BKK the EZ Way
An International Production Economy with Recursive Preferences

Ric Colacito   Max Croce   Steven Ho   Philip Howard[*]

Abstract

We characterize an international production economy in which (1) agents have Epstein and Zin (1989) preferences, (2) international productivity frontiers are exposed to both short- and long-run shocks, and (3) consumption features a larger degree of home bias relative to investment. Under our recursive risk-sharing scheme, good long-run news for domestic productivity creates a net outflow of domestic investments. This response accounts for the Backus, Kehoe and Kydland (1994) anomaly concerning the lower degree of correlation of international consumption relative to output. We document that our model is strongly consistent with novel empirical evidence on both international quantities and prices.

JEL classification: C62; F31; G12.
First draft: March 31, 2012. *This draft*: April 24, 2013.

1 Introduction

Does capital always flow to the most productive countries? Does it matter whether productivity improvements are deemed to be short lived or long lasting? In this paper we answer these questions by investigating the impact of short- and long-term productivity risk on international risk-sharing and capital flows in the context of a general equilibrium model in which agents have recursive preferences.

To assess the relevance of international long-run productivity, we study different risk and production structures and show that the introduction of asset pricing considerations into the design of the production activity delivers a rich set of novel and testable implications. One of the most important theoretical and empirical findings of our analysis is that countries receiving good long-run productivity news experience capital outflows in the short run. We obtain this result starting from a frictionless Backus, Kehoe, and Kydland (1994) (henceforth BKK) two-good and two-country production economy modified along three key dimensions.

First, we add Epstein and Zin (1989) (henceforth EZ) recursive preferences and long-run growth shocks in the spirit of the recent long-run risk literature on exchange rates. As long as the intertemporal elasticity of substitution (IES) is larger than the reciprocal of their relative risk aversion (RRA), investors dislike both low expected levels of wealth and increasing uncertainty about their future utility profiles. As shown by Colacito and Croce (2012), in this setting agents look for a risk-sharing arrangement that allows them to smooth future utility, and equivalently wealth, in addition to short-term consumption.

Second, we follow Erceg, Guerrieri, and Gust (2008) (henceforth EGG) and assume a larger degree of home bias in consumption than in investment. This is a key difference relative to BKK who assume that 88% of both the consumption and the investment bundles consist of domestic goods, following the empirical observation that total U.S. imports represent about 12% of total output. In contrast, EGG show that this approach is inconsistent with U.S. data, as foreign consumption goods represent only 3%–5% of the U.S. consumption bundle, whereas foreign investment goods represent about 40% of U.S. aggregate investment.

Third, we modify the basic BKK model by adding heterogenous exposure of capital

vintages to aggregate productivity, as in Ai, Croce, and Li (2012) (henceforth ACL). Using U.S. firm data, ACL document that young capital vintages have lower exposure to aggregate productivity risk than older capital vintages. We show that the introduction of this feature enhances all our results and allows us to produce an average annual equity premium of about 3.6%, a number three hundred times larger than that obtained by BKK.

As shown in prior work (Colacito and Croce 2012), in a frictionless two-good endowment economy featuring complete markets the optimal recursive risk-sharing scheme produces endogenous time variation in the distribution of wealth, consumption, and currency risk. This result shows that recursive preferences and long-run risk can simultaneously resolve several exchange rate anomalies (see Brandt et al. 2006, Backus and Smith 1993, and Backus et al. 2001). Thanks to a moderate investment home bias, our fully fledged recursive production economy also addresses the Backus et al. (1992) quantity anomaly. In our model, indeed, the cross-country correlation of consumption is smaller than that of output, as in the data.

Our resolution of the quantity anomaly relies on our most important prediction on international capital flows: that good long-run productivity news produces an immediate outflow of investment. The key economic insight underlying this prediction is the existence of a tension between two channels. On the one hand, the *productivity channel* suggests that resources should move from the least productive to the most productive country; on the other hand, the *risk-sharing channel* suggests that resources should flow from the low-marginal-utility country to the high-marginal-utility country. The relative intensity of these two channels depends on whether the economy is affected by short- or long-run shocks.

With respect to short-run shocks, the productivity channel always dominates, i.e., the most productive country receives resources from abroad and invests more. This result is well known, as it holds also in the BKK model with standard preferences. To explain our ability to turn the quantity anomaly into a general equilibrium regularity we must focus on the role of long-run shocks. Specifically, with time-additive preferences, the country that is expected to be more productive in the long-run receives a net inflow of investment, as in the case of short-run shocks. With recursive preferences, in contrast, the opposite is true, as the risk-sharing channel dominates the productivity channel.

To better understand this result, assume that the home country receives good news for the long run while the foreign country receives no shock. At this point, the marginal utility of the home country drops substantially because even small amounts of positive news for the long run can produce a substantial increase in domestic continuation utility. In order to equalize marginal utilities across domestic and foreign agents (the risk-sharing channel), resources have to flow abroad so that foreign consumption can immediately increase. Because of the lower home bias in investment, the most efficient way to help the foreign country is to export investment goods. As investment goods of the home country are used more effectively to boost foreign investment, more foreign goods are freed up to support foreign consumption.

Our empirical analysis is consistent with these theoretical findings. We follow Colacito and Croce (2011) and Bansal et al. (2010) in identifying short- and long-run innovations to productivity by regressing Solow residuals on a set of predictive variables ranging from asset prices to quantities. These estimations are performed under the retained assumption that the United States is the home country, and they consider a set of foreign countries that includes Canada, France, Germany, Italy, Japan, the United Kingdom, and a rest of the world (ROW) aggregate that comprises G-7 countries, excluding of the U.S.

Our empirical results confirm the model's prediction that investment and net exports of investment respond with opposite signs to short- and long-run innovations. Indeed, the signs and magnitudes of the estimated coefficients are always in line with the prediction of the model. Additionally, we document that the response of the net exports of consumption to long- and short-run news in the data is consistent with our model. This constitutes a major point of departure from BKK, and thus we regard our model as a noteworthy step forward in the international macroeconomics literature. By unveiling a new long-run risk-based trading channel that is consistent with the data, we propose a novel way to think about both international capital flows and production frontier dynamics. This framework may be of great interest for both long-term fiscal and monetary policy considerations.

In the next section we discuss other related literature. In sections 3 and 4 we present our model and our equilibrium conditions, respectively. In section 5 we discuss our results. Section 6 summarizes our empirical evidence and section 7 concludes.

2 Related Literature

Using the recursive methods in Anderson (2005) and Colacito and Croce (2012), Tretvoll (2012) is the first to study a production economy with capital accumulation and recursive preferences. We differ from Tretvoll (2012) in several respects. First of all, Tretvoll (2012) does not consider long-run shocks, which constitute the main element of our theoretical and empirical investigations. The present paper is therefore the first to highlight the existence of a relevant long-run risk-based investment channel.

Second, Tretvoll (2012) takes into consideration neither the EGG nor the ACL observations about investment composition and capital accumulation. For this reason, the quantitative performance of our model represents a substantial improvement relative to the existing literature. Third, Tretvoll (2012) uses a calibration in the spirit of the RBC literature with an IES smaller than 1 and an RRA of 100. We adopt a calibration in the spirit of Bansal and Yaron (2004), with a RRA of 10 and an IES slightly larger than 1.

We use Greenwood et al. (1988) preferences to bundle consumption and leisure in order to address the critique by Raffo (2008) regarding the sources of countercyclicality of net exports. We also use evidence from EGG on the composition of imports and exports to highlight the relevance of the long-run recursive risk-sharing channel. However, our long-run risk approach with recursive preferences differs from both EGG and Raffo (2008). ACL do not address international dynamics. Colacito and Croce (2012) address international dynamics, abstracting away from production activity and international investment flows.

Several studies have highlighted the role of real and financial frictions (among others, see Stockman and Tesar 1995, Baxter and Crucini 1995, Kehoe and Perri 2002, Heathcote and Perri 2004, Bai and Zhang 2010, Petrosky-Nadeau 2011, and Alessandria et al. 2011). Our analysis differs from these due to its the emphasis on risk and recursive preferences in the context of a frictionless economy.

From an empirical point of view, we expand the methodology used in previous work (Colacito and Croce 2011, 2012) to show that country-specific long-run shocks have a well identified negative impact on contemporaneous investment flows, consistent with our model. Our findings are broadly consistent with the international empirical

4

investigation of Kose et al. (2003, 2008), as we do find evidence of a highly correlated economic productivity factor across G-7 countries in our post-1970 sample. From a finance perspective, we provide a productivity-based general equilibrium explanation of the findings in Lustig and Verdelhan (2007), Colacito (2008), Lustig et al. (2011a, b), and Bansal and Shaliastovich (2013).

3 The Economy

We study a two-country and two-good economy similar to BKK. We first describe the technology used to produce consumption goods and the role played by recursive preferences, and we then turn our attention to the international production structure. In what follows, we denote foreign variables by * and use small letters for log-units, i.e., $x_t = \log X_t$.

Consumption aggregate. Let $\{X_t, Y_t\}$ and $\{X_t^*, Y_t^*\}$ denote the time t consumption of goods X and Y in the home and foreign countries, respectively. The consumption aggregates in our two countries take the following CES form:

$$C_t = \left[\lambda X_t^{1-\frac{1}{\vartheta}} + (1-\lambda)Y_t^{1-\frac{1}{\vartheta}}\right]^{\frac{1}{1-\frac{1}{\vartheta}}}, \quad C_t^* = \left[(1-\lambda)X_t^{*1-\frac{1}{\vartheta}} + \lambda Y_t^{*1-\frac{1}{\vartheta}}\right]^{\frac{1}{1-\frac{1}{\vartheta}}}. \tag{1}$$

We assume that the home (foreign) country produces good X (Y) and set $\lambda > 1/2$ to build consumption home bias into our model. This is a standard assumption in the international macrofinance literature (see Lewis 2011).

Consumption bundle. The domestic (foreign) country consumes a composite bundle, $\widetilde{C}$ ($\widetilde{C}^*$), of consumption and leisure. As in Raffo (2008), we adopt Greenwood et al. (1988) (henceforth GHH) preferences to avoid counterfactual adjustments of the terms of trade:

$$\tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{f}} A_{t-1}, \quad \tilde{C}_t^* = C_t^* - \varphi N_t^{*1+\frac{1}{f}} A_{t-1}^*,$$

where N and N^* denote the share of hours worked in the home and foreign country, respectively, and A and A^* measure both the productivity level and the standards of

living in the home and foreign countries. This specification of the GHH preferences guarantees balanced growth.

Preferences. In each country, the representative agent has Epstein and Zin (1989) recursive preferences. For the home country, we have the following expression:

$$U_t = \left[(1 - \delta) \cdot \widetilde{C}_t^{1-1/\psi} + \delta E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}. \tag{2}$$

The preferences of the foreign country are defined in the same manner over the consumption bundle $\widetilde{C}_t^*$. The coefficients γ and ψ measure the RRA and the IES, respectively. We assume that the two countries have the same RRA and IES, as well as the same subjective discount factor.

With these preferences, agents are risk averse in future utility as well as future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty, measured by $\gamma - 1/\psi > 0$. To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1 - 1/\psi}$$

and obtain the approximation

$$V_t \approx (1 - \delta) \frac{\widetilde{C}_t^{1-1/\psi}}{1 - 1/\psi} + \delta E_t \left[V_{t+1} \right] - (\gamma - 1/\psi) Var_t \left[V_{t+1} \right] \kappa_t, \tag{3}$$

where $\kappa_t \equiv \frac{\delta}{2E_t \left[U_t^{1-1/\psi} \right]} > 0$. When $\gamma = 1/\psi$, the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, i.e., $\gamma > 1/\psi$, uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility, $E_t \left[V_{t+1} \right]$, for future utility risk, $Var_t \left[V_{t+1} \right]$. This trade-off drives international consumption and investment flows, and it represents one of the most important elements of our analysis. Our study is the first to fully characterize trade with Epstein and Zin (1989) preferences in a production economy with long-run shocks.[1]

---

[1]Equation (3) is reported for explanatory purposes only. The rest of the analysis is conducted with

Since there is a one-to-one mapping between utility, U_t, and lifetime wealth, i.e., the value of a perpetual claim to consumption, the optimal risk-sharing scheme can also be interpreted in terms of mean-variance trade-off of wealth. For this reason, in what follows we will use the terms "wealth" and "continuation utility" interchangeably.

Aggregate productivity. We model productivity growth in the spirit of the long-run risk literature. Specifically, we introduce country-specific long-run productivity components (Croce 2008), z and z^*, and assume that the domestic and foreign productivity processes, A and A^*, are co-integrated (Colacito and Croce 2012):

$$
\begin{aligned}
\log A_t &= \mu + \log A_{t-1} + z_{t-1} + \tau \cdot (\log A_{t-1} - \log A^*_{t-1}) + \varepsilon_{a,t} \quad (4) \\
\log A^*_t &= \mu + \log A_{t-1} + z^*_{t-1} - \tau \cdot (\log A_{t-1} - \log A^*_{t-1}) + \varepsilon^*_{a,t} \\
z_t &= \rho z_{t-1} + \varepsilon_{z,t} \\
z^*_t &= \rho z^*_{t-1} + \varepsilon^*_{z,t}.
\end{aligned}
$$

Consistent with previous literature, $\tau \in (0,1)$ is calibrated to a small number to generate moderate co-integration. In contrast, the autoregressive coefficient ρ is calibrated to a high number to capture low-frequency productivity adjustments.

