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SHOULD DESIGNERS WORRY ABOUT MARKET SYSTEMS?

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ABSTRACT

Engineering approaches for optimizing designs within a market context generally take the perspective of a single producer, asking what design and price point will maximize producer profit predicted by consumer choice simulations. These approaches treat competitors and retailers as fixed or nonexistent, and they take business-oriented details, such as the structure of distribution channels, as separate issues that can be addressed *post hoc* by other disciplines.

It is well established that the structure of market systems influences optimal product pricing. In this paper, we investigate whether two types of these structures also influence optimal product design decisions; specifically, 1) consumer heterogeneity and 2) distribution channels. We first model firms as players in a profit-seeking game that compete on product attributes and prices. We then model the interactions of manufacturers and retailers in Nash competition under alternative market structures and compare the equilibrium conditions for each case. We find that when consumers are modeled as homogeneous in their preferences, optimal design can be decoupled from the game, and design decisions can be made without regard to price, competition, or channel structure. However, when consumer preferences are heterogeneous, the behavior of competitors and retailers is key to determining which designs are profitable. We examine the extent of this effect in a vehicle design case study from the literature and find that the presence of heterogeneity leads different market structures to imply significantly different profit-maximizing designs.

Keywords: Product Design; New Product Development; Market Structure; Channel Structure; Game Theory; Nash Equilibrium; Optimization; Heterogeneity; Design for Market

1 INTRODUCTION

Methods for profit maximization in design require the designer to model not only physical and technical attributes of the product, but also to predict cost and demand resulting from design decisions. To do this, researchers have drawn upon quantitative methods from marketing and econometrics to model consumer choice as a function of the design's attributes using survey data or past purchase data. A variety of consumer utility approaches have been employed, including deterministic utility functions [1,2] and random utility functions that account for unobservables, such as logit [3,6], latent class logit [7], nested logit [8], mixed logit [9], and Bayesian mixture models [6,10]. While econometricians have used these models primarily for *estimation*, to understand the structure of preferences in the marketplace, engineers have used these models for *prediction* to simulate market demand and optimize products for profitability.

In contrast to the active research on market demand modeling in design optimization, there has been only limited attention paid to the role of *competition* in product design. Table 1 classifies the prior literature on product design using random utility discrete choice models for consumer choice simulation. These approaches differ in their models of 1) manufacturers and 2) retailers. On the manufacturer dimension there are three main classes: **Class I** models treat the focal manufacturer as the only decision-maker, where competitors are either not present or they are treated as fixed entities that will not react to the presence of a new design entrant. **Class II** models assume that competitors will respond to a new design entrant by adjusting pricing strategy, but competitor designs will remain fixed. **Class III** models assume that competitors will respond by both repricing and redesigning their products. Most prior studies do not account for the presence of retailers, instead assuming that manufacturers sell directly to consumers. When the retailer is accounted for, the model is said to incorporate the product's distribution *channel structure* [11,12]. Studies that account for retailers either assume the retailer to impose an exogenously-determined fixed margin over the manufacturer's wholesale price, or the retailer is treated a decision maker who will set margin in order to maximize profit.

Table 1: Literature on product design optimization using random utility discrete choice models

			Retailer		
Manufacturer	**Class**	**Competitors**	**None**	**Fixed**	**Decide margin**
	I	**None**	Wassenaar *et al*. [3] Michalek *et al*. [5] Michalek *et al*. [6] Kumar *et al.* [8]	–	–
		Fixed	Besharati *et al*. [7]	Williams *et al*. [22]	–
	II	**Decide price**	Choi *et al.* [25] Shiau and Michalek [26]	–	Luo *et al*. [27]
	III	**Decide price and design**	Choi and Desarbo [28] Michalek *et al*. [4]	–	**This paper**

In this paper, we pose a class III model with all manufacturers and retailers as decision-makers, we derive general equilibrium equations for each channel scenario, we propose a numerical solution approach, and we use the resulting models to investigate the following questions:

1) *How does consumer demand heterogeneity affect optimal product design?* We compare the use of the standard logit model, where differences among consumers are modeled only as noise, against the random coefficient mixed logit model, where the structure of consumer heterogeneity is modeled directly, and we examine the resulting effects on optimal design.

2) *How does channel structure affect optimal product design?* Research in marketing and management science has shown that channel structure has a significant effect on optimal pricing decisions [13-21]; we investigate whether channel structure also has a significant effect on optimal design decisions.

2 LITERATURE REVIEW

Class I formulations are most common in the profit maximization design literature. These approaches take the perspective of a single firm and assume there are no other decision-makers. Most models have taken the firm to be a monopolist in the product class with no competition other than the outside good, so that consumers are modeled to either buy from the firm or not buy at all [1-6,8]. Besharati *et al*. [7] included static competitor products and proposed an approach to generate optimal robust-design sets considering utility variations in both the new design and competing products. Williams *et al.* [22] also included fixed competitors and went further to incorporate retailer decisions in their model. Rather than model the retailer as a margin-setting profit maximizer, they assume a fixed margin and predict the channel acceptance rate – i.e., the probability that a retailer will agree to sell the new product through its distribution channel, which depends on the product attributes, wholesale price, and slotting allowance paid to the retailer. The primary limitation of class I methods is that they ignore competitor reactions. In differentiated oligopoly markets, competitors can be expected to react to a new product entry by changing prices in the short term and by changing designs in the long term. Thus, models that ignore competitor reactions will tend to overestimate profitability of a new entrant.

Class II formulations assume that competitor designs are fixed but attempt to account for competitor pricing reactions using game theory [23]. A core concept of game theory is the *Nash equilibrium*: a point at which no player (decision maker) can make a unilateral change to its decision (price, in this case) without decreasing its payoff (profit) [24]. Such a point represents a stable market equilibrium. In class II models, price is modeled in Nash equilibrium, whereas product design is optimized by single firm conditional on the static attributes of other products in the market. Since the time needed to design and deploy a new product is substantial for many product classes, most firms are not able to change their product designs in the short term, but pricing decisions can be changed rapidly. Thus, class II formulations may be a good description of short-term firm behavior for many product classes. Choi *et al.* [25] posed a class II problem by iteratively optimizing the design and pricing of the new entrant followed by price optimization of competitors, repeating this sequence until convergence to an equilibrium point. Shiau and Michalek [26] proposed an alternative efficient single-step approach based on Nash necessary conditions and showed that ignoring competitor reactions can result in significant overestimation of profits and suboptimal design variables. Lou *et al.* [27] applied a different approach: They first performed a heuristic product selection by combining discrete product attributes to reduce the optimal candidates to a manageable number. Then the optimal price and design solution were determined by exhaustive enumeration to find the alternative with the highest profit at price equilibrium, given fixed competitor product attributes.

Class III formulations assume that competitors are able to change both prices and product designs in reaction to a new product entry. As the lead time of new product development becomes shorter due to advancements in CAD, CAE, concurrent engineering, rapid prototyping, flexible manufacturing, supply chain management, and streamlined processes, it may be overly restrictive to assume that competitor product lines will remain fixed. Class III formulations search for combinations of design and pricing decisions that are in equilibrium, therefore product design variables and price must be solved simultaneously. Choi and Desarbo [28] studied an attribute selection (integer nonlinear programming) and competition problem for automotive tires using sequential iterative optimization among firms to find the Nash equilibrium solution. Michalek *et al.* [4] proposed a vehicle design problem with multiple automobile manufacturers competing on vehicle design and price under alternative government policy scenarios, and Shiau and Michalek [29] posed a direct method for locating equilibria of the problem. These prior methods do not address channel structures, assuming instead that manufacturers set retail prices directly.

Channel structure models have been used widely in management and marketing science to model manufacturer-retailer, manufacturer-manufacturer, and retailer-retailer interactions in a competitive market. These studies focus on price competition and treat design as fixed. Jeuland and Shugan [13] introduced a bilateral channel structure model with two separate manufacturer-retailer channels competing in the market. Later McGuire and Staelin [14] proposed a model with two competing manufacturers selling products through a company store[1] and a franchised retailer[2]. Choi [15] presented a channel structure model for a common retailer[3], systematically defining several game rules to describe the interactions between manufacturers and retailers based on the concepts of Nash and Stackelberg (leader-follower) games. Lee and Staelin [16] extended Choi's single common retailer framework to include multiple common retailers. While these prior approaches used simple linear or nonlinear demand functions, Besanko *et al.* [17] incorporated the logit demand function into Choi's common retailer model, and Sudhir [20] extended Besanko's work by deriving an array of analytical equilibrium equations using various profit maximization strategies[4] under both vertical Nash and manufacturer Stackelberg game rules[5].

The proposed approach fills a gap in the prior literature by posing a class III formulation under alternative channel structures and examining the impact of each structure on design and pricing decisions in a continuous variable space. The remainder of the paper is organized as follows: In Section 3, we derive equations for an integrated model of design and pricing equilibrium under alternative channel structures and demand heterogeneity, and we examine the structure of the results, posing several propositions on the role of heterogeneity in competitive design. In Section 4 a vehicle design example is implemented as a case study to demonstrate our methodology and test the degree to which channel structure and demand heterogeneity influence optimal design in a practical example. We then conclude and outline future work in Section 5.

3 METHODOLOGY

We develop our methodology by first examining models for consumer choice and channel structures. We then derive equilibrium conditions for each case and make several observations.

