ENHANCING MARKETING WITH ENGINEERING: OPTIMAL PRODUCT LINE DESIGN FOR HETEROGENEOUS MARKETS

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Conditionally accepted at *International Journal of Research in Marketing*. Please do not quote or distribute without prior consent.

The authors wish to thank Shapour Azarm, Abbie Griffin, Aida Khajavirad, Jin Gyo Kim, Peter Lenk, Laura Stojan, Michel Wedel, and Carolyn Yoon for their help and comments, and to acknowledge support from the National Science Foundation CAREER Award #0747911 and the University of Michigan's Rackham Antilium interdisciplinary project, Reconfigurable Manufacturing Systems Engineering Research Center, and Ross School of Business.

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ABSTRACT

Successful product line design and development often requires balancing technical and market tradeoffs. Quantitative methods for optimizing product attribute levels using preference elicitation (e.g., conjoint) data are useful for many product types. However, products with substantial engineering content involve critical tradeoffs in the ability to *achieve* those desired attribute levels. Technical tradeoffs in a product's design must be made with an eye toward market consequences, particularly when heterogeneous market preferences make differentiation and strategic positioning critical to capturing a range of market segments and avoiding cannibalization.

We present a unified methodology for product line optimization that coordinates positioning and design models to achieve realizable firm-level optima. The approach overcomes several shortcomings of prior product line optimization models by incorporating a general Bayesian account of consumer preference heterogeneity, managing attributes over a continuous domain to alleviate issues of combinatorial complexity, and avoiding solutions that are impossible to realize. The method is demonstrated for a line of dial-readout scales, using physical models and conjoint-based consumer choice data. Results show that the optimal number of products in the line is not necessarily equal to the number of market segments; that an optimal single product for a heterogeneous market differs from that for a homogeneous one; and that the representational form for consumer heterogeneity has a substantial impact on the design and profitability of the resulting optimal product line – even for the design of a single product. The method is managerially valuable, as it yields product line solutions efficiently, accounting for marketing-based preference heterogeneity as well as engineering-based constraints on which product attributes can be realized.

KEYWORDS: *Product Line Design; Heterogeneity; Decomposition; Analytical Target Cascading; Hierarchical Bayes; Conjoint Analysis; Discrete Choice Analysis; Design Optimization.*

1. Introduction

Marketplace globalization, the proliferation of niche markets driven by heterogeneity of preferences, increased competitive pressures, and demand for differentiated products have rendered isolated design and production of individual products essentially obsolete in many instances. Across industries, standard practice involves *lines* of product variants that reduce cost via economies of scale and scope, reaching multiple market segments and deterring competitors (Urban & Hauser 1993, Simpson 2004). Product line planning requires care, as each product vies not only with competitors, but also with other products in the same line.

The scope and applicability of current methods in product line optimization have known limitations, both in Engineering and in Management Science. Engineering-based approaches focus on the tradeoff between increased commonality among products and the resulting decreased ability to meet (usually hypothetical and exogenous) performance targets for each product variant. Most of these approaches lack data-driven models of market preferences and consequently focus on reducing cost by increasing part commonality, designing platforms, or increasing modularity for mass customization. In contrast, product line optimization methods in the management science and marketing literatures rarely address product design details not directly observable by consumers. These approaches typically presume that any combination of product attributes in a conjoint study can somehow be attained by engineering designers *post hoc*. While this may be so for many simple or well-established products, it is questionable for those with even moderately complex engineering tradeoffs where value to consumers cannot be balanced against cost and constraints without joint consideration of marketing and engineering factors. Furthermore, existing approaches have not taken full advantage of recent advances in econometric modeling of consumer preference heterogeneity, in particular hierarchical Bayesian methods, and often require exogenous individual-level or homogeneous segment-level preference data (see Kaul & Rao 1995 for a review of the early literature).

In this article, we develop a novel, general method for designing lines of products for markets with heterogeneous preferences when technical complexity restricts the attainable space of product attributes. This method is modular and scalable, able to handle various sorts of product attributes, and is agnostic about optimization methods selected to solve the marketing and engineering subproblems.

The method makes powerful use of formal decomposition and coordination methods via *analytical target cascading* to take advantage of the structure underlying the complete product line design problem to improve computational efficiency, algorithmic stability, and model organization and coordination. We proceed by reviewing relevant literature on product line optimization in the marketing and engineering domains, suggesting how the proposed approach fills a number of extant gaps. Because the scopes, perspectives, modeling methods, and objectives differ substantially among product development disciplines (Krishnan & Ulrich 2001 provide a detailed overview), it is inevitable that some conflicts of terminology will exist: In this article, we take product *positioning* to be the process of choosing values for physical (as opposed to perceptual) product attributes observed directly by the consumer, whereas product *design* involves decisions about the product that are not observed directly, but that nevertheless influence product attributes observed by the customer.

Let us first consider a concrete example, fleshed out in detail in our application. Figure 1 depicts a dial-readout bathroom scale with the cover removed. In the marketplace, consumers observe only external product attributes such as readability (number size), and weight capacity. For product *positioning*, we need not be concerned with how these attributes are realized, but only how consumers respond to them. However, in order to specify product attribute profiles that are realizable, it is necessary to account for engineering tradeoffs in the product *design*. For example, the dial in Figure 1 must be small enough to fit within the scale housing, so there is a limit to how large numbers on the dial can be. For a given housing, increasing the size of numbers on the dial requires reducing the weight capacity, while increasing weight capacity requires smaller numbers. Increasing both these consumer preferred attributes, number size and weight capacity, cannot therefore be achieved simultaneously.

[INSERT FIGURE 1 HERE]

1.1. Product line optimization literature

Among the earliest conceptualizations for product line optimization in the marketing/management literature was that of Green and Krieger (1985), who posed the product line *selection* problem as a binary programming problem involving selection of products from a candidate set to be included in the line in order to maximize the seller's (or buyers') welfare. Here the set of candidate products with their associated utility values is determined exogenously, and product demand is predicted using a *first choice* model, where each individual is assumed to choose deterministically the alternative with the highest associated utility. Variants of the original model were later proposed by Dobson & Kalish (1988) and by McBride & Zufryden (1988), who offer alternative integer programming techniques and heuristics for solving the problem. Dobson & Kalish (1993) also introduce fixed and variable costs for each candidate product.

While these initial methods assumed each product's utility had been determined exogenously, Kohli & Sukumar (1990) instead used conjoint part-worths, and introduced a single stage binary programming formulation to select product lines based on their attribute levels. Chen & Hausman (2000) made use of *choice-based* conjoint analysis, arguably most similar to the choice task consumers perform in practice and often claimed to be the best method for extracting individual-level consumer preferences (Green & Krieger 1996). Chen & Hausman proposed a binary programming formulation solvable by nonlinear programming techniques. Because their approach requires homogeneous preferences, it cannot be used to design product lines meeting the disparate needs of most real consumer populations. Among the more recent contributions are applications of genetic algorithms (Steiner & Hruschka 2002, 2003) and particle swarm algorithms (Tsafarakis et al. 2010) to locate a population of near-optimal product line designs. Other approaches have also been proposed to model products qualitatively in terms of abstract "quality levels", although these are primarily used to analyze structural properties, rather than offer computational decision support tools (Krishnan & Zhu 2006 provide a recent review).

The bulk of the engineering literature on product line design focuses on product families and platforms and is generally designed around stochastic methods or gradient-based constrained nonlinear programming techniques to handle continuous formulations. The focus on continuous variables increases applicability for practical engineering problems, avoids combinatorial complexity found in many positioning approaches, and manages complex relationships among attainable combinations of product attributes that cannot be easily handled with attribute discretization. Most models focus on the tradeoff between increased commonality among products in a line and the resulting decreases in the ability to meet distinct performance targets set exogeneously for each product variant (see Simpson 2004, Simpson, Siddique & Jiao 2005 for a review).

One difficulty with integrating models from various product development disciplines is that the combined model can be large and complex, causing optimization difficulties. Recent efforts have linked engineering optimization models to market demand models for single products (e.g.: Gu et al. 2002, Wassenaar & Chen 2003, Li & Azarm 2000, Michalek, Feinberg & Papalambros 2005). Gu et al. (2002) proposed a method for maintaining separate models for marketing and engineering decisions, coordinating them using the collaborative optimization (CO) technique for multidisciplinary design optimization, although they do not propose details for modeling and data collection for the marketing component. Michalek et al. (2005) proposed a similar decomposition approach using analytical target cascading (ATC) to coordinate marketing and engineering models for a single product, assuming consumer preferences to be homogeneous. They point out a preference for the ATC approach over CO because ATC is defined for an arbitrary hierarchy of subsystems, and convergence proofs ensure coordination will lead to a solution that is optimal for the firm (Li, Lu & Michalek 2008). Shiau and Michalek (2009a,b) identify equilibria for single-product firms in Nash competition on price and product design decisions under mixed logit demand, finding that engineering design decisions are separable from strategic positioning and retail distribution channels only when consumer preference parameters are heterogeneous.

