Dynamic Costly Disclosure*

very preliminary and incomplete.

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Abstract

This paper studies disclosure dynamics and its implications for stock returns. Because disclosure is costly, the firm may withhold information for some time even when information is favorable. In equilibrium, the firm adopts a regular time-pattern of disclosure. Breaking this regularity, by failing to issue a disclosure when expected, leads to a sharp drop in the stock price and to a period of relatively low asymmetry of information.

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1 Introduction

Managers often disclose private information in a voluntary fashion. These discretionary disclosures constitute a significant source of information to capital markets whose relative importance has been growing over time.[1] Yet, not all the managers' private information is disclosed in a timely fashion. Since the seminal contributions of Grossman (1981); Milgrom (1981) the literature has recognized that, on the one hand, managers may strategically withhold private information that is likely to have negative price consequences and, on the other hand, that markets understand the strategic behavior of managers thus penalizing their silence.

This disclosure game between managers and markets is in essence dynamic. In reality, a manager who wishes to maximize his firm's stock price must select not just whether to disclose information but also when to do it and how often. After all, the manager's private information sooner or later will become public information even if he withholds it forever.

Unfortunately, very little is known about the dynamics of disclosure and it is fair to say that, in this area, measurement is ahead of theory. While a vast number of empirical papers study disclosure dynamics and its implications for the time-series of stock returns, (see e.g., Kothari, Shu and Wysocki (2009)) disclosure theories are for the most part static.

This paper follows the lead of Acharya, DeMarzo and Kremer (2011) in an attempt to fill in this gap. As Acharya, DeMarzo and Kremer (2011) we suppose that the firm's manager maximizes the present value of the firm's future stock prices, perhaps because his compensation at each point in time is proportional to the market value of the firm. The evolution of asset values is described by a continuous time Markov chain that fluctuates between two possible states: low asset value and high asset value. The distribution of asset values is known but the manager privately observes the evolution of actual values, namely he only can tell whether the asset has experienced a temporary impairment or, on the contrary, has recovered from one. Yet, the manager can disclose his private information to the market at any point in time and as many time as he so wishes. Unraveling is however not possible in equilibrium because –as in Jovanovic (1982); Verrecchia (1983)– disclosing information (specially, good news) is costly.

The baseline model generates the following dynamics. At the beginning of the game, or for that matter after any disclosure, there is a *blackout period* where no disclosure is made. During that period, stock prices experience a downward drift (driven by the possibility of

[1] Earnings guidance explains a large portion of the variation in stock returns (Ball and Shivakumar 2008; Beyer, Cohen, Lys, and Walther 2009). More than 15 percent of the variation in quarterly stock returns occurs around guidance announcements, compared to less than three percent for earnings announcements and about six percent for analyst forecasts (Beyer, Cohen, Lys and Walther 2009).

an undisclosed impairment) up to a point where the undervaluation of the asset could be so severe that a disclosure becomes profitable. At this point, the manager discloses his private information if favorable, in which case the price jumps upwards and the game restarts. If, on the contrary, the manager decides to withhold his information, the market infers the asset must have low value and the stock price experiences a drastic drop. From that point onwards the stock price stays flat (at the lowest level) until the asset recovers its value and the manager discloses good news.

The length of the blackout period is affected both by the cost of disclosure and, more importantly, by the time-series properties of asset values, specially the cash flows' mean reversion. A higher mean reversion means that information becomes more transitory. Consequently, the stock price drifts faster toward its long term value. That, in turn, gives the manager an incentive to accelerate his disclosure so as to mitigate the undervaluation the asset experiences in the hight state. This effect would seem to strengthen the incentives of the manager to disclose good news. However, a higher mean reversion, also means the price effect of disclosing good news will be shorter-lived which weakens the incentives for such disclosures. The interaction between these two effects results in the length of the blackout period being non-monotonic in the cash flows' mean reversion.

We then consider how the presence of a public news process, correlated with asset values but observed at random times, affects the manager's disclosure incentives. Specifically, we model the public news as a Poisson process whose arrival intensity depends on the value of assets. If arrivals are more likely in the bad state, a news arrival conveys bad news. The arrival thus triggers a price drop whose magnitude depends on the information quality of the news. Conversely, the absence of arrivals mitigates the price's downward drift –relative the case without public news– because the absence of arrivals is perceived by the market as good news. At first, this suggests that the presence of the public news process should moderate the propensity of the manager to disclose his information. But, public news also have an opposing effect: the observation of a news arrival by the market induces a drastic price drop which naturally stimulates the manager to disclose good news as soon as the asset recovers its value. We show however that the former effect dominates, so that the higher the frequency of news arrivals the lower is the frequency of managerial disclosures. In this setting, public news substitute managerial disclosures.

In the previous model, the manager may only disclose good news; bad news are eventually observed but only from the public news process. In the real world, however, bad news disclosures are prevalent (see e.g., Kothari, Shu and Wysocki (2009)) perhaps because the realization that the manager withheld adverse information has important legal implications (see e.g., Skinner (1997)). To capture this feature of the disclosure environment, we consider the possibility that a news arrival may give rise to litigation costs when the manager fails to disclose bad news and the news reveal the asset was overpriced. The presence of litigation risks, one might think, should stimulate the manager's bad news disclosures as a means to preempt the litigation costs: the manager should sometimes reveal bad news, specially when the stock price is relatively low. Yet, this idea presents a conundrum: if the manager revealed bad news with probability one, at any given stock price, then the absence of such disclosures would be perceived as perfect evidence the asset value is high, thus inducing a sharp increase in the stock price. This jump in the stock price would destroy the manager's incentives to disclose bad news in the first place. To overcome this conundrum, the equilibrium must entail disclosure randomization. When prices are sufficiently low, the manager randomizes between disclosing and not disclosing the bad news. At that point, the price remains constant over time, up until the bad news are either disclosed by the manager or revealed by the public news.

The manager's decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut-down (i.e., disclose bad news) or restart a project (i.e., disclose good news), based on the project's observed profitability.[2] In our setting, when the stock price is low and the value of the asset is also low, disclosing bad news becomes profitable for the same reason shutting down a project that is making losses is optimal in Dixit's model. Also, as in Dixit's model, the decision to disclose bad news today is inherently liked with the value of the option to disclose good news in the future: if the cost of disclosing good news is higher, then the benefit from disclosing bad news today goes down, which naturally delays such disclosures. This speaks to a certain complementarity between bad and good news disclosures in the presence of legal liability.

In the presence of legal liability, the public news process no longer substitutes managerial disclosures, but actually complements them. This is natural: a higher frequency of public news means that the expected litigation cost from withholding information goes up. Managers feel therefore more compelled to reveal bad news, since they are subject to a tighter public scrutiny. They thus accelerate the release of bad news.

Our model is simple but flexible enough to extend to areas currently unexplored by the disclosure literature. Indeed, the disclosure literature focuses mostly on financial disclosures taking place in capital markets. There are, however, other settings where disclosures have perhaps even greater relevance. Think, for example, of pharmaceutical firms' disclosures about the health risks of their drugs.[3] These drugs often enjoy monopoly rents while their

[2]But in our setting the profitability of the project is endogenous because it is determined by the Bayesian beliefs of the market about the asset value.

[3]An example is the case of Avandia, GlaxoSmithKline's main diabetes drug. The firm could face up to \$6

patents are in vigor, but the existence of these rents critically relies on there not being evidence the drug poses health risks to its users, as this evidence could trigger a commercialization ban. On the other hand, pharmaceutical companies constantly receive information from clinical trials in regards to the side effects of of their drugs. Upon receiving bad news, these companies may face the temptation to "time" their disclosures based on the length of the drug's patent, so as to prevent the value of the patent from sinking before expiry. In this context, we show that bad news tend to be delayed and that they cluster around the expiration of the patent. Furthermore, we show that longer patents exacerbate the tendency of the manager to delay the bad news' disclosures. This result is important from a normative point of view because it suggests that a patent design system that ignores manager's disclosure incentives may result in excessively long patents which delay disclosures whose timeliness have large social impact. In the same vein, we show that for assets where post-patent competition is expected to be weaker, the frequency and timeliness of disclosure will also be lower. The length of the patent and the extent of post-patent competition are thus two regulatory tools that can be used to stimulate disclosures.

1.1 Related Literature

This paper extends Jovanovic (1982) and Verrecchia (1983, 1990) to a continuous time setting. The most closely related paper is Acharya, DeMarzo and Kremer (2011). They consider a dynamic version of Dye (1985) where the manager may be privately informed about the asset value. When informed, the manager may disclose his private information at one of two points in time: at the start of the game or right after a public news signal is released, at a known date. If the manager's private information is not so favorable, waiting for news has positive option value since the public signal might induce a higher price in the absence of disclosure than in the presence of it. By contrast, if the public signal turned out to be unfavorable, the manager could mitigate the negative price effect of the public signal by dis-

billion in liability. According to the U.S. Food and Drug Administration (FDA), Avandia is linked to as many as 100,000 heart attacks. Clinical studies show that the drug increases the risk of heart attack by 43 percent and can double the risk of heart failure after one year of treatment. Despite these findings, Avandia's black-box warning label did not mention an increased risk of cardiovascular death until the FDA warned about the risks in 2007. A two-year investigation by the U.S. Senate Finance Committee revealed GlaxoSmithKline knew of the cardiovascular dangers associated with Avandia for years and tried to stifle concerns noted by several doctors about the medication. In July 2012, the company pleaded guilty to federal charges that it failed to report clinical data on Avandia. GlaxoSmithKline reached an agreement with the Department of Justice to pay a $3 billion settlement. The agreement, is the largest health care fraud settlement in U.S. history. GlaxoSmithKline has set aside funds in anticipation of the growing number of Avandia lawsuits. The company set aside $6.4 billion to pay for Avandia litigation and settlement costs. Avandia has been on the market since 1999. The patent for Avandia expired in March 2012.

closing his own private information. Their model is able to explain clustering of disclosure in bad times: the less favorable the public signal the higher the probability of disclosure.

Dye (2010) also studies the timing of disclosure but in a different setting. A risk averse manager must sell his shares among a number of risk neutral investors during several trading periods following a fixed trading profile. Also, in each period the manager must acquire and disclose a signal about the asset value. Ex-ante, the manager is allowed to choose the precision profile of the signals he will be releasing, but the the sum of their precisions is fixed. So the manager's choice regards the timing of disclosure: namely how much precision to allocate to each period. In equilibrium, the manager engages in *disclosure bunching*, namely he allocates all the precision to a single period instead of spreading the precision over time. This bang-bang solution is driven by optimal risk sharing between the manager and other traders: when the manager is too risk averse relative to other traders, very informative disclosures impose excessive risk on the manager's wealth, so the manager tends to delay them until a sufficiently high portion of his portfolio has already been off-loaded.

Beyer and Dye (2012) study a reputation model in which the manager may learn a single private signal in each of the two periods. The manager can be either "forthcoming' and disclose any information he learns or he may be "strategic.' At the end of each period, the firm's signal/cash flow for the period becomes public and the market updates beliefs about the value of the firm and the type of the agent.

Finally, Kremer, Guttman and Skrypacz (2012) consider the price consequences of the choice of disclosure timing. They study a two-period extension of Dye (1985) model, where in each period, the manager may observe any of two pieces of information (if previously unobserved) with some probability. They show that later disclosures are interpreted more favorably by the market because the probability that the manager is hiding information is perceived to be higher when partial disclosures are made earlier.

