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A GAME-THEORETIC APPROACH TO FINDING MARKET EQUILIBRIA FOR AUTOMOTIVE DESIGN UNDER ENVIRONMENTAL REGULATION

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ABSTRACT

Recent research has extended prior efforts to integrate firm-level objectives into engineering design optimization models by further enlarging the scope to investigate the effects of regulation on the design decisions of profit-seeking firms in competition. In particular, one study examined the effects of environmental policy on vehicle design decisions by integrating quantitative models of engineering performance, market demand, production cost and regulatory penalties in a joint optimization framework using game theory to model the effects of competition on design and pricing. Model complexity and the solution methods used to solve for market equilibria in prior research have led to a limitation where the prior approach is too computationally intensive to allow extensive parametric studies on the effects of policy changes on design. To address this issue, we present an alternative game-theoretic approach utilizing necessary and sufficient conditions with Nash conditions to find market equilibria in an oligopoly of automakers, and we use this approach to examine the resulting optimal design responses under various regulation scenarios.

Keywords: Design Optimization; Discrete Choice Analysis; Logit; Game Theory; Oligopoly; Non-cooperative Game; Nash Equilibrium; Environmental Policy; CAFE; Vehicle Design

1. INTRODUCTION

Automobile manufacturers aim to produce vehicle designs that earn market share and maximize their profits. However, a product equipped with 'good' engineering design does not guarantee that it will be successful in the marketplace. Consumers have preferences that may not be fully understood by vehicle designers and engineers. Market research and econometric methods, such as conjoint analysis and discrete choice models, provide useful quantitative tools for studying consumer preferences and supporting product planning. Most applications of these tools do not model engineering tradeoffs in product design; however, in recent years, researchers have proposed a number of approaches to integrating engineering design optimization with these marketing and econometric models of market performance to search for the most profitable product or product line [1-10].

Each of these approaches takes the perspective of an individual firm and coordinates market and engineering models in an effort to help the firm choose designs that will maximize its objectives (typically profit). It is well-established, however, that in cases where market externalities exist, an unregulated market of profit-seeking firms and rational consumers results in suboptimal social outcomes [11]. Regulation is routinely invoked to manage these cases, and such regulation has both direct and indirect influence on the decisions made by designers. In particular, environmental legislation aimed at reducing fuel consumption and greenhouse gas (GHG) emissions in the automotive industry shifts profit incentives and changes the set of designs that emerge as most-profitable in the regulated, competitive marketplace [12]. Studies of the effects of regulation on the economy are well-represented in the economics literature; however, study of the effects of policy on detailed engineering design decisions, where engineering variables, constraints and tradeoffs are accounted for in mapping the feasible domain of design alternatives, is an open research area.

1.1 Environmental Regulation

Corporate average fuel economy (CAFE) is one major environmental regulation influencing vehicle design in the United States. CAFE standards were developed at the direction of Congress by The National Highway Traffic Safety Administration (NHTSA) and the Environmental Protection Agency (EPA) in response to the oil crisis of the 1970s [13]. CAFE regulation plays an important role in the automobile industry: It forces automakers to produce automobile fleets with higher fuel efficiency by setting up specific fuel economy standards and penalties for violation. The primary purpose of the CAFE policy was originally to reduce domestic gasoline consumption and decrease US dependency on imported oil [14]. More recently, CAFE standards have also been discussed as an approach to reduce GHG emissions for environmental protection, though some researchers argue that the social cost of CAFE policies is too high for this purpose and less efficient than fuel taxes or carbon taxes [15]. Numerous studies have modeled the economic effects of CAFE standards. For example, Kwoka [16] established a simple linear demand model considering two vehicle types, small and large cars, to study the aggregate total fuel consumption changes under CAFE regulation, and Kleit [14] conducted a theoretical analysis with quadratic costs; concluding that CAFE standards encourage the production of small cars at the firm level, but that the CAFE standard is not efficient for energy saving purposes. These studies are not concerned with vehicle design, nor do they model the design changes that could result under changes in policy. Greene and Hopson [17] created a regression model to capture the relationship between retail price and fuel economy changes, and they used nonlinear programming (NLP) methods to maximize the automobile industry-wide fuel economy under alternative forms of fuel economy regulations. They concluded that a 25% to 30% fuel economy improvement for the time period from 2010 to 2015 would be beneficial to consumers. However, the maximization method of industry-wide fuel economy did not take producers' competition into account, and no vehicle engineering content was considered in the mathematical modeling.

1.2 Game Theory in Product Design

Since policy can influence the cost of producing different designs, the effect is to give firms incentive to change the set of designs they offer, and competitive reaction in design and pricing can be a critical component in determining the resulting designs. One well-established framework for modeling competitive behavior is game theory. Game theory has been applied extensively for studying competition between firms in the field of microeconomics [18]. However, most economic applications involve models that are constructed to be as simple as possible (e.g.: linear demand, quadratic cost) in order to study equilibria analytically and reach qualitative conclusions, rather than incorporating data-driven econometric models of consumer choice with physics-driven engineering performance constraints in a computational approach.

