The Misallocation of Finance

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Abstract

We ask whether financial assets are well-allocated in the cross-section of firms. Extending the framework of Hsieh and Klenow (2009) to the liabilities side of the balance sheet, we estimate the real losses that accrue from the cross-sectional misallocation of financial liabilities across firms. Using U.S. and Chinese data on manufacturing firms, we find significant misallocation of debt and equity. Although financial liabilities appear well-allocated in the United States, they are not in China. If China's debt and equity markets were as developed as those in the United States, China would realize gains of approximately 80% in terms of firm value added. We also back out the cost of debt and equity for each firm with our model, taking into account allocation distortions. We find that larger firms and firms located in more developed cities face markedly lower costs.

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1. Introduction

Over a decade of research in industrial organization, development, and macroeconomics has provided convincing evidence that misallocation of capital and labor is significant and can help explain why developing countries have lower total factor productivity (TFP).[1] The mere existence of such pervasive misallocation begs the question of whether the financial instruments used to purchase capital goods and fund payroll are also misallocated. Indeed, Hsieh and Klenow (2009) motivate their work on factor misallocation by appealing to distortions in access to external finance. This paper tackles the question of the cross-sectional allocation of finance directly, moving to the other side of the balance sheet to quantify the extent of financial or capital structure misallocation.

Why might there be gains in reallocating debt and equity? Under any trade-off theory of capital structure—either static or dynamic—firms weigh the benefits and costs of debt and equity to determine the optimal debt-equity ratio. Informational or agency frictions in raising funds via either security may then force firms to choose inefficient allocations. For example, a profitable firm may prefer debt to either internal or external equity finance at the margin in order to shield profits. However, the firm could face unreasonable loan covenants because it doesn't have an established relationship with a bank. Another firm might prefer equity to debt at the margin if it is already highly levered. However, potential new investors might not have sufficient information needed to offer a fair price on new equity issuance. Therefore, even keeping the total amount of debt and equity the same between these two

[1] Banerjee and Duflo (2005) offers an overview of the misallocation hypothesis in the development literature while Syverson (2011) surveys the literature from an industrial organization and macroeconomics perspective. Earlier works such as Cooley and Quadrini (2001), Hopenhayn (1992), Hopenhayn and Rogerson (1993), Robert E. Lucas (1978), and Olley and Pakes (1996) provide the theoretical underpinnings of misallocation. More recent papers such as Alfaro, Charlton, and Kanczuk (2009), Banerjee and Moll (2010), Bartelsman, Haltiwanger, and Scarpetta (2013), Buera, Kaboski, and Shin (2011), Chen and Song (2013), Hsieh and Klenow (2009), Hsieh and Klenow (2014), Jeong and Townsend (2007), Midrigan and Xu (2014), Petrin and Levinsohn (2012), Restuccia and Rogerson (2008), and Song, Storesletten, and Zilibotti (2011) use firm or establishment level microdata and heterogeneous firm models to investigate the quantitative importance of misallocation.

firms, gains are available if debt could somehow be shifted to the first firm from the second and equity could be shifted to the second firm from the first.

Another potential source of reallocation gains simply comes from moving debt or equity or both from less efficient firms to more efficient firms. This avenue for reallocation is available even if debt and equity are perfectly substitutable. There is no inherent theoretical prediction on whether inefficient allocations of the *scale* or *type* of financing is more important, but our framework is sufficiently flexible to inform this issue.

To provide quantitative evidence on the effects of financial misallocation, we turn to the empirical framework of Hsieh and Klenow (2009), who base their work on a model of cross-sectional factor allocation with differentiated products. In their model, the monopolistically competitive structure creates a downward sloping demand curve, which endogenously limits firm size even though firms have a constant returns to scale production function. More salient for their empirical investigations is the intuitive result that the marginal revenue products of each factor should be equalized across firms in an industry. Distortions in cross-sectional allocations then break this equality and adversely affect TFP. The greater the dispersion in factor marginal revenue products within a sector, the greater the potential reallocative gains.

Hsieh and Klenow (2009) use establishment level data on the manufacturing sectors in the United States, China, and India. They find that China and India could realize TFP gains of 30-50% and 40-60%, respectively, if these countries hypothetically reallocated their factors of production to achieve the U.S. level of efficiency.

Our model is directly analogous to the setup in Hsieh and Klenow (2009). While they model the factor mix that directly leads to potential distortions in TFP, we model the financial liabilities that back these factors and thus also potentially contribute to distortions in TFP. Of course, we do not literally think that different forms of finance are exactly equivalent to factors of production. However, it is quite reasonable to imagine that the stocks of debt and equity can be aggregated into a measure of benefit to the firm. Moreover, this aggregation is likely to exhibit decreasing marginal benefits for each different type of finance. *Any* model that derives an optimal interior solution for capital structure would lead to this type of structure.

In our framework, at an optimal allocation, the marginal benefits of debt and equity finance to total factor benefit should be equal across firms in a sector, and distortions in these allocations lower productivity. Given this observation, we infer distortions as deviations from the first best which is derived from first-order conditions for optimal allocations. Empirically, these deviations manifest themselves as large differences (relative to our model) in the debt-equity ratio across firms in a sector, and these large differences imply poorly developed financial markets and large gains from reallocation.

Using U.S. and Chinese data on manufacturing firms, we find significant misallocation of debt and equity. Although financial liabilities appear well-allocated in the United States, they are not in China. If China's debt and equity markets were as developed as those in the United States, 80% gains in real firm value would be available. We are also able to back out the cost of debt and equity for each firm with our model, and analyze the cross-sectional patterns. For instance, larger firms and firms located in more developed cities face markedly lower costs.

On the surface, the financial frictions aspect of our study appears most similar to Buera et al. (2011) and Midrigan and Xu (2014). However, there are substantive differences. For example, Buera et al. (2011) determine empirically that the manufacturing sector in general has a larger scale than the service sector across many different countries. This result is tangentially related to our work because larger scale industries such as manufacturing also tend to have more external financial dependence than smaller scale industries such as services. Buera et al. (2011) exploit this observation in a model in which financial frictions affect the manufacturing sector primarily on the extensive margin, as these frictions prevent talented agents from entering this sector. Midrigan and Xu (2014) is also closely related to our work because they also ask how financial frictions can affect TFP. However, their work is largely based on a calibrated model, while our work is mostly empirical. In their two sector model, they find that financial frictions lead to little intensive misallocation but substantial misallocation across sectors because the more productive sector requires a cost of entry which is difficult to pay if there are financial frictions.

Like Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), our model has nothing to say about the extensive margin. The particular type of misallocation we are after comes from the hypothesis that all forms of finance are not necessarily equivalent. Because we do recognize that more developed financial markets may cause new firms to enter, our claim is simply that we provide a lower bound on the extent of capital structure misallocation that a dynamic model with entry and exit may find.

In the finance literature, our work is related to Graham (2000), who also considers cross-sectional allocations of debt and equity. However, there are again substantive differences between our work and his. Graham (2000) computes firm-level estimates of the point at which the marginal tax benefits of debt begin to decline. A firm that incurs interest deductions to the left of this "kink" point has an inefficiently low level of debt. Estimates of this inefficiency imply large amounts of tax benefits left on the table by underleveraged firms: a puzzle. One notable feature of the framework in Graham (2000) is that he takes relative prices as given and then interprets deviations from the optimal responses to these prices as suboptimal behavior. In contrast, we assume that firms behave rationally and then use our framework to back out the price distortions that lead to the capital structure decisions that we observe in the data. This alternative perspective seems reasonable in light of the finding in Blouin, Core, and Guay (2010) that the marginal tax rate estimates of Graham (2000) imply rational behavior when the kink points are derived from more accurate estimates of future taxable income.

The rest of the paper is organized as follows. Section 2 outlines the model and shows how the model translates into an empirical framework for measuring misallocation. Section 3 describes the U.S. and Chinese data. Section 4 presents our empirical results, and Section 5 concludes. All proofs are contained in the appendix.

2. Model

Our model closely follows Hsieh and Klenow (2009), which develops a closed-economy version of Melitz (2003). This section sketches the model, with a description of the environment and technology, a statement of the optimality conditions, and a description of how to measure the benefits of reallocation. A full derivation of the model can be found in the appendix.

Environment and technology

Firms in our model are financed by debt and equity. In our model we do not distinguish between external and internal equity. Given the rarity of seasoned equity offerings (DeAngelo, DeAngelo, and Stulz 2010), and given that external equity constitutes a negligible source of funds over the last two decades in the Federal Reserve's Flow of Funds data, we view this simplification as innocuous for our purposes.

Firms use the proceeds from these financial assets to generate the real benefit of finance, and the total real benefit of finance in the economy is denoted by F. We assume that the economy consists of S sectors, and in each sector s, the real benefit of finance is given by F_s. These sectoral benefits are are combined using a Cobb-Douglas aggregator, as follows:

$$F = \prod_{s=1}^{S} F_s^{\theta_s}, \tag{1}$$

in which

$$\sum_{s=1}^{S} \theta_s = 1. \tag{2}$$

The Cobb-Douglas aggregator implies that increasing the size of any particular sector while holding the others constant has a decreasing marginal benefit.

