Identifying Accounting Quality

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Abstract

I develop an econometric strategy that allows identification of accounting quality. The strategy relies on a new way of characterizing the dynamics of accounting accruals. The characterization is intuitive and does not hinge on strong assumptions about the earnings and accrual processes, or about managerial preferences. The identifying assumptions derive from two accounting properties, namely, that both earnings and cash flows reflect the same underlying performance and that accruals and accounting errors must reverse over time. My approach discriminates between the accounting error and the part of accruals that captures the underlying economic performance. The proposed framework also offers a new way of testing for the presence of earnings management. Implementation issues and empirical evidence are discussed in a companion paper (Nikolaev 2014).

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1. Introduction.

In this paper, I develop an econometric framework that allows identification and estimation of accounting quality. I define accounting quality as the extent to which earnings measure the (unobservable) underlying economic performance without error. The identification strategy relies on a new way of modelling the dynamics of accounting accruals, which I am proposing. The proposed characterization of accruals is intuitive, captures institutional characteristics of accrual accounting, and does not hinge on strong assumptions about the earnings or accrual processes, managerial preferences, or other restrictive conditions. The key sources of identification that I exploit derive from our knowledge of the accounting process and, specifically, from the accounting properties that accruals and accounting errors must reverse over time, and that earnings and cash flows are both imperfect measures of the same economic performance.

The quality of accounting information (earnings/accruals) is one of the most widely researched areas in accounting literature. However, since the key contributions to this area by Jones (1991) and Dechow and Dichev (2002), the accounting literature has made little progress in advancing the measurement of accounting quality (e.g., Gerakos 2012). Existing models of earnings quality lack identification that would allow researchers to differentiate between the “noise” introduced into earnings by the use of accrual accounting from the volatility of the underlying economic performance (Hribar and Nichols 2007, Dechow, Ge and Schrand 2010). As a result, the measures of accounting quality based on proxies for discretionary accruals comingle the variance of earnings explained by accounting error, which is introduced when measuring economic performance, and the variance explained by the underlying performance itself, and ultimately the performance volatility or risk.

Another important limitation of existing accruals models is that they attribute all variation in discretionary (or unexplained) accruals to poor accounting quality and do not take into consideration the fact that discretionary accruals are also used to facilitate performance measurement. Indeed, it is widely
accepted that the primary role of accrual accounting is to improve performance measurement by shifting the recognition of cash flows over time in a way that reduces cash flow timing problems (AICPA 1973, Dechow 1994). This implies that a portion of accruals variance is explained by this performance measurement role and, as such, can be viewed as the “high quality” component of accruals. The existing literature has not made progress in this regard either and, hence, does not offer a way of identifying the portion of accruals that facilitates performance measurement. At the same time, identifying such a component of accruals seems imperative to understanding the economic rationale for accrual accounting.

In this paper, I offer a way to directly address the issues discussed above. I am proposing that accounting accruals can be described in a parsimonious way by two fundamentally different components, both of which are distinct from the underlying performance. The purpose of the first component is to capture the true accrued and deferred portions of the economic performance that are not reflected in cash flows and thus to facilitate performance measurement (i.e., reduce timing error in cash flows). The second component of accruals represents “accounting error” or noise introduced into earnings (and accruals) when measuring the underlying performance. The accounting error can be further decomposed into intentional and unintentional components (I come back to this issue later).

The simple knowledge that accruals consist of two components does not imply that these components (and hence characterization of accounting quality) are identifiable. However, it is possible to exploit the knowledge that, in the longer run, both cash flows and earnings generally agree on the magnitude of the underlying economic performance, and that both accrual components (performance component and error component) must reverse over time. This, in turn, motivates a simple dynamic representation of the accounting accrual process.

The key advantage of this representation of accruals is that, unlike, for example, the model in Dechow and Dichev (2002), it offers a way of identifying the variance of the two components of accruals and separating them from the underlying performance, by exploiting information contained in the time series of accruals, earnings, and cash flows. More specifically, identifying assumptions underlying the proposed representation, impose structure on a set of moment conditions based on auto- and cross-
correlations in these three time series. Applying General Method of Moments (Hansen 1982), now available in most statistical packages, to such moment conditions allows for direct estimation of the variance of the performance measurement components of accruals, i.e., cash flow timing error being addressed by accrual accounting, the variance of the accounting (or estimation) error in the measurement of current performance, and the variance and persistence of unobservable economic performance itself.

The key advantage of using GMM estimation over other time series models is that it relies on minimal distributional assumptions (e.g., it can readily accommodate skewness in earnings). Nevertheless, before the model can be taken to the data, it is important to make several assumptions about the correlations among variables in the model. For example, is the accounting error correlated with the underlying economic performance, ceteris paribus? Can we allow for accounting conservatism to be present in accruals? Can we allow for earnings management to smooth out shocks to performance?

The framework for modeling accruals and accounting quality offered in this paper is flexible, as it allows for a number of generalizations. I make a baseline set of assumptions, which are generally consistent with prior literature, discuss the restrictiveness of these assumptions, and demonstrate how they can be tested and relaxed in various directions. For example, I discuss how the proposed characterization can accommodate the asymmetric timeliness of gains and losses (Basu 1997). I also discuss how the framework can accommodate the modeling income smoothing proposed by Gerakos and Kovrijnykh (2013) and earnings management more generally.

I contribute to the literature in the following ways. First, I propose an intuitive dynamic characterization of accruals. Such characterization is compelling due to its simplicity and flexibility, which does not come at a cost of making restrictive assumptions about the accrual process. To my knowledge, such representation has not been used in the prior literature. Second, I offer a flexible identification strategy that can be used to identify the quality of earnings and accruals. To my knowledge, prior studies do not exploit the idea that cash flows and earnings reflect the same performance to identify earnings quality. Additionally, prior studies generally do not take advantage of the reversal in accruals to
identify accounting quality.\footnote{Two prior studies that exploit reversal are Dechow et al (2012), and Gerakos and Kovrijnykh (2013). Dechow et al use reversal primarily as a way to gain statistical power to detect earnings management rather than as an identification tool (identification comes from the specification of exogenous partitioning variable), although they also point out that reversal improves model specification. Gerakos and Kovrijnykh focus on the reversal of earnings management and do not model accruals or earnings quality more generally.} As a result, models of earnings/accrual quality (e.g., Jones 1991, Dechow and Dichev 2002) do not offer identification, and comingle the accounting error, the high quality accruals component, and the underlying economic performance, as I discuss in more detail in Section 5. Third, I offer an internally consistent framework to identify the variance of performance measurement component of accruals, i.e., the portion of “high quality” accruals. In contrast to the estimation error, this component of accruals is central to understanding the quality of accounting information as it reflects the “benefits” from using accrual basis accounting.

Fourth, I offer several generalizations of the model that allow for the incorporation of various covariance structures in accrual components. In particular, I offer a way of decomposing the “accounting error” into unmanaged and managed components and of identifying the variance of these components. Additionally, I offer a way of modeling the time series of accruals that takes into account the asymmetric timeliness of gains vs. losses. This is important because Ball and Shivakumar (2006) identify the failure to accommodate the asymmetry in accruals as an important limitation to the existing models of discretionary accruals. Fifth, the proposed framework allows researchers to test for the presence of earnings management and overcomes limitations of alternative approaches that rely on proxies for discretionary accruals (Gerakos 2012).

Finally, the study is related to several recent papers that offer ways to overcome some limitations of standard discretionary accruals models. Gerakos and Kovrijnykh (2012) advocate indirect inference about earnings management by testing for negative second-order auto-correlation in the error term in an earnings auto-regression. Beyer, Guttman and Marinovic (2014) rely on a structural model that makes assumptions about managerial preferences as a function of stock-based compensation to characterize dynamic equity valuation. The model does not account for the reversal in accruals but instead relies on
structural equations and the effect of earnings management on stock returns to identify accounting quality.

