Maturity Rationing*

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PRELIMINARY AND INCOMPLETE

Abstract

We propose a model of *maturity rationing.* When there is asymmetric information about default probabilities, firms may be unable to obtain financing for long-term projects. This is because asymmetric information worsens with the maturity of the project, such that lending markets for long maturities can break down (*maturity rationing*). Worse, firms whose first-best projects cannot get financed may react by adopting second-best projects of shorter maturities. This worsens the pool of financed projects, further amplifying rationing: a *rationing spiral* can emerge. Maturity rationing is stronger during recessions and, through its knock-on effects, may amplify the business cycle.

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1 Introduction

The recent financial crisis has underscored the importance of financial market frictions for real activity. In particular, frictions in financial markets may propagate and ampflify shocks to the economy, and thus may magnify the amplitude and increase the persistence of the business cycles. In this paper, we focus on one particular friction, asymmetric information, and analyze its effect on the both the amount and composition of real investment undertaken by firms. More specifically, we propose a simple model of *maturity rationing*, which can be seen as the generalization of credit rationing (e.g., Stiglitz and Weiss, 1981; DeMeza and Webb, 1987) to the maturity structure: In our model, asymmetric information frictions lead to credit rationing for longer maturities, while financing may be available for shorter maturity projects. Starting from this insight, our paper thus generates a theory of maximum equilibrium debt maturity and the real investment distortions that follow.

The main idea behind our model is that, under fairly general conditions, frictions stemming from asymmetric information are more severe at longer horizons compared to shorter horizons. As a result, beyond a certain maturity asymmetric information is too severe for financing to be sustained in equilibrium; credit is rationed beyond that point. More specifically, in our model there is asymmetric information about the riskiness and default probabilities of firms. Some firms have "good" projects which are risk-free and have positive NPV, whereas other firms have "bad" projects which are risky, have negative NPV, and default over time. Firms seek debt financing from a financial sector that cannot observe the type of the firms' projects. Since bad firms can always mimic good firms, the only way for firms to attract funding is through pooling contracts. Hence, in order for the financier to break even, the interest rate on the pooling debt contract has to increase with maturity in order to reflect higher default risk at long horizons. However, beyond a certain point this increase in interest rates causes firms with good projects not to seek financing. Good firms drop out of the market and, with only bad firms left to seek financing, lending breaks down beyond a certain maturity – *maturity rationing*.

After establishing our baseline model, we then show that maturity rationing can lead to important knock-on effects. In particular, to escape rationing, firms whose first-best projects cannot get financed may react by adopting second-best projects of shorter maturities, for which financing is available. In fact, even firms with longer maturity projects that can attract funding may prefer to adopt second-best projects with shorter maturities, because the adverse selection discount is smaller at shorter maturities. However, the resulting inflow of second-best projects worsens the pool of funded projects. This leads to a negative externality that exacerbates rationing by further shortening the maximum maturity that can be funded and worsening the financing terms for firms that can receive funding. But as financing terms worsen and more firms are rationed, this again leads to an additional inflow of second-best projects into the funded region. The process repeats and a *rationing spiral* emerges. When the negative externality from the adoption of second-best project in response to rationing is strong enough, it can lead to a complete breakdown of financing across *all* maturities.

In general, the adoption of shorter maturity projects by rationed firms can increase or decrease surplus, depending on the severity of the externality. At one extreme, when second-best projects are (almost) as good as first-best projects, the privately optimal decision of rationed firms to seek shorter maturity projects increases surplus. In this case, rationed firms that adopt shorter maturity projects in order to obtain financing do not impose an externality on financed firms, and thus the only consequence from the firms' maturity adjustement is an increase in output. On the other hand, when second-best projects are worse than firm's initial, first-best projects, the dilution of the financed pool by firms who change their maturity can lead to a decrease in surplus. While it is privately optimal for each individual firm to adopt a second-best shorter maturity project, the dilution externality on the pool leads to an overall reduction in surplus.

Our theory generates a number of empirical predictions. First, the model predicts a link between maturity rationing and the business cycle. In particular, a decrease in the fraction of good projects or an increase in the relative default probability of bad projects execarbates maturity rationing and thus reduces investment. This suggests that maturity rationing is stronger during recessions. Hence, the worsening of maturity rationing during downturns, and the related knock-on effects, may amplify the business cycle. Second, through firms' endogenous adjustment to maturity rationing, our model predicts short-termism as a result of asymmetric information frictions. Particularly during downturns unattractive funding terms at longer horizons may force firms to adopt inferior investment projects of shorter maturity.

Our paper relates to and extends the extensive literature on credit rationing. For a summary of this literature, a good starting point is the discussion in Bolton and Dewatripont (2005, Chapter 2) or the survey on financial contracting by Harris and Raviv (1992). The classic contributions on credit rationing are Jaffee and Modigliani (1969), Jaffee and Russell (1976), Bester (1985), Stiglitz and Weiss (1981), and DeMeza and Webb (1987). Suarez and Sussman (1997) develop an overlapping generations model in which credit rationing can lead to endogenous business cycles. Kurlat (2010) builds a dynamic model in which firms sell projects that are subject to adverse selection and shows that the resulting friction responds to aggregate shocks, thus amplifying the repsonse of prices and investment.

The main innovation of our paper relative to this literature extend the concept of credit rationing to a framework in which firms have investment projects of differing maturities. This additional element allows us to investigate the link between credit rationing and the maturity of a firm's investment and financing. In addition, the model allows us to analyze investment distortions and endogenous short-termism that arise from asymmetric information frictions and rationing of longer maturities. This endogenous adjustment pushes firms to shorter maturities, but unlike the papers by Flannery (1986) and Diamond (1991), shortening of maturity is not an attempt by a good firm to signal its type. Rather, maturity shortening stems from a firm's desire to pool with other firms at maturities where asymmetric information frictions are less severe.

2 Maturity Rationing: Baseline Model

In this section we develop our baseline model of *maturity rationing.* Consider a competitive, risk-neutral financial sector that provides debt financing to firms that borrow funds in order undertake investment projects. There is a continuum of such firms, and each firm is born with an investment project of a particular maturity. The maturity of a project, which is drawn uniformly from the interval $[0, \overline{T}]$, indicates how long it takes for the project to pay off. A project of maturity $t \in [0, \overline{T}]$ generates cash flow only at date t and no cash flows beforehand. Project maturity is observable to both firms and financiers, such that there is no asymmetric information about when a particular project pays off. The financial sector posts a funding schedule that details the applicable interest rate (or face value) for a project of maturity t.

While there is symmetric information about project maturity, there is asymmetric information about the quality of projects. Some firms, which we will refer to as "good firms", have positive NPV projects ("good" projects), while "bad firms" have negative NPV (or "bad") projects. Whether a project is good or bad is only observable to the firm (or the entrepreneur—we will use those terms interchangeably), but not to the financier. All the financier knows is that a fraction β of firms have good projects, while a fraction $1-\beta$ of firms have bad projects.

Both good and bad projects cost 1 dollar to set up, but they differ in the payoffs they generate. Good projects are risk-free and pay off a certain amount $e^{rt}R$ at maturity t, where r denotes the constant, exogenous risk-free rate. The present value of the cash flows from a good project is thus given by R and is independent of project maturity. We assume that good projects have positive NPV:

$$NPV_G = R - 1 > 0. \tag{1}$$

Bad projects, on the other hand, are riskier than good projects and have a constant negative NPV. In particular, we assume that once off the ground, the payoffs from a bad project are a mean-preserving spread of the payoffs from a good project: bad projects default with intensity λ, but when they are successful they pay off $e^{(\lambda+r)t}R$. In addition, bad projects only 'get off the ground' with probability $\Delta < 1/R$, while with probability $1-\Delta$ they are worthless from the start. One interpretation of Δ is to think of it as the part of default risk that is independent of the maturity of the project. Together, these assumptions imply that bad projects have a constant negative of NPV of[1]

$$NPV_B = \Delta R - 1 < 0. \tag{2}$$

Finally, in line with Stiglitz and Weiss (1981), we assume that in the case of default, the entrepreneur incurs a private cost of default. We assume that this cost is proportional to the return generated in the default state. This assumption is a reinterpretation of the cost of the loss of collateral in Stiglitz and Weiss (1981) which also affects good and bad types differentially.

Given these assumptions, we can now investigate which projects can receive financing. To do so, we assume that firms match maturities, i.e., they finance their investments via debt contracts that match the maturity of their projects.[2] In the appendix we show that this assumption is without loss of generality: When project maturity is observable, a firm that cannot finance by matching maturities can also not raise financing using rollover debt. Hence, rollover contracts do not add anything in our framework.

Since firms with bad projects are indistinguishable contractually from firms with good projects, financing is possible whenever the financier can break even on a debt contract that pools good and bad firms for a given maturity t. This is an outcome of our assumption that

[1]Most of our results are robust to variations in these assumptions. For example, it is not necessary to assume that the drift of bad projects compensates for the default intensity λ. However, this assumption is convenient because it guarantees that the NPV of bad projects is independent of the project maturity. Hence, our results are driven by differences in asymmetric information frictions across different maturities as opposed to differences in NPV across different maturities.

[2]As in the original models by Stiglitz and Weiss (1981) and DeMeza and Webb (1987) we assume that the firm uses debt contracts to finance the investment. While debt financing is optimal in DeMeza and Webb (1987), debt is not the optimal contract in Stiglitz and Weiss (1981). Our models is a hybrid, such that whether debt is optimal depends on parameter restrictions.

financiers are competitive, and thus they cannot cross-subsidize accross maturities. Futher, it should be clear that the financier cannot set the face value of debt $\tilde{D}(t)$ higher than $e^{rt}R$. If the face value of debt exceeded $e^{rt}R$, good firms would default with probability one, and, because default is costly for entrepreneurs, they would not participate. Hence, the debt contract would only attract bad firms. The maximum face value that supports pooling at any particular maturity t is thus given by given by $\tilde{D}^{max}(t) = e^{rt}R$.

Whether projects of maturity t can receive financing thus depends on whether the face value of debt that the financier requires to break even lies below the maximum face value that attracts both good and bad firms. Given a pool quality of β, the investor breaks even by setting the face value of debt to $\tilde{D}$ such that

$$e^{-rt}\left[\beta\tilde{D} + (1-\beta)\,\Delta e^{-\lambda t}\tilde{D}\right] = 1. \tag{3}$$

Defining the present value of the face value of debt as $D \equiv e^{-rt}\tilde{D}$, we can rewrite this breakeven constraint as

$$D(t,\beta) = \frac{1}{\beta + (1-\beta)\,\Delta e^{-\lambda t}}. \tag{4}$$

Note that the breakeven face value is increasing in time to maturity. This reflects that the riskiness of bad projects increases with their horizon (even though their NPV remains fixed).

The maximum funded maturity M is then given by the maximum maturity for which the breakeven debt contract still attracts both good and bad firms, i.e., characterized by $\tilde{D}(M,\beta) = \tilde{D}^{max}(M)$. When this maximum maturity lies below the maximum maturity

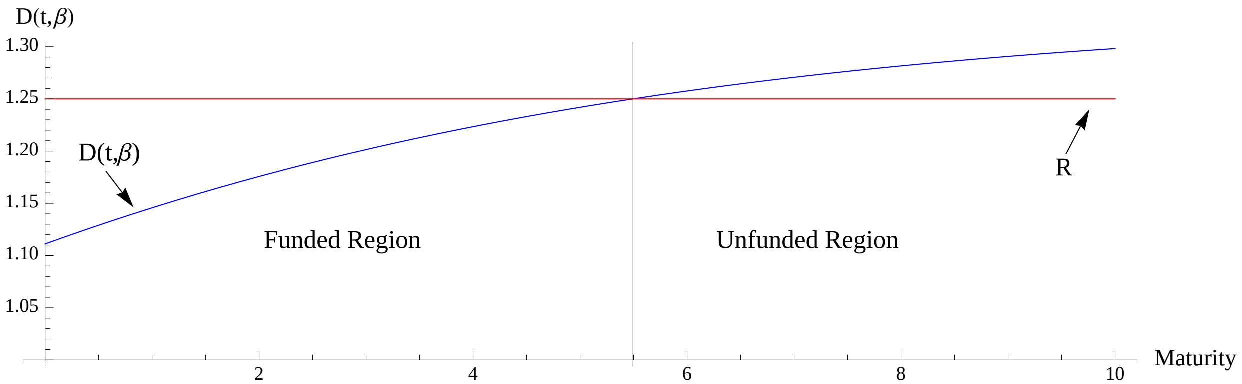

Figure 1: The Figure illustrates maturity rationing in our baseline model. For maturities below 5.5 years, the required face value $D(t,\beta)$ lies below R, such that a pooling equilibrium exists. Beyond a maturity of 5.5 years, the required face value exceeds R, such that good firms drop out and no pooling equilibrium exists. These maturities are not funded.

