Banks’ Voluntary Adoption of Fair Value Accounting and Interbank Competition∗

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Abstract

We study the role of interbank competition in shaping banks’ incentives to use fair-value accounting voluntarily and the impact of fair-value discretion on banks’ competing strategies. We find that allowing banks discretion to use fair-value accounting may give rise to multiple equilibria in situations where the equilibrium would be otherwise unique. When the profitability in the banking industry is sufficiently large or bank competition is not intense, a “mild” equilibrium stands as the unique equilibrium; when the profitability is sufficiently small or bank competition is very intense, an “aggressive” equilibrium sustains as the unique equilibrium; when the profitability is intermediate or bank competition is moderate, both mild and aggressive equilibria exist. Furthermore, we show factors that may lead to a shift between the two equilibria. The analysis in this study may provide insights and help us to better understand the role of accounting in the recent crisis.

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1 Introduction

The recent initiatives toward an accounting reporting system that centers on fair value have drawn a lot of attention from both practitioners and academia. This is especially true in the banking industry, one of the most affected by fair value accounting. In fact, the effects of the adoption of fair value accounting on the banking industry have been a long-time debate. The commonly argued trade-off at stake is that, although the fair-value measurement arguably provides information in a timelier manner than the historical-cost measurement, the fair-value measurement may introduce excessive volatility, and lead to huge losses for banks that hold a large amount of financial instruments measured at the fair value. In this paper, we examine how the higher volatility introduced by the use of fair value accounting interacts with banks’ competitive decisions. More specifically, we focus on the role that interbank competition plays in shaping banks’ incentives to use fair-value accounting voluntarily and the impact of fair-value discretion on banks’ competing strategies.

In contrast to that in most other industries, competition in the banking industry is scrutinized and heavily restrained by regulators in order to preserve financial and economic stability. Despite that, however, the role of the interbank competition in shaping banks’ attitudes toward fair-value accounting remains an under-explored topic in academic research. In this paper, we intend to examine this role by considering an oligopolistic setting in which banks chose both the extent to which they use fair value accounting and the size of their loan investments. Our findings may provide some incremental insight on banks’ behavior before and after the recent financial crisis. In particular, we find that an intense competition in the banking industry may coordinate banks into an aggressive equilibrium in which they focus on short-term payoffs, neglecting the imminent risk of insolvency. A softened competition, on the other hand, may lead the industry into a mild equilibrium in which banks weigh more on the long-term survival than on current profits. We contend that these predictions may be consistent with what we observed around the recent crisis.
and may help us understand one possible incremental role that accounting played in the recent financial crisis.

We study a setting in which banks invest in loans and have discretion to determine the fraction of their investment that is reported at fair value and the fraction that is reported at historical cost. In practice, the Fair Value Option defined in SFAS 159 (for U.S. GAAP) or IAS 39 (for international standards) grants reporting entities with discretion between fair-value measurement and historical-cost measurement in reporting most financial assets such as commercial loans, derivatives and mortgages. This option is of particular importance for banks, which hold a significant amount of these financial instruments. To examine banks’ inherent incentives to use the fair-value measurement, in our model we assume banks compete with each other in a Cournot fashion and choose their reporting methods. After a bank makes the accounting and investment decisions, some uncertainty about the value of the loan investment is resolved, and the subsequent accounting adjustment recognized according to the bank’s accounting choices may affect the bank’s solvency status. If the accounting value of the bank’s equity is negative, the bank goes bankrupt and its assets are turned over to creditors (depositors). In addition, the bank forfeits its charter value and loses all future cash flows. Otherwise, if the bank’s equity is positive, the bank survives and accrues the value of future cash flows. In our model, therefore, banks make investment and accounting decisions by weighing the trade-off between boosting short-term profits at the expense of a larger bankruptcy risk versus retaining long-term future value.

We show that allowing banks’ discretion to use fair-value accounting may give rise to multiple equilibria in situations where the equilibrium would be unique in the absence of the fair value option. Two equilibria exist: a "mild" equilibrium with less investment and less use of fair-value accounting, in which banks concentrate on long-term value, and an "aggressive" equilibrium with more investment and more use of fair-value accounting, in which banks focus on short-term profits.
We also find that the competitiveness of the banking industry may influence which equilibrium prevails. When the banking industry is not very competitive, only the mild equilibrium exists. When the competitiveness of the banking industry is intermediate, the mild and the aggressive equilibria coexist, and the mild equilibrium Pareto dominates the aggressive one. However, as the competitiveness of the banking industry increases, the mild equilibrium prevails in a smaller set of parameters, while the aggressive equilibrium becomes more prevalent. In fact, when the banking industry is sufficiently competitive, only the aggressive equilibrium is sustained.

We think our model may make an incremental contribution in understanding three dramatic changes in the banking industry around the onset of the recent financial crisis: massive lending contraction, banks’ resistance to fair-value accounting, and the spike in the interest spread. Many compelling explanations for these facts have been argued, and it is not our goal to discredit them. Instead, we maintain that these changes may be incrementally explained by a shift between equilibria. Indeed, our model suggests that around the onset of the crisis the prevailing equilibrium in the banking sector could have moved from the aggressive equilibrium to the mild equilibrium. This shift of equilibrium may have contributed to the discontinuities in bank lending, fair-value measurement usage and interest spread. Our model also suggests that the shift of equilibrium may have been partly triggered by a decrease of the competitiveness in the banking industry. This seems to be consistent with the decrease in the number of competing banks that resulted from the massive amount of bank failures during the crisis.\(^1\) Certainly, several previous studies argue that during the crisis, bank failures as well as bailouts and mergers greatly strengthened the market power of big banks and softened the competition among banks (Barth, Caprio and Levine, 2004; Caprio and Peria, 2002; Nicolo and Loukoianova, 2007; Vives, 2010). Our work complements this stream of literature by revealing one of the consequences brought by the decrease in the competitiveness of

\(^1\)See http://www.fdic.gov/bank/individual/failed/banklist.html for a complete list of bank failures.
the banking sector.

The paper is organized as follows. Section 2 provides a literature review. In Section 3, we describe the main model and analyze the resulting equilibria. Section 4 discusses the empirical implications of our results. Section 5 provides a couple of robustness checks to our main results. Section 6 concludes the paper.

2 Literature Review

The extant literature has examined the effect of the fair-value accounting measurement on the reporting entities’ solvency status quite extensively. However, the connection between this effect and the market competition has often been ignored. Some studies along this line argue that the fair-value measurement introduces excessive volatility and hence leads to huge losses for institutions that hold a large amount of financial instruments measured at the fair value (e.g., banks). Therefore, a more extensive usage of fair value measurement may substantially increase the likelihood of insolvency and bank failures. For example, Allen and Carletti (2008) examine a model in which a distressed bank is forced to sell assets in an illiquid market. Because of fair-value measurement, the resulting fire-sale prices cause huge write-downs and solvency problems for otherwise sound banks. Cifuentes et al. (2005) and Heaton et al. (2010) explore the implications of fair-value measurement for the stability of the banking system and show that mark-to-market accounting may contribute to contagious bank failures and lead to economic inefficiencies. The related empirical evidence along this line of literature is mixed. Barth et al. (1995) find that fair-value measurement introduces additional volatility to banks’ earnings and causes more frequent violation of regulatory capital requirements than under historical cost accounting. Bernard et al. (1995) examine the Danish banking system and find that earnings are three to four times more volatile after mark-to-market is adopted than before. However, Laux and Leuz (2009) find little evidence for the casual relation
between excessive write-downs of banks' assets and fair value accounting.

A second stream of related literature examines the strategic use of the fair-value option by financial institutions. Most studies concentrate on investigating the opportunistic adoption of fair-value measurements. For instance, Song (2008) finds that banks' decisions to use the fair-value option are associated with opportunistic motivations. In particular, Song reports that banks remove available-for-sale assets with loss positions to boost earnings. This is, in fact, a source of concern for some regulatory institutions. Indeed, before the adoption of the fair-value option, both the Securities and Exchange Commission (SEC) and the Center for Audit Quality (CAQ) warned that choice decisions without economic merits would be inconsistent with the intent and spirit of SFAS 159 (Guthrie et al., 2011). Nevertheless, other empirical studies reach different conclusions. For instance, Chang et al. (2011) find that regular adopters' use of fair-value option is not driven by opportunistic behavior. In addition, Guthrie et al. (2011) study a sample of 72 adopters and find no systematic evidence to support the opportunistic use of the fair value option. Our study supplements previous research on fair-value option and provides another motivation for banks to choose fair-value option in a competitive environment.

