Proceedings of the 2008 ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference
IDETC/CIE 2008
August 3-6, 2008, New York City, New York, USA

DETC2008/DAC-49176

OPTIMAL PRODUCT DESIGN UNDER PRICE COMPETITION

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ABSTRACT

Engineering optimization methods for new product development model consumer demand as a function of product attributes and price in order to identify designs that maximize expected profit. However, prior approaches have ignored the ability of competitors to react to a new product entrant; thus these methods can overestimate expected profit and select suboptimal designs that perform poorly in a competitive market. We propose an efficient approach to new product design accounting for competitor pricing reactions by imposing Nash and Stackelberg conditions as constraints, and we test the method on three product design case studies from the marketing and engineering design literature. We find that a Stackelberg leader strategy generates higher profit than a Nash strategy. Both strategies are superior to ignoring competitor reactions: In our case studies, ignoring price competition results in overestimation of profits by 12%-79%, and accounting for price competition increases realized profits by up to 3.4%. The efficiency, convergence stability, and ease of implementation of the proposed approach enables practical implementation for new product design problems in competitive markets.

Keywords: Design Optimization; Nash Equilibrium; Stackelberg Competition; Game Theory; Demand Model; Logit; Design for Market; New Product Development

1. INTRODUCTION

Product design optimization problems that account for competitive market decisions can be categorized into two groups: Short-run price equilibrium and long-run design equilibrium [1] [1-3]. The long-run scenario represents competition over a sufficiently long time period[2] that all firms in the market are able to redesign their products as well as set new prices competitively [3,5-8]. Short-run competition assumes that the design attributes of competitor products are fixed[3], but that competitors will adjust prices in response to a new entrant [2,9-11]. We focus here on new product design problems in short-run price competition[4].

Table 1 lists prior studies for price competition in product design and distinguishes them by solution approach, demand model type, equilibrium type, case studies, and presence of design constraints. The solution approach is the method used to find the design solution under price competition. The demand model type specifies the market demand function formulation. Equilibrium type distinguishes Nash and Stackelberg strategies [13]: Nash equilibrium refers to a point at which no firm can achieve higher profit by unilaterally selecting any decision other than the equilibrium decision (i.e. price). The Stackelberg case, also known as the *leader-follower* model, assumes that the leader is able to *predict* the response of followers, in contrast with the Nash model, which assumes that each firm only *observes* competitor responses [13]. The Stackelberg case is appropriate for cases where one player is able to "move first", and introduction of a new product entrant is a case where the firm can exploit this first-move advantage. Finally, the last column in Table 1 identifies whether the model incorporates design constraints representative of tradeoffs typically present in engineering design.

[1] Unlike long-run equilibrium concepts in economics, we do not account for firm entry and exit [4].

[2] Advances in CAD tools, concurrent engineering, rapid prototyping, and production technologies that reduce lead time can decrease the relevant timeframe of "long-run" equilibria [8].

[3] The situation of fixed competitor attributes under market competition is alternatively described as "sticky" [1].

[4] We came across an article [12] discussing short-term price competition of multiple products with fixed product attributes, but it did not consider new product entry, thus excluded from the list.

Table 1: Literature on new product design optimization under price competition

Literature	Solution approach	Demand model	Price equilibrium	Case study	Design constraints
Choi *et al.* (1990) [2]	Iterative variational inequality algorithm	Ideal point logit	Stackelberg	Pain reliever	Yes
Horsky and Nelson (1992) [9]	Discrete selection from FOC solutions	Logit	Nash	Automobile	No
Rhim and Cooper (2005) [10]	Two-stage genetic algorithm	Ideal point logit	Nash	Liquid detergents	No
Lou *et al.* (2007) [11]	Discrete selection and iterative optimization	HB mixed logit	Nash	Angle grinder	No
This paper	One-step NLP/MINLP with FOC constraints	Ideal point logit and latent class model	Nash and Stackelberg	1) Pain reliever 2) Angle grinder 3) Weight scale	Yes

Choi *et al.* [2] (henceforth CDH) proposed an algorithm for solving the new product design problem under price competition while treating the new product entrant as Stackelberg leader, and they tested the method on a pain reliever example with ingredient levels as decision variables and an ideal point logit demand model[5] with linear price utility. The study applied the variational inequality diagonalization algorithm [14] to solve the follower Nash price equilibria. In Section 3, we use CDH's problem as a study case and show that the method can have convergence difficulties, and as a result the Stackelberg solution found by their algorithm is not fully converged.

In contrast to the continuous decision variables used by CDH, other prior approaches restrict attention to discrete decision variables that reflect *product attributes* observed by consumers, as opposed to *design variables* controlled by designers under technical tradeoffs. We refer to the focus on product attributes as *product positioning*, in contrast to *product design*. These prior product positioning problems assume that all combinations of discrete variables are feasible, thus no additional constraint functions are considered. Horsky and Nelson [9] used historic automobile market data to construct a logit demand model and cost function using four product attribute decision variables. With five levels for each of their four variables, they applied exhaustive enumeration to solve for equilibrium prices of all 625 possible new product entrant combinations using first-order condition equations. Rhim and Cooper [10] used a two-stage method incorporating genetic algorithms and first-order conditions to find Nash solutions for new product positioning problems. The model allows multiple new product entries to target different user market segments. The product in the study is liquid detergent with two attributes. Recently, Lou *et al.* [11] conducted a study for optimal new product positioning of a handheld angle grinder under Nash price competition in a manufacturer-retailer channel structure[6]. There are six product attributes with various levels in the problem, resulting in 72 possible combinations. Similar to Horsky and Nelson [9], the study also used a discrete selection method, but the design candidates were pre-screened to a smaller number in order to avoid full exhaustive enumeration, and the profits of a few final candidates at Nash price equilibrium were calculated through a sequential iterative optimization approach.

Prior approaches to product design and positioning under price competition suffer from inefficient computation and convergence issues due to iterative strategies to identify equilibria and combinatorial limitations of discrete attribute models. We propose an alternative approach to find optimal design and equilibrium competition solutions in single step. Our approach poses a nonlinear programming (NLP) or mixed-integer nonlinear programming (MINLP) formulation for new product profit maximization with respect to prices and design variables subject to first-order necessary conditions for competitor Nash price equilibrium[7]. We examine three case studies from the literature and show that accounting for competitor price competition can result in different optimal design decisions than those determined under the assumption that competitors will remain fixed. The approach is well-suited to engineering design optimization problems, requiring little additional complexity and offering greater efficiency and convergence stability than prior methods, particularly for the highly-constrained problems found in engineering design.

The remainder of the article is organized as follows: In Section 2, we explain the detailed formulation of the proposed approach with Nash and Stackelberg competition strategies, and

[5] The ideal point model assumes that each consumer has an ideal point in the product attribute space, and utility of a product is calculated as a function of the Euclidean distance between the ideal point and the product.

[6] Lou *et al.*'s [11] study assumed the channel reaction follows a manufacturer Stackelberg scenario, but price setting at the retailer level is a Nash solution. This is distinct from taking the new product entrant as a Stackelberg leader.

[7] Such formulation is also called mathematical programming with equilibrium constraints (MPEC) [15]. The first-order price equilibrium equations can also be expressed in variational inequality [14] or mixed complementarity form [16], but in this paper we maintain the system equation form to represent equilibrium conditions.

we introduce a modified Lagrangian formulation to accommodate cases with price bounds. In Section 3, we demonstrate the proposed approach by solving three product design examples from the literature, and we conclude in Section 4.

