Robotic Deployment for Environmental Sampling Applications

Dan O. Popa, Koushil Sreenath and Frank L. Lewis
Automation and Robotics Research Institute (ARRI)
University of Texas at Arlington, Arlington, Texas, USA, Email: {popa@arri.uta.edu}

Abstract: The use of robotics in environmental monitoring applications requires distributed sensor systems optimized for effective estimation of relevant models subject to energy and environmental constraints. The sensor carrying robots are in fact agents that facilitate the repositioning of network nodes in order to increase their coverage and accuracy. Wireless or acoustic network communication is an essential technology in transmitting the sensed as well as telemetry information between robots. Each mobile sensing node (robot) is characterized by sensing or measurement noise, localization uncertainty, and navigates in a region with a parameterized field model.

This paper addresses a problem of great importance to the SN robotic deployment: adaptive sampling by selection and repositioning of mobile sensing nodes in order to optimally estimate the parameters of distributed variable field models. We present simulation results using deployment scenarios that will be experimentally validated at a future date.

Keywords: Environmental Sampling, Adaptive Sampling, Sensor Networks, Kalman Filter.

I. INTRODUCTION

Recently, there has been renewed interest in using mobile robots on land, under water and in air, as sensor carrying platforms in order to perform sampling missions, such as searching for harmful biological and chemical agents, search and rescue in disaster areas, or environmental mapping and monitoring [Mataric2002, Cortes2002, Kumar2003, Popab2004]. Even though mobility introduces additional degrees of complexity in managing an untethered collection of vehicles, it allows the repositioning of the on-board sensors, thus expanding the coverage and operational lifetime of the sensor network. In the context of autonomous vehicles, many important issues regarding the deployment architectures await to be fully addressed, including the selection of appropriate information measures to guide and evaluate the mission, and the distribution of computation and communication among the autonomous vehicles. Sampling is a broad methodology for gathering statistics about physical and social phenomena, and it provides a data source for predictive modeling in oceanography and meteorology [Bennet2002]. The term adaptive sampling has been used to denote sampling strategies that can change depending on prior measurements or analysis, and thus allow for adaptation to dynamic or unknown scenarios. One such scenario involves the deployment of multiple underwater vehicles for the environmental monitoring of large bodies of water, such as oceans, harbors, lakes, rivers and estuaries. Predictive models and maps can be created by repeated measurements of physical characteristics such as water temperature, dissolved oxygen, current strength and direction, or bathymetry. However, because the sampling volume could be quite large, only a limited number of measurements are usually available. Intuitively, a deliberate sampling strategy based on models will be more efficient than just a random sampling strategy.

There has been wide interest in the development of distributed sensor networks that incorporate local communications capability, changing of the network topology, protocols, and routing dependent on the task and constraints [Estrin, Yu 2001, Ye2002]. Problems addressed in this work include object tracking [Chen2003, Zhao2003], target classification [Mataric1995], distributed control [Sinopoli2003], and sensor validation [Aarabi2002].

Robotics technology provides for mobile sensing nodes in a distributed sensor network using prior research on cooperative mobile robots including localization methods and mapping [for instance Sanderson1998, Newman1999, Fennwick2002]. Many robot team tasks have been investigated for example foraging, ant colony behavior, robotic soccer, map making, area searching, mine sweeping, etc. In some instances, team behaviors have been implemented using potential fields, in particular for obstacle avoidance, goal attainment, and sensor network deployment [Mataric2002, Popab2004].

While multiple vehicle localization and sensor fusion are classic problems in robotics, the problem of distributed field variable estimation is typically relevant to charting, prediction and inverse modeling in oceanography and meteorology [Curtin1993, Brink1997, Creed1998]. In this context, measurement uncertainty has been addressed using Kalman-Filter Estimation [Lewis1986, Bennet2002].

In this paper we use both closed form formulas of parameter variance, as well as the Kalman Filter to select sampling locations for vehicles navigating inside a parameterized field. In the future, this work will be extended to include potential field deployment algorithms for mobile sensors, energy and sampling time constraints, physical constraints, as well as a limited communication bandwidth between multiple vehicles.

II. EKF-BASED SAMPLING FOR PARAMETRIZED FIELDS

A number of fundamental issues arise in the coordination of N vehicles used to sample a field distribution. Figure 1 depicts an environmental sampling mission using underwater vehicles, in which the AUVs must achieve cooperative localization, form a network for communication of the
sensory data, and maximize the information acquired through planning and control of the vehicles paths and their sampling locations.