3.1 Consumer Choice

Market equilibrium conditions for profit maximizing firms depend upon consumer choice behavior. We adopt the random utility discrete choice model, which is ubiquitous in marketing and econometrics [30] and has seen recent application in engineering design [3-6]. Random utility models presume that each consumer i gains some utility $u_{ij}\in\Re$ from each product alternative j. Consumers are taken as rational, selecting the alternative that provides the highest utility, but each consumer's utility is only partly observable. Specifically,

$$u_{ij} = v_{ij} + \varepsilon_{ij} \tag{1}$$

where v_{ij} is the observable component and ε_{ij} is the unobservable component. The observable term v_{ij} is a function of the observable parameters of a choice scenario: in this case, the attributes $\mathbf{z}_j$ and price p_j of each product j, so that $v_{ij} = v(p_j, \mathbf{z}_j, \boldsymbol{\beta}_i)$, where $\boldsymbol{\beta}_i$ is a vector of coefficients specific to individual i. The product attributes $\mathbf{z}_j$ are functions of the design variables $\mathbf{x}_j$ for each product, therefore $\mathbf{z}_j=\mathbf{z}(\mathbf{x}_j)$. By assuming the error term ε_{ij} follows the IID extreme value distribution $f_\varepsilon(\varepsilon)=\exp(-\exp(-\varepsilon))$, which is close to Gaussian but more convenient, the probability s_{ij} of consumer i choosing product j is given by the logit model [31]:

$$s_{ij} = \frac{\exp(v_{ij})}{\exp(v_0) + \sum_{k\in K}\sum_{j\in J_k}\exp(v_{ij})} \tag{2}$$

where K is the set of manufacturers, J_k is the set of products sold by manufacturer k, and the utility of the outside good[6] v_0 represents the utility value of the individual choosing none of the alternatives in the choice set. To obtain the total share of choices, we can integrate over consumers i. If $f_\beta(\boldsymbol{\beta})$ represents the joint probability density function of $\boldsymbol{\beta}$ coefficients across the consumer population i, and $s_{j|\boldsymbol{\beta}}$ is s_{ij} calculated conditional on $\boldsymbol{\beta}_i=\boldsymbol{\beta}$ (i.e.: as a function of $v_{ij}=v(p_j,\mathbf{z}_j,\boldsymbol{\beta})$), then the *share of choices* for product j (i.e.: the probability of a randomly selected individual choosing product j) is:

$$s_j = \int s_{j|\boldsymbol{\beta}}\, f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} \tag{3}$$

The integral form of Eq. (3) is called the *mixed logit* or *random coefficients* model [31]. The mixed logit model has been

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[1] A company store (also called factory store) is a retail store owned by a specific manufacturer, so that wholesale price and retail price are equal. Such a channel configuration is also referred to as vertical integration [14].

[2] A franchised retailer (also called exclusive store) is a retail store owned by a private company that sells products from only one manufacturer.

[3] A common retailer is a retailer who sells products produced by multiple manufacturers.

[4] Sudhir [20] used three retailer strategies in his paper: 1) category profit maximization, which includes profit from different brands and also margins from the outside good; 2) brand profit maximization, which considers single brand profit only; and 3) constant margin, where retailer holds a fixed retail margin. We focus exclusively on the category profit maximization retail strategy.

[5] Vertical Nash, first defined by Choi [15], is the Nash competition scenario for manufacturer and retailer players. Similarly, a manufacturer Stackelberg game treats manufacturer players as Stackelberg leaders and retailer players as Stackelberg followers.

[6] The outside good utility is obtained by estimation, the same as attribute coefficients.

demonstrated to be capable of approximating any random utility discrete choice model [32]. In practical applications, the mixed logit choice probability is approximated using numerical simulation by taking a finite number of draws $r = 1, 2, \ldots, R$ from the distribution $f_{\beta}(\boldsymbol{\beta})$ [31]:

$$\hat{s}_j = \frac{1}{R}\sum_{r=1}^{R} s_{rj} = \frac{1}{R}\sum_{r=1}^{R}\frac{\exp(v_{rj})}{\exp(v_0)+\sum_{k\in K}\sum_{j\in J_k}\exp(v_{rj})} \tag{4}$$

where R is the number of random draws, s_{rj} is the logit choice probability for product j in the r-th draw, and v_{rj} is the corresponding simulated observable utility. The random coefficients of the mixed logit coefficients are able to model systematic taste variations, i.e.: heterogeneity, across individuals.

The standard logit model, also known as the *multinomial logit model* when more than two choice alternatives are present, is a special case where the coefficients $\boldsymbol{\beta}$ are taken as deterministic, aggregate parameters during estimation, and variation across consumers is accounted for only in the unobservable error term ε. When heterogeneity of consumer preferences is negligible, the logit model may be sufficient for estimation while requiring less data and offering lower complexity and computational cost. When heterogeneity is significant, the mixed logit model is capable of capturing the structure of heterogeneity. For these reasons, both logit and mixed logit models are compared in this study.

3.2 Channel Structures

Figure 1 shows the vertical price interaction paths of four distribution channels with different retailer types, where w is the manufacturer's wholesale price and p is the retail price. The four channel scenarios are:

1) *Company store* (CS): A company store sells only products from a single manufacturer, and the retail prices are directly controlled by the correspondent manufacturer (w=p) [14]. There is no vertical interaction between a manufacturer and its company-owned retailer because of integration.
2) *Franchised retailer* (FR): A franchised store is privately-owned but has a contract with the corresponding manufacturer. It sells only the products produced by the specific manufacturer. However, the manufacturer does not control retail prices directly, and the retailer is able to determine its own retail margins [14].
3) *Single common retailer* (SCR): A common retailer sells mixed products from all available manufacturers, and it has control of its margins [15]. The SCR case represents a powerful retailer dominating a regional market with no other equal-powered competitors in the region.
4) *Multiple common retailers* (MCR): This scenario represents more than one medium-sized retailer in the regional market [16]. These common retailers compete with one another for pursuing maximum profits.

Manufacturer and retailer profit depend on demand q_j, which can be predicted by multiplying the total size of the market Q by the share of choices s_j taken by product j, so that q_j=Qs_j. We consider the product cost in two components, 1) the variable cost per unit, which is a function of the design $\mathbf{x}_j$, and 2) the total fixed investment cost c^{F}_j, so that total cost for product j is $q_jc_j(\mathbf{x}_j) + c^{\mathrm{F}}_j$. We derive first the general multiple common retailer case with a set of retailers $t\in T$ and then examine alternative channel structures as special cases. The profit function for manufacturer k is a sum over the retailers T and the set of products J_k:

$$\Pi_k^{\mathrm{M}} = \sum_{t\in T}\sum_{j\in J_k}\left[q_{jt}(w_{jt}-c_j)-c_j^{\mathrm{F}}\right] \tag{5}$$

where w_{jt} is the wholesale price of product j when sold to retailer t.[7] The manufacturer profit functions for the other three channel structure scenarios can be simplified from Eq. (5) by removing the retailer index t, as shown in Table 2.

The profit function for retailer t in the MCR scenario is given by:

$$\Pi_t^{\mathrm{R}} = \sum_{k\in K}\sum_{j\in J_k} q_{jt}(p_{jt}-w_{jt}) = \sum_{k\in K}\sum_{j\in J_k} q_{jt}m_{jt} \tag{6}$$

where m_{jt} is retailer t's margin for product j. The SCR scenario is a special case of MCR with a unique t. In the FR scenario, the profit function of a franchised store can be simplified from Eq. (6) by indexing each retailer with its corresponding manufacturer k and limiting the product category to the corresponding manufacturer source. For the CS scenario, the company store has no retail profit. The manufacturer and retailer profit functions for the four channel structure scenarios are listed in Table 2.

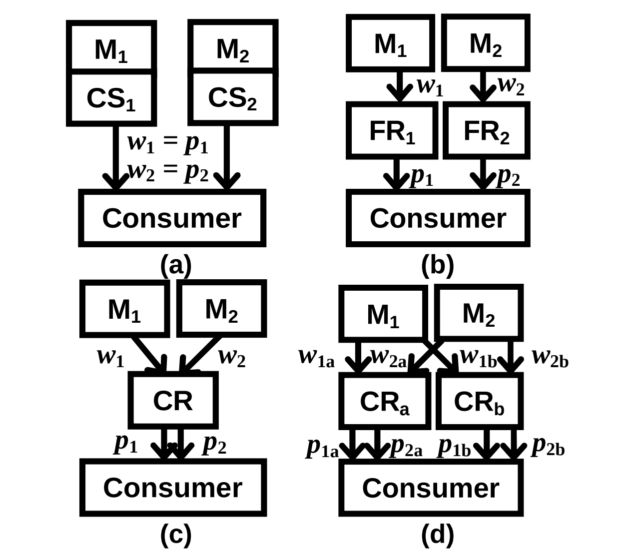

Figure 1 Channel structure scenarios:
(a) CS: Company Store (b) FR: Franchised Retailer
(c) SCR: Single Common Retailer
(d) MCR: Multiple Common Retailers

[7] We assume manufacturers are able to offer different wholesale prices to different retailers.