2. PROPOSED METHODOLOGY

We first construct price equilibrium optimization models for Nash and Stackelberg strategies for unconstrained prices. We then examine the special case where prices are constrained and develop a Lagrangian extension for this case. The assumptions for the proposed approach are: 1) The focal firm will design a set of differentiated products that will enter into a market with existing products sold by competitors; 2) competitors are Nash price setters for profit maximization with fixed products; 3) competitor product attributes and costs are known; and 4) price is continuous, and each firm's profit function is differentiable with respect to its corresponding price.

2.1 Profit Maximization under the Nash Strategy

The proposed formulation for new product design optimization under Nash price competition is:

$$
\begin{aligned}
&\text{maximize} \quad \Pi_k = \sum_{j\in J_k} q_j \left(p_j - c_j \right) \\
&\text{with respect to} \quad \mathbf{x}_j, p_j, p_{j'} \\
&\text{subject to} \quad \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0};\ \mathbf{g}\left(\mathbf{x}_j\right) \le \mathbf{0}; \\
&\qquad \frac{\partial \Pi_{j'}}{\partial p_{j'}} = 0 \\
&\text{where} \quad q_j = Qs_j\left(p_j, \mathbf{z}_j, p_{j'}, \mathbf{z}_{j'}\right) \\
&\qquad \mathbf{z}_j = f\left(\mathbf{x}_j\right) \\
&\forall j \in J_k;\ \forall j' \in J_{k'};\ \forall k' \in K \setminus k
\end{aligned}
\tag{1}
$$

In the above formulation, the objective function is the total profit Π_k of producer k, which is the sum of all its products J_k. Each new product j has design vector $\mathbf{x}_j$, attribute vector $\mathbf{z}_j$ (as a function of the design $\mathbf{z}_j = f(\mathbf{x}_j)$), price p_j, predicted market share s_j, and predicted demand q_j. The total size of the market is Q. The equality and inequality constraints, $\mathbf{h}(\mathbf{x}_j)$ and $\mathbf{g}(\mathbf{x}_j)$, define the feasible domain of the engineering design. Each competitor $k' \in K \backslash k$ has price decisions $p_{j'}$ with fixed design attributes $\mathbf{z}_{j'}$ for all its products $\forall j' \in J_{k'}$.

In the Nash game, each producer observes the prices and attributes of other products as exogenous parameters. The proposed formulation determines the profit-maximizing new product design $\mathbf{x}_j$ and price p_j that are in Nash equilibrium with competitor prices $p_{j'}$, $\forall j' \in J_{k'}$, $\forall k' \in K \backslash k$. More precisely, for each competing product j', price p_j must satisfy the first-order necessary conditions for Nash price equilibrium. If the profit function is concave with respect to price, which is common[8], the first-order condition is sufficient. However, in the case of nonconcavity, the solutions found by the proposed method must be verified as Nash *post hoc*. The mathematical expression of a Nash equilibrium is given by [13]:

$$
\begin{aligned}
&\Pi_k(p_1^*, \ldots, p_j^*, \ldots, p_J^*) \ge \Pi_k(p_1^*, \ldots, p_j, \ldots, p_J^*) \\
&\quad \forall j \in J_k,\ \forall k \in K
\end{aligned}
\tag{2}
$$

where the * denotes the decisions at Nash equilibrium. This formulation states that no unilateral change to a single firm's price decision can result higher profit for that firm than its Nash price, or, alternatively, each firm is responding optimally to the others. To test this condition, we take the FOC solution and optimize each individual producer's profit with respect to its own pricing decisions while holding other producer decisions fixed. If no higher profit is found throughout the test, the price solutions are Nash prices.

2.2 Profit Maximization under the Stackelberg Strategy

For the proposed Stackelberg competition strategy, it is assumed that the new product enters the market as a leader, while other competitors react as followers. Followers observe one others' price decisions, including the new product price, as exogeneous variables and compete with one another to reach a Nash price equilibrium. The new product leader is able to predict its followers' Nash price settings within its optimization, giving it an advantage[9]. The formulation for new product design optimization with the Stackelberg pricing strategy is:

$$
\begin{aligned}
&\text{maximize} \quad \Pi_k = \sum_{j\in J_k} q_j \left(p_j - c_j \right) \\
&\text{with respect to} \quad \mathbf{x}_j, p_j \\
&\text{subject to} \quad \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0};\ \mathbf{g}\left(\mathbf{x}_j\right) \le \mathbf{0} \\
&\text{where} \quad q_j = Qs_j\left(p_j, \mathbf{z}_j, p_{j'}, \mathbf{z}_{j'}\right);\ \ \mathbf{z}_j = f\left(\mathbf{x}_j\right) \\
&\qquad p_{j'} \text{ satisfies} \frac{\partial \Pi_{j'}}{\partial p_{j'}} = 0 \\
&\forall j \in J_k;\ \forall j' \in J_{k'};\ \forall k' \in K \setminus k
\end{aligned}
\tag{3}
$$

The Stackelberg formulation appears similar to the Nash case in Eq. (1), but the meaning is largely different. It can be seen that the Stackelberg formulation contains only new product design and price as decision variables. Competitor prices satisfying Nash equilibrium equations are included in the

[8] Anderson *et al.* [17] proved that there exists a unique price equilibrium under logit demand when decision sets are convex and the profit function is strictly quasi-concave.

[9] CDH [2] used a duopoly game to prove that a Stackelberg leader strategy can always receive at least as high a payoff as a Nash strategy if a Stackelberg equilibrium exists.

objective function for giving predicted price and market share information to the new product leader. Computationally, this can be thought of as the Nash condition enforcing price conditions only at the solution, whereas the Stackelberg condition calculates reaction prices at each intermediate iteration of the algorithm.

2.3 Incorporating Price Variable Bounds

Furthermore, we consider the special situation in our proposed formulation when finite price equilibrium solutions do not exist within the domain of the demand model's trusted region (i.e.: the region based on interpolation of measured survey or past purchase data). For example, in a general sense, increasing price induces decreasing utility, holding all other factors constant. However, some consumers may assume, within some range, that products with higher prices have higher quality or better non-visible characteristics [18]. A model built on such data will predict that higher prices result in greater demand, and thus higher profit if no other tradeoff exists. As a result, no price equilibrium exists within the measurable price range, and extrapolation leads to infinite prices. In order to account for the ability to restrict firm reactions to the domain covered by the demand model, we incorporate variable bounds and introduce Lagrange multipliers to the Nash equilibrium conditions.

$$\begin{aligned}
&\text{maximize} \quad \Pi_k = \sum_{j \in J_k} q_j \left(p_j - c_j \right) \\
&\text{with respect to} \quad \mathbf{x}_j, p_j, p_{j'} \\
&\text{subject to} \quad \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0};\ \mathbf{g}\left(\mathbf{x}_j\right) \le \mathbf{0} \\
&\qquad p^{\mathrm{L}} \le p_j \le p^{\mathrm{U}} \\
&\qquad \frac{\partial \Pi_{j'}}{\partial p_{j'}} + \mu_{j'}^{\mathrm{U}} - \mu_{j'}^{\mathrm{L}} = 0; \\
&\qquad p_{j'} - p^{\mathrm{U}} \le 0; \quad p^{\mathrm{L}} - p_{j'} \le 0; \\
&\qquad \mu_{j'}^{\mathrm{L}}\left(p_{j'} - p^{\mathrm{U}} \right) = 0; \quad \mu_{j'}^{\mathrm{L}}\left(p^{\mathrm{L}} - p_{j'} \right) = 0; \\
&\qquad \mu_{j'}^{\mathrm{U}} \ge 0; \quad \mu_{j'}^{\mathrm{L}} \ge 0; \\
&\text{where} \quad q_j = Qs_j\left(p_j, \mathbf{z}_j, p_{j'}, \mathbf{z}_{j'} \right) \\
&\qquad \mathbf{z}_j = f\left(\mathbf{x}_j\right) \\
&\forall j \in J_k;\ \forall j' \in J_{k'};\ \forall k' \in K \setminus k
\end{aligned} \tag{4}$$

This formulation introduces lower bounds p^{L} and upper bounds p^{U} on the prices of all firms, and the associated Lagrange multipliers μ enforce the first order Karush-Kuhn-Tucker (KKT) conditions [19]. Note that any solution with an active price-bounding constraint implies that more data is needed to extend the domain of trusted predictions made by the demand model.

