*Artificial Intelligence for Engineering Design, Analysis and Manufacturing* (1997), *11*, 379–394. Printed in the USA.


# Innovative dome design: Applying geodesic patterns with shape annealing

KRISTINA SHEA AND JONATHAN CAGAN
Computational Design Lab, Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A.
(RECEIVED November 27, 1996; ACCEPTED April 21, 1997)

**Abstract**

Shape annealing, a computational design method applied to structural design, has been extended to the design of traditional and innovative three-dimensional domes that incorporate the design goals of efficiency, economy, utility, and elegance. In contrast to deterministic structural optimization methods, shape annealing, a stochastic method, uses lateral exploration to generate multiple designs of similar quality that form a structural language of solutions. Structural languages can serve to enhance designer creativity by presenting multiple, spatially innovative, yet functional design solutions while also providing insight into the interaction between structural form and the trade-offs involved in multi-objective design. The style of the structures within a language is a product of the shape grammar that defines the allowable structural forms and the optimization model that provides a functional measure of the generated forms to determine the desirable designs. This paper presents an application of geodesic dome patterns that have been embodied in a shape grammar to define a structural language of domes. Within this language of domes, different dome styles are generated by changing the optimization model for dome design to include the design goals of maximum enclosure space, minimum surface area, minimum number of distinct cross-sectional areas, and visual uniformity. The strengths of the method that will be shown are 1) the generation of both conventional domes similar to shape optimization results and spatially innovative domes, 2) the generation of design alternatives within a defined design style, and 3) the generation of different design styles by modifying the language semantics provided by the optimization model.

**Keywords:** Topological Layout; Structural Optimization; Dome Design; Simulated Annealing; Shape Grammars

## 1. INTRODUCTION

Structural design can be described as the ability to create innovative and highly appropriate structural solutions to both familiar and new problems that balance efficiency, economy, elegance,[1] and utility (Billington, 1983; Addis, 1990). The motivation for this work is to create the foundation for a computational tool for structural design that embodies this definition of structural design. The development of computational structural design tools has largely focused on deterministic structural optimization methods that provide one design solution for a specified problem that maximizes functional efficiency subject to behavioral and geometric constraints. This limited scope of structural design goals does not consider utility or appropriateness of the design, economic factors in constructing the design, and visual impact of the structural form. Because structural design is more than iterative analysis, an effective computational tool to aid the designer that models the practical design goals of efficiency, economy, elegance, and utility is presented. In addition, the method presented could enhance designer creativity by providing alternative innovative solutions of similar quality and insight into the relationship between form and function.

The discrete topology layout of three-dimensional structures is a complex problem that has been approached by using

Reprint requests to: Dr. Jonathan Cagan, Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A. Tel: (412) 268-3713; Fax: (412) 268-3348; E-mail: jon.cagan@cmu.edu.

[1] For the present work **elegance** is used in the sense of formal elegance or elegance derived from formal geometric properties such as rhythm, symmetry, or ratios. A discussion of formalist critiscism and the distinctions between formal, material, and associative aesthetic values can be found in Mitchell (1990). This interpretation of elegance only uses a portion of what was meant by Billington (1983) who used the word *elegance* to mean the expressive power of a form that is a combination of all parts of aesthetic value. Because material and associative aesthetic values are dependent on personal interpretation, this portion of aesthetic value is left to be evaluated by the designer when judging the resulting designs within a structural language.

deterministic methods for both continuous and discrete layouts. Shape annealing has been presented as a stochastic method that combines discrete topology changes with continuous shape and sizing of topologies to generate optimally directed planar trusses (Reddy & Cagan, 1995). The dome design problem presents an intermediate step between planar layout and three-dimensional layout because a dome can be designed as a two-dimensional truss that is projected onto a curved surface, thereby requiring the third dimension to be a dependent variable of the planar layout. The shape annealing method will be shown to be capable of generating two-dimensional topologies in three-dimensional space to form dome designs.

The shape annealing method is a stochastic method that allows for lateral exploration of the design space to generate a variety of design styles to a given structural design problem. As Simon (1975) wrote,

> A style is one way of doing things, chosen from a number of alternative ways. Since design problems generally do not have unique optimal solutions style may enter in choosing any one of many satisfactory solutions.

In shape annealing, style is incorporated in the design process through multiobjective optimization of design goals. As a stochastic method, shape annealing provides a means of discovering alternative design styles that satisfy the global design goals and are functionally feasible, that is, a valid truss structure that meets the required behavioral and geometric constraints. While stochastic optimization sacrifices finding exact global optima, it benifits from design exploration that can generate good quality but suboptimal designs with interesting qualities and side effects not explicitly optimized but that are beneficial in satisfying the designer's needs. In addition, stochastic optimization is suited to searching an ill-defined design space and, in combination with a discrete representation, can be used to incorporate practical design goals in addition to structural efficiency.

## 2. BACKGROUND

Related work in structural topology optimization focuses on deterministic methods for finding one functionally efficient solution, that is the minimum amount of material required for a specified performance. A presentation of deterministic methods for structural topology optimization can be found in Bendsoe (1995). The general structural layout problem has been investigated as early as Michell's (1904) analytical work. According to Kirsch (1989), structural topology optimization methods can be broken down into two categories: distributed material optimization and discrete topology optimization. Distributed material optimization treats the layout problem as a continuum of material broken down into a grid of elements. Among the distributed parameter methods, much work has been done with the homogenization method that discretizes a specified space into finite elements. The optimal density of each element is then determined from the stress limit resulting on the principal stresses (Bendsoe & Kikuchi, 1988; Bremicker et al., 1991; Diaz & Belding, 1993). The homogenization method has been adapted for practical structural design (Chirehdast et al., 1994) through the development of a three-phase method that combines homogenization, vision algorithms, and shape optimization to generate parameterized manufacturable objects. Two continuous material methods that vary from homogenization are a simulation of adaptive bone mineralization used to generate structural topologies (Baumgartner et al., 1992) and a skeleton-based method (Stal & Turkiyyah, 1996).

Discrete topology optimization methods use either a ground structure on which the design is based or heuristics to introduce new members. Discrete methods have been formulated as a sizing optimization problem by using a highly connected ground structure where topology changes only occur as the removal of members with a minimum area based on stress limits (Hemp, 1973; Rozvany & Zhou, 1991; Bendsoe et al., 1994). The discrete layout problem also has been formulated as a grid of allowable nodes to lay out members (Dorn et al., 1964; Pederson, 1992). The primary disadvantage of these methods is the strong dependence of the resulting design on the ground structure on which it was based because often the only design variables are the size of members with no allowance for changes in the grid point locations (Bendsoe, 1995). A method that utilizes exhaustive topological search based on a heuristic to introduce new members was presented by Spillers (1975). A review of discrete topology optimization problems can be found in Kirsch (1989). Additional approaches to both discrete and continuous topology design can be found in Bendsoe and Soares (1993). Multiobjective approaches to structural topology design also have been developed that trade off behavioral attributes including stress, buckling, displacement, and frequency while searching for the best compromise design (Eschenauer et al., 1990; El-Sayed & Jang, 1991; Grandhi et al., 1993).

The methods described previously are all deterministic methods that focus on the optimization of functional efficiency based on behavioral and geometric constraints. Stochastic methods that focus on design exploration rather than on optimization have been investigated by using an approach similar to homogenization to generate optimal topologies from a genetic algorithm (Chapman et al., 1994) and a simulated annealing-based method for part design that considers performance, manufacturing, and material cost (Anagnostou et al., 1992).