The approach proposed by Popa and Sanderson [Popa2004] focuses on the combination of uncertainty in localization as well as in the sensor measurements to achieve effective adaptive sampling. Localization uncertainties are especially relevant for underwater vehicles, since position estimates are often inaccurate due to navigational errors from dead-reckoning. We integrate model parameter estimation for the field variable with estimation of the uncertainty in the mobile sensor localization. Sampling missions using a Solar AUV are currently being planned using this approach [Bildberg 2000].

Assuming that the field variable distribution is stationary, the combined nonlinear vehicle dynamics and sensor model with Gaussian noise assumptions can be generally represented by:

- **Sensor node state dynamics:**
  \[
  x_{k+1} = x_k + h(x_k, a_k) + w_k, \]
  where \( x_k \) is the vehicle state, with noise covariance matrix \( E[w_k w_k^T] = Q_k \).

- **Sensor node position output:**
  \[
  y_k = f(x_k) + \xi_k, \]
  where the output noise covariance is \( E[\xi_k \xi_k^T] = R_k \).

- **Distributed field variable parameterized model:**
  \[
  z_k = g(x_k, a_k) + \nu_k, \]
  where \( z_k \) is the field variable with measurement noise covariance \( E[\nu_k \nu_k^T] = R_2 \), and \( a_k \) is a vector of unknown coefficients describing the field.

If the set of coefficients is unknown but constant, we add the vector \( a \) to the overall system state, with an evolution governed by \( a_{k+1} = a_k \). [Popa2004] describes the simultaneous sampling and navigation estimation problem formulated using the extended Kalman Filter (EKF), using an overall state vector \( X_k = (x_k, a_k) \).

**A. Closed-form estimation for a parameter-linear field without localization uncertainty**

If the parametric form of a measurement field is known, as might be the case, for example, with a bottom profile or systematic variations in temperature or salinity, the field estimation can be integrated with localization in order to improve the estimation process. The assumption that the field distribution is linear in its parameters allows us to compute a closed form solution for the information measure used by the sampling algorithm. If there is no uncertainty in the vehicle localization, the unknown field coefficient covariance can be calculated directly using a simple least square estimation. After \( n \) measurements taken at location \( X_i \), the field model depends linearly on the coefficients \( a_j \) via position-dependent functions, and we can directly estimate the unknown coefficients from the least-square solution:

\[
Z_1 = a_1 + a_1 g_1(X_1) + \ldots + a_n g_n(X_1),
\]

\[
Z_2 = a_1 + a_1 g_1(X_2) + \ldots + a_n g_n(X_2),
\]

\[
Z_n = a_1 + a_1 g_1(X_n) + \ldots + a_n g_n(X_n).
\]

\[
A_n = \begin{pmatrix}
1 & g_1(X_1) & \ldots & g_n(X_1) \\
\vdots & \vdots & \ddots & \vdots \\
1 & g_1(X_n) & \ldots & g_n(X_n)
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
\vdots \\
Z_n
\end{pmatrix}
\]

Furthermore, because the pseudo-inverse has a closed form \( M_n^+ = (M_n^T M_n)^{-1} M_n^T \), we obtain:

\[
\hat{A}_n = \sum_{i=1}^n \begin{pmatrix}
1 & \ldots & g_i(X_i) & \ldots & g_n(X_i)
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
\vdots \\
Z_n
\end{pmatrix}
\]
The covariance matrix of \( \hat{A}_k \) can now be related directly to the (constant) measurement uncertainty as:
\[
\text{var}(\hat{A}_k) = \text{var}(Z_k)(M_n^T M_n)^{-1},
\]
and the adaptive sampling algorithm will move the vehicle from location \( X_{m} \) to \( X_{m+1} \), such that the following p-norm is maximized over the search space \( \Theta \):
\[
m(X) = \left| M_n^T g_n(X) \right|.
\]
\[
m(X_{m+1}) \geq m(X), \forall X \in \Theta.
\]

B. Kalman filter estimation for a parameter-linear field without localization uncertainty

The state and output equations can be written as:
\[
\dot{X}_k = A_k X_k + B U_k + \xi_k, \quad Y_k = C X_k + \nu_k,
\]
where \( A_k, B, C \) are time-varying matrices, \( X_k \) is the state vector, \( Y_k \) is the observation, \( U_k \) is the input, \( \xi_k \) is the process noise, \( \nu_k \) is the observation noise.

