# Information, Misallocation and Aggregate Productivity<sup>\*</sup>

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#### Abstract

We explore the implications of imperfect information about firm-level fundamentals for the misallocation of factors of production across heterogeneous firms and the consequences for aggregate productivity and output. In our setup, firms learn from both private sources and stock market prices, the latter determined in a noisy rational expectations model of asset markets. We devise a novel calibration strategy that uses a combination of crosscountry stock market and firm-level production data to pin down the information structure in the economy and apply it to data from the US and two emerging market economies -China and India. We find that significant productivity and output losses due to informational frictions are substantial - even when only one of the factors (namely, capital) is affected by the friction. Our estimates for these losses range from 8-16% for TFP and 12-24% for GDP in India and China. The numbers are even higher when labor decisions are also made under imperfect information. We also find that learning from financial markets contributes little to overall allocative efficiency.

> Preliminary and Incomplete Not for General Circulation

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## 1 Introduction

The optimal allocation of resources across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of such resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent literature empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent regarding the underlying factors driving misallocation, that is, regarding the precise source of the frictions that result in marginal product dispersion.

In this paper, we propose just such a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose productive inputs under imperfect information about fundamentals. Importantly, firms learn not only from their own private sources, but also from stock market prices. We explicitly model this source of information with a fully specified financial market in which informed investors and noise traders interact to trade the firm's shares. The presence of noise traders prevents prices from perfectly aggregating private information so that stock market prices provide firms with a noisy signal of fundamentals, which is combined with the firms' own information to guide input decisions.

The degree of misallocation then is a function of the firm's residual uncertainty at the time of this input choice decision, which in turn depends on the volatility of the fundamental shocks it faces and the quality of the information to which it has access. The parsimonious nature of our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational parameters at the micro-level to aggregate outcomes. Our general equilibrium framework also embeds an amplification mechanism - informational frictions and the resulting misallocation reduce economy-wide incentives to accumulate factors of production, specifically capital. As a result, the allocative inefficiency translates into even greater declines in output above and beyond those in TFP.

In the second part of our analysis, we assign values to key parameters in order to quantitatively assess the importance of informational frictions. In principle, this can be challenging because firm-level information is not directly observable. One of the contributions of this paper is a novel empirical strategy that overcomes this difficulty and thus leads to robust conclusions about the quantitative effects of imperfect information. The key insight here is that combining firm-level financial market and production data allows us to directly observe a subset of agents' information set (stock returns), infer the noisiness of such information (by measuring the correlation of returns with fundamentals) and gauge agents' responsiveness to this information (by measuring the sensitivity of investment to returns). Specifically, we exploit the model-implied relationship between firm-level production variables and equity market returns as a function of the fundamental and signal noise processes. Intuitively, the cross-sectional variability of stock returns and their ability to forecast fundamentals provides information on the magnitude of the noise in financial markets. The extent to which firms adapt their decisions to this noisy signal then allows us to infer the quality of information available to firms from other (unobserved) sources. The less precise is the latter, the greater the reliance on financial markets for information and therefore, the higher the correlation between investment decisions and stock market returns, although we show that this correlation is not a sufficient statistic for assessing the informativeness of markets.

We apply our empirical methodology to data from 3 countries - the US, China and India. The latter are two of only three emerging market economies with sizeable financial markets, i.e., in the top ten markets worldwide measured by total market capitalization.<sup>1</sup> Our results point to substantial uncertainty at the micro level, with very little learning about contemporaneous changes in fundamentals. The resulting losses in productivity and output are quite large, particularly in India and China, where TFP losses in our baseline case range from 7-16% (depending on a few key parameters), with corresponding output losses of 10-24%. The corresponding numbers for the US are significant yet noticeably smaller - 4-11% for TFP and 5-17% for output, implying that the informational frictions we measure are more severe in emerging markets. Importantly, these baseline figures can be viewed as conservative estimates of the impact of informational frictions, as they assume that only capital decisions are made under imperfect information, while labor is free to adjust to contemporaneous conditions. Assuming that the friction affects labor inputs as well leads to losses that are substantially higher. In a sense, these two cases provide bounds on the potential impact of imperfect information, with reality likely somewhere in between.

The presence of learning from financial markets thus serves two roles in our analysis: first, it is at the core of our empirical strategy and enables us to identify the severity of otherwise unobservable informational frictions in the economy; second, it allows to quantitatively assess the role of an important mechanism through which financial markets are thought to contribute to allocative efficiency, that is, by providing decision-makers in firms with better information. A number of studies in empirical finance document the so-called "feedback effect" of market prices on real economic activity (see Bond et al. (2012) for an recent survey). Our analysis is, to the best of our knowledge, the first to shed light on the aggregate consequences of this effect in a standard macroeconomic context.

<sup>&</sup>lt;sup>1</sup>The remaining is Brazil, but sufficient firm-level data are not available.

Our approach also enables us to quantify the extent to which stock markets ameliorate information frictions. Interestingly, the information in stock prices contributes very little to overall learning and therefore, the aggregate consequences of eliminating this channel are negligible, even in relative well-functioning financial markets like the US.<sup>2</sup> Moreover, delivering access to US-quality financial markets to emerging market firms in a counterfactual experiment generates only small aggregate gains through better information, again suggesting that the informational role of these markets is quite limited. Our findings suggest then that to the extent the severity of informational frictions varies significantly across countries, it is primarily disparities in the ability of firms to learn from private sources that are to blame, not a lack of access to well-functioning financial markets. Additionally, we show that differences in the volatility of the shocks to fundamentals that firms face also play an important role in generating cross-country differences in the severity of these frictions. Specifically, we find that firms in India and China are subject to larger shocks to their fundamentals than their US counterparts, making the inference problem harder in those countries even without the effect of differences in signal qualities.

Our paper relates to several existing branches of literature. We bear a direct connection to recent studies on the aggregate implications of misallocation resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008) and Bartelsman et al. (2013). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013), who study a model with financial frictions and Asker et al. (2012), who introduce capital adjustment costs. Our focus on the role of imperfect information parallels that of Jovanovic (2013), who studies an overlapping generating model where informational frictions impede the efficient matching of entrepreneurs and workers.

Our modeling of financial markets follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). We rely particularly on recent work by Albagli et al. (2011b) for our specific modeling structure. Our contribution here is in placing information aggregation through financial markets into a more standard macroeconomic framework and focusing on the implications for aggregate outcomes.

Finally, the informational role of stock markets has been the focus of a large body of work in empirical finance. One strand of this literature focuses on measuring the information content of stock prices. Durnev et al. (2003) show that firm-specific variation in stock returns (i.e., so-called price-nonsynchronicity) is useful in forecasting future earnings and Morck et al. (2000) find that this measure of price informativeness is higher in richer countries. A related

 $<sup>^{2}</sup>$ It is worth emphasizing that our findings do not rule out other channels through which well-functioning stock markets can play an important role in the real economy - e.g. to provide better incentives to managers, takeover risk or in the IPO market. Our point is that the information they provide to decision makers within firms does not appear to be very significant.

body of work closer to our own and recently surveyed by Bond et al. (2012) looks at the feedback from stock prices to investment and other decisions. Chen et al. (2007), Luo (2005) and Bakke and Whited (2010) are examples of work that find evidence of managers learning from markets while making investment decisions. Bai et al. (2013) combines a simple investment model with a noisy rational expectations framework to assess whether US stock markets have become more informative over time. Our analysis complements these papers by quantifying the macroeconomic implications of this so-called "feedback effect" within a fully specified general equilibrium model. Our results - that stock market informativeness has a negligible effect on aggregate allocative efficiency - are similar in spirit to the conclusions reached by Morck et al. (1990).

The remainder of the paper is organized as follows. Section 2 describes our model of production and financial market activity under imperfect information. Section 3 outlines our calibration strategy and presents our quantitative results, while we summarize our findings and discuss directions for future research in Section 4. Details of our derivations and data work are provided in the Appendix.

### 2 The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy and derive sharp relationships linking the extent of uncertainty at the micro level to aggregate outcomes. Next, we flesh out the information environment and in particular, lay out a fully specified financial market, where dispersed private information of investors and noise trading interact to generate imperfectly informative price signals.

#### 2.1 Production

We consider an infinite-horizon economy set in discrete time. The economy is populated by a representative large family endowed with a fixed quantity of labor that is supplied inelastically to firms. The aggregate labor endowment is denoted by N. The household has preferences over consumption of a final good and accumulates capital that is then rented to firms. We purposely keep households simple as they play a limited role in our analysis.

