Growth Opportunities, Technology Shocks, and Asset Prices*

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Abstract

We explore the impact of investment-specific technology (IST) shocks on the cross-section of stock returns and firms' investment using a production-based asset pricing model. The key property of our model is that the present value of growth opportunities has higher beta with respect to IST shocks than the value of assets in place, which leads to three main implications. First, firms with a higher fraction of growth opportunities in the firm value (high-growth firms) exhibit risk premia different from those of firms with fewer growth opportunities (low-growth firms). Second, high-growth firms co-move with each other, giving rise to a systematic factor in stock returns distinct from the market portfolio and related to the value factor. Third, stock return betas with respect to the IST shocks reveal cross-sectional heterogeneity in firms' growth opportunities. We find empirical support for qualitative predictions of the model. We calibrate our model and show that its main predictions for investment dynamics, cashflows and expected returns are quantitatively consistent with the data.

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1 Introduction

The recent literature on real determinants of economic growth has emphasized the role of changes in the technology for producing and installing new capital goods as an important driver of growth and fluctuations. We explore the implications of investment-specific technology (IST) shocks for the dynamics of stock returns. We base our analysis on the idea that IST shocks have a larger impact on firms demanding new capital goods, which are likely firms relatively rich with growth opportunities. Accordingly, we treat the firm value as a sum of two fundamental parts, the value of assets in place and the value of future growth opportunities, with the latter having higher exposure to IST shocks.

As a consequence of the heterogeneous exposure to IST shocks, three insights emerge. First, if risk premia on growth opportunities and assets in place are different, firms with a higher fraction of growth opportunities in the firm value (high-growth firms) exhibit risk premia different from those of firms with fewer growth opportunities (low-growth firms). Second, high-growth firms co-move with each other, giving rise to a systematic factor in stock returns distinct from the market portfolio and related to the value factor. Third, stock return betas with respect to the IST shocks reveal cross-sectional heterogeneity in firms' growth opportunities.

We develop a structural model of firm investment and stock price dynamics. In our model, firms are exposed to an exogenous sequence of neutral and IST shocks. In addition, each firm is endowed with a stochastic sequence of investment opportunities which it can implement by purchasing and installing new capital. The present value of the firm's growth opportunities rises in response to a positive neutral shock or a positive IST shock, where the latter corresponds to a decline in the price of new capital goods. In contrast, the value of assets in place responds only to the neutral productivity shock. Thus, in the model, high-growth firms exhibit higher stock return betas with respect to IST shocks than low-growth firms. Therefore, heterogeneity in firms' growth opportunities creates cross-sectional differences in risk premia. These differences are not captured by the market risk alone, as long as the two types of technology shocks are not perfectly correlated.

Our model generates heterogeneity in expected returns across firms with different book-to-market ratios. This happens because firms with higher growth opportunities tend to have lower book-to-market ratios. Since high-growth firms load more on the IST shocks, our model predicts that stocks with lower book-to-market ratios should have higher IST exposure. Thus, our model generates co-movement of firms with similar book-to-market ratios, giving rise to a value factor.

In addition to its implications for stock returns, our model offers novel predictions for the dynamics of firm investment. Specifically, firms with more growth opportunities should invest relatively more in response to a favorable IST shock since they have more potential projects to invest in. This prediction for firm investment behavior offers a natural direct test of the model's mechanism for generating dispersion in risk premia.

Our model suggests a natural observable proxy for IST shocks: the difference between stock returns of investment-good producers and consumption-good producers. Thus, in our empirical tests, we employ a zero-investment portfolio long the stocks of investment-good producers and short the stocks of consumption-good producers (IMC). The key benefit of our stock-return based measure of IST shocks is that it is available at high frequency. We use this high-frequency measure to estimate the conditional stock return betas with respect to IST shocks, capturing time variation in the share of growth opportunities in firm value.

We sort firms on their stock return betas with respect to IMC returns (β^{imc}). This results in a declining pattern in average returns and an increasing pattern of return volatility and stock market betas, which implies a declining pattern of CAPM alphas. The difference in average annualized returns and CAPM alphas between the high- and low-β^{imc} decile portfolios is -3.2% and -7.1% respectively. These empirical results suggest that the risk premium for the IST shocks is negative.

We find that firms with lower book-to-market ratios have higher IMC betas. This confirms that heterogeneous exposure to IST shocks generates co-movement among stocks with similar book-to-market ratios. Cross-sectional heterogeneity in IMC betas across the book-to-market portfolios contributes to the spread in their average returns, although the historical premium on the IMC portfolio is not sufficient to fully account for the observed value premium.

Our model generates differences in risk premia across the β^{imc} portfolios through cross-sectional differences in growth opportunities. To verify this mechanism empirically, we evaluate our model's implications for firms' investment behavior in relation to β^{imc}. We find that β^{imc} portfolios exhibit a number of patterns consistent with cross-sectional differences in growth opportunities. In particular, high-β^{imc} firms tend to have higher Tobin's Q, hold more cash, pay less in dividends, and invest more in R&D.

As predicted by our model, IMC betas identify heterogeneity in firms' investment responses to the IST shocks. High-β^{imc} firms not only invest more on average, but their investment increases more in response to a positive investment shock, as measured by high returns on the IMC portfolio. This pattern is statistically and economically significant. The difference in investment-goods price sensitivity between the high-beta and the low-beta firms is two to three times larger than the sensitivity of an average firm. Our model replicates these patterns quantitatively.

We perform a number of robustness tests. In particular, we find that IMC betas identify heterogeneous investment responses to other aggregate shocks affecting the firms' cost of capital, including the shocks to the price of new equipment and credit spread shocks. We find that IMC portfolio returns have the largest impact on the cross-section of firms' investment rates, suggesting that IMC returns are a useful empirical proxy for IST shocks.

Our model closely replicates many quantitative patterns in stock returns, including the failure of the CAPM to price the β^{imc} portfolios. Assuming a negative risk premium for the IST shocks, the model implies that high-growth firms have relatively low average returns. This, in turn, implies a positive premium on the value factor. Our model also produces a strong increasing pattern in market betas across the β^{imc} portfolios. This pattern is formed because growth firms have relatively high exposure to IST shocks, resulting in higher beta with respect to the market portfolio. Consequently, our model generates a negative relationship between market betas and average returns, as documented by Fama and French (1992).

In addition to its implications for stock returns, our model replicates the dynamics of cash flows and profitability of value and growth firms documented by Fama and French (1995). In the year of portfolio formation, growth firms have higher average profitability than value firms. In the years following portfolio formation, the average profitability of growth firms declines, whereas the average profitability of value firms rises. Despite the fall in average profitability, in the model, as in the data, the earnings of growth firms grow faster than value firms, controlling for size. This pattern of mean reversion in profitability is driven partly by the fact that growth firms invest relatively more on average and, as they accumulate capital, become similar to value firms.

In summary, our analysis highlights heterogeneous exposure of firms to IST shocks as a source of cross-sectional heterogeneity in risk premia. This mechanism has a number of implications for stock returns and firm investment behavior which we confirm empirically. Finally, we verify that a parsimonious structural model is able to account for several key empirical patterns quantitatively, providing additional support for our theory.

The rest of the paper is organized as follows. Section 2 relates our work to existing literature. Section 3 develops the theoretical model. In Section 4, we calibrate the model and test it empirically. Section 5 concludes.

2 Related Research

Our paper bridges and complements two distinct strands of the finance and macroeconomic literature. The first argues that differences in a firm's mix between growth options and assets are important in understanding the cross-section of risk premia, and the second argues for the importance of investment-specific shocks for aggregate growth and fluctuations.

In financial economics, the idea that growth opportunities may have different risk characteristics than assets in place is not new (e.g., Berk, Green, and Naik (1999); Gomes, Kogan, and Zhang (2003); Carlson, Fisher, and Giammarino (2004); Zhang (2005)). Earlier studies have argued that decomposing firm value into assets in place and growth opportunities may be useful in understanding the cross-section of risk premia. In these models, assets in place and growth opportunities have different dynamics of systematic risk. Our work complements this literature by illustrating how a different mechanism can generate differences in risk premia between assets in place and growth options. Most of these models feature a single aggregate shock and thus imply that the value factor is conditionally perfectly correlated with the market portfolio. The model of Berk et al. (1999) is one of the few exceptions, it incorporates both aggregate productivity and discount rate shocks. Santos and Veronesi (2009) argue that discount rate shocks lead to value firms having lower risk premia than growth firms since growth firms' cash flows have longer duration and thus higher exposure to discount rate shocks. Papanikolaou (2010) shows that in a two-sector general equilibrium model, investment shocks can generate a value premium as well as the value factor. There is no firm-level heterogeneity within the sectors in his model, while we explicitly model firm heterogeneity within the consumption-good sector in terms of the mix between growth opportunities and assets in place.

In macroeconomics, a number of studies have shown that IST shocks can account for a large fraction of the variability of output and employment, both in the long run and at business cycle frequencies (e.g., Greenwood, Hercowitz, and Krusell (1997, 2000); Christiano and Fisher (2003); Fisher (2006); Justiniano, Primiceri, and Tambalotti (2010)). Investment shocks can be modelled as either shocks to the marginal cost of capital as in Solow (1960) or as shocks to the productivity of a sector producing capital goods as in Rebelo (1991) or Christiano and Fisher (2003). Given that investment shocks lead to an improvement in the real investment opportunity set in the economy, they naturally have a differential impact on growth opportunities versus assets in place.

Our work is also connected to the literature relating asset prices and firm investment. In this literature, Tobin's Q is commonly used as a stock-market based predictor of investment (e.g., Hayashi (1982); Abel (1985); Abel and Eberly (1994, 1996, 1998); Eberly, Rebelo, and Vincent (2008)). Tobin's Q measures the valuation of capital installed in the firm relative to its replacement cost. Thus, Tobin's Q is commonly considered an observable proxy for growth opportunities. In our paper, we use an alternative empirical measure of growth opportunities introduced in Kogan and Papanikolaou (2010), which relies on the stock return betas with respect to IST shocks. Our tests demonstrate that this measure is incrementally informative when controlling for Tobin's Q and other standard empirical predictors of investment.

A growing branch of asset pricing literature in finance relates Q-based theories of investment to stock return behavior (e.g., Cochrane (1991, 1996); Lyandres, Sun, and Zhang (2008); Liu, Whited, and Zhang (2009); Li, Livdan, and Zhang (2009); Chen, Novy-Marx, and Zhang (2010); Li and Zhang (2010)). This literature focuses on the relationship between expected stock returns and firms' investment decisions, which follows from firm's optimizing behavior. Our focus is instead on the mechanism behind the joint determination of investment behavior and risk premia. Thus, our work complements the existing studies and offers a potentially fruitful way of improving our understanding of the links between real investment and stock returns.

3 The Model

In this section we develop a structural model of investment. We show that the value of assets in place and the value of growth opportunities have different exposure to the IST shocks. Thus, the relative weight of growth opportunities in a firm's value can be identified by measuring the sensitivity of its stock returns to IST shocks.

There are two sectors in our model: the consumption-good sector, and the investment-good sector. IST shocks manifest as changes in the cost of new capital goods. We focus on heterogeneity in growth opportunities among consumption-good producers.

3.1 Consumption-Good Producers

There is a continuum of measure one of infinitely lived firms producing a homogeneous consumption good. Firms behave competitively, and there is no explicit entry or exit in this sector. Firms are financed only by equity, hence the firm value is equal to the market value of its equity.

Assets in Place

Each firm owns a finite number of individual projects. Firms create projects over time through investment, and projects expire randomly.[1] Let $\mathcal{F}$ denote the set of firms and $\mathcal{J}^{(f)}$ the set of projects owned by firm f.

Project j managed by firm f produces a flow of output equal to

$$y_{fjt} = \varepsilon_{ft} u_{jt} x_t K_j^{\alpha}, \tag{1}$$

where K_j is physical capital chosen irreversibly at project j's inception date, u_{jt} is the project-specific component of productivity, ε_{ft} is the firm-specific component of productivity, such as managerial skill of the parent firm, and x_t is the economy-wide productivity shock affecting output of all existing projects. We assume decreasing returns to scale at the project level, $\alpha \in (0, 1)$. Projects expire independently at rate δ.

[1]Firms with no current projects may be seen as firms that temporarily left the sector. Likewise, idle firms that begin operating a new project can be viewed as new entrants. Thus, our model implicitly captures entry and exit by firms.

The three components of projects' productivity evolve according to

$$d\varepsilon_{ft} = -\theta_\varepsilon(\varepsilon_{ft} - 1)\,dt + \sigma_\varepsilon\sqrt{\varepsilon_{ft}}\,dB_{ft} \tag{2}$$

$$du_{jt} = -\theta_u(u_{jt} - 1)\,dt + \sigma_u\sqrt{u_{jt}}\,dB_{jt} \tag{3}$$

$$dx_t = \mu_x\, x_t\, dt + \sigma_x x_t\, dB_{xt}, \tag{4}$$

where dB_{ft}, dB_{jt} and dB_{xt} are independent standard Brownian motions. All idiosyncratic shocks are independent of the aggregate shock: $dB_{ft} \cdot dB_{xt} = 0$ and $dB_{jt} \cdot dB_{xt} = 0$. The firm and project-specific components of productivity are stationary processes, while the process for aggregate productivity follows a Geometric Brownian motion, generating long-run growth.

Investment

Firms acquire new projects exogenously according to a Poisson process with a firm-specific arrival rate λ_{ft}. The firm-specific arrival rate of new projects is

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{ft} \tag{5}$$

where $\tilde{\lambda}_{ft}$ follows a two-state, continuous-time Markov process with transition probability matrix between time t and $t + dt$ given by

$$P = \begin{pmatrix} 1 - \mu_L\, dt & \mu_L\, dt \\ \mu_H\, dt & 1 - \mu_H\, dt \end{pmatrix}. \tag{6}$$

We label the two states as $[\lambda_H, \lambda_L]$, with $\lambda_H > \lambda_L$. Thus, at any point in time, a firm can be either in the high-growth $(\lambda_f \cdot \lambda_H)$ or in the low-growth state $(\lambda_f \cdot \lambda_L)$, and $\mu_H\, dt$ and $\mu_L\, dt$ denote the instantaneous probability of entering each state respectively. We impose that $E[\tilde{\lambda}_{ft}] = 1$, which translates to the restriction

$$1 = \lambda_L + \frac{\mu_H}{\mu_H + \mu_L}(\lambda_H - \lambda_L) \tag{7}$$

When presented with a new project at time t, a firm must make a take-it-or-leave-it decision. If the firm decides to invest in a project, it chooses the associated amount of capital K_j and pays the investment cost $z_t x_t K_j$. The cost of capital relative to its average productivity, z_t, is assumed to follow a Geometric Brownian motion

$$dz_t = \mu_z z_t \, dt + \sigma_z z_t \, dB_{zt} \tag{8}$$

where $dB_{zt} \cdot dB_{xt} = 0$. The z shock represents the component of the price of capital that is unrelated to its current level of average productivity (x), and is the IST shock in our model. A positive realization of z increases the cost of new capital goods and is thus considered a negative IST shock.[2] Finally, at the time of investment, the project-specific component of productivity is at its long-run average value, $u_{jt} = 1$.

Valuation

Let π_t denote the stochastic discount factor. The time-0 market value of a cash flow stream C_t is then given by $\mathrm{E}\left[\int_0^\infty (\pi_t/\pi_0) C_t \, dt\right]$. For simplicity, we assume that the aggregate productivity shocks x_t and z_t have constant prices of risk, β_x and β_z respectively, and the risk-free interest rate r is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r \, dt - \beta_x \, dB_{xt} - \beta_z \, dB_{zt}. \tag{9}$$

This form of the stochastic discount factor is motivated by a general equilibrium model with IST shocks in Papanikolaou (2010). In Papanikolaou (2010), states with low cost of new capital (positive IST shock or low z) are high marginal valuation states because of improved investment opportunities. This is analogous to a positive value of β_z. Our analysis below shows that empirical properties of stock returns imply a positive value of β_z. Finally, we choose a positive price of risk of the aggregate productivity shock x, which is consistent with

[2] An alternative modeling strategy is to model IST shocks as productivity shocks in the production of new capital goods, as in Papanikolaou (2010). In that case,a positive IST shock translates into a larger quantity of investment goods produced and thus a lower equilibrium price.

most equilibrium models and empirical evidence.