Table 2 Manufacturer and retailer profit functions

Case	Manufacturer profit	Retailer profit
CS	$\Pi_k^{\mathrm{M}} = \left[\sum_{j \in J_k} q_j (w_j - c_j)\right] - c_j^{\mathrm{F}}$	–
FR	$\Pi_k^{\mathrm{M}} = \left[\sum_{j \in J_k} q_j (w_j - c_j)\right] - c_j^{\mathrm{F}}$	$\Pi_k^{\mathrm{R}} = \sum_{j \in J_k} q_j m_j$
SCR	$\Pi_k^{\mathrm{M}} = \left[\sum_{j \in J_k} q_j (w_j - c_j)\right] - c_j^{\mathrm{F}}$	$\Pi^{\mathrm{R}} = \sum_{k \in K} \sum_{j \in J_k} q_j m_j$
MCR	$\Pi_k^{\mathrm{M}} = \left[\sum_{t \in T} \sum_{j \in J_k} q_{jt} (w_{jt} - c_{jt})\right] - c_{jt}^{\mathrm{F}}$	$\Pi_t^{\mathrm{R}} = \sum_{k \in K} \sum_{j \in J_k} q_{jt} m_{jt}$

3.3 Equilibrium Conditions

In a non-cooperative game with K players where each player k chooses a strategy $\mathbf{y}_k$ in order to maximize its payoff function Π_k, the Nash equilibrium represents a set of strategies $\{\mathbf{y}_1^*, \mathbf{y}_2^*, ..., \mathbf{y}_k^*, ..., \mathbf{y}_K^*\}$, one for each player, such that no player is able to obtain higher profit Π_k by unilaterally choosing any strategy $\mathbf{y}_k'$ other than the equilibrium strategy $\mathbf{y}_k^*$ [24]. The mathematical expression is given by:

$$\Pi_k(\mathbf{y}_1^*, \mathbf{y}_2^*, \ldots, \mathbf{y}_k^*, \ldots, \mathbf{y}_K^*) \geq \Pi_k(\mathbf{y}_1^*, \mathbf{y}_2^*, \ldots, \mathbf{y}_k', \ldots, \mathbf{y}_K^*) \quad \forall k, \mathbf{y}_k' \tag{7}$$

The above equation also implies that a Nash equilibrium is a simultaneous stationary point of each player's best response function. If the strategy vector $\mathbf{y}$ is continuous and unconstrained, the necessary first-order condition (FOC) for a Nash equilibrium is:

$$\frac{\partial \Pi_k}{\partial \mathbf{y}_k} = \mathbf{0}; \ \forall k \tag{8}$$

When we consider channel structures in a game-theoretic framework, manufacturers and retailers are both players (decision makers) in the game. The strategy (decisions) of a manufacturer includes wholesale price w and product design variables $\mathbf{x}$, and the strategy of a retailer is retail margin[8] m. Choi defines this game as a *vertical Nash* game for price competition [15]. We extend the model by including design competition[9]. As shown in Figure 2, the manufacturer makes wholesale price and design decisions to maximize its profit based on the retail margin observed. Accordingly manufacturer profit is calculated as a function of wholesale price, cost, and market demand, which is a function of retail prices. The retailer makes its retail margin decision independently from manufacturer decisions (except in the CS case). Each retailer observes manufacturer wholesale prices and product attributes, as well as any competitor retailer prices. At market equilibrium, no manufacturer or retailer can reach higher profit by changing

[8] We follow the prior literature and treat retail margin, rather than retail price, as the retailer's strategic decision. The manufacturer observes retail margin and thus calculates retail price based on its own wholesale price.

[9] We assume *a static game of complete information* [24] where each player in the game is able to know the others' strategies and payoff functions.

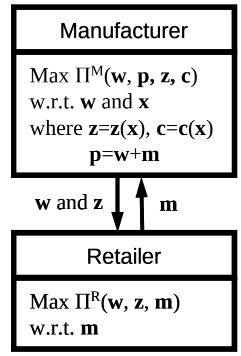

Figure 2 Interaction between manufacturer and retailer in the vertical Nash game

decisions unilaterally. For a vertical Nash game, all channel members act non-cooperatively.

The FOC necessary conditions for the vertical Nash game produce a system of nonlinear equations (one equation for each unknown) given by:

$$\begin{aligned} \frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} &= f_{\mathrm{w}}\left(\mathbf{x}_j, w_{jt}, m_{jt}; \forall j, t\right) = 0 \quad \forall k, t, j \in J_k \\ \frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} &= \mathbf{f}_{\mathrm{x}}\left(\mathbf{x}_j, w_{jt}, m_{jt}; \forall j, t\right) = \mathbf{0} \quad \forall k, j \in J_k \\ \frac{\partial \Pi_t^{\mathrm{R}}}{\partial m_{jt}} &= f_{\mathrm{m}}\left(\mathbf{x}_j, w_{jt}, m_{jt}; \forall j, t\right) = 0 \quad \forall k, t, j \in J_k \end{aligned} \tag{9}$$

where t is replaced by k in the FR case. These FOC conditions are necessary but not sufficient. Hence, any candidate FOC solution must be checked to see if it is a Nash equilibrium (Eq. (7)) by globally optimizing each player *post hoc* while holding all other players constant at the FOC solution[10] [23]. Similar to finding the optimal solution in a general optimization problem, the existence and uniqueness of an equilibrium solution in a market competition problem depends on the equations describing the model. Anderson *et al.* [33] demonstrated that a quasi-concave profit function with logit demand and price as the only variable results a unique Nash equilibrium. However, when design variables are included, the logit profit function may become non-concave, and multiple local optima may exist [34]. Therefore, convergence properties and the existence and uniqueness of equilibria are problem dependent. In our case study, necessary conditions in each case revealed either a unique solution or a small set of solutions that were easy to check *post hoc* to identify the unique Nash equilibrium.

To derive FOC equation sets for all channel structure scenarios, we first consider the general MCR mixed logit case and then derive other scenarios as special cases.

[10] The FOC approach is more efficient than the sequential iteration method used in the prior study of Michalek *et al.* [4]. The sequential iteration method requires iterative solution of a series of NLP problems for each manufacturer until Nash equilibrium is reached, while the FOC approach is a single step NLP execution for a local solution. The differences between two algorithms are discussed by Shiau and Michalek [29].

3.3.1 Wholesale Price

The wholesale price FOC equation is taken for each manufacturer k with respect to the wholesale price that manufacturer sets for each of its products $j\in J_k$ to sell to each retailer t. Under the mixed logit demand, the condition is[11]:

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \int_{\boldsymbol{\beta}} s_{jt|\boldsymbol{\beta}} \left(\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} \left(\left(w_{jt} - c_j \right) - \sum_{j'\in J_k} \sum_{t'\in T} s_{j't'|\boldsymbol{\beta}} \left(w_{j't'} - c_{j'} \right) \right) + 1 \right) f_{\beta}(\boldsymbol{\beta}) d\boldsymbol{\beta} = 0 \quad \forall t, k, j \in J_k \tag{10}$$

where $s_{jt|\boldsymbol{\beta}}$ is shorthand for the share of choices predicted by the logit model, given $\boldsymbol{\beta}$: in this case $\exp(v(p_{jt},\mathbf{z}_j,\boldsymbol{\beta}))[\exp(v_0) + \Sigma_k\Sigma_{t'}\Sigma_{j'\in Jk}\exp(v(p_{jt'},\mathbf{z}_{j'},\boldsymbol{\beta}))]^{-1}$, following Eq. (2). For the standard logit, the integral in Eq. (10) collapses, and the equation becomes:

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \frac{\partial v_{jt}}{\partial p_{jt}} \left(\left(w_{jt} - c_j \right) - \sum_{j'\in J_k} \sum_{t'\in T} s_{j't'} \left(w_{j't'} - c_{j'} \right) \right) + 1 = 0 \quad \forall k, t, j \in J_k \tag{11}$$

In the case of a single common retailer and a single product per manufacturer, Eq. (11) can be further simplified and rearranged as:

$$w_j = c_j + \left(-\frac{\partial v_j}{\partial p_j} \left(1 - s_j \right) \right)^{-1} \quad \forall j \in J_k \tag{12}$$

Eq. (12) illustrates that wholesale price at equilibrium is comprised of product cost plus a manufacturer margin, which is determined by the sensitivity of consumer observable utility to price and the corresponding share of choices. The same result was obtained by Besanko *et al.* [17] in the case of price only (with no design decisions).

3.3.2 Design

For the case of an unconstrained design space, the design variable FOC equations for MCR are obtained similarly by setting the derivative of the manufacturer profit function with respect to each design variable to zero. Without loss of generality, we assume all designs are carried by all retailers (potentially with q=0):

$$\frac{\partial \Pi_k}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} \sum_{t\in T} \left[\left(\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{z}_j} \frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} \right) \left(s_{jt|\boldsymbol{\beta}} \left(w_{jt} - c_j \right) - \left(\sum_{\bar{t}\in T} s_{j\bar{t}|\boldsymbol{\beta}} \right) \sum_{j'\in J_k} s_{j't|\boldsymbol{\beta}} \left(w_{j't} - c_{j'} \right) \right) - \frac{\partial c_j}{\partial \mathbf{x}_j} \right] f_{\beta}(\boldsymbol{\beta}) d\boldsymbol{\beta} = 0 \quad \forall k, t, j \in J_k \tag{13}$$

Under logit demand, Eq.(13) reduces to:

$$\frac{\partial \Pi_k}{\partial \mathbf{x}_j} = \sum_{t\in T} \left[\left(\frac{\partial v_{jt}}{\partial \mathbf{z}_j} \frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} \right) \left(s_{jt} \left(w_{jt} - c_j \right) - \left(\sum_{\bar{t}\in T} s_{j\bar{t}} \right) \left(\sum_{j'\in J_k} s_{j't} \left(w_{j't} - c_{j'} \right) \right) \right) - \frac{\partial c_j}{\partial \mathbf{x}_j} \right] = 0 \quad \forall k, t, j \in J_k \tag{14}$$

When equality constraints $\mathbf{h}(\mathbf{x})=\mathbf{0}$ and inequality constraints $\mathbf{g}(\mathbf{x})\leq\mathbf{0}$ exist in the design domain, additional constraint handling is needed. To account for constraints, we implement the Lagrangian FOC method [29] and re-formulate Eq. (13) as:

$$\frac{\partial L_k}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} \sum_{t\in T} \left[\left(\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{z}_j} \frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} \right) \left(s_{jt|\boldsymbol{\beta}} \left(w_{jt} - c_j \right) - \left(\sum_{\bar{t}\in T} s_{j\bar{t}|\boldsymbol{\beta}} \right) \left(\sum_{j'\in J_k} s_{j't|\boldsymbol{\beta}} \left(w_{j't} - c_{j'} \right) \right) \right) - \frac{\partial c_j}{\partial \mathbf{x}_j} \right] f_{\beta}(\boldsymbol{\beta}) d\boldsymbol{\beta} - \boldsymbol{\lambda}_j^T \frac{\partial \mathbf{h}_j}{\partial \mathbf{x}_j} - \boldsymbol{\mu}_j^T \frac{\partial \mathbf{g}_j}{\partial \mathbf{x}_j} = \mathbf{0} \quad \forall k, t, j \in J_k \tag{15}$$

$$\boldsymbol{\mu}_j{}^T \mathbf{g}\left(\mathbf{x}_j\right) = 0; \quad \boldsymbol{\mu}_j \geq \mathbf{0}; \quad \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0}; \quad \mathbf{g}\left(\mathbf{x}_j\right) \leq \mathbf{0}$$

where $\boldsymbol{\lambda}_j$ and $\boldsymbol{\mu}_j$ are Lagrange multiplier vectors for product j. Eq. (15) corresponds to the Karush-Kuhn-Tucker (KKT) necessary conditions for optimality of a constrained NLP [35].