**Technology.** A continuum of firms, indexed by i, produce intermediate goods using capital and labor according to:

$$Y_{it} = K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}, \qquad \hat{\alpha}_1 + \hat{\alpha}_2 \le 1$$
(1)

Intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left(\int A_{it} Y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

where  $A_{it}$  is the idiosyncratic quality or productivity component of product *i* and represents the only source of uncertainty in this economy (i.e., we abstract from aggregate risk). We assume that  $A_{it}$  follows an AR(1) process in logs:

$$a_{it} = (1 - \rho) \overline{a} + \rho a_{it-1} + \mu_{it}, \qquad \mu_{it} \sim \mathcal{N} \left( 0, \sigma_{\mu}^2 \right)$$

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g.,  $a_{it} = \log A_{it}$ . In this specification,  $\bar{a}$  represents the unconditional mean of  $a_{it}$ ,  $\rho$  the persistence, and  $\mu$  an i.i.d. innovation with variance  $\sigma_{\mu}^2$ .

Market structure and revenue. The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good i

$$Y_{it} = P_{it}^{-\theta} A_{it} Y_t \qquad \Rightarrow \quad P_{it} = \left(\frac{Y_{it}}{Y_t}\right)^{\frac{-1}{\theta}} A_{it}$$

where  $P_{it}$  denotes the relative price of good i in terms of the final good, which serves as numeraire.

The elasticity of substitution  $\theta$  indexes the market power of intermediate goods producers. Our specification nests various market structures. In the limiting case of  $\theta = \infty$ , we have perfect competition, i.e. all firms produce a homogeneous intermediate good. In this case, the survival of heterogenous firms requires decreasing returns to scale in production to limit firm size, that is  $\hat{\alpha}_1 + \hat{\alpha}_2 < 1$ . When  $\theta < \infty$ , we have monopolistic competition, with constant or decreasing returns to scale. No matter the assumption here, firm revenue can be expressed as

$$P_{it}Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} M_{it}^{\alpha_3}$$

$$\tag{2}$$

where  $\alpha_j \equiv \left(1 - \frac{1}{\theta}\right) \widehat{\alpha}_j$ .

Our framework accommodates two alternative interpretations of the idiosyncratic component  $A_{it}$ , either as a firm-specific level of demand or productive efficiency. The analysis is identical under both interpretations, though one could argue that learning from markets is likely more plausible for demand-side factors. In any event, neither the theory nor our empirical strategy requires us to differentiate between the two, so we will generally refer to  $A_{it}$  simply as a firm-specific fundamental. **Factor markets.** In our theory, the key decision affected by imperfect information is the firm's choice of productive inputs, that is capital and labor, which is an otherwise frictionless static decision. Specifically, firms rent capital and/or hire labor without full knowledge of the fundamental  $A_{it}$ . Needless to say, the impact of the friction will vary depending on whether it affects both inputs or just one.<sup>3</sup> Rather than take a particular stand on this important issue regarding the fundamental nature of the production process, we present results for two cases: in case 1, both factors of production are chosen simultaneously under the same (imperfect) information set; in case 2, only capital is chosen under imperfect information, whereas labor is freely adjusted after the firm perfectly learns the current state.

*Case 1: All factors chosen under the same information set.* In this case, the firm's profitmaximization problem is :

$$\max_{K_{it},N_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left[ A_{it} \right] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it} \tag{3}$$

where  $\mathbb{E}_{it}[A_{it}]$  denotes the firm's expectation of fundamentals conditional on its information set  $\mathcal{I}_{it}$ . Standard optimality and market clearing conditions imply

$$\frac{N_{it}}{K_{it}} = \frac{\alpha_2 R}{\alpha_1 W} = \frac{N}{K_t}$$

i.e., capital-labor ratios are constant across firms.

In our empirical strategy, we focus on second moments of firm-level investment data as a way to put discipline on the information structure. With this mind, we make use the optimality conditions characterized above and express the firm's expected revenues purely as a function of its capital input and the relevant aggregates:

$$Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left[ A_{it} \right] K_{it}^{\alpha_1} N_{it}^{\alpha_2} = \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left[ A_{it} \right] K_{it}^{\alpha_2}$$

where

$$\alpha = \alpha_1 + \alpha_2 = \left(\widehat{\alpha}_1 + \widehat{\alpha}_2\right) \left(\frac{\theta - 1}{\theta}\right)$$

Thus, the firm's expected revenues depend only on the aggregate capital-labor ratio, its (conditional) expectation of  $A_{it}$  and the chosen level of its capital input. The curvature param-

 $<sup>^{3}</sup>$ With a more general production function, the impact will also depend on the substitutability of the inputs. Intuitively, the impact is lower the greater the substitutability between inputs that can be perfectly adjusted to current conditions and those that are chosen under imperfect information.

eter  $\alpha$  depends both on the returns to scale in production as well as on elasticity of demand and will play an important role in our quantitative analysis.

Case 2: Only capital chosen under imperfect information. The firm's problem now is

$$\max_{K_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left( \max_{N_{it}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} \right) - R_t K_{it}$$

and maximizing over  $N_{it}$ , we can rewrite the firm's capital choice problem as

$$\max_{K_{it}} \quad (1 - \alpha_2) \left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left(\tilde{A}_{it}\right) K_{it}^{\tilde{\alpha}} - R_t K_{it}$$

where  $\tilde{A}_{it} \equiv A_{it}^{\frac{1}{1-\alpha_2}}$  and

$$\tilde{\alpha} \equiv \frac{\alpha_1}{1 - \alpha_2}$$

Thus, the firm's capital choice problem here has the same structure as in case 1, but with a transformation of the fundamental shock and the curvature parameter. This will make the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different.

To complete our characterization of the firm's problem and therefore, of the full productionside equilibrium in the economy, it remains to explicitly spell out the information set  $\mathcal{I}_{it}$ . We defer this discussion to the following section and for now, directly make conjectures about firm beliefs, which we will later verify under the specific information structure we put forth. In particular, we assume the conditional distribution of the fundamental to be log-normal

$$a_{it}|\mathcal{I}_{it} \sim \mathcal{N}\left(\mathbb{E}_{it}\left[a_{it}\right], \mathbb{V}\right)$$

with  $\mathbb{E}_{it}[a_{it}]$  and  $\mathbb{V}$  denoting, respectively, the posterior mean and variance of  $a_{it}$ . The variance  $\mathbb{V}$  indexes the severity of informational frictions in our economy and will turn out to be a sufficient statistic for misallocation and the associated productivity/output losses. It is straightforward to show that  $\mathbb{V}$  is closely related to commonly used measures of allocative efficiency. For example, it maps exactly into the dispersion of the marginal product of capital (measured along the lines of Hsieh and Klenow (2009)), i.e.,  $\sigma_{MPK}^2 = \mathbb{V}$ . Similarly, it has a negative effect on the covariance between fundamentals and firm activity (as in Bartelsman et al. (2013) and Olley and Pakes (1996)), for example, it is straightforward to show that the covariance between firm fundamentals and capital satisfies  $\sigma_{ak} = \frac{\sigma_a^2 - \mathbb{V}}{1-\alpha}$ .

Further, we will show that the cross-sectional distribution of the posterior mean  $\mathbb{E}_{it}[a_{it}]$  is

also normal, centered around the true mean,  $\bar{a}$  with associated variance  $\sigma_a^2 - \mathbb{V}$ . Similarly, under case 2,

$$\tilde{a}_{it}|\mathcal{I}_{it} \sim \mathcal{N}\left(\mathbb{E}_{it}\left[\tilde{a}_{it}\right], \tilde{\mathbb{V}}\right)$$

**Aggregation.** We now turn to the aggregate economy, and in particular, measures of total factor productivity and output. Given our focus on misallocation, we abstract from aggregate risk and restrict attention to steady states, where all aggregate variables are constant. To begin, we show in the Appendix that aggregate output has the following simple representation:

$$\log Y = a + \hat{\alpha}_1 \log K + \hat{\alpha}_2 \log N \tag{4}$$

The endogenously determined aggregate TFP measure  $a_t$  is given by

Case 1: 
$$a = a^* - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\mathbb{V}}{1 - \alpha} \alpha$$
(5)

Case 2: 
$$a = a^* - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\mathbb{V}}{1 - \alpha} \alpha_1 \left( 1 - \alpha_2 \right)$$
(6)

where  $a^*$  is aggregate TFP under full information.<sup>4</sup> These equations reveal a sharp connection between the micro-level uncertainty captured by  $\mathbb{V}$  and aggregate productivity. Directly, in case 1,

$$\frac{da}{d\mathbb{V}} = -\frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \left( \frac{\alpha}{1 - \alpha} \right) < 0 \tag{7}$$

i.e., aggregate productivity monotonically decreases with  $\mathbb{V}$ , with the magnitude of the effect depending on the aggregate curvature parameters. The higher is  $\alpha$ , that is, the closer to the constant returns/perfect competition, the more severe are the losses from misallocated resources. Similarly, in case 2,

$$\frac{da}{d\tilde{\mathbb{V}}} = -\frac{1}{2} \left(\frac{\theta}{\theta - 1}\right) \left(\frac{1}{1 - \alpha}\right) \alpha_1 \left(1 - \alpha_2\right)$$