2 Baseline Model

We study a dynamic model of voluntary disclosure that extends Jovanovic (1982); Verrecchia (1983, 1990). The value of assets V_t follows a continuous time Markov chain with state space $\{0, 1\}$ and $V_0 = 1$. The value of the asset jumps from 0 to 1 with intensity λ_1 while it jumps back from 1 to 0 with intensity λ_0. We can think of λ_0 as the intensity with which the asset suffers an impairment. When $\lambda_1 = 0$, the impairment is permanent.

At the beginning of the game, the asset value is known to be 1. From that point onwards, the manager privately observes any shock to the asset value. However, at any point in time, the manager can disclose the value of the asset at a cost c. As in Jovanovic (1982); Verrecchia

(1983), this cost may arise from proprietary costs, the need to certify the asset (by for example hiring an auditor to credibly convey its value to the market), or simply from the opportunity cost of the time the manager employs to prepare and present the information.[4]

We assume risk neutral pricing. So if $d_t \in \{0, \emptyset, 1\}$ denotes the disclosure decision at time t and σ the disclosure strategy then the price of the stock given the history of disclosures $\mathcal{F}_t^d$ is set as

$$p_t = E^\sigma(V_t|\mathcal{F}_t^d) \tag{1}$$

where $E^\sigma(\cdot)$ denotes the expectation operator based on the measure induced by the disclosure strategy σ. Following Acharya, DeMarzo and Kremer (2011) and Benmelech, Kandel and Veronesi (2010), we assume the manager chooses a disclosure strategy that maximizes the present value of future prices net of certification expenses:

$$U_t(\sigma) := E^\sigma\left[\int_t^\infty e^{-r(s-t)} p_s ds - c\sum_{s\geq t} e^{-r(s-t)}\middle|\mathcal{F}_t^d, V_t\right] \tag{2}$$

This objective function is somewhat non-standard and thus requires some discussion. Why would the manager be concerned about the present value of his firm's future stock price? On the one hand there is strong empirical support for this idea.[5] But at a more conceptual level, the manager's concern for his firm's stock price may arise from his compensation or reputation being linked to the stock price of his firm.

Definition 1. An equilibrium is a disclosure strategy $\sigma = \{d_t\}_{t\geq 0}$ and price process $p = \{p_t\}_{t\geq 0}$ such that

1. For all t, the market price is $p_t = E^\sigma(V_t|\mathcal{F}_t^d)$
2. For all t, the disclosure strategy maximizes the manager utility, that is $\sigma \in \arg\max_{\hat{\sigma}} U_t(\hat{\sigma})$

Both conditions are standard. At every point in time, the price is set according to Bayes' rule, given the manager's strategy and the history of the game. The manager's disclosure strategy σ maximizes his payoff given the asset value, the equilibrium price function, and the history of the game.

[4] A number of large investors such as Warren Buffett (1996) and analysts such as Candace Browning (2006), head of global research at Merrill Lynch, have called for managers to give up quarterly earnings guidance and hence avoid the myopic managerial behavior caused by attempts to meet market expectations.

[5] For example, Graham, Harvey and Rajgopal (2005) note, in their famous survey, that because of the severe market reaction to missing an earnings target, firms are willing to sacrifice economic value in order to meet a short-run earnings target. They find that managers make voluntary disclosures to reduce information risk associated with their stock but try to avoid setting a disclosure precedent that will be difficult to maintain.

We consider equilibria with disclosure strategies σ characterized by a disclosure threshold p_* such that

$$d_t = \mathbf{1}_{\{p_{t-} \leq p_*\}} V_t.$$

That is, the seller discloses at time t if and only if the price is lower than or equal to p_* and the value of assets is high. So, as long as p_t remains above p^*, the market expects the manager won't disclose the asset value. Given this strategy, and using standard results, we get that when $p_t > p_*$ the price must evolve according to

$$dp_t = \kappa(\bar{p} - p_t)dt \tag{3}$$

where

$$\bar{p} := \frac{\lambda_1}{\lambda_0 + \lambda_1}$$

is the stationary probability that the value of the asset is 1 and $\kappa := \lambda_0 + \lambda_1$ represents the cash flow's mean reversion, which determines the speed at which the asset converges to its stationary point $\overline{p}$ in the absence of disclosure. Let

$$\phi_t(p) = \bar{p} + e^{-\kappa t} (p - \bar{p}).$$

be the solution to equation (3) given an initial condition $p_0 = p$. In a no disclosure equilibrium, namely when the manager never discloses the asset value, the price at time t is given by $p_t = \phi_t(p_0) = \phi_t(1)$.[6] Accordingly, in a no disclosure equilibrium the manager's payoff is given by

$$U^{ND}(p_0) = \int_0^\infty e^{-rt}\phi_t(p)dt = \frac{\bar{p}}{r} + \frac{p_0 - \bar{p}}{r+\kappa}.$$

By contrast, for $p_t \leq p_*$, we have that $p_t = d_t$. That is, if the manager does not disclose his information when the price hits the threshold p_*, then the market infers that the asset must be of low value. As a consequence, the price falls sharply from p_* to zero and remains there until the manager discloses good news (i.e., $V_t = 1$) which happens immediately after the asset value returns to the high state.

Note that the failure to disclose at $p_t = p_*$ is followed by a period where (i) the price remains flat up until a disclosure is observed and (ii) the information becomes symmetric. By contrast, the period following a disclosure is characterized by the price (mean) reverting towards its stationary level p_*, and by the manager being privately informed about the asset value.

[6] The market uncertainty about the asset value, as measured by $p_t(1-p_t)$, peaks when $p_t = \max(\frac{1}{2}, \bar{p})$.

The dynamics of market beliefs are noteworthy: at the beginning of the game, market beliefs drift downward until a point where disclosure has positive probability. At that point, the price jumps upward if high value is disclosed or downwards if no disclosure is observed. Kothari, Shu and Wysocki (2009) empirically document a similar pattern. They find evidence consistent with the view that managers withhold bad news to investors and that prices tend to drift downward absent disclosure, and jump upward upon the announcement of good news.

For the price behavior described above to be consistent with the manager's equilibrium strategy, it must be optimal for the manager to actually withhold information if either the price is above the threshold p_* or if the asset value is low, and to disclose the information otherwise.

With some abuse of notation, let $U_\theta(p)$ be the manager profits given the market beliefs are p and an the asset value is $\theta \in \{0,1\}$. The manager's payoff in equilibrium can be represented by the Hamilton-Jacobi-Bellman (HJB) equation:

$$rU_\theta(p) = p + \frac{1}{dt}\mathbb{E}\left[dU_\theta\right].$$

when $p > p_*$. On the other hand,

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}\left[dU_1\right] &= \lambda_0[U_0\left(p\right) - U_1\left(p\right)] + U_1'\left(p\right)\frac{dp}{dt} \\
\frac{1}{dt}\mathbb{E}\left[dU_0\right] &= \lambda_1[U_1\left(p\right) - U_0\left(p\right)] + U_0'\left(p\right)\frac{dp}{dt}
\end{aligned}$$

The interpretation of the value function is standard; we can think of the manager's job as an asset, whose cost of capital in a competitive market $rU_\theta\left(p\right)$ must equal the rate of return on the asset, as given by its instantaneous flow p, and its capital gains $\frac{1}{dt}\mathbb{E}\left[dU_\theta\right]$. The latter may come in two forms: the deterministic evolution of investors' beliefs, as described by (3), and the possibility the asset experiences an impairment.

We obtain the following HJB equations:

$$rU_1(p) = p + \kappa(\bar{p} - p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{4}$$

$$rU_0(p) = p + \kappa(\bar{p} - p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] \tag{5}$$

with boundary conditions

$$U_1(p_*) = U_1(1) - c \tag{6}$$

$$U_0(p_*) = \frac{\lambda_1}{r+\lambda_1}[U_1(1) - c]. \tag{7}$$

Moreover the following parametric restrictions are required for an equilibrium where the probability of disclosure is positive:

$$\begin{aligned} U_1(1) - c &\geq 0 \\ U_1(p) \geq U_1(1) - c & \text{ for } \; p > p_* \\ U_1(p) \leq U_1(1) - c & \text{ for } \; p \leq p_*. \end{aligned}$$

In essence, the manager must solve an optimal stopping problem where the stopping time must be consistent with the market's rational expectations.

The following proposition provides the solution in closed form.

Proposition 1. *Suppose that $p_* \in (\bar{p}, 1)$ satisfies*

$$U_1(1) - c \geq 0 \tag{8}$$

and

$$U_1'(p_*) \geq 0 \tag{9}$$

Then, there exists an equilibrium with threshold p_*. The manager's payoff is given by

$$U_0(p) = U_1(p) - \frac{r}{r+\lambda_1}\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{1+\frac{r}{\kappa}}\Big(U_1(1) - c\Big) \tag{10}$$

$$U_1(p) = \int_0^{T(p)} e^{-rt}\phi_t(p)dt + \delta(p)\Big(U_1(1) - c\Big), \tag{11}$$

where

$$U_1(1) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c$$

and

$$\delta(p) := \left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{\frac{r}{\kappa}}\left[\frac{r\bar{p} + \kappa\bar{p}}{r + \kappa\bar{p}} + \frac{r(1-\bar{p})}{r+\kappa\bar{p}}\frac{p_* - \bar{p}}{p - \bar{p}}\right],$$

$$T(p) = -\frac{1}{\kappa}\log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right).$$

It is instructive to consider the manager's payoff at the start of the game, namely when the market beliefs are $p = 1$. Let's define

$$\mathcal{C}(c) := \frac{\delta(1)}{1-\delta(1)}c.$$

Hence, the manager's payoff at time zero is given by

$$U_1(1) = U^{ND}(1) - \mathcal{C}(c).$$

The first component, $U^{ND}(1)$, is the payoff the manager would obtain had he been able to commit to never disclose.[7] The second component $\mathcal{C}(c)$ is the present value of the disclosure expense the manager expects to bear over his lifetime, given his lack of commitment (As a mirror image, one can think of this term as the profits of a certifier who, at the outset, commits to sell his certification services for a fee c). The manager's payoff is thus bounded by the no disclosure payoff $U^{ND}(1)$. This is natural: in our setting information has no (social) value, hence the disclosure expense is a deadweight loss, which the manager bears ex-post only because he cannot avoid disclosing asset values when market beliefs are severely depressed. But ex-ante, the average trajectory of future prices is not affected by the manager's disclosure policy: although in equilibrium the event of disclosure drives the price up, the failure to disclose drives it down.

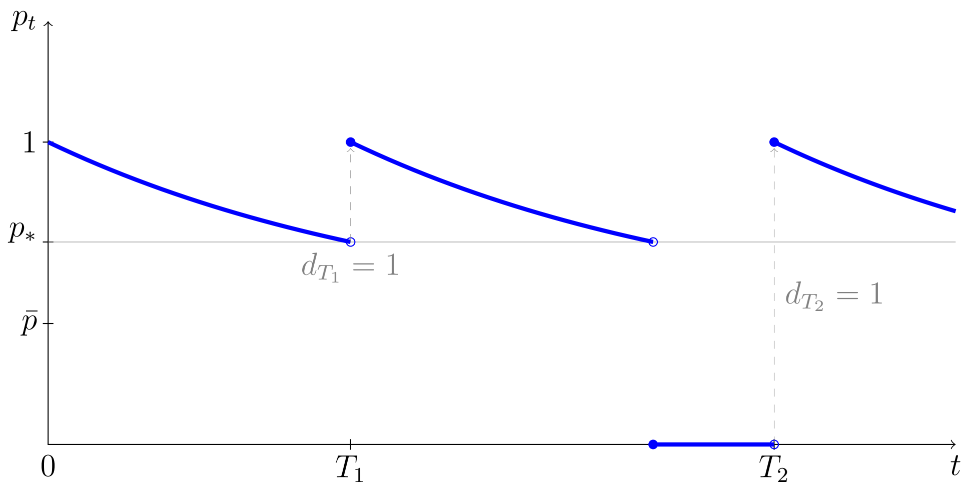


Figure 1: Example of a sample path of the share price.