Our paper is also related to the studies on the effect of limited liability and solvency concerns on firms' competitive strategies. Brander and Lewis (1986) study the limited liability effect of debt financing and find that a highly-leveraged firm may choose a very aggressive competing strategy. In a follow-up paper, Brander and Lewis (1988) further examine a strategic bankruptcy effect and suggest that firms might choose strategies to raise the chance of driving their opponents into insolvency. Several empirical studies investigate the strategic exploration of the solvency constraint. For example, Mackay and Phillips (2005) find that a firm’s financial structure depends on its position within the industry. Their results suggest that a firm’s financial structure, technology and output market decisions must be jointly determined. Similarly, Pichler et al. (2008) study the effects of
leverage on pricing and find that firms with higher leverage set higher prices. In addition, Matsa (2010) analyzes the strategic use of debt financing to improve a firm’s bargaining position and finds that strategic incentives substantially influence corporate financing decisions. Our paper contributes to this stream of literature by examining the effect of fair-value measurement on banks’ solvency status and investigating the implication of this solvency effect for banks’ competitive decisions.

3 Model

3.1 Setup

We examine a three-date setting in which $N$ risk-neutral banks compete in the loan market. The return of the loan is stochastic and subject to the realization of two successive shocks, $z_i$ and $\epsilon_i$. At date 0, each bank simultaneously determines how much to invest in loans, $q_i$, $i \in \{1, 2, ..., N\}$. In addition, each bank also decides on the portion of its loans to be recognized on a fair-value accounting basis (denoted by $\mu_i$). At date 1, each bank privately observes a shock $z_i$ before the realization of its investment return. The bank recognizes the effect of $z_i$ on the loans that are measured at fair value; the value of the loans that are measured at the historical cost is not affected.\(^2\) The recognition of shock $z_i$ may affect the bank’s status of solvency. At date 2, another shock, $\epsilon_i$, is realized, and the final outcome of the investment is fully realized which is also determined by the Cournot competition among the $N$ banks. A bank that has stayed solvent receives the profit from current operations and also accrues the value of future cash flows (i.e., the charter value), $V$. The time line of the model is as follows.

\(^2\)See SFAS 157, SFAS 159 and IAS 39.
Each bank $i$ chooses $\mu_i$ and $q_i$. Bank $i$ privately observes a shock $z_i$, and recognizes it accordingly. If a bank fails to remain solvent, it is out of business and receives 0. The shock $\epsilon_i$ is realized and the return of investment is accordingly. Banks that remain solvent receive their returns and a future value, $V$.

Time line.

We now describe and explain banks’ decisions and events at each date in more details. In the following analysis, we often refer to the expected cash flow generated by a bank’s investment in the loan assets as the value of the bank’s assets, the net expected cash flow earned by a bank as the profit, and the expected cash flow generated by each dollar of a bank’ investment net of the liability as the investment return.

**Date 0**

At date 0, bank $i$ determines how much to invest in loans, $q_i$, and the proportion of the loan investment that will be recognized on the fair-value accounting basis, $\mu_i \in [0, 1]$. In other words, $1 - \mu_i$ share of the investment is recognized on the historical-cost basis and its accounting value is affected only by events that have occurred as of the initial measurement date; the remaining $\mu_i$ share of the investment, on the contrary, is recognized on the fair-value basis and its accounting value can be affected even before the investment return is fully realized. In practice, choices between fair-value accounting and historical-cost accounting are usually granted by the Fair-Value Option.
defined in SFAS 159 (for U.S. GAAP) or IAS 39 (for international standards). According to these rules, a bank has a sufficient degree of discretion to choose between historical cost and fair value accounting when measuring financial assets and liabilities. The eligible items include, for example, commercial loans, derivatives or mortgages, etc. (see SFAS 159, paragraph 7).

In accordance with the accounting choices, banks issue their initial reports of their loan investment values. Since there is no new information on the return of the investment at date 0, the reported value based on fair-value accounting is the same as that based on the historical-cost accounting.

The value of each bank’s loan investment, $R_i$, will be realized at date 2. It will be determined by a Cournot competition among the $N$ banks, and will also be subject to the two shocks, $z_i$ and $\epsilon_i$. For simplicity, we assume a quadratic functional form for the value of the loan assets,

$$R_i(q_i, q_{-i}, z_i, \epsilon_i) = (a - q_i - b q_{-i} + z_i)q_i + \epsilon_i,$$

where $b > 0$ represents the effect of the other banks’ investments on the value of bank $i$’s assets, and $q_{-i}$ represents the other $N-1$ banks’ aggregate investment. At date 0, no one has information about the coming shocks, but the distributions of the two shocks are public information. Specifically, $z_i$ follows a uniform distribution over the interval $[-c, c]$. For simplicity, we assume that the second shock, $\epsilon_i$, is binary, where $\epsilon_i \in \{-d, d\}$, with $d > \hat{d}$. (The detailed derivation of $\hat{d}$ is in the Appendix.) We exclude the cases of small $d$ to avoid trivial scenarios because when $d$ is too small, the second shock does not play any role in a bank’s solvency status at date 2, and the bank’s solvency status is solely determined by the shock $z_i$ on date 1 regardless of the bank’s accounting choices between the fair value and the historical cost. In other words, the timing of information is not relevant. By focusing on cases with larger $d$, we consider the scenarios in which the timing of
information matters and a significant amount of uncertainty remains unresolved until date 2. We further assume that \( E[\epsilon_i] = 0 \), that is, the chance of a positive shock is the same as the chance of a negative shock.\(^3\)

In addition, we assume that a portion of the bank’s total investment, \( \gamma \in (0, 1) \), is financed with debt (mostly through deposits). In accordance with the U.S. GAAP, the bank recognizes its liabilities (deposits) at date 0 at their historical cost, \( \gamma q_i \).

With this setup, bank \( i \)’s expected assets value at date 0, \( A_i^{t=0} \), can be expressed as follows:

\[
A_i^{t=0} = E_{z_i}[E_{\epsilon_i}[R_i|z_i]] = (a - q_i - b q_{-i})q_i.
\]

We assume that banks are subject to a solvency examination upon every accounting report and must sustain solvency. Otherwise, the bank is insolvent and bankrupt.

**Date 1**

At date 1, we assume that bank \( i \) privately observes the first shock to its investment, \( z_i \). As the bank receives new information about the forthcoming realization of its assets value, the bank recognizes the change by the observed \( z_i \) in the loan investment’s expected value in its accounting report for the \( \mu_i \) proportion of its investment. This adjustment may affect the bank’s status of solvency. If the bank retains a positive equity value after recognizing the accounting changes, the bank remains solvent; otherwise, the bank is out of business and receives zero.

In accordance with accounting rules, the arrival of the new piece of information, \( z_i \), has different implications for the investment reported on a fair-value basis and the investment reported on a historical-cost basis. For the investment measured at the fair value, since learning \( z_i \) changes the bank’s expectations of future investment value, bank \( i \) is required to report a change in the value

\(^3\)We also examined a setting in which \( \epsilon_i \) is uniformly distributed in the interval \([-d, d]\) for a robustness check. We find our main results hold qualitatively when \( d \) is sufficiently large.
of the investment. As a result, the new accounting value for the fair-valued investment is:

\[ A^FV_i = \mu_i E_{\epsilon_i} [R_{\epsilon_i} | z_i] = \mu_i (a - q_i - b q_{-i} + z_i) q_i. \]

For the investment measured at the historical cost, however, at date 1 the new piece of information does not affect the accounting value of the investment measured under the historical-cost accounting, because the investment return is not fully realized until date 2. Therefore, the investment measured on the historical-cost basis remains unchanged at the value as of the recognition date \( t = 0 \):

\[ A^{FC}_i = (1 - \mu_i) E_{z_i} [E_{\epsilon_i} [R_{\epsilon_i} | z_i]] = (1 - \mu_i) (a - q_i - b q_{-i}) q_i. \]

The accounting value of bank \( i \)'s total assets at date 1 is \( A_i^{1=1} = A_i^{FC} + A_i^{FV} \). Notice that we assume the discount factor to be zero for simplicity.

