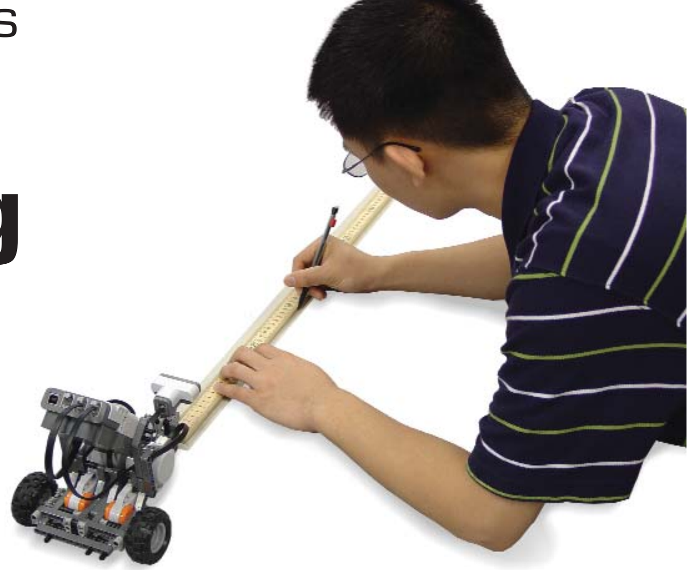
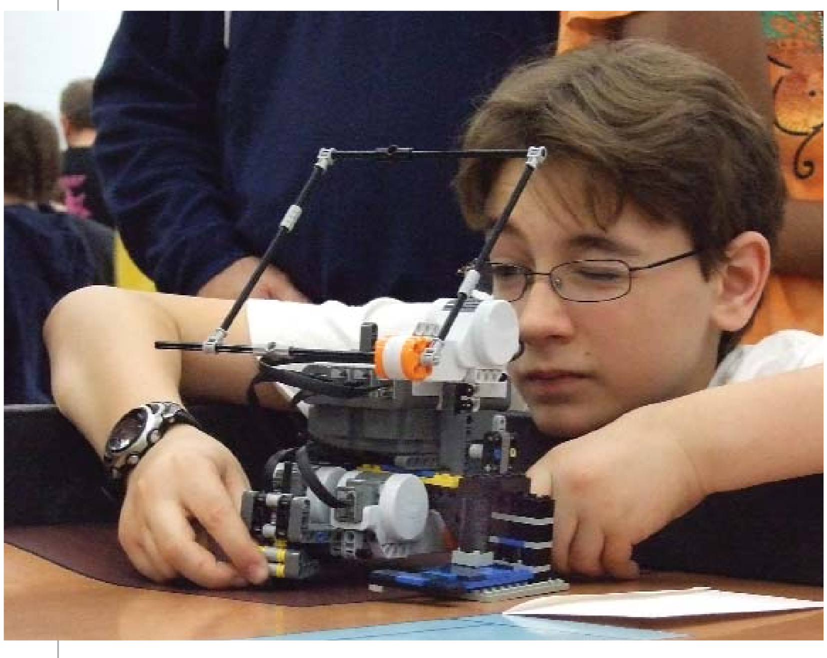
by Eli Silk, Christian Schunn & Robin Shoop

Motivating efficiency & meaning in problem solving with robotics

Synchronized Robot Dancing

Hobbyists and professionals regularly take advantage of math to solve interesting robot problems. In doing so, they speed up the problem-solving process and are able to create general solutions they can adapt and reuse in new situations. Because they do this with relative ease, it is common to overlook the challenges that young students have in trying to do the same thing. Beginners often develop their own creative solutions that work but may be inefficient, difficult to explain and hard to modify for other similar situations. We can help this by developing robot problems that take advantage of the engaging quality of robots and have strong connections to important mathematical ideas. The primary goal of the Robot Algebra project, a collaboration between Carnegie Mellon University's Robotics Academy and the Design-Based Learning group at the University of Pittsburgh's Learning Research and Development Center, is to do exactly that.





