Strategic Informed Trades, Diversification, and Expected Returns*

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September, 2012

* We appreciate the helpful comments of Ray Ball, Pingyang Gao, Christian Leuz, Doug Skinner, and other workshop participants at University of Chicago, Carnegie Mellon University, and University of Texas where earlier versions of this study have been presented. We also thank Robert Verrecchia for spurring us to model imperfectly competitive uninformed traders.
Strategic Speculation and Expected Returns for a Large Economy

Abstract

We consider effects of strategic speculation on expected returns in a noisy rational expectations equilibrium for the large economy limit of a set of finite economies in which all traders consider their impact on price. In our model, a large, informed trader profits from exploiting private information, and from capturing some of the systematic risk premium by covering the demands of noise traders. Similar to the case of competitive informed traders, we show that factor loadings (betas) explain all cross-sectional differences in expected returns. In contrast to the case with competitive informed traders, we show that private information increases the expected returns required by uninformed traders. While informative prices allow uninformed investors to glean some of the large trader’s information, a privately informed large trader diminishes her absorption risk, which dominates the effect of informative prices and leads to higher factor risk premiums. In other words, asymmetric information does not create a new, priced risk factor, but does impact the pricing of systematic payoff risks. An implication of the results in the large economy limit is that firm-specific accounting is likely to affect firm value only through cash flow (numerator) effects.
1 Introduction

Accounting research has devoted considerable interest to the effects of asymmetric information on expected returns. Prior theoretical studies show that, in economies comprised of perfectly competitive traders who can fully diversify their holdings, asymmetric information only affects expected returns via its impact on premia for systematic risks (Hughes et al. 2007; Lambert et al. 2007). In particular, cross-sectional differences in factor loadings (betas) explain any cross-sectional differences in expected returns.\(^1\) In a CARA/normal setting, the premium for systematic risk depends on the average precision of traders’ beliefs (Hughes et al. 2007; Lambert et al. 2012). Compared to a no-information benchmark, asymmetric information reduces expected returns by increasing the average precision of traders’ beliefs. For a given set of informative signals, expected returns are higher if a subset of traders observes them privately, because uninformed traders only observe a noisy version of private information via prices.

We examine whether similar results hold when modeling the economy as the limit of finite economies in which traders participate in imperfectly competitive markets, and therefore take into account the price impact of their trades. In addition, we develop new insights into how private information impacts returns. Our study relates to Lambert at al. (2012), who consider a finite economy where imperfectly competitive informed traders interact with an unbounded number of perfectly competitive uninformed investors. We derive an equilibrium that can be viewed as a multi-asset extension of Kyle (1989), where both informed and uninformed investors consider their impact on price. We then take large economy limits in order to distinguish between idiosyncratic and systematic risks, and to approximate the behavior of developed stock markets where a large number of traders exchange shares in a large number of securities.

\(^1\) When we speak of expected returns, we refer to expected return conditioned only on price. Privately informed traders may, and often do, expect returns in excess of the compensation for systematic risk.
Before examining economies with imperfect competition, we apply the results from Chamberlain (1983) and Chamberlain and Rothschild (1983) to show that the pricing of only systematic risk holds under very weak conditions, once one has assumed perfect competition among uninformed traders. We also show that a factor representation of risk entails virtually no loss of generality. This general setting does not address whether the economy represents the limit of finite economies comprised of imperfectly competitive traders, nor does it demonstrate whether or not information asymmetry creates a distinct systematic risk. We therefore develop a CARA/normal model to explore these two issues, and to more fully characterize expected returns and trading behavior. In the CARA/normal setting, we first derive a finite-economy equilibrium. We then take limits with respect to the numbers of uninformed traders and assets. Last, assuming price taking by uninformed traders in the limit and finite assets, we derive closed-form solutions for several special cases.

In order to show the maximum impact of imperfect competition among informed traders, we model the informed trader as a risk-neutral monopolist who can observe prices (i.e., places limit orders as in Kyle 1989). The informed trader plays two roles in our model: she exploits private information and provides liquidity to noise traders in order to capture the systematic risk premium. Risk neutrality maximizes the informed trader’s incentive for information-based trades, but also positions her to capture a larger portion of the systematic risk premium. Sans exploitation of private information, she absorbs half of the liquidity demand as the profit maximizing quantity. Otherwise, as in Lambert, Leuz, and Verrecchia (2012), she curbs her demands to limit the revelation of her private information to uninformed traders and, by doing

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2 A risk-averse informed trader yields qualitatively similar results in a finite economy so long as risk aversion is not too large. The similarity stems from the fact that the uninformed traders drive the unconditional expected returns. Risk-neutrality becomes crucial in the large economy limit because no trader with finite risk tolerance will bear a non-negligible amount of systematic risk in the limiting large economy.

3 Their model assumes a finite number of informed traders who anticipate the effect of their demands on price and
so, absorbs less of the liquidity demand and, hence, less of the systematic risk. In turn, uninformed traders bear greater systematic risk for which they require a higher expected return. Information asymmetry does not create a separate risk factor; rather, it increases systematic risk premia by reducing the large trader’s willingness to bear risk.\(^4\)

Diversification by uninformed traders plays a key role in driving our large economy predictions on expected returns. In the limit, uninformed traders diversify and trades in a given security represent a small fraction of an individual uninformed trader’s portfolio and a small fraction of overall trade in the security.\(^5\) This causes uninformed traders to behave as price takers. At the margin, prices depend on diversified uninformed traders’ demands and, therefore, include risk premiums only for non-diversifiable risk. If, at the margin, an uninformed trader were to infer that an individual asset is mispriced, he could initiate small trades in that security without affecting his overall risk exposure due to local risk-neutrality. Because all uninformed will draw the same inference that the asset is mispriced, all would adjust their demands. In equilibrium, perfect competition among uninformed traders would dissipate any distortions from full diversification.

We develop further insights by considering special cases. When uninformed investors do not learn from price, either because the large trader has no private information or because noise trade is so volatile as to preclude learning from prices, the risk-neutral informed trader absorbs half of the liquidity demands as the profit maximizing share of the systematic risk premium. At

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\(^4\) Diamond and Verrecchia (1991) predict a similar phenomenon in a single-firm setting where expected future market illiquidity increases large traders’ expected costs of unwinding large positions. While the settings differ, in both cases, illiquidity increases the risk that must be borne by uninformed traders and drives up expected returns.

\(^5\) In principle, uninformed investors could have some undiversified holdings so long as they do not all overweight the same individual stocks. In our model, each firm represents a small fraction of the economy so that a nontrivial fraction of the population can have a undiversified position in a given firm only if other investors take the opposite side of the trade.
the other extreme, when the variance of noise trades approaches zero, trading costs become infinitely large and the informed trader only takes infinitesimal positions. After imposing further structure, we predict that the impact of risk-sharing dominates the effect of information revealed through price. In particular, expected returns decrease in the variance of noise trades and expected returns are lower when the large trader, and therefore all traders, learn nothing beyond their prior beliefs based on public information. This contrasts with the case of perfectly competitive informed investors, where the introduction of private information reduces expected returns.

We also predict that noise traders’ portfolios play a key role in the pricing of systematic risk. If the informed trader has private information on systematic risks, then she needs to trade in fully diversified portfolios to exploit that information. She can only do so if noise traders also transact in diversified portfolios; otherwise, uninformed traders can infer her trades, similar to zero-noise-trade-variance case. As the variance of noise trades in factor portfolios increases, the informed trader’s information-based demands absorb more systematic risks causing expected returns required by uninformed traders to decrease. Even though greater noise trade variance reduces the uninformed traders’ learning about systematic risks, expected returns decline. This occurs because the informed trader’s larger positions reduce the risk borne by uninformed traders and the corresponding factor risk premium. The effect of the informed trader’s absorption of risk again dominates the effect of the information revealed to uninformed traders.

Recent empirical studies by Armstrong, Core, Taylor, and Verrecchia (2011) and Akins, Ng, and Verdi (2012) show a positive association between imperfect competition among informed traders and cost of capital. The theoretical basis is the prediction that when the number

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6 We can only examine the zero noise-trade variance case with competitive uninformed investors. The market breaks down if uninformed investors consider their price impact and there is no noise in supply (Kyle 1989, p. 335).
of informed traders is small, they trade less aggressively on their private information so as to limit the information that uninformed traders can learn from price. This lowers the average precision of information and increases the risk borne by uninformed traders, which raises cost of capital. In our model, however, returns depend solely on systematic risks, so that any cross-sectional variation in expected returns will vanish after controlling for factor loadings. Of course, it is also possible that uninformed investors are not well diversified, in which case firm-specific risks may be priced. This would allow for cross-sectional variations in cost of capital even after controlling for betas. However, this poses a conundrum in that it is unlikely that uninformed traders would irrationally under diversify and, yet, be sufficiently rational to extract information from prices. While not leading to a (within economy) cross-sectional prediction, we note that our model predicts that a decrease in liquidity from noise trades increases marginal trading costs, which leads to higher expected returns as the informed trader absorbs less of the systematic risks, suggesting an alternative direction to empirical inquiries on the effects of imperfect competition.