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity  $a_t$  is also the effect on aggregate output  $y_t$ . However, the aggregate capital stock is not invariant to the severity of the information friction. Uncertainty has two effects on capital demand - at the micro level, it induces firms to hold more capital. To see this note that  $\mathbb{E}_{it}[A_{it}] = \exp\left(\mathbb{E}_{it}[a_{it}] + \frac{1}{2}\mathbb{V}\right)$  is increasing in  $\mathbb{V}$ . But, there is a stronger offsetting effect working through general equilibrium forces - uncertainty and the resulting misallocation drives down the marginal product of labor and therefore the aggregate capital stock (recall that the

<sup>&</sup>lt;sup>4</sup>Note that  $a^*$  is the same in both cases - under full information, timing of input choices is irrelevant.

aggregate capital-labor ratio in steady state is decreasing in the wage).

Incorporating this additional effect of uncertainty amplifies the effect of allocative inefficiencies and leads to the following simple relationship, which we derive in detail in the Appendix:

$$\frac{dy}{d\mathbb{V}} = \frac{da}{d\mathbb{V}} \left(\frac{1}{1-\hat{\alpha}_1}\right) \tag{8}$$

$$\frac{dy}{d\tilde{\mathbb{V}}} = \frac{da}{d\tilde{\mathbb{V}}} \left(\frac{1}{1-\hat{\alpha}_1}\right) \tag{9}$$

that is, the impact of informational frictions on output are subject to a "multiplier" that depends only on the capital share in the production function<sup>5</sup>.

Thus far, we have derived a sharp relationship between the severity of informational frictions, the degree of misallocation, and aggregate productivity and output. Uncertainty at the firm level is summarized by  $\mathbb{V}$  (or, in case 2, by  $\tilde{\mathbb{V}}$ ), i.e., the variance of the firm's posterior beliefs. We now make explicit the information structure in the economy, that is, the elements of the firm's information set  $\mathcal{I}_{it}$ , which in turn will allow us to characterize the extent of uncertainty in terms of the primitives of the economy - specifically, the variances of shocks and signal noises.

#### 2.2 Information

The firm's information set  $\mathcal{I}_{it}$  is comprised of three elements. First, the firm observes the entire history of its fundamental shock realizations, i.e.,  $\{a_{it-s}\}_{s=1}^{\infty}$ . Given the AR(1) specification, only the most recent realization,  $a_{it-1}$  is relevant for the firm's forecasting problem. Second, each firm also observes a noisy private signal of its contemporaneous fundamental

$$s_{it} = a_{it} + e_{it}, \qquad e_{it} \sim \mathcal{N}\left(0, \sigma_e^2\right)$$

where  $e_{it}$  is an i.i.d. mean-zero and normally distributed noise term. The third and last element of the information available to the firm is the price of its own stock. The final piece of our theory then is to outline how this price  $p_{it}$  is determined and characterize its informational content.

The stock market Our formulation of the stock market and its informational properties follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). For specific modeling assumptions, we draw heavily from recent work by Albagli et al. (2011a) and Albagli et al. (2011b). For each firm i, there is a unit measure of outstanding stock or

<sup>&</sup>lt;sup>5</sup>This is analogous to the total effect on steady state output from a change in TFP in a standard neoclassical model with fixed labor supply.

equity, representing a claim on the firm's profits. These claims are traded by two groups of traders - investors and noise traders.<sup>6</sup> As we will see, the presence of these noise traders will prevent prices from perfectly aggregating investors' information.

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of equity in firm *i* at the current market price  $p_{it}$ . The market is also populated by noise traders, who purchase a random quantity  $\Phi(z_{it})$  of stock *i* each period, where  $z_{it} \sim \mathcal{N}(0, \sigma_z^2)$  and  $\Phi$  denotes the standard normal CDF.

Like firms, investors also observe the entire history, that is, they know  $a_{it-1}$  at time t. They also see the current stock price  $p_{it}$ . Finally, each investor j is endowed with a noisy private signal about the firm's contemporaneous fundamental  $a_{it}$ 

$$s_{ijt} = a_{it} + v_{ijt}, \qquad v_{ijt} \sim \mathcal{N}\left(0, \sigma_v^2\right)$$

The total demand of informed traders for stock i is given by

$$D(a_{it-1}, a_{it}, p_{it}) = \int d(a_{it-1}, s_{ijt}, p_{it}) dF(s_{ijt}|a_{it})$$

where  $d(a_{it-1}, s_{ijt}, p_{it}) \in [0, 1]$  is the demand of investor j and F is the conditional distribution of investors' private signals. The expected payoff to investor j from purchasing the stock is given by

$$\mathbb{E}_{ijt}\left[\Pi_{it}\right] \equiv \int \left[\pi\left(a_{it-1}, a_{it}, p_{it}\right) + \beta \overline{p}\left(a_{it}\right)\right] dH\left(a_{it}|a_{it-1}, s_{ijt}, p_{it}\right)$$

where  $H(a_{it}|a_{it-1}, s_{ijt}, p_{it})$  is the investor's posterior belief over  $a_{it}$  and  $\overline{p}(a_{it})$  is the expected future (period t + 1) stock price of a firm with realized fundamental  $a_{it}$ , defined by

$$\overline{p}(a_{it}) = \int p(a_{it}, a_{it+1}, z_{it+1}) \, dG(a_{it+1}, z_{it+1}|a_{it})$$

where G is the joint distribution of  $(a_{it+1}, z_{it+1})$ , conditional on  $a_{it}$ . The term  $\pi$   $(a_{it-1}, a_{it}, p_{it})$  denotes the expected profit of the firm which is a function of the elements of the firm's information set that are observable to investors, namely  $a_{it-1}$  and  $p_{it}$ . Clearly, optimality implies:

$$d(a_{it-1}, s_{ijt}, p_{it}) = \begin{cases} 1 & \text{if } \mathbb{E}_{ijt} [\Pi_{it}] > p_{it} \\ \in [0, 1] & \text{if } \mathbb{E}_{ijt} [\Pi_{it}] = p_{it} \\ 0 & \text{if } \mathbb{E}_{ijt} [\Pi_{it}] < p_{it} \end{cases}$$

<sup>&</sup>lt;sup>6</sup>One interpretation of these financial market participants are intermediaries investing on behalf of households. The details of such intermediation are not particularly crucial for our analysis, so we do not discuss them in greater detail here.

We begin by conjecturing that both  $p(\cdot)$  and  $\pi(\cdot)$  are monotonically increasing in  $a_{it}$ . Combined with the fact investors' posterior beliefs are first-order stochastically increasing in  $s_{ijt}$ , we can then show that the trading decisions of informed investors are characterized by a threshold rule - in equilibrium, there is a signal  $\hat{s}_{it}$  such that only investors observing signals higher than  $\hat{s}_{it}$  choose to buy.<sup>7</sup> Aggregating the demand decisions of all investors, market clearing implies

$$1 - \Phi\left(\frac{\widehat{s}_{it} - a_{it}}{\sigma_v}\right) + \Phi\left(z_{it}\right) = 1$$

which leads to a simple characterization of the threshold signal

$$\widehat{s}_{it} = a_{it} + \sigma_v z_{it}$$

Next, note that the marginal investor, i.e. the one with the  $s_{ijt} = \hat{s}_{it}$  is, by construction, exactly indifferent between buying and not buying. It then follows that the price,  $p_{it}$ , must equal her expected payoff from holding the stock, i.e.

$$p_{it} = \int \left[ \pi \left( a_{it-1}, a_{it}, p_{it} \right) + \beta \overline{p} \left( a_{it} \right) \right] dH \left( a_{it} | a_{it-1}, \widehat{s}_{it}, p_{it} \right)$$

Since *H* is first-order stochastically increasing in  $\hat{s}_{it}$ , the above relationship translates into a monotonic relationship between  $p_{it}$  and  $\hat{s}_{it}$ , which implies that observing the price is informationally equivalent to observing  $\hat{s}_{it}$ . To put it differently, the price serves as an additional noisy signal of firm fundamentals. The precision of this signal is  $\frac{1}{\sigma_v^2 \sigma_z^2}$ , i.e. it is decreasing both in the variance of the noise in investors' private information and the size of the noise trader shock.

