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Use of a Quality Loss Function to Select Statistical Tolerances

In this paper, we present a method for the selection of processes to manufacture various parts of an assembly by establishing a compromise between product quality and part manufacturing cost. We quantify the impact the precision of a part characteristic has on the overall quality of a product by using a standard Taguchi loss function. Part manufacturing cost is modeled as a function of process precision (i.e., standard deviation of the output characteristic) as opposed to previous models where manufacturing cost is a function of part tolerance. This approach is more realistic and does not assume, a priori, a relationship between conventional tolerance and process spread. Rather than allocating conventional tolerances on the assembly parts, we use statistical tolerances that are more pertinent when using a quality loss function. The model adopted makes it possible to investigate the relationship between optimum quality loss and tolerance variations. As expected, the optimum quality loss generally decreases when the tolerance increases. Exceptions may be encountered when changes of process occur. The manufacture of a simple three component assembly is studied to illustrate the findings.

Introduction

Designing a product for mass production requires numerous complex decisions pertaining to manufacturing options. These decisions include quantitative factors such as performance, cost and precision, but also vaguely defined qualitative characteristics such as quality and customer satisfaction. Further, when employing mass production, economic factors become primary concerns since slight variations in cost can have a substantial impact on corporate profits. This implies that the manufacturing process becomes an important concern of the design process and should be optimized for minimum cost.

In this paper, we present a method to select the processes best suited to manufacture the components of an assembly, given a performance requirement. We select the optimum precision of a part feature which in turn determines the suitable process, chosen from a predefined set of possible processes that differ by their precision and cost. We use a standard Taguchi loss function to quantify the tradeoff between product quality and process precision. The model presented makes it possible to investigate the relationship between optimum quality loss and tolerance variations.

Sections 1 and 2 briefly recall the concepts of conventional and statistical tolerances and their relationship with manufacturing quality. In connection with tolerances, Section 3 exposes a new method to model manufacturing costs. The tradeoff between cost and quality with statistical tolerances as variables is presented in Section 4. A resolution method is described in Section 5 and illustrated by an example in Section 6.

1 Conventional Tolerances: A First Step Toward Design Quality

Since the beginning of mass production, manufacturers have been struggling with the variability inherent in any manufacturing operation. For a given design, the performance or characteristics of the assemblies produced may vary and are sometimes unacceptable because it is impossible to manufacture parts with perfect nominal specification. The degradation of the assembly characteristics can be reduced if the variations of the part characteristics are limited. It is therefore common to permit constrained variations in part characteristics by allocating tolerances and removing out-of-tolerance parts. Although the imperfections are sometimes better characterized by other means (Requicha, 1983; Srinivasan and Wood, 1992), a widely used format for these specifications is

$$\text{Nominal Value} \pm \text{Tolerance.}$$

The first concern in allocating tolerances is to guarantee the proper functioning of the product and, therefore, to satisfy technical constraints. More precisely, the possibility to assemble the product or the product performance level requirement are often expressed by a function of the part characteristics. This function is denoted by F and is known as a *design function* (Martino and Gabriele, 1989):

$$\text{Performance} = F(\text{part characteristics}).$$

The variations of F have to be restricted to guarantee that the product can be assembled or that the product performance level is adequate.

For a known requirement on F (usually defined in terms of tolerance), the problem of *tolerance synthesis* consists of choosing tolerances on the part characteristics in such a way that F satisfies the requirements when the part characteristics vary within permitted tolerances. Tolerance allocation is therefore a way to sort a process output by quality level into two different categories: acceptable and rejects. This classification may be somewhat simplistic, as exposed in the next section.