We next assume that the real benefit of finance F_s for each sector comes from a CES aggregate of I differentiated firms, as follows:

$$F_s = \left(\sum_{i=1}^{I} F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

in which σ is the elasticity of substitution of the real benefit of finance between firms in a sector.

Finally, we assume that within an individual firm, debt and equity finance can be aggregated using a constant elasticity of substitution (CES) function that combines debt and equity to determine the real benefit of finance:

$$F_{si} = A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \tag{4}$$

In (4), A_{si} represents financial total factor benefit (TFB), $\alpha_s \in (0,1)$ provides the weight on the importance of debt in generating this benefit, and γ is the elasticity of substitution between debt and equity. It is worth noting that using a CES aggregator represents an important departure from Hsieh and Klenow (2009), who use a Cobb-Douglas production function. As will be seen below, the CES aggregator gives us flexibility to distinguish between reallocation gains that come from the amount of finance and the type. Nonetheless, this equation constitutes a strong functional form assumption on how debt and equity generate a real benefit to the firm. This equation also models the generation of benefits without explicitly modeling a production function with capital and labor. Implicitly, if firms ultimately finance their purchases of factors of production using debt and equity, then the proximate factors—capital, materials, labor, and energy—can be thought of as unmodeled intermediate inputs.

Note that certain variables are firm-specific, while others are sector-specific. For example, TFB, A_{si}, depends on both the sector and firm while the weight α_s only depends on the sector. An important feature of the real benefit function is that there is a decreasing marginal benefit to each individual financial factor input. A functional form with this property suggests a trade-off model of capital structure. Naturally, (4) also allows for perfect substitutability between different forms of finance, but even in this case, reallocation gains within a sector are possible by moving debt or equity from lower TFB firms to higher TFB firms.

Optimal allocations

Next, we define the prices that enter the firm's optimization problem. First, we let r and λ be the costs associated with using debt and equity, respectively. Second, to the extent that financial market frictions distort these costs, we also need to define "taxes" that represent reduced-form cost distortions. Specifically, $\tau_{D_{si}}$ is a "tax" on debt and $\tau_{E_{si}}$ is a "tax" on equity. Positive values indicate that firms face additional costs of finance. As noted in the introduction, these costs can arise from such frictions as informational asymmetry, agency problems, or financial sector underdevelopment. Negative values, on the other hand, suggest favorable financial relationships and/or government subsidies. We do not model the explicit mechanisms behind the distortions and assume that the they are well-encapsulated by $\tau_{D_{si}}$ and $\tau_{E_{si}}$.

Given these definitions, the nominal net benefit of finance π_{si} is given by:

$$\pi_{si} = P_{si}F_{si} - (1 + \tau_{D_{si}})\, rD_{si} - (1 + \tau_{E_{si}})\, \lambda E_{si} \tag{5}$$

Because the right side of (5), $(1 + \tau_{D_{si}})\, rD_{si} + (1 + \tau_{E_{si}})\, \lambda E_{si}$, is the cost of capital, π_{si} can be interpreted as economic value added (EVA), which is a sensible quantity to maximize in a static model. The one component of (5) that requires more explanation is the interpretation of P_{si}. It is somewhat unconventional to specify a price as a choice variable that determines the nominal benefit of finance, but this feature of our model can be justified as follows. If a firm has differentiated products, it sets prices in the product market and the benefit of finance should be related to how well it does on the productive side. Price setting for the financial side should then be related to price setting for the productive side. In other words, P_{si} is the price a differentiated firm would ask for the real benefits it is generating.

An individual firm aims to maximize π_{si} by choosing P_{si}, D_{si}, and E_{si}, taking r, λ, $\tau_{D_{si}}$, and $\tau_{E_{si}}$ as given. To solve the optimization problem, the firm first minimizes the cost of capital $(1+\tau_{D_{si}})\, rD_{si} + (1+\tau_{E_{si}})\,\lambda E_{si}$ by choosing D_{si} and E_{si} subject to a fixed real benefit $\bar{F}_{si}$. Then the firm chooses P_{si} to maximize the nominal net benefit π_{si}. The solution to the firm problem gives:

$$\begin{aligned} P_{si} &= \frac{\sigma}{\sigma-1}\frac{1}{A_{si}}\left((1+\tau_{D_{si}})\, r\left(\alpha_s + (1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}}\right)^{-\frac{\gamma}{\gamma-1}}\right. \\ &+ \left.(1+\tau_{E_{si}})\,\lambda\left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)\right)^{-\frac{\gamma}{\gamma-1}}\right), \end{aligned} \tag{6}$$

in which

$$Z_{si} = \left(\frac{\alpha_s}{1-\alpha_s}\frac{(1+\tau_{E_{si}})\,\lambda}{(1+\tau_{D_{si}})\, r}\right)^{\gamma}. \tag{7}$$

The optimality condition (6) naturally shows that price is a markup over marginal cost.

Next, we solve for the sector price P_s as a function of firm price P_{si}, by defining P_s to be the minimum price of acquiring a unit of the sector benefit. The solution is:

$$P_s = \left(\sum_{i=1}^{I} P_{si}^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \tag{8}$$

Finally, cost minimization of the Cobb-Douglas aggregator across sectors gives:

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s} \tag{9}$$

in which θ_s are the weights on each industry, and P is similarly defined to be the minimum price of acquiring a unit of the aggregate benefit. We assume that the nominal benefit of finance satisfies value additivity at the sector level and firm level, such that:

$$\sum_{s=1}^{S} P_s F_s = PF$$

and

$$\sum_{i=1}^{I} P_{si} F_{si} = P_s F_s.$$

From the derivation of P, the industry weights, θ_s, are found to be the fractions of the portfolio allocated to each industry, that is:

$$P_s F_s = \theta_s PF. \tag{10}$$

Up to this point, we have made no mention of preferences. Although preferences are not modeled explicitly, the implicit preferences that produce the results above are CES preferences over the benefit from firms in a sector and Cobb-Douglas preferences over the benefit from sectors in the economy.

Reallocation

We now demonstrate how to calculate the gains from reallocation using this framework. To find the reallocation gains, we first note that we can express financial total factor benefit,

A_{si}, as:

$$A_{si} = \eta_s \frac{(P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}}{\left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}}, \tag{11}$$

in which

$$\eta_s = \frac{1}{P_s(P_sF_s)^{\frac{1}{\sigma-1}}}. \tag{12}$$

The real benefit of finance is unobservable because prices are difficult to measure with any accuracy. However, the nominal benefit is, in principle, observable. Therefore, A_{si} can be measured using available data when written in the form of (11). As shown in the appendix, the reallocation gains are not affected if η_s is normalized to one for every sector s. Intuitively, the purpose of reallocation in a sector is to achieve the highest real benefit of finance while keeping the total amount of debt and equity the same in the sector. As a result, $D_s = \sum_i D_{si}$ and $E_s = \sum_i E_{si}$ for every sector s both before and after reallocation.

The efficient levels of debt $\hat{D}_{si}$ and equity $\hat{E}_{si}$ can be found from the first order conditions obtained from differentiating the expression for the aggregate benefits in a sector (3) with respect to these two variables. These optimality conditions are given by:

$$\hat{D}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} D_s \tag{13}$$

$$\hat{E}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} E_s. \tag{14}$$

In (13) and (14), a hat above a variable indicates the efficient level after reallocation. Once optimal debt and equity are determined, we can write the optimal real benefit of finance for an individual firm, sector, and economy respectively as:

$$\hat{F}_{si} = A_{si}\left(\alpha_s \hat{D}_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)\hat{E}_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{15}$$

$$\hat{F}_s = \left(\sum_i \hat{F}_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{16}$$

$$\hat{F} = \prod_s \hat{F}_s^{\theta_s}. \tag{17}$$

The original, prior to reallocation, real benefit of finance can be computed by replacing $\hat{D}_{si}$ and $\hat{E}_{si}$ by the original debt D_{si} and equity E_{si} in (15) above. Therefore, we can quantify the gains to reallocation by calculating the observed allocation as a fraction of the efficient allocation. Letting F denote the observed benefit of finance, these gains are given simply by $F/\hat{F}$.

It is worth discussing the role of the parameters σ and γ in the quantification of these gains. We first discuss σ, the elasticity of substitution of the real benefit of finance between firms in a sector. Potential reallocation gains depend positively on σ. To see this point, consider a case in which firms in a sector are all the same size but their allocations of debt and equity imply wide dispersion in the benefit of finance, A_{si}. In this case, moving to the efficient allocation would result in a great deal of dispersion in firm size, with the much more productive firms receiving more finance. Thus, overall reallocation gains are greater when σ is higher. Conversely, a low value for σ implies that reallocating debt and equity efficiently would result in the most productive firms getting only a modest amount of finance, so reallocation gains would also be modest.

Turning to γ, the elasticity of substitution between debt and equity, it is intuitive to see that when γ approaches infinity, debt and equity are perfect substitutes, so the potential gains from changing the debt-equity mix are zero.