Another avenue for future empirical inquiries is to consider the extent of uninformed shareholders’ diversification when examining the effects of information asymmetries on expected returns. For example, Faccio, Marchica, and Mura (2011) find empirical evidence that firms with shares held by investors with well diversified portfolios care less about firm-specific risk taking. Controlling for betas in such a context, we suggest consideration of numerator, or cash flow, effects as drivers of cross-sectional differences in expected return. For an example in a moral hazard setting, Gao and Verrecchia (2012) show that idiosyncratic risk increases the risk premium paid to managers as part of a performance-based compensation package (a numerator effect), while the expected return to investors (denominator effect) depends entirely on
systematic risk.\textsuperscript{7} In their setting, better firm-specific information reduces premiums required to induce managerial effort, while better economy-wide information reduces systematic risk and, hence, expected return.\textsuperscript{8}

The remainder of the paper is organized as follows: Section 2 characterizes equilibrium expected returns for the case with minimal structure assuming competitive uninformed traders; section 3 derives equilibrium expected returns and informed demands for the finite and large economy cases; section 4 considers special cases that reinforce insights; and section 5 concludes.

2 Expected returns in a large economy with competitive uninformed traders

We begin by characterizing equilibrium prices under minimal restrictions on preferences and distributions employing principal components \textit{a la} Chamberlain (1983) and Chamberlain and Rothschild (1983). The ensuing analysis establishes the generality of a factor structure for asset payoffs and demonstrates the power of full diversification in eliminating effects of idiosyncratic risks on expected returns, notwithstanding imperfect competition with respect to exploitation of private information. While for purposes of this analysis we assume uninformed traders are competitive, we later show that uninformed traders behave constructively as price takers in the large economy limit given a factor structure and standard assumptions of CARA utility and Normal distributions.

The market consists of $N$ risky assets, each with a payoff given by the random vector $\mathbf{v}$, and traded at an equilibrium price vector $\mathbf{p}$. There is an informed trader with demand vector $\mathbf{y}$ and a large number of price-taking, uninformed traders. Each uninformed trader has utility $u(\cdot)$

\textsuperscript{7} Also, see Ou-Yang (2005) for a related moral hazard setting. See Christensen, Feltham, and Wu (2002) for a setting in which the imposition of idiosyncratic risk on managers impacts the rate of return used in residual income compensation metrics, again resulting in numerator effects of idiosyncratic risk.

\textsuperscript{8} In some cases, concerns over cash flow effects can provide incentives to reduce the information provided by the accounting system. For example, Caskey and Hughes (2012) show that conservative accounting measures, which provide distorted information, have beneficial cash flow effects because they mitigate asset substitution problems in levered firms. In their case, more informative (unbiased) accounting reports can reduce firm value.
that is monotone increasing in wealth and concave. Assuming homogeneity in trading, all uninformed traders have the same demand vector \( d \). Recasting the representative uninformed trader’s objective function for a finite number of assets, conditional on whatever information can be inferred from price, we have expected utility of:

\[
E[u(d'(v - p)) \mid p],
\]

which gives the following first-order condition expressed in terms of a pricing kernel with a stochastic discount factor \( \xi \):

\[
p = E[\xi v \mid p] = E[v \mid p] + \text{cov}(\xi, v \mid p), \quad \xi \equiv \frac{u'(d'(v - p))}{E[u'(d'(v - p)) \mid p]}.
\]

Chamberlain (1983) and Chamberlain and Rothschild (1983) show that under no-arbitrage and mild restrictions on the structure of asset payoffs, the conditional variance-covariance matrix \( \Sigma_{v|p} \) of the uninformed trader’s payoffs, \( v - p \), will have an approximate \( K \)-factor structure:

\[
BB' + \Sigma_e,
\]

where \( B \) is a conditional (on price) \( N \times K \) covariance matrix for which the \( nk^{th} \) element is the covariance of the payoff of asset \( n \) and factor \( k \), and \( \Sigma_e \) has eigenvalues that remain bounded as the number of assets \( N \to \infty \). The matrix \( \Sigma_e \) represents risks that remain small relative to the economy as the economy expands. The \( K \) factors can be normalized to have a mean zero and variance of one. The following proposition lends a useful perspective on when the restrictions described in the appendix are met including a formal definition of no arbitrage:

**Proposition 1:**

*If there is no arbitrage and the equally weighted market portfolio has positive,* but

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9 Given that price is known to the trader, the conditional variance of \( (v - p) \) equals the conditional variance of \( v \).
bounded variance, then $\Sigma_{\nu'|p}$ has an approximate factor structure.

Given $\Sigma_{\nu'|p}$ has an approximate factor structure, the set of possible portfolios (i.e., linear combinations of net asset payoffs) can be separated into a set of fully diversified portfolios and a set of undiversified portfolios. A portfolio $d$ of an uninformed trader is fully diversified if its weights on individual assets become infinitesimally small in the large economy limit:

$$\lim_{N \to \infty} \sum_{n=1}^{N} d_n^2 = 0$$

The approximate $K$-factor structure ensures that the unexpected return in any portfolio with payoff $d'(v - p)$ can be decomposed into a fully diversified portfolio that has only systematic (factor) risk due to correlation with the $K$ factors, and an undiversified portfolio that has only idiosyncratic risk and is uncorrelated with both the $K$ factors and the payoff of the fully diversified portfolios. The decomposition follows from a projection theorem similar to a least squares regression in which a dependent variable is decomposable into a predictable component and an orthogonal residual.

Returning to the uninformed trader’s first-order conditions, it must be the case that expected returns arise from the covariance of payoffs with the trader’s marginal utility:


If the uninformed trader holds a diversified portfolio in equilibrium as we expand the numbers of assets and uninformed traders $(N, M \to \infty)$, then the stochastic discount factor only includes the effect of systematic risk on marginal utility:\[^{10}]

**Proposition 2:**

If the conditions of Proposition 1 are satisfied and the equilibrium portfolio of

[^10]: The number of assets $N$ and traders $M$ must expand at the same rate; otherwise, per capita risk either approaches zero or becomes unbounded.
uninformed traders is fully diversified, then the uninformed trader’s payoff can be written as a linear combination of \( K \) diversified portfolios that span systematic risks (i.e.,
\[
d'(v - p) = d' E[v - p | p] + \beta f
\]
and prices can be written as:
\[
p = E[v | p] + B p_f + E[\xi E[e | f, p] | p], \quad \xi = \frac{u'(d'(E[v | p] - p) + \beta' f)}{E[u'(d'(E[v | p] - p) + \beta' f) | p]}, \tag{6}
\]
where \( e \) is a vector of idiosyncratic risks orthogonal to \( f \). Moreover, if \( e \) is mean-independent of \( f \), then price can be written as
\[
p = E[v | p] + B p_f \tag{7}
\]

Our last proposition identifies sufficient conditions for uninformed traders to take fully diversified positions.

**Proposition 3:**

*If noisy supply \( x \) and informed trader’s demand \( y \) are bounded with probability 1, then the uninformed traders’ equilibrium portfolio is fully diversified.*

It follows from the above propositions that the dependence of expected returns on solely systematic risks holds in fairly general settings. In particular, after controlling for betas, there are no cross-sectional effects on expected returns of imperfect competition with respect to the exploitation of private information. The ability of competitive (price-taking) uninformed traders to fully diversify across a large number of assets in a pure exchange economy ensures that only systematic risks are priced.

While we show that expected returns depend only on systematic risk, the analysis does not rule out information asymmetry creating an additional source of systematic risk. In the next section, we show that the factor pricing results as a limit of finite economies in which investors consider the price impact of their trades. In the resulting equilibrium, information asymmetry
does not create a new source of systematic risk, but does impact the price of systematic risk.

3 Equilibrium in finite and large economies with strategic uninformed traders

3.1 Finite economy

Model setup

In this section, we derive a linear equilibrium for asset prices. As in Kyle (1989), we assume that all traders observe prices when choosing their demands. We assume a payoff structure for $N$ assets as follows:

$$\mathbf{v} = \mathbf{v} + B\mathbf{f} + \mathbf{e},$$  \hspace{1cm} (8)

where $\mathbf{v}$ is an $N \times 1$ constant vector, $\mathbf{f}$ is a $K \times 1$ vector of mean zero, standard normal random variables, $B$ is an $N \times K$ constant matrix, $\mathbf{e}$ is an $N \times 1$ vector of zero-mean normal random variables uncorrelated with $\mathbf{f}$ and with covariance matrix $\Sigma_e$, which we assume has bounded eigenvalues.\(^{11}\) This implies that $\mathbf{v}$ has the covariance matrix $\Sigma_v = BB' + \Sigma_e$.\(^{12}\) The supply of risky assets, net of noise trades, $\mathbf{x}$ is a $N \times 1$ vector of normally distributed random variables with mean $\mathbf{x}$, covariance matrix $\Sigma_x$, and independent of $\mathbf{f}$ and $\mathbf{e}$. As we showed in section 2, there is very little loss in generality from assuming a factor model structure for asset payoffs.

We assume that a risk neutral informed trader has private information yielding the posterior belief that $\mathbf{v}$ has mean $\mathbf{v}_i$ and covariance matrix $\Sigma_i = \Sigma_v - \Sigma_{\tilde{v}_i}$. We assume that $\mathbf{v}_i$ is joint normally distributed with $\mathbf{f}$, $\mathbf{e}$, and $\mathbf{x}$, and that the informed trader’s posterior does not induce a conditional correlation between $\mathbf{f}$ and $\mathbf{e}$. The informed trader places

\(^{11}\) For example, if $\Sigma_e$ is diagonal with bounded elements, then it has bounded eigenvalues. Our assumption of bounded eigenvalues assumes that the $BF$ term reflects any source of risk shared by a nontrivial fraction of firms.

