Optimal Product Design Under Price Competition

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Engineering optimization methods for new product development model consumer demand as a function of product attributes and price in order to identify designs that maximize expected profit. However, prior approaches have ignored the ability of competitors to react to a new product entrant. We pose an approach to new product design accounting for competitor pricing reactions by imposing Nash and Stackelberg conditions as constraints, and we test the method on three product design case studies from the marketing and engineering design literature. We find that new product design under Stackelberg and Nash equilibrium cases are superior to ignoring competitor reactions. In our case studies, ignoring price competition results in suboptimal design and overestimation of profits by 12–79%, and we find that a product that would perform well in today's market may perform poorly in the market that the new product will create. The efficiency, convergence stability, and ease of implementation of the proposed approach enable practical implementation for new product design problems in competitive market systems.
[DOI: 10.1115/1.3125886]

Keywords: design for market systems, game theory, Nash equilibrium, Stackelberg, demand model, logit, mathematical programs with equilibrium constraints, new product development

1 Introduction

Product design optimization problems that account for competitive market decisions can be categorized into two groups: short-run price equilibrium and long-run design equilibrium [1–4]. The long-run scenario represents competition over a sufficiently long time period that all firms in the market are able to redesign their products as well as set new prices competitively [4–8]. Short-run competition assumes that the design attributes of competitor products are fixed, but that competitors will adjust prices in response to a new entrant [9–11]. We focus here on new product design problems in short-run price competition.

Table 1 lists prior studies for price competition in product design and distinguishes them by solution approach, demand model type, equilibrium type, case studies, and presence of design constraints. The solution approach is the method for finding the design solution under price competition. The demand model type specifies the market demand function formulation. Equilibrium type distinguishes Nash and Stackelberg models [12]: Nash equilibrium refers to a point at which no firm can achieve higher profit by unilaterally selecting any decision other than the equilibrium decision. The Stackelberg case, also known as the leader-follower model, assumes that the leader is able to predict the response of followers, in contrast with the Nash model, which assumes that each firm only observes competitor responses. The Stackelberg case is appropriate for cases where one player is able to "move first," and the introduction of a new product entrant is a case where the firm could exploit this first-move advantage. Finally, the penultimate column in Table 1 identifies whether the model incorporates design constraints representative of tradeoffs typically present in engineering design.

Choi et al. [2] (henceforth CDH) proposed an algorithm for solving new product design problem under price competition while treating the new product entrant as Stackelberg leader. They tested the method on a pain reliever example with ingredient levels as decision variables and an ideal point logit demand model with linear price utility. The study applied the variational inequality relaxation algorithm [13] to solve the follower Nash price equilibria. In Sec. 3, we use CDH's problem as a case study and show that the method can have convergence difficulties, and as a result the approximate Stackelberg solution found by their algorithm was not fully converged.

In contrast to the continuous decision variables used by CDH, other prior approaches restrict attention to discrete decision variables that reflect product attributes observed by consumers, as opposed to design variables controlled by designers under technical tradeoffs. We refer to the focus on product attributes as product positioning, in contrast to product design. These product positioning problems assume that all combinations of discrete variables are feasible, thus no additional constraint functions are considered. Horsky and Nelson [9] used historic automobile market data to construct a logit demand model and cost function using four product attribute decision variables. With five levels for each of their four variables, they applied exhaustive enumeration to solve for equilibrium prices of all 625 possible new product entrant combinations using first-order condition (FOC) equations. Rhim and Cooper [10] presented a two-stage method incorporating genetic algorithms and FOCs to find Nash solutions for new product positioning problems. The model allows multiple new product entries to target different user market segments. The product in the study is a liquid detergent with two attributes. Recently, Lou et al. [11] conducted a study for optimal new product positioning of a handheld angle grinder under Nash price competition in a manufacturer-retailer channel. There are four product attributes with various levels in the problem, resulting in 72 possible combinations. Similar to Ref. [9], the study also used a discrete selection method, but the design candidates were prescreened to a smaller number in order to avoid full exhaustive enumeration, and the profits of a few final candidates at Nash price equilibrium were calculated through a sequential iterative optimization approach. Prior approaches to product design and positioning under price competition suffer from inefficient com-

Contributed by the Design Automation Committee of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received October 9, 2008; final manuscript received March 23, 2009; published online June 4, 2009. Review conducted by Shapour Azarm. Paper presented at the ASME 2008 Design Engineering Technical Conferences and Computers and Information in Engineering Conference (DETC2008), Brooklyn, NY, August 3–6, 2008.

Table 1 Literature on new product design optimization under price competition

Literature	Solution approach	Price equilibrium	Demand model	Design constraints	Case study
Choi et al. [2]	Iterative variational inequality algorithm	Stackelberg	Ideal point logit	Yes	Pain reliever
Horsky and Nelson [9]	Discrete selection from FOC solutions	Nash	Logit	No	Automobile
Rhim and Cooper [10]	Two-stage genetic algorithm	Nash	Ideal point logit	No	Liquid detergents
Lou et al. [11]	Discrete selection and iterative optimization	Nash	HB mixed logit	No	Angle grinder
This paper	NLP/MINLP with KKT constraints	Nash/Stackelberg	Logit and latent class models	Yes	Pain reliever, weight scale, and power grinder

putation and convergence issues due to iterative strategies to identify equilibria and combinatorial limitations of discrete attribute models.

We propose an alternative approach to find optimal design and equilibrium competition solutions without iterative optimization of each firm. Our approach poses a nonlinear programming (NLP) or mixed-integer nonlinear programming (MINLP) formulation for new product profit maximization with respect to prices and design variables subject to first-order necessary conditions for the Nash price equilibrium of competitors. We examine three case studies from the literature and show that accounting for competitor price competition can result in different optimal design decisions than those determined under the assumption that competitors will remain fixed. The approach is well-suited to engineering design optimization problems, requiring little additional complexity and offering greater efficiency and convergence stability than prior methods, particularly for the highly-constrained problems found in engineering design.

The remainder of the article is organized as follows. In Sec. 2, we explain the detailed formulation of the proposed approach with Nash and Stackelberg competition models, and we introduce a modified Lagrangian formulation to accommodate cases with variable bounds. In Sec. 3, we demonstrate the proposed approach by solving three product design examples from the literature, and we conclude in Sec. 4.

2 Proposed Methodology

For a new product design problem under short-run competition, there are three sets of decision variables to be determined—new product design variables, new product price, and prices of competitor products. Since price competition is one of the key elements affecting profit outcomes of a new product design, a reliable and efficient Nash price solution method is necessary. A Nash equilibrium problem can be solved by different numerical approaches, including the relaxation method [13], projection method [14], nonlinear complementarity problem approaches [15], fixed-point iteration method [16], and FOC method [17]. Each approach has its strengths and disadvantages. The relaxation method, which is derived from variational inequality theory, is a sequentially iterative optimization approach where each firm is optimized in turn while holding all other firms fixed, and the process is repeated sequentially over all firms until convergence [14]. The relaxation method has been used in solving practical equilibrium problems [2,7,13]; however, it requires an optimization process at each iteration, which results in long computational time and slow convergence. The projection method, another variational inequality-based approach, does not require an optimization loop and is computationally efficient, but it may not always converge [18]. The nonlinear complementarity approaches have been considered powerful tools to solve equilibrium problems [19]. However, they require specific solvers, e.g., the PATH solver [20] and a reformulation of the equilibrium problem into complementarity form. The fixed-point iteration method also requires derivations for the specific fixed-point equations [16].