$\overline{T}$, some maturities cannot attract funding, such that there is *maturity rationing.* This is illustrated in Figure 1.

Proposition 1 *The maximum project maturity that can attract financing is given by*

$$M(\beta) = \frac{1}{\lambda} \log \left[\frac{(1-\beta)\,\Delta R}{1-\beta R} \right]. \tag{5}$$

There is maturity rationing whenever $M(\beta) < \overline{T}$.

By competition, the lending sector will never ration at a lower maturity than $M(\beta)$, as each non funded maturity below $M(\beta)$ is a profit opportunity.

Proposition 1 shows that asymmetric information may lead to credit market breakdown beyond certain maturities. In particular, whenever $M(\beta) < \overline{T}$, projects with maturities on the interval $(M(\beta), \overline{T}]$ cannot receive financing and are rationed. Maturity rationing thus arises whenever the proportion of good firms lies below a critical quality level $\overline{\beta}$. Using

$M\left(\overline{\beta}\right) = \overline{T}$ we find that this cutoff is given by

$$\overline{\beta} = \frac{1 - e^{-\lambda \overline{T}} \Delta R}{\left(1 - e^{-\lambda \overline{T}} \Delta\right) R}. \tag{6}$$

Note that $\lim_{\overline{T} \to \infty} \overline{\beta} = \frac{1}{R} \equiv \overline{\overline{\beta}}$. Thus, when the pool of firms is of high enough pool quality, any maturity can attract financing. On the other hand, when β is sufficiently low, financing can break down across *all* maturities, leading to extreme maturity rationing. Solving $M\left(\underline{\beta}\right) = 0$, we find that this is the case whenever

$$\beta < \underline{\beta} \equiv \frac{1 - \Delta R}{(1 - \Delta) R}. \tag{7}$$

When β lies in between $\underline{\beta}$ and $\overline{\beta}$ there is partial rationing. Projects of maturity $t \leq M(\beta)$ can be financed, whereas projects whose maturity exceeds $M(\beta)$ cannot attract financing and are rationed. Similar to Stiglitz and Weiss (1981), this rationing arises because of the additional risk of bad projects at long horizons. This additional risk forces the financier to raise the face value of debt, but at some point this causes good firms to drop out of the market, which leads to the market breakdown beyond $M(\beta)$.

Proposition 1 generates a number of comparative statics. First, we see that the funded interval is increasing in the proportion of good projects β. Second, the funded interval is decreasing in the default intensity of bad projects λ. Both of these comparative statics suggest a link of maturity rationing to the business cycle. In particular, if during downturns either the proportion of positive NPV project drops, or if downturns are associated with and increase in asymmetric information about firms' default risk, our model predicts that the maximum funded maturity decreases during recessions. In addition, the funded interval is increasing the the NPV of the good project R and the NPV of the bad project ΔR.[3]

By looking at the total surplus that is generated in equilibrium, we can also use Proposition 1 to examine the welfare implications of the model. When $\beta \geq \overline{\beta}$, all maturitites 0 to $\overline{T}$ are funded. This outcome is thus akin to the result on DeMeza and Webb (1987): asymmetric information leads to overinvestment because both good and bad projects are funded, while in a first-best equilibrium only good projects would be financed. Hence, even when $\beta \geq \overline{\beta}$, welfare is less than first-best because negative NPV projects are funded.

$$W = [\beta NPV_G + (1-\beta) NPV_B] < \beta NPV_G = W^{FB} \tag{8}$$

Once β drops below $\overline{\beta}$, we move from a fully funded equilibrium to a rationing equilibrium in which not all maturities can attract financing. This leads to an unambiguous welfare loss relative to the fully funded case: as long as the expected NPV, $\beta NPV_G + (1-\beta) NPV_B$, is positive, any rationing reduces welfare. To formalize this, denote the mass of firms on the interval $[0, T]$ by $m(T) = \frac{T}{\overline{T}}$. When $\beta < \overline{\beta}$ firms on the interval $(M(\beta), \overline{T}]$ cannot attract financing, such that overall surplus drops to

$$W(\beta) = m(M(\beta)) [\beta NPV_G + (1-\beta) NPV_B] = \frac{M(\beta)}{\overline{T}} W < W. \tag{9}$$

Finally, it is instructive to examine the expected profits of good and bad firms. Because investors break even in expectation, on average firms receive the entire NPV from their

[3]Explicit calculations for these comparative statics are in the appendix.

investment projects. Hence, the unconditional expected profit for a funded firm with project maturity t is given by

$$\mathbb{E}\left[\pi^i\left(t,\beta\right)\right] = \beta NPV_G + \left(1-\beta\right)NPV_B. \tag{10}$$

However, while the average firm just earns its NPV, this is not true conditional on firm type. This is the case because in the presence of asymmetric information good firms subsidize bad firms. The expected discounted profit for a good firm with a project of maturity t given pool quality β is given by

$$\pi^G\left(t,\beta\right) = e^{-rt}\left[e^{rt}R - \tilde{D}\left(t,\beta\right)\right] = R - D\left(t,\beta\right). \tag{11}$$

This is less than the good firm's NPV, which is equal to $R-1$. Moreover, the profits of firms with good projects are decreasing in project maturity t because the face value of debt, $D\left(t,\beta\right)$, is increasing in maturity. Essentially, because adverse selection is more severe at longer horizons, good firms with projects of long maturities are losing more surplus in the form of a cross-subsidy to bad firms. Note also that, for any maturity, π^G is increasing in the quality of the pool of firms β because $D\left(t,\beta\right)$ is decreasing in β. This is the case because cross-subsidization become less severe the better the average quality of the pool.

Turn now to the profit of a firm with a bad project. We know that since the total surplus at every maturity t is fixed at $\beta NPV_G + \left(1-\beta\right)NPV_B$, the expected discounted profit for a bad firm is increasing in t. Writing out the profit for a bad firm at maturity t we find that

$$\begin{aligned} \pi^B(t,\beta) &= e^{-rt}\Delta e^{-\lambda t}\left[e^{rt}e^{\lambda t}R - \tilde{D}(t,\beta)\right] = \Delta\left[R - e^{-\lambda t}D(t,\beta)\right] \\ &= \Delta R - \underbrace{\frac{\Delta}{\beta e^{\lambda t} + (1-\beta)\Delta}}_{<1}, \end{aligned} \tag{12}$$

which is strictly positive and increasing in t. The reason that the bad firm's profit increases in t is the increasing volatility of the bad project, which allows bad firms to capture more surplus at longer project maturities: the volatility raises the value of a bad firm's equity claim and more than compensates for the increase in face value. As with good firms, the profit for a bad firm π^B is increasing in pool quality β because $D(t,\beta)$ is decreasing in the pool quality β.

Finally, our assumptions on the project characteristics give us the following implications on variance.

Lemma 1 *The good project's variance of its discounted cash flows is*

$$Var_t^G\left(e^{-rt}\cdot Return\right) = 0, \tag{13}$$

and the bad project's variance of its discounted cash flows is

$$Var_t^B\left(e^{-rt}\cdot Return_t\right) = \Delta R^2\left(e^{\lambda t} - \Delta\right) \tag{14}$$

which is increasing in t. Additionally, the average variance of projects at maturity t is given *by*

$$Var_t^{ExAnte}\left(e^{-rt}\cdot Return_t\right)=\left(1-\beta\right)R^2\left[\beta\left(1-\Delta\right)^2+\Delta\left(e^{\lambda t}-\Delta\right)\right], \tag{15}$$

which is decreasing in β *for* $\Delta\in\left[\frac{1}{2},1\right]$.

Lemma 1 etablishes that while the variance of the good project is constant at zero, the variance of the bad project is positive and increasing with maturity. In addition, as long as $\Delta\in\left[\frac{1}{2},1\right]$ the average volatility of firms for any maturity $t\geq 0$ is decreasing with pool quality β.

3 Second-best Projects, Dilution and Knock-on Effects

In this setion we extend our baseline model to allow firms to adjust their investment decision in response to the asymmetric information friction. More specifically, we assume that, while firms are born with a first-best project with some maturity drawn uniformly from $\Omega=\left[0,\overline{T}\right]$, they can adjust their project maturity in response to asymmetric information frictions. However, this adjustment in maturity comes at a cost to the firm: when a firm changes the maturity of its first-best investment opportunity, this reduces the attractiveness of the investment, such that the resulting project is only second-best. This assumption captures the intution that it is costly for firms to distort their investments away from the first-best investment strategy given to them by nature.

To make the analysis tractable, we make the following assumptions: After a firm learns the maturity of its first-best project, but before the quality of the project is revealed to the firm, firms can decide to search for a new project with a different and potentially shorter maturity. To search for a new project, firms effectively play a bandit: If a firm decides to redraw its project in order to change its project maturity, the firm receives a new project drawn from the original maturity distribution (i.e., uniform on $[0, \overline{T}]$). Moreover, when playing the bandit, the firm loses access to its original project.

Drawing a new project has no direct cost for firms (i.e., the firm can play the bandit for free). However, changing project maturity comes at an indirect cost because, as mentioned above, the average quality of newly obtained projects is lower. Specifically, the probability that a newly obtained project is of high quality drops from β to $\alpha\beta$, where $\alpha \in [0, 1]$. In this sense, we refer to these projects as second best.

On the credit supply side, we assume that financiers compete by posting schedules of financing terms. Specifically, financiers are competitive and post schedules, in the form of face values for every financed maturity, simultaneously. After the schedules of financing terms have been posted, entrepreneurs fund themselves at the best rate they can find if funding is available for the maturity of their investment project.

The introduction of investment distortions in response to asymmetric information gives rise to two feedback mechanisms. The first is a direct externality: firms that adopt second-best projects in response to asymmetric information frictions dilute the pool of funded projects. As this pool is diluted, debt becomes more expensive for the firms originally located in the funded region, possibly changing their decision between staying at their original maturity or searching for a new, second-best projects. Second, because of adverse selection at any maturity the face value of debt can rise up to a certain point before good firms decide not to seek financing. Hence, at some point the inflow of second-best projects that results from firms' redrawing their maturities means that investors will stop financing maturities that were funded before. As we will see, this second effect can potentially lead to rapid changes in equilibrium funding for small changes in parameters, up to complete unraveling of funding at all maturities.

Once we allow firms to redraw the maturity of their projects, an equlibrium is given by a a set of project redrawal decisions by firms and offered funding terms by financier that meet the following conditions:

Definition 1 *An equilibrium is given by redrawing decisions by firms and funding conditions offered by the financier sector such that:*

1. *Given the funding terms offered, no funded firms have an incentive to redraw the maturity of their projects*

2. *Investors break even at each funded maturity.*

In the next subsection we discuss existence and uniqueness of equilibrium and characterize the properties of the equilibrium that arises when firms can adjust their investment decisions in response to asymmetric information. After characterizing the equilibrium, we then first consider the two extreme cases, (i) $\alpha = 1$ (no dilution from second-best project) and (ii) $\alpha = 0$ (maximum dilution from second-best projects). These two cases are useful to build intuition about the knock-on effects of maturity rationing when firms can adjust their real investment decisions in reponse to adverse selection frictions. After covering the two extreme cases, we treat the general case $\alpha \in (0, 1)$.