Structural optimization methods can be categorized according to their representation, either discrete or continuous material, and their optimization technique, either deterministic or stochastic. Homogenization, a deterministic continuous method, has been applied successfully to the generation of monolithic parts in design domains such as the automo-

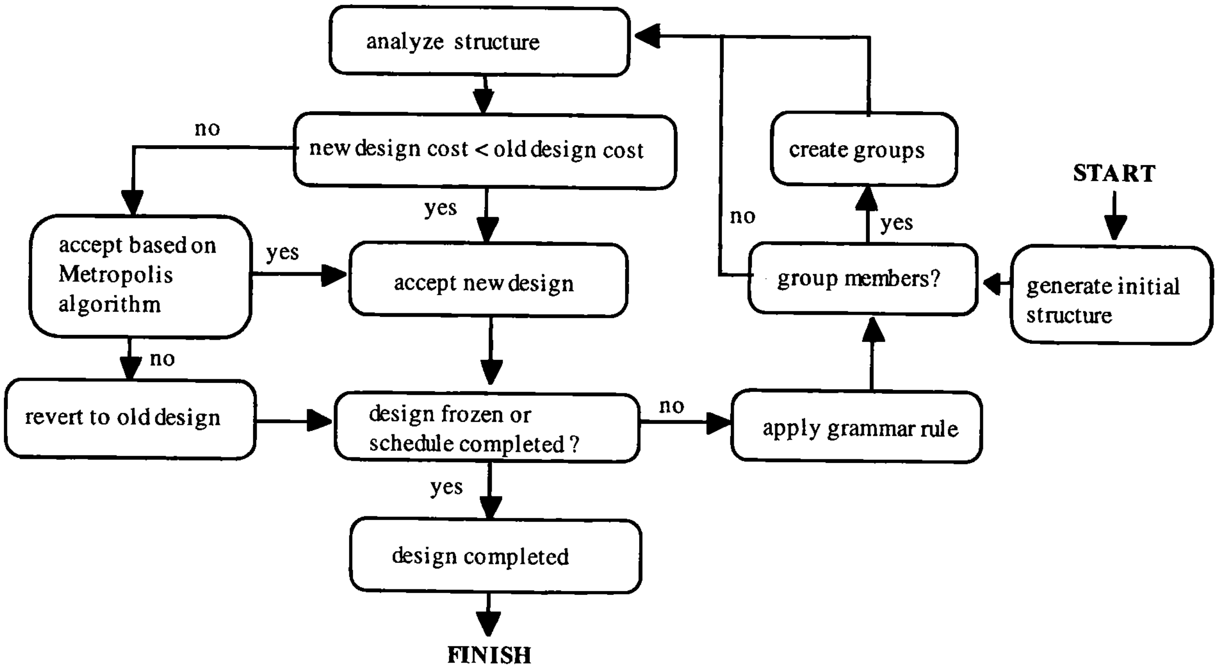


**Fig. 1.** Shape annealing algorithm for structures.

tive and aerospace industries, where structural efficiency is the primary goal. In the design of structures that have both functional and visual interactions with humans, the design goals are broader and cannot be modeled solely by physical laws, making discrete methods advantageous for adding problem-specific design knowledge to the representation and optimization. Using a stochastic optimization technique in combination with discrete design parameters, such as those in shape annealing, allows for optimally directed exploration of a design space consisting of multiple discontinuous objectives. The common ground for homogenization and shape annealing is truss design for maximal efficiency, as shown in a comparison by Reddy and Cagan (1993). Shape annealing has been extended in the present work to the generation of designs that not only take into account structural efficiency but also explore the trade-off among the additional design goals of utility, economy, and elegance.

## 3. METHOD

Shape annealing, a simulated annealing optimization on a shape grammar representation, was introduced as a design technique for the generation of optimally directed designs of shape (Cagan & Mitchell, 1993). Optimally directed design is an approach to design optimization that directs the design generation toward the numeric range of a global optimum. The shape annealing method was applied to the structural topology layout problem and was shown to be capable of generating optimally directed structural topologies that avoid geometric obstacles and satisfy buckling and stress criteria (Reddy & Cagan, 1995). This approach was extended to include a simple model of economy in the optimization through a search for optimal groupings of members by cross-sectional area (Shea et al., 1996). Given a problem specification, the optimal number of groups and the cross-sectional area of each group were determined during the design process. By limiting the number of distinct member sizes through groups, a trade-off was created between achieving the minimum weight of the structure and having fewer distinct member cross-sectional areas with a corresponding economy of scale.

### 3.1. Algorithm: Shape annealing

The shape annealing method, as applied to structural design, builds structures by using a shape grammar (Stiny, 1980), optimizes the structures with the stochastic optimization method of simulated annealing (Kirkpatrick et al., 1983; Swartz & Sechen, 1990; Ochotta, 1994), and analyzes the structures by using the finite element method.[2] The algorithm used follows the diagram shown in Figure 1. First, an initial structure is generated from a minimal connection of truss members between the applied loads and support points of the problem specification. If economy is included as a design goal, the members in a design are grouped accordingly. Second, the structure is loaded and analyzed with the finite element method. The cost function is then evaluated by using the optimization model that is based on the

[2] Interfaces for MSC/NASTRAN™ and FElt (Gobat & Atkinson, 1994) have been built. FElt was used as the FEA tool for the examples presented in this article.

design goals of the problem, the structural behavior of the design, and the geometric constraints. The initial design is automatically accepted. Third, a rule from the shape grammar is applied to the structure to create a new design that is then analyzed, and its cost is calculated. The costs of the new design and the previous design are used in the simulated annealing algorithm to determine whether to accept the new design or revert to the previous design. A better design is always accepted, but a worse design may be accepted based on a probability function (Metropolis et al., 1953). A rule from the shape grammar is applied again to the structure to create a new design, and the process continues iteratively until the annealing schedule terminates or the design has converged, or frozen. At the end of the design process, some members in the design may have the minimum allowable cross-sectional area and can be removed at the designer's discretion as long as the structure remains stable. However, a designer may choose to leave the members in the design because they add negligible weight but may provide additional visual benefits. Details of the shape annealing method implementation can be found in Shea et al. (1996).

The cost function used in the optimization process is a weighted sum of the computational models of the design goals (i.e., efficiency, economy, elegance, and utility) and the constraint violations (i.e., stress, Euler buckling, and overlap of geometric obstacles). The objective weights are set a priori by the designer and represent the relative trade-off of each objective to the weight of the structure, which has a weighting factor of one. Extending the shape annealing method to support multiple independent loading conditions results in the following cost function:

$$\text{cost} = \sum_{i=1}^{l} (\text{objective\_weight}_i \cdot \text{objective}_i) + \sum_{j=1}^{m} \left[ \text{constraint\_weight}_j \cdot \left( \sum_{k=1}^{n} \text{constraint\_violation}_{j,k} \right) \right], \qquad (1)$$

where

$l$ = number of objectives,

$m$ = number of constraints,

$n$ = number of loads

For each independent loading condition, the structure is analyzed for stress and buckling violations, these violations are summed, and the sum is then weighted by the corresponding constraint weight.

### 3.2. Representation: Shape grammars

A shape grammar is a means of defining a spatial language of designs that can be generated from the allowable transformations of shape embodied in the grammar rules (Stiny, 1980). A shape grammar is defined by a four-tuple: $S$, a finite set of shapes; $L$, a finite set of symbols; $R$, a finite set of shape transformations; and $I$, the initial shape. This four-tuple defines the language of spatial designs that can be generated from the grammar. For truss design, the only allowable shape is a triangle in which each line in the shape represents a truss member. The rules of the grammar are then formulated as shape transformations of triangles by dissection, addition, and modification such that the topology of the design still consists of triangles. The rules of the shape grammar are fully parametric where the lengths of lines and the angles between lines are allowed to change. The initial shape is determined from the minimal connection of members between the applied load points and the support points. The topology of the initial structure has a minimal influence on the final design because, with the simulated annealing method, there is a high probability of accepting inferior designs at the beginning of the exploration process such that the design always moves away from the initial structure. However, through the use of reverse grammar rules, which are discussed in Section 4, the initial structure may be revisited during the design process. The current implementation of the shape grammar, a labeled boundary graph, is loosely based on the representation of a solid boundary graph grammar (Heisserman & Woodbury, 1994), with the difference that our representation does not ensure manifold designs.