Recent studies have examined aspects of the product line design problem and its attendant optimization issues, although without fully addressing modeling and estimation of preference heterogeneity for horizontally differentiated products (i.e., those that differ in attributes valued by various consumer groups) and engineering constraints that impose tradeoffs in the ability to achieve desirable combinations of product attributes (e.g. Kokkolaras et al. 2002, Li & Azarm 2002, Belloni et al. 2008, Wang et al. 2009, Kim & Chhajed 2002, Chhajed & Kim 2004, Fruchter et al. 2006). One exception is the work of Luo (2010), who considers joint engineering and marketing solutions with consumer heterogeneity using stochastic algorithms, and who additionally addresses "robustness" (whether products in the line break down under known usage situations). Our approach addresses a continuous design space, offering improved computational efficiency and stability, a structure for organizing and coordinating models from different disciplines, and provable convergence to local minima[1]. The proposed ATC approach is preferred over prior methods when (1) the attribute space is significantly constrained by physical restrictions on the engineering design (e.g.: weight capacity is limited by number size), (2) when the design decisions and product attributes exist primarily along a continuum (e.g. number size or weight capacity), and/or (3) when modelers will develop engineering and market models separately and benefit from well-defined interfaces for coordination and modularity to support model additions.

Among the main points of the present paper is simply that, if a firm is going to design multiple products as part of a line, it needs to understand not only which market niches they will serve, but also whether they can be realized. That is, despite the advances of recent literature, a number of key problems remain. Specifically, current approaches to the product line problem: lack coordination with engineering in terms of product feasibility; do not easily accommodate a sophisticated account of preference heterogeneity; entail substantial computational problems; and require changes from the ground up to deal with new structures and phenomena. Our proposed methodology resolves each of these issues, as we discuss in the following sections.

1.2 Proposed Methodology

In this article, we propose a comprehensive methodology for product line design, using ATC to coordinate attribute selection for each of the products desired by a heterogeneous market, while ensuring they can each be realized by a feasible engineered design. The proposed approach avoids the combinatorial complexity of binary/integer formulations in the marketing literature while extending applicability to continuous formulations and avoiding the need to assume monotonic preferences. Importantly, our approach allows for a general representation of consumer tastes through the use of Bayesian mixture models. The decomposition-based ATC approach further offers the organizational and computational benefits of maintaining separate subsystems for positioning and design of each product in the line, reducing the dimensionality of each subspace and allowing each subsystem to be efficiently solved in parallel.

We adopt a random utility framework (Train 2003) for estimating market demand for the product line, where the utility of each product to each consumer depends on the product's attributes, the

[1] Luo (2010) tests the "all at once" solution approach against ATC, finding the latter to offer computational efficiencies.

consumer's idiosyncratic preferences for those attributes, and a random error component. A random utility framework allows for uncertainties owing to factors unobservable to the analyst and it avoids the discontinuities intrinsic to a deterministic framework (e.g., Dobson & Kalish 1988, 1993, Green & Krieger 1985, Kohli & Sukumar 1990, Li & Azarm 2002, McBride & Zufryden 1988) , enabling the use of efficient gradient-based nonlinear programming optimization tools. Importantly, a random utility framework allows for explicit modeling of consumer taste distributions, or heterogeneity. As our results illustrate, the various representations available to model taste differences can have a substantial, and substantive, impact on the final optimal product line and its profitability; this is especially so if one chooses an overly parsimonious heterogeneity representation. The impact of preference heterogeneity on line configuration has not received much attention in the product line literature. Even though models with continuous and discrete heterogeneity representations can predict choices about equally well (Andrews *et al*. 2002), the resulting optimized product lines from different heterogeneity representations may be very different[2].

The proposed product line design methodology entails four stages: First, consumers choose among products in a conjoint setting; second, heterogeneous preference coefficients in the model are estimated; third, demand models are formulated by interpolating preference coefficients using splines; and fourth, ATC coordinates optimization over the space of feasible product designs to yield optimal product attributes. The first three stages are viewed as preprocessing for the ATC model, as shown schematically in Figure 2, with symbols defined later in the text (see also appendix 1). In the context of the illustrative example in Figure 1, these three preprocessing stages concern the product positioning, i.e., determining consumers' preferences for weight capacity and number size. Then, given these preferences, ATC coordinates product positioning and design so that the dial (whose size is limited by weight capacity and number size) is small enough to fit inside the scale housing, until firm level goals (typically, concerning profitability) are achieved. We proceed by defining the ATC methodology in Section 2, conditional on a model to predict demand; next we describe alternative discrete choice model specifications for demand prediction in Section 3; and finally, we demonstrate

[2] Like Chen & Hausman (2000), we invoke a number of assumptions to tightly focus on product line optimization issues: (1) total market size is exogenously determined; (2) each customer purchases zero or one product; (3) customers do not directly influence one another; and (4) production can be scaled up or down to suit demand. As such, our formulation is well-suited to stable, differentiated durables and is less appropriate for commodities or rapidly-developing product classes.

the methodology with an application to dial-readout scales using models and data from the literature (Michalek et al. 2005) in Section 4, and discuss results and their marketing implications in Section 5.

[INSERT FIGURE 2 HERE]

2. ATC Coordination of Product Positioning and Design

ATC requires a mathematical or computational model of each subsystem, and in practice these can be numerous. The modeler's task is to organize the various subsystem models into a *hierarchy*, where each element in the hierarchy represents a (sub)system that is optimized to match targets passed from the parent (super)system while setting targets that are attainable by subsystem child elements (Kim et al. 2003). In our application, the joint product line positioning and design problem is decomposed into two types of interrelated subsystems, or subproblems: (1) a product positioning subproblem that sets product attribute targets for each product in the line, and (2) a set of product design subproblems, one for each product in the line, that each aims to achieve its targets subject to constraints. It has been shown that iteratively solving ATC subproblems under specific coordination strategies will converge to the solution of the joint problem under typical conditions (Li, Lu & Michalek 2008). In the present case, market positioning and the engineering design of each product in the line can be solved separately and in parallel, producing a solution that is optimal for the joint problem. In practice, the joint problem can be far more difficult to formulate and solve, sometimes impractically so, owing to high dimensionality, scaling difficulties and the need for modeler expertise in all core disciplines.

In general, ATC can accommodate an arbitrarily large hierarchy, where parent elements set targets for child elements. For example, the methodology has been demonstrated for large hierarchical systems such as vehicle design (Kim, Rideout, Papalambros & Stein 2003) and architectural design (Choudhary, Malkawi & Papalambros 2005). In the product line case, each design in the line could be decomposed into a set of subsystems and components, or additional marketing models could be included, say, for promotion and distribution. We focus on the case of a single marketing model for product positioning and a set of engineering models for design of each product. A schematic depiction of the process appears in Figure 3 (all symbols are introduced and discussed in detail below): The positioning subsystem involves determining price and (target) product attributes for the full product line to maximize a known objective function, which can be profit or some other measure of interest to the firm, while each design subsystem requires determining a feasible design – one conforming to known constraints – that exhibits product attributes as close as possible to the targets set in the positioning subsystem. Decomposition into the ATC structure can be even more important in the product line case than in the single product case because including engineering models for the design of multiple products in a single optimization statement creates a high-dimensional, highly constrained space; by contrast, with ATC decomposition of the line, the space of each individual product design remains unchanged as new products are added to the line. Another chief organizational benefit of ATC is that it segregates models by discipline: Marketers can build positioning models based on, say, conjoint analysis and new product demand forecasting; engineers can formulate models for product design and production; and other functional groups can focus on what they know how to do well. No functional area need become an expert in modeling the others, since ATC coordinates models with well-defined interfaces. The following sections lay out the design and positioning subsystems, as illustrated in Figure 3, explicitly. A summary of the various symbols used appears in Appendix 1.