This means that we have a complete characterization of the firm's information set<sup>8</sup> and hence the posterior variance  $\mathbb{V}$  even without a explicit solution for the price function. Formally,  $\mathcal{I}_{it} = (a_{it-1}, s_{it}, \hat{s}_{it})$ , which in turn yields a simple expression for  $\mathbb{V}$ :

$$\mathbb{V} = \left(\frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_v^2}\right)^{-1} \tag{10}$$

We will need the price function for numerical analysis, however. With a slight abuse of notation, we can express the indifference condition of the marginal investor in recursive form,

<sup>&</sup>lt;sup>7</sup>See Albagli et al. (2011a) for more details.

<sup>&</sup>lt;sup>8</sup>It is straightforward to show that conditional and cross-sectional distributions will be log-normal under this information set, exactly as conjectured.

yielding a fixed-point characterization of the price function

$$p(a_{-1}, a, z) = \int \pi (a_{-1}, a, a + \sigma_v z) dH(a|a_{-1}, a + \sigma_v z, a + \sigma_v z) +\beta \int \left[ \int p(a_{-1}, a, z) dG(a, z|a_{-1}) \right] dH(a|a_{-1}, a + \sigma_v z, a + \sigma_v z)$$

Given the profit function  $\pi$  (which we know from the production side), this problem can be solved using a standard iterative procedure.

# 3 Quantitative Analysis

In this section, we use the theory laid out in the previous sections to quantify the extent of informational frictions (and the resulting misallocation) across a number of countries. Our results also shed light on the informativeness of financial markets and their role in improving allocative efficiency through this channel.

#### 3.1 Calibration

A key hurdle in assessing the quantitative effects of imperfect information is imposing discipline on the information structure, given that we do not directly observe signals at the firm level. A novel feature of our analysis is a calibration strategy that uses moments of both firm production and stock market data to pin down the key informational parameters of our model. The presence of learning from markets is key to this strategy.

Before discussing this strategy and the resulting estimates in detail, we begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we will maintain the assumption that these are constant across countries - i.e. the only differences across countries will come from the parameters governing the stochastic processes on firm fundamentals and learning. Obviously, in practice, country-specific differences in technologies/preferences do play a role, but this approach is a natural starting point and allows us to focus on the role of imperfect information.

An important issue here is the choice of a period length. We argue that the right application of our analysis is to longer run decisions, which points to a longer period length. For one, we model input choice as a simple static decision, primarily to preserve analytical tractability on the production side. In doing so, we abstract from a number of features - e.g. adjustment lags/costs and irreversibility - that are probably very relevant for investment decisions. A longer time period partly mitigates the concerns raised by these omissions<sup>9</sup>. Moreover, it seems reasonable to conjecture that planning horizons for major investment decisions are relatively long. In light of this, we set the period length in our model to 3 years.

In accordance with the period length choice, we set the discount rate  $\beta$  to 0.90. We assume constant returns to scale in production. Firm scale is then limited by the curvature in demand, captured by the elasticity of substitution  $\theta$ . This will be a key parameter in governing the quantitative impact of the informational friction. The relevant literature contains a wide range of estimates for this parameter. In light of this, we report our results for two values of  $\theta$ : 4 and 10. Under case 1, in which all inputs are chosen under imperfect information, it is not necessary to calibrate the individual production parameters

 $hat\alpha_1$  and  $\hat{\alpha}_2$ , as only the aggregate returns to scale plays a role. In contrast, under case 2, in which only capital is subject to the informational friction, the relative magnitudes of these parameters is important, and we set them to standard values of  $\hat{\alpha}_1 = 0.33$  and  $\hat{\alpha}_2 = 0.67$ .

Next, we turn to the country-specific parameters and begin with those governing the process on firm fundamentals  $a_{it}$ , that is, the persistence  $\rho$  and variance of the innovations  $\sigma_{\mu}^2$ . We estimate these directly by first constructing a series of this object for each firm as  $a_{it} = rev_{it} - \alpha k_{it}$ . We then regress  $a_{it}$  on  $a_{it-1}$  as well as a time fixed-effect to isolate the idiosyncratic component of the innovations. The coefficient from this autoregression delivers  $\rho$  and the variance of the residuals  $\sigma_{\mu}^2$ . We report our results in Table 4 below.

Finally, it remains to pin down the three informational parameters, i.e., the variances of the error terms in firm and investor signals  $\sigma_e^2$  and  $\sigma_v^2$ , and the variance of the noise trader shock  $\sigma_z^2$ . The moments we target here are the correlations of investment growth and revenue growth with stock returns (lagged by one period), and the variance of stock returns.<sup>10</sup> Formally, we search over the parameter space to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the targets. The results of this procedure are summarized in Table 1.

The combination of firm-level production and financial market data is a key feature of our approach and it makes sense here to give a clear sense of how our model parameters map into these moments. Of course, the parameters are calibrated jointly and so there is not a one-to-one mapping between moments and parameters, but it is possible to make a heuristic argument in order to gain some intuition about the connection between specific moments and parameters.

<sup>&</sup>lt;sup>9</sup>Morck et al. (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g. 1 year) are quite low.

<sup>&</sup>lt;sup>10</sup>We lag returns by one period to avoid feedback from investment and sales to stock returns, the reverse of the relationship in which we are interested. We follow Morck et al. (1990) by focusing on changes in variables, i.e., sales and investment growth, rather than levels, in an effort to cleanse the data of firm fixed-effects, which tend to be the dominant influence in cross-sectional differences in these variable across firms.

Parameter	Description	Target/Value
Common		
	Time period	3 years
eta	Discount rate	0.90
$\hat{lpha}_1$	Capital share	0.33
$\hat{lpha}_2$	Labor share	0.67
heta	Elasticity of substitution	4, 10
Country-specific		
ho	Persistence of fundamentals	Estimates of
$\sigma_{\mu}$	Shocks to fundamentals	$\begin{cases} \text{Estimates of} \\ a_{it} = (1-\rho)\overline{a} + \rho a_{it-1} + \mu_{it} \end{cases}$
$\sigma_{e}$	Firm private info	) $\sigma(\Delta p)$
$\sigma_{z}$	Noise trading	$ \begin{cases} \sigma \left( \Delta p \right) \\ \rho \left( \Delta i, \Delta p \right) \\ \rho \left( \Delta y, \Delta p \right) \end{cases} $
$\sigma_{ u}$	Investor private info	$\int \rho\left(\Delta y, \Delta p\right)$

 Table 1: Calibration Summary

To illustrate this connection, Table 2 reports these moments from the US data (described in more detail below) and the implied values from various informational scenarios from our model, as well as the the resulting implications for the extent of micro-level uncertainty in the economy.<sup>11</sup> For example, the data show the standard deviation of stock returns (denoted by  $\Delta p$  in the table) of 0.53 in the US and the correlations of returns with investment growth and revenue growth to be 0.26 and 0.29 respectively.<sup>12</sup> The full information version of our model would imply substantially lower return volatility and much higher correlations, with the one between revenue growth and returns is approaching 1. The next row shows the implied moments when firms have full information, but the market is  $noisy^{13}$ . The volatility of returns increases to within a close range of its value in the data, pointing to the connection between market noisiness and volatility. Perhaps more strikingly, the correlations of returns with both investment and revenue growth fall drastically, the former actually becoming slightly negative, a pattern to which we return in a moment. Finally, the last row shows the moments when firms have no private information and markets are noisy, such that all learning takes place from imperfectly revealing market prices. In this case, while the variance of returns changes slightly, the correlations fall close to the midpoint of their values in the first two cases. Intuitively, the last two cases hold fixed the noisiness of markets and vary the precision of the firm's private

<sup>&</sup>lt;sup>11</sup>For illustration purposes, we display results where from case 2 where we have set  $\theta = 4$  and hence  $\tilde{\alpha} = 0.50$ . <sup>12</sup>The standard deviation of returns is subject to an adjustment for financial leverage, discussed in more detail below.

<sup>&</sup>lt;sup>13</sup>More precisely, we fix the noise in investor's private signal and the noise trading shock at our baseline estimates.

information. When the firm has full information, it relies not at all on the market, and these correlations fall very close to zero. In the opposite extreme, when the firm has no information, it relies heavily on the market, and these correlations rise. In sum then, we can intuitively map the volatility of returns into market noise, and correlations of returns with firm investment decisions and revenue growth into the quality of firm private information<sup>14</sup>.