3.1 Existence and uniqueness of equilibrium in cutoff strategies

In this section shows that there is a unique equilibrium in cutoff strategies, in which financiers fund maturities from 0 up to some cutoff T.

Before we derive the equilibrium, let us first introduce some important concepts we will use repeatedly. First, we define the set of cutoff strategies T that can be funded. For this, we denote by $x(T)$ the average pool quality on $[0, T]$ under the assumption that firms on $\left[T, \overline{T}\right]$ redraw their maturities. If everyone on $\left[T, \overline{T}\right]$ redraws, the average pool quality $x(T)$ on $[0, T]$ is given by

$$x(T) = \beta \frac{\overline{T} + \alpha\left(\overline{T} - T\right)}{2\overline{T} - T} \leq \beta, \tag{16}$$

whereas the average pool quality on $\left[T, \overline{T}\right]$ is simply given by $\hat{x} = \alpha\beta < x(T)$ (the only firms in this interval are ones who after searching for a second-best project ended up on $\left[T, \overline{T}\right]$).[4]

What are the incentives for entrepreneurs to change their maturities? Clearly, an unfunded firm always has an incentive to draw a new project, since redrawing is costless and hence a free option. We will thus concentrate on the incentives of funded firms to redraw their maturities. First, let us establish that given any set of firms that redraw their maturities $\mathcal{R}$, such that the quality on $\Omega/\mathcal{R}$ is $x(\mathcal{R})$ (with some slight abuse of notation), and the the quality on $\mathcal{R}$ is $\alpha\beta$, among the entrepreneurs who have not redrawnt their maturity, the entrepreneur with the highest maturity, i.e. $\sup \Omega/\mathcal{R}$, has the largest incentive to redraw his project maturity. Let us further define the term "funded properly" to mean funded at an average quality $x > \alpha\beta$. We use the term funded properly to distinguish financing at

[4]The derivative of pool quality is

$$x'(T) = \beta \frac{\overline{T}(1-\alpha)}{\left(2\overline{T} - T\right)^2} > 0 \tag{17}$$

an average quality $x > \alpha\beta$ from funding at the worst possible pool quality $\alpha\beta$, which is sometimes possible when $M(\alpha\beta) > 0$.

Lemma 2 *Suppose entrepreneurs on the arbitrary set* $\mathcal{A} \subset \Omega = \left[0, \overline{T}\right]$ *redraw maturities. Then the entrepreneur with the hightest maturity that is still properly funded and has not redrawn his maturity, i.e.* $\sup\{\Omega/\mathcal{A} \cap [0, M[x(\mathcal{A})]]\}$*, has the strongest incentive to redraw.*

To see this, first note that the payoff from drawing a new project is independent of current maturity t. Thus, we only have to concentrate on the payoff from staying put. Consider a firm with a project of maturity t under the conjecture that all firms on $\mathcal{A}$ decide to redraw. What is the firm's tradeoff between redrawing or keeping its original maturity? The payoff from staying and its maturity derivative are given by

$$\begin{aligned} stay(t,\beta) &= \beta[R - D(t, x(\mathcal{A}))] + (1-\beta)\Delta\left[R - e^{-\lambda t} D(t, x(\mathcal{A}))\right], && (18) \\ \frac{\partial stay(t,\beta)}{\partial t} &= -\beta D_t(t,x) + (1-\beta)\Delta\left[\lambda e^{-\lambda t} D(t,x) - e^{-\lambda t} D_t(t,x)\right] && (19) \\ &= (1-\beta)\Delta\lambda e^{-\lambda t} D(t,x)\left[1 - D(t,x)\left\{\beta + (1-\beta)\Delta e^{-\lambda t}\right\}\right] && (20) \\ &= (1-\beta)\Delta\lambda e^{-\lambda t} D(t,x)\left[1 - \frac{D(t,x)}{D(t,\beta)}\right] < 0 && (21) \end{aligned}$$

where we used the fact that $D_t = D^2(1-\beta)\Delta\lambda e^{-\lambda t}$, $D_\beta < 0$ and $\beta > x$. This immediately implies that incentives to keep the original project are larger for firms with projects of shorter maturity. The main mechanism behind this results is that adverse selection increases with project maturity t. An entrepreneur of quality β will have to pay a rate in excess of the applicable fair rate (as it is calculated on the basis of $x(\mathcal{A}) < \beta$) at any t. However, this gap between the actual rate and the fair rate increases with t, so that the benefits of staying put decrease.[5]

When changing maturity, with a certain probability, the entrepreneur will draw a maturity that is less than $\max\left[T, M\left(\alpha\beta\right)\right]$ and thus receive funding under the conjecture that $[0,T]$ is funded. We can thus write out the payoff to redrawing maturity as:

$$\begin{aligned} redraw\left(\beta,T\right) &= \frac{1}{\overline{T}}\int_0^T \alpha\beta\left[R - D\left(t, x\left(T\right)\right)\right] + \left(1-\alpha\beta\right)\Delta\left[R - e^{-\lambda t}D\left(t,x\left(T\right)\right)\right]dt \\ &\quad + \mathbf{1}_{\{M(\alpha\beta)>T\}}\frac{\min\left[M\left(\alpha\beta\right),\overline{T}\right]-T}{\overline{T}}\left(\left[\alpha\beta+\left(1-\alpha\beta\right)\Delta\right]R-1\right) \end{aligned} \tag{22}$$

Let us define the net value of redrawing as the difference between the payoffs from redrawing and staying put, $NVR\left(T\right) = redraw\left(x\left(T\right),T\right) - stay\left(x\left(T\right),T\right)$.

Second, let us now look at the additional adverse selection dimension of the problem: the possibility that the price mechanism is not flexible enough to accomodate the change in pool quality at a given maturity. To this end, we define the set of cutoff strategies T that are funded under the conjecture that everyone on $\left[T,\overline{T}\right]$ decides to draw a new project as

$$\mathcal{F} = \left\{T : T \leq M\left(x\left(T\right)\right)\right\} \cap \left[0,\overline{T}\right] \tag{23}$$

$T \in \mathcal{F}$ means that a conjectured cutoff T implies a pool quality $x\left(T\right)$ that leads to a maximum funded maturity $M\left(x\left(T\right)\right)$ that is weakly larger than this cutoff. The set $\mathcal{F}$ is thus determined by the investors' break-even condition.

[5]Note that the payoff from staying is less than $\left[\beta+\left(1-\beta\right)\Delta\right]R-1$ because $\beta > x\left(T\right)$ so that

$$\left[\beta+\left(1-\beta\right)\Delta e^{-\lambda T}\right]D\left(T,x\left(T\right)\right) = \frac{D\left(T,x\left(T\right)\right)}{D\left(T,\beta\right)} > 1$$

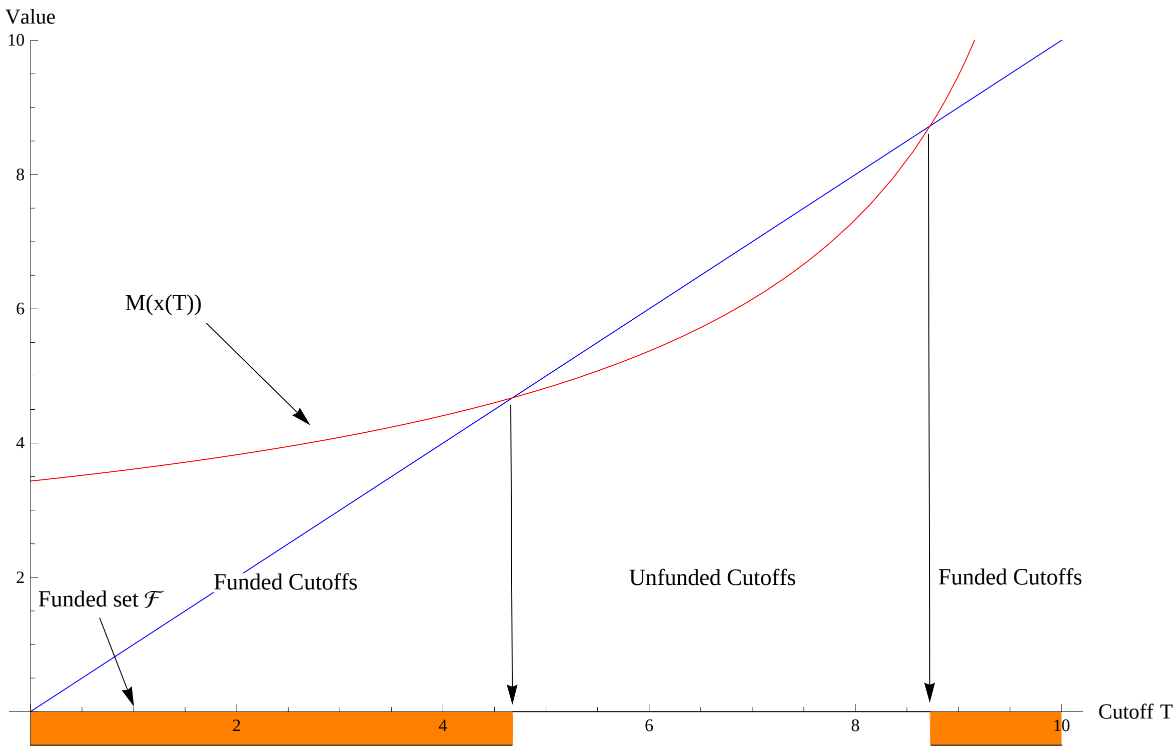

Figure 2: The funded set $\mathcal{F}$. The funded set indicates whether a given maturity cutoff can be financed, given that all firms beyond that cutoff redraw their maturity. For example, in the graph $4 \in \mathcal{F}$, this means that zero to four years can be an equilibrium funding interval. The same is true for the interval zero to nine years , as $9 \in \mathcal{F}$. However, note that $6 \notin \mathcal{F}$, which means that zero to six years cannot be an equilibrium funding interval.

The funded set $\mathcal{F}$ is is illustrated in Figure 2. The boundaries of the set $\mathcal{F}$ are fixed points defined by $T = M(x(T))$. Hence, $\mathcal{F}$ maps into a set of monotone sets, in that for each $T \in \mathcal{F}$ we know that $[0, T]$ can be funded in equilibrium. However, $\mathcal{F}$ itself need not be monotone and can have holes, as discussed in Section 3.3.

The reader can visualize the set $\mathcal{F}$ in the following manner. Suppose that initially some firms are rationed: $M(\beta) < \overline{T}$. Then we know that, even though $[0, M(\beta)]$ would be funded in the baseline model in which firms cannot redraw maturities, $[0, M(\beta)]$ cannot be funded in a setting that allows redrawing if $\alpha < 1$. This is because all unfunded firms (i.e., those with maturities on $\left[M(\beta), \overline{T}\right])$ will redraw their maturity for sure. But these firms lower the pool quality on $[0, M(\beta)]$ to $x(M(\beta)) < \beta$, which implies that the firms that were just getting funding in the baseline model are now rationed out of the market. To be exact, the originally funded maturities $[M(x(M(\beta))), M(\beta)]$ become unfunded, and firms on this interval will look for second best projects with lower maturities. This process repeats itself until we arrive at a fixed point $T = M(x(T))$. This fixed point says that if everyone on $\left[T, \overline{T}\right]$ redraws all projects on $[0, T]$ can (just) receive proper funding.