[INSERT FIGURE 3 HERE]

2.1. Market Positioning Subsystem

The market positioning objective is to maximize profit Π with respect to the price p_j and the vector of product attribute targets $\mathbf{z}^{M}_{j}$ for each product j in the product line $j = \{1,2,...,J\}$. Although firms can specify arbitrarily sophisticated profit functions based on their experience, internal accounting and historical demand, we use a simple profit (Π) formulation here – revenue minus cost – so that

$$\Pi = \sum_{j=1}^{J}\left(\left(p_j - c_j^{\mathrm{V}}\right)q_j - c_j^{\mathrm{I}}\right),$$

where p_j is the (retail) price of product j, c^{V}_{j} is the unit variable cost of product j, c^{I}_{j} is the investment cost for product j, which represents all costs of setting up a manufacturing line for product j, and q_j is quantity of product j sold (demand), which is a function of the product attributes $\mathbf{z}^{M}_{k}$ and price p_k of all products $k \in \{1,2,...,J\}$. We presume that product commonalities enabling investment cost sharing and improving economies of scale do not exist, so each new product design requires new manufacturing investment, though this can readily be relaxed, given appropriate cost-specific information. In general, c^{V}_{j} and c^{I}_{j} can be considered functions of market conditions or engineering decisions, although in the example they are taken as constants. Alternative models for calculating the quantity of product j sold (q_j) for each product j will be developed in the Section 3. To account for the need to match product attribute targets $\mathbf{z}^{\mathrm{M}}_j$ with the attributes of realizable engineering designs $\mathbf{z}^{\mathrm{E}}_j$, the consistency condition $\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j = \mathbf{0}$ is relaxed and moved into the objective function using a consistency constraint relaxation $\pi(\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j)$. This relaxation can be handled in a variety of ways (see Li et al. 2008 for a review of approaches and mathematical properties), and we adopt the augmented Lagrangian approach with diagonal quadratic approximation, described below. Finally, the positioning subsystem for a single-producer scenario, conditional on a model for demand, is written as:

$$\begin{aligned} &\underset{p_j, \mathbf{z}_j^{\mathrm{M}} \forall j \in \{1,\ldots,J\}}{\text{maximize}} \sum_{j=1}^{J} \left(\left(p_j - c_j^{\mathrm{V}} \right) q_j - c_j^{\mathrm{I}} - \pi\left(\mathbf{z}_j^{\mathrm{M}} - \mathbf{z}_j^{\mathrm{E}} \right) \right) \\ &\text{where } q_j = SP_j \left(p_k, \mathbf{z}_k^{\mathrm{M}} \ \forall k \in \{1, 2, \ldots, J\} \right) \end{aligned}$$

where S is the (exogeneous) market size and P_j is the share of choices for product j. In Section 3, we address how conjoint analysis, discrete choice modeling and Bayesian (MCMC) methods can be used to represent the functional relationship between P_j and the variables $\mathbf{z}^{\mathrm{M}}$ and p for positioning a product line. The market positioning subsystem is summarized in the top part of Figure 3.

2.2. Engineering Design Subsystems

Conceptually, the objective of each engineering subsystem involves finding a feasible design that exhibits product attributes matching the targets set by the market positioning subproblem as closely as possible. This is schematically depicted in the bottom part of Figure 3. Here the vector of product attributes $\mathbf{z}^{\mathrm{E}}_j$ for product j represents a set of objective, measurable aspects of the product, observable by the customer, resulting from engineering design decisions. In each engineering design subsystem j, search is conducted with respect to a vector of design variables $\mathbf{x}_j$, which represents decisions made by the designer that are not directly observable by consumers but that affect the attributes that consumers do observe, i.e. $\mathbf{z}^{\mathrm{E}}_j$. An engineering analysis simulation response function $\mathbf{r}(\mathbf{x}_j)$ is used to calculate attributes $\mathbf{z}^{\mathrm{E}}_j$ as a function of $\mathbf{x}_j$. The design variable vector $\mathbf{x}_j$ is restricted to feasible values by a set of constraint functions $\mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}$ and $\mathbf{h}(\mathbf{x}_j) = \mathbf{0}$, and so values for product attributes $\mathbf{z}^{\mathrm{E}}_j = \mathbf{r}(\mathbf{x}_j)$ are implicitly restricted to values that can be achieved by a feasible design. While construction of $\mathbf{x}$, $\mathbf{r}(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ to represent a particular product is necessarily case-specific, general principals and guidelines are well established in the literature (Papalambros & Wilde 2000, Ravindran & Reklaitis 2006). Each engineering design subsystem minimizes the consistency constraint relaxation function, which works to minimize deviation between the positioning targets $\mathbf{z}^{\mathrm{M}}_j$ set by marketing, which are held constant in each engineering design subsystem, and the attributes achieved by engineering $\mathbf{z}^{\mathrm{E}}_j$. The engineering optimization problem for product j can then be written as

$$\underset{\mathbf{x}_j}{\text{minimize}} \;\; \pi\left(\mathbf{z}_j^{\mathrm{M}} - \mathbf{z}_j^{\mathrm{E}}\right)$$
$$\text{subject to } \mathbf{g}\left(\mathbf{x}_j\right) \leq \mathbf{0}, \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0} \,,$$
$$\text{where } \mathbf{z}_j^{\mathrm{E}} = \mathbf{r}\left(\mathbf{x}_j\right)$$

2.3. Complete ATC Formulation

Figure 3 summarizes the mathematical description of the complete formulation, showing the flow of the ATC-based product line optimization model for a single producer, where the number of products in the line J is determined through a parametric study: i.e., J is held fixed during optimization, separate optimization solutions are found for each value of $J \in \{1,2,...\}$, and the value of J that produces the solution with the highest profit is selected. In this way, we determine not only the optimal designs of products in a given line, but the optimal line *size* as well.

As stated above, coordination of the subsystems can be handled through a variety of approaches for relaxing the consistency constraint $\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j = \mathbf{0}$. Here we use the augmented Lagrangian approach with diagonal quadratic approximation for improved stability and computational efficiency, where $\pi(\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j) = \boldsymbol{\lambda}^{\mathrm{T}}(\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j) + \|\mathbf{w} \bullet (\mathbf{z}^{\mathrm{M}}_j - \mathbf{z}^{\mathrm{E}}_j)\|_2^2$, $\boldsymbol{\lambda}$ is the Lagrange multiplier vector, $\mathbf{w}$ is a weighting coefficient vector, $\bullet$ is the Hadamard product (i.e.: $(\mathbf{A}\bullet\mathbf{B})_i = \mathbf{A}_i\mathbf{B}_i$), and the quadratic term is linearized at each iteration to enable separability of the subsystems for parallel processing (Li, Lu & Michalek 2008). The coordination procedure is:

1. Initialize all variables
2. Solve the market positioning subproblem and each engineering design subproblem in parallel using an NLP solver

3. Update $\boldsymbol{\lambda}$ using the method of multipliers $\boldsymbol{\lambda}^{(\kappa+1)} = \boldsymbol{\lambda}^{(\kappa)} + 2\mathbf{w}^{(\kappa)}\bullet\mathbf{w}^{(\kappa)}\bullet(\mathbf{z}^{\mathrm{M}}{}_{j}^{(\kappa)} - \mathbf{z}^{\mathrm{E}}{}_{j}^{(\kappa)})$ and update $\mathbf{w}$ using $\mathbf{w}^{(\kappa+1)} = \gamma\mathbf{w}^{(\kappa)}$ with $\gamma > 1$ as needed to provide stability and reduce duality gaps, where (κ) represents the iteration number.
4. If converged, stop, else return to step 2

3. Models of Product Demand

Green and Krieger's comparative study of alternative conjoint methods for eliciting consumer preferences concluded that choice-based conjoint offers the best method for the extraction of individual-level consumer preferences (Green & Krieger 1996). We use it as follows: Respondents are presented with a series of questions or "choice sets" $t = \{1,2,...,T\}$. In each choice set t, the respondent is presented a set of product alternatives $j \in \mathcal{J}_t$, with attributes set at discrete levels and systematically varied. The resulting data are each respondent's observed choices in each choice set: Φ_{ijt}, where Φ_{ijt} = 1 if respondent i chooses alternative j in choice set t, and $\Phi_{ijt} = 0$ otherwise. These data $\{\Phi_{ijt}\}$ are then used to estimate the parameters of the choice model for the positioning subsystem, as illustrated in Figure 2.