After bank \( i \) recognizes the effect of \( z_i \) on the value of its total assets, bank \( i \) is subject to a solvency examination. That is, to be able to remain solvent and continue operating, bank \( i \) is required to maintain a positive value of equity. The solvency constraint at date 1 can be written as

\[ A_i^{1=1} \geq \gamma q_i. \]

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4 Notice that in our setup, we model a full form of fair-value accounting which allows banks to recognize a positive change of expectation (a positive \( z_i \) ) in the value of the investments. In practice, another possible form of fair-value accounting is the Lower of Cost or Market (LCM) which, in contrast to the full form of fair-value accounting, restraints banks from recognizing a positive \( z_i \). Our results still hold if we consider LCM in the model. The reason is that in our model, accounting reports can affect banks' utilities only by changing banks' solvency probabilities. In the two equilibria \( SE_1 \) and \( SE_2 \), a bank in equilibrium attains a non-negative profit (equity value) for an average shock \( z_i = 0 \), which implies that the bank can become insolvent only if \( z_i < 0 \). Hence when a positive \( z_i \) is realized, although a bank that follows LCM can not recognize the fair-value gain, it remains solvent, just as if the bank had followed the full form of fair-value accounting and recognized the positive \( z_i \) in its accounting report. As a result, allowing LCM will not alter a bank's solvency probability and will yield the same utility for the bank. Therefore, our results are still valid under LCM.

5 Notice that since the bank's liabilities are usually measured by their historical costs, the value of these liabilities are unaffected by \( z_i \) and still reported at the initial recognition value \( \gamma q_i \).
It can be rewritten to be:

$$z_i \geq -\frac{a - \gamma - q_i - b q_i}{\mu_i}.$$ 

For convenience we denote the term on the right-hand side of the above inequation to be $\hat{z}_i$ (i.e., a bank remains solvent if and only if $z_i \geq \hat{z}_i$).\(^6\)

If a bank violates the solvency constraint, the bank files for bankruptcy. As a result of the limited liability, the bank attains a payoff of 0 instead of a loss on its investment but loses its future value $V$.

**Date 2**

At date 2, the second shock $\epsilon_i$ is realized and the total investment return is fully realized. The bank then recognizes the realized value in its accounting report accordingly. The accounting value of bank $i$'s total assets at date 2 is equal to:

$$A_{t=2}^i = R_i(q_i, q_{-i}, z_i, \epsilon_i) = (a - q_i - b q_{-i} + z_i)q_i + \epsilon_i.$$ 

The bank is solvent at date 2 if and only if it maintains a non-negative value of equity. The solvency constraint at date 2 can be written as

$$A_{t=2}^i \geq \gamma q_i.$$ 

This solvency constraint can also be simplified to:

$$\epsilon_i \geq \hat{\epsilon}_i = -(a - \gamma - q_i - b q_{-i} + z_i)q_i.$$ 

It can be verified that when $d > \hat{d}$, this solvency requirement is satisfied if $\epsilon_i = d$ and violated if $\epsilon_i = -d$. A bank that passes the solvency test at both date 1 and date 2 receives the investment

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\(^6\)Since $z_i$ is distributed in the support $[-c, c]$, we conveniently set $\hat{z}_i = -c$ if $\hat{z}_i < -c$, and set $\hat{z}_i = c$ if $\hat{z}_i > c$. 

profits after fully repaying the debt holders (depositors). In addition, the bank is also entitled to a future value (a charter value), \( V \), which represents the sum of all future expected discounted cash flows. Therefore, bank \( i \)'s utility, \( u_i \), is equal to

\[
u_i = \frac{1}{4c} \int_{\hat{z}_i}^{c} [(a - \gamma - q_i - b q_{-i} + z_i)q_i + d] dz_i + \frac{1}{2} \Pr(z_i \geq \hat{z}_i) V.
\]

In bank \( i \)'s utility function, the first term is the bank’s investment profit and the second term is the expected future value conditional upon bank \( i \)'s solvency. \( \Pr(z_i \geq \hat{z}_i) \) is the probability of solvency, which is equal to \( \frac{c - \hat{z}_i}{2c} \) given that \( z_i \) is uniformly distributed over \([-c, c]\).

\[3.2 \text{ The Equilibrium}\]

We now derive the equilibrium. We first define the equilibrium as follows:

**Definition 1 Equilibrium:** An equilibrium in this game is defined as a pair of actions for each bank \( \{q_i^*, \mu_i^*\}, i \in \{1, 2, \ldots, N\} \), such that at date 0, each bank chooses the optimal \( \{q_i^*, \mu_i^*\} \) to maximize its utility given other banks choose \( \{q_{-i}^*, \mu_{-i}^*\} \):

\[
\max_{q_i, \mu_i} \frac{1}{4c} \int_{\hat{z}_i(q_i, \mu_i, q_{-i}^*)}^{c} [(a - \gamma - q_i - b q_{-i}^* + z_i)q_i + d] dz_i + \frac{1}{2} \Pr(z_i \geq \hat{z}_i(q_i, \mu_i, q_{-i}^*)) V.
\]

Now we characterize the symmetric equilibria of the model. Proposition 1 summarizes banks' equilibrium decisions.

**Proposition 1** Denote two interior equilibria, \( SE_1 = \{q_{SE1}^*, \mu_{SE1}^*\} \) and \( SE_2 = \{q_{SE2}^*, \mu_{SE2}^*\} \). Denote a corner equilibrium, \( SE_{\mu=0} \) in which banks choose \( \mu^* = 0 \). Denote \( K = \frac{a - \gamma}{c} \). The symmetric equilibria of the model are as follows:7

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7 Notice that in some region, we also have another corner solution in which banks choose \( \mu^* = 1 \). However, in this equilibrium, at date 0, the bank is already insolvent since the accounting value of its equity is negative. Therefore, this equilibrium is no longer valid. See the proof the Proposition for details.
1) when $N \leq \frac{2K-1}{b} + 1$, for $V \in [0, V_2]$, $SE_1$ is the unique equilibrium;

2) when $\frac{2K-1}{b} + 1 < N \leq \frac{5K-1}{b(1-K)} + 1$, $SE_1$ exists if $V \in [V_3, V_2]$, $SE_2$ exists if $V \in [0, V_1]$;

3) when $\frac{5K-1}{b(1-K)} + 1 < N$, for $V \in [0, V_1]$, $SE_2$ is the unique equilibrium;

4) for $V > \max(V_1, V_2)$, $SE_{\mu=0}$ is the unique equilibrium.

$SE_1$ is composed of investment decision $q_{SE_1}^*$ and fair-value accounting decision $\mu_{SE_1}^*$, where

$$q_{SE_1}^* = \frac{a + c - \gamma + 2 \sqrt{(a + c - \gamma)^2 - 4[b(N - 1) + 3](V + d)}}{2[b(N - 1) + 3]},$$

$$\mu_{SE_1}^* = \frac{\{a - \gamma - [b(N - 1) + 1]q_{SE_1}^*\}q_{SE_1}^*}{V + d + \{a - \gamma - [b(N - 1) + 1]q_{SE_1}^*\}q_{SE_1}^*};$$

$SE_2$ is composed of investment decision $q_{SE_2}^*$ and fair-value accounting decision $\mu_{SE_2}^*$, where

$$q_{SE_2}^* = \frac{a - \gamma}{b(N - 1) + 1},$$

$$\mu_{SE_2}^* = \frac{V + d}{V_1}.$$

$V_1, V_2, V_3$ are functions of $(a, b, c, \gamma, N)$ as shown in the appendix.

In our model, using fair value measurement is costly to a bank. This is because as the bank measures more assets at fair value, the accounting value of its assets become more volatile. As a result, the bank’s insolvency risk increases, which increases the chance that the bank loses its future continuation value $V$. In addition, when the bank fails to be solvent, it also loses the proceeds generated from the loan investment realized at date 2.\textsuperscript{8} This is consistent with the common explanation for banks’ objections to the fair-value measurement. However, interestingly, our analysis shows that in equilibrium, as long as the future value is not too high, banks voluntarily choose to measure a non-zero portion of their assets at fair value.