In this study, we select the FOC method as our primary Nash price solution tool for several reasons. First, its solution process does not require iterative optimization loops. Second, the method only requires first-order derivatives of the profit function with respect to the decision variables, and no further reformulation needed. The first-order equations can usually be derived analytically, and when closed-form expressions are not available, numerical differentiation can apply. Finally, the system of first-order equations can be solved by general-purpose NLP algorithms, and no specialized solver is required. It is worthy of note that solutions satisfying FOCs only satisfy necessary conditions. If the profit function is concave with respect to price, the FOCs become sufficient[1] [17]. However, in the case of nonconcavity, the solutions found by the proposed method must be verified using the Nash equilibrium definition post-hoc. Taking price as an example, the mathematical expression of a Nash equilibrium is given by

$$\Pi_k(p_1^*, \ldots, p_j^*, \ldots, p_J^*) \geq \Pi_k(p_1^*, \ldots, p_j, \ldots, p_J^*)$$

$$\forall j \in J_k, \quad \forall k \in K \tag{1}$$

where Π_k is the payoff (profit) function of firm k, p_j is the price decision of product j of firm k, and the (*) denotes the decisions at Nash equilibrium [12]. This formulation states that no unilateral change to a single firm's price decision can result in higher profit for that firm than its Nash price, or, alternatively, each firm is responding optimally to the decision of the others.

In Secs. 2.1–2.3, we describe the proposed product design optimization models under Nash and Stackelberg games incorporating the FOC method. We then examine the special cases where prices are constrained and develop a Lagrangian extension for this case. The major assumptions for the proposed approaches are the following: (1) the focal firm designs a set of differentiated products that will enter into a market with existing products sold by competitors; (2) competitors are Nash price setters for profit maximization with fixed product attributes; (3) competitor product attributes are observed by the focal firm; and (4) price is continuous, and each firm's profit function is differentiable with respect to its price variable.

2.1 Profit Maximization Under the Nash Model. The necessary condition for an unconstrained Nash price equilibrium (Eq. (1)) can be expressed using the FOC equation $\partial\Pi_k/\partial p_j=0$ for all products j produced by each firm k [17]. For short-run Nash competition, new product design variables, new product price, and

[1] For a noncooperative game with complete information, a Nash equilibrium exists if: (1) the strategy set is nonempty, compact, and convex for each player; (2) the payoff function is defined, continuous, and bounded; and (3) each individual payoff function is concave with respect to individual strategy [17]. More specifically, Anderson et al. [21] proved that there exists a unique price equilibrium under logit demand when the profit function is strictly quasiconcave.

competitors' prices follow the Nash framework, which forms three sets of simultaneous equations. If there are no additional constraints on design variables $\mathbf{x}_j$ and prices p_j, the FOC equations are

$$\frac{\partial \Pi_k}{\partial \mathbf{x}_j} = \mathbf{0}; \quad \frac{\partial \Pi_k}{\partial p_j} = 0; \quad \frac{\partial \Pi_{k'}}{\partial p_{j'}} = 0$$

where

$$\Pi_k = \sum_{j \in J_k} q_j(p_j - c_j), \quad \Pi_{k'} = \sum_{j \in J_{k'}} q_{j'}(p_{j'} - c_{j'})$$

$$q_j = Qs_j, \quad s_j = f_{\mathrm{S}}(p_j, \mathbf{z}_j, p_{\hat{j}}, \mathbf{z}_{\hat{j}} \quad \forall \hat{j} \neq j)$$

$$q_{j'} = Qs_{j'}, \quad s_{j'} = f_{\mathrm{S}}(p_{j'}, \mathbf{z}_{j'}, p_{\hat{j}'}, \mathbf{z}_{\hat{j}'} \quad \forall \hat{j}' \neq j')$$

$$\mathbf{z}_j = f_{\mathrm{Z}}(\mathbf{x}_j), \quad c_j = f_{\mathrm{C}}(\mathbf{x}_j, q_j)$$

$$\forall j \in J_k, \quad \forall j' \in J_{k'}, \quad \forall k' \in K \setminus k \tag{2}$$

where Π_k is the net profit of all new products J_k from producer k, and $\Pi_{k'}$ is the net profit sum of the products of competitor k'. Each new product j has design vector $\mathbf{x}_j$, attribute vector $\mathbf{z}_j$ (as a function of the design $\mathbf{z}_j = f_{\mathrm{Z}}(\mathbf{x}_j)$), price p_j, unit cost c_j (as a function of the design and production volume $c_j = f_{\mathrm{C}}(\mathbf{x}_j, q_j)$), estimated market share s_j (as a function of the attributes and prices of all products in the market $s_j = f_{\mathrm{S}}(p_j, \mathbf{z}_j, p_{\hat{j}}, \mathbf{z}_{\hat{j}} \ \forall \hat{j} \neq j)$) and estimated demand q_j. The total size of the market is Q. Each competitor $k' \in K \backslash k$ has price decisions $p_{j'}$ with fixed design attributes $\mathbf{z}_{j'}$ for all its products $\forall j' \in J_{k'}$. In the equation set, the $\partial \Pi_k / \partial \mathbf{x}_j$ and $\partial \Pi_k / \partial p_j$ equations represent the FOCs of the Nash design and price decisions of the new product, and the $\partial \Pi_{k'} / \partial p_{j'}$ equations are the FOCs for the price decisions of competitor products.

While Eq. (2) shows the fundamental structure of the Nash equation set, it does not account for design constraints and price bounds. Constraints on design variables are typical in engineering design problems. Bounds on price may be imposed by manufacturer, retailer, consumer, or government policies, and they may also be used to indicate model domain bounds. To account for these cases, we propose a generalized formulation incorporating Lagrange multipliers into Eq. (2) and present it in an NLP form as follows:

$$\text{maximize} \ \ \Pi_k = \sum_{j \in J_k} q_j(p_j - c_j)$$

$$\text{with respect to} \ \ \mathbf{x}_j, p_j, \boldsymbol{\lambda}_j, \boldsymbol{\mu}_j, \overline{\boldsymbol{\mu}}_j, p_{j'}, \overline{\boldsymbol{\mu}}_{j'}$$

$$\text{subject to} \ \ \frac{\partial L_k}{\partial \mathbf{x}_j} = \mathbf{0}, \quad \frac{\partial L_k}{\partial p_j} = 0, \quad \frac{\partial L_{k'}}{\partial p_{j'}} = 0$$

$$\mathbf{h}(\mathbf{x}_j) = \mathbf{0}, \quad \mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}, \quad \boldsymbol{\mu}_j \geq \mathbf{0}, \quad -\boldsymbol{\mu}_j^T \mathbf{g}(\mathbf{x}_j) \leq t$$

$$\overline{\mathbf{g}}(p_j) \leq \mathbf{0}, \quad \overline{\boldsymbol{\mu}}_j \geq \mathbf{0}, \quad -\overline{\boldsymbol{\mu}}_j^T \overline{\mathbf{g}}(p_j) \leq t$$

