Quantum Computer

Jaewan Kim jaewan@kias.re.kr

School of Computational Sciences Korea Institute for Advanced Study



KIAS (Korea Institute for Advanced Study)

- Established in 1996
- Located in Seoul, Korea
- Pure Theoretical Basic Science
- Schools
 - Mathematics
 - Physics
 - Computational Sciences
 - By the computation, For the computation
- 20 Prof's, 2 Distinguished Prof's, 20 KIAS Scholars, 70 Research Fellows, 20 Staff Members



Twenty Questions

- Animal ...
- Four legs?
- Herbivorous?

Yes(1) or No(0) Yes(1) or No(0) Yes(1) or No(0)



It from Bit

John A. Wheeler

Yoot = Ancient Korean Binary Dice





- Thank you. = 감사합니다(kam sa hap ni da > kam sa ham ni da) 고맙습니다(ko map sup ni da > ko map sum ni da)
- greetings = 안녕하세요 (an nyeong ha se yo)
- good bye = 안녕히계세요 (an nyeong hi gye se yo) (Stay with peace.) 안녕히 가세요 (an nyeong hi ga se yo) (Go with peace.)

Taegeukgi (Korean National Flag)



Qubit = quantum bit (binary digit)

Businessweek 21 Ideas for the 21st Century



DEMOGRAPHICS

17 POLITICS

2 20

Ī

0

side effects. And there may be pressure--from insurance companies, even from employers--to take the preventive medicine even if we don't want to. Our chances of survival may be greater.

The costs of survival may be as well 👁Video interview with Alexandra Heerdt, director of Memorial

Sloan-Kettering Cancer Center's special surveillance breast program

15 MONEY

In the new financial cosmos, it will be safer to take a dare.

Imagine a market where hedgers and speculators meet to trade futures, similar to today's betting on the value of corn or soybeans. The wagering will concern the future value of a career, a neighborhood, or even a country. If the risk of a stick-your-neck-out choice is hedged, it's suddenly a whole lot

easier to take the plunge 17 POLITICS

Democracy goes direct--again.

In many ways, it's back to the 1830s. Candidates will canvass voters in their homes; citizens will question politicians in public forums. The big difference: It will all take place on the Internet. The danger: Net-based splinter groups could factionalize public life

Explore Business Week Online >> 💌

thousands of PUs working in concert have already tackled complex computing problems. In the not-so-distant future, some scientists expect spontaneous computer networks to emerge, forming a "huge digital creature"

SVideo interview with Cherry Murray, head of Physical Research Lab. Bell Labs

16 DEMOGRAPHICS

The 'little emperors' can save the world's aging population.

How will a shrunken generation of fewer, more pampered children worldwide support their retired elders? By using their extra education, ambition, and advantages to become more productive than those who came before them

18 EDUCATION

Kids were right all along: High school is obsolete.

Should kids head for college when they're 15 or 16? Some experts think so, and some kids agree. They argue that the last two years of high school just keep students in a holding pattern, when many are independent enough to be starting their advanced education

SVideo interview with Nathan Myrhvold, Chief Technology Officer. Microsoft

QUANTUM COMPUTERS The toughest problems will be solved with a roll of the dice. Physicists hope to use subatomic particles' imprecise nature to answer questions beyond the reach of today's computers



Astounding tales that might come true!

What high-tech marvels will materialize in the next 100 years? Science fiction is a wellspring of predictions for the next century. Drawing on predictions in the following 30 astounding tales, illustrator David B. Mattingly created this vision of the future

Q&As, Web links, and video interviews

Additional online-only items found throughout this package are collected here.

Transition to Quantum

- Mathematics: Real \rightarrow Complex
- Physics: Classical \rightarrow Quantum
- Digital Information Processing
 - Hardwares by Quantum Mechanics
 - Softwares and Operating Systems by Classical Ideas
- Quantum Information Processing
 All by Quantum Ideas

Quantum Information Processing

- Quantum Computer
 - Quantum Algorithms: Softwares
 - Simulation of quantum many-body systems

NT

IT

BT

- Factoring large integers-
- Database search
- Experiments: Hardwares
 - Ion Traps
 - NMR
 - Cavity QED, etc.

