A structural analysis of vehicle design responses to Corporate Average Fuel Economy policy

Ching-Shin Norman Shiau, Jeremy J. Michalek, Chris T. Hendrickson

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ABSTRACT

The US Corporate Average Fuel Economy (CAFE) regulations are intended to influence automaker vehicle design and pricing choices. CAFE policy has been in effect for the past three decades, and new legislation has raised standards significantly. We present a structural analysis of automaker responses to generic CAFE policies. We depart from prior CAFE analyses by focusing on vehicle design responses in long-run oligopolistic equilibrium, and we view vehicles as differentiated products, taking demand as a general function of price and product attributes. We find that under general cost, demand, and performance functions, single-product profit maximizing firm responses to CAFE standards follow a distinct pattern: firms ignore CAFE when the standard is low, treat CAFE as a vehicle design constraint for moderate standards, and violate CAFE when the standard is high. Further, the point and extent of first violation depends upon the penalty for violation, and the corresponding vehicle design is independent of further standard increases. Thus, increasing CAFE standards will eventually have no further impact on vehicle design if the penalty for violation is also not increased. We implement a case study by incorporating vehicle physics simulation, vehicle manufacturing and technology cost models, and a mixed logit demand model to examine equilibrium powertrain design and price decisions for a fixed vehicle body. Results indicate that equilibrium vehicle design is not bound by current CAFE standards, and vehicle design decisions are directly determined by market competition and consumer preferences. We find that with increased fuel economy standards, a higher violation penalty than the current stagnant penalty is needed to cause firms to increase their design fuel economy at equilibrium. However, the maximum attainable improvement can be modest even if the penalty is doubled. We also find that firms’ design responses are more sensitive to variation in fuel prices than to CAFE standards, within the examined ranges.

1. Introduction

When people drive vehicles, they generate negative externalities that impact society, including congestion, national security implications, and environmental impact such as greenhouse gas (GHG) emissions that contribute to global warming (Porter, 1999). While economists generally advocate Pigovian taxes to efficiently correct these negative externalities (Lesser et al., 1997; Kolstad, 2000), the vast majority of the US public and lawmakers object to increased gasoline taxes (Uri and Boyd, 1989; Chernick and Reschovsky, 1997; Hammar et al., 2004; Decker and Wohar, 2007), and the government has...
instead relied on mandated restrictions for the average characteristics of vehicles sold by automakers. Among such policies are (1) the Corporate Average Fuel Economy (CAFE) standards in the US, which penalize automakers whose sales-weighted average of fleet fuel economy drops below a government-determined standard and (2) similar policies in California and in Europe that set standards on average fleet carbon dioxide (CO2) emissions per mile. Rather than addressing driving patterns or fuel consumption directly, these policies create incentives for automakers to produce more efficient fleets. However, vehicle design responses to government policies are complicated by trade-offs in available technology, consumer preferences, and competition in the marketplace. Integrated analysis is required to understand and predict vehicle design responses to transportation policies.

1.1. Background of CAFE

The CAFE standard regulates the average fuel economy of new vehicles sold in the United States. It requires the fleet-wide sales-weighted average fuel economy of automobiles sold by each manufacturer to achieve a prescribed standard. Manufacturers who do not achieve the CAFE standard are penalized based on their annual vehicle sales and fuel economy shortfalls. The origin of CAFE regulation can be traced to the 1973 oil crisis, when soaring crude oil prices drew the government and public’s attention to the inefficiency of automobiles. The Energy Policy and Conservation Act (EPCA) of 1975 established separate CAFE standards for passenger cars and light trucks (US Congress, 1975). The executive responsibilities for implementing CAFE policy are distributed between the Environmental Protection Agency (US EPA, 2007) and the Department of Transportation (DOT) National Highway Traffic Safety Administration (NHTSA, 2006). EPA is responsible for determining the test procedures for measuring vehicle fuel economy (and emissions) and calculating the CAFE for automobile manufacturers (US EPA, 2007). NHTSA is in charge of establishing, amending, and enforcing fuel economy standards and regulations. In addition to the fuel economy criterion, NHTSA is also authorized to determine the financial penalty for violating the CAFE standard. The initial penalty value set in 1978 was $5.00 per vehicle per 0.1 mpg ($50 per mpg). In 1997, NHTSA raised the penalty to $5.50 per vehicle per 0.1 mpg ($55 per mpg)1 (US GAO, 1990). The penalty has not been changed since then and has not been adjusted for inflation. Fig. 1 shows the historic change of CAFE standards and average vehicle fuel efficiency. Note that during the 1990s combined fuel economy decreased even as the fuel economy in the separate car and truck categories increased due to consumers switching from cars to light trucks. As of December 2007, the total collected fines on CAFE violations reached $745 million, not adjusting for inflation (NHTSA, 2007). Historically, only European automobile manufacturers have paid CAFE fines, while Japanese automakers have consistently exceeded the regulatory standard, and US automakers have made it a policy to treat the CAFE standard as a constraint, using the CAFE credit system2 when necessary to avoid paying penalties.

In 2007, Congress passed the Energy Independence and Security Act of 2007 (EISA), which increased the target fleet-wide average fuel economy standard to 35 mpg in 2020 with combining cars and light trucks into a single category (US Congress, 2007). The act is the first legislation since the 1974 EPCA that directly regulates US fleet fuel economy. The legislation also requires NHTSA to annually reform the separate fuel economy standards for cars and light trucks in order to achieve the joint 2020 goal of 35 mpg. In April 2008, NHTSA initiated an attribute-based CAFE proposal by using vehicle footprint to determine the 2011–2015 standards (NHTSA, 2008). In April 2009, NHTSA announced the formal 2011 CAFE standards 30.2 mpg and 24.1 mpg for cars and light trucks, respectively (combined 27.3 mpg) (NHTSA, 2009), which are slightly lower than the values in the 2008 proposal. The announcement delivered two important messages. First, NHTSA will reform and adjust their regulation decisions every year based on the status of national fleet average and technology feasibility. The dash

1 NHTSA has the authority to raise CAFE penalties to $10 per 0.1 mpg ($100 per mpg) (US GAO, 2007).
2 CAFE regulation allows automakers to earn credits for exceeding fuel economy standards in one year and apply them to the prior or subsequent three model years to neutralize the violation penalty (NHTSA, 2006).
lines in Fig. 1 show that unreformed and predicted standards for 2011–2020. Second, the significant jump in car fuel economy standard means that CAFE standards will catch up to the national fleet average, which has exceeded the standard substantially in recent years, while the regulation has merely served as a lower-bound requirement in the past 10 years. In May 2009, President Obama announced a higher fuel economy regulatory target, combined 35.5 mpg by 2016 to be implemented as a CO2 regulation by the EPA (The White House, 2009), which aggressively exceeds the 2020 35 mpg target set by EISA.