There exist a few applications of game theory in the engineering product design literature. For example, Lewis and Mistree [19] modeled the design interactions in a multidisciplinary aircraft design problem as a sequential game, where the players are members of a design team using CAD tools to create an optimal design. These studies generally focus on interactions of designers and design teams, rather than of profit seeking firms in competition. In the marketing literature, some researchers have used game theory to find optimal product positioning[1] at market equilibrium. For example, Choi and Desarbo [20] applied branch and bound to solve the nonlinear integer programming problem of optimal product positioning and employed the sequential iterative method[2] to find Nash equilibria in an oligopolistic automobile tire market. Choi and Desarbo concluded that the forward and reversed orders of sequential iteration reached different Nash equilibrium solutions, which imply existence of multiple equilibria in their model. Green and Krieger [21] also implemented the sequential iterative approach to find the Nash equilibrium in an oligopolistic cellular phone market comprising of three firms, and they compare results under scenarios of equal and unequal costs. Finally, Besanko *et al*. [22] demonstrate that using game theory to model price endogeneity can improve estimates of price coefficients when fitting a demand model to revealed preference data in the marketplace. These prior marketing approaches tend to neglect the importance of engineering tradeoffs in the design. Michalek *et al*. [7] review critical differences between product attributes used in marketing studies and product design variables used in engineering optimization, and Krishnan and Ulrich [23] highlight the need for methods that can integrate the two.

One recent approach integrating models of engineering and market performance examines the effects of environmental policy on vehicle design in the context of game theory [12,24]. The study invokes game theory to examine profitable designs under oligopolistic competition at market equilibrium: The Nash equilibrium in this non-cooperative game is solved using nonlinear programming methods with the sequential iterative approach. It turns out that the approach suffers from two important limitations: First, it is too computationally intensive to allow extensive parametric studies on the effects of policy changes, and second, the approach failed to discover the existence of multiple local equilibria in the study. In this paper,

[1] Here the term *positioning* is used to highlight the fact that the models do not incorporate engineering detail, but rather use only target marketing attributes observed by the customer to describe the products.

[2] In the sequential iterative method, each producer selects its optimal designs given the fixed decisions of competitors, and this process is iterated across producers until equilibrium is reached such that no design change for any single producer can generate higher individual profit. Such a point is called a Nash equilibrium.

we develop an alternative computational approach to locating equilibria in the model by searching directly for points that satisfy Nash necessary and sufficient conditions [18], and we use the approach to execute parametric studies and examine trends.

2. MODEL FRAMEWORK

This study follows the automobile optimization design scheme established by Michalek *et al.* [12]. The overview of the modeling framework for a single producer is shown in Figure 2, integrating models of vehicle performance, market demand, vehicle cost, regulation cost and profit. The optimization algorithm controls decision variables of vehicle design and pricing, and producer profit is used as the objective function.

2.1 Vehicle Performance Model

The vehicle performance data are obtained using the vehicle simulator ADVISOR [25], which was originally developed by U.S. Department of Energy's National Renewable Energy Laboratory (NREL) and later commercialized by AVL [26]. The simulator can predict fuel economy z_1 and 0-60 mph acceleration time z_2 under a specified driving cycle as a function of engine power x_1^{P} and final gear drive ratio x_2. The engine power is used to extrapolate engine maps of existing engines for small increases or decreases in size, so engine power is represented by the base engine power b_M and engine design scale x_1:

$$x_1^{\mathrm{P}} = b_M x_1 \tag{1}$$

A specific spark ignition (gasoline) engine module SI102 in ADVISOR is selected for this research. The base power output b_M of SI102 engine is 102 kW and the engine scaling variable x_1 is allowed to vary between 0.75 and 1.50, the range described as acceptable for reasonable predictions in ADVISOR. The range of final drive ratio x_2 is constrained between 0.4 and 1.3. The performance responses of a mid-size passenger car carrying variants of the SI102 base engine are shown in Figure 3.