In the random utility choice model, individuals $i = \{1, 2, ..., I\}$ derive from each product $j = \{1, 2, ..., J\}$ some utility value u_{ij} that is composed of an observable, deterministic component v_{ij} and an unobservable random error component ε_{ij}, so that $u_{ij} = v_{ij} + \varepsilon_{ij}$. Each individual will choose the alternative that gives rise to the highest utility (i.e., alternative j is chosen by individual i if $u_{ij} > u_{ik}$ for all $k \neq j$). The deterministic utility v_{ij} derived by individual i from product j is written as

$$v_{ij} = \sum\nolimits_{\zeta=0}^{\mathrm{Z}} \sum\nolimits_{\omega=1}^{\Omega_\zeta} \beta_{i\zeta\omega}\delta_{j\zeta\omega} \ ,$$

where the binary dummy $\delta_{j\zeta\omega} = 1$ indicates alternative j possesses attribute ζ at level ω, and $\beta_{i\zeta\omega}$ is the part-worth coefficient of attribute ζ at level ω for individual i, which is estimated from the conjoint choice data Φ. The model thus accords with the typical main-effects conjoint set-up dominant in the literature, although interaction effects may also be included as needed. In $\delta_{j\zeta\omega}$ the elements of the product attribute vector $\mathbf{z}^{\mathrm{M}}{}_{j}$ are enumerated $\zeta = \{1, 2, ... \mathrm{Z}\}$, and price p is included in $\delta_{j\zeta\omega}$ and labeled as element $\zeta = 0$. Each product attribute ζ is either intrinsically discrete or is discretized into Ω_ζ levels,

$\omega = \{1, 2, \ldots \Omega_\zeta\}$; thus it does not presume linearity with respect to the underlying continuous variables. For an alternative approach to discrete attributes, see Luo (2010)[3].

The probability P_{ij} that alternative j is chosen by individual i depends on the assumed error distribution. The most common distributions for ε_{ij} are the normal and double exponential, resulting in the standard probit and logit models, respectively (Train 2003)[4]. We index the "no choice option" (the outside good) as alternative 0, with error ε_{i0} and observable utility v_{i0} for individual i, where $v_{i0} = 0$; $\forall i$ for identification. The inclusion of an outside good allows overall demand contraction when the set of products offered fails to match the market's preferences well.

The representation of differences in consumer tastes, as given by $\boldsymbol{\beta}_i$, where $\boldsymbol{\beta}_i$ contains the elements $\beta_{i\zeta\omega}$, can be expected to be important in product line optimization, as heterogeneity in preferences should give rise to differentiated product offerings. Failure to correctly model this heterogeneity can lead to biased parameter estimates, inaccurate predictions (Rossi, Allenby & McCulloch 2005, Andrews, Ansari & Currim 2002, Otter et al. 2004) and, consequently, suboptimal product line designs. Furthermore, when heterogeneity is not adequately accounted for it is well-known that the independence from irrelevant alternatives (IIA) problem is exacerbated (Train 2003). We therefore specify a very general continuous distributional form for $\boldsymbol{\beta}_i$ by using a mixture of normal distributions (Lenk & DeSarbo 2000). The approach assumes that there are a finite number of groups or segments, in which individuals are similar – though, importantly, not identical – with respect to their preferences and tastes. To be more specific, we have

$$\boldsymbol{\beta}_i \sim \sum_{b=1}^{B} s_b N\left(\boldsymbol{\theta}_b, \boldsymbol{\Lambda}_b\right),$$

where s_b is the fraction of the market in "segment" (or mixing component) $b \in \{1,\ldots,B\}$. Here $\boldsymbol{\theta}_b$ is the vector of means and $\boldsymbol{\Lambda}_b$ is a full variance-covariance matrix. This model provides a flexible

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[3] For quantitative attributes that exist in a continuous space, it is appropriate to measure preference at discrete points and optimize over a continuous space using interpolation. Moreover, when constraints restrict feasible combinations of attributes, it is sometimes the case that none of the combinations of discretized attributes are feasible (this is true in the case study presented in this paper). Interpolation is critical in these situations. For categorical attributes (e.g., brand name), decomposition can be used in conjunction with global MINLP search algorithms (Khajavirad & Michalek 2009), or stochastic approaches can be used (Luo 2010).

[4] Each error distribution confers distinct advantages for the problem at hand: normal errors (probit) offer conjugacy in Bayesian MCMC estimation, allowing all Gibbs draws and avoiding Metropolis steps, but require numerical integration in the product line optimization phase; Gumbel errors (logit) require Metropolis steps in Bayesian estimation, but entail closed-form expressions for gradients used in optimization. Amemiya (1985) shows that very large samples are required to distinguish results produced by the Gumbel and normal error specifications, and suggests a way to translate between coefficient estimates to take advantage of the former's optimization advantages and the latter's estimation efficiencies.

specification that combines both discrete and continuous heterogeneity and includes several well known heterogeneity models as special cases: (i) when B=1 the well-known standard random-effects model arises, which, in combination with Bayesian estimation, enables individual-level estimates by pooling information among individuals via "shrinkage" (Rossi, Allenby & McCulloch 2005); (ii) when $\mathbf{\Lambda}_b = \mathbf{0}$ for all $b \in \{1,...,B\}$ the standard latent class or finite mixture model arises (Kamakura & Russell 1989), and individuals within a segment b are assumed to have identical preferences $\mathbf{\theta}_b$; and (iii) when $\mathbf{\Lambda}_b = \mathbf{0}$ and B = 1 it is assumed that all individuals have the same preference parameters $\mathbf{\theta}_1$. The last, homogeneous case (iii) is overly restrictive for markets with heterogeneous preferences, and demand models that assume homogenous tastes can be expected to perform poorly in terms individual specific part-worth recovery and market predictions. Andrews *et al*. (2002) suggest that models with continuous (case i) and discrete (case ii) representations of heterogeneity recover parameter estimates and predict choices about equally well, except when the number of choices J is small, in which case discrete heterogeneity (ii) outperforms the continuous model (i). We examine whether the optimized product lines *conditional* on each of these models produce similar results.

For the general case, model parameters are estimated via standard Markov chain Monte Carlo (MCMC) techniques (Gelman et al. 2003, Rossi, Allenby & McCulloch 2005)[5]. We generally specify conjugate priors, and the full conditional distributions for the MCMC sampler can be derived straightforwardly (e.g. Lenk & DeSarbo 2000). In order to choose the number of mixture components B in the mixture representation for $\mathbf{\beta}_i$, we use the Deviance Information Criterion (DIC) statistic proposed by Spiegelhalter et al. (2002). DIC is particularly suited to complex hierarchical (Bayesian) models, because the DIC statistic determines the "effective number of parameters" entailed by the model specification itself, unlike measures such as AIC that are not suitable in a Bayesian setting.

Once the model parameters are estimated, we compute market demand for the positioning subproblem (Figure 3) in three steps: First, we generate a large set of $\boldsymbol{\beta}_i$ (say $i = 1,...,I_D$) from the hierarchical model $\{s_b, \mathbf{\theta}_b, \mathbf{\Lambda}_b\}$, which describes the mixture distribution.[6] Second, we use *natural*

[5] We are indebted to Peter Lenk for sharing both his GAUSS code and expertise.

[6] Estimating the model provides a set of draws from the posterior distribution of $\beta_{i\zeta\omega}$ for each survey respondent. One could then estimate market demand using this specific set of individuals. We take a Bayesian perspective and use the hyperparameters describing the mixture distribution (after the MCMC chain has converged), as these can be viewed as 'giving rise' to the individual-level β_i values. Specifically, an arbitrarily large sample of *new* β_i values from this distribution can be drawn to describe the market.

cubic splines (Boor 2001) to flexibly interpolate $\boldsymbol{\beta}_i$ for intermediate values of product attributes and price. Because all the levels in our conjoint application are made explicit, are modest in number, and do not vary across respondents, there is no need to resort to complex methods designed to deal with *latent* knot configurations, although such methods are available in the Bayesian choice modeling literature (Kim, Menzefricke, and Feinberg 2007). Specifically, natural cubic spline functions $\Psi_{i\zeta}$ are fit through the discrete part-worth coefficients $\beta_{i\zeta\omega}$ for each i and ζ, where $\omega = \{1,2,...,\Omega_\zeta\}$ to interpolate the deterministic component of utility[7]. Indexing attributes as $\zeta = 1,\ldots,$ Z and price as $\zeta = 0$, the interpolated value of the observable component of utility is

$$\hat{v}_{ij} = \Psi_{i0}\left(\beta_{i0\omega}, p_j\right) + \sum\nolimits_{\zeta=1}^{Z} \Psi_{i\zeta}\left(\beta_{i\zeta\omega}, \mathbf{z}^{M}_{j\zeta}\right),$$

where $\mathbf{z}^{M}_{j\zeta}$ indicates the ζ^{th} element of the vector $\mathbf{z}^{M}_{j}$. These interpolated $\hat{v}_{ij}$ give rise, through the random utility specification, to expected individual choice probabilities P_{ij}, which are computed using either a logit or a probit distribution for the errors. Finally, the individual choice probabilities are used to compute market demand for each product, q_j (Figure 3). Calculating market demand for product j involves multiplying the probability P_{ij}, by the market potential S for each individual i=1,..., I_D, and averaging the resulting quantities across the individuals. Market potential is assumed to be exogenously determined through pre-market forecasting techniques (Lilien, Kotler & Moorthy 1992).