\(^{12}\) The assumption that $\mathbf{f}$ is a vector of standard normal random variables is without further loss of generality because the factors can be recast. For example, if the vector of factors is $\mathbf{g}$, with loadings $B_g$ and covariance matrix $\Sigma_g$, then we can write a new factor $\mathbf{f} = \Sigma_g^{-1/2} \mathbf{g}$ with $\text{var}(\mathbf{f}) = I$ and loadings $B = B_g \Sigma_g^{1/2}$, giving $BF = B_g \Sigma_g^{1/2} \Sigma_g^{-1/2} g = B_g g$. 
limit orders $y_n, n = 1, \ldots, N$ to maximize her expected payoff:

$$E_i \left[ \sum_{n=1}^{N} y_n (v_n - p) \right] = E_i \left[ y'(v - p) \right] = y'(\bar{v}_i - E_i[p]),$$

(9)

where, without loss of generality, we have normalized the risk-free rate to equal one and $y'p$ is the opportunity cost of holding those shares.

There are $M$ uninformed traders with CARA utility and risk-aversion $\lambda$. In deriving a linear equilibrium, we assume that it is common knowledge that the uninformed traders conjecture that the informed trader follows a strategy of the form:

$$y = q_0 + Q\nu(\bar{v}_i - \bar{v}) - Q\rho p,$$

and that the informed trader and traders $m' \neq m$ conjecture that trader $m$ follows a strategy of the form:

$$d_m = c_0 - C_p p.$$

(11)

These conjectures imply that uninformed trader $m$’s posterior belief about $v$ is normally distributed, allowing us to write the objective function in terms of certainty equivalents:

$$\max_{d_m} d_m' (E[v \mid p] - p) - \frac{1}{2} d_m' \Sigma_{\nu p} d_m.$$

(12)

The market clearing condition is:

$$y + \sum_{m=1}^{M} d_m = x.$$

(13)

Trading strategies

We assume that the informed trader and uninformed trader $m$ optimize against the following two residual supply curves, respectively:

$$p = \mu_i + A_i y, \quad p = \mu_u + A_u d_m,$$

(14)

where $\mu_i$ ($\mu_u$) does not depend on $y$ ($d_m$). Substituting the informed trader’s residual supply curve (14) into the objective (9) yields the following two equivalent expressions of the informed
trader’s first-order condition, $0 = \bar{v}_i - \mu_i - (A_i + A'_i)y$:

\[ y = (A_i + A'_i)^{-1}(\bar{v}_i - \mu_i), \quad \text{and} \quad y = (A'_i)^{-1}(\bar{v}_i - p), \quad \text{(15)} \]
given the second-order condition that $A_i + A'_i$ is positive definite.

When extracting information from price, uninformed trader $m$ can apply the conjectured uninformed strategy (11) and informed strategy (10) to the market clearing condition (13) to extract a noisy signal $s$ of the informed trader’s information $\bar{v}_i - \bar{v}$:

\[
 q_0 + Q_v(\bar{v}_i - \bar{v}) - Q_p p + d_m + (M - 1)(c_0 - C_p p) = x \\
 \Rightarrow s = Q_v^{-1}(Q_p p + \bar{x}_q - q_0 - d_m - (M - 1)(c_0 - C_p p)) = \bar{v}_i - \bar{v} - Q_v^{-1}(x - \bar{x}).
\]

This yields the posterior beliefs:

\[
 E[v | p] = E[v | s] = \bar{v} + \Sigma_{\eta}(\Sigma_{\eta} + \Sigma_x)^{-1}s, \quad \Sigma_{v|p} = \Sigma_v - \Sigma_{\eta}(\Sigma_{\eta} + \Sigma_x)^{-1}\Sigma_{\eta},
\]

where $\Sigma_x = Q_v^{-1}\Sigma_x(Q_v^{-1})'$ and $E[v | p]$ do not vary with $d_m$. Recall from (14) the uninformed trader $m$’s residual demand curve $p = \mu_u + A_u d_m$. The first-order condition for (12) implies that $d_m$ satisfies the following equivalent expressions:

\[
 d_m = (A_u + A'_u + A\Sigma_{v|p})^{-1}(E[v | p] - \mu_u), \quad \text{and} \quad d_m = (A'_u + A\Sigma_{v|p})^{-1}(E[v | p] - p),
\]

where $p$ depends on $d_m$, and the second-order condition is $(A_u + A'_u + A\Sigma_{v|p})^{-1}$ positive definite.

Substituting the conjectured informed strategy (10) and uninformed strategy (11) into the market clearing condition (13), and rearranging yields $p = \mu_u + A_u d_m$, where $\mu_u = A_u(q_0 + Q_v(\bar{v}_i - \bar{v}) - Q_p p + (M - 1)c_0 - x)$ and $A_u = (Q_p + (M - 1)C_p)^{-1}$.

**Equilibrium**

The following proposition summarizes the equilibrium:

**Proposition 4:**

The equilibrium price, expected returns, informed trade, and uninformed trade are:
\[ p = \mathbb{E}[v \mid p] - (A'_i + A \Sigma_{v|p}) d_m \]
\[ = \bar{v} - \frac{1}{M} (A'_i + A \Sigma_{v|p}) (A_i + A')^{-1} (I + \Sigma_{\bar{v}} \Sigma_{s}^{-1}) A' \bar{x} + A (A_i + A')^{-1} (\bar{v}_i - \bar{v} - A'(x - \bar{x})) \], \hspace{1cm} (19)

\[ \mathbb{E}[v - p] = \frac{1}{M} (A'_i + A \Sigma_{v|p}) (A_i + A')^{-1} (I + \Sigma_{\bar{v}} \Sigma_{s}^{-1}) A' \bar{x}, \hspace{1cm} (20) \]

\[ y = (A_i + A')^{-1} \left( A_i - \Sigma_{\bar{v}} \Sigma_{s}^{-1} A' \right) \bar{x} + (A_i + A')^{-1} \left( \bar{v}_i - \bar{v} + A'(x - \bar{x}) \right), \hspace{1cm} (21) \]

\[ d_m = \frac{1}{M} (x - y) = \frac{1}{M} (A_i + A')^{-1} \left( (I + \Sigma_{\bar{v}} \Sigma_{s}^{-1}) A' \bar{x} - (\bar{v}_i - \bar{v} - A'(x - \bar{x})) \right). \hspace{1cm} (22) \]

The first expression for price in (19) differs slightly from the expression in (2). With CARA utility and normally distributed beliefs, the adjustment for risk is:

\[ \text{cov}(\xi, v \mid p) = -A \Sigma_{v|p} d_m, \quad \xi = \frac{u'(d_m(v - p))}{\mathbb{E}[u'(d_m(v - p))|p]}, \hspace{1cm} (23) \]

giving \( \mathbb{E}[v \mid p] + \text{cov}(\xi, v \mid p) = \mathbb{E}[v \mid p] - A \Sigma_{v|p} d_m \). The extra \( A'_i \) term in (19) arises from uninformed traders considering their impact on prices. The \( \Sigma_{\bar{v}} \Sigma_{s}^{-1} \) term in the informed trader’s absorption of the average supply \( \bar{x} \) reflects the informed trader’s share of sensitivity to revealing information. This can also be written in terms of the two trader types’ impacts on prices:

\[ (A_i + A')^{-1} (A_i - \Sigma_{\bar{v}} \Sigma_{s}^{-1} A') \bar{x} = (A')^{-1} \left( (A')^{-1} + M \left( A'_i + A \Sigma_{v|p} \right)^{-1} \right)^{-1} \bar{x}, \hspace{1cm} (24) \]

where the absorption depends on the magnitude of informed trader’s responsiveness \( (A')^{-1} \) to expected profits from (15), relative to the total of the informed trader’s responsiveness and the uninformed traders’ responsiveness \( M \left( A'_i + A \Sigma_{v|p} \right)^{-1} \) to expected profits from (18).

### 3.2 Large economy limit

We now derive an equilibrium for a large economy by letting \( N, M \to \infty \), with \( N / M \)
approaching a finite constant which, without loss of generality, we assume to be one.\textsuperscript{13} The following proposition characterizes expected returns:

**Proposition 5:**

Assume that the informed trader’s second-order condition is satisfied in the limiting economy and $\Lambda$ has bounded eigenvalues. Then, the risk premium approaches the following as $N, M \to \infty$, and depends only on systematic risks:

$$ E[v - p] \to A \frac{1}{M} B \Sigma_{f|p} B' (A_i + \Lambda')^{-1} \left( I + \Sigma_{\pi} \Sigma_{s}^{-1} \right) \Lambda' \bar{x}, $$

where $B$ is the $N \times K$ matrix of factor loadings and the $K \times 1$ vector of factor risk premia is

$$ A \frac{1}{M} \Sigma_{f|p} B' (A_i + \Lambda')^{-1} \left( I + \Sigma_{\pi} \Sigma_{s}^{-1} \right) \Lambda' \bar{x}. $$

It is clear from (25) that information asymmetry affects expected returns only via the factor risk premia (26). This implies that differences in firms’ betas explain any cross-sectional variation in expected returns. Information asymmetry also does not create any new factors, but instead impacts the pricing of the systematic components of the firms’ fundamental payoffs.