When financiers compete by posting funding schedules, we make the following two observations that will guarantee uniqueness. First, we note that a lower cutoff T leads to lower quality on the set $[0, T]$, that is $x'(T) < 0$, as was already noted in footnote 4. Second, we note that the posted interest rate (or face value), if everyone offers a lower cutoff T and thus expects a lower pool quality $x(T)$, is higher for each funded t. This is because $\frac{\partial D(\beta,t)}{\partial \beta} < 0$. Thus, for any $T < T^*$ a single firm, by posting funding up to T' with $T < T' \leq T^*$, can capture the whole market $[0, T']$ even when charging an ε amount more than the break-even face value $D(x(T'), t)$. What remains to be established is a sufficient condition for the indifference or weak preference of firms to stay put instead of redrawing their maturity. We state this condition in the following proposition:

Proposition 2 *Assume parameters such that $NVR(0) < 0 < NVR\left(\overline{T}\right)$. Further, let n be the number of roots of $NVR(T)$ on $\left[0, \overline{T}\right]$, such that root i is denoted by T_i. Then these roots induces an entrepreneur no-redrawing set $\mathcal{E} = [0, T_1] \cup [T_2, T_3] \cup ... \cup [T_{n-1}, T_n] \subset \left[0, \overline{T}\right]$. Then the unique equilibrium cutoff is given by*

$$T^* = \max\{\mathcal{F} \cap \mathcal{E}\} \tag{24}$$

when we restrict the analysis to cutoff equilibria.

Proof. By assumption, we have $NVR(0) < 0 < NVR\left(\overline{T}\right)$ so that by continuity of $NVR(T)$ we know that it crosses from negative into positive territory *at least once.* Suppose for generality that $NRV(T)$ crosses zero multiple times. It turns out that competition among investors will still guarantee uniqueness. To see this, suppose that wlog $NVR(T)$ crosses zero 3 times, at T_1, T_2, T_3. First it is easy to establish that investors would not all offer financing schedules with a maximum maturity of $t_2 \in [T_1, T_2]$. If an investor offered such a schedule, he would surely make losses for some small set $[t_2 - \varepsilon, t_2]$ if he offerd the highest maturity funding amongst all investors, because it is optimal for entprepreneurs on this set to redraw their maturities. Hence, all entrepreneurs in this region are of low quality $\alpha\beta$ but funding is based on $x(t_2) > \alpha\beta$. Next, suppose that an investor offers a competitive schedule $[0, t_1]$ with $t_1 \in [0, T_1]$. This schedule is internally consistent in the sense that if no one else posts schedules the investor will not make losses. But when an investor posts a schedule on $[0, t_1]$, a competing investor can offer a schedule $[0, T_1]$. In doing so, this competitor will be able to offer better rates on $[0, t_1]$ while not making losses on $[t_1, T_1]$ due to the fact that the newly funded entrepreneurs on $[t_1, T_1]$ will not redraw. This makes sure that funding will be provided at least up to T_1. But by a similar reasoning, funding will be offered up to the highest root of $NVR(T)$ still on $\left[0, \overline{T}\right]$. The reason is that if financing is offered up to T_1, there is again a profitable deviation to the funding schedule: a competitor could post a schedule $[0, t_3]$ with $t_3 \in [T_2, T_3]$. This way, the competitor still offers a schedule that does not lead to redrawing of maturities (since we know from the Lemma above that incentives to redraw are strongest for the highest funded type), and again is able to capture

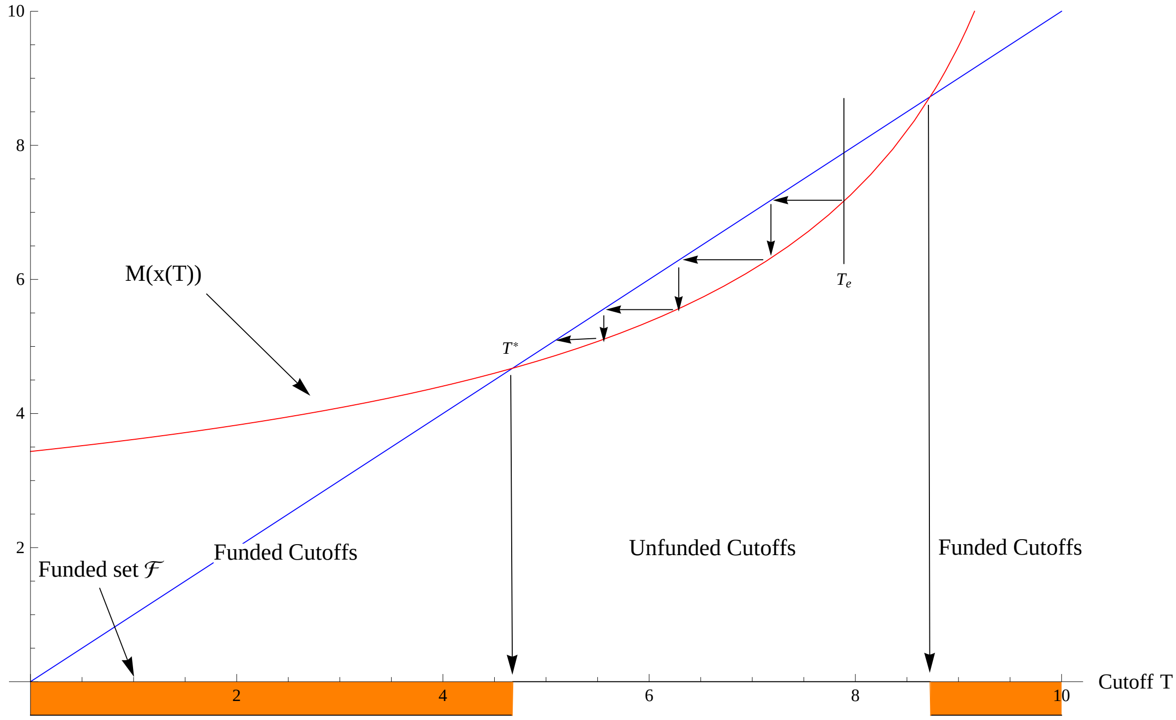

Figure 3: The Figure illustrates the case in which T_e cannot be sustained as an equilibrium. The maturity cutoff T_e is not part of the funded set, which means that if all firms with maturities beyond T_e redraw their maturities, the interval $[0, T_e]$ cannot be financed. Firms that cannot receive funding redraw their maturities, meaning that now the originally funded maturities $[M(x(M(\beta))), M(\beta)]$ become unfunded, such firms on this interval will look for second best projects with lower maturities. This process repeats itself until we arrive at a fixed point $T* = M(x(T*))$, as indicated in the Figure.

the whole market. Competition thus drives the equilibrium posted funding schedules all the way to T_3, or in general to the highest root of $NVR(T)$. ■

The parameter restrictions needed for the proof are simply derived by plugging in 0 and $\overline{T}$ as funding cutoffs:

$$NVR(0) = -[\beta + (1-\beta)\Delta][R - D(0, x(0))] + \frac{M(\alpha\beta)}{\overline{T}}\{[\alpha\beta + (1-\alpha\beta)\Delta]R - 1\} < 0$$

where we note that $x(0) = \beta^{\frac{1+\alpha}{2}} < \beta$. If $M(\alpha\beta) = 0$, this condition always holds as

$R - D(0, x(0)) \geq 0$. Further, we have at $\overline{T}$:

$$
\begin{aligned}
NVR\left(\overline{T}\right) &= [\alpha\beta + (1-\alpha\beta)\Delta] R - \frac{1}{\overline{T}} \int_0^{\overline{T}} \frac{D(t,\beta)}{D(t,\alpha\beta)} dt - \{[\beta + (1-\beta)\Delta] R - 1\} \\
&= -\beta(1-\alpha)(1-\Delta) R + 1 - \frac{1}{\overline{T}} \int_0^{\overline{T}} \frac{D(t,\beta)}{D(t,\alpha\beta)} dt \\
&= -\beta(1-\alpha)(1-\Delta) R + 1 - \frac{1-\alpha\beta}{1-\beta} + \frac{1-\alpha}{\lambda\overline{T}(1-\beta)} \log\left[\frac{e^{\lambda\overline{T}}\beta + (1-\beta)\Delta}{\beta + (1-\beta)\Delta}\right] > 0
\end{aligned}
$$

Note that $\lim_{\overline{T}\to\infty} \frac{1-\alpha}{\lambda\overline{T}(1-\beta)} \log\left[\frac{e^{\lambda\overline{T}}\beta+(1-\beta)\Delta}{\beta+(1-\beta)\Delta}\right] = \frac{1-\alpha}{1-\beta} > 0$, which reduces the above condition to $1 + \alpha - \beta(1-\alpha)(1-\Delta) R > 0$.

Thus, if T^* coincides with one of the roots of $\mathcal{E}$ then we call the equilibrium entrepreneur driven. If T^* coincides with a subset edge in $\mathcal{F}$, then we call the equilibrium investor driven. For the rest of paper, if not explicitly mentioned otherwise, we assume that we $NVR(T)$ only has one root on $\left[0, \overline{T}\right]$, which we denote by T_e.

3.2 No dilution – $\alpha = 1$

First, let us analyze the case in which there is no dilution from firms' changing their maturities, which is the case second best projects are of the same quality as first best projects ($\alpha = 1$). In this case, when firms redraw maturities, there is no dilution of the pool – irrespective of how many entrepreneurs change their maturity, the pool quality stays constant at β. We thus conclude that the funding horizon $M(\beta)$ stays constant as well, i.e., the funded set is always given by $\mathcal{F} = \left[0, \min\left[M(\beta), \overline{T}\right]\right]$. Thus, the only effect from firms' changing their maturities is that the mass of firms at each t changes.

The expected payoff from changing maturity is given by

$$\begin{aligned} redraw\left(T,\beta\right) &= \frac{1}{\overline{T}}\int_{0}^{\min\left[M(\beta),\overline{T}\right]} \beta\left[R-\tilde{D}\left(t,\beta\right)\right]+\left(1-\beta\right)\Delta\left[R-e^{-\lambda t}\tilde{D}\left(t,\beta\right)\right]dt \\ &= \min\left[\frac{M\left(\beta\right)}{\overline{T}},1\right]\left(\left[\beta+\left(1-\beta\right)\Delta\right]R-1\right). \end{aligned} \tag{25}$$

Thus, if $M\left(\beta\right)\geq\overline{T}$ the firm is indifferent between redrawing and staying as the entrepreneur always receives the expected NPV of the project regardless of maturity. If, on the other hand, there is some rationing to start with, i.e., $M\left(\beta\right)<\overline{T}$, no one in the funded area will want to change their maturities for fear of losing funding, since the probability of finding a new project in the funded range is strictly less than one, $\frac{M(\beta)}{\overline{T}}<1$, and the payoff to funding is again just the average NPV of the project. What does this impy for the funded set and the entrepreneur driven cutoff? It is clear that T_e is a corner solution $T_e=\min\left[\overline{T},M\left(\beta\right)\right]$. The equilibrium for $M\left(\beta\right)<\overline{T}$ is given by $T^{*}=\max\mathcal{F}\cap\left[0,M\left(\beta\right)\right]=M\left(\beta\right)$, so that even after redrawing maturities $\left[0,M\left(\beta\right)\right]$ are funded in equilibrium.

This leads to straightforward welfare implications for this special case. As some previously rationed firms will find projects in the funded maturity range, the redrawing of maturities leads to an unambiguous welfare improvement. To quantify this gain, it is easy to show that everyone on $\left[M\left(\beta\right),\overline{T}\right]$ redraws, and thus a mass $\frac{\overline{T}-M(\beta)}{\overline{T}}\frac{M(\beta)}{\overline{T}}$ of firms that didn't have funding before successfully finds financing with their second best projects, increasing surplus.