Throughout the paper, we refer to $\varepsilon_{z,t}$ and $\varepsilon^*_{z,t}$ as long-run shocks, due to their long-lasting impact on the growth rates of the two goods. Similarly, we denote $\varepsilon_{a,t}$ and $\varepsilon^*_{a,t}$ as the short-run shocks. Shocks are jointly log-normally distributed:

$$
\xi_t \equiv \begin{bmatrix} \varepsilon_{z,t} & \varepsilon^*_{z,t} & \varepsilon_{a,t} & \varepsilon^*_{a,t} \end{bmatrix} \sim \quad i.i.d.N(\mathbf{0}, \Sigma),
$$

where

$$
\Sigma = \begin{bmatrix}
\sigma^2_x & \rho_{lrr}\sigma^2_x & 0 & 0 \\
\rho_{lrr}\sigma^2_x & \sigma^2_x & 0 & 0 \\
0 & 0 & \sigma^2 & \rho_{srr}\sigma^2 \\
0 & 0 & \rho_{srr}\sigma^2 & \sigma^2
\end{bmatrix}.
$$

Our economy features a large correlation of long-run components (large ρ_{lrr}) and a low correlation of short-run shocks (low ρ_{srr}) across countries, in the spirit of Backus et al. (1994), and Colacito and Croce (2011, 2012).

---

the preference specification in equation (2).

Production function and resource constraints. In each country, output is a Cobb-Douglas aggregation of country-specific capital and labor. Output can be used for consumption or investment:

$$
\begin{aligned}
X_t^{Tot} &= K_t^{\alpha}(A_t N_t)^{1-\alpha} = X_t + X_t^* + I_{x,t} + I_{y,t} \\
Y_t^{Tot} &= K_t^{*\alpha}(A_t^* N_t^*)^{1-\alpha} = Y_t^* + Y_t + I_{y,t}^* + I_{x,t}^*.
\end{aligned}
$$

From a home (foreign) country perspective, $I_{x,t}$ ($I_{y,t}^*$) measures real local investment, while $I_{y,t}$ ($I_{x,t}^*$) measures investment abroad. Even though capital stocks and labor services are country-specific, agents can trade both consumption and investment goods without any friction in every period and state of the world. We link our resource constraints to quantities recorded in the national accounts as follows:

$$
\begin{aligned}
X_t^{Tot} &= \underbrace{(X_t + P_t Y_t)}_{C_{m,t}} + \underbrace{(I_{x,t} + P_t I_{x,t}^*)}_{I_{m,t}} + \underbrace{(X_t^* + I_{y,t})}_{Exp_{m,t}} - \underbrace{P_t(Y_t^* + I_{x,t}^*)}_{Imp_{m,t}} \\
Y_t^{Tot} &= \underbrace{(Y_t^* + X_t^*/P_t)}_{C_{m,t}^*} + \underbrace{(I_{y,t}^* + I_{y,t}/P_t)}_{I_{m,t}^*} + \underbrace{(Y_t + I_{x,t}^*)}_{Exp_{m,t}^*} - \underbrace{(X_t^* + I_{y,t})/P_t}_{Imp_{m,t}^*},
\end{aligned}
$$

where $P_t = \frac{1-\lambda}{\lambda}\left(\frac{X_t}{Y_t}\right)^{\frac{1}{\eta}}$ denotes the terms of trade, and the subscript m indicates that we are referring to accounting aggregates measured in local units. To be consistent with our data source, we report results in local output units. Our results continue to hold also in the case in which we choose the consumption bundle as numeraire. Because of home bias, $C_{m,t} = X_t + P_t Y_t$ and $C_t = \left[\lambda X_t^{1-\frac{1}{\eta}} + (1-\lambda)Y_t^{1-\frac{1}{\eta}}\right]^{1/(1-\frac{1}{\eta})}$ in fact have very similar dynamics.

Capital accumulation. In each country, the stock of physical capital is a productivity-based weighted average of new and old investments. ACL document that exposure to aggregate productivity risk is increasing in investment age. Specifically, they show that the exposure of newly created capital vintages, ϕ_0, is statistically zero and that the exposure of older vintages is about one. Working with a continuum of overlapping vintages of capital, ACL prove that aggregate physical capital follows the dynamics

reported below:

$$K_{t+1} = (1 - \delta_k)K_t + \varpi_{t+1}G_t, \quad K_{t+1}^* = (1 - \delta_k)K_t^* + \varpi_{t+1}^* G_t^*,$$

where δ_k takes into account depreciation; G_t and G_t^* measure the mass of the new vintage of capital; and K_t and K_t^* measure the total mass of all older vintages of capital. The endogenous processes ω_t and ω_t^* take into account productivity differences across new and old vintages and take the following form:

$$\varpi_{t+1} = e^{-(1-\phi_0)\frac{1-\alpha}{\alpha}(\Delta a_{t+1} - \mu)}, \quad \varpi_{t+1}^* = e^{-(1-\phi_0)\frac{1-\alpha}{\alpha}(\Delta a_{t+1}^* - \mu)}.$$

When $\phi_0 = 0$, these processes are a negative transformation of the productivity shocks; i.e., good news for the productivity of existing capital is relatively bad news for the new vintages of capital. The reason is that new vintages do not immediately pick up the productivity gain, and hence they contribute relatively less to the formation of aggregate capital. When $\phi_0 = 1$, heterogeneity in productivity exposure is shut down and capital accumulation evolves as in the BKK setting.

We consider capital vintages with heterogenous exposure to productivity risk because they improve the asset pricing performance of production-based long-run risk models. Specifically, this channel allows the model to simultaneously produce sizeable fluctuations in investment and the marginal value of capital. Our framework, therefore, can function as a new benchmark for both international macroeconomic and financial studies.

New capital formation. New capital is a CES aggregation of domestic and foreign goods:

$$G_t = \left[\nu I_{x,t}^{1-\frac{1}{\xi}} + (1-\nu) I_{x,t}^{*1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}, \quad G_t^* = \left[(1-\nu) I_{y,t}^{1-\frac{1}{\xi}} + \nu I_{y,t}^{*1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}$$

When $\nu = \lambda$ and $\xi = \beth$, our technology for the production of new capital is identical to BKK. EGG, however, point out that under this restriction the share of imported consumption goods is identical to the share of imported investment goods. This is counterfactual, since a substantial share of the imports in the U.S. are related to capital goods, as opposed to consumption goods.

4 Risk-Sharing Rules and Asset Prices

We assume that markets are complete both domestically and internationally, so the allocation of the competitive equilibrium can be found by solving the Pareto problem associated with our economy (see the appendix). Prices can then be recovered using the planner's shadow valuations. Below we report the equilibrium conditions for consumption and investment.

Consumption allocations. The optimal allocation of the two goods devoted to consumption can be characterized using the following first-order necessary conditions:

$$S_t \cdot \frac{\partial C_t}{\partial X_t} \cdot \frac{1}{C_t} = \frac{\partial C_t^*}{\partial X_t^*} \cdot \frac{1}{C_t^*}$$
$$S_t \cdot \frac{\partial C_t}{\partial Y_t} \cdot \frac{1}{C_t} = \frac{\partial C_t^*}{\partial Y_t^*} \cdot \frac{1}{C_t^*},$$

where S_t is the ratio of the pseudo-Pareto weight of the home and foreign countries, respectively. The dynamics of the additional state variable S_t are given by the process

$$S_t = S_{t-1} \frac{M_t}{M_t^*} \frac{e^{\Delta c_t}}{e^{\Delta c_t^*}},$$

where M_t denotes the home stochastic discount factor expressed in units of the local consumption aggregate, C_t,

$$M_{t+1} = \beta \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma},$$

and M_t^* takes the same form but refers to the foreign country.

Asset prices. The stochastic discount factors in local output units can be specified as follows:

$$M_{t+1}^X = \left(\frac{X_t}{X_{t+1}} \frac{C_{t+1}}{C_t} \right)^{\frac{1}{\Xi}} M_{t+1},$$

$$M_{t+1}^Y = \left(\frac{Y_t^*}{Y_{t+1}^*} \frac{C_{t+1}^*}{C_t^*} \right)^{\frac{1}{\Xi}} M_{t+1}^*.$$

Let $Q_{k,t}$ and $P_{k,t}$ denote the ex- and cum-dividend price of domestic capital expressed in local output units, respectively. International capital prices satisfy the following equations:

$$P_{k,t} = \alpha \frac{X_t^{Tot}}{K_t} + (1-\delta)Q_{k,t}, \qquad P_{k,t}^* = \alpha \frac{Y_t^{Tot}}{K_t^*} + (1-\delta)Q_{k,t}^* \tag{5}$$

$$Q_{k,t} = E_t \left[M_{t+1}^X P_{k,t+1} \right], \qquad Q_{k,t}^* = E_t \left[M_{t+1}^Y P_{k,t+1}^* \right]. \tag{6}$$

The returns of capital in the domestic and foreign countries are:

$$R_{k,t+1} = \frac{P_{k,t+1}}{Q_{k,t}}, \quad R_{k,t+1}^* = \frac{P_{k,t+1}^*}{Q_{k,t}^*},$$

and the real risk-free rates are computed as follows:

$$1/R_{f,t} = E_t[M_{t+1}^X], \quad 1/R_{f,t}^* = E_t[M_{t+1}^Y].$$

Since markets are complete, the log-growth of the real exchange rate in consumption units is

$$\Delta e_{t+1} = m_{t+1} - m_{t+1}^*.$$

Optimal investment. Similarly to ACL, optimal investment of each agent in its own country satisfies the following conditions:

$$\frac{1}{\nu} \left(\frac{I_{x,t}}{G_t} \right)^{\frac{1}{\xi}} = E_t[M_{t+1}^X P_{k,t+1} e^{\varpi_{t+1}}], \quad \frac{1}{\nu} \left(\frac{I_{y,t}^*}{G_t^*} \right)^{\frac{1}{\xi}} = E_t[M_{t+1}^Y P_{k,t+1}^* e^{\varpi_{t+1}^*}].$$

Heterogenous exposure to productivity shocks creates a stochastic wedge between the prices of new and old capital vintages (ϖ_{t+1} and ϖ_{t+1}^*), which is not known when the

investment decision is made at time t. For this reason, the agent equalizes the known marginal cost of capital ($\partial G/\partial I_x$ and $\partial G^*/\partial I_y^*$) to the expected discounted present value of a marginal unit of new capital adjusted by its relative productivity.

In an analogous way, investments abroad are determined by the following no-arbitrage equations:

$$\frac{1}{1-\nu}\left(\frac{I_{y,t}}{G_t^*}\right)^{\frac{1}{\xi}} = E_t[M_{t+1}^X(P_{k,t+1}^*e^{\varpi_{t+1}^*})P_{t+1}], \quad \frac{1}{1-\nu}\left(\frac{I_{x,t}^*}{G_t}\right)^{\frac{1}{\xi}} = E_t[M_{t+1}^Y(P_{k,t+1}e^{\varpi_{t+1}})/P_{t+1}],$$

which take into account exchange rate risk through the terms-of-trade, P_t.

5 Inspecting the Mechanism

In this section we explore the relevance of the key elements of our model. To do so, we start from a pure BKK economy and move toward our benchmark model by adding one modification at a time. We compare six models whose key elements are summarized in table 1. Our six calibrations are detailed in table 2.

Model 1 features a pure BKK economy with co-integrated productivity processes. In model 2 we add long-run risk to the standard BKK model and show that it plays no significant role with standard preferences. Models 3 and 4 highlight the relevance of higher RRA and higher IES, respectively. In the last three calibrations, we modify the technology structure and show that when the EGG and the ACL observations are combined together, the response of international investment flows to long-run news changes radically. In what follows, we refer to model 1, 4 and 6 as the BKK, EZ-BKK, and Benchmark models, respectively.

5.1 From BKK to EZ-BKK

A comparison of the first two columns of table 3 shows that long-run risk does little in an economy with standard preferences and BKK technology. Except for the increase in both the volatility and the cross-country correlation of the interest rates, nothing else changes in a significant way.

TABLE 1: **Main Components of Our Economy**

Model	1	2	3	4	5	5b	6
Long-run risk		✓	✓	✓	✓	✓	✓
High RRA			✓	✓	✓	✓	✓
High IES				✓	✓	✓	✓
Milder investment home bias						✓	✓
Heterogenous productivity of vintage capital					✓		✓

Notes: This table summarizes the main components active in each of our models. All parameter values are reported in table 2. Model 1 refers to the original BKK economy. We denote model 4 as EZ-BKK. Model 6 is our Benchmark.

In figure 1, we show the response of macroeconomic quantities to both short-run shocks (left panels) and long-run shocks (right panels) across models 2, 3 and 4. First, we note that moving from standard time-additive preferences to recursive preferences with higher RRA and IES alters only marginally the response of quantities to short-run shocks. In economies with just short-run uncertainty, therefore, recursive preferences alone will not be able to explain the data as in the case of standard preferences.

