A STUDY OF EMISSION POLICY EFFECTS ON OPTIMAL VEHICLE DESIGN DECISIONS

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ABSTRACT

A methodology is presented for studying the effects of automobile emission policies on the design decisions of profit-seeking automobile producers in a free-entry oligopoly market. The study does not attempt to model short-term decisions of specific producers. Instead, mathematical models of engineering performance, consumer demand, cost, and competition are integrated to predict the effects of design decisions on manufacturing cost, demand, and producer profit. Game theory is then used to predict vehicle designs that producers would have economic incentive to produce at market equilibrium under several policy scenarios. The methodology is illustrated with three policy alternatives for the small car market: corporate average fuel economy (CAFE) regulations, carbon dioxide emissions taxes, and diesel fuel vehicle quotas. Interesting results are derived, for example, it is predicted that in some cases a stiffer regulatory penalty can result in lower producer costs because of competition. This mathematical formulation establishes a link between engineering design, business, and marketing through an integrated optimization model that is used to provide insight necessary to make informed environmental policy.

Keywords: environmental policy, optimal design, game theory, oligopoly, emissions, CAFE, discrete choice analysis, logit, green engineering

NOMENCLATURE

\( c_k \) Total cost for producer \( k \)
\( c_b \) Base manufacturing cost per vehicle (without engine)
\( c_j \) Engine manufacturing cost for design \( j \)
\( c^I \) Investment cost
\( c_p \) Total production cost for design \( j \)
\( c^R_k \) Total regulation cost for producer \( k \)
\( c^V_j \) Variable manufacturing cost per vehicle for design \( j \)
\( d \) Lifetime vehicle miles traveled
\( f_M \) ADVISOR simulation for engine type \( M \)
\( j \) Vehicle design index
\( J \) Set of all vehicle designs produced
\( J_k \) Set of all vehicle designs produced by producer \( k \)
\( k \) Producer index
\( K \) Total number of producers in the market
\( M_j \) Index of vehicle engine type for design \( j \)
\( n_k \) Number of designs produced by producer \( k \) (size of \( J_k \))
\( p_j \) Selling price of design \( j \)
\( q_j \) Demand for design \( j \)
\( u_j \) Utility of design \( j \)
\( v_j \) Production volume of design \( j \)
\( x \) Design variable vector \((x_1, x_2)^T\)
\( x_{1j} \) Engine scaling parameter for design \( j \)
\( x_{2j} \) Final drive ratio for design \( j \)
\( z \) Product characteristics vector \((z_1, z_2)^T\)
\( z_{1j} \) Fuel economy of design \( j \)
\( z_{2j} \) Acceleration time (0-60mph) of design \( j \)
\( z_{CAFE} \) CAFE fuel economy limit
\( \alpha_{M} \) Tons CO2 produced per gallon of fuel for engine type \( M \)
\( \beta \) Regression coefficient parameter
\( \Pi_k \) Total profit for producer \( k \)
\( \rho \) Penalty parameter for regulation violation
\( \phi \) Minimum diesel sales percentage required by quota
\( \upsilon \) Societal cost valuation per ton of CO2 in U.S. dollars

1. INTRODUCTION

Optimal design studies have typically considered tradeoffs among engineering performance metrics. Multiple conflicting objectives can be combined to a Pareto-optimal scheme, but the scalarization preferences (e.g. weights) are often difficult to evaluate, and typically the problem must be reformulated iteratively [1]. Alternatively, objective conflicts can be resolved with a “higher level” criterion, such as maximization of profit for the producer of the designed artifact [2]. In automotive
manufacturing such profit maximization will depend on the vehicle’s appeal to the consumer and the regulatory environment resulting from government policies in addition to proper engineering functionality and cost. In the present work, we adopt this viewpoint and we explore how regulatory policies on emissions may impact design decisions by manufacturers.

Automobile producers provide private goods (vehicles) for private profit (investors), but emissions are generated in the process, and the effects are publicly shared. Vehicle emissions have a large impact on air quality, accounting for up to 95% of city CO emissions, 32% of NOx emissions, and 25% of volatile organic compound emissions in the U.S. [3]. These emissions create smog, increase greenhouse gas forcings, and create human health risks. The market in which goods are traded does not automatically provide individual incentives to reduce publicly shared environmental damage because the environment is treated as an externality (the “tragedy of the commons” [4]), so government regulatory policies have been imposed on vehicles at both national and state levels in order to provide incentive to reduce emissions. Examples include the Clean Air Act [5], which regulates tailpipe emissions, and the corporate average fuel economy (CAFE) standards [6], which require vehicle fleets to meet target average fuel efficiencies. A balance between reducing vehicle emissions, maintaining current vehicle markets, and meeting consumer transportation needs is necessary. For example, alternatives to gasoline engines, such as diesel, hybrid, fuel cell, and electric systems, are more expensive to manufacture compared to traditional gasoline systems. Government policies can provide incentives to bring these alternative choices into the market.

Testing the effects of emission policies requires consideration of the interactions of technology, corporate objectives, consumer choices, and competition among manufacturers. The link between engineering decisions and business decisions, including models of cost and demand, is not typically considered in relation to policy. In this article, each of these factors is represented by a separate analysis model, and their interaction is represented by an integrated design decision model. Solving the resulting optimization problem allows us to explore the effects that air emission policies have on consumers, manufacturers, total air quality, and the design decisions that a particular policy encourages.