3.3.3 Retailer Margin

The retailer margin FOC equation for the MCR case is taken for each retailer with respect to its margin. The condition for a common retailer t under mixed logit demand is:

$$\frac{\partial \Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \int_{\boldsymbol{\beta}} s_{jt|\boldsymbol{\beta}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} \left(m_{jt} - \sum_{k'\in K} \sum_{j'\in J_{k'}} s_{j't|\boldsymbol{\beta}} m_{j't} \right) + 1 \right] f_{\beta}(\boldsymbol{\beta}) d\boldsymbol{\beta} = 0 \quad \forall k, t, j \in J_k \tag{16}$$

For the logit model, the equation is simplified to:

$$\frac{\partial \Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \frac{\partial v_{jt}}{\partial p_{jt}} \left(m_{jt} - \sum_{k\in K} \sum_{j'\in J_k} s_{j't} m_{j't} \right) + 1 = 0 \quad \forall k, t, j \in J_k \tag{17}$$

In the case of a single product per manufacturer and a single common retailer, Eq. (17) can be simplified as:

$$m_j = \frac{1}{1 - s_j} \left[\left(-\frac{\partial v_j}{\partial p_j} \right)^{-1} + \sum_{k\in K} \sum_{\substack{j'\in J_k \\ j'\neq j}} s_{j'} m_{j'} \right] \quad \forall j \in J_k \tag{18}$$

[11] Detailed derivations of all FOC equations for the MCR scenario are shown in the appendix.

Table 3 FOC equations under each channel structure

	Logit	Mixed Logit
Company Store	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \frac{\partial v_j}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right) + 1 = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \left(\frac{\partial v_j}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left[\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right] - \frac{\partial c_j}{\partial \mathbf{x}_j} = \mathbf{0}$ $\forall k, j \in J_k$	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \int_{\boldsymbol{\beta}} s_{j\vert\boldsymbol{\beta}}\left[\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} s_{jt\vert\boldsymbol{\beta}}\left[\left(\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) - \frac{\partial c_j}{\partial \mathbf{x}_j}\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = \mathbf{0}$ $\forall k, j \in J_k$
Franchised Retailer	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \frac{\partial v_j}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right) + 1 = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \left(\frac{\partial v_j}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left[\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right] - \frac{\partial c_j}{\partial \mathbf{x}_j} = \mathbf{0}$ $\frac{\partial \Pi^{\mathrm{R}}}{\partial m_j} = \frac{\partial v_j}{\partial p_j}\left(m_j - \sum_{j' \in J_k} s_{j'} m_{j'}\right) + 1 = 0$ $\forall k, j \in J_k$	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \int_{\boldsymbol{\beta}} s_{j\vert\boldsymbol{\beta}}\left[\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} s_{jt\vert\boldsymbol{\beta}}\left[\left(\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) - \frac{\partial c_j}{\partial \mathbf{x}_j}\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = \mathbf{0}$ $\frac{\partial \Pi^{\mathrm{R}}}{\partial m_j} = \int_{\boldsymbol{\beta}} s_{j\vert\boldsymbol{\beta}}\left[\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial p_j}\left(m_j - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}} m_{j'}\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\forall k, j \in J_k$
Single Common Retailer	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \frac{\partial v_j}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right) + 1 = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \left(\frac{\partial v_j}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left[\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'}\left(w_{j'} - c_{j'}\right)\right] - \frac{\partial c_j}{\partial \mathbf{x}_j} = \mathbf{0}$ $\frac{\partial \Pi^{\mathrm{R}}}{\partial m_j} = \frac{\partial v_j}{\partial p_j}\left(m_j - \sum_{k' \in K}\sum_{j' \in J_{k'}} s_{j'} m_{j'}\right) + 1 = 0$ $\forall k, j \in J_k$	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_j} = \int_{\boldsymbol{\beta}} s_{j\vert\boldsymbol{\beta}}\left[\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial p_j}\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} s_{jt\vert\boldsymbol{\beta}}\left[\left(\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left(\left(w_j - c_j\right) - \sum_{j' \in J_k} s_{j'\vert\boldsymbol{\beta}}\left(w_{j'} - c_{j'}\right)\right) - \frac{\partial c_j}{\partial \mathbf{x}_j}\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = \mathbf{0}$ $\frac{\partial \Pi^{\mathrm{R}}}{\partial m_j} = \int_{\boldsymbol{\beta}} s_{j\vert\boldsymbol{\beta}}\left[\frac{\partial v_{j\vert\boldsymbol{\beta}}}{\partial p_j}\left(m_j - \sum_{k' \in K}\sum_{j' \in J_{k'}} s_{j'\vert\boldsymbol{\beta}} m_{j'}\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\forall k, j \in J_k$
Multiple Common Retailers	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \frac{\partial v_{jt}}{\partial p_{jt}}\left(\left(w_{jt} - c_j\right) - \sum_{j' \in J_k}\sum_{t' \in T} s_{j't'}\left(w_{j't'} - c_{j'}\right)\right) + 1 = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \sum_{t \in T}\left[\left(\frac{\partial v_{jt}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left(s_{jt}\left(w_{jt} - c_j\right) - \left(\sum_{\bar{t} \in T} s_{j\bar{t}}\right)\sum_{j' \in J_k} s_{j't}\left(w_{j't} - c_{j'}\right)\right) - s_{jt}\frac{\partial c_j}{\partial \mathbf{x}_j}\right] = \mathbf{0}$ $\frac{\partial \Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \frac{\partial v_{jt}}{\partial p_{jt}}\left(m_{jt} - \sum_{k' \in K}\sum_{j' \in J_{k'}} s_{j't} m_{j't}\right) + 1 = 0$ $\forall t, k, j \in J_k$	$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \int_{\boldsymbol{\beta}} s_{jt\vert\boldsymbol{\beta}}\left[\frac{\partial v_{jt\vert\boldsymbol{\beta}}}{\partial p_{jt}}\left(\left(w_{jt} - c_j\right) - \sum_{j' \in J_k}\sum_{t' \in T} s_{j't'\vert\boldsymbol{\beta}}\left(w_{j't'} - c_{j'}\right)\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}}\sum_{t \in T}\left[\left(\frac{\partial v_{jt\vert\boldsymbol{\beta}}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j}\right)\left(s_{jt\vert\boldsymbol{\beta}}\left(w_{jt} - c_j\right) - \left(\sum_{\bar{t} \in T} s_{j\bar{t}\vert\boldsymbol{\beta}}\right)\sum_{j' \in J_k} s_{j't\vert\boldsymbol{\beta}}\left(w_{j't} - c_{j'}\right)\right) - s_{jt\vert\boldsymbol{\beta}}\frac{\partial c_j}{\partial \mathbf{x}_j}\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = \mathbf{0}$ $\frac{\partial \Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \int_{\boldsymbol{\beta}} s_{jt\vert\boldsymbol{\beta}}\left[\frac{\partial v_{jt\vert\boldsymbol{\beta}}}{\partial p_{jt}}\left(m_{jt} - \sum_{k' \in K}\sum_{j' \in J_{k'}} s_{j't\vert\boldsymbol{\beta}} m_{j't}\right) + 1\right] f_\beta(\boldsymbol{\beta})d\boldsymbol{\beta} = 0$ $\forall t, k, j \in J_k$

Combining Eq. (12) and Eq. (18), the retail price of product j selling through common retailer t satisfies:

$$p_j = w_j + m_j = c_j + \left[\frac{1}{1-s_j}\left(-\frac{\partial v_j}{\partial p_j}\right)^{-1}\right] + \frac{1}{1-s_j}\left[\left(-\frac{\partial v_j}{\partial p_j}\right)^{-1} + \sum_{k \in K}\sum_{\substack{j' \in J_k \\ j' \neq j}} s_{j'} m_{j'}\right] \quad \forall j \in J_k \tag{19}$$

Equation (19) illustrates that retailer price at market equilibrium is composed of manufacturing cost, manufacturer margin, and retailer margin. From the general FOC equations for the MCR case under mixed logit demand, the equations for the other seven cases can be obtained through simplifications. The equations are shown in Table 3. Results for logit produce closed form expressions and provide intuition, while the mixed logit model accommodates heterogeneity by modeling its structure directly.