2 Design Quality as a Continuum: Statistical Tolerances

As seen in the previous section, the first definition of quality is conformance to specifications where manufacturers measure the quality of their production by the percentage of out of specifications items produced. This approach is reasonable when there is a significant probability of shipping unacceptable items to customers. When the rate of unacceptable items is reduced significantly, or when the final products are inspected, quality

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cannot be assessed solely by the percentage of unacceptable items produced; this measurement does not differentiate between items that possess characteristics near to the design nominal values and those that, despite being within specifications, deviate significantly from the target values. In many cases, the nominal value is that which provides the desired performance and, thus, the best product (Taguchi et al., 1989; Smith, 1993). Conventional tolerances exist to limit the degradation of the performance. In some cases, an item off target is more likely to break down than a product that has parameters closer to the target values. It is important to determine if a product is within specification, and if so, how far it is from the target value. Thus, the quality of a product is actually a continuous measure and conformance to conventional tolerances may not be the best means to measure quality.

In the case of mass production, product characteristics are more often than not statistically distributed in a continuous manner, and the parameters of the statistical distribution (e.g., mean, standard deviation) are the real quality indicators. Therefore, for mass production, it is more appropriate for a designer to use *statistical tolerances* for each part dimension rather than a conventional tolerance. Statistical tolerances characterize the statistical distributions of the part characteristics in order to determine the distribution of the assembly characteristics. In the general case, the relationship between the distributions of the part characteristics and that of the assembly characteristics is complex and cannot be described analytically. However, as described by Evans (1974) in his state of the art of statistical tolerancing, the problem is often simplified by assuming a linear relationship between part and assembly characteristics (i.e., a linear design function) and normal distributions for part characteristics. Given these assumptions, selecting statistical tolerances consists only of selecting the standard deviation of the part characteristic distributions.

Given the large quantities manufactured, it is possible to characterize the distributions of part characteristics through sampling techniques, for instance. Note that this distribution may vary due to many factors (e.g., temperature change, tool wear). In this paper, we refer to the long term average (constant) distribution parameters that matter for process selection. Short term variations of these parameters are assumed to be corrected by on-line quality control techniques (i.e., the process is "in statistical control") and do not significantly affect process selection. We assume that part characteristics are normally distributed about their nominal values, as is often the case. Therefore, selecting the standard deviation completely characterizes the distribution, assuming the process is on target on average. Cases where the normal assumption does not hold can be handled with minor modifications in the method. In particular, cases where the part characteristic distributions are not symmetric require an additional step to select the distribution mean.

The design function is used for the determination of the statistical distribution of the assembly characteristics during the production stage. The designer has to (*i*) select a quality level for the assembly (i.e., decide which distribution of the assembly characteristics is good enough for his purpose), (*ii*) select among many possibilities the statistical tolerances that produce the desired assembly quality. In the following sections, we present a methodology to address these questions, based on manufacturing cost and quality considerations.

3 Redefining the Manufacturing Cost

Several authors have investigated the economic aspect of design precision specifications. Various representations of the cost of an individual item have been used. Generally, the cost is decomposed into the sum of a constant part (cost of raw material, F_i) and a variable part (cost of holding the tolerance Δ_i). The functions most commonly used are:

$$C_i(\Delta_i) = F_i + \frac{\alpha_i}{\Delta_i^2},$$

$$C_i(\Delta_i) = F_i + \frac{\alpha_i}{(\beta_i - \Delta_i)}, \text{ or}$$

$$C_i(\Delta_i) = F_i + \lambda_i e^{-\Delta_i/\tau_i},$$

where α_i, β_i, λ_i and τ_i are process dependent parameters. From these relationships and from industrial data, we see that cost increases with precision. The assembly cost may be computed as the sum of the costs of the individual components. Tolerances are usually allocated by minimizing this cost under various design constraints. Several authors have presented a solution based on linear programming (Bjorke, 1989), on Lagrange's multipliers method (Speckhart, 1972; Spotts, 1973; Wilde, 1978; Chase and Greenwood, 1988), on nonlinear programming (Michael and Sidall, 1981; Parkinson, 1985; Lee and Woo, 1989), on geometric programming (Wilde and Prentice, 1975), or on a simulated annealing technique (Cagan and Kurfess, 1992).