Formal grammars define the manipulation of shapes that must be interpreted through the use of semantics to generate functional designs. Semantics have been used in spatial grammars to represent features in designs (Brown et al., 1995), and applied to the domain of structural design, will be used to incorporate functional and behavioral meaning in the design generation. For the spatial design of functional systems, the set of labels, $L$, are used to incorporate semantics in the grammar that require the rule applications to make functional sense. For example, a label attached to a point can represent the boundary condition of the joint for analysis purposes or a constraint on the allowable spatial transformation of the point to meet functional design constraints. Labels are also used to indicate the behavior of a member based on feedback from the design analysis. Geometric constraints are currently included in the rule formulation to ensure the generation of designs with a specified minimum angle between members and could additionally be used to constrain shapes to those that have the desired formal aesthetic values. Labels in the shape grammar allow for the incorporation of semantics based on known form and function relations that constrain the set of structural forms generated to those that are meaningful. Through the use of labels, semantics based on functional, behavioral, and aesthetic design requirements can be incorporated into the shape grammar, thus creating a functional grammar. Functional grammars enable the quantification of relations between form and function in the spatial layout of functional systems.

## 4. DOME DESIGN

Geodesic domes, originally invented by R. Buckminster Fuller (1954), have been used for a wide range of purposes including temporary exposition structures, scientific test centers, and housing (Baldwin, 1996). Domes incorporate many interesting design goals because they are considered the strongest, lightest, and most efficient building system (Prennis, 1973). As domical structures, geodesic domes enclose the maximum space with minimum surface area in which the layout of the structure is influenced by both the desired span and height as well as the method of layout. Economic design goals include minimizing the number of different strut lengths and the number of distinct cross-sectional areas as well as minimizing the surface area to present the least possible surface to the weather and thus minimizes heat loss (Baldwin, 1996). The objective function formulated for dome design minimizes both the weight of the structure and the surface area and maximizes the enclosed volume. In addition, economy is added to the design optimization model through dynamic grouping of members by cross-sectional area (Shea et al., 1996) or length. Area groups are a group of members with the same cross-sectional area, and length groups are a group of members with lengths that fall within a small range based on a specified tolerance. The number of groups is then used to calculate a group penalty that is added to the objective function. This group penalty is formulated to increase in magnitude throughout the annealing schedule and to have a strong influence with an increase in the number of groups.

A computational model of elegance can be based on formal elegance, which is elegance derived from formal qualities such as symmetries, proportion, rhythm, uniformity, and variety that are based purely on geometric properties. Elegance derived from material or associative aesthetic values (Mitchell, 1990) is not considered in the design generation because this type of elegance is founded in designer preferences and interpretation. The assessment of this type of elegance is left to the designer when viewing the structural language of solutions. In previous work, the form of a structure was based on functional considerations alone through models for structural efficiency and economy except when a geometric constraint on symmetry was imposed. In this work, an aesthetic model of visual uniformity is incorporated into the optimization model to reflect the geodesic dome design goal of a uniform breakdown of a sphere. Thus, an aesthetic value is calculated for a design from the standard deviation of the length of all members in a design. A design with a more uniform breakdown and thus a lower standard deviation of length is considered to be of greater aesthetic value than a design with a more random breakdown. The addition of an aesthetic design goal motivates the design of aesthetically pleasing structures that are not based solely on behavioral properties and geometric constraints.

The objective function consists of a summation of the design goals in which each design goal has a corresponding weighting factor that is set a priori by the designer to represent the relative trade-off with structural weight, which has a weighting factor of one. The objective function using all design goals for dome design is

$$\text{objective} = \text{mass} + \left(\text{weight}_1 \cdot \frac{1}{\text{enclosed volume}}\right) + (\text{weight}_2 \cdot \text{surface area}) + (\text{group\_weight} \cdot \text{group\_penalty}), \tag{2}$$

where the group penalty is defined as

$$\text{group\_penalty} = e^{(\text{iteration/max\_iterations})} \cdot \text{number\_of\_groups}^2. \tag{3}$$

The layout of Fuller's geodesic domes consists of regulated patterns of self-bracing triangles to produce a maximally efficient spherical structure (Figure 2). Geodesic domes are fractional parts of spherical tensegrities that consist of a planar truss system created by subdividing the faces of one of the five platonic solids and projecting these subdivisions onto a spherical plane (Prennis, 1973). A study of the standard rules used to generate geodesic patterns will be shown to quantify these rules in the form of a shape grammar for use in a nonconventional sequence to generate innovative geodesic-like forms.

The dome grammar presented is based on two standard methods for subdividing the faces of platonic solids, called the triacon (class I), and alternate (class II) methods (Figure 3). The triacon method derives its name from the subdivision of the faces of a triacontahedron from which it was first developed. Breakdowns are referred to by the frequency with which the original sides of the triangle are subdivided, represented as $nv$ for $n$ divisions on one side. For example, in Figure 3, $2v$ indicates that each original side was divided in two. With the triacon breakdown, the original sides of the triangle can be removed, although it is not necessary, and the new lines extended to the adjacent triangles; this is indicated in Figure 3 by the lines that extend past the original sides. In the alternate breakdown, the original sides of the triangle remain intact. Further discussion of standard breakdowns and general dome design techniques can be found in Prennis (1973) or Sheppard et al. (1974) and at numerous sites on the world wide web.[3] Calculating the angles and strut lengths for a geodesic design is a tedious task for which a computer program, DOME (Bono, 1996), has been developed that generates standard dome breakdowns for class I and II domes based on geodesic math. The dome designs presented in the present work are quite

[3] Web sites on geodesic domes include: www.bfi.org, www.teleport.com/~pdx4d/dome.html, www.netaxs.com/~cjf/fuller-faq-4.html, www.wnet.org/bucky/dome.html, www.crisis.com/~rjbono.

(a)

(b)

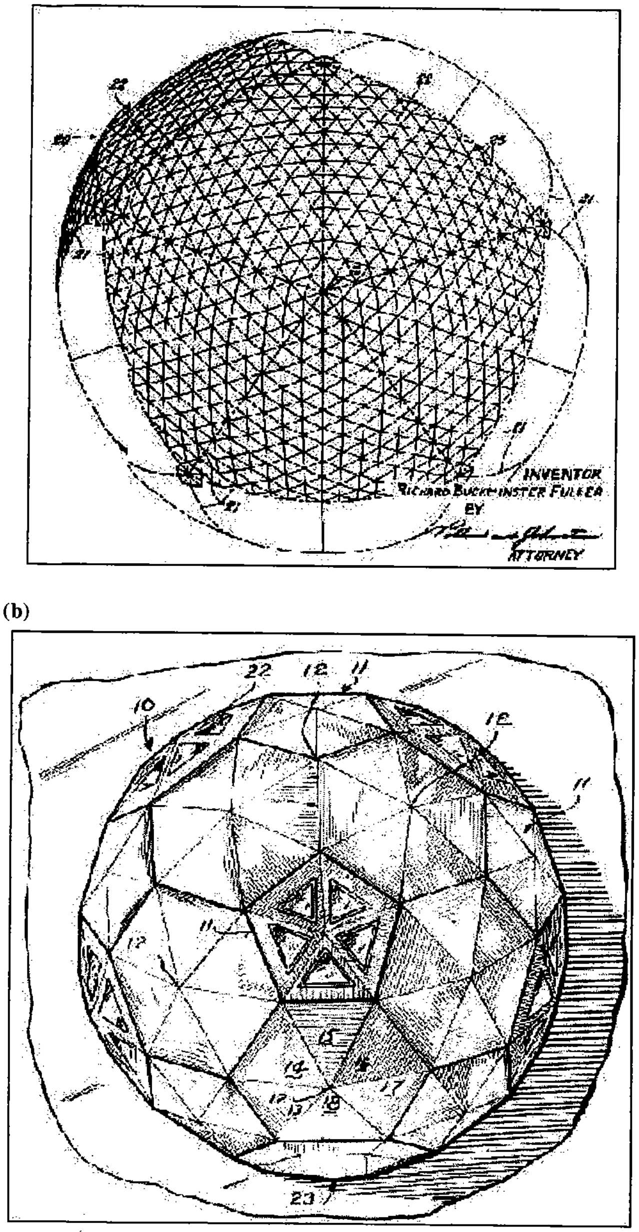

Fig. 2. Two of Buckminster Fuller's geodesic dome patents: Building Construction, 1954 (a) and Hex-Pent, 1970 (b).