1.2. Carbon dioxide emission regulations

Carbon dioxide (CO2) emission standards that are measured on a fleet average per mile basis can be seen as structurally equivalent policies to CAFE for regulating automobile fuel efficiency, since technology is not available on the market to separate and store CO2 emissions from vehicles. The estimated CO2 emissions per gallon of gasoline burned are roughly 8788 g (US EPA, 2005), without including CO2 emissions arising from the petroleum supply chain. We review the two most well-known standards: the European Union CO2 emission standard and the California CO2 emission standards.

The European carbon dioxide standards were issued by the European Commission (1995) to establish a voluntary CO2 emission standard for automobile manufacturers selling vehicles in Europe as a response to the Kyoto Protocol (1997), which requires 8% reduction in greenhouse gas emissions in all economic sectors relative to 1990 levels by 2008–2012. Expecting automobile manufacturers to improve vehicle emissions voluntarily, the 1995 regulation defined an intermediate target of 140 g/km by 2008–2009 and an ultimate target of 120 g/km for 2012. However, it was found that automakers had not been reducing vehicle CO2 emissions effectively, making the 2012 target less likely to be reached (European Parliament, 2005). Hence, in 2007, the European Commission issued a proposal for a new regulation to replace the original voluntary target with a mandatory standard of 130 g/km (European Commission, 2007). In December 2008, after automakers cited infeasibility of regulatory targets in the 2007 proposal, a resolution was made by European Commission for changing the firm 130 g/km target into gradually adaptive standards; 65% of automaker’s fleet reaches the 130 g/km requirement in 2012, 75% in 2013, 85% in 2014, and 100% in 2015 (European Parliament, 2008). Moreover, a new long-term target is set to 95 g/km in 2020. The resolution also revealed the step-size penalty structure of the regulation.3

The California greenhouse gas emission proposal set CO2 emission requirements for new vehicles sold in California (CARB, 2004). The program required a CO2 emission standard of 323 g/mile (201 g/km) for the model year 2009 with annual reductions to 205 g/mile (127 g/km) for the model year 2016. The program did not define a direct penalty parameter for the automotive manufacturers who violate the standards. Instead, the law implements a credit and debit system to monitor each manufacturer’s annual average fleet CO2 emission. If emission debits are not neutralized within five years, the manufacturer is issued a civil penalty according to the Health and Safety Code (CA Code, 2008). The California CO2 standards were rejected by Bush administration in 2007 since the standards are stricter than federal regulations. In June 2009, the Obama administration granted a waiver for California’s request beginning from model year 2009 (US EPA, 2009).

Fig. 2 shows the future CAFE standards, including the Obama administration’s 2009 new proposal (Broder, 2009), and the two CO2 emission regulatory standards for passenger cars. European emission standards are close to California emission levels, whereas the US CAFE regulation is the weakest criterion.4 However, the three regulations have similar slopes for equivalent annual carbon emission reductions. The common mechanism of the three regulations is to set increasing standards for vehicle characteristics (fuel consumption or emissions) and expect automobile manufacturers to respond with revised vehicle

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3 The resolution proposed €5 for the first gram/km over the target, €15 for the second g/km over the target, €25 for the third g/km over the target and €95 for the fourth g/km and subsequent. After 2019, any violation will be €95 per g/km (European Parliament, 2008).

4 Note that European standards are based on a different fleet composition from the car and light fleet in the US. Moreover, European standards use a different test cycle to measure vehicle fuel efficiency.
lines and pricing strategies that achieve the standards. We propose an integrated structural analysis to understand and predict vehicle design responses to transportation policies.

The paper proceeds as follows: Section 2 reviews the relevant literature on analysis of CAFE policy; Section 3 introduces the proposed model and analysis of vehicle design responses to CAFE policy; Section 4 presents a case study using vehicle simulation models and a mixed logit demand model from the literature; and finally, Section 5 discusses conclusions and policy implications.

2. Review of literature on CAFE impacts

Studies of CAFE effects follow two primary branches: econometric estimation and economic modeling. Econometric estimation studies use automobile sales data to examine the past impacts of CAFE policy on fuel economy (Crandall et al., 1986; Godek, 1997; Goldberg, 1998; Espey and Nair, 2005; Small and Van Dender, 2007) or on vehicle safety (Crandall et al., 1986; Crandall and Graham, 1989; Yun, 2002; Ahmad and Greene, 2005). In contrast, economic modeling studies draw on economic theory to simulate hypothetical manufacturer decision-making in response to CAFE or other policies with the aim to predict automaker responses to alternative regulation scenarios and understand structural policy implications.

The literature on economic modeling of CAFE policy can be categorized along two major dimensions where vehicles are viewed either as commodities or as differentiated products. If firms view vehicles as commodities, they control only price or production volume, while firms with differentiated products also control vehicle design attributes, such as fuel economy or performance. If consumers view vehicles as commodities, they react only to price; however, consumers of differentiated products also react to vehicle attributes, such as fuel economy or performance. Table 1 summarizes the prior literature with respect to this categorization.

Several studies treat vehicles entirely as commodities: Kwoka (1983) and Biller and Swann (2006) examined a single firm, using linear models of demand and treating the CAFE standard as a constraint. Kleit (1990) posed a model with two vehicle commodities (small car and large car) and examined perfect competition and oligopoly models by taking firms as price takers or price setters, respectively. Kleit argues that CAFE policy is not only inefficient, but also counterproductive by encouraging drivers to drive more in response to the reduced operation costs of higher fuel efficiency vehicles (the rebound effect). He argues for elimination of CAFE in favor of Pigovian gasoline taxes; however, Gerard and Lave argue that CAFE is potentially an effective complement to gasoline taxes (Gerard and Lave, 2003, 2004).