Firms' investment decisions are based on a tradeoff between the market value of a new project and the cost of physical capital. The time-t market value of an existing project j, $p(\varepsilon_{ft}, u_{jt}, x_t, K_j)$, is computed using the discounted cash flow formula:

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) = \mathrm{E}_t \left[\int_t^\infty e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} \varepsilon_{fs} u_{js} x_s K_j^\alpha \, ds \right] = A(\varepsilon_{ft}, u_{jt}) x_t K_j^\alpha, \tag{10}$$

where

$$\begin{aligned} A(\varepsilon, u) = & \frac{1}{r + \delta - \mu_X} + \frac{1}{r + \delta - \mu_X + \theta_\varepsilon}(\varepsilon - 1) + \frac{1}{r + \delta - \mu_X + \theta_u}(u - 1) \\ & + \frac{1}{r + \delta - \mu_X + \theta_\varepsilon + \theta_u}(\varepsilon - 1)(u - 1) \end{aligned} \tag{11}$$

Firms' investment decisions are straightforward because the arrival rate of new projects is exogenous and does not depend on their previous decisions. Thus, optimal investment decisions are based on the NPV rule. Firm f chooses the amount of capital K_j to invest in project j to maximize

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) - z_t x_t K_j \tag{12}$$

Proposition 1 *The optimal investment K_j in project j undertaken by firm f at time t is*

$$K^*(\varepsilon_{ft}, z_t) = \left(\frac{\alpha A(\varepsilon_{ft}, 1)}{z_t} \right)^{\frac{1}{1-\alpha}}. \tag{13}$$

The scale of a firm's investment depends on firm-specific productivity, ε_{ft}, and the price of investment goods relative to average investment-specific productivity, z_t. Because our economy features decreasing returns to scale at the project level, it is always optimal to invest a positive and finite amount.

The value of the firm can be computed as a sum of market values of its existing projects and the present value of its growth opportunities. The former equals the present value of cash flows generated by existing projects. The latter equals the expected discounted NPV of future investments. Following the standard convention, we call the first component of firm value *the value of assets in place*, VAP_{ft}, and the second component *the present value of growth opportunities*, $PVGO_{ft}$. The value of the firm then equals

$$V_{ft} = VAP_{ft} + PVGO_{ft} \tag{14}$$

The value of a firm's assets in place is simply the value of its existing projects:

$$VAP_{ft} = \sum_{j \in \mathcal{J}_f} p(e_{ft}, u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_f} A(\varepsilon_{ft}, u_{j,t}) K_j^{\alpha}. \tag{15}$$

The present value of growth options is given by the following proposition.

Proposition 2 *The value of growth opportunities for firm f is*

$$PVGO_{ft} = z_t^{\frac{\alpha}{\alpha-1}} \, x_t \, G(\varepsilon_{ft}, \lambda_{ft}) \tag{16}$$

where

$$\begin{aligned} G(\varepsilon_{ft}, \lambda_{ft}) &= C \cdot E_t \left[\int_t^{\infty} e^{-\rho(s-t)} \, \lambda_{fs} \, A(\varepsilon_{fs})^{\frac{1}{1-\alpha}} \, ds \right] \\ &= \begin{cases} \lambda_f \left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_H \\ \lambda_f \left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_L, \end{cases} \end{aligned} \tag{17}$$

and

$$\rho = r + \frac{\alpha}{1-\alpha} (\mu_z - \sigma_z^2/2) - \mu_x - \frac{\alpha^2 \sigma_z^2}{2(1-\alpha)^2}, \tag{18}$$

and

$$C = \alpha^{\frac{1}{1-\alpha}} \left(\alpha^{-1} - 1 \right). \tag{19}$$

The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ solve

$$C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - \rho \, G_1(\varepsilon) - \theta_{\varepsilon}(\varepsilon - 1) \frac{d}{d\,\varepsilon} \, G_1(\varepsilon) + \frac{1}{2} \sigma_e^2 \, \varepsilon \, \frac{d^2}{d\,\varepsilon^2} \, G_1(\varepsilon) = 0 \tag{20}$$

$$C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - (\rho + \mu_H + \mu_L) \, G_2(\varepsilon) - \theta_{\varepsilon}(\varepsilon - 1) \frac{d}{d\,\varepsilon} \, G_2(\varepsilon) + \frac{1}{2} \sigma_e^2 \, \varepsilon \, \frac{d^2}{d\,\varepsilon^2} \, G_2(\varepsilon) = 0. \tag{21}$$

In addition to aggregate and firm-specific productivity, the present value of growth opportunities depends on the IST shock, z, because the net present value of future projects depends on the cost of new investment. In summary, the firm value in our model is

$$V_{ft} = VAP_{ft} + PVGO_{ft} = x_t \sum_j A(\varepsilon_{ft}, u_{jt}) K_j^{\alpha} + z_t^{\frac{\alpha}{\alpha-1}} x_t G(\varepsilon_{ft}, \lambda_{ft}) \tag{22}$$

Risk and Expected Returns

Both assets in place and growth opportunities have constant exposure to systematic shocks dB_{xt} and dB_{zt}. However, their betas with respect to the productivity shocks are different. The value of assets in place is independent of the IST shock and loads only on the aggregate productivity shock. The present value of growth option depends positively on aggregate productivity and negatively on the unit cost of new capital. Thus, firms' stock return betas with respect to the aggregate shocks are time-varying and depend linearly on the fraction of firm value accounted for by growth opportunities. Since, by assumption, the price of risk of aggregate shocks is constant, the expected excess return of a firm is an affine function of the weight of growth opportunities in firm value, as shown in the following proposition:

Proposition 3 *The expected excess return on firm f is*

$$ER_{ft} - r_f = \beta_x \sigma_x - \frac{\alpha}{1-\alpha} \beta_z \sigma_z \frac{PVGO_{ft}}{V_{ft}} \tag{23}$$

Many existing models of the cross-section of stock returns generate an affine relationship between expected stock return and firms' asset composition similar to (23) (e.g., Berk et al. (1999), Gomes et al. (2003)). It is easy to see, in the context of our model, how the relationship (23) can give rise to a value premium. Assume that both prices of risk β_x and β_z are positive, which we justify in the following sections. Then growth firms, which derive a relatively large fraction of their value from growth opportunities, have relatively low expected excess returns because of their exposure to IST shocks. To the extent that firms' book-to-market (B/M) ratios are partially driven by the value of firms' growth opportunities, firms with high B/M ratios tend to have higher average returns than firms with low B/M ratios.

3.2 Investment-Good Producers

There is a continuum of firms producing new capital goods. We assume that these firms produce the demanded quantity of capital goods at the current unit price z_t. We assume that profits of investment firms are a fraction ϕ of total sales of new capital goods. Consequently, profits accrue to investment firms at a rate of $\Pi_t = \phi\, z_t\, x_t\, \overline{\lambda} \int_{\mathcal{F}} K_{ft} df$, where $\overline{\lambda} = \int_{\mathcal{F}} \lambda_{ft}$ is the average arrival rate of new projects among consumption-good producers. Even though λ_{ft} is stochastic, it has a stationary distribution, so $\overline{\lambda}$ is a constant.

Proposition 4 *The price of the investment firm satisfies*

$$V_{It} = \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{\rho_I} \tag{24}$$

where we assume

$$\rho_I \equiv r - \mu_X + \frac{\alpha}{1-\alpha}\, \mu_Z - \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 - \frac{1}{2}\frac{\alpha^2\, \sigma_Z^2}{(1-\alpha)^2} > 0 \tag{25}$$

and

$$\Gamma \equiv \phi\, \overline{\lambda}\, \alpha^{\frac{1}{1-\alpha}} \left(\int A(e_f, 1)^{\frac{1}{1-\alpha}} df \right). \tag{26}$$

The value of the investment firms will equal the present value of their cash flows. If we assume that these firms incur proportional costs of producing their output, and given that the market price of risk is constant for the two shocks, their value will be proportional to cash flows or the aggregate investment expenditures in the economy. The stock returns of the investment firms will then load on the IST shock (z) as well as the common productivity shock (x).

Here, we note that a positive IST shock, defined as a drop in z, benefits the investment-good producers. Even though the price of their output drops, the elasticity of investment demand with respect to price is greater than one, so their profits increase. An equivalent formulation of the model would be to specify a production function for investment firms subject to IST shocks as in Papanikolaou (2010). A positive productivity shock in the investment sector would lead to a drop in the price of new equipment (z) and an increase in profitability of investment goods producers. In Papanikolaou (2010), profits of investment-good producers are a constant fraction of total sales of capital goods.

We define an IMC portfolio in the model as a portfolio that is long the investment sector and short the consumption sector. The beta of firm f with respect to the IMC portfolio return is given by

$$\beta_{ft}^{imc} = \frac{cov_t(R_{ft}, R_t^I - R_t^C)}{var_t(R_t^I - R_t^C)} \tag{27}$$

where $R_t^I - R_t^C$ is the return on the IMC portfolio.

Proposition 5 *The beta of firm i with respect to the IMC portfolio return is given by*

$$\beta_{ft}^{imc} = \beta_{0t} \left(\frac{PVGO_{ft}}{V_{ft}} \right) \tag{28}$$

where

$$\beta_{0t} = \frac{\overline{V}_t}{\overline{VAP}_t} \tag{29}$$

Proposition 5 is the basis of our empirical approach to measuring growth opportunities. This approach was first used in Kogan and Papanikolaou (2010). The beta of firm f's return with respect to the IMC portfolio return is proportional to the fraction of firm f's value represented by its growth opportunities. Firms that have few active projects but expect to create many projects in the future derive most of their value from their future growth opportunities. These firms are anticipated to increase their investment in the future, and their stock price reflects that. There is also an aggregate term in (28) that depends on the fraction of aggregate value that is due to growth opportunities, which affects the IMC portfolio's beta with respect to the z-shock.

4 Empirical Analysis and Calibration

4.1 Data and Procedures

Investment-specific shocks

Following the model developed in Section 3, we use the IMC portfolio as an observable proxy for IST shocks. We first classify industries as producing either investment or consumption goods according to the NIPA Input-Output Tables. We then match firms to industries according to their NAICS codes. Gomes, Kogan, and Yogo (2009) and Papanikolaou (2010) describe the details of this classification procedure.

Our model also implies that a value factor, defined as a portfolio long firms with high book-to-market and short firms with low book-to-market in the consumption sector, also loads on the IST shock. We construct the equivalent of the HML portfolio in Fama and French (1993) using only firms in the consumption industry. We construct a 2×3 sort, sorting firms first on their market value of equity (CRSP December market capitalization) and then on their ratio of Book-to-Market (Compustat item ceq). We construct the breakpoints using NYSE firms only. We construct our value factor in the consumption sector ($CHML$) as $1/2(SV - SG) + 1/2(LV - LG)$, where SG, SV, LG and LV refer to the corner portfolios. Our $CHML$ portfolio has a correlation of -47% with the IMC portfolio and a correlation of 92% with the Fama and French (1993) HML factor.

Estimation of β^{imc}

We use the firm's stock return beta with respect to the IMC portfolio returns as a measure of this firm's IST shock sensitivity. For every firm in Compustat with sufficient stock return data, we estimate a time-series of (β_{ft}^{imc}) from the following regression

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^{imc} r_{tw}^{imc} + \varepsilon_{ftw}, \qquad w = 1 \ldots 52. \tag{30}$$

Here r_{ftw} refers to the (log) return of firm f in week w of year t, and r^{imc}_{ftw} refers to the log return of the IMC portfolio in week w of year t. Thus, β^{imc}_{ft} is constructed using information only in year t.

We omit firms with fewer than 50 weekly stock-return observations per year, firms in their first three years following the first appearance in Compustat, firms in the investment sector, financial firms (SIC codes 6000-6799), utilities (SIC codes 4900-4949), firms with missing values of CAPEX (Compustat item capx), PPE (Compustat item ppent), Tobin's Q, CRSP market capitalization, firms whose investment rate exceeds 1 in absolute value, firms with Tobin's Q greater than 100, firms with negative book values and firms where the ratio of cash flows to capital exceeds 5 in absolute value. Our final sample contains 6,831 firms and 62,495 firm-year observations and covers the 1965-2007 period.

Our estimates of β^{imc} are persistent even though they are estimated using non-overlapping data. Table 1 reports transition probabilities among the β^{imc} quintiles. Estimates of β^{imc} also exhibit sufficient variation over time, so we can distinguish the effect of β^{imc} from firm-level fixed effects.

[Table 1]

Summary statistics

We focus our analysis on firms in the consumption-good sector, following our theoretical analysis above. Every year we split the universe of consumption-good producers into 10 portfolios based on their estimate of β^{imc}. We summarize data definitions in the Appendix. The top panel of Table 2 reports the summary statistics for firms in different β^{imc} deciles. The table shows a declining pattern of risk premia across the β^{imc} deciles. At the same time, there is a clear positive relationship between β^{imc} and market betas. Thus, cross-sectional differences in risk premia among high- and low-growth stocks are not captured by their market risk. Moreover, as we show below, the increasing profile of market betas across the β^{imc} deciles is consistent with cross-sectional heterogeneity in growth opportunities and is present in our model.

The patterns of investment and firm characteristics across the deciles are also consistent with our interpretation of β^{imc} as measuring heterogeneity in growth opportunities. The portfolio of high-β^{imc} firms has a higher investment rate (22.4%) than the low-β^{imc} firms (16.5%). Moreover, high-β^{imc} firms tend to have higher Tobin's Q (1.38 vs 1.13), higher R&D expenditures (6.0% vs 1.4%), and pay less in dividends (2.8% vs 9.0%) than low-β^{imc} firms, although the latter relationship is hump-shaped. Furthermore, high β^{imc} firms tend to be smaller, both in terms of market capitalization as well as book value of capital. The highest β^{imc} portfolio accounts for a fraction of 3.9% and 2.8% of the total market capitalization and book value of capital versus 8.8% and 9.8% for the low β^{imc} portfolio.

[Table 2]

Calibration

We calibrate our model to approximately match moments of aggregate dividend growth and investment growth, accounting ratios, and asset returns. Thus, most of the parameters are chosen jointly based on the behavior of financial and real variables. Table 4 summarizes our parameter choices.

[Table 4]

We pick $\alpha = 0.85$, the parameters governing the projects' cash flows ($\sigma_\varepsilon = 0.2, \theta_\varepsilon = 0.35, \sigma_u = 1.5, \theta_u = 0.5$) and the parameters of the distribution of λ_f jointly, to match the average values and the cross-sectional distribution of the investment rate, the market-to-book ratio, and the return to capital (ROE).

We model the distribution of mean project arrival rates $\lambda_f = E[\lambda_{ft}]$ across firms as

$$\lambda_f = \mu_\lambda \delta - \sigma_\lambda \delta \log(X_f) \quad X_f \sim U[0,1], \tag{31}$$

We pick $\sigma_\lambda = \mu_\lambda = 2$. Regarding the dynamics of the stochastic component of the firm-specific arrival rate, $\tilde{\lambda}_{ft}$, we pick $\mu_H = 0.075$ and $\mu_L = 0.16$. We pick $\lambda_H = 2.35$, which according to (7) implies $\lambda_L = 0.35$. These parameter values ensure that the firm grows at about twice than the average rate in its high growth phase and about a third as fast than average in the low growth phase.

We set the project expiration rate δ to 10%, to be consistent with commonly used values for the depreciation rate. We set the interest rate r to 2.5%, which is close to the historical average risk-free rate (2.9%). We choose the parameters governing the dynamics of the shocks x_t and z_t to match the first two moments of the aggregate dividend growth and investment growth. We choose $\phi = 0.07$ to match the relative size of the consumption and investment sectors in the data.

Finally, the parameters of the pricing kernel, $\beta_x = 0.69$ and $\beta_z = 0.35$ are picked to match approximately the average excess returns on the market portfolio and the IMC portfolio. Given our calibration, the model produces a somewhat lower average return on the IMC portfolio equal to -3.9% vs -1.9% in the $1965 - 2008$ sample. However, investment firms tend to be quite a bit smaller than consumption firms, so the size effect may bias the estimated return of the IMC portfolio upwards. Two pieces of evidence support this: when excluding the month of January, which is when the size effect is strongest, the average return on the IMC portfolio is -3.5%; in addition, its α with respect to the Small-minus-Big (SMB) portfolio of Fama and French (1993) is -3.7%.