When we turn our attention to long-run news, in contrast, the responses look quite different across models over the first few periods. Specifically, when the IES is set to 1/2 (models 2 and 3), the agent has a strong incentive to consume more upon the realization of good long-run news. This is a reflection of the fact that the income effect dominates the substitution effect: as good long-run news increases wealth, the agent reduces savings and investment. In contrast, when the IES is set above unity, the substitution effect becomes stronger and both consumption and investment growth adjust by a moderate amount.

Another difference across models 2 and 4 is related to the response of the net exports–output ratio to a positive long-run shock. When the IES is set to 1/2, the home country is a net importer; i.e., it finances part of its consumption through foreign resources. When instead the IES is set to 1.1, the home country becomes a net exporter. In the first case, resources flow from the country with relatively poorer growth prospects to the country that is expected to be most productive for the long-run. In the second case, in contrast, resources flow away from the most productive country.

TABLE 2: Calibrated Parameter Values

Preferences		CRRA			EZ			
Model		1	2	3	4	5	5b	6
Subjective discount factor	β	0.985	0.985	0.985	0.985	0.985	0.985	0.9873
Risk aversion	γ	2	2	10	10	10	10	10
IES	ψ	0.5	0.5	0.5	1.1	1.1	1.1	1.1
Consumption home bias	λ	0.76	0.76	0.76	0.76	0.76	0.97	0.97
Consumption-bundle elasticity	ℶ	1.5	1.5	1.5	1.5	1.5	1	1
Consumption-labor elasticity	f	1	1	1	1	1	1	1
Capital income share	α	0.36	0.36	0.36	0.36	0.3	0.3	0.3
Depreciation rate of capital	δ	0.1	0.1	0.1	0.1	0.06	0.06	0.06
Investment home bias	ν	0.76	0.76	0.76	0.76	0.76	0.53	0.57
Investment-bundle elasticity	ξ	1.5	1.5	1.5	1.5	1.5	1	1
Exposure of young vintages	ϕ_0	1	1	1	1	0	1	0
Long-run mean of productivity	μ	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Persistence of long-run shock	ρ	0.9859	0.9859	0.9859	0.9859	0.9859	0.9859	0.9859
Co-integration parameter	τ	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05
Short-run shock volume	σ	0.027	0.027	0.027	0.027	0.027	0.027	0.027
Long-run shock volume	σ_x	0	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$
Short-run shocks correlation	ρ_{srr}	0	0.027	0.027	0.027	0.027	0.027	0.027
Long-run shocks correlation	ρ_{lrr}	–	0.85	0.85	0.85	0.85	0.85	0.85

Notes: This table reports the parameter values used for our calibrations. All models are calibrated at an annual frequency. Model 1 refers to the original BKK economy. We denote model 4 as EZ-BKK. Model 6 is our Benchmark.

Preferences	CRRA			EZ	
Model	1	2	3	4	Data
Quantities					
$E[I_m/X^{Tot}]$	27.60	28.34	30.52	35.21	20.13
$E[(I_y^* + Y)P/X^{Tot}]$	15.22	15.28	15.39	15.25	10.90
$E[I_x^*P/I_m]$	15.21	15.16	15.18	15.18	40.00
$E[P \cdot Y/C_m]$	15.22	15.29	15.32	15.28	5.00
$vol(\Delta x^{Tot})$	2.70	3.09	3.08	3.15	3.49
$vo(\Delta c_m)$	2.08	2.66	2.73	2.63	2.53
$vol(\Delta i_m)$	7.85	8.27	8.08	7.16	16.40
$vol(\Delta n)$	2.20	2.24	2.25	2.24	2.07
$corr(\Delta c, \Delta n)$	0.77	0.59	0.58	0.62	0.28
$corr(\Delta c_m, \Delta i_m)$	0.80	0.64	0.56	0.77	0.39
$vol(NX/X^{Tot})$	1.08	1.38	1.35	1.51	2.40
$corr(\Delta NX/X^{Tot}, \Delta x^{Tot})$	-0.64	-0.56	-0.56	-0.56	-0.27
$corr(\Delta NXQ/X^{Tot}, \Delta x^{Tot})$	-0.61	-0.51	-0.51	-0.53	0.00
$corr(\Delta x^{Tot}, \Delta y^{Tot})$	0.04	0.21	0.20	0.23	0.52
$corr(\Delta c_m, \Delta c_m^*)$	0.25	0.44	0.48	0.41	0.33
$corr(\Delta i_m, \Delta i_m^*)$	-0.76	-0.59	-0.54	-0.65	0.65
$corr(\Delta n, \Delta n^*)$	0.07	0.10	0.10	0.08	0.52
Asset Prices					
$E[r_f]$	5.48	5.27	4.27	2.21	0.86
$E[r_k^{ex}]$	0.01	0.01	0.08	0.08	5.71
$vol[r_f]$	0.38	2.44	2.59	1.29	0.97
$vol[r_k^{ex}]$	1.34	1.39	1.33	1.14	20.51
$vol(m)$	1.90	6.76	141.90	74.34	–
$corr(m, m*)$	0.97	0.997	0.99	0.99	–
$corr(m_x, m_y)$	0.86	0.99	0.99	0.99	–
$corr(r_k^{ex}, r_k^{ex*})$	-0.29	-0.34	-0.35	-0.34	–
$corr(r_f, r_f^*)$	0.29	0.97	0.98	0.92	65.00
$vol(\Delta e)$	0.48	0.56	0.56	0.54	11.20

Notes: All figures are multiplied by 100, except contemporaneous correlations. Empirical moments are computed using U.S. annual data from 1930 to 2008. International moments are from Raffo (2008). Returns are in log units and are levered using a coefficient of 3 (Garca-Feijo and Jorgensen (2010)). All the parameters are calibrated as in table 2. The entries for the models are obtained by repetitions of small-sample simulations.
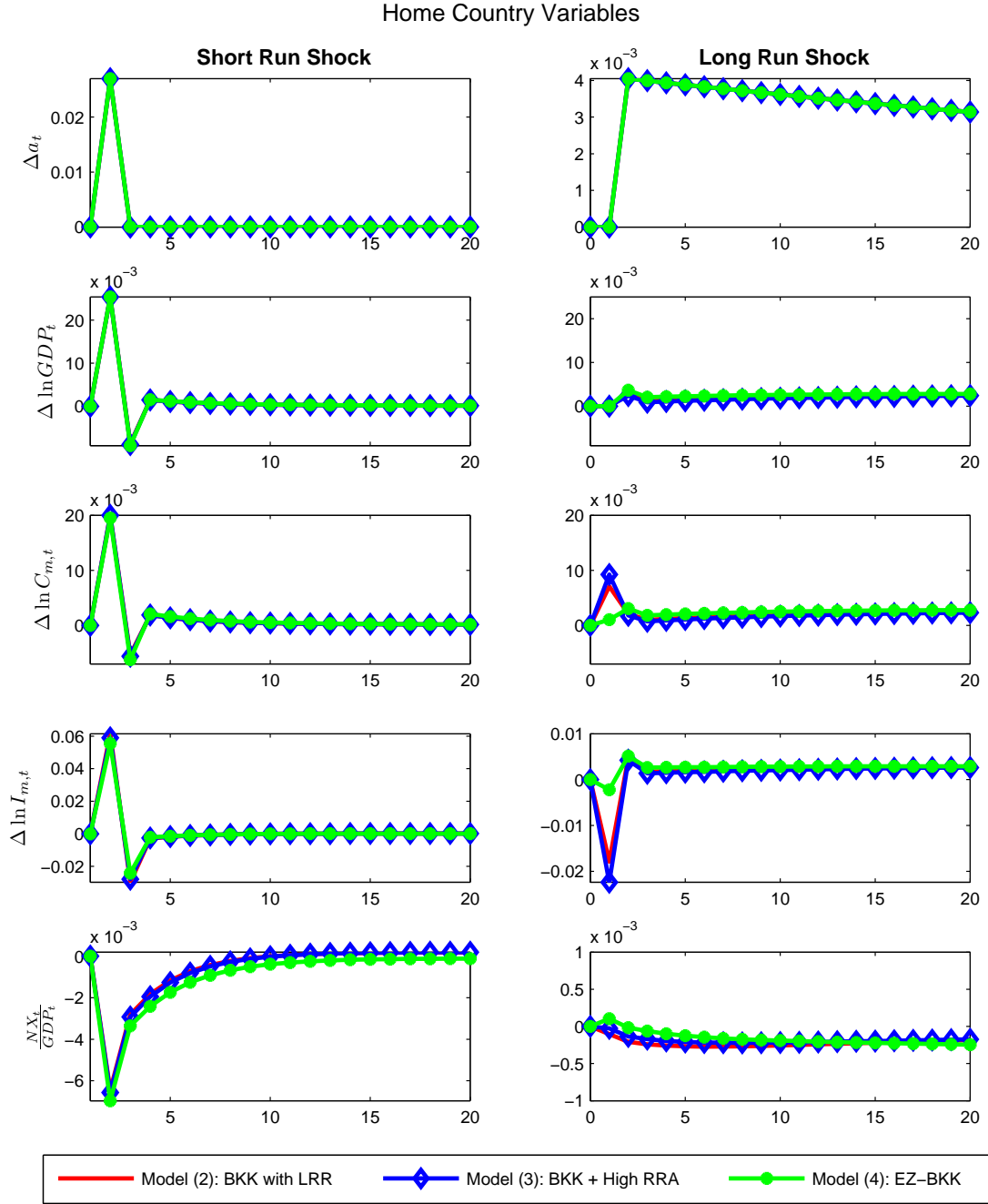
Home Country Variables

FIG. 1. Quantities with and without EZ preferences. This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home country and has a magnitude σ. The long-run shocks affect both the home country with magnitude σ_x and the foreign country with magnitude $\rho_{lrr}\sigma_x$, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$.
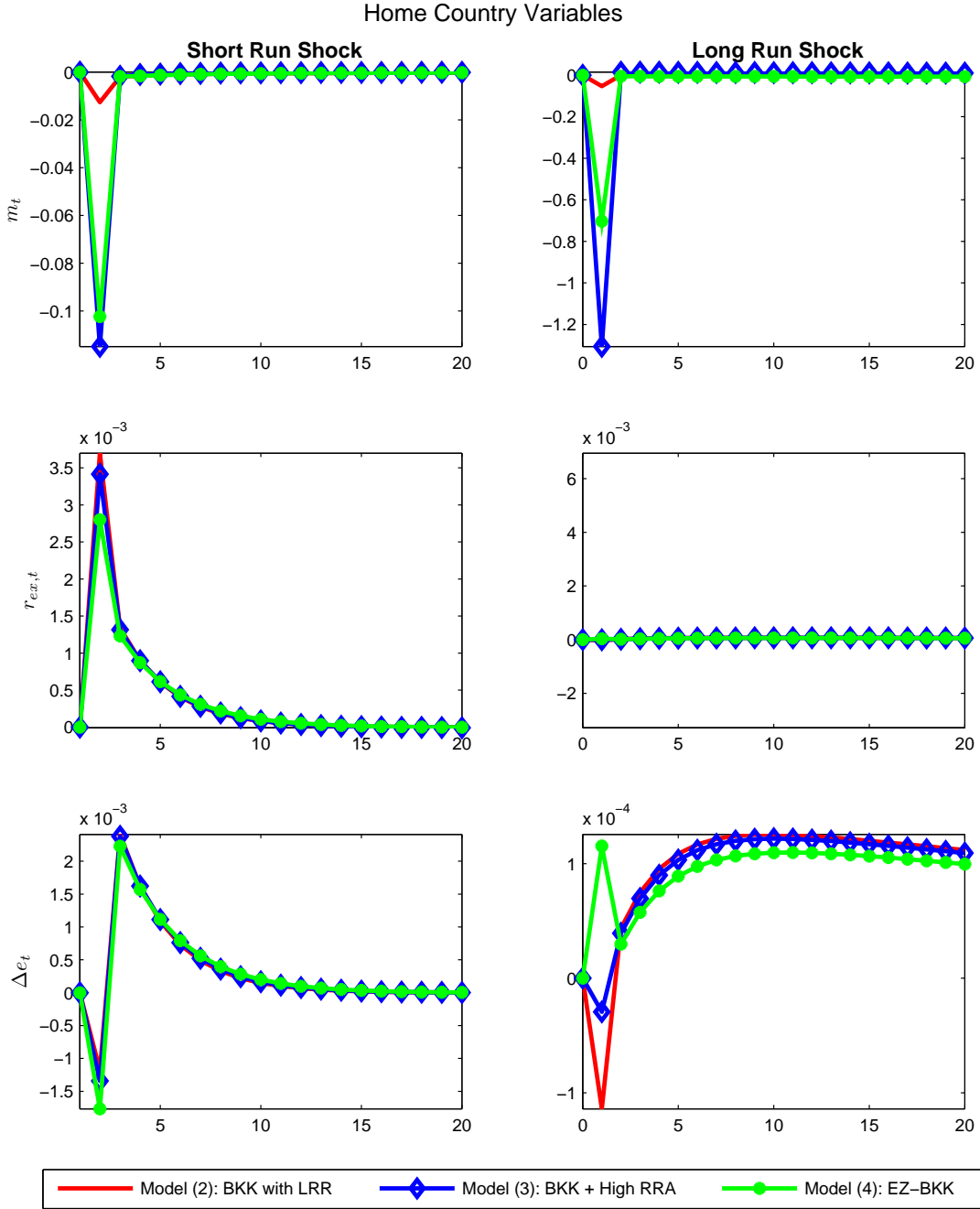
16

FIG. 2. **Prices with and without EZ preferences.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home country and has a magnitude σ. The long-run shocks affect both the home country with magnitude σ_x and the foreign country with magnitude $\rho_{lrr}\sigma_x$, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$.