MOTIVATING MATHEMATICS IN ROBOT PROBLEMS

Introductory robot curricula often start by teaching students about simple robot control: How do I get my robot to move forwards *X* centimeters? And how can I get my robot to turn *Y* degrees? The culminating activity is often a maze that students can solve by hard-coding wheel rotation values for each segment. How can this problem be modified to encourage students to think about math? One option is to add constraints, such as providing the measurements for the maze at the last second and restricting the number of test trials. But that is artificial, and in the end, the best solutions for maze problems almost always rely on sensors to detect lines or walls and not on hard-coding values.

A better option would be to reframe the problem within an authentic context where precise measured control is the actual goal. It was with that in mind that we developed *Synchronized Robot Dancing*! Having a set dance routine with specified measurements is authentic to synchronization. In addition, students can be encouraged to generalize their solutions to different dance routines and different robots; this provides an incentive to connect to the math.

Synchronized Robot Dancing is a careful blend that is fun and accessible to young students while still being appropriately challenging and targeted to learning mathematics. Dance has always been popular but is even more so today with well-known television programs such as "Dancing with the Stars." *Synchronized Robot Dancing* attempts to capture that experience and interest and connect it to the programming of basic robot movements. The students take on the role of a knowledgeable dance choreographer who designs their own dance routines that they then program on provided robots. In a short time, students are able to build creative and individualized dance routines for a single robot. They begin to realize some of the difficulties in the task when they are challenged to get all of the robots, all with different physical characteristics, to complete the dance in a synchronized manner. That is where the mathematical challenge begins—a challenge that is especially suited to helping young students to think through the foundational math concept of proportionality.

PHOTOS BY MATT KAMBIC, CARNEGIE MELLON ROBOTICS ACADEMY

PROPORTIONALITY IN SYNCHRONIZING ROBOT MOVEMENTS

Being able to reason proportionally is a culmination of elementary school math focused on arithmetic. At the same time, it is a critical building block for high school math and science, beginning with algebra and extending far beyond. As a result, proportional reasoning problems are especially suited to middle school students but can be accessible to students in upper elementary school and can also be challenging for high school students and adults. Proportional reasoning is conceptually demanding because it requires one to think carefully about what is changing and what is staying the same from one situation to the next, to describe relationships between quantities in multiplicative terms rather than in additive terms and to keep track of multiple pieces of information at one time.

The differences in robots included the track width (distance between wheels) and wheel circumference (distance traveled in a single rotation).

Depending on the type of robot being used, there are potentially a great many relationships involved in controlling a robot's movement that are proportional in nature, e.g., the effect of gear ratios on the relationship between motor rotations and wheel rotations is a very common one. But a few simple relationships on a basic robot design offer sufficient challenge and complexity to think about proportionality.

We use the standard LEGO® MINDSTORMS® Education NXT Base Set. The robots we build have a differentially steered drive system in which two wheels are independently powered and controlled by separate motors. A third caster wheel in front is used for balance. We use a 1:1 gear ratio between the motor and wheels so that the correspondence between motor rotations and wheel rotations is simple; more advanced configurations could take advantage of different gear ratios for an additional layer of mathematical complexity. Each motor has a built-in rotation sensor to measure the number of rotations.

The robot movements can be controlled in a program by setting the number of motor rotations and the motor speed. As the number of motor rotations is *increased* in the program, how much the robot moves straight or turns *increases*. Similarly, as the motor speed is *increased* in the program the speed at which the robot does the movement *increases*. Both of these are direct proportional relationships. The physical parameters of the robot also play a role: in particular, the wheel circumference (distance around the wheels) and track width (distance between the wheels) are critical. As the circumference of the wheel *increases* the amount the robot moves for each motor rotation *increases*. This is another direct proportional relationship. There are also inverse proportional relationships. In particular, as the track width *increases* the amount the robot turns for each motor rotation *decreases*.