We close this section by showing how to compute the price distortions, $\tau_{D_{si}}$ and $\tau_{E_{si}}$. In the case of a firm level Cobb-Douglas real benefit of finance function, there is an analytical expression for taxes as in Hsieh and Klenow (2009). However, with a more general CES function, finding the taxes involves numerically solving the nonlinear system:

$$D_{si} = \alpha_s \frac{\sigma-1}{\sigma} \frac{P_{si}F_{si}}{\left(1+\tau_{D_{si}}\right)r} \frac{1}{\alpha_s + (1-\alpha_s)\left(\frac{\alpha_s}{1-\alpha_s}\frac{\left(1+\tau_{E_{si}}\right)\lambda}{\left(1+\tau_{D_{si}}\right)r}\right)^{-(\gamma-1)}} \quad (18)$$

$$E_{si} = (1-\alpha_s) \frac{\sigma-1}{\sigma} \frac{P_{si}F_{si}}{\left(1+\tau_{E_{si}}\right)\lambda} \frac{1}{\alpha_s \left(\frac{\alpha_s}{1-\alpha_s}\frac{\left(1+\tau_{E_{si}}\right)\lambda}{\left(1+\tau_{D_{si}}\right)r}\right)^{\gamma-1} + (1-\alpha_s)} \quad (19)$$

for $\tau_{D_{si}}$ and $\tau_{E_{si}}$. After the taxes are calculated, we can then back out estimates for the cost of debt $\left(1+\tau_{D_{si}}\right)r$ and equity $\left(1+\tau_{E_{si}}\right)\lambda$ that individual firms face.

3. Data

Most of the variables in our model are readily observable, except the nominal benefit of finance, $P_{si}F_{si}$. To the extent that the proceeds from security offerings or loans are used to buy or indirectly pay for factors of production, a natural measure is value-added. One advantage of this particular measure is that it is readily computable for both public and private firms in China.

The Chinese data comes from the National Bureau of Statistics (NBS) of China and contains a panel of firms from 1999 to 2007. Firms with more than 5 million Chinese Yuan (CNY) in sales, or approximately 600,000 U.S. Dollars (USD) during this time period, are required to provide detailed financial information for the survey. The information provided includes statistics such as employment, income statement items, balance sheet items, and after 2004, cash flow items.

The data only contain firms from the mining, manufacturing, and utility sectors. We focus on the manufacturing sector because the mining sector is relatively small and operationally different from manufacturing while the utility sector is highly regulated in China. In addition, we also remove state-owned and collective corporations, which are also known as Township and Village Enterprises (TVEs). Each firm-year observation is classified as a private-corporation operating-year if the total state and collective paid-in capital is less than 50%.[2] We drop firms with negative and missing industrial value added, total liabilities, and shareholders' equity. We also drop firms with less than 5 million 1999 CNY in sales because the lack of reporting requirements for this group likely results in significant selection bias and undersampling. After applying these screens, we are left with 1,318,327 remaining firm-year observations.

We use industrial added value as our measure of the nominal benefit of finance, $P_{si}F_{si}$, total liabilities for D_{si}, and shareholders' equity for E_{si}. This measure of equity is the stock of book equity and thus external equity finance and retained earnings. These variables are all directly available in the NBS data. Note that we use total liabilities instead of debt, and the reason behind this choice is twofold. First, debt is not a separately available data item in the NBS survey, and second, using total liabilities can offer more robust estimation because there are almost no firms with zero liabilities. For a CES function without an infinite elasticity of substitution, the marginal benefit of a factor input is unbounded at zero, and this property of the CES aggregator would present omitted observation problems in the estimation. These choices for debt and equity imply that the sum of the two equals total assets.

Summary statistics for this sample are in Table 1. Panel A reports various statistics in the sample stratified by size, where we use a density breakdown of 5%, 10%, 15%, 20%, 20%, 15%, 10%, 5%, so the cumulative density breakdown is 0-5%, 5-15%, 15-30%, 30-50%, 50-70%, 70-85%, 85-95%, 95-100%. This partition is quite attractive because the mean of total assets roughly doubles for each size group, with the exception of the largest size group,

[2]This type of classification is often used since official corporate ownership registrations can lag several years behind actual ownership changes. For instance, see Guariglia, Liu, and Song (2011) for a similar approach.

which is reflective of the well-known right skewness of the firm size distribution.

In Panel A, we find two patterns in the data of interest. First, the ratio of liabilities to assets varies little across the different size classes, with the larger firms having only slightly lower leverage. Second, it is clear from comparing the value-added and assets columns, that the smaller firms use their assets far more efficiently to produce value-added. This pattern, juxtaposed with the similarity in leverage across size classes, points strongly to potential misallocation of capital structure, as firms with different productivities ought to have different capital structures (Hennessy and Whited 2005).

Panel B presents summary statistics by year. Here, we see that the ratio of liabilities to assets is little changed over the sample period. Interestingly, the average size of firms has shrunk somewhat from the beginning to the end of the sample. However, these slightly smaller firms are creating 60% more value added at the end of the sample than at the beginning.

For U.S. data, we use Compustat and correspondingly keep only the manufacturing sector with Standard Industrial Classification (SIC) codes between 2000 and 3999. We also drop firms with missing and negative data, keep only the years from 1999 to 2007 inclusive.

Value-added is computed in the same manner as in Imrohoroglu and Tuzel (2014). First, labor costs are estimated from the NBER-CES Manufacturing Industry Database by multiplying the number of employees Compustat variable (EMP) by the mean wage per employee (PAY/EMP) in the firm's 3-digit SIC industry. Value-added is operating income before depreciation (OIBDP) plus the imputed wages. For D_{si} and E_{si}, we use total liabilities and shareholders' equity.

We partition the firms by size according to the same densities as before. Of course, Compustat is a data set of U.S. public firms, so the average firm is much larger. However, the observed patterns by firm size can still be informative. Table 2 provides summary statistics by firm size and by year. In Panel A, we again present eight firm size categories, and the last three Chinese firm size categories are approximately equivalent to the first three U.S. firm size categories. In addition to the results above, we find a hump-shaped relation between size and leverage for U.S. firms, with the medium-sized firms having the highest leverage. Also in contrast to the Chinese firms, small and large U.S. firms have approximately the same ratio of value-added to assets.

Panel B shows two more differences between the Chinese and U.S. firms. First, the U.S. firms grow from the beginning to the end of our sample. Also, firms become more leveraged over time.

4. Results

We now use the framework developed in Section 2 to quantify the extent of capital structure misallocation. Before we present our results, we need to discuss the normalization of several parameters. First, we set the elasticity of substitution for the real benefit of finance between firms in an industry to $\sigma = 1.77$. Although this choice provides conservative estimates of the reallocation gains, we also explore below the robustness of our results to this assumption. Next, the pre-distortion cost of debt and equity is set to $r = \lambda = 0.1$. It should be emphasized that our reallocation results are not sensitive to this normalization because firms only care about the after tax cost of debt $(1 + \tau_{D_{si}})\, r$ and equity $(1 + \tau_{E_{si}})\, \lambda$. Changing r and/or λ only changes the interpretation of the tax distortions relative to the base cost of debt and equity. The weight on the importance of debt in a sector is set to $\alpha_s = rD_s^{1/\gamma}/(rD_s^{1/\gamma} + \lambda E_s^{1/\gamma})$ which is the value when there are no tax distortions. This assumption is innocuous, as raising r or λ by 0.05 has a less than 1% impact on the overall reallocation gains. Next, we set the elasticity of substitution between debt and equity, $\gamma = 2$, and again we explore below the robustness of our results to varying the value of this parameter. Finally, we need to define sectors. Here, we use 3-digit industry classifications from the Chinese NBS and 3-digit Standard Industrial Classification (SIC) industries from Compustat.

Table 3 contains estimates of the potential gains from the reallocation of finance across firms in a sector. Each row corresponds to a separate year. The first column shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal US allocation: $F_{\mathrm{US}}/\hat{F}_{\mathrm{US}}$. The second column shows the corresponding percentage gain from moving from the observed to the optimal allocation. The next two columns present analogous calculations for Chinese firms. The two columns after that show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\mathrm{China}}/\hat{F}_{\mathrm{China}})(\hat{F}_{\mathrm{US}}/F_{\mathrm{US}})$, and the corresponding percentage gains, in other words, the percentage gains available if China's debt and equity markets were as developed as those in the United States. The last two columns provide a breakdown of misallocation into the misallocation due to scale and due to misallocation of factors, holding scale fixed.

Bootstrapped standard errors are in parentheses under the parameter estimates. All of the standard errors are quite small. This result makes sense inasmuch as the figures that we present are all essentially means, which can be estimated with a great deal of precision with several thousand data points.

In the first two columns, we see that U.S. public firms stand to gain about 13-18% in moving to an optimal allocation. The potential gains appear to be less during the boom periods and greater during the recession during the early part of our sample. This result makes sense inasmuch as financial frictions are generally regarded to be more severe in recessions. In the next two columns, we see, somewhat surprisingly, the efficiency of the allocation of debt and equity appears to worsen over our sample period in China. This phenomenon can be mostly attributed to the expansion of the NBS survey in the 2004 Industrial Census, which picked up firms that were left out in previous annual surveys.[3] When we restrict our sample to firms that are in the NBS survey before and after the 2004 Industrial Census, the pattern of increasing misallocation is lessened.