Note that, due to a potentially large number of ways to relax the baseline set of assumptions, I leave the discussion of implementation issues and empirical evidence for a companion paper (Nikolaev 2014). The current study proceeds as follows. The next section develops a dynamic representation of accounting accruals and discusses accounting quality in this context. Section 3 discusses the identification strategy and lays out a baseline set of assumptions. Section 4 considers ways to relax these assumptions. Section 5 more formally discusses the advantages of the current approach to identifying accounting quality over conventional approaches. Section 6 considers the effect of accounting conservatism. Section 7 discusses explicit ways to account for earnings management. Section 8 concludes the study.


In this section, I propose a simple dynamic representation of accounting accruals. This representation is based on the idea that cash flows and earnings can be viewed as noisy measures of the underlying economic performance (e.g., Stickney and Weil 2007). This view, in turn, implies a simple dynamic structure for accounting accruals, which can be exploited to understand and identify earnings (accruals) quality. Consistent with the Statement of Financial Accounting Concepts No. 1 and Dechow, Ge and Schrand (2010), I define the quality of accounting earnings (accruals) in the following way:

**Definition:** Accounting quality is the extent to which accrual accounting facilitates the measurement of the underlying economic performance.

In the rest of this section, I develop this concept more formally.

2.1. *Accounting error in periodic performance measures.*

How well did a company perform over a period of time? There is no perfect way to answer this question. Indeed, the need to measure economic performance over relatively short time periods, such as quarters or years, is ultimately at the root of measurement problems in accounting. Different performance measures, such as accounting earnings, cash flows, or stock returns, will generally give the same or
similar answer about a firm’s performance in the longer run (e.g., Trueblood study, AICPA 1977, Easton, Harris and Ohlson 1992). In the short run, however, these measures will exhibit a different degree of measurement error. The larger the measurement error, the lower the quality of an accounting performance measure.

The measurement error across different performance measures has a key similarity: it arises because of the difficulties in attributing (allocating) the underlying economic performance to a particular time period. Thus, it can be thought of as a “timing error” that cancels out over time and thus exhibits negative autocorrelation. To make things more concrete, suppose \( y_t \) is the true underlying economic performance over a period \( t \). One way to measure this unobservable economic performance is to rely on cash flow from operations, \( C_t \). However, such performance measure will contain noise due to timing and matching problems present in cash flows (Dechow 1994). These timing and matching problems result in measurement error \( w_t \) that reverses in the future. This idea is modeled econometrically using the following representation of cash flows:

\[
C_t = y_t + w_t - w_{t-1}, \tag{1}
\]

which captures the idea that cash flows are a noisy measure of a firm’s performance. As the performance measurement error reverses over time, long run cash flow will approximate the long-run economic performance \( y_t \). To the extent that accounting earnings are designed to improve performance measurement over that implied by cash flows, the structure of the error term in equation (1) yields a natural role for accounting accruals. Specifically, to measure the underlying performance properly, accountants need to book an accrual \( A_t^{\text{perfect}} = -w_t + w_{t-1} \) to offset the timing error in cash flows. However, even if accountants observe the true performance of the firm and know \(-w_t + w_{t-1}\), in order to account for this component they will generally have to follow a certain set of procedures that introduce assumptions and require the use of estimates (often based on prior history). As a result, accruals will

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3 For example, a simple aggregation of cash inflows less cash outflows over a number of periods will also do a good job in reflecting the underlying economic performance. At one extreme, the total net cash inflow equals the true economic performance over the life of the firm.
introduce an accounting error into earnings, which will, presumably, be smaller than the timing error in cash flows. Nevertheless, the error will share the same econometric properties and, hence, the accounting earnings will have a similar econometric structure:

\[ E_t = y_t + v_t - v_{t-1}. \]  

(2)

As in the case of cash flows, the measurement error \( v_t \) also exhibits reversal over time. However, the reversal arises due to a different reason, namely because accrual accounting relies on estimates and assumptions to measure the underlying performance. Over longer horizons, earnings will also measure the economic performance with a high degree of accuracy because the timing errors cancel out over time.

2.2. The implied characterization of accruals.

The econometric representation of earnings and cash flows implies a simple (and intuitive) dynamic characterization of accounting accruals. Since accounting accruals \( A_t \) are equal earnings less cash flows, we have:

\[ A_t = -w_t + w_{t-1} + v_t - v_{t-1}. \]  

(3)

I refer to the first component of accruals, \( -w_t + w_{t-1} \equiv W_t \), as a “performance measurement component” of accruals because the role of this component is to facilitate the measurement of underlying economic performance relative to cash flows. In other words, because \( C_t + W_t = y_t \), we can also think of \( W_t \) as “high quality” portion of accruals (in the sense that \( W_t = A_t^{\text{perfect}} \)), i.e., a portion of economic performance not captured by cash flows. Note that \( W_t \) should not be viewed as non-discretionary accruals, as a firm’s management may have the full discretion to choose the level of accruals that capture the underlying performance correctly but will generally do so with error.

The second component of accruals, \( v_t - v_{t-1} \equiv V_t \), arises because of imperfections in accrual accounting. I refer to this component as the “accounting error” or the “estimation error,” which is in line with the terminology in Dechow and Dichev (2002). Empirically, I expect the variance of \( v_t - v_{t-1} \), on average, to be smaller than the variance of \( w_t - w_{t-1} \), as otherwise the rationale for accrual accounting is unclear.
The above representation of accruals captures the salient institutional properties of accrual accounting. First, using conventional accounting terminology, the component $w_t$ represents deferred performance (deferred revenue less deferred expenses) less accrued performance (accrued revenue less accrued expenses) in the absence of accounting noise. This is intuitive, as deferred revenue (expenses) implies higher (lower) cash flow relative to underlying performance, whereas accrued revenue (expenses) implies lower (higher) cash flows relative to the underlying performance. In turn, the component $w_{t-1}$ represents the reversals of accruals or deferrals previously created.\footnote{For example, if a customer makes a $1 advance payment in the current period (for work performed in the future), operating cash flow overstates performance by $1 in the current period and will understate performance by the same amount in the future. Subsequently, a negative $1 accrual is booked upon a receipt of a $1 advance payment.} Second, accruals exhibit reversal over time. Over longer horizons, both cash flow and earnings are equivalent in measuring economic performance. Third, the role of accruals is to mitigate the known problems in cash flows and hence to facilitate performance measurement (Dechow 1994). Fourth, any gain in performance measurement from accruals comes at the cost of using estimates and assumptions, which introduce an estimation error in accruals and earnings (Dechow and Dichev 2002).

2.3. Understanding accounting quality.

The above representation reveals that the variances of cash flows, earnings, and accruals are jointly determined by the following parameters: $\sigma^2_y$, $\sigma^2_w$, and $\sigma^2_{\nu}$. The first component $\sigma^2_y$ is the volatility of the underlying economic performance. This component can be thought of as the riskiness of the business in which a company operates or as operating volatility (a portion of this risk can be idiosyncratic and the rest can be systematic, i.e., exhibit links to market beta, etc.).

The parameters $\sigma^2_w$ and $\sigma^2_{\nu}$ are the variance of performance component of accruals (variance of high quality accruals), and the variance of the estimation error, respectively. These components, jointly, determine accounting quality. In particular, the component $\sigma^2_w$ captures the benefit of accrual accounting, as discussed previously, whereas the component $\sigma^2_{\nu}$ reflects the cost of accrual accounting. Note that the
former is equally important as the latter in understanding “earnings quality” as well as the demand for accrual accounting more generally.