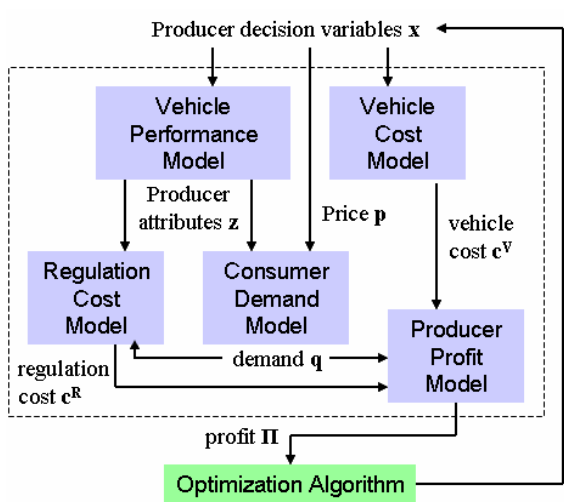

Figure 2: Modeling Framework Diagram

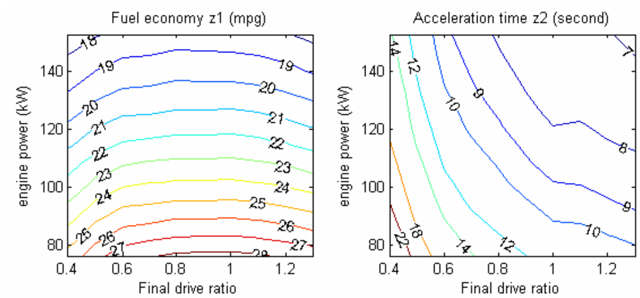

Figure 3: Vehicle performance simulation results of SI102 engine

2.2 Market Demand Model

The demand model used in this paper is based on the multinomial logit model and the data from a discrete choice survey conducted by Boyd and Mellman [27][3]. The standard multinomial logit model assumes that the utility a consumer gains from a particular product is partly observed and partly unobserved, and that the unobserved random term is assumed to follow the independent and identically-distributed (iid) extreme value distribution [28]. Under these assumptions, the probability of a consumer choosing one product out of a set of alternatives reduces to the well-known explicit form:

$$P_j = \frac{\exp(v_j)}{\sum_{j \in J} \exp(v_j)} \tag{2}$$

where P_j is the probability of product j being chosen by the consumer, v_j is the observable utility of product j, and J is the set of product alternatives in the choice situation. The observable component of utility is assumed to be a function of the observed characteristics: in this case vehicle price, fuel economy and acceleration time [12,27]:

$$v_j = \beta_1 p_j + \beta_2 \left(\frac{100}{z_{1j}} \right) + \beta_3 \left(\frac{60}{z_{2j}} \right) \tag{3}$$

where the three attributes for vehicle j are the price p_j, the fuel economy in miles per gallon (mpg) z_{1j}, and the 0-to-60 mph acceleration time z_{2j}. The coefficients were found[4] by fitting the choice model to data on past consumer choices: $\beta_1 = -2.86\text{x}10^{-4}$, $\beta_2 = -0.339$ and $\beta_3 = 0.375$. The price is scaled by a scaling parameter s_p:

$$p_j = s_p x_{pj} \tag{4}$$

where s_p=10,000 and x_{pj} is the scaled price variable of product j. Since the fuel economy and acceleration time are both functions of engine power and final drive ratio, the utility of product j can be expressed as the function of x_{pj}, x_{1j} and x_{2j}:

[3] Because of the age of the data in this study, and because of the broader scope of vehicle classes, interpretation of numerical results using this data should be made with caution.

[4] This aggregate logit model assumes a single "average" set of coefficients over the consumer population, and it is not able to model consumer heterogeneity, implying that design solutions will be identical for all producers. More advanced models can be used to relax this limitation [28].

$$v_j(x_{pj}, x_{1j}, x_{2j}) = \beta_1 s_p x_{pj} + \beta_2\left(\frac{100}{z_1(x_{1j}, x_{2j})}\right) + \beta_3\left(\frac{60}{z_2(x_{1j}, x_{2j})}\right) \quad (5)$$

Demand q_j for product j is taken as market share P_j times market size S, and S is assumed fixed at $(11/7)\times10^6$ [12][5].

2.3 Vehicle Cost Model

In this study, the vehicle cost is taken to be composed of an investment cost for manufacturing equipment c^{I}, paid once per vehicle design, and a variable cost per vehicle produced, which is comprised of two major cost items: the engine cost c^{E} and the manufacturing cost c^{B} to make all other parts of the vehicle. The cost terms c^{I} and c^{B} are taken to be \$550M and \$7,500 respectively, and the spark-ignite engine cost is given by the following formulation [12]:

$$c^{\mathrm{E}}(\mathbf{x}) = \beta_4 \,\mathbf{exp}(\beta_5 b_M x_1) \quad (6)$$

where β_4=670.51 and β_5=0.0063. It can be seen that cost is an exponential function of the engine power $b_M x_1$. The variable cost can be expressed as:

$$c^{\mathrm{V}} = c^{\mathrm{B}} + c^{\mathrm{E}} = c^{\mathrm{B}} + \beta_4 \,\mathbf{exp}(\beta_5 b_{\mathrm{M}} x_1) \quad (7)$$

2.4 Regulation Cost Model

Two policies are considered in the regulation cost model. The first policy is CAFE, which penalizes producers whose average fleet fuel economy is below the CAFE standard. The second policy is a hypothetical carbon dioxide (CO_2) emission tax, which taxes the producer for the estimated externality cost of the lifetime CO_2 emissions produced from a vehicle. The formulations of these two policies are described in the following sections.