Fig. 3. Classes of geodesic dome breakdowns.

different from standard layouts because, although they are based on geodesic breakdowns, the designs themselves are not required to adhere to geodesic patterns. However, designs generated from the grammar can approach geodesic patterns.

Subdividing the members of a dome serves to achieve behavioral, economic, and utilitarian design goals. For example, dividing a member in half decreases the length of each individual member, which in turn decreases the required cross-sectional area of compression members. However, the total length of the two new members is greater than the single old member because the bisection point is projected onto the specified ellipsoid, thus forming two legs of a triangle above the old member. Therefore, to reduce the weight of the structure and increase the efficiency, subdividing creates a trade-off between decreasing cross-sectional area and increasing member length. Adding the utilitarian goals of maximizing enclosure space and minimizing surface area create additional trade-offs in subdividing members. A higher frequency of subdivisions allows the structure to approximate a sphere more closely, which will lead to a maximum enclosure space for minimum surface area.

A shape grammar for dome design based on geodesic patterns for the standard breakdowns discussed is presented in Figure 4. This grammar was developed for an open dome without a covering skin, which implies that the only allowable shapes are triangles to ensure structural stability. The lines in the shape grammar represent truss members, and thus only truss topologies can be generated using this grammar. The rules of the grammar described below are divided into shape and topology modification rules. The shape modification rules take a free point and move it some distance or change the diameter of a member cross section based on the labels + or −, which indicate the direction the change is made. These labels are set based on the behavior of each member: If a member is below both the stress and buckling limits, the label is set to − to decrease the cross-sectional area; otherwise, the label is set to +. Shape modification rules and the shape annealing algorithm perform shape and sizing optimization of a structure with a fixed topology. The topology modification rules take a shape within the design and transforms it into a different configuration of shapes based on geodesic patterns. A free line, denoted by the label *f*, is an exterior line in the design. Note that the topology rules are created in pairs so that any modification can be reversed, except for rule 5, which is its own reverse. The application of a topology reversal rule is monotonic, and in theory any previous design state can be reached through the sequential application of reverse rules.[4] The grammar rules are fully parametric and use the following geometric constraints based on considerations of good structural design:

[4] Spatial emergence does not exist in the representation presented, so that any rule is always monotonic.

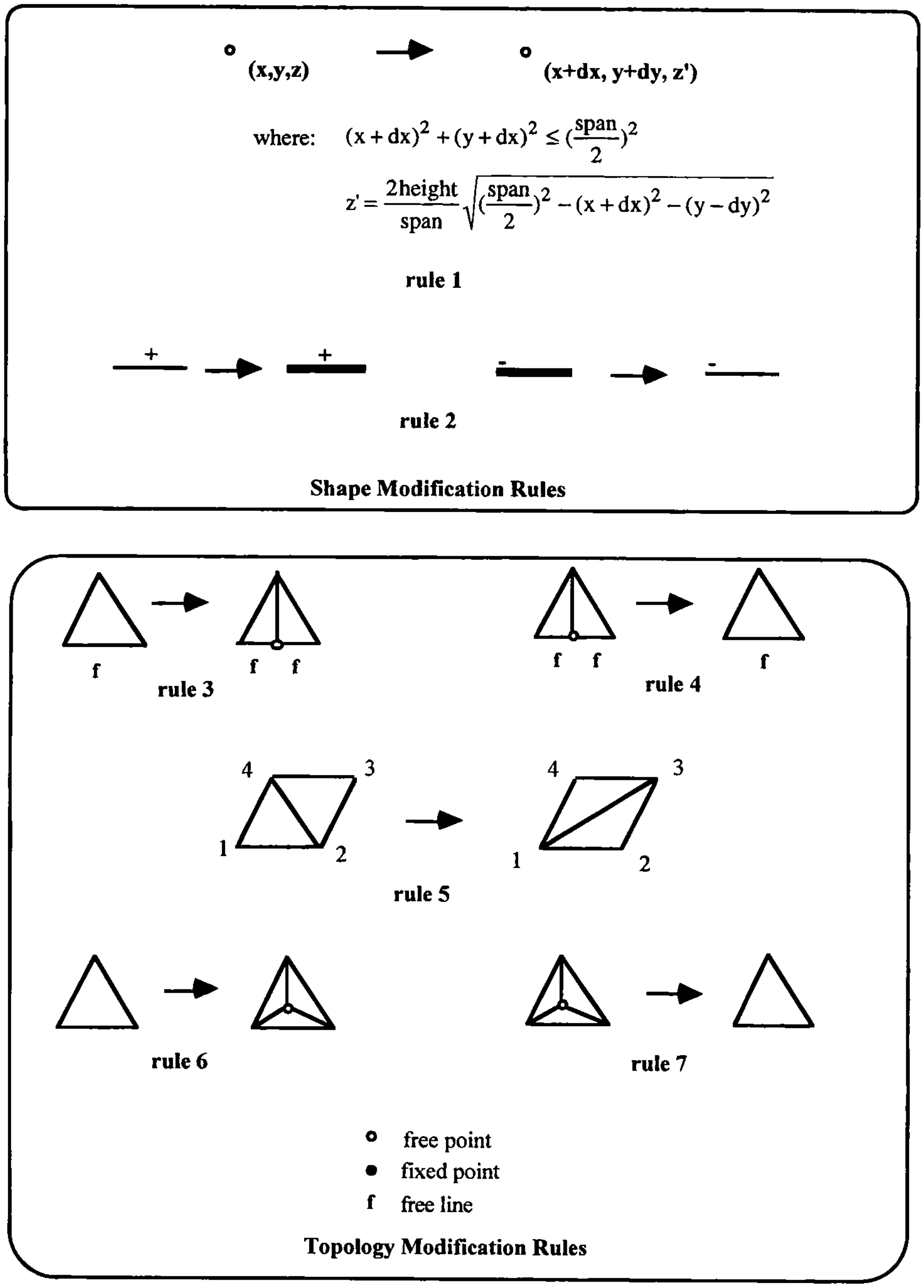


**Fig. 4.** Dome grammar.

1. no two members may intersect without a joint,
2. members cannot overlap, and
3. the minimum angle between members is 10°.