The above discussion reveals a gap in the literature. Specifically, prior research, following Dechow and Dichev (2002), focused on quantifying and understanding the variance of $v_t - v_{t-1}$ component and not the variance of $w_t - w_{t-1}$. Indeed, the literature has so far neglected the benefits of accrual accounting, i.e., the variance of the $w_t$ component of accruals. More importantly, the above suggests that focusing on the variance of the estimation error results in an incomplete picture of accounting quality, as it does not take into account the benefit side of accruals. Consistent with the definition of earnings quality, one way of constructing an overall measure of accounting quality would be as follows:

$$\text{Accounting Quality} \equiv \frac{\sigma_w^2 - \sigma_v^2}{\sigma_v^2}.$$  

The numerator of this ratio captures the net benefit of using accruals, which is scaled by the magnitude of the error introduced by accrual accounting. Intuitively, earnings are of high quality when the variance of the performance portion of accruals (i.e., high quality accruals) is high relative to the variance of the estimation error and vice versa. Depending on the research objective, one may also be interested in the following two signal-to-noise ratios:

$$r_{s/n}(E) \equiv \frac{\sigma_w^2}{\sigma_v^2}, \text{ and } r_{s/n}(C) \equiv \frac{\sigma_w^2}{\sigma_w^2}.$$  

These ratios reflect the intuition that the performance of riskier companies is harder to measure.

3. Identification of Accounting Quality.

My identification strategy relies on the time series properties of the above model. Intuitively, the primary source of identification comes from the fundamental properties of the accounting process, and namely, from our knowledge that (1) accounting accruals and estimation error reverse over time, and (2) that both earnings and cash flows are noisy measures of the same underlying performance. To illustrate
this, first, consider a hypothetical example where earnings process is \( E_t = y_t + V_t \) and \( y_t, V_t, \) and \( W_t \) are independently distributed random variables. In such a case, it would not be possible to identify the variances of \( y_t, W_t \) or \( V_t \) unless one used additional information to separate the measurement error from the performance. Such information is available in the structure of the error term and in cash flows. Indeed, if the error process is \( V_t = v_t - v_{t-1} \), the reversal can be used as an identifying assumption based on which the variance of \( y_t \) and \( v_t \) components can be recovered. Specifically, the resulting moment conditions \( E[E_tE_t] = \sigma_y^2 + 2\sigma_v^2 \), and \( E[E_tE_{t-1}] = -\sigma_v^2 \) represent two equations with two unknowns that can be solved for using GMM (or other minimum distance methods). Similarly, our knowledge that \( C_t = y_t + W_t \) offers a moment condition, \( E[E_tC_t] = \sigma_y^2 \), that, combined with \( E[E_tE_t] \), can be used to identify the variance of estimation error, etc. Note that, while in this simple example either of the identifying assumptions will suffice, both will contribute towards identifying the parameters in more general cases.

Following the intuition above, I choose moment conditions that exploit reversal in both the performance component of accruals (high quality accruals) and the estimation error, and also that combine the information in earnings and cash flows. Constructing moment conditions from multiple time series comes in handy when time series are relatively short, as it exploits more information.

3.1. Identifying assumptions.

The model (1) does not rely on any restrictive assumptions about the accrual process. The identifying assumption made so far is that the reversal of accruals happens in the next reporting period. In this section, I state more formally a flexible set of assumptions that allow taking the model to the data. While these assumptions are consistent with prior literature, they should be viewed as a benchmark case.

Assumption 1. (i) \( y_t \) is a stationary process; (ii) timing errors \( v_t \) and \( w_t \) reverse within subsequent periods.

Assumption 2. (i) \( \text{cov}(v_t, v_{t-s}) = 0 \), and (ii) \( \text{cov}(w_t, w_{t-s}) = 0 \) for all \( s > 1 \).

Assumption 3. \( \text{cov}(w_t, v_{t-s}) = 0 \) for all \( s \).
Assumption 4. (i) $\text{cov}(y_t, w_{t-s}) = 0$, and (ii) $\text{cov}(y_t, v_{t-s}) = 0$ or all $s$.

This baseline set of assumptions warrants some discussion. The assumption (1) is generally descriptive of working capital accruals on an annual basis (Dechow and Dichev 2002). These accruals are of primary interest to the literature on earnings quality. The assumption (2) is warranted because the error structure (e.g., $V_t = v_t - v_{t-1}$) directly incorporates the expected negative autocorrelation in accruals. Assumption (3) is also plausible institutionally. Recall that accountants measure the true accrued/deferred portion of economic performance $w_t$ to offset the timing error in cash flows and in doing so introduce an expectation error $v_t$. If accruals measure performance in an unbiased way, there should not be a systematic relation between the error $w_t$ being corrected and the error $v_t$ that arises in doing so. Assumption (4) also seems plausible. For example, Dechow and Dichev (2002) make an implicit assumption that the estimation error and performance are uncorrelated: $\text{cov}(y_t, v_t) = 0$. Further, it is not clear why the timing error in cash flows should be expected to systematically correlate with performance. For example, in a steady-state firm one would not expect that a positive shock to economic performance is more likely to result in the need for accrued performance (negative $w_t$), as opposed to deferred performance (positive $w_t$).

The main advantage of using the assumptions (1)-(4) is the parsimony of the model. It is important to understand how these assumptions can be relaxed because a violation of these assumptions will be generally informative of accounting quality. In subsequent sections of the paper, I will consider generalizations of the model and offer ways to relax the baseline assumptions in a number of ways.

3.2. Moment conditions.

Recall that the processes for earnings, accruals, and cash flows have the following dynamic representation (for convenience stated in one place):

\begin{align*}
C_t &= y_t + w_t - w_{t-1} \quad (4a) \\
A_t &= -w_t + w_{t-1} + v_t - v_{t-1} \quad (4b) \\
E_t &= y_t + v_t - v_{t-1}. \quad (4c)
\end{align*}
This representation, in conjunction with assumptions (1)-(4) stated above, allows for identification of the variance of the performance component of accruals, $\sigma_w^2$, the variance of accounting estimation error, $\sigma_v^2$, and the variance of shocks to economic performance, $\sigma_y^2$. Doing so requires selecting moment conditions that are informative about the parameters of interest. Note that the choice of moments is not guided by econometric theory and is more of an art than science. The natural principles that should guide the choice of moment conditions: (1) moments should provide substantial information about the parameters of interest, i.e., changing a parameter should have a first-order effect on the moment; (2) moments that rely on assumptions that are unlikely to be met should be avoided, and (3) moments that are linearly dependent should not be used (e.g., note that moment $E(E_t E_t)$ does not add information if moments $E(E_t C_t)$ and $E(E_t A_t)$ are used). I suggest the following simple set of moments conditions, however, note that other moments can be used as well (for example, using the moment $E(E_t E_{t-2})$ requires additional assumptions about the process $y_t$). Note that all variables are measured as deviations from their means.

\begin{align*}
m(1): & \quad E(E_t E_t) = \sigma_y^2 + 2\sigma_v^2, \\
m(2): & \quad E(C_t C_t) = \sigma_y^2 + 2\sigma_w^2, \\
m(3): & \quad E(A_t A_t) = 2\sigma_v^2 + 2\sigma_w^2, \\
m(4): & \quad E(E_t E_{t-1}) = \rho_y \sigma_y^2 - \sigma_v^2, \\
m(5): & \quad E(C_t C_{t-1}) = \rho_y \sigma_y^2 - \sigma_w^2, \\
m(6): & \quad E(A_t A_{t-1}) = -\sigma_v^2 - \sigma_w^2,
\end{align*}

where $\rho_y$ is the first-order autocorrelation of economic performance $y_t$, i.e., persistence of the underlying economic performance.