Quantum Information Processing

- Quantum Communication
 - Quantum Cryptography-
 - Absolutely secure digital communication
 - Generation and Distribution of Quantum Key
 - ~100 km through optical fiber (Toshiba)
 - -23.4 km wireless \rightarrow secure satellite communication

IT

- Quantum Teleportation
 - Photons
 - Atoms, Molecules
- Quantum Imaging and Quantum Metrology

Cryptography and QIP

Giving disease, Giving medicine.

Out with the old, In with the new.

- Public Key Cryptosystem (Asymmetric)
 - Computationally Secure
 - Based on unproven mathematical conjectures
 - Cursed by Quantum Computation
- One-Time Pad (Symmetric)
 - Unconditionally Secure
 - Impractical
 - Saved by Quantum Cryptography



Classical Computation

- Hilbert (1900): 23 most challenging math problems
 - The Last One: Is there a mechanical procedure by which the truth or falsity of any mathematical conjecture could be decided?
- Turing
 - Conjecture ~ Sequence of 0's and 1's
 - Read/Write Head: Logic Gates
 - Model of Modern Computers

Turing Machine

Finite State Machine: Head



Infinitely long tape: Storage



DNA Computing

Adleman

- Bit { 0, 1 } → Tetrit (?) { A, G, T, C}
- Gate → Enzyme
- Parallel Ensemble Computation
 - Hamiltonian Path Problem

Complexity

N= (# bits to describe the problem, size of the problem)

(#steps to solve the problem) = Pol(N) \rightarrow "P(polynomial)": Tractable, easy

(#steps to verify the solution) = Pol(N) \rightarrow "NP (nondeterministic polynomial)": Intractable



Quantum Information

- Bit {0,1} \rightarrow Qubit $a|0\rangle+b|1\rangle$ with $|a|^2+|b|^2=1$
- N bits → 2^N states, One at a time
 Linearly parallel computing AT BEST
- N qubits → Linear superposition

of 2^N states at the same time

Exponentially parallel computing

→ Quantum Parallelism Deutsch

But when you extract result,

you cannot get all of them.

Quantum Algorithms

- [Feynman] Simulation of Quantum Physical Systems with HUGE Hilber space (2^N-D) e.g. Strongly Correlated Electron Systems
- 2. [Peter Shor] Factoring large integers, period finding $t_q = Pol(N) = t_{cl} = Exp(N^{1/3})$
- 3. [Grover] Searching $t_q \sqrt{N} \qquad \langle t_{cl} \rangle \sim N/2$





Hadamard Gate

$$\begin{split} H_1 \otimes H_2 \otimes ... \otimes H_N |0\rangle_1 \otimes |0\rangle_2 \otimes ... \otimes |0\rangle_N \\ &= \frac{1}{\sqrt{2}} (|0\rangle_1 + |1\rangle_1) \otimes \frac{1}{\sqrt{2}} (|0\rangle_2 + |1\rangle_2) \otimes ... \frac{1}{\sqrt{2}} (|0\rangle_N + |1\rangle_N) \\ &= \frac{1}{\sqrt{2^N}} (|0_1 0_2 ... 0_N\rangle + |1_1 0_2 ... 0_N\rangle + ... + |1_1 1_2 ... 1_N\rangle) \\ &= \frac{1}{\sqrt{2^N}} \sum_{k=0}^{2^{N-1}} |k(\text{binary expression})\rangle \end{split}$$

Universal Quantum Gates

General Rotation of a Single Qubit



 X_c : CNOT (controlled - NOT) or XOR

æ	0ö, æl	0ö a	9 Oö.	. a0	1 ö
Č O				Å Ç ₁	0÷
= 0]	$\rangle \langle 0 \ddot{\mathbf{A}} I +$	$ 1 angle\langle 1 $ $\ddot{\mathbf{A}}$.	X		