We simulate the model at a weekly frequency ($dt = 1/52$) and time-aggregate the data to form annual observations. Each simulation sample contains 2,500 firms for 100 years. We drop the first half of each simulated sample to eliminate the dependence on initial values. We simulate 1,000 samples and report medians of parameter estimates and t-statistics across simulations. In the simulated data, we estimate firm-level β^{imc} using the same methodology as in our empirical results, namely by estimating equation (30) using weekly data every year. In the model, our estimates of β^{imc} are slightly more persistent than the data.

[Table 3]

The bottom panel of Table 2 reports the summary statistics for different β^{imc} deciles using simulated model output. Similarly to the empirical results in the top panel of the table, the model generates a declining pattern of risk premia across the β^{imc} deciles accompanied by an increasing pattern of market betas. In the model, market betas are higher for firms with more growth opportunities. The aggregate stock market value consists of the aggregate value of assets in place and the aggregate value of growth opportunities. Both assets in place and growth opportunities have unit beta with respect to neutral technology shocks. At the same time, only growth opportunities load on the IST shocks. Thus, firms with higher fraction of growth opportunities in their value have higher market betas in the model.

The patterns of investment and firm characteristics across the deciles are also similar to the empirical patterns. High-β^{imc} firms tend to have higher investment rates as measured by the aggregate investment rate of the portfolio. High-β^{imc} firms tend to have higher Tobin's Q and smaller size, measured either by their market capitalization or capital stock.

Next, we test the predictions of our model for the dynamics of stock returns and firms' investment. We compare our empirical findings to the output of the calibrated model and evaluate both the qualitative features and the magnitude of the patterns generated by the model.

4.2 Stock Returns

As we show in Proposition 3, cross-sectional differences in the fraction of growth opportunities in firms' values lead to cross-sectional differences in equity risk premia. Furthermore, we show in Proposition 5 that unobservable growth opportunities can be measured empirically using the firms' stock return betas with respect to the IMC portfolio returns. We now verify that our model implies empirically realistic behavior of stock returns in relation to the differences in growth opportunities (captured by β^{imc}) across firms.

IMC-beta sorted portfolios

We sort firms annually into 10 value-weighted portfolios based on the past value of β^{imc}. Both in the actual and simulated data, we estimate β^{imc} using weekly returns and rebalance the portfolios at the end of every year. In each simulation and for each portfolio p, we estimate average excess returns $E[R_{pt}] - r_f$, return standard deviations $\sigma(R_{pt})$, and regressions

$$R_{pt} - r_f = \alpha_p + \beta_p^{mkt}(R_{Mt} - r_f) + \varepsilon_{pt}, \tag{32}$$

$$R_{pt} - r_f = \alpha_p + \beta_p^{mkt}(R_{Mt} - r_f) + \beta_p^{imc}(R_t^{imc}) + \varepsilon_{pt}, \tag{33}$$

$$R_{pt} - r_f = \alpha_p + \beta_p^{mkt}(R_{Mt} - r_f) + \beta_p^{chml}(R_t^{chml}) + \varepsilon_{pt}, \tag{34}$$

where R_t^{imc} and R_t^{chml} denote returns on the IMC and $CHML$ portfolios respectively.

[Table 5]

Table 5 compares the properties of returns in historical and simulated data. The top panel replicates the findings of Papanikolaou (2010), who shows that sorting firms into portfolios based on β^{imc} results in i) a declining pattern in average returns; ii) an increasing pattern of return volatility and market betas; and iii) a declining pattern of CAPM alphas. The difference in average returns and CAPM alphas between the high- and low-β^{imc} portfolios is -3.2% and -7.1% respectively. The high-β^{imc} portfolio has a standard deviation of 29.7% and a market beta of 1.6 versus 15.8% and 0.75 respectively for the low-β^{imc} portfolio. Furthermore, a two-factor model that includes either the IMC portfolio or the $CHML$ factor in addition to the market portfolio reduces the difference in alphas to -3.0% and -2.6% respectively. In addition, there is a monotonically decreasing pattern in β^{chml} across the β^{imc} deciles, suggesting that IMC and $CHML$ capture common sources of co-movement in the data.

The bottom panel of Table 5 contains the same regressions performed in simulated data. The difference in average returns and CAPM alphas between the high- and low-β^{imc} portfolios is −3.5% and −5.7% respectively. Moreover, as in the data, the high-β^{imc} portfolio has both a higher standard deviation (20%) and market beta (1.2) than the low-β^{imc} portfolio (14% and 0.8 respectively). In simulated data, a two factor model that includes market portfolio returns and IMC or $CHML$ successfully prices the spread across decile portfolios. The estimates of α_p in (33) and (34) are both close to zero and not statistically significant.

Book-to-market sorted portfolios

We now assess the ability of our model to replicate the empirical relationship between stock returns and the book-to-market ratio. As we show in Section 3, the book-to-market ratio in our model is correlated with expected stock returns because it is inversely related to $PVGO/V$. We perform the same exercise as Fama and French (1993) and sort firms in the consumption industry on their ratio of Book Equity (Compustat item ceq) to Market Equity (CRSP December market capitalization) into 10 portfolios.

[Table 6]

The top panel in Table 6 estimates equations (32-34) in historical data. In the consumption-good sector, the difference in average returns and CAPM alphas between value firms and growth firms is 6.1% and 5.9% respectively. When estimating equation (33), with the exception of the extreme value portfolios, there is a declining pattern in β^{imc} across book-to-market deciles. Similarly, when estimating equation (34), there is an increasing pattern in β^{chml} across deciles. As before, these results suggest that IMC and $CHML$ may be capturing common sources of co-movement among stocks with similar book-to-market ratios. The difference in estimated alphas across the decile portfolios in equations (33-34) is 6.0% and 0.9% with t-statistics of 2.4 and 0.5 respectively.

The bottom panel in Table 6 presents estimates of equations (32-34) in simulated data. The difference in average returns and CAPM alphas between the two extreme book-to-market portfolios is 4.3% and 6.3% respectively. Moreover, the CAPM betas decline across the book-to-market deciles. Thus, our model replicates the failure of the CAPM to price the cross-section of book-to-market portfolios. When estimating equations (33-34), the difference in alphas across the extreme book-to-market deciles is 0.8% and -0.1% respectively. Interestingly, even though the two-factor unconditional specification works very well in simulated data, $CHML$ works marginally better than IMC in pricing the cross-section of book-to-market portfolios. The reason why this happens in the model is that both IMC and $CHML$ exhibit time-variation in their loading on the IST shock, even though both are perfectly conditionally correlated. By construction, $CHML$ explains the cross-section of book-to-market portfolios slightly better unconditionally.

Size and book-to-market sorted portfolios

Next we assess the model's ability to match the properties of portfolio returns sorted first on market equity and then on book-to-market. In the model, this is an informative sort because controlling for firm size increases the informativeness of the book-to-market ratio about $PVGO/V$. Focusing on firms in the consumption industry, we first sort stocks into five size quintiles on their market capitalization, as in Fama and French (1993). Then, we further divide each size quintile into five book-to-market quintiles. We use NYSE breakpoints. In the data, the interquintile spread in book-to-market in the small quintile portfolio is roughly two times the interquintile spread in book-to-market in the large portfolio. In the model, the proportional difference in interquintile book-to-market spreads across the extreme size portfolios is about one and a half. In the model, there is greater dispersion in $PVGO/V$ among small firms. Part of the reason is that small firms tend to have fewer projects, which implies lower diversification of the project specific shocks (u), and thus tend to have greater heterogeneity in assets in place relative to total firm value. In addition, small firms in the model tend to have low productivity and low value of assets in place. As a result, their growth opportunities can account for a large fraction of firm value, creating larger dispersion in $PVGO/V$ than large firms.

[Table 7]

Table 7 shows the properties of the portfolios obtained by the two-dimensional sort of firms in the data and in the model. The top panel summarizes empirical statistics for the lowest and the highest size quintile of firms. For both the small and large quintile portfolios there is a positive relationship between average returns and book-to-market, but the difference is larger for smaller firms. Moreover, conditional on size, there is a declining relationship between CAPM betas and book-to-market, more so for small firms than for large. In equation (32), the estimated CAPM alphas are 9.5% and 2.5% for small and large firms respectively. When we estimate equations (33-34), we find that, conditional on size, there is a monotone pattern between the book-to-market sort and either β^{imc} or β^{chml}. The two-factor model with IMC and $CHML$ works well in pricing the cross-section of book-to-market among large firms, but not among small firms. The difference in estimated α_p coefficients across book-to-market for the smallest quintile is 8.3% and 5.4% respectively. Interestingly, the dispersion in both the book-to-market characteristic and β^{imc} is two times larger in the smallest quintile than in the largest, but risk premia are four times larger.

The bottom panel contains the corresponding numbers based on model output. The model is able to reproduce the pattern of co-movement in the data rather well. First, the model accurately captures the profile of declining market betas across book-to-market sorts very accurately, as well as the level and dispersion of β^{imc} across the size and book-to-market portfolios. In the model, the cross-sectional spread in risk premia and market betas is much larger among small stocks because they exhibit larger dispersion in the ratio of growth opportunities to firm value. This can be seen from the cross-section dispersion in β^{imc}, which is higher among small stocks.

The two-shock structure of the model implies that conditional expected returns are perfectly explained by the conditional betas with respect to two non-degenerate aggregate risk factors. In the model, the combination of the market portfolio with either the IMC portfolio or the $CHML$ portfolio also explains the cross-section of expected returns on the doublesorted portfolios very well unconditionally. In the data, the two factors lead to statistically significant pricing errors, particularly for small size portfolios. Moreover, the fraction of portfolio return variance captured by the factors is significantly lower than in the model. Finally, with the exception of the extreme value portfolio, the CAPM beta tend to be negatively related to the book-to-market ratio.

4.3 Response of firm-level investment to IST shocks

Since growth opportunities are not observable directly, we base our empirical tests on observable differences between firms with high and low growth opportunities. In particular, our model makes an intuitive prediction that firms with high growth opportunities, being better positioned to take advantage of positive IST shocks, should increase investment more in response to a positive investment shock than firms with low growth opportunities. While this prediction is easy to verify in our model, one would expect it to hold much more generally.

Investment response to IMC returns

We compare quantitative implications of the model for firm-level investment sensitivity to IST shocks to the empirical patterns. A subset of the following empirical results has been reported in Kogan and Papanikolaou (2010). We reproduce these results here to facilitate a comparison with the model's output.

We use returns on the IMC portfolio as our benchmark measure of IST shocks. Empirically, we estimate the sensitivity of firms' investment to IST shocks using the econometric specification

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}^{imc}_t + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_t + cX_{f,t-1} + \gamma_f + u_t, \quad (35)$$

where $i_t \equiv I_t/K_{t-1}$ is the firm's investment rate, defined as capital Expenditures (Compustat item capx) over Property Plant and Equipment (Compustat item ppent), $\tilde{R}^{imc}_t = R^{imc}_t + R^{imc}_{t-1}$ refers to accumulated log returns on the IMC portfolio and $D(x)_d$ is a β^{imc}-quintile dummy variable ($D(\beta^{imc}_{i,t-1})_n = 1$ if the firm's β^{imc} belongs to the quintile n in year $t-1$). $X_{f,t-1}$ is a vector of controls which includes the firm's Tobin's Q, its lagged investment, leverage, cash flows and log of its capital stock relative to the aggregate capital stock. Definitions of these variables are standard and are summarized in the Appendix. We standardize all independent variables to zero mean and unit standard deviation using unconditional moments. The sample covers the 1962-2007 period.

The coefficients $(a_1, \ldots, a_5)$ and $(b_1, \ldots, b_5)$ on the dummy variables measure differences in the level of investment and response of investment to IST shocks respectively. We estimate the investment response both with and without firm- and industry-level fixed effects, and both with and without controlling for commonly used predictors of firm-level investment. When computing standard errors, we account for the fact that investment may contain an unobservable firm and time component. Following Petersen (2009), we cluster standard errors both by firm and time.[3]

[Table 8]

We summarize the results in Table 8. For all specifications, firms with high β^{imc} invest more on average, and their investment rate responds more to an investment-specific shock. Thus stock-return betas with respect to IMC translate into investment-rate betas. A single-standard-deviation IMC return shock changes firm-level investment by 0.096 standard deviations on average. This number varies between 0.053 for the low-β^{imc} firms and 0.176 for the high-β^{imc} firms. The spread between quintiles is economically significant and equal to 0.123 standard deviations, which is larger than the average sensitivity of investment rate to IST shocks. In response to a single-standard-deviation IMC return shock, the level of the investment rate of low-β^{imc} firms changes by 0.9% compared to the 3.1% response by the high-β^{imc} firms. Fluctuations of this magnitude are substantial compared to the

[3]Petersen (2009) suggests following Cameron, Gelbach, and Miller (2006) and Thomson (2006) who estimate the variance-covariance matrix by combining the matrices obtained by separately clustering by firm and by time.

unconditional volatility of the aggregate investment rate changes in our sample, which is 2.4%.

When controlling for industry fixed effects and Tobin's Q, lagged investment rate, leverage, cash flows and log capital, the difference in coefficients on our proxy for the IST shock between the extreme β^{imc} quintiles of firms diminishes somewhat to 0.086, and it is at 0.089 once firm fixed effects are included in the specification.

In the model, we define firm-level investment during year t as a sum of the investment expenses incurred throughout that year, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K^*_{fs}$, where K^*_{fs} refers to the scale of a project acquired by firm f at time s.[4]

We define the book value of the firm as the replacement cost of its capital, $B_{ft} = z_t\, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t.[5]

Table 9 shows that in simulated data, a single-standard-deviation investment shock leads to an increase in firm-level investment of 0.053 standard deviations. However, as in the actual data, the impact of investment shocks varies in the cross-section of firms from 0.026 to 0.110 between the low- and high-β^{imc} firms respectively. The difference in coefficients between the high- and low-β^{imc} firms drops to 0.029 when we include Tobin's Q and cash flows in the specification. Thus, the magnitude of investment response to IST shocks (approximated by IMC returns) in the model is very similar to the empirical estimates in Table 8.

[Table 9]

Investment response to CHML returns

Our model implies that IST shocks give rise to a value factor in stock returns. Thus, it is natural to ask whether $CHML$ also predicts heterogenous investment behavior among firms

[4] We simulate the model at a weekly frequency and aggregate to form annual observations. Thus, firms can acquire multiple projects in a year.

[5] As a robustness check, we also perform simulations with the book value of the firm defined as the cumulative historical investment cost of its current portfolio of projects. Our results are essentially the same under the two definitions.

with different growth opportunities. We estimate equation (35) while replacing $\tilde{R}_t^{imc}$ with $-\tilde{R}_t^{chml} = -R_t^{chml} - R_{t-1}^{chml}$. We summarize the results in Table 10.

[Table 10]

For all specifications, the investment rate of firms with high β^{imc} responds more to $CHML$. $CHML$ does not predict average investment very well, but it does predict heterogenous investment rates in the cross-section. The spread between quintiles is economically and statistically significant, and ranges from 0.046 to 0.073 standard deviations, depending on controls. The magnitude of the difference in investment responses to $CHML$ is smaller than that to IMC. In response to a single-standard-deviation $CHML$ return shock, the level of the investment rate of low-β^{imc} firms changes by -0.3%, compared to a 1.6% response for the high-β^{imc} firms. Given that the unconditional volatility of the aggregate investment rate change in our sample is 2.4%, the difference in investment responses is economically significant. Replacing $CHML$ with the HML factor of Fama and French (1993) results in very similar results. Finally, given that in our model the unconditional correlation between $CHML$ and IMC is over 95% in absolute terms, the equivalent of Table 10 in the model is very similar to Table 9.

The results of this section provide direct support for the model's mechanism behind the value factor in stock returns and behind the differences in risk premia between value and growth firms. The fact that both IMC and $CHML$ returns forecast firm-level investment in agreement with the model's predictions highlights the impact of IST shocks on the cross-section of stock returns.