2.4.1 Corporate Average Fuel Economy (CAFE)

The current CAFE standard is 27.5 mpg for passenger cars and 21.6 mpg for light duty trucks (under 8,500 lb gross vehicle rating) [13]. The standard for passenger cars has not been changed since 1985. In this study, we assume each producer manufactures passenger cars only. The average fleet fuel economy z_{1k}^{AVG} of producer k is calculated by the harmonic mean[6] [13]:

$$z_{1k}^{\mathrm{AVG}} = \left(\sum_{j\in J_k} q_j\right)\left(\sum_{j\in J_k} \frac{q_j}{z_{1j}}\right)^{-1} \quad (8)$$

where the numerator is total production of the fleet and the denominator is the sum of the production quantity of each model divided by the individual fuel economy. Then the total CAFE regulation cost for producer k is calculated by the following equation:

$$c_k^{\mathrm{R}} = \mathbf{max}\left(0, \sum_{j\in J_k} q_j \rho (z_{\mathrm{CAFE}} - z_{1k}^{\mathrm{AVG}})\right) \quad (9)$$

where ρ is the penalty per vehicle per mpg under the standard fuel economy z_{CAFE}[7]. The current CAFE penalty is \$5.50 per 0.1 mpg (\$55 per mpg).

2.4.2 Carbon Dioxide Emission Tax

A regulation scenario of CO_2 emission taxation has been presented in prior research [12]. The formation of CO_2 tax cost is:

$$c_k^{\mathrm{R}} = \sum_{j\in J_k} q_j \frac{v d \alpha}{z_{1j}} \quad (10)$$

where v is the dollar (externality) valuation of a ton of CO_2, d is total mileage traveled in the vehicle lifetime, and α is CO_2 emission per gallon of gasoline. The defined values are d=150,000 miles, $\alpha = 9.94\text{x}10^{-3}$ per gallon and v is varied from \$2 to \$22 per ton, based on valuation estimates [29].

2.5 Profit Model

The profit of each producer k is calculated as revenue less costs due to investment c^{I}, variable cost c^{V}, and regulation c_k^{R}:

$$\Pi_k = \left(\sum_{j\in J_k} q_j (p_j - c_j^{\mathrm{V}}) - c^{\mathrm{I}}\right) - c_k^{\mathrm{R}} \quad (11)$$

The overview of the modeling framework integrated with optimization algorithm is shown in Figure 1.

3. SOLUTION METHODOLOGY

In this paper, the producers in the market are competing with their product prices and vehicle designs for attracting demand from consumers. Following the prior research [12], we model this situation as a static non-cooperative game with perfect information and only pure strategies - meaning no random decision-making is involved [30].

From the Nash equilibrium definition in a non-cooperative game, assuming that there are K producers (players) in the market and each producer k has his individual strategy (price and product design set) $\mathbf{x}_k$ with payoff function (profit) Π_k, the set of strategies are in Nash equilibrium when each producer k cannot obtain better profit Π_k by choosing any new design other than the equilibrium design $\mathbf{x}_k$. The mathematical expression is:

$$\Pi_k(\mathbf{x}_1, \ldots \mathbf{x}_k, \ldots, \mathbf{x}_K) \geq \Pi_k(\mathbf{x}_1, \ldots \mathbf{x}'_k, \ldots, \mathbf{x}_K) \quad \forall k, \mathbf{x}'_k \quad (12)$$

for all feasible $\mathbf{x}_k$'. A direct approach to find the equilibrium solution for the game is optimize the decision variables of each producer sequentially and iteratively while holding all other producers constant at each iteration. The process is continued

[5] No data were available on consumers who chose not to purchase, so the model contains no “outside good,” implying that total demand is constant (the logit model determines only market share among competitors). Such a model is inappropriate for a monopolist case, and results must in interpreted with care in the oligopoly case.

[6] Note that the arithmetic mean was incorrectly used in the reference study [12]; however CAFE standards specify use of the harmonic mean [13].

[7] CAFE credits are ignored for the study.

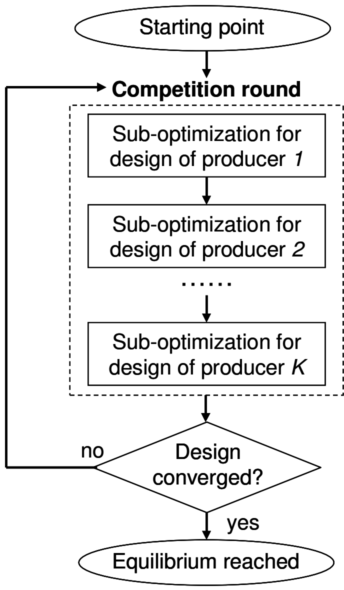

Figure 5: Flowchart of the sequential optimization algorithm

until no producer can increase its profit by changing the decisions under its control (product design and price). The process flowchart of this approach is shown in Figure 5.