These simple moment conditions present six equations with four unknowns (i.e., the model is overidentified). The first three equations help identification by exploiting the information in two different performance measures, whereas the last three moments also exploit the reversal in the error components. GMM estimation can be used to identify and consistently estimate the parameters $\sigma_w^2$, $\sigma_v^2$, $\sigma_y^2$, and $\rho_y$ at
firm or industry level. Maximum likelihood can also be sued to estimate these parameters but requires strong distributional assumptions, such as normality, which is an important advantage of using GMM.

As an alternative, we can use changes in the time series of earnings, cash flows, and accruals. In this case, it is important to account for differences in the dynamics of the three time series. For example, the differenced version of the accrual process given by equation (4b) is as follows:

\[ A_t - A_{t-1} = -w_t + 2w_{t-1} - w_{t-2} + v_t - 2v_{t-1} + v_{t-2}. \]

Differenced formulation implies the following set of moment conditions:

\[ m(1): \quad E(\varepsilon_t - \varepsilon_{t-1})(\varepsilon_t - \varepsilon_{t-1}) = \sigma_y^2 + 6\sigma_v^2, \]
\[ m(2): \quad E(C_t - C_{t-1})(C_t - C_{t-1}) = \sigma_y^2 + 6\sigma_w^2, \]
\[ m(3): \quad E(A_t - A_{t-1})(A_t - A_{t-1}) = 6\sigma_y^2 + 6\sigma_w^2, \]
\[ m(4): \quad E(\varepsilon_t - \varepsilon_{t-1})(\varepsilon_{t-1} - \varepsilon_{t-2}) = \rho_y\sigma_y^2 - 4\sigma_v^2, \]
\[ m(5): \quad E(C_t - C_{t-1})(C_{t-1} - C_{t-2}) = \rho_y\sigma_y^2 - 4\sigma_w^2, \]
\[ m(6): \quad E(A_t - A_{t-1})(A_{t-1} - A_{t-2}) = -4\sigma_y^2 - 4\sigma_w^2, \]

where \( \rho_y \) is the first-order autocorrelation of changes in the economic performance, \( y_t - y_{t-1} \).

Such formulation has two principal advantages over the levels formulation above. It no longer requires that the process \( y_t \) is stationary (only its changes must be stationary), i.e., this formulation allows for a very weak assumptions about economic performance. Second, differencing effectively eliminates highly persistent, i.e., long-term accrual components, such as depreciation or amortization (e.g., of bond premium).

The proposed empirical strategy has a principle advantage over the models based on Jones (1991) or Dechow and Dichev (2002). Namely, these models do not allow for identifying and separating the variance of estimation error and the variance of high quality accruals while controlling for operating volatility. I discuss this in more detail in Section 5, where I also offer reconciliation between the representation of accruals offered here and that in Dechow and Dichev (2002).

In this section, I discuss how assumptions (1) and (4) can be relaxed to incorporate potentially more realistic scenarios. Relaxing these assumptions should be decided on a case-by-case basis as it presents a trade-off: a potentially better fit of the model with the data comes at a risk of over-parameterizing the model (without a significant gain in explanatory power). Over-parameterizing the model is of particular concern when the number of observations is relatively small, e.g., when the estimation is done at a firm level.

Several directions to relax the baseline assumptions require particular consideration. First, the timing errors $w_t$ and $v_t$ may be auto-correlated. For example, a positive auto-correlation in $w_t$ may arise because cash flow tends to systematically fall short of economic performance in the growth phase. The estimation error $v_t$ may be auto-correlated because consistent application of accounting procedures and assumptions is likely to result in a correlated error in performance measurement. Second, the components $w_t$ and $v_t$ may be cross-correlated if accounting error tends to be correlated with the timing error. This may take place due to the presence of accounting conservatism to the extent that unrealized profits are conservatively deferred until the future. Finally, the shocks to the underlying economic performance may be correlated with accounting error (Gerakos and Kovrijnykh 2013) or cash flow timing error. To allow for these possibilities, I consider an alternative set of assumptions, discussed next.

4.1. An alternative set of assumptions.

Relaxing assumptions (1) and (4) generally requires imposing more structure on the process $y_t$ (so far we have only assumed stationary). For example, modeling non-zero covariance between the accrual components and lagged values of economic performance makes it necessary to specify process $y_t$. One way to do this issue is to modify assumption (1) as follows:

Assumption 1A. (i) $y_t$ follows $ARMA(1,0)$ process, $y_t = \alpha y_{t-1} + \epsilon_t$, where $\epsilon_t$ is a shock to performance; (ii) timing errors $v_t$ and $w_t$ reverse within the subsequent period.
This assumption is not particularly important here but is also made in Gerakos and Kovrijnykh (2013) and Beyer, Guttman and Marinovic (2014). I use the following convenient notation (recall that under the initial set of assumptions, \( v_t \) and \( w_t \) are uncorrelated with \( \epsilon_t \)):

\[
\varphi_{v_s} \equiv \text{cov}(v_t, v_{t-s}), \quad \varphi_{w_s} \equiv \text{cov}(w_t, w_{t-s}), \quad \varphi_{vw_s} \equiv \text{cov}(v_t, w_{t-s}),
\]

\[
\varphi_{\epsilon v_s} \equiv \text{cov}(\epsilon_t, v_{t-s}), \quad \text{and} \quad \varphi_{\epsilon w_s} \equiv \text{cov}(\epsilon_t, w_{t-s}).
\]

The alternative set of assumptions that accounts for issues discussed above is as follows:

**Assumption 2A.** (i) \( \varphi_{v_1} = \rho_v \sigma_v^2 \), and \( \varphi_{v_s} = 0 \) for \( s > 1 \); (ii) \( \varphi_{w_1} = \rho_w \sigma_w^2 \), and \( \varphi_{w_s} = 0 \) for \( s > 1 \).

**Assumption 3A.** \( \varphi_{vw_0} = \rho_{vw} \sigma_v \sigma_w \), and \( \varphi_{vw_s} = 0 \) for \( s > 0 \).

**Assumption 4A.** (i) \( \varphi_{\epsilon v_0} = \rho_{\epsilon v} \sigma_v \sigma_{\epsilon} \), and \( \varphi_{\epsilon v_s} = 0 \) for \( s > 0 \); (ii) \( \varphi_{\epsilon w_0} = \rho_{\epsilon w} \sigma_{\epsilon} \sigma_w \), and \( \varphi_{\epsilon w_s} = 0 \) for \( s > 0 \).

Under this set of assumptions, we have nine parameters of interest (\( \sigma_v^2, \sigma_w^2, \sigma_{\epsilon}^2, \alpha, \rho_v, \rho_w, \rho_{\epsilon v}, \rho_{vw}, \rho_{\epsilon w} \)) and thus we require at least nine moment conditions that are linearly independent. For example, one could use an analogous set of equations \( E(E_t E_{t-s}) \), \( E(C_t C_{t-s}) \), and \( E(A_t A_{t-s}) \), with \( s \) taking values from 0 to 3, which amounts to 12 moments. Ideally, we would also want to exploit the time series of \( y_t \) but this is not feasible since the underlying performance cannot be observed directly.