None of these models provide a designer with the cost information necessary to allocate statistical tolerances on a design. Indeed, the attempts to allocate statistical tolerances based on cost considerations are still influenced by conventional tolerance allocation. Typical models are based on an implicit arbitrary relationship between the tolerance and the yield of the process, the famous relationship $\Delta = 3\sigma$, where Δ is the tolerance and σ the standard deviation of the process output. Three standard deviations is sometimes referred to as the *natural tolerance* (Speckhart, 1972), but there is no reason why there should be a constant linear relationship between Δ and σ. Moreover, this relationship has been sometimes questioned, and practices may vary, depending on the industry considered. Harry (1987) recommends 6σ to accommodate possible process mean shifts.

The importance of the relationship between process spread and conventional tolerances is illustrated by the extensive use of process capability indices in the quality control area (e.g., C_p, C_{pk}). These indices are meant to monitor the actual process spreads, compared with the conventional tolerances allocated on the parts. With the notations employed above, C_p is defined as

$$C_p = \frac{\Delta}{3\sigma},$$

and C_{pk}, meant to take into account the location of the process mean as well as the process spread, is defined as

$$C_{pk} = \text{Min}\left(\frac{\text{USL} - \mu}{3\sigma}; \frac{\mu - \text{LSL}}{3\sigma}\right),$$

where μ is the actual process mean, USL and LSL are the upper and lower specification limits, respectively. Note that the definitions of the indices take 3σ as a reference point. The assumption made in this paper is that no a priori value is assumed for the process capability indices.

For a model using an a priori relationship, statistical tolerance allocation is not fundamentally different from conventional tolerance allocation since they are based on the same manufacturing costs viewed as functions of conventional tolerances. We believe that this cost representation based on tolerances rather than process output distribution may camouflage part of the design decision problem because the tolerance on a part is often an arbitrary number, chosen by the designer that has nothing to do with the process itself.

A better model would view manufacturing cost as a function of the process output spread, rather than the tolerance on the part. Under the normality assumption we made for the process output distributions, part characteristics are symmetrically distributed and are approximately spread about a nominal target value: unavoidable sporadic trends are eventually detected, corrected and averaged to the nominal value. The standard deviation of the part population can then be easily estimated (through sampling techniques) and constitutes an intrinsic representation of the process quality. Therefore, our model treats the cost of manufacturing a part with a given process as a convex function of the *standard deviation* of the part population produced by the process.

Note that the manufacturing cost function presented above is a function of process *precision* (or repeatability), as opposed to process *accuracy* (see (Slocum, 1992) for a more detailed presentation of the difference between accuracy and precision.) What this cost function takes into account is how tight the process output distribution is regardless of the distribution mean (i.e., the precision of the process). However, manufacturing quality also depends significantly on the average deviation of part characteristics from the nominal target value (i.e., process accuracy). We assume in the rest of the paper that the process is in statistical control. More precisely, we assume that the output distribution is constant and that the output mean coincides with the nominal value of the part characteristic. Under this assumption, precision and accuracy are synonymous.

Selecting the process precision required to meet a given specification involves a tradeoff between manufacturing cost and precision. The selection is performed by using a quality loss function as described in the next section.

4 Loss Function

We have seen that in many cases, quality can be related to the manufacturing imperfections that occur during the production stage. It is the designer's responsibility to determine what level of imperfection is acceptable. Here we analyze the particular case of an assembly whose performance is measurable. We assume that there is a nominal design value for the performance. This target value is accompanied with a tolerance corresponding to a customer requirement or to a technical constraint (e.g., voltage of a generator to be hooked on appliances). This tolerance on the overall assembly performance is assumed to be given, as opposed to the distribution of the manufactured assembly performances.