The sequential algorithm is straightforward, but it can be computationally costly to find the Nash equilibrium, especially when the number of producers is large and each sub-optimization task is complex and each sub-optimization must be solved globally. The method is also not guaranteed to converge in general. Instead of solving the optimization of each producer repeatedly, an alternative optimization scheme is proposed here to find solutions that satisfy the first-order necessary condition (FOC) and second-order sufficient condition (SOC) for Nash equilibrium, as shown in Figure 6. If the problem has no constraint, the FOC can be expressed in a mathematical form as:

$$\text{Find } \mathbf{X} \text{ such that } \frac{\partial \Pi_k}{\partial x_{ik}} = 0;\ \forall i,k \qquad \text{where } \mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_k \cdots \mathbf{x}_K] \tag{13}$$

where Π_k is the profit of producer k, x_{ik} is the i-th variable (including price and design variables) of producer k, $\mathbf{x}_k$ is the variable vector of product k and $\mathbf{X}$ is variable vector of all K producers. When constraints $\mathbf{h}(\mathbf{x}_k)=\mathbf{0}$ and $\mathbf{g}(\mathbf{x}_k)\leq\mathbf{0}$ are included in model, the Lagrangian function L for each producer k is given by:

$$L_k = \Pi_k - \boldsymbol{\lambda}_k^T \mathbf{h}(\mathbf{x}_k) - \boldsymbol{\mu}_k^T \mathbf{g}(\mathbf{x}_k) \tag{14}$$

where $\boldsymbol{\lambda}_k$ and $\boldsymbol{\mu}_k$ are the vectors of Lagrange multipliers for the equality and inequality constraints, respectively. Therefore, the first-order necessary KKT conditions for a Nash equilibrium in a general constrained formulation can be expressed in Lagrangian form as the system of equations (and inequality relations):

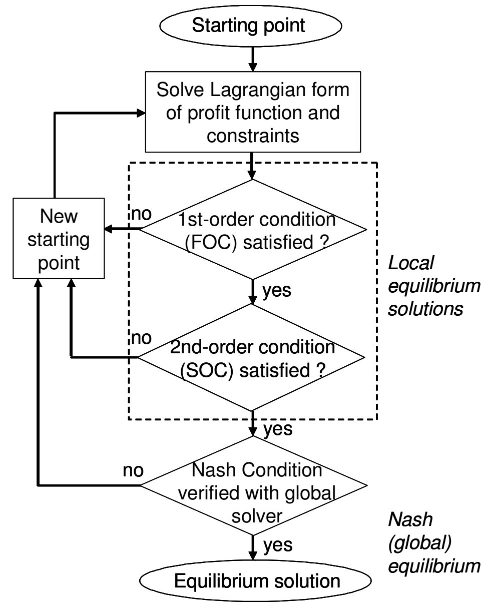

Figure 6: Flowchart of Lagrangian FOC algorithm

$$\begin{aligned}
&\nabla_{\mathbf{x}_k} L_k = \nabla_{\mathbf{x}_k} \Pi_k - \boldsymbol{\lambda}_k{}^T \nabla_{\mathbf{x}_k} \mathbf{h}(\mathbf{x}_k) - \boldsymbol{\mu}_k{}^T \nabla_{\mathbf{x}_k} \mathbf{g}(\mathbf{x}_k) = \mathbf{0} \\
&\boldsymbol{\mu}_k{}^T \mathbf{g}(\mathbf{x}_k) = 0; \quad \boldsymbol{\mu}_k \geq \mathbf{0}; \\
&\mathbf{h}(\mathbf{x}_k) = \mathbf{0}; \quad \mathbf{g}(\mathbf{x}_k) \leq \mathbf{0}; \quad \forall k
\end{aligned} \tag{15}$$

The second-order sufficiency condition (SOC) for local optimality of each producer is verified after a FOC solution is obtained. The Hessian matrix of each producer's profit function with respect to the variables under its control is:

$$\mathrm{H}_k = \begin{bmatrix}
\frac{\partial^2 \Pi_k}{\partial^2 x_{1k}} & \cdots & \frac{\partial^2 \Pi_k}{\partial x_{1k} \partial x_{ik}} & \cdots & \frac{\partial^2 \Pi_k}{\partial x_{1k} \partial x_{Ik}} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\frac{\partial^2 \Pi_k}{\partial x_{ik} \partial x_{1k}} & \ddots & \frac{\partial^2 \Pi_k}{\partial^2 x_{ik}} & \ddots & \frac{\partial^2 \Pi_k}{\partial x_{ik} \partial x_{Ik}} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\frac{\partial^2 \Pi_k}{\partial x_{Ik} \partial x_{1k}} & \cdots & \frac{\partial^2 \Pi_k}{\partial x_{Ik} \partial x_{ik}} & \cdots & \frac{\partial^2 \Pi_k}{\partial^2 x_{Ik}}
\end{bmatrix} \tag{16}$$

where I is the number of design variables. Note that the Hessian matrix for each producer k is not the Hessian matrix of the complete optimization scheme. Since the Nash equilibrium is a point such that the profit of each producer is maximized given the fixed decisions of other producers, a solution to the first order necessary conditions where each producer's Hessian matrix $\mathbf{H}_k$ is negative definite on the subspace tangent to the producer's active constraints will imply that the point is locally optimal for each producer, and the SOC solution is a local equilibrium point[8]. The last step in the algorithm is to verify that the SOC solution is a (global) Nash equilibrium. With all other producers' solutions fixed, a single producer should not be

[8] Specifically: each producer cannot improve profit given the decisions of other producers for (small) changes in design variable. This condition is not yet sufficient for a Nash equilibrium.

able to generate higher profit by changing its design variables with global solver. If the test is failed, that means the SOC solution is just a local optimum but not a truly global Nash equilibrium. If an SOC solution passes the global verification for each producer, equilibrium is confirmed.

