictly positive. Assume ex post the issuer purchases only $r_i$, so that equilibrium profits upon disclosing $r_i$ equal

$$E(X|\, r_i) = \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}}(r_i - \mu_X). \tag{B.29}$$

Conditional on $r_i$, the expected payoff from deviating, purchasing also $r_{-i}$ and disclosing $r_{-i}$ but not $r_i$ equal

$$-f_{-i} + E(E(X|\, r_{-i})|\, r_i) = -f_{-i} + \mu_X + \left(\frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}}\right)^2 (r_i - \mu_X). \tag{B.30}$$

Comparing the R.H.S. of both (B.29) and (B.30), it is immediate that there is a low enough value of $r_i$ that the issuer has an incentive to deviate. ■

**Proof of Proposition 5.** Economic surplus in the equilibrium of Proposition 3 equals

$$\kappa_L + \frac{\Delta}{2} - c_1 - \frac{q^*}{2} c_2. \tag{B.31}$$

Since Proposition 2 holds, Lemma B.2 implies that the largest economic surplus generated by a full disclosure equilibrium of the overall game amounts to

$$\kappa_L + \frac{\Delta}{2} - \frac{c_1 + c_2}{2}, \tag{B.32}$$

which obtains when issuers of one type purchase and disclose $r_1$ while issuers of the other type purchase and disclose $r_2$. It is immediate that (B.31) exceeds (B.32)for

$$c_1 < c_2 \left(1 - q^*\right). \tag{B.33}$$

Defining

$$\bar{c}_1 := c_2 \left(1 - q^*\right),$$

then (B.33) holds for all $c_1 < \bar{c}_1$. ■

**Proof of Proposition 6 Part-(i).** Recall that in the equilibrium of Proposition 1 $f_1^* = f_2^* = c_2$. Furthermore, $c_2 < \min\{\kappa_L, \widehat{v}\}$ by assumption. Suppose CRA$_2$ deviates from $f_2^* = c_2$ by setting its fee equal to some $f_2 \in (c_2, \min\{\kappa_L, \widehat{v}\})$ and by making its rating opaque. We first show that on the subgame induced by $f_1 = c_2$ and $f_2 \in (c_2, \min\{\kappa_L, \widehat{v}\})$ (such that $r_1$ is transparent and $r_2$ is opaque) there is no equilibrium in which no rating is purchased. Since $r_1$ is transparent, there is no asymmetric information about $r_1$ if $r_1$ is not purchased by the issuer. Hence, no-trade (i.e., the issuer's retaining the asset) is not an equilibrium on the subgame for either $\kappa = \kappa_L$ or $\kappa = \kappa_H$ because the issuer can deviate and purchase and disclose $r_2$, which gives ex ante utility of $\mu_X - f_2$, which exceed the payoff from holding the asset because $f_2 < \kappa_\tau$ for $\tau \in \{L, H\}$. The same argument used in the proof of Proposition 2 shows that there is no equilibrium on the subgame in which $r_2$ is purchased with probability zero while $r_1$ is purchased with positive probability. Finally, since $r_1$ is transparent and $f_2 < \kappa_L$, purchasing and disclosing $r_2$ (but not $r_1$) is an equilibrium on the subgame; the equilibrium is supported by worst-case off-equilibrium beliefs on $r_2$ if $r_2$ is not disclosed and by worst-case off-equilibrium beliefs on $r_1$ if $r_1$ is purchased but not disclosed. Since $f_2 > c_2$, CRA$_2$ makes positive profit in this equilibrium and the deviation is profitable. ■

**Proof of Proposition 6 Part-(ii).** We first prove that no CRA has an incentive to deviate from the equilibrium of Proposition 3 by making its rating transparent. Recall that $f_1^* = \widehat{v}$ and $f_2^* > \widehat{v}$ in the equilibrium of Proposition 3.

Assume CRA$_2$ makes its rating transparent for some $f_2 \geq c_2$. For all $f_2 < \widehat{v}$, the proof of Proposition 3 shows that the deviation is not profitable. For all $f_2 \geq \widehat{v}$, there exists an equilibrium of the subgame (for both $\kappa = \kappa_L$ and $\kappa = \kappa_H$) in which the issuer purchases and discloses $r_1$ (but not $r_2$); the equilibrium is supported by worst-case off-equilibrium beliefs on $r_1$ if $r_1$ is not disclosed and worst-case off-equilibrium beliefs on $r_2$ if $r_2$ is purchased but not disclosed. (Since $f_1^* = \widehat{v}$ and $\kappa_L > \widehat{v}$, the issuer's participation constraint is trivially satisfied.)

If CRA$_1$ deviates by making its rating transparent, it can only make higher profits if it sets a higher fee. But then Lemma B.2 implies that there is an equilibrium on the subgame in which no rating is purchased. This concludes the proof. ■