\textsuperscript{8} Notice that from the bank’s point of view (as an equity holder), the profit from the loan investment is always non-negative since the bank is protected by the limited liability rules.
To understand the non-zero fair-value measurement in equilibrium, notice that a bank’s usage of fair-value accounting is determined by its competition with others.\textsuperscript{9} In particular, the bank’s choices of the fair value measurement $\mu_i$ and the investment $q_i$ are bundled such that the bank’s investment level is strictly increasing in its usage of fair value accounting. This is because measuring more of its assets at fair value (increasing $\mu_i$) increases the bank’s insolvency risk at date 1. The increase of the insolvency risk, in return, forces the bank to increase its investment. More specifically, when the states of the world are bad (i.e., $z_i < \hat{z}_i$), the bank is unable to repay the depositors and becomes insolvent. That is, in these states of the world, the bank is liquidated and hence the marginal profit is irrelevant. In other words, the bank chooses the investment level only by considering the marginal profits of the states in which the bank stays solvent ($z_i > \hat{z}_i$). As increasing $\mu_i$ raises the insolvency risk, the region in which the bank is insolvent becomes larger (\(\hat{z}_i\) increases). As we illustrated in Figure 1, the states of negative marginal profits between \(\hat{z}\) and \(\hat{z}'\), which were in the solvent region, get into the insolvent region as $\mu_i$ increases. Therefore, more low-marginal-profit states become irrelevant to the bank’s investment decision, which increases the weighted average of the marginal profits and motivates the bank to invest more. In other words, by choosing more fair-value accounting, banks substitute negative marginal profits with zero marginal profit. We sometime refer to this effect the \textit{substitution effect}. Because of this substitution effect, a higher $\mu_i$ has a one-to-one correspondence to a more aggressive investment strategy.\textsuperscript{10} In our model, the Cournot competition among banks induces them to overinvest in loans. Because of the “bundling” between the fair value accounting and the investment choice, each bank measures a non-zero portion

\textsuperscript{9}In our model, the use of fair-value measurement is not driven by ex post opportunistic motivations (e.g., Song [2008] shows that banks measure assets with gains at fair value while measuring assets with losses at historical cost to report an accounting profit). In fact, banks make the fair-value measurement decision before knowing the return of the underlying assets.

\textsuperscript{10}An alternative way to explain this solvency effect is to regard the bank’s average investment profit as a call option. The bank earns a positive profit only in good states ($z_i > \hat{z}_i$); otherwise, the bank earns 0. That is, the bank’s payoff is convex in the state, $z_i$. As increasing $\mu_i$ causes $\hat{z}_i$ to rise, the bank’s payoff becomes more convex, which motivates the bank to take a more aggressive strategy and expand its investment.
of its assets at fair value in accordance with its overinvestment strategy. That is, in equilibrium, the bank uses fair value accounting as a complimentary tool to better compete with other banks and pursue short-term investment profits, despite the increase in the risk of insolvency and the likely loss of the long-term value.

We find that allowing banks discretion to use fair-value accounting may give rise to multiple equilibria in situations where the equilibrium would be otherwise unique. Two sets of factors determine the equilibria in our model: 

\( (b, N) \) and \( K \equiv \frac{a-\gamma}{c} \). \( (b, N) \) represents the competitiveness of the banking industry. \( K \), in some sense, can be regarded as a measurement of the net profitability of loan investment, normalized by the degree of interim uncertainty in the investment. When the bank competition is less intense (i.e., \( N \leq \frac{2K-1}{b} + 1 \)) or the profitability \( K \) is sufficiently large, \( SE_1 \) stands as the unique equilibrium; when the bank competition is more intense (i.e., \( \frac{5K-1}{b(1-K)} + 1 < N \)) or the profitability \( K \) is sufficiently small, \( SE_2 \) becomes the only sustainable equilibrium; when the bank competition is moderate (i.e., \( \frac{2K-1}{b} + 1 < N \leq \frac{5K-1}{b(1-K)} + 1 \)) or the profitability \( K \) is intermediate, two equilibria, \( SE_1 \) and \( SE_2 \) coexist; when the future value \( V \) is sufficiently large, banks choose not to use fair-value accounting at all.

Figure 1: The shift of the solvency threshold \( \hat{z}_i \) and the increase in the usage of fair value measurement

In the diagram, the x-axis represents the solvency threshold \( z \), and the y-axis represents the usage of fair value measurement. The shift caused by an increase in the usage of fair value measurement is shown as the movement from \( \hat{z} \) to \( \hat{z}' \). The diagram illustrates the region of insolvent and solvent banks, with the shift indicating the change in the usage of fair value accounting.
We illustrate how the distribution of the equilibria is jointly determined by the number of banks $N$ in the competition and the future value $V$ in Figure 2. (The analysis for $K$ and $V$ is similar.) In the left dark-shaded region of Figure 2, only equilibrium $SE_1$ exists, while in the light-shaded region on the right hand side, only equilibrium $SE_2$ exists. In the middle overlapped region, the two equilibria, $SE_1$ and $SE_2$, coexist. In the upper blank region, the corner equilibrium, denoted by $SE_{\mu=0}$, in which banks choose no fair value ($\mu_i = 0$) prevails, because when the future value $V$ is sufficiently high, banks would choose an extremely conservative strategy, using no fair-value accounting to avoid any insolvency risk and to secure the precious future value. To make our analysis interesting, in later analysis we only consider the nontrivial scenario in which the future value $V$ is not too high and banks choose a non-zero amount of fair-value accounting.

3.3 Properties of the Equilibria

We now characterize the properties of the equilibria, $SE_1$ and $SE_2$. Analysis of the characteristics of these equilibria also helps us to better understand banks’ strategies in the competition. We
Proposition 2

1) Banks originate more loans in $SE_2$ than in $SE_1$;

2) Banks voluntarily measure more assets at fair value in $SE_2$ than in $SE_1$;

3) In $SE_2$, the expected return of each bank’s investment is zero; in $SE_1$, the expected return of each bank’s investment is positive;

4) $SE_1$ Pareto dominates $SE_2$.

In $SE_2$, banks focus more on short-term profits than on long-term value. They adopt a more aggressive and risky strategy. Banks tend to invest more in loans and thereby supply more credits to borrowers. In addition, banks also voluntarily report more assets at fair value. By doing this, banks expose themselves to a higher risk of reaching insolvency and losing future continuation value. As a result, the fierce competition destroys profits, leading to a zero expected return on each bank’s investment. We refer to this zero-expected-return equilibrium, $SE_2$, as the aggressive equilibrium. On the contrary, in $SE_1$, banks concentrate more on long-term value. They choose a more conservative and safer strategy. Banks invest less in loans, and are more reluctant to use the fair-value measurement. To the banks, it seems unnecessarily risky to put themselves on the verge of insolvency. In this “mild” competition environment, each bank’s investment project yields a positive expected return. We refer to this positive-expected-return equilibrium, $SE_1$, as the mild equilibrium. Proposition 2 also suggests that $SE_1$ is the Pareto-dominant equilibrium when both the mild equilibrium and the aggressive equilibrium coexist. Therefore, if banks were able to coordinate on the Pareto dominant equilibrium, $SE_1$ would prevail in the region in which the two equilibrium coexist.

As illustrated in Figure 2, in the middle overlapping region, we have two equilibria. The multiplicity of equilibria is due to the existence of a strategic complementarity between banks’
decisions. According to the theoretical prediction by Cooper and John (1988), under very general conditions, existence of strategic complementarity is a necessary condition for the multiplicity of equilibria. Consistent with their prediction, we find numerically that in our model banks’ decisions sometimes are strategic complements.\footnote{We are able to show analytically that when the bank’ choice of fair value accounting is below a threshold, banks’ decisions are supermodular to each other, which is a stronger notion than the strategic complementarity in a vector space (see Foldes and Hammer, 2005). The explicit conditions for the existence of the strategic complementarity are not analytically tractable, but we are able to verify the existence of complementarity numerically.} Intuitively, as opponents’ investments increase, they affects bank $i$’s investment decision in two ways. On one hand, they directly reduce bank $i$’s marginal profit for any realized $z_i$ and motivate bank $i$ to invest less. That is, this effect alone would induce banks’ investment decisions to be strategic substitutes. This is a direct consequence of the Cournot competition and we call this effect the Cournot effect. On the other hand, an increase in $q_{-i}$ also raises bank $i$’s insolvency risk at date 1. This is because more fierce competition (higher $q_{-i}$) lowers bank $i$’s expected assets value for any realized $z_i$ and thereby leads to a higher chance of bank $i$’s insolvency. As explained before, the increase of the insolvency risk, in return, forces bank $i$ to increase its investment and leads banks’ investment decisions toward strategic complementarity. For our convenience, we call this effect the solvency effect. Overall, the relationship between banks’ investment decisions is determined by the trade-off between the strategic substitutability induced by the Cournot effect and the strategic complementarity induced by the solvency effect. When the solvency effect dominates, banks’ decisions are strategic complements.