Similarly, the price-constrained Stackelberg pricing strategy is:

$$\begin{aligned}
&\text{maximize} \quad \Pi_k = \sum_{j \in J_k} q_j \left(p_j - c_j \right) \\
&\text{with respect to} \quad \mathbf{x}_j, p_j \\
&\text{subject to} \quad \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0};\ \mathbf{g}\left(\mathbf{x}_j\right) \le \mathbf{0} \\
&\qquad p^{\mathrm{L}} \le p_j \le p^{\mathrm{U}} \\
&\text{where} \quad q_j = Qs_j\left(p_j, \mathbf{z}_j, p_{j'}, \mathbf{z}_{j'} \right) \\
&\qquad \mathbf{z}_j = f\left(\mathbf{x}_j\right) \\
&\qquad p_{j'} \text{ satisfies:} \\
&\qquad \left\{ \begin{array}{c}
\dfrac{\partial \Pi_{j'}}{\partial p_{j'}} + \mu_{j'}^{\mathrm{U}} - \mu_{j'}^{\mathrm{L}} = 0 \\
p_{j'} - p^{\mathrm{U}} \le 0; \quad p^{\mathrm{L}} - p_{j'} \le 0 \\
\mu_{j'}^{\mathrm{L}}\left(p_{j'} - p^{\mathrm{U}} \right) = 0; \quad \mu_{j'}^{\mathrm{L}}\left(p^{\mathrm{L}} - p_{j'} \right) = 0 \\
\mu_{j'}^{\mathrm{U}} \ge 0; \quad \mu_{j'}^{\mathrm{L}} \ge 0
\end{array} \right\} \\
&\forall j \in J_k;\ \forall j' \in J_{k'};\ \forall k' \in K \setminus k
\end{aligned} \tag{5}$$

Compared to the solution approaches in literature, the proposed method has significant advantages in several aspects. First, the approach is able to solve the problem in a single step if a unique design solution with price equilibrium exists[10]. Second, since the approach employs first-order condition equations to find equilibrium prices, the convergence of the whole formulation is faster and more stable than prior approaches that use iteration loops. Third, the formulations can be solved using commercially-available NLP solvers with minimum additional programming effort. When discrete design variables exist, the NLP model becomes a MINLP problem. However, the price equilibrium constraints remain in the continuous domain, and commercial MINLP solvers can be used to solve Eq. (4) and Eq. (5) [20-22].

2.4 Strategy Evaluation

In order to compare profitability of the new product design arrived at under different modeling assumptions, we define three profit terms:

1) *Predicted profit*: Profit of the design and price solution to a particular model as predicted by that model.

2) *Realized profit*: Profit of the design and price solution to a particular model as predicted via post-hoc computation of competitor price equilibrium. The realized profit represents the

[10] For the cases of multiple local optima and price equilibria, multi-start can be implemented to identify solutions.

profit that a particular design and pricing solution would realize if competitors adjust prices in response to the new entrant. Realized profit is equal to predicted profit for the Nash and Stackelberg case, but if the new entrant is optimized while assuming fixed competitors, the difference between predicted and realized profit measures the impact of ignoring competitor reactions.

3) *Price-adjusted profit*: Profit of the design solution to a particular model as predicted via post-hoc computation of price equilibrium of *all* firms (including the new entrant). The price-adjusted profit represents the profit that a particular design solution would realize if all firms adjust prices in response to the new entrant. Price-adjusted profit is equal to predicted profit for the Nash and Stackelberg case, but if the new entrant is optimized while assuming fixed competitors, the difference between predicted and price-adjusted profit measures the impact of ignoring competitor reactions on the *design of the product*, assuming that poor pricing choices can be corrected in the marketplace after product launch.

3 CASE STUDIES

We examine three product design case studies from the literature to test the proposed approach and examine the improvement that Stackelberg and Nash strategies can make with respect to methods that ignore competitive reactions. Each case study involves different product characteristics, utility functions, demand models, variable types, and design constraints. For each case, we solve the problem using the traditional fixed competitor approach and compare to Nash and Stackelberg competition strategies.

3.1 Case study 1: Pain Reliever

The pain reliever problem was introduced by CDH [2]: Price and product attributes of a new pain reliever product are to be determined for maximizing profit in the presence of fourteen existing competitor products in the market. Each product has four attributes of pharmaceutical ingredient weight (unit in mg), including aspirin z_1, aspirin substitute z_2, caffeine z_3 and additional ingredients z_4. The product specifications[11] and initial prices of competitor products are listed in Table 2. There are two highlights in the model. First, the product H is assumed a generic brand, which has a fixed price of $1.99 [2]. The generic brand does not participate in the price competition. Second, there are five products, A, C, I, K and L, with identical product attributes and costs. The demand model is an ideal point model with observable utility v, given by:

$$v_{ij} = -\left(\sum_{n=1}^{N} \beta_{in} \left(z_{nj} - z_{in}^{\mathrm{C}} \right)^2 + \beta_i^{\mathrm{P}} p_j + b_i \right) \quad \forall i, j \qquad (6)$$

[11] The values of aspirin substitute are the weighted combination of acetaminophen and ibuprofen. The numbers are not provided in the original paper [2]. We obtained the attribute data from the mixed complementarity programming library (MCPLIB) [23] and verified with original author. The data of consumer preference weightings (30 individuals) are also included in the data file.

Table 2: Specification of existing pain reliever products in the market

Prod-uct	Aspn. (mg)	Aspn. sub. (mg)	Caff. (mg)	Add. ingd. (mg)	Cost	Initial price
	z_1	z_2	z_3	z_4	c	p
A	0	500	0	0	$4.00	$6.99
B	400	0	32	0	$1.33	$3.97
C	0	500	0	0	$4.00	$5.29
D	325	0	0	150	$1.28	$3.29
E	325	0	0	0	$0.98	$2.69
F	324	0	0	100	$1.17	$3.89
G	421	0	32	75	$1.54	$5.31
H	500	0	0	100	$1.70	$1.99
I	0	500	0	0	$4.00	$5.75
J	250	250	65	0	$3.01	$4.99
K	0	500	0	0	$4.00	$7.59
L	0	500	0	0	$4.00	$4.99
M	0	325	0	0	$2.60	$3.69
N	227	194	0	75	$2.38	$4.99
Cost	0.3	0.8	0.4	0.2	cost unit: $/100mg	

where z_{nj} is the value of product attribute n on product j, z^{C}_{in} is consumer i's desired value for attribute n, β_{in} is consumer i's sensitivity of utility to deviation from the ideal point, β^{P}_i is consumer i's sensitivity of utility to price, and b is a constant utility term. In this formulation, product attributes that deviate from ideal attributes cause reduced utility, which is less preferred by consumers. Under the standard assumption that utility u_{ij} is partly observable v_{ij} and partly unobservable ε_{ij} so that $u_{ij}=v_{ij}+\varepsilon_{ij}$, and that the unobservable term ε_{ij} is an IID random variable with an extreme value distribution, the resulting choice probability is defined in logit form with a unit dummy outside good [24]:

$$s_{ij} = \frac{\mathbf{exp}(\chi v_{ij})}{1 + \sum_{j=1}^{J} \mathbf{exp}(\chi v_{ij})} \quad \forall i, j \qquad (7)$$