This behavior of the net exports is consistent with the risk-sharing motives high-lighted in an endowment economy by Colacito and Croce (2012). Agents with high IES and RRA are adverse to utility risk, $Var_t(U_{t+1})$, and are willing to give up current resources in exchange for wealth insurance. In this class of models, if the domestic country receives good news for the long-run, it finds it optimal to give up more resources to the rest of the world in order to have better access to insurance assets in the financial markets, thereby reducing conditional wealth volatility. This finding is relevant in our production economy because it rationalizes the less-than-perfect correlation between cross-country investment flows and relative productivity. That is, resources do not always immediately flow toward the country that is expected to be the most productive.

These responses of net exports to long-run shocks explain the different adjustments of the exchange rate highlighted in the bottom right panel of figure 2. When the IES is set to 1/2, goods flow toward the home country and its currency appreciates. When the IES is set to 1.1, conversely, goods flow toward the foreign country and the domestic currency becomes weaker.

Overall, figure 2 documents that adding recursive preferences to a BKK economy has very few consequences for exchange rates and excess returns. Even though the pricing kernel becomes more volatile because of the higher aversion to utility risk (given by $\gamma - 1/\psi$), models 2, 3, and 4 are hard to distinguish. Model 4 features an overly smooth exchange rate and excess returns, as in BKK (see table 3). On the quantities side, model 4 also delivers consumption growth rates that are more cross-country correlated than are output growth rates, which is at odds with the data.

We conclude this section by observing that all models studied thus far predict an appreciation of the home currency upon the realization of good short-run news to domestic productivity. This result is very different from that obtained by BKK and is driven by our assumption concerning labor preferences. Indeed, with GHH preferences labor responds only to changes in productivity through the wage channel. This implies that upon the realization of good short-run news to the home country, labor does not increase abroad. Overall, the domestic consumption bundle falls relative to the foreign one due to the drop in leisure, leading to an appreciation of the domestic currency. We discuss this point further in the next sections.

5.2 From EZ-BKK to Our Benchmark Model

In this section we change the technology side of the economy along two key dimensions. First, we take seriously the empirical evidence documented by EGG and introduce stronger home bias in consumption and weaker home bias in investment. We argue that this is essential to obtain better results on the quantity side. Second, we assume that younger vintages of capital are less exposed to aggregate productivity than older vintages, consistent with ACL. We show that this friction is relevant in capturing a significantly higher degree of risk for investment.

5.2.1 Heterogenous home bias across consumption and investment

In table 4, we report all relevant moments for models 4 through 6. We start our discussion by comparing model 4 and model 5b, i.e., by addressing the role of heterogenous home bias across consumption and investment. In model 5b, we use the same consumption aggregator adopted by Colacito and Croce (2012) in an exchange economy. We do so to better compare our result to theirs and to highlight the role of production and investment. Specifically, we adopt a simple Cobb-Douglas aggregator ($\beth = 1$) and decrease the share of consumption imports ($\mathrm{E}[P \cdot Y/C_m]$) by increasing the consumption home bias ($\lambda = 0.97$), which is consistent with the data.

On the investment side, we retain the BKK assumption that the degree of substitution between foreign and domestic goods is the same across the investment and consumption sectors, i.e., $\xi = \beth = 1$. To capture openness in trade of investment goods, we adjust ν and allow the imports of capital goods to increase up to about 40% of total investment, which is again consistent with the data.

The joint analysis of table 4 and figure 3 reveals three important implications. First, by allowing more substitution among capital goods, we obtain a much higher level of investment volatility than in model 4. This is explained by the fact that the G aggregator generates decreasing marginal return of investment similarly to an adjustment cost function. By allowing more cross-country substitution, we reduce the intensity of the adjustment costs and obtain more sizeable investment fluctuations both within each country and across countries. As a reflection, net exports become more volatile as well. Specifically, the volatility of our net exports–to–output ratio is now higher

Model	4	5	5b	6	Data
Milder investment home bias			✓	✓	
Vintage capital		✓		✓	
Quantities					
$\mathrm{E}[I_m/X^{Tot}]$	35.21	29.21	30.09	30.20	20.13
$\mathrm{E}[(I_y^* + Y)P/X^{Tot}]$	15.25	15.33	16.33	15.22	10.90
$\mathrm{E}[I_x^* P/I_m]$	15.18	15.19	47.31	43.49	40.00
$\mathrm{E}[P \cdot Y/C_m]$	15.28	15.29	3.00	3.00	5.00
$\mathrm{vol}(\Delta x^{Tot})$	3.15	3.31	3.54	3.31	3.49
$\mathrm{vo}(\Delta c_m)$	2.63	2.66	3.02	2.82	2.53
$\mathrm{vol}(\Delta i_m)$	7.16	9.76	27.66	25.81	16.40
$\mathrm{vol}(\Delta n)$	2.24	2.38	2.46	2.37	2.07
$\mathrm{corr}(\Delta c, \Delta n)$	0.62	0.67	0.63	0.52	0.28
$\mathrm{corr}(\Delta c_m, \Delta i_m)$	0.77	0.72	0.43	0.41	0.39
$\mathrm{vol}(NX/X^{Tot})$	1.51	1.78	6.67	6.98	2.40
$\mathrm{corr}(\Delta NX/X^{Tot}, \Delta x^{Tot})$	-0.56	-0.59	-0.15	-0.23	-0.27
$\mathrm{corr}(\Delta NXQ/X^{Tot}, \Delta x^{Tot})$	-0.53	-0.56	-0.06	-0.14	0.00
$\mathrm{corr}(\Delta x^{Tot}, \Delta y^{Tot})$	0.23	0.19	0.16	0.18	0.52
$\mathrm{corr}(\Delta c_m, \Delta c_m^*)$	0.41	0.40	0.04	0.13	0.33
$\mathrm{corr}(\Delta i_m, \Delta i_m^*)$	-0.65	-0.69	-0.73	-0.68	0.65
$\mathrm{corr}(\Delta n, \Delta n^*)$	0.08	0.07	0.04	0.07	0.52
Asset Prices					
$\mathrm{E}[r_f]$	2.21	1.31	2.04	0.99	0.86
$\mathrm{E}[r_k^{ex}]$	0.08	2.73	0.22	3.46	5.71
$\mathrm{vol}[r_f]$	1.29	1.17	1.61	1.66	0.97
$\mathrm{vol}[r_k^{ex}]$	1.14	2.80	12.11	13.99	20.51
$\mathrm{vol}(m)$	74.34	67.81	72.55	72.11	–
$\mathrm{corr}(m, m*)$	0.99	0.99	0.99	0.99	–
$\mathrm{corr}(m_x, m_y)$	0.99	0.99	0.99	0.99	–
$\mathrm{corr}(r_k^{ex}, r_k^{ex*})$	-0.34	0.67	-1.00	-0.93	–
$\mathrm{corr}(r_f, r_f^*)$	0.92	0.90	0.21	0.17	65.00
$\mathrm{vol}(\Delta e)$	0.54	0.63	8.73	10.27	11.20

Notes: All figures are multiplied by 100, except contemporaneous correlations. Empirical moments are computed using U.S. annual data from 1930 to 2008. International moments are from Raffo (2008). Returns are in log units and are levered using a coefficient of 3 (Garca-Feijo and Jorgensen (2010)). All the parameters are calibrated as in table 2. The entries for the models are obtained by repetitions of small-sample simulations.

than in the data. This suggests that the volatility of international trade can be significant in this class of models even after adding trading costs or financial frictions. Models with standard preferences are subject to the opposite problem, as they are not able to generate enough trade volatility.

Second, we can see that in model 5b the recursive risk sharing motive is amplified (figure 3, bottom right panels). Upon the realization of good long-run news for domestic productivity, the home country finds it optimal to further decrease aggregate investment, I_m, in order to export a greater fraction of output. Under model 5b an even more sizeable flow of resources moves from the country that is expected to be more productive to the less productive one.

We examine this response from a foreign-country perspective. For the foreign economy, receiving more investment goods is very convenient. Because of substitutability, investment in the foreign country, I_m^*, can be supported with home-investment goods, I_y, even though foreign investment, I_y^*, drops. Under this strategy, more foreign output, Y^*, can be used to support foreign consumption, C^*. This increase in foreign consumption enables marginal utilities across countries to be equalized according to the risk-sharing channel.[2] With respect to short-run shocks, in contrast, net exports are driven by the productivity channel: more productive countries run negative current accounts, as they are net investment receivers.

Third, upon the realization of long-run shocks, investment drops, whereas both net exports and consumption growth increase. This helps us better match the data on the co-movements of these variables. Under model 5b, the correlation of the net exports–to–output ratio and output growth is slightly negative, as it is in the data, in contrast to what is observed under model 1. Following Raffo (2008), we construct a measure of net exports under the assumption that the terms of trade are constant, NXQ, to test whether our net exports are driven by quantities or relative prices. We find that our results are driven by the adjustment of international quantities, consistent with the

---

[2]The analysis that we conduct in this section takes into account the large degree of correlation of long-run shocks that we have assumed in our calibration. This means that the relative long-run shock experienced by a country is small, due to the large probability that the other country has also been exposed to such a shock. We report in the appendix the analysis for the case in which shocks are orthogonalized, which can be interpreted as a manifestation of a very large relative long-run shock in one of the two countries. We document that the response of consumption in this case brings the model closer to the endowment economy analysis of Colacito and Croce (2012).
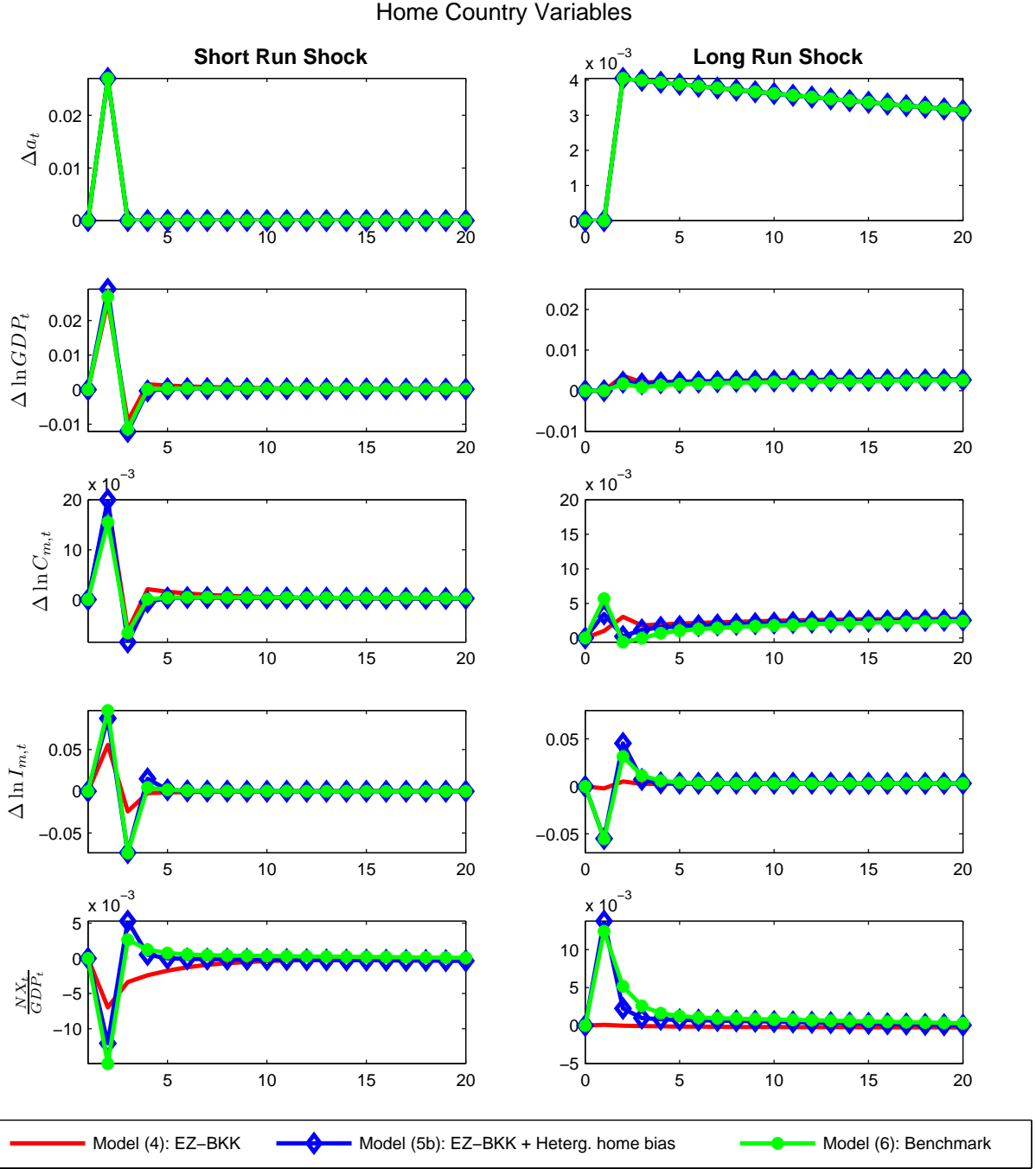
FIG. 3. Quantities with and without modified investment technology. This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home country and has a magnitude σ. The long-run shocks affect both the home country with magnitude σ_x and the foreign country with magnitude $\rho_{lrr}\sigma_x$, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$.