$$\overline{\mathbf{g}}(p_{j'}) \leq \mathbf{0}, \quad \overline{\boldsymbol{\mu}}_{j'} \geq \mathbf{0}, \quad -\overline{\boldsymbol{\mu}}_{j'}^T \overline{\mathbf{g}}(p_{j'}) \leq t \tag{3}$$

where

$$L_k = \Pi_k + \sum_{j \in J_k} (\boldsymbol{\lambda}_j^T \mathbf{h}(\mathbf{x}_j) + \boldsymbol{\mu}_j^T \mathbf{g}(\mathbf{x}_j) + \overline{\boldsymbol{\mu}}_j^T \overline{\mathbf{g}}(p_j))$$

$$L_{k'} = \Pi_{k'} + \sum_{j' \in J_{k'}} (\overline{\boldsymbol{\mu}}_{j'}^T \overline{\mathbf{g}}(p_{j'})), \quad \Pi_{k'} = \sum_{j \in J_{k'}} q_{j'}(p_{j'} - c_{j'})$$

$$q_j = Qs_j, \quad s_j = f_{\mathrm{S}}(p_j, \mathbf{z}_j, p_{\hat{j}}, \mathbf{z}_{\hat{j}} \quad \forall \hat{j} \neq j)$$

$$q_{j'} = Qs_{j'}, \quad s_{j'} = f_{\mathrm{S}}(p_{j'}, \mathbf{z}_{j'}, p_{\hat{j}'}, \mathbf{z}_{\hat{j}'} \quad \forall \hat{j}' \neq j')$$

$$\forall j \in J_k, \quad \forall j' \in J_{k'}, \quad \forall k' \in K \setminus k$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and $\overline{\boldsymbol{\mu}}$ are the Lagrange multiplier vectors for the design equality constants $\mathbf{h}$, design inequality constraints $\mathbf{g}$, and price bounds $\overline{\mathbf{g}}$, respectively. The above formulation determines the profit-maximizing new product design $\mathbf{x}_j$ and price p_j that are in Nash equilibrium with competitor prices $p_{j'}$, $\forall j' \in J_{k'}$ and $\forall k' \in K \backslash k$. The objective function[2] is the total profit Π_k of producer k. The equality and inequality constraints $\mathbf{h}(\mathbf{x}_j)$ and $\mathbf{g}(\mathbf{x}_j)$ define the feasible domain of the engineering design, and the inequality constraint $\overline{\mathbf{g}}$ accounts for the price bounds. The FOCs of the Lagrangian equations with additional inequality constraints represent the Karush-Kuhn-Tucker (KKT) necessary condition [22] of Nash equilibrium for regular points [4]. Such a formulation has been known as mathematical programs with equilibrium constraints (MPECs)[3] [24]. Since MPECs do not satisfy constraint qualifications, it can induce numerical instability in convergence [25,26]. For resolving the issue, various algorithms and reformulation approaches have been proposed [24,27,28]. In this study, we follow a regularization scheme presented by Ralph and Wright [28] and introduce a positive relaxation parameter t into the KKT complementary slackness conditions (Eq. (3)). The regularized NLP formulation can avoid the constraint qualification failures of MPECs and result in strong stationarity and second-order sufficient condition near a local solution of the MPEC [28]. The competitors' prices obtained from Eq. (3) are solutions based on necessary conditions. If the profit function is nonconcave, the solutions need to be tested based on Eq. (1) for verifying sufficient conditions. We take the FOC solution and optimize each individual producers' profit with respect to its own pricing decisions while holding other producer's decisions fixed. If no higher profit is found throughout the test, the price solutions are Nash prices.

2.2 Profit Maximization Under the Stackelberg Model. For the proposed Stackelberg competition model, it is assumed that the new product enters the market as a leader, while other competitors react as followers. Followers observe others' price decisions, including the new product price, as exogenous variables and compete with one another to reach a Nash price equilibrium. The new product leader is able to predict its followers' Nash price settings within its optimization, giving it an advantage. The constrained formulation using a Stackelberg model is expressed in the following NLP form:

$$\text{maximize} \ \ \Pi_k = \sum_{j \in J_k} q_j(p_j - c_j)$$

$$\text{with respect to} \ \ \mathbf{x}_j, p_j, p_{j'}, \overline{\boldsymbol{\mu}}_{j'}$$

$$\text{subject to} \ \ \mathbf{h}(\mathbf{x}_j) = \mathbf{0}, \quad \mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}, \quad \overline{\mathbf{g}}(p_j) \leq \mathbf{0}$$

$$\frac{\partial L_{k'}}{\partial p_{j'}} = 0, \quad \overline{\mathbf{g}}(p_{j'}) \leq \mathbf{0}, \quad \overline{\boldsymbol{\mu}}_{j'} \geq \mathbf{0}, \quad -\overline{\boldsymbol{\mu}}_{j'}^T \overline{\mathbf{g}}(p_{j'}) \leq t$$

where

$$L_{k'} = \Pi_{k'} + \sum_{j' \in J_{k'}} (\overline{\boldsymbol{\mu}}_{j'}^T \overline{\mathbf{g}}(p_{j'})), \quad \Pi_{k'} = \sum_{j \in J_{k'}} q_{j'}(p_{j'} - c_{j'})$$

$$q_j = Qs_j, \quad s_j = f_{\mathrm{S}}(p_j, \mathbf{z}_j, p_{\hat{j}}, \mathbf{z}_{\hat{j}} \quad \forall \hat{j} \neq j)$$

[2] Note that the objective function of the NLP form is not needed to identify points that satisfy Nash necessary conditions; however, in practice including the objective of producer k can help to also enforce (local) sufficiency conditions for producer k. Sufficiency for competitors must be determined post-hoc.

[3] The formulation should be distinguished from equilibrium problems with equilibrium constraints (EPECs) [23] since no separate upper and lower level equilibria exist and the focal firm is in Nash price competition with competitors.

$$q_{j'} = Qs_{j'}, \quad s_{j'} = f_S(p_{j'}, \mathbf{z}_{j'}, p_{\hat{j}'}, \mathbf{z}_{\hat{j}'} \quad \forall \hat{j}' \neq j')$$

$$\mathbf{z}_j = f_Z(\mathbf{x}_j), \quad c_j = f_C(\mathbf{x}_j, q_j)$$

$$\forall j \in J_k, \quad \forall j' \in J_{k'}, \quad \forall k' \in K \setminus k \tag{4}$$

Nash sufficiency conditions for followers must be verified post-hoc as previously described. Comparing Eq. (4) to Eq. (3), the Stackelberg case relaxes the constraint requiring the focal firm to be in Nash equilibrium. Stated as a relaxation, it is clear that the focal firm's profit will be at least as large with the Stackelberg case as with the Nash case.[4]

Compared with the solution approaches in literature, the proposed method has significant advantages in several aspects. First, the approach is able to solve the problem in a single step if a unique design solution with price equilibrium exists.[5] Second, since the approaches employ FOC equations to find equilibrium prices, the convergence of the whole formulation is faster and more stable than prior approaches that use iteration loops. Third, the formulations can be solved using general-purpose NLP solvers with minimum additional programming effort. When discrete design variables exist, the NLP model becomes a MINLP problem. With the price equilibrium constraints remaining in the continuous domain, MINLP solvers [29–31] can be used to solve the Stackelberg formulation (Eq. (4)).[6] MPEC problems with discrete-constraints have been studied in the literature [32–35], but we do not pursue them here.