 $X_{c} |a\rangle |b\rangle = |a\rangle |a \mathbf{A} b\rangle$

Quantum Circuit/Network



Quantum Key Distribution [BB84,B92]	Single-Qubit Gates	: 3
QKD[E91] Quantum Repeater Quantum Teleportation	Single- & Two-Qubit Gates	: 3
Quantum Error Correction Quantum Computer 7-Qubit QC	Single- & Two-Qubit Gates	з 40 з 100

Physical systems actively considered for quantum computer implementation

- Liquid-state NMR
- NMR spin lattices
- Linear ion-trap spectroscopy
- Neutral-atom optical lattices
- Cavity QED + atoms
- Linear optics with single photons
- Nitrogen vacancies in diamond

- Electrons on liquid He
- Small Josephson junctions
 - "charge" qubits
 - "flux" qubits
- Spin spectroscopies, impurities in semiconductors
- Coupled quantum dots
 - Qubits: spin,charge, excitons
 - Exchange coupled, cavity coupled

 $15 = 3 \times 5$

Chuang

Nature 414, 883-887 (20/27 Dec 2001) OR QP/0112176



Concept device: spin-resonance transistor R. Vrijen et al, Phys. Rev. A 62, 012306 (2000)



Quantum-dot array proposal:

Loss & DiVincenzo, Phys. Rev. A 57, 120 (1998).

side gates

- -quantum dots defined in 2DEG by side gates
- -Coulomb blockade used to fix electron number at magnetized one per dot
- -spin of electron is qubit
- -gate operations: controllable coupling of dots by point-contact gate voltage
- -readout by gatable magnetic barrier

Quantum-dot array proposal

high g-factor

layer

Ion Traps

• Couple lowest centre-of-mass modes to internal electronic states of N ions.



Quantum Error Correcting Code Three Bit Code





Sculpturing a Quantum State

- Cluster state quantum computing -

- One-way quantum computing --



- 1. Initialize each qubit in |+ñstate.
- 2. Contolled-Phase between the neighboring qubits.
- 3. Single qubit manipulations and single qubit measurements only [Sculpturing]. No two qubit operations!



Controlled-NOT





 $X_{AB} = \left| 0 \right\rangle_{AA} \left\langle 0 \right| \otimes I_{B} + \left| 1 \right\rangle_{AA} \left\langle 1 \right| \otimes X_{B}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{A} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{P} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_{A} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{P}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{AB}$

Qubit Copying Circuit?



Entanglement by Two-Qubit Gates



Single Particle Entanglement



Quantum Teleportation using a single particle entanglement



No Cloning Theorem

An Unknown Quantum State Cannot Be Cloned.

Zurek, Wootters

<Proof>

$$U(|\mathbf{a}\rangle|0\rangle) = |\mathbf{a}\rangle|\mathbf{a}\rangle$$
$$U(|\mathbf{b}\rangle|0\rangle) = |\mathbf{b}\rangle|\mathbf{b}\rangle \qquad |\mathbf{a}\rangle \neq |\mathbf{b}\rangle$$
$$Let |\mathbf{g}\rangle = \frac{1}{\sqrt{2}}(|\mathbf{a}\rangle + |\mathbf{b}\rangle).$$
Then $U(|\mathbf{g}\rangle|0\rangle) = \frac{1}{\sqrt{2}}(|\mathbf{a}\rangle|\mathbf{a}\rangle + |\mathbf{b}\rangle|\mathbf{b}\rangle) \neq |\mathbf{g}\rangle|\mathbf{g}\rangle$

"If an unknow quantum state can be cloned ...

