Firm Fundamentals and Variance Risk Premiums

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Abstract

We develop and empirically test an accounting-based model that ties two firm characteristics, book-to-market (*bm*) and return on equity (*roe*), to risk. The model predicts a negative relation between these characteristics and expected variance returns embedded in option prices (variance risk premiums). We confirm the predictions of the model using a variety of empirical specifications. Our results show that accounting data can be used to forecast the returns of assets other than stocks and that accounting data simultaneously inform investors about cash flows as well as the risk of those cash flows.

JEL: G12, G14, G27

Keywords: Stock returns, Variance returns, Accounting-based Valuation, Risk

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I. Introduction

Investors face at least two sources of uncertainty when choosing a security: the uncertainty about the return as captured by the return variance, and the uncertainty about the return variance itself. This latter source of uncertainty introduces an additional source of risk from holding assets. The so-called variance risk premium arises because investors generally dislike uncertainty about the return variance and, in equilibrium, demand a premium for accepting this risk (Bakshi et al., 2003; Todorov, 2009). Variance risk is an integral component of many financial assets. As a result, how investors price variance risk has fundamental implications for asset allocation decisions, the pricing of hedge derivative securities, and the behavior of financial asset prices in general (Cochrane, 2011).

Despite its importance to financial markets, we know very little about the cross-sectional determinants of variance risk premiums. Many prior studies have examined the time-series properties of the aggregate market level variance risk premium. For example, Bollerslev et al. (2009) show that the variance risk premium of the S&P 500 explains a nontrivial fraction of the time-series variation in post-1990 aggregate stock market returns, with high (low) premia predicting high (low) future returns. A smaller number of studies have examined the cross-sectional properties of the variance risk premium at the firm level using data extracted from option prices. These studies have generally shown that variance risk premiums exhibit large cross-sectional variation (Carr and Wu, 2009; Di Pietro and Vainberg, 2006). However, despite the intuitive connection between stock return volatility and stock returns, common factor models such as the CAPM and the Fama-French factors do not explain the excess returns on variance (Carr and Wu, 2009). Carr and Wu (2009) point out that this implies either a large inefficiency in the market for index variance or else that the majority of variance risk is generated by an independent risk factor that the market prices heavily.

We take a first step toward understanding whether cross sectional firm-level characteristics are associated with variance risk premiums by developing and empirically testing a model that expresses the variance risk premium as a function of accounting-based firm fundamentals. This approach contrasts with prior studies on the cross-sectional determinants of variance risk in at least three ways. First, we use a simple theoretically motivated partial equilibrium model to motivate our empirical analyses. Second, our approach uses accounting data which allows us to not only identify whether such data is associated with the variance risk premium, but also to develop a strategy that uses accounting data to trade variance. Prior studies have generally used market-based statistical models to examine variance risk premiums (e.g., Todorov, 2009). Third, our approach allows us to explicitly examine whether variance risk premiums are driven by an independent risk factor by examining whether the same accounting fundamentals are associated with expected stock returns.

The relation between the variance risk premium and accounting-based firm fundamentals is not obvious. Bollerslev and Todorov (2011) argue that realized variance is priced due to its correlation with large negative jumps, suggesting that the variance risk premium is likely to be uncorrelated with historical accounting data.[1] Moreover, studies which have examined the relation between firm characteristics and bond returns have found little evidence that *bm* and profitability measures are significantly associated with bond returns, despite bond returns being highly correlated with stock returns (Crawford et al., 2014). Thus, while a number of studies have linked accounting-based fundamentals with expected stock returns (e.g., Lyle et al., 2013; Kelly and Pruitt, 2013), it is unclear whether these same fundamentals are linked to other types of assets, and in particular, variance risk premiums. This is particularly so given the lack of an association between variance risk premiums and traditional factor models commonly used to explain variation in stock returns (Carr and Wu, 2009).

We develop a parsimonious partial equilibrium model that expresses the variance risk premium as a linear function of book-to-market (*bm*) and return-on-equity (*roe*) using

[1] Our reference to accounting data is to the levels of simple items derived from the financial statements, such as return on equity. Prior research has shown that more complicated metrics derived using financial statement data, such as conservatism (Kim and Zhang, 2015) or the variance of accruals (Hutton, Marcus and Tehranian, 2009) are sometimes associated with crash risk.

three assumptions. First, we assume that *bm* is a covariance-stationary process, consistent with prior empirical studies (e.g., Chattopadhyay et al., 2015). Second, we assume that the growth rate in book follows an auto-regressive "variance-in-mean" process. This process assumes that growth rates are persistent, which is a common feature of this literature (e.g., Campbell, 1991; Lyle and Wang, 2015; Nissim and Penman, 2001). This assumption also allows for book growth to depend on conditional variance, similar to a (G)ARCH-in-mean model (e.g., Engle et al., 1987; Glosten et al., 1993). Lastly, and similar to other cross-sectional studies, we assume the existence of a stochastic discount factor that prices all assets in the economy (e.g., Armstrong et al., 2013; Johnson, 2004; Pástor and Veronesi, 2003, 2006).

The model we derive predicts a positive (negative) relation between equity risk (variance risk) premiums and both *bm* and *roe*. Even though prior work has established empirically that there is a negative association between the aggregate equity and aggregate variance risk premiums, the relations with *bm* and *roe* that we derive do not dependent on any assumed relation between the equity and variance risk premiums. In fact, the relation is only revealed once equity and variance prices are derived in equilibrium. Moreover, even if we did assume a negative relation between equity and variance risk premiums, it is not clear why both *bm* and *roe* would necessarily have the opposite relation across the equity and variance risk premiums. For example, Fama and French (1992) and many subsequent papers find strong empirical evidence that *bm* has a strong positive relation with stock returns, which might suggest that variables which have positive (negative) correlations with *bm* will have positive (negative) correlations with stock returns. However, *roe* has a strong negative relation with *bm*, and yet *roe* has been shown to have a strong and robust positive relation with stock returns. Thus, it could certainly be the case that the firm fundamentals associated with variance risk premiums are distinct from those associated with the equity risk premium, especially given the Carr and Wu (2009) findings. In addition, our model uniquely predicts that *bm* and *roe* work in tandem to explain cross sectional variation in the variance risk premium. Again, it is not clear how this prediction would follow from assuming a negative association between the aggregate equity and variance risk premiums.

Our empirical analyses proceed in three steps. First, we investigate the cross-sectional relation between variance risk premiums and *bm* and *roe* from January 1996 to December 2013. We find that the predicted negative relation between variance risk premiums and *bm* and *roe* requires that both variables be included in the specification. This suggests that *bm* and *roe* work together, and emphasizes the importance of our model-based approach, as an *ad-hoc* set of empirical analyses that does not include both *bm* and *roe* might potentially generate different conclusions.

We find that our results are not sensitive to the inclusion of standard factor model controls. When we include the slope coefficients from the Fama and French (1993) three factor model as well as a set of firm-specific control variables, the coefficients on *bm* and *roe* and virtually unchanged.[2] Our results are also not sensitive to the holding period, as we find similar results using 60 day-ahead variance returns. In addition, our conclusions are unchanged when we limit our sample to S&P 500 firms. This provides assurance that our results are not attributable to small firms or noise in our estimation procedures, as liquidity or other market imperfections are less likely to affect these firms because options for these firms are actively traded.

We provide additional assurance that our results are not driven by the negative association between variance returns and stock returns by including a set of firm-specific control variables: size (log of market capitalization), historical 30 day stock return variance, and both contemporaneous and lagged stock returns. Once again, we find the predicted negative relation between variance risk premiums and *bm* and *roe*. This specification indicates that there is a relation between variance returns and both *bm* and *roe* and is independent of the association between variance returns and stock returns.

Second, we examine whether there is an association between the variance risk premium and *bm* and *roe* in time series data to investigate whether *bm* and *roe* carry information

[2]The control variables are: log of market capitalization (*size*), historical 30 day stock return variance (*lvar*), and both contemporaneous (R_{t+1}) and lagged (R_t) stock returns.

about systematic risk. In addition, this approach allows us to investigate whether these characteristics are economic drivers of aggregate volatility, something that Engle and Rangel (2008) suggest is sorely missing from this body of research. We conduct this analysis in two ways. First, we run a time series regression of the median variance return on the median *bm* and the median *roe* where each variable is calculated from the cross-sectional data on a monthly basis. Second, we run a similar regression, but replace the median variance return with the return on the variance of the S&P 500 index from Bollerslev et al. (2009). As with our first set of analyses, we find a strong negative relation between variance returns and *bm* and *roe* for both approaches, consistent with the model. In addition, we find that the predicted associations between variance returns and *bm* and *roe* depend on the inclusion of both variables, suggesting once again that these variables work in tandem to explain variation in variance risk premiums.

Third, we construct a simple trading strategy of writing put options based on *bm* and *roe* to show that the realized returns to this strategy are consistent with the predictions of our model. The model predicts that stock returns are positively associated with *bm* and *roe*, whereas variance risk premiums are negatively associated with *bm* and *roe*. Writing puts is equivalent to going long the stock and short the variance. Therefore, writing puts generates high realized returns to a strategy based on *bm* and *roe* since it maximizes the exposure to the price of risk of both equity and variance. Our results indicate that once we condition on *bm*, the realized returns are lowest for the portfolios that contain the lowest quintile of *roe*, consistent with the model's predictions. We repeat the above analysis using only firms which are constituents of the S&P 500, as options on S&P 500 firms are actively traded, highly liquid and have low transaction costs. Our conclusions are unchanged. Future returns increase in *roe* within each *bm* quintile and tend to generate the highest returns for firms which have high *roe* and high *bm*.