The shape grammar shown generates a two-dimensional truss layout that is projected onto a curved plane defined by an ellipsoid of a given height and span. The base of the dome is circular, defined by a uniform span in both the $x$ and $y$ directions. These parameters specified by the designer act as functional constraints on the shape grammar because they define the desired enclosure space. An ellipsoid rather than a sphere is used because much vertical enclosure space often is wasted with spherical domes; nevertheless, an ellipsoid can represent a sphere if so desired. An additional parametric constraint is placed on the shape modification rule to keep all points of the design within the circular base of the defined ellipsoid. Figure 5 shows a top-down view of two dome layouts generated by hand from the grammar; the view illustrates that the grammar is capable of generating standard geodesic forms through the sequential application

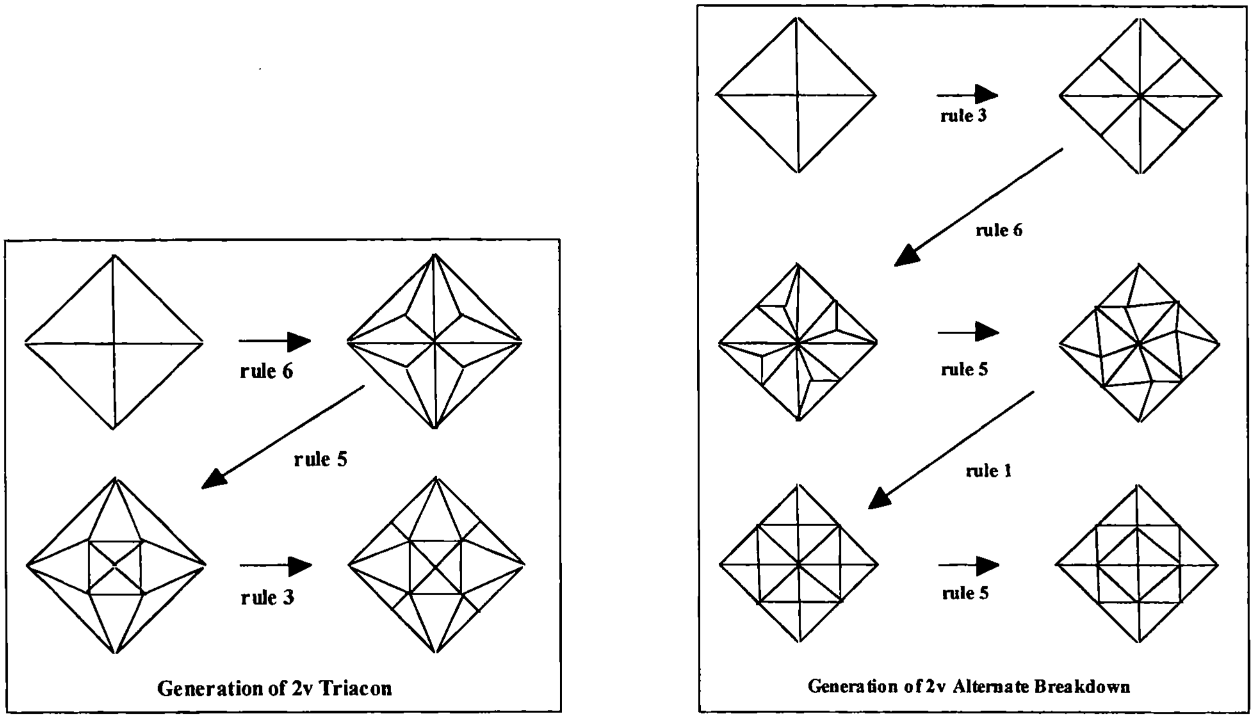


**Fig. 5.** Generation of standard forms.

of shape grammar rules. For the triacon pattern layout, the specified rule is applied under reflected quarter symmetry; for the alternate pattern layout, the specified rule is applied under rotational quarter symmetry.

## 5. RESULTS

The dome grammar presented in Section 4 will now be used to generate solutions for two dome design problems. The first problem compares the shape annealing method with traditional shape optimization, and the second problem explores innovative dome design at both a spatial and functional level based on geodesic patterns. The shape annealing method is a stochastic method that does not guarantee global optimality but rather uses design exploration to produce a variety of good designs of essentially equal quality, that is, objective function value. Spatial dome layout is investigated through an optimally directed search by using the geometric design goals of geodesic structures, that is, to approximate closely the defined ellipsoid by maximizing enclosure space and minimizing surface area. This problem then will be extended to the functional layout of domes and will explore the effect of changing the semantics included in the objective function, or design goals, on the spatial and functional styles and quality of solutions generated. For all designs, generated points in the planar layout are projected onto the surface defined by an ellipsoid to generate three-dimensional structures. For all designs presented, a top-down view and a three-dimensional view are shown in which the width of the lines represents the diameter of the members uniformly scaled across all figures.

### 5.1. Comparison with structural optimization

The first problem is based on the shape optimization of a space truss presented by Pederson (1973). Figure 6 shows the fixed layout for Pederson's shape optimization problem and the corresponding initial layout for shape annealing. The objective of the design is to maximize efficiency of the structure subject to four independent loading conditions applied in the $z$ direction:

1. $-3 \times 10^5$ N at joint 1,
2. $-3.9 \times 10^5$ N divided equally among all free joints,
3. $-1.5 \times 10^5$ N at joint 1 and $-1 \times 10^5$ N at joints 4 and 5, and
4. $-1.5 \times 10^5$ N at joint 1 and $-.7 \times 10^5$ N at joints 2–4.

Parametric constraints on the problem are as follows: Joint 1 is restricted to move only in $z$, which changes the overall height of the dome and thus the ellipsoid on which the planar layout is projected. Joints 2–5 are restricted to move along the orthogonal axis on which they lie in the initial layout. In addition, the design is required to maintain 1/8 symmetry. The material properties are modulus of elasticity ($E$, $2.1 \times 10^{11}$ N/m$^2$), specific weight ($v$, 77,008.5 N/m$^3$), allowable tensile stress ($\sigma^t$, $1.3 \times 10^8$ N/m$^2$), allowable compressive stress ($\sigma^c$, $-1.04 \times 10^8$ N/m$^2$), proportional stress limit ($\sigma^L$, $-1.3 \times 10^8$ N/m$^2$), radius of gyration ($\alpha = 1.0$), and a factor of safety against buckling ($n = 2.5$). The allowable force before buckling for each member is calculated with the following formulae:

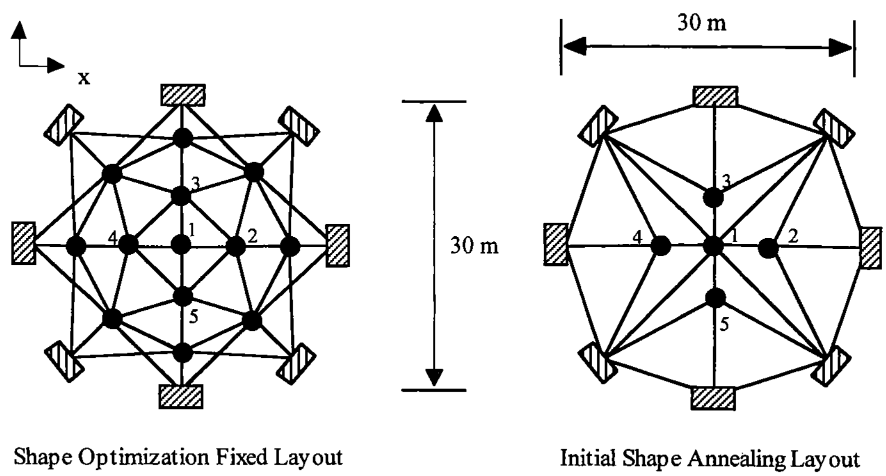


**Fig. 6.** Structural optimization comparison.

| | Tensile | Compressive Elastic | Compressive Plastic |
|---|---|---|---|
| Allowable Force | $\sigma \cdot a$ | $-\left(\frac{a}{c}\right)^2$ | $p^L + \sigma^c(a - c\sqrt{-p^L})$ |

where

$a \equiv$ cross-sectional area,

$$c \equiv \frac{1\sqrt{n}}{\pi\alpha\sqrt{E}};$$

$l \equiv$ member length; and $\sigma \equiv$ member stress,

$$p^L \equiv \frac{-1^2 s^{L^2}}{\pi^2\alpha^2 En}. \tag{4}$$

Shape annealing designs have been generated for two cases: 1) those requiring 1/8 symmetry as in Pederson's shape optimization problem and 2) those allowing asymmetric designs. The designs requiring 1/8 symmetry were allowed a maximum of 20 members in the 1/8 segment of the design, and the asymmetric designs were allowed a maximum of 50 members in the entire design. An upper bound is placed on the number of members in a design to make the topology design space finite, although still large, while allowing for sufficient exploration of design concepts. Three designs generated for the symmetric problem are shown in Figures 7a–c with weights of 66,757 N, 67,679 N, and 68,503 N, respectively. Shape optimization of the topology shown in Figure 6 results in weights of 55,162–65,482 N for different parametric constraints[5] (Pederson, 1973). The designs generated from shape annealing are not the exact optima but are close to the range of the optimal solutions without using prior topological knowledge of optimal designs.