22

U.S. data. The correlation between national consumption and investment growth is moderate as well, which is again consistent with the data.

Turning our attention to the bottom portions of table 4 and figure 4, we make three relevant points about the implications of the EGG observation for asset prices. First, thanks to more sizeable international trade, the terms of trade and hence the exchange rate are much more volatile in model 5b than in any of the models previously analyzed. Specifically, the growth rate of the exchange rate becomes an order of magnitude larger than before.

Second, thanks to a larger inflow of investment goods, the home country anticipates a more pronounced accumulation of capital upon the realization of positive short-run shocks. This means that the home future utility increases more than in model 4. Since agents are averse to continuation utility risk in addition to consumption risk, the marginal utility of the home country falls more than that of the foreign country. For this reason, under model 5b short-run shocks produce a depreciation of the home currency, as in standard international RBC models.

Third, looking at domestic capital excess returns and risk-free rates, we can see that lowering the investment home bias produces very small differences. The EGG observation helps us on the international quantity side but has no effect on local returns. In the next section, we show that the introduction of heterogenous productivity across capital vintages can improve the performance of the model exactly in this direction.

5.2.2 Heterogenous productivity risk across capital vintages

In model 5, we augment the EZ-BKK model (model 4) with heterogenous productivity risk across capital vintages. In our Benchmark model (model 6), we add the ACL friction to model 5b, i.e., the EZ-BKK economy with lower investment home bias. By comparing our simulated results in table 4, we note that the vintage capital has a very powerful effect on asset prices, even though it does not seem to significantly influence most of the international quantities. The same conclusion can be obtained by comparing the responses depicted in figures 3 and 4.

A closer look at international investment flows helps us reveal the impact of this friction on capital dynamics and excess returns. In figure 5, we plot the investment–
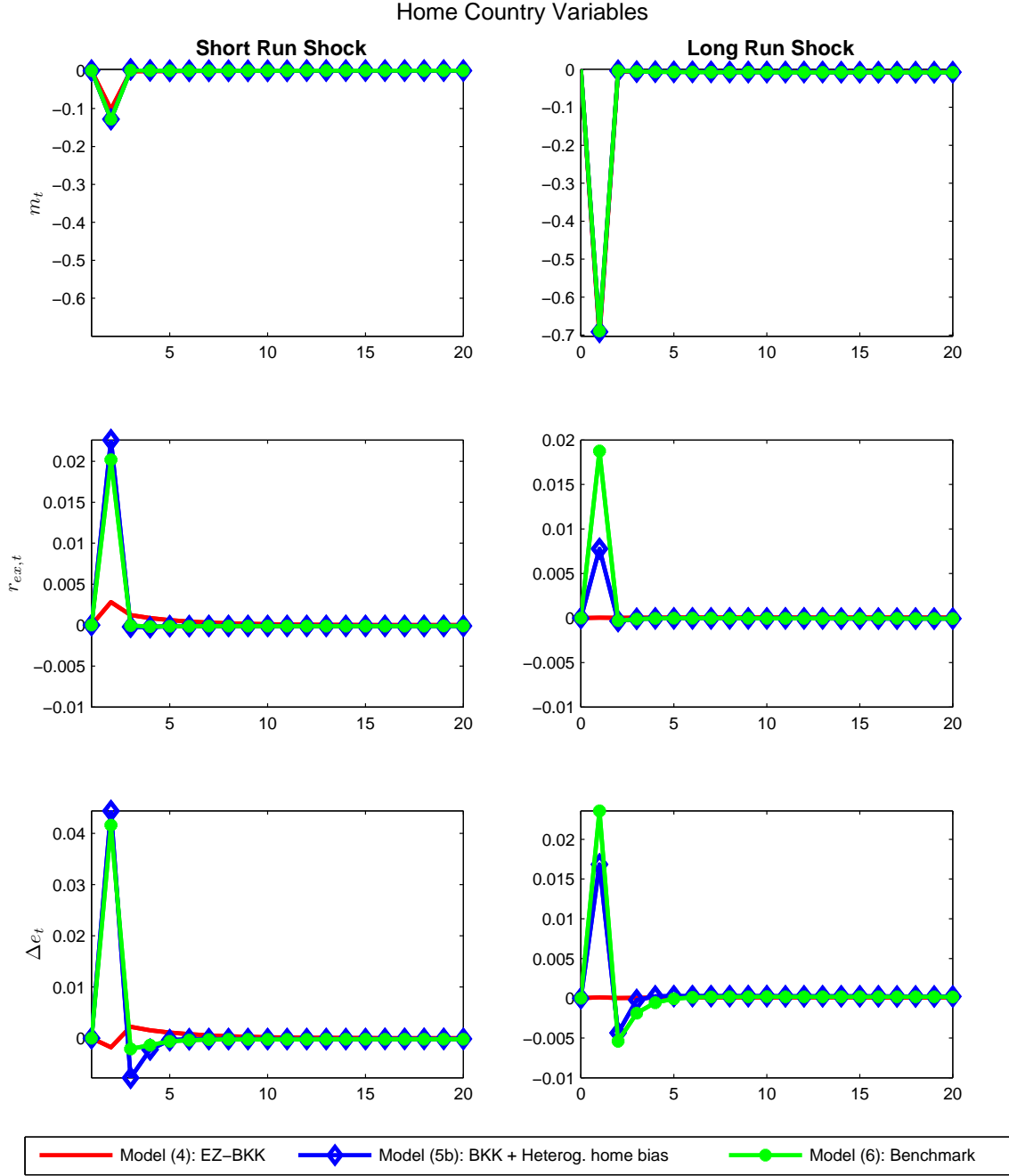
FIG. 4. Prices with and without modified investment technology. This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home country and has a magnitude σ. The long-run shocks affect both the home country with magnitude σ_x and the foreign country with magnitude $\rho_{lrr}\sigma_x$, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$.
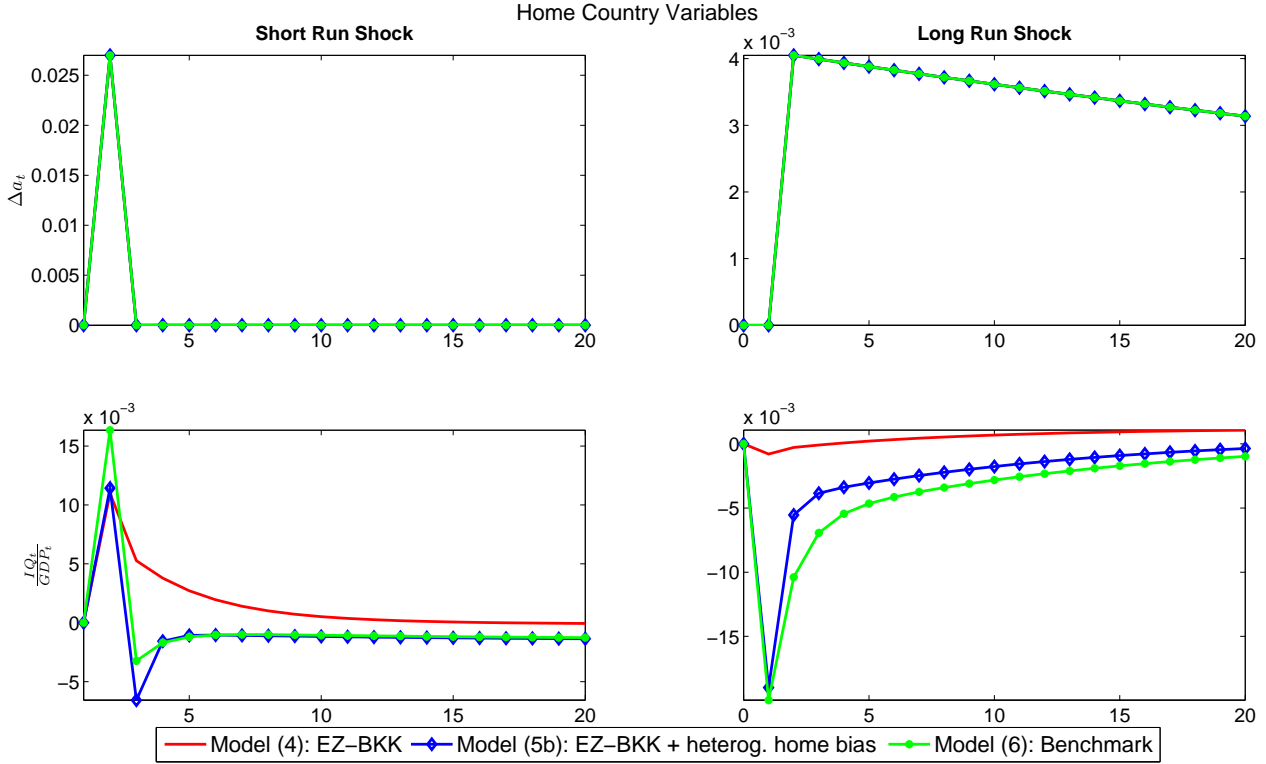
24

FIG. 5. **Investment share and capital vintages.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home country and has a magnitude σ. The long-run shocks affect both the home country with magnitude σ_x and the foreign country with magnitude $\rho_{lrr}\sigma_x$, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$. The variable IQ_t is defined as $I_{x,t} + \overline{P}I_{x,t}^*$, where $\overline{P}$ is the terms of trade at the steady state.

output share in the home country, keeping constant the terms of trade. When long-run shocks materialize, the *quantity* of investment declines more under our Benchmark model than under model 5b, consistent with the ACL analysis (bottom right panel).

This difference in the behavior of investment is not visible in figure 3, in which we focus on the *value* of investment in local units. Upon the realization of long-run shocks, the value of investment is almost the same across models 5b and 6, simply because the terms of trade worsen and make foreign investment more expensive. The fact that young vintages of capital do not immediately pick up the long-run productivity

shock makes them less valuable, implying a delay in investment and a slow-down in capital accumulation.

By solving forward equation (5), we see that the value of capital, Q_t, is the expected present value of capital marginal productivity. When capital accumulation declines, the marginal productivity of capital increases because of decreasing marginal returns. The expected increase of capital productivity over the long horizon leads to a substantial increase in Q. Consequently, as shown in figure 4, the excess return of capital sharply increases precisely when the agent's marginal utility is low, creating a sizeable equity premium.

We also point out that heterogenous exposure to productivity shocks across capital vintages makes the exchange rate depreciate more upon the realization of good domestic long-run news. The reason this happens is that capital accumulation slows down in the home country more than in the foreign one. As a result, short-run consumption increases relatively more in the home economy than it does abroad, thus resulting in a larger decrease in the pricing kernel in the home country. Under no-arbitrage, the domestic currency becomes weaker than under the EZ-BKK model.

5.3   Anomalies

We conclude this section by reporting the performance of our model with respect to three very well-known anomalies in international finance. In table 5, the first row refers to the Backus and Smith (1993) puzzle, that is, the lack of correlation between exchange rate growth and consumption growth cross-country differentials. We point out that all our models resolve the puzzle. Under GHH preferences, there is a substantial difference between the behavior of the pricing kernels and the consumption aggregate growth rates.

In the second row of table 5, we report the OLS coefficient of the uncovered interest parity regression, a typical measure of the so-called forward premium anomaly, that is the tendency of high interest rate currency to appreciate. In the data this coefficient is negative and is explained by countercyclical currency risk (see, among others, Lustig et al. 2011b).

Model	1	2	3	4	5	5b	6	Data
corr$(\Delta e, \Delta c_m - \Delta c_m^*)$	-0.31	-0.20	-0.29	-0.19	-0.41	0.24	-0.07	-0.02
β_{UIP}	1.01	1.03	0.90	1.04	1.06	0.81	0.51	-0.72
$\rho_{\Delta c} - \rho_{\Delta y}$	0.20	0.15	0.17	0.10	0.14	-0.12	-0.06	-0.19

Notes: Empirical moments are computed taking the U.S. as home-country. Data are annual from 1930-2008. The entries for the models are obtained by a long-sample simulation. $\rho_{\Delta c} - \rho_{\Delta y}$ is equal to corr$(\Delta c_m, \Delta c_m^*)$ - corr$(\Delta y_m, \Delta y_m^*)$.

Even though our productivity shocks are homoscedastic, our model features endogenous countercyclical time-varying volatility in consumption and pricing kernels. This is a general feature of recursive risk-sharing schemes that generates time-varying currency risk premia (Colacito and Croce 2012). Only our Benchmark calibration features enough time-varying volatility to generate a β_{UIP} significantly lower than one.

Note also the difference between the cross-country correlations of consumption and output (table 5, row 3). BKK was the first paper to point out that with standard preferences consumption is more correlated than output, while in the data the opposite is true. BKK denote this fact as the quantity anomaly.