2. BACKGROUND

Vehicle emissions have long been recognized as a major contributor to air quality problems; however, much of the research in the area of emissions modeling has been performed in recent years. Research related to the automotive industry has focused primarily on the effects of changing CAFE standards. One such study by the National Academy of Sciences [7] identified technologies that could be included in all vehicles today, along with estimated costs and potential effects on fuel economy for each. Specifically, the effects of incremental changes in CAFE standards on vehicle price, performance, demand and product mix were evaluated. External factors, such as gasoline price were also included in assessing the impact of standards. While this study was very thorough, producer-level decisions focused on inclusion of new technology in existing engines rather than new vehicle design decisions.

Another study by Greene and Hopson [8] examined the impacts of various fuel economy regulatory strategies using a mathematical programming model. Regulatory options included raising the CAFE level, making a fuel-economy standard voluntary, and creating a weight-based metric. The effects of each strategy on producers and consumers were evaluated in terms of monetary costs or savings; however, the market positions of manufacturers were taken as constant, and few technology upgrades were considered.

Policy effects on net automobile emissions have also been evaluated. A recent European Union report [9] describes the Auto-Oil II Programme, which included development of an emissions model, forecast studies to evaluate future vehicle emissions levels, research on the effect of fuel quality on emissions, and development of cost-effectiveness measures to compare policy options. However, this study did not model producer design decisions in response to policy.

While these previous models analyze important aspects of emissions policies, there are opportunities for further steps of improvement. Previous models assume each manufacturer will maintain their current product mix, making only incremental technology improvements to existing products (direct injection, VVT, etc). In contrast, this article provides an economic oligopoly analysis where each firm designs its product mix, changing design variables in response to regulations and competition. Previous models also rely on assumptions about consumer willingness to pay for increased fuel economy rather than using attribute-based consumer choice models derived from past purchase data. This article uses an optimization framework to integrate models for each component, including emissions, engineering design, cost, consumer demand, and producer profit. The framework is modular and hence allows for the substitution of other models for any of the various parts for expansion of this study. Producers in this context are abstract; that is, the results obtained do not apply to a specific producer’s actions, but rather represent the general market trend created by government incentives. In this manner, the model created here is able to simulate the effects of new policy scenarios for the reduction of automobile emissions and improvement of fuel economy.

The remainder of this article proceeds as follows. The proposed methodology is detailed in Section 3, which integrates individual models of engineering performance, consumer demand, cost, producer profit, and regulation. The Nash equilibrium solution strategy used to study market equilibrium is then described, and implementation issues are noted. Results of the study are summarized in Section 4 for each policy scenario, and conclusions are provided in Section 5.

3. METHODOLOGY

The general modeling framework used to capture producer and consumer behavior in this study is shown in Figure 1, where individual analysis models are shown in gray. The producer is assumed to make product design and production decisions to maximize profit. Consumers are assumed to purchase products that maximize their benefit (utility) based on each individual’s preferences. Policy can influence these decisions by imposing penalties and incentives as drivers for the modification of producer and consumer behavior.

In this framework, each producer $k$ decides on a set of designs to produce $J_k$ including design decisions, prices, and production volumes for each design. Design topology $M$ and design variables $x$ (such as engine size) determine product
characteristics \( z \) (such as fuel economy), calculated using the engineering analysis model. Design variables, production volume \( v \), and regulation penalties \( c^k \) also determine producer cost \( c \) calculated by the cost analysis model. The set of competitor designs, \( J_{-J_k} \), are modeled as static parameters, and consumers make purchasing choices among all designs \( J \) (producer and competitor products) based on product characteristics and prices \( p \). The purchasing choices that the consumers make determine demand for each design \( q \) calculated by the demand model, and resulting profits \( \Pi \) are calculated in terms of \( p, q, \) and \( c \). The optimization model represents each producer’s attempt to optimize profit by making the best design, pricing, and production decisions. Government regulation can influence this process by providing penalties to producers, which affect production cost and provide incentive to produce different designs.

![Figure 1 Overview of modeling framework](image)

In the present study, all producers are profit driven, so production volume will equal product demand at an optimum. This assertion is valid for continuous demand functions with negative price elasticities in this formulation because any producer who wishes to produce a lower volume of a product (because of capacity constraints or marginal cost curves, for example) can simply raise the price until demand is lowered to the desired production volume. For a profit-maximizing producer, there is no incentive to produce less volume than there is demand for; instead, the price can always be increased. So, it is assumed that \( v_j = q_j \) from this point forward.

Each producer is modeled as maximizing profit \( \Pi \) (revenue minus cost) subject to engineering constraints.

Maximize \( \Pi_k = \left( \sum_{j \in J_k} q_j p_j \right) - c_k \)

with respect to \( \left\{ M_j, x_j, p_j \right\} \forall j \in J_k \)

subject to engineering constraints.

Profit for each producer is calculated as a function of the producer decision variables by combining the engineering performance, consumer demand, cost, profit, and regulation models described in Sections 3.1–3.5. The size \( n_k \) of the set \( J_k \) is a variable in this formulation. For a fixed \( n_k \) and fixed engine types \( M_j \) for each vehicle, the model, Eq.(1), is a smooth, continuous optimization formulation that can be solved with gradient-based methods. To take advantage of this property, separate optimization runs are formulated for each combinatorial set of \( n_k \in \{1,2,\ldots,n_{max}\} \) and \( M_j \forall j \in J_k \) for each value of \( n_k \). The largest optimum of these cases is then taken as the maximum.