- Quantum States can be measured as accurately as possible ???
 jyñÞ jyñ, jyñ, jyñ, jyñ...
 measure, measure, ...
- Communication Faster Than Light? $|\mathbf{y}\tilde{\mathbf{n}} = |\mathbf{0}\tilde{\mathbf{n}}_{A}|\mathbf{1}\tilde{\mathbf{n}}_{B} - |\mathbf{1}\tilde{\mathbf{n}}_{A}|\mathbf{0}\tilde{\mathbf{n}}_{B} \text{ for "0"}$ $= |+\tilde{\mathbf{n}}_{A}|-\tilde{\mathbf{n}}_{B} - |-\tilde{\mathbf{n}}_{A}|+\tilde{\mathbf{n}}_{B} \text{ for "1"}$

Communication Faster Than Light? "If" an unknown quantum state can be copied; $|\mathbf{y}\tilde{\mathbf{n}} = |0\tilde{\mathbf{n}}_k|1\tilde{\mathbf{n}}_k - |1\tilde{\mathbf{n}}_k|0\tilde{\mathbf{n}}_k$ for "0" $= |+\tilde{\mathbf{n}}_k|-\tilde{\mathbf{n}}_k - |-\tilde{\mathbf{n}}_k|+\tilde{\mathbf{n}}_k$ for "1"

- Alice wants to send Bob "1." $|\mathbf{y}\tilde{\mathbf{n}} = |+\tilde{\mathbf{n}}_{A}| \tilde{\mathbf{n}}_{B} |-\tilde{\mathbf{n}}_{A}| + \tilde{\mathbf{n}}_{B}$ for "1" Alice meaures her qubit in {|+>,|->}.
- Alice's state will become |+> or |->.
- Bob's state will become |-> or |+>.
 Let's assume it is |+>.
- Bob makes many copies of this. |+ñ |+ñ |+ñ |+ñ |+ñ ...
 He measures them in {|+>, |->}, and gets 100% |+>.
 He measures them in {|0>, |1>}, and gets 50% |0> and 50% |1>.

Thus Bob can conclude that Alice measured her state in {|+>, |->}.

Mysterious Connection Between QM & Relativity

- Weinberg: Can QM be nonlinear?
- Experiments: Not so positive result.
- Polchinski, Gisin:

If QM is nonlinear, communication faster than light is possible.

Cluster State

1. Qubits on lattice sites are initialized to be $|+\rangle$.

$$\left|+\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}} = H\left|0\right\rangle$$

2. Operate Control-Z on neighboring qubits.

$$\begin{aligned} \text{Cont-Z} &= |0\rangle \langle 0| \otimes I + |1\rangle \langle 1| \otimes X \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{aligned}$$

$$\operatorname{Cont} - \operatorname{Z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} e^{0i} & 0 & 0 & 0 \\ 0 & e^{0i} & 0 & 0 \\ 0 & 0 & e^{0i} & 0 \\ 0 & 0 & 0 & e^{-pi} \end{pmatrix}$$

 $\operatorname{Cont} - \operatorname{Z} = e^{i\frac{p}{4}H}$







One-way quantum computing



Optical Lattice, Quantum Dots, Superconductors, etc.

Single qubit manipulations and single qubit measurements only [Sculpturing]. No two qubit operations!

Quantum computing with quantum-dot cellular automata



$$\hat{H} = -\sum_{j=1}^{N} \gamma_j \hat{\sigma}_x(j) - \sum_{j=1}^{N-1} E_j \hat{\sigma}_z(j) \hat{\sigma}_z(j+1)$$

$$+\sum_{j=1}^{n} E_0 P_{\text{bias},j} \hat{\sigma}_z(j).$$

Ν

Some comments

- 1-D cluster state quantum computing can be efficiently simulated by digital computing.
- 2-D cluster state quantum computing is equivalent to quantum circuit quantum computing, but needs more(just polynomially more) qubits.

"Proposal": 2-D array of vertical QDs

- 2-D cluster state does not seem to be a ground state of some Hamiltonian.
- Once the cluster state is prepared, only single qubit measurements are needed.

QUIPU [kí:pu]

1. Quantum Information Processing Unit [JK]

2. A device consisting of a cord with knotted strings of various colors attached, used by the ancient Peruvians for **recording events**, **keeping accounts**, etc.