This approach is significantly faster than the sequential optimization method since it handles all producers' variables at once without running through multiple iterations of sub-optimization loops. The flowchart of this approach is shown in Figure 4. Furthermore, if multiple equilibria exist in the model, the algorithm is sufficiently efficient to enable a multistart loop using an array of different starting points to find all equilibria.

4. RESULTS AND DISCUSSIONS

According to analysis results in the prior research [12], all producers in the oligopolistic market make the same design decisions at equilibrium to produce a single product instead of a product line for pursuing their own maximum profit. This is a direct result of the use of an aggregate logit model to predict demand; however, the relative simplicity of the model and relative ease of analyzing results make this assumption desirable for initial studies to develop improved understandings of the system before heterogeneity is introduced. The conclusion indicates that the spark-ignition (gasoline fuel) engine always creates more profit than the compression-ignition (diesel fuel) engine. Further, the maximum number of producers to yield positive profit (ten) is found in the prior study. Therefore, the oligopoly model in this paper contains ten producers, each making a mid-size passenger vehicle with the SI102 engine module. In the following sections, the Nash equilibria under different regulation scenarios are found using the direct first-order optimality algorithm described in Section 3. The sequential quadratic programming (SQP) code in the Matlab Optimization Toolbox [31] is used as the nonlinear programming solver. Since the solution of the gradient-based optimization method is dependent on the starting point, multiple starting points on a grid with interval 0.1 in the normalized design variable range are tested in order to discover all the possible equilibrium solutions in the market.

4.1 Scenario of No Regulation

The first scenario assumes all producers make their engine design decisions under no environmental regulation. There are seven SOC solutions found, as shown in Table 1. For each SOC solution, the variables of all ten producers converge to an identical design with equal market share (10%) and profit ($60.5M)[10]. By using global optimization solver[11], only the fourth solution satisfies the Nash criterion, which is the same solution obtained by using iterative optimization method [12]. The Lagrangian FOC/SOC algorithm provides a more efficient tool to search equilibrium solution in decision domain, especially for a complex system with nonlinear engineering constraints.

Table 1: SOC solutions under no regulation scenario

#	Eng. scale x_1	FD ratio x_2	Price p	Gas mile. z_1	Acc. time z_2	Observ. utility v	Variable cost c^V	Profit Π
1	1.14	1.10	12776	22.1	8.22	-2.45	8891	60.5M
2	1.18	1.10	12817	21.6	8.09	-2.45	8932	60.5M
3	1.25	1.11	12884	20.8	7.80	-2.43	8999	60.5M
4	*1.25*	*1.29*	*12886*	*20.2*	*7.46*	*-2.34*	*9001*	*60.5M*
5	1.28	1.01	12912	20.6	7.63	-2.39	9027	60.5M
6	1.32	1.29	12955	19.5	7.27	-2.35	9070	60.5M
7	1.43	1.27	13063	18.5	6.98	-2.35	9178	60.5M

The #4 solution is the Nash equilibrium.

4.2 Scenario of CAFE Policy Alternatives

There are two parameters, standard mpg z_{CAFE} and unit penalty ρ, in CAFE regulation. We studied three fixed CAFE mpg standards of 27.5 mpg (the current CAFE standard for passenger cars), 20 mpg (low standard) and 35 mpg (high standard) with applying a range of penalty parameter scenarios from $0 to $110 per vehicle per mpg (twice the current CAFE penalty $55 for violation). The resulting utilities and fuel economy at equilibrium for each penalty parameter are plotted in Figure 5 and Figure 6. With the 27.5 mpg standard, a clear trend of decreasing consumer utility with increasing penalty is observed while the gas mileage in vehicle deign is improved. At the low CAFE mpg standard of 20 mpg, producers' vehicle designs meet the criterion easily and have no incentive to improve fuel economy. The decrease in observable utility response is primarily due to producers passing the regulatory cost on to consumers[12]. At the high CAFE mpg standard of 35 mpg, which is beyond producers' design capability in the engineering model, the results at market equilibrium show that producers maintain their design and transfer CAFE cost into price, as consumer utility drops.

4.3 Scenario of CO_2 Emission Tax Changes

The utility and gas mileage responses to CO_2 emission taxes are shown in Figure 7 and Figure 8 respectively. The utility plot shows a linearly decreasing trend over the increasing CO_2 valuation levels. The gas mileage is not improved significantly with the increasing CO_2 valuation parameter until $14/ton is reached. However, the highest CO_2 tax still does not result in vehicles with fuel economy better than 22 mpg.

[10] Because of using the aggregate demand model, each producer makes the same design decision to reach maximum (identical) profit at market equilibrium. The resulting market share of each producer is also equal when no outside goods in the utility is considered [12]. When preference heterogeneity is considered (e.g. mixed logit model), the design, price, market share and maximum profit of each producer will generally be different at market equilibrium [32].

[11] We use multistart as the global optimization algorithm to verify Nash conditions for each producer.