[3]Brandt, Van Biesebroeck, and Zhang (2014) discuss the impact of the 2004 Industrial Census on the NBS survey sample.

The available reallocation gains appear enormous. We find that value-added could potentially be increased by over 100% if the Chinese firms were to move to an efficient allocation. Although these figures seem large, they are of the same order of magnitude as the estimated gains found in Hsieh and Klenow (2009) regarding capital and labor allocations.

To put these results in more perspective, we now examine the last four columns in Table 3. The column labeled "relative fractional benefit" shows the efficiency gains in China *relative* to the efficiency gains in the United States. This comparison is motivated by the observation in Hsieh and Klenow (2009) that because a simple static model based on the framework in Melitz (2003) is likely to be misspecified, a researcher is likely to observe positive potential gains even when allocations are efficient. This observation is particularly applicable in our context of financial misallocation because U.S. financial markets are highly developed. Thus, by comparing the potential gains in China relative to the observed gains in the United States, we isolate the potential gains in China relative to an assumed efficient allocation. Here, the results are more modest. We find potential gains of approximately 70% before the expansion of the NBS survey, and of approximately 100% after the expansion.

To understand whether these gains come from the amount of finance available to Chinese firms or to the type of finance, we compare the relative fractional benefit to a case in which we set $\gamma = \infty$. If $\gamma = \infty$, then the type of finance does not matter for the aggregate benefit of finance because debt and equity are perfect substitutes. This exercise produces an interesting result. We find that the majority of the potential reallocation gains come from the misallocation of scale. Before the expansion of the NBS survey, we find that only approximately 8% of the gains could be realized by reallocating the type of finance. After the expansion, this figure rises to between 11.7% to 14.1%. This result is interesting because it means that the access to finance in general is behind the large potential TFP losses in China.

We next examine the robustness of the reallocation results to the calibration of the parameters γ and σ, the elasticity of substitutions between debt and equity and elasticity of substitution between firms in an industry, respectively. These results are in Table 4. First, we find that allowing γ to range between 1.5 and 10 has a negligible effect our estimates of percent gains in both the United States and China. By construction, γ has no effect on the misallocation of scale. However, changing γ does materially alter our estimates of the percent of of gains that come from reallocating the type of finance, with lower levels of γ corresponding to more gains. Thus, our estimates in Table 3 can be thought of as upper bounds, and this interpretation leaves intact our general qualitative result that the vast majority of potential gains from the the misallocation of scale.

While varying γ has little effect on our estimated reallocation gains, varying σ does. We find that the estimated gains increase sharply when we increase σ. Intuitively, the firm size distribution becomes excessively skewed if σ is too large because, in this case, all resources flow to the most productive firms. In other words, if one were to pick the most productive Chinese firms in a sector and give them all the resources, the gains would be large because of the substantial dispersion in productivity.

However, as we have argued above, the calibration of σ is conservative if σ is chosen to be on the low end. We now expand on these arguments. One way to discipline the choice of σ is to calculate the distributions of debt and equity when the allocation is efficient and compare these distributions to the realized distributions in the data. For the United States, when $\sigma = 1.77$, the standard deviation of the efficient size distribution is exactly the same as the observed standard deviation, that is, the standard deviation of $\hat{D}_{si} + \hat{E}_{si}$ equals the standard deviation of $D_{si} + E_{si}$. We argue that this choice of σ is very conservative, because if σ is lower, the efficient size distribution would be more compressed than the observed. However, when σ rises above 2, the efficient size distribution becomes more and more stretched out and the reallocation gains become quite large.

We now examine the implications of our estimates for the cross-sectional distribution of firm size. Recall that because of downward sloping demand, each firm has a well-defined optimal size, with an optimal financing mix. Deviations of the financing amount and mix from the optimal allocation therefore impact firm size, so comparing the distributions of firm size under the actual and efficient allocations is a useful way to quantify misallocation. Figure 1 illustrates this idea with plots of the observed and efficient firm size distributions for the United States and China. We compute observed firm size as $\log(D_{si} + E_{si})$ and efficient firm size as $\log(\hat{D}_{si}+\hat{E}_{si})$. Panel A shows that the efficient U.S. firm size distribution exhibits approximately as much dispersion as the actual distribution. Of course, this result is to be expected, given our calibration of σ. Nonetheless, we do observe somewhat less dispersion in the actual distribution than in the efficient distribution, especially in the left tail. This result points to inefficient quantities of financing for the smallest U.S. firms. In contrast, in Panel B, we see that the efficient firm size distribution for China has a significantly fatter left tail, with far too few small Chinese firms. These size distortions in turn stem from misallocation of either the amount or mix of financing to the affected firms.

Although the plots in Figure 1 show the firm size distributions before and after reallocation, they do not illustrate the individual changes in firm size that happens with reallocation. Figure 2 shows these movements via heat maps. Panel A contains the heat map of a three-dimensional histogram in which the observed U.S. firm size distribution is on the x-axis and the efficient U.S. firm size distribution is on the y-axis. The legend for the z-axis heat map is located at right of the map and represents the number of observations in each bin. Similarly, Panel B contains the heat map for China. From the heat maps, we can see that U.S. firms are concentrated along the 45 degree line, where firm size before and after reallocation is the same. In contrast, Chinese firms are much more spread out, reflecting the substantial efficiency gains available from reallocation. Interestingly, both heat maps are more concentrated towards the top right than towards the bottom left. This pattern indicates that small firms are more likely to suffer from financial misallocation than large firms.

Next, we move on to the distortions in the prices of debt and equity that we can back out of our estimation. Table 5 summarizes the post-distortion cost of debt $(1 + \tau_{D_{si}})\,r$ and equity $(1 + \tau_{E_{si}})\,\lambda$ by year, again under the assumptions that $\gamma = 2$ and $\sigma = 1.77$. Panel A contains means and Panel B contains medians. In Panel A, we find that the costs of debt and equity fall over the sample period in the United States. In contrast, these costs rise in China over the same time period. This pattern reinforces the result in Table 3 that points to greater misallocation after 2004, when the NBS survey samples more firms. These extra firms exhibit more misallocation and consequently greater costs of debt and equity. Finally, the figures in Panel B are uniformly much smaller than those in Panel A, especially for the Chinese firms. This result points to extreme right skewness in the distribution of the cost of finance, implying that some firms are likely effectively barred from financial markets.

Table 6 is structured exactly as Table 5, except the sample is stratified by size instead of year. In the United States, the average cost of debt is substantially lower for large firms, while the cost of equity displays no clear pattern across firm size. This second result is consistent with the well-documented lack of a size premium in equity markets in recent years. In contrast, both the costs of debt and equity are dramatically lower for large Chinese firms in comparison to small Chinese firms.

Beyond analyzing the cost of debt and equity by year and firm size, we run two descriptive OLS regressions on our sample of Chinese firms to examine how these costs vary by firm characteristics. Specifically, we regress the cost of debt and equity respectively on location, state investment, firm size, time, and firm age. *Location* is a dummy variable that equals 1 if a firm is located in Beijing, Shanghai, Shenzhen, or Guangzhou and 0 otherwise. These four Chinese cities are also known as first tier cities and are the most developed in China. *State investment* is a dummy variable that equals 1 if a firm has a non-zero percentage of paid-in-capital from state sources and 0 otherwise. However, note that all firms in our sample are private. The dummy variable just indicates whether there is any state investment. Next, *size* is the log of total assets measured in 2005 CNY. Finally, *time* is a simple linear time trend, and *age* is firm age in years and is censored at 100 years.

Table 7 presents the results. We find that costs are significantly lower for larger firms, and this result confirms our cross-sectional sorts by size in Table 6. Firms operating in first tier Chinese cities also face lower costs. Surprisingly, firms with non-zero state investment actually face slightly higher costs on average. It is important to note that this result is *conditional* on firm size. If we break down the total set of firms into those with and without state paid-in-capital, we find that firms with state paid-in-capital have lower costs. However, these firms are also significantly larger, so the effect of state investment on costs reverses once we control for size. Next, the positive coefficient on the time trend reflects the increasing costs also evident in Table 5. Finally, firm age is associated with a statistically significant but tiny decrease in the cost of debt, as well as a tiny increase in the cost of equity.

Table 8 offers another robustness check of the model. In all of our work thus far, we have measured the nominal benefit of finance using value-added. A natural alternative measure is the sum of the market values of debt and equity. Of course, we cannot use this measure in our sample of Chinese firms, as most of these firms are not publicly traded. However, we look at the measure in our sample of U.S. firms. We find that overall reallocation gains are similar in magnitude to those in Table 3. One exception can be found during the dot-com boom, in which we find more misallocation.