Observe that there are multiple equilibria, given the discrete support of V_t. In particular, when the cost of disclosure is not so high, there exists a continuum of thresholds p_* satisfying conditions (8) and (9). The following proposition characterizes the set of equilibrium

[7]Weak commitments are sometimes observed in the real world. On December 13, 2002, the Coca Cola Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hopes the move would focus investor attention on long-run performance.

thresholds. We refer to an equilibrium in which disclosure happens with probability zero (at any point in time and for any history) as a no disclosure equilibrium.

With some abuse of notation, we let $U_\theta(p|p_*)$ be the manager's expected payoff in an equilibrium with disclosure threshold p_*.

Proposition 2. *Let*

$$\overline{c} := \frac{\lambda_1 + r}{r\,(r+\kappa)}.$$

If $c < (1-\bar{p})\overline{c}$, then any equilibrium has a positive probability of disclosure. In particular:

1. *If $c < (1-\bar{p})\overline{c}$, there are disclosure thresholds $p_*^- < p_*^+$ satisfying the boundary conditions*

$$U_1\left(1|p_*^+\right) - c = 0 \tag{12}$$

$$U_1'\left(p_*^-|p_*^-\right) = 0, \tag{13}$$

   such that, for any $p_* \in [p_*^-, p_*^+]$, there is an equilibrium with disclosure threshold p_*.

2. *If $(1-\bar{p})\overline{c} \le c < \overline{c}$, then for any $p_* \in [\bar{p}, p_*^+]$, where p_*^+ satisfies* (12)*, there is an equilibrium with disclosure threshold p_*.*

3. *If $c \ge \overline{c}$, the only equilibrium has no disclosure.*

Hence, the *most transparent* equilibrium, in terms of the probability of disclosure, arises when condition (8) is binding. By contrast, the *most opaque* equilibrium arises when condition (9) is binding. Confronted with this multiplicity of equilibria, it is natural to focus on the Pareto dominant one.

Definition 2. The equilibrium threshold $p_*^\dagger$ is Pareto dominant if and only if $U_\theta(p|p_*^\dagger) \ge U_\theta(p|p_*)$ for all $p \in [0,1], p_* \in [p_*^-, p_*^+]$ and $\theta \in \{0,1\}$.

This selection criterion is natural but somewhat arbitrary because in practice there is no guarantee that the manager and the market will coordinate in any particular equilibrium. On the other hand, one can think of the Pareto dominant equilibrium as the natural outcome when, at the outset, the manager informally announces the firm's disclosure policy to the market. Though the manager could not fully commit to disclose information regularly he could issue a cheap talk message along the lines of "we will try to provide guidance on a quarterly basis".[8]

[8]For example, Chen, Matsumoto and Rajgopal (2011) note that on December 13, 2002, the Coca Cola

Proposition 3. *Suppose that* $c < (1-\bar{p})\overline{c}$*, then the Pareto dominant equilibrium is the least transparent equilibrium, that is,* $p_*^\dagger = p_*^-$*. On the other hand, if* $c \geq (1-\bar{p})\overline{c}$*, then the Pareto dominant equilibrium has no disclosure.*

This is intuitive. Given that disclosure is a deadweight cost, the most efficient equilibrium and the one the manager prefers, is the equilibrium that minimizes the frequency of disclosure, since this equilibrium also minimizes the present value of the disclosure expense. Notice that the least transparent equilibrium is the preferred equilibrium for the manager for any initial belief p, and any asset value. Hence the manager's incentives to coordinate in the least transparent equilibrium will remain constant for all the histories of the game.

Non-Markov Equilibrium

Through out the paper, we focus attention on Markov equilibrium. In this section we briefly discuss the effect of considering non-Markov equilibrium. The main result is that there is a non-Markov equilibrium without disclosure. Moreover, a direct implication of Proposition 1 is that this equilibrium yields the maximum profit for the manager.

Corollary 1. *For all* $c > 0$*, there is a non-Markov equilibrium with no disclosure.*

Proof. If $c \geq \overline{c}$, then the only equilibrium involves no disclosure and there is nothing to prove. If $c < \overline{c}$, consider the following trigger strategy: the firm never discloses unless it has disclosed in the past, in which case it uses a disclosure strategy with threshold p_*^+. Accordingly, the market expectations are that the firm never disclose private information unless it has disclosed in the past. Then, for any $p_t \geq 0$ we have

$$U^{ND}(p_t) \geq U(1|p_*^+) - c = 0.$$

Hence, there is no incentive to deviate and disclose. Moreover, from Proposition 1, $U(1|p_*^+)$ is the equilibrium payoff in the continuation game following disclosure. ∎

Corollary 1 shows the existence of a discontinuity with respect to c in the disclosure game. If there is no cost of disclosing information there is unraveling and we have full disclosure. However, for any arbitrarily small cost of disclosure there is an equilibrium without disclosure. According to Graham, Harvey and Rajgopal (2005), managers limit voluntary disclosures to prevent setting a disclosure precedent that will be difficult to satisfy in the future. The

Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hopes the move would focus investor attention on long-run performance. Shortly thereafter, several other prominent firms such as AT&T and McDonalds made similar announcements renouncing quarterly earnings guidance.

previous equilibrium reflects how in a non-Markov equilibrium this concern may eliminate the incentives to disclose altogether. However, a major limitation of the previous equilibrium is the degree of coordination required to sustain no-disclosure.[9] For this reason, we only focus on Markov equilibria hereafter.

2.1 The Frequency of Disclosure

Dye (2012) notes that Robert Elliott (former KPMG partner and a one-time candidate for the chairmanship of the FASB) asserted that to make accounting relevant in the 21st century, financial reporting must move toward continuous frequency. But what is the firm's optimal voluntary frequency?

Given the Markov structure of the problem, the sequence of disclosure times is a renewal process. Hence, it suffices to focus on the expected time of the first disclosure to derive the frequency of disclosure. If we let $T = \inf\{t > 0 | d_t = 1\}$ be the timing of the first disclosure, we are interested in computing $\bar{T}_\theta(p) := E(T|p_0 = p, \theta)$. Given $\bar{T}_\theta(\cdot)$, the frequency of disclosures is simply given by $1/\bar{T}_1(1)$.

Note that T is a random variable with support $[T(1; p_*), \infty)$ where $T(1; p_*) > 0$. Here, $T(1; p_*)$ is then the minimum time that must elapse until the first disclosure is observed. We find $T(p; p_*)$ by solving $\phi_T(p) = p_*$, which yields

$$T(p; p_*) = -\frac{1}{\kappa} \log \left(\frac{p_* - \bar{p}}{p - \bar{p}} \right). \tag{14}$$

Before time $T(1; p_*)$, disclosure has probability zero. On the other hand, if the manager does not disclose the value of the asset at time $T(1; p_*)$, then the expected time spell before the next disclosure is released has an exponential distribution with mean $1/\lambda_1$, that is, $\bar{T}_0(p_*) = 1/\lambda_1$. Noting that $\bar{T}_1(p_*) = 0$, we can compute $\bar{T}_\theta(\cdot)$ directly. The frequency of disclosures is given by $1/\bar{T}_1(1)$ where

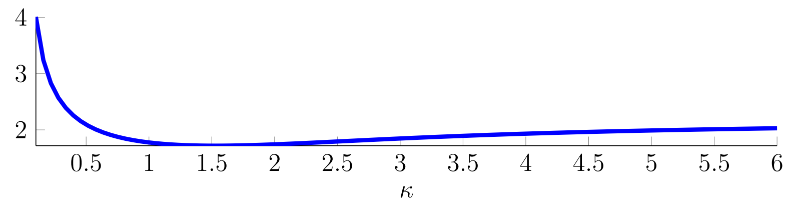
$$\bar{T}_1(1) = T(1; p_*) + \frac{1 - p_*}{\lambda_1}. \tag{15}$$

For a given threshold p_*, the frequency of disclosure increases in cash flows' mean reversion κ. This is natural: a higher mean reversion exacerbates the downward drift in market beliefs which in turn shortens the time until the market beliefs hits p_*. Of course, this is only part of the story because the disclosure threshold also depends on κ.The following proposition studies how κ affects the disclosure threshold $p_*^\dagger$.

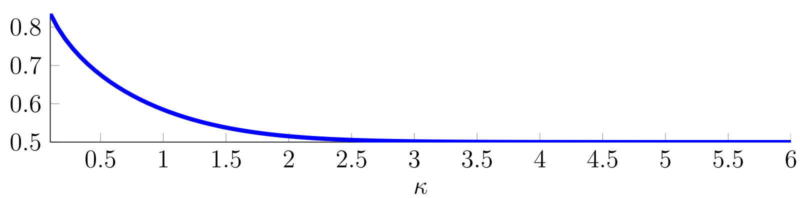
[9]This is a standard critiques in the dynamic games literature to the use of non-Markov equilibria.

Proposition 4. *The equilibrium threshold $p_*^\dagger$ decreases in the cash flows' mean reversion κ.*

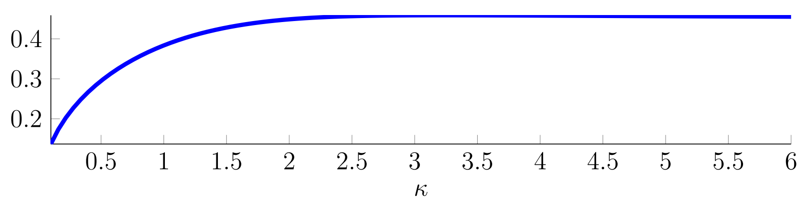
The price benefit of disclosure is weaker when mean reversion is stronger, since the effect of a disclosure on future prices becomes more transitory. This creates an incentive for the manager to reduce the frequency of disclosure. The effect of a higher κ on the frequency of disclosure could therefore be ambiguous: on the one hand, the manager has an incentive to stop the price drift by disclosing good news earlier. On the other hand, the effect of disclosures on the stock price is less long-lasting, which reduces the benefit of disclosing good news. This makes the overall effect ambiguous: indeed, Figure 2 shows that the duration of the period in which no disclosures are expected, $T(p; p_*)$, is non monotonic in κ. Yet, the frequency of disclosure increases monotonically in κ.



(a) Duration of "quiet period," $T(1, p_*^\dagger)$



(b) Pareto dominating equilibrium disclosure threshold, $p_*^\dagger$



(c) Expected frequency of disclosures, $1/\bar{T}_1(1)$

Figure 2: Effect of cash flow persistence, κ, on disclosures in the baseline model. Parameters: $r = 0.1$, $\bar{p} = 0.5$ and $c = 0.5$.

In the next section, we add a public information process to the baseline setting as an intermediate step toward analyzing the case of litigation costs in Section 4.

3 Public Information

In practice, managers' incentives to disclose information at a given point in time depend on the speed the information is expected to leak into the market via external sources (e.g., via peer firm disclosures, media coverage, analysts recommendations, etc.) and the way the market interprets the absence of public information, which by itself can be very informative. An interesting question in this context is whether a more transparent market, as characterized by a higher intensity of public news, reduces the frequency of managerial disclosures thus mitigating the firm's disclosure expense.