**4.2. Using information in the time series process of economic performance.**

If performance \( y_t \) was observable, it would be possible to use the time series of \( y_t \) to construct additional moment conditions and gain additional information. For example, using moments \( E(y_t y_{t-s}) \) or running an OLS regression \( y_t = \alpha y_{t-1} + \epsilon_t \) would be a powerful way of identifying the parameters \( \alpha \) and \( \sigma^2_{\epsilon} \). While this is not feasible, it is possible to take advantage of this idea by substituting \( y_t = E_t - v_t + v_{t-1} \) into equation \( y_t = \alpha y_{t-1} + \epsilon_t \) to express earnings as a time series process with autoregressive components (an analogous substitution is used in Gerakos and Kovrijnykh 2013). Similarly, we can substitute \( y_t = C_t - w_t + w_{t-1} \) into the same equation to obtain the following set of equations that no longer depend on \( y_t \).

\[
E_t - \alpha E_{t-1} = \epsilon_t + v_t - (1 + \alpha)v_{t-1} + \alpha v_{t-2}, \quad (5a)
\]
\[ C_t - \alpha C_{t-1} = \epsilon_t + w_t - (1 + \alpha)w_{t-1} + \alpha w_{t-2}, \quad (5b) \]

\[ A_t = -w_t + w_{t-1} + v_t - v_{t-1}. \quad (5c) \]

There are a number of ways how these equations can be used to formulate moments. For example, if we multiply both sides of equations (5a) and (5b) by \( E_{t-s} \) or \( C_{t-s} \) and take expectations we will obtain a set of moments representing analogues of Yule-Walker equations. Note, further that equation (5c) is no longer a linear combination of the other two equations for parameters \( \alpha > 0 \), which generally increases the number of moments. One convenient way to formulate moment conditions is as follows:

**Auto-covariance of earnings.**

m(1): \( E(E_t - \alpha E_{t-1})(E_t - \alpha E_{t-1}) = \sigma_{\epsilon}^2 + 2 \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} + 2 \sigma_{\sigma}^2 (1 + \alpha + \alpha^2) - 2 \rho_{\sigma} \sigma_{\sigma}^2 (1 + \alpha)^2, \)

m(2): \( E(E_t - \alpha E_{t-1})(E_{t-1} - \alpha E_{t-2}) = -\rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \sigma_{\sigma}^2 (1 + \alpha^2) + \rho_{\sigma} \sigma_{\sigma}^2 (2(1 + \alpha)^2 - \alpha), \)

m(3): \( E(E_t - \alpha E_{t-1})(E_{t-2} - \alpha E_{t-3}) = \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \sigma_{\sigma}^2 (1 + \alpha^2). \)

**Auto-covariance of cash flows.**

m(4): \( E(C_t - \alpha C_{t-1})(C_t - \alpha C_{t-1}) = \sigma_{\epsilon}^2 + 2 \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} + 2 \sigma_{\sigma}^2 (1 + \alpha + \alpha^2) - 2 \rho_{\sigma} \sigma_{\sigma}^2 (1 + \alpha)^2, \)

m(5): \( E(C_t - \alpha C_{t-1})(C_{t-1} - \alpha C_{t-2}) = -\rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \sigma_{\sigma}^2 (1 + \alpha^2) + \rho_{\sigma} \sigma_{\sigma}^2 (2(1 + \alpha)^2 - \alpha), \)

m(6): \( E(C_t - \alpha C_{t-1})(C_{t-2} - \alpha C_{t-3}) = \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \sigma_{\sigma}^2 (1 + \alpha^2). \)

**Cross-covariance of earnings and cash flows.**

m(7): \( E(E_t - \alpha E_{t-1})(C_t - \alpha C_{t-1}) = \sigma_{\epsilon}^2 + \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} + \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} + 2 \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} (1 + \alpha + \alpha^2), \)

m(8): \( E(E_t - \alpha E_{t-1})(C_{t-1} - \alpha C_{t-2}) = -\rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} (1 + \alpha^2), \)

m(9): \( E(E_t - \alpha E_{t-1})(C_{t-2} - \alpha C_{t-3}) = \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} \sigma_{\sigma} + \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} \alpha, \)

m(10): \( E(E_{t-1} - \alpha E_{t-2})(C_t - \alpha C_{t-1}) = -\rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} (1 + \alpha) - \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} (1 + \alpha^2), \)

m(11): \( E(E_{t-2} - \alpha E_{t-3})(C_t - \alpha C_{t-1}) = \rho_{\epsilon \sigma} \sigma_{\epsilon} \sigma_{\sigma} \alpha + \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} \alpha. \)

**Auto-covariance of accruals.**

m(12): \( E(A_t A_t) = 2 \sigma_{\sigma}^2 + 2 \sigma_{\sigma}^2 - 4 \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} - 2 \rho_{\sigma} \sigma_{\sigma}^2 - 2 \rho_{\sigma} \sigma_{\sigma}^2, \)

m(13): \( E(A_t A_{t-1}) = -\sigma_{\sigma}^2 - \sigma_{\sigma}^2 + 2 \rho_{\epsilon \sigma} \sigma_{\sigma} \sigma_{\sigma} + 2 \rho_{\sigma} \sigma_{\sigma}^2 + 2 \rho_{\sigma} \sigma_{\sigma}^2, \)

16
m(14): $E(A_t A_{t-2}) = -\rho_w \sigma_w^2 - \rho_w \sigma_w^2.$

It is important to note that, while assumption about the process $y_t$ adds statistical power and complements identification strategy, it is also a potential source of misspecification. This assumption can be relaxed by, for example, including more lags into the autoregressive process or adding a moving average component. This, however, comes at a cost of adding more parameters and algebraic complexity. In such a case, the moment conditions should be altered accordingly. Another way to relax assumption (1A) is to explicitly model non-stationary $y_t$ (note that non-stationarity is less likely to be an issue for earnings scaled by total assets; the analogy would be stock returns). A conventional way to do this is to model the first difference $y_t - y_{t-1}$ as ARMA process. In such case, the moment conditions considered above need to be modified in order to accommodate for the differenced time series (see p. 13).

4.3. Short-term vs. long-term accruals.

In accordance with Assumption 1, the focus so far has been on working capital accruals, which are expected to reverse within one year. It is also useful to consider a possibility of a different reversal patterns. For example, quarterly accruals will generally exhibit reversals over several quarters. Non-working capital accruals, such as capital expenditures (capital expenditures are not a part of operating cash flows also reverse over a number of periods in the form of depreciation or future write-offs.

One approach to deal with long-term accruals in the literature is to try to exclude them from earnings (e.g., use earnings before depreciation). Nevertheless, my proposed framework can be used to model more complex reversal patterns. To make things more concrete, suppose a portion of accruals reverses in period 1 and the remainder reverses in period 2. In that case, we have the following structure of the model:

\begin{align}
C_t &= y_t + w_t - (1 - \delta)w_{t-1} - \delta w_{t-2}, \\
A_t &= -w_t + (1 - \delta)w_{t-1} + \delta w_{t-2} + v_t - (1 - \delta)v_{t-1} - \delta v_{t-2}, \\
E_t &= y_t + v_t - (1 - \delta)v_{t-1} - \delta v_{t-2}.
\end{align}
This model adds an additional parameter \( \delta \) that governs the reversal process and can be estimated. These equations can be used to construct moment conditions. For example, under the baseline set of assumptions the following moments will hold:

\[
\begin{align*}
\text{m}(1): \quad & E(A_t A_t) = 2(\sigma^2_w + \sigma^2_\nu)(1 - \delta + \delta^2), \\
\text{m}(2): \quad & E(A_t A_{t-1}) = - (\sigma^2_w + \sigma^2_\nu)(1 - \delta)^2, \\
\text{m}(3): \quad & E(A_t A_{t-2}) = - (\sigma^2_w + \sigma^2_\nu)\delta.
\end{align*}
\]

The model can be generalized further by allowing \( v_t \), reverse over \( k \) periods at a certain rate or with certain probability. For example, one reversal pattern to consider is \( v_t - (1/k)v_{t-1} \ldots - (1/k)v_{t-k} \), which is applicable to non-current accruals (e.g., improper capitalization of PPE and subsequent depreciation). Notice that the distribution of \( v \) (and other variables in the model) can be asymmetric (e.g., as in the case of capital expenditures) and the estimation method does not make distributional assumptions. A slightly different approach to dealing with long term accruals would be to allow for accrual components that reverse in \( k \) periods, i.e., \( v^k_T - v^k_{T-k} \). I view these extensions of the baseline model as opportunities for future research.