5. Conclusion

This paper entertains the possibility that finance may be misallocated in the cross-section of firms. We explore this hypothesis using a tractable model of differentiated firms based on Hsieh and Klenow (2009). In our framework, the optimal allocation of debt and equity equates the marginal benefit of these two securities within an industry. Thus, observed dispersion in the marginal benefit of debt and marginal benefit of equity is symptomatic of misallocation.

Our evidence points only to modest potential reallocation gains in the United States, with American firms standing to gain only 13-18% in terms of aggregate real firm value if they were to move to an efficient allocation. Our results are much more dramatic for China, where firms stand to gain over 100% from moving to the efficient allocation. If China was able to achieve the more reasonable U.S. level of efficiency, gains of 70-100% would still be possible. When we break this figure down by the amount versus the type of finance, we find that nearly all of this figure can be attributed to the amount of finance and little to the mix of securities used to fund a firm's operations.

Our work sheds light on the interaction between productive and financial allocation and the puzzling persistence of productive misallocation. Here, Banerjee and Moll (2010) show in a model that productive misallocation along the intensive margin should disappear within several years. Yet Hsieh and Klenow (2009) show that this type of misallocation has not dissipated over time. The financial misallocation we investigate in this paper may be related to productive misallocation and can help explain this puzzle. For instance, debt financing might be more conducive to capital investment, and if financial frictions are persistent, the misallocation of productive factors should be as well. Overall, we believe that productivity losses can result both from the misallocation of debt and equity and from the misallocation of capital and labor. We leave to further work for an analysis of these forms of misallocation together in a unified framework.

Appendix

Aggregate price

We begin by solving for the aggregate price P as a function of sector price P_s, where P is defined to be the minimum price of acquiring a unit of the aggregate benefit. The minimization problem is mathematically stated as:

$$\min_{F_s} \left\{ \sum_s P_s F_s \right\}, \tag{20}$$

subject to:

$$\prod_s F_s^{\theta_s} = \bar{F}. \tag{21}$$

The Lagrangian is:

$$\mathcal{L} = -\sum_s P_s F_s + M \left[\prod_s F_s^{\theta_s} - \bar{F} \right], \tag{22}$$

where M is the Lagrange multiplier. The first-order condition with respect to F_s gives:

$$P_s = M\theta_s \frac{\prod_s F_s^{\theta_s}}{F_s}, \tag{23}$$

which simplifies to:

$$\frac{P_s F_s}{\theta_s} = PF \tag{24}$$

because $M = P$. After aggregation of sectors in the economy, we can write the aggregate price as a function of sector price:

$$P = \prod_s \left(\frac{P_s}{\theta_s}\right)^{\theta_s}. \tag{25}$$

Sector price

In a similar fashion, we can solve for the sector price P_s as a function of firm price P_{si}, where P_s is defined to be the minimum price of acquiring a unit of the sector benefit. The minimization problem is mathematically stated as:

$$\min_{F_{si}} \left\{ \sum_i P_{si} F_{si} \right\}, \tag{26}$$

subject to:

$$\left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \bar{F}_s. \tag{27}$$

The Lagrangian is:

$$\mathcal{L}_s = -\sum_i P_{si} F_{si} + M_s \left[\left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \bar{F}_s \right] \tag{28}$$

where M_s is the Lagrange multiplier. The first order condition with respect to F_{si} gives:

$$P_{si} = M_s \left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} F_{si}^{-\frac{1}{\sigma}} \tag{29}$$

which simplifies to:

$$P_{si}^{\sigma} F_{si} = P_s^{\sigma} F_s \tag{30}$$

because $M_s = P_s$. After aggregation of firms in a sector, we can write the sector price as a function of firm price:

$$P_s = \left(\sum_i P_{si}^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \tag{31}$$

Firm's problem

A firm i in sector s chooses price P_{si}, debt D_{si}, and equity E_{si} to maximize the nominal net benefit of finance π_{si}. The debt and equity decision aims to minimize the total cost of finance for a given level of real benefit $\bar{F}_{si}$, and can be separated from the price decision. Formally, the minimization problem is:

$$\min_{D_{si}, E_{si}} \left\{ \left(1 + \tau_{D_{si}}\right) r D_{si} + \left(1 + \tau_{E_{si}}\right) \lambda E_{si} \right\}, \tag{32}$$

subject to:

$$A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1 - \alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} = \bar{F}_{si}. \tag{33}$$

After setting up the Lagrangian and taking the first order conditions respect to D_{si} and E_{si}, we arrive at the following optimal debt-equity ratio:

$$\frac{D_{si}}{E_{si}} = \left(\frac{\alpha_s}{1 - \alpha_s} \frac{\left(1 + \tau_{E_{si}}\right) \lambda}{\left(1 + \tau_{D_{si}}\right) r} \right)^{\gamma}. \tag{34}$$

To simplify notation, let:

$$Z_{si} = \left(\frac{\alpha_s}{1 - \alpha_s} \frac{\left(1 + \tau_{E_{si}}\right) \lambda}{\left(1 + \tau_{D_{si}}\right) r} \right)^{\gamma} \tag{35}$$

so that the optimal ratio can be rewritten as:

$$\frac{D_{si}}{E_{si}} = Z_{si}. \tag{36}$$

Debt and equity can thus be expressed as linear functions of the real benefit, as follows:

$$\begin{aligned} D_{si} =& \frac{\bar{F}_{si}}{A_{si}} \left(\alpha_s + (1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}} \right)^{-\frac{\gamma}{\gamma-1}} \\ E_{si} =& \frac{\bar{F}_{si}}{A_{si}} \left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) \right)^{-\frac{\gamma}{\gamma-1}} \end{aligned} \tag{37}$$

Then using the above expressions for debt and equity, the minimum cost function becomes a function of the fixed real benefit $\bar{F}_{si}$:

$$\begin{aligned} C(\bar{F}_{si}) &= (1+\tau_{D_{si}})\, r D_{si} + (1+\tau_{E_{si}})\, \lambda E_{si} \\ &= C_{si} \bar{F}_{si}, \end{aligned} \tag{38}$$

where

$$\begin{aligned} C_{si} =& \frac{1}{A_{si}} \Bigg((1+\tau_{D_{si}})\, r \left(\alpha_s + (1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}} \right)^{-\frac{\gamma}{\gamma-1}} \\ & + (1+\tau_{E_{si}})\, \lambda \left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) \right)^{-\frac{\gamma}{\gamma-1}} \Bigg). \end{aligned} \tag{39}$$

Next, we choose P_{si} to maximize the nominal net benefit of finance, that is:

$$\max_{P_{si}} \{\pi_{si}\} = \max_{P_{si}} \{P_{si} F_{si} - C_{si} F_{si}\}. \tag{40}$$

Recall from the sector price derivation that firm real benefit is a function of sector price, firm price, and sector real benefit, $F_{si} = \left(\frac{P_s}{P_{si}}\right)^{\sigma} F_s$. Therefore, the firm's real benefit is just a function of price once the optimal debt-equity ratio is computed, and the firm faces a downward sloping demand curve. The maximization problem is bounded due to downward sloping demand even though the firm has constant returns to scale. From the first order condition on price we find:

$$P_{si} = \frac{\sigma}{\sigma - 1} C_{si}. \tag{41}$$

Note that the price is a fixed markup over marginal cost and a higher elasticity of substitution between firms in a sector lowers the price the firm can charge for the real benefit it is generating.

Taxes

To solve for the tax distortions, the nominal benefit of finance should first be written as:

$$P_{si} F_{si} = P_s F_s^{\frac{1}{\sigma}} F_{si}^{\frac{\sigma-1}{\sigma}}. \tag{42}$$

The marginal nominal benefit of debt must equal the marginal nominal cost of debt for the maximizing firm, so the first order condition with respect to D_{si} gives:

$$P_s F_s^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} F_{si}^{-\frac{1}{\sigma}} A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1 - \alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1} - 1} \alpha_s D_{si}^{-\frac{1}{\gamma}} = (1 + \tau_{D_{si}}) r \tag{43}$$

which simplifies to:

$$D_{si} = \alpha_s \frac{\sigma - 1}{\sigma} \frac{P_{si} F_{si}}{(1 + \tau_{D_{si}}) r} \frac{1}{\alpha_s + (1 - \alpha_s) \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_{E_{si}})\lambda}{(1+\tau_{D_{si}})r} \right)^{-(\gamma-1)}}. \tag{44}$$

Similarly, the first order condition with respect to E_{si} simplifies to:

$$E_{si} = (1 - \alpha_s) \frac{\sigma - 1}{\sigma} \frac{P_{si} F_{si}}{(1 + \tau_{E_{si}}) \lambda} \frac{1}{\alpha_s \left(\frac{\alpha_s}{1-\alpha_s} \frac{(1+\tau_{E_{si}})\lambda}{(1+\tau_{D_{si}})r} \right)^{\gamma-1} + (1 - \alpha_s)}. \tag{45}$$

The taxes for each firm can be backed out by solving the nonlinear system of two equations

(44) and (45) and two unknowns $\tau_{D_{si}}$ and $\tau_{E_{si}}$.