Our study is the first to formally link accounting-based valuation models to equity *and* variance risk premiums. We offer direct evidence that accounting numbers simultaneously inform investors about future cash flows as well as the risk of those future cash flows. Barth and So (2014) find that the variance risk premium is higher around earnings announcements for larger firms, industry leaders, and firms whose earnings are both more sensitive to aggregate earnings factors and convey more news. Han and Zhou (2012) find that stocks whose returns tend to be low when systematic volatility increases have higher variance risk premiums. We extend these studies by using a parsimonious model to identify firm-level characteristics that are associated with the time series and cross-sectional variation in variance risk premiums.

We also contribute to the literature that examines the relation between characteristics and asset returns (e.g., Ball et al., 2015; Van Binsbergen and Koijen, 2010; Daniel and Titman, 1997; Kelly and Pruitt, 2013; Lettau and Van Nieuwerburgh, 2008; Novy-Marx, 2013; Sloan, 1996; Piotroski, 2000) in two ways. First, we show that accounting-based characteristics can be used to systematically trade variance risk using two easily obtainable ratios. This extends prior studies by formally showing that accounting information is useful for forecasting the returns of financial assets other than stocks. Second, because the model we derive shows that firm fundamentals impact both the equity and variance risk components simultaneously, our study is also related to the extensive literature that examines the drivers of volatility. Prior studies have generally predicted volatility using time series information, rather than contemporaneous economic variables (Engle and Rangel, 2008). In a recent paper, David and Veronesi (2013) derive a model that relates variation in aggregate stock and bond prices to the earnings-to-price ratio. We add this line of work by showing that accounting-based valuation models can be used to predict the returns of financial assets whose prices are based on measures of stock return volatility.

The remainder of the paper is organized as follows. SectionII presents the accounting-based model for estimating variance risk premiums. Section III discusses the estimation of the model and outlines our data. Section III also provides our empirical analyses. Section IV concludes the paper.

II. The Model

In this section, we derive models that express expected stock and variance returns as linear combinations of *bm* and *roe*. Our derivation is similar to Van Binsbergen and Koijen (2010); Kelly and Pruitt (2013); Lyle and Wang (2015), but differs on an important dimension. These prior papers are agnostic about risk and assume that expected log-returns follow an exogenous AR(1) process. In contrast, we endogenize expected rates of return by solving a partial equilibrium model. This approach allows us to tie firm characteristics to the priced risk embedded in stock returns and stock return variance.

A. Main Assumptions

Our model relies on three main assumptions. First, we make the assumption that the log book-to-market ratio (*bm*) has a long-run mean that is time independent, i.e., it is a covariance-stationary process.[3]

$$\lim_{j\to\infty} E_t[\log(\frac{B_{t+j}}{M_{t+j}})] = \overline{bm} < \infty \tag{1}$$

where B_{t+j} and M_{t+j} represent the book value and market value, respectively, of equity at time $t+j$. The notion that *bm* does not "blow up" (or go to ∞) in expectation is largely consistent with prior research. For example, Pástor and Veronesi (2003, 2006) assume that at some time in the future, market values and book values become equal because of competitive market forces. Additionally, implicit in the assumptions of the popular Ohlson (1995) model is that abnormal earnings eventually erode through time, which implies that market values and book values will be unconditionally connected in expectation. Moreover, if a firm is expected to remain a going concern and accounting systems are expected to become closer to "mark-to-market" through time, then a relation similar to (1) would be expected.

[3]Chattopadyay et al. (2015) find strong statistical evidence in support of this assumption using data from 29 countries.

Second, we assume that the growth rate in book, $\log(\frac{B_{t+1}}{B_t}) \equiv g_{t+1}$, follows an autoregressive "variance-in-mean" process,

$$g_{t+1} = \bar{g} + \kappa g_t + \eta \sigma_{g,t}^2 + \sigma_{g,t}\epsilon_{t+1}. \tag{2}$$

Here κ is the persistence of book growth and η is the variance-in-mean coefficient which we solve for endogenously based on no-arbitrage conditions. The innovation term, ϵ_{t+1}, is assumed to be normally distributed with mean zero and variance one. This process assumes that growth rates are persistent, which is a common feature of this literature (e.g., Campbell, 1991; Nissim and Penman, 2001; Penman, 1991). We include η to allow for the fact that book growth rates may depend on conditional variance. This approach is similar to the "(G)ARCH in mean" models that have been used extensively in the finance literature to model asset returns (e.g., Engle et al., 1987; Glosten et al., 1993). Thus, if firm profitability is related to the fluctuations in the prices of the assets that the firms sells, and if those prices evolve in a manner that is consistent with a variance in mean process, then profitability will also follow a variance in mean process. An example of this is an oil production company, where the producer's profitability would exhibit dynamics that resemble oil price dynamics. Moreover, Arif et al. (2015) find that accruals (a component of book growth) have a significant negative relation with stock return volatility. In addition, we find strong empirical evidence that book growth is indeed related to book growth variance in our sample.[4] Thus, the assumption of a variance in mean process is consistent with both economic intuition and is largely supported empirically.

To allow for time variation in the conditional variance of book growth, we assume that $\sigma_{g,t}$ follows the discrete time version of the popular Heston (1993) volatility model. Specifically,

[4]Under no arbitrage, we find that the predicted relation between expected growth in book and conditional book growth variance is negative ($\eta = -\frac{1}{2}(\frac{1}{1-\kappa}) < 0$). Moreover, when we regress future roe on lagged roe, bm and future stock return variance (which in equation (53) of the appendix) using the following specification: $roe_{t+1} = A_0 + A_1 roe_t + A_2 bm_t + A_3\sigma_{m,t}^2 + \epsilon_{t+1}$. We find that $A_1 = 0.604$, $A_2 = -0.001$ and $A_3 = -0.718$ and all are highly significant.

$$\sigma_{g,t+1} = \omega\sigma_{g,t} + \gamma z_{t+1} \tag{3}$$

where γ is a non-negative constant and represents the "volatility of volatility", z_{t+1} is normally distributed with mean zero and variance one, and the covariance between z_{t+1} and ϵ_{t+1} is assumed to be q (i.e., $E_t[\epsilon_{t+1}z_{t+1}] = q$). By allowing z_{t+1} to be correlated with ϵ_{t+1}, we implicitly assume that investors use realizations in book growth provided in financial reports to update their estimates of the conditional variance of book growth. While not formally modeled in the paper, this relation, like a traditional GARCH model, is similar to a standard Bayesian learning model where investors learn about the volatility of book growth by observing realizations through time. In such a case, large innovations in book growth lead to large revisions in beliefs about the variance of book growth.[5]

Third, we assume the existence of a stochastic discount factor, Λ_t, (the marginal rate of consumption for a representative agent in the economy) that prices all assets in the economy (e.g., Armstrong et al., 2013; Bakshi et al., 2003; Bakshi and Kapadia, 2003; Johnson, 2004; Pástor and Veronesi, 2003).

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \exp(-r_f - \frac{\sigma_\Lambda^2}{2} - \sigma_\Lambda w_{t+1}), \tag{4}$$

where r_f is the continuously compounded risk-free rate, σ_Λ is the volatility of the discount factor, and $w_{t+1} \sim N(0,1)$ represents random shocks to the state of the economy. The covariance between w_{t+1} and ϵ_{t+1} is assumed to be ρ (i.e., $E_t[\epsilon_{t+1}w_{t+1}] = \rho$) which we assume is positive. This implies that market values must then satisfy the no-arbitrage condition $M_t = E_t[\frac{\Lambda_{t+1}}{\Lambda_t}M_{t+1}]$. Our assumption about the dynamics of the discount factor are identical to that used by Armstrong et al. (2013) as well as (in discrete time) Johnson

[5] To see that conditional variance depends on the history of book growth realizations, note that because $z_{t+1} \sim N(0,1)$ we can write it as $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$ where ϵ_{t+1} and ξ_{t+1} are uncorrelated IID normal distributions. So an update in investors estimate of conditional variance is then given by $\sigma^2_{g,t+1} = (\omega\sigma_{g,t} + \gamma(q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}))^2 = (\omega\sigma_{g,t} + \gamma(q\frac{(g_{t+1}-E_t[g_{t+1}])}{\sqrt{\sigma^2_{g,t}}} + \sqrt{1-q^2}\xi_{t+1}))^2$, where the $\frac{g_{t+1}-E_t[g_{t+1}]}{\sqrt{\sigma^2_{g,t}}}$ term represents the normalized information in the "growth (i.e. earnings) surprise" that investors use to update their expectations about conditional variance.

(2004) and Pástor and Veronesi (2003) and it generates expected returns consistent with the traditional (consumption) CAPM.

While the assumptions presented above are necessarily parsimonious to obtain closed form solutions, as we show in the next section, they are realistic enough to generate stock return behavior observed in empirical data.

B. Market Values

As we show in the appendix, the above assumptions imply that the fair market value of a non-dividend paying firm in the economy is given by:

$$m_t = b_t + \alpha_0 + \alpha_1 roe_t - \alpha_2 \sigma_{g,t}, \tag{5}$$

where $\alpha_0 = -\overline{bm} - \alpha_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$, $\alpha_1 = \frac{\kappa}{1-\kappa} > 0$, and $\alpha_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-q\gamma}$.[6] This equation captures the intuition that the market value is equal to book value of equity plus a linear combination of a constant, return on book equity, and the conditional volatility of book growth. Higher return on book equity increases market value (α_1 is positive for all $\kappa \in (0,1)$), whereas volatility in book growth, $\sigma_{g,t}$, decreases market value (α_2 is positive for all $q < \frac{(1-\kappa)(1-\omega)}{\gamma}$ which is guaranteed if $q < 0$). The correlation coefficients ρ and q which tie growth and volatility to the state of the economy show how risk in book growth and the volatility of book growth impact market value. Intuitively, firms with innovations in profitability and risk that move more with the economy will have lower market values, because, all else equal, they have higher exposure to systematic risk. Indeed (5) says firms with book growth that is more highly correlated (ρ) with the state of the economy have lower market values. The same is true for the correlation coefficient q. Firms with conditional volatility that varies more with the state of the economy have lower market values. While the equity pricing equation offers reasonable economic intuition, it does not tell us how stock returns behave or how fundamentals relate to expected stock and

[6] A similar solution exists for dividend paying firms if dividends over the interval t to $t+1$ are proportional to either book value or market value. See for example, Chattopadhyay et al. (2015).

other asset returns. Our next section shows the dynamics of stock returns and how they relate to firm fundamentals.