3.4 Observations

We now examine several useful observations about equilibrium conditions under the standard logit case when the utility function v is linear in price. The linear price assumption is important because models with nonlinear utility for price may contain interaction terms that imply consumers' sensitivity to price varies with the value of other attributes, thus coupling price to attributes. However, if interaction terms are negligible, as is most commonly assumed, then the standard main-effects logit model has utility linear in price, and consumers make choices via typical compensatory tradeoffs between price and other attributes. The first two propositions show that manufacturers and retailers set identical margins for all products.

Proposition 1: In the logit case with utility linear in price, the Nash equilibrium requires that manufacturer margins are equal for all products and all manufacturers.

Proof: From the wholesale price FOC equation for the general MCR case under the logit model in Table 3, the equation can be rearranged to:

$$w_{jt} - c_j = -\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1} + \sum_{j' \in J_k} \sum_{t' \in T} s_{j't'}\left(w_{j't'} - c_{j'}\right) \quad \forall j \in J_k \tag{20}$$

For the case where v_j is linear in price, $\partial v_j/\partial p_j = \beta_{\mathrm{p}}$, and the right hand side of the equation is identical for all $j \in J_k$. Therefore, each product produced by manufacturer k has the identical manufacturing margins w_{jt}-c_j. This result holds for the other channel types, which are special cases of Eq. (20).

Proposition 2: In the logit case with utility linear in price, the Nash equilibrium requires that retail margins are equal for all products and all retailers.

Proof: From the retail margin FOC equation for the general MCR case under the logit model in Table 3, the retail margin of product j selling at retailer t is:

$$m_{jt} = -\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1} + \sum_{k' \in K} \sum_{j' \in J_{k'}} s_{j't} m_{j't} \quad \forall j \in J_k \tag{21}$$

For the case where v_j is linear in price, $\partial v_j/\partial p_j = \beta_{\mathrm{p}}$, and the right hand side of the equation is identical for all products sold by retailer t. Therefore, the retail margins of all products of retailer t are equal. This result holds for the other channel types (FR and SCR), which are special cases of Eq. (21).

The third proposition shows that design is independent of pricing and competition under the linear logit model. This implies that design can successfully be undertaken independently when consumers are homogeneous (or, more precisely, when variation among consumers is taken as IID random noise). However, heterogeneity couples the problems, making necessary joint consideration of design with pricing and competition.

Proposition 3: In the logit case with utility linear in price, the Nash equilibrium requires that all designs satisfy a system of equations that is independent of price and competitor designs. When this system of equations has a unique solution, it implies that a) all designs are identical across all producers and b) optimal design is independent of price, competition, and market structure.

Proof: By substituting Eq. (20) from Proposition 1 into Eq. (13) for the general MCR case under the logit model, we obtain:

$$\begin{aligned}\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \frac{\partial v_{jt}}{\partial \mathbf{x}_j}\Bigg[&\left(-\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1} + \sum_{j' \in J_k}\sum_{t' \in T} s_{j't'}\left(w_{j't'} - c_{j'}\right)\right) \\ &\left(\sum_{\bar{t} \in T} s_{j\bar{t}}\right) - \left(\sum_{\bar{t} \in T} s_{j\bar{t}}\right)\left(\sum_{t \in T}\sum_{j' \in J_k} s_{j't}\left(w_{j't} - c_{j'}\right)\right)\Bigg] \\ &- \frac{\partial c_j}{\partial \mathbf{x}_j}\left(\sum_{\bar{t} \in T} s_{j\bar{t}}\right) = \mathbf{0}\end{aligned} \tag{22}$$

This equation can be further simplified to:

$$\left(\sum_{t \in T} s_{jt}\right)\left(-\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1}\frac{\partial v_{jt}}{\partial \mathbf{x}_j} - \frac{\partial c_j}{\partial \mathbf{x}_j}\right) = \mathbf{0} \tag{23}$$

Because $s > 0$ (for all finite values of the decision variables):

$$\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1}\frac{\partial v_{jt}}{\partial \mathbf{x}_j} + \frac{\partial c_j}{\partial \mathbf{x}_j} = \mathbf{0} \tag{24}$$

For the case where v_j is linear in price, $\partial v_j/\partial p_j = \beta_{\mathrm{p}}$, the function can be presented as:

$$\frac{\partial v_{jt}}{\partial \mathbf{z}_j}\frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} + \beta_{\mathrm{p}}\frac{\partial c_j}{\partial \mathbf{x}_j} = \mathbf{0} \quad \forall t \in T, j \in J_k \tag{25}$$

Satisfaction of this system of equations is a necessary condition for a Nash equilibrium. If Eq. (25) has a unique solution and if a Nash equilibrium exists, then Eq. (25) specifies the equilibrium design. Implication a) follows from noting that Eq. (25) is identical for each j and is independent of all other $j' \neq j$.[12] Implication b) follows from noting that Eq. (25) is independent of p_j, $p_{j'}$, $\mathbf{x}_{j'}$ $\forall j' \neq j$. In other words, the equilibrium design can be calculated as a function of consumer utility functions and manufacturer cost functions without regard to price or competitor decisions, and design is decoupled from the game.

While we do not derive conditions under which Eq. (25) has a unique solution, we observe that in practical applications Eq. (25) typically has a unique solution or a small finite number of candidate solutions that can be checked *post hoc* for satisfaction of the Nash definition.

The final two propositions show the necessity of incorporating an outside good to establish finite equilibria in the case of a manufacturer or retailer monopoly.

Proposition 4: In the logit case with utility linear in price and a monopolist manufacturer, an outside good is required for existence of a finite Nash equilibrium.

Proof: Considering a single manufacturer with multiple common retailers (MCR case), the outside good market share s_0 is:

[12] Note also that for the special case of traditional profit maximization of a product line for a single producer with fixed competitors (outside good) and no retail structure (CS case), this implies that under logit linear in price all products in the line will be identical at the optimum.

$$s_0 = 1 - \sum_{j \in J} \sum_{t \in T} s_{jt} \tag{26}$$

For the case where v_j is linear in price, $\partial v_j / \partial p_j = \beta_p$. Following Proposition 1 and substituting Eq. (26) into Eq. (11), the retail margin solution at equilibrium becomes a function of s_0:

$$w_{jt} - c_j = -\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1} \left(1 - \sum_{j' \in J} \sum_{t' \in T} s_{j't'}\right)^{-1} = \frac{-1}{\beta_p s_0} \quad \forall t \in T, j \in J \tag{27}$$

When the outside good is not included in the demand model, s_0=0, and the Eq. (27) is undefined, implying no finite solution. This result holds true for all four channel types.

Proposition 5: In the logit case with utility linear in price and a monopolist retailer, an outside good is required for existence of a finite Nash equilibrium.

Proof: In the SCR case, the market share of the outside good s_0 is:

$$s_0 = 1 - \sum_{k \in K} \sum_{j \in J_k} s_j \tag{28}$$

With utility linear in price, $\partial v_j / \partial p_j = \beta_p$. Following Proposition 2 and substituting Eq. (28) into Eq. (17), the retail margin solution at equilibrium becomes a function of s_0:

$$m_j = -\left(\frac{\partial v_{jt}}{\partial p_{jt}}\right)^{-1} \left(1 - \sum_{k' \in K} \sum_{j' \in J_{k'}} s_{j'}\right)^{-1} = \frac{-1}{\beta_p s_0} \quad \forall j \in J_k \tag{29}$$

When the outside good is not included in the demand model, s_0=0, and Eq. (29) is undefined, implying no finite solution. Since the retail price is decided by the single common retailer's profit maximization behavior, the absence of an outside good implies that consumers have no other choice and must purchase one of the products from the retailer. For the estimation studies of single common retailer pricing behavior in the marketing science literature, the outside good is usually included in the logit choice model to represent the consumer's no-purchase choice [17,20].

4 CASE STUDY

Theoretical results show that design is decoupled from competition and channel structures when heterogeneity is not present. However, it does not necessarily follow that designs will differ substantially at equilibrium under alternative channel structures for representative problems in the engineering design domain when heterogeneity is present. To demonstrate the methodology and test the sensitivity of design solutions to channel structure, we adopt the vehicle design model proposed by Michalek *et al*. [4], which integrated engineering simulations of vehicle performance with logit models of consumer choice to study vehicle design of profit seeking firms in competition under the CS channel structure. Following [4], we take the firm's decision variables [13] to be the relative size of the vehicle's engine x_1, final drive ratio x_2, and price p. We examine only the default small car equipped with a SI-102 spark-ignition engine (base engine power 102 kW) and use the ADVISOR-2004 vehicle simulator [36] to simulate performance data. Specifically, two attributes, gas mileage z_1 and required time to accelerate from 0-60mph z_2, are simulated as a function of x_1 and x_2. To calculate z_1, two EPA-regulated drive cycles, for city (Federal Test Procedure, FTP75) and highway (Highway Fuel Economy Test, HFET) driving, were simulated, with z_1 = 1/(0.55/city+0.45/highway) [37]. The acceleration performance is calculated through simulated full throttle acceleration. To simplify calculations, simulation points were taken over a range of variable values, and curve-fitting was used to create a meta-model for each:

$$\begin{aligned} z_1(x_1, x_2) &= 2.34x_1^2 - 6.72x_2^2 - 0.81x_1x_2 - 16.0x_1 \\ &\quad + 11.2x_2 + 38.6 \\ z_2(x_1, x_2) &= 2.22 \cdot \exp(-1.85x_1 + 2.25) + 4.39x_2^2 \\ &\quad - 10.6x_2 + 12.2 \end{aligned} \tag{30}$$

Over the points in the sample, the curves deviate from simulator predictions by no more than 0.3 mpg and 0.7 seconds. Each design variable has associated lower and upper bounds: $1.0 \le x_1 \le 3.0$ and $0.8 \le x_2 \le 1.3$. The cost function, built from a regression on engine sales data [4], is given by c^V=7500+670.5·exp(0.643x_1).

