OPTIMAL FEASIBLE PRODUCT LINE DESIGN FOR HETEROGENEOUS MARKETS

Jeremy J. Michalek
Assistant Professor
Mechanical Engineering
Carnegie Mellon University

Fred M. Feinberg
Associate Professor
Ross School of Business
University of Michigan

Peter Ebbes
Assistant Professor
Smeal College of Business
Pennsylvania State University

Feray Adigüzel
Assistant Professor
Economics and Business Administration
Vrije Universiteit Amsterdam

Panos Y. Papalambros
Professor
Mechanical Engineering
University of Michigan

Under review at *Management Science*. Please do not quote or distribute without prior consent.

The authors wish to thank Abbie Griffin, Jin Gyo Kim, Peter Lenk, Laura Stojan, Michel Wedel and Carolyn Yoon for their help and comments, and to acknowledge the sponsorship of the Department of Mechanical Engineering at Carnegie Mellon University and the Rackham Antilium interdisciplinary project, the Reconfigurable Manufacturing Systems Engineering Research Center and the Ross School of Business, all at the University of Michigan.

OPTIMAL FEASIBLE PRODUCT LINE DESIGN FOR HETEROGENEOUS MARKETS

ABSTRACT

Successful product line design and development requires balancing technical and market tradeoffs. Quantitative methods for 'optimizing' product attribute levels based on conjoint survey data are useful, taken on their own, for many product types. However, products with substantial engineering content involve critical tradeoffs in the ability to actually *achieve* those desired levels. These technical tradeoffs must be made with an eye toward their market consequences, particularly when heterogeneous market preferences make differentiation and strategic positioning critical to capturing a range of market segments and avoiding cannibalization.

We present a methodology for product line optimization that efficiently coordinates positioning and design models to achieve realizable firm-level optima. The approach leverages prior methods where possible, overcoming several shortcomings of current positioning models by incorporating a general (Bayesian) account of consumer preference heterogeneity, alleviating known issues of combinatorial complexity, and avoiding infeasible solutions. The method is demonstrated for a line of dial-readout scales, using physical models and web-collected consumer choice data from the literature. Results show that the optimal number of products is not necessarily equal to the number of market segments, and the representational form for consumer heterogeneity has a substantial impact on the design and profitability of the resulting optimal product line – even for the design of a single product. The method is managerially valuable, as it yields product line solutions efficiently, accounting for marketing-based preference heterogeneity as well as engineering-based constraints on which products can actually be built.

KEYWORDS: *Product Line Design; Heterogeneity; Analytical Target Cascading; Hierarchical Bayes; Conjoint Analysis; Discrete Choice Analysis; Design Optimization.*

1. Introduction

Marketplace globalization, the proliferation of niche markets driven by heterogeneity of preferences, increased competitive pressures, and demand for differentiated products have rendered isolated design and production of individual products essentially obsolete. Across industries, standard practice involves *lines* of product variants that reduce cost via economies of scale and scope, reaching multiple market segments and deterring competitors. Product line planning requires care, as each product vies not only with competitors, but also with other products in the same line. The scope and applicability of current methods in product line optimization are limited: "engineering-based" approaches focus on the tradeoff between increased commonality among products and the resulting decreased ability to meet (usually hypothetical and exogenous) performance targets for each product variant. These approaches generally lack data-driven models of market preferences, and consequently focus on reducing cost by increasing part commonality, designing platforms, or increasing modularity for mass customization.

In contrast, product line optimization methods in the management science and marketing literatures do not, by-and-large, address product design details that are not directly observable by consumers. These approaches typically presume that any combination of product attributes in a conjoint study can somehow be attained by engineering designers *post hoc*. While this may be so for many simple or well-established products, it is questionable for those with even moderately complex engineering tradeoffs. Furthermore, existing approaches have not taken advantage of recent advances in econometric modeling of consumer preference heterogeneity, in particular hierarchical Bayes methods, and generally require exogenous individual-level or homogeneous segment-level preference data. A few cross-disciplinary approaches have recently emerged that design engineering products under business and marketing objectives; however, most focus on the design of a single product.

In this article, we develop a novel method for designing lines of products with engineering complexity and preference heterogeneity. We proceed by reviewing relevant literature on product line optimization from engineering, management and marketing, suggesting how the proposed approach fills a number of extant gaps. Because the scopes, perspectives, modeling methods, and objectives differ substantially among product development disciplines (Krishnan and Ulrich provide a detailed overview [1]), it is inevitable that some conflicts of terminology will exist: In this article, we take product *positioning* to be the process of choosing values for physical (as opposed to perceptual) product attributes observed directly by the consumer, whereas product *design* involves decisions made by the engineer that are not observed directly, but that nevertheless exert influence on the product attributes observed by the customer.

1.1. Product Line Optimization in Engineering

The bulk of the engineering literature relevant to product line design focuses on studying product commonality and product platforms [2, 3]. These efforts generally focus on the tradeoff between increased commonality among products in a line and the resulting decreases in the ability to meet distinct performance targets for each product variant. Authors use the terms "product family" and "product platform" rather than "product line" to emphasize the focus on commonality among the set of products. These approaches tend to be engineering-centric and do not invoke econometric models of consumer choice built on market data. Instead, most focus on hypothetical targets set exogenously for each product in the family. Those that do reference marketing pose hypothetical market segments [4] or define an intuitive market segment grid by enumerating levels of product performance [5].

Recent efforts have attempted to link engineering optimization models to conjoint preference models; however, these approaches generally focus on a single-product [6-9]. A notable exception is Li and Azarm [10], who pose an approach that addresses design of a product line by first generating a set of designs that are Pareto-optimal in engineering performance and then selecting a product line from that set based on a conjoint model of demand using genetic algorithms. This sequential approach can be effective for product line design cases where preferences for product attributes are strictly monotonic across the consumer population (e.g., fuel economy, reliability, quality, price, etc.) and individual consumers vary only in their preferred tradeoffs among desirable attributes. However, it is not clear how this method might be extended for non-monotonic cases, such as the dial-readout scale examined in Section 4, where individuals have different ideal points and a single Pareto set cannot be defined common to all consumers.

Engineering approaches are generally designed around gradient-based constrained nonlinear programming techniques to handle continuous or mixed formulations with continuous and discrete variables. The focus on continuous variables increases applicability for practical engineering problems and avoids combinatorial complexity found in many positioning approaches. One difficulty with integrating models from various product development disciplines is that the combined model can be quite large and complex, causing optimization difficulties. Gu *et al*. [11] proposed a method for maintaining separate models for marketing and engineering decisions, coordinating them using the collaborative optimization (CO) technique for multidisciplinary design optimization, although they do not propose details for modeling and data collection for the marketing component. Michalek *et al*. [9] proposed a similar decomposition approach using analytical target cascading (ATC) to coordinate marketing and engineering models for a single product, assuming consumer preferences to be homogeneous. They point out a preference for the ATC approach over CO because ATC is defined for an arbitrary hierarchy of subsystems, and convergence proofs ensure coordination will lead to a solution that is optimal for the firm [12-17]. Most of the research on ATC has been applied to the design of a single product; however, Kokkolaras *et al*. [18] proposed a method using the ATC formulation to coordinate the design of a family of products. This approach offers a distinct advantage, as the design of each product is handled in a separate subsystem, and the subsystems are coordinated. However, as is common in engineering approaches, Kokkolaras *et al*. use a pre-specified hypothetical objective function for positioning and do not explicitly model market preferences for the variants. As such, their approach cannot be expected to be broadly useful in managerial decision-making.

Engineering methods for product line design are powerful, but their main drawback is that the vast majority fail to incorporate consumer demand. Those that do are dramatically simpler than prevalent approaches in marketing; for example, they sidestep preference heterogeneity entirely. Hence, a product deemed ‘optimal’ may be so with respect to performance, cost or efficiency, but not in terms of profitability.

1.2. Product Line Optimization in Management Science and Marketing

Table 1 summarizes several historical and recent major contributions to the product line optimization literature, focusing on those in marketing and management science, but including two especially on-point engineering approaches. Among the earliest conceptualizations for product line optimization was that of Green and Krieger [19], who posed the product line *selection* problem as a binary programming problem involving selection of products from a candidate set to be included in the line in order to maximize the seller's (or buyers') welfare. Here the set of candidate products with their associated utility values is determined exogenously, and product demand is predicted using a *first choice* model, where each individual is assumed to choose deterministically the alternative with the highest associated utility. Variants of the original model were later proposed by Dobson and Kalish [20] and by McBride and Zufryden [21], who offer alternative integer programming techniques and heuristics for solving the problem. Dobson and Kalish [22] also introduce fixed and variable costs for each candidate product.