When agents have recursive preferences, they have an incentive to share utility risk as opposed to short-run consumption risk. That is, agents can equate their marginal utilities by keeping their continuation utilities highly correlated. In our production economy, equating utility dynamics is equivalent to equating long-run production dynamics. Ultimately, this is accomplished by having highly cross-country-correlated capital accumulation dynamics. When investment home bias is strong, equating long-run capital dynamics is relatively difficult. Hence the optimal allocation can be achieved only by keeping the correlation of short-run consumption bundles sufficiently high. This explains why models 1–5 fail to reproduce the quantity anomaly, while both model 5b and our Benchmark model succeed in this.

Finally, in figure 6 we contrast the dynamics of consumption and investment goods within and across countries across the BKK economy and our Benchmark model. We note first that under our Benchmark model, the net exports of consumption goods co-moves with both short- and long-run productivity shocks, consistent with the results
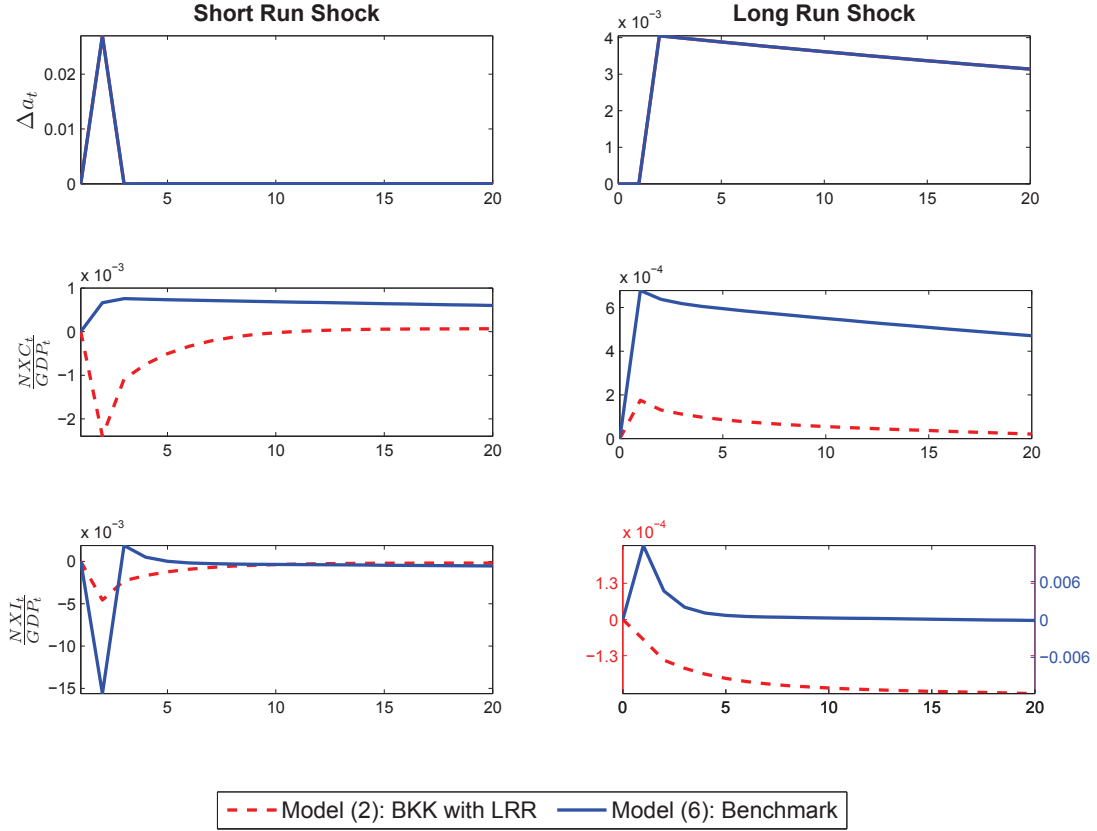
FIG. 6. **International flows in the short- and long-run.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home-country productivity, ϵ_a and ϵ_x, materialize at time 2. The short-run shock affects only the home-country, with magnitude σ, and the long-run shock affects the home and foreign countries with magnitudes σ_x and $\rho_{lrr}\sigma_x$ respectively, where $\rho_{lrr} = corr(\epsilon_x, \epsilon_x^*)$.

obtained by Colacito and Croce (2011) in an exchange economy.

Second, with respect to short-run shocks, a BKK economy predicts a net inflow of both consumption and investment goods. In our Benchmark economy, in contrast, there is a substantial inflow of investment goods (the productivity channel) that dominates the outflow of consumption goods (the risk-sharing channel).

Third, upon the realization of positive long-run news, the risk-sharing channel is so strong in our Benchmark model that it determines a net outflow of both consumption and investment goods. This result sharply contrasts with the predictions of a standard BKK economy. With standard preferences, indeed, the home country should be

a net receiver of investment goods. In the next section, we test these responses in the data and obtain positive results in support of our recursive risk-sharing channel.

6 Empirical Findings

In this section, we provide direct empirical evidence supporting the implications of our model for the response of investment and net exports to both short- and long-run news. For a cross-section of G-7 countries starting in 1972, we find that investment, net exports of investment, and net export of consumption co-move with productivity shocks as prescribed by our complete markets mechanism.[3] We report our results in table 6. A detailed description of our data sources is reported in the appendix.

6.1 Identification of Short- and Long-run Shocks

We follow Colacito and Croce (2011) and Bansal et al. (2010) in identifying short- and long-run innovations to productivity by regressing Solow residuals on a set of predictive variables. These estimations are performed for Canada, France, Germany, Italy, Japan, the United Kingdom, and the U.S. We adopt the convention of denoting the U.S. as the home country. To study the robustness of our empirical results, we employ the following five sets of variables commonly used in the long-run risk literature to identify long-run components:

$$
\begin{aligned}
F_{1,t}^i &= \left[pd_t^i\right] \\
F_{2,t}^i &= \left[pd_t^i, rf_t^i\right] \\
F_{3,t}^i &= \left[pd_t^i, rf_t^i, \Delta c_t^i\right] \\
F_{4,t}^i &= \left[pd_t^i, rf_t^i, \Delta I_t^i\right] \\
F_{5,t}^i &= \left[pd_t^i, rf_t^i, \Delta c_t^i, \Delta I_t^i\right],
\end{aligned}
$$

---

[3]The choice of 1972 as the starting date of our empirical investigation is determined by the fact that in August 1971 the United States unilaterally terminated convertibility of the US dollar to gold, effectively bringing the Bretton Woods regime to an end. Since in our analysis we abstract from the role of nominal rigidities, we focus the empirical analysis on the post-1971 sample.

where pd, rf, Δc, and ΔI denote the price-dividend ratio, the risk-free rate, the consumption growth rate, and the investment growth rate, respectively. The index i denotes each of the aforementioned G-7 countries, and a rest-of-the-world (henceforth ROW) aggregate that features the G-7 countries, excluding the U.S. We denote this ROW group as the G-6.

We employ three different methods of constructing the ROW aggregate productivity: (i) a GDP weighted average of the G-6 countries' productivity, (ii) an investment weighted average of the G-6 countries' productivity, and (iii) a world Solow residual calculated directly from the aggregated GDP, investment, and labor data of the G-6 countries. Specifically, world productivity in the last construction is calculated as $\frac{GDP}{K^{.36}L^{.64}}$, where GDP is G-6 aggregated output, K is the capital stock computed from G-6 aggregated investment, and L measures the population-weighted average of hours worked per worker in the G-6 countries. We identify short- and long-run shocks by estimating the system of equations (4) in conjunction with the projection restrictions $z_{i,t,j} = \beta_{i,j} F_{j,t}^i$, for all countries i reported above.

6.2   Testable Implications

Response of investment. The model predicts that the difference between home and foreign investment should respond negatively (positively) to home (foreign) long-run news and positively (negatively) to home (foreign) short-run news (figure 3). We test this prediction by regressing investment growth differentials on short-run (ε_a) shock differentials, long-run (ε_x) shock differentials, and lagged long-run risk differentials. We summarize our results in panel A of table 6.

The sets of estimated coefficients labeled *GDP*, *Investments*, and *Solow* refer to the response of U.S. investments relative to the ROW aggregate computed with the three methodologies discussed above. The rows labeled *System* refer to a panel estimation in which the dependent variables in the cross-section are the differentials between the U.S. investment and that of each of the other G-7 countries, and in which all the loadings on the short- and long-run news are restricted to be the same for each country pair. We perform this last estimation exercise in order to gain statistical power from the cross-section of countries.

<center>**TABLE 6: Empirical Analysis**</center>

Panel A: Response of Investments

		Benchmark	pd	pd,rf	pd,rf,dc	pd,rf,di	pd,rf,dc,di
GDP	ε_a	2.529	2.32***	2.37***	2.80***	2.96***	3.08***
			[0.81]	[0.74]	[0.70]	[0.74]	[0.67]
	ε_x	−0.850	−4.95	−3.33	−2.50	−5.17***	−4.89***
			[6.78]	[5.64]	[2.45]	[1.59]	[1.43]
Investments	ε_a	2.529	2.02***	2.16***	2.67***	2.93***	2.92***
			[0.77]	[0.72]	[0.65]	[0.69]	[0.66]
	ε_x	−0.850	−2.94	−2.44	−2.84*	−4.95***	−4.48***
			[6.26]	[4.44]	[1.97]	[1.45]	[1.39]
Solow	ε_a	2.529	2.38***	2.28***	2.57***	2.60***	2.87***
			[0.91]	[0.93]	[0.90]	[1.02]	[0.84]
	ε_x	−0.850	−10.7	−7.61	−3.23	−6.93***	−6.52***
			[10.1]	[10.3]	[3.71]	[1.87]	[1.70]
System	ε_a	2.529	1.23***	1.21***	1.47***	1.43***	1.50***
			[0.04]	[0.04]	[0.03]	[0.03]	[0.04]
	ε_x	−0.850	−0.49**	−1.53***	−1.35***	−4.62***	−3.05***
			[0.28]	[0.28]	[0.16]	[0.10]	[0.12]

Panel B: Response of Net Exports of Investments

		Benchmark	pd	pd,rf	pd,rf,dc	pd,rf,di	pd,rf,dc,di
GDP	ε_a	−0.347	−0.06	−0.00	−0.04	−0.03	−0.04
			[0.19]	[0.01]	[0.17]	[0.14]	[0.15]
	ε_x	0.345	−0.19	−0.12	0.27	0.44	0.18
			[0.46]	[0.36]	[0.48]	[0.38]	[0.27]
Investments	ε_a	−0.347	0.07	0.02	−0.01	−0.00	−0.01
			[0.15]	[0.11]	[0.20]	[0.01]	[0.33]
	ε_x	0.345	−0.47	−0.18	0.22	0.38	0.13
			[0.43]	[0.34]	[0.45]	[0.35]	[0.24]
Solow	ε_a	−0.347	−0.27	−0.04	−0.07	−0.05	−0.07
			[0.24]	[0.15]	[0.19]	[0.17]	[0.19]
	ε_x	0.345	0.49	0.03	0.32	0.54	0.27
			[0.60]	[0.60]	[0.50]	[0.47]	[0.36]
System	ε_a	−0.347	−0.16***	−0.05***	−0.09***	−0.05***	−0.07***
			[0.01]	[0.00]	[0.01]	[0.01]	[0.01]
	ε_x	0.345	0.24***	0.14***	0.21***	0.21***	0.12***
			[0.03]	[0.02]	[0.02]	[0.04]	[0.02]

Panel C: Response of Net Exports of Consumption

		Benchmark	pd	pd,rf	pd,rf,dc	pd,rf,di	pd,rf,dc,di
GDP	ε_a	0.014	0.38	0.23	0.26	0.32	0.27
			[0.35]	[0.31]	[0.32]	[0.31]	[0.30]
	ε_x	0.0002	1.03	1.10*	0.80*	0.96**	0.84**
			[0.96]	[0.85]	[0.55]	[0.55]	[0.42]
Investments	ε_a	0.014	0.39*	0.21	0.23	0.27	0.23
			[0.27]	[0.28]	[0.30]	[0.29]	[0.26]
	ε_x	0.0002	1.06	1.14*	0.79*	0.91**	0.80**
			[0.93]	[0.75]	[0.55]	[0.52]	[0.38]
Solow	ε_a	0.014	0.29	0.25	0.28	0.36	0.30
			[0.42]	[0.36]	[0.36]	[0.38]	[0.36]
	ε_x	0.0002	1.34	1.04	0.77	0.96*	0.89**
			[1.15]	[1.07]	[0.61]	[0.65]	[0.51]
System	ε_a	0.014	0.04***	0.05***	0.09***	0.17***	0.16***
			[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
	ε_x	0.0002	1.76***	1.24***	0.73***	0.64***	0.62***
			[0.10]	[0.08]	[0.04]	[0.04]	[0.04]

Notes: The top panel reports the response of the difference of investments to the difference of short-run shocks (ε_a) and long-run shocks (ε_x). The column labeled *Benchmark* reports the coefficients estimated by simulating the Benchmark version of the model and regressing the difference of investments between the home and the foreign country on the difference of short-run shocks, of long-run shocks, and of the lagged predictive components (not reported in the table). Long-run risks were estimated by regressing Solow residuals on the corresponding set of predictive variables indicated by the column titles. pd, rf, dc, and di correspond to price-dividend, risk-free rate, consumption growth, and investment growth, respectively. The rows labeled *GDP*, *Investments*, and *Solow* refer to the cases in which ROW investments were aggregated by weighting each country respectively by its GDP, Investments, and Solow residuals relative to the other G-6 countries. The rows labeled *System* show the results for the case of the panel estimation in which the dependent variables in the cross-section are the differentials between the investments in the U.S. and each of the other G-7 countries, and those in which all the loadings on the short- and long-run news are restricted to be the same for each country pair. The numbers in brackets underneath each point estimate are heteroskedasticty-adjusted standard errors. One, two, and three asterisks denote 10%, 5%, and 1% significance, respectively, of a one-tailed test that the sign of the corresponding estimated coefficient is different from the sign predicted by the model. Panels B and C repeat the same analysis for the case in which the dependent variables are the *Net Exports of Investments* and *Net Exports of Consumption*, respectively.