For a given dome topology, an optimal solution exists in which all members are at their limit for at least one loading condition and thus have an optimal height. By allowing the topology to change, a distinct topology may be found that decreases the optimal height of the structure because the objective is to minimize weight subject to Euler buckling. The heights of the designs generated by shape annealing (6.33–7.62 m) are considerably less than the shape optimization solutions for which the height ranges from 9.25 m to 11.24 m. This difference could be advantageous depending on the purpose of the design as vertical space is often wasted and increases the surface area that must be covered, which in turn increases the energy costs of maintaining the building. This result demonstrates the benefit of design exploration over deterministic optimization in presenting solutions that may have secondary benefits not explicitly modeled in the optimization objective function.

Asymmetric designs for this problem generated by shape annealing are shown in Figure 8, with weights of 65,129 N and 80,334 N, respectively. It is interesting and somewhat expected that the most efficient design generated was asymmetric because the design can take advantage of the asymmetries to compensate for the asymmetric loading. Comparing the symmetric and asymmetric designs shows that both sets of designs provide the required function but have different visual effects. Because aesthetic value is partly dependent on designer interpretation, it is left to the designer to evaluate the relative importance between beauty and functional efficiency. This example illustrates that shape annealing is capable of designing comparable efficient solutions to traditional shape optimization problems under multiple loading conditions when well constrained, but, given more latitude, can generate visually interesting solutions that may be more suited to the problem specification.

[5] Three constrained conditions were used: 1) linking all vertical coordinates of the joints and not allowing horizontal coordinates to change, resulting in a weight of 65,482 N with a height of 9.25 m; 2) allowing the horizontal coordinates and the vertical coordinates to change, resulting in a weight of 55,162 N and a height of 11.34 m; and 3) constraining the height to 9.25 m in addition to horizontal and vertical coordinate changes, resulting in a weight of 56,241 N.

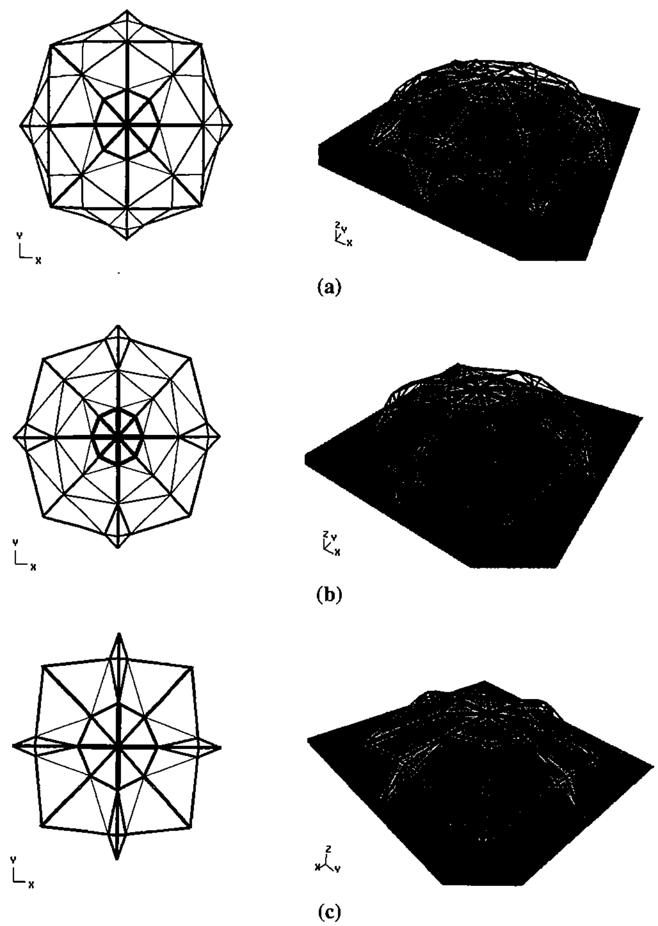


**Fig. 7.** Symmetric dome designs: (a) weight = 66,757 N, height = 7.62 m; (b) weight = 67,679 N, height = 7.45 m; (c) weight = 68,503 N, height = 6.33 m.

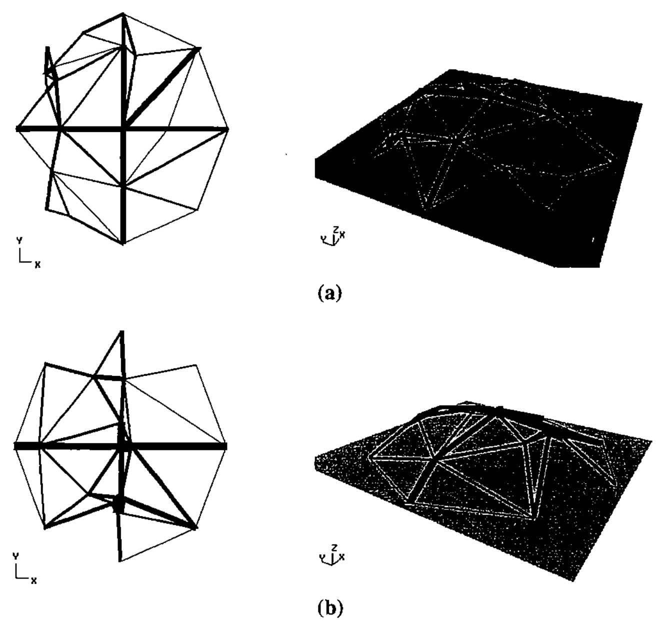


**Fig. 8.** Asymmetric dome designs: (a) weight = 65,129 N, height = 5.43 m; (b) weight = 80,334 N, height = 5.31 m.

## 5.2. Innovative geodesic dome design

This section explores the application of geodesic patterns with shape annealing for both spatial and functional dome design.

### *5.2.1. Spatial dome design*

The spatial breakdown of an ellipsoid for maximum enclosure space and minimum surface area will now be investigated. The purpose of this investigation is 1) to verify that the grammar, through random, iterative application of rules, is capable of generating geodesiclike spatial designs and 2) to determine the achievable spatial limits of enclosure space and surface area with a maximum of 50 members in the design. The problem dimensions are shown in Figure 9, where point 1 at the center of the structure is fixed at a height of 9.25 m, thus defining an ellipsoid with a span, or circular base, of 30 m diameter and a height of 9.25 m. The design shown in Figure 10a has an enclosure space of 3555 $m^3$ and a surface area of 10.07 $m^2$. The design shown in Figure 10b has an enclosure space of 3528 $m^3$ and a surface area of 9.77 $m^2$. Geodesic patterns can be seen in these designs as highlighted in Figure 10a, thus verifying the ability of the dome grammar to generate geodesic-like dome designs under random application of rules.