While these initial methods assumed each product's utility had been determined exogenously, Kohli and Sukumar [23] instead used conjoint part-worths, and introduced a single stage binary programming formulation to select product lines based on their attribute levels. Chen and Hausman [24] made use of *choice-based* conjoint analysis, arguably most similar to the choice task consumers perform in practice, and often claimed to be the best method for extracting individual-level consumer preferences [25]. Chen and Hausman proposed a binary programming formulation solvable by nonlinear programming techniques. Because their approach requires homogeneous preferences, it cannot be used to design product lines meeting the disparate needs of most real consumer populations. Among the most recent contributions is that of Steiner and Hruschka [26], who use genetic algorithms to efficiently locate near-optimal product line designs. Other approaches have also been proposed to model products qualitatively in terms of abstract "quality levels", although these are primarily used to analyze structural properties, rather than offer computational decision support tools (Krishnan and Zhu provide a recent review [27]).

Despite these advances, a number of key problems remain. Specifically, current approaches to the product line problem: lack coordination with engineering terms of product feasibility; do not easily accommodate a sophisticated account of preference heterogeneity; entail substantial computational problems; and require changes from the ground up to deal with new structures and phenomena (e.g. channel structure, models of competition). Our proposed methodology resolves each of these issues, as we discuss in the following sections.

1.3. Conjoint-Based Choice Models

As is typical in marketing and econometrics, we adopt a random utility framework [28] for estimating market demand for the product line, where the utility of each product to each consumer depends on the product's attributes, the consumer's idiosyncratic preferences for those attributes and a random error component. Most of the previous product line optimization formulations adopt a demand model where utility is written as a *deterministic* function of the product and consumer attributes, which offers computational benefits for rating or ranking conjoint data (e.g., [10, 19-23]). A random utility framework, however, can be expected to provide a more realistic representation of the consumer decision process, allowing as it does for uncertainties and factors unobservable to the analyst. Furthermore, it avoids the discontinuities intrinsic to a deterministic framework, allowing the use of efficient gradient-based nonlinear programming optimization algorithms. Importantly, a random utility framework allows for explicit modeling of consumer taste distributions, or heterogeneity. As our results illustrate, the various representations available to model taste differences can have a substantial, and substantive, impact on the final optimal product line and its profitability; this is especially so if one chooses an overly parsimonious representation. Surprisingly, the impact of preference heterogeneity on line configuration has not received much attention in the product line literature. To our knowledge, the present paper is the first to explore the issue in depth.

1.4. Proposed Methodology

Prior approaches to product line optimization can work well for certain types of products and markets but have identifiable gaps for a wide range of real-world product lines. In this article, we propose a coordinated methodology to overcome these gaps, using Analytical Target Cascading (ATC) to coordinate attribute selection for each of the products desired by a heterogeneous market, while ensuring they can actually be *achieved* by a set of realizable designs. In demonstrating the methodology, we compare explicitly throughout to a prior, single-product design example under homogeneous demand [9], adopting its product topology model. We can therefore not only demonstrate the superiority of the derived product line results, but also show that even for the single product case, assumption of preference homogeneity can be problematic. The proposed approach avoids the combinatorial complexity of binary/integer formulations in the marketing literature while extending applicability to continuous or mixed formulations and avoiding the need to assume monotonic preferences. The decomposition-based ATC approach offers the organizational and computational benefits of maintaining separate subsystems for positioning and design of each project, reducing the dimensionality of each subspace and allowing each subsystem to be efficiently solved in parallel [16, 17]. ATC also facilitates scaling up to large problems with many products.

Like Chen and Hausman [24], we invoke a number of assumptions to tightly focus on product line optimization issues: (1) total market size is exogenously determined; (2) each customer purchases zero or one product; (3) customers do not directly influence one another; and (4) production can be scaled up or down to suit demand. As such, our formulation is well-suited to stable durables and is less appropriate for rapidly-developing product classes. Unlike prior research in the area, we invoke only mild parametric assumptions about how to represent consumer preferences.

The proposed product line design methodology entails four stages: First, consumers choose among products in a conjoint setting; second, heterogeneous preferences models are estimated; third, demand models are formulated by interpolating preference coefficients using splines; and fourth, ATC coordinates optimization over the space of feasible product designs to yield optimal product attributes. The first three stages are viewed as preprocessing for the ATC model, as shown schematically in Figure 1, with symbols rigorously defined later in the text. We proceed by defining the ATC methodology in Section 2, conditional on a model to predict demand; next we describe alternative discrete choice model specifications for demand prediction in Section 3; and finally, we demonstrate the methodology with an application to dial-readout scales using models and data from the literature in Section 4, and discuss results and their marketing implications in Section 5.

2. ATC Coordination of Product Positioning and Design

The ATC-based methodology presented here calls on established modeling traditions in engineering design and marketing. It is innovative in terms of formally coordinating them, and does not seek to "reinvent the wheel" when unnecessary. Indeed, this modularity is among its chief strengths. ATC was conceived as a broad platform for large-scale engineering systems optimization. By viewing a system as a decomposable hierarchy of interrelated subsystems, ATC allows each subsystem to be modeled and optimized separately, coordinated, and then iteratively re-optimized to reach a system solution [29].

ATC requires a mathematical or computational model of each subsystem, and in practice these can be numerous. The modeler's task is to organize the various subsystem models into a *hierarchy*, where each element in the hierarchy represents a (sub)system that is optimized to match targets passed from the parent (super)system while setting targets that are attainable by subsystem child elements. For example, a vehicle design could be decomposed into systems such as body, chassis, and powertrain; the powertrain system could be decomposed into subsystems such as the transmission and engine; and the engine could be further decomposed into components such as the piston, crankshaft, etc. In our application of ATC, the joint product line positioning and design problem is (formally) decomposed into interrelated subsystems, which can then intercommunicate and algorithmically iterate. It is known that the iterative solution of decomposed ATC subsystems under specific coordination strategies will converge to the solution of the joint non-decomposed problem [13-17]. In the present case, market positioning and the engineering design of each product in the line can be solved separately and in parallel, producing a solution that is optimal for the joint problem [16, 17, 30]. In practice, the joint problem can be far more difficult to solve, sometimes impossibly so, owing to high dimensionality, scaling difficulties and the need for modeler expertise in all areas.

In general, ATC can accommodate an arbitrarily large hierarchy, where parent elements set targets for child elements. For example, the methodology has been demonstrated for large hierarchical systems such as vehicle design [31] and architectural design [32]. In the product line case, each design in the line could be decomposed into a set of subsystems and components, or additional marketing models could be included, say, for promotion and distribution. In this article, we address product line optimization by introducing a set of engineering design subsystems, one for each product in the line, along with a positioning formulation that sets targets for all products in the line using a heterogeneous-preference-based demand model. A schematic depiction of the process appears in Figure 2: The positioning subsystem involves determining price and (target) product attributes for the full product line to maximize a known objective function, which can be profit or some other measure of interest to the firm, while each design subsystem requires determining a feasible design – one conforming to known constraints – that exhibits product attributes as close as possible to the targets set in the positioning subsystem. Decomposition into the ATC structure can be even more important in the product line case than in the single product case because including engineering models for the design of multiple products in a single optimization statement creates a high-dimensional, highly constrained space; by contrast, with ATC decomposition of the line, the space of each individual product design remains unchanged as new products are added to the line. The chief organizational benefit of ATC is that it segregates models by discipline: Marketers can build positioning models based on, say, conjoint analysis and new product demand forecasting; engineers can formulate models for product design and production; and other functional groups can focus on what they know how to do well. No functional area need become an expert in modeling the others, since ATC coordinates models with well-defined interfaces. The following sections lay out the design and positioning subsystems explicitly.