Investment response to price of equipment shocks

As an additional robustness check, we consider the quality-adjusted series of new equipment prices as an alternative proxy for IST shocks. This provides an additional test of the model's mechanism as well as a test of β^{imc} as an empirical measure of growth opportunities. One-sector macroeconomic models with IST shocks often imply that the relative price of new equipment is directly inversely related to the investment shock (e.g., Greenwood et al. (1997, 2000); Fisher (2006)). In more elaborate models (e.g., Fisher (2009); Papanikolaou (2010); Justiniano et al. (2010)) the price of new equipment is endogenous and responds negatively to positive IST shocks.

As a measure of the price of new equipment, we use the quality-adjusted price series of new equipment constructed by Gordon (1990), Cummins and Violante (2002) and Israelsen (2010).[6] To compute the price of equipment relative to consumption goods, we normalize the price of new equipment by the NIPA consumption deflator. As Fisher (2006) points out, the real equipment price experiences an abrupt increase in its average rate of decline in 1982, which could be due to the effect of more accurate quality adjustment in more recent data (e.g., Moulton (2001)). To address this issue, we remove the time trend from the series of equipment prices and define investment-specific technological changes as changes in the detrended log relative price of new equipment goods.

Specifically, we construct a de-trended equipment price series z_t by regressing the logarithm of the quality-adjusted price of new equipment relative to the NIPA personal consumption deflator on a piece-wise linear time trend:

$$p_t = a_0 + b_0 \mathbf{1}_{1982} + (a_1 + b_1 \mathbf{1}_{1982}) \cdot t + z_t \tag{36}$$

where $\mathbf{1}_{1982}$ is an indicator function that takes the value 1 post 1982. We then define investment-specific technology shocks as increments of the de-trended series

$$\Delta z_t = z_t - z_{t-1}, \tag{37}$$

Innovations in investment technology lead to a decline in the quality-adjusted price of new equipment, so we refer to a negative realization of Δz_t as a positive IST shock. Our results remain similar when, instead of using first log differences of the z_t series, we compute AR(1)

[6] Cummins and Violante (2002) extrapolate the quality adjustment of Gordon (1990) to construct a price series for the period 1943-2000. Israelsen (2010) extends the price series through 2006.

residuals, use simple first differences or apply the HP-filter to the relative price series. The series Δz_t is weakly positively correlated with the series of returns on the IMC portfolio. The historical correlation between the two series is 22.3% with a HAC-t-statistic of 2.31.

Using the new measure of investment-specific technological changes, we estimate equation (35) with Δz_t replacing $\tilde{R}^{imc}$. We present the results in Table 11. A one-standard deviation shock to Δz_t increases firm-level investment on average by 0.035 standard deviations, but the response differs in the cross-section and ranges from 0.007 to 0.069 for the low- and high-β^{imc} quintiles respectively. Thus, high-β^{imc} firms invest more in response to a decline in equipment prices, which further supports our interpretation of these firms as having more growth opportunities.

[Table 11]

In the model, z-shocks are perfectly conditionally correlated with IMC returns, and the absolute unconditional correlation is over 95%. Thus, the impact of shocks to the price of capital on investment is very similar to the impact of IMC returns. We therefore compare the model implications from Table 9 to the empirical patterns in Table 11. The two are very similar qualitatively and quantitatively.

Investment response to credit shocks

We have shown that three theoretically motivated measures of IST shocks, namely IMC, $CHML$ and the relative price of equipment, predict cross-sectional differences in investment rates between firms with high and low β^{imc}. Using the same methodology, one can estimate investment response to a variety of economic shocks affecting the willingness of firms to invest. Here we consider one important example: investment response to unexpected changes in aggregate credit or liquidity conditions, measured by aggregate credit spreads. Tightening credit conditions should have a similar effect on investment as a negative investment-specific shock, effectively leading to increased cost of investment. Thus, states with tight credit are effectively states with low real investment opportunities. This is related to the findings in Justiniano, Primiceri, and Tambalotti (2009), who estimate a medium-scale dynamic stochastic general equilibrium model (DSGE) allowing for two types of investment shocks. One of their investment shocks is a shock to the marginal efficiency of investment (MEI), which affects the marginal rate of transformation of investment goods to installed capital. Justiniano et al. (2009) find that the filtered MEI shock exhibits negative correlation with credit spreads. Their finding is related to Philippon (2009), who finds that a measure of Tobin's Q constructed using bond market data, which to a first order is equal to the corporate bond spread, predicts aggregate investment quite well. The economic source of this MEI shock is unclear, but it seems to capture time-variation in aggregate investment opportunities.

Given the above, we consider the innovation in the spread between Baa and Treasury bonds as an additional measure of the investment shock. Specifically, we use an AR(1) model of credit spread dynamics to define innovations (Δs_t) in credit spreads:

$$\Delta s_t = cr_t - 0.784\, cr_{t-1}, \tag{38}$$

where cr_t is the yield spread between Baa and Treasury bonds. We consider an increase in the aggregate credit spread ($\Delta s > 0$) as an adverse investment shock. The correlation between Δs_t and our two measures of investment shocks, $\tilde{R}^{imc}$ and Δz, is equal to -0.36 and 0.14 respectively in the 1963-2008 sample.

We estimate cross-sectional differences in the firm-level investment response to changes in credit spreads across the β^{imc}-quintiles. Specifically, we estimate equation (35) with Δs_t replacing $\tilde{R}^{imc}$. Table 12 reports the results. On average, firms increase investment when credit spreads fall, and the sensitivity of investment rate to credit shocks increases across the β^{imc}-quintiles. A single-standard-deviation positive credit shock increases the average firm-level investment rate by 0.078 standard deviations. The difference in investment rate responses between high- and low-β^{imc} quintiles of firms is statistically significant and equal to

0.064 standard deviations. With various additional controls, the latter estimate falls between 0.049 and 0.061.

[Table 12]

Tobin's Q and growth opportunities

Next, we investigate how well Tobin's Q performs as an alternative measure of growth opportunities. Tobin's Q, defined as the market value of the firm divided by the replacement cost of its capital, is commonly used as an empirical proxy for growth opportunities. The underlying intuition is well known: firms with abundant growth opportunities have relatively high market value compared to their physical assets and thus tend to have high Tobin's Q.

We estimate equation (35) using Tobin's Q instead of β^{imc}. When doing so, we drop Tobin's Q as a control. The results are qualitatively similar but noticeably weaker than those obtained with β^{imc}, as we show in Table 13. The difference in the response of the investment rate to the IMC return between high- and low- Tobin's Q firms is between 0.054 and 0.068, depending on the controls.

[Table 13]

In the model, Tobin's Q, or the market-to-book ratio, also contains information about growth opportunities. To verify this, we estimate equation (35) in simulated data. We report simulation averages of coefficients and t-statistics in Table 14. In simulated data, the effect of a single-standard-deviation investment shock on firm investment varies from 0.12 for the top Q-quintile to 0.02 for the bottom quintile. From this, we conclude that in the model Tobin's Q is a good proxy for growth opportunities. Of course, this is partly because it is measured accurately in simulations, whereas in the data it might be contaminated by measurement errors.

[Table 14]

We conclude that our model replicates the key empirical properties of firms' investment, both qualitatively and quantitatively. In particular, we find that firm's IMC beta and Tobin's Q contain information about its growth opportunities, and IMC returns predict significant heterogeneity in firms' investment rates, thus providing a useful observable proxy for IST shocks.

Additional robustness checks

We perform a number of additional robustness checks. First, it is possible that β^{imc} captures firms' financial constraints and not the differences in their real production opportunities. This possibility is consistent with our approach, since financially constrained firms, defined as firms with insufficient cash holdings and limited access to external funds, cannot take advantage of investment opportunities and as such have effectively low growth opportunities. Thus, future growth opportunities depend both on the firm's financial constraints and its real investment opportunities. To sharpen the interpretation of our empirical results, we attempt to distinguish financial constraints from real effects. We replicate our empirical analysis on a sample of firms relatively less likely to be constrained, namely firms that have been assigned a credit rating by Standard and Poor's. This restricts our sample to $1,336$ firms and $13,456$ firm-year observations. We find that our results hold in this sample, with the difference in the response of investment to $\tilde{R}^{imc}$ between the extreme β^{imc}-quintiles of 0.205. This estimate is in fact greater than the one obtained for the entire sample of firms, indicating that our findings are unlikely to be explained by financial constraints alone.

Second, we estimate β^{imc} using stock return data, while the theory suggests using returns on the total firm value. Our findings could be explained by investment of highly levered firms being relatively sensitive to investment shocks. The results in Table 2 suggest that this is not likely to be the case, as there does not seem to be systematic differences in leverage across portfolios. Furthermore, as a robustness check, we approximate β^{imc} at the *asset* level (de-lever the equity-based estimates) under the assumption that firms' debt is risk-free. We re-estimate equation (35) using de-levered β^{imc}. We find that the difference in investment responses between the high- and low-β^{imc} firms is statistically significant and equal to 0.097 and 0.118, depending on whether we use book or market leverage.

Third, we consider whether β^{imc} may be capturing inter-industry differences in technology instead of capturing meaningful differences in growth opportunities.[7] We investigate this possibility by defining β^{imc} quintiles based on the firm's intra-industry β^{imc} ranking, where we use the 30 industry classification of Fama and French (1997). We find that our results are driven by intra- rather than inter-industry variation. The difference in investment responses between the firms in high- and low-β^{imc} quintiles relative to their industry peers is statistically significant and equal to 0.121.

Finally, we check whether the sort on β^{imc} is informative because it indirectly generates dispersion in market betas. As we show in Table 2, there is a large difference in market betas between the β^{imc} decile portfolios, ranging from 0.75 for the low-β^{imc} decile to 1.61 for the high-β^{imc} decile. This dispersion in market betas is consistent with the model. Empirically, we find that intra-industry ranking of firms on market betas leads to largely insignificant response differences between the β^{mkt} quintiles. This shows that IMC betas are superior to market betas at identifying cross-sectional differences in growth opportunities.

To conserve space, we do not report the full details of the above robustness checks and refer the reader to the web Appendix.

4.4 Cash flow properties of value and growth firms

In this section we analyze the dynamics of cash flows implied by our model.[8] In particular, we compare model output to the empirical properties of cash flows patterns of value and growth firms studied by Fama and French (1995). Fama and French (1995) find that, at the time

[7]This possibility is not addressed by the controls we use in estimation, since we do not allow the loadings on quintile dummies to interact with the industry fixed effects.

[8]We focus on firms' cash flows rather than their dividends because the payout policy is not uniquely determined in the model.

of portfolio formation, growth firms are more profitable in terms of return on equity (ROE) than value firms. In the years following portfolio formation, profitability of growth firms declines whereas the profitability of value firms increases. Post-portfolio formation, earnings of growth firms grow faster than those of value firms; however, their book value grows faster as well. As growth firms acquire more capital, their average profitability, measured by the earnings-to-book ratio, declines. We thus observe mean-reversion in profitability: the profitability gap between value and growth firms is at its highest value in the year of portfolio formation, and declines steadily in the following years.

We first reproduce the findings of Fama and French (1995) for the subset of firms producing consumption goods. We sort firms into size portfolios based on their market capitalization and the ratio of book-to-market equity using NYSE breakpoints. For each portfolio, we compute the portfolio's earnings-to-book ratio and the ratio of portfolio earnings to market earnings for a period of five years before and five years after the year of portfolio formation. The portfolio's earnings-to-book ratio is defined as the sum of cash flows of firms in the portfolio divided by their lagged book value of equity. The ratio of the portfolio's earnings to market is defined as the sum of cash flows of all firms in the portfolio divided by the sum of all cash flows of firms in the consumption sector. We normalize this ratio by its value in the formation year. We construct cash flows as the sum of income before extraordinary items (Compustat item ib) plus depreciation (Compustat item dp) minus preferred dividends (Compustat item dvp).

We present our empirical findings in the top two panels of Table 15. Similar to Fama and French (1995), growth firms have a higher ratio of earnings-to-book than value firms at the time of portfolio formation (0.25 for small growth firms and 0.35 for large growth firms versus 0.11 and 0.18 for small and large value firms respectively). Over the next five years the average profitability of growth firms declines to 0.23 and 0.31 for small and large firms respectively, while the average profitability of value firms increases to 0.17 and 0.22 for small and large firms respectively. In addition, controlling for size, the earnings of growth firms grow faster than value firms, implying that the fall in the average profitability of growth firms is driven by an increase in book value rather than a fall in earnings. In the 5 years before portfolio formation, the average profitability of value firms drops, whereas the average profitability of growth firms increases.

Our model captures the above empirical patterns. In the bottom two panels of Table 15 we apply the same empirical procedures as above to simulated model output. In the model, growth firms have higher earnings-to-book than value firms at the time of portfolio formation (0.32 for small growth firms and 0.31 for large growth firms, 0.17 for small value firms and 0.21 for large value firms). Over the next five years the average profitability of growth firms declines to 0.27 and 0.26 for small and large firms respectively, whereas the average profitability of value firms rises to 0.23 and 0.24 for small and large firms respectively.

In the model, this convergence in profitability between growth and value firms arises for two reasons. First, it is due the mean-reversion in project productivity. Projects of growth firms tend to be more productive in the formation period, but this productivity gap dissipates over time. Second, growth firms tend to invest at a relatively high rate, accumulating book value relatively fast and becoming value-like over time. The end result is that, despite the declining productivity of growth firms, conditional on firm size, earnings of growth firms grow faster than those of value firms following portfolio formation. The reverse patterns are observed for value firms.

In summary, our model closely replicates the dynamics of earnings and profitability of value and growth firms. We view this as additional evidence supporting the model's mechanism for stock returns and firm investment dynamics.

5 Conclusion

In this paper we show that investment-specific technology shocks are an important driver of the cross-section of stock returns. Our theoretical model predicts that IST shocks generate heterogeneity in risk premia and co-movement in the cross-section of stock returns. The key property of the model is that firms with abundant growth opportunities benefit more from positive investment-specific shocks than firms with few growth opportunities, and therefore stock returns of high-growth firms have higher exposure to IST shocks. Thus, cross-sectional differences in growth opportunities generate differences in risk premia and co-movement among stock returns. In particular, our model gives rise to a value factor in returns and a positive relationship between expected returns and the book-to-market ratio.

Our empirical findings confirm the model's predictions. High-growth firms have lower returns on average. Moreover, investment rates of high-growth firms, as identified by our measure, are relatively high on average and more sensitive to investment-specific shocks than investment rates of low-growth firms. We calibrate our structural model and show that it can quantitatively replicate the observed empirical patterns in expected returns, firm investment behavior and cashflow dynamics.

References

Abel, A. B. (1985). A stochastic model of investment, marginal q and the market value of the firm. *International Economic Review 26*(2), 305–322.

Abel, A. B. and J. C. Eberly (1994). A unified model of investment under uncertainty. *American Economic Review 84*(5), 1369–1384.

Abel, A. B. and J. C. Eberly (1996). Optimal investment with costly reversibility. *Review of Economic Studies 63*(4), 581–593.

Abel, A. B. and J. C. Eberly (1998). The mix and scale of factors with irreversibility and fixed costs of investment. *Carnegie-Rochester Conference Series on Public Policy 48*, 101–135.

Berk, J. B., R. C. Green, and V. Naik (1999). Optimal investment, growth options, and security returns. *Journal of Finance 54*(5), 1553–1607.

Cameron, A. C., J. B. Gelbach, and D. L. Miller (2006). Robust inference with multi-way clustering. Technical working paper 0327, National Bureau of Economic Research.

Campbell, J. Y. and J. Cochrane (1999). Force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(2), 205–251.

Carlson, M., A. Fisher, and R. Giammarino (2004). Corporate investment and asset price dynamics: Implications for the cross-section of returns. *Journal of Finance 59*, 2577–2603.

Chen, L., R. Novy-Marx, and L. Zhang (2010). An alternative three-factor model. Working paper.

Christiano, L. J. and J. D. M. Fisher (2003). Stock market and investment goods prices: Implications for macroeconomics. Technical report.

Cochrane, J. H. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance 46*(1), 209–37.

Cochrane, J. H. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy 104*(3), 572–621.

Cummins, J. G. and G. L. Violante (2002). Investment-specific technical change in the U.S. (1947-2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics 5*(2), 243–284.