C. Risk Premiums

In this section, we first derive stock return dynamics and then use these dynamics to determine priced risk in both equity and variance markets.

C.1. Stock Returns

In the appendix, we show that stock returns (changes in log-stock prices) exhibit the following dynamics:

$$r_{t+1} = \mu_t - \frac{1}{2}\sigma_t^2 + (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \alpha_2\gamma z_{t+1}, \tag{6}$$

$$\mu_t = \underbrace{r_f}_{\text{risk free rate}} + \underbrace{\rho\sigma_\Lambda[(1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma]}_{\text{Equity risk premium}}. \tag{7}$$

Here μ_t is the expected rate or return on equity and $\sigma_t^2 (= E_t[(r_{t+1} - E_t[r_{t+1}])^2])$ is the conditional variance of the stock return. The coefficients α_1 and α_2 are defined above. The $-\frac{1}{2}\sigma_t^2$ term is an "adjustment" term because r_{t+1} represents a log (not a simple) return. (6) suggests that the assumptions used to solve for market values deliver stock return behavior that is broadly consistent with how stock returns are believed to behave. Both discount rates, μ_t, and variances, σ_t^2, are time varying, which is consistent with the large literature in finance and economics (Cochrane, 2011; Tsay, 2005). Expected rates of return embody the intuition that higher risk, $\sigma_{g,t}$, in book growth increases the rate of return demanded by investors for holding the equity. The innovation terms are composed of shocks in book growth, $(1+\alpha_1)\sigma_{g,t}\epsilon_{t+1}$, and shocks in the volatility of book growth, $-\alpha_2\gamma z_{t+1}$. As a result, the model delivers return behavior that is consistent with the return decomposition literature (e.g., Campbell, 1991; Vuolteenaho, 2002). It shows that stock returns are a function of expected returns (μ_t), "cash flow news" ($(1+\alpha_1)\sigma_{g,t}\epsilon_{t+1}$)

and "discount rate news" ($-\alpha_2\gamma z_{t+1}$). In addition, equation (6) captures the economic intuition outlined in Ball et al. (1993), that book growth (approximately earnings deflated by book) carry information about both cash flows and discount rates and stock returns move in response to both of these pieces of information.

Moreover, equity risk premiums, $\mu_t - r_f$, are a function of the priced risk in both cash flow and discount rate news. Specifically, because $\mu_t - r_f = -cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), (1 + \alpha_1)\sigma_{g,t}\epsilon_{t+1}) - cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), -\alpha_2\gamma z_{t+1})$ this implies that expected equity returns carry information about priced risk to cash flows, $-cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), (1 + \alpha_1)\sigma_{g,t}\epsilon_{t+1})$, and the priced risk in the risk of those cash flows, $-cov_t(\ln(\frac{\Lambda_{t+1}}{\Lambda_t}), -\alpha_2\gamma z_{t+1})$. This non-trivial result has an important implication. In a rational market, characteristics that carry information about priced risk in equities should also carry information about the priced risk in discount rates (or in the above model, stock return volatility itself).

C.2. Equity Risk Premiums as a Function of Fundamentals

Equation (7) shows that equity risk premiums are a function of the variance of book growth, an unobservable variable that must be estimated. In this section, we show that this latent variable can be substituted out using the market value equation and allows equity risk premiums to be expressed as a linear combination of firm fundamentals. We define the firm specific equity risk premium from the period t to $t+1$ as the continuously compounded return on equity minus the risk free rate: $ERP_{t,t+1} = \mu_t - r_f$. We show in the appendix that, under no-arbitrage, the expected equity risk premium can be written as:

$$ERP_{t,t+1} = \theta_0 + \theta_1 bm_t + \theta_2 roe_t, \tag{8}$$

where $\theta_0 = \theta_1(\alpha_0 - \alpha_2 q\gamma)$, $\theta_1 = (1 - \omega) - \frac{\gamma q}{(1-\kappa)}$, $\theta_2 = \alpha_1\theta_1$. All of the constant terms are predicted to be positive if the correlation coefficient between book growth volatility and the state of the economy, q, is negative. This result is important because it extends the findings of Lyle et al. (2013) and formally shows that equity risk premiums are rationally associated with firm characteristics, and in particular they are increasing in both bm_t and roe_t. This suggests that prior studies which have documented a strong relation between future stock returns and these and other correlated variables are consistent with traditional asset pricing theory (e.g., Ball et al., 2015; Fama and French, 1992; Harvey et al., 2014; Novy-Marx, 2013; Kelly and Pruitt, 2013; Subrahmanyam, 2010).

While we do not formally model the accounting system that generates book values and earnings, the rationale for bm_t and roe_t carrying information about priced risk can be explained in the following way. Market values represent realized cash flows (realized earnings) plus assets in place (the sum of these represents book value) plus expected discounted future cash flows. Combining book value and market values gives discounted future cash flows. By "adding back" roe, this provides information about future cash flows because of the persistence of roe. Thus, once market values are combined with book values and roe, the remainder represents discounted future cash flows (expected returns).

C.3. Variance Risk Premiums as a Function of Fundamentals

The above result offers a rational explanation for the findings of prior empirical studies which link bm and/or profitability measures to future stock returns, but it does not tell us whether firm characteristics carry information about priced variance risk. Therefore, we next show that firm fundamentals are related to the risk embedded in stock return variance and that an expression for the firm's variance risk premium is also a linear combination of firm fundamentals. We define the expected variance risk premium as $VRP_{t,t+1} = E_t[\sigma^2_{t+1}] - R_f v_{t,t+1}$, where $v_{t,t+1}$ is the fair price for holding variance from t to $t+1$ and $E_t[\sigma^2_{t+1}]$ is the expected return variance over the interval t to $t+1$. In the appendix, we show that this expression can be combined with the stock return dynamics and the equation (5) such that the expected variance risk premium can also be written as a linear combination of bm and roe:

$$VRP_{t,t+1} = \phi_0 + \phi_1 bm_t + \phi_2 roe_t, \tag{9}$$

where $\phi_0 = \alpha_0\phi_1 + \eta_0$, $\phi_1 = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega)-\gamma q)$, $\phi_2 = \alpha_1\phi_1$ and $\eta_0 = -[(1+\alpha_1)\gamma q(1+\rho^2(\sigma_\Lambda^2+1))-2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$. Here, all of the coefficients, (ϕ_0, ϕ_1, ϕ_2), are predicted to be negative if the correlation coefficient, q, is also negative. Equation (9) offers an important and empirically testable prediction: if the two characteristics bm_t and roe_t have information about priced risk, then their relation with variance risk should be negative as long as the correlation between the volatility of book growth and the state of the economy (q) is negative. Given the large empirical evidence mentioned above which documents that the relation between *bm*, *roe* and future stock returns is positive, evidence of a negative relation between these characteristics and variance risk premiums would offer new empirical evidence that these characteristics carry information about "priced" risk.

III. Data and Empirical Analyses

This section describes the data collection process and the empirical implementation of the model presented in Section II. Our empirical analyses proceed in three steps. First, we examine whether the variance risk premium has a cross-sectional relation with *bm* and *roe*. We then examine whether there is an association between aggregate measures of variance risk and *bm* and *roe* in time series data to investigate whether *bm* and *roe* carry information about systematic risk. Finally, we construct a simple trading strategy of writing put options based on *bm* and *roe* to show that the realized returns to this strategy are consistent with the predictions of our model.

A. Data

We collect stock price information from The Center for Research in Security Prices (CRSP), financial statement data from Compustat quarterly files, and option data from OptionMetrics. Our sample includes all firms with fiscal year ends of March, June, September, and December from January 1996 to December 2013. We require firms to have positive book values, at least four quarters of historical accounting information and beginning-of-month stock prices greater than \$5. Our final sample consists of 312,229 firm-month observations for one-month ahead returns.

At the end of each month, we match a firm's most recently reported quarterly book value of equity and return on book equity to the price of variance contracts with standardized expiration of 30 and 60 calendar days ahead. Variance contracts are calculated using the model free method outlined in the appendix.[7] We then calculate variance risk premiums following Carr and Wu (2009) as the difference between future realized variance and the cost of purchasing a variance contract. Realized variance is calculated as the sum of squared daily log returns. All estimated and independent variables are winsorized at the 1% level. We use variance contracts on the S&P 500 Index in some of our tests. For these contracts, we obtain both the price of the variance contract and the realized variances on the S&P 500 Index employed by Bollerslev et al. (2009).[8]

Table's I and II provide descriptive statistics of key variables used in the analysis as well as other firm-level variables commonly used in cross-sectional asset pricing studies. Table I shows that the price of a 30 day ahead variance contract $v_{t,t+1}$ is on average greater than future realized 30 day variance as well as lagged variance, consistent with variance carrying a negative risk premium. Moreover, the economic magnitude of this premium is large with the excess return on a 30 day variance contract averaging -16.72 percent. Realized stock returns in our sample average 0.77 percent per month, the log

[7]To ensure that the financial statement data is publicly available at the end of the month, we use the firm's report date in Compustat (the RDQ variable) and add an additional month of time before the firm obtains a new book or earnings value.