Notice the key role that learning from prices plays in our empirical approach. It is precisely the strength of the firm's response to this publicly observed signal that allows us to infer the precision of the firm's private signal. In the full info information benchmark, this correlation does not mean much - it is high irrespective of whether firms learn the true state of the world from their own sources or through market signals. When firms are uncertain, however, a strong response to the price signals indicates the poor quality of the other information available to the firm; conversely, a weak response to this signal by the firm indicates that the firm does not have much to learn from this source - precisely because its own private information is very good.

				20101101 800110
		$\sigma\left(\Delta p\right)$	$\rho\left(\Delta i,\Delta p\right)$	$\rho\left(\Delta y,\Delta p\right)$
US Data		0.28	0.26	0.29
Firm Info	Price Info			
Full	Full	0.11	0.63	0.98
Full	Noisy	0.25	-0.03	0.13
None	Noisy	0.24	0.39	0.36

 Table 2: Implied Moments from Various Informational Scenarios

The intuition for our identification strategy is illustrated in Figure 1. Each panel plots a moment of interest against changes in each of the informational parameters, holding everything else constant. The x-axis in each graph is the change in the parameter of interest, normalized so that 1 represents the estimated value in the baseline US calibration. For example, in the first panel, the blue (solid) line shows that halving the standard deviation of the noise in the firm's private signal ( $\sigma_e$ ), holding the other two constant, reduces the correlation between lagged returns and investment growth from 0.30 to a little over 0.10. The other two panels reveal that this change also lowers the correlation of returns with revenue growth (from 0.3 to 0.25), but has very little effect on the variability of returns. The other two parameters ( $\sigma_v, \sigma_z$ ) have similar (negative) effects on the correlation of returns with revenues and investment growth, but the noise trader shock  $\sigma_z$  has a much more pronounced effect on return variability.

<sup>&</sup>lt;sup>14</sup>One possible criticism of this approach is that the returns and investment might be correlated for reasons other than learning - e.g. managerial incentive contracts. To address this concern, we evaluated the robustness of our quantitative findings to significant changes in the correlation targets. Obviously, the parameter estimates change with the targets, but our overall conclusions about the severity of informational frictions and the role of financial markets are unaffected.

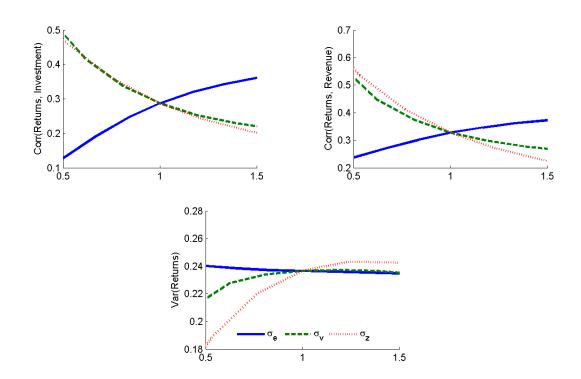


Figure 1: Effect of Variance Parameters

#### 3.2 Data and Targets

We use data on firm-level production variables, i.e., firm revenues, capital stock, and investment, as well as stock returns from Compustat North America for the US and Compustat Global for China and India. Investment is computed as the change in the firm's capital stock, measured by its property, plant and equipment (PP&E). Stock returns are constructed as the change in the firm's stock price adjusted for splits and dividend distributions. In order to be comparable to the (unlevered) returns in our model, stock returns in the data need to be adjusted for financial leverage. To account for this, we assume that claims to firm profits are sold to investors in the form of debt and equity claims in a constant proportion. This then implies that observed return variances have to be divided by a constant factor to make them comparable to the model. The factor we use is 2, which corresponds to assuming a debt-equity ratio of approximately 2:1 (the figures we report below reflect this adjustment). To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period. Further details of how we select the data and construct these variables are described in the Appendix. We report the target moments across countries for our internally calibrated parameters in Table 3. The moments exhibit significant cross-country variation, which as we will see, translate to differences in the severity of informational frictions and the extent of misallocation.

Table 3: Calibration Targets								
Country	$\sigma\left(\Delta p\right)$	$\rho\left(\Delta i,\Delta p\right)$	$\rho\left(\Delta y, \Delta p\right)$					
US	0.28	0.26	0.29					
China	0.13	0.12	0.14					
India	0.28	0.21	0.12					

#### 3.3 Results

We begin by presenting results for case 2, in which only capital is subject to the informational friction. This is the more conservative scenario and may be closer to reality than case 1, particularly given our 3-year planning horizon. Table 4 presents our parameter estimates, the implied value for  $\mathbb{V}$ , both in absolute terms and as a percentage of the underlying uncertainty  $\sigma_{\mu}^2$  and dispersion in fundamentals  $\sigma_a^2$ , as well as the implied aggregate TFP and output losses relative to the full information benchmark. As mentioned above, we report results for two values of the elasticity of substitution  $\theta$  (namely, 4 and 10) and note that our results depend quantitatively to a great extent on this value. As we would expect given the cross-country variation in the targets listed in Table 3, the parameter estimates also vary markedly across countries. The US has smaller fundamental shocks and lower levels of noise trading. Although US firms are better informed than their Chinese and Indian counterparts, there is a substantial degree of uncertainty across the board. Firm-level information eliminates at most about 30% of the uncertainty about contemporaneous innovations to fundamentals.<sup>15</sup> The resulting productivity and output losses are substantial: the former range from 4% to 11% in the US and from 7%to 16% in India and China; the latter are even higher, ranging from 6% to 17% in the US and from about 10% to 24% in China and India. Recall that productivity losses stem solely from the misallocation of resources while output losses take into account the additional effect of such "static" misallocation on the level of the aggregate capital stock. Finally, these results suggest that the impact of differences in the severity of informational frictions across countries are significant, ranging from 3% to 5% for productivity and about 5% to 7% for output (subtract  $a^* - a$  and  $y^* - y$  for the US from the corresponding values for China and India), although these are modest in comparison to standard measures of differences in TFP and income per-capita.

<sup>&</sup>lt;sup>15</sup>However, because  $a_{it-1}$  is perfectly known, uncertainty about total fundamentals is much lower.

				<u>able 4</u>	: <u>Kesun</u>	<u>.s - Ca</u>	<u>se z</u>			
	ρ	$\sigma_{\mu}$	$\sigma_e$	$\sigma_{\nu}$	$\sigma_z$	$ $ $\tilde{\mathbb{V}}$	$\frac{\tilde{\mathbb{V}}}{\sigma_{\mu}^2}$	$\frac{\tilde{\mathbb{V}}}{\sigma_a^2}$	$a^*-a$	$y^* - y$
$\theta = 4$										
US	0.91	0.42	0.63	0.47	3.93	0.12	0.67	0.07	0.04	0.06
China	0.90	0.52	1.02	0.23	18.57	0.21	0.78	0.09	0.07	0.10
India	0.87	0.51	1.23	0.78	6.79	0.22	0.85	0.07	0.07	0.11
$\theta = 10$										
US	0.91	0.42	2.17	0.62	5.49	0.17	0.95	0.08	0.11	0.17
China	0.90	0.52	2.76	0.69	11.49	0.26	0.96	0.18	0.16	0.24
India	0.87	0.51	6.05	1.08	9.37	0.26	0.99	0.09	0.16	0.24

Table 4: Results - Case 2

Notice that we have tied our hands to a great extent in case 2 by assuming that only capital is subject to the information friction. Table 5 reports our results from case 1, in which both capital and labor are subject to the friction. It is likely that the truth lies somewhere between the two, and so we can think of the two scenarios as providing some bounds on the effect of these frictions on aggregate performance. The general patterns in fundamentals and uncertainty pointed out in case 2 continue to hold in case 1, but the quantitative impact of informational frictions is now much larger. Productivity and output losses compared to the full information benchmark range between 34% and 130% for the former and between 51% and almost 200% for the latter. Additionally, differences in the severity of informational frictions account for large disparities in TFP and income across countries, ranging from 18% to 45% for the former and 27% to 67% for the latter.

Under either scenario then, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the calibrated economies to their full information benchmarks and when comparing the effects of such frictions in emerging markets compared to the US. The quantitative magnitude of these effects depend to a great extent on the nature of the firm's input decisions, that is, are all inputs chosen subject to the friction or only capital. Although we do not take a firm stand on this issue, our belief is that the truth likely lies in between the two extreme cases we have examined here, putting bounds, although clearly quite wide ones, on the detrimental effects of imperfect information. It would be fruitful for future work to shed light on which of these cases is empirically more relevant in order to pin down more accurately the ramifications of the friction we propose.