C. Kalman filter estimation for a linear field with localization uncertainty

The state and observation equations can be written as:
\[
\dot{X}_k = A_k X_k + B U_k + \xi_k, \quad Y_k = C X_k + \nu_k,
\]
where \( A_k, B, C \) are time-varying matrices, \( X_k \) is the state vector, \( Y_k \) is the observation, \( U_k \) is the input, \( \xi_k \) is the process noise, \( \nu_k \) is the observation noise.

The state update equation can be written as:
\[
\hat{X}_k = \hat{X}_{k-1} + P_{k-1} R^{-1}(Z_k - G_k \hat{X}_{k-1}),
\]
where \( \hat{X}_k \) is the estimate of the state, \( P_k \) is the covariance matrix of the estimate, \( G_k \) is the gain matrix, \( R \) is the covariance of the observation noise.

The observation update equation can be written as:
\[
\hat{Y}_k = Y_k - C \hat{X}_k,
\]
where \( \hat{Y}_k \) is the estimate of the observation, \( Y_k \) is the observation, \( C \) is the observation matrix, \( \hat{X}_k \) is the estimate of the state.

The Kalman gain can be computed as:
\[
K_k = P_{k-1} R^{-1},
\]
where \( K_k \) is the Kalman gain, \( P_k \) is the covariance matrix of the estimate, \( R \) is the covariance of the observation noise.

The estimation residual can be computed as:
\[
\hat{\epsilon}_k = Y_k - C \hat{X}_k,
\]
where \( \hat{\epsilon}_k \) is the estimation residual, \( Y_k \) is the observation, \( C \) is the observation matrix, \( \hat{X}_k \) is the estimate of the state.

The covariance matrix of the estimation residual can be computed as:
\[
\text{var}(\hat{\epsilon}_k) = R + C P_{k-1} R^{-1} C^T,
\]
where \( \text{var}(\hat{\epsilon}_k) \) is the covariance matrix of the estimation residual, \( R \) is the covariance of the observation noise, \( C \) is the observation matrix, \( P_{k-1} \) is the covariance matrix of the estimate.

The nonlinear Kalman filter equations become:
\[
P_{k+1} = P_k + Q_k (X_{k+1} - A_{k+1} \hat{X}_k) = P_k + B U_k
\]
\[
P_{k+1} = (P_k + G_k^T R^{-1} G_k)^{-1} G_k [Y_{k+1} - G_k \hat{X}_k] + B U_k
\]
where \( P_k \) is the covariance matrix of the estimate, \( Q_k \) is the covariance of the process noise, \( R \) is the covariance of the observation noise, \( G_k \) is the gain matrix, \( Y_{k+1} \) is the observation, \( \hat{X}_k \) is the estimate of the state.

III. SIMULATION RESULTS

We have run sets of Monte Carlo simulation for the sampling problem in both 1D and 2D. The first set of simulations were obtained using a linear field distribution without localization uncertainty. The simulation conditions and the various measures used for comparison are shown in Table 1. Measurement samples were generated by using Gaussian random numbers with a set variance of 10% of the nominal field value. Different measures were utilized in order to select the sampling sequence, and the averaged results over 1000 field sample instantiations were compared to another using a common measure. This common performance measure is the same as the sampling criterion used in simulation set 5, namely the 2-norm of the error between the “true” field measurement value, and the predicted values at locations in space not yet sampled. For simulation sets 1 and 2 we used the closed form variance formula and the SVD of the covariance matrix \( \hat{A}_k \), and evaluated the infinity and the 2-norm of the singular values. For simulation sets 3 and 4 we used the same next point sampling criterion as the common performance measure, and for sample 5 we used the default linear (for 1D) or “lawn mower” (for 2D) sampling criteria.

<table>
<thead>
<tr>
<th>Set</th>
<th>Iterations</th>
<th>Next point sampling criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>Closed form variance, infinity norm</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>Closed form variance, 2-norm</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>2-norm of prediction error</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>Infinity norm of prediction error</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>Default 1D step or 2D “lawn mower”</td>
</tr>
</tbody>
</table>

Table 1: Summary of Monte Carlo simulation results, using various sampling measures.