We have mentioned in Section 2 that the closer a product characteristic is to its target value, the higher the quality of the product. Relating the quality of a part to the deviation of its characteristics from their nominal values provides a continuous measure of quality, as opposed to the pass/fail test corresponding to simple conformance verification. More precisely, the deviation measures quality degradation. Taguchi et al. (1989) proposed the use of a loss function to quantify quality degradation. This approach is appealing because it provides a *monetary* indication of part quality by relating the deviation of a part characteristic from its target value to the monetary loss related to a lower quality level. The methods advocated by Taguchi et al. (1989) to calculate a loss function are briefly presented below.

We denote by $L(D)$ the quality loss corresponding to the deviation, D, of a product characteristic. We denote by Δ the tolerance allocated on the product characteristic and by A the loss incurred when the product is defective (out of specification). $L(D)$ is evaluated as

$$L(D) = \frac{A}{\Delta^2} D^2. \qquad (1)$$

When considering an entire population of products, the expected value of the quality loss, $E[L]$, is a function of the variance of the product characteristic, σ_F^2:

$$E[L] = \frac{A}{\Delta^2} \operatorname{Var}(D) = \frac{A}{\Delta^2} \sigma_F^2. \qquad (2)$$

Supposing that defective products are removed before being shipped to the customers, the manufacturer incurs a loss depending on the percentage of products that have to be scrapped or reworked. We denote by P the proportion of the production that meets the specification. To simplify the model, we also assume that the loss caused by out of specification products consists only of the cost to make these products. We assume that they are disposed of without any attempt to rework them. Additional related costs such as the cost of sorting and disposing of these products are not taken into account. We denote by C the cost of making a product, whether it is defective or not. Consequently, the cost of making a product that meets the specification is C/P. The Total Cost, TC, associated with making the product is the sum of the production cost and the quality loss:

$$TC = \frac{C}{P} + \frac{A}{\Delta^2} \sigma_F^2. \qquad (3)$$

TC includes three components: the cost of making a product, whether it meets the specification or not, the cost associated with the fact that a fraction of the production has to be thrown away and a loss corresponding to the fact that even products that meet the specification are not perfect.

The value of A in Eq. (3) depends on the problem considered. A may represent the consequences incurred by the manufacturer when the product is found out of specifications by the customer (e.g., warranty costs, liabilities). Then, the analysis may lead to the determination of *safety factors* reflecting the cost difference between locating a defect inside and outside the production facility (Taguchi et al., 1989, pp. 37–39). The product considered may also be a part of an assembly to be delivered to an assembly plant. Then, if the product cannot be assembled, it may have to be scrapped and A can be estimated by the production cost of the sub-assembly. That is the assumption we make in the rest of this paper. Consequently, the optimum statistical tolerances to be allocated on the components of the assembly considered are the process precisions that minimize TC defined by Eq. (3). Kapur et al. (1990) present a similar problem as a constrained minimization problem. Here, rather than upper bounding the overall variability, we obtain its value through the minimization of the Total Cost.

5 Statistical Tolerance Allocation Algorithm

As described in the previous section, the optimal statistical tolerances are the ones that minimize the Total Cost. Since

```
Begin Statistical_Tolerance_Allocation
      Initialize Current_Statistical_Tolerances
      Current_Total_Cost = Total_Cost(Current_Statistical_Tolerances)
      For Prespecified_Number_of_Iterations do Begin
            Select New_Statistical_Tolerances
            New_Total_Cost = Total_Cost( New_Statistical_Tolerances )
            If New_Total_Cost < Current_Total_Cost then Begin
                  Current_Total_Cost = New_Total_Cost
                  Current_Statistical_Tolerances = New_Statistical_Tolerances
            Else
                  Generate Random_Unit_Variable
                  If Random_Unit_Variable < Φ(Number_of_Iterations) then Begin
                        Current_Total_Cost = New_Total_Cost
                        Current_Statistical_Tolerances = New_Statistical_Tolerances
                  End
            End
            Shrink Selection_Range
      End
End
```

Fig. 1 Statistical tolerance allocation algorithm

the cost functions are sometimes ill-behaved, we introduce an algorithm for performing the optimal allocation which is general enough to handle most cost functions; simpler methods may be more efficient for certain classes of cost functions.