2.1. Engineering Design Subsystems

The objective of each engineering subsystem is to find a feasible design that exhibits product attributes matching the targets set during market positioning as closely as possible. Here the vector of product attributes $\mathbf{z}_j$ for product j represents a set of objective, measurable aspects of the product, observable by the customer, resulting from engineering design decisions. In each engineering design subsystem j, search is conducted with respect to a vector of design variables $\mathbf{x}_j$, which represents decisions made by the designer that are not directly observable by consumers but that affect the attributes that consumers do observe $\mathbf{z}_j$. An engineering analysis simulation response function $\mathbf{r}(\mathbf{x}_j)$ is used to calculate attributes $\mathbf{z}^{\mathrm{E}}_j$ as a function of $\mathbf{x}_j$. The design variable vector $\mathbf{x}_j$ is restricted to feasible values by a set of constraint functions $\mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}$ and $\mathbf{h}(\mathbf{x}_j) = \mathbf{0}$, and so values for product attributes $\mathbf{z}^{\mathrm{E}}_j = \mathbf{r}(\mathbf{x}_j)$ are implicitly restricted to values that can be achieved by a feasible design. While construction of

x, **r**(**x**), **g**(**x**) and **h**(**x**) to represent a particular engineering design system is necessarily case-specific, general principals and guidelines are well established in the literature [33, 34]. The objective of each engineering design subsystem is to minimize deviation between the attributes achieved by engineering $\mathbf{z}^{E}_{j}$ and the positioning targets $\mathbf{z}^{M}_{j}$ set by the positioning subsystem, which are held constant in each engineering design subsystem. This deviation objective originates from a relaxation of the consistency condition ($\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j} = \mathbf{0}$) by instead introducing a consistency constraint relaxation function $\pi(\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j})$. This relaxation can be handled in a variety of ways including penalty functions [12, 14, 29], Lagrangian relaxation [15] and augmented Lagrangian relaxation [13, 16, 17]. In particular, the diagonal quadratic approximation approach applied by Li *et al*. [16] produces separable subsystems and allows each design in the product line to be optimized in parallel, dramatically improving computational efficiency. The engineering optimization problem for product j can then be written as

$$\begin{aligned} &\underset{\mathbf{x}_j}{\text{minimize}}\ \pi\left(\mathbf{z}_j^{M} - \mathbf{z}_j^{E}\right) \\ &\text{subject to } \mathbf{g}\left(\mathbf{x}_j\right) \le \mathbf{0}, \mathbf{h}\left(\mathbf{x}_j\right) = \mathbf{0}\,, \\ &\text{where } \mathbf{z}_j^{E} = \mathbf{r}\left(\mathbf{x}_j\right) \end{aligned} \qquad (1)$$

2.2. Market Positioning Subsystem

The market positioning objective is to maximize profit Π with respect to the price p_j and the vector of product attribute targets $\mathbf{z}^{M}_{j}$ for each product j in the product line $j = \{1,2,...,J\}$. Although firms can specify arbitrarily sophisticated profit functions based on their experience, internal accounting and historical demand, we use a simple profit (Π) formulation here – revenue minus cost – so that

$$\Pi = \sum_{j=1}^{J}\left(\left(p_j - c_j^{V}\right)q_j - c_j^{I}\right), \qquad (2)$$

where p_j is the (retail) price of product j, c^{V}_{j} is the unit variable cost of product j, c^{I}_{j} is the investment cost for product j, which represents all costs of setting up a manufacturing line for product j, and q_j is quantity of product j sold (demand), which is a function of the product attributes $\mathbf{z}^{M}_{j'}$ and price $p_{j'}$ of all products $j' = \{1,2,...,J\}$. We presume that product commonalities enabling investment cost sharing and improving economies of scale do not exist, so each new product design requires new manufacturing investment, though this can readily be relaxed, given appropriate cost-specific information. In general,

c^{V}_{j} and c^{I}_{j} can be considered functions of market conditions or engineering decisions, although in the example they are taken as constants. Given a demand model to calculate q_j for each product j as a function of $\mathbf{z}^{M}_{j'}$ and $p_{j'}$ for all products j', which will be developed in the Section 3, the profit function is fully defined. The objective function also involves the consistency constraint relaxation function $\pi(\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j})$ for minimizing deviation from attributes achievable by engineering $\mathbf{z}^{E}_{j}$, which are held constant in the positioning subsystem. Finally, the positioning subsystem for a single-producer scenario, conditional on a model for demand, is written as:

$$\begin{aligned} &\underset{\mathbf{z}^{M}_{j},\, p_j;\forall j\in\{1,2,\ldots,J\}}{\text{maximize}} \ \sum_{j=1}^{J}\left(\left(p_j - c^{V}_{j}\right)q_j - c^{I}_{j} - \pi\left(\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j}\right)\right) \\ &\text{where } q_j = q_j\left(\mathbf{z}^{M}_{j'}, p_{j'};\forall j'\right) \end{aligned} \quad (3)$$

In Section 3, we address how conjoint analysis, discrete choice modeling and Bayesian (MCMC) methods can be used to represent the functional relationship between demand q and the variables $\mathbf{z}^{M}$ and p for positioning a product line.

2.3. Complete ATC Formulation

Figure 2 conveys both a mathematical and verbal description of the complete formulation, showing the flow of the ATC-based product line optimization model for a single producer, where the number of products in the line J is determined through a parametric study: i.e., J is held fixed during optimization, separate optimization solutions are found for each value of $J = \{1,2,...\}$, and the value of J that produces the solution with the highest profit is selected. Coordination of the subsystems can be handled in a variety of ways. The most efficient method according to a recent study [16, 17] is the truncated diagonal quadratic approximation augmented Lagrangian approach. This method uses $\pi(\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j}) = \boldsymbol{\lambda}^{T}(\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j}) + \|\mathbf{w} \bullet (\mathbf{z}^{M}_{j} - \mathbf{z}^{E}_{j})\|_{2}^{2}$, where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector, $\mathbf{w}$ is a weighting coefficient vector, and $\bullet$ is the Hadamard product (i.e.: $(\mathbf{A}\bullet\mathbf{B})_i = \mathbf{A}_i\mathbf{B}_i$). The coordination procedure is:

1. Initialize all variables
2. Solve the positioning subsystem and each design subsystem in parallel
3. Update linearization of the cross component of the augmented term at the new point
4. Update $\boldsymbol{\lambda}$ and $\mathbf{w}$ using the method of multipliers
5. If converged, stop, else return to step 2

A review of alternative coordination methods for ATC is provided in [16, 17].

3. Models of Product Demand

Green and Krieger's comparative study of alternative conjoint methods for eliciting consumer preferences concluded that choice-based conjoint offers the best method for the extraction of individual-level consumer preferences [25]. We use it as follows: Respondents are presented with a series of questions or "choice sets" $t = \{1,2,...,T\}$. In each choice set t, the respondent is presented a set of product alternatives $j \in \mathcal{J}_t$, with attributes set at discrete levels and systematically varied [35, 36]. The resulting data are each respondent's observed choices in each choice set: Φ_{ijt}, where $\Phi_{ijt} = 1$ if respondent i chooses alternative j in choice set t, and $\Phi_{ijt} = 0$ otherwise. These data $\{\Phi_{ijt}\}$ are then used to estimate the parameters of the choice model for the positioning subsystem, as illustrated in Figure 1.

In the random utility choice model, individuals $i = \{1, 2, ..., I\}$ derive from each product $j = \{1, 2, ..., J\}$ some utility value u_{ij} that is composed of an observable, deterministic component v_{ij} and an unobservable random error component ε_{ij}, so that $u_{ij} = v_{ij} + \varepsilon_{ij}$. Each individual will choose the alternative that gives rise to the highest utility (i.e., alternative j is chosen by individual i if $u_{ij} > u_{ij'}$ for all $j' \neq j$). The deterministic utility v_{ij} derived by individual i from product j is written as

$$v_{ij} = \sum\nolimits_{\zeta=0}^{Z} \sum\nolimits_{\omega=1}^{\Omega_\zeta} \beta_{i\zeta\omega} \delta_{j\zeta\omega} \,, \qquad (4)$$

where the binary dummy $\delta_{j\zeta\omega} = 1$ indicates alternative j possesses attribute ζ at level ω, and $\beta_{i\zeta\omega}$ is the part-worth coefficient of attribute ζ at level ω for individual i, which is estimated from the conjoint choice data Φ. The model thus accords with the typical main-effects conjoint set-up dominant in the literature. In $\delta_{j\zeta\omega}$ the elements of the product attribute vector $\mathbf{z}^{M}_{j}$ are enumerated $\zeta = \{1, 2, ...\, Z\}$, and price p is included in $\delta_{j\zeta\omega}$ and labeled as element $\zeta = 0$. Note that each product attribute ζ is either intrinsically discrete or is discretized into Ω_ζ levels, $\omega = \{1, 2, ...\, \Omega_\zeta\}$; this is crucial, as it does not presume linearity with respect to the underlying continuous variables.

The probability P_{ij} that alternative j is chosen by individual i depends on the assumed error distribution. The most common distributions for ε_{ij} are the normal and double exponential, resulting in the standard probit and logit models, respectively [28]. It is well-known that very large samples are required to distinguish results produced by the logit and probit specifications [37]. Finally, we index the "no choice option" (the outside good) as alternative 0, with error ε_{i0} and attraction value v_{i0} for individual i, where $v_{i0} = 0$; $\forall i$ for identification purposes. The inclusion of a "no choice option" is critical, as it allows overall demand contraction when the set of products offered fails to match the market's preferences well.