After describing the individual models in Sections 3.1-3.5, the full producer optimization formulation is summarized in Section 3.6, where game theory is used to model competition among the profit-maximizing producers.

### 3.1 Engineering Performance Model

The engineering model takes design decisions \( x_j \) as input and predicts performance characteristics \( z_j \) that can be calculated for each design \( j \). Several analysis models were explored for vehicle modeling, and ADVISOR was chosen because of availability, simplicity, and compatibility with the study. For a review of ADVISOR, see [10],[11]. ADVISOR contains models for conventional, electric, hybrid electric, compressed natural gas, and fuel cell vehicles. Empirically-derived engine maps are used to estimate fuel economy and emissions characteristics across engine operating conditions. The vehicle is simulated through a driving cycle, and fuel economy, performance characteristics and vehicle emissions are calculated for the cycle. Engine, motor, and energy storage systems can be scaled in size, and the effects of size changes on performance, fuel economy, and emissions can be computed. The ADVISOR model provides an appropriate level of detail for this study.

Vehicles here are assumed to differ only by engine design, so the default small car vehicle parameters were used in all simulations (based on the 1994 Saturn SL1), and only the engine variables were changed. ADVISOR has a set of nine gasoline and eleven diesel engine types from which to choose. Each engine type has a base size \( b_{Min} \) corresponding to the size of the empirically tested engine, which can be scaled to predict performance of larger or smaller engines. Good predictions are claimed for scaling parameters between 0.75 and 1.50. The EPA Federal Test Procedure (FTP) driving cycle was used for all simulations. In this study two engine types, \( M = \{SI102, CI188\} \), were used with two design variables: the engine scaling parameter \( x_1 \) in the range \([0.75, 1.50]\), and the final drive ratio \( x_2 \) in the range \([0.2, 1.3]\). The computed outputs (performance criteria) were the gas mileage (gasoline equivalent) \( z_1 \) in mpg and the time to accelerate from 0 to 60 mph, \( z_2 \) in seconds. The engine type \( M = SI102 \) refers to a spark ignition (gasoline) engine with \( b_{SI102} = 102kW \) based on the 1991 Dodge Caravan...
where $z = (z_1, z_2)^T$, and $x = (x_1, x_2)^T$. Simulations were computed for a set of points in the range of the design variables for each engine type, and the responses were used to create a set of surface splines as surrogate models for ease of computation during optimization. Sample contour plots of the simulation results are shown in Table 1.

### Table 1 ADVISOR simulation result contour plots

<table>
<thead>
<tr>
<th>Engine Type</th>
<th>M = SI102</th>
<th>M = CI88</th>
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<td>Engine size (kW)</td>
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</table>

#### 3.2 Consumer Demand Model

The consumer demand model is based on discrete choice analysis (DCA), an area of rational choice theory (RCT), which assumes that users make purchasing decisions based on the utility value of each product option. Utility is composed of a deterministic component, which is a function of product characteristics, and a stochastic error component. The probability of choosing a particular product is calculated as the probability that the product has a higher utility value than all alternatives.

Various probabilistic choice models follow the DCA approach, including multinomial logit models [20], probit models [21], and the BLP model [22]. The multinomial logit model, developed by McFadden to study transportation choices, was used here because of its simplicity, transparency of interpretation, capability to extend predictions to new designs, and the availability of existing models for automotive demand. Logit models have been used extensively in the marketing literature and have recently been applied to engineering design problems [23]. The logit model assumes the unobserved error component to the utility function is independently identically distributed (iid) for each alternative and follows the extreme value (double exponential) distribution. Under the logit assumptions, the probability of choosing alternative $j$ from set $J$ can be derived as

$$
Pr(j|J) = \frac{e^{\beta_j}}{\sum_{j=1}^{J} e^{\beta_j}}
$$

Each utility function $u_j$ depends on the characteristics $z$ and the price $p_j$ of design $j$ by a functional form that is linearly separable. Given data used to fit $u_j(z)$, regression coefficients are chosen that maximize the likelihood of generating the sample data with the model.

The specific demand model used here was a non-nested multinomial logit model developed by Boyd and Mellman [24] based on automotive choice data. Several other models were also explored [25]-[31], but the Boyd and Mellman model was chosen for the following reasons:

- The regression is performed on product characteristics as opposed to consumer demographics.
- The independent variables in the regression include the vehicle’s price, fuel economy, and performance characteristics, which match the characteristics predicted by the ADVISOR engineering model.
- The regression was performed on a large volume of annual market data and validated using data from a subsequent year.