#### 3.3.1 Decomposing $\mathbb{V}$

The role of financial markets: To explore the importance of stock prices in delivering new information to the firm, we recompute firm-level uncertainty, i.e.,  $\mathbb{V}$ , under the assumption

			,	<u>able 5</u>	<u>: Result</u>	<u>.s - Ca</u>	se 1			
	ρ	$\sigma_{\mu}$	$\sigma_e$	$\sigma_{\nu}$	$\sigma_z$	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_{\mu}^2}$	$\frac{\mathbb{V}}{\sigma_a^2}$	$a^*-a$	$y^* - y$
$\theta = 4$										
US	0.91	0.42	2.17	0.62	5.49	0.17	0.95	0.08	0.34	0.51
China	0.90	0.52	2.76	0.69	11.49	0.26	0.96	0.18	0.52	0.78
India	0.87	0.51	6.05	1.08	9.37	0.26	0.99	0.09	0.52	0.78
$\theta = 10$										
US	0.91	0.42	2.31	0.61	5.69	0.17	0.95	0.08	0.85	1.27
China	0.90	0.52	1.74	0.27	19.29	0.25	0.96	0.18	1.30	1.94
India	0.87	0.51	6.29	0.68	8.97	0.26	0.99	0.09	1.30	1.94

Table 5. Deculta Case

that firms learn nothing from stock prices, i.e., that there is no information in these prices. This simply involves sending  $\sigma_{\nu}^2$  and  $\sigma_z^2$  to infinity, and denoting the change in  $\mathbb{V}$  as  $\Delta \mathbb{V}$ , it is straightforward to see

$$\Delta \mathbb{V} = \left(\frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_e^2}\right)^{-1} - \mathbb{V}$$

where  $\Delta \mathbb{V}$  then measures the contribution of stock market informativeness to firm learning and we report this value in the first column of Table 6, along with the associated output ramifications of this increase in uncertainty.<sup>16</sup> As the table shows, across all countries, there is virtually no difference between the two levels of uncertainty and so the aggregate consequences of financial market informativeness are quite small, on the order of 0.2% for the US, which has the highest degree of investment-return correlation in our sample. Thus, our results suggest that the contribution of the stock market to allocative efficiency through an informational channel is negligible.

It is worth emphasizing this result and placing it in the context of the existing empirical work on this topic. Our model suggests that the contribution of learning from financial markets is extremely small even in the US, despite the fact that the data show a significantly positive correlation between returns and investment growth, the same relationship studied by much of the empirical work cited in the Introduction. That a high correlation here is not necessarily indicative of a strong learning channel is best highlighted in the full information example from Table 2, in which the correlation was quite high, yet firms were perfectly informed and learned nothing from markets. Now compare this high correlation to the case where firms are perfectly informed but markets are noisy: the correlation drops to zero. In both cases, the firms is learning nothing from markets, but the correlation changes dramatically. Clearly, the correlation

<sup>&</sup>lt;sup>16</sup>The table is computed under case 2 using  $\theta = 4$  and hence  $\alpha = 0.50$ . Examining  $\frac{\mathbb{V}}{\sigma_{\mu}^2}$  in Tables 4 and 5 shows that this case delivers the highest level of learning and so gives markets their best shot at playing a quantitatively significant role.

between returns and investment growth is not a sufficient statistic to capture the importance of this potential learning channel and our structural model has enabled us to quantitatively tease this out.

	No N	Iarket		US Market			
	$\Delta \mathbb{V}$	$\Delta y$		$\Delta \mathbb{V}$	$\Delta y$		
US	0.004	-0.002		*	*		
China	0.002	-0.001	-	-0.010	0.005		
India	0.002	-0.001	-	-0.012	0.006		

Table 6: Financial Market Informativeness

A second way to assess the informational role of financial markets is to look at the gains to India and China from having access to financial markets with a quality of information like that in the US. To answer this, we recompute  $\mathbb{V}$  under the assumption that each country has a  $\sigma_{\nu}^2$  and  $\sigma_z^2$  equal to that in the US, leaving the other country-specific parameters fixed. The results are reported in the last two columns of Table 6 and confirm that even delivering USquality markets to emerging economies would have a small effect on the aggregate economy, with output gains of about 0.5%.

The role of private learning: We have seen that financial markets play a negligible role in improving allocative efficiency through the transmission of information. In this section, we examine the importance of firm learning and its contribution to economic performance. Towards this end, we recompute  $\mathbb{V}$  under the assumption that firms obtain no private signal and only learn from the market, i.e., we send  $\sigma_e^2$  to infinity and calculate the resulting change in  $\mathbb{V}$ . The results are reported in Table 7 and reveal that firm private information plays a significant role in reducing uncertainty and improving aggregate performance, contributing between 2% and 3% to aggregate output. By way of comparison, we report in the two columns titled 'No info' the impact of removing all learning about contemporaneous fundamentals. We note that the overall effect of learning is significant, but is quite similar to that of just private information, a result that is not surprising given the negligible role of market informativeness found above.

Finally, in the last two columns of Table 7, we recompute  $\mathbb{V}$  for China and India under the assumption that firms' private information is as precise as in the US, that is, we assume that firms in India and China have the same  $\sigma_e^2$  as US firms. The results suggest that access to US-quality private information would have a significant effect in these markets, increasing aggregate output by about 3%. Compare this to Table 6, where we showed that giving these firms access to a financial market of US-quality would increase output only about 0.5%. These findings imply that to the extent informational frictions drive cross-country differences in aggregate productivity and output, these disparities are primarily due to a lack of high quality firm

private information, rather than lack of access to a well-functioning (in an informational sense) financial market.

	Table 7. The Role of Learning							
	No Private Info		No	Info	US Fir	US Firm Info		
	$\Delta \mathbb{V}$	$\Delta y$	$\Delta \mathbb{V}$	$\Delta y$	$\Delta \mathbb{V}$	$\Delta y$		
US	0.05	-0.02	0.06	-0.03	*	*		
China	0.05	-0.03	0.06	-0.03	-0.05	0.03		
India	0.04	-0.02	0.04	-0.02	-0.06	0.03		

Table 7: The Role of Learning

The role of shocks to fundamentals: Finally, we also study the role of differences in the fundamental processes in generating the cross-country differences highlighted above. In this last counterfactual exercise then, we recompute  $\mathbb{V}$  in China and India giving them the fundamental shocks that US firms experience, that is, the  $\sigma_{\mu}^2$  of the US. This raises output in both countries by about 3%, suggesting that in addition to disparities in learning, differences in fundamentals also play a significant role in explaining differences across countries<sup>17</sup>.

## 4 Conclusion

The previous sections lay out a theory of informational frictions that distort the allocation of factors across heterogeneous firms. This misallocation of productive resources reduces aggregate productivity and output. A central element of the theoretical framework is an information structure in which firms learn from both private sources and imperfectly revealing stock market prices. This allows us to use observed second moments of returns and firm-level variables to infer the severity of informational frictions.

Our analysis uncovers substantial uncertainty at the micro level, particularly in India and China. More interestingly, it attributes much of the cross-country variation to differences in the size of shocks and the quality of the firms' own information sources. Learning from financial markets seem to contribute little to allocative efficiency, even for the US. This suggests that policies aimed at directly improving firm-level information would be more fruitful than any meant to improve financial market performance.

Our modeling of information is rather abstract, so we are left to speculate on the exact form of such policies. For example, one interpretation of cross-country differences in private information is as the result of better information collection/processing systems within firms

 $<sup>^{17}</sup>$ Asker et al. (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.

and/or the skill of managers.<sup>18</sup> A thorough investigation of these issues is an important direction for future research.

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 $<sup>^{18}</sup>$ This is reminiscent of Bloom et al. (2013), who highlight role of better management practices and/or manager skill in explaining cross-country productivity differences.