This table contains the parameters that need to be coordinated in order to successfully program a synchronized robot dance performance using robots with different physical characteristics

CHART 1

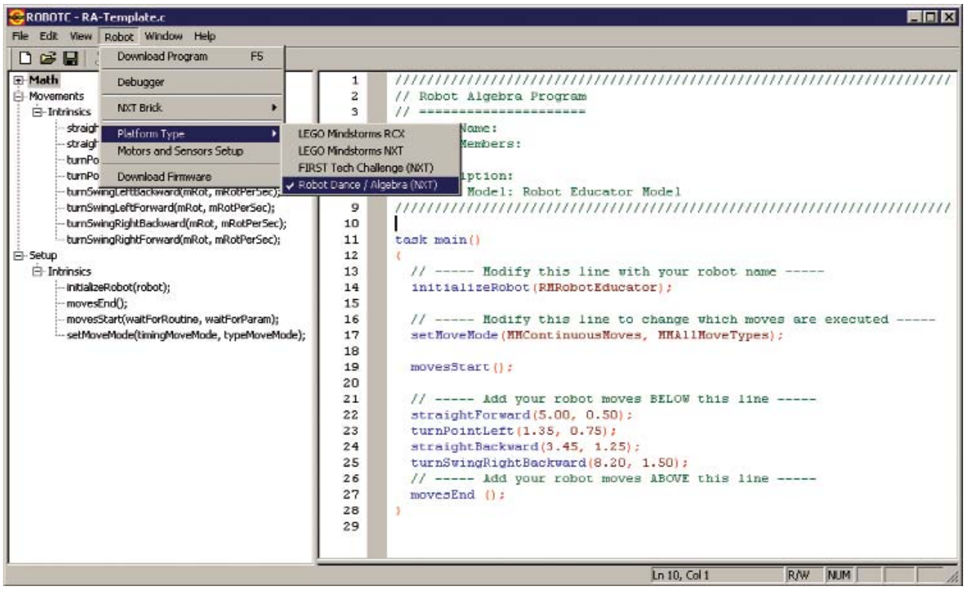
Programming parameters	Physical parameters	Straight movement targets	Turning movement targets
Motor rotations	Wheel circumference	Straight distance	Turn angle
Motor speed	Track width*	Straight speed	Turn speed

*Distance between the drive wheels

By providing different robot types on which these physical parameters vary, we give students the opportunity to explore their effects. The education base set comes with medium-size wheels (5.6cm diameter) and smaller wheels (3cm diameter). The track width varies with robot design, including the design from the instructions given in the base set, which has a relatively narrow track width of about 11cm. Damien Kee's Domabot is an example of a robot with a wider track of about 17cm.

CHART 2 Dance step		Robot 1		Robot 2*		Robot 3**	
Movement	**Target**	**Motor rotations**	**Measured movement**	**Motor rotations**	**Measured movement**	**Motor rotations**	**Measured movement**
Straight forward	100cm	10.61	100cm	10.61	186.7cm	5.68	100cm
Point turn right	45 deg.	0.71	45 deg.	0.71	129.8 deg.	0.25	45 deg.
Straight forward	50cm	5.31	50cm.	5.31	93.3cm	2.84	50cm