The remaining studies view vehicles as differentiated from the manufacturer’s perspective and account for long-run vehicle design changes made by firms in response to CAFE policy. Using technology-cost and technology-demand models from a prior study (Greene and DeCicco, 2000), Greene and Hopson (2003) constructed a non-linear programming framework using an industry-wide net value of fuel economy improvement as the objective function and treating the CAFE standard as a constraint. Kleit (2004) adopted Greene and Hopson’s cost-technology model to extend his previous study (Kleit, 1990) to include manufacturer fuel economy responses to CAFE standard increases under perfect competition using a price-elasticity demand matrix based on General Motors conjoint analysis data with multiple market segments. Kleit assumes that firms must pay for increased fuel efficiency, but changes in fuel economy do not affect demand. The study concluded that a 3.0 mpg increase in the CAFE standard can be replaced by an 11 cent gasoline tax to save the same amount of gasoline annually at only one-fourteenth of the social welfare cost. Adopting Kleit’s (2004) demand elasticity model, Austin and Dinan (2005) modeled manufacturer pricing and fuel economy improvement decisions treating CAFE as a constraint. Jacobsen (2008) identifies and models the heterogeneous responses of different manufacturer groups to the CAFE regulation; domestic automakers bind with CAFE standards, but foreign manufacturers treat the regulations as inactive lower bounds or associated taxes. His market equilibrium simulation also shows that gasoline taxes would be much cheaper than CAFE in welfare cost even if technologies for fuel efficiency improvement in response to CAFE regulation are taken into account. Fischer et al. (2007) found that the efficiency and benefits of tightening CAFE standards are difficult to quantify, but they recommend that fuel economy standards should be raised gradually over time. Finally, Michalek et al. (2004) conducted a numerical study of firm responses to CAFE standards accounting for logit consumer responses to vehicle fuel economy, performance, and price. They modeled firms as players in a Nash equilibrium who decide engine size and price in response to CAFE policy, and they used physics simulators to model performance, fuel economy, and cost complications of engine size. They argue that CAFE standards can result in greater fuel economy improvements at lower cost to the manufacturer; however, they did not account for government revenue generated.

The bulk of prior studies treat vehicles as commodities to consumers; however, there exists a rich literature on econometric measurement of consumer responses to (differentiated) vehicle attributes (Boyd and Mellman, 1980; Train, 1980; Bunch et al., 1993; Berry et al., 1995, 2004; Goldberg, 1995; Brownstone and Train, 1999; Brownstone et al., 2000; McFadden and Train, 2000; Sudhir, 2001; Choo and Mokhtarian, 2004; Train and Winston, 2007). We argue that vehicles are not commodities, and accounting for consumer preferences and technical capabilities is important to understand firm responses to CAFE. As such, we follow Greene and Hopson (2003) and Michalek et al. (2004) in viewing the vehicle as a differentiated product from the perspective of the firm and the consumer, where firms control vehicle design variables and consumers react to vehicle attributes as well as price. While Greene and Hopson (2003) and Michalek et al. (2004) provide specific numerical anal-

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5 Greene and DeCicco (2000) created the technology-cost model via regression of retail price increases in technologies that offer fuel efficiency improvements. Similarly, they estimated the market penetration of fuel economy technology using regression on market data.
yses, we instead develop a general structural analysis of long-run oligopoly Nash responses (Tirole, 1988) to CAFE policy under general assumptions for cost functions, technical trade-offs, and consumer demand, and we identify a distinct pattern in Nash responses to CAFE. We then instantiate the model with specific data and examine policy implications.

3. Model

We define firm $k$’s profit function as

$$
\Pi_k = \left( \sum_{j \in J_k} q_j(p_j - c_j) - c_1 \right) - \left( \rho \delta(z_{AVG}^k) \sum_{j \in J_k} q_j \right)
$$

(1)

where $p_j$, $q_j$, and $c_j$ are the price, demand, and variable cost, respectively, of vehicle $j$; $J_k$ is the set of vehicle models produced by firm $k$; $c_1$ is the fixed investment cost per vehicle model; $\rho$ is the penalty for CAFE violation in dollars per vehicle per mpg$^6$; $\delta$ is the CAFE violation function; and $z_{AVG}^k$ is the CAFE achieved by firm $k$. According to NHTSA’s CAFE formulation definition, the fleet-wide average fuel economy for manufacturer $k$ (cars and light trucks are currently calculated separately) is$^7$

$$
z_{AVG}^k = \frac{\sum_{j \in J_k} q_j}{\sum_{j \in J_k} q_j / c_j}
$$

(2)

The function $\delta$ is defined as

$$
\delta(z_{AVG}^k, \kappa) = \begin{cases} 
0 & \text{if } z_{AVG}^k > \kappa \text{ (case 1)} \\
0 & \text{if } z_{AVG}^k = \kappa \text{ (case 2)} \\
\kappa - z_{AVG}^k & \text{if } z_{AVG}^k < \kappa \text{ (case 3)}
\end{cases}
$$

(3)

where $\kappa$ is the CAFE standard. We take the fuel economy $z_{fj}$ and variable cost $c_j$ of each vehicle $j$ to each be a function of a vector of vehicle design variables $x_j$ so that $z_{fj} = f_j(x_j)$ and $c_j = f_c(x_j)$. We further take the demand $q_j$ for each vehicle $j$ to be a function of the design $x'_j$ and price $p'_j$ of all vehicles $f$ in the market, so that $q_j = q_j(p'_j, x'_j)$. Finally, we assume that each firm sets the price $p_j$ and design $x_j$ of its vehicle, and the investment cost $c_1$ and policy parameters $\kappa$ and $\rho$ are taken as exogenous.

The three cases in Eq. (3) are classified by the relationship between fleet fuel economy design decisions and the CAFE fuel economy standard: In case 1 the fleet fuel economy surpasses the standard ($z_F > \kappa$); in case 2 the fleet fuel economy matches the standard ($z_F = \kappa$); and in case 3 the fleet fuel economy violates the standard ($z_F < \kappa$). The derivative of $\delta$ with respect to firm’s average fuel economy is

$$
\frac{\partial \delta}{\partial z_{AVG}^k} = \begin{cases} 
0 & \text{if } z_{AVG}^k > \kappa \text{ (case 1)} \\
\text{undefined} & \text{if } z_{AVG}^k = \kappa \text{ (case 2)} \\
-1 & \text{if } z_{AVG}^k < \kappa \text{ (case 3)}
\end{cases}
$$

(4)

$^6$ We ignore violation cost other than the government fee, such as public relations and litigation costs.

$^7$ We examine only the basic CAFE penalty structure here and leave study of attribute-based standards and year to year credits for future study.