### 3.3 Risk-neutral uninformed traders

A useful special case for characterizing price sensitivities to informed demands in closed form is to assume uninformed traders’ are risk neutral; i.e., $A = 0$:

**Corollary 4.1:**

If uninformed traders are risk-neutral, then the following $A_i$ satisfies the equilibrium conditions:

$$ A_i = \sqrt{\frac{M}{M-1}} \Sigma_x^{-1/2} \left( \Sigma_{\Delta}^{1/2} \Sigma_{\pi} \Sigma_{s}^{1/2} \right) \Sigma_x^{-1/2}, $$

which implies the following expected returns and informed trade:

\textsuperscript{13} The assumption that $N / M$ approaches a constant means that both $N$ and $M$ grow at the same rate; otherwise, the ratio $N / M$ approaches either zero or infinity. Hughes, Liu, and Liu (2007), Lambert, Leuz and Verrecchia (2007), and Ou-Yang (2005) employ similar assumptions.
\[ E[v - p] = \frac{1}{2M} A \bar{x}, \quad y = \frac{1}{2M} \bar{x} + \frac{1}{2} A^{-1} \left( \bar{v} - \bar{v} + A (x - \bar{x}) \right). \]  

The solution for \( A \) resembles the solution from Caballé and Krishnan (1994).\(^{14}\) The first term in the informed trade \( y \) in (28) reflects the informed trader’s average holdings. In the large economy limit, expected returns approach zero and the informed trader bears none of the average supply \( \bar{x} \). When uninformed traders are risk-averse, the informed trader extracts some of the risk premium by bearing some of the average supply even in the limit.

4 Further analysis of equilibrium demands and expected returns

4.1 Extreme cases of market liquidity with price taking uninformed traders

Price taking by the uninformed enables removal of the price impact matrix, \( A_u \), from the expressions (19) and (20) for prices and expected returns, and from the equilibrium relation that determines \( A_i \).\(^{15}\) This eases the analysis allowing us to examine the impact of noise trade variance as a disguise for the informed trader’s actions by comparing the extremes of uninformative prices versus certain noise trade. We cannot examine the certain noise trade case when uninformed investors consider their price impact because the market breaks down as in Kyle (1989, p. 335). When there is no uncertainty about noise trade, the price impact matrices \( A_i \) and \( A_u \) become unbounded and both informed and uninformed traders hold positions approaching zero, as can be seen by the inversion of the price impact matrices in (15) and (18).

In the case of price-taking uninformed investors, we have the following corollary:

**Proposition 6:**

\(^{14}\) Informed traders in Caballé and Krishnan (1994) submit market orders to perfectly competitive market makers. Their expression for \( A_i \) is equivalent to expression (27) after removing the \( \sqrt{\frac{M}{M+1}} \) term and multiplying by one half.

\(^{15}\) As in Caballé and Krishnan (1994), it is possible that there exists another, non-symmetric, matrix \( A_i \) satisfying the equilibrium conditions.

Alternatively, we could assume that risk-aversion \( A \) increases in proportion to the number \( M \) of uninformed traders, and take limits as \( M \to \infty \). This is analogous to the Kyle’s (1989, section 8) analysis of a market with free entry of uninformed traders.
If uninformed traders are price-takers, then as noise trade variance becomes unbounded and price becomes uninformative ($\Sigma_x \to \infty$):

\[
\Sigma_{v|p} \to \Sigma_v, \quad E[v - p] \to \frac{1}{2M} A \Sigma_v \bar{x}, \quad y \to \frac{1}{2} x + \frac{1}{2} \left( \frac{1}{M} A \Sigma_v \right)^{-1} (\bar{v}_i - \bar{v}).
\] (29)

As noise trade becomes certain ($\Sigma_x \to 0$):

\[
\Sigma_{v|p} \to \Sigma_v - \frac{1}{2} \Sigma_\eta, \quad E[v - p] \to \frac{1}{2M} A (2 \Sigma_v - \Sigma_\eta) \bar{x}, \quad y \to \frac{1}{2} (x - \bar{x}) \approx 0.
\] (30)

The extreme noise-trade-variance cases of Proposition 6 provide closed-form solutions for $A$. Note that uninformed traders learn more in the zero-variance case, where the informed trader reveals half of her information. Despite this, the expected returns are higher in the zero-variance case. This occurs because the informed trades approach zero, leaving virtually all of the payoff risk to be borne by uninformed traders. As in Kyle (1989), the informed trader reduces the size of her positions in such a way that they approach zero, but the price reaction to her trades become large so that there is an impact on what uninformed traders learn. Comparing expected returns in (30) to (29), we see that expected returns are lower in the case where uninformed traders learn nothing. This is because greater noise trade variance induces the informed trader to absorb more risk, which more than compensates for the diminished learning from prices. In the next subsection, we provide a setting in which we demonstrate a monotonic relation between noise trade variance and expected returns.

4.2 Trading on information about systematic versus idiosyncratic risks

In section 3, we showed the existence of an equilibrium conditional on the informed trader’s price impact matrix $A$. In this section, we impose some structure on the nature of the informed trader’s information and noise trade that enables us to both show the existence and uniqueness of the linear equilibrium and perform additional analysis of expected returns and
informed trader demands.

If there are no redundant assets, we can, without loss of generality, write the covariance matrix of \( \mathbf{v} \) as \( \Sigma_v = \mathbf{T} \Theta \mathbf{T}' \) where \( \mathbf{T} \) is an orthonormal matrix of eigenvectors \( (\mathbf{T} \mathbf{T}' = \mathbf{I}) \) and \( \Theta \) is a diagonal matrix containing the \( N \) strictly positive eigenvalues of \( \Sigma_v \), all of which are strictly positive. Denoting the \( k^{th} \) largest eigenvalue by \( \theta_k \), \( \theta_1 > \theta_2 > \cdots > \theta_N > 0 \), we can then write \( \Sigma_v = \sum_{k=1}^{N} \theta_k \mathbf{t}_k \mathbf{t}_k' \), where \( \mathbf{t}_k \) denotes the \( k^{th} \) column of \( \mathbf{T} \). A portfolio is a linear combination of the individual assets’ payoffs. Investors can trade a portfolio \( f_k = \mathbf{t}_k' \mathbf{v} \) with price \( p_{jk} = \mathbf{t}_k' \mathbf{p} \) that isolates the risk represented by the \( k^{th} \) eigenvalue, with

\[
\text{var}(f_k) = \text{var}(\mathbf{t}_k' \mathbf{v}) = \sum_{j=1}^{N} \theta_j \mathbf{t}_j' \mathbf{t}_j \mathbf{t}_k = \theta_k. ^{16}
\]

The \( N \) portfolio payoffs of the form \( \mathbf{t}_k' \mathbf{v} \) are the principal components of \( \mathbf{v} \), which we refer to as ‘factors’ for the sake of brevity.

We make two assumptions that allow us to derive an expression for \( \Lambda \) in terms of trade in each factor. This facilitates the separate analysis of trade and information on systematic versus idiosyncratic risks. First, we assume that the covariance matrix of the informed trader’s posterior mean \( \mathbf{v} \) has the same eigenvectors as \( \Sigma_v \), \( \text{var} (\mathbf{v}) = \mathbf{T} \Theta \mathbf{T}' \), where \( \Theta \preceq \Theta' \) is a diagonal matrix. In other words, the informed trader’s posterior variance, \( \text{var} (\mathbf{v}) = \Sigma_v - \Sigma_{\mathbf{v}} = \mathbf{T} (\Theta - \Theta') \mathbf{T}' \). This assumption implies that the informed trader’s information does not alter the covariance structure between assets. It applies, for example, if the informed trader obtains a signal \( \mathbf{v} + \eta \) where \( \eta \) is normally distributed, independent of \( \mathbf{v} \), with a covariance matrix having eigenvectors \( \mathbf{T} \). Second, we assume that the noise trade covariance matrix has the eigenvectors \( \mathbf{T} \) so that \( \Sigma_x = \mathbf{T} \Theta \mathbf{T}' \), consistent with noise traders transacting directly in the factor portfolios. We denote by \( \theta_k \) the \( k^{th} \) diagonal element of \( \Theta \), corresponding to the informed trader’s information on the \( k^{th} \) factor,

\(^{16}\text{The equality follows because the eigenvectors are orthonormal or, equivalently, } \mathbf{t}_k' \mathbf{t}_k = 1 \text{ and } \mathbf{t}_j' \mathbf{t}_j = 0 \text{ for } j \neq k.\)
whose prior variance is $\theta_k$. We denote by $\theta_{k\nu}$ the $k^{th}$ diagonal element of $\Theta_x$, corresponding to the variance of noise trade in the portfolio with payoffs $f_k = t_t'v$ that represents the $k^{th}$ factor.