To explain the pattern of the equilibria illustrated in Figure 2 and Proposition 1, we focus on the results related to the number of banks $N$, since the explanations for $K$ and $b$ are similar. When the competition among banks is very intense ($N$ is very large), the return on each bank’s investment mostly depends on the aggregate investment of others instead of the investment of its own. In other words, each bank behaves almost like a price taker in a competitive equilibrium. As suggested by the theory of competitive equilibrium, when the competition is near perfect, the underlying equilibrium
is unique under very general conditions.\textsuperscript{12} In this equilibrium, each bank invests aggressively until the expected return for each dollar of the investment reaches zero.\textsuperscript{13} That is, it is the aggressive equilibrium that sustains. On the other hand, when the number of competing banks is small, the investment decision by one bank can induce a strong adverse effect on others’ investment decisions. That is, the Cournot effect is strong and dominates the solvency effect. As a result, banks’ investment decisions are strategic substitutes, leading to a unique competitive equilibrium in which each bank’s investment yields a positive expected return because of the limited intensity of competition. That is, when the number of competing banks is small, it is the mild equilibrium that prevails. Now consider the case when the competition among banks is of an intermediate level. Both the mild and the aggressive equilibria may coexist because in this region the trade-off between the Cournot and the solvency effect is more balanced. As a result, banks’ investment decisions can be either strategic complements or substitutes. As Cooper and John (1988) predict, in our model a combination of strategic complementarity and substitutability leads to the emergence of multiple equilibria of distinct characteristics.

3.4 Comparative Statics

We now examine how the investment level $q_i$ and the accounting choice $\mu_i$ vary with the primitives $(a, b, c, \gamma, V)$ in the equilibria $SE_1$ and $SE_2$, respectively. We summarize the comparative statics in the proposition below.

**Proposition 3** The equilibrium strategies, $q^*_{SE_2}$, $\mu^*_{SE_2}$, $q^*_{SE_1}$ and $\mu^*_{SE_1}$, vary with the primitives $(a, b, c, \gamma, V, N)$ as follows:

\textsuperscript{12}In particular, Mas-Colell et al (1995) show that when products are gross substitute to each other, which are satisfied in our Cournot setting, the competitive equilibrium is unique.

\textsuperscript{13}Notice that in the aggressive equilibrium of our model, each bank earns a small yet positive expected profit in spite of the zero net expected investment return. This is because, by the virtue of limited liability, the bank is not liable for its outstanding debts when it is insolvent. Therefore, the bank earns a profit when the realized investment return is positive but bypasses the loss when the realized return is negative. Overall, the bank earns a positive expected profit.
1. \( q_{SE2}^*, \mu_{SE2}^*, q_{SE1}^* \) and \( \mu_{SE1}^* \) vary with the primitives monotonically as described in the table below, where “+”, “-”, “ind” and “nm” denote “positive association,” “negative association,” “being independent,” and “non-monotonic,” respectively:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( q_{SE2}^* )</th>
<th>( \mu_{SE2}^* )</th>
<th>( q_{SE1}^* )</th>
<th>( \mu_{SE1}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>( nm )</td>
</tr>
<tr>
<td>( b )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( nm )</td>
</tr>
<tr>
<td>( N )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( nm )</td>
</tr>
<tr>
<td>( c )</td>
<td>ind</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( nm )</td>
</tr>
<tr>
<td>( V )</td>
<td>ind</td>
<td>+</td>
<td>-</td>
<td>( nm )</td>
</tr>
</tbody>
</table>

2. if \( N > \frac{2K}{b} + 1 \), \( \mu_{SE1}^* \) increases with \( V \), while if \( N \leq \frac{2K}{b} + 1 \), \( \mu_{SE1}^* \) decreases with \( V \);

3. if \( V > \hat{V} \), \( \mu_{SE1}^* \) is decreasing in \( a \), increasing in \( b \), \( N \) and \( \gamma \), while if \( V < \hat{V} \), \( \mu_{SE1}^* \) is increasing in \( a \), decreasing in \( b \), \( N \) and \( \gamma \). \( \hat{V} \) is defined as the following:

\[
\hat{V} = \frac{c^2}{4} \left[ \frac{a - \gamma}{c} - \frac{b(N - 1) - 1}{4} - d \right].
\]

Most of the comparative statistics listed in Proposition 3 appear intuitive except the non-monotonic variations of \( \mu_{SE1}^* \) with the primitives \((V, a, b, \gamma)\). To understand the intuition of the non-monotonic variations of \( \mu_{SE1}^* \), we first show the expression of \( \mu_{SE1}^* \):

\[
\mu_{SE1}^* = \frac{\left\{a - \gamma - \left[ b(N - 1) + 1 \right] q_{SE1}^* \right\} q_{SE1}^* + \frac{V}{\text{The future value}}}{\text{The average investment profit}}.
\]

The expression suggests that in equilibrium \( SE1 \), the portion of assets that is measured at the fair value, \( \mu_{SE1}^* \), is equal to the ratio of the average investment profit to the sum of the average
investment profit and the future value. Intuitively, in equilibrium \( SE_1 \), a bank determines the accounting policy \( \mu \) by optimally comparing the long-term continuation value \( V \) and the short-term investment profit. On the one hand, if the future value \( V \) contributes more to the bank’s objective function, the bank chooses to measure a smaller amount of assets at the fair value to avoid insolvency and losing the future value; on the other hand, if the short-term investment profit weighs more in the bank’s objective function, the bank instead exercises the fair-value option more aggressively to exploit the substitution effect of fair-value accounting, which in turn increases the bank’s investment profit. Note that since \( \mu^*_{SE_1} \) is concave in the investment profit, the investment profit plays a more important role in determining \( \mu^*_{SE_1} \) when the investment profit is small or the future value \( V \) is large.

Now consider how a change of \( V \) may affect the trade-off. On the one hand, an increase of the future value \( V \) encourages a bank to apply a smaller portion of fair-value accounting to retain the future value. This is the direct effect of \( V \). On the other hand, a larger \( V \) reduces the investment level \( q^*_{SE_1} \), which in turn changes a bank’s short-term investment profit.\(^{14}\) Note that because of the substitution effect of fair-value accounting, banks are encouraged to choose a higher investment level and in equilibrium all banks over-invest compared with the optimal level that maximizes a bank’s utility when there is no fair-value option. Therefore, a larger \( V \) reduces the investment level, which ameliorates the over-investment and in turn boosts the investment profit. This is the indirect effect of \( V \). When \( N > \frac{2K}{\rho} + 1 \), a fierce competition (a large \( N \)) or a lower investment profitability (a small \( K \)) leads to a smaller investment profit.\(^{15}\) When the investment profitability is very low, the investment profit becomes more important in determining \( \mu^*_{SE_1} \), which makes the indirect effect of \( V \) dominant. Therefore, when \( N > \frac{2K}{\rho} + 1 \), a higher future value encourages banks

\(^{14}\)Intuitively, a higher future value motivates a bank to compete less aggressively.

\(^{15}\)Note that a fierce competition (a large \( N \)) or a poor investment return (a small \( K \)) also would indirectly reduce a bank’s investment level \( q^*_{SE_1} \), which in turn increases a bank’s investment profit. However, the direct effect more than offsets the indirect effect and leads to a smaller investment profit.
to adopt the fair-value option more aggressively. The intuition for the case of $N \leq \frac{2K}{b} + 1$ can be derived analogously.

The intuition for the variations of $\mu_{SE_1}^*$ with $(a, b, N, \gamma)$ is similar. On the one hand, an increased profitability (increased $\alpha$ and/or decreased $\gamma$) or a milder competition (decreased $N$ and/or $b$) directly boosts the investment profit for banks. This is the direct effect of $(a, \gamma, b, N)$. On the other hand, a higher profitability or a less intensive competitive environment also encourages banks to invest more (by choosing a larger $q_{SE_1}^*$), which reduces the investment profit. This is the indirect effect of $(a, \gamma, b, N)$. When $V < \hat{V}$, a small future value can no longer restrain a bank’s over-investment incentive, which entails a large investment amount $q_{SE_1}^*$. As a result, the direct effect of $(a, \gamma, b, N)$ becomes dominant. Hence, an increased profitability (increased $\alpha$ and/or decreased $\gamma$) or a milder competition (decreased $N$ and/or $b$) leads to an overall larger investment profit, which in turn implies more use of fair-value accounting. The intuition for the case of $V > \hat{V}$ can be derived analogously.