Proposition 3 *If $\alpha=1$, there is an unambiguous welfare gain from redrawing. The maturities that are funded in equilibrium are simply $\left[0,\min\left[M\left(\beta\right),\overline{T}\right]\right]$. For $M\left(\beta\right)<\overline{T}$, it is a dominant strategy for unfunded firms to redraw, and also for funded firms to stay.*

3.3 Maximal dilution – $\alpha = 0$

Consider now the case in which second best projects are always of the bad type, that is $\alpha = 0$. In this case we will be able to prove results simply based on the funded set $\mathcal{F}$ and thus can sidestep the possible messy analysis of the set in which firms do not redraw their maturities, $\mathcal{E}$.

Let us first consider $M(\beta) < \overline{T}$, i.e., even before potential redrawing of maturities some firms are rationed out of the market. The average pool quality $x(T)$ with $\alpha = 0$ then becomes

$$x(T) = \beta \frac{\overline{T}}{2\overline{T} - T} \tag{26}$$

We can now establish that the externality of firms that adopt second-best projects can be sufficiently strong to unravel funding at all maturities irrespective of the entrepreneur cutoff T_e, i.e. the set of fundable cutoffs $\mathcal{F}$ is empty.

Proposition 4 *The set of fundable cutoffs $\mathcal{F}$ is empty for $\beta < 2\underline{\beta}$ and $M(\beta) < \overline{T}$. Funding completely unravels because rationed firms adopt second-best projects.*

Second, let us consider the case where $\beta \geq \overline{\beta}$ so that $M(\beta) > \overline{T}$. We will now derive a sufficient condition for a no-redrawing equilibrium to exist. Let us start by conjecturing that everyone decides not to redraw. In this case, is there an individual incentive for any entrepreneur to redraw? We know from above that the entrepreneur's incentives are summarized by $NVR\left(\overline{T}\right)$. If $NVR\left(\overline{T}\right) < 0$, then indeed we have a no-redrawing equilibrium. If however, we have $NVR\left(\overline{T}\right) > 0$, we have some redrawing at $\overline{T}$, and combining it with the result in the preceding proposition we know that then if the set $\mathcal{F}$ is empty after this cycle of redrawings is over, such that financing becomes impossible across all maturities.

3.4 Intermediate dilution – $\alpha \in (0,1)$

Finally, let us now look at the general case where $\alpha \in (0,1)$. Our first step is to investigate set of funded cutoffs $\mathcal{F}$. We first establish the following Lemma.

Lemma 3 *Suppose $\beta \leq \frac{1+R}{2R}$ so that the function defining $\mathcal{F}$, $M(x(T))$, is increasing and convex in T. Then the set $\mathcal{F}$ contains an interval starting at 0 iff $\beta > \frac{2}{1+\alpha}\underline{\beta}$ so that*

1. *if $M(\beta) < \overline{T}$ we have $\mathcal{F} = [0, T_1]$ for some T_1, and*
2. *if $M(\beta) > \overline{T}$ we have $\mathcal{F} = [0, T_1] \cup \left[T_2, \overline{T}\right]$ for some $T_1 \leq T_2$ (i.e. there is at most one hole).*

If $\beta < \frac{2}{1+\alpha}\underline{\beta}$ we have $0 \notin \mathcal{F}$ so that

1. *if $M(\beta) < \overline{T}$ we have $\mathcal{F} = \emptyset$ (i.e. complete unraveling), and*
2. *if $M(\beta) > \overline{T}$ we have that $\mathcal{F} = \left[T_1, \overline{T}\right]$ for some $T_1 > 0$.*

Suppose $\beta \leq \frac{1+R}{2R}$ and $\beta < \frac{2}{1+\alpha}\underline{\beta}$. Suppose $\overline{T}$ is such that $M(\beta) < \overline{T}$. Then we know that the funded set is empty – funding is just possible up to $M(\beta)$ without redrawing, which implies that $T = M(x(T))$ has a unique solution greater than $\overline{T}$. Again, there is *complete unraveling of all funding solely due to redrawing.*[6]

Second, suppose $\beta \in \left(\frac{2}{1+\alpha}\underline{\beta}, \frac{1+R}{2R}\right)$ so we know that the funded set $\mathcal{F}$ includes some interval that starts at 0. A sufficient condition for the funded set $\mathcal{F}$ to be equal to $\left[0, \overline{T}\right]$ is

[6]Note that $\beta < \frac{2}{1+\alpha}\underline{\beta}$ implies $\alpha\beta < \frac{2\alpha}{1+\alpha}\underline{\beta} < \underline{\beta}$ which in turn implies that $M(\alpha\beta) = 0$.

that $M'(x(0))\,x'(0) > 1$ and that $M(\beta) > \overline{T}$. It is also intuitive that if $M(\beta) < \overline{T}$ that the interval $\left[M(\beta), \overline{T}\right] \notin \mathcal{F}$ – if a project is not funded in the absence of redrawing, then surely an inferior second best project of the same maturity will not be funded either.

3.5 Implications

We will now numerically investigate the properties of the equilibrium when we allow firms to change their investment behavior in response to adverse selection. As we will see, the endogenous response of firms to the adverse selection can create additional amplification relative to the analysis in our baseline model.

Recall that by assumption there is a unique equilibrium if the net value to redrawing, i.e. $NVR(T)$, cuts 0 uniquely from below at T_e. Hence the equilibrium maximum funded maturity is given by $T^* = \max \mathcal{F} \cap [0, T_e]$.

The maximum funded maturity T^* has interesting dynamics. In particular, because of the endogenous adjustment of firms' investment decisions to adverse selection, the maximum funded maturity displays "falling of the cliff" dynamics: a small change in fundamentals can lead to a large change in funded maturities.

This is illustrated in Figure 4, which summarizes both the model without redrawing of maturities and the additional effects that result from the endogenous adjustment in firms' investment decisions. The black dashed line is the equilibrium maximum maturity absent redrawing of maturities. This line shows that already absent knock-on effects from the distortion of firms investment decisions, a deterioration in the average quality β leads to a non-linear decrease in the maximum funded maturity. Now consider, in addition, the effects resulting from fims' privately optimal decision to change their maturities. To do this, for simplicity we consider the case in which $\alpha = .7$, which means that original projects have an average quality of β, whereas second-best projects have a qualtiy of $.7 \times \beta < \beta$. The red line shows T_e, the cutoff that results from firms' privately optimal redrawing behavior. Firms located at maturities beyond T_e would decide to change their project maturity, even if funded. We find that in equilibrium no firm outside of this cutoff is funded (i.e. $T^* > M(\alpha\beta)$ for all β we consider). However, to determine whether a candidate redrawing cutoff T_e is actually funded by investors, we need to check whether it is part of the funded set, i.e., $T_e \in \mathcal{F}$.

The blue line in Figure 4 illustrates the equilibrium T^*. If this line does not coincide with the red line T_e, then the funding cutoff imposed by the investors is strictly tighter than the entrepreneur-driven cutoff T_e. In particular, the Figure illustrates that as the economy deteriorates and the average project quality β declines, the entrepreneur-driven maximum maturity T_e declines. For high β we see that $T^* = T_e$. Over that region, a deterioration in β leads to a moderate decline in the maximum funded maturity. This effect is driven by the pure pecuniary externality as entrepreneurs that redraw impose higher average debt financing costs on entrerpeneurs that were already being funded and did not redraw. Entrepreneurs redraw in the hope of getting better funding terms, even if the absence of redrawing would result in funding for everyone, i.e., $M(\beta) \geq 10$.

However, as β drops below .75 we see the second effect kick in. As more and more firms redraw, funding terms become sufficiently unattractive for firms with good projects. Since investors can no longer raise the face value of debt in order to break even, they start pulling the funding even from those entrepreneurs that were just indifferent between redrawing and

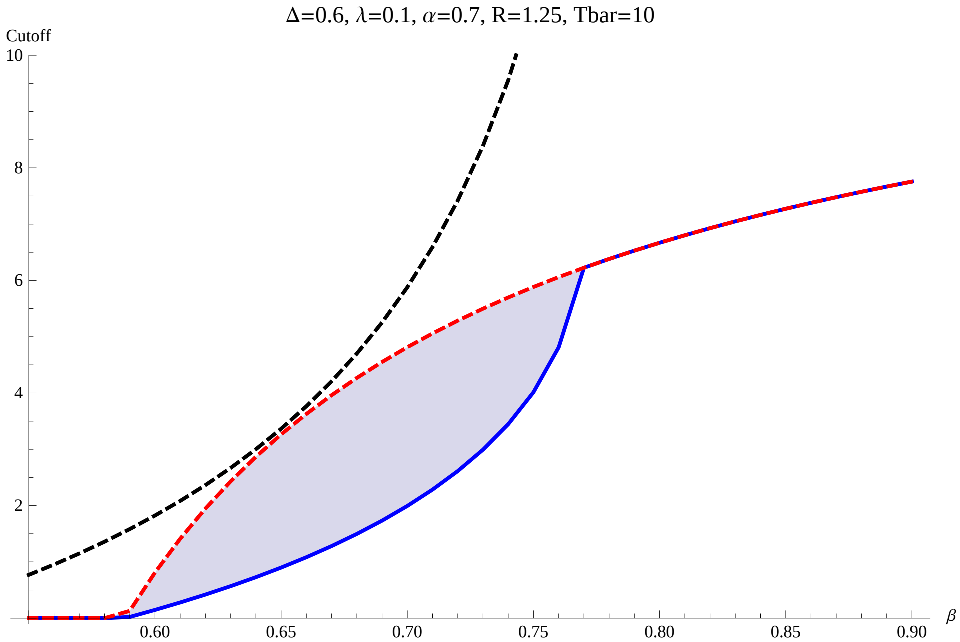

Figure 4: Static vs redrawing equilibrium. The black dashed line is the equilibrium maximum maturity in the static model. The red line shows T_e, the cutoff that results from firms' privately optimal redrawing behavior. The blue line illustrates the region for which the funding cutoff imposed by the investors is strictly tighter than the entrepreneur-driven cutoff T_e.

staying. As these firms lose their funding, they search for shorter maturity projects by redrawing. Through this feedback loop the maximum funded maturity deteriorates. In the figure, this is the case when the red line lies above the blue line – although entrepreneurs are willing to stop redrawing their maturities at the red line, investors are unwilling to fund up to this threshold. This additional effect leads to a downward spiral all the way down to the blue line.

Figure 4 allows us to make two observations about the effects of firm's endogenous response to adverse selection. First, the maximum funded maturity when firms can change the maturity of their projects is lower than in the case in which they do not have this option. Second, in comparison to the baseline case, the equilibrium in which firms can redraw their projects is fragile, in the sense that a switch from an entrepreneur-driven equilibrium to an investor-driven equilibrium can lead to ‘falling off the cliff’ dynamics.

3.6 Welfare in the general case

We are now in a position the discuss the additional welfare effects that result from firms’ adjusting their maturities in response to adverse selection frictions. The two extreme case $\alpha = 1$ and $\alpha = 0$ demonstrated that there are two counteracting forces to consider. On one hand, as firms change their maturities, they shift project mass to lower maturities, which results in an inflow of new projects into the funded region. In isolation, as is the case for $\alpha = 1$, this will lead to a welfare improvement.

On the other hand, as firms distort their investment policy, they dilute the pool in the funded region whenever their second-best projects have lower net present value. This effect was at work in isolation for $\alpha = 0$: all additional firms that receive funding via maturity adjustment are negative NPV firms, thus reducing surplus. Even worse, the presence of these additional negative NPV firms will reduce the mass of good firms that can receive funding. This is because the dilution of pool quality leads to a shortening of the maximal funding horizon. Thus, the dilution will lead to welfare losses via a direct channel – individual firm quality drops – and via an indirect channel – pool quality drops that leads to less firms being funded.

For intermediate values of α, these two effects will both be present and thus have to be traded off against each other. The overall welfare effect of maturity adjustment is thus a combination of the welfare gains from a possible increase in the mass of funded firms and the welfare losses from funding worse projects and restricting funding to good projects at some maturities.