2.3 Evaluation. In order to compare profitability of the new product design arrived at under different modeling assumptions, we define the following three profit terms:

(1) *Model-estimated profit*. Profit of the design and price solution to a particular game model, i.e., fixed, Nash, or Stackelberg, as estimated by that model.
(2) *Competitor-reacted profit*. Profit of the design and price solution to a particular game model via post-hoc computation of competitor price equilibrium. The profit represents the market performance of a particular design and pricing solution if competitors adjust prices in response to the new entrant. Competitor-reacted profit is equal to model-estimated profit for the new product using the Nash and Stackelberg games, but if the new entrant is optimized while assuming fixed competitors, the difference between model-estimated and competitor-reacted profit measures the impact of ignoring competitor price adjusting reactions.
(3) *Price-equilibrium profit*. Profit of the design solution as estimated via post-hoc computation of the price equilibrium of *all* firms (including the new entrant).[7] The equilibrium profit represents the profit that a particular design solution would realize if all firms adjust prices in response to the new entrant and reach a market equilibrium. Equilibrium profit is equal to model-estimated profit for new product design using the Nash and Stackelberg games, but if the new entrant is optimized while assuming fixed competitors, the difference between model-estimated and price-equilibrium profit measures the impact of ignoring competitors' price reactions on the *design of the product*, assuming that poor pricing choices can be corrected in the marketplace after product launch.

[4] CDH [2] used a duopoly game to prove that a Stackelberg leader model can always receive at least as high a payoff as a Nash model if a Stackelberg equilibrium exists.

[5] For the cases of multiple local optima and price equilibria, multistart can be implemented to identify solutions.

[6] Discrete decision variables cannot be implemented in the Nash formulation (Eq. (3)) since KKT conditions assume continuity.

[7] For Stackelberg, price-equilibrium profit is calculated from Stackelberg pricing.

3 Case Studies

We examine three product design case studies from the literature to test the proposed approach and examine the improvement that the Stackelberg and Nash approaches can make with respect to methods that ignore competitive reactions. Each case study involves different product characteristics, utility functions, demand models, variable types, and design constraints. For each case, we solve the problems using the traditional fixed-competitor approach and compare with the proposed Nash and Stackelberg approaches. We also compare the computational efficiency and convergence of the proposed methods with the relaxation methods [14].

3.1 Case Study 1: Pain Reliever. The pain reliever problem was introduced by CDH [2]; price and product attributes of a new pain reliever product are to be determined for maximizing profit in the presence of 14 existing competitor products in the market. This new product design case is a product positioning problem, and thus the attributes of a product are identical to its decision variables ($\mathbf{z}=\mathbf{x}$). Each product has four attributes of pharmaceutical ingredient weight (unit in mg), including aspirin z_1, aspirin substitute z_2, caffeine z_3, and additional ingredients z_4. The product specifications[8] and initial prices of competitor products are listed in Table 2. There are two highlights in the model. First, the product H is assumed to be a generic brand, which has a fixed price of \$1.99 [2]. The generic brand product does not participate in the price competition. Second, there are five products, A, C, I, K, and L, with identical product attributes and costs.

The demand model is an ideal point model with observable utility v, given by

$$v_{ij} = -\left(\sum_{n=1}^{N} \beta_i (z_{nj} - \theta_{in})^2 + \bar{\beta}_i p_j + b_i\right) \quad \forall i,j \tag{5}$$

where z_{nj} is the value of the product attribute n on product j, θ_{in} is consumer i's desired value for attribute n, β_i is consumer i's sensitivity of utility to deviation from the ideal point, $\bar{\beta}_i$ is consumer i's sensitivity of utility to price, and b_i is a constant utility term estimated from consumer i. In this model, product attributes that deviate from ideal attributes cause reduced utility, which is less preferred by consumers. Under the standard assumption that utility u_{ij} is partly observable v_{ij} and partly unobservable ε_{ij} so that $u_{ij}=v_{ij}+\varepsilon_{ij}$, and that the unobservable term ε_{ij} assumed to be an independent and identically-distributed (IID) random variable with a standard Gumbel distribution, the resulting choice probability is defined in logit form with an outside good of utility $v_{i0}=0$ [36]:

$$s_{ij} = \frac{\exp(\chi v_{ij})}{1 + \sum_{j' \in J} \exp(\chi v_{ij'})} \quad \forall i,j \tag{6}$$

The weighting coefficient χ is equal to 3, given by CDH. The profit function is

$$\Pi_j = q_j(p_j - c_j) = \left(Q\frac{1}{I}\sum_{i=1}^{I} s_{ij}\right)(p_j - c_j) \quad \forall j \tag{7}$$

In this problem, the market demand and profit are based on a simulated market size of 30 consumers. The FOC equation for the price is[9]

[8] The values of aspirin substitute are the weighted combination of acetaminophen and ibuprofen. The numbers are not provided in the original paper [2], and we obtained the attribute data from the mixed complementarity programming library (MCPLIB) [37] and verified with the original author. The data of consumer preference weightings (30 individuals) are also included in that library.

[9] The derivations of all FOC equations in this paper are included in a separate supporting information document that is available by contacting the authors.

Table 2 Specifications of existing pain reliever products in the market

Product	Aspn. z_1 (mg)	Aspn. sub. z_2 (mg)	Caff. z_3 (mg)	Add. ingd. z_4 (mg)	Cost c ($)	Initial price p ($)
A	0	500	0	0	$4.00	$6.99
B	400	0	32	0	$1.33	$3.97
C	0	500	0	0	$4.00	$5.29
D	325	0	0	150	$1.28	$3.29
E	325	0	0	0	$0.98	$2.69
F	324	0	0	100	$1.17	$3.89
G	421	0	32	75	$1.54	$5.31
H	500	0	0	100	$1.70	$1.99
I	0	500	0	0	$4.00	$5.75
J	250	250	65	0	$3.01	$4.99
K	0	500	0	0	$4.00	$7.59
L	0	500	0	0	$4.00	$4.99
M	0	325	0	0	$2.60	$3.69
N	227	194	0	75	$2.38	$4.99
Cost	0.3	0.8	0.4	0.2	(unit: $/100 mg)	

Table 3 New product design and competitor price solutions for the pain killer problem