Figures 2 (a) – (c) compare the results of simulation sets 2, 4, and 5 for the 1D case. While the resulting sampling sequences are different for each sampling set, the value of the estimation residuals are similar in all cases, and much smaller than the estimation residuals obtained in the default sampling sequence 5.
The second set of simulations uses a 2D non-linear field (though still linear in the parameters), consisting of a sum of two Gaussian distributions:

\[ G(x, y) = a_0 + a_1 g_1(x, y) + a_2 g_2(x, y), \]

where

\[ g_1(x, y) = e^{-\frac{(x-x_1)^2 + (y-y_1)^2}{2\sigma^2}} \]

\[ g_2(x, y) = e^{-\frac{(x-x_2)^2 + (y-y_2)^2}{2\sigma^2}} \]

The centers of Gaussians for \( g_1(x, y) \) and \( g_2(x, y) \) for either field are \( (30,30), (65,45) \) at \( \sigma = 10 \), and the sampling sequences generated are shown in Figure 3 (a)-(d).
The third set of simulations was performed for the case of simultaneous vehicle localization and linear parameter field estimation. In the 1D case, we considered data sets generated under Gaussian noise assumptions, using nominal coefficient values $a_0 = 2, a_1 = 0.5$ (intercept and slope), and measurement noise covariance of 0.5, state measurement noise covariance of 0.1, and state transition noise covariance of 0.1. The convergence of the coefficient estimates to their nominal values is shown in Figures 4 (a)-(d) for two different sampling sequences. For figures 4(a) and (c), the sampling sequence is the default one (1,2,…), while figures 4(b) and (d) were obtained by minimizing the error covariance, e.g. the adaptive sampling algorithm moves the vehicle from location $X_n$ to $X_{n+1}$, such that the following infinity norm is maximized over the search space $\Theta$:

$$m(X) = \| P_m(X_m(\cdot), m(X_m(\cdot))) \|_{\infty}, \forall X \in \Theta.$$  

Note that the slope estimate of the second sampling sequence convergences faster to its nominal value. The optimal sampling sequence is similar to the one in Figure 2(a) because the localization error is still smaller than the measurement error.

![Image](attachment:image_url)

**Fig 4:** (a-d): The convergence of slope and intercept in a combined EKF field estimation and localization problem for the default (a,c) and optimal (b,d) 1D sampling sequence.

### IV. CONCLUSION AND FUTURE WORK

The algorithms described in this paper provide an initial demonstration of a strategy for systematic exploration of environmental sensing methods based on information measures. Mobile sensors are directed to sample at locations that most reduce the uncertainty in our knowledge of the field distribution. The goal is to develop generalized methods for deployment of mobile sensing nodes taking advantage of broad process and flow models of regions and environments.

Future work includes expanding the simulation and experimental work to include non-linear and time-varying field distributions, proper kinematic and dynamic vehicle models, multiple vehicles, and secondary optimization objectives such as energy resources and communication bandwidth.

The research described here is coordinated with larger efforts in environmental sensing and monitoring related to rivers, estuaries, and coastal areas [Popa2004]. Furthermore, a fleet of 20 inexpensive (below $1,000 per unit) mobile
sensor network nodes is currently being built at ARRI’s WSN lab (Figure 5), as part of a larger testbed composed of larger CyberGuard SR2/ESP robots and wireless Xbow motes (Figure 6). The inexpensive robot units are equipped with wheel encoders for localization, and share this data through a mote that lacks the antenna. The absence of the antenna makes it possible for two robot units to go out of range if the distance between them is greater than 1ft. The rovers are controlled by a Javelin Stamp CPU and are also equipped with a color sensor that can measure 256 RGB values on the lab floor. By using the color sensor and the limited range Xbow motes, we can test the proposed adaptive sampling algorithms, as well as other algorithms maximizing the communication bandwidth in the wireless network.

Fig 5: Inexpensive Rover Unit for color-based indoor adaptive sampling algorithm validation testbed.

Fig 6: CyberGuard SR2/ESP, inexpensive Rovers, and MICA Xbow motes on the lab floor at ARRI’s WSN Lab.

ACKNOWLEDGEMENTS
This work has been conducted with support in part from the Automation and Robotics Research Institute at UTA and in part by grant IIS-0329837 from the National Science Foundation. We wish to thank Prof. Arthur Sanderson at Rensselaer Polytechnic Institute for his helpful insight.

REFERENCES