4 Empirical Implications

Our model may provide implications on some observed phenomena in the banking industry and help us better understand banks’ use of fair value option. First, the result in Propositions 1 and 2 regarding the equilibria may provide us insights about the dramatic changes in the banking industry around the onset of the recent crisis. The first is the massive contraction in the lending market. In the pre-crisis period, the credit market was booming, partly due to the trend of deregulation. Many empirical studies document that, before the crisis, banks originated abundant amounts of mortgage and commercial loans. For example, Keys, Mukherjee, Seru, and Vig (2010) document that the total amount of securitized mortgage loans reached $3.6 trillion in 2006. Berger and Bouwman (2009) also document the massive size of lending by banks before the crisis. After the burst of
the crisis, however, banks suddenly became very selective in financing loans and as a result, at the aggregate level, the lending market was substantially contracted. For example, Ivashina and Scharfstein (2010) report that in U.S., new loans to large borrowers contracted by 79% relative to the credit boom in 2007 (see also Cetorelli and Goldberg [2011] for international evidence). Second, during the crisis, banks became very resistant to fair-value accounting rules. For example, in an investigation report on mark-to-market accounting prepared by the SEC, at the first quarter-end in 2008, only 4% of total assets were reported at fair value on a voluntary basis. Partly because of the intensive lobbying by large banks, the Financial Accounting Standards Board was forced to relax fair-value accounting rules. Third, during the crisis, there was a jump in the interest spread earned by banks. Before the crisis, the interest spread for banks stayed very low, with an average around 0.5%. After 2008, the interest spread increased to about 1% and stayed at around 1.3% which is close to the record high. Our model may provide an explanation for these changes as a result of an equilibrium shift. It may be the case that, around the onset of the crisis, the prevailing equilibrium in the banking sector moved from the aggressive equilibrium to the mild equilibrium. It may be the shift of equilibrium that causes discontinuities in bank lending, fair-value measurement usages and the interest spread. In addition, our model’s prediction is also consistent with the observation of the change in the competition environment of banking industry around the recent crisis. Several studies document that, during the crisis, the competitiveness of the banking industry has been substantially reduced (Vives, 2010; Barth, Caprio and Levine, 2004; Caprio and Peria, 2002; Nicolo and Loukoianova, 2007). Our model predicts that when the number of banks \( N \) dramatically decreases, the mild equilibrium may replace the aggressive one as the prevailing

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18 In our model, we define interest spread as the difference between the expected return of investment on loans and the deposit cost.
19 http://online.wsj.com/article/SB10001424127887324660404578197782701079650.html
equilibrium. This seems to be consistent with what we observed in the crisis. That is, as the competitiveness in the banking industry greatly declines, banks shift from aggressive investing and more use of fair-value accounting to conservative investing and less use of fair-value accounting.

We illustrate these empirical implications of our analysis in the following figure.

<table>
<thead>
<tr>
<th>Before Crisis</th>
<th>After Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive Equilibrium</td>
<td>Mild Equilibrium</td>
</tr>
<tr>
<td>Banks are aggressive in lending, use more fair-value accounting, and earn a low interest spread.</td>
<td>Banks are conservative in lending, reluctant to use fair-value accounting, and earn a high interest spread.</td>
</tr>
</tbody>
</table>

Empirical Implications.

In addition, our results in Proposition 3 regarding the comparative statics may also provide some empirical implications on banks’ use of fair value accounting. Conventional wisdom believes that banks tend to use more fair value accounting when the profitability of the banking industry is high. This is consistent with our prediction for the aggressive equilibrium. As shown in Proposition 3, in equilibrium $SE_2$, when the profitability of banking industry (represented by $a - \gamma$) is high, or when the competition is softened (represented by $(b, N)$), banks indeed use more fair value accounting. However, our analysis also shows that in the mild equilibrium, it is possible that we observe less usage of fair value accounting even though the profitability of banks is higher or the competition is less fierce. This happens when the future value is more important ($V > \hat{V}$) and banks concentrate...
on long-term value. This prediction may be consistent with the casual observation that after the onset of the financial crisis, although the competition is less fierce due to the massive amount of bank failures, banks are more reluctant to use fair value accounting. For example, in October 2008, intensive lobbying activities by banks forced the Financial Accounting Standard Board (FASB) to relax and suspend fair value accounting rules for selected financial assets (Bowen et al, 2010; Bischof et al, 2010). Although there are other factors that may influence banks’ usage of fair value accounting, the effects we show in our model provide one possible explanation on banks’ reduced usage of fair value after the onset of the financial crisis. We leave more sophisticated empirical tests of our predictions for future studies.

5 Robustness Analyses

5.1 Endogenous Future Value

Recall that in our main setting, we assume an exogenous future value for a bank, $V$, to maintain the tractability of our analysis. In this section, we numerically examine a model with an endogenous future value and show that our main results remain valid.

We expand the main setting by assuming that banks that pass the solvency test at date 1 will repeatedly play the Cournot game at date 2 and choose a new investment amount. In this way, the future value $V$ in the main setting is replaced by the outcome of the second investment determined by the Cournot competition in the second round. We denote the investment amount choices in the first and the second periods to be $q_{i1}$ and $q_{i2}$, respectively. For simplicity, we assume that in the second period, the value of the bank’s investment is not stochastic and is completely determined by the Cournot competition among banks.

We solve the model by backwards induction. As in the main setting, if bank $i$ fails the solvency
test, it goes out of the business and receives 0. Now suppose bank \( i \) passes the solvency test. Consider first the scenario that \( M \) banks, including bank \( i \), remain in the business and compete in the second round in a Cournot game. Standard cournot results imply that each bank chooses \( q_{i2} \) equal to
\[
q_{i2}^M = \frac{a - \gamma}{2 + b(M - 1)},
\]
and each bank receives a payoff for the second round
\[
u_{i2}^M = \left[ \frac{a - \gamma}{2 + b(M - 1)} \right]^2.
\]
To compute bank \( i \)'s expected payoff \( u_{i2} \) before the bank makes the second-round investment decision, we need to compute the probability that \( M \) banks, including bank \( i \), pass the solvency test. Recall that bank \( i \) passes the solvency test if and only if it recognize a \( z_i \) that is higher than the threshold \( \hat{z}_i \) (\( z_{i1} > \hat{z}_i \)) and a \( c_i = d \). Therefore, the probability that bank \( i \) passes the solvency test is:
\[
Prob(z_i > \hat{z}_i) = \frac{c - \frac{\hat{z}_i}{2}}{c - \frac{1}{2}}.
\]
Conditional on bank \( i \) being solvent, we now compute the probability that the other \( M - 1 \) banks out of \( N - 1 \) banks are solvent. Since \( z_i \)s for each bank are independent, this probability is equal to
\[
Prob(M - 1 \text{ banks stay solvent}) = C_{N-1}^{M-1} [Prob(z_i > \hat{z}_i)]^{M-1}[1 - Prob(z_i > \hat{z}_i)]^{N-M},
\]
where \( C_{N-1}^{M-1} \) is a combination number.

Now bank \( i \)'s expected utility in the second period can be computed as:
\[
u_{i2} = Prob(z_i > \hat{z}_i) \sum_{M=1}^{N} [Prob(M - 1 \text{ banks stay solvent}) u_{i2}^M].\]
The first-round game can be formulated similarly as in the main setup. Each bank chooses a set of decisions \((q_{i1}^*, \mu_{i1}^*)\) to maximize the total utility in the two rounds:

\[
\max_{q_{i1}, \mu_{i1}} \frac{1}{4c} \int_{\bar{z}_i(q_{i1}, \mu_{i1}, q_{-i1})}^{c} [(a - \gamma - q_{i1} - b q^*_{-i1} + z_i)q_{i1} + d] dz_i + u_2(q_{i1}, \mu_{i1}, q^*_{-i1}).
\]

By parameterizing the model, we solve numerically for the equilibrium \((q_{i1}^*, \mu_{i1}^*)\). We find that the main results are still valid: when the interbank competition is restrained, only the mild equilibrium exists; when the competitiveness is intermediate, the mild and the aggressive equilibrium coexist. In addition, as the competition becomes more intensive, the mild equilibrium prevails in a smaller set of parameters, while the aggressive equilibrium becomes more prevalent.

5.2 Uniformly Distributed \(\epsilon_i\)

In our main setting, we assume a binary second shock, \(\epsilon_i\), to maintain the tractability of our analysis. In this section, we numerically examine a setting in which the second shock \(\epsilon_i\) is uniformly distributed in the interval \([-d, d]\) and show that our main results hold qualitatively if \(d\) is sufficiently large.