Under these assumptions, we obtain the following expression for expected returns:

**Proposition 7:**

*If the covariance matrix of the informed trader’s posterior mean and noise trade satisfy $\Sigma_i = T\Theta_i T'$ and $\Sigma_x = T\Theta_x T'$, $\Theta_i$ and $\Theta_x$ diagonal, then the expected return on the $k^{th}$ factor and the vector of expected returns are:

$$E[f_k - p_{jk}] = \left(\frac{1}{2M-1} \lambda_k + A \frac{1}{M} \theta_{k/p}\right) \frac{1}{2} \left(1 + \frac{\theta_i}{\lambda^2 \theta_x}\right) \bar{x}_{jk},$$

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$$E[v - p] = T E[f - p_f],$$

(31)

where $\theta_{k/p} = \theta_k - \frac{\theta_i}{\theta_i + \lambda^2 \theta_x}$ is the uninformed traders’ posterior variance for the $k^{th}$ factor, and $\lambda_k$ is the $k^{th}$ element of the diagonal matrix $\Theta_\lambda = T'A'T$ and $\bar{x}_{jk} = t_t' \bar{x}$. The informed trade in the $k^{th}$ factor is:

$$y_{jk} = \frac{1}{2} \left(1 - \frac{\theta_i}{\lambda^2 \theta_x}\right) \bar{x}_{jk} + \frac{1}{2} t_t' (\bar{\nu} - \bar{v}) + \frac{1}{2} t_t' (x - \bar{x}).$$

(32)

Consistent with $K$ systematic factors, we assume that the $K$ largest eigenvalues of $\Sigma_i$ become unbounded as the economy grows ($N,M \to \infty$). Before taking the large economy limit, we state the following corollary that gives the effect of noise trade variance on expected returns and $\lambda_k$. This effect plays a role in deriving the large-economy expected returns.

**Corollary 7.1:**

*The price impact $\lambda_k > \sqrt{\frac{M}{M-1} \theta_{k\nu}}$. Expected returns $E[f_k - p_{jk}]$ and the price impact $\lambda_k$ are both decreasing in the variance $\theta_{k\nu}$ of noise trades. Informed trade $y_{jk}$ is increasing in $\theta_{k\nu}$.***
As $\theta_{kx} \to \infty$ ($\theta_{kx} \to 0$), $\lambda_k \to A \frac{2M-1}{2(M-1)} \frac{1}{M} \theta_k$ ($\lambda_k$ becomes unbounded at rate $\theta_{kx}^{-1/2}$).

$$E[f_k - p_{fr}] \to \frac{1}{2} \frac{2M-1}{2(M-1)} \frac{1}{M} \lambda_k \bar{x}_k$$

(E[$f_k - p_{fr}$] becomes unbounded), and

$$y_{fr} \to \frac{1}{2} \bar{x}_{fr} + \frac{M-1}{2M-1} \frac{1}{M} \lambda_k x^T_i (\bar{v} - \bar{v}) + \frac{1}{2} x^T_i (x - \bar{x})$$

$$y_{fr} \to \frac{1}{2} \bar{x}_{fr} + \frac{1}{2} t^T_i (x - \bar{x})$$

Corollary 7.1 implies that, although noise trades reduce information available to uninformed investors, the increase in the informed trader’s risk-absorption dominates. Expected returns therefore decline as the variance of noise trade increases.

In this setting, the effect of distorted risk-sharing is so strong that any private information increases expected returns, as we state in the following corollary:

**Corollary 7.2**

Any private information ($\theta_{ki} > 0$) yields higher expected returns than no private information. Formally, $E[f_k - p_{fr}; \theta_{ki} \neq 0] > E[f_k - p_{fr}; \theta_{ki} = 0]$ for any $\theta_{ki} \in (0, \theta_k]$.

The relation between private information $\theta_{ki}$ and expected returns is non-monotonic; however, the above corollary shows that expected returns are lower if there was no private information at all. This stands in contrast to settings where all investors are perfectly competitive and any information that increases the average precision of investors’ beliefs yields a reduction in expected returns (Lambert et al. 2012).

The following proposition characterizes the expected returns and informed holdings in a large economy ($N, M \to \infty$).

**Proposition 8:**

If the covariance matrix of the informed trader’s posterior mean and of noise trade satisfy $\Sigma_{\bar{v}} = T\Theta T'$ and $\Sigma_x = T\Theta_x T'$, $\Theta_i$ and $\Theta_x$ diagonal, then, as the economy expands ($N, M \to \infty$), the expected return and informed trade $y_k = t^T_i y$ on the $k^{th}$ systematic factor are:
assuming that the variance $\theta_{ki}$ of noise trades for the $k^{th}$ factor grows with the economy (at order $N$). Otherwise, relaxing the above assumption, the expected return and informed trade on the $k^{th}$ systematic factor are:

\begin{align*}
\mathbb{E}[f_k - p_{jk}] \rightarrow \frac{1}{2} \left(1 + \frac{\theta_{ki}}{\lambda_k^2 \theta_{ki}}\right) A \frac{1}{M} \theta_{k|p} \bar{x}_{jk}, \\
y_{jk} \rightarrow \frac{1}{2} \left(1 - \frac{\theta_{ki}}{\lambda_k^2 \theta_{ki}}\right) \bar{x}_{jk} + \frac{1}{2 \lambda_k} t_k' (\bar{v}_i - \bar{v}) + \frac{1}{2} t_k' (x - \bar{x}).
\end{align*}

(33)

The expected returns on idiosyncratic factors all approach zero and the corresponding informed trade approaches

\begin{align*}
y_{jk} \rightarrow \frac{1}{2 \lambda_k} t_k' (\bar{v}_i - \bar{v}) + \frac{1}{2} t_k' (x - \bar{x}), \\
\lambda_k \rightarrow \sqrt{\frac{\theta_{ki}}{\theta_{ki}}} \quad (\lambda_k \rightarrow 0)
\end{align*}

where the information-based portion of informed trade approaches $\frac{1}{2} \sqrt{\frac{\theta_{ki}}{\theta_{ki}}} t_k' (\bar{v}_i - \bar{v})$ (infinity) if the corresponding variance $\theta_{ki}$ of noise trades remains bounded (becomes unbounded).

Analogous to Proposition 5, Proposition 8 implies that only systematic risks are priced in a large economy. Furthermore, it highlights the key role played by noise traders. In particular, if the set of noise traders who transact in systematic factor portfolios does not grow with the economy, the informed trader has no disguise for her trades. Similar to Corollary 7.1, this drives down the magnitude of informed trades. The proof of Corollary 7.1 shows that the noise in price, $\lambda_k^2 \theta_{ki}$, is increasing in $\theta_{ki}$ so that noise trades reduce the information available to uninformed traders; however, the effect of reduced risk-absorption by the informed trader dominates. This implies that the sharing of systematic risks depends heavily on the extent to which noise traders transact in diversified portfolios.

5 Conclusion

Our analysis makes a strong case for concluding that, in a large economy, where idiosyncratic risks are fully diversifiable, effects of imperfect competition with respect to the exploitation of private information on expected returns do not extend beyond systematic risk
premia. Borrowing from the principal components approach of Chamberlain (1983) and Chamberlain and Rothschild (1983), we showed that only systematic risks are priced when competitive uninformed traders are able to fully diversify. We then imposed a standard factor structure for asset payoffs, CARA utility, and normal distributions and derived characterizations of a linear equilibrium in which uninformed traders as well as an informed trader are strategic in setting their demands. Taking the large economy limit, we again found that only systematic risks are priced. Although our characterizations of equilibrium expected returns and informed trades are up to a matrix of price sensitivities to the informed trader’s demands, we derived closed-form solutions for a series of special cases lending further efficacy to results in the more general cases. Notably, adding structure to the nature of the informed trader’s private information and noise trades, we again show that only systematic risks are priced. Further insights on the tension between risk absorption and exploitation of private information by the informed trader and implications for expected returns are exhibited by these cases.

A relevant implication for accounting is that in such an idealized setting where the numbers of securities and uninformed investors become very large, strategic speculation on private information and the portion inferred from prices lead to no cross-sectional predictions concerning the impact of imperfect competition with respect to that information. Rather, we suggest that researchers interested in the consequences of accounting policies on cost of capital might find it useful to focus more on cash flow effects of accounting information, so-called “numerator effects”.
References


Appendix

Notation for Section 2

In an exchange economy with \( N \) securities, denote portfolio weights by the \( N \)-vector \( d \) with payoff \( r_N = d'v \) with \( \lim_{N \to \infty} r_N = r \), portfolio prices \( p_{rN} = d'p \to p_r \) and portfolio expected payoffs \( \bar{v}_{rN} = d'\bar{v} \to \bar{v}_r \).

We define the following conditions as in Chamberlain and Rothschild (1983) and Chamberlain (1983):

**Condition A: No arbitrage**

1) There are no riskless, costless portfolios with nonzero payoffs. Formally, if \( \text{var}(r_N) \to 0 \) and \( p_{rN} \to 0 \), then \( \bar{v}_{rN} \to 0 \).

2) There are no riskless portfolios with positive cost and non-positive payoffs. Formally, if \( \text{var}(r_N) \to 0 \) and \( p_{rN} \to p_r \neq 0 \), then \( \bar{v}_r \neq 0 \) and \( \bar{v}_r \propto p_r \).

If Condition A1 were violated by some limit portfolio \( \hat{r} \), then every trader would gain by adding long (short) positions in \( \hat{r} \) if \( \bar{v}_r > 0 \) (\( \bar{v}_r < 0 \)). If Condition A2 were violated by some limit portfolio \( \hat{r} \) with \( p_\hat{v} > 0 \) (\( p_\hat{v} < 0 \)) and \( \bar{v}_r \leq 0 \) (\( \bar{v}_r \geq 0 \)), then every trader would gain by adding short (long) positions in \( \hat{r} \) without incurring any future obligations.