4. Empirical Application

In demonstrating the methodology, we adapt a product topology model for dial-readout bathroom scales developed previously for a simple single-product, homogeneous market example by Michalek et al. (2005) for comparison. This example was originally posed for the design of a single product under the assumption that consumer preferences are homogeneous. While expedient, neither assumption accords well with managerial practice. Furthermore, as we will see, even in those rare cases where firm does seek to enter the market with a single product, the presumption of homogeneity

[7] It is possible to use piecewise linear interpolation as an alternative to cubic splines; however, piecewise linear formulations are non-smooth in the continuous space, requiring a mixed integer formulation and solution approach that will often produce corner solutions at the discrete levels measured in the conjoint task.

is troublesome; in fact, the single-product solution obtained in Michalek et al. (2005) is substantially changed once *any* form of preference heterogeneity is allowed.

The inherent modularity of the proposed methodology for product line design circumvents the need to build a joint model of the full product line for each case $J = \{1,2,...\}$. Instead, a model of only a single product need be developed, and a duplicate can be created for each product j constituting the line[8], as illustrated in Figure 3. It is important to note that the design space **x** for this product does *not* map one-to-one with the attributes **z** communicated to consumers. This comes about because the engineering design model specifies some product attributes as functions of interactions among design variables; that is, different designs may exhibit identical product attributes, as observed by the customer. A manager could enact any number of criteria *post hoc* to choose from among such a continuum, or detailed cost data and preferences for commonality could drive selection of a single engineering design among the set of possibilities, although we do not pursue such strategies here.

The conjoint attributes came from a major scale manufacturer who had conducted extensive marketing research into what consumers care about. From this list, "decorative" attributes were removed, ones that were not part of the engineering model, had no cost implications, and could be switched-out postproduction (e.g., color, packaging). The product attributes **z** "seen" by consumers are weight capacity z_1, aspect ratio z_2, platform area z_3, tick mark gap z_4, and number size z_5, in addition to price p. For the conjoint study, the range of values for each attribute was captured by five (discrete) levels. Each respondent (n = 184) made choices from 50 consecutive sets in a choice-based conjoint task, identical across respondents, each with three options (plus "no choice"), implemented through a web browser. Interestingly, with the ATC approach, it is neither necessary nor practical to pre-restrict choice sets to include only realizable products, so long as unrealizable product attribute specifications describe product profiles that are meaningful to the respondent. The goal of the conjoint task is the effective and unbiased measurement of consumer preference drivers. Infeasible combinations of product attributes are implicitly avoided during optimization through coordination with the engineering design subsystem.

[8] All models and results are available from the authors upon request.

The demand/profit function requires (exogenous) estimates of several quantities, which are based here on manufacturer and publicly-available figures: c^{V}_{j} = \$3 cost per unit, c^{I}_{j} = \$3 million for initial investment, and market size S = 5 million, the approximate yearly US dial-readout scale market. Being completely exogenous, these values are easily altered. The special cases of discrete mixture ($\mathbf{\Lambda}=\mathbf{0}$) and homogeneous ($B$=1, $\mathbf{\Lambda}=\mathbf{0}$) models are straightforward to estimate using maximum likelihood techniques (Train 2003). For the mixture of normal distributions, estimates from a classical mixture of probits were used as starting values, and the Gibbs sampler was iterated until a stationary posterior was obtained. To mitigate autocorrelation, the data were thinned by retaining every 10th draw, after a burn-in of 50,000 iterations. Convergence was examined through iteration plots; posterior marginals revealed no convergence problems.

In order to optimize over this posterior surface, Monte Carlo integration was applied, as follows: When the chain has stabilized, *new* values of $\boldsymbol{\beta}_i$ are generated as the chain continued to run, allowing hyperparameters to vary across the generated values. These are thinned to reduce serial correlations; specifically, 10,000 values are generated, and every 10th is retained. The resulting set of 1000 $\boldsymbol{\beta}_i$ draws, with splines fit through the part-worth attribute levels of each draw, is used to represent the population (the posterior surface) throughout the optimization. Accuracy can be enhanced, if need be, by generating additional $\boldsymbol{\beta}_i$ values, although in the case study, tests of solution sensitivity to additional draws (up to 24,000) show that 1000 draws is sufficient to represent the "demand surface".

5. Results

There are two main methodological components to the approach advocated here: 1) econometric, for the extraction of individual-level preferences and generation of the preference splines, and 2) optimization-based, for the determination of the best number of products, their positioning and design conditional on the preference splines. We examine these in turn.

5.1. Demand Model Results

Table 1 lists DIC results for the normal mixture model and BIC results for the discrete mixture and homogeneous cases as well as classical log-likelihood values for reference. The latent class model identified by BIC consists of seven segments, while the mixture model with a diagonally-restricted covariance matrix identified by DIC has three mixing components, and the full-covariance mixture model has two. It is apparent that: (1) continuous heterogeneity (normal mixture) alone is superior to discrete heterogeneity (latent class) alone, up through a fairly large number of segments (Rossi, Allenby & McCulloch 2005); (2) a correlated (random) coefficients specification for the normal mixture is superior to an uncorrelated one; and (3) more than one segment in the normal mixture model is supported. In short, the most general specification fares best, and each of its attributes – correlated coefficients, and both discrete and continuous heterogeneity – is useful in accurately representing consumer preferences[9]. In the following sections, we will refer primarily to this full model, calling on others peripherally to compare the "optimal product lines" they entail.

[INSERT TABLE 1 HERE]

For illustration and a 'reality check' we briefly examine the posterior means of part-worth coefficient vectors, $\boldsymbol{\beta}_i$. The resulting splines are shown graphically in Figure 4, along with analogous splines for the discrete mixture and homogeneous cases. For identification purposes these values are scaled so that the sum in each set of attributes is zero, making for easier visual comparison. In each of the six attribute spline graphs the heterogeneous model is most "arched" or highly sloped, suggesting the presence of some consumers with relatively strong preference differentials across attribute levels. Of course, part-worth values have a nonlinear mapping onto choice probabilities, so an "averaged part-worth" is only a rough guide to comparing across heterogeneity specifications.

Although it is not our main focus here, a number of trends are apparent across these mean estimated coefficient values. Unsurprisingly, price appears to exert the strongest influence, and is decisively downward-sloping (this is true of the posterior means for each of the n = 184 original participants). One might have expected similarly monotonic preferences for number size and weight capacity, but this is only true for the former; apparently, too high a capacity was viewed as "suboptimal" by the respondents, on average. Note that these $\boldsymbol{\beta}_i$ values reflect pure consumer preference, and *not* any sort of constraint resulting from infeasible designs, which can only arise from the engineering design subsystem. Preferences for the other three variables (platform area, aspect ratio (i.e., shape) and interval mark gap) all have interior maxima.

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[9] The posterior predictive distribution (Gelman et al. 2003) may be used to further assess whether the various demand models and different heterogeneity representations provide adequate fit to the observed data.

[INSERT FIGURE 4 HERE]

5.2. Product Line Optimization Results

Conditional on the generated splines arising from the HB conjoint estimates (using the full normal mixture model), the design and positioning subsystems are solved iteratively until convergence. Optimization was carried out with each subsystem solved using sequential quadratic programming. The ATC hierarchy is solved for a set of fixed product line size J; the value of J producing the most profitable overall product line is determined post hoc. As is typical, local optima are possible, so global optima are sought using multi-start (Martí 2003). Figure 5 depicts the highest resulting profit levels, using ten runs with random starting points for $J = \{1,2,...,7\}$. It is clear that a product line with four products is most profitable.