		Fixed competitor	Nash	Stackelberg	CDH solution
New product design and price	$x_1=z_1$	124.0	102.7	101.5	102.1
	$x_2=z_2$	201.0	222.3	223.5	222.9
	$x_3=z_3$	0	0	0	0
	$x_4=z_4$	0	0	0	0
	Price	$3.74	$3.85	$3.74	$3.77
	Cost	$1.98	$2.09	$2.09	$2.38
Model-estimated profit		$8.60 (16.3%)	$7.78 (14.7%)	$7.80 (15.7%)	$8.16 (16.1%)
Competitor-reacted profit		$7.68 (14.5%)	$7.78 (14.7%)	$7.80 (15.7%)	$7.80 (15.5%)
Price-equilibrium profit		$7.68 (14.5%)	$7.78 (14.7%)	$7.80 (15.7%)	$7.80 (15.7%)
Price, market share %, profit of competitors at price equilibrium	A	$6.25, 3.55%, $2.39	$6.27, 3.45%, $2.35	$6.29, 3.41%, $2.34	$6.29, 3.41%, $2.34
	B	$2.26, 6.15%, $1.71	$2.26, 6.18%, $1.73	$2.26, 6.16%, $1.72	$2.26, 6.16%, $1.72
	C	$6.25, 3.55%, $2.39	$6.27, 3.45%, $2.35	$6.29, 3.41%, $2.34	$6.29, 3.41%, $2.34
	D	$2.27, 7.73%, $2.31	$2.28, 7.78%, $2.34	$2.28, 7.73%, $2.32	$2.28, 7.73%, $2.32
	E	$1.97, 11.4%, $3.39	$1.97, 11.5%, $3.42	$1.97, 11.3%, $3.39	$1.97, 11.3%, $3.39
	F	$2.18, 9.10%, $2.75	$2.18, 9.16%, $2.78	$2.19, 9.08%, $2.76	$2.19, 9.08%, $2.76
	G	$2.47, 4.60%, $1.28	$2.47, 4.63%, $1.29	$2.47, 4.62%, $1.29	$2.47, 4.62%, $1.29
	H	$1.99, 7.52%, $0.65	$1.99, 7.57%, $0.66	$1.99, 7.56%, $0.66	$1.99, 7.56%, $0.66
	I	$6.25, 3.55%, $2.39	$6.27, 3.45%, $2.35	$6.29, 3.41%, $2.34	$6.29, 3.41%, $2.34
	J	$4.76, 3.37%, $1.77	$4.76, 3.36%, $1.76	$4.77, 3.29%, $1.74	$4.77, 3.29%, $1.74
	K	$6.25, 3.55%, $2.39	$6.27, 3.45%, $2.35	$6.29, 3.41%, $2.34	$6.29, 3.41%, $2.34
	L	$6.25, 3.55%, $2.39	$6.27, 3.45%, $2.35	$6.29, 3.41%, $2.34	$6.29, 3.41%, $2.34
	M	$4.29, 11.4%, $5.80	$4.26, 11.5%, $5.70	$4.26, 11.2%, $5.59	$4.27, 11.2%, $5.60
	N	$3.90, 6.36%, $2.90	$3.93, 6.33%, $2.93	$3.95, 6.11%, $2.88	$3.95, 6.11%, $2.88

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{i=1}^{I} s_{ij}[1 - \chi\bar{\beta}_i(p_j - c_j)(1 - s_{ij})] = 0 \quad \forall j \tag{8}$$

Two constraint functions on the new product design are given by the ingredient weight limitations [2]:

$$g_1 = 325 - x_{1j} - x_{2j} \leq 0, \quad g_2 = x_{1j} + x_{2j} - 500 \leq 0 \tag{9}$$

By applying the above equations into the Nash and Stackelberg formulations of Eqs. (3) and (4),[10] the model was solved using the sequential quadratic programming (SQP) active-set solver in the MATLAB Optimization Toolbox. The solutions to the pain reliever problem with fixed competitors, Nash, and Stackelberg approaches are presented in Table 3, with CDH's Stackelberg solution shown in the last column. Several interesting observations are found from the results. First, the fixed-competitor solution has overestimated profit and market share predictions by presuming that competitors will not act. When competitors are allowed to react by altering prices under Nash competition, the competitor-reacted profit shows a significant profit reduction from estimated. Second, the competitor-reacted profit and price-equilibrium profit are nearly identical (to significant digits). The equilibrium profit from the fixed-competitor case is lower than the Nash and Stackelberg cases, implying that the attribute decisions determined by assuming fixed competitors are suboptimal, even if the new entrant's price is adjusted optimally in response to market competition. Third, we found that the solution under the Stackelberg model has a different design and price point, resulting in slightly higher profit than the Nash solutions,[11] which supports the claim that Stackelberg is a better approach when promoting new product development [2]. Fourth, CDH's Stackelberg solution is not fully converged since our Nash test (Eq. (1)) results showed that competitors (followers) can find alternative price decisions that result

[10]We use $t=10^{-9}$ for all the cases.

[11]CDH [2] compared their Stackelberg solution to the optimal new product solution with competitors fixed at Nash prices (suboptimal solution) and concluded Stackelberg resulted in higher profit. However, the comparison for the two models should base on fully converged equilibrium solutions.

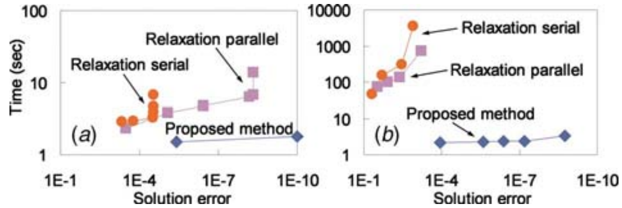

Fig. 1 Computational time versus solution error for the painkiller problem: (*a*) Nash case and (*b*) Stackelberg case

Table 4 Attribute and engineering constraint functions

Product attribute functions
$z_1=\dfrac{4\pi x_6 x_9 x_{10}(x_1+x_2)(x_3+x_4)}{x_{11}(x_1(x_3+x_4)+x_3(x_1+x_5))},\ 200\le z_1\le 400$
$z_2=x_{13}x_{14}^{-1},\ 0.75\le z_2\le 1.33$
$z_3=x_{13}x_{14},\ 100\le z_3\le 400$
$z_4=\pi x_{12}z_1^{-1},\ 2/32\le z_4\le 6/32$
$z_5=\dfrac{(2\tan(\pi y_{11}z_1^{-1}))(0.5x_{12}-y_{10})}{(1+2y_{12}^{-1}\tan(\pi y_{11}z_1^{-1}))},\ 0.75\le z_5\le 1.75$
Design constraint functions
$g_1: x_{12}-(x_{14}-2y)_1\le 0$
$g_2: x_{12}-(x_{13}-2y_1-x_7-y_9)\le 0$
$g_3: (x_4+x_5)-(x_{13}-2y_1)\le 0$
$g_4: x_5-x_2\le 0$
$g_5: x_7+y_9+x_{11}+x_8-(x_{13}-2y_1)\le 0$
$g_6: (x_{13}-2y_1)-(0.5x_{12}+y_7)-x_7-y_9-x_{10}-x_8\le 0$
$g_7: (x_1+x_2)^2-(x_{13}-2y_1-x_7)^2-0.25(x_{14}-2y_1)^2\le 0$
$g_8: (x_{13}-2y_1-x_7)^2+y_{13}^2-(x_1+x_2)^2\le 0$

in higher profits. In other words, the follower prices do not reach a Nash equilibrium in their solution and fail the Nash best response definition. Moreover, the competitor-reacted profit has a significant gap from CDH's model-estimated profit, which again shows that their solution is not a stable equilibrium. The fixed-competitor approach has the worst performance when market competition is present, while Stackelberg leads to a higher profit than Nash, and the competitor-reacted profit upon CDH's solution does not reach the true Stackelberg equilibrium due to incomplete convergence. As a result, CDH's suboptimal Stackelberg design solutions overestimate the profit and have a lower equilibrium profit than the true Stackelberg profit solved by our proposed method.[12] Overall, the proposed methods using the Nash and Stackelberg models result in an equilibrium profit of 1.2% and 1.5% higher than the fixed-competitor case, respectively, and prevent the suboptimal design decisions.