Table 1

<table>
<thead>
<tr>
<th>Demand modeling</th>
<th>Differentiated</th>
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<tr>
<td>Commodities</td>
<td>as a function of price only</td>
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<tr>
<th>Market structure</th>
<th>Commodities</th>
<th>Single firm optimization</th>
<th>Perfect competition</th>
<th>Oligopolistic competition</th>
</tr>
</thead>
<tbody>
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<td>Perfect competition</td>
<td>Kleit (1990)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Design change considered (long run)</td>
<td>Industry-wide optimization</td>
<td>Greene and Hopson (2003)</td>
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<td>–</td>
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</tr>
</tbody>
</table>

| Literature categorization on firm decision and CAFE regulation modeling. |
|--------------------------|--------------------------|--------------------------|--------------------------|
| Market structure | Commodities | Single firm optimization | Perfect competition | Oligopolistic competition |
| Differentiated   | Perfect competition | Kleit (1990) | – |
| Design change considered (long run) | Industry-wide optimization | Greene and Hopson (2003) | – |
The function $\delta$ has continuity, but its derivative is discontinuous at $z_{AVG}^k = \kappa$. Fig. 3 illustrates Eqs. (3) and (4).

In the long-run scenario, manufacturers alter price and vehicle design under competition and CAFE policy. We consider price and vehicle design as endogenous, while the CAFE standard and penalty are applied to the competitive market as exogenous variables. We assume the market is described by Nash equilibrium, where all manufacturers compete non-cooperatively in an oligopoly market (Fudenberg and Tirole, 1991). Also, for simplicity and to facilitate closed-form solutions each manufacturer is assumed to produce a single vehicle model only. We examine first-order conditions (FOC) for Nash equilibrium in each of the three cases below.

**Case 1: vehicle gas mileage surpasses the CAFE standard**

In this case the first-order condition with respect to price $p_j$ from Eq. (1) is

$$\frac{\partial \Pi_k}{\partial p_j} = \frac{\partial q_j}{\partial p_j} (p_j - c_j) + \frac{q_j}{C_0} = 0$$

(5)

Therefore, the price at market equilibrium can be expressed as:

$$p_j = c_j + \frac{q_j}{C_0} \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1}$$

(6)

Here, the equilibrium price is comprised of vehicle cost plus manufacturer markup, where the markup depends on total demand (itself a function of price) and the price elasticity. Assuming that the design variable space is unconstrained, the first-order condition with respect to the design variables $x_j$ is

$$\frac{\partial \Pi_k}{\partial x_j} = \frac{\partial q_j}{\partial x_j} (p_j - c_j) - q_j \frac{\partial c_j}{\partial x_j} = 0$$

(7)

Inserting Eq. (6) and assuming positive demand, the equation is simplified as

$$\frac{\partial q_j}{\partial x_j} \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1} + \frac{\partial c_j}{\partial x_j} = 0$$

(8)

Here, the equilibrium design is a balance between the marginal cost of a design change and the marginal price that can be charged for the design change without changing demand (Shiau and Michalek, 2009b).

**Case 2: vehicle design gas mileage is equal to the CAFE standard**

In this case the FOC condition for price is the same as Eq. (6). Since vehicle fuel economy equals the CAFE standard in this case, the design solution satisfies the design function:

$$f_k(x_j) = \kappa$$

(9)

If the function has an inverse, then $x_j = f_k^{-1}(\kappa)$.

**Case 3: vehicle design gas mileage violates the CAFE standard**

In this case the first-order condition with respect to price $p_j$ is

$$\frac{\partial \Pi_k}{\partial p_j} = \frac{\partial q_j}{\partial p_j} (p_j - c_j) + q_j - \rho \delta(z_{AVG}^k) \frac{\partial q_j}{\partial p_j} = 0$$

(10)

If design constraints are present, a Lagrangian formulation can be included (Shiau and Michalek, 2009a).
The price solution becomes

$$p_j = c_j + q_j \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1} + \rho \delta(z_{ij})$$  \hspace{1cm} (11)$$

Here, the equilibrium price is comprised of vehicle cost, manufacturer markup and the CAFE penalty per vehicle. The manufacturer markup depends on demand and the price elasticity, and the CAFE penalty is passed to the consumer. The first-order condition with respect to the design variable vector (again assuming no constraints) \(x_j\) is

$$\frac{\partial \Pi_j}{\partial x_j} = \frac{\partial q_j}{\partial x_j} (p_j - c_j - \rho \delta_j) + q_j \left( \rho \frac{\partial z_{ij}}{\partial z_{ij}} - \frac{\partial c_j}{\partial x_j} \right) = 0$$  \hspace{1cm} (12)$$

Plugging in Eq. (11), the equation is simplified to

$$\frac{\partial q_j}{\partial x_j} \left( \frac{\partial q_j}{\partial p_j} \right)^{-1} + \left( \frac{\partial c_j}{\partial x_j} - \rho \frac{\partial z_{ij}}{\partial x_j} \right) = 0$$  \hspace{1cm} (13)$$

Here, the equilibrium design is a balance between the marginal cost of a design change due to direct cost and regulation cost and the marginal price that can be charged for the design change without changing demand.

The FOC equations for Nash pricing and design solutions for representing each firm’s decisions are summarized in Table 2. For each case, the fuel economy of vehicle design shows different characteristics and variable dependencies. For case 1, the vehicle design is independent of CAFE parameters; for case 2, vehicle design has a fuel economy equal to the CAFE standard \(\kappa\); and for case 3 vehicle price and design are functions of the CAFE penalty \(\rho\), but not the CAFE standard \(\kappa\). So \(z_{ij}\) is independent of \(\kappa\) in cases 1 and 3. For any given \(f_c, f_k, f_q\) and \(\rho\) such that \(z_{ij}^* > z_{ij}\), which is the case for practical markets, at most two adjacent cases will have equilibrium conditions that are consistent with case assumptions for a given \(\kappa\).