The weighting coefficient χ is arbitrarily given by χ=3.[12] The profit function is:

$$\Pi_j = Q(p_j - c_j) \frac{1}{I} \sum_{i=1}^{I} s_{ij} \quad \forall j \qquad (8)$$

In this problem, the market demand and profit are based on a simulated market size of 30 people. The first-order condition for the price equilibrium is (the detailed derivations shown in Appendix A.1):

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{i=1}^{I} s_{ij} \left[1 - \chi \beta_i^{\mathrm{P}} (p_j - c_j)(1 - s_{ij}) \right] = 0 \qquad (9)$$

Two constraint functions on the new product design are given by the ingredient weight limitations [2]:

[12] The weighting coefficient affects the degree of competition, where χ=3 is defined by CDH [2].

$$g_1 = 325 - z_{1j} - z_{2j} \le 0$$
$$g_2 = z_{1j} + z_{2j} - 500 \le 0 \qquad (10)$$

By applying the above equations into Nash and Stackelberg formulations of Eq. (4-5), the model was solved using Matlab[13]. The solutions to the pain reliever problem with fixed competitors, Nash, and Stackelberg strategies are presented in Table 3, with CDH's Stackelberg solution shown in the last column. Several interesting observations are found from the results. First, the fixed competitor solution has overestimated profit and market share predictions by presuming that competitors will not act. When competitors are allowed to react by altering prices under Nash competition, the realized profit shows a profit reduction from predicted. Second, the price-adjusted and realized profits are nearly identical (to significant digits). The price-adjusted profit from the fixed competitor case is lower than the Nash and Stackelberg cases, implying the attribute decisions determined by assuming fixed competitors are suboptimal, even if the new entrant's price is adjusted optimally in response to market competition. Third, we found the solution under the Stackelberg strategy has a different design and price point, resulting in slightly higher profit than Nash[14], which supports the claim that Stackelberg is a better strategy when promoting new product development [2]. Fourth, we found the CDH's Stackelberg solution is not fully converged since our Nash test (Eq. (2)) results showed that producers are able to find alternative price decisions that have higher profit. Moreover, the realized profit has a significant gap from the CDH's predicted profit, which shows the solution is not a stable one. Table 4 lists the price, market share and profit details of all products in market. A further evidence that CDH's solutions had not converged is that CDH has different solutions[15] among products A, C, I, K and L, while our proposed method converged to a consistent answer[16]. Since these five products have identical attributes, their solutions should be identical at market equilibrium.

We compare the computational time and convergence stability of the proposed method vs. the variational inequality diagonalization algorithm [14] used in the CDH paper. The results are shown in Table 5. The computer system setup is comprised of Pentium D 2.80Hz processor with 1.0 GB RAM. When solving the competition problem under the Nash scenario, the proposed approach is three times faster than the iterative method on CPU time benchmarking. It has superior convergence precision at 10^{-12}, while the iterative method cannot find the equilibrium solution when the convergence tolerance tightened to 10^{-8}. For the Stackelberg case, the proposed formulation is able converge to a stable equilibrium solution at tolerance 10^{-12} and iterative method fails to reach a valid solution even with fairly loose convergence tolerances (10^{-3}). Furthermore, the proposed approaches are less sensitive to the choice of starting point.

Table 3: Design attribute and pricing solutions of the new product entrant for the pain reliever problem

	Fixed competitors	Nash	Stackelberg	CDH
Price	$3.74	$3.86	$3.74	$3.77
Aspirin z_1	124.0	101.8	101.5	102.1
Aspirin sub. z_2	201.0	223.2	223.5	222.9
Caffeine z_3	0	0	0	0
Add. ingd. z_4	0	0	0	0
Cost	$1.98	$2.09	$2.09	$2.38
Market share	16.26%	14.69%	15.74%	16.13%
Predicted profit	$8.60	$7.78	$7.80	$8.16
Realized profit	**$7.68**	**$7.78**	**$7.80**	**$7.80**
Adjusted profit	$7.68	$7.78	$7.80	$7.78

Table 4: Comparison of solution strategies for the pain reliever problem on realized profits

Prod-uct	Price			Realized Market Share			Realized Profit		
	Nash	Stkg.	CDH	Nash	Stkg.	CDH	Nash	Stkg.	CDH
A	$6.27	$6.29	$6.28	3.45%	3.41%	2.34%	$2.35	$2.34	$2.34
B	$2.26	$2.26	$2.26	6.18%	6.16%	1.72%	$1.73	$1.72	$1.72
C	$6.27	$6.29	$6.28	3.45%	3.41%	2.34%	$2.35	$2.34	$2.34
D	$2.28	$2.28	$2.28	7.79%	7.73%	2.33%	$2.34	$2.32	$2.33
E	$1.97	$1.97	$1.97	11.47%	11.3%	3.40%	$3.42	$3.39	$3.40
F	$2.18	$2.19	$2.19	9.16%	9.08%	2.76%	$2.78	$2.76	$2.76
G	$2.47	$2.47	$2.47	4.63%	4.62%	1.29%	$1.29	$1.29	$1.29
H	$1.99	$1.99	$1.99	7.57%	7.56%	0.66%	$0.66	$0.66	$0.66
I	$6.27	$6.29	$6.28	3.45%	3.41%	2.34%	$2.35	$2.34	$2.34
J	$4.76	$4.77	$4.77	3.36%	3.29%	1.74%	$1.76	$1.74	$1.74
K	$6.27	$6.29	$6.28	3.45%	3.41%	2.34%	$2.35	$2.34	$2.34
L	$6.27	$6.29	$6.28	3.45%	3.41%	2.34%	$2.35	$2.34	$2.34
M	$4.26	$4.26	$4.26	11.46%	11.2%	5.62%	$5.70	$5.59	$5.62
N	$3.93	$3.95	$3.95	6.33%	6.11%	2.89%	$2.93	$2.88	$2.89
New	**$3.86**	**$3.74**	**$3.77**	**14.7%**	**15.7%**	**7.80%**	**$7.78**	**$7.80**	**$7.80**

Table 5: Comparison of computational time and convergence accuracy

	Nash		Stackelberg	
	CPU time (sec)	Convg. tolerance	CPU time (sec)	Convg. tolerance
Proposed methods	2.547	10^{-12}	6.969	10^{-12}
CDH methods	9.969	10^{-7}	Unstable	

[13] We use the sequential quadratic programming (SQP) solver, fmincon, in the Matlab Optimization Toolbox.

[14] CDH [2] compared their Stackelberg solution with optimal new product solution of competitor fixed at Nash prices (suboptimal solution) and concluded Stackelberg resulted higher profit. However, the comparison for the two strategies should base on fully converged equilibrium solutions.

[15] The Stackelberg price solutions for the five identical-attribute products reported by CDH are A=$2.41, C=$2.39, I=$2.39, K=$2.41, and L=$2.39, which are not consistent values. To be noticed, the CDH prices listed in Table 4 are the prices calculated for realized profit based on CDH's optimal product attributes, not the original price solutions from CDH's paper.

[16] The proposed approach is able to reach 12 consistently significant digits for the five identical products at market equilibrium. This shows the superior convergence of the proposed approach in solving this case study.