The utility equation developed by Boyd and Mellman is:

$$
u_j = \beta_1 p_j + \beta_2 \left( \frac{100}{z_{1j}} \right) + \beta_3 \left( \frac{60}{z_{2j}} \right)
$$

where $\beta_1=-2.86 \times 10^{-4}$, $\beta_2=-0.339$, $\beta_3=0.375$, $p_j$ is the price of vehicle $j$, $z_{1j}$ is the gas mileage of vehicle $j$, and $z_{2j}$ is the 0-60 time of vehicle $j$. Several other variables were included in the regression, such as vehicle style, noise and reliability; however, in this study these qualities were assumed constant across all vehicles. The logit choice predictions depend on the differences in utility values, so factors that are constant across alternatives do not affect choice predictions, and they can be ignored. Other factors, such as advertising, promotions, aesthetics, and brand image were also assumed equal across alternatives. While this demand model is adequate for a preliminary analysis, it does introduce several sources of error:

- The regression was performed on 1977-1978 purchase data.
- The model was regressed on purchase data only, so all consumers in the model are purchasers, and the model does not consider consumers who do not purchase a vehicle. Therefore, the purchasing population is fixed, independent of vehicle prices in the market (no outside good).
- This is an aggregate model, and therefore it does not account for different segments or consumer groups.
- The logit model has a property called “independence from irrelevant alternatives” (IIA), which implies that as one product’s market share increases, the shares of all competitors are reduced in equal proportion. This further implies, for example, that BMW competes equally with

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1 The coefficients $\beta_1$ and $\beta_2$ were assumed to be negative, even though they are listed as positive in the Boyd and Mellman article. In the text the authors describe the variables as having a negative relationship even though all coefficients are listed as positive in the regression summary.
Mercedes as with Chevrolet, and this issue is often referred to as the “red bus / blue bus problem” [29]. In reality, different vehicles attract different kinds of consumers, and competition is not equal; however, the simplifications in the logit model do not allow for measurement of these differences. Predictive limitations of the IIA property are mitigated in this study since we apply the model only to the small car market, a relatively homogeneous market, rather than the entire spectrum of vehicles.

The model above was developed to study the effects of fuel economy standards on the market and should be sufficient in capturing the trends important in our general analysis, even if the numbers vary for today’s consumers. For the purposes of this study the assumption was made that the size of the car-buying population $s$ is 1.57 million people. This figure is based on data that 11 million people bought cars in 1977 [33]. In the absence of better information, we assumed that the size of the small car market was 1/7 of the total market, and we applied the demand model to this sub-market. Further research indicated that a better estimate of the size of the small car market may have been 2/7 of the total market [34], and the demand model was derived for the entire population, not only for the small car market. These are not significant limitations here because the primary intention of this analysis is to study trends rather than to calculate precise predictions.

Using the logit model, the demand $q_j$ for product $j$ is

$$q_j = s \Pr(j \mid J) = s \sum_{j \in J} e^{x_j}$$

where $u_j$ is defined by Eq.(4).

### 3.3 Cost Model

Production cost is modeled as a function of the vehicle design, and all producers are assumed to have the same manufacturing cost structure. In practice, differences in equipment, assets, suppliers, and expertise affect manufacturing costs; however, this assumption is appropriate for oligopoly analysis, and it is useful to analyze trends even if individual numbers differ between firms. The total cost to manufacture a vehicle $c$ is decomposed into two components: the investment cost to set up the production line $c^I$ and variable cost per vehicle $c^V$. The variable cost is composed of engine cost $c^E$ and cost to manufacture the rest of the vehicle $c^M$. The investment cost to manufacture $q$ units of a vehicle with topology $M$ design variables $x$ is then

$$c^M(M, x) = c^I + qc^V(M, x) = c^I + q(c^E + c^V(M, x))$$

where $c^B =$ US 7500 for all vehicles, based on data for the Ford Taurus [12], and $c^I =$ US 550 million per vehicle design for all manufacturers, based on an average of two figures for new production lines [13]. The cost to manufacture an engine is modeled as a function of engine power. Functions were obtained by regression analysis of data obtained from manufacturing, wholesale, and rebuilt engine cost data [14]-[19]. Wholesale and rebuilt engine prices were assumed to be close to manufacturing prices, and these data fit the curve well. The resulting functions are

$$c^E(M, x) = \begin{cases} \beta_1 \exp(\beta_2 b_j x_j) & \text{if } M \in \text{SI} \\ \beta_3 (b_j x_j) + \beta_4 & \text{if } M \in \text{CI} \end{cases}$$

where $\beta_1=670.51$, $\beta_2=0.0063$, $\beta_3=26.23$ and $\beta_4=1642.8$. These functions are plotted in Figure 2. All designs in this study fall within the range of the data. The diesel engine function is linear, but the diesel data represent a smaller range of engines than for gasoline. As expected, the cost associated with manufacturing diesel engines is higher than for gasoline engines. It is possible that increased diesel production volumes would change this cost structure, but this possibility was not explored. Although both cost regression models rely on max engine power as the only dependant variable, Figure 2 demonstrates that the regressions fit the data well and predict realistic cost trends.

![Figure 2 Manufacturing Cost for SI and CI Engines](image)

The total cost to producer $k$ is the sum of the production cost of each vehicle in $k$’s product line and regulation cost $c^R_k$, as described in Section 3.5.

$$c_k = \sum_{j \in J_k} c_j + c^R_k$$

### 3.4 Profit Model

The profit model for each producer, $k$, is assumed to be calculated simply as revenue minus cost, with no account for the time value of money, opportunity costs, or changes in production loads over time. Demand is predicted over the course of one year, with all costs and revenue occurring during that year. The inclusion of dynamic time considerations brings a plethora of uncertainties and issues difficult to model, and it is left for future consideration. In

$$\Pi_k = \sum_{j \in J_k} q_j p_j - c_k = \left(\sum_{j \in J_k} q_j (p_j - c_j) - c^I - c^E \right) - c^R_k$$

where $c^R_k$ is the regulation cost for producer $k$ (see Section 3.5). The model assumes that all transactions happen instantaneously.
practice, the investment cost associated with designing and building production lines and planning supply chains is spread over several years with only minor changes to the vehicles during those few years, implying that this model will tend to over-predict investment cost.