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# Appendix

## A Detailed derivations

#### A.1 Case 1: Both factors chosen under imperfect information

Firm's Problem:

$$\max_{K_{it}} \qquad \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left[A_{it}\right] K_{it}^{\alpha} - R_t K_{it}$$

Solution:

$$\alpha \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} K_{it}^{\alpha-1} \mathbb{E}_{it} \left[A_{it}\right] = R_t$$
$$\Rightarrow \left[\alpha \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \frac{\mathbb{E}_{it} \left[A_{it}\right]}{R_t}\right]^{\frac{1}{1-\alpha}} = K_{it}$$

Market clearing:

$$\int K_{it} di = \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \frac{\alpha Y_t^{\frac{1}{\theta}}}{R_t} \right]^{\frac{1}{1-\alpha}} \int \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di = K_t$$

$$\Rightarrow \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \frac{\alpha Y_t^{\frac{1}{\theta}}}{R_t} \right]^{\frac{1}{1-\alpha}} = \frac{K_t}{\int \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di}$$

$$\Rightarrow \qquad K_{it} = \frac{\left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di}{\int \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di} K_t$$

Employment and output:

$$K_{it} = \frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} K_{t}$$

$$P_{it}Y_{it} = \left(\frac{N}{K_{t}}\right)^{\alpha_{2}} Y_{t}^{\frac{1}{\theta}} A_{it} \left(\frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} K_{t}\right)^{\alpha}$$

$$= K_{t}^{\alpha_{1}} N^{\alpha_{2}} Y_{t}^{\frac{1}{\theta}} A_{it} \left(\frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di}\right)^{\alpha}$$

$$Y_{t} = \int P_{it}Y_{it} di$$

$$= K_{t}^{\alpha_{1}} N^{\alpha_{2}} Y_{t}^{\frac{1}{\theta}} \frac{\int A_{it} \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}} di}{\left(\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di\right)^{\alpha}}$$

In logs,

$$y_t = \frac{1}{\theta} \log Y_t + \alpha_1 \log K + \alpha_2 \log N + \log \int A_{it} \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di$$

Note that, under conditional log-normality,

$$a_{it}|\mathcal{I}_{it} \sim \mathcal{N}\left(\mathbb{E}_{it}\left[a_{it}\right], \mathbb{V}\right) \quad \Rightarrow \quad \mathbb{E}_{it}\left[A_{it}\right] = \exp\left(\mathbb{E}_{it}a_{it} + \frac{1}{2}\mathbb{V}\right)$$

The true fundamental  $a_{it}$  and its conditional expectation  $\mathbb{E}_{it}a_{it}$  are also jointly normal, i.e.

$$\begin{bmatrix} a_{it} \\ \mathbb{E}_{it}a_{it} \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} \bar{a} \\ \bar{a} \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_a^2 - \mathbb{V} \\ \sigma_a^2 - \mathbb{V} & \sigma_a^2 - \mathbb{V} \end{bmatrix} \right)$$

Then,

$$\log \int A_{it} \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} = \log \int \exp \left( a_{it} + \frac{\alpha}{1-\alpha} \mathbb{E}_{it} a_{it} + \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V} \right) di$$
$$= \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \sigma_a^2 - \mathbb{V} \right)$$
$$+ \frac{\alpha}{1-\alpha} \left( \sigma_a^2 - \mathbb{V} \right) + \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V}$$
$$= \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \alpha \frac{2-\alpha}{(1-\alpha)^2} \left( \sigma_a^2 - \mathbb{V} \right)$$
$$+ \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V}$$

 $\quad \text{and} \quad$ 

$$\log \int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} = \frac{1}{1-\alpha}\bar{a} + \frac{1}{2}\left(\frac{1}{1-\alpha}\right)^{2}\left(\sigma_{a}^{2} - \mathbb{V}\right) + \frac{1}{2}\frac{1}{1-\alpha}\mathbb{V}$$

Combining,

$$\log \int A_{it} \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left( \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1}{1-\alpha}} di$$
$$= \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1-\alpha} - \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V}$$

Substituting and re-arranging, we obtain the expression in (5) in the text.

$$y_{t} = \frac{1}{\theta} y_{t} + \alpha_{1} \log K_{t} + \alpha_{2} \log N + \bar{a} + \frac{1}{2} \frac{\sigma_{a}^{2}}{1 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V}$$
  

$$\Rightarrow y_{t} = \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_{a}^{2}}{1 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \mathbb{V}$$
  

$$+ \hat{\alpha}_{1} \log K_{t} + \hat{\alpha}_{2} \log N$$
(11)

with

$$a = \underbrace{\frac{\theta}{\theta - 1}\bar{a} + \frac{1}{2}\left(\frac{\theta}{\theta - 1}\right)\frac{\sigma_a^2}{1 - \alpha}}_{a^*} - \frac{1}{2}\frac{\alpha}{1 - \alpha}\left(\frac{\theta}{\theta - 1}\right)\mathbb{V}$$

Next, we endogenize  $K_t$ . The rental rate in steady state

$$R = \frac{1}{\beta} - 1 + \delta$$

Then, from the market clearing conditions, we have

$$\frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \qquad \Rightarrow \qquad K \propto W$$

To characterize wages,

$$\max \ Y^{\frac{1}{\theta}} E_{it} (A_{it}) K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2} - W N_{it} - R K_{it}$$

Maximizing over capital, the objective becomes

$$(1 - \alpha_1) \left\{ \left[ Y^{\frac{1}{\theta}} E_{it} \left( A_{it} \right) N_{it}^{\alpha_2} \right]^{\frac{1}{1 - \alpha_3}} \left( \frac{\alpha_1}{R} \right)^{\alpha_1} \right\}^{\left( \frac{1}{1 - \alpha_1} \right)} - W N_{it}$$
$$= (1 - \alpha_1) \left( \frac{\alpha_1}{R} \right)^{\frac{\alpha_1}{1 - \alpha_1}} \left[ Y^{\frac{1}{\theta}} E_{it} \left( A_{it} \right) N_{it}^{\alpha_2} \right]^{\frac{1}{1 - \alpha_1}} - W N_{it}$$

Optimality and labor market clearing lead to

$$\frac{\alpha_2}{W} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1}} \left[Y_t^{\frac{1}{\theta}} E_{it}\left(A_{it}\right)\right]^{\frac{1}{1-\alpha_1}} = N_{it}^{1-\frac{\alpha_2}{1-\alpha_1}} \left(\frac{\alpha_2}{W}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \int \left[Y_t^{\frac{1}{\theta}} E_{it}\left(A_{it}\right)\right]^{\frac{1}{1-\alpha_1-\alpha_2}} di = N$$

As before, letting  $\alpha = \alpha_1 + \alpha_2$ , we see that

$$W \propto \left( \int [E_{it}A_{it}]^{\frac{1}{1-\alpha}} di \right)^{\frac{1-\alpha}{1-\alpha_1}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_1}}$$
  
= 
$$\left[ \left( \int e^{\left(E_{it}a_{it}+\frac{1}{2}\mathbb{V}\right)\frac{1}{1-\alpha}} di \right)^{1-\alpha} Y^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}} = \left[ e^{\bar{a}+\frac{1}{2}\left(\frac{\sigma_a^2-\mathbb{V}}{1-\alpha}\right)+\frac{1}{2}\mathbb{V}} Y^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}}$$
  
= 
$$\left[ e^{\bar{a}+\frac{1}{2}\left(\frac{\sigma_a^2-\alpha\mathbb{V}}{1-\alpha}\right)} Y^{\frac{1}{\theta}} \right]^{\frac{1}{1-\alpha_1}}$$

or in logs,

$$w = \bar{a} + \frac{1}{1 - \alpha_1} \frac{1}{2} \left( \frac{\sigma_a^2 - \alpha \mathbb{V}}{1 - \alpha} \right) + \frac{1}{\theta} \frac{1}{1 - \alpha_1} y_t + \text{Constants}$$

Recall that

$$K \propto W \qquad \Rightarrow \qquad \qquad \frac{dk}{d\mathbb{V}} = \frac{dw}{d\mathbb{V}}$$

which implies

$$\frac{dy}{d\mathbb{V}} = \hat{\alpha}_1 \left(\frac{dk}{d\mathbb{V}}\right) - \frac{1}{2} \left(\frac{\theta}{\theta - 1}\right) \frac{\alpha}{1 - \alpha} \\ = \frac{\alpha_1}{1 - \hat{\alpha}_1 - \hat{\alpha}_3} \left[ -\frac{1}{2} \frac{\alpha}{1 - \alpha} + \frac{1}{\theta} \frac{dy}{d\mathbb{V}} \right] - \frac{1}{2} \left(\frac{\theta}{\theta - 1}\right) \frac{\alpha}{1 - \alpha}$$

Collecting terms,

$$\begin{aligned} \frac{dy}{d\mathbb{V}} \left[ 1 - \frac{\hat{\alpha}_1}{1 - \alpha_1} \frac{1}{\theta} \right] &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left[ \frac{\hat{\alpha}_1}{1 - \alpha_1} + \frac{\theta}{\theta - 1} \right] \\ \frac{dy}{d\mathbb{V}} \left[ 1 - \frac{\alpha_1}{1 - \alpha_1} \frac{1}{\theta - 1} \right] &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left[ \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha_1}{1 - \alpha_1} + \frac{\theta}{\theta - 1} \right] \\ &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \left( \frac{1}{1 - \alpha_1} \right) \\ \Rightarrow \frac{dy}{d\mathbb{V}} &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \frac{\frac{1}{1 - \alpha_1}}{1 - \frac{\alpha_1}{1 - \alpha_1} \frac{1}{\theta - 1}} \\ &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \frac{1}{1 - \alpha_1 - \frac{\alpha_1}{\theta - 1}} \\ &= -\frac{1}{2} \frac{\alpha}{1 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \frac{1}{1 - \alpha_1} \\ &= \frac{da}{d\mathbb{V}} \frac{1}{1 - \alpha_1} \end{aligned}$$