The representation of differences in consumer tastes, as given by $\boldsymbol{\beta}_i$, where $\boldsymbol{\beta}_i$ contains the elements $\beta_{i\zeta\omega}$, can be expected to be important in product line optimization, as heterogeneity in preferences should give rise to differentiated product offerings. Failure to correctly model this heterogeneity can lead to biased parameter estimates, inaccurate predictions [38, 39] and, consequently, suboptimal product line designs. Furthermore, when heterogeneity is not adequately accounted for it is well-known that the independence from irrelevant alternatives (IIA) problem is exacerbated [40]. We therefore specify a very general continuous distributional form for $\boldsymbol{\beta}_i$ by using a mixture of normal distributions [41, 42]. The approach assumes that there are a finite number of groups or segments, in which individuals are similar – though, importantly, not identical – with respect to their preferences and tastes. To be more specific, we have

$$\boldsymbol{\beta}_i \sim \sum_{b=1}^{B} s_b N\left(\boldsymbol{\theta}_b, \boldsymbol{\Lambda}_b\right), \tag{5}$$

where s_b is the fraction of the market in "segment" (or mixing component) b; $b = 1,...,B$. Here $\boldsymbol{\theta}_b$ is the vector of means for $\boldsymbol{\beta}_i$ and $\boldsymbol{\Lambda}_b$ is a full variance-covariance matrix. This model provides a very general specification that combines both discrete and continuous heterogeneity and includes several well known heterogeneity models as special cases: (i) when B=1 the well-known standard random-effects model arises, which, in combination with Bayesian estimation, enables individual-level estimates by pooling information among individuals via "shrinkage" [42]; (ii) when $\boldsymbol{\Lambda}_b = \mathbf{0}$ for all $b = 1,...,B$ the standard latent class or finite mixture model arises [43], and individuals within a segment b are assumed to have identical preferences $\boldsymbol{\theta}_b$; and (iii) when $\boldsymbol{\Lambda}_b = \mathbf{0}$ and $B = 1$ it is assumed that all individuals have the same preference $\boldsymbol{\theta}_1$. The last, homogeneous case (iii) is overly simplistic, and demand models that assume homogenous tastes can be expected to perform poorly in terms individual specific part-worth recovery and market predictions. Andrews *et al.* [44] suggest that models with continuous (case i) and discrete (case ii) representations of heterogeneity recover parameter estimates and predict choices about equally well, except when the number of choices J is small, in which case discrete heterogeneity (ii) outperforms the continuous model (i). One objective of the proposed research is to examine whether the optimized product lines *conditional* on each of these models produce similar results.

For the general case, model parameters are estimated via standard Markov chain Monte Carlo (MCMC) techniques [45, 46][1]. We generally specify conjugate priors, and the full conditional distributions for the MCMC sampler can be derived straightforwardly (e.g. [41, 42]). In order to choose the number of mixture components B in the mixture representation for $\boldsymbol{\beta}_i$, we use the Deviance Information Criterion (DIC) statistic proposed by Spiegelhalter *et al.* [47]. DIC is particularly suited to complex hierarchical (Bayesian) models in which the number of parameters is "not clearly defined" [48], because the DIC statistic determines the "effective number of parameters" entailed by the model specification itself, unlike measures such as AIC, which in any case is inapplicable in a Bayesian setting.

Once the model parameters are estimated, we compute market demand for the positioning subsystem (Figure 2) in three steps: First, we generate a large set of $\boldsymbol{\beta}_i$ (say i = 1,...,I_D) from the hierarchical model $\{s_b, \theta_b, \Lambda_b\}$, which describes the mixture distribution.[2] Second, we use *natural cubic splines* [9, 49] to flexibly interpolate $\boldsymbol{\beta}_i$ for intermediate values of product attributes and price. Specifically, natural cubic spline functions $\Psi_{i\zeta}$ are fit through the discrete part-worth coefficients $\beta_{i\zeta\omega}$ for each i and ζ, where $\omega = \{1,2,...,\Omega_\zeta\}$ to interpolate the deterministic component of utility. Indexing attributes as ζ = 1,…, Z and price as ζ = 0, the interpolated value of the observable component of utility is

$$\hat{v}_{ij} = \Psi_{i0}\left(\beta_{i0\omega}, p_j\right) + \sum_{\zeta=1}^{Z} \Psi_{i\zeta}\left(\beta_{i\zeta\omega}, \mathbf{z}_{j\zeta}^{M}\right), \quad (6)$$

[1] We are indebted to Peter Lenk for sharing both his GAUSS code and expertise.

[2] Estimating the model provides a set of draws from the posterior distribution of $\beta_{i\zeta\omega}$ for each survey respondent. One could then estimate market demand using this specific set of individuals. We take a Bayesian perspective and use the hyperparameters describing the mixture distribution (after the MCMC chain has converged), as these can be viewed as 'giving rise' to the individual-level β_i values. Specifically, an arbitrarily large sample of *new* β_i values from this distribution can be drawn to describe the market.

where $\mathbf{z}^{M}_{j\zeta}$ indicates the ζ^{th} element of the vector $\mathbf{z}^{M}_{j}$. These interpolated $\hat{v}_{ij}$ give rise, through the random utility specification, to expected individual choice probabilities P_{ij}, which are computed using either a logit or a probit distribution for the errors. Finally, the individual choice probabilities are used to compute total market demand (Figure 2). The logit formulation increases optimization speed considerably, since choice probabilities can be calculated exactly and efficiently in each step of the ATC optimization, as opposed to the probit case, where choice probabilities must be approximated using numerical methods [37]. Calculating market demand for product j involves multiplying the probability P_{ij}, by the market potential S for each individual i=1,..., I_D, and averaging the resulting quantities across the individuals. Market potential is assumed to be exogenously determined through pre-market forecasting techniques [50]. In conflating conjoint choice shares and projected market shares, we have enacted the usual *ceteris paribus* assumption that differences in advertising, promotion and distribution (etc.) across our hypothetical products can be disregarded for modeling purposes.

4. Empirical Application

We apply the proposed methodology to design a line of dial-readout bathroom scales using engineering models and conjoint choice data from [9] for comparison. This example was originally posed in a rather limited and unrealistic context: for the design of a *single* product under the assumption that consumer preferences are *homogeneous*. While expedient, neither assumption accords well with actual managerial practice. Furthermore, as we will see, even in those rare cases where firm does seek to enter the market with a single product, the presumption of homogeneity is troublesome; in fact, the single-product solution obtained in [9] fails once *any* form of preference heterogeneity is allowed.

The inherent modularity of the proposed methodology for product line design circumvents the need to build a joint model of the full product line for each case J = {1,2,...}. Instead, a model of only a single product need be developed, and a duplicate can be created for each product j constituting the line[3], as illustrated in Figure 2. It is important to note that the design space **x** for this product does *not* map one-to-one with the attributes **z** communicated to consumers. This comes about because the engineering design model specifies some product attributes as functions of interactions among design variables; that is, different designs may exhibit identical product attributes, as observed by the customer. A manager could enact any number of criteria *post hoc* to choose from among such a continuum, or detailed cost data and preferences for commonality could drive selection of a single engineering design among the set of possibilities, although we do not pursue such strategies here.

The product attributes **z** seen by consumers are weight capacity z_1, aspect ratio z_2, platform area z_3, tick mark gap z_4, and number size z_5, in addition to price p. For the conjoint study, the range of values for each attribute was captured by five (discrete) levels. Each respondent (n = 184) made choices from 50 consecutive sets in a choice-based conjoint task, identical across respondents, each with three options (plus "no choice"), implemented through a web browser. With the ATC approach, it is neither necessary nor practical to pre-restrict choice sets to include only realizable products. The goal of the conjoint task is the effective and unbiased measurement of consumer preference drivers. Infeasible combinations of product attributes are implicitly avoided during optimization through coordination with the engineering design subsystem.

The demand/profit function requires (exogenous) estimates of several quantities, which are based here on manufacturer and publicly-available figures: c^{V}_j = \$3 cost per unit, c^{I}_j = \$3 million for initial investment, and market size S = 5 million, the approximate yearly US dial-readout scale market. Being completely exogenous, these values are easily altered. The entire marketing subproblem is formulated as in Eq.(3), with the demand model specified in Eq.(4)-(6). The special cases of discrete mixture ($\mathbf{\Lambda}$=$\mathbf{0}$) and homogeneous (B=1, $\mathbf{\Lambda}$=$\mathbf{0}$) models are straightforward to estimate using maximum likelihood techniques [28]. For the mixture of normal distributions, estimates from a classical mixture of probits were used as starting values, and the Gibbs sampler was iterated until a stationary posterior was obtained. To mitigate autocorrelation, the data were thinned by retaining every 10$^{\text{th}}$ draw, after a burn-in of 50,000 iterations. Convergence was examined through iteration plots; posterior marginals revealed no convergence problems.

[3] All models and results are available from the authors upon request.

In order to optimize over this posterior surface, Monte Carlo integration was applied, as follows: When the chain has stabilized, *new* values of $\boldsymbol{\beta}_i$ are generated as the chain continued to run, allowing hyperparameters to vary across the generated values. These are thinned to reduce serial correlations; specifically, 10,000 values are generated, and every 10^{th} is retained. The resulting set of 1000 $\boldsymbol{\beta}_i$ draws, with splines fit through the part-worth attribute levels of each draw, is used to represent the population (the posterior surface) throughout the optimization. Accuracy can be enhanced, if need be, by generating additional $\boldsymbol{\beta}_i$ values, although in the case study, tests of solution sensitivity to additional draws (up to 24,000) show that 1000 draws is sufficient to represent the "demand surface".