The logit model utility form was adopted from a study by Boyd and Mellman [38], where $v_j = \beta_p p_j + 100\beta_1/z_{1j} + 60\beta_2/z_{2j}$, and β_p, β_1 and β_2 are the coefficients of each attribute. The study provided the coefficients for both logit and mixed logit [14] models according to their vehicle demand study. For aggregate logit, $\beta_p = -2.84\times10^{-4}$, $\beta_1 = -0.339$ and $\beta_2 = 0.375$. For mixed logit, each beta coefficient is taken as following an independent lognormal distribution. The random coefficients are given by $\boldsymbol{\beta}=\exp(\boldsymbol{\eta}+\boldsymbol{\Phi\sigma})$, where $\boldsymbol{\Phi}$ is the standard normal distribution and $\boldsymbol{\eta}$ and $\boldsymbol{\sigma}$ are the lognormal parameters. The parameters for the three vehicle attributes are $\mu_p = -7.94$, $\mu_1 = -1.28$, $\mu_2 = -1.75$, σ_p= 1.18, σ_1= 0.001 and σ_2= 1.34. The means [15] of $\boldsymbol{\beta}$ are thus -7.15×10^{-4}, -0.278 and 0.426, respectively. Compared to the logit coefficients, the mean mixed logit preferences are more sensitive to price and acceleration time, but less sensitive to fuel economy. However, the standard deviations [16] of the mixed logit coefficients, 1.24×10^{-3}, 2.78×10^{-4} and 0.956, disclose that consumer taste variation for acceleration performance is relatively larger than the other two attributes. One thousand

[13] We assume that automotive manufacturers are capable of adjusting engine power and gear ratio on their existing engines and gearboxes without completely re-design from scratch. Therefore automakers compete on both vehicle design and price in a static timeframe.

[14] The mixed logit model is called "hedonic demand model" in Boyd and Mellman's paper [38].

[15] The mean of a lognormal distribution is $\exp(\eta+\sigma^2/2)$.

[16] The standard deviation of a lognormal distribution is $[(\exp(\sigma^2)-1)\cdot\exp(2\eta+\sigma^2)]^{1/2}$.

Table 4 Vehicle price and design solutions at market equilibrium

			Price and Cost					Design				Market Performance		
			Wholesale price	**Vehicle cost**	**Mfgr. margin**	**Retailer margin**	**Retail price**	**Eng. scale**	**FD ratio**	**MPG**	**Acc. time**	**Mkt. share**	**Mfgr. profit**	**Retailer profit**
			w	c^V	w-c^V	m	p	x_1	x_2	z_1	z_2	s	Π^M	Π^R
Logit	**CS**	M1 M2	$13,168	$9,301	$3,867	N/A	$13,168	1.54	1.12	22.2	7.11	9.6%	$583M	N/A
	FR	M1 M2	$12,947	$9,301	$3,646	$3,646	$16,593	1.54	1.12	22.2	7.11	4.1%	$235M	$235M
	SCR	M1 M2	$12,941	$9,301	$3,640	$16,737	$29,678	1.54	1.12	22.2	7.11	3.9%	$225M	$470M
	MCR	M1-R1 M1-R2 M2-R1 M2-R2	$13,066	$9,301	$3,765	$3,765	$16,831	1.54	1.12	22.2	7.11	3.6%	Π^M_1= $422M Π^M_2= $422M	Π^R_1= $422M Π^R_2= $422M
Mixed Logit	**CS**	M1 M2	$17,083	$10,167	$6,916	N/A	$17,083	2.15	1.16	16.9	6.26	11.9%	$1,155M	N/A
	FR	M1 M2	$18,713	$10,364	$8,349	$8,349	$27,062	2.26	1.16	16.1	6.19	7.3%	$952M	$952M
	SCR	M1 M2	$58,044	$11,441	$46,603	$246,564	$304,608	2.76	1.17	13.5	6.00	0.3%	$255M	$2,702M
	MCR	M1-R1	$42,899	$10,327	$32,572	$32,572	$75,471	2.24	1.16	16.2	6.20	0.3%	Π^M_1= $1,066M	Π^R_1= $1,066M
		M1-R2	$18,490	$10,327	$8,163	$8,164	$26,654					7.2%		
		M2-R1	$18,490	$10,327	$8,163	$8,164	$26,654	2.24	1.16	16.2	6.20	7.2%	Π^M_2= $1,066M	Π^R_2= $1,066M
		M2-R2	$42,899	$10,327	$32,572	$32,572	$75,471					0.3%		

random draws (R=1000) are used for the mixed logit probability simulation. Further, we assume the outside good utility v_0 is equal to zero throughout the case study in order to avoid the monopoly pricing issue revealed in Proposition 5, although estimation of the outside good was not included in the original study. In particular, if an outside good were included during the initial maximum likelihood data fitting procedure[17], we would expect the relative utility of the outside good to differ in the logit and mixed logit model fits, so attaching an arbitrary outside good utility *post hoc* should not be expected to yield accurate share of choices predictions for the auto market. Still, the example serves well to illustrate the structure of the problem and the method and principles outlined here.

We examine the case of two manufacturers for all four scenarios and two common retailers in the MCR scenario. The total market size Q is given by 1.57×10^6 [4]. We solve the FOC equations for each scenario using the sequential quadratic programming (SQP) implementation in the Matlab Optimization Toolbox and verify that solutions are Nash by globally optimizing each player separately *post hoc* using a multistart loop. The results at market equilibrium under all eight scenarios are shown in Table 4.[18] In all cases except the mixed logit MCR case, competing firms have identical solutions to one another at equilibrium, so only the solution of one manufacturer and one retailer is reported[19]. The mixed logit MCR case results in firms selecting distinct strategies, so all solutions are reported. Specifically, the first two rows in the mixed logit MCR scenario show manufacturer M1's products sold through the two retailers R1 and R2. M1's profit is the sum of M1-R1 and M1-R2, and similarly R1's profit is the sum of M1-R1 and M2-R1.

Results verify that the equilibrium design is unchanged under alternative channel structures in the logit case, although wholesale price and retail price vary. This is expected since the conditions satisfy Proposition 3. In this case the optimal design is independent of the game, and the resulting wholesale prices and retail margins can be interpreted as the outcomes of pure price competition.

In the CS scenario, manufacturers are the only decision makers and thus have the highest wholesale price and profit, due to the integrated retailer (profits need not be split among manufacturers and retailers). For the SCR scenario, the single retailer has the highest unit retail margin and also the highest profit because of its dominative power among channel members. Since consumers can only choose between the product offered by the retailer and the outside good, lack of

[17] Besanko [17] and Sudhir [20] used zero utility as outside good in their estimations for the market data.

[18] There is no active constraint for the solutions in all cases.

[19] Under assumptions of constant marginal cost and identical fixed cost, Anderson *et al.* [33] proved that under multinomial logit in an oligopolistic model there exists a unique and symmetric price equilibrium when the profit function is strictly quasi-concave.

price competition leads to high prices. For the FR and MCR scenarios, neither the manufacturer nor the retailer has dominative power in the market channel. However, for the same outside good, the MCR scenario is able to gain higher total market share (7.2% vs. 4.1%) and higher profits ($422M vs. $235M) than the FR. The MCR channel provides the manufacturer with higher market share than a single franchised retailer. Furthermore, we expect that the logit model will tend to overestimate demand for similar products in a competitive market because the logit's independence from irrelevant alternatives (IIA) property restricts substitution patterns and underestimates the degree to which similar (or in this case, identical) products draw market share from one another [31].

In contrast to the identical designs under the logit model, the mixed logit model results in substantially different design solutions under different channel structure scenarios. The CS case results in the highest manufacturer profit and market share among the four channel types, as might be expected because there is no retailer competing with the manufacturer [20]. Compared to the other three scenarios, a smaller engine is chosen and greater fuel economy is achieved. The FR case results in equal margins for manufacturers and retailers, and an intermediate design result at the market equilibrium. The SCR case shows an extreme solution with high retail margin, which results in high retail price and low market share. In this case, each manufacturer's profit is drastically reduced due to low demand, though wholesale price is increased significantly at market equilibrium. The equilibrium strategy in this case appears to target those few consumers willing to pay high a price premium for the product when no alternative is available except the outside good. As such, the solution is sensitive to the utility of the outside good. Figure 3 shows the retail price and the wholesale price at equilibrium in the SCR mixed logit case as a function of v_0. It can be seen that the retail price (retail margin) is more sensitive to the utility of the outside good, while the manufacturer wholesale price is less affected. The mixed logit MCR case shows an interesting result. The solution indicates that the best strategy for manufacturers is to offer different wholesale prices for the same product to different retailers[21]. On the other hand, a common retailer's best margin decision is to set a higher margin on the high price product and lower margin on low price product. Therefore each product has a high-low price pair, causing significant market share differences. The two manufacturers and two common retailers have similar profits, and the vehicle design solutions in this case are close to the FR design solutions. This solution appears to set low prices that target the general population but also offer the *same design* at higher prices in order to target a very small

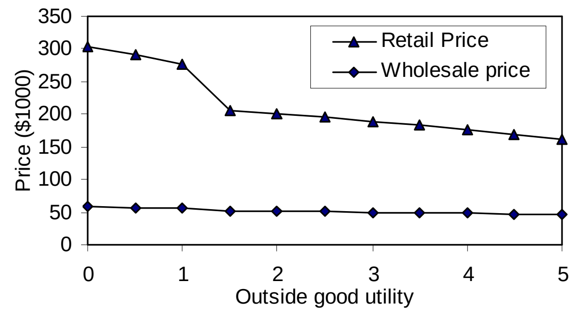

Figure 3 Effect on pricing of outside good utility in the SCR scenario

segment of the market (0.3%) that is insensitive to price. Although the lognormal distribution insures that all consumers prefer lower prices (β_p<0), the price-insensitive consumers (with β_p≈0) will choose the higher priced product with some nonzero probability and thus provide high profit per consumer to the manufacturer and retailer. Thus, this particular result may be an artifact of the assumed shape of the utility distribution in the mixed logit demand model (for example, the assumed independence of beta distributions for each attribute).