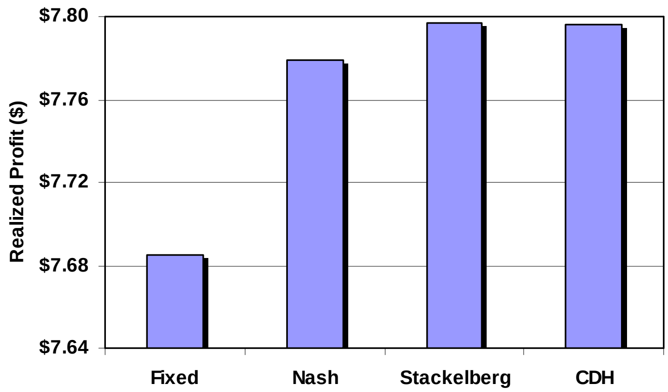

Figure 1: Comparison of four strategies for the pain reliever problem

Figure 1 presents a visual comparison of the realized profits for the four different approaches. The fixed-competitor strategy has the worst performance when market competition is present, while Stackelberg leads to a higher profit than Nash, and CDH's realized profit does not quite reach the true Stackelberg equilibrium due to incomplete convergence. Compared to the fixed competitor case, Nash and Stackelberg strategies result in a realized profit 1.22% and 1.46% higher than the fixed competitor case, respectively, in this problem.

3.2 Case study 2: Angle Grinder

The angle grinder case study, introduced by Luo *et al*. [26], determines the optimal attributes and price of a hand held power grinder [11,25-28]. The market demand model is a latent class model [17] with four market segments and six discrete attributes, including price (3 levels, unit: dollar), current rating (3 levels, unit: ampere), product life (3 levels, unit: hour), switch type (4 levels) and girth type (2 levels). The part-worth utilities of product attributes and price at each level are shown in Table 6. The utility of the no-purchase option and the market size ratio in each segment are given in the last two rows of the table.

In order to derive analytical expressions for price utility, we interpolate the discrete price part-worths into the underlying continuous space. Therefore, the observable utility component for product j in market segment m is given by:

$$v_{mj} = v^{\mathrm{P}}_{mj}(p_j) + \sum_{d=1}^{D_n} w_{mnd} z_{ndj} \tag{11}$$

where m is the market segment index, v^{P}_{mj} is the interpolated price utility for market segment m as function of price p_j, w_{mnd} is the part-worth utility at level d of attribute n in market segment m, and z_{ndj} is a binary indicator variable that is equal to 1 if product j contains attribute n at level d and 0 otherwise. Further, M is the number of segments and D_n is the number of levels for

[17] The original demand model was presented in latent class model form with four market segments [23-24,26], while later fit with a hierarchical Bayesian method in mixed logit form [14].

Table 6: Conjoint part-worths in the angle grinder latent class model

Attribute	Level	Seg. 1	Seg. 2	Seg. 3	Seg. 4
		Market segment			
Price p	$79	-0.11	-0.09	0.005	-0.02
	$99	-0.89	-1.15	1.92	-0.24
	$129	1.00	1.25	-1.92	0.26
Brand z_1	New	-0.55	0.45	2.21	-0.17
	A	0.18	1.06	-2.37	-0.20
	B	0.83	0.11	-1.59	1.15
	C	-0.47	-1.63	1.74	-0.79
Current rating z_2	6 amps.	1.25	0.45	-1.48	-0.46
	9 amps.	0.13	-1.42	-0.65	-2.38
	12 amps.	-1.39	0.97	2.13	2.84
Product life z_3	80 hrs	-0.86	-0.13	-4.72	0.80
	110 hrs	1.34	-0.47	-5.83	0.74
	150 hrs	-0.47	0.60	10.5	-1.55
Switch type z_4	Paddle	0.43	0.30	-3.29	-0.65
	Top slider	-1.02	-0.65	-3.05	0.42
	Side slider	2.39	-0.07	2.46	0.56
	Trigger	-1.81	0.43	3.87	-0.33
Girth size z_5	Small	1.51	0.72	1.51	0.41
	Large	-1.51	-0.72	-1.51	-0.41
No-purchase		-0.02	-0.02	-0.02	-0.02
Market size ratio		37.8%	24.8%	12.1%	25.3%

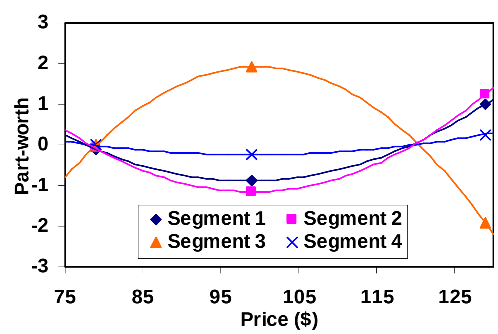

Figure 2: Price part-worth fitting functions for the angle grinder demand model

attribute n. The price utility function in each segment is fit through the discrete levels with a quadratic function $v^{\mathrm{P}}_{mj} = a_{2m}p_j^2 + a_{1m}p_j + a_{0m}$, where a_{2m}, a_{1m} and a_{0m} are coefficients determined via ordinary least squares regression. The four resulting price utility curves are plotted in Figure 2. It can be seen that the price responses in each segment are not monotonically decreasing when price increases within the range of $75-$130. This implies that the data will predict an unusual increase in demand with increasing price in segments 1, 2, and 4, providing incentive for firms to charge high prices.

The share of choice captured by product j in segment m in logit form is given by:

$$s_{mj} = \frac{\exp\left(v_{mj}\right)}{\exp\left(v_{m0}\right) + \sum_{j'} \exp\left(v_{mj'}\right)} \quad \forall j, m \tag{12}$$

Thus the profit function of product j is:

$$\Pi_j = \sum_{m=1}^{M} q_m s_{mj} \left(p_j - c_j \right) \;\; \forall j \qquad (13)$$

where q_m is the market size of segment m. The first-order condition for the Nash price equilibrium (derived in Appendix A.2) is:

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} q_m s_{mj} \Big[\left(2a_{2m} p_j + a_{1m} \right) \left(1 - s_{mj} \right) \left(p_j - c_j \right) + 1 \Big] = 0 \;\; \forall j \qquad (14)$$

Based on the available price part-worth utility in the demand model, we confine the price decisions within a range of p^L=$75 to p^U=$130. Furthermore, the specifications of competing products in the market are shown in Table 7. The estimated costs of product A, B and C are $68.15, $100.94 and $49.58 respectively [14], and the new product cost is assumed to be $75, independently of the design. The total market size is 9 million units.

The profit maximization problem for this case forms a MINLP model because of the discrete design attributes and continuous price variables. The problem is solved by MINLP solver GAMS/DICOPT[18] [20]. For the fixed competitor case, the optimal design attributes of the new product are current rating 12 amperes (level 3), product life 110 hours (level 2), side slider switch (level 3), and small girth size (level 1) with price at the upper bound of $130. The market shares and profits of all products are shown in the left block of Table 8. It can be seen that new product and product B dominate the market with relatively high shares and profits, while product A and C have low market shares. Solutions to the Nash price competition case are shown in the right block of Table 8. The design attributes and price of the new product are identical to the fixed competitor case, but it can be seen that all competitors revised their price decisions in response to the new entrant to increase profitability. As a result, all product prices reached the upper bound ($130) of the model, and the predicted market shares and profits of product A, B and C are higher than the no competition case. Stackelberg solutions are identical to the Nash solution because prices are constrained by the upper bound. If the upper bound is removed, no solution exists because all producers will tend toward prices of infinity in order to maximize profit.