[8]We thank Hau Zhou for making this data publicly available. The data can be found at: https://sites.google.com/site/haozhouspersonalhomepage/ .

book-to-market ratio, is -0.89 and quarterly rate of return on book equity is 1.38 percent. Log market cap (*size*) is 7.28 and β is 1.31, consistent with firms in our sample being large and having a high covariance with the overall market.

Consistent with intuition, Table II shows that the univariate correlation between the variance contract and realized variance (both future and lagged) is large and exceeds 0.5. The correlation between stock returns and the price and returns of variance is negative, but positively associated with both *bm* and *roe*. The return on variance is negatively related to *bm* and *roe*, and positively related to β, *size* and lagged variance *lvar*. Consistent with prior research, larger firms have on lower stock return variance (both future and lagged).

B. Empirical Tests

B.1. Cross-sectional Tests

Our cross-sectional analyses follows directly from equation (9). We first write equation (9) in terms of a "traditional" risk premium as follows:

$$E_t[R^v_{t,t+1} - R_f] = \frac{E_t[\sigma^2_{t+1}]}{v_{t,t+1}} - R_f = \frac{\phi_0}{v_{t,t+1}} + \phi_1 \frac{bm_t}{v_{t,t+1}} + \phi_2 \frac{roe_t}{v_{t,t+1}}. \tag{10}$$

This leads directly to the following empirical specification:

$$R^v_{t,t+1} - R_f = a_0 + a_1 \frac{1}{v_{t,t+1}} + a_2 \frac{bm_t}{v_{t,t+1}} + a_3 \frac{roe_t}{v_{t,t+1}} + w_{t+1}. \tag{11}$$

The book-to-market ratio is calculated as $bm_t = \log(\frac{B_t}{M_t})$, where B_t is book value of equity from Compustat and M_t represents market capitalization calculated as stock price multiplied by shares outstanding from CRSP divided by 1,000. The return on equity is calculated as $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$, where x_t is income before extraordinary items from Compustat. $v_{t,t+1}$, as stated above, is calculated using the model free approach outlined in the appendix using a cross-section of firm level options from the OptionMetrics volatility surface file. w_{t+1} is a mean zero error term.

For our first set of analyses, we estimate equation (11) monthly using the Fama-MacBeth approach. Table III provides the results of regressing variance risk premiums on each right had side variable separately, and then all simultaneously as specified in equation (11). Moving from left to right across the table it becomes clear that *bm* and *roe* work together to deliver the predicted relation for *roe*. While the coefficient on *bm* is negative and statistically significant when it is the only independent variable in the regression, the coefficient on *roe* is insignificant when it is the only independent variable in the regression. When both variables are combined as prescribed by equation (11), the predicted negative relation emerges in the data. Both coefficients are highly significant. In addition, the explanatory power of the regression goes up considerably when the full set of variables are included. We test whether our results are sensitive to the holding period by repeating our analysis with 60 calendar day ahead variance returns. The results in Columns (4) of Table III show that the coefficients of interest remain unchanged when the holding period is extended. The coefficients are statistically significant and the signs of the coefficients match the predictions of the model.

We next examine whether the conclusions in Table III are sensitive to the inclusion of standard factor model controls. We augment equation (11) to include the slope coefficients from the Fama and French (1993) three factor model as well as a set of firm-specific control variables that may be associated with the variance risk premium (Carr and Wu, 2009). The slope coefficients from the Fama and French (1993) three factor model are estimated at the firm level using the full sample (i.e., they contain significant look-ahead information). The empirical specification is as follows: $R^v_{t,t+1} - R_f = \alpha + \beta_m R_{m,t+1} + \beta_H R_{H,t+1} + \beta_S R_{S,t+1} + \epsilon_{t+1}$, where the factor returns $R_{m,t+1}, R_{H,t+1}, R_{S,t+1}$ represent the excess return on the value weighted market portfolio, the return on a portfolio of high minus low book-to-market firms, and the return on a portfolio of small minus large firms, respectively. These factor returns were downloaded from Ken French's online data library. The firm-specific control variables we include are the log of market capitalization (*size*), historical 30 day stock return variance (*lvar*), and both contemporaneous (R_{t+1}) and lagged (R_t) stock returns. We include the latter two variables to ensure that our results are not simply driven by the negative correlation between variance returns and stock returns.

The results in Table IV indicate that our conclusions are unchanged by the inclusion of these additional variables. Moving from left to right, the first column regresses variance returns on the full slope coefficients using the Fama and French (1993) three factor model, the second column adds the additional firm-specific control variables, and the third column adds the variables from equation (11). The results in Column (1) show that each of the slope coefficients from the Fama and French (1993) three factor model are associated with variance returns. The coefficients on full sample slopes, β_m, β_S are both negative and statistically significant, and the coefficient on β_H is positive and statistically significant. These associations are unaffected by the inclusion of the additional firm-specific control variables in column (2) and the variables from equation (11).

The coefficients on the firm-specific control variables are generally statistically significant in column (2). Consistent with intuition, there is a high correlation between variance returns and historical stock return variance ($lvar$), and a negative association with both contemporaneous (R_{t+1}) and lagged (R_t) stock returns. Once again, these associations are unaffected by the inclusion of the variables from equation (11). In contrast, the coefficient on $size$ is positive and significant in column (2) but negative and significant in column (3). The results in column (2) suggest that larger firms have higher variance returns. This is consistent with findings in Barth and So (2014), who find that the variance risk premium is higher around earnings announcements for larger firms and industry leaders. However, the results in column (3) suggest that the relation between $size$ and variance returns depends on the inclusion of the firm-specific drivers of variance returns. More specifically, holding constant bm and roe, we find that there is a negative association between $size$ and variance returns.

The results in column (3) shows that the coefficients on both bm and roe are virtually unchanged when compared with Table III. This indicates that while significant, the additional explanatory variables do not affect the cross-sectional relation predicted by the model. We provide additional support for this conclusion by repeating our analysis using 60 calendar day ahead variance returns. The results in column (5) show that the signs of the coefficients match the results in column (4).

B.2. Cross-Sectional Results Using S&P 500 firms

One potential concern with using the entire cross-section of firms with traded options is that option activity in smaller firms may be limited and the price of the variance contracts we extract from these options may be noisy. Therefore, we repeat the above cross-sectional analysis after limiting our sample to firms which are constituents of the S&P 500 Index. Examining the relation between variance risk premiums and firm fundamentals for S&P 500 firms serves two purposes. First, they are large and highly liquid stocks. Therefore, if our full sample results are driven by liquidity constraints or another market imperfection, then the results would be different for the S&P 500 firms relative to our full sample. Second, options on S&P 500 firms are actively traded and thus the price of variance extracted from these options will contain less noise than the prices of variance extracted from options on firms with lower option trading activity.

Table V provide the results of the cross-sectional analysis using only S&P 500 firms. Despite the significantly reduced sample size, the empirical results conform with the findings from the full sample of firms. Both bm and roe have a strong negative association with variance returns and this association is not subsumed when we include coefficients from the Fama and French (1993) three factor model and a set of firm-specific control variables that may be associated with the variance risk premium. The fact that the predicted relation between bm, roe and variance returns is preserved for the biggest and most liquid firms in the economy is noteworthy because many empirical relationships between firm characteristics and stock returns either vanish or are significantly attenuated when samples are constrained to large firms (Fama and French, 2008).

C. Time Series Tests

The above tests document a strong cross-sectional relation between variance returns and *bm* and *roe*, consistent with our model. However, because these tests are conducted at the firm level, it is not clear whether our results are attributable to latent systematic risk. To examine whether this is the case, we investigate whether the predicted relation is preserved in the aggregate using time series data. The use of time series data to examine the drivers of aggregate volatility is a very common approach Engle and Rangel (2008).

We conduct this analysis in two ways. First, we simply calculate the median variance return, median *bm* and median *roe* in the cross-section over our 1996-2013 sample period. We then run a time series regression of the median variance return on the median *bm* and the median *roe*. Second, we run a time series regression where we use the return on the variance of the S&P 500 index from Bollerslev et al. (2009)as an aggregate measure of the variance return. This extends our sample period back to 1990, as we are no longer constrained to only those firms with actively traded options. We then regress the return on the variance of the S&P 500 index on median *bm* and median *roe*.

Obtaining identical results in both cross-sectional and time series tests is not obvious. For example, Kothari et al. (2005) find that earnings surprises have a positive cross-sectional association with stock returns, but a negative association in aggregate. Similarly, Hirshleifer et al. (2009) find a strong positive relation between aggregate accruals and aggregate stock returns, which is the exact opposite to the findings of the firm-level results documented by Sloan (1996).

Table VI presents the time series regression results. As with our analysis in Table III, we show results separately and together for *bm* and *roe*. As with our main analysis, we find that the predicted associations between variance returns and *bm* and *roe* depend on the inclusion of both variables. In column (1), the coefficient on *bm* is positive and statistically significant. In contrast, the coefficient on *bm* is negative and statistically significant in both Column (4) and (5) when *roe* is included in the specification. This suggests that *bm* and *roe* work together to provide the predicted relation. The full specification in column (5) has negative and statistically significant coefficients for each variable, consistent with our model.

The analysis in columns (6) through (10) use the return on the variance of the S&P 500 index from Bollerslev et al. (2009) as the dependent variable. The results closely mirror those in columns (1) through (5). In column (6), the coefficient on *bm* is positive and insignificant. In contrast, the coefficient on *bm* is negative and statistically significant in both Column (9) and (10) when *roe* is included in the specification. Once again, this implies that *bm* and *roe* work together to provide the predicted relation. The full specification in column (10) has negative and statistically significant coefficients for each variable, consistent with predictions of the model.