## A.2 Case 2

The firm's objective,

$$\max_{N_{it}} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}$$
$$\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} = N_{it}^{1-\alpha_2}$$

Optimality

Labor market clearing implies

$$\int \left(\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}} di = N$$
$$\frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di} N = N_{it}$$
$$\frac{\alpha_2}{N^{1-\alpha_2}} Y_t^{\frac{1}{\theta}} \left(\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di\right)^{1-\alpha_2} = W$$

The maximized objective for capital choice then becomes

$$(1 - \alpha_2) \left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} \left(\tilde{A}_{it}\right) K_{it}^{\tilde{\alpha}} - R_t K_{it}$$

Imposing optimality and capital market clearing, we have

$$K_{it} = \frac{\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}}{\int \left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di}K_t$$

Then, the labor input decision becomes

$$N_{it} = \frac{\tilde{A}_{it}K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}di}N = \frac{\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}N$$

Combining,

$$P_{it}Y_{it} = Y_{t}^{\frac{1}{\theta}}A_{it}\left\{\frac{\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}}{\int\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di}K_{t}\right\}^{\alpha_{1}}\left\{\frac{\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}{\int\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}N\right\}^{\alpha_{2}}\right.$$
$$= Y_{t}^{\frac{1}{\theta}}K_{t}^{\alpha_{1}}N^{\alpha_{2}}\frac{A_{it}\tilde{A}_{it}^{\alpha_{2}}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di\right]^{\alpha_{1}}\left\{\int\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right]^{\alpha_{2}}}{\left\{\int\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di\right]^{\alpha_{1}}\left\{\int\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right]^{\alpha_{2}}}\right.$$
$$= Y_{t}^{\frac{1}{\theta}}K_{t}^{\alpha_{1}}N^{\alpha_{2}}\frac{\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di\right]^{\alpha_{1}}\left\{\int\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right]^{\alpha_{2}}}{\left\{\int\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{1}{1-\tilde{\alpha}}}di\right]^{\alpha_{1}}\left\{\int\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di\right\}^{\alpha_{2}}}\right\}$$

Aggregate output is defined as before

$$y \equiv \log Y = \log \int P_{it} Y_{it} \quad di$$

Then,

$$y = \frac{1}{\theta}y + \alpha_1 \log K_t + \alpha_2 \log N + \alpha_1 \log \int \left[ E_{it} \left( \tilde{A}_{it} \right) \right]^{\frac{1}{1-\tilde{\alpha}}} di$$
$$+ (1-\alpha_2) \log \int \tilde{A}_{it} \left[ E_{it} \left( \tilde{A}_{it} \right) \right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di$$

Again, we exploit log-normality to obtain

$$\log \int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} = \log \int \exp \left( \tilde{a}_{it} + \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \mathbb{E}_{it} \tilde{a}_{it} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}} \right) di$$
$$= \frac{1}{1-\alpha} \tilde{a} + \frac{1}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \tilde{\alpha} \frac{2-\tilde{\alpha}}{\left(1-\tilde{\alpha}\right)^2} \left( \sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}} \right)$$
$$+ \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}}$$

Similarly,

$$\log \int \left[ E_{it} \left( \tilde{A}_{it} \right) \right]^{\frac{1}{1-\tilde{\alpha}}} di = \log \int e^{\left( \tilde{a}_{it} + \frac{1}{2} \tilde{\mathbb{V}} \right) \frac{1}{1-\tilde{\alpha}}} di$$
$$= \frac{1}{1-\alpha} \tilde{a} + \frac{1}{2} \frac{\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}}}{\left(1-\tilde{\alpha}\right)^2} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}}$$

Substituting and collecting terms, we obtain the expression in (6) in the text:

$$y = \frac{\theta}{\theta - 1}\bar{a} + \alpha_1 \log K + \alpha_2 \log N + \frac{1}{2} \left(\frac{\theta}{\theta - 1}\right) \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \left(\frac{\theta}{\theta - 1}\right) \frac{\tilde{\alpha}}{1 - \alpha} \tilde{\mathbb{V}}$$

where we also make use of the fact that  $\tilde{a}_{it} = \frac{a_{it}}{1-\alpha_2}$ . To endogenize K, we solve the following system of equations for Y and W.

$$Y = \int P_{it}Y_{it} \quad di$$

$$P_{it}Y_{it} = Y^{\frac{1}{\theta}}A_{it}\left\{\left(\frac{1-\alpha_2}{R}\right)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}Y^{\frac{1}{\theta}}\mathbb{E}_{it}\left(\tilde{A}_{it}\right)\right\}^{\frac{\alpha_1}{1-\tilde{\alpha}}} \left\{\frac{\tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it}\left[E_{it}\left(\tilde{A}_{it}\right)\right]^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}di}N\right\}^{\alpha_2}$$

$$W = \frac{\alpha_2}{N^{1-\alpha_2}}Y^{\frac{1}{\theta}}\left\{\int \tilde{A}_{it}\left[\left(\frac{1-\alpha_2}{R}\right)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}Y^{\frac{1}{\theta}}\mathbb{E}_{it}\left(\tilde{A}_{it}\right)\right]^{\tilde{\alpha}} \quad di\right\}^{1-\alpha_2}$$

The solution strategy is the same as before - we solve the last equation for W in terms of  $\int \tilde{A}_{it} \left[ \mathbb{E}_{it} \left( \tilde{A}_{it} \right) \right]^{\tilde{\alpha}} di$  and  $Y^{\frac{1}{\theta}}$ . We then plug this back into the expressions for revenue and total output, leaving us with an expression for Y only in terms of second moments of  $\tilde{A}_{it}$  and  $\left[ \mathbb{E}_{it} \left( \tilde{A}_{it} \right) \right]$ . The algebra is more tedious but straightforward and leads to the expression in the text.

## B Data

All data are from Compustat North America (for the US) and Compustat Global (for China, Mexico and India). For each country, we limit the sample to firms incorporated within that country and firms reporting in that country's currency. Stock returns are constructed as the change in the firm's stock price adjusted for splits and dividend distributions and follows the computation outlined in the Compustat manual. In Compustat terminology, and using  $\Delta p_{it}$  as shorthand for returns, log returns for the US are computed as

$$\log\left(\Delta p_{it}\right) = \log\left(\frac{\text{PRCCM}_{it} * \text{TRFM}_{it}}{\text{AJEXM}_{it}}\right) - \log\left(\frac{\text{PRCCM}_{it-1} * \text{TRFM}_{it-1}}{\text{AJEXM}_{it-1}}\right)$$

where periods denote years, PRCCM is the firm's stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the relevant months. The calculation is analogous for the remaining countries, except that global securities data come daily so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where "D" now denotes days. Again, we extract the data for the last trading day of the relevant months.

Three year periods are constructed using the sum of firm sales and log returns and the average capital stock over non-overlapping 3-year horizons. We measure the capital stock  $k_{it}$  as gross property, plant and equipment (PPENT), defined by Compustat as the tangible fixed

assets used in the production of revenue, and investment as  $i_{it} = \Delta k_{it}$ . It is then straightforward to compute revenue and investment growth as  $\Delta y_{it} = y_{it} - y_{it-1}$  and  $\Delta i_{it} = i_{it} - i_{it-1}$ .

As described in the text, we construct  $a_{it}$  as  $y_{it} - \alpha k_{it}$ . We then run the autoregression of  $a_{it}$  on  $a_{it-1}$  and a time fixed-effect to estimate  $\rho$ . Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The residuals from this regression correspond to  $\mu_{it}$ .

To extract the firm-specific variation in our variables,  $\Delta y_{it}$ ,  $\Delta i_{it}$ , and  $\Delta p_{it}$ , we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. We trim the 1% tails of the data following the following algorithm: for each series, we identify if an observation falls within the 1% tails of that particular series. We then drop observations that fall into the 1% tail on any dimension. This simultaneous procedure is more appropriate than a sequential one as our calibration relies heavily on correlations between the series. Finally, we remove all observations without the requisite data for all series.<sup>19</sup> It is then straightforward to construct the target moments, i.e.,  $\sigma (\Delta p)$ ,  $\rho (\Delta i, \Delta p)$ , and  $\rho (\Delta y, \Delta p)$ .

 $<sup>^{19}</sup>$ Other adjustments include removing duplicate observations and firms that change fiscal year reporting dates or SIC codes within 3-year periods.