### 3.5 Regulation Policy

Three specific producer penalty regulation policy scenarios are modeled: CAFE, CO2 emission tax, and diesel fuel vehicle sales quotas. Each of these policies applies to the producer a penalty cost associated with vehicle emissions.

**Corporate Average Fuel Economy (CAFE)**

CAFE regulations set minimum average fuel economy standards that each producer’s vehicle fleet must meet to avoid penalties. Both the fuel economy standard and the penalty must be specified to define the policy. The current CAFE fuel economy standard for cars, \( z_{\text{CAFE}} = 27.5 \text{ mpg} \), was used here, and two different penalty charges were explored. The first penalty charge is the current standard: \( \rho = 55 \text{ per vehicle per mpg under the limit. The total cost incurred by design } k \text{ is therefore } \rho q_j (z_{\text{CAFE}} - z_{ij}) \), where \( \rho \) is the penalty ($55), \( q_j \) is the number of vehicles of type \( j \) that are sold, \( z_{\text{CAFE}} \) is the CAFE limit, and \( z_{ij} \) is the fuel economy of vehicle \( j \). The total regulation cost to producer \( k \) is then

\[
\epsilon_k^R = \max \left( 0, \sum_{j \in J} \rho q_j (z_{\text{CAFE}} - z_{ij}) \right)
\]

(10)

Additional future credit for vehicle fleets with average fuel economies greater than the standard is not modeled.

The second penalty explored is a hypothetical “strict” CAFE policy where the penalty charge is greatly increased (\( \rho = 10000 \)), and the deviation from the CAFE standard is squared:

\[
\epsilon_k^R = \max \left( 0, \sum_{j \in J} \rho q_j (z_{\text{CAFE}} - z_{ij})^2 \right)
\]

(11)

The squared term was introduced to smooth the objective function for ease of computation and to penalize larger violations more strongly. The “strict” CAFE policy is intended to force producers to adhere to the fuel economy standard. To get the entire picture of how the CAFE policy affects the producers and the types of vehicles produced, a parametric study on the fuel economy standard and the penalty value could be performed.

**CO2 Emission Tax**

A CO2 valuation study [35] was used to estimate the economic cost to society associated with environmental damage due to the release of each ton of carbon dioxide. Using this valuation, a tax can be imposed on the manufacturer based on the estimated lifetime emissions of each vehicle sold. Valuations of other pollutants were included in the study; however, CO2 is the only pollutant considered here because its impacts are globally distributed rather than dependent on local conditions. Tax per vehicle sold can be calculated as \( \nu d \alpha_d z_1 \), where \( \nu \) is the dollar valuation of a ton of CO2, \( d \) is the number of miles traveled in the vehicle’s lifetime, \( \alpha_d \) is the number of tons of CO2 produced by combusting a gallon of fuel, and \( z_1 \) is the fuel economy of the vehicle. The total regulation cost to the producer in this study is

\[
\epsilon_k^R = \sum_{j \in J} q_j \frac{\nu d \alpha_d z_1}{z_{ij}}
\]

(12)

where \( d = 150,000 \text{ miles}, \alpha_d = 9.94 \times 10^{-3} \text{ tons CO2 per gallon for gasoline or 9.21 \times 10^{-3} tons CO2 per gallon for diesel fuel [36], and the value of } \nu \text{ ranges from } \$2/\text{ton} \text{ to } \$23/\text{ton} \text{ with a median estimation of } \$14/\text{ton}. While there is much variability in the estimated cost of CO2 emissions, analyzing the range of valuations is still useful in aiding policy makers to weigh the relative social costs.

**Diesel Fuel Vehicle Sales Quotas**

As a regulation method, quotas can be used to force alternative fuel vehicles into the market. A policy that introduces a large penalty cost for violation of a minimum diesel ratio quota was modeled as a way to enforce use of fuel alternatives to gasoline. The regulation cost is modeled as

\[
\epsilon_k^R = \max \left( 0, \rho \left( q_{\text{SI}}^{\text{CL}} - (1 - \phi) (q_{\text{SI}}^{\text{CL}} + q_{\text{CI}}^{\text{CL}}) \right) \right)
\]

(13)

where \( \rho \) is the penalty per vehicle over quota ($10,000), \( \phi \) is the minimum diesel percentage required by the quota (40%), \( q_{\text{SI}}^{\text{CL}} \) is the total number of spark ignition engines sold by producer \( k \), and \( q_{\text{CI}}^{\text{CL}} \) is the total number of compression ignition engines sold by producer \( k \).

### 3.6 Nash Equilibrium Solution Strategy

In a free market, manufacturers have economic incentive to produce and sell products only if there is opportunity to make profit. Each producer has the option to produce multiple designs (a product line); however, component and equipment sharing between designs (commonality) is not considered.

To account for competition, a methodology was developed to find market (Nash) equilibria for \( k \) competing producers. In game theory, a set of actions is in Nash equilibrium if, given the actions of its rivals, a firm cannot increase its own profit by choosing an action other than its equilibrium action [37]. In the absence of a cartel agreement, the market will stay stable at this point. It is assumed that this market equilibrium point can provide a reasonable prediction of which designs manufacturers are driven to produce under various regulation scenarios. It should be noted however that the Nash equilibrium does not model preemptive competitive strategies by producers. Instead, it assumes that each producer will move to increase its profit while treating competitor decisions as constant.