We further compare the computational time and solution error of the proposed method with two other approaches, the relaxation parallel method (the CDH method) [2,13,14] and the relaxation serial method [7,14]. We use infinity norm $\delta=|\mathbf{Z}^*-\mathbf{Z}|_\infty$ to define solution errors, where $\mathbf{Z}^*$ is the target solution vector, including prices and new product design attributes, and $\mathbf{Z}$ is the solution vector found by each algorithm.[13] The benchmarking results are presented in Fig. 1.[14] For the Nash case, while the two relaxation methods have difficulty in reaching a solution with error less than 10^{-6}, the proposed approach is able to find more accurate solutions with relatively shorter computational time. For the Stackelberg case, the proposed formulation shows a surpassing performance on both computational time and solution error.

3.2 Case Study 2: Weight Scale. The weight scale case study was introduced by Michalek and co-workers [37–39]. Compared with the first case study, this model has more complex engineering design constraints and product attributes with higher-order nonlinear equations. There are 14 design variables x_1-x_{14}, and 13 fixed design parameters y_1-y_{13}, where detailed definitions are included in Ref. [37]. The five product attributes z_1-z_5 and engineering constraint functions g_1-g_8 are shown in Table 4 as functions of the design variables. Table 5 shows the part-worth utility of the latent class model presented in Ref. [38]. There are seven market segments, where no-choice utility in each segment is fixed at zero during estimation. The discrete part-worths are interpolated by using fourth-order polynomials, and the utility is expressed as a continuous function ψ. Thus the observable utility of product j in market segment m is given by

$$v_{mj}=\bar{\psi}_{mj}(p_j)+\sum_{n=1}^{N}\psi_{mnj}(z_{nj}) \tag{10}$$

where $\bar{\psi}_{mj}$ is the price utility polynomial and ψ_{mnj} is utility polynomial for attribute n for product j in segment m. The logit choice probability of product j in segment m is

$$s_{mj}=\frac{\exp(v_{mj})}{\exp(v_{m0})+\sum_{j'\in J}\exp(v_{mj'})}\quad \forall j \tag{11}$$

with outside good utility $v_{m0}=0$. The profit function of product j is given by

$$\Pi_j=\sum_{m=1}^{M}Q_m s_{mj}(p_j-c_j)-c^{\mathrm{F}}\quad \forall j \tag{12}$$

where the segment market size Q_m is calculated by multiplying the total market size, 5×10^6 units, by the corresponding market size ratio listed in the bottom row of Table 5. The unit cost c_j is \$3.00, and the fixed investment cost c^{F} is \$1 million dollars [29]. The FOC equation for the Nash price equilibrium is

$$\frac{\partial \Pi_j}{\partial p_j}=\sum_{m=1}^{M}Q_m s_{mj}\left[\frac{\partial \bar{\psi}_{mj}}{\partial p_j}(1-s_{mj})(p_j-c_j)+1\right]=0\quad \forall j \tag{13}$$

Table 6 shows the specifications of four competing products C1, R2, S3, and T4 in the market, where each product has a unique combination of product characteristics. We used MATLAB SQP active-set solver with multistart and found multiple solutions that satisfy FOCs. After verifying post-hoc with the Nash definition (Eq. (1)) the unique market equilibrium was identified. The optimal price and attribute solutions under the fixed competitors, Nash, and Stackelberg cases and competitor solutions at price equilibrium are presented in Table 7. The fixed-competitor case produces a distinct design solution from the other two, while Nash and Stackelberg cases have similar design attributes but significantly different price decisions. The design variables (not shown) vary arbitrarily within the space of feasible designs that produce

[12]We use multi-start to search for all stationary points in the feasible domain and perform post hoc Nash best response verification (Eq. (1)). We found only one unique Stackelberg solution.

[13]The elements in the $\mathbf{Z}^*$ and $\mathbf{Z}$ vectors are dimensionless and normalized to upper and lower bounds of each variable. $\mathbf{Z}^*$ is obtained by using the proposed method with a convergence tolerance 10^{-15}.

[14]The computer system setup comprises of OS: Windows XP; CPU: Intel Core2 2.83Hz; RAM: 2.0 Gbyte; and solver: active-set SQP algorithm in MATLAB R2008a.

Table 5 Latent class model of the weight scale problem

Attribute	Level	Market segment 1	2	3	4	5	6	7
Weight Capacity z_1 (lb)	200	−1.34	−0.60	−0.38	−0.34	−0.92	−0.70	−1.19
	250	−0.36	−0.11	0.03	0.34	0.50	0.02	0.55
	300	0.06	0.21	0.08	0.70	0.37	0.04	0.34
	350	−0.21	0.05	−0.14	0.70	0.57	−0.09	−0.20
	400	−0.13	−0.15	0.20	0.51	0.55	−0.12	−0.19
Aspect Ratio z_2	0.75	−0.79	0.20	−0.04	0.44	0.10	−0.18	−1.40
	0.88	0.07	0.70	0.15	0.50	0.32	0.23	−0.62
	1	0.38	0.79	0.20	0.55	0.51	0.29	−0.02
	1.14	−0.09	−0.07	0.12	0.54	0.16	−0.10	0.57
	1.33	−1.34	−1.73	−0.56	−0.08	0.09	−0.89	0.39
Platform Area z_3 (in.^{2})	100	0.01	−0.45	0.19	0.36	0.17	0.45	−0.45
	110	−0.04	−0.21	−0.02	0.28	0.09	0.10	−0.49
	120	−0.41	−0.03	0.00	0.50	0.05	−0.05	−0.01
	130	−0.68	0.10	−0.12	0.46	0.30	−0.48	0.00
	140	−0.86	0.00	−0.27	0.31	0.45	−0.87	0.25
Gap size z_4 (in.)	2/32	−1.56	−0.55	−3.49	0.18	0.32	−0.39	−0.06
	3/32	−0.89	−0.21	−0.65	0.39	0.28	−0.15	−0.08
	4/32	−0.07	0.22	0.92	0.66	0.22	0.15	−0.13
	5/32	0.18	−0.02	1.48	0.49	0.00	−0.13	−0.28
	6/32	0.37	−0.03	1.56	0.20	0.23	−0.33	−0.14
Number size z_5 (in.)	0.75	−0.96	−1.20	−0.73	−0.35	−0.40	−1.24	−1.13
	1	−0.44	−0.51	−0.18	0.15	0.17	−0.72	−0.26
	1.25	0.12	0.34	0.25	0.58	0.22	0.17	0.07
	1.5	−0.30	0.32	0.21	0.72	0.60	0.48	0.17
	1.75	−0.39	0.47	0.24	0.81	0.48	0.46	0.46
Price p	\$10	0.47	0.13	0.43	0.70	3.19	1.64	0.24
	\$15	−0.08	0.13	0.41	0.64	1.92	1.28	0.19
	\$20	−0.22	0.02	0.03	0.52	0.40	0.36	0.03
	\$25	−0.79	−0.02	−0.29	0.25	−1.48	−1.12	−0.34
	\$30	−1.35	−0.86	−0.79	−0.20	−2.97	−3.02	−0.81
Outside good		0	0	0	0	0	0	0
Segment size		7.1%	19.2%	14.2%	19.8%	13.6%	15.8%	10.3%