**Condition B: Covariance matrix restrictions**

For finite \( N \), denote the uninformed posterior covariance matrix of \( v_n \) by \( \Sigma_{vlpN} \) and denote the \( k \)th largest eigenvalue of \( \Sigma_{vlpN} \) by \( \delta_{kN} \).

1) In the limiting economy, there are \( K < \infty \) unbounded eigenvalues: \( \sup_N \delta_{kN} = \infty \) and \( \sup_N \delta_{k+1,N} = \delta_{k+1} < \infty \).

2) In the limiting economy, there are no redundant securities: \( \inf_N \delta_{NN} = \delta_\infty > 0 \).
Chamberlain and Rothschild (1983) and Chamberlain (1983) prove the following:

**Lemma A1:**

If Condition A holds and if Condition B holds with respect to the uninformed information set, then:

\[
\Sigma_{v|pN} = B_N B_N^T + \Sigma_{eN}, \tag{A1}
\]

where the nkth element of the \( N \times K \) matrix \( B_N \) is \( \text{cov}(v_n, f_k \mid p) \) and \( \{ \Sigma_{eN} \} \) is a sequence of positive semi-definite matrices with bounded eigenvalues. The set \( P_1 \) of fully-diversified portfolios, from the perspective of uninformed traders, is spanned by the \( K \) orthonormal portfolios \( f_1, \ldots, f_K \). The set \( P_2 \) of undiversified portfolios is orthogonal to \( P_1 \) and any portfolio payoff \( r \) can be decomposed as \( r = n + r_2 \) with \( n \in P_1 \) and \( r_2 \in P_2 \) with \( \text{var}(r \mid p) = \text{var}(n \mid p) + \text{var}(r_2 \mid p) \).

**Proof of Proposition 1**

Denote the payoff from the equally-weighted market portfolio by \( r_{ew} \) and diagonalize the matrix \( \Sigma_{v|pN} = \text{var}(v_N \mid p_N) \) as \( T_N D_N T_N^T = \sum_{k=1}^{N} \delta_k t_k t_k^T \):

\[
\text{var}(r_{ew} \mid p) = \lim_{N \to \infty} \text{var}\left( \frac{1}{N} 1' v_N \mid p \right) = \lim_{N \to \infty} \frac{1}{N^2} 1' \Sigma_{v|pN} 1 = \lim_{N \to \infty} \frac{1}{N^2} \sum_{k=1}^{N} \delta_k (1' t_k)^2. \tag{A2}
\]

The Cauchy-Schwarz inequality implies that \( (1' t_k)^2 \leq (1' 1)(t_k' t_k) = N \), where \( t_k' t_k = 1 \) follows from the eigenvectors being orthonormal. Thus, each summand \( \frac{1}{N^2} \delta_k (1' t_k)^2 \leq \frac{1}{N} \delta_k \), which approaches zero for any bounded eigenvector \( \delta_k = o(N) \).\(^{17}\) The elements of the eigenvectors associated with unbounded eigenvalues, \( \delta_k \sim N \), have elements of order \( N^{-1/2} \), which follows because \( t_k' t_k = 1 \) and the eigenvectors have nonzero elements for a nontrivial fraction of the

\(^{17}\) We use the notation \( x_N = O(y_N) \) to denote that \( x_N/y_N \) is bounded, \( x_N = o(y_N) \) denotes that \( x_N/y_N \to 0 \), and \( x_N - y_N \) denote that both \( x_N/y_N \) and \( y_N/x_N \) are bounded and thus increase at roughly the same rate as \( N \to \infty \).
assets. Thus, the summand \( \frac{1}{N^2} \delta_k (1't_k)^2 \sim 1 \) for any eigenvalue \( \delta_k \sim N \). If there are \( K \) such unbounded eigenvalues, then the sum is of order \( K \), \( \text{var}(r_{ew} \mid p) \sim K \). If there are no unbounded eigenvalues, then \( \text{var}(r_{ew} \mid p) \to 0 \) while if there are infinitely many, then \( \text{var}(r_{ew} \mid p) \to \infty \).

If the equally-weighted portfolio has positive but bounded variance, then Condition B holds. In conjunction with the assumption that the no-arbitrage Condition A holds, this implies that \( \Sigma_{v'p} \) has an approximate \( K \)-factor structure as implied by Lemma A.

**Proof of Proposition 2**

If the assumptions of Proposition 1 are satisfied, the unexpected variation
\[
d'(v - p) - E[d'(v - p) \mid p] = d'(v - E[v \mid p])
\]
well-diversified portfolio can be written as a linear combination \( \beta'f \) of the \( K \) portfolios that span the set \( P_1 \) of well-diversified portfolios. In other words, \( d'(v - E[v \mid p]) = \beta'f \) so that \( \xi = \frac{u(d'(E[v\mid p] - p) + \beta'f)}{E[u(d'(E[v\mid p] - p) + \beta'f) \mid p]} \). We can also divide \( v \) into the component correlated with \( f \) and an orthogonal component \( e \):
\[
v = E[v \mid p] + Bf + e.
\]
(A3)

From the equilibrium condition (2), we then have:
\[
p = E[\xi v \mid p] = E[v \mid p] + BE[\xi f \mid p] + E[E[\xi E[f \mid p] \mid p].
\]
(A4)

The pricing condition (2) holds for all securities, and therefore it must hold for the spanning portfolios \( f \), so that the price vector \( p_f \) of the \( K \) spanning portfolios must be \( p_f = E[\xi f \mid p] \), giving:
\[
p = E[v \mid p] + Bp_f + E[E[\xi E[f \mid p] \mid p].
\]
(A5)

Expression (7) follows after putting \( E[e \mid f, p] = 0 \) for the case where \( e \) is mean-independent of

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18 This can also be seen by noting that the variance of asset \( n \) is \( \sum_{k=1}^{N} \delta_k t_{nk}^2 \), which is unbounded for any individual asset that bears a fraction greater than order \( N^{-1/2} \) of the risk of associated with an eigenvector of order \( N \).
Given the conditioning information in prices \( p \).

**Proof of Proposition 3**

The market clearing condition given \( M \) uninformed traders is:

\[
x = Md + y \quad \Rightarrow \quad d = \frac{1}{M}(x - y) \rightarrow 0 \quad \text{as} \quad M \rightarrow \infty,
\]

where the limits hold if \( x \) and \( y \) are bounded.

**Proof of Proposition 4**

The conjectured coefficients we need to specify are \( q_0, Q, \) and \( Q_p, \) from the uninformed traders’ conjecture (10) of informed trade; \( c_0 \) and \( C_p \) from the uninformed traders’ conjecture (11) of other uninformed traders’ positions; \( \mu_i (\mu_u) \) and \( \Lambda_i (\Lambda_u) \) from the informed (uninformed) trader’s residual supply curve. We express the equilibrium up to the solution for the informed trader’s liquidity matrix \( \Lambda_i \). Matching coefficients between the informed trader’s conjectured strategy (10) and chosen strategy (15) yields \( Q_v = Q_p = (A_i')^{-1} \) and \( q_0 = (A_i')^{-1}v \).

Substituting the conjectured uninformed strategy (10) into the market clearing condition (13) and rearranging yields \( p = \Lambda_x (Mc_0 - x) + \Lambda_y y, \) where \( \mu_i = \Lambda_i (Mc_0 - x) \) and \( \Lambda_i = \frac{1}{M}C_p^{-1}. \)

When solving the coefficients for the uninformed, we first impose homogenous strategies in the expression for the signal \( s \) in (16) by setting \( d_m = c_0 - C_p p, \) which gives the following after substituting \( Q_v = Q_p, q_0 = Q_v v \) and rearranging:

\[
s = (I - MQ_v^{-1}C_p)p - v + Q_v^{-1}(\overline{x} - Mc_0).
\]

Substituting from (A7) into the uninformed demand (18) and rearranging gives:
\[ d_m = (A'_u + A\Sigma_{\nu\nu}')^{-1}\left(\bar{\nu} + \Sigma_{\nu\nu}(\Sigma_{\nu\nu} + \Sigma_v)^{-1}s - p\right) \]

\[ = (A'_u + A\Sigma_{\nu\nu}')^{-1}\left( (I - \Sigma_{\nu\nu}(\Sigma_{\nu\nu} + \Sigma_v)^{-1})\bar{\nu} + \Sigma_{\nu\nu}(\Sigma_{\nu\nu} + \Sigma_v)^{-1}Q_v^{-1}(\bar{x} - Mc_0) \right) \]

\[ - (A'_u + A\Sigma_{\nu\nu}')^{-1}\left( (I - \Sigma_{\nu\nu}(\Sigma_{\nu\nu} + \Sigma_v)^{-1})(I - MQ_v^{-1}C_p) \right) p. \]  

(A8)

The expression for \( C_p \) in (A8), to which return later when solving for \( A_i \), implies that:

\[ C_p = (A'_u + A\Sigma_{\nu\nu} + MQ_v^{-1}(\Sigma_{\nu\nu} + \Sigma_v)^{-1}Q_v^{-1})^{-1}\Sigma_v(\Sigma_{\nu\nu} + \Sigma_v)^{-1}, \]  

(A9)

which implicitly defines \( A_i \). Solving (A8) for \( c_0 \) and substituting from (A9) and the solutions for the informed strategy yields:

\[ c_0 = C_p(\bar{\nu} + \Sigma_{\nu\nu}\Sigma_v^{-1}Q_v^{-1}\bar{x}) = \frac{1}{M}A_i^{-1}\left(\bar{\nu} + \Sigma_{\nu\nu}(A_i\Sigma_{A}^{-1})^{-1}A_i\bar{x}\right). \]  

(A10)

Expression (A10) yields \( \mu_i \) from \( \mu_i = A_i(Mc_0 - x) \). Substituting \( \mu_i \) and (A10) into the first expression in (15) yields (21).