In order to search for the equilibrium point, a strategy was employed in which each producer separately optimizes for profit with respect to its own decisions while holding all other producer decisions constant. Each producer’s optimization model is solved sequentially, and the process is iterated across producers, updating each producer’s decisions after optimization. Each producer may decide to produce zero
products (drop out of the market) if competitor conditions are such that it is not profitable to produce. In this way, the number of producers in the market is variable, and the strategy is used to determine how many producers result under each regulation scenario in an oligopoly free-entry market at Nash equilibrium. The strategy did not generate strictly converged market equilibrium in most cases; however, the decision variables \((M_j, x_{i1}, x_{i2}, p_j)\) quickly stabilize to a relatively small range of values. Figure 3 shows the average vehicle selling price over fifty iterations of the Nash equilibrium solution strategy. Although the system has not strictly converged, the price has quickly settled to a narrow range that is acceptable for the purposes of this analysis. Tests show that the method predicts the number of producers \((K)\) with a fidelity of ±1, prices \((\rho)\) within ±$100, engine size \((z_1)\) within ±1kW, and final drive ratio \((z_2)\) within ±0.1.

\[
\begin{align*}
\text{maximize} & \quad \sum_{j \in J_k} q_j \left( p_j - c^*_j \right) - c^*_k \\
\text{subject to} & \quad p_j \geq 0.75 \\
& \quad x_{i1} \leq 1.50 \\
& \quad x_{i2} \geq 0.2 \\
& \quad x_{i2} \leq 1.3 \\
& \quad q_j = s \sum_{j \in J_k} e^{x_j} \\
& \quad u_j = \beta_1 p_j + \beta_2 \left( \frac{100}{z_{i1}} \right) + \beta_3 \left( \frac{60}{z_{i2}} \right) \\
& \quad z_j = f_u(x_j) \\
& \quad c^*_j = c^* + \left\{ \begin{array}{ll}
\beta_4 \exp \left( \beta_5 b_{\text{SI}} x_{i1} \right) & \text{if } M_j = \text{SI102} \\
\beta_6 b_{\text{CI}} x_{i1} + \beta_7 & \text{if } M_j = \text{CI188}
\end{array} \right.
\end{align*}
\]

where \(s = (11.7) \times 10^6\), \(\beta_1 = -2.86 \times 10^{-4}\), \(\beta_2 = -0.339\), \(\beta_3 = 0.375\), \(\beta_4 = 670.51\), \(\beta_5 = 0.0063\), \(\beta_6 = 26.23\), \(\beta_7 = 1642.8\), \(b_{\text{SI102}} = 102\text{kW}\), \(b_{\text{CI88}} = 90.5\text{kW}\), \(c^* = 7500\), \(c^* = 550(10^6)\), and \(c^*_k\) is defined by Eq. (10), Eq. (11), Eq. (12), Eq. (13) or zero, depending on which regulation scenario is used. For each producer, competitor products are represented in the set \(J-J_k\) as fixed parameters that affect demand (Eq. (5)). The first four constraints represent limits on the ability to model variables outside these ranges rather than physical feasibility limits. An active constraint in this case would represent inability to model the optimum solution [38]. None of the constraints were active in any of the results, the optima are all interior optima, and the solutions are valid.

Despite the computational savings gained by creating metamodels of the engineering simulations, the computational burden is still significant. For each producer, separate optimization runs must be computed to determine which combination of vehicles is best for the product line. This combinatorial set of optimization problems is computed for each producer, and each producer model is then iterated several times in the Nash equilibrium solution strategy. Each iteration required about one hour on a 1GHz machine for the problem analyzed here. In order to reduce computational burden in this study, the number of designs per producer was limited to two \((n_{\text{max}} = 2)\). This limitation is reasonable because results of all runs indicate that each producer manufactures only one design (except for the quota regulation case where each producer manufactures both an SI and a CI engine), implying that there is a lack of incentive to produce multiple designs.

\[\text{Figure 3 Average Price Over Fifty Iterations of the Nash Equilibrium Solution Strategy}\]

Using the models developed in Sections 3.1-3.5, each producer \(k\) will individually attempt to maximize profit by solving the following optimization problem.

\[\text{maximize} \quad \sum_{j \in J_k} q_j \left( p_j - c^* \right) - c^*_k\]

with respect to \(M_j, x_{i1}, x_{i2}, p_j \forall j \in J_k\)

subject to \(x_{i1} \geq 0.75\)

\(x_{i1} \leq 1.50\)

\(x_{i2} \geq 0.2\)

\(x_{i2} \leq 1.3\)

\(q_j = s \sum_{j \in J_k} e^{x_j}\)

\(u_j = \beta_1 p_j + \beta_2 \left( \frac{100}{z_{i1}} \right) + \beta_3 \left( \frac{60}{z_{i2}} \right)\)

\(z_j = f_u(x_j)\)