D. Portfolio Sorts

The prior tests examine whether the relation between variance returns and *bm* and *roe* are statistically significant and robust to alternative specifications. However, they do not offer insight into the economic magnitude of this relation. Our next set of analyses investigate the economic magnitude of the relation between variance returns and *bm* and *roe* by determining whether economically meaningful returns to a variance trading strategy based on *bm* and *roe* are present in the data. The results in Table VII show that the variance return two-way portfolio sorts based on *bm* and *roe* map well into the predicted relation. The returns to this strategy are, on average, decreasing in both *bm* and *roe*. The variance return is -8.6 percent per month when both *bm* and *roe* are in the lowest quintile, compared with -21.3 percent when both *bm* and *roe* are in the highest quintile. The returns on a hedged portfolio within each *bm* quintile are all negative, and the magnitudes increase as we move into the higher *bm* quintiles. The Fama-French three factor α's are also high in magnitude and significant, suggesting that the inclusion of classic risk factors have virtually no impact on the average variance returns based on our strategy. This finding is consistent with Carr and Wu (2009), who also document that traditional risk characteristics do not explain variance risk premiums.

E. Writing Puts

For our final set of analyses, we construct a simple trading strategy of writing put options based on *bm* and *roe*. The model predicts that stock returns are positively associated with *bm* and *roe*, whereas the variance risk premium is negatively associated with *bm* and *roe*. Writing puts is equivalent to going long the stock and short the variance, which based on the predictions of the model, should maximize exposure to both prices of risk and generate high realized returns to a strategy based on *bm* and *roe*. Tables VIII and IX present the average monthly returns to a strategy that writes a 30 day put at the end of each month. The first set of results in Table VIII use the full sample of firms. These results indicate that once you condition on *bm*, the realized returns are lowest for the portfolios that contain the lowest quintile of *roe*. For firms in the lowest quintile of *bm*, the realized returns increase from 15.2 percent to 21.2 percent as you move from the lowest to the highest quintile of *roe*. Similarly, for firms in the highest quintile of *bm*, the realized returns increase from 11.9 percent to 18.0 percent as you move from the lowest to the highest quintile of *roe*. The hedged returns to this strategy are economically large and statistically significant within each *bm* quintile. Moreover, the α's that are generated from the strategy are also large and highly significant, and suggest that the returns are not driven by variation in classic risk factors.

We repeat the above analysis using only firms which are constituents of the S&P 500. We do this to mitigate the concern that our results are driven by liquidity or other market imperfections, as options on S&P 500 firms are actively traded. The results in Table IX are very similar to those presented in Table VIII. Once again, conditional on *bm*, realized returns are lowest for the portfolios that contain the lowest quintile of *roe*. For firms in the lowest quintile of *bm*, the realized returns increase from 19.3 percent to 24.1 percent as you move from the lowest to the highest quintile of *roe*. Similarly, for firms in the highest quintile of *bm*, the realized returns increase from 9.3 percent to 24.7 percent as you move from the lowest to the highest quintile of *roe*. Like the full sample results, the hedged returns and the α using only S&P 500 firms are economically large and highly significant.

IV. Conclusion

Our study formally links accounting-based valuation models to variance risk premiums. Our empirical analyses provide evidence that accounting numbers inform investors about the priced risk of future cash flows. This analysis extends several prior studies (e.g., Barth and So, 2014; Han and Zhou, 2012,) by identifying firm-level characteristics that are associated with the time series and cross-sectional variation in variance risk premiums identified in those studies. It also represents an important contribution to the literature that examines the relation between firm characteristics and asset returns (e.g., Daniel and Titman, 1997; Haugen and Baker, 1996; Lewellen, 2014 among others) because it shows that accounting information is useful for forecasting the returns of financial assets other than stocks.

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A. Derivations

A. Book-to-Market Derivation

Given the stationary assumption of bm we have that for the book-to-market ratio to be finite, we have the following:

$$bm_t = \overline{bm} + \sum_{i=1}^{\infty} E_t[r_{t+i} - g_{t+i}], \tag{12}$$

where $r_{t+i} = m_{t+i} - m_{t+i-1}$ is the ex dividend return on market equity and book growth. As outlined in our text we have that the book growth process is given by,

$$g_{t+1} = \bar{g} + \kappa g_t + \eta \sigma_{g,t}^2 + \sigma_{g,t}\epsilon_{t+1}, \tag{13}$$

$$\sigma_{g,t+1} = \omega \sigma_{g,t} + \gamma z_{t+1}, \tag{14}$$

where $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$. $q \in [-1, 1]$ is a correlation coefficient and both ϵ_{t+1} and ξ_{t+1} are IID standard normal distributions. The ξ_{t+1} term is assumed to be uncorrelated ϵ_{t+1} and the shocks to the discount factor, w_{t+1}. To solve for bm_t we use the same approach as Bansal and Yaron (2004) and conjecture that the ratio is linear in the state variables g_t and $\sigma_{g,t}$ and verify that the solution satisfies the no-arbitrage condition

$$1 = E_t[\frac{\Lambda_{t+1}}{\Lambda_t} e^{m_{t+1} - m_t}], \tag{15}$$

$$= E_t[e^{\lambda_{t+1} + r_{t+1}}], \tag{16}$$

where $\lambda_{t+1} = \log(\frac{\Lambda_{t+1}}{\Lambda_t})$. We conjecture that that the log book-to-market ratio is $bm_t = A_0 + A_1 g_t + A_2 \sigma_{g,t}$,which implies that $g_{t+1} = r_{t+1} + A_1(g_{t+1} - g_t) + A_2(\sigma_{g,t+1} - \sigma_{g,t})$. Thus $r_{t+1} = g_{t+1}(1 - A_1) + A_1 g_t - A_2(\sigma_{g,t+1} - \sigma_{g,t})$. Since both λ_{t+1} and r_{t+1} are conditionally normal, then this implies that

$$E_t[r_{t+1}] + \frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1}), \tag{17}$$

where

$$E_t[r_{t+1}] = (1 - A_1)(\bar{g} + \kappa g_t + \eta\sigma_{g,t}^2) + A_1 g_t - A_2(\omega - 1)\sigma_{g,t}, \tag{18}$$

$$V_t[r_{t+1}] = ((1 - A_1)\sigma_{g,t} - A_2 q\gamma)^2 + (1 - q^2)A_2^2\gamma^2, \tag{19}$$

$$cov_t(\lambda_{t+1}, r_{t+1}) = -((1 - A_1)\sigma_{g,t} - A_2 q\gamma)\sigma_\Lambda\rho. \tag{20}$$

Collecting like terms, we have:

$$g_t:\ (1 - A_1)\kappa + A_1 = 0 \tag{21}$$

$$\sigma_{b,t}:\ -A_2(\omega - 1) - (1 - A_1)A_2 q\gamma = (1 - A_1)\rho\sigma_\Lambda \tag{22}$$

$$\sigma_{g,t}^2:\ (1 - A_1)\eta + \frac{1}{2}(1 - A_1)^2 = 0 \tag{23}$$

$$(1 - A_1)\bar{g} + \frac{1}{2}(A_2^2 q^2 + (1 - q^2)A_2^2)\gamma^2 = r_f + \rho\sigma_\Lambda A_2 q\gamma \tag{24}$$

Solving the above set of equations simultaneously implies that $A_1 = -\frac{\kappa}{1-\kappa}$, $A_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}$, and the "variance-in-mean" parameter is $\eta = -\frac{1}{2}\frac{1}{(1-\kappa)}$. To solve for A_0 we have $\overline{mb} = A_0 + A_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$ which implies $A_0 = \overline{mb} + \frac{\kappa}{1-\kappa}(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$. Using this and writing market values as stated in the text gives,

$$m_t = b_t + \alpha_0 + \alpha_1 g_t - \alpha_2\sigma_{g,t}, \tag{25}$$

where $\alpha_0 = -\overline{bm} - \alpha_1(\frac{\bar{g}}{1-\kappa} + \frac{\gamma^2}{(1-\kappa)(1-\omega^2)})$, $\alpha_1 = \frac{\kappa}{1-\kappa}$, $\alpha_2 = \frac{\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}$ and $roe_t \equiv g_t$ since the firm does not pay dividends.

B. Derivation of Stock Returns

Armed with the market value equation (25) and that $E_t[r_{t+1}] + \frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1})$ we now can calculate market returns as

$$r_{t+1} = E_t[r_{t+1}] - (r_{t+1} - E_t[r_{t+1}]), \tag{26}$$

$$= r_f - ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)\rho + \frac{1}{2}\sigma_t^2$$

$$+ (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \gamma\alpha_2 z_{t+1}. \tag{27}$$

which is the equation in the text.

C. Stock Return Variance

The conditional variance of the stock return is given by $V_t = E_t[(r_{t+1} - E_t[r_{t+1}])^2] = \sigma_t^2$. From (6) this implies

$$\sigma_t^2 = ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)^2 + (1-q^2)\alpha_2^2\gamma^2. \tag{28}$$

D. Expected rates of equity returns

By the no arbitrage condition we have that $\mu_t = \log(E_t[e^{r_{t+1}}]) = E_t[r_{t+1}] + \frac{1}{2}V_t[r_{t+1}] = r_f - cov_t(\lambda_{t+1}, r_{t+1})$. From (6) we arrive at

$$\mu_t = r_f + \rho\sigma_\Lambda[(1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma]. \tag{29}$$

To express μ_t in terms of accounting-based variables we can use (25) to write the volatility of book growth as $\sigma_{g,t} = \frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t]$. Substituting this into (29) we obtain

$$\mu_t = r_f + \rho\sigma_\Lambda[(1+\alpha_1)\frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t] - \alpha_2 q\gamma].$$

After some algebra, gives

$$\mu_t = r_f + \theta_0 + \theta_1 bm_t + \theta_2 g_t, \tag{30}$$

where $\theta_0 = \theta_1(\alpha_0 - \alpha_2 q\gamma)$, $\theta_1 = (1-\omega) - \frac{\gamma q}{(1-\kappa)}$, $\theta_2 = \alpha_1\theta_1$and $roe_t \equiv g_t$ since the firm does not pay dividends.