One reason for the high price results in the SCR and MCR cases is the limited number of retailers in the case study. If there were only one or two retailers controlling sales channel to consumers, retailer would own dominating market power to raise prices, and consumers would have no other place to purchase automobile. With an increased number of retailers, as observed in practice, the retail margins and prices will decrease due to competition[22].

Overall, the case study verifies that optimal design decisions depend on competition and channel type when heterogeneity is taken into account. Only under the logit model linear in price can the problem can be reduced to pure pricing competition and independent design optimization. In the case study, the company store is an integrated channel that takes no retailer profit, and the manufacturer gains the highest profit in this case. The franchised retailer and manufacturer have equal "power" in our case study of two manufacturers and two retailers, and each makes equal profit at equilibrium. The single common retailer has the highest retail margin due to domination of the regional market and reduced competition. The multiple common retailer case presents the results of two-level competition: The optimal decisions show different price decisions for the same product design at market equilibrium.

5 CONCLUSIONS

We pose a game theoretic model for determining equilibrium design and pricing decisions of profit seeking firms in competition, and we examine the influence of two factors: (1)

[20] In the Nash game, the number of players in game affects the price and profit at equilibrium. For example, a monopoly results in higher profit and prices than an oligopoly [39].

[21] A saddle point is found in the MCR model, which has identical solutions across manufacturers and retailers (w=$19,275, m=$8,990, x_1=2.22, x_2=1.16). It satisfies the first-order criterion but fails in Nash equilibrium verification.

[22] Anderson *et al.* [33] showed that under standard logit a producer's margin is proportional to the inverse of number of producers minus one (section 7.2). Therefore, including more producers would reduce the margin and price.

the structure of manufacturer-retailer interactions in the market and (2) the structure of heterogeneity in the consumer population. We find that the influence these factors are coupled: If consumer preferences are homogeneous and if preferences for price are linear, then the optimal design can be determined independently of price and competition. However, consumer heterogeneity couples the two problems, bringing design into the competitive game. Empirical results from a vehicle design case study show that profit-maximizing designs can change substantially under alternative market structures for practical problems. Thus, as consumer heterogeneity becomes increasingly important to modeling market phenomena for guiding design, it will also become more important to effectively coordinate product planning decisions with engineering design decisions.

Future work will investigate the effect of manufacturer-retailer decision timing on equilibrium designs by examining Stackelberg leader-follower game rules. Additionally, we aim to collect data on past firm design behavior in order to understand in what domains and over what time scale design decisions may be best modeled as a game. Structural models for econometric estimation commonly incorporate price endogeneity because firms are known to set prices competitively and adjust them quickly under changing conditions [40]. Design decisions for differentiated products are likewise made competitively, with awareness of the product attributes offered by other firms, yet this design endogeneity has not been studied. A better understanding of the role of competitive design behavior may lead to advances in econometric estimation of firm behavior as well as supporting strategic engineering design.

Finally, the results of this study suggest the need for more interdisciplinary modeling work that account for interactions among decisions in engineering design, marketing and management disciplines in order to produce competitive and profitable differentiated designs.

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APPENDIX

A.1 Derivation of Wholesale Price FOC Equation for MCR Scenario

From the manufacturer profit Eq. (4), the total profit of manufacturer k is the sum of product $j\in J_k$ and all other products $j'\in\{J_k/j\}$:

$$\Pi_k^{\mathrm{M}} = \Pi_{jt}^{\mathrm{M}} + \sum_{\substack{t'\in T\\ t'\neq t}} \Pi_{jt'}^{\mathrm{M}} + \sum_{\substack{j'\in J_k\\ j'\neq j}} \sum_{\bar{t}\in T} \Pi_{j'\bar{t}}^{\mathrm{M}} \quad \forall t,k, j\in J_k$$

$$\Pi_{jt}^{\mathrm{M}} + \sum_{\substack{t'\in T\\ t'\neq t}} \Pi_{jt'}^{\mathrm{M}} = \left(q_{jt}\left(w_{jt}-c_{jt}\right) + \sum_{\substack{t'\in T\\ t'\neq t}} q_{jt'}\left(w_{jt'}-c_{jt'}\right) \right) - c_j^{\mathrm{F}}$$

$$\sum_{\substack{j'\in J_k\\ j'\neq j}} \sum_{\bar{t}\in T} \Pi_{j'\bar{t}}^{\mathrm{M}} = \sum_{\substack{j'\in J_k\\ j'\neq j}} \left[\left(\sum_{\bar{t}\in T} q_{j'\bar{t}}\left(w_{j'\bar{t}}-c_{j'\bar{t}}\right) \right) - c_{j'}^{\mathrm{F}} \right]$$

$$\frac{\partial p_{jt}}{\partial w_{jt}} = \frac{\partial\left(w_{jt}+m_{jt}\right)}{\partial w_{jt}} = 1 + \frac{\partial w_{jt}}{\partial m_{jt}} = 1$$

$$\frac{\partial s_{ijt}}{\partial w_{jt}} = \frac{\partial s_{ijt}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial w_{jt}} = \frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt}\left(1-s_{ijt}\right)$$

$$\left.\frac{\partial s_{ijt'}}{\partial w_{jt}}\right|_{t'\neq t} = \frac{\partial s_{ijt'}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial w_{jt}} = -\frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt} s_{ijt'}$$

$$\left.\frac{\partial s_{ij't}}{\partial w_{jt}}\right|_{j'\neq j} = \frac{\partial s_{ij't}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial w_{jt}} = -\frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt} s_{ij't}$$

Taking the derivative with respect to w_{jt}:

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \frac{\partial \Pi_j^{\mathrm{M}}}{\partial w_{jt}} + \sum_{\substack{j' \in J_k \\ j' \neq j}} \frac{\partial \Pi_{j'}^{\mathrm{M}}}{\partial w_{jt}}$$

$$= \left[\frac{\partial q_{jt}}{\partial w_{jt}} \left(w_{jt} - c_j \right) + q_{jt} \right] + \sum_{\substack{t' \in T \\ t' \neq t}} \left[\frac{\partial q_{jt'}}{\partial w_{jt}} \left(w_{jt'} - c_j \right) \right] +$$

$$\sum_{\substack{j' \in J_k \\ j' \neq j}} \left[\sum_{t' \in T} \frac{\partial q_{j't'}}{\partial w_{jt}} \left(w_{j't'} - c_{j'} \right) \right]$$

$$= Q \int_{\boldsymbol{\beta}} \left[\frac{\partial s_{jt|\boldsymbol{\beta}}}{\partial w_{jt}} \left(w_{jt} - c_j \right) + s_{jt|\boldsymbol{\beta}} \right] + \sum_{\substack{t' \in T \\ t' \neq t}} \left[\frac{\partial s_{jt'|\boldsymbol{\beta}}}{\partial w_{jt}} \left(w_{jt'} - c_j \right) \right]$$

$$+ \sum_{\substack{j' \in J_k \\ j' \neq j}} \left[\sum_{t' \in T} \frac{\partial s_{j't'|\boldsymbol{\beta}}}{\partial w_{jt}} \left(w_{j't'} - c_{j'} \right) \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta}$$

$$= Q \int_{\boldsymbol{\beta}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} s_{jt|\boldsymbol{\beta}} \left(1 - s_{jt} \right) \left(w_{jt} - c_j \right) + s_{jt|\boldsymbol{\beta}} \right]$$

$$- Q \sum_{\substack{t' \in T \\ t' \neq t}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} s_{jt|\boldsymbol{\beta}} s_{jt'|\boldsymbol{\beta}} \left(w_{jt'} - c_j \right) \right]$$

$$\sum_{\substack{j' \in J_k \\ j' \neq j}} \sum_{t' \in T} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} s_{jt|\boldsymbol{\beta}} s_{j't'|\boldsymbol{\beta}} \left(w_{j't'} - c_{j'} \right) \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta}$$

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial w_{jt}} = \int_{\boldsymbol{\beta}} s_{jt|\boldsymbol{\beta}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial p_{jt}} \left(\left(w_{jt} - c_j \right) - \sum_{j' \in J_k} \sum_{t' \in T} s_{j't'|\boldsymbol{\beta}} \left(w_{j't'} - c_{j'} \right) \right) \right.$$

$$\left. + 1 \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta} = 0$$

A.2 Derivation of Design Variable FOC Equation for MCR Scenario

From the manufacturer profit Eq. (4), the total profit of manufacturer k is the sum of product $j \in J_k$ and all other products $j' \in \{J_k/j\}$:

$$\Pi_k^{\mathrm{M}} = \sum_{t \in T} \Pi_{jt}^{\mathrm{M}} + \sum_{\substack{j' \in J_k \\ j' \neq j}} \sum_{t \in T} \Pi_{j't}^{\mathrm{M}} \quad \forall t, k, j \in J_k$$

$$\sum_{t \in T} \Pi_{jt}^{\mathrm{M}} = \left(\sum_{t \in T} q_{jt} \left(w_{jt} - c_j \right) \right) - c_j^{\mathrm{F}}$$

$$\sum_{\substack{j' \in J_k \\ j' \neq j}} \sum_{t' \in T} \Pi_{j't'}^{\mathrm{M}} = \sum_{\substack{j' \in J_k \\ j' \neq j}} \left[\left(\sum_{t \in T} q_{j't} \left(w_{j't} - c_{j'} \right) \right) - c_{j'}^{\mathrm{F}} \right]$$

$$\frac{\partial s_{ijt}}{\partial \mathbf{x}_j} = s_{ijt} \frac{\partial v_{ijt}}{\partial \mathbf{x}_j} \left(1 - \sum_{\bar{t} \in T} s_{ij\bar{t}} \right)$$

$$\left. \frac{\partial s_{ij't}}{\partial \mathbf{x}_j} \right|_{j' \neq j} = -s_{ij't} \frac{\partial v_{ijt}}{\partial \mathbf{x}_j} \sum_{\bar{t} \in T} s_{ij\bar{t}}$$