Let us assume that a product has a design function F (as defined in Section 1), depending on the part characteristics $x_1, x_2, \ldots x_n$. The designer must choose the σ_i's, statistical tolerances on the x_i's (i.e., the standard deviations of the manufactured part characteristics). An efficient selection of process precision is based on manufacturing cost information; namely, for each part, it is necessary to know the processes available for the manufacture, their precision, and their cost. The precision of many processes can be adjusted over a certain range; for each process, the range of possible precision has to be specified.

We denote by $C(\sigma)$ the cost function of the process considered. $C(\sigma)$ represents the relationship between the cost of a part and the precision of the process employed to manufacture the part. Because the use of statistical tolerances is recent, little information is available on these cost functions. Presently, the collection of information has to be done by each company on its own machines. One can expect that statistical tolerance cost information will be made available in the future. Such information exists for conventional tolerances (Peat, 1968), and the growing interest for statistical tolerances will lead to the development of similar standard tables.

The purpose of the algorithm is to minimize the Total Cost of the assembly defined as the sum of the manufacturing costs of the components. The process precisions for which the Total Cost is minimum are the selected statistical tolerances. The optimization technique of simulated annealing is employed (Kirkpatrick et al., 1983). The choice of the optimization technique is dictated by the potentially ill-behaved objective function. The algorithm proceeds as follows (also see Fig. 1). A statistical tolerance is selected randomly for each part. Then, the algorithm determines the best (cheapest) process to use to produce each part with the assigned precision. This process is selected by computing the unit production cost for each process available, and choosing the cheapest process.

By summing the manufacturing costs of all components, the assembly cost is computed, which in turn yields the Total Cost, computed from Eq. (3). The variance of the assembly performance, σ_F^2, is computed by using a linear approximation of the design function. Then, alternative process precisions are selected for each part. For each part, the new process precision is selected randomly, within a given range of the previous process precision. Similarly to the previous iteration, the Total Cost of the assembly is computed with the new process precisions. If the new value of the Total Cost is lower than the previous one, the new process precisions are retained and replace the old ones; otherwise, the new precisions are retained with a probability (denoted by Φ (Number of Iterations) in Fig. 1) that decreases as the number of iterations increases. This feature helps to avoid the convergence of the algorithm towards a local minimum. When a combination of precisions is retained, the process precisions become the centers of the ranges over which the new precisions are to be randomly selected. The width of the ranges decreases as the number of iterations increases. The algorithm has been implemented in C on a Mac IIci; results are presented in the next section.

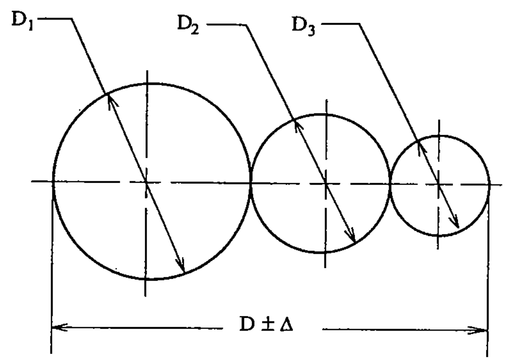

Fig. 2 System of friction wheels

6 Example Application: Friction Wheel Assembly

6.1 Description of the Problem. To illustrate the previous result, we examine the particular example of a capstan drive assembly of three friction wheels. This mechanism is used in precision machines to avoid the backlash that regular gears may produce. The characteristic of importance of the assembly is its length. For a given tolerance allocated on the assembly length, we select the optimum manufacturing processes to produce the components by using a quality loss function. We study the variations of the quality loss when the tolerance on the assembly length varies.

