

**CMU Mathematical Sciences**  
**21-259 Sample Prerequisite Waiver Exam Booklet #2**  
**Answer Key**

1. Let  $A = (1, 0, -1)$ ,  $B = (2, 2, 0)$ , and  $C = (0, 1, 3)$  be three points in space. Which of the following points lies in the *same plane* as  $A$ ,  $B$ , and  $C$ ?

- A.  $(1, 4, 5)$
  - B.  $(3, 1, -2)$
  - C. None of these choices
  - D.  $(4, 0, -8)$**
  - E.  $(2, 3, 1)$
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2. Let  $C$  be the curve of intersection of the plane  $x + y + z = 2$  and the elliptic cylinder  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ . Which of the following is a smooth parametrization of the entire curve  $C$ ?

- A.  $\mathbf{r}(t) = \langle 2 \cos t, 3 \sin t, 2 \cos t + 3 \sin t \rangle$
  - B.  $\mathbf{r}(t) = \langle 2 \cos t, 3 \sin t, 2 - 2 \cos t - 3 \sin t \rangle$**
  - C.  $\mathbf{r}(t) = \langle \cos t, \sin t, 2 - \cos t - \sin t \rangle$
  - D. None of these choices
  - E.  $\mathbf{r}(t) = \langle 2 \cos t, 3 \sin t, 2 + 2 \cos t + 3 \sin t \rangle$
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3. Determine which of the following parametrizations is parametrized *with respect to arc length*.

- A.  $\mathbf{r}(t) = \left\langle \frac{t^2}{2}, t, \frac{t^2}{2} \right\rangle$
  - B.  $\mathbf{r}(t) = \left\langle \frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}}, 0 \right\rangle$**
  - C.  $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$
  - D. None of these choices
  - E.  $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), 0 \rangle$
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4. Evaluate the limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^2}{x^8 + y^4}$ .

- A. None of these choices
  - B.  $\infty$
  - C.  $\frac{1}{2}$
  - D. Does Not Exist**
  - E. 0
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5. Let  $f(x, y) = 2\sqrt{x} - y^2$ . Find the directional derivative of  $f$  at the point  $(1, 4)$  in the direction of the vector  $\mathbf{v} = \langle 3, -4 \rangle$ .

A. None of these choices

B.  $-5$

C.  $\langle \frac{3}{5}, -\frac{4}{5} \rangle$

D.  $\langle 1, -8 \rangle$

**E. 7**

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6. Find the equation of the tangent plane to the graph  $z = f(x, y)$  at the point  $(1, 0, 1)$ .

$$f(x, y) = \sqrt{x^2 + y^2} + \sin(xy)$$

A. None of these choices

B.  $z = x + y + 1$

**C.  $z = x + y$**

D.  $z = x - y$

E.  $z = x + 2y$

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7. Let  $f(x, y) = x^3 - 3xy + y^3$ . Which of the following statements is true?

A.  $(0, 0)$  is a local minimum of  $f(x, y)$ .

B.  $(0, 0)$  is a local maximum of  $f(x, y)$ .

C.  $(0, 0)$  is not a critical point of  $f(x, y)$ .

**D.  $(0, 0)$  is a saddle point of  $f(x, y)$ .**

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8. Let  $C$  be the curve of intersection of the surfaces  $xy = 1$  and  $y^2 + z^2 = 1$ . Determine the minimum value of  $f(x, y, z) = yz + xy$  on  $C$ .

A.  $-\frac{1}{2}$

B. 0

C. None of these choices

D. 1

**E.  $\frac{1}{2}$**

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9. Evaluate  $\int_0^2 \int_{y\sqrt{3}}^{\sqrt{16-y^2}} \sqrt{x^2 + y^2} dx dy$ .

A.  $\frac{32\pi}{3}$

**B. None of these choices**

C.  $\frac{64\pi}{3}$

D.  $\frac{128\pi}{9}$

E.  $\frac{64\pi}{9}$

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10. Let  $E$  be the solid region inside the sphere  $x^2 + y^2 + z^2 \leq 9$  and above the cone  $z = \sqrt{x^2 + y^2}$ . Assume  $E$  has constant density. Find the centroid of  $E$ .

A.  $\left(0, 0, \frac{9(2 + \sqrt{2})}{8}\right)$

B. None of these choices

C.  $\left(0, 0, \frac{9\sqrt{2}}{16}\right)$

**D.  $\left(0, 0, \frac{9(2 + \sqrt{2})}{16}\right)$**

E.  $\left(0, 0, \frac{9(2 - \sqrt{2})}{16}\right)$

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11. Let  $C$  be the curve parametrized by  $\mathbf{r}(t) = \langle \cos t, \cos^2 t, \sin t \rangle$  for  $0 \leq t \leq 2\pi$ . Evaluate

$$\int_C -z dx + z dy + x dz.$$

A.  $\pi$

B. None of these choices

C. 0

**D.  $2\pi$**

E.  $-2\pi$

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12. Let  $C$  be the curve parametrized by  $\mathbf{r}(t) = \langle t - 1, e^{t^4}, t^2 + 1 \rangle$ , for  $0 \leq t \leq 1$ , and let

$$\mathbf{F}(x, y, z) = \left\langle e^{-x} \ln y, -\frac{e^{-x}}{y}, z \right\rangle.$$

Calculate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .

A. 0

B.  $\frac{3}{2}$

**C. None of these choices**

D. 1

E.  $\frac{5}{2}$

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13. Let  $C$  be the positively oriented rectangle with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 2)$ , and  $(0, 2)$ . Let

$$\mathbf{F}(x, y) = \left\langle -e^{xy} + x^2 y, \frac{x^3}{3} \right\rangle.$$

Compute  $\oint_C \mathbf{F} \cdot d\mathbf{r}$ .

A.  $\frac{e^2 - 1}{2}$

**B.  $\frac{e^2 - 3}{2}$**

C.  $e^2 - 3$

D. None of these choices

E.  $\frac{3 - e^2}{2}$

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14. Let  $S$  be the surface defined by  $z = 10 - x^2 - y^2$  with  $z \geq 1$ , oriented upward. Let

$$\mathbf{F}(x, y, z) = \langle 2xyz - 5y, x^2z, e^x \cos(yz) \rangle.$$

Calculate  $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ .

A.  $90\pi$

B. None of these choices

**C.  $45\pi$**

D.  $9\pi$

E.  $20\pi$

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15. Let  $S$  be the closed surface consisting of the frustum of the cone  $z^2 = x^2 + y^2$  for  $1 \leq z \leq 2$ , the bottom disk  $z = 1$ ,  $x^2 + y^2 \leq 1$ , and the top disk  $z = 2$ ,  $x^2 + y^2 \leq 4$ , oriented outward. Let

$$\mathbf{F}(x, y, z) = \langle 2y + e^z, \sin(xz^2), z(x^2 + y^2) \rangle.$$

Find the flux of  $\mathbf{F}$  across  $S$ .

- A.  $\frac{31\pi}{10}$
  - B.  $\frac{31\pi}{20}$
  - C. 0
  - D. None of these choices
  - E.  $\frac{31\pi}{5}$
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