In this section, we model the interaction between public information and managerial disclosures. We model public information as a Poisson process $N = \{N_t\}_{t \geq 0}$ with the following characteristics. If the value of assets has a low value, N has arrival rate μ. If the value of assets is high, then N has arrival rate 0. Observing an arrival is thus perfect bad news as it provides perfect evidence that the value of the asset is low.[10]

This information process has intuitive features. As a preliminary analysis, consider how market beliefs evolve during periods where the probability of disclosure is perceived to be zero. Using Bayes' rule, the evolution of beliefs – in the absence of news arrivals – (aka belief drift) must obey

$$dp_t = f(p_t)dt, \tag{16}$$

where

$$f(p) = \kappa(\bar{p} - p) + \mu p(1 - p). \tag{17}$$

In the absence of disclosures and news, beliefs experience a downward drift toward the stationary level $\hat{p}$ as defined by $f(\hat{p}) = 0$, where

$$\hat{p} = \frac{1}{2}\left(1 - \frac{\kappa}{\mu}\right) + \sqrt{\frac{1}{4}\left(1 - \frac{\kappa}{\mu}\right)^2 + \frac{\kappa}{\mu}\bar{p}}.$$

From these conditions, we can see that the mere presence of the news process affects not only the drift but also the stationary belief. The stationary belief $\hat{p}$ increases in the intensity of news arrivals μ. Since arrivals can only take place when the underlying state is low, the absence of arrivals is perceived as good news by the market.

It is easy to verify that for $p \geq \hat{p}$ we have $f(p) < 0$, and vice versa. Of course, in the event of a news arrival, beliefs drop abruptly to zero. The belief $\hat{p}$ has the same role in the

[10] When public information is noisy, managerial disclosures may be triggered by a news arrival, and be used by the manager as a means to counteract the sometimes adverse price effect of noisy news. This reactive-like disclosures generate clustering of disclosure in bad times (see Acharya, DeMarzo and Kremer (2011)). For simplicity, we abstract away from this effect and instead focus on the the case where a news arrival reveal the underlying state perfectly, without noise.

model with news as the one $\bar{p}$ had in the baseline model.

Consider how the presence of the public news process affects the manager's incentives to disclose his information. Assuming that the cost of disclosure is not too high, there exists a disclosure threshold, above the stationary point, i.e., $p_* > \hat{p}$. The HJB equations are

$$
\begin{aligned}
rU_1(p) &= p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \\
rU_0(p) &= p + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)]
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
U_1(p) &= U_1(1) - c, \;\; p \leq p_* \\
U_0(p) &= \frac{\lambda_1}{r + \lambda_1}[U_1(1) - c], \;\; p \leq p_*.
\end{aligned}
$$

The public news process adds uncertainty to the manager's payoff, specially in the low state. By perfectly revealing the asset value, the news would trigger a significant drop in market beliefs, which other things equal, would reduce the manager's payoff in the low state and increase it in the high state. Of course, ex-ante, at time zero, the news process can only affect the manager's payoff if it modifies the frequency of disclosure and the disclosure expenses.

The presence of public news has the following effect on the manager's disclosure incentives. First, both the belief drift $f\left(p\right)$ and the stationary belief $\hat{p}$ are altered by the presence of public news. A higher intensity of news arrivals μ generally leads to a slower drift and more favorable market beliefs. This naturally reduces the manager's disclosure incentives. Hence, even in the absence of news arrivals, the mere presence of the public news process, has the capacity to affect the frequency of managerial disclosures. However, a news arrival results in a sharp price drop which stimulates disclosure as soon as the asset returns to the high state. The former effect is stronger: more public information, represented by a higher μ, leads reduces the speed/frequency of disclosure. One can thus say that public information substitutes managerial disclosures. In the following section we examine whether this result holds in the presence of litigation costs.

4 Legal Liability and Disclosure of Bad News

The empirical disclosure literature shows that litigation costs are an important driver of firm's voluntary disclosures. For example, Lev (1992) and Skinner (1994) document that managers can reduce stockholder litigation costs by voluntarily disclosing adverse earnings news "early", namely before the mandated release date. Consistent with this view, Skinner (1994) finds that managers use voluntary disclosures to preempt large, negative earnings surprises more often than other types of earnings news.[11]

In this section, we analyze the effect of litigation on the incentives of the manager to disclose bad news. The presence of litigation costs fundamentally alters the structure of the analysis and the equilibrium. Technically, litigation costs introduce a signaling component: the firm can signal high value by not disclosing its information (for example, see Bar-Isaac (2003); Kremer and Skrzypacz (2007); Daley and Green (2012); Gul and Pesendorfer (2012)).

For simplicity, we assume that the cost of disclosure varies with the asset value disclosed. That is, the cost of disclosing information is given by c_θ. In particular, we assume that $c_1 = c$ and $c_0 = 0$.[12] If c is interpreted as the proprietary cost associated to revealing a strategic advantage in a market, then it is natural to assume that only revealing positive information is costly.As in the previous section, we assume that there is public news process N_t that arrives with intensity $\mu \mathbf{1}_{\{V_t=0\}}$. That is, bad news only arrive when the firm is in the low state.

The manager is subject to legal liability if the bad news process arrives and the manager has not disclosed the firm was in the low state. Let ℓ_t be a random variable that takes the value one in the event that $dN_t = 1$ if the manager is found liable for the lack of disclosure, and zero otherwise. The manager's cost associated with the legal liability is c_ℓ while the probability of suffering this cost is q if the last time the firm disclosed information it disclosed favorable information and zero otherwise. The idea is that if the firm's last disclosure was negative, the manager can claim he had already disclosed that the value of the assets were low in the event of a public negative news. We denote by $\theta := c_\ell q$ the expected legal cost of not disclosing negative information.

Consequently, the manager expected payoffs, given disclosure strategy σ, can be written as

$$U_t(\sigma) := E^\sigma \left[\int_t^\infty e^{-r(s-t)} p_s ds - \sum_{\{s \geq t : d_s = 1\}} e^{-r(s-t)} c d_s - c_\ell \int_t^\infty e^{-r(s-t)} \ell_s dN_s \middle| \mathcal{F}_t^d, V_t \right] \quad (18)$$

In this setting, the manager might chose to disclose bad news to preempt litigation

[11] Also, Skinner (1997) finds that voluntary disclosures occur more frequently in quarters that result in litigation than in quarters that do not, because managers' incentives to predisclose earnings news increase as the news becomes more adverse, presumably because this reduces the cost of resolving litigation that inevitably follows in bad news quarters.

[12] Note that this assumption is not necessary for the results; assuming that $c_0 = c_1 = c$ would generate the same results but would give rise to some non-monotonicity in the value function of the manager.

costs if the cost of legal liability outweighs the benefit of keeping the price "inflated". On the surface, it might appear that the presence of litigation costs will drive the manager to "spontaneously" disclose bad news, even when prices are relatively high. But on closer inspection, the equilibrium is less clear: if the market expected the manager to disclose low asset value at a given point in time, then the absence of such announcement would be perceived as strong evidence of high asset values. In turn, this would result in a sharp upward jump in the stock price. This would destroy the manager's incentives to disclose bad news in the first place. This conundrum suggests that the equilibrium must entail mixed strategies, as shown below.

4.1 Equilibrium Description

Depending on the cost of disclosure c, three classes of equilibria may emerge. All of them are characterized by a threshold p_* for the market's belief such that whenever market beliefs hit the threshold p_* the manager discloses some of his information with positive probability.[13] Which information the manager discloses, depends nonetheless on the relative magnitude of the disclosure cost.

First, if the disclosure cost is sufficiently low, $c < c^o$, the manager discloses his information when the price reaches p_* regardless of the value of assets (and, off equilibrium, the market assumes the manager is withholding low asset values if no disclosure is observed). In essence, this is the equilibrium characterized in previous sections, except for the fact that sometimes the manager must bear the litigation cost. So, in the sequel, we assume $c \geq c^o$.

Second, at the other extreme of the spectrum, when the cost of disclosure is very high, $c > c^+$, the manager may disclose low asset values with positive probability, whenever the market beliefs hit the threshold p_*, but he never discloses high asset values, because such disclosures are unaffordable. One can think of this case as the one arising in settings where certification is too expensive or the proprietary nature of the information makes it too costly for the firm to reveal good news.

Third, for intermediate disclosure costs, i.e., when $c \in [c^o, c^+]$, the manager discloses bad news with positive probability when the price hits the threshold p_* but he discloses good news only when beliefs are severely depressed, in particular, when $p_t = 0$. This is the equilibrium we focus on in the sequel. Interestingly, as shown below, the presence of litigation costs results in a unique equilibrium, so no selection criterion is necessary.

[13] Throughout, we assume that the cost of litigation is neither too low nor too high, so that the threshold p_* is interior, namely it belongs to $(\hat{p}, 1)$.

We conjecture and verify that if the cost of legal liability lies in $[c^o, c^+]$ then the equilibrium is given by

1. If $V_t = 1$, then $d_t = \mathbf{1}_{\{p_t < p_*\}}$.
2. If $V_t = 0$, then:
   (a) If $p_t > p_*$ we have $d_t = \emptyset$.
   (b) If $p_t = p_*$ then the manager discloses with a mean arrival rate

$$\zeta = \kappa \frac{p_* - \bar{p}}{p_*(1 - p_*)} - \mu.$$

   (c) If $p_t < p_*$ then the manager discloses immediately with probability[14]

$$\frac{p_t}{1 - p_t} \frac{1 - p_*}{p_*}.$$

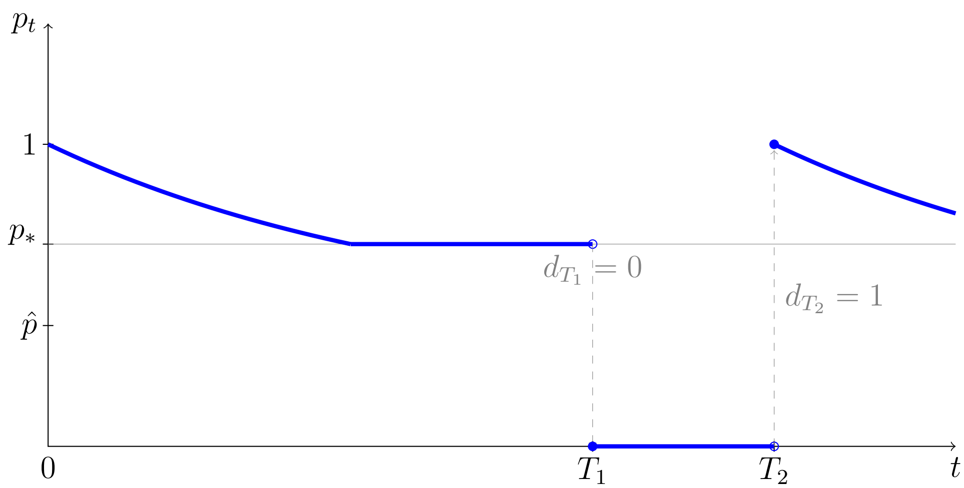


Figure 3: Example of a sample path of the share price with litigation cost.

Figure 3 shows a sample path of the stock price in equilibrium. At the beginning, the price drifts downward until it hits the threshold p_*. The initial drift is driven purely by the perception of an undisclosed impairment. Then, the price remains flat until the manager reports bad news, at time T_1. Naturally, this disclosure causes the price to drop to zero and stay there until the situation of the firm improves, at time T_2, and the manager discloses good news. The presence of litigation naturally leads to bad news disclosures but, more

[14]This is an out-off-equilibrium event as with perfect bad news beliefs never enter the interval $(0, p_*)$ on the equilibrium path.

interestingly, it also renders unnecessary the good news disclosures, except in the case of very severe undervaluation, namely when $p_* = 0$.