The assembly is made of wheels with different diameters and, therefore, different production costs. Each wheel, from largest to smallest, has a given nominal diameter of: 4, 3, and 1 inches. The customer specifications require that the total length of the assembly, D, be the sum of the three nominal wheel diameters (D_i), with a given overall tolerance Δ (inches) as shown in Fig. 2.

The wheels can be manufactured with three different processes: sawing/cutting bar stock, turning on a lathe, or grinding. For each of those processes, there is a best possible machining precision involving the highest cost. Decreasing the processing times decreases the precision as well as the cost per part. Each machine is best suited for a given precision range over which the manufacturing cost per part can be represented as a hyperbolic function of precision (output standard deviation) as shown in

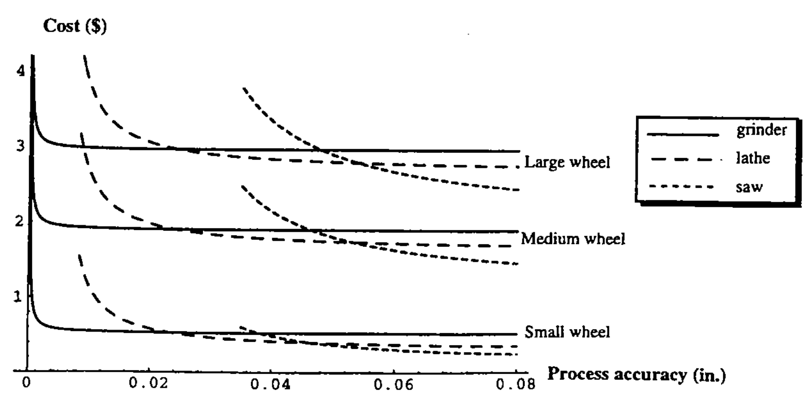

Fig. 3 Cost functions of the components for different processes

Table 1 Manufacturing cost functions used in the example (σ in in., and cost in $)

	Large Wheel	Medium Wheel	Small Wheel
Saw	$\frac{0.02661}{\sigma - 0.02} + 2.0$	$\frac{0.02014}{\sigma - 0.02} + 1.13$	$\frac{0.00719}{\sigma - 0.02} + 0.13$
Lathe	$\frac{0.00568}{\sigma - 0.005} + 2.67$	$\frac{0.00519}{\sigma - 0.005} + 1.63$	$\frac{0.00421}{\sigma - 0.005} + 0.3$
Grinder	$\frac{0.0003}{\sigma - 0.0002} + 2.95$	$\frac{0.00029}{\sigma - 0.0002} + 1.89$	$\frac{0.00026}{\sigma - 0.0002} + 0.52$

Fig. 3 and Table 1. See Cagan & Kurfess (1992) for an explanation of the methodology used to determine the cost functions. Each wheel has its own cost function due to the different sizes of the wheels: the larger the diameter is, the more expensive the part is and the more expensive quality improvement is (due to a longer machined surface).

We determine the manufacturing cost, C, of an assembly by summing the costs of machining the individual wheels, $C_i(\sigma_i)$, with a process of precision σ_i. The quality loss function is denoted by L. The populations of parts are assumed to be normally distributed about their nominal or target diameters. Note that we do not specify conventional tolerances on individual components. The only tolerance is on the overall length of the assembly and is customer defined. Given this tolerance Δ, we determine the optimal distribution of the assembly length, as well as that of the component diameters. We study the same problem for different values of Δ. Then, we study the variations of the quality loss for the different values of Δ selected.