When a unique oligopolistic symmetric market equilibrium exists, Fig. 4 shows Nash vehicle fuel economy responses \(z_{ij}\) as a function of the CAFE standard \(\kappa\) under a fixed penalty \(\rho\), which forms three regions. Case 1 and case 3 are independent of \(\kappa\), so they appear as horizontal lines. Case 2 follows the 45° line passing through (0, 0). Case 1 is valid for \(z_{ij}^* < \kappa\), and case 3 is valid for \(z_{ij}^* > \kappa\). Case 2 is valid for all \(\kappa\), such that \(x_j \cdot f_c(x_j) = \kappa\). However, because case 2 is a border case for case 1 and case 3, it is not an equilibrium solution to the relaxed problem where \(z_{ij}\) is not restricted to \(\kappa\), and we consider case 2 only when

<table>
<thead>
<tr>
<th>Condition</th>
<th>Case 1 (z_i(x_i) &gt; \kappa)</th>
<th>Case 2 (z_i(x_i) = \kappa)</th>
<th>Case 3 (z_i(x_i) &lt; \kappa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_j) (\partial x_j)</td>
<td>(0)</td>
<td>(0)</td>
<td>(\kappa - z_{ij})</td>
</tr>
<tr>
<td>Price</td>
<td>(p_j = c_j + q_j \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1})</td>
<td>(p_j = c_j + q_j \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1})</td>
<td>(p_j = c_j + q_j \left( -\frac{\partial q_j}{\partial p_j} \right)^{-1} + \rho \delta_j)</td>
</tr>
<tr>
<td>Design</td>
<td>(\frac{\partial q_j}{\partial x_j} \left( \frac{\partial q_j}{\partial p_j} \right)^{-1} + \frac{\partial c_j}{\partial x_j} = 0)</td>
<td>(x_j \cdot f_c(x_j) = \kappa)</td>
<td>(\frac{\partial q_j}{\partial x_j} \left( \frac{\partial q_j}{\partial p_j} \right)^{-1} + \left( \frac{\partial c_j}{\partial x_j} - \rho \frac{\partial z_{ij}}{\partial x_j} \right) = 0)</td>
</tr>
<tr>
<td>Fuel economy</td>
<td>(z_{ij}) depends on (f_c, f_k, f_q)</td>
<td>(z_{ij}^*) depends on (\kappa)</td>
<td>(z_{ij}^*) depends on (f_c, f_k, f_q, \rho)</td>
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</tbody>
</table>

Fig. 4. Three regions of fuel economy design responses.
the other two cases are invalid. Therefore, case 1 is valid for \( k < z_{ij} \), case 3 is valid for \( k > z_{ij} \), and case 2 is valid for \( z_{ij} < k < z_{ij} \). For the three regions, the policy implications are:

Region 1: low CAFE standards do not affect firms’ design decision, and fuel economy and pricing decisions are determined by oligopolistic competition directly.

Region 2: moderate CAFE standards result in fuel economy responses that follow the standard exactly.

Region 3: high CAFE standards result in fuel economy responses that violate the standard, and firms ignore further increases in the standard, instead transferring the regulation penalty cost to consumers in the retail price. The point of first violation and the resulting fuel economy response depends on the penalty for violation.

These results imply that the performance of CAFE standards is affected by both the fuel economy criteria and the penalty: setting too high a standard without a corresponding increase in violation penalties will result in firms ignoring further increases and passing costs onto consumers. Moreover, the difference between the solution equation set of case 1 and case 3 suggests that the width of region 2 \( (z_{ij} - z_{ij}) \) is a function of the CAFE penalty \( \rho \) and the marginal change in fuel economy with respect to the design variables \( \left( \partial z_i / \partial x \right) \).

4. Case study

We next examine a case study by using automotive market data, vehicle performance simulation, costs, and fuel economy technology from the literature. In the following subsections, we detail our manufacturer design decision model and market demand model, results, and sensitivity analyses.

4.1. Supply side

We consider a mid-size car equipped with a gasoline engine in our supply side modeling. The vehicle design decision is represented by two design variables, an engine-scaling variable \( x_e \), and a technology implementation \( x_T \). The former determines the size and power of engine, and the latter represents implementation of fuel-saving technologies. We use the vehicle physics simulator ADVISOR-2004 (AVL, 2004)\(^1\) to evaluate fuel economy with the standard EPA city driving cycle (FTP) and highway driving cycle (HWFET). The combined fuel economy is then calculated by the harmonic mean of 55% city and 45% highway (US EPA, 2007). A meta-model is established over the fuel economy simulation data as a function of the engine-scaling variable \( x_e \), as shown in Fig. 5. Thus, vehicle’s fuel consumption \( z_i \) (gallon per mile), fuel economy \( z_{ij} \) (mile per gallon), and power-to-weight ratio \( z_{ij} \) (hp per 100 lbs) can be defined as functions of \( x_e \) and \( x_T \):\(^2\)

\[
\begin{align*}
Z_{ij} &= (a_{22} x_e^2 + a_{11} x_T + a_{10})^{-1} (1 - x_T) \\
Z_{ij} &= a_{12} x_e^2 + a_{11} x_T + a_{10} \\
Z_{ij} &= z_{ij}^{-1}
\end{align*}
\]

The meta-model coefficients are \( a_{22} = 4.90, a_{11} = -24.7, a_{10} = 48.8, a_{12} = -0.44, a_{11} = 3.87, \) and \( a_{10} = 0.12 \). The vehicle cost model comprises the vehicle base cost \( c_b \), engine cost \( c_e \), and fuel-saving technology cost \( c_f \) so that \( c_j = c_b + c_e + c_f \). The engine cost is modeled as an exponential function \( c_e = b_1 \exp (b_2 b_M x_e) \) (Michalek et al., 2004). According to the technology options and cost data in NHTSA’s report (NHTSA, 2008), we construct a technology-cost model by combining various fuel economy improvement technologies,\(^3\) as shown in Table 6. The thick and dashed curves represent the upper and lower estimates, respectively, where the technology cost function is given by \( c_j = b_1 x_{ij}^2 + b_2 x_{ij} + b_3 \). With all costs converted into year 2007 dollars using consumer price index (US Census Bureau, 2008), the coefficients of the vehicle manufacturing cost function are \( b_M = 95 \) (base engine power 95 kW), \( c_g = 7836, b_1 = 701, \) and \( b_2 = 0.0063 \). The coefficients for the technology cost curves are upper estimate: \( b_3 = 85936 \) and \( b_4 = 24.7 \), mean: \( b_3 = 34121 \) and \( b_4 = -847 \), and lower estimate: \( b_3 = 16699 \) and \( b_4 = -639 \). For the simulation study in Section 4.3, we use the upper estimate cost curve as a base case since the medium and lower estimate may be both optimistic and underestimate the costs of fuel-saving technology.\(^4\)

4.2. Demand side

In this study, we estimate market demand based on Ward’s Auto 2007 sales data using a mixed logit specification in order to account for consumer preference heterogeneity (Train, 2003). We consider four random coefficients: manufacturer suggested retail price\(^5\) (MSRP) (unit: $10,000), operation cost (unit: cent per mile), power-to-weight ratio (hp per 100 lbs), and

\(^1\) The configurations of the vehicle in ADVISOR are mid-size car body, 95 kW spark-ignition engine (SI95) with engine power scale ranging from 0.8 to 2.0, and an empirical automatic 4-speed transmission module (TX-AUTO4-4L60E) with default control strategies.