5. **Distinctions from Prior Approaches.**

In this section, I discuss the main distinctions of the approach to modeling accounting quality proposed here from more conventional approaches in the literature. I discuss why the conventional approaches do not identify the variance of the estimation error or accounting quality.

The two primary approaches to modeling accruals and estimating earnings/accruals quality available in the literature are based on important contributions by Jones (1991) and Dechow and Dichev (2002). Both the Jones model and Dechow-Dichev model use a linear regression model to separate accruals into their expected and unexpected components, often loosely referred to as non-discretionary and discretionary accruals, respectively. The explained variable in both models is accruals and the explanatory variables include change in revenue and PPE (Jones model), and current, future, and past
cash flow from operations (Dechow-Dichev model). The standard deviation of residuals from these models (or their absolute values) are used in the literature to proxy for accruals/earnings quality (e.g., Francis et al., 2004). The primary concern with these models is that they do not properly control the (unmanipulated) economic performance when determining the “discretionary” portion (e.g., Dechow et al., 2010) and as a result are picking up the volatility of the working capital and ultimately the riskiness of the firm. The approach here directly overcomes this issue by explicitly separating risk and the performance component of accruals from estimation error. To be more concrete, I discuss this in more detail in the context of the Dechow-Dichev model.

The main objective of the Dechow-Dichev model is to estimate the variance of the estimation error (which may or may not be subject to manipulations). The model is based on the following set of equations (Dechow and Dichev 2002, pp. 38-40), using my notation:

\begin{align*}
C_t &= C_{t-1}^c + C_t^c + C_{t+1}^c, \\
E_t &= C_{t-1}^c + C_t^c + C_{t+1}^c + v_t - v_{t-1}, \\
A_t &= C_{t-1}^c - (C_{t+1}^c + C_t^{c-1}) + C_{t+1}^c + v_t - v_{t-1}.
\end{align*}

(7a) (7b) (7c)

where $C_t^c$ is cash received in period $t$ and recognized in earnings in period $s$, and other variables are defined previously. Intuitively, the model states that earnings are the sum of current, past, and future cash flows recognized in current period earnings, plus an estimation error $v_t$ and its reversal $v_{t-1}$. In contrast, cash flows represent a portion of past, current, and future earnings realized in the current period. Accruals are thus the difference between the earnings and cash flows.

Note that such representation of accruals appears quite different from the representation advocated in this paper, which raises the question of which representation is more realistic. However, it is straightforward to show that the two representations reconcile. Indeed, one can add and subtract terms $C_{t-1}^c$ and $C_{t+1}^c$ to equation (7a) so that both earnings and cash flows include the portion $C_{t-1}^c + C_t^c + C_{t+1}^c$.

This portion represents ultimate cash implications of events that occur in the current period and hence is

\begin{figure}
\centering
\includegraphics[width=\textwidth]{dechow_dichev.png}
\caption{Dechow-Dichev Model Diagram}
\end{figure}

\footnote{A disadvantage of representation offered in Dechow and Dichev (2002) is that the variance of component $C_t^c$ does not appear to be feasible to identify, unlike the variance of $w_t$.}
equal to true performance in the current period, i.e., \( y_t = C^t_{t-1} + C^t_t + C^t_{t+1} \). Further, we can define the following variable \( w_t \equiv C^{t+1}_t - C^t_{t+1} \) and its lagged counterpart \( w_{t-1} \equiv C^t_{t-1} - C^t_t \). It is straightforward to show that substituting these definitions into equations (7a), (7b), and (7c) and rearranging them obtains the set of equations (4a) (4b) and (4c). The accruals component \(-w_t + w_{t-1}\) has an analogous interpretation in this context. The portion \(-w_t\) is a portion of current earnings realized (collected/paid) in the future less a portion of current cash flow recognized (earned/incurred) in the next period. The second component, \(+w_{t-1}\), is the cash flow collected in the prior period but recognized currently less cash recognized previously but collected in the current period.

To estimate the variance of the estimation error, i.e., \( \text{var}(v_t - v_{t-1}) \), characterized by parameter \( \sigma^2_v \), Dechow and Dichev (2002) use the variance of the residual from the following regression model (constant is omitted) motivated by equation (7b):

\[
A_t = a_1 C_{t-1} + a_2 C_t + a_3 C_{t+1} + u_t. \tag{8}
\]

It not difficult to see that the variance of the residual \( u_t \) falls short of identifying the estimation error and, in fact, comingles \( \sigma^2_v \) with the variance of high quality accruals, \( \sigma^2_w \), as well as the variance of the underlying economic performance, \( \sigma^2_y \). To see this more explicitly, it is convenient to express equation (8) in terms of the underlying performance \( y_t \) by substituting equations (1) and (2) into equation (8):

\[
-w_t + w_{t-1} + v_t - v_{t-1} = a_1 (y_{t-1} + w_{t-1} - w_{t-2}) + a_2 (y_t + w_t - w_{t-1})
\]

\[
+ a_3 (y_{t+1} + w_{t+1} - w_t) + u_t. \tag{9}
\]

Examination of the dependent variable reveals that to isolate the variance of the estimation error \( v_t - v_{t-1} \), it is necessary to control for the performance component of accruals \(-w_t + w_{t-1}\) (in which case \( \text{var}(u_t) = \text{var}(v_t - v_{t-1}) \)). For this to happen, it must be the case that:

\[
-w_t + w_{t-1} = a_1 (y_{t-1} + w_{t-1} - w_{t-2}) + a_2 (y_t + w_t - w_{t-1}) + a_3 (y_{t+1} + w_{t+1} - w_t).
\]

This is a contradiction as the equality (9) does not hold under any set of parameters in the linear model. As a result, the residual, \( u_t \), will contain an additional term, \( \pi_t \), so that:
Because the variance of \( \pi_t \) is a function of \( \sigma_x^2 \) and \( \sigma_y^2 \), the regression (9) does not isolate high quality accruals: the variance of the residual thus becomes a function of risk, \( \sigma_x^2 \), the performance portion of accruals, \( \sigma_w^2 \), and the variance of the estimation error \( \sigma_e^2 \). This mimics the conclusion in Wysocki (2009) in that the current regression model does not separate “high quality” and “low quality” accruals.

A similar logic applies to the Jones model as, by controlling for change in revenue and the level of PPE, this model will generally not fully control for the performance component of accruals. In sum, the existing models do not identify the variance of the estimation error in earnings/accruals. Furthermore, these models are not suitable to identify the variance of the performance measurement component of accruals.


Ball and Shivakumar (2006) argue that, in addition to mitigating the transitory variation in cash flows (Dechow 1994), the (performance measurement) role of accruals also includes the accelerated recognition of unrealized economic losses and gains that arise due to changes in expectations of future cash flows. Note that unrealized gains and losses can also be viewed as timing errors in cash flows and hence fall under \( w_t \). The key difference, however, is that the unrealized losses and gains follow asymmetric recognition under the conservative accounting rules (Basu 1997). Failure to take this into account constitutes an important criticism of the existing models of discretionary accruals (Ball and Shivakumar 2006) which, for example, are likely to classify write-offs and impairment charges as “discretionary accruals.” Such a classification is problematic because these charges facilitate the recognition of adverse economic performance and in this sense are a desirable property of financial reporting. It is necessary to point out, however, that asymmetrically recorded gains and losses are less
likely to be a part of working capital accruals (because write-offs and impairments usually primarily affect non-current accruals, e.g., PPE, and are reported separately).