E. Variance risk premiums

The price of a variance contract is it's discounted payoff, and for no-arbitrage, must satisfy the standard condition:

$$v_{t,t+1} = E_t[e^{\lambda_{t+1}}\sigma^2_{t+1}] = e^{-r_f}E_t[\sigma^2_{t+1}] + cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}). \tag{31}$$

This implies that the variance risk premium is given by

$$E_t[\sigma^2_{t+1}] - e^{r_f}v_{t,t+1} = -e^{r_f}cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}). \tag{32}$$

From (28) we have:

$$\begin{aligned}\sigma^2_t &= ((1+\alpha_1)\sigma_{g,t} - \alpha_2 q\gamma)^2 + (1-q^2)\alpha_2^2\gamma^2 \\ &= (1+\alpha_1)^2\sigma^2_{g,t} - 2\omega(1+\alpha_1)\alpha_2\sigma_{g,t}\gamma \\ &+ \alpha_2^2 q^2\gamma^2\end{aligned} \tag{33}$$

$$+ (1-q^2)\alpha_2^2\gamma^2. \tag{34}$$

Thus next period expected variance is given by:

$$\sigma_{t+1}^2 = (1+\alpha_1)^2\sigma_{g,t+1}^2 - 2\omega\gamma(1+\alpha_1)\alpha_2\sigma_{g,t+1} + \alpha_2^2\gamma^2, \tag{35}$$

and

$$E_t[\sigma_{t+1}^2] = (1+\alpha_1)^2 E_t[\sigma_{g,t+1}^2] - 2\omega\gamma(1+\alpha_1)\alpha_2 E_t[\sigma_{g,t+1}] + \alpha_2^2\gamma^2, \tag{36}$$

$$= (1+\alpha_1)^2(\omega^2\sigma_{g,t}^2 + \gamma^2)$$

$$- 2\omega^2(1+\alpha_1)\alpha_2\gamma\sigma_{g,t} + \alpha_2^2\gamma^2. \tag{37}$$

We need to determine the covariance term,

$$cov_t(e^{\lambda_{t+1}}, \sigma_{t+1}^2) = (1+\alpha_1)^2 cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}^2)$$

$$+ -2\omega\gamma(1+\alpha_1)\alpha_2 cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}). \tag{38}$$

In order to solve this we need to determine $cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}^2)$ and $cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1})$, where $\sigma_{g,t+1}^2 = (\omega\sigma_{g,t} + \gamma z_{t+1})^2$ and $\sigma_{g,t+1} = \omega\sigma_{g,t} + \gamma z_{t+1}$. Given that λ_{t+1} and $z_{t+1} = q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}$are normal, we have

$$cov_t(e^{\lambda_{t+1}}, \sigma_{g,t+1}) = \gamma cov_t(e^{\lambda_{t+1}}, q\epsilon_{t+1} + \sqrt{1-q^2}\xi_{t+1}), \tag{39}$$

$$= -\gamma e^{-r_f} q\rho\sigma_\Lambda. \tag{40}$$

To calculate the second covariance term, note that $(\omega\sigma_{g,t}+\gamma z_{t+1})^2 = \omega\sigma_{g,t}^2 + 2\omega\gamma z_{t+1}\sigma_{g,t} + \gamma^2 z_{t+1}^2$, thus

$$cov(e^{\lambda_{t+1}}, \sigma^2_{g,t+1}) = -e^{-r_f}\rho q \sigma_\Lambda 2\omega\gamma\sigma_{g,t} + \gamma^2 cov(e^{\lambda_{t+1}}, \gamma z^2_{t+1}). \tag{41}$$

To solve for the second term, we have

$$\gamma^2 cov(e^{\lambda_{t+1}}, z^2_{t+1}) = \gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1} + 2q\sqrt{1-q^2}\epsilon_{t+1}\xi_{t+1} + (1-q^2)\xi^2_{t+1})), \tag{42}$$

$$= \gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1}). \tag{43}$$

Since ϵ_{t+1} is normal, we can decompose it into $\epsilon_{t+1} = \rho w_{t+1} + \sqrt{1-\rho^2} w^*_{t+1}$ where w^*_{t+1} is a independent normal distribution. This implies

$$\gamma^2 E_t((e^{\lambda_{t+1}} - e^{-r})(q^2\epsilon^2_{t+1}) = -\gamma^2(e^{-r_f}q^2 + q^2 E_t[e^{\lambda_{t+}}\epsilon^2_{t+1}]) \tag{44}$$

$$= -\gamma^2 e^{-r_f} q^2(1 + \rho^2(\sigma^2_\Lambda + 1)). \tag{45}$$

Thus

$$cov_t(e^{\lambda_{t+1}}, \sigma^2_{g,t+1}) = -e^{-r_f}\rho\sigma_\Lambda 2\omega\gamma\sigma_{g,t} - \gamma^2 e^{-r_f} q^2(1 + \rho^2(\sigma^2_\Lambda + 1)). \tag{46}$$

Plugging this back into (41), we obtain

$$\begin{aligned} e^{r_f} cov_t(e^{\lambda_{t+1}}, \sigma^2_{t+1}) &= -(1+\alpha_1)^2(\rho\sigma_\Lambda 2\omega\gamma\sigma_{g,t} + \gamma^2 q^2(1 + \rho^2(\sigma^2_\Lambda + 1))) \\ &\quad + 2\omega^2(1+\alpha_1)\alpha_2\gamma q\rho\sigma_\Lambda, \qquad (47) \\ &= -2(1+\alpha_1)^2\rho\sigma_\Lambda\omega\gamma\sigma_{g,t} \end{aligned}$$

$$- (1+\alpha_1)^2\gamma^2 q^2(1+\rho^2(\sigma_\Lambda^2+1)) + 2\omega^2(1+\alpha_1)\alpha_2\gamma q\rho\sigma_\Lambda, \tag{48}$$

$$= \eta_1\sigma_{g,t} + \eta_0. \tag{49}$$

where $\eta_0 = [-(1+\alpha_1)\gamma q(1+\rho^2(\sigma_\Lambda^2+1)) + 2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$ and $\eta_1 = -(1+\alpha_1)^2 q\rho\sigma_\Lambda 2\omega\gamma$.

Thus the variance risk premium is given by

$$E_t[\sigma_{t+1}^2] - e^{r_f}v_t = \eta_0 + \eta_1\sigma_{g,t}. \tag{50}$$

But we can use the fact that $\sigma_{g,t} = \frac{1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t]$ to obtain

$$E_t[\sigma_{t+1}^2] - e^{-r_f}v_t = -\eta_0 + -\frac{\eta_1}{\alpha_2}[bm_t + \alpha_0 + \alpha_1 g_t] \tag{51}$$

But $\frac{\eta_1}{\alpha_2} = \frac{-(1+\alpha_1)^2 q\rho\sigma_\Lambda 2\omega\gamma}{\frac{-\rho\sigma_\Lambda}{(1-\kappa)(1-\omega)-\gamma q}} = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega)-\gamma q) = \phi_1$ which is negative if $q < 0$. Thus

$$E_t[\sigma_{t+1}^2] - e^{-r_f}v_t = \phi_0 + \phi_1 bm_t + \phi_2 g_t, \tag{52}$$

where $\phi_0 = \alpha_0\phi_1 + \eta_0$, $\phi_1 = q(1+\alpha_1)^2\omega\gamma((1-\kappa)(1-\omega)-\gamma q)$, $\phi_2 = \alpha_1\phi_1$, $\eta_0 = -[(1+\alpha_1)\gamma q(1+\rho^2(\sigma_\Lambda^2+1)) - 2\omega^2\alpha_2\rho\sigma_\Lambda]q\gamma(1+\alpha_1)$, and $roe_t \equiv g_t$ since the firm does not pay dividends delivers the equation in the main body of the text.

F. A relation between book growth and market volatility

Our model of book growth depends upon $\sigma_{g,t}^2$ which is difficult to estimate do to times series data limitations. In her we derived an expression that relates expected book growth to stock returns variance, σ_t^2 as well as lagged book growth and the book-to-market ratio. Given that, $g_{t+1} = \bar{g} + \kappa g_t + \eta\sigma_{g,t}^2 + \sigma_{g,t}\epsilon_{t+1}$, where $\eta = -\frac{1}{2}\frac{1}{(1-\kappa)}$. We can substitute out $\sigma_{g,t}^2$ by using (33). Combining this with (25) implies that future book growth can be written as a function of current growth, expected market variance and bm_t:

$$g_{t+1} = \varphi_0 + \varphi_1 g_t + \varphi_2 bm_t + \varphi_3 \sigma_t^2 + \sigma_{g,t}\epsilon_{t+1}, \tag{53}$$

where $\varphi_0 = \bar{g} + \omega\gamma\alpha_0 + \frac{1}{2}\frac{1}{1+\alpha_1}\alpha_2^2\gamma^2$, $\varphi_1 = (1 - \frac{\omega\gamma}{1-\kappa})\kappa$, $\varphi_2 = -\omega\gamma$, and $\varphi_3 = -\frac{1}{2}(1+\kappa)$. The α_0, α_1, and α_2 terms are defined above.