\(^2\) We assume that implementation of fuel-saving technology does not affect engine horsepower.

\(^3\) NHTSA’s analysis report points out that synergy or dissynergy (overlapping effectiveness) can exist when implementing multiple fuel-saving technologies into a vehicle (NHTSA, 2008). For instance, when 5-speed auto-transmission is used with variable valve timing with coupled cam phasing (VVT), there is 1% overlapping in fuel consumption reduction. Our technology-cost model has taken this factor into account.

\(^4\) We also examined the medium cost curve and found that firms fully implement the maximum technology (20% reduction) in this case.

\(^5\) Use of MSRP as a proxy for transaction price is a potential source of error; however, transaction price varies by consumer, and the data are unavailable.
footprint (100 square-feet). The base vehicle type is a domestic mid-size vehicle. There are eight dummy variables included for distinguishing different vehicle types. The utility $u_{ij}$ for vehicle $j$ and consumer $i$ under the mixed logit framework is

$$
    u_{ij} = v_{ij} + e_{ij} = (\mu_p + \sigma_p \Phi_H) p_j + (\mu_c + \sigma_c \Phi_C) c_{ij} + (\mu_H + \sigma_H \Phi_H) H_{ij} + (\mu_S + \sigma_S \Phi_S) S_{ij} + \beta_{LP} L_{ij} + \beta_{LC} C_{ij} + \beta_{LS} S_{ij} + b_2 S_{ij} + b_{SC} C_{ij} + b_{CP} P_{ij} + b_{LG} G_{ij} + b_{SP} P_{ij} + b_{IM} M_{ij} + b_{HY} Y_{ij}
$$

where $v_{ij}$ is the observable utility, $e_{ij}$ is the unobservable random utility component, $\mu$ is the mean coefficient, $\sigma$ is the standard deviation, and each $\Phi$ is an i.i.d. normal distribution. The subscripts $P, C, H, S$ represent price, operation cost, horse-power-to-weight ratio, and vehicle size (footprint), respectively. $c$ is gas price in dollar per gallon. The remaining terms $z_{2S}, z_{SC}, z_{CP}, z_{LG}, z_{3S}, z_{SP}, z_{IM},$ and $z_{HY}$ are binary variables for two-seater, subcompact, compact, large, luxury, sports, imported, and hybrid vehicles, respectively, and the betas are corresponding coefficients. Assuming $e_{ij}$ as i.i.d. type I extreme distribution, the mixed logit choice probability for vehicle $j$ becomes (Train, 2003)

$$
    s_j = \frac{\exp(v_{j0})}{\sum_{k=1}^{K} \sum_{j=1}^{J} \exp(v_{j0})} f_\Phi(\Phi) d\Phi \approx \frac{1}{R} \sum_{r=1}^{R} \sum_{k=1}^{K} \sum_{j=1}^{J} \frac{\exp(v_{j0})}{\exp(v_{j0})}
$$

where $f_\Phi(\Phi)$ is the probability density function of the set of distributions $\Phi$. Eq. (16) shows that numerical simulation is required to estimate mixed logit probability since no closed-form expression is available for integration. We use 1000 pseudo-random normal draws ($R = 1000$) and the maximum likelihood method\(^{14}\) for our mixed logit estimation. The results are shown in Table 3. While the mean coefficients of price and operation cost imply that consumers generally prefer lower purchase price and operation cost, there is significant heterogeneity in the degree of importance placed on the attributes. The positive coefficients of power-to-weight ratio (a proxy for acceleration performance) and footprint with small deviations represent consistent preferences for cars with faster acceleration and larger size. The negative coefficients for two-seater and subcompact, and the positive coefficient for large cars matches our expectation of people’s preferences for car class. One exception is that compact

\(^{14}\) We did not account for price endogeneity here.
vehicle has a slightly higher utility than that of mid-size. Since vehicle classes are taken into account in the estimation, the footprint preference implies spacious cars are preferred in every vehicle class. The last four coefficients show that luxury, sports, and hybrid vehicles are appreciated by consumers, whereas imported vehicles are less preferred, all else being equal.

We further estimate the vehicle class own- and cross-elasticities of price by increasing all vehicle prices by 1% in the corresponding segment and observing the change in predicted demand. The price-elasticity matrix is shown in Table 4. The mid-size and compact vehicle segments have lower own-elasticities than other vehicle segments. Furthermore, the price variation of mid-size vehicle has stronger cross-demand influence than other vehicle segments. The demand for sports cars and two-seaters are strongly correlated. The only hybrid vehicle in our 2007 sales data is the Toyota Prius. It can be seen that the price of the hybrid vehicle, as a unique vehicle segment, has less influence on the demand for other vehicles in the market. Since mid-size vehicle is our main focus in the study, we further verify the attribute elasticities of regular domestic mid-size vehicle. The elasticity values with respect to price, operation cost, and power-to-weight ratio are \(1.194\), \(1.349\), and \(0.870\), respectively. The result indicates that operation cost has a higher elasticity than price and power-to-weight ratio.

4.3. Results and discussions

By integrating the demand estimation results in Section 4.2 with the vehicle design model in Section 4.1, the manufacturer’s vehicle design decisions are solved using the framework proposed in Section 3. For the base case study, we use a gasoline price \(\gamma = $2.50\) per gallon and a CAFE penalty of $55 per mpg per car. We simulate 10 generic domestic manufacturers competing in the market. The fuel economy design responses to various CAFE standards are shown in Fig. 7. The solid line represents the result of the base case under different levels of CAFE fuel economy standards. For the 2007 passenger car standard 27.5 mpg, the manufacturer’s fuel economy responses are not binding with CAFE regulation. The 34.6 mpg in region 1 matches the general trend of the passenger car fuel economy at 2007–2008 levels in Fig. 1, where manufacturers are designing cars with higher fuel efficiency than the regulatory level. The situation implies that automakers’ vehicle design decisions are direct responses to market demand and consumer preferences. Thus, CAFE regulation is inactive. The CAFE-binding range (region 2) is between 34.6 and 36.5 mpg. When the regulatory standard rises beyond 36.5 mpg, the optimal equilibrium response is to ignore further increases and pass the CAFE penalty cost along to consumers (region 3).