Eberly, J., S. Rebelo, and N. Vincent (2008). Investment and value: A neoclassical benchmark. Working paper 13866, National Bureau of Economic Research.

Fama, E. F. and K. R. French (1992). The cross-section of expected stock returns. *Journal of Finance 47*(2), 427–65.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (1995, March). Size and book-to-market factors in earnings and returns. *Journal of Finance 50*(1), 131–55.

Fama, E. F. and K. R. French (1997). Industry costs of equity. *Journal of Financial Economics 43*(2), 153–193.

Fisher, J. D. M. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy 114*(3), 413–451.

Fisher, J. D. M. (2009). Comment on 'letting different views about business cycles compete'. Technical report.

Gomes, J. F., L. Kogan, and M. Yogo (2009). Durability of output and expected stock returns. *Journal of Political Economy 117*(5), 941–986.

Gomes, J. F., L. Kogan, and L. Zhang (2003). Equilibrium cross section of returns. *Journal of Political Economy 111*(4), 693–732.

Gordon, R. J. (1990). *The Measurement of Durable Goods Prices.* University of Chicago Press.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-run implications of investment-specific technological change. *American Economic Review 87*(3), 342–362.

Greenwood, J., Z. Hercowitz, and P. Krusell (2000). The role of investment-specific technological change in the business cycle. *European Economic Review 44*(1), 91–115.

Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica 50*(1), 213–224.

Israelsen, R. D. (2010). Investment based valuation. Technical report, Indiana University.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2009). Investment shocks and the relative price of investment. Discussion Paper 7598, Centre for Economic Policy Research.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Karlin, S. and H. M. Taylor (1975). *A First Course in Stochastic Processes, Second Edition.* Academic Press.

Kogan, L. and D. Papanikolaou (2010). Growth opportunities and technology shocks. *American Economic Review 100*(2), 532–536.

Li, D. and L. Zhang (2010). Does q-theory with investment frictions explain anomalies in the cross-section of returns? *Journal of Financial Economics forthcoming*.

Li, E. X. N., D. Livdan, and L. Zhang (2009). Anomalies. *Review of Financial Studies 22*(11), 4301–4334.

Liu, L. X., T. M. Whited, and L. Zhang (2009). Investment-based expected stock returns. *Journal of Political Economy 117*(6), 1105–1139.

Lyandres, E., L. Sun, and L. Zhang (2008). The new issues puzzle: Testing the investment-based explanation. *Review of Financial Studies 21*(6), 2825–2855.

Moulton, B. R. (2001). The expanding role of hedonic methods in the official statistics of the united states. Technical report.

Papanikolaou, D. (2010). Investment shocks and asset prices. Working paper, Northwestern University.

Petersen, M. A. (2009). Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches. *Review of Financial Studies 22*(1), 435–480.

Philippon, T. (2009, August). The bond market's q. *The Quarterly Journal of Economics 124*(3), 1011–1056.

Rebelo, S. (1991). Long-run policy analysis and long-run growth. *Journal of Political Economy 99*(3), 500–521.

Santos, T. and P. Veronesi (2009). Habit formation, the cross section of stock returns and the cash flow risk puzzle. *Journal of Financial Economics forthcoming*.

Solow, R. M. (1960). Investment and technical progess. In K. J. Arrow, A. Karlin, and P. Suppes (Eds.), *Mathematical Methods in the Social Sciences*, pp. 89–104. Stanford, CA: Stanford University Press.

Thomson, S. (2006). Simple formulas for standard errors that cluster by both firm and time. Working paper, Harvard University.

Zhang, L. (2005). The value premium. *Journal of Finance 60*(1), 67–103.

Table 1: Portfolio Transition Probabilities: 5 Portfolios sorted on β^{imc}

		Sort(t-1)				
		Lo	2	3	4	Hi
	Lo	30.4%	23.1%	18.8%	15.0%	12.5%
	2	24.2%	25.2%	23.1%	18.7%	11.7%
Sort(t)	3	18.7%	23.3%	22.6%	22.3%	14.7%
	4	15.1%	17.9%	21.9%	24.3%	21.7%
	Hi	11.7%	10.5%	13.6%	19.7%	39.5%

Table 1 reports the transition probabilities across portfolio quintiles. Stocks are sorted into 5 portfolios based on β_{t-1}^{imc}. β_t^{imc} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 2: Summary Statistics: β^{imc} sorted portfolios

	DATA									
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi
$E(R_i) - r_f$	5.62	5.51	6.36	6.72	5.43	5.15	4.84	4.83	4.15	2.42
β^{mkt}	0.75	0.77	0.79	0.85	0.92	1.02	1.06	1.2	1.4	1.61
$\bar{I}/\bar{K}$	16.5%	17.0%	16.6%	17.5%	17.3%	18.6%	18.7%	19.3%	20.1%	22.4%
Tobin's Q	1.13	1.08	1.09	1.10	1.10	1.11	1.14	1.19	1.23	1.38
k/K	9.8%	16.0%	15.9%	13.0%	11.9%	10.0%	8.9%	7.0%	4.9%	2.8%
m/M	8.8%	15.7%	14.4%	12.6%	10.8%	11.0%	9.2%	7.6%	6.0%	3.9%
CASH/ASSETS	6.6%	6.0%	6.0%	6.1%	6.0%	6.3%	6.6%	7.3%	8.9%	11.4%
DEBT/ASSETS	16.1%	17.2%	17.5%	17.5%	17.6%	17.7%	17.7%	17.3%	17.2%	14.6%
R&D/SALES	1.4%	1.2%	1.2%	1.3%	1.5%	1.5%	1.8%	2.4%	3.7%	6.0%
DIV/CF	9.0%	16.6%	18.4%	18.1%	17.4%	16.9%	13.7%	10.3%	7.3%	2.8%
	MODEL									
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi
$E(R_i) - r_f$	7.50	7.29	7.03	6.78	6.50	6.20	5.83	5.41	4.84	3.99
β^{mkt}	0.82	0.87	0.89	0.92	0.95	0.98	1.02	1.06	1.11	1.19
$\bar{I}/\bar{K}$	7.0%	7.5%	7.8%	8.1%	8.4%	8.8%	9.2%	9.8%	10.8%	14.0%
Tobin's Q	1.05	1.09	1.15	1.21	1.30	1.40	1.54	1.74	2.11	3.30
k/K	18.2%	17.2%	14.9%	12.7%	10.6%	8.7%	7.0%	5.3%	3.6%	1.7%
m/M	14.3%	14.6%	13.5%	12.1%	10.8%	9.5%	8.3%	7.1%	5.8%	3.9%

Table 2 shows summary statistics for 10 portfolios of firms sorted by β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. I/K is investment over capital, CASH/A refers to cash holdings over assets, D/A is debt over assets, Q refers to Tobin's Q, k/K refers to the sum of property plant and equipment (PPE) within each portfolio as a fraction of the total PPE, m/M refers to each portfolio's market capitalization as a fraction of total market capitalization, R&D/A refers to research and development over sales, DIV/CF refers to dividends plus share repurchases over cashflows, and β^{mkt} refers to the portfolio's market beta estimated using monthly returns. When computing DIV/CF, we drop firms with negative cashflows. For the firm-level variables we report the time series averages of a portfolio's median characteristic, except for the investment rate I/K where we report the average investment rate of the portfolio defined as $\frac{\bar{I}}{\bar{K}} \equiv = \frac{\sum_{i \in P} I_i}{\sum_{i \in P} K_i}$. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 3: Model: Portfolio Transition Probabilities: 5 Portfolios sorted on β^{imc}

		Sort(t-1)				
		Lo	2	3	4	Hi
	Lo	49.1%	28.3%	14.4%	6.2%	1.9%
	2	27.6%	32.6%	24.4%	12.0%	3.4%
Sort(t)	3	14.0%	23.8%	30.7%	23.7%	8.0%
	4	6.4%	11.4%	22.8%	36.6%	22.9%
	Hi	2.7%	3.7%	7.7%	21.4%	63.6%

Table 1 plots the estimated transition probabilities across β^{imc} portfolio quintiles in the model. We simulate 2,500 firms for 50 years and repeat the procedure 1,000 times. We report median estimates of the transition probabilities across simulations. Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. β_t^{imc} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t.

Table 4: Parameter values and Calibration

Moment	Data	Model			Parameter	Value
		Median	5%	95%		
$\mu(dD/D)$	0.025	0.017	-0.056	0.073	μ_x	0.010
$\sigma(dD/D)$	0.118	0.150	0.105	0.476	σ_x	0.13
$\mu(dI/I)$	0.047	0.035	-0.042	0.067	μ_z	-0.005
$\sigma(dI/I)$	0.157	0.223	0.169	0.253	σ_z	0.035
$\rho(dI/I, dD/D)$	0.201	-0.055	-0.319	0.270	δ	0.10
r_f	0.025	0.025			r	0.025
$E(R_M) - r_f$	0.059	0.056	0.047	0.117	β_x	0.69
$\sigma(R_M)$	0.161	0.165	0.132	0.205	β_z	0.40
$E(R_{IMC})$	-0.019	-0.039	-0.090	-0.012	α	0.85
$\sigma(R_{IMC})$	0.112	0.115	0.088	0.155	σ_ε	0.20
$\rho(R_{IMC}, R_M - r_f)$	0.267	0.522	0.261	0.734	θ_ϵ	0.35
V_I/V_C	0.149	0.140	0.088	0.197	ϕ	0.07
I/K (mean)	0.202	0.128	0.073	0.251	θ_u	0.50
I/K (IQR)	0.187	0.168	0.074	0.200	σ_u	1.50
CF/K (mean)	0.293	0.248	0.187	0.281	μ_λ	2.00
CF/K (IQR)	0.361	0.223	0.161	0.252	σ_λ	2.00
Market-to-Book Eq (median)	1.569	1.988	1.558	2.627	μ_H	0.075
Market-to-Book Eq (IQR)	1.437	1.564	0.722	1.937	λ_H	2.35
$\hat{\beta}^{imc}$ (median)	0.683	0.731	0.457	1.074	μ_L	0.160
$\hat{\beta}^{imc}$ (IQR)	0.990	0.639	0.378	0.846		
$V_i/\bar{V}$ (median)	0.201	0.701	0.669	0.721		
$V_i/\bar{V}$ (IQR)	0.830	0.882	0.858	0.940		

The top panel of Table 4 shows the parameters in our calibration. The bottom panel shows sample moments. We report mean and standard deviation of dividend growth [$\mu(D_t)$, $\sigma(D_t)$], mean and standard deviation of investment growth [$\mu(I_t), \sigma(I_t)$], mean and standard deviation of excess returns on the market portfolio [$E(R_M) - r_f$, $\sigma(R_M)$], mean and standard deviation of the investment minus consumption portfolio [$E(R^{imc})$, $\sigma(R^{imc})$], and the ratio of the market capitalization of the investment sector relative to the consumption sector. Investment is real private nonresidential investment in equipment and software. We report time series averages of the mean and inter-quintile range (IQR) of the investment rate and cashflows over capital, and the median and inter-quintile range of the market to book ratio. Stock return moments are estimated over the sample 1963-2008. The moments of investment growth are estimated over the sample 1927-2008. Moments of firm-specific variables are estimated using Compustat data over the 1963-2007 period. Moments of dividend growth are from the long sample in Campbell and Cochrane (1999).

Table 5: 10 portfolios sorted on IMC beta

					Data						
β^{imc}	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$ (%)	5.62	5.51	6.36	6.72	5.43	5.15	4.84	4.83	4.15	2.42	-3.20
	(2.31)	(2.51)	(2.89)	(2.96)	(2.26)	(1.98)	(1.76)	(1.52)	(1.10)	(0.53)	(-0.80)
σ (%)	15.78	14.23	14.27	14.74	15.61	16.86	17.80	20.56	24.36	29.70	25.88
β^{mkt}	0.75	0.77	0.79	0.85	0.92	1.02	1.06	1.20	1.40	1.61	0.86
	(17.74)	(27.77)	(29.86)	(36.37)	(41.10)	(59.01)	(54.44)	(50.65)	(34.57)	(27.40)	(9.81)
α(%)	2.22	2.01	2.78	2.88	1.26	0.55	0.04	-0.61	-2.19	-4.88	-7.10
	(1.40)	(1.74)	(2.56)	(2.96)	(1.48)	(0.68)	(0.04)	(-0.53)	(-1.37)	(-2.10)	(-2.13)
R^2(%)	56.75	73.75	77.31	83.30	87.62	91.44	89.05	85.77	82.99	74.00	27.87
β^{mkt}	0.86	0.86	0.88	0.92	0.99	1.04	1.06	1.14	1.27	1.39	0.53
	(21.17)	(34.96)	(42.91)	(54.68)	(56.58)	(58.23)	(56.46)	(62.21)	(44.73)	(36.52)	(8.28)
β^{imc}	-0.48	-0.39	-0.41	-0.33	-0.29	-0.08	-0.01	0.28	0.59	1.00	1.48
	(-9.71)	(-10.67)	(-14.66)	(-7.16)	(-11.05)	(-2.66)	(-0.17)	(4.42)	(10.86)	(10.99)	(17.40)
α(%)	0.88	0.92	1.63	1.97	0.45	0.31	0.02	0.16	-0.55	-2.11	-2.99
	(0.61)	(0.97)	(2.03)	(2.56)	(0.64)	(0.40)	(0.02)	(0.13)	(-0.45)	(-1.26)	(-1.25)
R^2(%)	67.56	82.62	87.02	89.03	91.65	91.73	89.05	87.87	89.82	87.06	65.73
β^{mkt}	0.84	0.82	0.84	0.88	0.96	1.04	1.07	1.18	1.34	1.53	0.69
	(21.27)	(38.61)	(31.74)	(39.52)	(50.16)	(62.38)	(52.81)	(49.10)	(37.96)	(28.57)	(8.97)
β^{chml}	0.56	0.34	0.31	0.22	0.22	0.13	0.04	-0.15	-0.41	-0.55	-1.11
	(7.38)	(5.87)	(5.03)	(3.77)	(4.67)	(3.74)	(0.74)	(-3.28)	(-5.20)	(-6.03)	(-7.87)
α(%)	-0.05	0.64	1.53	2.01	0.38	0.02	-0.10	0.01	-0.50	-2.63	-2.58
	(-0.04)	(0.59)	(1.50)	(2.20)	(0.47)	(0.02)	(-0.12)	(0.01)	(-0.33)	(-1.25)	(-0.90)
R^2(%)	66.91	78.30	81.03	85.03	89.19	91.92	89.08	86.22	85.33	76.80	42.80
					Model						
β^{imc}	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$(%)	7.52	7.30	7.04	6.78	6.50	6.20	5.83	5.40	4.84	3.97	-3.55
	(3.72)	(3.51)	(3.30)	(3.10)	(2.89)	(2.67)	(2.42)	(2.14)	(1.81)	(1.34)	(-2.50)
σ(%)	14.36	14.81	15.16	15.55	15.99	16.47	17.04	17.75	18.72	20.39	10.53
β^{mkt}	0.83	0.87	0.89	0.92	0.95	0.98	1.02	1.06	1.11	1.19	0.36
	(23.51)	(30.68)	(38.90)	(49.83)	(68.62)	(94.21)	(105.80)	(77.51)	(49.93)	(31.73)	(5.15)
α(%)	2.71	2.25	1.82	1.38	0.93	0.44	-0.13	-0.79	-1.65	-2.99	-5.70
	(4.82)	(5.00)	(4.93)	(4.65)	(4.07)	(2.49)	(-0.85)	(-3.46)	(-4.57)	(-4.89)	(-5.05)
R^2(%)	91.27	94.69	96.56	97.81	98.70	99.21	99.30	98.91	97.63	94.49	34.59
β^{mkt}	0.96	0.98	0.99	0.99	1.00	1.01	1.01	1.02	1.02	1.03	0.07
	(53.74)	(70.54)	(82.43)	(89.78)	(96.72)	(95.19)	(91.74)	(90.18)	(86.67)	(86.23)	(2.54)
β^{imc}	-0.33	-0.27	-0.22	-0.17	-0.11	-0.05	0.02	0.10	0.21	0.38	0.71
	(-11.69)	(-12.75)	(-12.44)	(-11.09)	(-8.27)	(-3.97)	(1.03)	(6.15)	(12.03)	(21.72)	(18.42)
α(%)	0.29	0.28	0.21	0.14	0.09	0.03	-0.04	-0.08	-0.11	-0.08	-0.37
	(0.97)	(1.12)	(0.98)	(0.67)	(0.43)	(0.09)	(-0.22)	(-0.39)	(-0.48)	(-0.26)	(-0.84)
R^2(%)	97.79	98.77	99.13	99.30	99.36	99.38	99.35	99.31	99.21	99.18	91.73
β^{mkt}	0.98	0.99	0.99	0.99	1.00	1.00	1.01	1.01	1.02	1.03	0.05
	(74.21)	(85.55)	(88.05)	(87.92)	(90.66)	(90.29)	(91.70)	(95.04)	(89.51)	(65.15)	(2.11)
β^{chml}	0.75	0.59	0.47	0.36	0.23	0.10	-0.05	-0.23	-0.46	-0.81	-1.56
	(21.22)	(19.60)	(15.74)	(11.74)	(7.75)	(3.33)	(-1.73)	(-7.88)	(-14.41)	(-16.14)	(-25.47)
α(%)	-0.18	-0.05	-0.02	-0.01	0.02	0.04	0.05	0.08	0.12	0.18	0.36
	(-0.77)	(-0.35)	(-0.20)	(-0.15)	(0.02)	(0.13)	(0.25)	(0.44)	(0.63)	(0.71)	(0.97)
R^2(%)	98.74	99.11	99.24	99.29	99.32	99.35	99.37	99.39	99.29	98.92	93.43

The top panel of Table 5 reports asset-pricing tests on 10 portfolios sorted on β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. The construction of the IMC portfolio is detailed in Papanikolaou (2010). We construct the value factor in the consumption sector (CHML) as $1/2(LV - LG) + 1/2(SV - SG)$ where LV, LG, SV, LG refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t-statistics are reported in parenthesis. Estimation is done using monthly data. We report annualized estimates of mean returns and alphas by multiplying the monthly estimates by 12. The bottom panel reports the corresponding estimates for simulated data. Each simulation sample contains 2,500 firms and has a length of 50 years. We simulate 1,000 samples and report medians across simulations of coefficients and t statistics (in parenthesis). The market portfolio includes the investment and the consumption sector.