[INSERT FIGURE 5 HERE]

Table 2 presents the resulting product attributes for various heterogeneity specifications. Several of the resulting scale designs are bounded by active engineering design constraints; this is necessary to ensure that the scale is physically tenable, e.g., that the dial, spring plate and levers fit in the case. Furthermore, all the scales in the line lie well within the range available through online retailers, although resulting prices migrate to the upper bound due to the single-producer scenario. Looking across the table, and considering primarily marketing attributes, we might term the resulting products "large high-capacity, small-numbered square scale" (27.4% of the market), "large-number portrait scale" (21.0%), "small, low-capacity landscape scale" (18.6%) and "high-priced, middle-of-the-road" scale (11.3%). The estimated market shares do not add to 100%, given the presence of the "no choice" option, which allows some portion of the potential market to prefer no scale at all to any of the four in the final line configuration.

Although the four scales may appear not to differ tremendously, they do in fact cover a wide swath of the attribute space bounded by the original conjoint levels; thus, they are quite different *relative* to the dial-readout scales available in the market. Note that extrapolation beyond the extreme conjoint levels was disallowed. This means that scale 4, the "high-priced, middle-of-the-road" scale, may command an even higher price than indicated ($30). This situation was exacerbated for the latent class model, for which *four* of the six scales in the best solution (as per Table 2) were at this upper price limit. As discussed more fully in Conclusions, whether this represents an untapped surplus, an insufficient upper limit on Price in the conjoint design, or is an artifact of all competitive interaction being subsumed by the "outside good", is an issue that should be teased apart by further studies.

[INSERT TABLE 2 HERE]

5.3. Effectiveness of ATC Coordination

To demonstrate the importance of the proposed approach, the ATC solution was compared to the solution obtained through a disjoint sequential approach, which has been referred to as analytical target setting (Cooper et al. 2006). In the disjoint scenario, price and product attribute positioning targets are set based on consumer preference data *without* engineering feasibility information (the positioning subproblem), and these are passed to the engineering design subproblems. Each engineering subproblem is then solved to design a feasible product that meets its targets as closely as possible[10] (the engineering subsystem) without further iteration. This can be viewed as a 'single pass' through ATC, similar to actual practice, where marketing studies precede engineering design, and subsequent iteration is costly and time-consuming.

In this disjoint scenario, marketing produces a plan for a line of four scales with a predicted combined market share (relative to full potential) of 83.4% and resulting profit of $81.2 million. There is no reason to believe these target product attributes will be attainable, as they are based on consumer preferences alone. In the disjoint case, these targets are passed to the engineering subproblems, which each design a feasible product to achieve product attributes as close as possible to the targets requested by marketing without further iteration. The resulting products differ significantly from the initial plan and so have attributes less preferred by consumers, resulting in combined 70.5% market share and $67.9 million profit: 16% less than marketing's original (unachievable) prediction. If ATC is instead used to iteratively coordinate positioning and design, the resulting joint solution is a line of four different products, resulting in 78.2% market share and $72.4 million profit. In this case, coordination resulted in a feasible product line with a predicted 6% improvement in profitability relative to disjoint decision-making.

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[10] We choose zero initial values for Lagrange multipliers, reducing the consistency constraint relaxation function to a quadratic penalty Euclidean distance metric.

In the disjoint scenario, marketing “leads” by developing the original plan and engineering design “follows” by attempting to meet product attribute targets. The reverse situation, where engineering “leads”, is possible when all consumers have monotonic preferences for product attributes by first designing a set of products that are Pareto-optimal in performance and then allowing marketing to pick a line from that set of products (Li & Azarm 2002). However, in this example, preferences for attributes are non-monotonic, so no such common Pareto set exists, and without preference information, engineering design has no single well-defined optimization objective.

5.4. Heterogeneity Representation

We examine the impact of the heterogeneity specification on the joint solution and whether simpler forms might have sufficed for optimal feasible line design. The simple homogeneous demand model is obviously ill-suited for generating product lines; moreover, because the IIA property is greatly exacerbated by preference homogeneity, the well-known “red bus, blue bus” problem can lead to lines with duplicate products (Train 2003, Shiau et al., 2009a). We thus compare the discrete mixture (latent class) model with the normal mixture model. Because the discrete mixture is natively supported in many statistical packages, it might prove convenient for line optimization; recent literature suggests that discrete and continuous heterogeneity can often represent preferences about equally well (Rossi, Allenby & McCulloch 2005, Andrews et al. 2002). Though fit statistics (Table 1) alone argue that the discrete mixture is inferior to the normal mixture specification in terms of representing *preferences*, this does not necessarily mean that, conditional on the resulting estimates, the resulting optimal line will be similarly inferior in terms of *profitability*.

Table 2 lists a comparison between the resulting profitability of the best locally-optimal solutions found using the discrete and continuous mixture demand models over ten multi-start runs with random starting points for each value of *J*. Not only do the different heterogeneity specifications result in different product line solutions (a line of six products under the seven-class discrete mixture vs. four under the two-component normal mixture), but the former suffers a profit decrement of 18% relative to the later using the continuous mixture model to evaluate solutions post hoc. The discrete mixture solution also entails little variation in the values *across* segments for a given attribute. Furthermore, because the discrete mixture specification models all individuals within a segment as having identical preferences, the remaining within-segment IIA property can result in solutions with duplicate or near-duplicate products, such as the one reported in Table 2. It is important to note that such solutions are artifacts of the econometric model and may be difficult to interpret for managerial use. For example, simply taking the solution resulting from the model and eliminating product duplicates will not, in general, produce a locally optimal solution in the reduced space. Furthermore, the within-segment homogeneity of preferences results in a profit surface containing pronounced local minima, which impedes the optimization process and makes global search difficult. Thus, even a relatively sophisticated heterogeneity representation can offer very different, and potentially sub-optimal, product line results.

While it may be unsurprising that simpler heterogeneity representations can lead to suboptimal product lines, it is less obvious whether a homogeneous model is sufficient for the design of a *single* product (as assumed, for example, in Michalek et al. 2005). Our analysis strongly suggests that it is not. Table 2 lists single-product solutions under the three demand model scenarios. Although in this case the more restrictive models do a fairly good job predicting some of the optimal product attributes, this is not so for price, which is notably exaggerated (relative to the normal mixture model), resulting in a solution with a loss of 7% market share using the discrete mixture model and 14% using the homogeneous one (again, using the continuous mixture model to evaluate solutions post hoc). These results make sense because continuous heterogeneity allows for some consumers that are highly price sensitive, so that a single price need be lower to avoid losing them entirely. Simply put, even when making a 'one size fits all' product, a manufacturer should not presume that all customers have the same preferences. It had not initially been anticipated, based on any prior literature of which we are aware, that preference heterogeneity would be so important when only a single product is being produced. How heterogeneity specification affects contingent optimization results is surely worthy of further study on its own.

6. Conclusions and future research

Firms work to position and design lines of products that best suit their market and profitability goals. Different functional entities within the firm can interpret this imperative idiosyncratically: measuring customer preferences and positioning new products for marketers; maximizing performance under technological constraints for engineers. Considered independently, these goals often lead to conflict, both in practice and with respect to optimization models in each discipline; moreover, disjoint sequential approaches can lead to suboptimal decision-making.

The product line design methodology advocated in this article draws on a wide array of techniques – in product line optimization, analytical target cascading, discrete choice analysis, preference heterogeneity and Bayesian econometrics – to model various subsystems separately, coordinating them via ATC. The resulting product line is a solution to the "joint" marketing and engineering problem, and produces results superior to a sequential approach. The product line problem is especially well suited to ATC decomposition. The separation of the subsystems is advantageous both for organizational purposes, since each modeling group can focus on what it knows best and need not be an expert in all areas, and for computational purposes, since the individual subsystems can be solved in parallel within low dimensional spaces and with fewer constraints then the full, non-decomposed system. ATC decomposition can improve scalability of the problem to complex products (which can be represented as (sub)hierarchies themselves), or to large numbers of products, simply by adding more subsystems. The intrinsic modularity of the approach accommodates additions, variations, and extensions.

A number of concrete conclusions emerged from applying the proposed method. One for which we know of no precedent is that accounting for preference heterogeneity can be critical even for a "one-size fits all" product. That is, the single-product solution looks quite different when the market itself is assumed (incorrectly) to be homogeneous. In our case study, this was so particularly for price: \$26.41 (homogeneous) vs. \$22.61 (heterogeneous). Homogeneity presumes that all consumers have the same preference coefficients, so the design can be tuned exactly for those preferences. When heterogeneity is incorporated, the optimal design will often be a compromise between conflicting preferences of different consumers, and the compromise product cannot command as high a price in our example. Identification of generalizable relationships between heterogeneity specification and optimal price is worthy of further research.