To compare overall welfare before and after redrawing, let us first establish the mass of people on $[0, T]$ when everyone on $\left[T, \overline{T}\right]$ redraws. Before redrawing, this mass is simply given by $m(T) = \frac{T}{\overline{T}}$. After everyone on $\left[T, \overline{T}\right]$ redraws, this mass is given by

$$m^s(T) = \frac{T}{\overline{T}} \frac{2\overline{T} - T}{\overline{T}} + 1_{\{T < M(\alpha\beta)\}} \frac{M(\alpha\beta) - T}{\overline{T}} \frac{\overline{T} - T}{\overline{T}}. \tag{27}$$

This means that we can decompose the welfare after redrawing, $W^s(\alpha, \beta)$ into its separate effects:

$$\begin{aligned} W^s(\alpha, \beta) &= m^s(T)\left[x(T) NPV_G + (1 - x(T)) NPV_B\right] && (28) \\ &= m^0(M(\beta))\left[\beta NPV_G + (1 - \beta) NPV_B\right] \\ &\quad + \left[m^s(T) - m^0(M(\beta))\right]\left[\beta NPV_G + (1 - \beta) NPV_B\right] \\ &\quad - m^s(T)\left[\beta - x(T)\right]\left[NPV_G - NPV_B\right] && (29) \\ &= W(\beta) + Add.Funding \times Avg.Initial.NPV \\ &\quad - New.Mass \times Quality.Diff \times NPV.Diff && (30) \end{aligned}$$

where we know that $Avg.Initial.NPV > 0$ and also that $NPV.Diff > 0$. This decomposition states that welfare after firms adjust their maturities is the sum of the welfare before maturity adjustments, plus the additional mass of funded entrerpreneurs each contributing their initial average NPV, less the new total mass overall multiplied by the deterioration of the NPV brought about by the usage of second best project. Recall the two extreme cases discussed above. When $\alpha = 1$ we have $Quality.Diff = 0$ and $T = M(\beta)$, so $Add.Funding > 0$.

Thus, there is an unambiguous welfare gain. For $\alpha = 0$, on the other hand, we know from the above arguments that there is either a total welfare loss or at least a partial one – each project that arises out of redrawing is negative NPV, and some good projects will be squeezed out of the market by the investor's zero profit condition. While the two extreme cases show that the welfare difference relative to the static case can go either way, we can derive the following sufficient condition under which firms' privately optimal response to maturity rationing leads to additional welfare losses:

Proposition 5 *A sufficient condition for an equilibrium with redrawing to lead to a welfare loss relative to the static case is that $T < T_0$ where T_0 is given by*

$$T_0 = \begin{cases} \overline{T}\left(1 - \sqrt{1 - \frac{M(\beta)}{\overline{T}}}\right) & if\, T_0 < M(\alpha\beta) \\ \overline{T}\left(\frac{M(\beta) - M(\alpha\beta)}{\overline{T} - M(\alpha\beta)}\right) & if\, T_0 > M(\alpha\beta) \end{cases} \tag{31}$$

$T_0 > M(\alpha\beta)$ only holds iff α and β are such that

$$\left(2 - \frac{M(\alpha\beta)}{\overline{T}}\right)\frac{M(\alpha\beta)}{\overline{T}} \geq \frac{M(\beta)}{\overline{T}} \tag{32}$$

Proposition 5 states that there is an unambigous welfare loss if less people are funded after redrawing than in the static equilibrium, i.e. whenever $Add.Funding = [m^s(T) - m^0(M(\beta))] < 0$. This is inuitive: When after maturity adjustments fewer firms are financed overall, and if the average quality of those firms is lower than before firms adjust their maturities, *a fortiori* surplus must decrease.

Figure 5 depicts the total welfare, i.e. the total funded NPV, in the benchmark case

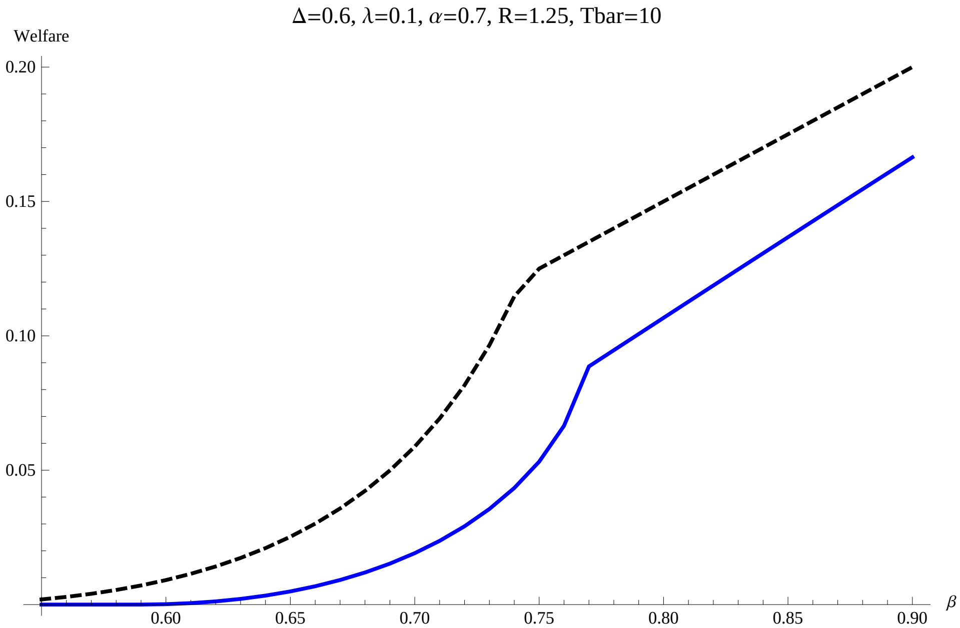

Figure 5: Total funded NPV in benchmark case (black dashed) and in case with redrawing (blue solid) for $\alpha = .7$, $\lambda = .1$, $\Delta = .6$ and $R = 1.25$.

without redrawing (black dashed line) and in the case in which redrawing is allowed (blue solid line). We see that the enodgenous maturity choices of the agent can substantially reduce overall welfare by distorting market rates and funding cutoffs. The kink in the black curve occurs at $M(\beta) = 10$, at which point the expansion of funding ceases, and further increases in β simply lead to an increase in the average quality of funded projects. The kink in the blue curve occurs at the point at which the T_e starts coinciding with T^* in Figure 4. For higher α, the welfare in the redrawing case can lie above the no redrawing case, as discussed in Section 3.2.

4 Conclusion

This paper proposes a model of maturity rationing. We build on the credit rationing literature to illustrate how asymmetric information frictions lead to a theory of maximum funded maturities and the distortions that follow from it. In our model, firms may be unable to obtain financing for long-term projects because asymmetric information worsens with the maturity of the project. Beyond some maturity the lending market can break down (*maturity rationing*). Firms whose first-best projects cannot get financed may react by adopting second-best projects of shorter maturities. This generates enodgenous adverse selection in this model. Redrawing worsens the pool of financed projects, further amplifying rationing. Hence, a *rationing spiral* emerges. Our model suggests that maturity rationing is stronger during recessions and, through its knock-on effects, may amplify the business cycle. In addition, the model shows how asymmetric information frictions generate short-termism in investment decisions.

References

Bester, H. (1985): "Screening vs. Rationing in Credit Markets with Imperfect Information," *American Economic Review*, 75(4), pp. 850–855.

Bolton, P., and M. Dewatripont (2005): *Contract Theory*. MIT Press.

DeMeza, D., and D. C. Webb (1987): "Too Much Investment: A Problem of Asymmetric Information," *Quarterly Journal of Economics*, 102(2), pp. 281–292.

Diamond, D. W. (1991): "Debt Maturity Structure and Liquidity Risk," *The Quarterly Journal of Economics*, 106(3), 709–737.

Flannery, M. J. (1986): "Asymmetric Information and Risky Debt Maturity Choice," *Journal of Finance*, 41(1), 19–37.

Harris, M., and A. Raviv (1992): "Frinancial Contracting Theory," in *Advances in economit theory: Sixth World Congress*, ed. by J.-J. Laffont, vol. II. Cambridge University Press.

Jaffee, D. M., and F. Modigliani (1969): "A Theory and Test of Credit Rationing," *American Economic Review*, 59(5), pp. 850–872.

Jaffee, D. M., and T. Russell (1976): "Imperfect Information, Uncertainty, and Credit Rationing," *Quarterly Journal of Economics*, 90(4), pp. 651–666.

Kurlat, P. (2010): "Lemons, Market Shutdowns and Learning," Working Paper, Stanford University.

Stiglitz, J. E., and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71(3), pp. 393–410.

Suarez, J., and O. Sussman (1997): "Endogenous Cycles in a Stiglitz-Weiss Economy," *Journal of Economic Theory*, 76(3), pp. 47–71.

A Appendix

A.1 Comparative statics of $D(t,\beta)$ and $M(\beta)$

The derivatives of $D(t,\beta)$ are

$$\frac{\partial D(t,\beta)}{\partial t} = D(t,\beta)^2(1-\beta)\Delta\lambda e^{-\lambda t} > 0 \tag{A.1}$$

$$\frac{\partial D(t,\beta)}{\partial \beta} = -D(t,\beta)^2\left(1-\Delta e^{-\lambda t}\right) < 0 \tag{A.2}$$

The derivatives of $M(\beta)$ are

$$\frac{\partial M(\beta)}{\partial \beta} = \frac{R-1}{(1-\beta)(1-\beta R)\lambda} > 0 \tag{A.3}$$

$$\frac{\partial M(\beta)}{\partial \lambda} = -\frac{1}{\lambda}T(\beta) < 0 \tag{A.4}$$

$$\frac{\partial M(\beta)}{\partial R} = \frac{1}{\lambda R(1-\beta R)} > 0 \tag{A.5}$$

$$\frac{\partial M(\beta)}{\partial \Delta} = \frac{1}{\lambda\Delta} > 0 \tag{A.6}$$

A.2 Unobservable maturity

Suppose maturity is unobservable, so that the financiers offer a maturity un-contingent contract $D(\beta) \leq R$. The maximum value that financiers can possibly recoup per investment then is $D = R$ which gives

$$er\left(\overline{T}\right) = \frac{1}{\overline{T}}\int_0^{\overline{T}} \beta R + (1-\beta)\Delta e^{-\lambda t}R dt = R\left[\beta + (1-\beta)\Delta\frac{1}{\lambda\overline{T}}\left(1-e^{-\lambda\overline{T}}\right)\right]$$

We can now show that there is a unique $\overline{T}$ beyond which this expected return per project falls below the initial investment of 1. To this end, let us investigate the function

$$f(x) = \frac{1}{x}\left(1-e^{-x}\right)$$

$$f'(x) = \frac{e^{-x}(1+x-e^x)}{x^2}$$

First, by L'Hopital we know that $\lim_{x\to 0} f(x) = 1$ and $\lim_{x\to 0} f'(x) = -\frac{1}{2}$. Also, since $1+x < e^x$ for all $x > 0$ we have $f'(x) < 0$ for all $x \geq 0$. Further, note that $\lim_{x\to\infty} f(x) = 0$. We thus know that

$$\lim_{\overline{T}\to\infty} er\left(\overline{T}\right) = R\beta.$$

Thus, only if $\beta > \frac{1}{R}$ will there be funding without observable maturity in the limit. For any $\beta < \frac{1}{R}$ there is a unique finite $\overline{T}^*(\beta)$ so that if the distribution extends beyond this value that the unobservable maturity market (without redrawing) completely collapses. However, for $\beta < \frac{1}{R}$ we note that there exists a finite $\overline{T}^*$ such that the whole market breaks down. Funding, however, would still be available in the observable maturity case for low levels of $t < M(\beta)$.