Table 6: 10 portfolios sorted on BE/ME (consumption firms)

	Data										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$ (%)	3.41	5.61	4.44	5.51	5.63	6.07	5.37	7.96	7.88	9.53	6.12
	(1.27)	(2.24)	(1.76)	(2.36)	(2.35)	(2.57)	(2.20)	(3.04)	(2.88)	(3.10)	(2.62)
σ (%)	17.36	16.26	16.35	15.12	15.53	15.31	15.84	16.95	17.71	19.91	15.12
β^{mkt}	1.01	0.97	0.97	0.87	0.88	0.85	0.86	0.91	0.95	1.05	0.04
	(42.71)	(47.36)	(49.48)	(33.94)	(33.33)	(26.10)	(23.34)	(24.28)	(23.45)	(22.54)	(0.67)
α(%)	-1.16	1.21	0.06	1.56	1.64	2.23	1.45	3.83	3.57	4.77	5.93
	(-1.06)	(1.46)	(0.08)	(1.53)	(1.59)	(1.94)	(1.11)	(2.63)	(2.35)	(2.67)	(2.41)
R^2(%)	84.92	90.01	87.94	83.34	81.05	76.97	74.84	72.66	72.52	69.90	0.18
β^{mkt}	1.04	1.01	1.02	0.93	0.95	0.92	0.92	0.99	1.01	1.07	0.03
	(42.30)	(55.17)	(56.68)	(38.83)	(35.58)	(29.38)	(27.02)	(28.67)	(24.68)	(23.92)	(0.50)
β^{imc}	-0.14	-0.17	-0.23	-0.26	-0.31	-0.30	-0.26	-0.35	-0.27	-0.09	0.04
	(-4.79)	(-7.69)	(-5.94)	(-7.60)	(-7.32)	(-7.39)	(-5.08)	(-6.51)	(-4.72)	(-1.62)	(0.59)
α(%)	-1.55	0.73	-0.57	0.85	0.79	1.39	0.74	2.86	2.82	4.51	6.06
	(-1.44)	(0.94)	(-0.76)	(0.93)	(0.86)	(1.39)	(0.61)	(2.21)	(1.99)	(2.54)	(2.44)
R^2(%)	85.66	91.30	90.22	86.68	85.56	81.53	77.89	77.52	75.23	70.16	0.29
β^{mkt}	0.95	0.96	0.99	0.91	0.94	0.92	0.97	1.03	1.07	1.18	0.23
	(46.09)	(47.59)	(52.01)	(36.98)	(38.18)	(36.72)	(45.86)	(46.07)	(40.79)	(32.56)	(5.30)
β^{chml}	-0.42	-0.10	0.17	0.27	0.38	0.50	0.66	0.74	0.76	0.82	1.24
	(-6.00)	(-2.33)	(2.87)	(5.24)	(6.64)	(11.24)	(15.61)	(19.63)	(17.64)	(9.52)	(8.73)
α(%)	0.54	1.63	-0.61	0.48	0.09	0.21	-1.21	0.81	0.48	1.44	0.90
	(0.55)	(1.99)	(-0.71)	(0.49)	(0.10)	(0.24)	(-1.45)	(0.86)	(0.47)	(1.10)	(0.51)
R^2(%)	89.59	90.34	88.77	85.84	85.89	85.51	88.72	88.14	87.42	83.65	54.43
	Model										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$(%)	3.62	4.65	5.26	5.72	6.12	6.46	6.78	7.06	7.40	7.90	4.28
	(1.21)	(1.76)	(2.12)	(2.40)	(2.66)	(2.89)	(3.11)	(3.31)	(3.53)	(3.83)	(2.98)
σ(%)	20.49	18.49	17.48	16.83	16.30	15.87	15.50	15.18	14.91	14.67	10.65
β^{mkt}	1.19	1.09	1.04	1.00	0.97	0.94	0.92	0.90	0.87	0.84	-0.34
	(29.75)	(48.67)	(75.39)	(98.94)	(87.67)	(64.14)	(48.75)	(38.70)	(31.12)	(24.01)	(-4.71)
α(%)	-3.35	-1.76	-0.85	-0.17	0.42	0.92	1.40	1.83	2.31	2.98	6.34
	(-5.16)	(-4.88)	(-3.70)	(-1.01)	(2.22)	(3.85)	(4.60)	(4.94)	(5.18)	(5.33)	(5.41)
R^2(%)	93.81	97.65	98.93	99.29	99.14	98.59	97.71	96.56	94.90	91.60	31.02
β^{mkt}	1.02	1.01	1.01	1.00	1.00	1.00	0.99	0.99	0.98	0.98	-0.04
	(78.45)	(76.73)	(81.33)	(88.22)	(92.69)	(93.95)	(90.01)	(81.18)	(69.01)	(54.13)	(-1.30)
β^{imc}	0.41	0.20	0.09	0.00	-0.06	-0.12	-0.17	-0.22	-0.26	-0.33	-0.74
	(20.54)	(9.92)	(4.84)	(0.11)	(-4.58)	(-8.78)	(-11.19)	(-11.97)	(-12.06)	(-11.42)	(-17.24)
α(%)	-0.23	-0.30	-0.23	-0.17	-0.07	0.01	0.13	0.23	0.38	0.57	0.80
	(-0.92)	(-1.22)	(-1.09)	(-0.89)	(-0.43)	(0.01)	(0.63)	(1.06)	(1.47)	(1.83)	(1.69)
R^2(%)	99.22	99.13	99.27	99.34	99.39	99.37	99.27	99.12	98.77	97.88	91.17
β^{mkt}	1.01	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	1.00	-0.01
	(70.85)	(92.98)	(89.78)	(88.13)	(85.68)	(86.75)	(87.95)	(90.54)	(95.92)	(85.91)	(-0.72)
β^{chml}	-0.89	-0.46	-0.22	-0.04	0.11	0.25	0.37	0.48	0.60	0.76	1.66
	(-19.83)	(-15.76)	(-7.33)	(-1.35)	(3.42)	(7.89)	(12.03)	(16.27)	(21.73)	(23.92)	(32.79)
α(%)	0.13	0.03	-0.01	-0.02	-0.02	-0.05	-0.02	-0.02	-0.00	0.02	-0.10
	(0.51)	(0.13)	(-0.08)	(-0.14)	(-0.17)	(-0.32)	(-0.21)	(-0.18)	(-0.10)	(0.22)	(-0.29)
R^2(%)	99.19	99.43	99.42	99.37	99.33	99.31	99.28	99.28	99.30	99.09	95.87

The top panel of Table 6 reports asset-pricing tests on 10 portfolios sorted on Book to Market Equity. The data come from Kenneth French's website. The construction of the IMC portfolio is detailed in Papanikolaou (2010). We construct the value factor in the consumption sector (CHML) as $1/2(LV - LG) + 1/2(SV - SG)$ where LV, LG, SV, LG refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t-statistics are reported in parenthesis. Estimation is done using monthly data. We report annualized estimates of mean returns and alphas. The bottom panel reports the corresponding estimates for simulated data. Market Equity equals the value of the firm, V_{ft}, and book to market equals Book Value divided by Market Equity. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. Each simulation sample contains 2,500 firms and has a length of 50 years. We simulate 1,000 samples and report medians across simulations of coefficients and t-statistics (in parenthesis). The market portfolio includes the investment and the consumption sector.

Table 7: 25 portfolios sorted on ME and BE/ME (consumption firms)

	Data											
	SMALL (20 pctile)						LARGE (80 pctile)					
BE/ME	Lo	2	3	4	Hi	Hi-Lo	Lo	2	3	4	Hi	Hi - Lo
$E(R) - r_f$ (%)	4.05	9.85	10.74	11.32	12.41	8.36	3.38	5.03	4.48	4.92	5.63	2.25
	(1.03)	(3.15)	(3.57)	(3.57)	(3.65)	(3.97)	(1.31)	(1.98)	(1.91)	(2.16)	(2.22)	(1.10)
σ (%)	25.38	20.24	19.52	20.54	22.06	13.63	16.66	16.48	15.20	14.80	16.44	13.22
β^{mkt}	1.31	1.03	0.98	1.04	1.05	-0.26	0.93	0.97	0.87	0.79	0.88	-0.05
	(27.29)	(22.26)	(19.49)	(20.45)	(18.56)	(-5.63)	(32.28)	(37.99)	(37.16)	(30.95)	(25.41)	(-1.02)
α(%)	-1.89	5.17	6.31	6.63	7.64	9.53	-0.83	0.66	0.53	1.34	1.67	2.50
	(-0.81)	(2.70)	(3.33)	(3.27)	(3.19)	(4.31)	(-0.67)	(0.67)	(0.55)	(1.13)	(1.18)	(1.18)
R^2(%)	67.20	65.54	63.34	64.07	57.26	9.09	78.19	86.29	82.92	72.14	71.22	0.43
β^{mkt}	1.21	1.01	0.97	1.02	1.05	-0.16	0.97	1.02	0.93	0.87	0.97	-0.00
	(20.96)	(19.81)	(18.58)	(19.82)	(18.88)	(-3.43)	(31.36)	(42.91)	(41.87)	(32.05)	(28.49)	(-0.04)
β^{imc}	0.47	0.12	0.05	0.08	0.03	-0.44	-0.18	-0.24	-0.27	-0.33	-0.41	-0.23
	(3.52)	(1.20)	(0.63)	(1.18)	(0.35)	(-3.53)	(-4.63)	(-7.17)	(-9.95)	(-7.52)	(-8.06)	(-3.50)
α(%)	-0.59	5.51	6.44	6.85	7.72	8.31	-1.34	-0.00	-0.23	0.40	0.52	1.86
	(-0.25)	(2.83)	(3.35)	(3.34)	(3.19)	(3.84)	(-1.11)	(-0.00)	(-0.26)	(0.37)	(0.42)	(0.90)
R^2(%)	71.14	65.96	63.41	64.24	57.28	21.13	79.59	88.72	86.66	78.09	78.58	3.96
β^{mkt}	1.27	1.06	1.05	1.12	1.17	-0.10	0.87	0.96	0.90	0.87	1.00	0.13
	(21.85)	(21.14)	(22.07)	(25.52)	(24.44)	(-3.06)	(34.51)	(36.79)	(33.50)	(37.21)	(60.53)	(4.49)
β^{chml}	-0.27	0.19	0.46	0.57	0.75	1.02	-0.41	-0.02	0.18	0.48	0.79	1.20
	(-1.41)	(1.30)	(3.87)	(5.05)	(5.96)	(10.77)	(-4.78)	(-0.33)	(3.05)	(9.09)	(21.06)	(16.82)
α(%)	-0.79	4.40	4.44	4.33	4.59	5.37	0.84	0.74	-0.21	-0.62	-1.55	-2.39
	(-0.32)	(2.22)	(2.47)	(2.43)	(2.27)	(3.38)	(0.71)	(0.73)	(-0.21)	(-0.63)	(-1.91)	(-1.92)
R^2(%)	68.13	66.25	67.81	70.19	66.64	54.65	83.07	86.30	84.08	80.70	90.04	67.32

	Model											
	SMALL (20 pctile)						LARGE (80 pctile)					
BE/ME	Lo	2	3	4	Hi	Hi-Lo	Lo	2	3	4	Hi	Hi - Lo
$E(R) - r_f$(%)	3.09	4.18	4.96	5.75	6.80	3.71	5.61	6.53	6.99	7.35	7.86	2.25
	(0.96)	(1.47)	(1.88)	(2.32)	(2.92)	(3.02)	(2.35)	(2.95)	(3.27)	(3.52)	(3.84)	(2.88)
σ(%)	21.81	19.72	18.48	17.48	16.52	8.90	16.92	15.74	15.19	14.84	14.58	6.02
β^{mkt}	1.25	1.15	1.09	1.04	0.98	-0.27	1.01	0.93	0.89	0.87	0.83	-0.17
	(24.34)	(33.78)	(44.85)	(52.84)	(42.39)	(-4.34)	(70.14)	(52.97)	(37.59)	(29.25)	(22.51)	(-3.97)
α(%)	-4.21	-2.57	-1.43	-0.32	1.09	5.30	-0.29	1.06	1.76	2.30	3.00	3.29
	(-4.91)	(-4.48)	(-3.36)	(-0.83)	(2.93)	(5.23)	(-1.43)	(3.63)	(4.58)	(4.89)	(5.06)	(4.77)
R^2(%)	90.70	94.87	96.89	97.80	97.07	27.53	98.74	97.89	96.25	94.15	90.34	26.24
β^{mkt}	1.02	1.01	1.01	1.01	1.01	-0.01	1.00	0.99	0.99	0.98	0.98	-0.03
	(64.94)	(53.61)	(49.98)	(46.57)	(41.31)	(-0.41)	(67.93)	(78.12)	(71.55)	(61.44)	(48.67)	(-0.92)
β^{imc}	0.54	0.33	0.19	0.07	-0.08	-0.62	0.00	-0.15	-0.22	-0.27	-0.34	-0.35
	(25.59)	(13.11)	(7.23)	(2.42)	(-2.28)	(-16.01)	(0.11)	(-8.79)	(-11.31)	(-11.58)	(-11.09)	(-7.40)
α(%)	-0.00	-0.00	0.08	0.26	0.60	0.60	-0.34	-0.05	0.14	0.29	0.48	0.83
	(-0.05)	(-0.03)	(0.20)	(0.68)	(1.43)	(1.25)	(-1.33)	(-0.28)	(0.56)	(1.04)	(1.43)	(1.62)
R^2(%)	99.38	98.91	98.58	98.23	97.47	88.06	98.89	99.07	98.88	98.39	97.26	65.74
β^{mkt}	1.03	1.02	1.02	1.02	1.02	-0.01	0.99	0.99	0.99	0.99	0.99	0.01
	(41.84)	(46.96)	(46.13)	(45.14)	(43.27)	(-0.20)	(68.98)	(69.00)	(69.71)	(73.45)	(68.59)	(0.09)
β^{chml}	-1.12	-0.70	-0.40	-0.13	0.20	1.32	-0.06	0.30	0.47	0.61	0.79	0.86
	(-13.75)	(-10.63)	(-6.17)	(-1.83)	(3.33)	(14.78)	(-2.00)	(7.16)	(12.27)	(17.24)	(20.08)	(14.35)
α(%)	0.17	0.16	0.13	0.18	0.32	0.15	-0.05	-0.09	-0.05	-0.07	-0.07	-0.02
	(0.46)	(0.43)	(0.32)	(0.44)	(0.76)	(0.25)	(-0.25)	(-0.41)	(-0.27)	(-0.36)	(-0.17)	(0.06)
R^2(%)	98.15	98.50	98.34	98.11	97.64	85.53	98.89	98.91	98.87	98.83	98.57	81.64

The top panel of Table 7 reports asset-pricing tests on 10 portfolios of firms in the consumption sector, sorted first on Market Equity (ME) and then on Book to Market (BE/ME), using NYSE breakpoints. Market Equity is December market capitalization from CRSP and Book Equity is item ceq from COMPUSTAT. We rebalance firms in June every year. We report results for the top and bottom quintile of Market Equity, (SMALL and LARGE). The construction of the IMC portfolio is detailed in Papanikolaou (2010). We construct the value factor in the consumption sector (CHML) as $1/2(LV - LG) + 1/2(SV - SG)$ where LV, LG, SV, LG refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t-statistics are reported in parenthesis. Estimation is done using monthly data. We report annualized estimates of mean returns and alphas. The bottom panel reports the corresponding estimates for simulated data. Market Equity equals the value of the firm, $V_{i,t}$, and book to market equals Book Value divided by Market Equity. Book Value is computed as the replacement cost of capital, $B_{it} = z_t\, x_t \sum_{j \in \mathcal{J}_{it}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{it}$ denotes the set of projects owned by firm i at the end of year t. Each simulation sample contains 2,500 firms and has a length of 50 years. We simulate 1,000 samples and report medians across simulations of coefficients and t-statistics (in parenthesis). The market portfolio includes the investment and the consumption sector.