### *5.2.2. Spatial and functional dome design*

We will now apply the dome grammar to the design of domes for maximum efficiency, economy, utility, and elegance. As discussed in Section 4, the purpose of a dome structure is to provide a maximum amount of enclosed space for an exposition structure or housing, for example, and provide low covering and energy costs by minimizing the surface area. We will now investigate how adding design goals to the optimization objective function influences the quality and appropriateness of the generated designs. The ma-

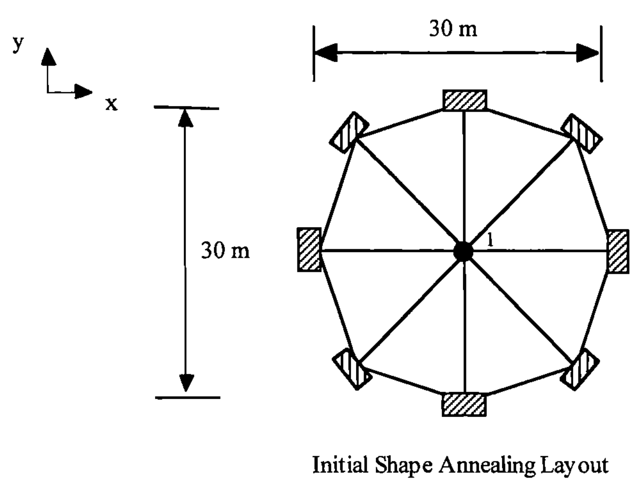


**Fig. 9.** Initial shape.

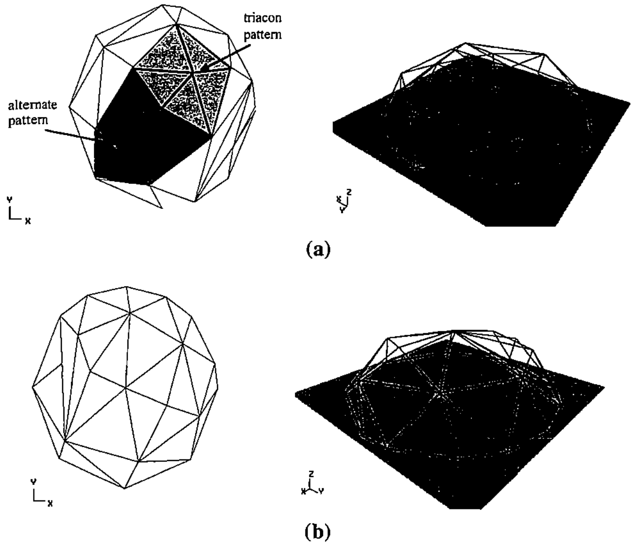


**Fig. 10.** Spatial layout of domes.

terial properties used for these designs are modulus of elasticity ($E$, $2.1 \times 10^{11}$N/m$^2$), specific weight ($\upsilon$, 7850 N/m$^3$), allowable tensile stress ($\sigma^t$, $1.3 \times 10^8$ N/m$^2$), and allowable compressive stress ($\sigma^c$, $-1.04 \times 10^8$ N/m$^2$). The buckling limit is calculated by using the Euler buckling formula, $P_{cr} = \pi Ea/l$. The initial layout for the problem is shown in Figure 9, where the two simultaneous loads applied are self-weight of structural members and −300,000 N applied at the center, joint 1, in the $z$ direction. A sequence of solutions will be shown in which each case expands the objective function to incorporate additional design goals, as described in Table 1. The designs are shown in Figures 11–18, with the corresponding design metrics listed in Table 2. Costs are listed for each design shown to compare multiple designs from the same case. Convergence statistics for 12 designs generated for each case, with standard deviations shown in parentheses, are shown in Table 3. Because simulated annealing is a stochastic method that can get trapped in local optima, it is standard practice to generate three designs and select the best design. Using this heuristic, a second set of statistics, shown in Table 4, is calculated from four sets of three designs using the same 12 designs generated for the statistics in Table 3. These designs were generated in approximately 80 min, each using a DEC alpha.

## 6. DISCUSSION

Shape annealing has been shown to be capable of generating traditional solutions to structural design problems provided the design generation is properly constrained. When these constraints are removed, shape annealing generates functional yet spatially innovative solutions for the same design problem. Comparing the solutions in Figure 7a and Figure 7b shows that the topologies of the two designs are only slightly different. Previous work on the generation of transmission towers (Shea et al., 1996) that compared shape annealing of planar trusses with shape optimization solutions also resulted in a small number of distinct topologies. The solution shown in Figure 7c has a different topology but results in an increase in weight, leaving the designer to decide which design is preferable. Although constraining the problem may lead to finding only a few distinct solutions, these solutions are still quite different from the standard geodesic forms. Constraining the problem generates solutions that a designer may expect, but the strength of this method is design exploration that allows the method to generate a variety of distinct novel solutions. When the symmetry constraints are removed to allow for asymmetric designs, functionally feasible solutions with drastically different topologies are generated.

When the layout problem is not well constrained, the design space increases, providing more possibilities of designs to explore. Semantics can be added to the generation process through both the optimization model and the grammar itself to increase the level of problem knowledge based on design goals that serve to focus the search for appropriate designs. Expanding the objective function to reflect the design problem more accurately, reinforces good design decisions. For

**Table 1.** *Dome design cases*

| Case | Design Goals | Loading |
|---|---|---|
| 1 | Utility = maximum enclosure space<br>Economy = minimum surface area | None |
| 2 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area | Self-weight |
| 3 | Efficiency = minimum weight | Self-weight and center applied load |
| 4 | Efficiency = minimum weight<br>Utility = maximum enclosure space | Self-weight and center applied load |
| 5 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area | Self-weight and center applied load |
| 6 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area<br>minimum member area groups | Self-weight and center applied load |
| 7 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area<br>minimum member length groups | Self-weight and center applied load |
| 8 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area<br>Elegance = visual uniformity | Self-weight and center applied load |
| 9 | Efficiency = minimum weight<br>Utility = maximum enclosure space<br>Economy = minimum surface area<br>minimum member area groups<br>Elegance = visual uniformity | Self-weight and center applied load |

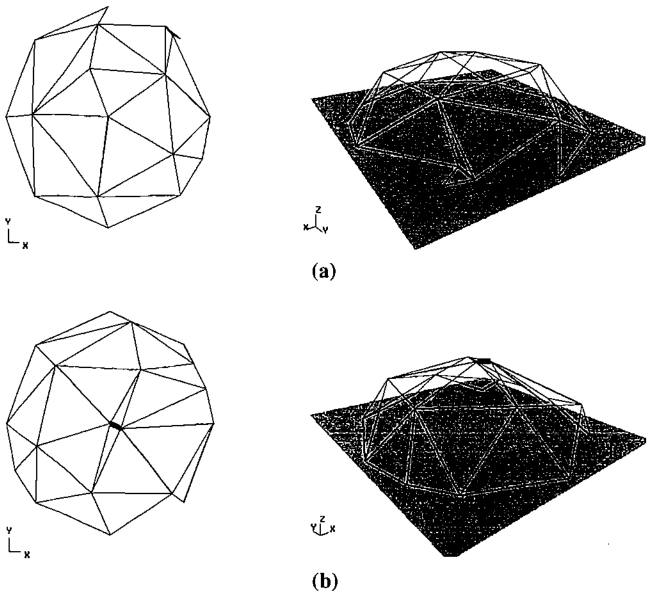

Fig. 11. Dome design for self-weight.

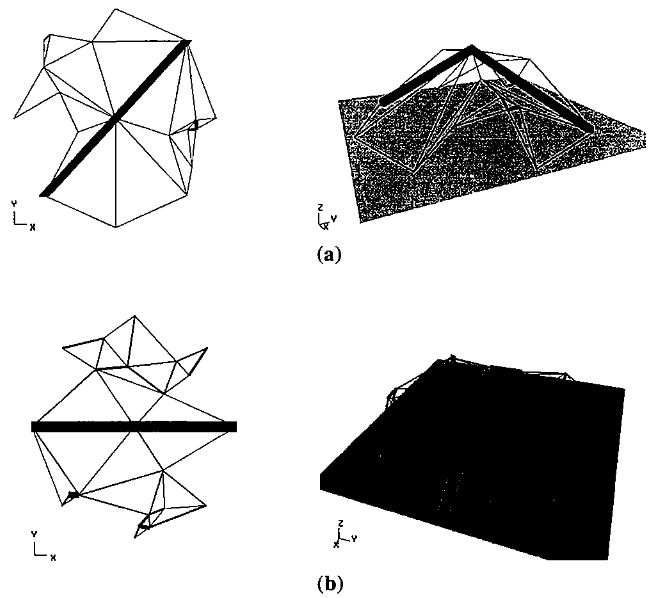

Fig. 12. Objective = weight.