5. Results

There are two main methodological components to the approach advocated here: 1) econometric, for the extraction of individual-level preferences and generation of the preference splines, and 2) optimization-based, for the determination of the best number of products, their positioning and design conditional on the preference splines. We examine these in turn.

5.1. Demand Model Results

Table 2 lists DIC results for the normal mixture model and BIC results for the discrete mixture and homogeneous cases as well as classical log-likelihood values for reference. The latent class model identified by BIC consists of seven segments, while the mixture model with a diagonally-restricted covariance matrix identified by DIC has three mixing components, and the full-covariance mixture model has two. It is apparent that: (1) continuous heterogeneity (normal mixture) alone is superior to discrete heterogeneity (latent class) alone, up through a fairly large number of segments [42]; (2) a correlated (random) coefficients specification for the normal mixture is superior to an uncorrelated one; and (3) more than one segment in the normal mixture model is supported. In short, the most general specification fares best, and each of its attributes – correlated coefficients, and both discrete and continuous heterogeneity – is useful in accurately representing consumer preferences. In the following sections, we will refer primarily to this full model, calling on others peripherally to compare the "optimal product lines" they entail.

For illustration and a "reality check" we briefly examine the posterior means of part-worth coefficient vectors, $\boldsymbol{\beta}_i$. The resulting splines are shown graphically in Figure 3, along with analogous splines for the discrete mixture and homogeneous cases. Recall that for identification purposes these values are scaled so that the sum in each set of attributes is zero, making for easier visual comparison. In each of the six attribute spline graphs the heterogeneous model is most "arched" or highly sloped, suggesting the presence of some consumers with relatively strong preference differentials across attribute levels. Of course, part-worth values have a nonlinear mapping onto choice probabilities, so an "averaged part-worth" is only a rough guide to comparing across heterogeneity specifications.

Although it is not our main focus here, a number of trends are apparent across these mean estimated coefficient values. Unsurprisingly, price appears to exert the strongest influence, and is decisively downward-sloping (this is true of the posterior means for each of the n = 184 original participants). One might have expected similarly monotonic preferences for number size and weight capacity, but this is only true for the former; apparently, too high a capacity was viewed as "suboptimal" by the respondents, on average. Note that these $\boldsymbol{\beta}_i$ values reflect pure consumer preference, and *not* any sort of constraint resulting from infeasible designs, which can only arise from the engineering design subsystem. Preferences for the other three variables (platform area, aspect ratio (i.e., shape) and interval mark gap) all have interior maxima.

5.2. Product Line Optimization Results

Conditional on the generated splines arising from the HB conjoint estimates (using the full normal mixture model), the design and positioning subsystems are solved iteratively until convergence. Optimization was carried out with each subsystem solved using sequential quadratic programming. The ATC hierarchy is solved for a set of fixed product line size J; the value of J producing the most profitable overall product line is determined post hoc. As is typical, local optima are possible, so global optima are sought using multi-start. Figure 4 depicts the highest resulting profit levels, using ten runs with random starting points for $J = \{1,2,...,7\}$. It is clear that a product line with four products is most profitable.

Table 3 presents the resulting product attributes for various heterogeneity specifications. Several of the resulting scale designs are bounded by active engineering design constraints; this is necessary to ensure that the scale is physically tenable, e.g., that the dial, spring plate and levers fit in the case. Note as well that all the scales in the line lie well within the range available through online retailers, although resulting prices migrate to the upper bound due to the single-producer scenario. Looking across the table, and considering primarily marketing attributes, we might term the resulting products "large high-capacity, small-numbered square scale" (27.4% of the market), "large-number portrait scale" (21.0%), "small, low-capacity landscape scale" (18.6%) and "high-priced, middle-of-the-road" scale (11.3%). Note that these do not add to 100%, given the presence of the "no choice" option, which allows some portion of the potential market to prefer no scale at all to any of the four in the final line configuration.

Although the four scales may appear not to differ tremendously, they do in fact cover a wide swath of the attribute space bounded by the original conjoint levels; thus, they are quite different *relative* to the dial-readout scales available in the market. Note as well that extrapolation beyond the extreme conjoint levels was disallowed. This means that scale 4, the "high-priced, middle-of-the-road" scale, may command an even higher price than indicated ($30). This situation was exacerbated for the latent class model, for which *four* of the six scales in the best solution (as per Table 3) were at this upper price limit. As discussed more fully in Conclusions, whether this represents an untapped surplus, an insufficient upper limit on Price in the conjoint design, or is an artifact of all competitive interaction being subsumed by the "outside good", is an issue that should be teased apart by further studies.

5.2.1. Effectiveness of ATC Coordination

A major contribution of the methodology presented here is to provide rigorous coordination between positioning and design models for a product line to find an optimal, *feasible* solution when customer preferences are heterogeneous. To demonstrate the importance of this coordination, the ATC solution was compared to the solution obtained through a disjoint sequential approach, which has been referred to as analytical target setting [9, 51]. In the disjoint scenario, price and product attribute positioning targets are set based on consumer preference data *without* engineering feasibility information (the positioning subsystem), and these are passed to engineering design teams. Each engineering team then designs a feasible product that meets its targets as closely as possible (the engineering subsystem) without further iteration. This can be viewed as a 'single pass' through ATC, similar to actual practice, where marketing studies precede engineering design, and subsequent iteration is costly and time-consuming.

In this disjoint scenario, marketing produces a plan for a line of four scales with a predicted combined market share (relative to full potential) of 83.4% and resulting profit of $81.2 million. There is no reason to believe these products will be *feasible*, as they are based on consumer preferences alone. In the disjoint case, these (unachievable) targets are passed to engineering teams who each design a feasible product to achieve product attributes as close as possible to the targets requested by marketing without further iteration. The resulting products differ significantly from the initial plan and so have attributes less preferred by consumers, resulting in combined 70.5% market share and $67.9 million profit: 16% less than marketing's original (unachievable) prediction. If ATC is instead used to iteratively coordinate positioning and design, the resulting joint solution is a line of four different products, resulting in 78.2% market share and $72.4 million profit. In this case, coordination resulted in a feasible product line with a predicted 6% improvement in profitability relative to disjoint decision-making. In the disjoint scenario, marketing "leads" by developing the original plan and engineering design "follows" by attempting to meet product attribute targets. The reverse situation, where engineering "leads", is possible when all consumers have monotonic preferences for product attributes by first designing a set of products that are Pareto-optimal in performance and then allowing marketing to pick a line from that set of products [10]. However, in this example, preferences for attributes are non-monotonic, so no such common Pareto set exists, and without preference information, engineering design has no single well-defined optimization objective.

5.2.2. Heterogeneity Representation

Among the main goals of our investigation is to assess what impact the heterogeneity specification has on the joint solution, and whether simpler forms might have sufficed for optimal feasible line design. The simple homogeneous demand model is obviously ill-suited for generating product lines; moreover, because the IIA property is greatly exacerbated by preference homogeneity, the well-known "red bus, blue bus" problem can lead to lines with duplicate products [28]. We thus compare the discrete mixture (latent class) model with the normal mixture model. Because the discrete mixture is natively supported in many statistical packages, it might prove convenient for line optimization; recent literature suggests that discrete and continuous heterogeneity can often represent preferences about equally well [38, 44]. Though fit statistics (Table 2) alone argue that the discrete mixture is inferior to the normal mixture specification in terms of representing *preferences*, this does not necessarily mean that, conditional on the resulting estimates, the resulting optimal line will be similarly inferior in terms of *profitability*.

Table 3 lists a comparison between the resulting profitability of the best locally-optimal solutions found using the discrete and continuous mixture demand models over ten multi-start runs with random starting points for each value of J. Not only do the different heterogeneity specifications result in different product line solutions (a line of six products under the seven-class discrete mixture vs. four under the two-component normal mixture), but the former suffers a profit decrement of 18.4%. Furthermore, because the discrete mixture specification models all individuals within a segment as having identical preferences, the remaining within-segment IIA property can result in solutions with duplicate or near-duplicate products, such as the one reported in Table 3. It is important to note that such solutions are artifacts of the econometric model and may be difficult to interpret for managerial use. For example, simply taking the solution resulting from the model and eliminating product duplicates will not, in general, produce a locally optimal solution in the reduced space. Furthermore, the within-segment homogeneity of preferences results in a profit surface containing pronounced local minima, which impedes the optimization process and makes global search difficult. Thus, even a relatively sophisticated heterogeneity representation can offer very different, and potentially sub-optimal, product line results.