The way of identifying accounting quality I am proposing can accommodate asymmetric timeliness of gain and loss recognition. It also allows identifying and isolating a portion of accruals explained by conservatism. Let me first illustrate this with a simple example. In particular, let’s hold other things equal and consider a loss $y_t = -$100 that happens in the current period but is not realized until a future period. Such a loss implies the timing error in cash flows $\omega_t = 100$ and requires a negative accrual of $-w_t = -$100 to offset the timing error. Note that accruals improve performance measurement and are not classified as “discretionary,” i.e., a part of accounting noise. By analogy, an unrealized gain $y_t = 100$ implies an equivalent timing error in cash flows $\omega_t = -$100 (again because the gain is not realized until the future). Without a conservative treatment of gains, an accrual $-w_t = +100$ should be recorded. Because a conservative treatment of gains does not consider $100$ as current income, it introduces a conservative accounting error $v_t = -$100 into earnings (and accruals). Intuitively, accounting conservatism introduces skewness into accruals and earnings as it implies that accountants are more likely to make negative errors than positive ones when measuring performance (and hence less likely to report high positive earnings).

The above example has two main implications. First, economic performance (through unrealized gains/losses) is now correlated with the timing error $\omega_t$. Second, and more importantly, the asymmetric treatment of gains and losses introduces a nonlinear relation between $v_t$ and $\omega_t$. To allow for such non-linearity in the accrual process we can generalize the model in the following way:

\[
\begin{align*}
\breve{y}_t &= y_t + z_t, \\
\breve{w}_t &= w_t - z_t, \\
\breve{v}_t &= v_t - g(z_t), \\
A_t &= -\breve{w}_t + \breve{w}_{t-1} + \breve{v}_t - \breve{v}_{t-1},
\end{align*}
\]
where $w_t$ and $v_t$ represent the timing error in a sense discussed previously (Dechow 1994), and exclude unrealized gains and losses that are treated asymmetrically. The component $z_t$ represents unrealized gains (losses) that happen in the current period and thus is subject to asymmetric treatment, which is a part economic performance $\tilde{y}_t$ but falls out from current cash flow due to its timing error $\tilde{w}_t$. Because of asymmetric treatment of gains and losses, $g(z_t)$ is expected to be close to 0 for negative news (because negative news is recognized immediately). In contrast, $g(z_t)$ is expected to be equal to $z_t$, as long as unrealized gains are deferred until realization (if gains are deferred only partially, we would expect $g(z_t) = az_t$, where $0 < a < 1$).

The natural candidate for the function $g(.)$ is $g(z_t) = \max(0, z_t)$. Intuitively, when $z_t$ is negative (the case when we have unrealized loss), the accrual $A_t = -z_t$ is recorded and the corresponding part of the estimation error, $\max(0, z_t)$, is zero. In contrast, when $z_t$ is positive (the case of unrealized gain), conservative accounting precludes recognition of $z_t$, which implies $\tilde{v}_t = w_t = -z_t$ and $A_t = 0$. To construct moment conditions, it necessary to add assumptions about the distribution of $z_t$. To keep the set of moment conditions simple, I keep assumptions (1)-(4) and make the following assumption:

**Assumption 5**: The distribution of $z_t$ is symmetric around 0, $\text{cov}(z_t, \epsilon_{t-s}) = 0$, $\text{cov}(z_t, v_{t-s}) = 0$ and $\text{cov}(z_t, w_{t-s}) = 0$ for all $s$.

In such case, the following conditions become relevant:

$$
\text{E}(\tilde{w}_t \tilde{w}_t) = \sigma^2_{\tilde{w}} + \sigma^2_z,
$$

$$
\text{E}(\tilde{v}_t \tilde{v}_t) = \sigma^2_v + \text{E}[\max(0, z_t)^2] = \sigma^2_v + 0.5\sigma^2_z,
$$

$$
\text{E}(\tilde{w}_t \tilde{v}_t) = \text{E}[z_t \max(0, z_t)] = 0.5\sigma^2_z,
$$

$$
\text{E}(\tilde{y}_t \tilde{w}_t) = -\text{E}[z_t z_t] = -\sigma^2_z,
$$

$$
\text{E}(\tilde{y}_t \tilde{v}_t) = -\text{E}[z_t \max(0, z_t)] = -0.5\sigma^2_z.
$$

These modifications introduce one additional parameter, $\sigma_z$, and can be easily incorporated into the moment conditions discussed above. These moment conditions allow for estimating and isolating the
portion of accruals explained by accounting conservatism. We are still able to identify the variance of the remaining components of accruals.

Note that the symmetry assumption can be dropped at the expense of one extra parameter, \( k_z \). In such a case the following modifications take place:

\[
E(\tilde{v}_t \tilde{v}_t) = \sigma_v^2 + E[\max(0, z_t)^2] = \sigma_v^2 + k_z,
\]

\[
E(\tilde{w}_t \tilde{w}_t) = E[z_t \max(0, z_t)] = k_z,
\]

\[
E(\tilde{y}_t \tilde{v}_t) = -E[z_t \max(0, z_t)] = -k_z.
\]

I view these extensions of the model as opportunities for future research.

7. **Identifying Earnings Management.**

So far I have not made assumptions about whether the estimation error is an unintentional error or whether it has a “managed” component. In this section, I discuss how the framework offered in this paper can be used to accommodate earnings management. I also discuss how one can use the model to test hypotheses about the presence of earnings management, while fully taking into account the dynamics accrual components.

To allow for earnings management, it is possible to extend the model to include the following structure of accounting error:

\[
\tilde{v}_t = v_t + m_t,
\]

where \( v_t \) is the unmanaged portion of the estimation error and \( m_t \) is its managed component. To separate the managed component, however, it is necessary to either (1) impose assumptions on the dynamic process through which earnings management is manifested in accruals (e.g., Gerakos and Kovrijnykh 2013), or (2) rely on additional information about the incentives to manage earnings (e.g., Dechow et al. 1995). I discuss these alternatives in the remainder of this section.

7.1. **Income smoothing.**

Gerakos and Kovrijnykh (2013) argue that earnings management is manifested via performance smoothing. They show that incentives to smooth out shocks to economic performance arise under a fairly
general set of assumptions about managerial preferences (e.g., meet-or-beat targets). In line with this intuition, they model earnings using the following dynamic structure:

\[ E_t = y_t - \gamma \epsilon_t + \gamma \epsilon_{t-1}, \]  

(11)

where true (unmanaged) economic performance, \( y_t \), is assumed to follow an AR(1) process, \( y_t = \alpha y_{t-1} + \epsilon_t \). Based on this assumption, the authors predict that earnings will also follow an autoregressive process, however, with a negative second-order autocorrelation of the error term. Their estimate of the second-order autocorrelation parameter confirms this prediction.\(^6\)

Note that the above earnings representation relies on a rather strong assumption that, effectively, \( \nu_t = 0 \) and \( \nu_t = m_t = -\gamma \epsilon_t \), i.e., the entire estimation error in accruals is explained by earnings management. The current framework allows for relaxing this assumption while at the same time incorporating the idea in Gerakos and Kovrijnykh (2013) that earnings management introduces a correlation between economic performance and accounting error. This can be achieved by allowing estimation error to incorporate both an unmanaged error as well as the income smoothing component:

\[ \nu_t = \epsilon_t - \gamma \epsilon_t. \]

Under the assumptions (1)-(4), it is necessary to revise the moment conditions to incorporate the following covariance structure:

\[ E(\nu_t \nu_t) = \sigma_{\nu}^2 + \gamma^2 \sigma_\epsilon^2, \]

\[ E(\nu_t \epsilon_t) = -\gamma \sigma_\epsilon^2. \]

As previously, such an approach allows for identifying the variance of the estimation error as well as other parameters of interest. The advantage of using this approach in conjunction with GMM estimation is that the parameter \( \gamma \) can be estimated directly (as opposed to being imputed from the second-order autocorrelation in the error term of the reported earnings), and without imposing a normality assumption necessary to estimate ARMA process. In addition, GMM takes into account the information about \( \gamma \) contained in the first-order autocorrelation of the error term in reported earnings.