Efficient allocation

We now turn to the derivation of the efficient allocation in a sector. Under the efficient allocation, total debt and total equity in a sector are kept the same, but debt and equity are reallocated across firms in a sector to maximize sector real benefit. The debt-equity ratio $Z_{si} = \frac{D_s}{E_s} = Z_s$ can be shown to be the same for all firms i in sector s when debt and equity are reallocated to achieve efficiency. The real benefit of finance can then be written as a function of $\hat{D}_{si}$:

$$\hat{F}_{si} = \left(\alpha_s + (1-\alpha_s)Z_s^{-\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} A_{si}\hat{D}_{si}, \tag{46}$$

where a hat above a variable indicates the efficient level after reallocation. The Lagrangian is:

$$\hat{\mathcal{L}}_s = \left(\sum_i \left(\left(\alpha_s + (1-\alpha_s)Z_s^{-\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} A_{si}\hat{D}_{si}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} + \hat{M}_s\left[\sum_i \hat{D}_{si} - D_s\right] \tag{47}$$

where $\hat{M}_s$ is the Lagrange multiplier. The first order condition with respect to $\hat{D}_{si}$ and $\hat{D}_{sj}$ for firms i and j respectively rearranges to:

$$\left(\frac{\hat{D}_{si}}{\hat{D}_{sj}}\right)^{-\frac{1}{\sigma}} = \left(\frac{A_{sj}}{A_{si}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{48}$$

After aggregation, the expression above can be simplified to:

$$\hat{D}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} D_s. \tag{49}$$

The optimal equity allocation can be similarly derived as:

$$\hat{E}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} E_s. \tag{50}$$

The real benefit F_{si} is assumed to be unobservable. However, A_{si} can be expressed as variables obtainable from data such as the nominal benefit $P_{si}F_{si}$, that is:

$$A_{si} = \eta_s \frac{(P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}}{\left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}} \tag{51}$$

where

$$\eta_s = \frac{1}{P_s (P_s F_s)^{\frac{1}{\sigma-1}}} \tag{52}$$

because

$$F_{si} P_s (P_s F_s)^{\frac{1}{\sigma-1}} = (P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}. \tag{53}$$

Reallocation gains are not affected if η_s is normalized to one for all sectors s.

Aggregation

The ultimate goal is to find the ratio of the aggregate real benefit computed from data over the efficient allocation. The real benefit computed from data is given by:

$$\begin{aligned} F_{si} &= A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \\ F_s &= \left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ F &= \prod_s F_s^{\theta_s}, \end{aligned} \tag{54}$$

while the efficient allocation is given by:

$$\begin{aligned}
\hat{F}_{si} &= A_{si}\left(\alpha_s \hat{D}_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)\hat{E}_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \\
\hat{F}_s &= \left(\sum_i \hat{F}_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
\hat{F} &= \prod_s \hat{F}_s^{\theta_s}.
\end{aligned} \tag{55}$$

Therefore, the ratio is $F/\hat{F}$.

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Table 1: Chinese Firm Summary Statistics

Panel A

Size percentile	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
0-5	65945	1.9	1.0	0.9	0.474	3.9
5-15	131838	3.8	2.1	1.7	0.447	4.4
15-30	197766	6.5	3.7	2.8	0.431	5.3
30-50	263614	11.3	6.5	4.8	0.425	6.9
50-70	263647	21.7	12.5	9.2	0.424	10.4
70-85	197736	46.7	26.8	19.9	0.426	17.6
85-95	131829	121.3	69.6	51.7	0.426	37.4
95-100	65908	851.2	498.5	352.3	0.414	222.2

Panel B

Year	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
1999	56247	76.3	44.6	31.7	0.415	16.6
2000	67530	77.8	45.2	32.6	0.419	18.2
2001	84524	73.0	41.4	31.6	0.433	18.1
2002	101559	70.4	40.0	30.4	0.432	18.7
2003	125626	72.1	41.8	30.3	0.420	20.6
2004	193306	59.1	35.0	24.1	0.408	—
2005	199743	68.9	40.3	28.5	0.414	22.4
2006	227377	70.7	41.1	29.6	0.419	24.6
2007	262371	71.5	41.4	30.0	0.420	26.5

Calculations are based on a sample of Chinese firms from the annual survey conducted by the National Bureau of Statistics of China (NBS) from 1999 to 2007. All firms in the manufacturing sector with more than 5 million Chinese Yuan (CNY) in sales are included. There are a total of 1,318,283 firm-year observations, and all variables are reported in millions of 2005 CNY. Value-added is not available for 2004. Panel A presents summary statistics broken down by firm size percentile. Panel B presents summary statistics by year.

Table 2: U.S. Firm Summary Statistics

Panel A

Size percentile	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
0-5	1005	7.8	3.6	4.2	0.538	0.4
5-15	2002	20.8	8.9	12.0	0.574	4.7
15-30	3003	51.7	20.1	31.6	0.611	12.4
30-50	4000	140.0	52.4	87.6	0.626	41.5
50-70	4005	464.4	213.0	251.6	0.542	158.5
70-85	3003	1583.2	875.3	708.2	0.447	545.9
85-95	2002	6114.3	3621.6	2492.7	0.408	1996.6
95-100	996	42037.1	24503.8	17533.4	0.417	11957.2

Panel B

Year	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
1999	2589	2126.7	1284.4	846.2	0.397	651.7
2000	2552	2409.8	1408.2	1006.2	0.417	781.6
2001	2392	2461.0	1437.9	1027.5	0.417	742.0
2002	2244	2730.7	1629.1	1107.9	0.405	793.1
2003	2121	3240.8	1904.3	1340.9	0.413	912.2
2004	2107	3460.1	1982.3	1481.7	0.428	1036.4
2005	2058	3643.6	2080.6	1569.7	0.430	1103.7
2006	2018	3972.5	2231.2	1752.1	0.440	1174.1
2007	1935	4206.2	2317.9	1900.2	0.450	1199.2

Calculations are based on a sample of manufacturing firms (SIC 2000 to 3999) from Compustat. The sample period is 1999 to 2007 and includes 20,016 firm-year observations. All variables are reported in millions of 2005 USD. Value-added is operating income before depreciation (OIBDP) plus imputed wages. Imputed wages are calculated by multiplying the employment of each firm with the mean wage per employee in the appropriate 3-digit SIC industry. Panel A presents summary statistics broken down by firm size percentile. Panel B presents summary statistics by year.

Table 3: Reallocation Gains by Year

	United States		China		United States vs. China			
Year	Fractional benefit	Percent Gain	Fractional benefit	Percent Gain	Relative fractional benefit	Percent Gain	Percent Scale	Percent Factor
1999	0.871	14.8	0.493	102.9	0.566	76.7	67.5	9.1
	(0.012)	(1.5)	(0.008)	(3.4)	(0.012)	(3.8)	(3.4)	(0.7)
2000	0.856	16.9	0.495	101.9	0.579	72.8	65.0	7.8
	(0.014)	(1.8)	(0.008)	(3.0)	(0.012)	(3.7)	(3.3)	(0.6)
2001	0.863	15.9	0.506	97.6	0.587	70.5	62.4	8.0
	(0.015)	(2.0)	(0.008)	(2.9)	(0.014)	(4.1)	(3.8)	(0.5)
2002	0.850	17.7	0.508	97.0	0.597	67.4	59.9	7.6
	(0.019)	(2.5)	(0.008)	(2.9)	(0.016)	(4.6)	(4.2)	(0.6)
2003	0.847	18.1	0.503	98.7	0.594	68.3	60.7	7.6
	(0.017)	(2.2)	(0.005)	(2.1)	(0.012)	(3.5)	(3.3)	(0.5)
2004	0.860	16.2	—	—	—	—	—	—
	(0.015)	(2.0)						
2005	0.878	13.9	0.446	124.1	0.508	96.8	85.1	11.7
	(0.013)	(1.7)	(0.007)	(3.3)	(0.011)	(4.3)	(3.8)	(0.7)
2006	0.876	14.2	0.441	126.7	0.504	98.6	86.2	12.4
	(0.011)	(1.4)	(0.005)	(2.5)	(0.007)	(3.0)	(2.6)	(0.6)
2007	0.888	12.6	0.429	133.2	0.483	107.2	93.1	14.1
	(0.009)	(1.1)	(0.004)	(2.4)	(0.007)	(3.2)	(2.8)	(0.7)

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007 inclusive. Value-added is not available in 2004 for Chinese firms. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma = 2$. The first column shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal US allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. The second column shows the corresponding percentage gain from moving from the observed to the optimal allocation. The next two columns present analogous calculations for Chinese firms. The two columns after that show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\text{China}}/\hat{F}_{\text{China}})(\hat{F}_{\text{US}}/F_{\text{US}})$, and the corresponding percentage gains, in other words, the percentage gains available if China's debt and equity markets were as developed as those in the United States. The last two columns provide a breakdown of misallocation into the misallocation due to scale and due to misallocation of factors, holding scale fixed. Below each estimate, the corresponding standard error is reported in parentheses.