The manager's payoffs satisfy the following HJB equation. For $p_t > p_*$,

$$\begin{aligned} rU_1(p) &= p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \\ rU_0(p) &= p - \mu\theta + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)]. \end{aligned}$$

These equations are analogous to those encountered in previous settings, except for the fact that in the low state the manager's instant payoffs must take into account the expected litigation costs, as given by $\mu\theta$. In this context, the manager's payoffs are exposed to two types of shocks. First, the asset value may experience a "real shock" which even when not observed by the market, affects the expected trajectory of future prices hence the manager's payoffs. Second, the manager may experience a "publicity" shock: a news arrival may reveal the manager was withholding information thus triggering both a drop in the stock price and litigation costs.

We need to deduce the boundary conditions. When $p_t = p_*$, we have

$$U_0(p_*) = E\left[\int_0^{\tau_N \wedge \tau_D \wedge \tau_1} e^{-rt}(p_* - \mu\theta)dt + e^{-r\tau_N \wedge \tau_D \wedge \tau_1}\left(U_0(0)\mathbf{1}_{\{\tau_N \wedge \tau_D < \tau_1\}} + U_0(0)\mathbf{1}_{\{\tau_N \wedge \tau_D > \tau_1\}}\right)\right],$$

where τ_N is the first arrival of bad news, τ_D is the time at which the manager discloses bad news, and τ_1 is the time at which the value of assets jump from 0 to 1. We can solve for the expected payoff of a low type manager, as given by

$$\begin{aligned} U_0(p_*) &= \int_0^\infty e^{-(r+\mu+\zeta+\lambda_1)t}\big(p_* - \mu\theta + (\mu+\zeta)U_0(0) + \lambda_1 U_1(p_*)\big)dt \\ &= \frac{p_* - \mu\theta}{r+\mu+\lambda_1+\zeta} + \frac{\mu+\zeta}{r+\mu+\lambda_1+\zeta}U_0(0) + \frac{\lambda_1}{r+\mu+\lambda_1+\zeta}U_1(p_*). \end{aligned} \tag{19}$$

Following similar steps as the ones above we get the boundary condition when for a high type manager as given by

$$U_1(p_*) = \frac{p_* + \lambda_0 U_0(p_*)}{r+\lambda_0}. \tag{20}$$

Finally, we have the following conditions when $p_t = 0$:

$$U_0(0) = \frac{\lambda_1}{r+\lambda_1}U_1(0) \tag{21}$$

$$U_1(0) = U_1(1) - c. \tag{22}$$

When $p_t = p_*$, the manager must be indifferent between disclosing negative information and not disclosing it, otherwise he would not be willing to randomize. Hence, the threshold p_* is pin-downed using the indifference condition for a mixed strategy:

$$U_0(p_*) = U_0(0). \tag{23}$$

The strategies above constitute an equilibrium as long as the following conditions are satisfied

1. $U_1(1) - c \geq 0$.

2. $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$.

3. $U_0(p) \geq U_0(0)$ for $p > p_*$.

We can solve for $U_0(p_*)$ by combining equations (19) and (23), which give us

$$U_0(p_*) = \frac{p_* - \mu\theta + \lambda_1 U_1(p_*)}{r + \lambda_1}. \tag{24}$$

Then combining (20) with (24) we get

$$U_0(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{r + \lambda_0}{r + \lambda_0 + \lambda_1} \tag{25}$$

$$U_1(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{\lambda_0}{r + \lambda_0 + \lambda_1}. \tag{26}$$

The value of p_* can thus be obtained from

$$U_0(p_*) = \frac{\lambda_1}{r + \lambda_1}\left[U_1(1) - c\right]. \tag{27}$$

A necessary condition for optimality of the disclosure strategy above is that $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$. If U_1 is increasing in p_*, then this condition is satisfied if

$$U_1(p_*) \geq U_1(1) - c = \left(1 + \frac{r}{\lambda_1}\right) U_0(p_*),$$

where we have used the equilibrium condition (27). Combining (25) and (26) we get the following upper bound for the disclosure threshold p_*

$$p_* \leq \mu\theta.$$

This means that the disclosure threshold is not greater than the myopic threshold (namely the one that a manager exclusively concerned with his current payoffs) would pick. This is natural, the more the manager cares about future prices the weaker his incentive to reveal information that would cause a drop in the stock price. Note also that waiting for the asset to recover its value has option value, for that event would render the disclosure unnecessary.

The manager's decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut-down a project (i.e., disclose bad news) or restart it (i.e., disclose good news), based on the project's observed profitability, except that here the cash flows are endogenous, since they are linked to market's equilibrium beliefs. When the stock price is low and the value of the asset is also low, disclosing bad news becomes profitable for the same reason shutting down a project that is making losses is optimal in Dixit's model. Also, as in Dixit's model, the decision to disclose bad news today is inherently liked with the value of the option to disclose good news in the future: if the likelihood of disclosing good news in the future is lower (perhaps because $\lambda_1 \to 0$), then the benefit from disclosing bad news today goes down, which naturally delays such disclosures. This speaks to a certain complementarity between bad and good news disclosures in the presence of legal liability.

The condition $U_1(1) - c \geq 0$ can only be satisfied if $U_0(p_*) \geq 0$. Thus, using (25) we obtain a lower bound for p_* given by

$$p_* \geq \frac{r + \lambda_0}{r + \lambda_0 + \lambda_1} \mu\theta.$$

This lower bound reveals an intuitive feature of the model: if litigation costs are too high the bound will hit 1 which means that no asymmetry of information can ever be experienced in equilibrium: negative information must be revealed immediately when litigation costs are prohibitive.

Unlike in the case without litigation, analyzed in Sections 2, here the manager strictly prefers not to disclose the asset value when it is high, unless the price is experiencing the most severe possible undervaluation, i.e., when $p_t = 0$. The reason for the manager's reluctance to disclose good news is that, in the presence of litigation costs, the absence of disclosures is not strongly penalized by the market because the market expects disclosures to convey bad news as well. Hence, in this case, no news is often good news.

Randomization is an essential component of the equilibrium in the presence of litigation costs. For randomization to be optimal, the manager must be indifferent between disclosing low values (so as to avoid the risk of litigation) and not disclosing it (so as to enjoy inflated prices). If, at any point, the manager disclosed his bad news with probability one then the absence of such disclosure would be interpreted by investors as an indication that the asset value was high, which would in turn cause the price to jump upward. This in turn would destroy the manager's incentives to disclose bad news in the first place. The manager's randomization allows the price not to jump upward at p_* but either to remain constant, in the absence of disclosure, or to drop down to zero in the presence of a disclosure (see Figure 3).

Finally, note that the equilibrium characterized above exhibits a nice feature: unlike the case without litigation costs the threshold p_* is unique.

4.1.1 An example: Permanent Shocks

A particularly tractable example arises when $\lambda_1 = 0$, namely when negative shocks are permanent. Since in that case the asset never recovers its value after an adverse shock we must have

$$U_0(0) = 0.$$

Using this condition along with (23) and (25) we get

$$p_* = \mu\theta,$$

Hence, the optimal disclosure strategy is the myopic policy. Recall that when the shock is less severe, i.e., $\lambda_1 > 0$, the threshold would be lower which suggests that the manager would tend to disclose his information later, so to take advantage of the option to wait for the asset's recovery. Intuitively, higher litigation costs θ accelerate the manager's disclosure by increasing the threshold p_*. On the other hand, a higher frequency of public news μ also increases the threshold p_*. But notice this does not per se means that disclosures will be more frequent, because a higher μ also implies that public news will arrive more often, thereby substituting the manager's disclosures. Hence, on average disclosures might end up being less frequent when μ increases.

When the price hits the threshold p_* the manager discloses low value of assets with intensity

$$\zeta = \max\left(\frac{\lambda_0}{1-\mu\theta} - \mu, 0\right).$$

Unlike the effect of θ, a higher frequency of arrivals μ may decrease the intensity of disclosure at the threshold p_*. So in principle it is not clear whether more public information complements or substitutes managerial disclosures.Consider now the effect of real shocks λ_0. The higher the frequency of adverse shocks, λ_0, the higher is ζ hence the higher the likelihood that the manager discloses low values at the threshold p_*. Yet note that the threshold p_* is not affected by λ_0.

Disclosing that the value of assets is high is never profitable if $c \geq c^+$ where

$$c^+ = U_1(1) - U_1(\mu\theta).$$

Otherwise, the manager has an incentive to disclose good news prior to the market beliefs reaching $\mu\theta$, which in turn means the manager must also disclose bad news at $p_* = \mu\theta$ to preempt litigation.

4.2 The Frequency of Disclosure

Naturally, the extent of the asymmetry of information between the manager and the market depends on the frequency of disclosure. The more frequently the manager discloses his information, the smaller will be the the asymmetry of information affecting the market.

In order to find the frequency of disclosure we must compute the expected time at which the firm will disclose. As before, the Markov structure of the problem allows us to focus on the expected time of the first disclosure. Let $T = \inf\{t > 0 | d_t = 1\}$. We want to compute $\overline{T}_\theta(p) := E(T|p_0 = p, \theta)$. By standard arguments, $\overline{T}_\theta$ satisfies:

$$-1 = f(p)\overline{T}'_1(p) + \lambda_0[\overline{T}_0(p) - \overline{T}_1(p)] \tag{28}$$
$$-1 = f(p)\overline{T}'_0(p) + \lambda_1[\overline{T}_1(p) - \overline{T}_0(p)] + \mu[\overline{T}_0(0) - \overline{T}_0(p)] \tag{29}$$

In order to find the right boundary conditions, we note that

$$\overline{T}_1(p_*) = \frac{1}{\lambda_0} + \overline{T}_0(p_*) \tag{30}$$
$$\overline{T}_0(p_*) = \frac{1}{\mu + \zeta + \lambda_1} + \frac{\lambda_1}{\mu + \zeta + \lambda_1}\overline{T}_1(p_*) + \frac{\mu}{\mu + \zeta + \lambda_1}\overline{T}_0(0). \tag{31}$$

A high type manager never disclose when $p_t = p_*$. Equation (30) simply says that the expected time until the next disclosure equals the expected time that it takes for the value of the firm to jump down to zero plus the expected time that it takes for a low type manager to disclose. Equation (31) has a similar interpretation. The expected time that it takes for a low type firm to disclose consider three possible events: 1) the firm disclose negative information, 2) the value of the assets jumps up to one, 3) there are negative public news that take beliefs down to zero. In addition, as in the case without litigation, we have that

$$\overline{T}_1(0) = 0$$
$$\overline{T}_0(0) = \frac{1}{\lambda_1}.$$

The solution to these differential equations characterize the frequency of disclosure as a function of market beliefs and asset values. Figure 4 studies the determinants of the frequency of disclosure. Intuitively, we see that the frequency of disclosure decreases in the cost of disclosure c. A higher disclosure cost delays the disclosure of bad news by making the disclosure of good news more costly hence less attractive to the manager. If the manager anticipates that overcoming a possible undervaluation by disclosing good news will be too costly, he might as well delay the bad news in the first place.

By contrast, the frequency of disclosure increases both in the intensity of public news μ and in the cash flows' mean reversion κ. The former effect is natural: a higher intensity of public news increases the risk of costly litigation thus stimulating preemptive disclosures. A higher mean reversion, on the other hand, makes the shocks less long-lasting. This means that disclosing bad news will have a weaker impact on the trajectory of future stock prices.