As described in Section 5, we select the process precision for each component by minimizing the sum of the quality loss and the manufacturing cost of the assembly. The variance of the length of the assembled products, σ^2, is simply the sum of the variances of the component diameters:

$$\sigma^2 = \sum_{i=1}^{3} \sigma_i^2 .$$

Consequently, it is possible to compute the proportion of assemblies, P, that meet the specification Δ. If defective products are removed, the variance of the remaining ones is (Taguchi et al., 1989)

$$\sigma_F^2 = -2 \frac{\sigma\Delta}{\sqrt{2\pi}P} e^{-\Delta^2/2\sigma^2} + \sigma^2 .$$

The manufacturing cost of the assemblies within specifications is simply the sum of the manufacturing cost of the three components over the proportion of assemblies that meet the specification Δ:

$$\frac{1}{P} \sum_{i=1}^{3} C_i(\sigma_i).$$

We select the precision of the manufacturing process for each component by minimizing the Total Cost defined by (3):

$$TC = \frac{1}{P} \sum_{i=1}^{3} C_i(\sigma_i) \left[1 + \frac{\sigma_F^2}{\Delta^2} \right] .$$

We study the friction wheel design for several values of the overall tolerance. In each case, the overall tolerance is a given parameter that we use to define the loss function. To different values of the overall tolerance correspond different uses of the assembly and therefore different products. Examples of products for which specifications may change the use of the product include electronic components such as resistors: the tolerance on the resistance value determines the type of circuits in which the resistor can be used. Other examples are some engine parts that are quite similar in diesel or gasoline engines except for the tolerances that are much tighter for diesel engines. The same deviation from a nominal dimension does not have the same impact whether the part is used in diesel or gasoline engines.

7.2 Results. The results of the minimization are presented in Fig. 4. We have studied the friction wheel design for several values of the overall tolerance (on the horizontal axis). The vertical axis represents the precision of the processes that minimize the sum of the manufacturing cost and the quality loss. The components with the steepest manufacturing cost functions (the largest wheels) are allocated the loosest statistical toler-

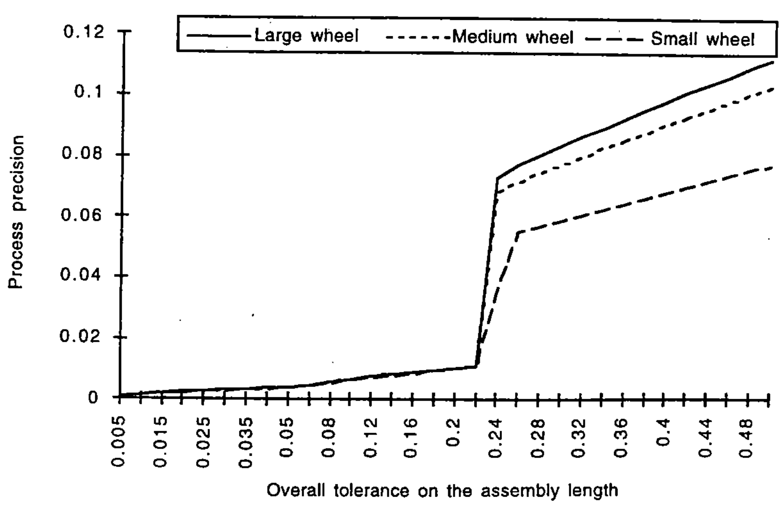

Fig. 4 Optimal process precision

ances (the cheapest processes), and the components with the lowest manufacturing costs (the smallest wheels) receive the tightest statistical tolerances (requiring expensive processes). This is due to the fact that it is cheaper to accurately manufacture smaller components; large components usually require a longer processing time since more material has to be removed. Note that the slope discontinuities in Fig. 4 relate to production process changes (from grinding to turning to saw cutting with increasing overall tolerance).

Figure 5 displays the manufacturing cost and quality loss for various overall tolerances. A high quality loss is experienced for very tight overall tolerances (0.005 in. to 0.01 in.). As the overall requirement is relaxed, the quality loss decreases. A significant quality loss is abruptly experienced for an overall tolerance equal to 0.24 in., and the quality loss decreases again as the overall tolerances increases. Increasing Δ beyond the range of Fig. 5 eventually leads to a negligible quality loss. Yet, imposing a tolerance of such magnitude becomes meaningless from an engineering standpoint. The difference between the quality loss experienced for $\Delta = 0.22$ in. and $\Delta = 0.24$ in. is counterintuitive: the tolerance is tightened (i.e., the requirement is made more difficult to meet) and yet the quality loss decreases (i.e., the requirement is better met.) Figure 6 displays the variations of the defect rate for the assembled products. There again, an abrupt change occurs for $\Delta = 0.22$ in.