Table 4: Reallocation Gains by Elasticities of Substitution

	United States		China		United States vs. China			
Year	Fractional benefit	Percent Gain	Fractional benefit	Percent Gain	Relative fractional benefit	Percent Gain	Percent Scale	Percent Factor
$\gamma = 1.5$	0.858	16.5	0.461	116.8	0.537	86.1	71.6	14.5
$\gamma = 2$	0.865	15.6	0.478	109.3	0.552	81.1	71.6	9.5
$\gamma = 3$	0.872	14.7	0.492	103.2	0.564	77.2	71.6	5.6
$\gamma = 5$	0.877	14.0	0.502	99.1	0.572	74.7	71.6	3.1
$\gamma = 10$	0.881	13.5	0.509	96.4	0.578	73.0	71.6	1.4
$\sigma = 1.5$	0.884	13.1	0.541	84.9	0.612	63.4	55.6	7.8
$\sigma = 1.77$	0.865	15.6	0.478	109.3	0.552	81.1	71.6	9.5
$\sigma = 2$	0.849	17.7	0.425	135.1	0.501	99.7	88.3	11.4
$\sigma = 2.5$	0.814	22.9	0.321	211.3	0.395	153.4	136.3	17.1
$\sigma = 3$	0.778	28.5	0.240	317.5	0.308	224.9	200.1	24.8

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007 inclusive. Value-added is not available in 2004 for Chinese firms. This table presents potential reallocation gains averaged across all years when we allow the elasticities of substitution, γ and σ, to vary. When γ is varied, σ is set to 1.77, and when σ is varied, γ is set to 2. The first column shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal US allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. The second column shows the corresponding percentage gain from moving from the observed to the optimal allocation. The next two columns present analogous calculations for Chinese firms. The two columns after that show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\text{China}}/\hat{F}_{\text{China}})(\hat{F}_{\text{US}}/F_{\text{US}})$, and the corresponding percentage gains, in other words, the percentage gains available if China's debt and equity markets were as developed as those in the United States. The last two columns provide a breakdown of misallocation into the misallocation due to scale and due to misallocation of factors, holding scale fixed.

Table 5: Costs of Debt and Equity by Year

Panel A:	United States		China	
Year	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
1999	0.219	0.182	0.307	0.281
	(0.0034)	(0.0070)	(0.0035)	(0.0029)
2000	0.217	0.183	0.314	0.289
	(0.0044)	(0.0034)	(0.0044)	(0.0023)
2001	0.202	0.167	0.322	0.301
	(0.0042)	(0.0034)	(0.0035)	(0.0023)
2002	0.206	0.172	0.334	0.313
	(0.0038)	(0.0043)	(0.0030)	(0.0018)
2003	0.209	0.179	0.403	0.321
	(0.0048)	(0.0114)	(0.0057)	(0.0017)
2004	0.210	0.171	—	—
	(0.0041)	(0.0033)		
2005	0.209	0.174	0.448	0.366
	(0.0042)	(0.0035)	(0.0037)	(0.0022)
2006	0.206	0.170	0.491	0.390
	(0.0042)	(0.0037)	(0.0049)	(0.0016)
2007	0.197	0.165	0.579	0.443
	(0.0051)	(0.0043)	(0.0041)	(0.0024)
Panel B:	**United States**		**China**	
Year	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
1999	0.181	0.154	0.146	0.158
	(0.0029)	(0.0021)	(0.0008)	(0.0008)
2000	0.178	0.156	0.153	0.167
	(0.0029)	(0.0023)	(0.0007)	(0.0008)
2001	0.162	0.139	0.163	0.175
	(0.0024)	(0.0024)	(0.0008)	(0.0007)
2002	0.167	0.141	0.171	0.182
	(0.0030)	(0.0024)	(0.0006)	(0.0008)
2003	0.162	0.136	0.179	0.188
	(0.0031)	(0.0023)	(0.0006)	(0.0006)
2004	0.167	0.140	—	—
	(0.0027)	(0.0023)		
2005	0.171	0.143	0.196	0.201
	(0.0027)	(0.0026)	(0.0005)	(0.0005)
2006	0.164	0.142	0.209	0.215
	(0.0041)	(0.0019)	(0.0006)	(0.0006)
2007	0.156	0.132	0.230	0.236
	(0.0032)	(0.0021)	(0.0006)	(0.0006)

This table displays the estimated mean (Panel A) and median (Panel B) costs of debt $(1+\tau_{D_{si}})\,r$ and equity $(1+\tau_{E_{si}})\,\lambda$ in the US and China by year. The elasticity of substitution between debt and equity is set at $\gamma = 2$. Below each estimate, the corresponding standard error is reported in parentheses.

Table 6: Costs of Debt and Equity by Firm Size

Panel A:	United States		China	
Percentile	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
0-5	0.313	0.205	1.666	1.109
	(0.0163)	(0.0108)	(0.0237)	(0.0095)
5-15	0.279	0.201	0.762	0.606
	(0.0087)	(0.0062)	(0.0054)	(0.0036)
15-30	0.243	0.186	0.556	0.451
	(0.0049)	(0.0040)	(0.0043)	(0.0015)
30-50	0.239	0.173	0.401	0.346
	(0.0034)	(0.0049)	(0.0025)	(0.0013)
50-70	0.203	0.170	0.310	0.274
	(0.0024)	(0.0042)	(0.0022)	(0.0012)
70-85	0.167	0.171	0.238	0.217
	(0.0017)	(0.0024)	(0.0016)	(0.0014)
85-95	0.150	0.165	0.180	0.173
	(0.0018)	(0.0021)	(0.0016)	(0.0011)
95-100	0.133	0.138	0.150	0.143
	(0.0018)	(0.0019)	(0.0097)	(0.0010)
Panel B:	United States		China	
Percentile	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
0-5	0.253	0.161	0.746	0.641
	(0.0073)	(0.0050)	(0.0041)	(0.0027)
5-15	0.214	0.155	0.385	0.385
	(0.0052)	(0.0024)	(0.0013)	(0.0011)
15-30	0.197	0.143	0.268	0.282
	(0.0029)	(0.0024)	(0.0006)	(0.0007)
30-50	0.201	0.137	0.198	0.213
	(0.0027)	(0.0018)	(0.0005)	(0.0005)
50-70	0.171	0.140	0.158	0.166
	(0.0018)	(0.0018)	(0.0004)	(0.0004)
70-85	0.155	0.148	0.130	0.134
	(0.0020)	(0.0018)	(0.0003)	(0.0003)
85-95	0.139	0.149	0.111	0.114
	(0.0016)	(0.0020)	(0.0004)	(0.0004)
95-100	0.122	0.128	0.097	0.101
	(0.0017)	(0.0017)	(0.0005)	(0.0004)

This table displays the estimated mean (Panel A) and median (Panel B) costs of debt $(1+\tau_{D_{si}})\,r$ and equity $(1+\tau_{E_{si}})\,\lambda$ in the US and China by firm size. Firm size is defined to be the 0-5%, 5-15%, 15-30%, 30-50%, 50-70%, 70-85%, 85-95%, 95-100% total asset percentiles of firms in each country. The elasticity of substitution between debt and equity is set at $\gamma = 2$. Below each estimate, the corresponding standard error is reported in parentheses.

Table 7: The Costs of Debt and Equity and Firm Characteristics

	$(1 + \tau_{D_{si}})\, r$	$(1 + \tau_{E_{si}})\, \lambda$
Location	-0.134	-0.092
	(-26.7)	(-39.2)
State investment	0.055	0.013
	(7.8)	(4.0)
Size	-0.192	-0.132
	(-153.3)	(-225.9)
Time	0.032	0.018
	(47.7)	(56.7)
Age	-0.001	0.001
	(-3.8)	(8.1)

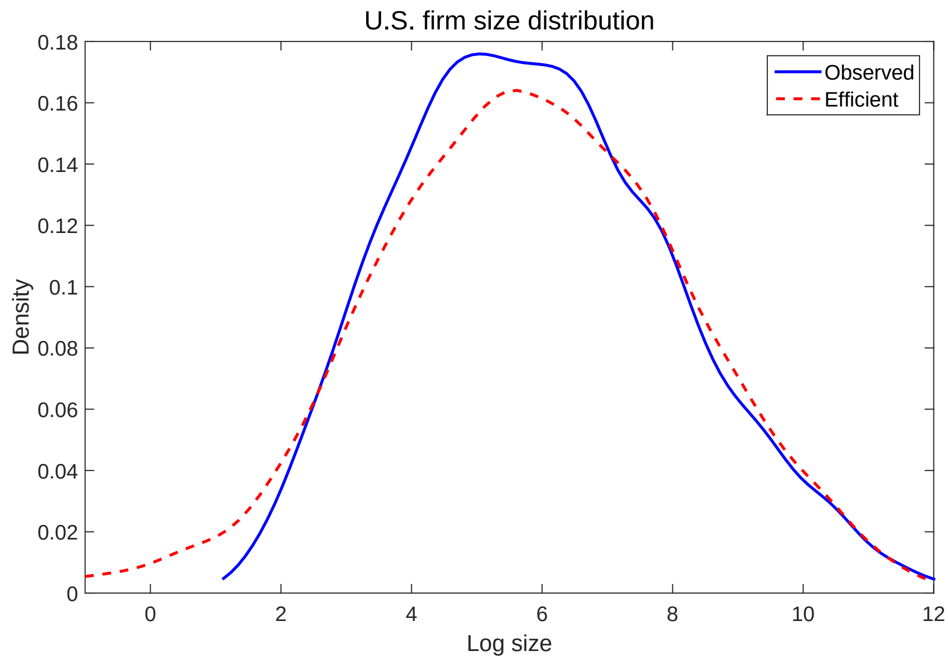
This table summarizes two OLS regressions on the costs of debt $(1 + \tau_{D_{si}})\, r$ and equity $(1 + \tau_{E_{si}})\, \lambda$ respectively. The regressors are location, state investment, firm size, time, and firm age. Location is a dummy variable that equals 1 if a firm is located in Beijing, Shanghai, Shenzhen, or Guangzhou and 0 otherwise. State investment is a dummy variable that equals 1 if a firm has a non-zero percentage of paid-in-capital from state sources and 0 otherwise. Size is log total assets measured in 2005 CNY. Time is a linear time trend, and age is firm age in years, which is censored at 100 years. Below each estimate, the corresponding t-statistic is reported in parentheses.