There are several practical observations for this case study. First, we demonstrate that the new product design with discrete variables under the proposed formulation can be easily solved by a MINLP solver without exhaustive search or heuristic selection used in prior methods [12,14]. Second, the fixed competitor model has significantly overestimated predicted profit by 22.5%. Third, since all product prices reach the upper bound, price competition becomes ineffective, so that the fixed competitor model and the Nash and Stackelberg strategies all result in the same design solutions. Thus all the realized and price-adjusted profits in Table 8 are all identical. Fourth, as the detailed market shares in each segment show in Table 9, product B dominates market segments 2 and 3, while the new product is designed to dominate segments 1 and 4, which are the two biggest segments. In a heterogeneous market, design and pricing decisions are often coupled, and the best solution depends on the positioning of competitors; therefore, accounting for competitor reactions can be critical to successfully locating new products in the market. And finally, without applying an upper bound to price, we find that all price decisions diverge, and no finite price equilibrium solution exists. As we can see in Figure 2, extrapolating the price utility curves of segments 1, 2, and 4 results in higher utility for higher prices. Applying an upper bound creates finite equilibria, but the bound activity clearly suggests that the data do not support the solution. This model is problematic for the optimization application, and results suggest that more data should be collected beyond the existing range in order to measure the eventually-decreasing utility associated with increased price. It is also possible in this case that survey respondents inferred high quality from high prices in the survey, since they tend to see such correlations in the marketplace.

Table 7: Specifications of existing angle grinder products in the market

Product brand z_1	Price p	Current rating z_2	Product life z_3	Switch type z_4	Girth size z_5
A	$99	9 amps	110 hrs	Side slider	Large
B	$129	12 amps	150 hrs	Paddle	Small
C	$79	6 amps	80 hrs	Paddle	Small

Table 8: Optimal new product solutions

Scenario	Fixed competitors			Nash / Stackelberg		
Product	Price	Mkt share	Predicted profit	Price	Mkt share	Realized profit
A	$99	1.9%	$5M	$130	10.0%	$56M
B	$129	34.4%	$87M	$130	34.0%	$89M
C	$79	2.4%	$6M	$130	6.0%	$43M
New	**$130**	**60.3%**	**$299M**	**$130**	**49.3%**	**$244M**
No-purchase	-	0.9%	-	-	0.7%	-
Predicted profit			$299M			$244M
Realized profit			$244M			$244M
Adjusted profit			$244M			$244M

Table 9: Market shares in each segment at equilibrium

Market segment	1	2	3	4	Total
Market size ratio	37.8%	24.8%	12.1%	25.3%	100%
A	25.6%	0.9%	0%	0.2%	10.0%
B	5.1%	70.8%	99.8%	10.0%	34.0%
C	13.1%	3.6%	0%	0.6%	6.0%
New product	**55.6%**	**23.6%**	**0.1%**	**88.5%**	**49.3%**
No-purchase	0.6%	1.1%	0.1%	0.8%	0.7%

[18] DICOPT implements the outer-approximation algorithm with equality relaxation methods to solve optimization problems with continuous and discrete variables [21]. It is a local MINLP solver, so that the solutions shown in the article are local optima found by the multi-start method.

3.3 Case study 3: Weight Scale

The weight scale case study was introduced by Michalek *et al*. [29-31]. Compared to the first two study cases, this model has more complicated design constraints and product attributes with a highly nonlinear formulation. The fourteen design variables x_1–x_{14}, thirteen fixed design parameters y_1–y_{13} and eight design constraint functions g_1–g_8 for the new weight scale design are shown in Table 10. The five product attributes z_1-z_5 and engineering constraint functions g_1-g_8 are shown in Table 11 as functions of the design variables. Table 12 presents the part-worth utility of five attributes and price from the latent class model constructed in [31]. There are seven market segments, where the no-choice utility in each segment is fixed at zero during regression. The discrete part-worths are interpolated by using cubic splines [29], so that the utility of each attribute is expressed as a continuous spline function ψ. Thus the observable utility of product j in market segment m is given by:

$$v_{mj} = \psi_{mj}^{\mathrm{P}}\left(p_j\right) + \sum_{n=1}^{5} \psi_{mnj}\left(z_{nj}\right) \tag{15}$$

And the logit choice probability of product j in segment m is:

$$s_{mj} = \frac{\exp\left(v_{mj}\right)}{1+\sum_{j'} \exp\left(v_{mj'}\right)} \quad \forall j, m \tag{16}$$

The profit function of product j is given by:

$$\Pi_j = \sum_{m=1}^{M} q_m s_{mj} (p_j - c_j) - c^{\mathrm{F}} \quad \forall j \tag{17}$$

where the segment market size q_m is calculated by multiplying the total market size, \$5 millions units, by the corresponding market size ratio listed in Table 13. The unit cost c_j is \$3, and the fixed investment cost c^{F} is \$1 million dollars [29]. The analytical expression of the first-order condition of Nash price equilibrium under the latent class model is obtained through the derivations in Appendix A.2.

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} q_m s_{mj} \left[\frac{\partial \psi_{mj}^{\mathrm{P}}}{\partial p_j} \left(1 - s_{mj}\right)\left(p_j - c_j\right) + 1 \right] = 0 \tag{18}$$

Table 13 shows the specifications of four competing products, C1, R2, S3 and T4, in the market, where each product has a unique combination of product characteristics. We used Matlab solver with multi-start method and found multiple solutions that satisfy first-order conditions. After verifying post-hoc with the Nash definition (Eq. (2)) the unique market equilibrium was identified. The optimal price and attribute solutions under the fixed competitors, Nash, and Stackelberg cases are presented in Table 14. The realized profits with price predictions and market shares under three strategies after new product entry are listed in Table 15. The fixed competitor case produces a distinct design solution from the other two, while Nash and Stackelberg cases have similar design attributes but significantly different price decisions. The design variables (not

Table 10: Design variables, parameters and constraint functions in the weight scale design problem

	Description	Unit	Up/lower bounds
x_1	Length from base to force on long lever	in.	[0.125, 36]
x_2	Length from force to spring on long lever	in.	[0.125, 36]
x_3	Length from base to force on short lever	in.	[0.125, 24]
x_4	Length from force to joint on short lever	in.	[0.125, 24]
x_5	Length from force to joint on long lever	in.	[0.125, 36]
x_6	Spring constant	lb/in	[1, 200]
x_7	Distance from base edge to spring	in.	[0.5, 12]
x_8	Length of rack	in.	[1, 36]
x_9	Pitch diameter of pinion	in.	[0.25, 24]
x_{10}	Length of pivot horizontal arm	in.	[0.5, 1.9]
x_{11}	Length of pivot vertical arm	in.	[0.5, 1.9]
x_{12}	Dial diameter	in.	[9, 13]
x_{13}	Cover length	in.	[9, 13]
x_{14}	Cover width	in.	[9, 13]
y_1	Gap between base and cover	in.	0.30
y_2	Min. distance between spring and base	in.	0.50
y_3	Internal thickness of scale	in.	1.90
y_4	Minimum pinion pitch diameter	in.	0.25
y_5	Length of window	in.	3.0
y_6	Width of window	in.	2.0
y_7	Distance from top of cover to window	in.	1.13
y_8	Number of lbs measured per tick mark	lb	1.0
y_9	Horizontal dist. spring to pivot	in.	1.10
y_{10}	Length of tick mark plus gap to number	in.	0.31
y_{11}	Number of lbs that number spans	lb	16
y_{12}	Aspect ratio of number (length/width)	-	1.29
y_{13}	Min. allow lever dist. base to centerline	in.	4.0