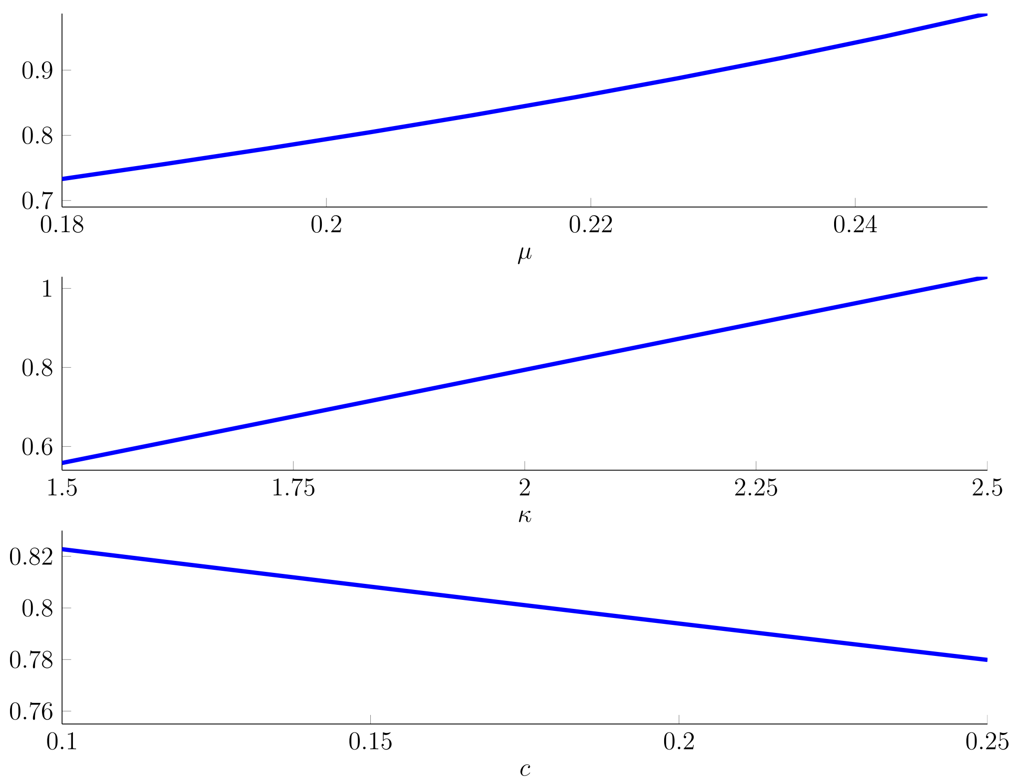


Figure 4: The Frequency of Disclosure ($1/\overline{T}_1(1)$)

5 Patent Length and Disclosure

Consider the case when the asset value is modified by an event taking place at a known date in the future. To be more concrete, assume the asset is protected by a patent that expires at a fixed time T that is known. Before the patent expires, the exploitation of the asset generates monopoly rents to its owner. After expiry the asset yields lower rents to the owner, perhaps due to the presence of stronger competition. In this section, we study how the length of the patent and the intensity of post-patent competition affects the propensity and timeliness of the manager's disclosures.

Formally we assume that in the high state the asset evolves in the following way

$$V_1(t) = \begin{cases} \overline{V} - e^{-\rho(T-t)}(\overline{V} - \underline{V}) & \text{if } t \leq T \\ \underline{V} & \text{if } t > T. \end{cases} \tag{32}$$

where T represents the date of patent expiration, which we refer to as patent length. Before expiration, the firm earns a flow of profits $\overline{v}$. After expiration, competition reduces the profits of the firm to $\underline{v}$. Under this specification, the value of the patent is given by (32), where $\overline{V} = \overline{v}/\rho$, $\underline{V} = \underline{v}/\rho$ and ρ is the market's discount rate. We refer to $\overline{V} - \underline{V}$ as the post-patent competition.

For simplicity we assume that $\lambda_1 = 0$. Also, in order to focus on the interesting case, we suppose that the cost of certification c is too high so that only bad news can eventually be disclosed in equilibrium. As before, we assume that the value of the asset in the low state is $V_0(t) = 0$ for all $t \geq 0$. If the asset is a drug, we can think of the low state as occurring when a serious side effect is discovered which —if known by the health authorities– would trigger an immediate recall of the drug from the market and a permanent ban of its commercialization. Consistent with the previous setting, the manager always knows which state the asset is in, though the market may learn it indirectly if a news arrival is observed before any disclosure is made.

We conjecture an equilibrium given by a threshold $p_*(t)$ such that the low type manager discloses with mean arrival rate $\zeta_t > 0$ when $p_t = p_*(t)$. Note that before expiration, namely for $t < T$, the asset evolves as follows

$$\frac{dV_1(t)}{dt} = -\rho\big(\overline{V} - V_1(t)\big),$$

In turn, the market's beliefs, absent disclosure, evolve as

$$\frac{dp_t}{dt} = \kappa(\bar{p} - p_t) + (\mu + \zeta_t)p_t(1 - p_t).$$

So unlike in the previous settings, here the manager's disclosure strategy may affect the drift of market beliefs p_t on a continuous basis, because the manager's disclosure intensity may depend continuously on t.

In principle, this may seem a very complicated problem. However, as mentioned above, when $\lambda_1 = 0$ there is no option value from waiting, hence the decision of the low type manager to disclose his information is a monotone optimal stopping problem, which means that the myopic stopping rule is optimal. Consequently, the manager does not disclose negative information if the instant price benefit from withholding the information exceeds the expected litigation cost, namely $p_t V_1(t) > \mu\theta$. The manager discloses with positive probability if $p_t V_1(t) = \mu\theta$. Let's define

$$T_* := \inf\{t > 0 : p_t V_1(t) = \mu\theta\},$$

and

$$T_{**} := \inf\{t > 0 : V_1(t) = \mu\theta\} \tag{33}$$

Naturally, when $t \le T_*$ we must have $\zeta_t = 0$. By contrast, when $t \ge T_{**}$ the manager must disclose his information immediately, hence $\zeta_t = \infty$. For $t \in (T_*, T_{**})$, the equilibrium condition that determines the disclosure intensity ζ_t is that $p_t V_1(t) = \mu\theta$, which requires that $dp_t V_1(t)/dt = 0$. Hence, we have the equilibrium condition

$$\frac{dp_t}{dt} V_1(t) + p_t \frac{dV_1(t)}{dt} = 0.$$

This condition formalizes the idea that the market's assessment of the asset value must be constant over the interval $t \in (T_*, T_{**})$. Using the equilibrium condition $p_t V_1(t) = \mu\theta$, we can reduce the condition above to

$$\kappa \frac{\bar{p} V_1(t)}{\mu\theta} + (\mu + \zeta_t)(1 - p_t) = \kappa + \rho \left(\frac{p_t \overline{V}}{\mu\theta} - 1 \right).$$

Solving for ζ_t from the above equation gives

$$\zeta_t = \frac{1}{1 - p_t} \left[\kappa - \rho + \frac{1}{\mu\theta} \big(\rho p_t \overline{V} - \kappa \bar{p} V_1(t) \big) \right] - \mu. \tag{34}$$

The condition $p_t V_1(t) \ge \mu\theta$ can be satisfied for all $t \le T$ if and only if $\underline{V} \ge \mu\theta$. Otherwise, there is T_{**}, as defined in (33), such that a low type manager discloses his private information with probability 1 when he is close to the the patent's expiration. Summarizing our previous discussion, the equilibrium disclosure strategy is:

$$\zeta_t = \begin{cases} 0 & \text{if } t \leq T_* \\ \frac{1}{1-p_t}\left[\kappa - \rho + \frac{1}{\mu\theta}\left(\rho p_t \overline{V} - \kappa \bar{p} V_1(t)\right)\right] - \mu & \text{if } T_* < t < T_{**} \\ \infty & \text{if } t \geq T_{**}, \end{cases} \tag{35}$$

where $\zeta_t = \infty$ indicates immediate disclosure and

$$T_{**} = \begin{cases} T - \frac{1}{\rho}\ln\left(\frac{\overline{V}-\underline{V}}{\overline{V}-\mu\theta}\right) & \text{if } \underline{V} \leq \mu\theta \\ \infty & \text{if } \underline{V} > \mu\theta. \end{cases}$$

This means that, conditionally on not disclosing negative information, $p_t = 1$ for $t \geq T_{**}$.

Figure 5 shows the evolution of beliefs, absent disclosure, for different levels of $\underline{V}$. An interesting aspect of these dynamics is that disclosures tend to cluster around patent expiration date. Also, notice that after the market beliefs hit the threshold p_*, they start to improve over time in the absence of disclosure. Delay is thus a powerful signal which has the potential to overcome even the downward drift in stock prices. The price benefits of disclosure delay in market beliefs has been recently pointed out out by Kremer, Guttman and Skrypacz (2012) in a different setting. They show that delayed disclosures often indicate a lower probability that the manager is withholding other pieces of information. The dynamic signaling (see e.g., Bar-Isaac (2003); Kremer and Skrzypacz (2007); Daley and Green (2012); Gul and Pesendorfer (2012)) literature also highlights the role of trading delay as the seller's means of signaling high quality in the presence of privately informed sellers.

The above analysis provides a unique opportunity for studying how patent length affect the manager's propensity to disclose adverse information timely (i.e., to reveal that the asset value is low). This is interesting from a theory standpoint alone, but it is also relevant from a patent design and regulatory perspective, given the social impact of some of the disclosures referring to assets protected by patents. Indeed, when the information that the manager may withhold concerns the health benefits or risks that the consumption of a drug may cause, timely disclosures are very important.

Proposition 5. *The propensity (ζ_t) with which the manager discloses bad news decreases in the patent's length (T)*

This result suggests that from a social standpoint it might be desirable to award shorter patents to inventions for which timely disclosures are particularly important, as is the case of drugs where safety is a major concern. To our knowledge this aspect of the patent design has been ignored by the literature studying the design of intellectual property rights.

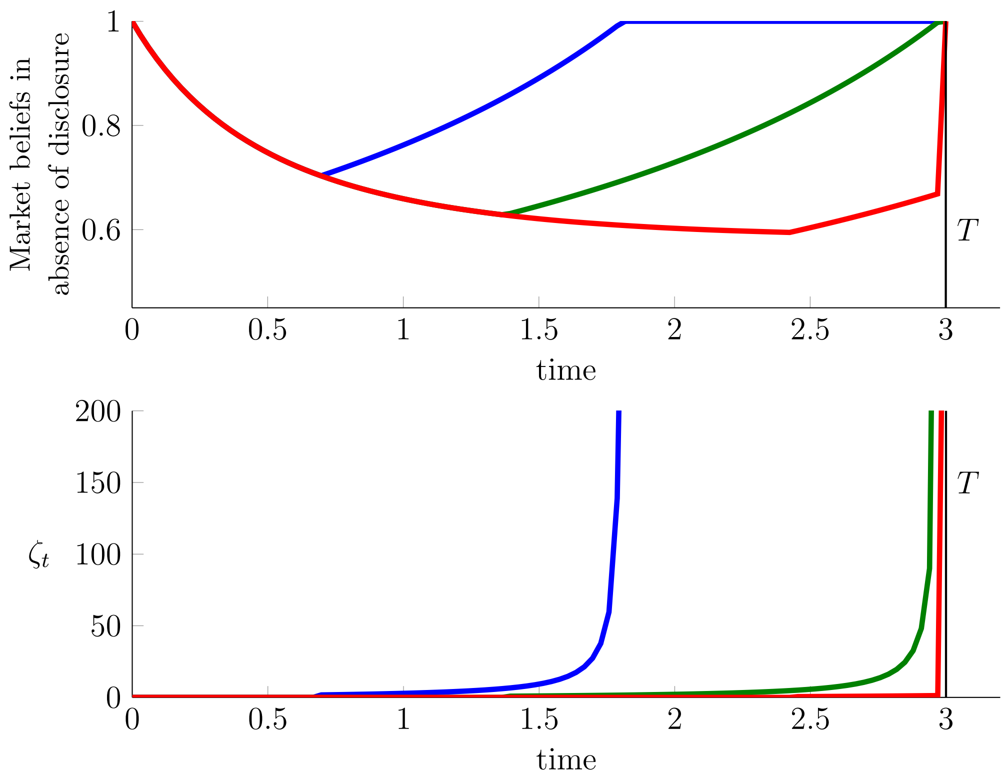


Figure 5: Effect of post-patent competition ($\underline{V}$) on timing and intensity of disclosure. $T = 3$. We assume $\underline{V}$=[0.09 0.18 0.27].