Several noteworthy observations emerge from our results. First, the signs and the magnitudes of the estimated coefficients are predominantly in line with the model. This confirms the prediction of our model that investments will respond with oppo-

site signs to short- and long-run innovations. Second, the results are robust to the alternative ways in which we aggregated ROW productivities. Third, as we enrich our set of predictive variables to include both prices and quantities, the statistical significance improves.

Finally, when we focus on the *System* estimation, all coefficients are strongly significant. We interpret this result as confirming that (i) the data line up extremely well with our model, and (ii) in the cases in which our coefficients are not significant the reason for this may be lack of statistical power due to cross-sectional aggregation.

Net export of investment. As shown in figure 7, our model suggests a novel mechanism according to which investment should flow away from countries that receive good long-run news, whereas the BKK model predicts the opposite result. We employ the same methodology described above to investigate the plausibility of this theoretical channel and find that the data confirm our prediction. Countries receiving good long-run productivity news typically experience an outflow of investment (table 6, panel B). Furthermore, the *System* estimation confirms the ability of the panel estimation to compensate for the relatively short time-series: indeed, in this case, all estimated coefficients are strongly statistically significant.

Net export of consumption. Following our discussion in section 5.3, we investigate the way in which net export of consumption responds to short- and long-run news. This is relevant because this dimension provides a sharp contrast between the predictions of our model (that net export of consumption goes up in response to both sources of uncertainty) and the BKK model (that net export of consumption goes up only when a long-run shock materializes). Our results show that the data line up with our model: the signs of the estimated coefficients are always positive and, in the case of the *System* estimation, strongly statistically significant (table 6, Panel C).

7 Concluding Remarks

In this article, we have investigated the effect of long-term productivity risk on international risk-sharing and capital flows in a frictionless BKK economy modified along the following three dimensions. First, we added Epstein and Zin (1989) recursive preferences and long-run growth shocks in the spirit of the recent long-run risk literature on exchange rates (Colacito and Croce 2011). Second, we used a higher home bias in consumption than in investment, as in EGG. Finally, we added heterogenous exposure of capital vintage to aggregate productivity, as in ACL.

The first two modifications enable our model to produce the correct amount of exchange rate volatility and resolve the Backus et al. (1992) quantity anomaly. Furthermore, they allow us to obtain our most important prediction on international capital flows: that under the optimal risk-sharing scheme, good long-run productivity news produces an immediate outflow of investment.

The introduction of heterogenous productivity across capital vintages, as in ACL, enables us to obtain high and volatile capital excess returns with an equity premium close to 4%, thereby setting a new quantitative benchmark in international macrofinance.

Future research should focus on the long-term fiscal and monetary policy implications of our model. It will also be important to take into consideration both private and sovereign credit shocks and market incompleteness. Studying the role of capital flows in the determination of long-term price and shock elasticities (Borovička et al. 2011 and Borovička and Hansen 2011) is relevant as well, especially because this could shed new light on the behavior of long-term currency risk premia (Froot and Ramadorai 2005, Engel 2012, and Evans 2012). Furthermore, our model should be extended to study international capital flows in economies with broader forms of heterogeneity in the spirit of Borovička (2012) and Ready et al. (2012).

References

Ai, H., M. M. Croce, and K. Li. 2012. Toward a Quantitative Genaral Equilibirum Asset Pricing Model with Intangible Capital. *Review of Financial Studies*. Forthcoming.

Alessandria, G., J. P. Kaboski, and V. Midrigan. 2011. Us trade and inventory dynamics. *American Economic Review* 101.

Anderson, E. 2005. The dynamics of risk-sensitive allocations. *Journal of Economic Theory* 125(2):93–150.

Backus, D., and G. Smith. 1993. Consumption and real exchange rates in dynamic exchange economies with nontraded goods. *Journal of International Economics* 35:297–316.

Backus, D., S. Foresi, and C. Telmer. Affine term structur models and the forward premium anomaly. Technical report 2001.

Backus, D. K., P. J. Kehoe, and F. E. Kydland. 1992. International real business cycles. *Journal of Political Economy* 100(4):745–775.

Backus, D. K., P. J. Kehoe, and F. E. Kydland. 1994. Dynamics of the trade balance and the terms of trade: The j-curve? *The American Economic Review* 84(1):pp. 84–103.

Bai, Y., and J. Zhang. 2010. Solving the Feldstein-Horioka Puzzle with Financial Frictions. *Econometrica*.

Bansal, R., and I. Shaliastovich. 2013. A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies* 26(4).

Bansal, R., and A. Yaron. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59:1481–1509.

Bansal, R., D. Kiku, and A. Yaron. 2010. Risks for the long run: Estimation and inference. *Working Paper*.

Barsky, B., and E. Sims. 2011. News Shocks and Business Cycles. *Journal of Monetary Economics* 58(3):273–289.

Baxter, M., and M. J. Crucini. 1995. Business cycles and the asset structure of foreign trade. *International Economic Review* 36.

Borovička, J. 2012. Survival and long-run dynamics with heterogeneous beliefs under recursive preferences. Working Paper, New York University.

Borovička, J., and L. Hansen. 2011. Examining macroeconomic models through the lens of asset pricing. Working Paper, Chicago University.

Borovička, J., L. Hansen, M. Hendricks, and J. Scheinkman. 2011. Risk-price dynamics. *Journal of Financial Econometrics* 9:3–65.

Brandt, W., J. Cochrane, and P. Santa-Clara. 2006. International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics* 53(4):671–698.

Campbell, J. 2003. Consumption-based asset pricing. *forthcoming in Handbook of the Economics of Finance*. George Constantinides, Milton Harris, and Rene Stulz eds., North-Holland, Amsterdam.

Colacito, R. 2008. Six anomalies looking for a model. a consumption based explanation of international finance puzzles. *Working Paper, Department of Finance, University of North Carolina, Chapel Hill NC*.

Colacito, R., and M. M. Croce. 2011. Risks for the Long Run and the Real Exchange Rate. *Journal of Political Economy* 119(1):153–182.

Colacito, R., and M. M. Croce. 2012. International Asset Pricing with Recursive Preferences. *Journal of Finance*. Forthcoming.

Croce, M. M. 2008. Long-Run Productivity Risk: A New Hope For Production-Based Asset Pricing? Working paper, Kenan-Flagler Business School, UNC.

Engel, C. 2012. The Real Exchange Rate, Real Interest Rates, and the Risk Premium. Working Paper, University of Wisconsin.

Epstein, L. G., and S. E. Zin. July 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57(4): 937–69.

Erceg, C., L. Guerrieri, and C. Gust. 2008. Trade Adjustment and the Composition of Trade. *Journal of Economic Dynamics and Control* 32(8).

Evans, M. 2012. Exchange-Rate Dark Matter. Working Paper, Georgetown University.

Froot, K. A., and T. Ramadorai. 2005. Currency Returns, Intrinsic Value, and Institutional-Investor Flows. *Journal of Finance* 60.

Garca-Feijo, L., and R. D. Jorgensen. 2010. Can Operating Leverage Be the Cause of the Value Premium? *Financial Management*.

Greenwood, J., Z. Hercowitz, and G. Huffman. 1988. Investment capacity utilization and the real business cycle. *American Economic Review* 78.

Heathcote, J., and F. Perri. 2004. Financial globalization and real regionalization. *Journal of Economic Theory* 119.

Kehoe, P. J., and F. Perri. 2002. International business cycles with endogenous incomplete markets. *Econometrica* 70:907–928.

Kose, M. A., C. Otrok, and C. H. Whiteman. 2003. International Business Cycles: World, Region, and Country-Specific Factors. *The American Economic Review* 93.

Kose, M. A., C. Otrok, and C. H. Whiteman. 2008. Understanding the evolution of world business cycles. *Journal of International Economics* 75.

Lewis, K. 2011. Global Asset Pricing. *Annual Reviews of Financial Economics*.

Lustig, H., and A. Verdelhan. 2007. The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review* 97:89–117.

Lustig, H., N. Roussanov, and A. Verdelhan. 2011a. Common risk factors in currency markets. *Forthcoming in the Review of Financial Studies*.

Lustig, H., N. Roussanov, and A. Verdelhan. 2011b. Counter-cyclical currency risk premia. *Working paper, Wharton*.

Petrosky-Nadeau, N. 2011. Endogenous flows of foreign direct investment and international real business cycles. Working paper, CMU.

Raffo, A. 2008. Net exports,consumption volatility and international business cycle models. *Journal of International Economics*.

Ready, R., N. Roussanov, and C. Ward. 2012. Commodity Trade and the Carry Trade: A Tale of Two Countries. Working Paper, the Wharton School.

Stockman, A. C., and L. L. Tesar. 1995. Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *The American Economic Review* 85.

Tretvoll, H. 2012. Real exchange rate variability in a two-country cycle model. Working paper, NYU.

Appendix A: Data Sources

Consumption, investment, exports, and import data are from the OECD and are PPP adjusted in 2005 U.S. dollars. Labor data are from Raffo (2008). Net exports of investment data are from BEA's NIPA tables 1.1.5 and 4.2.5. All quantities are deflated using the GDP deflator from NIPA table 1.1.9. The risk-free rate is calculated as the nominal risk-free rate minus the inflation rate, where both rates are obtained from the IMF (for the United Kingdom, the retail index is used to calculate inflation). Germany's and Italy's risk-free rate series calculated by the IMF begin in 1975 and 1976, respectively. To extend the data back to 1971, risk-free interest rates are obtained from Campbell (2003). The price-dividend ratio for the U.S. comes from Colacito and Croce (2011), and price-dividend ratios for the rest of the G-7 are calculated using Ken French's cum- and ex-dividend country value-weighted dollar index returns (using "All 4 Data Items Not Reqd" series). French's data begin in 1977; to extend data to 1971, price-divdend ratios from Campbell (2003) are included for 1971–1976.

Appendix B: Pareto Problem

For the sake of brevity, in this appendix we suppress notation to denote state and histories and retain only subscripts for time. We represent the Epstein and Zin (1989) utility preference in the following compact way:

$$U_t = W(\widetilde{C}_t, U_{t+1}),$$

so that the dependence of current utility on $j-$step ahead consumption can easily be denoted as follows:

$$\frac{\partial U_t}{\partial \widetilde{C}_{t+j}} = W_{2,t+1} \cdot W_{2,t+2} \cdots W_{2,t+j} W_{1,t+j}, \tag{B1}$$

where $W_{2,t+j} \equiv \frac{\partial U_{t+j-1}}{\partial U_{t+j}}$ and $W_{1,t+j} \equiv \frac{\partial U_{t+j}}{\partial \widetilde{C}_{t+j}}$. Given this notation, the intertemporal marginal rate of substitution between $\widetilde{C}_t$ and $\widetilde{C}_{t+1}$ is

$$IMRS_{\widetilde{C},t+1} = \frac{W_{2,t+1} W_{1,t+1}}{W_{1,t}} = M_{t+1}\pi_{t+1}, \tag{B2}$$

where M_{t+1} is the stochastic discount factor in $\widetilde{C}$ units and it has the following form:

$$M_{t+1} = \beta \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.$$

The consumption bundle, $\widetilde{C}$, depends on both the consumption aggregate, C, and labor, N:

$$\widetilde{C}_t = \widetilde{C}(C_t, N_t).$$

The consumption aggregate combines together two goods, x and y:

$$C_t = C(x_t, y_t).$$

The planner faces the following constraints:

$$F(A_t, K_t, N_t) \geq x_t + x_t^* + I_{x,t} + I_{y,t} \tag{B3}$$

$$F(A_t^*, K_t^*, N_t^*) \geq y_t + y_t^* + I_{x,t}^* + I_{y,t}^* \tag{B4}$$

$$K_t \leq (1-\delta)K_{t-1} + e^{\omega_t}G(I_{x,t-1}, I_{x,t-1}^*) \tag{B5}$$

$$K_t^* \leq (1-\delta)K_{t-1}^* + e^{\omega_t^*}G^*(I_{y,t-1}, I_{y,t-1}^*), \tag{B6}$$

where A_t and A_t^* are the exogenous stochastic productivity processes in equation (4).