While it may be unsurprising that simpler heterogeneity representations can lead to suboptimal product lines, it less obvious whether a homogeneous model is sufficient for the design of a *single* product (as assumed, for example, in [9]). Our analysis strongly suggests that it is not. Table 3 lists single-product solutions under the three demand model scenarios. Although in this case the more restrictive models do a fairly good job predicting some of the optimal product attributes, this is not so for price, which is notably exaggerated (relative to the normal mixture model), resulting in a solution with a loss of 7% market share using the discrete mixture model and 14% using the homogeneous one. These results make sense because continuous heterogeneity allows for some consumers that are highly price sensitive, so that a single price need be lower to avoid losing them entirely. Simply put, even when making a ‘one size fits all’ product, a manufacturer should not presume that all customers have the same preferences. It had not initially been anticipated, based on any prior literature of which we are aware, that preference heterogeneity would be so important when only a single product is being produced. How heterogeneity specification affects contingent optimization results is surely worthy of further study on its own.

6. Conclusions, Caveats and Extensions

Firms work to position and design lines of products that best suit their market and profitability goals. Different functional entities within the firm can interpret this imperative idiosyncratically: measuring customer preferences and positioning new products for marketers; maximizing performance under technological constraints for engineers. Considered independently, these goals often lead to conflict, both in practice and with respect to optimization models in each discipline; moreover, disjoint sequential approaches can lead to suboptimal decision-making.

The product line design methodology advocated in this article draws on a wide array of techniques – in product line optimization, analytical target cascading, discrete choice analysis, preference heterogeneity and Bayesian econometrics – to model various subsystems separately, coordinating them via ATC. The resulting product line is a solution to the “joint” marketing and engineering problem, and produces results superior to a simple sequential approach. The separation of the subsystems is advantageous both for organizational purposes, since each modeling group can focus on what it knows best and need not be an expert in all areas, and for computational purposes, since the individual subsystems can be solved in parallel within low dimensional spaces and with fewer constraints then the full, non-decomposed system. Iterative coordination of these decision-models serves to reduce the need for more costly human iteration.

The product line problem is well suited for ATC decomposition, facilitating scalability of the problem to complex products (which can be represented as (sub)hierarchies themselves), or to large numbers of products, simply by adding more subsystems. The intrinsic modularity of the approach also readily accommodates additions, variations, and extensions. Nor does the approach require the set of attributes of interest to have monotonic preferences in the population, so it can be useful for accommodating attributes of any sort.

A number of concrete conclusions emerged from applying the proposed method. One for which we know of no precedent is that accounting for preference heterogeneity can be critical even for a "one-size fits all" product. That is, the single-product solution looks quite different when the market itself is assumed (incorrectly) to be homogeneous. This was so even for price: $26.41 (homogeneous) vs. $22.61 (heterogeneous). Homogeneity presumes that *everyone wanted the same thing*, so the "optimal" product appears able to command a higher price; when heterogeneity is incorporated, willingness-to-pay is lower for many, if not most, in the market.

Conversely, we see that prices for the (heterogeneous) line can be higher than for one product alone: each product fits a 'segment' better, and so extracts a premium over the 'mass market' single-product case; this is true regardless of which type of heterogeneity representation the modeler selects. Overall, and unsurprisingly, multiple products turned out to perform better than single ones, depending on the cost structure of the market. Of course, the precise number of products to produce depends on the cost of adding additional product variants to the line.

We also saw that it is possible for one or a number of "optimal" products to hit a corner solution – this happened in our application for price ($30) – suggesting an untapped surplus. To explore this further, a broader conjoint study, with more levels or an adaptive algorithm, would need to be undertaken. Even without constraining all solutions to lie within the conjoint levels used in the preference study, it is possible that the products in the "optimal" line don't seem to differ enormously. However, in our application, they did indeed span a great deal of the 'attribute space' available in the market. Generally speaking, products in a line don't need to appear drastically different to 'cover' the market and offer greater profit than a smaller number.

It is crucial to realize the model does not advocate offering four (or any specific number) different consumer products – there are clearly far more in the market – but four different product *designs*: other elements (e.g., color, packaging, marketing mix variables like advertising and point-of-purchase inducements) can be altered 'on the fly', as they do not interact with the engineering model.

Comparing solutions achieved under different heterogeneity specifications indicates that the *form* of heterogeneity chosen by the modeler can exert non-trivial impact on the "optimal" solution obtained. This suggests that a suitably general heterogeneity specification should be used, where possible, for product line optimization. In our case, the full-covariance finite normal mixture was superior to the homogeneous and discrete mixture representations at capturing underlying preferences; a more restrictive model can lead to different solutions with substantial reductions in profitability.

There are a number of caveats, and related extensions, that should be noted before ATC or our specific set-up is widely applied in product planning. For example, we have assumed costs (i.e., for additional products in the line) to be linear. In practice, this is unlikely to be so. However, the modularity of ATC allows for any cost function to be accommodated; costs (e.g., for various product configurations or components) can also be included in the engineering model directly. A second caveat concerns situations in which another form of preference heterogeneity might be useful. In such cases, we need only alter the discrete choice model component; everything else is unchanged. Revising the product topology model is similarly straightforward: only that portion of the system need be altered. A third caveat concerns the nature of competition, which here, as in many other marketing models, is subsumed by an "outside good". This is a substantial, and as yet unsolved, problem, one that would require a demand system, including other firms' decisions. ATC would still be applicable, but data requirements would be demanding, and a game-theoretic analysis may come into play. We regard this as fertile ground for additional research. Finally, one might question how 'real' a durable dial-readout scales really are, and whether ATC can handle far more complex product line designs. Again, we point to ATC's scalability, and its prior application in automotive design.

In closing, several maxims are relevant for management, marketing and engineering design communities. First, although engineers are keenly aware, at every step of their work, of real, inviolable constraints, marketers tend to work to find desirable product attribute targets for exploring new markets. A tacit belief is that most, if not all, design constraints can be vanquished by ingenuity or sufficient capital. While this is sometimes true, often it is not. ATC encodes non-negotiable technological infeasibilities directly into its conceptual foundations. As such, marketers using their own models within an overarching ATC formulation can gain a gut feel for what will work, and what will not, in terms of actual, deliverable products, to supplement their intuitive understanding of the consumer marketplace. The flip side is that engineers can come to terms with the “consumer space”, every bit as real as the geometry and physics underlying their own models, and resolve tradeoffs among competing performance goals through coordination with marketing. Second, while it may appear simple to specify directly which product attribute combinations cannot co-exist, in practice it is often impractical: this “infeasible hull” can snake through the product attribute space in ways difficult to visualize or translate into meaningful consumer terms. ATC frees marketers from considering such issues when collecting consumer preferences; iterative coordination avoids infeasible product line configurations implicitly. Third, and most important, heterogeneity matters: it must be accounted for in sufficient generality, even for the design of a single product.

And finally, the question arises whether our proposed approach can be trusted alongside mainstays like conjoint analysis and discrete choice modeling to aid in product line design. The underlying ATC framework is proven, for a broad class of problems, to converge to joint optimality across its various subsystems. As such, it can literally guarantee better profitability, as in our application, than possible by sequentially optimizing the design and positioning subsystems. Given its scalability, efficiency, and ability to key into a wide variety of extant modeling techniques, we hope to see this framework widely adopted as a cross-disciplinary platform for the design of complex product lines.

FIGURES AND TABLES

Figure 1 Diagram of the modeling process

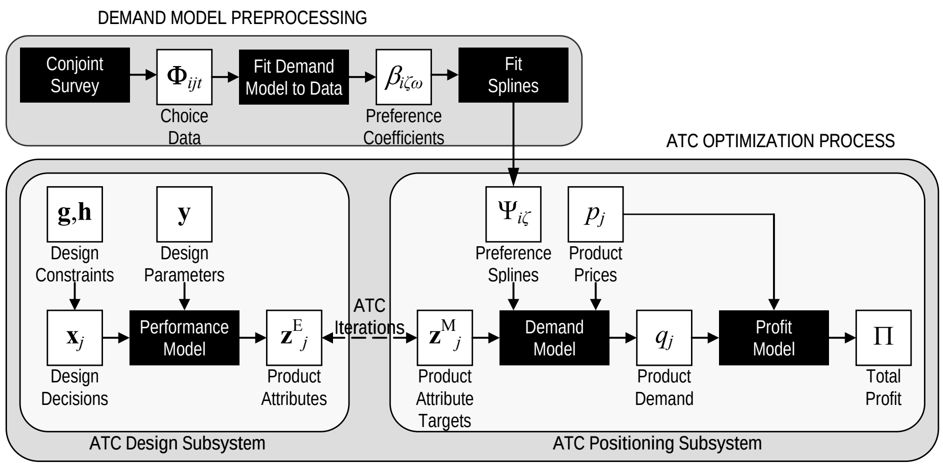

Figure 2 ATC Formulation of the Product Planning and Engineering Design Product Development Problem