The actual closed form solution for $\overline{T}^*$ involves a Lambert-W function, and is given by

$$\overline{T}^*(\beta) = \frac{W\left[-\frac{(1-\beta)\Delta}{\frac{1}{R}-\beta}e^{-\frac{(1-\beta)\Delta}{\frac{1}{R}-\beta}}\right] + \frac{(1-\beta)\Delta}{\frac{1}{R}-\beta}}{\lambda}$$

for $\underline{\beta} < \beta < \frac{1}{R}$. It is easy to check that $\overline{T}^*(\beta) > M(\beta)$. We additionally have $\overline{T}^*\left(\underline{\beta}\right) = 0$ and $\overline{T}^*\left(\frac{1}{R}\right) = \infty$.

A.3 Rollover Contracts

In this section we show that the assumption that firms match maturities is without loss of generality. The reason is that when project maturity is observable, a firm that cannot raise financing by matching maturities can also not raise financing through a rollover debt contract.

To see this, assume that some maturity T firm drawn out of a pool of quality β (not necessarily the static β) is rationed and that the longest available maturity is $M(\beta) = t < T$. We know show that a firm with a project of maturity T cannot finance via rollover if (i) project maturity is observed and (ii) the financier learns at rollover whether project has defaulted on $[0, t]$.

Proof: At the rollover date, the financier updates his belief on whether the financed firm is good or bad. Conditional on a firm having survived until the rollover date t, the probability that the firm is good is given by

$$\widehat{\beta}(t) = \frac{\beta}{\beta + \Delta(1-\beta)e^{-\lambda t}}, \tag{A.7}$$

and the probability that the firm is bad is

$$1 - \widehat{\beta}(t) = \frac{\Delta(1-\beta)e^{-\lambda t}}{\beta + \Delta(1-\beta)e^{-\lambda t}}. \tag{A.8}$$

This implies that when rolling over its debt from t to T, the maximum a firm can credibly promise to repay is

$$X = e^{-r(T-t)}\left[\widehat{\beta}(t)e^{rT}R + \left(1-\widehat{\beta}(t)\right)e^{-\lambda(T-t)}e^{rT}R\right] \tag{A.9}$$

$$e^{-rt}X = \widehat{\beta}(t)R + \left(1-\widehat{\beta}(t)\right)e^{-\lambda(T-t)}R \tag{A.10}$$

$$= \frac{\beta}{\beta + \Delta(1-\beta)e^{-\lambda t}}R + \frac{\Delta(1-\beta)e^{-\lambda t}}{\beta + \Delta(1-\beta)e^{-\lambda t}}e^{-\lambda(T-t)}R \tag{A.11}$$

$$= \frac{R}{\beta + \Delta(1-\beta)e^{-\lambda t}}\left[\beta + \Delta(1-\beta)e^{-\lambda T}\right] \tag{A.12}$$

where we used the fact that the maximum discounted face-value that a firm can credibly commit to is R. From 0 to t then the firm will be able to promise a maximal repayment of X if it stays alive until t, which gives the maximimum a firm can promise to repay at 0 as

$$e^{-rt}\left[\beta X + \Delta(1-\beta)e^{-\lambda t}X\right] \tag{A.13}$$

We can rewrite this as

$$e^{-rt}X\left[\beta + \Delta(1-\beta)e^{-\lambda t}\right]$$

$$= \frac{R}{\beta + \Delta(1-\beta)e^{-\lambda t}}\left[\beta + \Delta(1-\beta)e^{-\lambda T}\right]\left[\beta + \Delta(1-\beta)e^{-\lambda t}\right] \tag{A.14}$$

$$= R\left[\beta + \Delta(1-\beta)e^{-\lambda T}\right] \tag{A.15}$$

But this is the same amount the firm can raise by financing directly up to date T. Hence, if maturity T is rationed, a firm with a project of maturity T can also not obtain financing by using rollover finance.

A.4 Benefit of redrawing

Note the following integral that we will evaluate numerous times. If an agent redraws onto $[0,T]$, he expects to repay on average the following discounted face value for $x > \alpha\beta$:

$$-\frac{1}{\overline{T}}\int_0^T D(t,x)\left(\alpha\beta+(1-\alpha\beta)\Delta e^{-\lambda t}\right)dt = -\frac{1}{\overline{T}}\int_0^T \frac{\alpha\beta+(1-\alpha\beta)\Delta e^{-\lambda t}}{x+(1-x)\Delta e^{-\lambda t}}dt \tag{A.16}$$

$$= -\frac{T}{\overline{T}}\frac{(1-\alpha\beta)}{(1-x)}+\frac{x-\alpha\beta}{\lambda\overline{T}(1-x)x}\log\left[\frac{e^{\lambda T}x+(1-x)\Delta}{x+(1-x)\Delta}\right] \tag{A.17}$$

It is clear that for $\alpha = 0$ and for $\alpha = 1$ (and thus $x = \beta$) we have considerable simplifications. We can now write

$$\begin{aligned} switch(T,\beta) &= \frac{1}{\overline{T}}\int_0^T \alpha\beta\left[R-D(t,x(T))\right]+(1-\alpha\beta)\Delta\left[R-e^{-\lambda t}D(t,x(T))\right]dt \\ &\quad +\mathbf{1}_{\{M(\alpha\beta)>T\}}\frac{1}{\overline{T}}\int_T^{\min\left[M(\alpha\beta),\overline{T}\right]}\alpha\beta\left[R-D(t,\alpha\beta)\right]+(1-\alpha\beta)\Delta\left[R-e^{-\lambda t}D(t,\alpha\beta)\right]dt \quad (A.18)\\ &= \frac{1}{\overline{T}}\int_0^T \alpha\beta\left[R-D(t,x(T))\right]+(1-\alpha\beta)\Delta\left[R-e^{-\lambda t}D(t,x(T))\right]dt \\ &\quad +\mathbf{1}_{\{M(\alpha\beta)>T\}}\frac{\min\left[M(\alpha\beta),\overline{T}\right]-T}{\overline{T}}\left(\left[\alpha\beta+(1-\alpha\beta)\Delta\right]R-1\right) \quad (A.19)\end{aligned}$$

A.5 Uniform as limit of truncated exponential

All of the results of the paper go through under a truncated exponential. A uniform distribution on $\left[0,\overline{T}\right]$ is the limit of a truncated exponential distribution with intensity γ on $\left[0,\overline{T}\right]$ as the intensity $\gamma \to 0$. The pdf of a truncated exponential on $\left[0,\overline{T}\right]$ is simply $pdf_{\gamma,\overline{T}}(t) = \frac{\gamma e^{-\gamma t}}{1-e^{-\gamma\overline{T}}}$. As $\gamma \to 0$, we see that we have a situation $\frac{"0"}{"0"}$ and we have to use L'Hopital's rule:

$$\lim_{\gamma\to 0} pdf_{\gamma,\overline{T}}(t) = \lim_{\gamma\to 0}\frac{\gamma e^{-\gamma t}}{1-e^{-\gamma\overline{T}}} = \lim_{\gamma\to 0}\frac{e^{-\gamma t}-\gamma t e^{-\gamma t}}{\overline{T}e^{-\gamma\overline{T}}} = \frac{1}{\overline{T}}$$

which is just the pdf of a uniform distribution on $\left[0,\overline{T}\right]$. Thus, as the intensity shrinks to zero a truncated exponential distribution becomes a uniform distribution. Thus, we have

$$pdf_{\gamma,\overline{T}}(t) = \begin{cases}\frac{1}{\overline{T}} & \gamma = 0\\ \frac{\gamma e^{-\gamma t}}{1-e^{-\gamma\overline{T}}} & \gamma > 0\end{cases}$$

A.6 Omitted proofs in the main text

Proof of Lemma 1.

First, a good project has uniformly zero variance in its discounted cash flows as $Return_t = e^{rt}R$, that is

$$Var_t^G\left(e^{-rt}\cdot Return_t\right) = Var_t^G(R) = 0 \tag{A.20}$$

On the other hand, the bad project will have a positive variance – it returns $e^{(r+\lambda)t}R$ with probability $p = \Delta e^{-\lambda t}$ and 0 with probability $(1-p) = \left(1-\Delta e^{-\lambda t}\right)$. Via the Bernoulli distribution variance formula $Var_{Bernoulli} = p(1-p)$ we have the variance of the discounted cash flows

$$Var_t^B\left(e^{-rt}\cdot Return_t\right) = \left(e^{\lambda t}R\right)^2\Delta e^{-\lambda t}\left(1-\Delta e^{-\lambda t}\right) = \Delta R^2\left(e^{\lambda t}-\Delta\right) \tag{A.21}$$

and we can see that Var_t^B is *increasing* in t.

Ex-ante, that is before the project is revealed to be good or bad, the variance can be easily determined via the variance decomposition $Var(X) = Var(\mathbb{E}[X|Y]) + \mathbb{E}[Var(X|Y)]$ where we identify X with the outcome, i.e. $Return_t$, and Y with the project being good with probability β and bad with probability $(1-\beta)$. Thus, the ex-ante variance for a project of maturity t is

$$\begin{aligned}
& Var_t^{ExAnte}\left(e^{-rt} \cdot Return_t\right) \\
= \; & Var_t\left(PV_t^i\right) + \mathbb{E}\left[Var_t^i\right] && \text{(A.22)} \\
= \; & \beta\left(R - \mathbb{E}\left[PV_t^i\right]\right) + (1-\beta)\left(\Delta R - \mathbb{E}\left[NPV_t^i\right]\right) + \mathbb{E}\left[Var_t^i\right] && \text{(A.23)} \\
= \; & R^2\beta(1-\beta)(1-\Delta)^2 + \beta \cdot 0 + (1-\beta) Var_t^B && \text{(A.24)} \\
= \; & (1-\beta) R^2\left[\beta(1-\Delta)^2 + \Delta\left(e^{\lambda t} - \Delta\right)\right] && \text{(A.25)}
\end{aligned}$$

where we used PV_t^i as the present value of project of quality i so that $\mathbb{E}\left[PV_t^i\right] = \beta R + (1-\beta)\Delta R$, and $Var_t^B = \Delta R^2\left(e^{\lambda t} - \Delta\right)$. Again, the ex-ante variance is increasing in maturity t. How does this variance change with a shift in pool quality β?

$$\begin{aligned}
\frac{\partial Var_t^{ExAnte}}{\partial \beta} &= -R^2\left[\beta(1-\Delta)^2 + \Delta\left(e^{\lambda t} - \Delta\right)\right] + (1-\beta) R^2 (1-\Delta)^2 && \text{(A.26)} \\
&= R^2\left[(1-2\beta)(1-\Delta)^2 - \Delta\left(e^{\lambda t} - \Delta\right)\right] && \text{(A.27)}
\end{aligned}$$

We see that this derivative is clearly smaller the larger β, and is unambiguously negative for $\beta \geq \frac{1}{2}$. Thus, let us assume $\beta = 0$. For what values of Δ is this derivative non-positive?

$$\begin{aligned}
0 &\geq (1-\Delta)^2 - \Delta\left(e^{\lambda t} - \Delta\right) && \text{(A.28)} \\
\iff e^{\lambda t} &\geq \frac{(1-\Delta)^2 + \Delta^2}{\Delta} && \text{(A.29)} \\
\iff e^{\lambda t} - 1 &\geq \frac{1 - 3\Delta + 2\Delta^2}{\Delta} && \text{(A.30)}
\end{aligned}$$

The left hand side is always non-negative for $t \geq 0$, and the right hand side is unambiguously non-positive for $\Delta \in \left[\frac{1}{2}, 1\right]$ – the quadratic expression $2\Delta^2 - 3\Delta + 1$ has roots $\frac{1}{2}$ and 1 and is increasing and positive as $|\Delta|$ becomes large.

■

Proof of Proposition 3.