Table 11: Attribute design functions and engineering constraint functions

Design attribute functions	Engineering design constraint functions
$z_1 = \frac{4\pi x_6 x_9 x_{10}\left(x_1+x_2\right)\left(x_3+x_4\right)}{x_{11}\left(x_1\left(x_3+x_4\right)+x_3\left(x_1+x_5\right)\right)}$	$g_1: x_{12} \le x_{14} - 2y_1$
	$g_2: x_{12} \le x_{13} - 2y_1 - x_7 - y_9$
$z_2 = x_{13} x_{14}^{-1}$	$g_3: \left(x_4 + x_5\right) \le x_{13} - 2y_1$
$z_3 = x_{13} x_{14}$	$g_4: x_5 \le x_2$
$z_4 = \pi x_{12} z_1^{-1}$	$g_5: x_7 + y_9 + x_{11} + x_8 \le x_{13} - 2y_1$
$z_5 = \frac{\left(2\tan\left(\pi y_{11} z_1^{-1}\right)\right)\left(0.5 x_{12} - y_{10}\right)}{\left(1 + 2 y_{12}^{-1}\tan\left(\pi y_{11} z_1^{-1}\right)\right)}$	$g_6: x_8 \ge \left(x_{13} - 2y_1\right) - \left(0.5 x_{12} + y_7\right) - x_7 - y_9 - x_{10}$
	$g_7: \left(x_1 + x_2\right)^2 \le \left(x_{13} - 2y_1 - x_7\right)^2 + 0.25\left(x_{14} - 2y_1\right)^2$
	$g_8: \left(x_1 + x_2\right)^2 \ge \left(x_{13} - 2y_1 - x_7\right)^2 + y_{13}^2$

Table 12: Latent class model for the weight scale problem

Attribute	Level	Market Segments 1	2	3	4	5	6	7
Weight Capacity z_1 (lb)	200	-1.34	-0.60	-0.38	-0.34	-0.92	-0.70	-1.19
	250	-0.36	-0.11	0.03	0.34	0.50	0.02	0.55
	300	0.06	0.21	0.08	0.70	0.37	0.04	0.34
	350	-0.21	0.05	-0.14	0.70	0.57	-0.09	-0.20
	400	-0.13	-0.15	0.20	0.51	0.55	-0.12	-0.19
Aspect Ratio z_2	0.75	-0.79	0.20	-0.04	0.44	0.10	-0.18	-1.40
	0.88	0.07	0.70	0.15	0.50	0.32	0.23	-0.62
	1	0.38	0.79	0.20	0.55	0.51	0.29	-0.02
	1.14	-0.09	-0.07	0.12	0.54	0.16	-0.10	0.57
	1.33	-1.34	-1.73	-0.56	-0.08	0.09	-0.89	0.39
Platform Area z_3 (in.^{2})	100	0.01	-0.45	0.19	0.36	0.17	0.45	-0.45
	110	-0.04	-0.21	-0.02	0.28	0.09	0.10	-0.49
	120	-0.41	-0.03	0.00	0.50	0.05	-0.05	-0.01
	130	-0.68	0.10	-0.12	0.46	0.30	-0.48	0.00
	140	-0.86	0.00	-0.27	0.31	0.45	-0.87	0.25
Gap size z_4 (in.)	2/32	-1.56	-0.55	-3.49	0.18	0.32	-0.39	-0.06
	3/32	-0.89	-0.21	-0.65	0.39	0.28	-0.15	-0.08
	4/32	-0.07	0.22	0.92	0.66	0.22	0.15	-0.13
	5/32	0.18	-0.02	1.48	0.49	0.00	-0.13	-0.28
	6/32	0.37	-0.03	1.56	0.20	0.23	-0.33	-0.14
Number size z_5 (in.)	0.75	-0.96	-1.20	-0.73	-0.35	-0.40	-1.24	-1.13
	1	-0.44	-0.51	-0.18	0.15	0.17	-0.72	-0.26
	1.25	0.12	0.34	0.25	0.58	0.22	0.17	0.07
	1.5	-0.30	0.32	0.21	0.72	0.60	0.48	0.17
	1.75	-0.39	0.47	0.24	0.81	0.48	0.46	0.46
Price p	$10	0.47	0.13	0.43	0.70	3.19	1.64	0.24
	$15	-0.08	0.13	0.41	0.64	1.92	1.28	0.19
	$20	-0.22	0.02	0.03	0.52	0.40	0.36	0.03
	$25	-0.79	-0.02	-0.29	0.25	-1.48	-1.12	-0.34
	$30	-1.35	-0.86	-0.79	-0.20	-2.97	-3.02	-0.81
Outside good		0	0	0	0	0	0	0
Mkt. size ratio		7.1%	19.2%	14.2%	19.8%	13.6%	15.8%	10.3%

shown) vary arbitrarily within the space of feasible designs that produce optimal attributes in this model. Similar to the observations in the previous two cases, the fixed competitor assumption gives the highest predicted profit, but the realized profit demonstrates the prediction is actually overestimated when market competition is taken into account. Accounting for competition when designing the new product results in a 3.4% increase in realized profit. The price-adjusted profit is 1.6% higher than realized profit, which implies that poor pricing is a significant component of the realized profit loss in the fixed competitor case, but poor design is a larger component. Finally, Figure 3 compares the realized profits of three cases. It shows that the Stackelberg strategy leads to a higher expected profit than Nash. The Nash and Stackelberg strategies are able to produce 3.0% and 3.4% higher realized profits than the fixed competitor case. In this case, the new product Stackelberg leader has the lowest product price, but the strategy is able to gain the highest market share and profit. This case study again demonstrates that incorporating price competition in product design can not only avoid overestimation of profitability, but also help designer make the best strategic design decisions.

Table 13: Specifications of weight scale competitors

Product	Weight capacity z_1	Aspect ratio z_2	Platform area z_3	Gap size z_4	Number size z_5	Price p
C1	350	1.02	120	0.188	1.40	$29.99
R2	250	0.86	105	0.094	1.25	$19.99
S3	280	0.89	136	0.156	1.70	$25.95
T4	320	1.06	115	0.125	1.15	$22.95

Table 14: New product design solutions for the weight scale problem

	Fixed competitors	**Nash**	**Stackelberg**
Price	$18.24	$16.87	$15.80
z_1	258	261	261
z_2	1.046	1.039	1.040
z_3	132	140	140
z_4	0.117	0.119	0.119
z_5	1.350	1.385	1.385
Predicted Profit	$24.07M	$13.86M	$13.92M
Realized Profit	**$13.46M**	**$13.86M**	**$13.92M**
Adjusted Profit	$13.68M	$13.86M	$13.92M

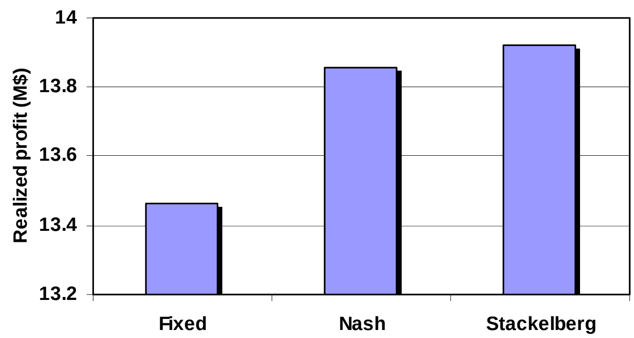

Figure 3: Realized profits under various strategies for the weight scale design problem

4. CONCLUSIONS

Prior profit maximization methods in engineering design ignore competitive reactions. We propose an approach to solve the new product design problem for profit maximization while accounting for competitive reactions under Nash and Stackelberg price competition strategies. Our approach accounts for competitive reactions through inclusion of equilibrium conditions as constraints in the optimization framework, and we propose a Lagrangian extension for cases with price bounds. This approach requires little additional complexity and offers greater efficiency and convergence stability than prior methods. Because the equilibrium conditions are set only with respect to competitor pricing decisions, it is not necessary to know competitor cost structures or internal competitor product engineering details, and the equilibrium conditions can be added to any existing product design profit optimization problem, including those with black box simulations or discrete variables, with appropriate solvers.