Table 6 Specifications of weight scale competitors

Product	Weight capacity z_1	Aspect ratio z_2	Platform area z_3	Gap size z_4	Number size z_5	Price p
C1	350	1.02	120	0.188	1.40	\$29.99
R2	250	0.86	105	0.094	1.25	\$19.99
S3	280	0.89	136	0.156	1.70	\$25.95
T4	320	1.06	115	0.125	1.15	\$22.95

Table 7 New product design solutions for the weight scale problem

		Fixed competitor	Nash	Stackelberg
New product design and price	z_1	258	261	260
	z_2	1.046	1.038	1.039
	z_3	132	140	140
	z_4	0.117	0.119	0.119
	z_5	1.350	1.383	1.386
	Price	\$18.24	\$17.14	\$15.87
Model-estimated Profit		\$24.0M (33.8%)	\$13.8M (21.0%)	\$13.9M (23.2%)
Competitor-reacted Profit		\$13.5M (19.0%)	\$13.8M (21.0%)	\$13.9M (23.2%)
Price-equilibrium Profit		\$13.7M (21.2%)	\$13.8M (21.0%)	\$13.9M (23.2%)
Price, market share %, profit of competitors at price equilibrium	C1	\$16.96, 21.3%, \$13.8M	\$17.26, 21.3%, \$14.2M	\$17.13, 20.7%, \$13.7M
	R2	\$15.00, 14.6%, \$7.75M	\$14.84, 14.7%, \$7.70M	\$15.11, 14.2%, \$7.60M
	S3	\$17.54, 20.2%, \$13.7M	\$16.99, 20.2%, \$13.1M	\$17.81, 19.6%, \$13.5M
	T4	\$17.69, 16.7%, \$11.2M	\$18.13, 16.8%, \$11.7M	\$17.93, 16.3%, \$11.1M
Share of no-choice		6.1%	6.1%	6.0%

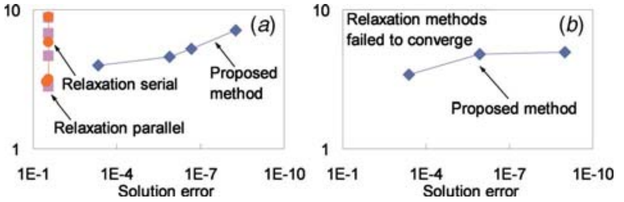

Fig. 2 Computational time versus solution error for the weight scale problem: (*a*) Nash case (*b*) Stackelberg case

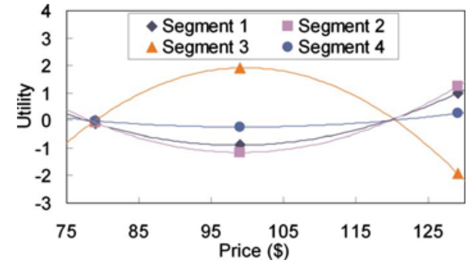

Fig. 3 Price part-worth fitting functions for the angle grinder demand model

optimal attributes in this model.

Similar to the observations in the previous case, the fixed-competitor assumption gives the highest model-estimated profit, but the competitor-reacted and price-equilibrium profits demonstrate that the prediction is overestimated when market competition is taken into account. The price-equilibrium profit is 1.6% higher than competitor-reacted profit, which implies that suboptimal pricing is a significant component of the competitor-reacted profit loss in the fixed-competitor case, but suboptimal design is a larger component. The Stackelberg approach leads to a higher expected profit than Nash. The Nash and Stackelberg approaches are able to produce 1.1% and 3.4% higher competitor-reacted profit, and 1.3% and 1.8% higher price-equilibrium profit than the fixed-competitor case, respectively. In this case, the new product Stackelberg leader has the lowest price, but the approach is able to gain the highest market share and profit. This case study again demonstrates that incorporating price competition in product design can not only avoid overestimation of profitability, but also help the designer to make the best strategic design decisions.

The computational benchmarking for this problem between the proposed method and the relaxation methods is shown in Fig. 2. For the Nash case, the relaxation methods cannot reach a solution with an error less than 10^{-2}. In the same amount of computational time, the proposed Nash formulation finds the solutions with significantly higher accuracy. For the Stackelberg case, the relaxation methods fail to converge, whereas the proposed Stackelberg formulation reaches the solutions in a relatively short computational time. These results once again show the limitation for the algorithms using iterative optimizations for handling an engineering design problem with higher-order nonlinearity and complexity.

3.3 Case Study 3: Angle Grinder. The angle grinder case study determines the optimal attributes and price of a hand held power grinder [11,40–42]. The market demand model is a latent class model with four market segments and six discrete attributes, including price (three levels: \$79, \$99, and \$129), current rating (three levels: 6, 9, and 12 A), product life (three levels: 80, 110, and 150 h), switch type (four levels: paddle, top slider, side slider, and trigger) and girth type (two levels: small and large). The part-worth utilities of the latent class model, brand dummy utility, market segment size, and outside good utility are reported in Ref. [41]. Since the new product design variables are identical to the product attributes, we categorize this case study as a product positioning problem ($\mathbf{z}=\mathbf{x}$).

The major difference of this case study from the previous two cases is its discrete decision variables. In order to derive analytical expressions for price utility, we interpolate the discrete price part-worths into the underlying continuous space using polynomial $\bar{\psi}$. Therefore, the observable utility component for product j in market segment m is given by

$$v_{mj} = \bar{\psi}_{mj} + \sum_{d=1}^{D_n} w_{mnd} z_{ndj} \tag{14}$$

where m is the market segment index, $\bar{\psi}_{mj}$ is the interpolated price utility for market segment m as a function of price p_j, w_{mnd} is the part-worth utility at level d of attribute n in market segment m, and z_{ndj} is a binary indicator variable that is equal to 1 if product j contains attribute n at level d and 0 otherwise. Furthermore, M is the number of segments and D_n is the number of levels for attribute n. The price utility function in each segment is fit through the discrete levels with a quadratic function $\bar{\psi}_{mj}=\bar{a}_{2m}p_j^2+\bar{a}_{1m}p_j+\bar{a}_{0m}$, where $\bar{a}_{2m}$, $\bar{a}_{1m}$, and $\bar{a}_{0m}$ are coefficients interpolated via least-squares regression. The four resulting price utility curves are plotted in Fig. 3. It can be seen that the price responses in each segment are not monotonically decreasing when price increases within the range of \$75-\$130. This implies that the data will predict an unusual increase in demand with increasing price in segments 1, 2, and 4, providing incentive for firms to charge high prices. The share of choice s_{mj} and profit Π_j are given by Eqs. (11) and (12), respectively. The FOC equation for the Nash price equilibrium is

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} Q_m s_{mj}[(2\bar{a}_{2m}p_j + \bar{a}_{1m})(1 - s_{mj})(p_j - c_j) + 1] = 0 \quad \forall j \tag{15}$$

Based on the available price part-worth utility in the demand model, we confine the price decisions within a range of the survey data ($\bar{g}_1: 75-p_j \leq 0,\ \bar{g}_2: p_j-130 \leq 0$) since unbounded prices in this model will encourage firms toward infinite prices and will result in no equilibrium solution. Furthermore, the specifications of three competing products in the market are shown in Table 8. The estimated costs of products X, Y, and Z are \$68.15, \$100.94 and \$49.58, respectively [11], and the new product cost is assumed \$75, independent of the design. The total market size is 9×10^6 units.