\(c^*_j = c^* + \left\{ \begin{array}{ll}
\beta_4 \exp \left( \beta_5 b_{\text{SI}} x_{i1} \right) & \text{if } M_j = \text{SI102} \\
\beta_6 b_{\text{CI}} x_{i1} + \beta_7 & \text{if } M_j = \text{CI188}
\end{array} \right.\)

\[\text{where } s = (11.7) \times 10^6, \beta_1 = -2.86 \times 10^{-4}, \beta_2 = -0.339, \beta_3 = 0.375, \beta_4 = 670.51, \beta_5 = 0.0063, \beta_6 = 26.23, \beta_7 = 1642.8, b_{\text{SI102}} = 102\text{kW}, b_{\text{CI88}} = 90.5\text{kW}, c^* = 7500, c^* = 550(10^6), \text{and } c^*_k \text{ is defined by Eq. (10), Eq. (11), Eq. (12), Eq. (13) or zero, depending on which regulation scenario is used. For each producer, competitor products are represented in the set } J-J_k \text{ as fixed parameters that affect demand (Eq. (5)). The first four constraints represent limits on the ability to model variables outside these ranges rather than physical feasibility limits. An active constraint in this case would represent inability to model the optimum solution [38]. None of the constraints were active in any of the results, the optima are all interior optima, and the solutions are valid.}\]

\[\text{Despite the computational savings gained by creating metamodels of the engineering simulations, the computational burden is still significant. For each producer, separate optimization runs must be computed to determine which combination of vehicles is best for the product line. This combinatorial set of optimization problems is computed for each producer, and each producer model is then iterated several times in the Nash equilibrium solution strategy. Each iteration required about one hour on a 1GHz machine for the problem analyzed here. In order to reduce computational burden in this study, the number of designs per producer was limited to two (n_{_{\text{max}}} = 2). This limitation is reasonable because results of all runs indicate that each producer manufactures only one design (except for the quota regulation case where each producer manufactures both an SI and a CI engine), implying that there is a lack of incentive to produce multiple designs.}\]

\[\text{4. RESULTS AND DISCUSSION}\]

The primary results of the study are summarized in Table 2. For each regulation scenario, the table shows the number of producers, the number of designs per producer, and the market share per producer resulting from five iterations of the Nash equilibrium solution strategy with free-entry and exit. Through the strategy the number of producers is predicted such that any additional producers entering the market would lose money on production, and any fewer number of producers would leave room for an additional entrant to make profit. The use of the aggregate demand model results in each producer making the same decisions at market equilibrium, so Table 2 summarizes the decision variables, product characteristics, and costs for a typical producer in each scenario. The fact that all producers are driven to produce the same vehicle design facilitates comparison of the trends that result from each regulation scenario. Another result is that each producer produces only a single design rather than a product line (except in the quota case) due to competition and the existence of substantial investment cost. Figure 4 summarizes the resulting fuel economy and regulation cost per vehicle for each regulation scenario.

It is important to take care when interpreting results of optimization that is based on a demand regression model. Even if the demand model succeeds in capturing important trends in consumer purchasing preferences according to measurable characteristics, the metrics do not capture purchasing criteria entirely. For example, the model used in this study predicts a preference for vehicles with faster acceleration. A vehicle that dramatically sacrifices unmeasured characteristics such as maximum speed in order to slightly improve acceleration time will be preferred according to the model; however, in practice a consumer would observe the unmeasured limitations during a road test, especially if they are extreme. In this study, all vehicle designs were able to follow the standard FTP driving cycle and achieve a speed of at least 110mph on a flat road.
Table 2  Nash equilibrium results for each regulation scenario

<table>
<thead>
<tr>
<th>Regulation Scenario</th>
<th>None</th>
<th>CAFE</th>
<th>Strict CAFE</th>
<th>Low CO₂</th>
<th>Med CO₂</th>
<th>High CO₂</th>
<th>Quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of producers</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>Designs per producer</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Market share per producer</td>
<td>9%</td>
<td>9%</td>
<td>9%</td>
<td>9%</td>
<td>9%</td>
<td>9%</td>
<td>12%</td>
</tr>
<tr>
<td>Engine type</td>
<td>SI102</td>
<td>SI102</td>
<td>SI102</td>
<td>SI102</td>
<td>SI102</td>
<td>SI102</td>
<td>SI102</td>
</tr>
<tr>
<td>Engine size</td>
<td>145.6 kW</td>
<td>113.4 kW</td>
<td>77.5 kW</td>
<td>145.1 kW</td>
<td>114.3 kW</td>
<td>110.4 kW</td>
<td>127.9 kW</td>
</tr>
<tr>
<td>FD ratio</td>
<td>1.27</td>
<td>1.28</td>
<td>1.30</td>
<td>1.27</td>
<td>1.28</td>
<td>1.27</td>
<td>1.29</td>
</tr>
<tr>
<td>Price</td>
<td>$13,000</td>
<td>$13,000</td>
<td>$12,500</td>
<td>$13,200</td>
<td>$13,700</td>
<td>$14,200</td>
<td>$13,400</td>
</tr>
<tr>
<td>MPG</td>
<td>18.5</td>
<td>22.0</td>
<td>27.4</td>
<td>18.5</td>
<td>21.8</td>
<td>22.4</td>
<td>20.2</td>
</tr>
<tr>
<td>Time 0-60</td>
<td>6.98 s</td>
<td>7.96 s</td>
<td>10.2 s</td>
<td>6.99 s</td>
<td>7.37 s</td>
<td>8.09 s</td>
<td>7.4 s</td>
</tr>
<tr>
<td>Variable cost per vehicle</td>
<td>$9,180</td>
<td>$8,870</td>
<td>$8,590</td>
<td>$9,170</td>
<td>$8,880</td>
<td>$8,840</td>
<td>$9,000</td>
</tr>
<tr>
<td>Reg. cost per vehicle</td>
<td>$0</td>
<td>$300</td>
<td>$15</td>
<td>$160</td>
<td>$960</td>
<td>$1530</td>
<td>$0</td>
</tr>
</tbody>
</table>

4.1 Base Case
A base case was analyzed with no regulation ($c^R = 0$) as a comparison for results with regulation. With no regulation the model predicts eleven producers in the small car market. Each producer manufactures a single vehicle with design variables, product characteristics and costs shown in Table 2 (regulation type “None”).