Table 8: Response of I/K to R^{imc}: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1039	-0.0909	-0.0815	-0.0440
		(-6.18)	(-5.52)	(-4.34)	(-4.06)	(-2.30)
$D(\beta^{imc})_2$		0.0380	0.0361	0.0361	0.0339	0.0275
		(2.85)	(2.84)	(3.00)	(2.83)	(2.59)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0691	0.0658	0.0411
		(5.26)	(5.28)	(5.02)	(4.99)	(3.15)
$D(\beta^{imc})_4$		0.1794	0.1572	0.1385	0.1247	0.0676
		(8.66)	(7.69)	(8.02)	(7.35)	(4.21)
$D(\beta^{imc})_5$		0.2908	0.2448	0.2113	0.1833	0.0841
		(11.00)	(9.45)	(9.26)	(8.41)	(4.10)
$\tilde{R}^{imc}_{t-1}$	0.0959	0.0532	0.0491	0.0634	0.0592	0.0571
	(4.90)	(4.52)	(4.01)	(4.10)	(3.88)	(4.13)
$D(\beta^{imc})_2 \times \tilde{R}^{imc}_{t-1}$		0.0014	0.0023	-0.0008	0.0004	0.0027
		(0.12)	(0.21)	(-0.07)	(0.04)	(0.28)
$D(\beta^{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.0256	0.0247	0.0125	0.0132	0.0144
		(1.68)	(1.79)	(0.83)	(0.96)	(1.09)
$D(\beta^{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.0641	0.0610	0.0411	0.0413	0.0394
		(2.76)	(2.85)	(2.10)	(2.25)	(2.48)
$D(\beta^{imc})_5 \times \tilde{R}^{imc}_{t-1}$		0.1226	0.1169	0.0862	0.0855	0.0885
		(4.88)	(5.59)	(4.38)	(5.13)	(6.20)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.009	0.022	0.077	0.162	0.192	0.438
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 8 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_f + u_{ft}, \tag{39}$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t-statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 9: Model: Response to R^{imc}: sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.122	-0.119	-0.063	0.072
		(-18.34)	(-19.01)	(-5.35)	(4.14)
$D(\beta^{imc})_2$		0.026	0.024	0.015	-0.032
		(7.41)	(7.23)	(3.84)	(-4.63)
$D(\beta^{imc})_3$		0.058	0.055	0.019	-0.071
		(12.31)	(11.84)	(3.16)	(-5.65)
$D(\beta^{imc})_4$		0.113	0.109	0.036	-0.125
		(15.85)	(15.12)	(3.63)	(-5.87)
$D(\beta^{imc})_H$		0.384	0.375	0.221	-0.152
		(15.65)	(14.86)	(11.53)	(-4.18)
$\tilde{R}^{imc}_{t-1}$	0.053	0.026	0.025	0.017	-0.022
	(4.40)	(4.07)	(4.05)	(2.25)	(-3.02)
$D(\beta^{imc})_2 \times \tilde{R}^{imc}_{t-1}$		0.006	0.006	0.004	-0.005
		(2.21)	(2.18)	(1.40)	(-1.42)
$D(\beta^{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.014	0.014	0.011	-0.008
		(3.16)	(3.13)	(2.74)	(-1.48)
$D(\beta^{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.026	0.025	0.023	-0.008
		(3.86)	(3.81)	(3.55)	(-1.13)
$D(\beta^{imc})_H \times \tilde{R}^{imc}_{t-1}$		0.084	0.083	0.082	0.029
		(3.74)	(3.71)	(3.73)	(1.91)
R^2	0.003	0.025	0.026	0.037	0.074
Controls (i_{t-1})	N	N	Y	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y	Y
Controls (Q_{t-1})	N	N	N	N	Y

Table 9 shows median coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=2}^{2} R^{imc}_{t-1}$ and a vector of controls X_t, which includes lagged values of log Tobin's Q, cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + u_{ft}, \tag{40}$$

Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of β^{imc}_{t-1}. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s\in t} x_s z_s K^*_{fs}$, where K^*_{fs} is the capital of project acquired by firm f at time s. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j\in\mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t.All variables have been standardized to zero mean and unit standard deviation. We report averages across simulations of coefficients and t-statistics (in parenthesis). Standard errors are clustered by firm and time. Each simulation sample contains 2,500 firms for 50 years. We simulate 1,000 samples.

Table 10: Response of I/K to R^{hml}: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1037	-0.0908	-0.0813	-0.0445
		(-5.72)	(-5.27)	(-3.82)	(-3.63)	(-2.06)
$D(\beta^{imc})_2$		0.0380	0.0360	0.0351	0.0330	0.0274
		(2.93)	(2.96)	(3.00)	(2.86)	(2.60)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0681	0.0649	0.0407
		(5.36)	(5.36)	(4.98)	(4.94)	(3.09)
$D(\beta^{imc})_4$		0.1794	0.1569	0.1383	0.1243	0.0677
		(8.08)	(7.17)	(7.62)	(6.96)	(4.02)
$D(\beta^{imc})_5$		0.2908	0.2443	0.2128	0.1844	0.0866
		(9.74)	(8.27)	(8.78)	(7.92)	(4.00)
$-\tilde{R}^{chml}_{t-1}$	0.0097	-0.0221	-0.0246	-0.0163	-0.0187	-0.0132
	(0.40)	(-1.01)	(-1.21)	(-0.68)	(-0.83)	(-0.58)
$D(\beta^{imc})_2 \times (-\tilde{R}^{chml}_{t-1})$		0.0187	0.0208	0.0147	0.0168	0.0095
		(1.55)	(1.91)	(1.26)	(1.54)	(0.89)
$D(\beta^{imc})_3 \times (-\tilde{R}^{chml}_{t-1})$		0.0297	0.0273	0.0106	0.0106	0.0075
		(1.64)	(1.69)	(0.70)	(0.77)	(0.61)
$D(\beta^{imc})_4 \times (-\tilde{R}^{chml}_{t-1})$		0.0377	0.0352	0.0197	0.0199	0.0179
		(1.81)	(1.90)	(1.25)	(1.38)	(1.11)
$D(\beta^{imc})_5 \times (-\tilde{R}^{chml}_{t-1})$		0.0730	0.0679	0.0526	0.0510	0.0466
		(2.92)	(3.03)	(3.27)	(3.45)	(2.82)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.000	0.012	0.068	0.153	0.183	0.431
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 10 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\,(-\tilde{R}^{chml}_{t-1}) + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times (-\tilde{R}^{chml}_{t-1}) + cX_{f,t-1} + \gamma_f + u_{ft}, \quad (41)$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock, on cumulative log returns on the CHML portfolio, $\tilde{R}^{chml}_{t-1} \equiv \sum_{l=1}^{2} R^{chml}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t-statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 11: Response of I/K to equipment price shocks: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1038	-0.0908	-0.0813	-0.0442
		(-5.63)	(-5.11)	(-3.84)	(-3.62)	(-2.05)
$D(\beta^{imc})_2$		0.0380	0.0361	0.0353	0.0332	0.0274
		(2.91)	(2.91)	(3.03)	(2.85)	(2.56)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0683	0.0650	0.0406
		(5.23)	(5.26)	(5.11)	(5.09)	(3.15)
$D(\beta^{imc})_4$		0.1794	0.1570	0.1383	0.1244	0.0673
		(8.02)	(7.15)	(7.88)	(7.22)	(4.18)
$D(\beta^{imc})_5$		0.2908	0.2445	0.2124	0.1843	0.0856
		(9.27)	(7.87)	(8.72)	(7.82)	(4.20)
$-\Delta z_{t-1}$	0.0355	0.0070	0.0029	0.0170	0.0131	0.0156
	(2.13)	(0.55)	(0.24)	(0.76)	(0.64)	(0.60)
$D(\beta^{imc})_2 \times (-\Delta z_{t-1})$		0.0169	0.0155	0.0173	0.0162	0.0099
		(1.66)	(1.78)	(2.22)	(2.32)	(0.99)
$D(\beta^{imc})_3 \times (-\Delta z_{t-1})$		0.0249	0.0241	0.0240	0.0236	0.0241
		(2.40)	(2.42)	(2.85)	(2.89)	(2.46)
$D(\beta^{imc})_4 \times (-\Delta z_{t-1})$		0.0382	0.0360	0.0354	0.0340	0.0361
		(2.66)	(2.57)	(3.74)	(3.59)	(2.96)
$D(\beta^{imc})_5 \times (-\Delta z_{t-1})$		0.0623	0.0541	0.0432	0.0386	0.0367
		(3.13)	(2.85)	(2.61)	(2.33)	(2.24)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.001	0.013	0.068	0.155	0.184	0.432
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 11 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{i,t-1})_d + b_1 \, (-\Delta z_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{i,t-1})_d \times (-\Delta z_{t-1}) + cX_{it-1} + \gamma_i + u_{it}, \tag{42}$$

$i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock. Δz_t is the first difference of the detrended quality-adjusted price of investment goods divided by the consumption deflator from Cummins and Violante (2002) and extended by Israelsen (2010). Vector of controls X_t includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. β^{imc}_t is the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t-statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 12: Response of I/K to Credit Spreads: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1038	-0.0911	-0.0816	-0.0911
		(-6.27)	(-5.74)	(-4.07)	(-3.88)	(-4.07)
$D(\beta^{imc})_2$		0.0380	0.0360	0.0356	0.0334	0.0356
		(2.85)	(2.85)	(3.00)	(2.83)	(3.00)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0687	0.0654	0.0687
		(5.15)	(5.15)	(5.02)	(4.98)	(5.02)
$D(\beta^{imc})_4$		0.1794	0.1570	0.1387	0.1248	0.1387
		(7.85)	(6.92)	(7.64)	(6.91)	(7.64)
$D(\beta^{imc})_5$		0.2908	0.2444	0.2128	0.1844	0.2128
		(9.34)	(7.78)	(8.82)	(7.77)	(8.82)
$-\Delta s_{t-1}$	0.0781	0.0565	0.0540	0.0476	0.0464	0.0476
	(3.56)	(3.27)	(3.33)	(1.96)	(2.03)	(1.96)
$D(\beta^{imc})_2 \times (-\Delta s_{t-1})$		0.0046	0.0052	0.0121	0.0120	0.0121
		(0.43)	(0.51)	(1.18)	(1.20)	(1.18)
$D(\beta^{imc})_3 \times (-\Delta s_{t-1})$		0.0145	0.0157	0.0163	0.0170	0.0163
		(0.99)	(1.15)	(1.52)	(1.67)	(1.52)
$D(\beta^{imc})_4 \times (-\Delta s_{t-1})$		0.0247	0.0241	0.0192	0.0194	0.0192
		(0.94)	(0.95)	(0.97)	(0.98)	(0.97)
$D(\beta^{imc})_5 \times (-\Delta s_{t-1})$		0.0643	0.0612	0.0503	0.0491	0.0503
		(2.12)	(2.27)	(2.33)	(2.48)	(2.33)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.006	0.018	0.073	0.158	0.188	0.158
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 12 shows estimates of the regression of the ratio of firm investment to its capital stock, $i_t \equiv I_t/K_{t-1}$, on the innovation in the spread between Baa and Aaa bonds, ξ_t, and a vector of controls X_{it} which includes lagged values of log Tobin's Q, cashflows over lagged capital, log Book Equity over Book Assets, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{f,t-1})_d + b_1 \, (-\Delta s_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{f,t-1})_d \times (-\Delta s_{t-1}) + cX_{f,t-1} + \gamma_f + u_{ft}, \quad (43)$$

The innovation Δs_t is computed as the innovation of an AR(1) model on the difference between Baa and Treasury bond yields. The data on bond yields are from the St. Louis Federal Reserve. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 13: Response of I/K to R^{imc}: firms sorted by Tobin's Q

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.2491	-0.2217	-0.2588	-0.2371	-0.3211
		(-12.01)	(-11.26)	(-12.02)	(-11.47)	(-13.85)
$D(Q)_2$		0.0202	0.0107	0.0666	0.0560	0.1320
		(1.54)	(0.85)	(4.66)	(4.10)	(7.92)
$D(Q)_3$		0.1495	0.1295	0.2036	0.1847	0.2841
		(8.42)	(8.22)	(11.73)	(11.53)	(13.08)
$D(Q)_4$		0.3607	0.3246	0.3675	0.3394	0.4430
		(16.88)	(14.41)	(18.18)	(15.89)	(15.59)
$D(Q)_5$		0.7158	0.6448	0.6573	0.6064	0.7477
		(25.81)	(21.72)	(25.93)	(21.91)	(22.72)
$\tilde{R}^{imc}_{t-1}$	0.0959	0.0688	0.0635	0.0621	0.0584	0.0611
	(4.90)	(3.16)	(3.01)	(2.86)	(2.78)	(3.10)
$D(Q)_2 \times \tilde{R}^{imc}_{t-1}$		0.0147	0.0139	0.0162	0.0153	0.0239
		(0.99)	(0.99)	(0.95)	(0.93)	(1.70)
$D(Q)_3 \times \tilde{R}^{imc}_{t-1}$		0.0364	0.0346	0.0338	0.0325	0.0321
		(3.69)	(3.78)	(3.33)	(3.37)	(2.84)
$D(Q)_4 \times \tilde{R}^{imc}_{t-1}$		0.0280	0.0302	0.0338	0.0351	0.0267
		(1.36)	(1.59)	(1.64)	(1.81)	(1.53)
$D(Q)_5 \times \tilde{R}^{imc}_{t-1}$		0.0563	0.0567	0.0677	0.0670	0.0536
		(2.02)	(2.05)	(2.30)	(2.30)	(2.30)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.009	0.080	0.125	0.174	0.202	0.447
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 13 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(Q_{f,t-1})_d + b_1\, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d\, D(Q_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_f + u_{ft}, \tag{44}$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over lagged capital, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(Q_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of Tobin's Q. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t-statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 14: Model: Response to R^{imc}: sorted by Tobin's Q

	(1)	(2)	(3)	(4)
Constant		-0.162	-0.159	-0.096
		(-28.15)	(-30.37)	(-7.19)
$D(Q)_2$		0.052	0.049	0.026
		(16.97)	(16.03)	(5.37)
$D(Q)_3$		0.096	0.091	0.041
		(21.61)	(20.46)	(5.02)
$D(Q)_4$		0.154	0.149	0.066
		(24.07)	(22.56)	(-0.01)
$D(Q)_H$		0.483	0.477	0.321
		(17.64)	(17.01)	(14.04)
$\tilde{R}^{imc}_{t-1}$	0.053	0.020	0.018	0.013
	(4.40)	(3.63)	(3.64)	(2.19)
$D(Q)_2 \times \tilde{R}^{imc}_{t-1}$		0.011	0.011	0.009
		(3.75)	(3.75)	(3.14)
$D(Q)_3 \times \tilde{R}^{imc}_{t-1}$		0.018	0.018	0.016
		(4.36)	(4.36)	(3.77)
$D(Q)_4 \times \tilde{R}^{imc}_{t-1}$		0.028	0.029	0.026
		(4.76)	(4.76)	(4.34)
$D(Q)_H \times \tilde{R}^{imc}_{t-1}$		0.100	0.102	0.100
		(4.00)	(3.98)	(3.95)
R^2	0.003	0.035	0.037	0.041
Controls (i_{t-1})	N	N	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y

Table 14 shows median coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=2}^{2} R^{imc}_{t-1}$ and a vector of controls X_t which includes cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(Q_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d \, D(Q_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + u_{ft}, \tag{45}$$

Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. Tobin's Q is computed as the ratio of the market value of the firm divided by Book Value. $D(Q_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of Tobin's Q. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K_{fs}$, where K_{fs} denotes the capital of project acquired by firm f at time s. Book value is computed as the replacement cost of capital, $B_{ft} = z_t \, x_t \, \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. All variables have been standardized to zero mean and unit standard deviation. Each simulation sample contains 2,500 firms for 50 years. We simulate 1,000 samples and report medians across simulations of coefficients and t-statistics (in parenthesis). Standard errors are robust to heteroscedasticity and clustered at the firm level.