G. Measuring the price of a variance contract

We want the price of a variance contract from time t to some future date τ, $v_{t,t+\tau} = E_t[\frac{\Lambda_{t+\tau}}{\Lambda_t}\sum_{i=1}^{\tau} E_t([r_{t+i} - E_t[r_{t+i}])^2]$. To recover this value, we can use the market price of a contract that pays off the logarithm of the stock price. From (6) we have,

$$\begin{aligned} m_{t+1} &= m_t + \mu_t - \frac{1}{2}\sigma_t^2 + (1+\alpha_1)\sigma_{g,t}\epsilon_{t+1} - \alpha_2\gamma z_{t+1}, &(54)\\ &= m_t + \mu_t - \frac{1}{2}\sigma_t^2 + \eta_{t+1}. &(55)\end{aligned}$$

The time $t+\tau$ log price is then given by

$$m_{t+\tau} = m_t + \sum_{i=1}^{\tau}\mu_{t+i-1} - \frac{1}{2}\sum_{i=1}^{\tau}\sigma_{t+i-1}^2 + \sum_{i=1}^{\tau}\eta_{t+i}. \tag{56}$$

The price of the log-contract is then given by

$$\begin{aligned} f_{t,t+\tau} &= E_t[\frac{\Lambda_{t+\tau}}{\Lambda_T}m_{t+\tau}], &(57)\\ &= e^{-r_f(t+\tau)}(m_t + r_f(t+\tau)) - \frac{1}{2}E_t[\frac{\Lambda_{t+\tau}}{\Lambda_t}\sum_{i=1}^{\tau}\sigma_{t+i-1}^2]. &(58)\end{aligned}$$

This implies that the price of a contract that pays the cumulative variance from time t to $t+\tau$ is $v_{t,t+\tau} = 2(e^{-r_f(t+\tau)}(m_t + r_f(t+\tau)) - f_{t,t+\tau})$.

H. The price of the log contract

We apply the model free equation provided by Bakshi and Madan (2000) where any twice differentiable function $F(S)$ can be expressed as:

$$F(S) = F(\bar{S}) + (S - \bar{S})F_S(\bar{S}) \tag{59}$$

$$+ \int_{\bar{S}}^{\infty} F_{SS}(K)(S-K)^+ dK + \int_0^{\bar{S}} F_{SS}(K)(K-S)^+ dK, \tag{60}$$

where $\bar{S}$ is an arbitrary real constant.

Let $F(S) = \log(M_{t+\tau}) = m_{t+\tau}$, then

$$m_{t+\tau} = \log(\bar{S}) + \frac{(S_{t+\tau} - \bar{S})}{\bar{S}} - \int_{\bar{S}}^{\infty} \frac{(S-K)^+ dK}{K^2} - \int_0^{\bar{S}} \frac{(K-S)^+ dK}{K^2}. \tag{61}$$

The value of the log contract is thus

$$\begin{aligned} f_{t,t+\tau} = E^Q[e^{-r_{t+\tau}} m_{t+\tau}] &= e^{-r_{t+\tau}} \log(F_{t+\tau}) \\ &- \int_{F_{t+\tau}}^{\infty} \frac{C(K, t+\tau)dK}{K^2} - \int_0^{F_{t+\tau}} \frac{P(K, t+\tau)dK}{K^2} \end{aligned} \tag{62}$$

where $F_{t,t+\tau}$ is a forward contract on the equity, while $C(K, t+\tau)$ and $P(K, t+\tau)$ represent call and put contracts respectively. This implies that the price of variance can be given by,

$$\begin{aligned} v_{t,t+\tau} = 2(e^{-r_f(t+\tau)}&(m_t + r_f(t+\tau)) - e^{-r_{t+\tau}} \log(F_{t+\tau}) \\ &+ \int_{F_{t+\tau}}^{\infty} \frac{C(K, t+\tau)dK}{K^2} + \int_0^{F_{t+\tau}} \frac{P(K, t+\tau)dK}{K^2}). \end{aligned} \tag{63}$$

We approximate this equation using OptionMetrics' volatility surface files along with their estimate of the forward contract, $F_{t+\tau}$.

B. Tables

Table I: Summary Statistics

Table I presents summary statistics of key variables used in the analysis and other common firm-level characteristics. $v_{t,t+1}$ represents the price of a 30 day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. σ^2_{t+1} represents 30 day ahead realized variance, which is calculated using the sum of squared daily log returns. R_f is the gross risk free rate obtained from OptionMetrics zero coupon rate file. $R^v_{t+1} - R_f$ is the excess return on a variance contract (in percent) on a 30 day ahead variance contract. R_{t+1} is the 30 day ahead net stock return (in percent). $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio. $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity. $size = \log(M_t)$ is the logarithm of market capitalization. β is a firm's rolling 5 year (60 months) historical "beta" estimated using the market model, and $lvar$ is the lagged 30 day variance.

	Mean	Std Dev	P10	P25	P50	P75	P90
$100\times v_{t,t+1}$	2.79	3.06	0.63	1.08	1.92	3.39	5.83
$100\times\sigma^2_{t+1}$	2.29	3.62	0.27	0.52	1.13	2.54	5.34
$100\times(\sigma^2_{t+1} - v_{t,t+1}R_f)$	-0.44	2.87	-2.58	-1.32	-0.50	0.02	1.32
$R^v_{t+1} - R_f$	-16.72	77.95	-78.20	-62.24	-37.24	1.23	61.88
R_{t+1}	0.77	14.98	-14.99	-6.47	0.52	7.36	16.03
bm_t	-0.89	0.77	-1.90	-1.35	-0.84	-0.38	0.01
$100\times roe_t$	1.38	9.14	-5.63	0.39	2.75	4.78	7.49
$size$	7.28	1.57	5.33	6.13	7.15	8.26	9.45
β	1.31	0.85	0.42	0.73	1.15	1.72	2.41
$100\times lvar$	2.40	3.50	0.31	0.60	1.26	2.74	5.54

Table II: Correlation Matrix

Table II presents the correlation matrix of key variables used in the analysis and other common firm-level characteristics. $v_{t,t+1}$ represents the price of a 30 day ahead variance contract, which is estimated using a cross-section of call and put options from OptionMetrics Volatility surface file following the model free method outlined in the appendix. σ^2_{t+1} represents 30 day ahead realized variance, which is calculated using the sum of squared daily log returns. R_f is the gross risk free rate obtained from OptionMetrics zero coupon rate file. $R^v_{t+1} - R_f$ is the excess return on a variance contract (in percent) on a 30 day ahead variance contract. R_{t+1} is the 30 day ahead net stock return (in percent). $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio. $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity. $size = \log(M_t)$ is the logarithm of market capitalization. β is a firm's rolling 5 year (60 months) historical "beta" estimated using the market model, and $lvar$ is the lagged 30 day variance.

$100\times v_{t,t+1}$		0.515	-0.361	-0.059	-0.025	0.030	-0.355	-0.463	0.363	0.542
$100\times\sigma^2_{t+1}$	0.515		0.492	0.658	-0.029	-0.011	-0.257	-0.366	0.368	0.499
$100\times(\sigma^2_{t+1} - v_{t,t+1}R_f)$	-0.361	0.492		0.767	-0.023	-0.049	0.111	0.144	-0.001	-0.065
$R^v_{t+1} - R_f$	-0.059	0.658	0.767		-0.025	-0.049	-0.008	-0.021	0.116	0.103
R_{t+1}	-0.025	-0.029	-0.023	-0.025		0.010	0.017	0.009	-0.012	-0.029
bm_t	0.030	-0.011	-0.049	-0.049	0.010		-0.083	-0.234	-0.026	-0.008
$100\times roe_t$	-0.355	-0.257	0.111	-0.008	0.017	-0.083		0.302	-0.205	-0.285
$size$	-0.463	-0.366	0.144	-0.021	0.009	-0.234	0.302		-0.272	-0.404
β	0.363	0.368	-0.001	0.116	-0.012	-0.026	-0.205	-0.272		0.406
$100\times lvar$	0.542	0.499	-0.065	0.103	-0.029	-0.008	-0.285	-0.404	0.406	

Table III: Cross-Sectional Regressions

Table III reports mean coefficients and t-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity (both deflated by beginning period $v_{t,t+1}$ as per equation (11). $v_{t,t+1}$ is the price of a 30 (60 in column 6) day ahead variance contract, which is estimated using the model free method outlined in the appendix from a cross-section of call and put options from OptionMetrics Volatility surface file. The t-statistics are calculated from Fama–MacBeth standard errors. *, **, and *** denote two- tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
bm_t	-0.046***			-0.069***	-0.044***	-11.224***
	(-3.995)			(-6.304)	(-3.771)	(-5.069)
roe_t		0.188		-0.580***	-0.706***	-1.688***
		(1.481)		(-6.471)	(-9.410)	(-10.554)
$\frac{1}{v_{t,t+1}}$			0.065***		0.061**	6.115*
			(2.989)		(2.355)	(1.763)
Intercept	-20.667***	-17.647***	-21.685***	-20.907***	-22.886***	-16.681***
	(-8.997)	(-7.366)	(-10.672)	(-9.100)	(-11.583)	(-6.730)
# Observations	312,229	312,229	312,229	312,229	312,229	312,167
R^2	0.011	0.006	0.013	0.014	0.020	0.021