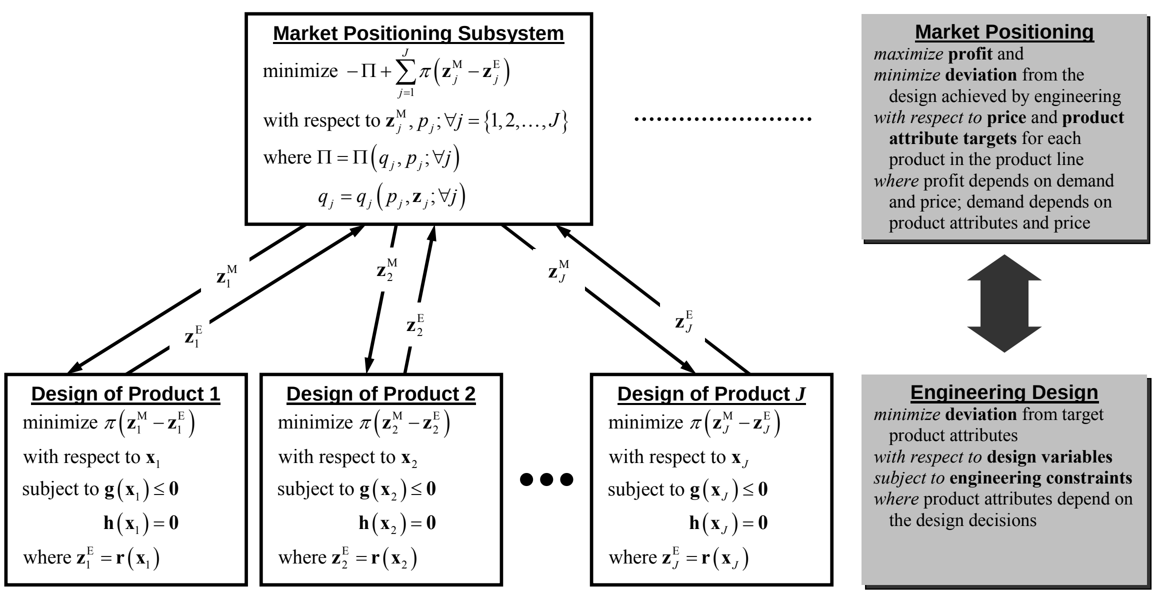

Figure 3 Plots of the average splines for each product attribute and price under the three demand models

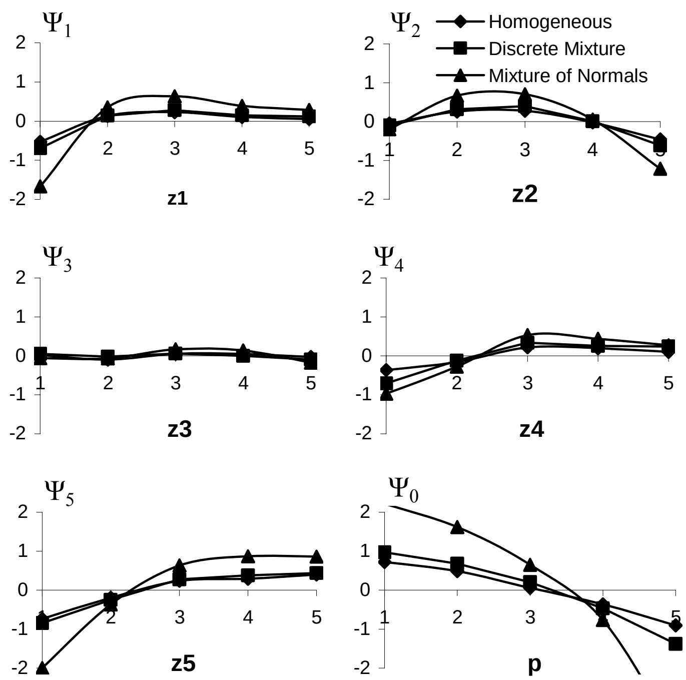

Figure 4 Resulting Profit as a Function of the Number of Products in the Line

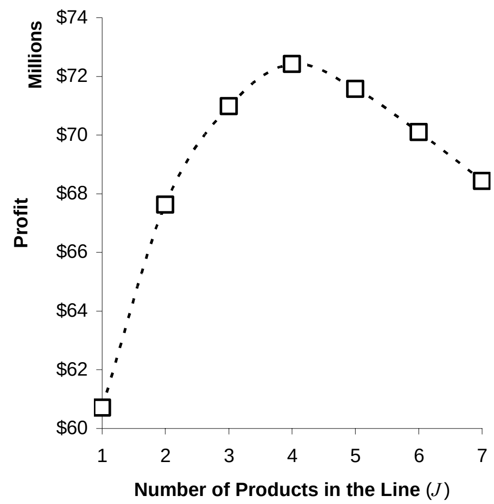

Table 1: A Summary of the Product Line Optimization Literature

	Green and Krieger (1985)	McBride and Zufryden (1988)	Dobson and Kalish (1988)	Kohli and Sukumar (1990)	Dobson and Kalish (1993)	Chen and Hausman (2000)	Steiner and Hruschka (2002)	Li and Azarm (2002)	Kokkolaras et al. (2002)	Proposed Method
Product Selection	B	B	B	-	B	B	-	B	-	-
Product Attributes	-	-	-	B	-	-	B	M	-	M
Product Design	-	-	-	-	-	-	-	M	C	M
Preference Elicitation	-	-	-	CA	-	CBC	CBC	CA	-	CBC
Preference Model	FC	FC	FC	FC	FC	L	L	FC	-	L/P
Preference Heterogeneity	-	-	-	-	-	-	-	-	-	BNM/LC
Solution Algorithm	H	H	H	H	H	NLP	GA	GA	ATC	ATC

Key: B=binary; C=continuous; M=mixed; CA=conjoint; CBC=choice-based conjoint; FC=first choice; L=logit; P=probit; LC=latent class
BNM=Bayesian normal mixture, H=heuristics; NLP=nonlinear programming; GA=genetic algorithms; ATC=target cascading

Table 2: Comparison of Heterogeneity Specifications: Discrete Latent Class vs. HB Random Parameters

	B	**Λ**	**LL**	**BIC**
Homo	**1**	**0**	**-10983**	**22194**
Latent Class Discrete Mixture	2	0	-10239	20944
	3	0	-9784	20271
	4	0	-9537	20014
	5	0	-9336	19850
	6	0	-9187	19788
	7	**0**	**-9059**	**19770**
	8	0	-8948	19785

	B	**Λ**	**LL***	**DIC**
Hierarchical Bayes Continuous Mixture	1	Diag	-3813	12432
	2	Diag	-3713	12073
	3	**Diag**	**-3656**	**11961**
	4	Diag	-3638	12029
	1	Full	-4051	11742
	2	**Full**	**-4016**	**11674**
	3	Full	-4017	11745

* Classical LL for the HB models was evaluated using posterior means as plug-in values and is included only for informal comparison to the Latent Class models.

Table 3: Optimal Product Line Solutions under Each Demand Specification

			Single Product Solutions			**Product Line Solutions**									
						Discrete Mixture (Latent Class)						**Normal Mixture**			
			Homo-geneous	**Discrete Mixture**	**Normal Mixture**	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**
Π	Profit (Millions)*	$	$54.10	$58.30	$60.70	$59.10						$72.40			
	Market share*	%	48.80%	57.80%	65.00%	25.10%	8.70%	8.70%	8.70%	6.90%	4.90%	27.40%	21.00%	18.60%	11.30%
z_1	Weight capacity	lbs.	255	254	256	238	257	257	257	253	248	292	262	200	255
z_2	Aspect ratio	- -	0.996	1.047	1.002	1.045	1.041	1.039	1.039	1.062	1.051	0.98	1.156	0.921	0.986
z_3	Platform area	in^2	134	127	130	100	131	131	131	123	114	140	122	105	135
z_4	Tick mark gap	in.	0.116	0.117	0.115	0.106	0.116	0.116	0.116	0.114	0.111	0.103	0.116	0.121	0.116
z_5	Number size	in.	1.334	1.339	1.315	1.193	1.341	1.337	1.337	1.316	1.268	1.221	1.351	1.293	1.331
p	Price	$	$26.41	$24.21	$22.61	$23.96	$30.00	$30.00	$30.00	$30.00	$29.37	$22.89	$24.53	$23.84	$30.00

* as calculated post-hoc using the normal mixture demand model

References:

1. Krishnan, V. and K.T. Ulrich, *Product development decisions: A review of the literature.* Management Science, 2001. **47**(1): p. 1-21.
2. Simpson, T.W., *Product platform design and customization: Status and promise.* Artificial Intelligence for Engineering Design Analysis and Manufacturing, 2004. **18**(1): p. 3-20.
3. Simpson, T.W., Z. Siddique, and J. Jiao, eds. *Product Platform and Product Family Design: Methods and Applications*. 2005, Kluwer Academic Publishers: New York.
4. Rai, R. and V. Allada, *Modular Product Family Design: Agent-based Pareto-Optimization and Quality Loss Function-based Post-Optimal Analysis.* International Journal of Production Research, 2003. **41**(17): p. 4075-4098.
5. Farrell, R. and T.W. Simpson, *Product Platform Design to Improve Commonality in Custom Products.* Journal of Intelligent Manufacturing, 2003. **14**(6): p. 541-556.
6. Li, H. and S. Azarm, *Product design selection under uncertainty and with competitive advantage.* Journal of Mechanical Design, 2000. **122**(4): p. 411-418.
7. Wassenaar, H.J. and W. Chen, *An approach to decision-based design with discrete choice analysis for demand modeling.* Journal of Mechanical Design, 2003. **125**(3): p. 490-497.
8. Wassenaar, H.J., et al., *Enhancing discrete choice demand modeling for decision-based design.* Journal of Mechanical Design, 2005. **127**(4): p. 514-523.
9. Michalek, J.J., F.M. Feinberg, and P.Y. Papalambros, *Linking marketing and engineering product design decisions via analytical target cascading.* Journal of Product Innovation Management, 2005. **22**(1): p. 42-62.
10. Li, H. and S. Azarm, *An approach for product line design selection under uncertainty and competition.* Journal of Mechanical Design, 2002. **124**(3): p. 385-392.
11. Gu, X.Y., et al., *Decision-based collaborative optimization.* Journal of Mechanical Design, 2002. **124**(1): p. 1-13.
12. Michalek, J.J. and P.Y. Papalambros, *An efficient weighting update method to achieve acceptable consistency deviation in analytical target cascading.* Journal of Mechanical Design, 2005. **127**(2): p. 206-214.
13. Tosserams, S., et al., *An augmented Lagrangian relaxation for analytical target cascading using the alternating direction method of multipliers.* Structural and Multidisciplinary Optimization, 2006. **31**(3): p. 176-189.
14. Michelena, N., H. Park, and P.Y. Papalambros, *Convergence properties of analytical target cascading.* Aiaa Journal, 2003. **41**(5): p. 897-905.
15. Lassiter, J.B., M.M. Wiecek, and K.R. Andrighetti, *Lagrangian coordination and analytical target cascading: Solving ATC-decomposed problems with Lagrangian duality.* Optimization and Engineering, 2005. **6**(3): p. 361-381.

16. Li, Y., Z. Lu, and J.J. Michalek, *Diagonal Quadratic Approximation for Parallelization of Analytical Target Cascading*. 2006, Carnegie Mellon University.
17. Li, Y., *Diagonal Quadratic Approximation for Parallel Computing with Analytical Target Cascading*, in *Department of Mathematical Sciences*. 2006, Carnegie Mellon University: Pittsburgh.
18. Kokkolaras, M., et al., *Extension of the target cascading formulation to the design of product families.* Structural and Multidisciplinary Optimization, 2002. **24**(4): p. 293-301.
19. Green, P.E. and A.M. Krieger, *Models and Heuristics for Product Line Selection.* Marketing Science, 1985. **4**(1): p. 1-19.
20. Dobson, G. and S. Kalish, *Positioning and Pricing a Product Line.* Marketing Science, 1988. **7**(2): p. 107-125.
21. McBride, R.D. and F.S. Zufryden, *An Integer Programming Approach to the Optimal Product Line Selection Problem.* Marketing Science, 1988. **7**(2): p. 126-140.
22. Dobson, G. and S. Kalish, *Heuristics for Pricing and Positioning a Product-Line Using Conjoint and Cost Data.* Management Science, 1993. **39**(2): p. 160-175.
23. Kohli, R. and R. Sukumar, *Heuristic for Product Line Design Using Conjoint Analysis.* Management Science, 1990. **36**(12): p. 1464-1477.
24. Chen, K.D. and W.H. Hausman, *Technical Note: Mathematical Properties of the Optimal Product Line Selection Problem Using Choice-Based Conjoint Analysis.* Management Science, 2000. **46**(2): p. 327-332.
25. Green, P.E. and A.M. Krieger, *Individualized hybrid models for conjoint analysis.* Management Science, 1996. **42**(6): p. 850-867.
26. Steiner, W. and H. Hruschka, *Genetic algorithms for product design: how well do they really work?* International Journal of Market Research, 2003. **45**(2): p. 229-240.
27. Krishnan, V. and W. Zhu, *Designing a family of development-intensive products.* Management Science, 2006. **52**(6): p. 813-825.
28. Train, K.E., *Discrete Choice Methods with Simulation*. 2003: Cambridge University Press.
29. Kim, H.M., et al., *Target cascading in optimal system design.* Journal of Mechanical Design, 2003. **125**(3): p. 474-480.
30. Bertsekas, D.P. and J.N. Tsitsiklis, *Parallel and distributed computation : numerical methods*. 1989, Englewood Cliffs, N.J.: Prentice Hall. xix, 715 p.
31. Kim, H.M., et al., *Analytical target cascading in automotive vehicle design.* Journal of Mechanical Design, 2003. **125**(3): p. 481-489.
32. Choudhary, R., A. Malkawi, and P.Y. Papalambros, *Analytic target cascading in simulation-based building design.* Automation in Construction, 2005. **14**(4): p. 551-568.
33. Papalambros, P.Y. and D.J. Wilde, *Principles of optimal design : modeling and computation*. 2nd ed. 2000, Cambridge ; New York: Cambridge University Press. xxi, 390 p.

34. Ravindran, A., K.M. Ragsdell, and G.V. Reklaitis, *Engineering Optimization: Methods and Applications*. Second Edition ed. 2006: Wiley.
35. Wu, C.-F. and M. Hamada, *Experiments : planning, analysis, and parameter design optimization*. Wiley series in probability and statistics. Texts and references section. 2000, New York: J. Wiley. xxviii, 630 p.
36. Kuhfeld, W.F. (2003) *Conjoint Analysis*, SAS Institute Publications.
37. Amemiya, T., *Advanced econometrics*. 1985, Cambridge, Mass.: Harvard University Press.
38. Andrews, R.L., A. Ansari, and I.S. Currim, *Hierarchical Bayes versus finite mixture conjoint analysis models: A comparison of fit, prediction, and partworth recovery.* Journal of Marketing Research, 2002. **39**(1): p. 87-98.
39. Allenby, G.M. and P.E. Rossi, *Marketing models of consumer heterogeneity.* Journal of Econometrics, 1999. **89**(1-2): p. 57-78.
40. Orme, B.K. *The Benefits of Accounting for Respondent Heterogeneity in Choice Modeling*. 1998 [cited; Available from: www.sawtoothsoftware.com/download/techpap/benefits.pdf.
41. Lenk, P.J. and W.S. DeSarbo, *Bayesian inference for finite mixtures of generalized linear models with random effects.* Psychometrika, 2000. **65**(1): p. 93-119.
42. Rossi, P.E. and G.M. Allenby, *Bayesian statistics and marketing.* Marketing Science, 2003. **22**(3): p. 304-328.
43. Kamakura, W.A. and G.J. Russell, *A Probabilistic Choice Model for Market-Segmentation and Elasticity Structure.* Journal of Marketing Research, 1989. **26**(4): p. 379-390.
44. Andrews, R.L., A. Ainslie, and I.S. Currim, *An empirical comparison of logit choice models with discrete versus continuous representations of heterogeneity.* Journal of Marketing Research, 2002. **39**(4): p. 479-487.
45. Rossi, P.E., G.M. Allenby, and R.E. McCulloch, *Bayesian statistics and marketing*. 2005, Hoboken, NJ: Wiley. x, 348 p.
46. Gelman, A., et al., *Bayesian Data Analysis*. 2nd ed. 2003, New York: Chapman & Hall/CRC.
47. Spiegelhalter, D.J., N.G. Best, and B.P. Carlin, *Bayesian Deviance, the Effective Number of Parameters, and the Comparison of Arbitrarily Complex Models*. 1998, Division of Biostatistics, University of Minnesota.
48. Spiegelhalter, D.J., et al., *Bayesian measures of model complexity and fit.* Journal of the Royal Statistical Society Series B-Statistical Methodology, 2002. **64**: p. 583-616.
49. Boor, D., *A Practical Guide to Splines (revised ed.)*. 2001, New York: Springer.
50. Lilien, Kotler, and Moorthy, *Marketing Models*. 1992: Prentice-Hall.
51. Cooper, A.B., et al., *Analytical target setting: An enterprise context in optimal product design.* Journal of Mechanical Design, 2006. **128**(1): p. 4-13.