**Table 2.** *Design metrics for Figures 10–18*

| Case | Figure | Objective to Minimize[a] | Cost | Weight (N) | Enclosed Volume ($m^3$) | Surface Area ($m^2$) | No. Groups |
|---|---|---|---|---|---|---|---|
| 1 | 10a | $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 3741 | N/A | 3658 | 10.07 | N/A |
| 1 | 10b | $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 3789 | N/A | 3555 | 9.77 | N/A |
| 2 | 11a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 4311 | 203 | 3101 | 8.82 | N/A |
| 2 | 11b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 4626 | 405 | 2980 | 8.66 | N/A |
| 3 | 12a | weight | 6584 | 6584 | 1933 | 6.85 | N/A |
| 3 | 12b | weight | 7477 | 7477 | 1792 | 5.78 | N/A |
| 4 | 13a | weight + $(w_1/\text{volume})$ | 10,591 | 6791 | 2641 | 8.34 | N/A |
| 4 | 13b | weight + $(w_1/\text{volume})$ | 10,597 | 6889 | 2699 | 8.56 | N/A |
| 5 | 14a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 10,877 | 6449 | 2860 | 9.06 | N/A |
| 5 | 14b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ | 11,305 | 6538 | 2566 | 8.69 | N/A |
| 6 | 15a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(area) | 18,371 | 9343 | 2751 | 8.94 | 4 |
| 6 | 15b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(area) | 33,687 | 13,940 | 2337 | 7.33 | 7 |
| 7 | 16a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(length) | 32,875 | 17,984 | 2317 | 7.89 | 6 |
| 7 | 16b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(length) | 43,637 | 21,211 | 2368 | 8.06 | 8 |
| 8 | 17a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + $\sigma$(length) | 29,018 | 11,203 | 3098 | 8.53 | $\sigma = 1.36$ m |
| 8 | 17b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + $\sigma$(length) | 30,261 | 10,756 | 2799 | 7.70 | $\sigma = 1.50$ m |
| 9 | 18a | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(area) + $\sigma$(length) | 42,489 | 14,726 | 2277 | 6.46 | 8 $\sigma = 1.96$ m |
| 9 | 18b | weight + $(w_1/\text{volume}) + (w_2 \cdot \text{surface area})$ + group_penalty(area) + $\sigma$(length) | 43,781 | 16,889 | 2475 | 7.48 | 7 $\sigma = 2.48$ m |

[a] $w_1 = \text{weight}_1$, $w_2 = \text{weight}_2$.

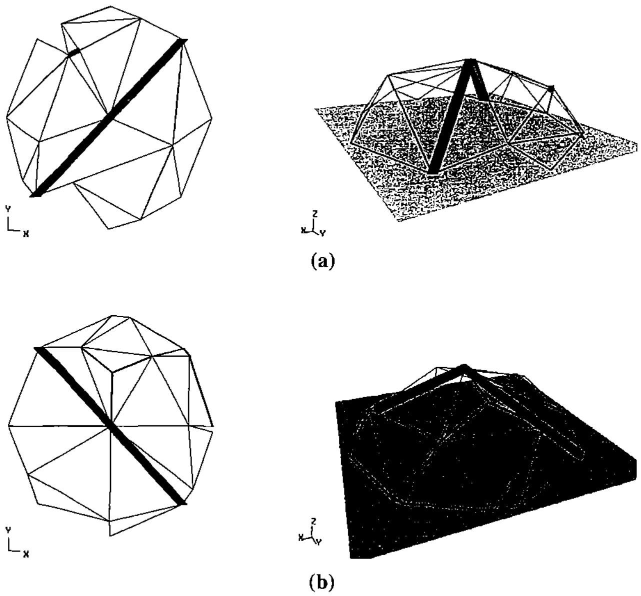


**Fig. 13.** Objective = weight + ($w_1$/enclosure space).

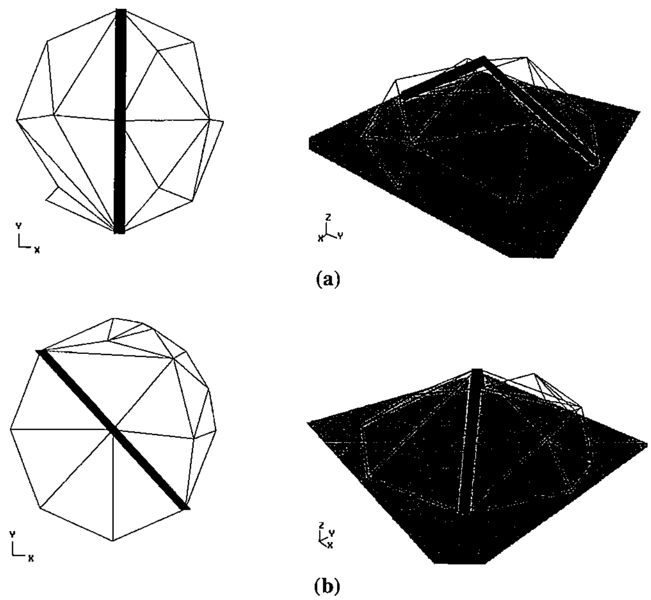


**Fig. 14.** Objective = weight + ($w_1$/enclosure space) + ($w_2 \cdot$surface area).

example, when comparing the results in Table 4 for cases 3 and 4, adding the goal of maximizing volume not only increases the enclosure space but also decreases the weight, implying that for this case these design goals cooperate. Design goals also can compete to create a trade-off, as illustrated when comparing cases 4 and 5. With the addition of minimizing surface area to the objective function, the weight increases while the enclosure space decreases, but the design goal of minimizing surface area was improved to only a small degree. This result may indicate to the designer that an explicit minimization of surface area may not be necessary if the previous solutions were found to be satisfactory. It should be noted that small declinations of the objective values also can be attributed to the increase in the difficulty of the optimization problem from the addition of design goals. Adding economy to the objective function through member grouping in cases 6 and 7 creates further competition that has adverse effects on weight and enclosure space and improves the surface area. The design goal of elegance based on an aesthetic value determined from spatial uniformity creates designs that are visually more familiar and intuitive to the designer and have the additional benefit of increasing the enclosure space at the expense of

**Table 3.** *Design statistics for 12 designs*

| Case | Objective to Minimize[a] | Weight (N)[b] | Enclosed Volume ($m^3$)[b] | Surface Area ($m^2$)[b] | No. Groups[b] |
|---|---|---|---|---|---|
| 1 | ($w_1$/volume) + ($w_2 \cdot$surface area) | N/A | 3408 (149) | 9.42 (0.36) | N/A |
| 2 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) | 1407 (1301) | 2733 (219) | 8.02 (0.5) | N/A |
| 3 | weight | 11659 (3425) | 1631 (337) | 5.91 (1.06) | N/A |
| 4 | weight + ($w_1$/volume) | 8983 (2046) | 2588 (127) | 8.34 (0.45) | N/A |
| 5 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) | 9722 (2784) | 2486 (145) | 7.98 (0.6) | N/A |
| 6 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) + group_penalty(area) | 15,826 (3269) | 2312 (247) | 7.37 (0.86) | 10 (4) |
| 7 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) + group_penalty(length) | 22,157 (4471) | 2286 (153) | 7.68 (0.52) | 13 (7) |
| 8 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) + $\sigma$(length) | 12,137 (1381) | 2844 (211) | 8.26 (0.53) | $\sigma$ = 2.01 m (0.48 m) |
| 9 | weight + ($w_1$/volume) + ($w_2 \cdot$surface area) + group_penalty(area) + $\sigma$(length) | 16,901 (2833) | 2455 (224) | 7.48 (0.80) | 8 (1) $\sigma$ = 2.82 m (0.63 m) |

[a] $w_1$ = $\text{weight}_1$, $w_2$ = $\text{weight}_2$.
[b] Standard deviations are shown in parentheses.