Table 15: Cashflows around portfolio formation

	DATA										
	-5	-4	-3	-2	-1	0	1	2	3	4	5
	Panel A: E_t/B_{t-1}										
SG	0.213	0.212	0.217	0.229	0.240	0.250	0.221	0.231	0.226	0.225	0.226
SV	0.158	0.151	0.144	0.131	0.116	0.112	0.127	0.153	0.162	0.170	0.171
LG	0.318	0.324	0.326	0.332	0.341	0.345	0.317	0.318	0.311	0.307	0.305
LV	0.200	0.201	0.199	0.194	0.188	0.182	0.183	0.201	0.204	0.209	0.215
	Panel B: E_t/E_t^m										
SG	0.620	0.626	0.658	0.717	0.780	1.000	1.042	1.076	1.118	1.125	1.177
SV	1.534	1.492	1.443	1.335	1.071	1.000	1.106	1.144	1.159	1.140	1.162
LG	0.895	0.895	0.919	0.926	0.968	1.000	1.007	1.019	1.060	1.081	1.101
LV	1.103	1.116	1.151	1.066	1.022	1.000	1.007	1.008	0.955	0.947	0.955
	MODEL										
	-5	-4	-3	-2	-1	0	1	2	3	4	5
	Panel A: E_t/B_{t-1}										
SG	0.240	0.243	0.246	0.254	0.280	0.320	0.296	0.284	0.276	0.271	0.269
SV	0.241	0.234	0.223	0.205	0.177	0.174	0.201	0.219	0.229	0.235	0.239
LG	0.242	0.246	0.254	0.268	0.300	0.311	0.288	0.276	0.269	0.265	0.263
LV	0.262	0.257	0.248	0.235	0.217	0.208	0.218	0.225	0.230	0.232	0.233
	Panel B: E_t/E_t^m										
SG	1.229	1.139	1.058	0.998	0.977	1.000	1.069	1.174	1.297	1.421	1.552
SV	1.354	1.327	1.279	1.183	1.023	1.000	1.124	1.181	1.207	1.222	1.226
LG	0.854	0.842	0.846	0.877	0.964	1.000	0.973	0.982	1.009	1.042	1.075
LV	1.115	1.131	1.129	1.105	1.036	1.000	1.007	0.992	0.969	0.945	0.918

In panel A, we calculate the evolution of portfolio cashflows to portfolio book equity $\bar{E}^p_{t+i}/\bar{B}^p_{t+i-1}$ for size-BM portfolios formed in June every year. The four portfolios LV, LG, SV, LG refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. For instance, for portfolio p, $\bar{E}^p_{t+i}$ equals the sum of earnings at time $t+i$ of firms assigned to portfolio p in year t. In panel B, earnings are measured relative to the values of the market portfolio constructed using only consumption firms (E^m_t), and then standardized so the ratios are 1.0 in the portfolio formation date. For instance, for portfolio p we compute $\bar{E}^p_{t+i}/\bar{E}^m_{t+i}$ and $\bar{E}^p_t/\bar{E}^m_t$ for each portfolio formation year t and lead/lag i using firms that have data in years t and $t+1$. The two ratios are then averaged separately across portfolio formation years. Our procedure closely mimics Fama and French (1995). In the data, cashflows equals operating income before extraordinary items plus depreciation minus preferred dividends. Book Equity is book common equity. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). In the model, we simulate at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. Earnings is computed as the sum of cashflows from existing projects, $E_{ft} = \sum_{j \in \mathcal{J}_{ft}} y_{jft}$. Book value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. All variables have been standardized to zero mean and unit standard deviation. We report means across simulations of coefficients and t-statistics (in parenthesis). Each simulation sample contains 2,500 firms for 50 years. In each simulation, we exclude firms with book values of zero (zero active projects).

6 Appendix

6.1 Data

Variable Definitions

Variable	Source
Investment (I)	Compustat item capx
capital (K)	Compustat item ppent
Book Assets (A)	Compustat item at
Book Debt (D)	Compustat item dltt
Book Preferred Equity (EP)	Compustat item pstkrv
Book Common Equity (EC)	Compustat item ceq
Operating cashflows (CF)	Compustat item dp + item ib
Inventories (INV)	Compustat item invt
Deferred Taxes (T)	Compustat item txdb
Market capitalization (MKCAP)	CRSP December market cap
R&D Expenditures (R&D)	Compustat item xrd
Cash Holdings (CASH)	Compustat item che
Dividends (DIV)	Compustat item dvc +item dvp
Share Repurchases (REP)	Compustat item prstkc
Tobin's Q (Q)	(MKCAP + EP + D - INV - T)/(EC+EP + D)
Quality Adjusted Price of Investment Goods	Israelsen (2010)
Consumption Deflator	NIPA

6.2 Proofs and Derivations

Proof of Proposition 1. That is, K_f is the solution to the problem:

$$\max_{K_f} A(\varepsilon_{ft}, 1) x_t K_f^{\alpha} - z_t\, x_t\, K_f. \tag{46}$$

The first order condition is

$$\alpha\, A(\varepsilon_{ft}, 1) K_f^{\alpha-1} = z_t. \tag{47}$$

■

Proof of Proposition 2. The value of growth options depends on the NPV of future projects. When a project is financed, the value added net of investment costs is

$$\left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] z_t^{\frac{\alpha}{\alpha-1}} x_t\, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}} = C z_t^{\frac{\alpha}{\alpha-1}} x_t\, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}}. \tag{48}$$

The value of growth options for firm f equals the sum of the net present value of all future projects

$$\begin{aligned} PVGO_{ft} &= E_t^{\mathcal{Q}}\left[\int_t^{\infty} e^{-r(s-t)} \lambda_{fs} C z_s^{\frac{\alpha}{\alpha-1}} x_s\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t\, E_t^{\mathcal{Q}}\left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t\, E_t\left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= z_t^{\frac{\alpha}{\alpha-1}} x_t\, G(\varepsilon_{ft}, \lambda_{ft}), \end{aligned}$$

where $E_t^{\mathcal{Q}}$ denotes expectations under the risk-neutral measure $\mathcal{Q}$, where

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(-\beta_x B_{xt} - \beta_z B_{zt} - \frac{1}{2}\beta_x^2\, t - \frac{1}{2}\beta_z^2\, t\right). \tag{49}$$

The second to last equality follows from the fact that λ_{ft} and ε_{ft} are idiosyncratic, and thus have the same dynamics under $\mathcal{P}$ and $\mathcal{Q}$.

Let $\mathbf{M}$ be the infinitesimal matrix associated with the transition density (Karlin and Taylor (1975)) of λ_{ft}:

$$\mathbf{M} = \begin{pmatrix} -\mu_L & \mu_L \\ \mu_H & -\mu_H \end{pmatrix} \tag{50}$$

The eigenvalues of $\mathbf{M}$ are 0 and $-(\mu_L + \mu_H)$. Let $\mathbf{U}$ be the matrix of the associated eigenvectors, and define

$$\Lambda(u) = \begin{pmatrix} 1 & 0 \\ 0 & e^{(-\mu_L + \mu_H)\, u} \end{pmatrix} \tag{51}$$

Then

$$E_t[\lambda_{fs}] = \lambda_f \cdot \mathbf{U}\,\Lambda(s-t)\,\mathbf{U}^{-1} \begin{bmatrix} \lambda_H \\ \lambda_L \end{bmatrix} = \lambda_f \cdot \begin{bmatrix} 1 + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H - \lambda_L) e^{-(\mu_L+\mu_H)(s-t)} \\ 1 - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H - \lambda_L) e^{-(\mu_L+\mu_H)(s-t)} \end{bmatrix} \tag{52}$$

and

$$G(\varepsilon_{ft}, \lambda_{ft}) = C \cdot E_t \left[\int_t^\infty e^{-\rho(s-t)}\, \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds \right] = C \cdot E_t \left[\int_t^\infty e^{-\rho(s-t)}\, E_t[\lambda_{fs}]\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds \right] \tag{53}$$

$$= \begin{cases} \lambda_f \left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H - \lambda_L)\, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_H \\ \lambda_f \left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H - \lambda_L)\, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_L \end{cases} \tag{54}$$

The second equality uses the law of iterated expectations and the fact that λ_{ft} is independent across firms. The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ are defined as

$$G_1(\varepsilon_t) = C \cdot E_t \int_t^\infty e^{-\rho(s-t)}\, A(\varepsilon_s, 1)^{\frac{1}{1-\alpha}}\, ds \tag{55}$$

$$G_2(\varepsilon_t) = C \cdot E_t \int_t^\infty e^{-(\rho+\mu_L+\mu_H)(s-t)}\, A(\varepsilon_s, 1)^{\frac{1}{1-\alpha}}\, ds. \tag{56}$$

$G_1(\varepsilon)$ and $G_1(\varepsilon)$ will satisfy the ODEs:

$$C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - \rho G_1(\varepsilon) - \theta_\varepsilon(\varepsilon - 1) \frac{d}{d\,\varepsilon} G_1(\varepsilon) + \frac{1}{2}\sigma_e^2\, \varepsilon \frac{d^2}{d\,\varepsilon^2} G_1(\varepsilon) = 0 \tag{57}$$

$$C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - (\rho + \mu_H + \mu_L) G_2(\varepsilon) - \theta_\varepsilon(\varepsilon - 1) \frac{d}{d\,\varepsilon} G_2(\varepsilon) + \frac{1}{2}\sigma_e^2\, \varepsilon \frac{d^2}{d\,\varepsilon^2} G_2(\varepsilon) = 0. \tag{58}$$

■

Proof of Proposition 3. The risk premium on assets in place will be determined by the covariance with the pricing kernel:

$$E_t R_{ft}^{vap} - r_f = -cov\left(\frac{dVAP_{ft}}{VAP_{ft}}, \frac{d\pi_t}{\pi_t} \right) = \beta_x \sigma_x \tag{59}$$

Similarly for growth options:

$$E_t R_{ft}^{gro} - r_f = -cov\left(\frac{dPVGO_{ft}}{PVGO_{ft}}, \frac{d\pi_t}{\pi_t} \right) = \beta_x \sigma_x - \frac{\alpha}{1-\alpha} \beta_z \sigma_z \tag{60}$$

The risk premium on growth options will be lower than assets in place as long as $\beta_z > 0$. Consequently, expected returns excess returns of the firm are a weighted average of the risk premia of its components

$$E_t R_{ft} - r_f = \frac{VAP_{ft}}{V_{ft}} (ER_{ft}^{vap} - r_f) + \frac{PVGO_{ft}}{V_{ft}} (ER_{ft}^{gro} - r_f) \tag{61}$$

■

Proof of Proposition 4. Profits accruing to the I-sector can be written as

$$\begin{aligned}
\Pi_t &= \phi\, z_t\, x_t \int K_{ft} df \\
&= \phi \left(\int A(e_{ft}, 1)^{\frac{1}{1-\alpha}} df \right) \bar{\lambda} \alpha^{\frac{1}{1-\alpha}}\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \\
&= \Gamma \cdot x_t\, z_t^{\frac{\alpha}{\alpha-1}}
\end{aligned}$$

K_{ft} is the solution to the first order condition 47. Because ε_{ft} has a stationary distribution, $\Gamma = \phi\, \bar{\lambda} \alpha^{\frac{1}{1-\alpha}} \left(\int A(e_{ft}, 1)^{\frac{1}{1-\alpha}} df \right)$ is a constant.

The price of the investment firm satisfies

$$\begin{aligned}
V_{It} &= E_t^{\mathcal{Q}} \int_t^\infty \exp\{-r(s-t)\}\; \phi \Pi_s ds \\
&= \Gamma\, E_t^{\mathcal{Q}} \int_t^\infty \exp\{-r(s-t)\}\; x_s\, z_s^{\frac{\alpha}{\alpha-1}} ds \\
&= \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} E_t^{\mathcal{Q}} \int_t^\infty \exp\left\{ \left(-r + \mu_X - \frac{1}{2}\sigma_X^2 - \frac{\alpha\, \mu_Z}{1-\alpha} + \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 \right)(s-t) + \right. \\
&\quad \left. + \sigma_X (B_{xs} - B_{xt}) + \frac{\alpha\, \sigma_Z}{\alpha - 1} (B_{zs} - B_{zt}) \right\} \\
&= \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \int_t^\infty \exp\left\{ \left(-r + \mu_X - \frac{\alpha}{1-\alpha}\, \mu_Z + \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 + \frac{1}{2}\frac{\alpha^2\, \sigma_Z^2}{(1-\alpha)^2} \right)(s-t) \right\} \\
V_{It} &= \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{\rho_I}
\end{aligned}$$

where

$$\rho_I \equiv r - \mu_X + \frac{\alpha}{1-\alpha}\, \mu_Z - \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 - \frac{1}{2}\frac{\alpha^2\, \sigma_Z^2}{(1-\alpha)^2} > 0 \tag{62}$$

■

Proof of Proposition 5. Returns on the IMC portfolio follow:

$$\begin{aligned}
R_t^I - R_t^C &= (\cdot)\, dt + \sigma_X dB_{xt} + \frac{\alpha}{\alpha-1} \sigma_Z dB_{zt} - \sigma_X dB_{xt} - \frac{\overline{PVGO}_t}{\overline{V}_t} \frac{\alpha}{\alpha-1} \sigma_Z dB_{zt} \quad (63) \\
&= (\cdot)\, dt + \frac{\overline{VAP}_t}{\overline{V}_t} \frac{\alpha}{\alpha-1} \sigma_Z dB_{zt} \quad (64)
\end{aligned}$$

whereas the return of firm f in the consumption sector is:

$$\begin{aligned} R_{ft} &= (\cdot)\,dt + \frac{VAP_{ft}}{V_{ft}}\,\sigma_X\,dB_{xt} + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\sigma_X dB_{xt} + \frac{\alpha}{\alpha-1}\sigma_Z dB_{zt}\right) + (\cdot)dB_{ft} + \sum_j (\cdot)dB_{jt} \\ &= (\cdot)\,dt + \sigma_X dB_{xt} + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\frac{\alpha}{\alpha-1}\sigma_Z dB_{zt}\right) + (\cdot)dB_{ft} + \sum_j (\cdot)dB_{jt} \end{aligned}$$

so

$$cov_t(R_{ft}, R_t^I - R_t^C) = \left(\frac{PVGO_{ft}}{V_{ft}}\right)\left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)\frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2 \qquad (65)$$

and

$$var_t(R_t^I - R_t^C) = \left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)^2 \frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2 \qquad (66)$$

which implies that the beta of firm f with the IMC portfolio is increasing in firm f's growth options. ■