As in the main setting, a bank is solvent if and only if it passes the solvency test at both date 1 and date 2. The solvency constraint at date 1 can be formulated as follows:

\[
z_i \geq \hat{z}_i = -\frac{a - \gamma - q_i - b q_{-i}}{\mu_i}.
\]

Similarly, the solvency constraint at date 2 is:

\[
\epsilon_i \geq \hat{\epsilon}_i = -(a - \gamma - q_i - b q_{-i} + z_i)q_i.
\]
Therefore, the bank stays solvent if and only if it receives a $z_i \geq \hat{z}_i$ and a $\epsilon_i \geq \hat{\epsilon}_i$. In this case, the bank receives accrued profits from current operations as well as the future value $V$. Otherwise, the payoff to the bank is zero. Each bank chooses a set of decisions $(q_{i1}^*, \mu_{i1}^*)$ to maximize the total utility in the two rounds:

$$\max_{q_{i1}, \mu_{i1}} \frac{1}{4cd} \int_{\hat{z}_i(q_{i1}, \mu_{i1}, q_{i1}^*)}^{c} \int_{\hat{\epsilon}_i(q_{i1}, \mu_{i1}, q_{i1}^*)}^{d} \left[(a - \gamma - q_{i1} - b q_{i1}^* + z_i)q_{i1} + \epsilon_i + V\right] dz_i d\epsilon_i.$$

By parameterizing the model, we solve numerically for the equilibrium $(q_{i1}^*, \mu_{i1}^*)$. We find that the main predictions are still valid as long as $d$ is above some threshold.

6 Conclusions

We study the role of interbank competition in shaping banks’ incentives to use fair-value accounting voluntarily and the impact of fair-value discretion on banks’ competing strategies. We find that allowing banks discretion to use fair-value accounting may give rise to multiple equilibria in situations where the equilibrium would be otherwise unique. When the profitability in the banking industry is sufficiently large or bank competition is less intense, a “mild” equilibrium stands as the unique equilibrium; when the profitability is sufficiently small or bank competition is very intense, only an “aggressive” equilibrium is sustained; when the profitability is intermediate or bank competition is moderate, both equilibria exist. Furthermore, we show factors that may lead to a shift between these two equilibria. The analysis in this study may provide insights for us and help us to better understand the role of accounting in the recent crisis.
References


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Appendix

The derivation of \( \hat{d} \):

\( \hat{d} \) is the threshold such that for \( d \geq \hat{d} \), in equilibrium the bank is solvent at date 2 if \( \epsilon_i = d \) and insolvent if \( \epsilon_i = -d \).
Recall the solvency constraint at date 2 is

\[ \epsilon_i \geq -(a - \gamma - q_i - bq_{-i} + z_i)q_i. \]

Notice that for the bank to be solvent at date 1, \( \hat{z}_i \leq z_i \leq c \). Therefore, the right hand side of the solvency constraint at date 2 is

\[-(a - \gamma - q_i - bq_{-i} + c)q_i \leq -(a - \gamma - q_i - bq_{-i} + z_i)q_i \leq -(a - \gamma - q_i - bq_{-i} + \hat{z}_i)q_i.\]

Therefore, to make the bank solvent when \( \epsilon_i = d \), it is necessary that

\[ d \geq -(a - \gamma - q_i - bq_{-i} + \hat{z}_i)q_i.\]

Similarly, to make the bank insolvent when \( \epsilon_i = -d \), it is necessary that

\[-d \leq -(a - \gamma - q_i - bq_{-i} + c)q_i,\]

which can be simplified as

\[ d \geq (a - \gamma - q_i - bq_{-i} + c)q_i.\]

These two inequalities need to hold at both \( SE_1 \) and \( SE_2 \). Notice that at both \( SE_1 \) and \( SE_2 \), the following holds:

\[
[a - \gamma - q_{SE_1}^* - b(N - 1)q_{SE_1}^* + c]q_{SE_1}^* \geq -[a - \gamma - q_{SE_1}^* - b(N - 1)q_{SE_1}^* + \hat{z}_i]q_{SE_1}^*,
\]

\[
[a - \gamma - q_{SE_2}^* - b(N - 1)q_{SE_2}^* + c]q_{SE_2}^* \geq -[a - \gamma - q_{SE_2}^* - b(N - 1)q_{SE_2}^* + \hat{z}_i]q_{SE_2}^*.
\]
Therefore, it suffices to require only

\[ d \geq \max([a - \gamma - q_{SE_1}^* - b(N - 1)q_{SE_1}^* + c]q_{SE_1}^*, [a - \gamma - q_{SE_2}^* - b(N - 1)q_{SE_2}^* + c]q_{SE_2}^*) \] 

Through a few algebra step, it can be verified that

\[ [a - \gamma - q_{SE_1}^* - b(N - 1)q_{SE_1}^* + c]q_{SE_1}^* \leq [a - \gamma - q_{SE_2}^* - b(N - 1)q_{SE_2}^* + c]q_{SE_2}^*, \]

where \( q_{SE_1}^* \) and \( q_{SE_2}^* \) are as defined in Proposition 1. Therefore, it suffices to require

\[ d \geq [a - \gamma - q_{SE_2}^* - b(N - 1)q_{SE_2}^* + c]q_{SE_2}^* = \frac{(a - \gamma)c}{b(N - 1) + 1}, \]

so that in equilibrium, at date 2, the bank is solvent if \( \epsilon_i = d \) and insolvent if \( \epsilon_i = -d \).

**Proof of Proposition 1**

**Proof.** Define the following expressions \( V_1, V_2, V_3 \) as follows:

\[
\begin{align*}
V_1 &= c\left(\frac{c}{2} - \frac{a - \gamma}{b(N - 1) + 1}\right) - d, \\
V_2 &= \frac{(a - \gamma)c[b(N - 1) + 2] - (a - \gamma)}{b(N - 1) + 2} - d, \text{ if } N \leq \frac{2(2K - 1)}{b(1 - K)} + 1 \\
\text{and } V_2 &= \frac{(a + c - \gamma)^2}{4[3 + b(N - 1)]} - d, \text{ if } N > \frac{2(2K - 1)}{b(1 - K)} + 1 \\
V_3 &= \frac{(a - \gamma)c[b(N - 1) + 1] - 2(a - \gamma)}{b(N - 1) + 1} - d.
\end{align*}
\]

We first solve for the situation when the corner solutions exist. First, for \( \mu_i = 1 \) to be qualified as a corner solution, the derivative \( \partial u_i / \partial \mu_i \) at \( \mu_i^* = 1 \) must be positive. However, the derivative
evaluated at $\mu_i = 1$, is equal to

$$-(a - \gamma - q^*_i - bq^*_{-i})V < 0,$$

The expressions of $q^*_i$ and $q^*_{-i}$ are given by the first-order condition on $q$. Substituting these expressions into the equation above gives:

$$V < V_1 \text{ and } N > \frac{2K - 1}{b} + 1.$$  

That is, the corner equilibrium exists when $V < V_1$ and $N > \frac{2K - 1}{b} + 1$. However, in this equilibrium, at date 0, the accounting value of the bank’s equity is already negative and the bank is insolvent. That is,

$$A^t=0_i - \gamma q_i = a - \gamma - q^*_i - bq^*_{-i} < 0.$$  

Hence, this equilibrium is not valid.

Similarly, for $\mu_i = 0$ to be qualified as a corner solution, the derivative $\partial u_i / \partial \mu_i$ at $\mu_i = 0$ must be negative. The derivative evaluated at $\mu_i = 0$ is equal to

$$\lim_{\mu_i \to 0} \frac{(a - \gamma - q^*_i - bq^*_{-i})}{\mu_i^2}[(a - c - q^*_i - bq^*_{-i})q^*_i + V + d] > 0.$$  

Substituting in the expressions of $q^*_i$ and $q^*_{-i}$, we have that the corner solution exists if

$$V > \max(V_1, V_2).$$  

Now we solve the interior equilibria, $SE_1$ and $SE_2$ as in the proposition. It is straightforward to

\footnote{The detailed expressions for $q^*_i$ and $q^*_{-i}$ are available upon request.}
derive the expression for \((q^*_{SE1}, \mu^*_{SE1})\) and \((q^*_{SE2}, \mu^*_{SE2})\) as in Proposition 1 by solving the first-order conditions. For \(SE2\), the first-order conditions give real roots if and only if \(V < V_1\). We verify that the second-order sufficient condition is satisfied and the investment levels under both equilibria, \(q^*_{SE1}\) and \(q^*_{SE2}\), are strictly positive. Hence it remains to show the conditions under which the accounting choices, \(\mu^*_{SE1}\) and \(\mu^*_{SE2}\), are between 0 and 1 and the thresholds for solvency, \(\hat{z}_i\), are between \(-c\) and \(c\) under both equilibria.