Table 15: Realized profits with product prices and market shares after new product entry

Product	Fixed competitors			Nash			Stackelberg		
	Price	Market share	Realized profit	Price	Market share	Realized profit	Price	Market share	Realized profit
C1	$16.86	21.8%	$14.13M	$17.00	21.2%	$13.85M	$17.13	20.7%	$13.64M
R2	$14.98	15.0%	$8.00M	$15.06	14.6%	$7.77M	$15.11	14.2%	$7.59M
S3	$17.37	20.9%	$14.00M	$17.59	20.1%	$13.68M	$17.81	19.6%	$13.48M
T4	$17.48	17.2%	$11.44M	$17.73	16.7%	$11.26M	$17.94	16.2%	$11.13M
New	**$18.24**	**19.0%**	**$13.46M**	**$16.87**	**21.4%**	**$13.86M**	**$15.80**	**23.3%**	**$13.92M**
No-purchase	–	6.1%	–	–	6.0%	–	–	6.0%	–

Prior profit maximization methods in engineering design ignore competitive reactions. We propose an approach to solve the new product design problem for profit maximization while accounting for competitive reactions under Nash and Stackelberg price competition strategies. Our approach accounts for competitive reactions through inclusion of equilibrium conditions as constraints in the optimization framework, and we propose a Lagrangian extension for cases with price bounds. This approach requires little additional complexity and offers greater efficiency and convergence stability than prior methods. Because the equilibrium conditions are set only with respect to competitor pricing decisions, it is not necessary to know competitor cost structures or internal competitor product engineering details, and the equilibrium conditions can be added to any existing product design profit optimization problem, including those with black box simulations or discrete variables, with appropriate solvers.

We show that failing to account for competitive reactions can result in suboptimal design and pricing solutions and significant overestimation of expected market performance. Application of the method to three case studies from the literature exhibits its ability to handle problems of interest in the engineering domain. The case study results indicate that the Stackelberg strategy is most preferred because of the capability to generate higher profits than Nash by anticipating competitor reactions. Both Nash and Stackelberg strategies avoid overestimation of market performance and potentially poor product design positioning resulting from the common fixed competitor model.

We have focused on accounting for competitor pricing reactions, assuming that in the short-run competitors will not be able to change their design decisions in response to new product entrants. However, an important topic for future research is to account for long-run competitor design changes made in reaction to a new product entrant in order to support long-run competitive strategy in design optimization.

ACKNOWLEDGMENTS

The authors would like to thank Professor S. Chan Choi, Professor Nathan Williams, and Professor Shapour Azarm for providing modeling data for their case studies. This research was supported in part by a grant from the National Science Foundation program for Material Use, Science, Engineering and Society: Award #0628084 and by a grant from Ford Motor Company.

NOMENCLATURE

a	=	Polynomial coefficient
b	=	Constant utility term
c	=	Cost
$\mathbf{g}$	=	Inequality constraint of new product design
$\mathbf{h}$	=	Equality constraint of new product design
i	=	Variable index
I	=	Number of individuals (observations)
j	=	Product index
J	=	Set of all products
k	=	Producer index
K	=	Set of all producers
m	=	Index of market segments
M	=	Number of market segments
n	=	Index of product attributes
N	=	Number of product attributes
p	=	Price
p^{L}	=	Price lower bound
p^{U}	=	Price upper bound
q_m	=	Market size within segment m
Q	=	Total market size
s	=	Share of choice
v	=	Observable utility
v^{P}	=	Price utility
v_0	=	Utility of outside good (no-purchase option)
x	=	Design variable
$\mathbf{x}$	=	Design variable vector
y	=	Design parameter
z	=	Product attribute
z^{C}	=	Consumer desired attribute
$\mathbf{z}$	=	Product attribute vector
β	=	Preference coefficient
Π	=	Profit
ψ	=	Spline utility function
ψ^{P}	=	Price utility in spline form
χ	=	Utility weight coefficient

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APPENDIX

A.1 Derivation of first-order conditions for Nash price equilibrium under the ideal point logit model

The first-order derivation of the choice probability in the disutility function is:

$$\frac{\partial s_{ij}}{\partial p_j} = \left(-\chi \frac{\partial v_{ij}^{\mathrm{D}}}{\partial p_j}\right) s_{ij} - s_{ij}\left(-\chi \frac{\partial v_{ij}^{\mathrm{D}}}{\partial p_j}\right) s_{ij}$$

$$= \left(-\chi\beta_{\mathrm{p}i}\right) s_{ij}\left(1-s_{ij}\right)$$

Therefore the first-order condition for profit maximization is:

$$\frac{\partial \Pi_j}{\partial p_j} = Q\frac{1}{I}\sum_{i=1}^{I} s_{ij} + Q(p_j - c_j)\frac{1}{I}\sum_{i=1}^{I}\frac{\partial s_{ij}}{\partial p_j}$$

$$= \frac{Q}{I}\left[\sum_{i=1}^{I} s_{ij} + (p_j - c_j)\sum_{i=1}^{I}\left(-\chi\beta_{\mathrm{p}i}\right) s_{ij}\left(1-s_{ij}\right)\right]$$

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{i=1}^{I} s_{ij}\left[1-\chi\beta_{\mathrm{p}i}(p_j - c_j)\left(1-s_{ij}\right)\right] = 0$$

A.2 Derivation of first-order conditions for Nash price equilibrium under the latent class model with multiple market segments

We derive the following equations as the general case for multiple market segments. A demand model with a market without segmentation can be considered as a special case of general equation. We consider that each producer k has one specific brand-product $j \in J_k$. The share of choices for the product j in segment m is:

$$s_{mj} = \frac{\mathbf{exp}(v_{mj})}{\mathbf{exp}(v_{m0}) + \sum_m \mathbf{exp}(v_{mj})} \quad \forall j \in J_k, m$$

The first-order derivative of choice probability with respect to price for each segment is:

$$\frac{\partial s_{mj}}{\partial p_j} = \frac{\partial v_{mj}}{\partial p_j} s_{mj}(1 - s_{mj})$$

The profit function of product j is:

$$\Pi_j = \left(\sum_{m=1}^{M} q_m s_{mj}(p_j - c_j)\right) - c_j^{\mathrm{F}}$$

The first-order condition equation is:

$$\frac{\partial \Pi_j}{\partial p_j} = \left[\sum_{m=1}^{M} q_m \frac{\partial s_{mj}}{\partial p_j}(p_j - c_j) + s_{mj}\right]$$

$$= \sum_{m=1}^{M} q_m s_{mj}\left[\frac{\partial v_{mj}}{\partial p_j}(1 - s_{mj})(p_j - c_j) + 1\right]$$

Therefore, the necessary condition for profit maximization at price equilibrium is:

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} q_m s_{mj}\left[\frac{\partial v_{mj}}{\partial p_j}(1 - s_{mj})(p_j - c_j) + 1\right] = 0$$

For the angle grinder problem, the price utility in each segment is a quadratic function. The first-order derivative has a closed form expression as $\partial v_{mj}/p_j = 2a_{2m}p_j + a_{1m}$. Therefore, the equilibrium equation becomes:

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} q_m s_{mj}\Big[\left(2a_{2m}p_j + a_{1m}\right)\left(1 - s_{mj}\right)\left(p_j - c_j\right) + 1\Big] = 0$$

For the weight scale problem, the part-worth price utility is interpolated with a piece-wise spline function ψ^{P}, which has first-order and second-order continuity. Thus the first-order condition is given by:

$$\frac{\partial \Pi_j}{\partial p_j} = \sum_{m=1}^{M} q_m s_{mj}\left[\frac{\partial \psi_{mj}^{\mathrm{P}}}{\partial p_j}(1 - s_{mj})(p_j - c_j) + 1\right] = 0$$