Rearranging the market clearing condition (13) yields:

\[ p = A_u\left(q_0 + Q_v(\bar{v} - \bar{v}) + (M - 1)c_0 - x\right) + A_u d_m, \]  

(A11)

where \( A_u = ((M - 1)C_p + Q_p)^{-1} = \left(\frac{M-1}{M}A_i^{-1} + (A_i')^{-1}\right)^{-1} \), and we have already given the expressions for \( q_0, Q_v, \) and \( c_0 \) in terms of \( A \). Substituting \( A_i = \frac{1}{M}C_p^{-1} \) and \( \Sigma_v = Q_v^{-1}\Sigma_v(Q_v^{-1})' = A_i\Sigma_{A} \) into (A9) and rearranging gives the equilibrium equation that defines \( A_i \):

\[ O = A_i\Sigma_{A}^{-1}\left(A_i - \frac{1}{M}A_i'\right) - A_i\Sigma_{A}^{-1}\Sigma_v - \frac{M}{M-1}\left(A_i' + \frac{1}{M}A_i'\right) - A_i\Sigma_{A}^{-1}(\Sigma_v - \Sigma_{\nu\nu}). \]  

(A12)

The uninformed trader’s first-order condition implies the first line of (19), while market clearing and (21) imply (22). Substituting for \( E[v | p] = E[v | s] \) and \( d_m \) yield the second line of (19), after a substitution from (A12).
Proof of Proposition 5

We first divide the expected return into the portions related to risk-aversion and imperfect competition among uninformed traders:

$$E[v - p] = \frac{1}{M} \Lambda_i (A_i + A_i')^{-1} (I + \Sigma_\pi \Sigma_s^{-1})A'_i \bar{x} + \frac{1}{M} A \Sigma_{vi,p} (A_i + A_i')^{-1} (I + \Sigma_\pi \Sigma_s^{-1})A'_i \bar{x}. \quad (A13)$$

Expanding the imperfect competition term gives:

$$\frac{1}{M} \Lambda_i (A_i + A_i')^{-1} (I + \Sigma_\pi \Sigma_s^{-1})A'_i \bar{x} = \frac{1}{M} \left( \frac{2M-1}{M} I + \frac{M-1}{M} A_i A_i'^{-1} + A_i (A_i')^{-1} \right)^{-1} (I + \Sigma_\pi \Sigma_s^{-1})A'_i \bar{x}. \quad (A14)$$

The matrix being inverted on the right-hand-side of (A14) has strictly positive eigenvalues so that its inverse has bounded eigenvalues. The matrix $I + \Sigma_\pi \Sigma_s^{-1}$ that multiplies $A'_i \bar{x}$ in (A14) has bounded eigenvalues when the informed trader’s second order condition is satisfied. In particular, the equilibrium relation (A12) can be expressed as:

$$O = \Sigma_s \Sigma_\pi^{-1} \left( A_i - \frac{1}{M} A_i' \right) - A \frac{1}{M} \Sigma_s \Sigma_\pi^{-1} \Sigma_v - \frac{M}{M-1} \left( A_i' + \frac{1}{M} A_i' \right) - A \frac{1}{M} (\Sigma_v - \Sigma_\pi). \quad (A15)$$

If $\Sigma_\pi \Sigma_s^{-1}$ becomes unbounded, then $\Sigma_s \Sigma_\pi^{-1}$ approaches the zero matrix. The expression $\frac{1}{M} \Sigma_v$ is bounded as $N, M \to \infty$, so that the right-hand-side of (A15) approaches $-A_i' - A \frac{1}{M} (\Sigma_v - \Sigma_\pi)$; but this implies that $A_i' = -A \frac{1}{M} (\Sigma_v - \Sigma_\pi)$, which violates the informed trader’s second-order condition. Last, $A_i'$ has bounded eigenvalues by assumption, so that the right-hand-side of (A14) approaches zero as $N, M \to \infty$.

Because $\Sigma_e > \Sigma_{\epsilon p}$ has bounded eigenvalues, $\frac{1}{M} \Sigma_{\epsilon p} = \frac{1}{M} B \Sigma_{f|p} B' + \frac{1}{M} \Sigma_{\epsilon p}$ approaches $\frac{1}{M} B \Sigma_{f|p} B'$. The matrix $(A_i + A_i')^{-1} (I + \Sigma_\pi \Sigma_s^{-1})A'_i$ that multiplies $\bar{x}$ in A(14) has eigenvalues less than $I$ when the informed trader’s second order condition is satisfied, as discussed above.

This yields (25). ■
**Proof of Corollary 4.1**

If \( \Lambda_i \) is symmetric, then \( \Lambda_i = \frac{M}{2M-1} \Lambda \). Setting \( A = 0 \) in (A12) and rearranging yields:

\[
\Sigma_x^{1/2} A_i \Sigma_x^{1/2} \Sigma_x^{1/2} A_i \Sigma_x^{1/2} = \frac{M}{2M-1} \Sigma_x^{1/2} \Sigma_{\pi} \Sigma_x^{1/2},
\]

(A16)

which implies (27). Expression (28) follows from substituting \( A = 0 \) and (27) into (20) and (21).

**Proof of Proposition 6**

Define \( C = A' \Sigma_x^{1/2} \). We first prove a preliminary result that the informed trader never reveals all of her information \( (C > 0) \) and that \( C \to \infty \) as \( \Sigma_x \to \infty \). The equilibrium equation (A12) that defines \( \Lambda_i \) can be written as:

\[
A_i \left( A_i + \frac{M}{2M-1} A_i' \right)^{-1} = \frac{1}{M} A \left( \Sigma_v - \Sigma_{\pi} (\Sigma_{\pi} + CC')^{-1} \Sigma_{\pi} \right) (A_i' + A_i)^{-1} + \Sigma_{\pi} (\Sigma_{\pi} + CC')^{-1} \tag{A17}
\]

If \( C \to \infty \), then (A17) can be written as \( A_i \left( A_i + \frac{M}{2M-1} A_i' \right)^{-1} = \frac{1}{M} A \Sigma_v \left( A_i' + A_i \right)^{-1} \), which is solved by \( A_i = \frac{1}{M} \frac{2M-1}{2M} A \Sigma_v \).\(^{19}\) In the price-taking uninformed case, we can write this as \( A_i = \frac{1}{M} A \Sigma_v \). If \( C \to 0 \), then (A17) can be written as \( A_i \left( A_i + \frac{M}{2M-1} A_i' \right)^{-1} (A_i' + A_i) = -\frac{1}{M} A (\Sigma_v - \Sigma_{\pi}) \), which has a negative definite right-hand-side. In the price-taking uninformed case, or for \( M \) sufficiently large, the left-hand-side approaches \( \Lambda_i \), which must be positive definite in order to satisfy the informed trader’s second-order condition. Thus, \( \Lambda_i \) must become unbounded at order \( \Sigma_x^{-1/2} \) as \( \Sigma_x \to 0 \).

If the uninformed traders are price-takers, we can reflect this by dropping \( A_i \) from the expression (A12) that defines \( \Lambda_i \) and from the expression (20) for expected returns. If \( \Sigma_x \to \infty \), then \( \Lambda_i \to \frac{1}{M} A \Sigma_v \) because any bounded, nonzero \( \Lambda_i \) yields \( C \to \infty \). This gives (29). If \( \Sigma_x \to 0 \), \( \Lambda_i \) becomes unbounded. Solving (A12) for \( \Sigma_x = A_i' \Sigma_x A_i \) and taking limits as \( \Lambda_i \to \infty \)

\(^{19}\) We cannot rule out additional solutions with a non-symmetric \( \Lambda_i \).
for the $A_i$ terms other than $\Sigma_i$ yields $\Sigma_i \to \Sigma_0$ as $\Sigma_x \to \mathbf{0}$. This gives (30), where $y \approx 0$ because the variance approaching zero implies that $x \approx \bar{x}$, in terms of the mean squared difference approaching zero.

Proof of Proposition 7

Given the assumptions on $\Sigma_0$ and $\Sigma_x$, we can pre- (post-) multiply (A12) by $T' (T)$ to and use $TT' = T'T = I$ to obtain:

$$
\mathbf{O} = \frac{2(M-1)}{2M-1} \Theta_x \Theta_x \Theta_x^{-1} \Theta_x = A \frac{1}{M} \Theta_x \Theta_x \Theta_x^{-1} \Theta_x = A \frac{1}{M} (\Theta_x - \Theta_i)
$$

$$
\Rightarrow \mathbf{O} = \Theta_x \Theta_x^{-1} \Theta_x^3 - \frac{2M-1}{2(2M-1) A} \Theta_x \Theta_x^{-1} \Theta_x^2 - \frac{M}{2M-1} \Theta_x - \frac{2M-1}{2(2M-1) A} \frac{1}{M} (\Theta_x - \Theta_i).
$$

(A18)

where we have used $T' \Lambda_i T = \frac{M}{2M-1} \Theta_i$ and the fact that diagonal matrices commute. Because the matrices in (A18) are diagonal, we can express it as $N$ independent scalar equations:

$$
0 = \frac{\partial \times}{\partial \theta_k} \lambda_k^3 - \frac{2M-1}{2(2M-1) A} \frac{\partial \times}{\partial \theta_k} \lambda_k^2 - \frac{M}{2M-1} \lambda_k - \frac{2M-1}{2(2M-1) A} \frac{1}{M} (\theta_k - \theta_i),
$$

(A19)

each of which is solved by a unique $\lambda_k > 0$ because only the leading $\lambda_k^3$ term has a positive coefficient.