Taking the derivative of retailer profit with respect to $\mathbf{x}_j$:

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = \sum_{t \in T} \left[\frac{\partial q_{jt}}{\partial \mathbf{x}_j} \left(w_{jt} - c_j \right) - q_{jt} \frac{\partial c_j}{\partial \mathbf{x}_j} \right]$$

$$+ \sum_{\substack{j' \in J_k \\ j' \neq j}} \sum_{t \in T} \left[\frac{\partial q_{j't}}{\partial \mathbf{x}_j} \left(w_{j't} - c_{j'} \right) - q_{j't} \frac{\partial c_{j'}}{\partial \mathbf{x}_j} \right]$$

$$= Q \int_{\boldsymbol{\beta}} \sum_{t \in T} s_{jt|\boldsymbol{\beta}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{x}_j} (w_{jt} - c_j) - \frac{\partial c_j}{\partial \mathbf{x}_j} \right]$$

$$- \sum_{j' \in J_k} \sum_{t \in T} s_{j't|\boldsymbol{\beta}} \left[\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{x}_j} \left(\sum_{\bar{t} \in T} s_{j\bar{t}|\boldsymbol{\beta}} \right) (w_{j't} - c_{j'}) \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta}$$

$$\frac{\partial \Pi_k^{\mathrm{M}}}{\partial \mathbf{x}_j} = Q \int_{\boldsymbol{\beta}} \sum_{t \in T} \left[\left(\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{z}_j} \frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} \right) \left(s_{jt|\boldsymbol{\beta}} (w_{jt} - c_j) - \left(\sum_{\bar{t} \in T} s_{j\bar{t}|\boldsymbol{\beta}} \right) \right. \right.$$

$$\left. \left. \left(\sum_{j' \in J_k} s_{j't|\boldsymbol{\beta}} (w_{j't} - c_{j'}) \right) \right) - s_{jt|\boldsymbol{\beta}} \frac{\partial c_j}{\partial \mathbf{x}_j} \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta} = 0$$

Taking Lagrange multipliers into account, the equation becomes:

$$\frac{\partial L_k}{\partial \mathbf{x}_j} = \int_{\boldsymbol{\beta}} \sum_{t \in T} \left[\left(\frac{\partial v_{jt|\boldsymbol{\beta}}}{\partial \mathbf{z}_j} \frac{\partial \mathbf{z}_j}{\partial \mathbf{x}_j} \right) \left(s_{jt|\boldsymbol{\beta}} (w_{jt} - c_j) - \left(\sum_{\bar{t} \in T} s_{j\bar{t}|\boldsymbol{\beta}} \right) \right. \right.$$

$$\left. \left. \left(\sum_{j' \in J_k} s_{j't|\boldsymbol{\beta}} (w_{j't} - c_{j'}) \right) \right) - s_{jt|\boldsymbol{\beta}} \frac{\partial c_j}{\partial \mathbf{x}_j} \right] f_\beta(\boldsymbol{\beta}) d\boldsymbol{\beta}$$

$$- \boldsymbol{\lambda}_j^T \frac{\partial \mathbf{h}_j}{\partial \mathbf{x}_j} - \boldsymbol{\mu}_j^T \frac{\partial \mathbf{g}_j}{\partial \mathbf{x}_j} = \mathbf{0} \quad \forall k, t, j \in J_k$$

Also, the additional equations for completing the Karush-Kuhn-Tucker conditions, $\mathbf{h}=\mathbf{0}$, $\mathbf{g}\leq\mathbf{0}$, $\boldsymbol{\mu}^T\mathbf{g}=0$, must be included in the system and solved simultaneously.

A.3 Derivation of Retail Margin FOC Equation

Starting from the retailer profit function Eq. (5), the total profit of common retailer t can be separated into three parts: 1) profit of the specific product j from manufacturer k; 2) the sum of profits from other products of manufacturer k; and 3) the sum of profits from all other manufacturers' products.

$$\Pi_t^{\mathrm{R}} = \sum_{k\in K}\sum_{j\in J_k}\Pi_{jt}^{\mathrm{R}} = \Pi_{jt}^{\mathrm{R}} + \sum_{\substack{j'\in J_k\\ j'\neq j}}\Pi_{j't}^{\mathrm{R}} + \sum_{\substack{k'\in K\\ k'\neq k}}\sum_{j''\in J_{k'}}\Pi_{j''t}^{\mathrm{R}}$$

$$\forall t, k, j\in J_{kt}$$

In the Nash game, manufacturer's wholesale price decision is made under a fixed retailer's retail margin decision, so:

$$\frac{\partial p_{jk}}{\partial m_{jk}} = \frac{\partial\left(w_{jk}+m_{jk}\right)}{\partial m_{jk}} = \frac{\partial w_{jk}}{\partial m_{jk}} + 1 = 1$$

$$\frac{\partial s_{ijk}}{\partial m_{jt}} = \frac{\partial s_{ijk}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial m_{jt}} = \frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt}\left(1-s_{ijt}\right)$$

$$\left.\frac{\partial s_{ij't}}{\partial m_{jt}}\right|_{j'\neq j} = \frac{\partial s_{ij't}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial m_{jt}} = -\frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt}s_{ij't}$$

$$\left.\frac{\partial s_{ijt'}}{\partial m_{jt}}\right|_{t'\neq t} = \frac{\partial s_{ijt'}}{\partial p_{jt}}\frac{\partial p_{jt}}{\partial m_{jt}} = -\frac{\partial v_{ijt}}{\partial p_{jt}} s_{ijt}s_{ijt'}$$

Taking the derivative of retailer profit with respect to m_{jt}:

$$\frac{\partial\Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \frac{\partial\Pi_j^{\mathrm{R}}}{\partial m_{jt}} + \sum_{\substack{j'\in J_k\\ j'\neq j}}\frac{\partial\Pi_{j'}^{\mathrm{R}}}{\partial m_{jt}} + \sum_{\substack{k'\in K\\ k'\neq k}}\sum_{j''\in J_{k'}}\frac{\partial\Pi_{j''}^{\mathrm{R}}}{\partial m_{jt}}$$

$$= \left[\frac{\partial q_{jt}}{\partial m_{jt}} m_{jt} + q_{jt}\right] + \left[\sum_{\substack{j'\in J_k\\ j'\neq j}}\frac{\partial q_{j't}}{\partial m_{jt}} m_{j't}\right] + \left[\sum_{\substack{k'\in K\\ k'\neq k}}\sum_{j''\in J_{k'}}\frac{\partial q_{j''t}}{\partial m_{jt}} m_{j''t}\right]$$

$$= Q\int_{\boldsymbol\beta}\left[\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}} s_{jt|\boldsymbol\beta}\left(1-s_{jt|\boldsymbol\beta}\right)m_{jt} + s_{jt|\boldsymbol\beta}\right] - Q\left[\sum_{\substack{j'\in J_k\\ j'\neq j}}\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}} s_{jt|\boldsymbol\beta}s_{j't|\boldsymbol\beta}m_{j't}\right] - Q\left[\sum_{\substack{k'\in K\\ k'\neq k}}\sum_{j''\in J_{k'}}\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}} s_{jt|\boldsymbol\beta}s_{j''t|\boldsymbol\beta}m_{j''t}\right] f_\beta(\boldsymbol\beta)d\boldsymbol\beta$$

$$= Q\int_{\boldsymbol\beta}\left[\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}} s_{jt|\boldsymbol\beta}m_{jt} + s_{jt|\boldsymbol\beta}\right] - \left[\sum_{k'\in K}\sum_{j'\in J_{k'}}\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}} s_{jt|\boldsymbol\beta}s_{j't|\boldsymbol\beta}m_{j't}\right] f_\beta(\boldsymbol\beta)d\boldsymbol\beta$$

$$= Q\int_{\boldsymbol\beta} s_{jt|\boldsymbol\beta}\left[\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}}\left(m_{jt} - \sum_{k'\in K}\sum_{j'\in J_{k'}} s_{j't|\boldsymbol\beta}m_{j't}\right) + 1\right] f_\beta(\boldsymbol\beta)d\boldsymbol\beta$$

$$\frac{\partial\Pi_t^{\mathrm{R}}}{\partial m_{jt}} = \int_{\boldsymbol\beta} s_{jt|\boldsymbol\beta}\left[\frac{\partial v_{jt|\boldsymbol\beta}}{\partial p_{jt}}\left(m_{jt} - \sum_{k'\in K}\sum_{j'\in J_{k'}} s_{j't|\boldsymbol\beta}m_{j't}\right) + 1\right] = 0$$

$$f_\beta(\boldsymbol\beta)d\boldsymbol\beta = 0$$