[12] Because there is no outside good in our model, the total demand for vehicles is constant, and only market share is affected by changing vehicle price and attributes. As such, the model predicts that all regulatory cost is passed on to consumers, although this is not generally observed in practice, where consumers have the ability to switch to other modes of transportation.

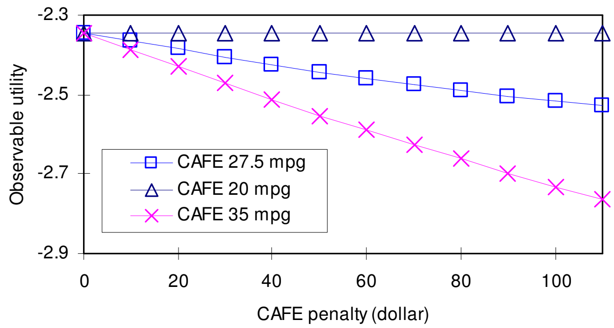

Figure 5: Utility responses to CAFE regulation

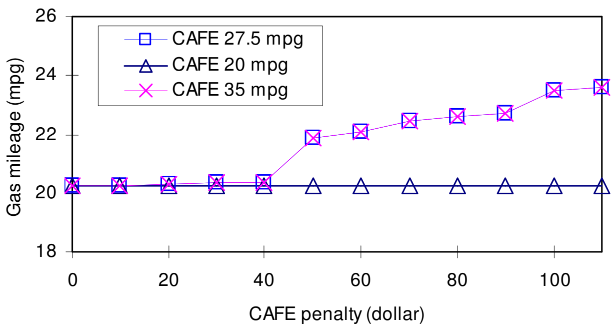

Figure 6: Fuel economy responses to CAFE regulation

Compared to responses of CAFE regulation in Figure 5 and Figure 6, the CO_2 emission tax has inferior performance on fuel economy improvement per unit of lost utility to the consumer.

4.4 The Fuel Economy / Utility Tradeoff

The tradeoff in the problem is interlaced with several variables. When we consider the engineering aspects, a tradeoff exists between two engineering attributes, gas mileage and acceleration time. For economic aspects, another tradeoff is formed in the profit function Eq. (11), where higher price results in lower market share when producers try to maximize their profits. We are particularly interested in the tradeoff between gas mileage and observable utility because increased fuel economy represents a social preference while utility represents individual consumer preferences. Higher fuel economy increases the observable utility, while the resulting slower acceleration causes utility decreased. When the CAFE penalty pressure is increased, the producers have to change their designs and prices to pursue maximum profit under non-cooperative competition. It should be noted that this tradeoff does not represent a revenue-neutral comparison. In particular, the dominant reason for decreasing utility with increasing regulation penalty in our model is due to the passing of regulatory costs to consumers, without accounting for how the increased government revenue is used. As such, the tradeoff curves should be interpreted with care.

The tradeoff between fuel economy and consumer utility in a market is expected from the observation of the utility and fuel economy responses under different regulation scenarios. Figure

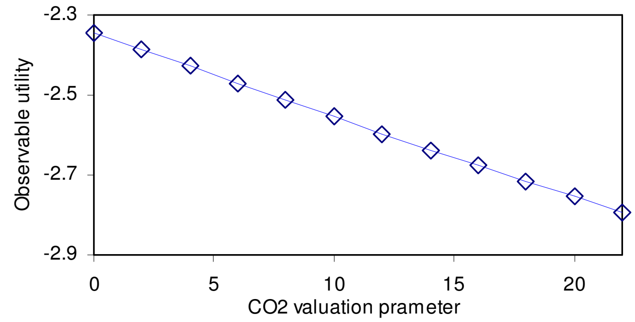

Figure 7: Utility responses to CO_2 tax

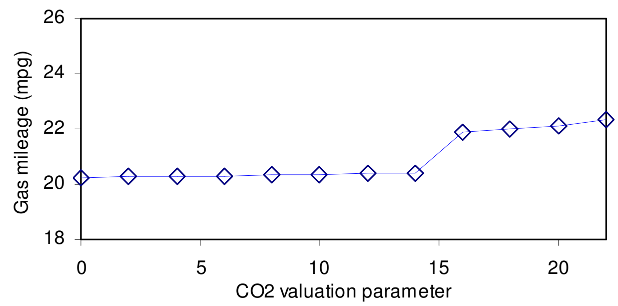

Figure 8: Fuel economy responses to CO_2 tax

9 presents the Nash equilibria resulting under different levels of CAFE regulation and CO_2 emission tax. The CAFE fuel economy z_{CAFE} is fixed at 27.5 mpg and the penalty parameter ρ is changed from \$0 to \$110. The valuation parameter v for CO_2 emissions is varied between \$0 and \$22 per ton. Considering the trade-off between fuel economy and utility, the CAFE standard appears to be a better tool than CO_2 tax for controlling fuel economy in the automobile market, ignoring government revenue generated. However, when the CAFE penalty is low (\$0-\$40), the fuel economy improvement is not significant, which suggests a minimum penalty parameter is required to maintain the CAFE performance.