Conversely, we see that prices for the products in a line can be higher than for one product alone: each product fits a 'segment' better, and so extracts a premium over the 'mass market' single-product case; this is true regardless of which type of heterogeneity representation the modeler selects. Overall, and unsurprisingly, multiple products turned out to perform better than single ones, depending on the cost structure of the market. Of course, the precise number of products to produce depends on the cost of adding additional product variants to the line.

Comparing solutions achieved under different heterogeneity specifications indicates that the *form* of heterogeneity chosen by the modeler can exert non-trivial impact on the solution obtained. This suggests the need for further research to characterize the relationship between model specification and resulting optimal solutions. Use of general heterogeneity specifications, such as the full covariance normal mixture presented here, is likely preferred; however, dedicated study is needed to establish this to a degree typical of the preference heterogeneity literature (e.g., Andrews et al. 2002).

Several elements of our proposed framework can be readily relaxed or extended to accommodate different engineering- or marketing-based problem settings. Although others certainly exist, let us briefly discuss six potential avenues for future research. First, while we assume linear cost functions and particular specifications for share and product architecture, each of these can be modified without affecting the basic methodological structure. Second, we have focused on product attributes that can be represented in a continuous space. For categorical attributes, like brand, similar decomposition schemes can be used for MINLP formulations, although further research on convexification and global optimization is needed to improve scalability for these highly nonconvex problems (Khajavirad & Michalek 2009). Third, while we study the case of a monopolist with an outside good, it is trivial to incorporate competitors with known fixed attribute positionings. If competitors are likely to react to a new entrant, solutions that fail to account for competitor reactions may be suboptimal. In many cases, it is possible to incorporate competitor reactions using game theoretic models (Shiau and Michalek 2009a,b, Luo 2010, Tsafarakis et al, 2010). Future research might consider dynamics resulting from line re-positioning by extant and entrant firms, and how the former could enact optimal defensive strategies. This might provide a natural framework to address the sort of endogeneity issues intrinsic to dynamic, multi-player marketing optimization problems. Fourth, consumer preference heterogeneity specification merits renewed attention in the domain of product and line design; specifically, how to best balance the revenue-enhancing benefits of differentiation (due to heterogeneity) against the cost-saving benefits of product commonality in manufacturing (Kumar et al., 2009). Fifth, incorporation of perceptual attribute transformations, which vary by consumer, could extend applicability to categories that rely more on image. Lastly, further research is also needed to characterize uncertainty of optimal design recommendations in relation to uncertainty in demand model parameters as well as demand model representation (Luo et al. 2005).

In closing, several maxims are relevant for management, marketing and engineering design communities. First, although engineers are keenly aware of real, inviolable constraints, marketers tend to work to find desirable product attribute targets for exploring new markets. A tacit belief is that most, if not all, design constraints can be vanquished by ingenuity or sufficient capital. While this is sometimes true, often it is not. ATC encodes non-negotiable technological infeasibilities directly into its conceptual foundations, which includes a “consumer space” that is driven by heterogeneity of consumer preferences. As such, marketers and engineers using their own models within an overarching ATC formulation can come to terms and resolve tradeoffs among competing performance goals through coordination in designing deliverable products. Second, while it may appear simple to specify directly which product attribute combinations cannot co-exist, in practice it is often impractical: the feasible domain can snake through the product attribute space in ways difficult to visualize or translate into meaningful consumer terms, and in many cases, such as our case study, the feasible domain does not include any of the discrete combinations of attributes used for conjoint, but rather intermediate combinations that satisfy physical constraints. ATC frees marketers from considering such issues when collecting consumer preferences; iterative coordination avoids infeasible product line configurations implicitly. Third, and most important, heterogeneity matters: it must be accounted for in sufficient generality, even for the design of a single product. And finally, ATC is proven, for a broad class of problems, to converge to joint optimality across its various subsystems. Given its scalability, efficiency, and ability to key into a wide variety of extant modeling techniques, we hope to see ATC-based frameworks widely adopted as a cross-disciplinary platform for the design of complex product lines.

FIGURES AND TABLES

Figure 1 Dial Readout Scale, Internal View

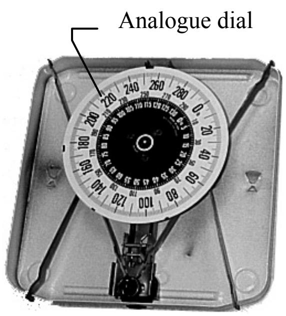

Figure 2 Diagram of the proposed modeling process.
TOP: A choice-based conjoint survey is designed and fielded; a part-worth utility function is fit to the data; splines are used to interpolate utility values of intermediate attribute levels. BOTTOM RIGHT: The market positioning subproblem is solved to determine price and product attribute values that maximize profit. BOTTOM LEFT: The product design subproblem is solved to determine the best feasible design that achieves the desired product attributes as closely as possible. Finally, these two subproblems iterate until they converge on a consistent solution that is optimal for the joint problem.

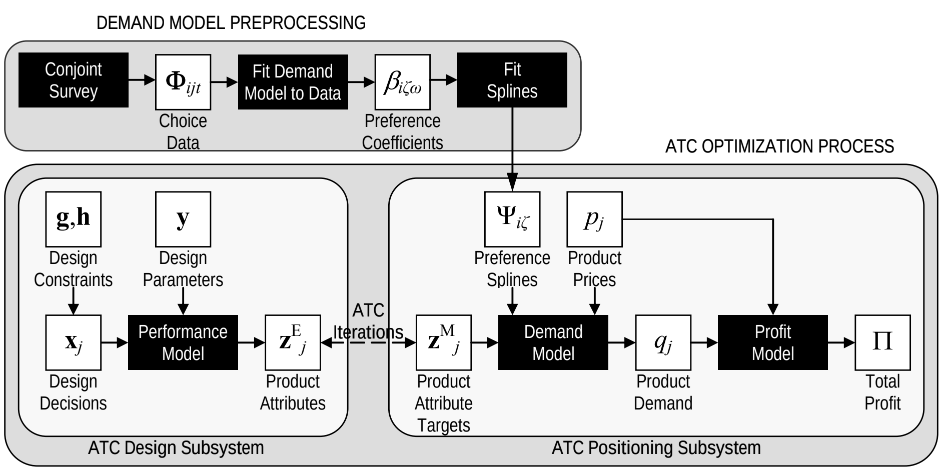

Figure 3 ATC Formulation of the Product Positioning and Engineering Design Product Development Problem

The market positioning subproblem (top) is to find the price (p_j) and product attributes ($z^{\mathbf{M}}_j$) for all products in the line that maximize profit (Π) with a relaxation function (π) to manage deviation from values achievable by engineering design ($z^{\mathbf{E}}_j$). The engineering design subproblem (bottom) is to find values for design variables (x_j) that minimize deviation from the product attribute targets ($z^{\mathbf{M}}_j$) set in the positioning subproblem, given engineering design constraints ***g*** and ***h***.

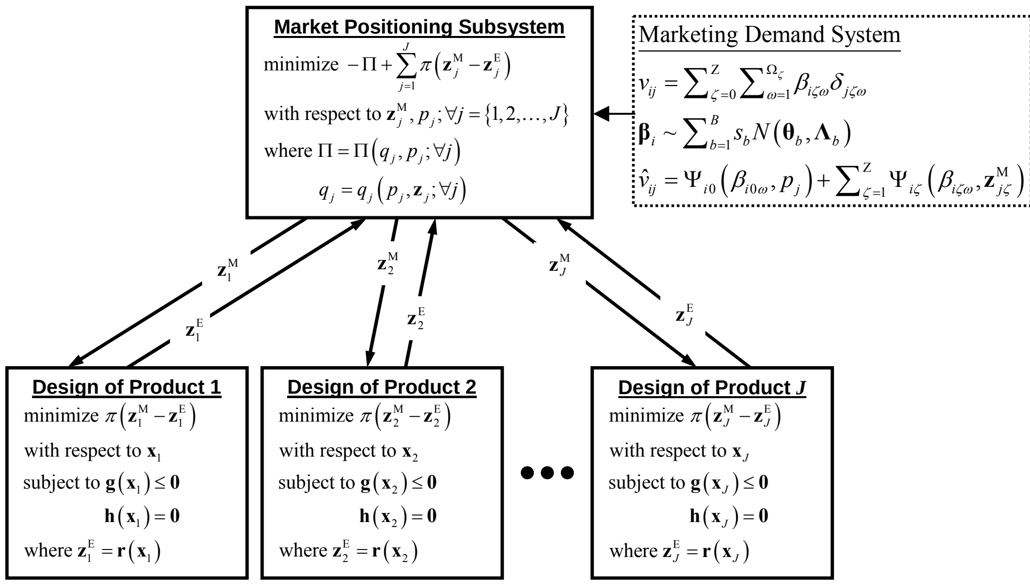

Figure 4 Plots of the average splines for each product attribute and price under the three demand models