The peculiar patterns of Fig. 5 can be explained by the slopes of the component cost functions. As the overall tolerance on the assembly is tightened, the total cost increases but the terms of the tradeoff between manufacturing cost and quality loss may change. When the slopes of the component cost functions are steep, the tolerance on the assembly is tightened faster than the precision of the components. As a result, the quality loss increases. When the component cost functions are almost flat, the component precision follows the overall requirement on the assembly, and the quality loss is kept low. Because of the process changes, the slope of each component cost function is discontinuous. At each process change, the slope decreases abruptly as the component precision gets better (i.e., smaller standard deviation). As a result, when processes change, the magnitude of the cost function varies greatly.

The meaning of an increase of the quality loss is simply that the requirement imposed on the assembly is not easily met by the processes available: for very tight overall tolerances, the quality loss increases because the processes available are simply not accurate enough. Around the overall tolerances that impose process changes, the quality loss increases because one process becomes too loose when the other one remains too expensive to be used.

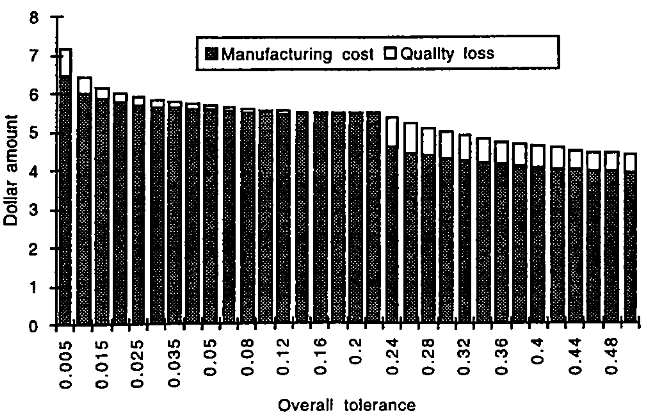

Fig. 5 Manufacturing cost and quality loss for various tolerances

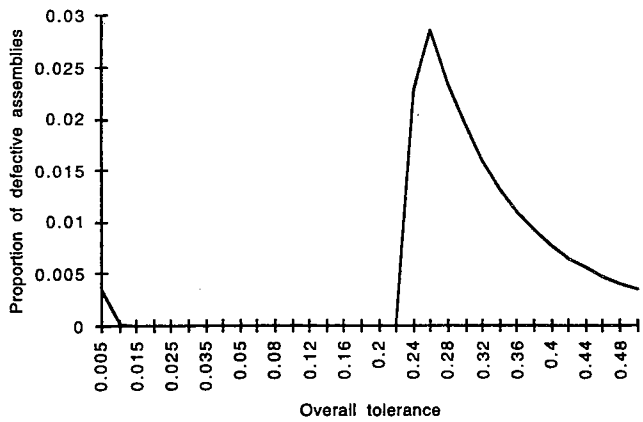

Fig. 6 Variations of the assembly defect rate

8 Conclusion

This paper has showed the pertinence of employing statistical tolerances when using quality loss functions. Also of significant importance is the need to model part manufacturing cost as a function of process precision (i.e., standard deviation of the output characteristic) as opposed to previous models where manufacturing cost is a function of the conventional tolerance allocated on the part characteristic. This approach is more realistic and eliminates the need to impose an arbitrary relationship between process precision and tolerance (e.g., $\Delta = 3\sigma$); the optimum relationship is deduced from our model by introducing manufacturing information into the design process.

The example presented shows a situation in which using a quality loss function identifies the potential quality problems associated with some product grades. In the case considered, the processes available are not suited to make all the product grades we studied.

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