*Programmed using the same values as Robot 1
** Adjusting to synchronize moves



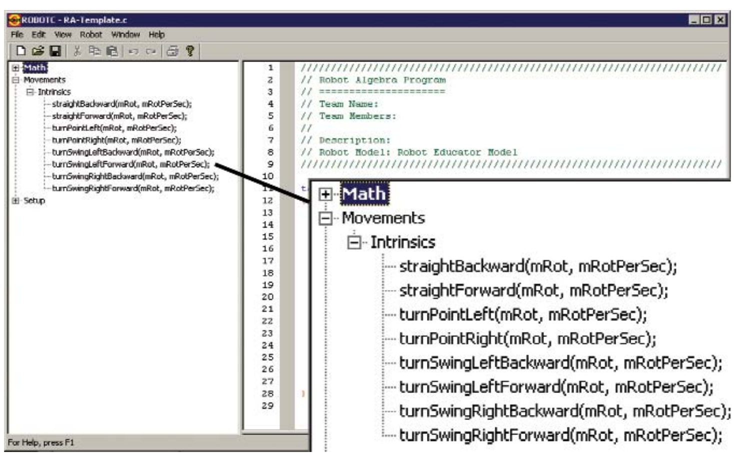
The chart above shows three types of movements: straight forward for 100cm, point turn right for 45 degrees and straight forward for 50cm. The first column, "Dance Step," shows the type of dance step and the target movement. The second column, "Robot 1," shows the calculated number of motor rotations and the measured movement. The third column shows the results the student will obtain if they program Robot 2 using the same values as Robot 1. The fourth column shows the adjusted numbers. Another proportional relationship is speed; this table accounts only for distance travelled, but as the wheel diameters change so does the speed at which the robot travels. Wheel diameter and distance travelled, wheel track and turning angle and wheel diameter and rate all form proportional relationships that a student must solve to successfully complete the Robot Dancing design problem

A PROGRAMMING TOOL FOR FOCUSING ON MOVEMENTS

In addition to framing the task and choosing robot designs to encourage students to think about the math, we also tailor the programming environment to highlight the programming parameters and functions that are most relevant. ROBOTC, developed by the Robotics Academy, is an excellent programming environment for this purpose. ROBOTC is software for a Windows platform and supports programming NXTs in a C-based language. It has a powerful interface that has many features, such as real-time debugging and a text-based drag-and-drop feature for easily adding function calls to a program.

The programming environment is customized to the goals of the curriculum through the inclusion of a Robot Algebra platform type. Since the task asks students to think at the level of individual movements, the Robot Algebra platform provides built-in function calls for each straight and turning movement that they will use. For example:

straightBackward(mRot, mRotPerSec)
turnPointRight(mRot, mRotPerSec)



The graphics show how the programming environment has been simplified for non-programmers. ROBOTC has built-in functions to control robot forward, backward and turning movements. This allows the student and teacher to focus on the math rather than learning to program.

Each movement function takes two parameters: the first specifies how many times to rotate the motors, and the second specifies how fast to rotate them. Students are then free to think in terms of motor rotations rather than in degrees or power levels.

ROBOTC is also ideal because of the ease with which we can use variables. Variables are a form of abstraction that can be connected to students' increasing generalization of the task.

Students may begin by entering a hard-coded concrete value for each movement:

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straightForward(3.45);
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They may then shift to a structural representation of the task based on the important physical parameters:

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straightForward(TargetDistance/WheelCircumference);
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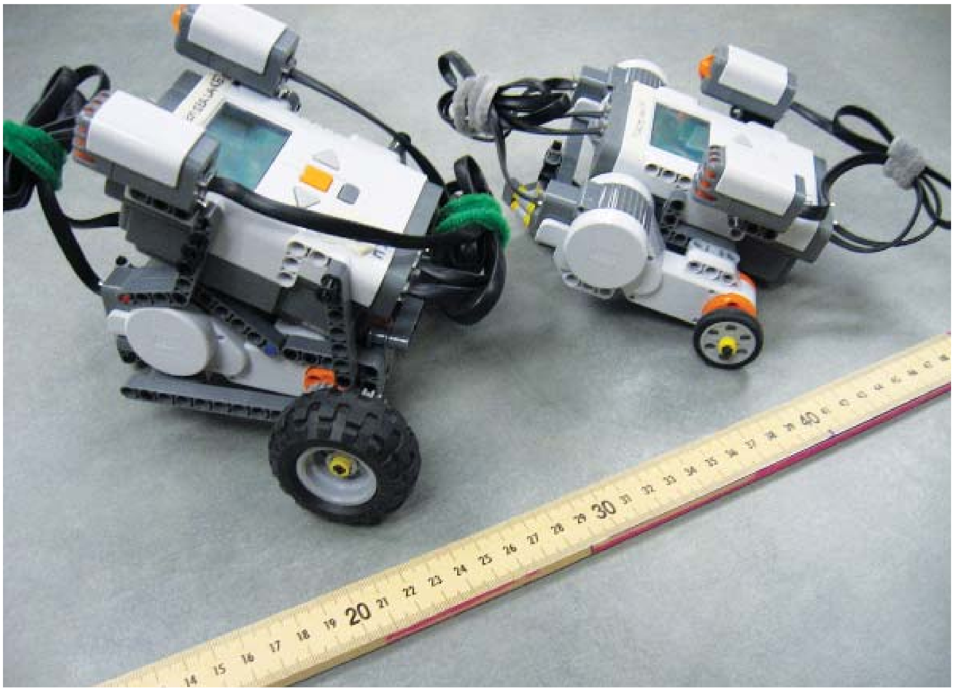
ROBOTC thus serves as an ideal environment in which to focus on the mathematically relevant aspects of synchronized robot dancing.