---

\( \text{\(^6\) They further impute the parameter } \gamma, \text{ which indicates significant earnings management (on average, the parameter } \gamma \text{ is } 0.32, \text{ which implies that roughly } 1/3 \text{ of performance shocks (and hence ultimately performance) is being "hidden" via earnings smoothing).} \)
7.2. Directional tests of earnings management.

Another way to isolate earnings management in accruals is to model it as a function of the underlying economic incentives to manage earnings. Suppose we have a variable $x_t$, given exogenously, which is informative about managerial incentives to overstate (understate) earnings and accruals (i.e., estimation error). For example, $x_t$ could be proximity to covenant violation or some other threshold or benchmark (e.g., earnings forecast). Alternatively, $x_t$ could be a dummy variable that takes the value of one in firm-years where the incentives for manipulation are present (e.g., Dechow, Sloan and Sweeney 1995). In this case, we can model that the estimation error has the following process:

$$v_t = \theta v_{t-1} + \nu_t,$$

(12)

where the first component captures the extent of earnings manipulation as a function of the economic incentives, i.e., $m_t = \theta x_t$, and the second component captures the portion of the estimation error orthogonal to incentives to manage earnings.

To start, I will assume that $x_t$ is independently distributed over time and is also independent of other variables in the model. To avoid unnecessary parameters, let’s normalize the mean of $x_t$ to zero and $\text{var}(x_t) = 1$.

Under the baseline set of assumptions, the following modifications to the moment conditions stated earlier are necessary to take into account:

$$E(\tilde{v}_t \tilde{v}_t) = \theta^2 v + \sigma^2,$$

$$E(\tilde{v}_t x_t) = \theta v.$$

Note that these two equations with two unknowns can be solved as a part of a larger set of moments to decompose accruals into both managed and unmanaged portions. Furthermore, it is possible to test whether $\theta$ and, hence, earnings management, is statistically significant and in the predicted direction. This could be done by pooling cross-sectional and time series variation (to the extent that a cross-sectional pool has similar firms, e.g., at industry level, or to the extent that we allow for heterogeneity in other examples of the model parameters). In this sense, this approach complements the approach in Dechow,
Sloan and Sweeney (1995) and Dechow, Hutton, Kim and Sloan (2012), who model discretionary accruals as a function of \( x_t \) to test for earnings management. In particular, testing for \( \theta > 0 \) explicitly accounts for reversal of earnings management on a different period, which is something that Dechow et al. (2012) have shown to increase statistical power.

However, the current approach has several advantages. Gerakos (2012) argues that discretionary accruals based on, e.g., Jones (1991) do not properly account for the dynamic properties of unmanaged accruals. In particular, my framework reveals the following two issues. The first issue arises if incentives to manage earnings and the timing error in cash flows are correlated, which can be expressed as follows:

\[
\bar{w}_t = \theta_w x_t + w_t, \quad (13)
\]

Indeed, there are reasons to expect that the incentives to manage earnings can be correlated with a set of factors, e.g., growth, that in turn are correlated with \( w_t \). Because the proxies for discretionary accruals do not properly isolate the \( w_t \) component, their use will generally result in confounded inferences with respect to earnings management. To simplify things, suppose one uses total accruals to test for earnings management and runs the following regression:

\[
A_t = \beta x_t + u_t.
\]

The estimated slope coefficient in this regression has the following expected value:

\[
E(\hat{\beta}) = \frac{\text{cov}(A_t, x_t)}{\text{var}(x_t)} = \theta_v - \theta_w.
\]

Thus, depending on the sign of the coefficient of \( \theta_w \), a regression framework can overstate or understate the estimated magnitude of earnings management, \( \theta_v \). In contrast, we can use the method of moments to estimate and isolate \( \theta_w \) by making use of following two moments:

\[
E(\bar{w}_t \bar{w}_t) = \theta_w^2 + \sigma_w^2, \quad \text{and} \quad E(\bar{w}_t x_t) = \theta_w.
\]

The second problem arises if the incentives to manage earnings persist for several periods, e.g.,

\[
x_t = \omega x_{t-1} + \theta_t.\]

Assuming for simplicity that \( \theta_w = 0 \), now we have:

\[
E(\hat{\beta}) = E((v_t - v_{t-1}) x_t) = \theta_v - \omega \theta_v.
\]
In other words, the more persistent earnings management is, the harder it is to detect it within a regression framework, i.e., without accounting for persistence in $x_t$. Intuitively, this takes place because it is difficult to consistently overstate accruals for a number of periods in a row. In contrast, the following moment conditions make use of lags of $x_t$ to overcoming this problem and identify the degree of earnings management $\theta_v$:

$$E(x_t x_{t-1}) = \omega, \text{ and } E(\tilde{v}_t x_{t-1}) = \theta_v \omega.$$ 

To sum up, the framework I am proposing, in contrast to those that rely on proxies for discretionary accruals, is a one-step estimation immune to the issues discussed above because it directly accounts for the dynamics of the managed and unmanaged accruals process.

8. Conclusions.

The quality of accounting earnings and accruals is one of the most actively researched areas in accounting. Over the past decades, however, the literature has made rather limited progress in measuring accounting quality. At the same time, there is a mounting criticism that the existing models of discretionary accruals do not properly control for economic performance as well as high quality accruals (e.g., Wysocki 2009, Dechow et al. 2010, Gerakos 2012). In this paper, I have presented a conceptual framework and an empirical strategy that enable identification of accounting quality (quality of earnings/accruals) and also directly addresses the existing criticisms. The strategy is based on our knowledge of accounting processes, which implies the reversal in both accounting error and high quality accruals. The information in accruals reversal, in turn, can be used as a source of identifying variation and naturally calls for the GMM estimation. This source of identification is rather general and does not depend on a particular set of assumptions about the model.

I also offered a parsimonious empirical model of accruals as a dynamic process. The proposed characterization is intuitive and reflects institutional features of accrual accounting. Specifically, accruals are characterized as a sum of two distinct components. This model is based on the idea that both cash flows and earnings can be viewed as noisy measures of the same underlying economic performance. The
first component plays a performance measurement role, in line with Dechow (1994), by resolving the
timing error in cash flows. The second component represents accounting error. Both components exhibit
reversal. Such a characterization makes it natural to characterize accounting quality both in terms of the
magnitude of accounting error introduced via the accrual process (cost of using accruals), and the
magnitude of high quality accruals (benefit of using accrual accounting). The later components have been
largely unaddressed in the literature on accounting quality.

I offer a number of generalizations to the baseline set of assumptions under which the model can
be estimated. Most notably, I offer a new approach to testing for earnings management and discuss how
the model can be modified to explicitly deal with the presence of accounting conservatism. The paper
offers a number of avenues for future research, which should determine which assumptions and model
generalizations are empirically important.
References