Because of the existence of discrete design variables, Eq. (3) is not valid for Nash solutions. On the other hand, the Stackelberg

Table 8 Specifications of existing angle grinder products in the market

Product brand	Current rating z_1 (A)	Product life z_2 (h)	Switch type z_3	Girth size z_4	Price p
X	9	110	Side slider	Large	\$99
Y	12	150	Paddle	Small	\$129
Z	6	80	Paddle	Small	\$79

Table 9 Optimal new product solutions for fixed and Stackelberg cases

		Fixed competitor	Stackelberg
New product design and price	$x_1 = z_1$	12 A	12 A
	$x_2 = z_2$	110 h	110 h
	$x_3 = z_3$	Side slider switch	Side slider switch
	$x_4 = z_4$	Small girth	Small girth
	Price	\$130	\$130
Model-estimated profit		\$299M (60.3%)	\$244M (49.3%)
Competitor-reacted profit		\$244M (49.3%)	\$244M (49.3%)
Price-equilibrium profit		\$244M (49.3%)	\$244M (49.3%)
Price, market share%, profit of competitors	X	\$99, 1.9%, \$5.0M	\$130, 10.0%, \$56M
	Y	\$129, 34.4%, \$87M	\$130, 34.0%, \$89M
	Z	\$79, 2.4%, \$6.0M	\$130, 6.0%, \$43M
Share of no-choice		0.9%	0.7%

formulation of this problem using only price FOC equations (Eq. (15)) can form a MINLP model without difficulty. The formulation is solved by using MINLP solver GAMS/BONMIN[15] [31] (CPU time: 0.860 s), and the solutions are presented in Table 9. For the fixed competition case, it can be seen that the new product price reaches the modeling upper bound. We find that the new product and product Y dominate the market with relatively high shares and profits, while product X and Z have low market shares. For the Stackelberg case, the design attributes and price of the new product are identical to the fixed-competitor case, but it can be seen that all competitors revised their price decisions in response to the new entrant to increase profitability. As a result, all prices reached the upper bound (\$130) of the demand model, and the estimated market shares and profits of products X, Y, and Z are higher than the fixed-competitor case.[16]

In this case, price bounds were added because finite price equilibrium solutions do not exist within the domain of the demand model's trusted region, i.e., the region based on interpolation of measured survey or past purchase data. For example, in a general sense, increasing price induces decreasing utility, holding all other factors constant. However, some consumers may assume, within some range, that products with higher prices have higher quality or better nonvisible characteristics [43]. A model built on such data will predict that higher prices result in greater demand, and thus higher profit if no other tradeoff exists. As a result, no price equilibrium exists within the measurable price range, and extrapolation leads to infinite prices.

There are several useful observations for this case study. First, we demonstrate that a Stackelberg product design and price competition problem containing discrete design variables can be solved by a MINLP solver without exhaustive search or heuristic selection used in prior methods [9,10]. Second, the fixed-competitor model has significantly overestimated profit by 22.5%. Third, this special case demonstrates the influence of concavity to the existence of equilibrium solutions. Due to the unique price utility responses, the individual profit function is not concave with respect to its price variable. Thus it is expected that a price equilibrium may not exist in the interior decision space but only boundary equilibrium exists [17]. Finally, product Y dominates market segments 2 and 3, while the new product is designed to dominate segments 1 and 4, which are the two biggest segments (Table 10). In a heterogeneous market, design and pricing decisions are often coupled, and the best solution depends on the

[15]The BONMIN MINLP solver implements multiple algorithms to solve optimization problems with continuous and discrete variables [31]. It is a local solver, and the solutions shown in the article are local optima found by multistart.

[16]We do not compare computational cost or test the CDH method in this case because active price bounds make price solutions trivial.

Table 10 Market shares in each segment at boundary equilibrium

Market segment	1 (%)	2 (%)	3 (%)	4 (%)	Total (%)
	37.8	24.8	12.1	25.3	100
X	25.6	0.9	0	0.2	10.0
Y	5.1	70.8	99.8	10.0	34.0
Z	13.1	3.6	0	0.6	6.0
New product	55.6	23.6	0.1	88.5	49.3
No-purchase	0.6	1.1	0.1	0.8	0.7

positioning of competitors [4]; therefore, accounting for competitor reactions can be critical to successfully locating new products in the market. Furthermore, without applying an upper bound to price, we find that all price decisions diverge, and no finite price equilibrium solution exists. As we can see in Fig. 3, extrapolating the price utility curves of segments 1, 2, and 4 results in higher utility for higher prices. Applying an upper bound creates finite equilibria, but the bound activity clearly suggests that the data do not support the solution. This model is problematic for the optimization application, and the results suggest that more data should be collected beyond the existing range in order to measure the eventually-decreasing utility associated with increased price. It is also possible in this case that survey respondents inferred high quality from high prices in the survey, since they tend to see such correlations in the marketplace; however, conjoint results should not exhibit these trends if respondents correctly treat all attributes not shown as equal across all profiles.

4 Conclusions

Prior profit maximization methods in engineering design ignore competitive reactions in market systems. We propose an approach to solve the new product design problems for profit maximization while accounting for competitive reactions under Nash and Stackelberg price competition. Based on the theory of mathematical programs with equilibrium constraints, our approach accounts for competitive reactions through inclusion of equilibrium conditions as constraints in the optimization framework. This approach requires little additional complexity and offers greater efficiency and convergence stability than prior methods. Because the equilibrium conditions are set only with respect to competitor pricing decisions, it is not necessary to know competitor cost structures or internal competitor product engineering details, and the equilibrium conditions can be added to any existing product design profit optimization problem.

We show that failing to account for competitive reactions can result in suboptimal design and pricing solutions and significant overestimation of expected market performance. Application of the method to three case studies from the literature exhibits its ability to handle problems of interest in the engineering domain. The case study results indicate that the Stackelberg approach is most preferred because of its capability to generate higher profits than Nash by anticipating competitor reactions. Both Nash and Stackelberg approaches can avoid overestimation of market performance and potentially poor product design positioning resulting from the common fixed-competitor model.

Acknowledgment

The authors would like to thank Professor S. Chan Choi, Dr. Nathan Williams, and Professor Shapour Azarm for providing modeling data of their case studies. We also want to thank to Dr. Brian Baumrucker and anonymous reviewers for their comments on our study. This research was supported in part by the National Science Foundation's CAREER Award No. 0747911, a grant from the National Science Foundation program for Material Use, Science, Engineering and Society: Award No. 0628084, and a grant from Ford Motor Co.

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