We then verify the design responses at different levels of CAFE penalty. Fig. 8 shows the history of the CAFE penalty compared to the inflation-adjusted value of the original $50 penalty set in 1978 using gross domestic product (GDP) price index adjustment (US BEA, 2008). Clearly, the CAFE penalty has lagged below inflation. We verify the vehicle design response at the higher penalty $124 level, and the result is shown as a dashed line in Fig. 7. The higher CAFE penalty extends the window of region 2.
Fig. 9 shows a contour plot of Nash responses for a range of CAFE regulatory fuel economy standards and penalty values. The structural effect of two CAFE regulatory parameters to firm’s vehicle design responses is visible: in region 1, when the CAFE standard is less than 34.6 mpg, manufacturer design responses are not affected by the CAFE standard or penalty. In region 2, fuel economy design responses are affected only by the CAFE standard but not the CAFE penalty parameter. In region 3, fuel economy design response is a function of the CAFE penalty but not the CAFE standard, and the border between region 2 and region 3 depends on both the CAFE standard and the CAFE penalty. There are several useful implications of our observations. When firms are binding with the CAFE regulation (region 2), changing mpg standard as a policy tool to urge automakers to improve their fuel economy is effective. However, changes in regulatory standards may not be useful when firms have no incentive to follow the standard (regions 1 and 3). Moreover, when firms violate the CAFE regulation (region 3), increasing the penalty can increase firms’ fuel economy responses, but the improvement may be modest: we find that a $100 increase in penalty would cause a fuel economy increase less than 1 mpg.15

Fig. 7. Design responses to various fuel economy regulatory levels.

Fig. 8. CAFE penalty level and inflation-adjusted value of $50 in 1978.

Fig. 9. Vehicle fuel economy responses under various fuel economy standards and penalty levels.

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15 Additional costs observed by the firm, such as public relations or litigation cost for violation, would extend the effective region of CAFE policy. In practice, domestic automakers hold a policy to treat CAFE as a constraint, due in part to fear of shareholder reaction.
We further analyze vehicle fuel economy responses under different gasoline prices and a fixed CAFE penalty of $55 per vehicle per mpg. We tested the fuel economy response by using two gasoline price levels, $4.20 per gallon and $1.60 per gallon. The former is the highest weekly retail gas price during 2007–2008, and the later is the lowest (EIA, 2009). The analysis results in Fig. 10 show that gasoline price variations offset the entire fuel economy response curve significantly: high fuel prices shift the Nash design responses upward, while low fuel prices shift the response curves to a lower fuel economy region. At high fuel price level, we find that automakers reduce their engine sizes to the lower bound and implement more fuel-saving technology. On the other hand, lower fuel prices create incentives for automakers to design vehicles with more powerful engines and fewer technology options implemented.

The response curves are relatively sensitive to fuel price (because of consumer demand for low operating cost) compared to CAFE standards, despite the fact that CAFE standards more directly address fuel economy (Vlasic, 2008). Thus, policies that influence fuel prices, such as fuel taxes or carbon taxes, may encourage greater vehicle fuel economy improvement than adjusting the CAFE standard if prices are sufficiently high. Indeed, historic data on CAFE (Fig. 1) shows that manufacturers have moved ahead of the CAFE standard in recent years with higher fuel prices.

We also examine the penalty amount required for the Obama administration’s new fuel efficiency target 39 mpg on passenger cars. Our simulation model shows that a high penalty up to $255 would be needed for reaching the target if gasoline price remains at the base level of $2.50 per gallon. In contrast, without CAFE regulations, the target can be reached with a gasoline price $4.40 per gallon. The required increase in CAFE penalty is significantly higher than the 72% increase in gasoline price. Furthermore, at such a high fuel economy level, the equilibrium vehicle design solutions have the smallest engine size at lower bound, and more fuel-saving technology options are implemented (18% fuel-saving), which are close to our model limits of mid-size conventional vehicle. The result implies that automakers should have more sales shift towards small vehicles, such as compact or subcompact cars and have more advanced technology implementations, such as alternative fuel vehicle, hybrids, and plug-in hybrids, when facing the arriving high fuel economy requirements.

5. Conclusions

We pose an oligopoly model of automaker responses to CAFE standards where vehicles are viewed as differentiated products. We find that Nash vehicle design responses to CAFE standards follow a distinctive pattern under general demand, cost, and performance functions and single-vehicle firms: firms ignore low CAFE standards, treat moderate CAFE standards as binding, and violate high CAFE standards, where the point and amount of violation depends on the penalty for violation, and increases in the CAFE standard beyond the violation point are ineffective. While the original penalty for CAFE violation set in 1978 has not been adjusted for inflation, other factors, such as public and government relations costs for violation of CAFE standards, may contribute to extending the range of effective CAFE standards.

Our case study results show that for current models of automotive demand, cost, and performance, vehicle fuel economy responses are more sensitive to fuel prices than to CAFE standards within the ranges examined, and fuel prices address driving patterns in addition to vehicle design. This result may partly support prior conclusions in the literature that Pigovian gasoline taxes are a lower cost option than CAFE policy for reducing gasoline consumption. The effects on vehicle design caused by the increases in CAFE standards set by the Obama administration to a combined 35.5 mpg by 2016 will depend on the path of gasoline prices and the penalties set for violation of CAFE standards. Responding to stricter fuel efficiency standards, such as the European and California corporate average standards on CO2 emissions per mile, will likely require both a shift to smaller, lighter vehicles and an inclusion of alternative technologies, such as cellulose-based ethanol vehicles, hybrid electric vehicles, and plug-in hybrid electric vehicles (Lave et al., 2000).

Fig. 10. Vehicle fuel economy responses under various gasoline prices.

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16 Our demand model assumes that consumer preference is for operating cost, rather than fuel economy, and that preference for operating cost does not vary with fuel price.