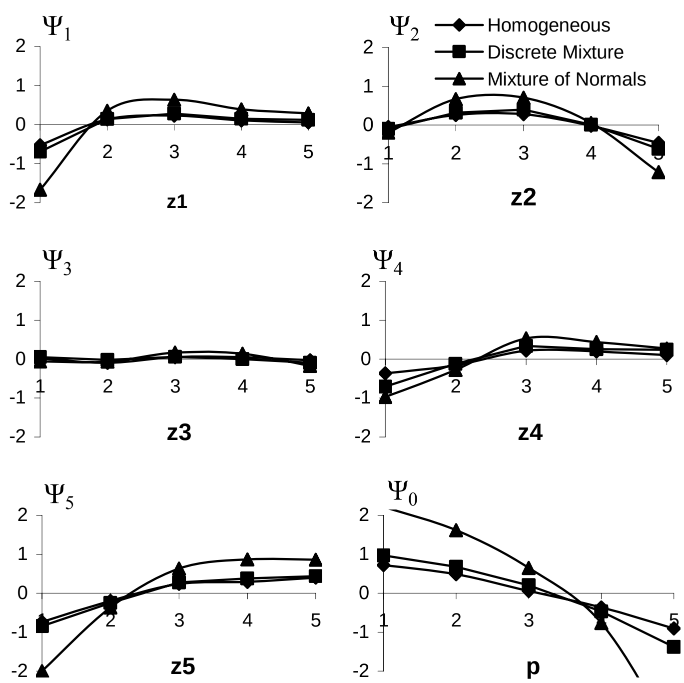

Figure 5 Resulting Profit as a Function of the Number of Products in the Line

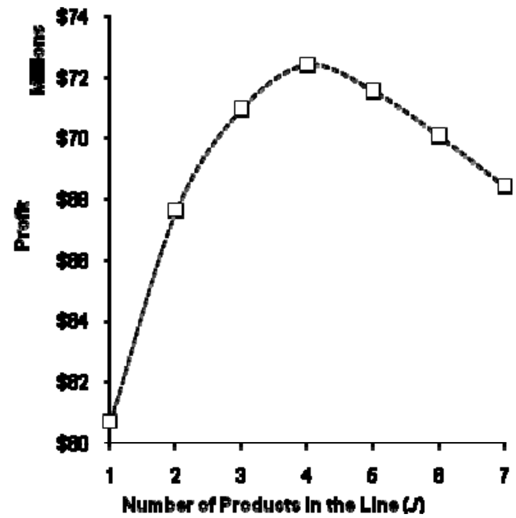

Table 1: Comparison of Heterogeneity Specifications for Demand: Discrete Latent Class vs. HB Random Parameters

	B	Λ	LL	BIC
Homo	**1**	**0**	**-10983**	**22194**
Latent Class Discrete Mixture	2	0	-10239	20944
	3	0	-9784	20271
	4	0	-9537	20014
	5	0	-9336	19850
	6	0	-9187	19788
	7	**0**	**-9059**	**19770**
	8	0	-8948	19785

	B	Λ	LL*	DIC
Hierarchical Bayes Continuous Mixture	1	Diag	-3813	12432
	2	Diag	-3713	12073
	3	**Diag**	**-3656**	**11961**
	4	Diag	-3638	12029
	1	Full	-4051	11742
	2	**Full**	**-4016**	**11674**
	3	Full	-4017	11745

* Classical LL for the HB models was evaluated using posterior means as plug-in values and is included only for informal comparison to the Latent Class models.

Table 2: Optimal Product Line Solutions under Each Demand Specification

			Single Product Solutions			Product Line Solutions									
			Homo-geneous	Discrete Mixture	Normal Mixture	Discrete Mixture (Latent Class) 1	2	3	4	5	6	Normal Mixture 1	2	3	4
?	Profit (Millions)*	$	$54.10*	$58.30*	$60.70*	$59.10*						$72.40			
	Market share*	%	48.80%	57.80%	65.00%	25.10%	8.70%	8.70%	8.70%	6.90%	4.90%	27.40%	21.00%	18.60%	11.30%
z_1	Weight capacity	lbs.	255	254	256	238	257	257	257	253	248	292	262	200	255
z_2	Aspect ratio	- -	0.996	1.047	1.002	1.045	1.041	1.039	1.039	1.062	1.051	0.98	1.156	0.921	0.986
z_3	Platform area	in^2	134	127	130	100	131	131	131	123	114	140	122	105	135
z_4	Tick mark gap	in.	0.116	0.117	0.115	0.106	0.116	0.116	0.116	0.114	0.111	0.103	0.116	0.121	0.116
z_5	Number size	in.	1.334	1.339	1.315	1.193	1.341	1.337	1.337	1.316	1.268	1.221	1.351	1.293	1.331
p	Price	$	$26.41	$24.21	$22.61	$23.96	$30.00	$30.00	$30.00	$30.00	$29.37	$22.89	$24.53	$23.84	$30.00

* as calculated post-hoc using the normal mixture demand model

Appendix 1: List of Symbols

$\bullet$	Hadamard product (element by element multiplication of vectors)

Roman characters

b	Index of mixing components (segments)
B	Number of mixing components
c_j^{I}	Investment cost for product *j*
c_j^{V}	Unit variable cost for product *j*
$\mathbf{g}$	Vector function of engineering design inequality constraints
$\mathbf{h}$	Vector function of engineering design equality constraints
i	Consumer index
I	Number of individual consumers
I_{D}	Number of individual consumer draws from the Bayesian mixture model
j	Product index
J	Number of products in the product line
p_j	Price of product *j*
P_{ijt}	Probability that individual *i* chooses product *j* from choice set *t*
q_j	Demand for product *j*
$\mathbf{r}$	Engineering design response function. Calculates product attributes as a function of design variables
S	Size of the market
s_b	Size of segment *b* (percentage)
t	Choice set index for conjoint survey
T	Number of choice sets in the conjoint survey
u_{ij}	Utility of alternative *j* for individual *i*
v_{ij}	Observable component of utility of product *j* for individual *i*
v_{i0}	Utility of the outside good for individual *i*
$\mathbf{w}$	Vector of weighting coefficients
$\mathbf{x}_j$	Vector of engineering design variables for product *j*
$\mathbf{z}_j^{\mathrm{M}}$	Vector of product characteristic targets set by marketing for product *j*

$\mathbf{z}_j^{\mathrm{E}}$	Vector of product characteristics achieved by the engineering design of product j

Greek characters

$\beta_{i\zeta\omega}$	Part-worth coefficient for consumer segment i for attribute ζ at level ω
$\boldsymbol{\beta}_i$	Vector of part-worth coefficients compiling $\beta_{i\zeta\omega}$ for all ζ and ω
$\delta_{j\zeta\omega}$	Binary dummy variable indicating whether product j possesses attribute ζ at level ω
ε_{ij}	Random error coefficient for individual i product j
ζ	Index of product characteristics = $\{1,2,....,Z\}$. ζ=0 refers to price.
Z	Number of product characteristics
$\boldsymbol{\theta}_b$	Vector of mean values for mixture component b
$\boldsymbol{\Lambda}_b$	Covariance matrix for mixture component b
$\boldsymbol{\lambda}$	Vector of Lagrange multipliers
Π	Profit
π	Consistency constraint relaxation function. Relaxes the constraint $\mathbf{z}_j^{\mathrm{M}} = \mathbf{z}_j^{\mathrm{E}}$
Φ_{ijt}	Binary dummy variable indicating observed choice of individual i with respect to alternative j on choice occasion t
$\Psi_{i\zeta}$	Spline function of β for individual i and product attribute ζ
ω	Product characteristic level index = $\{1,2,...,\Omega_\zeta\}$ for product characteristic ζ
Ω_ζ	Number of discrete levels for product characteristic ζ

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