Table IV: Cross-Sectional Regressions with Additional Controls

Table IV reports mean coefficients and t-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity (both deflated by beginning period $v_{t,t+1}$ as per equation (11). $v_{t,t+1}$ is the price of a 30 (60 in column 4) day ahead variance contract, which is estimated using the model free method outlined in the appendix from a cross-section of call and put options from OptionMetrics Volatility surface file. β_M , β_H , and β_S denote full sample firm-specific excess variance return betas obtained from the Fama-French (1993) three-factor model where the betas represent the firm specific slopes on the value weighted market portfolio, the portfolio of high minus low book-to-market firms (HML), and the portfolio of small minus large firms (SMB), respectively. $lvar$ is lagged 30 day variance and $size = \log(M_t)$ is the logarithm of market capitalization. R_{t+1} is the 30 (60 in column 4) day ahead net stock return and R_t is the lagged one-month return. The t-statistics are calculated from Fama–MacBeth standard errors. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
bm_t			-0.052***	-0.133***
			(-5.588)	(-7.408)
roe_t			-0.439***	-0.984***
			(-7.509)	(-8.297)
$\frac{1}{v_{t,t+1}}$			0.211***	0.474***
			(6.629)	(7.663)
β_m	-3.663***	-2.586***	-3.058***	-4.956***
	(-11.392)	(-9.282)	(-11.886)	(-13.779)
β_H	0.665**	0.275	0.476**	1.393***
	(2.572)	(1.345)	(2.368)	(4.684)
β_S	-1.408***	-1.173***	-1.369***	-2.628***
	(-6.730)	(-5.842)	(-7.358)	(-10.195)
$Size$		1.597***	-1.950***	-3.263***
		(4.572)	(-6.312)	(-7.536)
$lvar$		2.195***	3.354***	4.613***
		(10.918)	(19.105)	(19.322)
R_{t+1}		-0.284***	-0.298***	-0.445***
		(-3.743)	(-3.700)	(-6.534)
R_t		-0.050*	-0.122***	-0.070*
		(-1.679)	(-3.381)	(-1.810)
Intercept	-21.931***	-42.755***	-35.601***	-30.033***
	(-9.266)	(-17.265)	(-15.173)	(-8.221)
# Observations	312,229	312,223	312,223	292,915
R^2	0.020	0.074	0.102	0.137

Table V: Cross-Sectional Regressions with Controls for S&P 500 firms

Table V reports mean coefficients and t-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-period-ahead excess variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity (both deflated by beginning period $v_{t,t+1}$ as per equation (11). $v_{t,t+1}$ is the price of a 30 (60 in column 4) day ahead variance contract, which is estimated using the model free method outlined in the appendix from a cross-section of call and put options from OptionMetrics Volatility surface file. β_M , β_H , and β_S denote full sample firm-specific excess variance return betas obtained from the Fama-French (1993) three-factor model where the betas represent the firm specific slopes on the value weighted market portfolio, the portfolio of high minus low book-to-market firms (HML), and the portfolio of small minus large firms (SMB), respectively. $lvar$ is lagged 30 day variance and $size = \log(M_t)$ is the logarithm of market capitalization. R_{t+1} is the 30 (60 in column 4) day ahead net stock return and R_t is the lagged one-month return. The t-statistics are calculated from Fama–MacBeth standard errors. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
bm_t	-0.081***		-0.023***	-0.050***
	(-8.864)		(-3.233)	(-3.287)
roe_t	-0.349***		-0.303***	-0.748***
	(-4.115)		(-4.740)	(-5.972)
$\frac{1}{v_{t,t+1}}$	-28.924***		0.262***	0.613***
	(-11.099)		(8.939)	(10.363)
β_m		-3.170***	-4.235***	-9.947***
		(-6.331)	(-10.670)	(-16.614)
β_H		0.833*	1.647***	2.887***
		(1.903)	(4.357)	(4.723)
β_S		-1.693***	-2.446***	-4.764***
		(-3.938)	(-7.323)	(-11.950)
$Size$		2.827***	-0.152	-1.502***
		(6.969)	(-0.390)	(-3.399)
$lvar$		7.125***	15.060***	16.029***
		(10.676)	(17.024)	(18.124)
R_{t+1}		-0.511***	-0.519***	-0.584***
		(-3.831)	(-3.646)	(-5.589)
R_t		-0.057	-0.202***	-0.166**
		(-1.113)	(-3.482)	(-2.473)
Intercept	-34.355***	-63.247***	-71.426***	-70.737***
	(-14.589)	(-16.539)	(-18.385)	(-14.596)
# Observations	75,628	75,628	75,628	76,421
R^2	0.037	0.115	0.206	0.197

Table VI: Time Series regressions

Table VI reports the results of a time series regression of excess 30 day ahead variance returns on the variables shown. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio, $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity (both deflated by beginning period $v_{t,t+1}$ as per equation (11). $v_{t,t+1}$ is the price of a 30 day ahead variance contract, which is estimated using the model free method outlined in the appendix from a cross-section of call and put options from OptionMetrics Volatility surface file.. The t-statistics are calculated from robust standard errors corrected for heteroscedasticity. *, **, and *** denote two- tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Variance Returns					S&P 500 index Variance Returns		
bm_t	0.002**			-0.004**	-0.006***	0.001			-0.007***	-0.007**
	(2.436)			(-2.515)	(-3.751)	(1.177)			(-5.391)	(-2.473)
roe_t		-0.078***		-0.204***	-0.097*		-0.084***		-0.287***	-0.286***
		(-3.247)		(-3.703)	(-1.736)		(-3.283)		(-7.546)	(-3.736)
$\frac{1}{v_{t,t+1}}$			-0.003***		-0.006***			-0.001*		-0.000
			(-3.555)		(-3.400)			(-1.933)		(-0.019)
Intercept	-0.159***	-0.118**	-0.076	-0.113**	-0.042	-0.473***	-0.422***	-0.440***	-0.473***	-0.473***
	(-3.194)	(-2.237)	(-1.258)	(-2.121)	(-0.678)	(-9.396)	(-9.144)	(-8.041)	(-9.628)	(-7.722)
# Observations	205	205	205	205	205	287	287	287	287	287
R^2	0.027	0.048	0.063	0.064	0.096	0.004	0.030	0.012	0.070	0.070

Table VII: Variance Return Portfolio Sorts

Table VII reports portfolio returns based on a two-way sort of bm_t and roe_t using the full sample of firms. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio and $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity. Each month firms are first sorted on bm_t and then on roe_t. The (Hi-Low) column provides the return on a high roe portfolio minus a low roe portfolio. The α column represents the intercept, based on a Fama-French three factor model, of the return on the hedged portfolio. The t-statistics are calculated from robust standard errors corrected for heteroscedasticity. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

		$roe\,(\rightarrow)$								
		1 (Low)	2	3	4	5 (Hi)	(Hi-Low)	t-stat	α	t-stat
	1 (low)	-8.57	-10.81	-10.46	-8.77	-9.24	-0.67	(-0.59)	-0.79	(-0.69)
	2	-9.44	-9.82	-14.64	-16.78	-14.17	-4.72***	(-3.43)	-4.67***	(-3.36)
$bm(\downarrow)$	3	-12.51	-11.09	-17.77	-21.19	-16.77	-4.27***	(-3.24)	-4.05***	(-2.95)
	4	-16.74	-20.28	-24.74	-28.55	-22.24	-5.49***	(-3.68)	-5.44***	(-3.49)
	5 (High)	-12.89	-22.62	-28.90	-27.36	-21.29	-8.41***	(-6.57)	-8.78***	(-6.20)

Table VIII: Returns to Short Put Portfolios for Full Sample

Table VIII reports portfolio returns based on a two-way sort of bm_t and roe_t using the full sample of firms. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio and $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity. Each month firms are first sorted on bm_t and then on roe_t. The (Hi-Low) column provides the return on a high *roe* portfolio minus a low *roe* portfolio. The α column represents the intercept, based on a Fama-French three factor model, of the return on the hedged portfolio. The t-statistics are calculated from robust standard errors corrected for heteroscedasticity. *, **, and *** denote two-tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

		$roe\,(\rightarrow)$								
		1 (Low)	2	3	4	5 (Hi)	(Hi-Low)	t-stat	α	t-stat
	1 (low)	15.17	18.98	21.02	21.60	21.19	6.02***	(2.64)	5.87***	(2.59)
	2	10.76	15.42	16.12	18.35	17.10	6.33***	(2.38)	5.57***	(2.03)
$bm(\downarrow)$	3	10.88	11.74	16.17	17.45	17.88	7.03***	(2.60)	6.42***	(2.29)
	4	13.33	13.35	18.26	18.72	18.36	5.01***	(1.93)	4.68***	(1.75)
	5 (High)	11.88	16.91	17.88	19.15	18.05	6.15***	(2.46)	6.41***	(2.52)

Table IX: Returns to Short Put Portfolios for S&P 500 Firms

Table IX reports portfolio returns based on a two-way sort of bm_t and roe_t using only those firms that are constituents of the S&P500. $bm_t = \log(\frac{B_t}{M_t})$ is the book-to-market ratio and $roe_t = \log(1 + \frac{x_t}{B_{t-1}})$ is the quarterly return on equity. Each month firms are first sorted on bm_t and then on roe_t. The (Hi-Low) column provides the return on a high *roe* portfolio minus a low *roe* portfolio. The α column represents the intercept, based on a Fama-French three factor model, of the return on the hedged portfolio. The t-statistics are calculated from robust standard errors corrected for heteroscedasticity. *, **, and *** denote two- tailed statistical significance at the 10%, 5%, and 1% levels, respectively.

		$roe\,(\rightarrow)$								
		1 (Low)	2	3	4	5 (Hi)	(Hi-Low)	t-stat	α	t-stat
	1 (low)	19.28	18.53	20.72	21.71	24.10	4.81	(1.50)	4.69	(1.44)
	2	16.81	18.65	19.98	27.80	21.35	4.72	(1.39)	5.21	(1.56)
$bm(\downarrow)$	3	11.24	15.47	20.83	21.47	22.30	11.28***	(3.21)	10.73***	(3.03)
	4	12.37	19.02	20.54	21.45	19.77	7.46***	(2.21)	8.38***	(2.33)
	5 (High)	9.27	18.20	16.47	21.25	24.66	15.63***	(4.35)	14.76***	(4.13)