First, let us investigate if a conjectured cutoff $T = 0$ leaves any funding at $t = 0$. Noting that $x(0) = \frac{\beta}{2}$, we have the following conditions for a positive funding maturity

$$\begin{aligned}
\log[...] &> 0 && \text{(A.31)} \\
\iff \frac{\left(1 - \frac{\beta}{2}\right) R\Delta}{1 - \frac{\beta}{2} R} &> 1 && \text{(A.32)} \\
\iff \beta &> 2\frac{(1 - \Delta R)}{(1-\Delta) R} = 2\underline{\beta} && \text{(A.33)}
\end{aligned}$$

Thus, if $\beta < 2\underline{\beta}$ the cutoff $T = 0$ is not in the funded set. Second, we know that

$$\frac{dM\left(x\left(T_i\right)\right)}{dT_i} = \frac{\partial M\left(x\right)}{\partial x}\frac{dx}{dT_i} = \frac{R-1}{\left(1-x\right)\left(1-xR\right)\lambda}\frac{dx}{dT_i} \tag{A.34}$$

$$= \frac{R-1}{\left(1-x\right)\left(1-xR\right)\lambda}\frac{\beta}{\left(2-\frac{T}{\overline{T}}\right)^2\overline{T}} \tag{A.35}$$

$$= \frac{R-1}{\left(1-x\right)\left(1-xR\right)\lambda}\frac{x^2}{\beta\overline{T}} \tag{A.36}$$

$$= \frac{R-1}{\left(\frac{1}{x}-1\right)\left(\frac{1}{x}-R\right)\lambda}\frac{1}{\beta\overline{T}} > 0 \tag{A.37}$$

as $(1-x)$ and $(1-xR)$ are both positive by assumption. Furthermore, we see that as x increases the denominator shrinks, and thus the derivative increases – the function $M\left(x\left(T\right)\right)$ is convex in T. We conclude that in the case of $\beta < 2\underline{\beta}$ and $T\left(\beta\right) < \overline{T}$ the funded set $\mathcal{F}$ is empty. In other words, the *market completely unravels.*

■

Proof of Lemma 2. We will start at checking if $0 \in \mathcal{F}$ – under what conditions will a project of maturity $t = 0$ still get funding even if everyone else, i.e. $\left(0, \overline{T}\right]$, redraws? Plugging in $x\left(0\right) = \beta\frac{1+\alpha}{2} > \alpha\beta$, we see that

$$M\left(x\left(0\right)\right) > 0 \tag{A.38}$$

$$\iff \frac{1}{\lambda}\log\left[\frac{\left(1-\beta\frac{1+\alpha}{2}\right)\Delta R}{1-\beta\frac{1+\alpha}{2}R}\right] > 0 \tag{A.39}$$

$$\iff \frac{\left(1-\beta\frac{1+\alpha}{2}\right)\Delta R}{1-\beta\frac{1+\alpha}{2}R} > 1 \tag{A.40}$$

$$\iff \beta > \frac{2}{1+\alpha}\frac{1-\Delta R}{\left(1-\Delta\right)R} = \frac{2}{1+\alpha}\underline{\beta} \tag{A.41}$$

We see that this condition trivially holds for $\alpha = 1$ by assumption $\beta > \underline{\beta}$. If we do not have $\beta > \frac{2}{1+\alpha}\underline{\beta}$ we know that $0 \notin \mathcal{F}$.

Next, let us look at the derivative of $M\left(x\left(T\right)\right)$ w.r.t. T. Writing it out, we have

$$\frac{dM\left(x\left(T\right)\right)}{dT} = M'\left(x\right)x'\left(T\right) > 0 \tag{A.42}$$

$$\frac{d^2M\left(x\left(T\right)\right)}{dT^2} = M'\left(x\right)x''\left(T\right) + M''\left(x\right)x'\left(T\right)^2 \tag{A.43}$$

Taking derivatives, we know that $M'\left(x\right) = \frac{R-1}{(1-x)(1-xR)\lambda} > 0$, $x'\left(T\right) = \beta\frac{\overline{T}(1-\alpha)}{\left(2\overline{T}-T\right)^2} \geq 0$ and $x''\left(T\right) = \beta\frac{2\overline{T}(1-\alpha)}{\left(2\overline{T}-T\right)^3} \geq 0$ (where the inequalities are strict for $\alpha < 1$). The second derivative of the maturity cut-off function is

$$M''\left(x\right) = \left(R-1\right)\frac{1+R-2Rx}{\left(1-x\right)^2\left(1-Rx\right)^2\lambda} \tag{A.44}$$

which is positive for

$$1+R-2Rx > 0 \tag{A.45}$$

$$\iff \frac{1+R}{2R} > x \tag{A.46}$$

We know that $\alpha\beta < x \leq \beta$, so any $\beta \leq \frac{1+R}{2R}$ will lead to a positive second derivative $M\left(x\left(T\right)\right)$ or convexity of the function $M\left(x\left(T\right)\right)$ for all cutoffs T.

We can now state our first result for $\alpha \in (0,1)$. The function $M(x(T))$ is increasing, it is convex if $\beta < \frac{1+R}{2R}$ and it is smaller than 0 at $T = 0$ if $\beta < \frac{2}{1+\alpha}\underline{\beta}$. Suppose $\overline{T}$ is such that $M(\beta) < \overline{T}$. Then we konw that the funded set is empty – funding is just possible up to $M(\beta)$ without redrawing, which implies that $T = M(x(T))$ has a unique solution greater than $\overline{T}$. Again, there is *complete unraveling of all funding solely due to redrawing.*[7]

Secondly, the function $M(x(T))$ is increasing, convex if $\beta < \frac{1+R}{2R}$ and greater than 0 at $T = 0$ if $\beta > \frac{2}{1+\alpha}\underline{\beta}$. Thus, for $\beta \in \left(\frac{2}{1+\alpha}\underline{\beta}, \frac{1+R}{2R}\right)$ we know that the funded set $\mathcal{F}$ includes some interval that starts at 0. As we know that $M(x(T))$ is convex and increasing, we know that $\mathcal{F}$ consists either of one interval $[0, T_1]$ where $T_1 \leq \overline{T}$ or two disjoint intervals $[0, T_1]$ and $\left[T_2, \overline{T}\right]$ with $T_1 < T_2 < \overline{T}$. Two intervals or full funding can only exists in the case of $M(\beta) > \overline{T}$, whereas one interval exists for the case of $M(\beta) < \overline{T}$. A sufficient condition for the funded set $\mathcal{F}$ to be equal to $\left[0, \overline{T}\right]$ is if $M'(x(0))\, x'(0) > 1$ and that $M(\beta) > \overline{T}$. It is also intuitive that if $M(\beta) < \overline{T}$ that the interval $\left[M(\beta), \overline{T}\right] \notin \mathcal{F}$ – if a project is not funded in the absence of redrawing, then surely an inferior second best project of the same maturity will not be funded either.

■

Proof of Proposition 5.

What cutoff level T_0 would give the same mass of funded entrepreneurs as the static model? Let $y \equiv \frac{T}{\overline{T}}$ for ease of notation. Then, for $M(\beta) < \overline{T}$, we have

$$m^s(T_0) = m^0(\beta) \tag{A.47}$$

$$\iff y(2-y) + 1_{\left\{\frac{M(\alpha\beta)}{\overline{T}} > y\right\}} \left(\frac{M(\alpha\beta)}{\overline{T}} - y\right)(1-y) = \frac{M(\beta)}{\overline{T}} \tag{A.48}$$

$$\iff y(2-y) + 1_{\left\{\frac{M(\alpha\beta)}{\overline{T}} > y\right\}} \left(\frac{M(\alpha\beta)}{\overline{T}} - y\right)(1-y) - \frac{M(\beta)}{\overline{T}} = 0 \tag{A.49}$$

Let us investigate the roots of $F(y) = y(2-y) + 1_{\left\{\frac{M(\alpha\beta)}{\overline{T}} > y\right\}} \left(\frac{M(\alpha\beta)}{\overline{T}} - y\right)(1-y) - \frac{M(\beta)}{\overline{T}}$. Suppose now that $y > \frac{M(\alpha\beta)}{\overline{T}}$ so we have a simple quadratic equation. Note that both roots of $F(y) = -y^2 + 2y - \frac{M(\beta)}{\overline{T}}$ are of positive sign, and by $F(0) < 0$ and $F(1) = -1 + 2 - \frac{M(\beta)}{\overline{T}} > 0$ we know that only one root is in $(0,1)$. We conclude that the threshold we are looking for is

$$T_0 = \overline{T}\left(1 - \sqrt{1 - \frac{M(\beta)}{\overline{T}}}\right) \in \left(0, \overline{T}\right) \tag{A.50}$$

Suppose on the other hand that $y < \frac{M(\alpha\beta)}{\overline{T}}$, so that we have $F(y) = y + \frac{M(\alpha\beta)}{\overline{T}}(1-y) - \frac{M(\beta)}{\overline{T}}$ which has a unique root

$$T_0 = \overline{T}\left(\frac{M(\beta) - M(\alpha\beta)}{\overline{T} - M(\alpha\beta)}\right) \in \left(0, \overline{T}\right) \tag{A.51}$$

We are left with checking for what parameters α and β does the different conditions apply. The roots are continuous during the switch from quadratic to the linear root for small changes in α and β by the definition of $m^s(T)$. Setting the equations equal, we see that the $T_0 > M(\alpha\beta)$ only applies iff there is α and β such that

$$\left(2 - \frac{M(\alpha\beta)}{\overline{T}}\right)\frac{M(\alpha\beta)}{\overline{T}} \geq \frac{M(\beta)}{\overline{T}} \tag{A.52}$$

■

[7] Note that $\beta < \frac{2}{1+\alpha}\underline{\beta}$ implies $\alpha\beta < \frac{2\alpha}{1+\alpha}\underline{\beta} < \underline{\beta}$ which in turn implies that $M(\alpha\beta) < 0$.

B Notation

$\left[0,\overline{T}\right]$	range of initial project maturities
t	maturity of specific project
β	initial population share of good projects
$D\left(t,\beta\right)=\frac{1}{\beta+(1-\beta)\Delta e^{-\lambda t}}$	Discounted face value at maturity t and pool quality β
$M\left(\beta\right)=\frac{1}{\lambda}\log\left[\frac{(1-\beta)\Delta R}{1-\beta R}\right]$	max financed maturity for pool quality β
$x\left(T\right)=\beta\frac{\overline{T}+\alpha\left(\overline{T}-T\right)}{2\overline{T}-T}$	share of good projects on no redrawing interval $[0,T]$
$\hat{x}=\alpha\beta$	share of good projects on redrawing interval $\left[T,\overline{T}\right]$
$\overline{\beta}=\frac{1-e^{-\lambda\overline{T}}\Delta R}{\left(1-e^{-\lambda\overline{T}}\Delta\right)R}$	min share of good projects that leads to no rationing on $\left[0,\overline{T}\right]$
$\overline{\overline{\beta}}=\frac{1}{R}$	min share of good projects that leads to no rationing at any maturity
$\underline{\beta}=\frac{1-\Delta R}{(1-\Delta)R}$	min share of good projects to avoid market breakdown
α	decrease in probability of good project if switching
$R\in(1,2)$	discounted payoff of good projects
$NPV_G=R-1$	NPV good project
$NPV_B=\Delta R-1$	NPV bad project
λ	default intensity of bad projects
$\Delta\in\left(\frac{1}{2},\frac{1}{R}\right)$	probability that bad project gets off the ground
$\mathcal{F}=\{T:T\leq M\left(x\left(T\right)\right)\}\cap\left[0,\overline{T}\right]$	set of funded cutoffs strategies T: $T\in\mathcal{F}$ means $[0,T]$ funded
$m\left(T\right)$	funding in static model if cutoff is T
$m^s\left(T\right)$	funding in switching model if cutoff is T