Figure 6 shows that the intensity of disclosure monotonically increases over time and clusters around the date of patent expiration. Anecdotal evidence suggests, indeed, that in the pharmaceutical industry negative disclosures seem to cluster around patent expiration, but we are not aware of empirical evidence documenting this hypothesis (we hope to conduct this study in the future).

The intensity of disclosure is also affected by the extent of post patent competition. In practice the intensity of post-patent competition varies among patents.

Proposition 6. *Stronger competition post-expiration (i.e,. lower $\underline{V}$) increases the manager's disclosure propensity (ζ_t).*

The stronger the post-patent competition the earlier the manager will disclose his information. This is intuitive: stronger competition means the manager does not fully internalize the rents he obtains from withholding adverse information. His incentives to disclose the information, so as to preempt litigation costs, become stronger which accelerates the disclosure process. In a sense this result is related to the previous one and reflects, from a regulatory point of view, that restricting patent length and inducing stronger post market competition are complementary tools, when the regulator is concerned with accelerating disclosures.

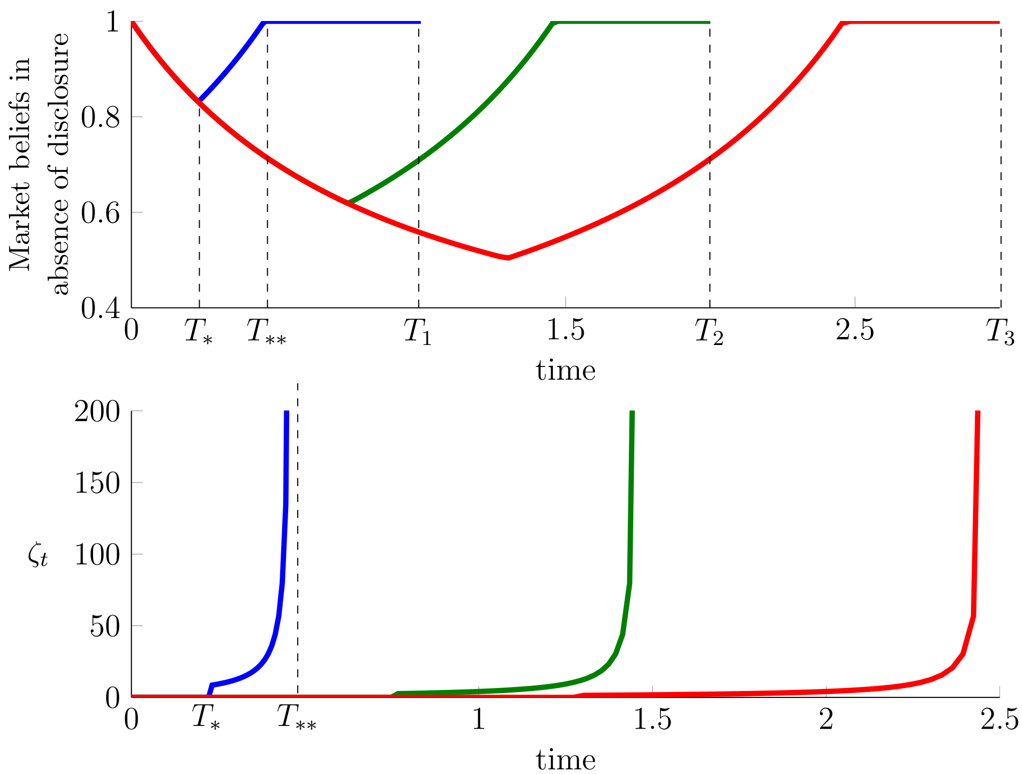


Figure 6: Effect of patent length (T) on timing and intensity of disclosure. $T = 1, 2, 3$.

6 Concluding Remarks

This paper studies a model of dynamic costly disclosure. We make the following contribution to the literature. To our knowledge, this is the first dynamic model of disclosure with the realistic feature that private and public information flows happen in an ongoing (continuous) basis. We characterize the dynamics of disclosure, and derive its implications for the time-series of stock returns. Our analysis is consistent with a number of stylized facts such as the clustering of announcements in bad times, the downward drift of stock prices prior to a disclosure, the negative market reaction to firm's breaking their disclosure (implicit) commitments, the higher volatility of prices given no disclosure.

Our model has several limitations. First, the state of nature is binary. One could consider the possibility of asset values that are continuously distributed. This is not just for the sake of elegance but because some properties of the equilibrium —such as the *blackout* period where no disclosure is observed– are purely an artifact of the binary setting. Moreover the multiplicity of equilibria is also driven by the discontinuous nature of the distribution of the asset value process.

Second, we have modeled the public information process as a Poisson process. An interesting but difficult extension is to consider the idea that the public information process follows a Brownian motion whose drift depends on the state of nature, along the lines of the information structure considered by Daley and Green (2012). Again, this would allow for a more realistic and elegant characterization of the dynamics of stock prices.

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A Proofs of Section 2

Proof of Proposition 1

We divide the proof of Proposition 1 in two steps. First, we show that the functions in the proposition solve the HJB equation with the required boundary conditions. Second, we show that the solution constitutes an equilibrium.

Step 1:

In the absence of any disclosure, the beliefs at time t are given by

$$\phi_t(p_0) = \bar{p} + e^{-\kappa t}(p_0 - \bar{p}).$$

Let's define $T(p; p_*)$ as the time that it takes the beliefs to reach p_* give that current beliefs are p. That is,

$$T(p; p_*) = -\frac{1}{\kappa} \log \left(\frac{p_* - \bar{p}}{p - \bar{p}} \right),$$

where $\frac{\partial T(p;p_*)}{\partial p} > 0$ and $\frac{\partial T(p;p_*)}{\partial p_*} < 0$. The results in **?**, pp. 92-93 imply that the solution to the HJB equation (4)-(5) satisfies

$$\begin{aligned}
U_0(p|p_*) = \int_0^{T(p;p_*)} & e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)} \Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 0)U_0(p_*|p_*) \\
+ \Pr(V_{T(p;p_*)} = 1|V_0 = 0)U_1(p_*|p_*)\Big] \\
U_1(p|p_*) = \int_0^{T(p;p_*)} & e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)} \Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 1)U_0(p_*|p_*) \\
+ \Pr(V_{T(p;p_*)} = 1|V_0 = 1)U_1(p_*|p_*)\Big].
\end{aligned}$$

Let $\bar{U}(p|p_*) := \bar{p}U_1(p|p_*) + (1 - \bar{p})U_0(p|p_*)$ and $\Delta U(p|p_*) := U_1(p|p_*) - U_0(p|p_*)$. Replacing $\Pr(V_{T(p;p_*)} = j|V_0 = i)$ for $i, j \in \{0, 1\}$, we can write the manager's expected payoff as

Using the boundary conditions we get

$$U_0(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r + \lambda_1} - \frac{r\bar{p}}{r + \lambda_1} e^{-(p;p_*)} \right] \Big(U_1(1|p_*) - c \Big) \quad (36)$$

$$U_1(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r + \lambda_1} + \frac{r(1 - \bar{p})}{r + \lambda_1} e^{-(p;p_*)} \right] \Big(U_1(1|p_*) - c \Big). \quad (37)$$

Using equation (36) we can write $U_1(1|p_*)$ as

$$U_1(1|p_*,\kappa) = \frac{\int_0^{T(1;p_*)} e^{-rt}\phi_t(1)dt}{1-\delta(1)} - \frac{\delta(1)}{1-\delta(1)}c, \tag{38}$$

where

$$\delta(1) = e^{-rT(1;p_*)}\Big[\frac{r\bar{p}+\kappa\bar{p}}{r+\kappa\bar{p}} + \underbrace{\frac{r(p_*-\bar{p})}{r+\kappa\bar{p}}}_{\frac{r(1-\bar{p})}{r+\kappa\bar{p}}e^{-\kappa T(1;p_*)}}\Big].$$

The first term in (38) can be written as

$$U^{ND}(1)\frac{\overbrace{1-e^{-rT(1;p_*)}\frac{U^{ND}(p_*)}{U^{ND}(1)}}^{1-\delta(1)}}{1-\delta(1)} = U^{ND}(1).$$

Hence,

$$U_1(1|p_*) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c, \tag{39}$$

Step 2:

The only step left is to show that (8) and (9) imply $U_1(p) \geq U_1(1) - c$ for all $p > p_*$ so a threshold policy is optimal. We first show that (8) and (9) imply $U_1'(p) \geq 0$ for all $p > p_*$. The derivative of U_1 is given by

$$U_1'(p) = e^{-rT(p;p_*)}\Phi(p)\frac{\partial T(p;p_*)}{\partial p} + \int_0^{T(p;p_*)} e^{-(r+\kappa)t}dt \tag{40}$$

where

$$\Phi(p) := p_* - re^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{(1-\bar{p})(r+\kappa)}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1)-c\Big).$$

From here we get that $U_1'(p_*) \geq 0$ if and only if $\Phi(p_*) \geq 0$. Moreover, $U_1(1) - c > 0$ implies $\Phi'(p) > 0$, which means that $\Phi(p) \geq 0$ for all $p > p_*$. Accordingly, $U_1'(p) \geq 0$ for all $p > p_*$, and

$$U_1(p) = U_1(p_*) + \int_{p_*}^p U_1'(y)dy = U_1(1) - c + \int_{p_*}^p U_1'(y)dy > U_1(1) - c.$$

Proof of Proposition 2

We begin proving two lemmas.

Lemma 1. *Suppose that conditions* (8) *and* (9) *are satisfied, then* $\frac{\partial}{\partial p_*} U_1(p|p_*) < 0$.

Proof. Differentiating (36) with respect to p_* we get

$$\frac{\partial}{\partial p_*} U_1(p|p_*) = e^{-rT(p;p_*)} \Phi(p;p_*) \frac{\partial T(p;p_*)}{\partial p_*} + e^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r + \lambda_1} + \frac{r(1-\bar{p})}{r + \lambda_1} e^{-\kappa T(p;p_*)} \right] \frac{\partial}{\partial p_*} U_1(1|p_*). \tag{41}$$

From here we get

$$\frac{\partial}{\partial p_*} U_1(1|p_*) = \frac{e^{-rT(1;p_*)} \Phi(1;p_*)}{1 - e^{-rT(1;p_*)} \left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1} e^{-\kappa T(1;p_*)} \right]} \left. \frac{\partial T(p;p_*)}{\partial p_*} \right|_{p=1} < 0,$$

so $\frac{\partial}{\partial p_*} U_1(p|p_*) \leq 0$ as $\Phi(p;p_*) \geq 0$ (see proof of Proposition 1). ∎

Lemma 2. *Suppose that conditions* (8) *and* (9) *are satisfied, then* $U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*} U_1'(p_*|p_*) > 0$.