SOLUTIONS BY TEACHERS & STUDENTS

We have tried out versions of *Synchronized Robot Dancing* with teachers in professional development sessions and with students during in-school and after-school settings. In all cases, an encouraging range of intuitive strategies was generated.

In school, students are often taught the *cross-multiplication* method. This is a highly efficient way to find unknown values in simple proportion problems, but there is an important distinction between efficiency and meaning. Research on learning proportions has found that students often apply the cross-multiplication method successfully to school-like problems but have little understanding. Students often misapply the strategy in situations where the structure of the problem is not immediately obvious, such as qualitative problems and real-world problems, and in situations where a proportional relationship is not appropriate, such as inverse proportion problems. Alternative methods may build better on intuitive ideas and may support the more flexible use of proportional reasoning. For example, when teachers were asked to synchronize the distance traveled by two robots, they generated a number of different strategies. Some used a scale-factor strategy in which they took a known combination of rotations for a certain distance, figured out how many of those distances fit into the target distance, and then multiplied the rotations by that number. Others used unit ratios such as the number of rotations required to go 1cm or the number of centimeters traveled in one rotation, and then they used that ratio to find the number of rotations required for the full distance. Recognizing that a combination is scaled multiplicatively and that the ratio is constant in proportional situations are key conceptual aspects of proportionality that the teachers used intuitively. These alternative strategies are connected to the real task of synchronizing robot movements in a meaningful way rather than being a mindless application of a procedure.



Students generate similar strategies, even though they are often less articulate and confident about their ideas. Their strategies may start with random guess-and-check actions, but they quickly advance to more sophisticated guess-and-check strategies in which guesses are more informed and based on multiplicative relationships; they then progress to more explicit mathematical strategies that are similar to the unit-rate methods used by teachers. We help students to build on their intuitive ideas by having them present their team strategies to the whole group, helping them to compare and contrast strategies and encouraging them to adopt and continue to improve on the most efficient ones. Although this approach takes much longer than either providing a ready-made math solution that they simply apply or letting them create their own strategies independently without improving on them using math, it provides a much better base for making problem-solving with robotics a meaningful learning activity.

CONCLUSION

There are still many opportunities for robot dance connected to math that we have not yet fully explored. Gear ratios, rounded turns, scaled dance floors and other physical robot designs are just some of the possibilities. Those who are interested in extending the creative component of the robot dancing task also have many possible directions to explore, including costumes for the robots, rotating heads, moving arms that allow more human-like movements, and symmetrical or opposite movements instead of synchronized ones. The important point is that robot problems have a wide variety of solutions, but many can be improved in efficiency and in meaning when they are explicitly connected to math. The problem-solving experiences can then have a positive impact on the quality of the robotics solutions themselves and also on conceptual understanding and the appreciation of math more generally. We think that is a win-win for all involved.

Links

Domabot Robot Design, www.domabotics.com/domabot.php

Robotics Academy's Robot Algebra Website, www.education.rec.ri.cmu.edu/content/educators/research/robot_algebra/

Robotics Academy's ROBOTC, www.robotc.net/

University of Pittsburgh's Design Based Learning Group, www.lrdc.pitt.edu/schunn/research/design.html

For more information, please see our source guide on page ___.