17 The equilibrium framework predicts static long run responses and does not account for responses to short run fuel price volatility or uncertainty.
While our results reveal general structural properties of CAFE policy in a differentiated automobile market, there are several limitations that provide opportunities for future work: first, the assumption of a single vehicle design per producer helps to produce closed-form results and general conclusions; however, it restricts the ability to predict sales shifts from one vehicle type to another and instead presumes that firms respond only through redesign. The role of consumer heterogeneity and differences in firm brand and cost structures must be better understood in order to predict product line design response in equilibrium. Second, uncertainties in market demand estimations may significantly affect the robustness of the automakers’ optimal design decisions. Characterization of uncertainty propagation in a market competition framework is needed. Attribute-based CAFE standards will imply different incentives for vehicle responses than single-target standards. Further study of attribute-based standards is warranted. Finally, non-regulation-fee costs for CAFE violation could be incorporated to examine the general effect and the effect when some firms observe higher costs than others. We see opportunity for a range of potential follow-up studies examining transportation policy while accounting for the effects of differentiated vehicle design, consumer responses to differentiated products, and technical trade-offs in the ability to achieve vehicle attributes that are competitive in the regulated marketplace.

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Appendix A. Mixed logit demand model

The logit model is a random utility model, by which the utility of an individual consumer $i$ selecting vehicle $j$ is comprised of an observable component $v_i$ and an unobservable random error component $\epsilon_i$:

$$u_i = v_i + \epsilon_i = v_i(p_i, z_i) + \epsilon_i \quad (A.1)$$

The observable utility is a function of vehicle price $p_i$, vehicle attributes $z_i$ (including fuel economy), and consumer $i$’s preference coefficients $\beta_i$. When the unobservable random component is assumed to be an independent and identically distributed (i.i.d.) standard Gumbel distribution, the probability that $u_i > u_j$ ($j \neq i$), i.e., the share of choices $s_j$ or the probability that a randomly selected consumer will choose product $j$ over the alternatives, can be simplified into an integral expression conditional on the $\beta$ coefficients:

$$s_j = \int_{\beta} \frac{\exp(v_{ij})}{\sum_k \sum_f \exp(v_{kj})} f_\beta(d\beta)$$

where $f_\beta(d\beta)$ is the probability density function that describes the distribution of consumer preference coefficients over the population. This model is called mixed logit or random coefficients logit, and it can approximate any random utility model (McFadden and Train, 2000). The standard multinomial logit can be seen as a special case of the mixed logit, where the coefficients are competitive in the regulated marketplace. There are several limitations that provide opportunities for future work: first, the assumption of a single vehicle design per producer helps to produce closed-form results and general conclusions; however, it restricts the ability to predict sales shifts from one vehicle type to another and instead presumes that firms respond only through redesign. The role of consumer heterogeneity and differences in firm brand and cost structures must be better understood in order to predict product line design response in equilibrium. Second, uncertainties in market demand estimations may significantly affect the robustness of the automakers’ optimal design decisions. Characterization of uncertainty propagation in a market competition framework is needed. Attribute-based CAFE standards will imply different incentives for vehicle responses than single-target standards. Further study of attribute-based standards is warranted. Finally, non-regulation-fee costs for CAFE violation could be incorporated to examine the general effect and the effect when some firms observe higher costs than others. We see opportunity for a range of potential follow-up studies examining transportation policy while accounting for the effects of differentiated vehicle design, consumer responses to differentiated products, and technical trade-offs in the ability to achieve vehicle attributes that are competitive in the regulated marketplace.

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$$q_j = Qs_j = Q \int_{\beta} \frac{\exp(v_{ij})}{\sum_k \sum_f \exp(v_{kj})} f_\beta(d\beta) \quad (A.3)$$

$$\frac{\partial q_j}{\partial p_j} = Q \frac{\partial s_j}{\partial p_j} = Q \int_{\beta} \frac{\partial v_{ij}}{\partial p_j} s_j(1 - s_j) f_\beta(d\beta) \quad (A.4)$$

$$\frac{\partial q_j}{\partial x_{nj}} = Q \frac{\partial s_j}{\partial x_{nj}} = Q \int_{\beta} \left( \frac{\partial v_{ij}}{\partial z_j} \right)^T \left( \frac{\partial z_j}{\partial x_{nj}} \right) s_j(1 - s_j) f_\beta(d\beta) \quad (A.5)$$

**Case. 1 (surpassing the CAFE standard)**

The price solution with mixed logit demand function is expressed as:

$$\int_{\beta} s_{ij} \left( \frac{\partial v_{ij}}{\partial p_j} (1 - s_{ij}) (p_j - c_j) + 1 \right) f_\beta(d\beta) = 0 \quad (A.6)$$

The design FOC equation with mixed logit demand function is:

$$\int_{\beta} s_{ij} \left( \frac{\partial v_{ij}}{\partial z_j} \right)^T \left( \frac{\partial z_j}{\partial x_{nj}} \right) (1 - s_{ij}) (p_j - c_j) - \frac{\partial c_j}{\partial x_{nj}} f_\beta(d\beta) = 0 \quad (A.7)$$
Case. 2 (equal to CAFE standard)

The price FOC equation is:

$$\int s_j \frac{\partial v_j \partial}{\partial p_j} (1 - s_j) (p_j - c_j) + 1) f_0(\beta) d\beta = 0$$  \hspace{1cm} (A.8)

The design solution is the inverse function of vehicle fuel economy equal to regulation standard $\kappa$:

$$x_j = f^{-1}_e(\kappa)$$  \hspace{1cm} (A.9)

Case. 3 (violating the standard)

The price solution equation is:

$$\int s_j \frac{\partial v_j \partial}{\partial p_j} (1 - s_j) (p_j - c_j - \rho x_j) + 1) f_0(\beta) d\beta = 0$$  \hspace{1cm} (A.10)

The design FOC equation is:

$$\int s_j \frac{\partial v_j \partial}{\partial z_j} (\frac{\partial z_j}{\partial x_j}) (1 - s_j) (p_j - c_j - \rho x_j) + \rho \frac{\partial z_j}{\partial x_j} - \frac{\partial c_j}{\partial x_j}) f_0(\beta) d\beta = 0$$  \hspace{1cm} (A.11)

References


European Commission, 2007. Results of the review of the community strategy to reduce CO2 emissions from passenger cars and light-commercial vehicles.


Regulatory Analysis 03-10.


US Census Bureau, 2008. Consumer Price Index History Table.