Substituting from $\Sigma_v = T \Theta_i T'$, $\Sigma_0 = T \Theta_i T'$, $\Sigma_x = T \Theta_i T'$, and $A = T \Theta_i' T'$ into the expected returns (19) yields:

$$
\mathbb{E}[\mathbf{v} - \mathbf{p}] = \frac{1}{2} T \left( \frac{1}{2M-1} \Theta_i + A \frac{1}{M} (\Theta_i - \Theta_i^2 (\Theta_i + \Theta_i^2 \Theta_i^{-1})) (I + \Theta_i \Theta_i^2 \Theta_i^{-1}) T \bar{x}. \right)
$$

(A20)

Because $\mathbb{E}[\mathbf{f} - \mathbf{p}_f] = T' \mathbb{E}[\mathbf{v} - \mathbf{p}]$, (A20) implies (31).

Proof of Corollary 7.1

The right-hand-side of (A19) can be rearranged as:

$$
0 = \left( \lambda_k \frac{\partial \times}{\partial \theta_k} - \frac{M}{M-1} \right) \lambda_k - \frac{2M-1}{2(2M-1) A} \frac{\partial \times}{\partial \theta_k} \left( \frac{\partial \times}{\partial \theta_k} + \frac{\partial \times}{\partial \theta_k} \right) \frac{1}{M} (\theta_k - \theta_i).
$$

(A21)
which implies that \( \lambda_k > \sqrt{\frac{M}{M-1} \frac{\partial g}{\partial \theta_k}} \).

For the first part of the corollary, define the right-hand-side of (A19) as \( g(\lambda_k) \). Because \( g'(\lambda_k) > 0 \) at the equilibrium \( \lambda_k \), \( \frac{d\lambda_k}{d\theta_k} \propto -\frac{\partial g}{\partial \theta_k} \). Direct computations give:

\[
\frac{\partial g}{\partial \theta_k} = \lambda_k^2 \frac{1}{\theta_k} - \frac{2M-1}{2(M-1)} A \frac{1}{M} \frac{\partial}{\partial \theta_k} \lambda_k^2 = \frac{1}{\theta_k} \left( \frac{M}{M-1} \lambda_k + \frac{2M-1}{2(M-1)} A \frac{1}{M} (\theta_k - \theta_{ki}) \right) > 0,
\]

(A22)

where the second equality follows from a substitution from (A19). Noise trade variance impacts expected returns via the noise term \( \lambda_k^2 \theta_{kx} \) and the ‘standalone’ effect of \( \lambda_k \) in the \( \frac{1}{2M-1} \lambda_k \) term in (31). This gives:

\[
\frac{d\text{E}[f_{i-p,k}]}{d\theta_k} = \frac{\partial \text{E}[f_{i-p,k}]}{\partial \lambda_k^2 \theta_{kx}} \frac{d\lambda_k^2 \theta_{kx}}{d\theta_k} + \frac{1}{2} \left( 1 + \frac{\theta_k}{\lambda_k^2 \theta_{kx}} \right) \frac{1}{2M-1} \frac{d\lambda_k}{d\theta_k} < 0.
\]

(A23)

The inequality \( \frac{\partial \text{E}[f_{i-p,k}]}{\partial \lambda_k^2 \theta_{kx}} < 0 \) follows from direct computations. We previously showed \( \frac{d\lambda_k}{d\theta_k} < 0 \).

The inequality \( \frac{d\lambda_k^2 \theta_{kx}}{d\theta_k} > 0 \) follows from computations that give \( \frac{d\lambda_k^2 \theta_{kx}}{d\theta_k} \propto \lambda_k^3 \theta_{kx} - \frac{M}{M-1} \frac{\lambda_k}{\theta_k} > 0 \), where the inequality follows from (A19). The change in the informed trade is:

\[
\frac{dy_{jk}}{d\theta_k} = \frac{1}{2} \frac{\theta_k}{(\lambda_k^2 \theta_{kx})^2} \lambda_k \frac{d\lambda_k^2 \theta_{kx}}{d\theta_k} > 0 \frac{1}{2} \frac{\lambda_k^2}{\lambda_k^2} t_k'(\theta_k - \theta_k) \frac{d\lambda_k}{d\theta_k} < 0.
\]

(A24)

Both terms in (A24) indicate the informed trader becomes more aggressive as noise trade variance increases. The actual trade size \( y_{jk} \) may decrease depending on the realization of \( t_k'(\theta_k - \theta_k) \).

For the second part of the corollary, if \( \lambda_k^2 \theta_{kx} \rightarrow \infty \), then (A19) implies that \( \lambda_k \rightarrow \frac{2M-1}{2(M-1)} A \frac{1}{M} \theta_k \). If \( \lambda_k^2 \theta_{kx} \rightarrow 0 \), then (A19) implies that \( \lambda_k \rightarrow -\frac{2M-1}{2M} A \frac{1}{M} (\theta_k - \theta_{ki}) < 0 \), which violates the informed trader’s second-order condition and implies that \( \lambda_k^2 \theta_{kx} \rightarrow 0 \). If
\( \theta_{kx} \to \infty \), then \( \lambda_k^2 \theta_{kx} \to \infty \), giving \( \lambda_k \to \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} \theta_k \) and the statements for \( \theta_{kx} \to \infty \). If \( \theta_{kx} \to 0 \), then \( \lambda_k \) becomes unbounded at a rate of \( \theta_{kx}^{-1/2} \); otherwise, \( \lambda_k^2 \theta_{kx} \) approaches infinity, implying a bounded \( \lambda_k \) and \( \lambda_k^2 \theta_{kx} \to 0 \), a contradiction, or \( \lambda_k^2 \theta_{kx} \to 0 \), which generates a violation of the informed trader’s second order condition. Expression (A19) can be rearranged as:

\[
\lambda_k^2 \theta_{kx} = \frac{\frac{M-1}{2(M-1)} \lambda_k + \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} (\theta_k - \theta_{kx})}{\lambda_k - \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} \theta_k} \theta_{kli} \to \frac{M}{M-1} \theta_{kli} \quad \text{as} \quad \lambda_k \to \infty. \tag{A25}
\]

This gives the statements regarding \( \theta_{kx} \to 0 \), where the statement regarding returns stems from the \( \frac{1}{2(M-1)} \lambda_k \) term that represents imperfect competition among uninformed investors in (31). ■

**Proof of Corollary 7.2**

If the monopolist trader has no private information \( (\theta_{kli} = 0) \), then (A19) implies that

\[
\lambda_k = \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} \theta_k,
\]

giving expected returns of

\[
E[f_k - p_k] = \frac{1}{2} \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} \theta_k \bar{x}_k.
\]

Comparing to the expected return (31), we have:

\[
\begin{align*}
&\frac{1}{2} \frac{2^{M-1}}{2(M-1)} A \frac{1}{M} \theta_k \left| \bar{x}_k \right| < \left( \frac{1}{2(M-1)} \lambda_k + A \frac{1}{M} \theta_k p \right) \frac{1}{2} \left( 1 + \frac{\theta_{kli}}{\lambda_k^2 \theta_{kx}} \right) \left| \bar{x}_k \right| \\
\iff & \frac{1}{M-1} \lambda_k + \frac{2^{M-1}}{2(M-1)} A \frac{\theta_k - \theta_{kli}}{M} > 0,
\end{align*}
\]

where the second line of (A26) follows after a substitution from (A19) and the inequality follows because \( \lambda_k > 0 \), by the informed trader’s second-order condition, and \( \theta_k > \theta_{kli} \). ■

**Proof of Proposition 8**

For the systematic factors, \( \theta_k, \theta_{kli} \sim N \). If \( \theta_{kx} \sim N \), then the coefficients in (A19) are all bounded, implying that \( \lambda_k \) is bounded, as well. If \( \theta_{kx} \sim N^\alpha, \ \alpha < 1 \), then an argument similar to
that used in proving Corollary 6.1 implies that $\lambda_k \sim N^{(1-\alpha)/2}$. In both cases, the term in expected returns (31) $\frac{1}{2M-1} \lambda_k \to 0$ because $M$ increases faster than $\lambda_k$. In the latter case ($\theta_k \sim N^\alpha, \alpha < 1$), rearranging (A19) and taking limits, using the fact that $\lambda_k \to \infty$, implies that $\lambda_k^2 \theta_{ks} \to \theta_{ki}$. Substituting into expected returns (31) and informed trade (32) yields (33) and (34).

For the idiosyncratic factors, if $\theta_{ks}$ remains bounded, then taking limits on (A19) implies that $\lambda_k \to \sqrt{\frac{\theta_{ks}}{\theta_{ki}}}$. If $\theta_{ks}$ becomes unbounded, then (A19) implies that $\lambda_k \to 0$ and $\lambda_k^2 \theta_{ks} \to \theta_{ki}$.

These two facts give the second part of the proposition.