4.2 Corporate Average Fuel Economy (CAFE)
The regulation type “CAFE” represents the current CAFE standards (Eq.(10)). Table 2 shows that the CAFE standard results in increased fuel efficiency at a lower manufacturing cost relative to the base case; however, performance is sacrificed, and regulatory costs are incurred (see Figure 4).

The “Strict CAFE” policy results in still lower manufacturing cost and price than the current CAFE standard, and fuel economy is very close to the CAFE standard. Under normal CAFE penalties it is profitable for manufacturers to violate CAFE and take the penalty in order to increase market share. Compliance with CAFE is dangerous for a producer because competitors can produce larger engines, which are in high demand, and capture market share. However, when CAFE penalties are increased substantially, producers are forced to meet the standard in order to stay in business. In this case there is no danger of losing market share to a competitor who sells more powerful engines; therefore all of the producers design smaller, cheaper engines.

In practice, many producers do not currently accrue CAFE penalties and instead treat the CAFE standard as a constraint [39]. One reason for this is the un-modeled extra costs to the producer caused by violation, such as damage to the producer’s reputation, which could affect demand, public relations and government relations as well as making future compliance more difficult. The results of this study predict companies to violate CAFE significantly, suggesting that these un-modeled aspects may be significant drivers worth further consideration.

4.3 CO₂ Emissions Tax
Comparing CO₂ valuation regulations to the base case, several trends can be observed. As the valuations increase, producers tend to design smaller, more fuel-efficient engines while transferring the added regulation cost to the consumer.
through an increased vehicle price (Figure 4). A low valuation penalty has little effect on fuel efficiency and only raises price slightly. The median valuation has a larger impact, increasing fuel-efficiency by 3.3 mpg, while the high valuation adds only slight improvement in fuel economy with a substantial price increase and performance loss over the median valuation. These trends predict reasonable real-world scenarios, since regulation provides incentive to produce smaller, more fuel-efficient engines. However, in practice such increases in vehicle costs could lower the demand and sales of vehicles with respect to other modes of transportation.

4.4 Diesel Fuel Sales Quota

In the quota policy, producers were forced to offer diesel engines as a minimum percentage of their vehicle fleet (\( \phi = 40\% \)). The results show that producers follow this regulation strictly to avoid expensive penalties, producing exactly the minimum regulated percentage of diesels (of the 20% total producer market share, 40% of those (8% of the total market) are diesels, see Table 2). Also, the SI vehicle produced in the quota scenario is more fuel-efficient than in the base case. This makes sense because the decreased number of producers in the market (5 vs. 11) and the diesel quota led to decreased competition for high performance SI engines, so smaller, cheaper engines could be built.

5. CONCLUSIONS

The formulation developed in this article provides a model for studying the effects of emissions policies on optimal vehicle design decisions. Several trends are observed in the policy scenarios examined in this study. Increased regulation penalties can result in cost savings for all parties, as in the CAFE scenarios. Without regulation, producers cannot afford to make smaller, cheaper engines because of competition; however, when all producers are subject to the same regulation costs, then all producers are driven to produce smaller engines. On the other hand, increased regulation penalties can also lead to diminishing returns, as in the CO\(_2\) regulation scenarios. Therefore, modeling the effects of regulation on design decisions is important for evaluating regulation concepts and choosing regulation parameters such as penalty values.

The study also shows that regulation is necessary to provide incentives for producers to design diesel or alternative fuel vehicles. While diesel engines have better fuel efficiency per power, gasoline engines are cheaper to manufacture and are therefore preferred. However, the increased cost of manufacturing diesels may be due in part to the relative lack of infrastructure for producing such engines, which could be improved if more diesels are manufactured.

Results indicate that each producer chooses to manufacture a single vehicle design rather than a product line, except in the case of the quota regulation. This result may be caused in part by the lack of models to predict cost savings due to economies of scope [40] and possible commonality among designs [41]. Inclusion of such models in this framework could therefore enrich the results. This result is also affected by the use of the aggregate model for demand, which ignores the fact that two very similar vehicles will compete more directly than two different vehicles. The consequence is that all producers have incentive to produce the same vehicle design. This result is expected to change if a demand model that accounts for consumer heterogeneity is used.

Overall, the models presented in this article were successful in predicting realistic trends resulting from several regulation scenarios. If individual cost structures can be formulated for existing manufacturers, the framework could be adapted to model specific short-term decisions of existing manufacturers. However, the abstract oligopoly analysis provided here yields a useful analytical perspective of long-term regulation effects in general. The analysis demonstrates the model’s predictive power and suggests that policy models that include design decisions can be used to improve understanding of the ultimate effects of regulation on industry, consumers, and the environment.

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