The social planner chooses $\left\{x_t, x_t^*, y_t, y_t^*, N_t, N_t^*, K_t, K_t^*, I_{x,t}, I_{y,t}, I_{x,t}^*, I_{y,t}^*\right\}_t$ to maximize

$$\mu_0 W_0 + (1-\mu_0)W_0^*,$$

subject to sequences of constraints (B3)–(B6). Specifically, let $\lambda_{i,t}$ be the Lagrangian multiplier for the time t constraint (Bi); then the Lagrangian is

$$
\begin{aligned}
\Omega = &\mu_0 W_0 + (1-\mu_0)W_0^* \\
&+ \dots \\
&+ \lambda_{1,t}(F(A_t, K_t, N_t) - x_t - x_t^* - I_{x,t} - I_{y,t}) \\
&+ \lambda_{2,t}(F(A_t^*, K_t^*, N_t^*) - y_t - y_t^* - I_{x,t}^* - I_{y,t}^*) \\
&+ \lambda_{3,t}((1-\delta)K_{t-1} + \omega_t G(I_{x,t-1}, I_{x,t-1}^*) - K_t) \\
&+ \lambda_{4,t}((1-\delta)K_{t-1}^* + \omega_t^* G^*(I_{y,t-1}, I_{y,t-1}^*) - K_t^*) \\
&+ \dots
\end{aligned}
$$

The optimality condition for the allocation of good X_t for $t = 1, 2, \dots$ in each possible state is

$$\mu_0 \cdot \left(\prod_{j=1}^{t} W_{2,j}\right) \cdot W_{1,t}\widetilde{C}_{C,t}C_{x,t} = \lambda_{1,t} = C_{x^*,t}^*\widetilde{C}_{C^*,t}^*W_{1,t}^* \cdot \left(\prod_{j=1}^{t} W_{2,j}^*\right) \cdot \mu_0^*, \tag{B7}$$

where $\mu_0^* = (1 - \mu_0)$, $\widetilde{C}_{C,t} = \partial \widetilde{C}_t / \partial C_t$, $C_{x,t} = \partial C_t / \partial x_t$, and the analogous partial derivatives for the foreign country are denoted by *.

Define μ_t as the date t Pareto weight for the home country. Using equation (B2), we get

$$
\begin{aligned}
\mu_t &= \mu_0 \cdot \left(\prod_{j=1}^{t} W_{2,j} \right) \cdot W_{1,t} C_t \\
&= \mu_{t-1} \cdot W_{2,t}^i \cdot \frac{W_{1,t}}{W_{1,t-1}} \cdot \frac{C_t}{C_{t-1}} = \mu_{t-1} \cdot M_t \cdot \frac{C_t}{C_{t-1}}.
\end{aligned}
$$

It follows that equation (B7) can be rewritten as

$$
\mu_t \cdot \widetilde{C}_{C,t} C_{x,t} \frac{1}{C_t} = \frac{1}{C_t^*} C_{x^*,t}^* \widetilde{C}_{C^*,t}^* \cdot \mu_t^* \tag{B8}
$$

Let $S_t \equiv \mu_t / \mu_t^*$, and note that with GHH preferences, $\widetilde{C}_{C,t} = 1$. Then the optimality condition in equation (B8) can be represented by the following system of recursive equations:

$$
\begin{aligned}
S_t \cdot C_{x,t} \cdot \frac{1}{C_t} &= C_{x^*,t}^* \cdot \frac{1}{C_t^*} \\
S_t &= S_{t-1} \frac{M_t e^{\Delta c_t}}{M_t^* e^{\Delta c_t^*}}.
\end{aligned} \tag{B9}
$$

In a similar fashion, the optimal allocation of good Y is determined by

$$
S_t \cdot C_{y,t} \cdot \frac{1}{C_t} = C_{y^*,t}^* \cdot \frac{1}{C_t^*}.
$$

Given our GHH preferences, the optimal allocation of labor implies the following standard intratemporal conditions:

$$
\begin{aligned}
\widetilde{C}_{N,t} &= -F_{N,t} \\
\widetilde{C}_{N^*,t}^* &= -F_{N^*,t}^*,
\end{aligned}
$$

where $\widetilde{C}_{N,t} = \partial \widetilde{C}_t / \partial N_t$ and $F_{N,t} = \partial F_t / \partial N_t$.

Let s_{t+1} index the possible states at time $t+1$. The first-order condition with respect to $I_{x,t}$ is

$$-\lambda_{1t} + \sum_{s_{t+1}}(\lambda_{3,t+1}e^{\omega_{t+1}}G_{I_x,t}) = 0$$

$$\Leftrightarrow \sum_{s_{t+1}}\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\frac{\lambda_{3,t+1}}{\lambda_{1,t+1}}\omega_{t+1}^h\right) = \frac{1}{G_{I_x,t}}.$$

By definition, $IMRS_{t+1|t}^x = \frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\partial U_0/\partial x_{t+1}}{\partial U_0/\partial x_t} = M_{t+1}^x\pi_{t+1|t}$ for $i \in \{h, f\}$, where M_{t+1}^x is the stochastic discount factor in X-units. Substituting the stochastic discount factor into the above equation, we have

$$\frac{1}{G_{I_x,t}} = E_t[M_{t+1}^x P_{k,t+1}e^{\omega_{t+1}}], \tag{B10}$$

where $G_{I_x,t} \equiv \frac{\partial G(I_{x,t}, I_{x,t}^*)}{\partial I_{x,t}}$, and $P_{k,t+1} \equiv \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}}$ is the cum-dividend price of capital in X-units. The optimal accumulation of K_t has to satisfy

$$-\lambda_{3,t} + \lambda_{1,t}F_{k,t} + \sum_{s_{t+1}}((1-\delta)\lambda_{3,t+1}) = 0$$

$$\Leftrightarrow E_t[M_{t+1}^x(1-\delta)P_{k,t+1}] + F_{k,t} = P_{k,t},$$

where $F_{k,t} \equiv \frac{\partial F}{\partial k_t}$. Define $Q_{k,t} \equiv E_t[M_{t+1}^x P_{k,t+1}]$ as the ex-dividend price of capital. Then we have

$$P_{k,t} = F_{k,t} + (1-\delta)Q_{k,t}$$

$$Q_{k,t} = E_t[M_{t+1}^x P_{k,t+1}]$$

and

$$R_{k,t+1} = \frac{P_{k,t+1}}{Q_{k,t}}.$$

The first-order condition with respect to $I_{y,t}$ states the following:

$$-\lambda_{1,t} + \sum_{s_{t+1}}\left(\lambda_{4,t+1}e^{\omega_{t+1}^*}G_{I_y,t}^*\right) = 0$$

$$\Leftrightarrow \sum_{s_{t+1}}\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\frac{\lambda_{4,t+1}}{\lambda_{2,t+1}}\frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}e^{\omega_{t+1}^*}\right) = \frac{1}{G_{I_y,t}^*},$$

where $G_{I_y,t}^* \equiv \frac{\partial G_t^*}{\partial I_{y,t}}$. Similarly to what done for the home country, define $P_{k,t+1}^* \equiv \frac{\lambda_{4,t+1}}{\lambda_{2,t+1}}$ as the cum-dividend price of capital in Y-units and note that $P_{t+1} = \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}$ measures the terms of

trade. It is then possible to obtain that

$$\frac{1}{G^*_{I_y,t}} = E_t[M^x_{t+1}P^*_{k,t+1}P_{t+1}e^{\omega^*_{t+1}}].$$ (B11)

Define $M^y_{t+1} \equiv \frac{\lambda_{2,t+1}}{\lambda_{2,t}}$ as the SDF in Y-units. The remaining first-order conditions imply

$$\frac{1}{G^*_{I^*_y,t}} = E_t[M^y_{t+1}P^*_{k,t+1}e^{\omega^*_{t+1}}]$$ (B12)

$$P^*_{k,t} = F^*_{k,t} + (1-\delta)Q^*_{k,t}$$

$$R^*_{k,t+1} = \frac{P^*_{k,t+1}}{Q^*_{k,t}}$$

$$Q^*_{k,t} = E_t[M^y_{t+1}P^*_{k,t+1}]$$

$$\frac{1}{G_{I^*_x,t}} = E_t\left[M^y_{t+1}P_{k,t+1}\frac{1}{P_t}e^{\omega_{t+1}}\right].$$

We use perturbation methods to solve our system of equations. We compute our policy functions using the `dynare++4.2.1` package. All variables included our dynare++ code are expressed in log-units.

Appendix C: Consumption Response to Long-Run News

Under the recursive risk-sharing mechanism studied by Colacito and Croce (2012), positive long-run news gives an incentive to the home country to export resources to the rest of the world in exchange for a reduction in wealth volatility. In Colacito and Croce's exchange economy, an increase in exports is feasible if and only if consumption growth slows down.

In our Benchmark economy, however, the connection between consumption growth and net exports becomes non-monotonic. In particular, consumption growth increases for small positive country-specific long-run news, and it declines for big news. This non-monotonicity is relevant because it reconciles our international-recursive approach with the empirical evidence in the macroeconomic news literature.

Specifically, as shown in figure 3, upon the realization of small positive country-specific long-run news, the home country finds it convenient to export more by reducing investment, as opposed to reducing consumption. In our setting, therefore, home consumption tends to in-
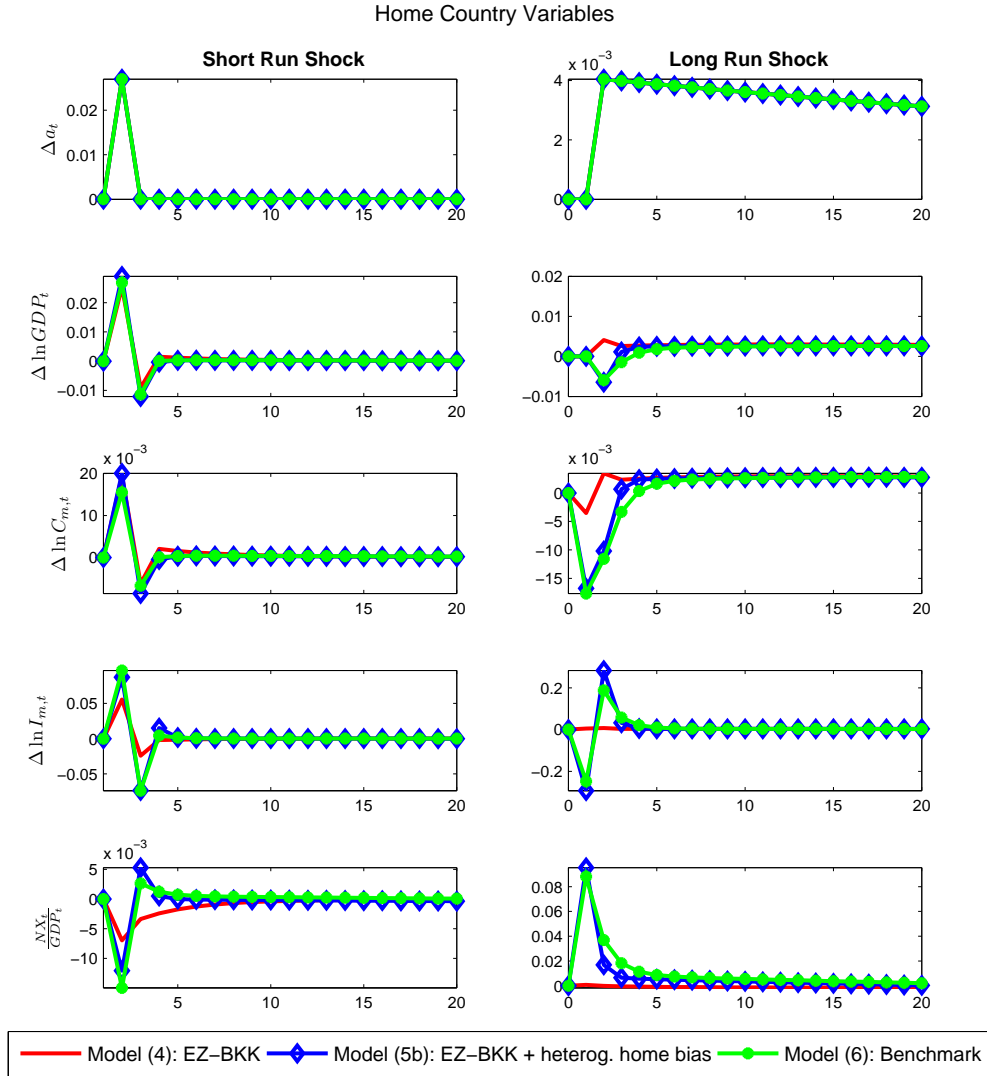
Home Country Variables

FIG. 7. **Consumption response to sizeable long-run news.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. Shocks to the home country productivity, ϵ_a and ϵ_x, materialize at time 2. Both short-run and long-run shocks affect only the home country, with magnitudes σ and σ_x, respectively.

crease upon the arrival of good news for the long run, consistent with the empirical evidence proposed in the macroeconomic news literature (see, among others, Barsky and Sims 2011).

In figure 7, we plot the response of quantities to a positive one-standard-deviation long-run shock to home productivity, assuming that the foreign country receives no news. Equivalently,

this means that the difference between figure 3 and figure 7 is that in one case long-run shocks are not orthogonalized (figure 3), while in the other case they are (figure 7). This case corresponds to the realization of sizeable country-specific news, and it implies a drop in consumption growth, as in Colacito and Croce (2012).

The intuition for this result is the following. On the one hand, because of the milder home bias in investment, the home country finds it initially optimal to export investment goods. On the other hand, this outflow comes at the cost of undermining capital accumulation and hence increasing the shadow cost of consumption. For this reason, upon the realization of sizeable country-specific long-run shocks, the agent finds it optimal to reduce consumption and bear a less severe drop in investment.