Note first, under \(SE2\), the threshold \(\hat{z}^{SE2}_i = 0\) and hence is always between \(-c\) and \(c\). Under \(SE1\), the threshold \(\hat{z}^{SE1}_i\) is equal to,

\[
\hat{z}^{SE1}_i = -\frac{(a - \gamma)[b(N - 1) + 4] + c - [b(N - 1) + 2]\sqrt{(a + c - \gamma)^2 - 4[b(N - 1) + 3](V + d)}}{3 + b(N - 1)}.
\]

It is straightforward to verify that \(\hat{z}^{SE1}_i < 0 < c\). Note also that \(\hat{z}^{SE1}_i\) is decreasing in \(V\). If \(\frac{a - \gamma}{c} < \frac{b(N - 1) + 2}{b(N - 1) + 4}\), \(\hat{z}^{SE1}_i \geq \hat{z}^{SE1}_i(V_1) = -c\). Hence for \(V < V_2\) and \(\frac{a - \gamma}{c} < \frac{b(N - 1) + 2}{b(N - 1) + 4}\), \(\hat{z}^{SE1}_i \geq -c\). If \(\frac{a - \gamma}{c} > \frac{b(N - 1) + 2}{b(N - 1) + 4}\), there exists a critical value \(V_2\) such that for \(V < V_2\), \(\hat{z}^{SE1}_i \geq \hat{z}^{SE1}_i(V_2) = -c\).

Now we need to find conditions under which \(\mu^*_{SE1}\) and \(\mu^*_{SE2}\) are between 0 and 1. Since \(\mu^*_{SE2} = \frac{V}{V_1}\), \(\mu^*_{SE2} \leq 1\) gives \(V < V_1\) while \(\mu^*_{SE2} \geq 0\) gives \(V_1 > 0\), which in turn implies that \(SE1\) exists only if \(\frac{a - \gamma}{c} < \frac{b(N - 1) + 1}{2}\).

As for \(\mu^*_{SE1}\), it is equal to,

\[
\frac{(a - \gamma - [b(N - 1) + 1]q^*_{SE1})q^*_{SE1}}{V + (a - \gamma - [b(N - 1) + 1]q^*_{SE1})q^*_{SE1}}.
\]

Hence \(\mu^*_{SE1} < 1\) if \(\{a - \gamma - [b(N - 1) + 1]q^*_{SE1}\}q^*_{SE1} > 0\), which holds because \(q^*_{SE1} > 0\) and \(a - \gamma - [b(N - 1) + 1]q^*_{SE1} > 0\). Therefore, \(\mu^*_{SE1} < 1\).

It remains to show the conditions under which \(\mu^*_{SE1} > 0\). For \(\frac{a - \gamma}{c} > \frac{b(N - 1) + 1}{2}\), it can be verified that \(\mu^*_{SE1}\) is continuous in \(V\) for \(V > 0\) and decreasing in \(V\). And at the point \(V = V_1\), \(\mu^*_{SE1}\)
takes its minimum, which is positive. Therefore, $\mu_{SE_1}^*$ is always positive for $0 < V < V_1$. For $\frac{\alpha-\gamma}{c} < \frac{b(N-1)+1}{2}$, $\mu_{SE_1}^*$ is increasing in $V$ and is non-continuous at the point $V = V_5$. For $V < V_5$, $\mu_{SE_1}^*$ takes its minimum at $V = 0$, where $\mu_{SE_1}^*(0) = 1$. Therefore, for $V > 0$, $\mu_{SE_1}^* > 1$ and cannot be a part of an equilibrium. For $V > V_5$, $\mu_{SE_1}^*$ takes its maximum at $V = V_2$, where $\mu_{SE_1}^*(V_2)$ is equal to

\[
\frac{(a-\gamma)[b(N-1)+5]-c[b(N-1)+1]}{2((a-\gamma)[b(N-1)+4]+c)}.
\]

If $\mu_{SE_1}^*$ can be positive for some value of $V$, it must be that $\mu_{SE_1}^* > 0$ at its maximum $\mu_{SE_1}^*(V_2)$ which implies that $\frac{\alpha-\gamma}{c} > \frac{b(N-1)+1}{b(N-1)+5}$. Moreover, when $\frac{\alpha-\gamma}{c} > \frac{b(N-1)+1}{b(N-1)+5}$, it can be verified that at the point $V = V_3$, $\mu_{SE_1}^* = 0$. Hence, if $\frac{\alpha-\gamma}{c} > \frac{b(N-1)+1}{b(N-1)+5}$ and $V \geq V_3$, $\mu_{SE_1}^* \geq 0$. ■

**Proof of Proposition 2**

**Proof.** In this proof, we will only show the comparison between $SE_1$ and $SE_2$. It is straightforward to show $q_{SE_1}^* < q_{SE_2}^*$. In $SE_1$, the threshold $\hat{z}_i < 0$ which is implied by the first order condition on $\mu_i$,

\[
(1-\mu_i)\hat{z}_i q_i + V + d = 0.
\]

That is, in equilibrium $SE_1$, $\hat{z}_i = \frac{a-\gamma q_{SE_1}^* - [b(N-1)+1]}{\mu_i} < 0$, which can be reduced into,

\[
q_{SE_1}^* < \frac{a-\gamma}{b(N-1)+1} = q_{SE_2}^*.
\]

To show $\mu_{SE_1}^* < \mu_{SE_2}^*$. Note that for $\frac{b(N-1)+1}{b(N-1)+5} < \frac{2-\gamma}{c} < \frac{b(N-1)+1}{2}$, both $\mu_{SE_1}^*$ and $\mu_{SE_2}^*$ are increasing in $V$.

Note also $SE_1$ and $SE_2$ exist if and only if $V_4 < V < V_2$. By a little algebra, we can show, $\mu_{SE_1}^*(V_2) < \mu_{SE_2}^*(V_4)$. Therefore, for $V_4 < V < V_2$, $\mu_{SE_1}^* < \mu_{SE_1}^*(V_2) < \mu_{SE_2}^*(V_4) < \mu_{SE_2}^*$, which concludes the proof of $\mu_{SE_1}^* < \mu_{SE_2}^*$.

---

21 By Proposition 1, this is the region in which both equilibria exist.
To show $SE_1$ Pareto dominates $SE_2$, we first compute bank $i$’s utilities under the two symmetric equilibria. By substituting $V = \frac{(a+c-x)^2-x^2}{4[3+b(N-1)]}$, we have:

$$u_i^{SE_1} = \frac{(a + c + x - \gamma)((a + c - \gamma)[4 + b(N - 1)] - [b(N - 1) + 2]x)^2}{8[b(N - 1) + 3]^3 c},$$

$$u_i^{SE_2} = \frac{c(a - \gamma)}{4[b(N - 1) + 1]} + \frac{V + d}{2}.$$

Note that since $V > V_3$, it must be that $0 < x < \frac{b(N-1)+5}{b(N-1)+1}(a - \gamma) - c$. First, by computing the first-order derivative, we can straightforwardly show that the difference $u_i^{SE_2} - u_i^{SE_1}$ is strictly increasing in $x$ for $0 < x < \frac{b(N-1)+5}{b(N-1)+1}(a - \gamma) - c$. Hence, $u_i^{SE_2} - u_i^{SE_1}$ takes its maximum when $x = \frac{b(N-1)+5}{b(N-1)+1}(a - \gamma) - c$. At the point $x = \frac{b(N-1)+5}{b(N-1)+1}(a - \gamma) - c$,

$$u_i^{SE_2} - u_i^{SE_1} = -\frac{(a - \gamma)\{2(a - \gamma) - [b(N - 1) + 1]c\}^2}{4[b(N - 1) + 1]^3 c} < 0.$$

Hence, for $0 < x < \frac{b(N-1)+5}{b(N-1)+1}(a - \gamma) - c$, namely, $V_3 < V < V_1$, $u_i^{SE_2} - u_i^{SE_1} < 0$. That is, $SE_1$ Pareto dominates $SE_2$. In addition, it is straightforward to verify that in $SE_1$, the average investment return is positive; while in $SE_2$, the average investment return is zero by substituting $q_{SE_1}^*$ and $q_{SE_2}^*$ into the average investment return function, $E[R_i(q_i, q_i)]$.

**Proof of Proposition 3**

**Proof.** It is straightforward to verify these comparative statics by computing the corresponding first-order derivatives. More detailed proofs are available upon request. ■