To summarize, the results from this integrated model suggest that 1) increasing regulatory penalties will generally result in increased fuel economy and decreased utility to the consumer as a tradeoff, 2) CAFE regulation has the best performance when a proper fuel economy standard and penalty are applied, and 3) compared to a CO_2 tax, the CAFE regulation would generally be expected to achieve higher fuel economy at lower cost in reduced consumer utility on average. Ignoring government revenue generated, it is also interesting to note that the model predicts that even doubling the CAFE penalty will not provide enough incentive for automakers to attain the 27.5 mpg standard. In practice, we observe that foreign automakers routinely violate the standard – deciding that it is more profitable to pay the associated penalties [33]; however, U.S. automakers, reluctant to be seen as lawbreakers, have treated the standard as a binding constraint [14]. Modeling the additional (political) cost of CAFE constraint violation would be necessary to predict the behavior of these firms. Despite the homogenous product assumption in the current model, our study

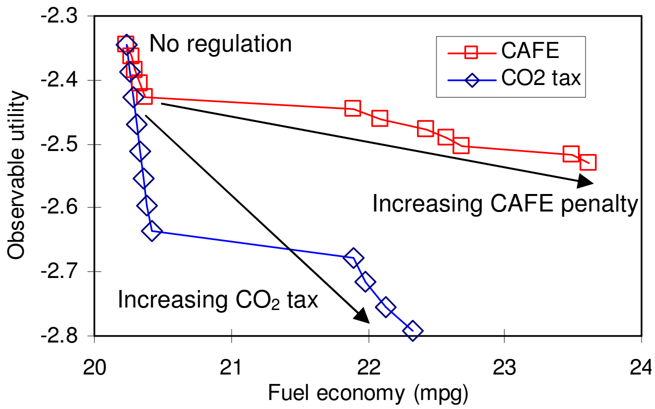

Figure 9: Utility-Fuel Economy Tradeoffs of CAFE and CO_2 tax

provides the fundamental knowledge for more complicated problems with multiple differential products in future development.

5. CONCLUSIONS

This paper presents a multi-stage approach to finding market equilibrium in an integrated model of engineering design, marketing demand and environmental policy. The algorithm first searches the solution satisfying the KKT first-order necessary conditions (FOC) of the Lagrangian function. The FOC solution is then verified by the second-order condition (SOC) to ensure a local equilibrium has been reached. Each point satisfying FOC and SOC is then checked for Nash conditions with a global solver. This method is more efficient than a sequential iterative approach and it is able to find multiple equilibria, if they exist. Following prior research, a vehicle design problem in an oligopolistic automobile market under two environmental policies has been revisited as a methodological demonstration. According to the utility and the fuel economy responses of market equilibria under Corporate Average Fuel Economy (CAFE) and carbon dioxide (CO_2) emission taxes, the results show that increased regulatory penalties cause increased gas mileage at the expense of decreased consumer utility primarily due to regulation costs being passed on to the consumer. The tradeoff between utility and fuel economy resulted in trends that indicate CAFE may be a more efficient tool than CO_2 taxes for achieving fuel economy improvement; however, further research is needed to account for government revenue generated and make revenue-neutral comparisons. Furthermore, the effectiveness of CAFE policy is driven by the proper combination of fuel economy standard and penalty decisions, whereas either excessively low or high standards result in no significant improvement in vehicle gas mileage.

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NOMENCLATURE

c^B	=	Base manufacturing cost per vehicle
c_j^E	=	Engine cost for design j
c^I	=	Investment cost
c_j^R	=	Total regulation cost of producer k
c_j^V	=	Variable cost of design j
d	=	Average vehicle lifetime mileage
$\mathbf{g}$	=	Inequality constraint vector
g_{ik}	=	Inequality constraint for variable i and producer k
$\mathbf{h}$	=	Equality constraint vector
h_{ik}	=	Equality constraint for variable i and producer k
$\mathbf{H}_k$	=	Hessian matrix of producer k
i	=	Variable index
j	=	Product (vehicle design) index
J	=	Set of all vehicle designs
k	=	Producer index
K	=	Total number of producers in the market
L	=	Lagrangian augment function
p_j	=	Price of product j
q_j	=	Demand for design j
v_j	=	Observed utility of product j
v	=	Valuation of CO_2 per ton
$\mathbf{x}_k$	=	Design variable vector of producer k
$\mathbf{X}$	=	Design variable vector of all producers
x_{ik}	=	Design variable i for producer k
$\mathbf{z}$	=	Product characteristic vector
z_{1j}	=	Fuel economy of design j
z_{1k}^{AVG}	=	Average fleet fuel economy of producer k
z_{2j}	=	Acceleration time (0-60 mph) of design j
z_{CAFE}	=	CAFE fuel economy limit
α	=	CO_2 emission per gallon of gasoline
ρ	=	CAFE penalty parameter
$\boldsymbol{\lambda}$	=	Lagrangian multiplier vector for equality constraints
$\boldsymbol{\mu}$	=	Lagrangian multiplier vector for inequality constraints

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