Proof. Rearranging the HJB equation (4) we can write

$$U_1'(p|p_*) = \frac{rU_1(p|p_*) - p - \lambda_0 [U_0(p|p_*) - U_1(p|p_*)]}{\kappa(\overline{p} - p)}$$

Evaluating at $p = p_*$ and using the boundary conditions, equations (6) and (7), yields

$$\begin{aligned} U_1'(p_*|p_*) &= \frac{rU_1(p_*|p_*) - p_* + U_1(p_*|p_*) \frac{r\lambda_0}{r+\lambda_1}}{\kappa(\overline{p} - p_*)} \\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_*}{\kappa(\overline{p} - p_*)} \end{aligned} \tag{42}$$

Now, we can show that

$$U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*} U_1'(p_*|p_*) > 0.$$

Differentiating equation (42) with respect to p_* yields

$$\begin{aligned} \frac{\partial}{\partial p_*} U_1'(p_*|p_*) &= \frac{\frac{r(r+\kappa)}{r+\lambda_1} \left. \frac{\partial U_1(p|p_*)}{\partial p_*} \right|_{p=p_*} - 1}{\kappa(\overline{p} - p_*)} + \frac{\kappa}{\kappa(\overline{p} - p_*)} U_1'(p_*|p_*) \\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1} \left. \frac{\partial U_1(p|p_*)}{\partial p_*} \right|_{p=p_*} - 1}{\kappa(\overline{p} - p_*)} > 0 \end{aligned}$$

But from Lemma 1 we know that $\frac{\partial U_1(p|p_*)}{\partial p_*} < 0$. This along with $\kappa(\overline{p} - p_*) < 0$ proves the lemma. ∎

Proof of Proposition 2. Suppose there exist $p_*^- < p_*^+$ such that

$$\begin{aligned} U_1'\left(p_*^-|p_*^-\right) &= 0 \\ U_1\left(1|p_*^+\right) - c &= 0 \end{aligned}$$

A direct consequence of Lemma 2 is that $U_1'\left(p_*|p_*\right)$ crosses 0 only once. Thus, $U_1'\left(p_*|p_*\right) \geq 0$ for $p_* \geq p_*^-$, and $U_1'\left(p_*|p_*\right) < 0$ for $p_* < p_*^-$. Moreover, from Lemma 1 we have that $U_1\left(p_*|p_*\right) - c \geq 0$ for all $p_* \leq p_*^+$. Hence, p_* satisfies conditions (8) and (9) if and only if $p_* \in [p_*^-, p_*^+]$.

The only step left is to show that if the cost of disclosure satisfy the conditions in the proposition then exist $p_*^-, p_*^+ \in (\bar{p}, 1)$ with the required properties.

Claim 1: If $c < \frac{r+\lambda_1}{r(r+\kappa)}$, then there is a threshold $p_*^+ \in (\bar{p}, 1)$ such that $U_1\left(1|p_*^+\right) - c = 0$.

First, from equation (36) we have that $U(1|\bar{p}) = U^{ND}(1)$. Hence, $U(1|\bar{p}) - c > 0$ if and only if

$$c < \frac{\lambda_1 + r}{r(r+\kappa)}.$$

Second, $U(1|1-\epsilon) - c < 0$ for ϵ close to zero. Let

$$\beta(\epsilon) := e^{-rT(1;1-\epsilon)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(1;1-\epsilon)}\right].$$

Using equation (36) we get that

$$(1-\beta(\epsilon))\big[U(1|1-\epsilon) - c\big] < T(1;1-\epsilon) - \beta(\epsilon)c,$$

where $T(1;1-\epsilon) - \beta(\epsilon)c <$ for ϵ close to zero. Hence, by continuity there exist $p_*^+ \in (\bar{p}, 1)$ such that $U_1\left(1|p_*^+\right) - c = 0$. Moreover, equation (40) implies $U_1'\left(p_*^+|p_*^+\right) > 0$.

Claim 2: If $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$, then there there is $p_*^- < p_*^+$ such that $U'(p_*^-|p_*^-) = 0$.

First, we verify that $\lim_{p_*\downarrow\bar{p}} U_1'(p_*|p_*) < 0$. Using the HJB equation

$$U_1'(p_*|p_*) = \frac{\frac{r(r+\kappa)}{r+\lambda_1}U_1(p_*|p_*) - p_*}{\kappa(\overline{p} - p_*)}$$

Noting that $\lim_{p_*\downarrow\bar{p}} U_1(p_*|p_*) = U_1^{ND}(1) - c$, it suffices to show that

$$\frac{r(r+\kappa)}{r+\lambda_1}\big(U_1^{ND}(1) - c\big) - \bar{p} > 0,$$

which, after straightforward algebra, is satisfied if and only if $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$. Second, we verify that $\lim_{p_*\uparrow 1} U_1'(p_*|p_*) > 0$. When $p_* \uparrow 1$ the firm starts disclosing infinitely often. Hence, the cost of disclosure grows without bound. Moreover, the benefit of disclosing is bounded. Accordingly

$$\lim_{p_*\uparrow 1} \frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_* < 0$$

so, from the HJB equation, $\lim_{p_*\uparrow 1} U_1'(p_*|p_*) > 0$. By continuity there is $p_*^- \in (\bar{p}, 1)$ with the required properties. Moreover, $U_1'(p_*^+|p_*^+) > 0$ implies that $p_*^- < p_*^+$.

■

Proof of Proposition 3

From Lemma 1, $\partial U_1(p|p_*)/\partial p_* < 0$. Hence, it only remains to show that $\partial U_0(p|p_*)/\partial p_* < 0$. Differentiating (**??**) with respect to p_*, we get

$$\frac{\partial}{\partial p_*} U_0(p|p_*) = e^{-rT(p;p_*)}\Gamma(p;p_*)\frac{\partial T(p;p_*)}{\partial p_*} + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} - \frac{r\bar{p}}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\frac{\partial}{\partial p_*}U_1(1|p_*), \tag{43}$$

where

$$\begin{aligned}
\Gamma(p;p_*) &= p_* - re^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} - \frac{\bar{p}(r+\kappa)}{r+\lambda_1}e^{-(p;p_*)}\right]\Big(U_1(1)-c\Big) \\
&= p_* - re^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{(1-\bar{p})(r+\kappa)}{r+\lambda_1}e^{-(p;p_*)} - \frac{(r+\kappa)}{r+\lambda_1}e^{-(p;p_*)}\right]\Big(U_1(1)-c\Big) \\
&= \Phi(p;p_*) + \frac{r(r+\kappa)}{r+\lambda_1}e^{-(r+\kappa)T(p;p_*)}\Big(U_1(1)-c\Big) \\
&\geq 0.
\end{aligned}$$

Thus, $\partial U_0(p|p_*)/\partial p_* < 0$ as both $\partial U_1(1|p_*)/\partial p_* < 0$ and $\partial T(p;p_*)/\partial p_* < 0$.

Proof of Proposition 4

We are interested in $p_*'(\kappa)$ for p_* solving $U_1'(p_*) = 0$. Using the HJB equation and the boundary conditions we have that

$$r(r+\kappa)U_1(p_*) = (r+\lambda_1)p_*.$$

We we want to change κ keeping $\bar{p}$ constant. Noting that $\lambda_1 = \kappa\bar{p}$ we have that p_* solves

$$r(r+\kappa)U_1(p_*) = (r+\kappa\bar{p})p_*.$$

The proof is going to be by contradiction. We are going to assume that $p_*'(\kappa) > 0$ and then arrive to a contradiction.

Lemma 3. *Suppose that $p_*'(k) > 0$, then $dU_1(1|p_*(\kappa),\kappa)/d\kappa < 0$.*

Proof. Let $U_1(1|p_*,\kappa)$ be the manager's expected utility given an equilibrium p_* and mean reversion κ. Then

$$\frac{d}{d\kappa}U_1(1|p_*(\kappa),\kappa) = \frac{\partial}{\partial p_*}U_1(1|p_*(\kappa),\kappa)p_*'(\kappa) + \frac{\partial}{\partial \kappa}U_1(1|p_*(\kappa),\kappa).$$

Lemma 1 and $p_*'(\kappa) > 0$ imply that the first term is negative. With some abuse of notation let's define $\delta(\kappa, p_*) := \delta(1)$ for $\delta(1)$ in Proposition 1 as a function of κ and p_*. Thus, we have from Proposition 1 that

$$U_1(1|p_*,\kappa) = U^{ND}(1) - \frac{\delta(\kappa,p_*)}{1-\delta(\kappa,p_*)}c. \tag{44}$$

Finally,

$$\delta_\kappa(\kappa,p_*) = \underbrace{-rT_\kappa(1;p_*)}_{>0}\delta(\kappa,p_*) + \frac{re^{-rT(1;p_*)}\bar{p}(1-p_*)}{(r+\kappa\bar{p})^2} > 0$$

implies that $\frac{\partial}{\partial\kappa}U_1(1|p_*,\kappa) < 0$ completing the proof of the Lemma. ∎

Lemma 4. *Suppose that $p_*'(k) > 0$, then $\partial\big(\kappa U_1(1)\big)/\partial\kappa > 0$.*

Proof. Using the HJB equation and the boundary conditions we get

$$\kappa U_1(1) = (r+\kappa)c + (1+\kappa\bar{p}/r)p_*(\kappa) - rU_1(1)$$

By Lemma 3, given the hypothesis $p_*'(\kappa) > 0$, we have that $U_1(1)$ is decreasing in κ. Hence, $\kappa U_1(1)$ is increasing in κ. ∎

Proof. We prove the proposition by contradiction. Take $\tilde{\kappa} > \kappa$ and let $\tilde{U}_\theta$ and U_θ be the respective solutions. Suppose that $p_*'(\kappa) > 0$ so $\tilde{p}_* > p_*$. Using the HJB equation and the boundary conditions we get

$$\begin{aligned} r(r+\kappa)U_1(p_*) - r(r+\tilde{\kappa})\tilde{U}_1(\tilde{p}_*) &= (r+\kappa\bar{p})p_* - (r+\tilde{\kappa}\bar{p})\tilde{p}_* && (45)\\ &= (r+\tilde{\kappa}\bar{p})(p_*-\tilde{p}_*) + \bar{p}(\kappa-\tilde{\kappa})p_* < 0. && (46)\end{aligned}$$

In order to establish the contradiction we need to show that

$$r(r+\kappa)U_1(p_*) - r(r+\tilde{\kappa})\tilde{U}_1(\tilde{p}_*) > 0 \tag{47}$$

We can rewrite the right hand side in (45) as

$$r(r+\kappa)(U_1(1)-c) - r(r+\tilde{\kappa})(\tilde{U}_1(1)-c) = r^2(U_1(1)-\tilde{U}_1(1)) + r(\kappa U_1(1) - \tilde{\kappa}\tilde{U}_1(1)) + r(\tilde{\kappa}-\kappa)c. \tag{48}$$

By Lemma 4 we have that

$$\tilde{\kappa}(U_1(1) - \tilde{U}_1(1)) \geq \kappa U_1(1) - \tilde{\kappa}\tilde{U}_1(1) \geq 0. \tag{49}$$

Equation (49) together with (48) give us (47) and yields the desired contradiction. ∎