Table 8: U.S. Reallocation Gains with Market Value Benefit

	United States	
Year	Fractional benefit	Percent Gain
1999	0.740	35.2
	(0.026)	(4.6)
2000	0.746	34.1
	(0.029)	(5.1)
2001	0.854	17.1
	(0.012)	(1.6)
2002	0.889	12.4
	(0.011)	(1.3)
2003	0.890	12.4
	(0.010)	(1.2)
2004	0.890	12.3
	(0.010)	(1.2)
2005	0.887	12.8
	(0.010)	(1.2)
2006	0.891	12.2
	(0.008)	(1.0)
2007	0.865	15.6
	(0.013)	(1.7)

Calculations are based on U.S. firms from Compustat. The sample period is from 1999 to 2007 inclusive. The nominal benefit of finance is measured as the market value of debt plus the market value of equity, instead of value-added. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma = 2$. The first column shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal US allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. The second column shows the corresponding percentage gain from moving from the observed to the optimal allocation. Below each estimate, the corresponding standard error is reported in parentheses.

Panel A:



Panel B:

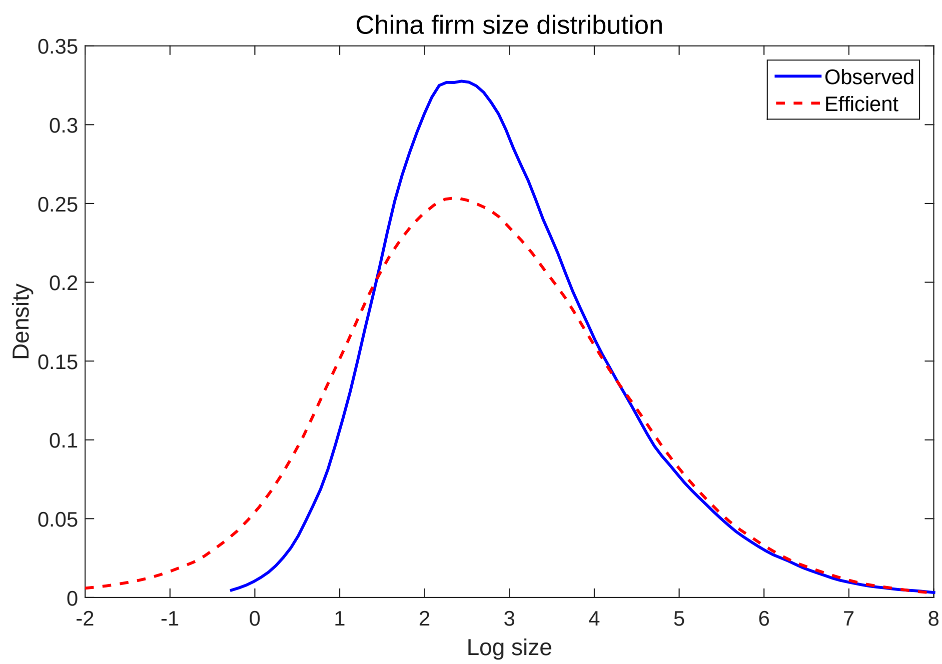
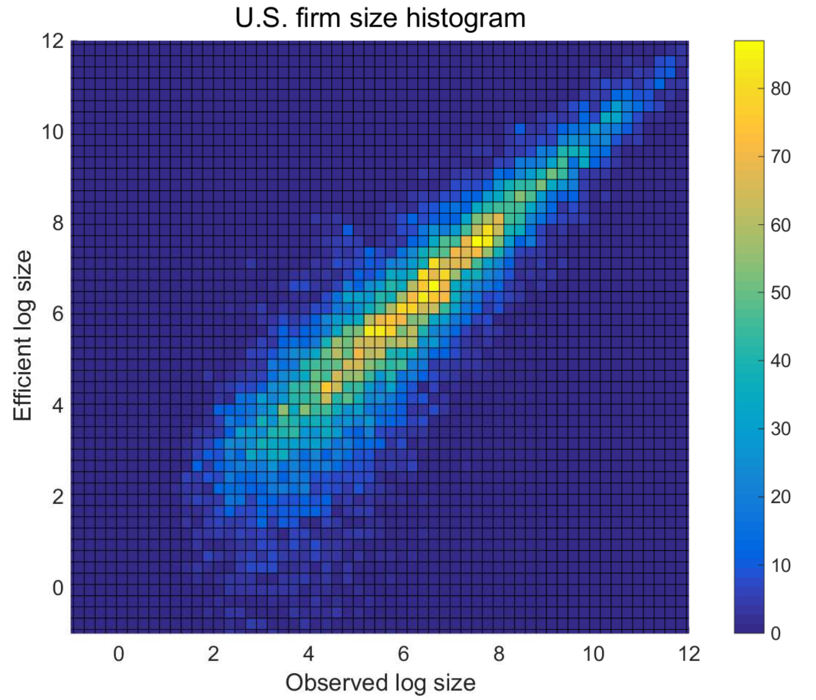


Figure 1: Panel A compares the U.S. observed and efficient firm size distributions using a kernel density estimator. Observed firm size is computed as $\log(D_{si} + E_{si})$, and efficient firm size is computed as $\log(\hat{D}_{si} + \hat{E}_{si})$, where D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 USD. Panel B similarly compares the observed and efficient firm size distributions in China. Firm size is computed in the same manner, but D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 CNY.

Panel A:



Panel B:

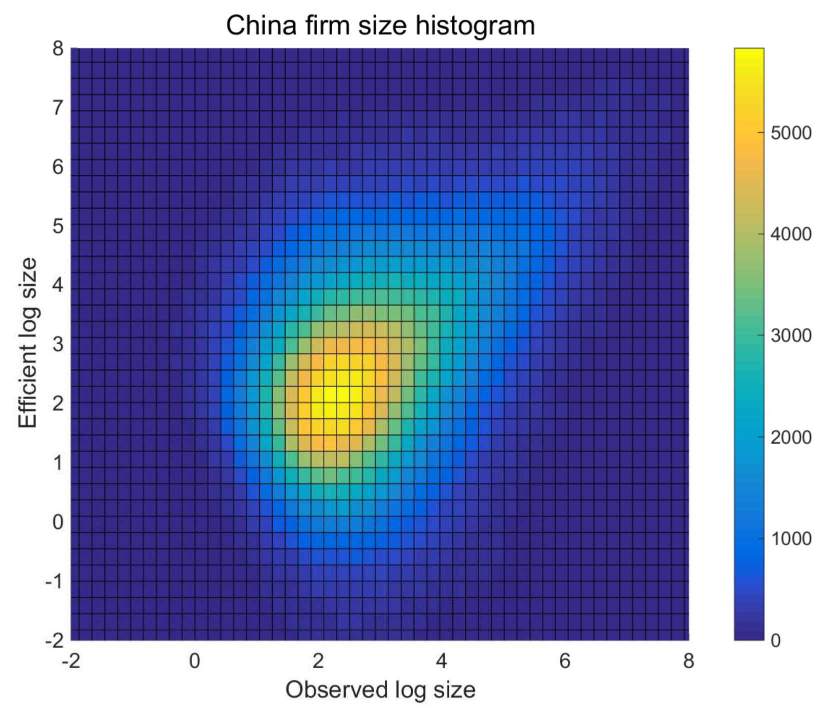


Figure 2: Panel A contains the heat map of a 3D histogram where the observed U.S. firm size distribution is on the x-axis and the efficient U.S. firm size distribution is on the y-axis. The legend for the z-axis heat map is located at right of the plot and represents the number of observations in each bin. Observed firm size is computed as $\log(D_{si} + E_{si})$, and efficient firm size is computed as $\log(\hat{D}_{si} + \hat{E}_{si})$, where D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 USD. Panel B similarly compares the observed and efficient firm size distributions in China. Firm size is computed in the same manner, but D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 CNY.